Computer algebra independent integration tests

Summer 2022 edition

1-Algebraic-functions/1.1-Binomial-products/1.1.3-General/26-1.1.3.3-a+b-x^n-^p-c+d-x^n-^q

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Contents

Chapter 1

Introduction

Local contents

This report gives the result of running the computer algebra independent integration test.

The download section in the appendix contains links to download the problems in plain text format used for all CAS systems.

The number of integrals in this report is [385]. This is test number [26].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.1 (June 29, 2022) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.0.1 on windows 10.
3. Maple 2022.1 (June 1, 2022) on windows 10.
4. Maxima 5.46 (April 13, 2022) using Lisp SBCL 2.1.11.debian on Linux via sagemath 9.6.
5. Fricas 1.3.8 (June 21, 2022) based on sbcl 2.1.11.debian on Linux via sagemath 9.6.
6. Giac/Xcas 1.9.0-13 (July 3, 2022) on Linux via sagemath 9.6.
7. Sympy 1.10.1 (March 20, 2022) Using Python 3.10.4 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly from Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (385)	0.00 (0)
Mathematica	99.48 (383)	0.52 (2)
Fricas	55.58 (214)	44.42 (171)
Maple	51.43 (198)	48.57 (187)
Mupad	44.16 (170)	55.84 (215)
Maxima	43.38 (167)	56.62 (218)
Sympy	35.84 (138)	64.16 (247)
Giac	32.47 (125)	67.53 (260)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

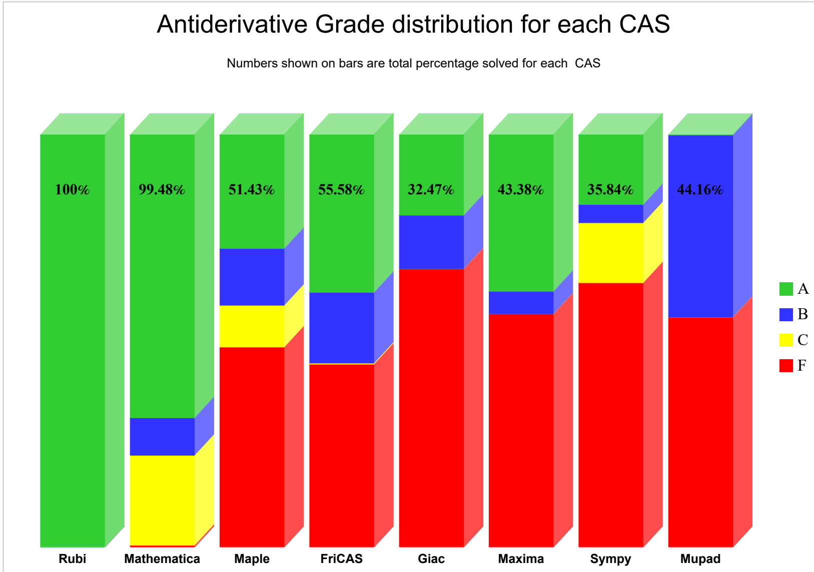
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

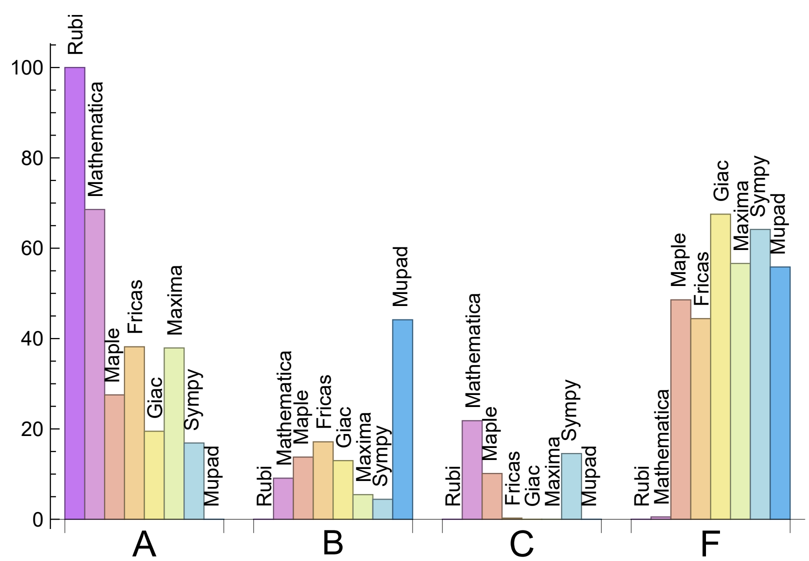
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.00	0.00	0.00	0.00
Mathematica	68.57	9.09	21.82	0.52
Fricas	38.18	17.14	0.26	44.42
Maxima	37.92	5.45	0.00	56.62
Maple	27.53	13.77	10.13	48.57
Giac	19.48	12.99	0.00	67.53
Sympy	16.88	4.42	14.55	64.16
Mupad	N/A	44.16	0.00	55.84

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and

Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	2	50.00 %	50.00 %	0.00 %
Maple	187	100.00 %	0.00 %	0.00 %
Fricas	171	54.97 %	42.11 %	2.92 %
Giac	260	86.92 %	0.00 %	13.08 %
Maxima	218	100.00 %	0.00 %	0.00 %
Sympy	247	55.06 %	34.82 %	10.12 %
Mupad	215	100.00 %	0.00 %	0.00 %

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

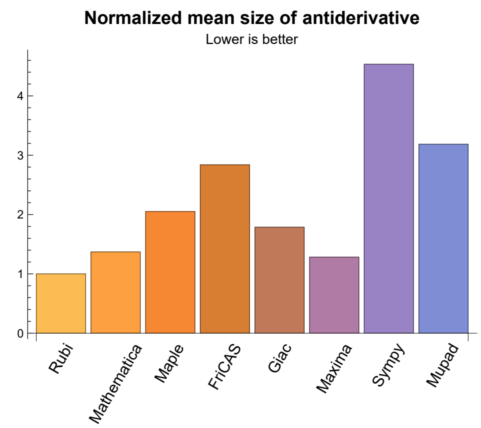
Similarly the **Normalized median** is relative to the median leaf size of the optimal.

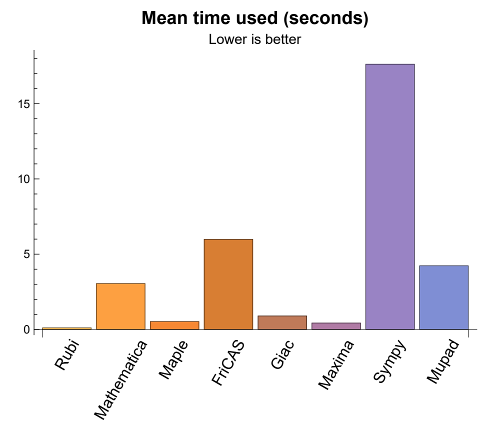
For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.10	170.35	1.00	122.00	1.00
Mathematica	3.04	198.28	1.37	142.00	0.97
Maple	0.52	357.44	2.05	168.50	1.05
Maxima	0.42	185.69	1.28	144.00	1.14
Fricas	5.98	554.38	2.84	232.00	2.11
Sympy	17.62	451.34	4.53	144.50	1.14
Giac	0.89	277.65	1.79	203.00	1.39
Mupad	4.23	588.76	3.19	148.50	1.22

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used from the above table.


Normalized mean size of antiderivative
Lower is better
0
1
2
3
4
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad


Mean time used (seconds)
Lower is better
0
5
10
15
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad

1.4 list of integrals that has no closed form antiderivative

{}

1.5 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {34, 35, 37, 38, 39, 79, 80, 81, 82, 83, 84, 93, 94, 95, 96, 97, 98, 104, 105, 106, 107, 108, 109, 110, 111, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 136, 137, 141, 142, 143, 144, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 199, 200, 201, 202, 203, 204, 211, 212, 213, 214, 215, 218, 221, 222, 269, 306, 307, 312, 317, 318, 319, 320, 328, 334, 335, 348, 350, 352, 353, 354, 355, 357, 378, 379, 384}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
```

```
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.9.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```python
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

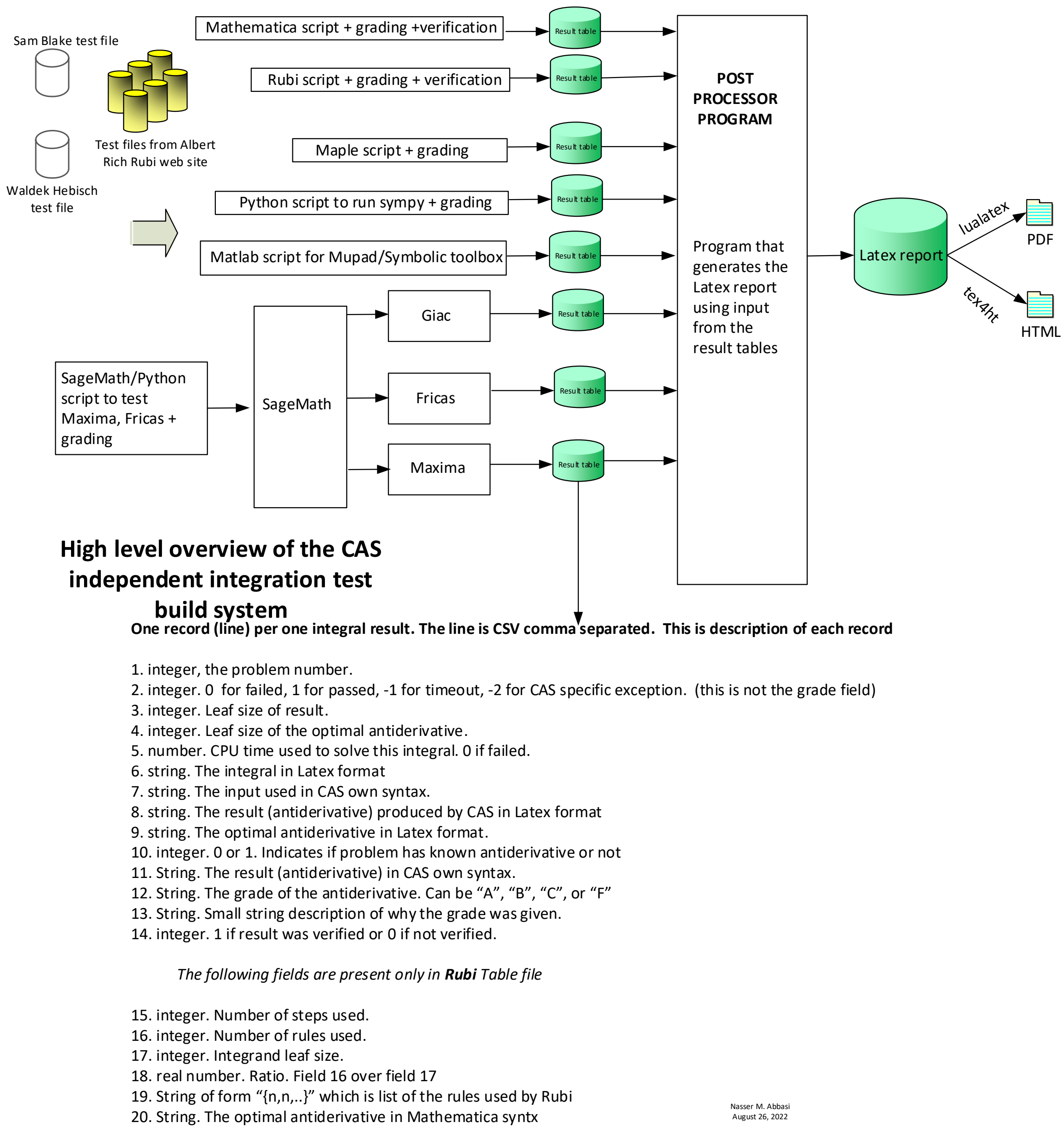
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

Local contents

2.1 List of integrals sorted by grade for each CAS

Local contents

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 36, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 83, 84, 85, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 134, 135, 138, 139, 140, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 191, 216, 217, 219, 220, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 268, 270, 272, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 301, 302, 303, 304, 305, 308, 309, 310, 311, 313, 314, 315, 316, 320, 321, 322, 324, 325, 326, 327, 329, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 381, 383, 385 }

B grade: { 82, 93, 94, 95, 96, 97, 104, 105, 106, 107, 108, 117, 118, 119, 120, 121, 133, 136, 137, 142, 143, 144, 218, 221, 222, 267, 269, 312, 317, 318, 319, 328, 334, 335, 384 }

C grade: { 34, 35, 37, 38, 39, 86, 87, 88, 89, 90, 91, 92, 98, 99, 100, 101, 102, 103, 109, 110, 111, 112, 113, 114, 115, 116, 141, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 223, 271, 273, 298, 299, 300, 306, 307, 323, 330, 331, 332, 333, 379, 380 }

F grade: { 336, 382 }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 30, 31, 32, 33, 44, 45, 46, 47, 60, 61, 62, 63, 75, 76, 77, 78, 85, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 191, 192, 224, 248, 270, 271, 272, 273, 275, 276, 277, 278, 279, 280, 284, 285, 286, 287, 292, 293, 294, 337, 338, 339, 349, 351, 353, 355, 357, 359, 361, 369, 371, 378, 379 }

B grade: { 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 324, 340, 341, 342, 363, 365, 367, 373, 375, 377 }

C grade: { 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 216, 343, 344, 345, 346, 347, 348, 350, 352, 354, 356, 358, 360, 362, 364, 366, 368, 370, 372, 374, 376 }

F grade: { 27, 28, 29, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 79, 80, 81, 82, 83, 84, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 217, 218, 219, 220, 221, 222, 223, 269, 274, 281, 282, 283, 288, 289, 290, 291, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 380, 381, 382, 383, 384, 385 }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 29, 30, 31, 44, 45, 59, 60, 61, 62, 63, 74, 75, 76, 77, 78, 85, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 224, 225, 226, 227, 231, 232, 233, 234, 238, 239, 240, 241, 245, 248, 252, 253, 255, 259, 260, 261, 262, 270, 275, 276, 277, 278, 279, 280, 284, 285, 286, 287, 292, 293, 294, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379 }

B grade: { 26, 27, 28, 32, 33, 40, 41, 42, 43, 46, 47, 56, 57, 58, 70, 71, 72, 73, 246, 247, 254 }

C grade: { }

F grade: { 34, 35, 36, 37, 38, 39, 48, 49, 50, 51, 52, 53, 54, 55, 64, 65, 66, 67, 68, 69, 79, 80, 81, 82, 83, 84, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 228, 229, 230, 235, 236, 237, 242, 243, 244, 249, 250, 251, 256, 257, 258, 263, 264, 265, 266, 267, 268, 269, 271, 272, 273, 274, 281, 282, 283, 288, 289, 290, 291, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 380, 381, 382, 383, 384, 385 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 14, 15, 16, 17, 18, 19, 24, 25, 30, 31, 32, 33, 40, 41, 44, 45, 46, 47, 56, 57, 58, 60, 61, 62, 63, 70, 71, 72, 75, 76, 77, 78, 85, 145, 146, 147, 148, 149, 153, 154, 155, 156, 224, 225, 226, 227, 228, 229, 231, 232, 233, 234, 235, 236, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 252, 253, 254, 255, 259, 260, 261, 262, 266, 267, 268, 270, 275, 276, 277, 278, 279, 280, 287, 322, 323, 324, 329, 330, 331, 332, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 373, 374, 375, 376, 377, 378, 379, 381, 385 }

B grade: { 7, 12, 13, 20, 21, 22, 23, 26, 27, 28, 29, 36, 42, 43, 59, 73, 74, 86, 87, 88, 98, 99, 110, 111, 150, 151, 152, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 191, 192, 193, 194, 205, 206, 216, 230, 237, 251, 256, 257, 258, 263, 264, 265, 284, 285, 286, 292, 293, 294, 321, 372 }

C grade: { 380 }

F grade: { 34, 35, 37, 38, 39, 48, 49, 50, 51, 52, 53, 54, 55, 64, 65, 66, 67, 68, 69, 79, 80, 81, 82, 83, 84, 89, 90, 91, 92, 93, 94, 95, 96, 97, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 207, 208, 209, 210, 211, 212, 213, 214, 215, 217, 218, 219, 220, 221, 222, 223, 269, 271, 272, 273, 274, 281, 282, 283, 288, 289, 290, 291, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 325, 326, 327, 328, 333, 334, 335, 336, 382, 383, 384 }

2.1.6 Sympy

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 20, 21, 22, 23, 24, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 168, 169, 170, 224, 225, 226, 227, 231, 232, 233, 238, 239, 240, 241, 245, 246, 247, 248, 255, 275, 276, 277, 278, 279, 280 }

B grade: { 30, 31, 60, 61, 234, 254, 261, 262, 270, 284, 285, 286, 287, 292, 293, 294, 324 }

C grade: { 27, 28, 29, 40, 41, 48, 49, 50, 56, 57, 58, 59, 64, 65, 66, 67, 68, 69, 70, 71, 72, 79, 80, 81, 82, 134, 135, 139, 140, 141, 219, 220, 288, 289, 291, 295, 298, 299, 300, 301, 308, 313, 314, 315, 316, 349, 351, 353, 354, 356, 359, 361, 363, 364, 366, 378 }

F grade: { 18, 19, 25, 26, 32, 33, 34, 35, 36, 37, 38, 39, 42, 43, 44, 45, 46, 47, 51, 52, 53, 54, 55, 62, 63, 73, 74, 75, 76, 77, 78, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 136, 137, 138, 142, 143, 144, 145, 164, 165, 166, 167, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 221, 222, 223, 228, 229, 230, 235, 236, 237, 242, 243, 244, 249, 250, 251, 252, 253, 256, 257, 258, 259, 260, 263, 264, 265, 266, 267, 268, 269, 271, 272, 273, 274, 281, 282, 283, 290, 296, 297, 302, 303, 304, 305, 306, 307, 309, 310, 311, 312, 317, 318, 319, 320, 321, 322, 323, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 350, 352, 355, 357, 358, 360, 362, 365, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 379, 380, 381, 382, 383, 384, 385 }

2.1.7 Giac

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 162, 163, 164, 165, 168, 169, 170, 171, 270, 275, 276, 277, 278, 279, 280, 340, 341, 346, 348, 349, 350, 351, 352, 353, 354, 358, 359, 360, 361, 362, 363, 364, 368, 370, 378 }

B grade: { 160, 161, 166, 167, 172, 227, 230, 237, 244, 247, 248, 250, 251, 253, 254, 255, 259, 260, 261, 262, 265, 284, 285, 286, 287, 292, 293, 294, 337, 338, 339, 342, 343, 344, 345, 347, 355, 356, 357, 365, 366, 367, 369, 371, 372, 373, 374, 375, 376, 377 }

C grade: { }

F grade: { 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 228, 229, 231, 232, 233, 234, 235, 236, 238, 239, 240, 241, 242, 243, 245, 246, 249, 252, 256, 257, 258, 263, 264, 266, 267, 268, 269, 271, 272, 273, 274, 281, 282, 283, 288, 289, 290, 291, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 379, 380, 381, 382, 383, 384, 385 }

2.1.8 Mupad

A grade: { }

B grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 30, 31, 32, 33, 44, 45, 46, 47, 60, 61, 62, 63, 75, 76, 77, 78, 85, 141, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 270, 275, 276, 277, 278, 279, 280, 284, 285, 286, 287, 292, 293, 294, 316, 324, 332, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 369, 371, 374, 376, 378, 379 }

C grade: { }

F grade: { 27, 28, 29, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 79, 80, 81, 82, 83, 84, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 142, 143, 144, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 268, 269, 271, 272, 273, 274, 281, 282, 283, 288, 289, 290, 291, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 317, 318, 319, 320, 321, 322, 323, 325, 326, 327, 328, 329, 330, 331, 333, 334, 335, 336, 368, 370, 372, 373, 375, 377, 380, 381, 382, 383, 384, 385 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column `N.S.` in the table below, which stands for **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To help make the table fit, `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	94	97	96	96	104	97	87
N.S.	1	1.00	1.00	1.03	1.02	1.02	1.11	1.03	0.93
time (sec)	N/A	0.048	0.016	0.296	0.281	3.450	0.016	1.039	0.051

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	70	73	70	70	80	74	66
N.S.	1	1.00	1.00	1.04	1.00	1.00	1.14	1.06	0.94
time (sec)	N/A	0.032	0.010	0.283	0.308	2.738	0.014	0.748	0.034

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	50	49	48	48	51	50	48
N.S.	1	1.00	1.00	0.98	0.96	0.96	1.02	1.00	0.96
time (sec)	N/A	0.021	0.007	0.313	0.265	2.649	0.012	1.527	0.048

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	25	24	24	26	26	25
N.S.	1	1.00	1.00	0.89	0.86	0.86	0.93	0.93	0.89
time (sec)	N/A	0.010	0.004	0.126	0.287	3.346	0.007	1.165	0.037

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	128	110	128	369	71	133	123
N.S.	1	1.00	0.89	0.76	0.89	2.56	0.49	0.92	0.85
time (sec)	N/A	0.066	0.056	0.259	0.488	3.094	0.231	0.881	1.383

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	145	134	158	537	97	160	143
N.S.	1	1.00	0.86	0.79	0.93	3.18	0.57	0.95	0.85
time (sec)	N/A	0.060	0.067	0.254	0.503	2.728	0.327	0.911	1.398

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	197	175	153	192	743	133	180	173
N.S.	1	1.00	0.89	0.78	0.97	3.77	0.68	0.91	0.88
time (sec)	N/A	0.069	0.096	0.257	0.482	2.544	0.459	0.836	1.396

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	122	125	124	124	139	132	116
N.S.	1	1.00	1.00	1.02	1.02	1.02	1.14	1.08	0.95
time (sec)	N/A	0.053	0.013	0.298	0.296	2.333	0.017	0.702	1.199

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	82	87	82	82	90	91	75
N.S.	1	1.00	1.00	1.06	1.00	1.00	1.10	1.11	0.91
time (sec)	N/A	0.034	0.009	0.318	0.264	2.789	0.014	0.847	0.042

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	50	49	48	48	51	50	48
N.S.	1	1.00	1.00	0.98	0.96	0.96	1.02	1.00	0.96
time (sec)	N/A	0.019	0.006	0.301	0.277	1.775	0.011	0.658	0.045

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	173	167	140	189	505	156	211	152
N.S.	1	1.00	0.97	0.81	1.09	2.92	0.90	1.22	0.88
time (sec)	N/A	0.085	0.064	0.270	0.505	4.262	0.356	0.942	1.386

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	203	210	167	226	771	189	233	191
N.S.	1	1.00	1.03	0.82	1.11	3.80	0.93	1.15	0.94
time (sec)	N/A	0.158	0.144	0.272	0.494	2.983	0.790	0.707	1.412

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	258	258	234	200	267	1067	233	264	249
N.S.	1	1.00	0.91	0.78	1.03	4.14	0.90	1.02	0.97
time (sec)	N/A	0.150	0.202	0.274	0.523	3.450	1.183	0.693	1.428

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	252	252	253	272	364	873	371	391	250
N.S.	1	1.00	1.00	1.08	1.44	3.46	1.47	1.55	0.99
time (sec)	N/A	0.129	0.091	0.331	0.572	2.883	0.974	0.708	1.430

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	208	208	203	193	273	700	257	296	192
N.S.	1	1.00	0.98	0.93	1.31	3.37	1.24	1.42	0.92
time (sec)	N/A	0.103	0.068	0.276	0.510	3.179	0.562	0.651	1.402

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	173	167	140	190	507	156	211	152
N.S.	1	1.00	0.97	0.81	1.10	2.93	0.90	1.22	0.88
time (sec)	N/A	0.084	0.085	0.280	0.491	5.212	0.429	0.691	1.375

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	129	110	128	390	71	133	123
N.S.	1	1.00	0.89	0.76	0.88	2.69	0.49	0.92	0.85
time (sec)	N/A	0.070	0.049	0.257	0.500	1.951	0.218	0.621	1.379

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	288	288	224	207	293	254	0	278	1364
N.S.	1	1.00	0.78	0.72	1.02	0.88	0.00	0.97	4.74
time (sec)	N/A	0.200	0.083	0.325	0.533	2.755	0.000	0.831	7.705

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	346	346	336	246	489	432	0	443	2589
N.S.	1	1.00	0.97	0.71	1.41	1.25	0.00	1.28	7.48
time (sec)	N/A	0.208	0.144	0.363	0.534	18.558	0.000	0.678	16.807

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	320	320	313	367	509	1619	546	529	416
N.S.	1	1.00	0.98	1.15	1.59	5.06	1.71	1.65	1.30
time (sec)	N/A	0.211	0.177	0.286	0.507	3.383	128.222	0.614	0.390

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	267	267	260	281	397	1316	405	412	302
N.S.	1	1.00	0.97	1.05	1.49	4.93	1.52	1.54	1.13
time (sec)	N/A	0.166	0.149	0.279	0.513	3.612	11.866	0.737	1.493

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	234	234	227	216	306	1027	291	319	240
N.S.	1	1.00	0.97	0.92	1.31	4.39	1.24	1.36	1.03
time (sec)	N/A	0.157	0.111	0.283	0.503	2.595	1.475	0.742	0.296

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	203	205	170	220	768	189	227	191
N.S.	1	1.00	1.01	0.84	1.08	3.78	0.93	1.12	0.94
time (sec)	N/A	0.164	0.149	0.281	0.487	2.148	0.699	0.667	1.467

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	145	134	158	537	97	160	143
N.S.	1	1.00	0.86	0.79	0.93	3.18	0.57	0.95	0.85
time (sec)	N/A	0.057	0.069	0.251	0.494	2.406	0.326	0.649	1.428

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	346	346	337	247	489	440	0	443	2492
N.S.	1	1.00	0.97	0.71	1.41	1.27	0.00	1.28	7.20
time (sec)	N/A	0.183	0.155	0.345	0.506	22.037	0.000	0.884	15.930

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	419	419	381	285	784	897	0	664	2500
N.S.	1	1.00	0.91	0.68	1.87	2.14	0.00	1.58	5.97
time (sec)	N/A	0.353	0.448	0.410	0.504	254.692	0.000	0.787	24.310

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	B	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	156	0	322	399	80	0	-1
N.S.	1	1.00	1.39	0.00	2.88	3.56	0.71	0.00	-0.01
time (sec)	N/A	0.024	0.336	0.007	0.501	4.427	2.260	0.000	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	B	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	140	0	244	363	76	0	-1
N.S.	1	1.00	1.54	0.00	2.68	3.99	0.84	0.00	-0.01
time (sec)	N/A	0.014	0.280	0.007	0.512	2.246	1.406	0.000	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	142	0	130	372	70	0	-1
N.S.	1	1.00	1.67	0.00	1.53	4.38	0.82	0.00	-0.01
time (sec)	N/A	0.009	0.243	0.032	0.523	4.349	3.392	0.000	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	28	25	50	44	190	0	27
N.S.	1	1.00	0.60	0.53	1.06	0.94	4.04	0.00	0.57
time (sec)	N/A	0.007	0.163	0.260	0.280	3.754	23.671	0.000	1.348

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	40	37	85	69	709	0	44
N.S.	1	1.00	0.73	0.67	1.55	1.25	12.89	0.00	0.80
time (sec)	N/A	0.010	0.212	0.285	0.290	3.005	117.232	0.000	1.425

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	51	48	119	91	0	0	58
N.S.	1	1.00	0.69	0.65	1.61	1.23	0.00	0.00	0.78
time (sec)	N/A	0.013	0.304	0.269	0.277	3.299	0.000	0.000	1.391

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	62	59	153	113	0	0	73
N.S.	1	1.00	0.67	0.63	1.65	1.22	0.00	0.00	0.78
time (sec)	N/A	0.020	0.442	0.266	0.267	1.948	0.000	0.000	1.371

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	483	483	232	0	0	0	0	0	-1
N.S.	1	1.00	0.48	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.403	10.221	0.006	0.000	0.000	0.000	0.000	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	464	464	217	0	0	0	0	0	-1
N.S.	1	1.00	0.47	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.292	10.107	0.005	0.000	0.000	0.000	0.000	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	398	398	428	0	0	644	0	0	-1
N.S.	1	1.00	1.08	0.00	0.00	1.62	0.00	0.00	-0.00
time (sec)	N/A	0.161	2.581	0.061	0.000	25.415	0.000	0.000	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	452	452	153	0	0	0	0	0	-1
N.S.	1	1.00	0.34	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.196	10.049	0.062	0.000	0.000	0.000	0.000	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	473	473	213	0	0	0	0	0	-1
N.S.	1	1.00	0.45	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.284	10.097	0.060	0.000	0.000	0.000	0.000	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	492	492	240	0	0	0	0	0	-1
N.S.	1	1.00	0.49	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.315	10.137	0.064	0.000	0.000	0.000	0.000	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	167	0	552	421	126	0	-1
N.S.	1	1.00	1.20	0.00	3.97	3.03	0.91	0.00	-0.01
time (sec)	N/A	0.036	0.467	0.010	0.506	4.418	6.128	0.000	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	165	0	436	399	121	0	-1
N.S.	1	1.00	1.38	0.00	3.63	3.32	1.01	0.00	-0.01
time (sec)	N/A	0.029	0.370	0.010	0.533	3.981	3.284	0.000	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	154	0	296	412	0	0	-1
N.S.	1	1.00	1.36	0.00	2.62	3.65	0.00	0.00	-0.01
time (sec)	N/A	0.025	0.402	0.009	0.539	3.924	0.000	0.000	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	137	0	180	521	0	0	-1
N.S.	1	1.00	1.25	0.00	1.64	4.74	0.00	0.00	-0.01
time (sec)	N/A	0.027	0.305	0.049	0.509	2.562	0.000	0.000	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	40	37	105	67	0	0	44
N.S.	1	1.00	0.53	0.49	1.38	0.88	0.00	0.00	0.58
time (sec)	N/A	0.015	0.251	0.296	0.286	2.840	0.000	0.000	1.432

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	51	48	155	91	0	0	56
N.S.	1	1.00	0.49	0.46	1.48	0.87	0.00	0.00	0.53
time (sec)	N/A	0.023	0.376	0.290	0.307	4.209	0.000	0.000	1.389

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	62	59	206	113	0	0	71
N.S.	1	1.00	0.63	0.60	2.10	1.15	0.00	0.00	0.72
time (sec)	N/A	0.023	0.497	0.271	0.290	2.441	0.000	0.000	1.436

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	73	70	257	135	0	0	86
N.S.	1	1.00	0.62	0.60	2.20	1.15	0.00	0.00	0.74
time (sec)	N/A	0.030	0.765	0.283	0.310	3.019	0.000	0.000	1.427

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	97	0	0	0	168	0	-1
N.S.	1	1.00	1.03	0.00	0.00	0.00	1.79	0.00	-0.01
time (sec)	N/A	0.024	7.522	0.013	0.000	0.000	2.720	0.000	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	85	0	0	0	126	0	-1
N.S.	1	1.00	0.90	0.00	0.00	0.00	1.34	0.00	-0.01
time (sec)	N/A	0.022	5.841	0.012	0.000	0.000	1.784	0.000	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	75	0	0	0	121	0	-1
N.S.	1	1.00	0.80	0.00	0.00	0.00	1.29	0.00	-0.01
time (sec)	N/A	0.024	10.035	0.010	0.000	0.000	1.844	0.000	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	62	0	0	0	0	0	-1
N.S.	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.025	10.034	0.009	0.000	0.000	0.000	0.000	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	70	0	0	0	0	0	-1
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.019	10.049	0.054	0.000	0.000	0.000	0.000	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	85	0	0	0	0	0	-1
N.S.	1	1.00	1.10	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.018	10.055	0.048	0.000	0.000	0.000	0.000	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	95	0	0	0	0	0	-1
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.023	10.077	0.049	0.000	0.000	0.000	0.000	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	106	0	0	0	0	0	-1
N.S.	1	1.00	1.14	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.023	10.085	0.053	0.000	0.000	0.000	0.000	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	208	0	406	482	170	0	-1
N.S.	1	1.00	1.20	0.00	2.33	2.77	0.98	0.00	-0.01
time (sec)	N/A	0.042	0.614	0.005	0.489	3.552	7.072	0.000	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	180	0	322	424	82	0	-1
N.S.	1	1.00	1.28	0.00	2.28	3.01	0.58	0.00	-0.01
time (sec)	N/A	0.030	0.569	0.006	0.512	3.690	1.964	0.000	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	163	0	244	362	78	0	-1
N.S.	1	1.00	1.47	0.00	2.20	3.26	0.70	0.00	-0.01
time (sec)	N/A	0.020	0.390	0.005	0.524	2.452	1.291	0.000	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	150	0	134	488	71	0	-1
N.S.	1	1.00	1.52	0.00	1.35	4.93	0.72	0.00	-0.01
time (sec)	N/A	0.016	0.326	0.031	0.495	3.929	3.081	0.000	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	37	34	51	54	190	0	33
N.S.	1	1.00	0.79	0.72	1.09	1.15	4.04	0.00	0.70
time (sec)	N/A	0.007	0.221	0.235	0.304	4.600	21.869	0.000	1.370

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	60	57	86	87	709	0	87
N.S.	1	1.00	0.66	0.63	0.95	0.96	7.79	0.00	0.96
time (sec)	N/A	0.018	0.311	0.247	0.274	6.958	111.050	0.000	1.424

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	80	81	120	121	0	0	105
N.S.	1	1.00	0.66	0.67	0.99	1.00	0.00	0.00	0.87
time (sec)	N/A	0.027	0.412	0.240	0.305	3.439	0.000	0.000	1.460

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	100	105	154	155	0	0	132
N.S.	1	1.00	0.66	0.70	1.02	1.03	0.00	0.00	0.87
time (sec)	N/A	0.033	0.607	0.272	0.301	6.313	0.000	0.000	1.453

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	77	0	0	0	265	0	-1
N.S.	1	1.00	0.91	0.00	0.00	0.00	3.12	0.00	-0.01
time (sec)	N/A	0.016	8.741	0.006	0.000	0.000	3.613	0.000	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	75	0	0	0	170	0	-1
N.S.	1	1.00	0.90	0.00	0.00	0.00	2.05	0.00	-0.01
time (sec)	N/A	0.016	7.488	0.005	0.000	0.000	2.235	0.000	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	72	0	0	0	82	0	-1
N.S.	1	1.00	0.88	0.00	0.00	0.00	1.00	0.00	-0.01
time (sec)	N/A	0.015	5.607	0.003	0.000	0.000	1.408	0.000	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	73	0	0	0	78	0	-1
N.S.	1	1.00	0.89	0.00	0.00	0.00	0.95	0.00	-0.01
time (sec)	N/A	0.015	10.042	0.003	0.000	0.000	1.133	0.000	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	66	0	0	0	78	0	-1
N.S.	1	1.00	0.71	0.00	0.00	0.00	0.84	0.00	-0.01
time (sec)	N/A	0.017	10.031	0.028	0.000	0.000	4.037	0.000	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	75	0	0	0	78	0	-1
N.S.	1	1.00	0.80	0.00	0.00	0.00	0.83	0.00	-0.01
time (sec)	N/A	0.019	10.035	0.032	0.000	0.000	41.838	0.000	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	262	262	293	0	672	717	270	0	-1
N.S.	1	1.00	1.12	0.00	2.56	2.74	1.03	0.00	-0.00
time (sec)	N/A	0.108	0.997	0.007	0.501	2.636	32.753	0.000	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	219	219	256	0	552	634	131	0	-1
N.S.	1	1.00	1.17	0.00	2.52	2.89	0.60	0.00	-0.00
time (sec)	N/A	0.115	0.762	0.006	0.551	3.405	5.807	0.000	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	223	0	436	554	126	0	-1
N.S.	1	1.00	1.27	0.00	2.49	3.17	0.72	0.00	-0.01
time (sec)	N/A	0.066	0.616	0.007	0.533	3.126	3.415	0.000	0.000

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	199	0	301	652	0	0	-1
N.S.	1	1.00	1.25	0.00	1.89	4.10	0.00	0.00	-0.01
time (sec)	N/A	0.069	0.763	0.009	0.489	5.782	0.000	0.000	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	198	0	190	719	0	0	-1
N.S.	1	1.00	1.30	0.00	1.25	4.73	0.00	0.00	-0.01
time (sec)	N/A	0.048	0.607	0.046	0.574	3.238	0.000	0.000	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	73	76	109	103	0	0	148
N.S.	1	1.00	0.94	0.97	1.40	1.32	0.00	0.00	1.90
time (sec)	N/A	0.015	0.423	0.255	0.276	3.731	0.000	0.000	1.427

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	106	115	159	152	0	0	176
N.S.	1	1.00	0.61	0.66	0.91	0.87	0.00	0.00	1.01
time (sec)	N/A	0.052	0.584	0.262	0.284	2.817	0.000	0.000	1.450

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	211	138	156	210	200	0	0	217
N.S.	1	1.00	0.65	0.74	1.00	0.95	0.00	0.00	1.03
time (sec)	N/A	0.087	0.824	0.282	0.286	2.790	0.000	0.000	1.430

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	253	253	169	197	261	246	0	0	257
N.S.	1	1.00	0.67	0.78	1.03	0.97	0.00	0.00	1.02
time (sec)	N/A	0.138	1.255	0.265	0.285	3.910	0.000	0.000	1.481

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	177	0	0	0	418	0	-1
N.S.	1	1.00	1.31	0.00	0.00	0.00	3.10	0.00	-0.01
time (sec)	N/A	0.047	10.963	0.010	0.000	0.000	5.124	0.000	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	176	0	0	0	270	0	-1
N.S.	1	1.00	1.32	0.00	0.00	0.00	2.03	0.00	-0.01
time (sec)	N/A	0.055	10.374	0.008	0.000	0.000	3.326	0.000	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	179	0	0	0	131	0	-1
N.S.	1	1.00	1.37	0.00	0.00	0.00	1.00	0.00	-0.01
time (sec)	N/A	0.046	7.690	0.007	0.000	0.000	1.814	0.000	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	C	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	304	0	0	0	126	0	-1
N.S.	1	1.00	2.30	0.00	0.00	0.00	0.95	0.00	-0.01
time (sec)	N/A	0.052	12.955	0.008	0.000	0.000	1.977	0.000	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	171	0	0	0	0	0	-1
N.S.	1	1.00	1.17	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.065	11.934	0.008	0.000	0.000	0.000	0.000	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	171	0	0	0	0	0	-1
N.S.	1	1.00	1.16	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.062	13.135	0.052	0.000	0.000	0.000	0.000	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	120	134	182	166	0	0	271
N.S.	1	1.00	1.10	1.23	1.67	1.52	0.00	0.00	2.49
time (sec)	N/A	0.027	0.729	0.260	0.296	2.646	0.000	0.000	1.556

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	331	331	525	0	0	643	0	0	-1
N.S.	1	1.00	1.59	0.00	0.00	1.94	0.00	0.00	-0.00
time (sec)	N/A	0.249	7.973	0.007	0.000	32.034	0.000	0.000	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	273	273	467	0	0	535	0	0	-1
N.S.	1	1.00	1.71	0.00	0.00	1.96	0.00	0.00	-0.00
time (sec)	N/A	0.146	4.466	0.006	0.000	6.329	0.000	0.000	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	233	233	423	0	0	469	0	0	-1
N.S.	1	1.00	1.82	0.00	0.00	2.01	0.00	0.00	-0.00
time (sec)	N/A	0.055	2.797	0.058	0.000	2.909	0.000	0.000	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	255	0	0	0	0	0	-1
N.S.	1	1.00	1.72	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.015	1.270	0.053	0.000	0.000	0.000	0.000	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	328	0	0	0	0	0	-1
N.S.	1	1.00	1.83	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.054	2.086	0.057	0.000	0.000	0.000	0.000	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	226	226	364	0	0	0	0	0	-1
N.S.	1	1.00	1.61	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.139	2.751	0.058	0.000	0.000	0.000	0.000	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	280	280	420	0	0	0	0	0	-1
N.S.	1	1.00	1.50	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.215	5.288	0.060	0.000	0.000	0.000	0.000	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	346	0	0	0	0	0	-1
N.S.	1	1.00	5.77	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.020	10.319	0.006	0.000	0.000	0.000	0.000	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	160	0	0	0	0	0	-1
N.S.	1	1.00	2.71	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.019	10.125	0.056	0.000	0.000	0.000	0.000	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	161	0	0	0	0	0	-1
N.S.	1	1.00	2.73	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.019	10.065	0.056	0.000	0.000	0.000	0.000	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	332	0	0	0	0	0	-1
N.S.	1	1.00	5.35	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.018	10.195	0.059	0.000	0.000	0.000	0.000	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	429	0	0	0	0	0	-1
N.S.	1	1.00	6.92	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.019	10.559	0.058	0.000	0.000	0.000	0.000	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	351	351	698	0	0	819	0	0	-1
N.S.	1	1.00	1.99	0.00	0.00	2.33	0.00	0.00	-0.00
time (sec)	N/A	0.458	10.700	0.010	0.000	11.008	0.000	0.000	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	301	301	510	0	0	631	0	0	-1
N.S.	1	1.00	1.69	0.00	0.00	2.10	0.00	0.00	-0.00
time (sec)	N/A	0.123	5.343	0.058	0.000	5.546	0.000	0.000	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	182	319	0	0	0	0	0	-1
N.S.	1	1.00	1.75	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.036	1.735	0.059	0.000	0.000	0.000	0.000	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	217	217	336	0	0	0	0	0	-1
N.S.	1	1.00	1.55	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.066	2.132	0.052	0.000	0.000	0.000	0.000	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	261	261	370	0	0	0	0	0	-1
N.S.	1	1.00	1.42	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.146	3.442	0.066	0.000	0.000	0.000	0.000	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	324	324	443	0	0	0	0	0	-1
N.S.	1	1.00	1.37	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.246	6.315	0.061	0.000	0.000	0.000	0.000	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	341	0	0	0	0	0	-1
N.S.	1	1.00	5.68	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.020	10.233	0.058	0.000	0.000	0.000	0.000	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	232	0	0	0	0	0	-1
N.S.	1	1.00	3.93	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.019	10.155	0.059	0.000	0.000	0.000	0.000	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	393	0	0	0	0	0	-1
N.S.	1	1.00	6.66	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.019	10.213	0.065	0.000	0.000	0.000	0.000	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	386	0	0	0	0	0	-1
N.S.	1	1.00	6.23	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.020	10.370	0.060	0.000	0.000	0.000	0.000	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	550	0	0	0	0	0	-1
N.S.	1	1.00	8.87	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.020	10.656	0.061	0.000	0.000	0.000	0.000	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	541	541	1171	0	0	0	0	0	-1
N.S.	1	1.00	2.16	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.500	11.571	0.009	0.000	0.000	0.000	0.000	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	458	458	908	0	0	1246	0	0	-1
N.S.	1	1.00	1.98	0.00	0.00	2.72	0.00	0.00	-0.00
time (sec)	N/A	0.368	11.332	0.007	0.000	59.206	0.000	0.000	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	391	391	651	0	0	954	0	0	-1
N.S.	1	1.00	1.66	0.00	0.00	2.44	0.00	0.00	-0.00
time (sec)	N/A	0.274	10.714	0.054	0.000	11.189	0.000	0.000	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	217	217	344	0	0	0	0	0	-1
N.S.	1	1.00	1.59	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.057	2.514	0.057	0.000	0.000	0.000	0.000	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	267	267	366	0	0	0	0	0	-1
N.S.	1	1.00	1.37	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.097	3.274	0.055	0.000	0.000	0.000	0.000	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	307	307	407	0	0	0	0	0	-1
N.S.	1	1.00	1.33	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.159	4.483	0.057	0.000	0.000	0.000	0.000	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	377	377	471	0	0	0	0	0	-1
N.S.	1	1.00	1.25	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.251	7.991	0.059	0.000	0.000	0.000	0.000	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	463	463	1991	0	0	0	0	0	-1
N.S.	1	1.00	4.30	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.422	15.096	0.067	0.000	0.000	0.000	0.000	0.000

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	316	0	0	0	0	0	-1
N.S.	1	1.00	5.27	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.019	10.351	0.056	0.000	0.000	0.000	0.000	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	431	0	0	0	0	0	-1
N.S.	1	1.00	7.31	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.018	10.405	0.056	0.000	0.000	0.000	0.000	0.000

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	442	0	0	0	0	0	-1
N.S.	1	1.00	7.49	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.019	10.472	0.059	0.000	0.000	0.000	0.000	0.000

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	531	0	0	0	0	0	-1
N.S.	1	1.00	8.56	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.019	10.679	0.063	0.000	0.000	0.000	0.000	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	515	0	0	0	0	0	-1
N.S.	1	1.00	8.31	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.020	11.078	0.058	0.000	0.000	0.000	0.000	0.000

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	90	0	0	0	0	0	-1
N.S.	1	1.00	0.58	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.044	5.802	0.072	0.000	0.000	0.000	0.000	0.000

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	90	0	0	0	0	0	-1
N.S.	1	1.00	0.58	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.041	5.535	0.066	0.000	0.000	0.000	0.000	0.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	89	0	0	0	0	0	-1
N.S.	1	1.00	0.73	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.027	5.808	0.056	0.000	0.000	0.000	0.000	0.000

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	89	0	0	0	0	0	-1
N.S.	1	1.00	0.73	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.026	3.836	0.061	0.000	0.000	0.000	0.000	0.000

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	86	0	0	0	0	0	-1
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.012	3.561	0.069	0.000	0.000	0.000	0.000	0.000

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	86	0	0	0	0	0	-1
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.014	5.636	0.069	0.000	0.000	0.000	0.000	0.000

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	89	0	0	0	0	0	-1
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.012	3.549	0.064	0.000	0.000	0.000	0.000	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	89	0	0	0	0	0	-1
N.S.	1	1.00	0.74	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.026	5.595	0.057	0.000	0.000	0.000	0.000	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	89	0	0	0	0	0	-1
N.S.	1	1.00	0.74	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.026	5.737	0.065	0.000	0.000	0.000	0.000	0.000

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	90	0	0	0	0	0	-1
N.S.	1	1.00	0.59	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.041	5.848	0.066	0.000	0.000	0.000	0.000	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	90	0	0	0	0	0	-1
N.S.	1	1.00	0.59	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.041	5.824	0.063	0.000	0.000	0.000	0.000	0.000

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	172	0	0	0	0	0	-1
N.S.	1	1.00	2.18	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.031	0.216	0.097	0.000	0.000	0.000	0.000	0.000

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	176	106	0	0	0	121	0	-1
N.S.	1	1.05	0.63	0.00	0.00	0.00	0.72	0.00	-0.01
time (sec)	N/A	0.089	0.158	0.063	0.000	0.000	109.741	0.000	0.000

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	85	90	0	0	0	75	0	-1
N.S.	1	1.01	1.07	0.00	0.00	0.00	0.89	0.00	-0.01
time (sec)	N/A	0.029	0.105	0.036	0.000	0.000	40.410	0.000	0.000

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	162	0	0	0	0	0	-1
N.S.	1	1.00	2.84	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.020	0.254	0.073	0.000	0.000	0.000	0.000	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	162	0	0	0	0	0	-1
N.S.	1	1.00	2.84	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.019	0.333	0.082	0.000	0.000	0.000	0.000	0.000

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	298	298	137	0	0	0	0	0	-1
N.S.	1	1.00	0.46	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.213	5.200	0.053	0.000	0.000	0.000	0.000	0.000

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	106	0	0	0	121	0	-1
N.S.	1	1.00	0.60	0.00	0.00	0.00	0.69	0.00	-0.01
time (sec)	N/A	0.087	5.677	0.061	0.000	0.000	105.619	0.000	0.000

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	85	90	0	0	0	75	0	-1
N.S.	1	0.91	0.97	0.00	0.00	0.00	0.81	0.00	-0.01
time (sec)	N/A	0.029	0.106	0.039	0.000	0.000	40.020	0.000	0.000

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	196	0	0	0	34	0	41
N.S.	1	1.00	4.45	0.00	0.00	0.00	0.77	0.00	0.93
time (sec)	N/A	0.007	0.132	0.035	0.000	0.000	6.024	0.000	1.346

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	162	0	0	0	0	0	-1
N.S.	1	1.00	2.84	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.017	0.261	0.072	0.000	0.000	0.000	0.000	0.000

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	162	0	0	0	0	0	-1
N.S.	1	1.00	2.84	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.018	0.345	0.079	0.000	0.000	0.000	0.000	0.000

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	162	0	0	0	0	0	-1
N.S.	1	1.00	2.84	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.018	0.475	0.054	0.000	0.000	0.000	0.000	0.000

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	52	71	0	91	0	0	131
N.S.	1	1.00	0.98	1.34	0.00	1.72	0.00	0.00	2.47
time (sec)	N/A	0.014	0.389	0.329	0.000	2.394	0.000	0.000	1.905

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	94	97	96	96	107	98	88
N.S.	1	1.00	1.00	1.03	1.02	1.02	1.14	1.04	0.94
time (sec)	N/A	0.049	0.015	0.277	0.281	2.965	0.018	0.541	1.303

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	70	73	70	70	76	74	66
N.S.	1	1.00	1.00	1.04	1.00	1.00	1.09	1.06	0.94
time (sec)	N/A	0.033	0.012	0.271	0.288	3.613	0.018	0.543	1.242

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	50	49	48	48	53	50	48
N.S.	1	1.00	1.00	0.98	0.96	0.96	1.06	1.00	0.96
time (sec)	N/A	0.021	0.009	0.269	0.275	3.432	0.013	0.502	0.048

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	25	24	24	26	26	25
N.S.	1	1.00	1.00	0.89	0.86	0.86	0.93	0.93	0.89
time (sec)	N/A	0.011	0.004	0.102	0.272	3.548	0.007	1.127	0.036

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	223	196	120	212	639	87	245	720
N.S.	1	1.00	0.88	0.54	0.95	2.87	0.39	1.10	3.23
time (sec)	N/A	0.107	0.090	0.251	0.496	3.361	0.291	1.538	1.480

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	245	245	212	140	236	711	112	266	740
N.S.	1	1.00	0.87	0.57	0.96	2.90	0.46	1.09	3.02
time (sec)	N/A	0.103	0.118	0.249	0.490	3.585	0.447	1.053	1.520

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	273	273	243	159	271	787	151	286	762
N.S.	1	1.00	0.89	0.58	0.99	2.88	0.55	1.05	2.79
time (sec)	N/A	0.120	0.137	0.245	0.497	3.023	0.612	1.108	1.581

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	154	163	158	158	185	173	146
N.S.	1	1.00	1.00	1.06	1.03	1.03	1.20	1.12	0.95
time (sec)	N/A	0.078	0.023	0.299	0.276	2.845	0.021	1.013	0.067

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	122	125	124	124	139	132	116
N.S.	1	1.00	1.00	1.02	1.02	1.02	1.14	1.08	0.95
time (sec)	N/A	0.056	0.017	0.276	0.280	4.281	0.024	0.613	1.297

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	82	87	82	82	97	91	75
N.S.	1	1.00	1.00	1.06	1.00	1.00	1.18	1.11	0.91
time (sec)	N/A	0.036	0.012	0.273	0.291	3.790	0.016	0.821	0.046

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	50	49	48	48	53	50	48
N.S.	1	1.00	1.00	0.98	0.96	0.96	1.06	1.00	0.96
time (sec)	N/A	0.020	0.006	0.277	0.272	3.860	0.013	0.739	0.045

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	253	253	231	150	286	1239	187	353	1081
N.S.	1	1.00	0.91	0.59	1.13	4.90	0.74	1.40	4.27
time (sec)	N/A	0.131	0.077	0.249	0.503	3.674	0.688	0.664	1.485

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	291	291	298	175	319	1335	219	376	1254
N.S.	1	1.00	1.02	0.60	1.10	4.59	0.75	1.29	4.31
time (sec)	N/A	0.383	0.121	0.267	0.483	3.250	2.013	0.509	1.536

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	349	349	319	206	361	1411	264	407	1401
N.S.	1	1.00	0.91	0.59	1.03	4.04	0.76	1.17	4.01
time (sec)	N/A	0.226	0.131	0.272	0.504	4.071	85.510	0.498	1.660

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	332	332	322	282	489	2477	435	617	1822
N.S.	1	1.00	0.97	0.85	1.47	7.46	1.31	1.86	5.49
time (sec)	N/A	0.187	0.137	0.289	0.499	3.647	22.785	0.620	1.515

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	288	288	271	203	385	1855	303	481	1433
N.S.	1	1.00	0.94	0.70	1.34	6.44	1.05	1.67	4.98
time (sec)	N/A	0.155	0.103	0.250	0.486	6.241	1.352	0.606	1.488

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	253	253	231	150	287	1240	187	353	1081
N.S.	1	1.00	0.91	0.59	1.13	4.90	0.74	1.40	4.27
time (sec)	N/A	0.133	0.074	0.257	0.485	4.407	0.584	0.545	1.466

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	223	196	120	212	639	87	245	720
N.S.	1	1.00	0.88	0.54	0.95	2.87	0.39	1.10	3.23
time (sec)	N/A	0.096	0.087	0.240	0.482	3.368	0.312	0.608	0.221

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	449	449	340	226	365	1354	0	437	2500
N.S.	1	1.00	0.76	0.50	0.81	3.02	0.00	0.97	5.57
time (sec)	N/A	0.185	0.088	0.327	0.492	4.276	0.000	0.622	2.755

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	513	513	498	262	481	3301	0	667	2500
N.S.	1	1.00	0.97	0.51	0.94	6.43	0.00	1.30	4.87
time (sec)	N/A	0.291	0.204	0.398	0.519	75.667	0.000	0.592	4.004

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	407	407	391	373	644	3222	0	798	2490
N.S.	1	1.00	0.96	0.92	1.58	7.92	0.00	1.96	6.12
time (sec)	N/A	0.273	0.250	0.275	0.505	3.548	0.000	0.630	1.707

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	357	357	341	288	521	2580	0	642	2043
N.S.	1	1.00	0.96	0.81	1.46	7.23	0.00	1.80	5.72
time (sec)	N/A	0.244	0.193	0.266	0.499	3.333	0.000	0.648	0.302

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	317	317	301	220	405	1937	337	496	1616
N.S.	1	1.00	0.95	0.69	1.28	6.11	1.06	1.56	5.10
time (sec)	N/A	0.210	0.154	0.273	0.509	2.659	82.858	0.642	1.530

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	291	291	297	176	319	1335	219	376	1254
N.S.	1	1.00	1.02	0.60	1.10	4.59	0.75	1.29	4.31
time (sec)	N/A	0.252	0.126	0.260	0.500	4.079	1.086	0.584	0.298

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	245	245	212	140	236	711	112	266	740
N.S.	1	1.00	0.87	0.57	0.96	2.90	0.46	1.09	3.02
time (sec)	N/A	0.100	0.114	0.241	0.554	2.683	0.398	0.668	1.533

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	513	513	499	263	470	3301	0	667	2500
N.S.	1	1.00	0.97	0.51	0.92	6.43	0.00	1.30	4.87
time (sec)	N/A	0.298	0.203	0.379	0.566	41.387	0.000	0.585	3.817

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	596	596	629	298	670	0	0	967	2500
N.S.	1	1.00	1.06	0.50	1.12	0.00	0.00	1.62	4.19
time (sec)	N/A	0.505	6.116	0.456	0.536	0.000	0.000	0.690	5.617

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	321	321	290	408	0	0	0	0	-1
N.S.	1	1.00	0.90	1.27	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.256	10.494	0.342	0.000	0.000	0.000	0.000	0.000

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	277	277	341	307	0	0	0	0	-1
N.S.	1	1.00	1.23	1.11	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.180	10.255	0.298	0.000	0.000	0.000	0.000	0.000

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	240	240	155	259	0	0	0	0	-1
N.S.	1	1.00	0.65	1.08	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.106	10.141	0.273	0.000	0.000	0.000	0.000	0.000

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	156	183	0	0	0	0	-1
N.S.	1	1.00	0.96	1.13	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.073	10.149	0.269	0.000	0.000	0.000	0.000	0.000

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	281	281	381	301	0	0	0	0	-1
N.S.	1	1.00	1.36	1.07	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.149	10.192	0.275	0.000	0.000	0.000	0.000	0.000

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	334	334	422	361	0	0	0	0	-1
N.S.	1	1.00	1.26	1.08	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.259	10.506	0.279	0.000	0.000	0.000	0.000	0.000

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	926	926	346	322	0	0	0	0	-1
N.S.	1	1.00	0.37	0.35	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.124	10.316	0.321	0.000	0.000	0.000	0.000	0.000

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	881	881	161	273	0	0	0	0	-1
N.S.	1	1.00	0.18	0.31	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.566	10.130	0.289	0.000	0.000	0.000	0.000	0.000

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	742	742	161	191	0	0	0	0	-1
N.S.	1	1.00	0.22	0.26	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.395	10.045	0.271	0.000	0.000	0.000	0.000	0.000

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	913	913	331	313	0	0	0	0	-1
N.S.	1	1.00	0.36	0.34	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.862	10.185	0.287	0.000	0.000	0.000	0.000	0.000

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	976	976	429	371	0	0	0	0	-1
N.S.	1	1.00	0.44	0.38	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.103	10.509	0.266	0.000	0.000	0.000	0.000	0.000

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	426	426	477	539	0	0	0	0	-1
N.S.	1	1.00	1.12	1.27	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.335	10.594	0.346	0.000	0.000	0.000	0.000	0.000

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	365	365	396	411	0	0	0	0	-1
N.S.	1	1.00	1.08	1.13	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.246	10.421	0.323	0.000	0.000	0.000	0.000	0.000

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	309	309	342	328	0	0	0	0	-1
N.S.	1	1.00	1.11	1.06	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.172	10.233	0.267	0.000	0.000	0.000	0.000	0.000

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	276	276	233	293	0	0	0	0	-1
N.S.	1	1.00	0.84	1.06	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.138	10.120	0.273	0.000	0.000	0.000	0.000	0.000

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	310	310	386	321	0	0	0	0	-1
N.S.	1	1.00	1.25	1.04	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.159	10.206	0.273	0.000	0.000	0.000	0.000	0.000

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	362	362	374	374	0	0	0	0	-1
N.S.	1	1.00	1.03	1.03	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.438	10.382	0.273	0.000	0.000	0.000	0.000	0.000

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	439	439	382	484	0	0	0	0	-1
N.S.	1	1.00	0.87	1.10	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.347	10.618	0.267	0.000	0.000	0.000	0.000	0.000

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	81	99	0	315	0	0	-1
N.S.	1	1.00	0.79	0.96	0.00	3.06	0.00	0.00	-0.01
time (sec)	N/A	0.037	0.305	0.294	0.000	4.377	0.000	0.000	0.000

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	88	165	0	339	0	0	-1
N.S.	1	1.00	0.76	1.42	0.00	2.92	0.00	0.00	-0.01
time (sec)	N/A	0.016	0.307	0.273	0.000	4.659	0.000	0.000	0.000

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	211	288	0	0	2381	0	0	-1
N.S.	1	1.00	1.36	0.00	0.00	11.28	0.00	0.00	-0.00
time (sec)	N/A	0.145	1.021	0.006	0.000	6.764	0.000	0.000	0.000

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	173	255	0	0	844	0	0	-1
N.S.	1	1.00	1.47	0.00	0.00	4.88	0.00	0.00	-0.01
time (sec)	N/A	0.070	0.742	0.057	0.000	3.585	0.000	0.000	0.000

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	178	0	0	0	0	0	-1
N.S.	1	1.00	1.70	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.039	0.586	0.055	0.000	0.000	0.000	0.000	0.000

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	229	0	0	0	0	0	-1
N.S.	1	1.00	1.71	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.064	1.171	0.054	0.000	0.000	0.000	0.000	0.000

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	264	0	0	0	0	0	-1
N.S.	1	1.00	1.47	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.130	2.620	0.055	0.000	0.000	0.000	0.000	0.000

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	233	233	320	0	0	0	0	0	-1
N.S.	1	1.00	1.37	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.186	3.796	0.060	0.000	0.000	0.000	0.000	0.000

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	316	316	294	0	0	0	0	0	-1
N.S.	1	1.00	0.93	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.211	10.466	0.007	0.000	0.000	0.000	0.000	0.000

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	274	274	346	0	0	0	0	0	-1
N.S.	1	1.00	1.26	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.103	10.239	0.006	0.000	0.000	0.000	0.000	0.000

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	160	0	0	0	0	0	-1
N.S.	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.056	10.125	0.057	0.000	0.000	0.000	0.000	0.000

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	259	259	161	0	0	0	0	0	-1
N.S.	1	1.00	0.62	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.097	10.041	0.053	0.000	0.000	0.000	0.000	0.000

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	304	304	332	0	0	0	0	0	-1
N.S.	1	1.00	1.09	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.153	10.192	0.056	0.000	0.000	0.000	0.000	0.000

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	357	357	430	0	0	0	0	0	-1
N.S.	1	1.00	1.20	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.261	10.470	0.056	0.000	0.000	0.000	0.000	0.000

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	280	280	349	0	0	3308	0	0	-1
N.S.	1	1.00	1.25	0.00	0.00	11.81	0.00	0.00	-0.00
time (sec)	N/A	0.231	1.986	0.010	0.000	41.735	0.000	0.000	0.000

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	230	230	341	0	0	1667	0	0	-1
N.S.	1	1.00	1.48	0.00	0.00	7.25	0.00	0.00	-0.00
time (sec)	N/A	0.112	1.264	0.057	0.000	4.175	0.000	0.000	0.000

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	238	0	0	0	0	0	-1
N.S.	1	1.00	1.76	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.053	0.884	0.055	0.000	0.000	0.000	0.000	0.000

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	251	0	0	0	0	0	-1
N.S.	1	1.00	1.55	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.072	1.622	0.052	0.000	0.000	0.000	0.000	0.000

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	205	285	0	0	0	0	0	-1
N.S.	1	1.00	1.39	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.131	2.924	0.060	0.000	0.000	0.000	0.000	0.000

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	266	266	357	0	0	0	0	0	-1
N.S.	1	1.00	1.34	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.217	4.682	0.063	0.000	0.000	0.000	0.000	0.000

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	353	353	392	0	0	0	0	0	-1
N.S.	1	1.00	1.11	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.225	10.353	0.008	0.000	0.000	0.000	0.000	0.000

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	298	298	341	0	0	0	0	0	-1
N.S.	1	1.00	1.14	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.153	10.250	0.056	0.000	0.000	0.000	0.000	0.000

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	308	308	233	0	0	0	0	0	-1
N.S.	1	1.00	0.76	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.133	10.163	0.055	0.000	0.000	0.000	0.000	0.000

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	330	330	337	0	0	0	0	0	-1
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.161	10.252	0.059	0.000	0.000	0.000	0.000	0.000

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	390	390	387	0	0	0	0	0	-1
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.350	10.370	0.059	0.000	0.000	0.000	0.000	0.000

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	44	211	0	208	0	0	-1
N.S.	1	1.00	0.83	3.98	0.00	3.92	0.00	0.00	-0.02
time (sec)	N/A	0.012	0.176	1.577	0.000	9.170	0.000	0.000	0.000

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	48	0	0	0	0	0	-1
N.S.	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.014	0.315	0.057	0.000	0.000	0.000	0.000	0.000

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	172	0	0	0	0	0	-1
N.S.	1	1.00	2.18	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.028	0.200	0.096	0.000	0.000	0.000	0.000	0.000

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	106	0	0	0	119	0	-1
N.S.	1	1.00	0.60	0.00	0.00	0.00	0.68	0.00	-0.01
time (sec)	N/A	0.086	0.188	0.051	0.000	0.000	83.876	0.000	0.000

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	85	90	0	0	0	75	0	-1
N.S.	1	0.91	0.97	0.00	0.00	0.00	0.81	0.00	-0.01
time (sec)	N/A	0.029	0.126	0.039	0.000	0.000	30.544	0.000	0.000

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	162	0	0	0	0	0	-1
N.S.	1	1.00	2.84	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.018	0.317	0.074	0.000	0.000	0.000	0.000	0.000

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	162	0	0	0	0	0	-1
N.S.	1	1.00	2.84	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.018	0.429	0.054	0.000	0.000	0.000	0.000	0.000

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	545	545	48	0	0	0	0	0	-1
N.S.	1	1.00	0.09	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.755	10.027	0.054	0.000	0.000	0.000	0.000	0.000

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	118	248	164	306	454	0	173
N.S.	1	1.00	0.83	1.73	1.15	2.14	3.17	0.00	1.21
time (sec)	N/A	0.082	0.213	0.072	0.495	3.244	30.666	0.000	2.593

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	84	191	126	208	121	0	99
N.S.	1	1.00	0.85	1.93	1.27	2.10	1.22	0.00	1.00
time (sec)	N/A	0.043	0.184	0.046	0.495	2.969	20.780	0.000	1.901

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	52	164	106	128	87	0	92
N.S.	1	1.00	0.70	2.22	1.43	1.73	1.18	0.00	1.24
time (sec)	N/A	0.030	0.109	0.038	0.485	2.442	24.707	0.000	1.962

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	39	74	50	99	42	64	58
N.S.	1	1.00	1.00	1.90	1.28	2.54	1.08	1.64	1.49
time (sec)	N/A	0.012	0.014	0.018	0.479	3.503	0.920	1.505	0.076

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	100	286	0	482	0	0	149
N.S.	1	1.00	0.96	2.75	0.00	4.63	0.00	0.00	1.43
time (sec)	N/A	0.075	0.221	0.102	0.000	3.448	0.000	0.000	1.631

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	122	945	0	801	0	0	1195
N.S.	1	1.00	0.83	6.43	0.00	5.45	0.00	0.00	8.13
time (sec)	N/A	0.133	0.365	0.076	0.000	4.224	0.000	0.000	2.263

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	213	189	1972	0	1749	0	820	1895
N.S.	1	1.00	0.89	9.26	0.00	8.21	0.00	3.85	8.90
time (sec)	N/A	0.221	1.033	0.091	0.000	3.458	0.000	2.001	3.729

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	159	353	190	380	1817	0	327
N.S.	1	1.00	0.97	2.15	1.16	2.32	11.08	0.00	1.99
time (sec)	N/A	0.097	0.259	0.069	0.494	2.456	74.174	0.000	3.878

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	115	260	152	268	534	0	197
N.S.	1	1.00	0.91	2.06	1.21	2.13	4.24	0.00	1.56
time (sec)	N/A	0.056	0.231	0.057	0.488	3.038	50.320	0.000	2.579

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	73	205	132	164	163	0	81
N.S.	1	1.00	0.73	2.05	1.32	1.64	1.63	0.00	0.81
time (sec)	N/A	0.040	0.172	0.047	0.490	2.795	32.839	0.000	2.512

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	B	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	46	100	63	100	92	0	34
N.S.	1	1.00	0.85	1.85	1.17	1.85	1.70	0.00	0.63
time (sec)	N/A	0.018	0.010	0.023	0.475	2.873	2.251	0.000	1.501

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	103	528	0	519	0	0	556
N.S.	1	1.00	0.97	4.98	0.00	4.90	0.00	0.00	5.25
time (sec)	N/A	0.084	0.267	0.061	0.000	2.554	0.000	0.000	1.684

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	144	836	0	769	0	0	448
N.S.	1	1.00	0.92	5.36	0.00	4.93	0.00	0.00	2.87
time (sec)	N/A	0.148	0.363	0.082	0.000	3.266	0.000	0.000	2.165

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	169	1817	0	1765	0	727	1664
N.S.	1	1.00	0.81	8.69	0.00	8.44	0.00	3.48	7.96
time (sec)	N/A	0.232	0.459	0.099	0.000	3.281	0.000	1.128	3.472

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	201	457	219	494	5513	0	487
N.S.	1	1.00	1.02	2.31	1.11	2.49	27.84	0.00	2.46
time (sec)	N/A	0.104	0.319	0.086	0.492	3.052	85.569	0.000	6.049

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	145	336	181	350	1841	0	271
N.S.	1	1.00	0.95	2.21	1.19	2.30	12.11	0.00	1.78
time (sec)	N/A	0.067	0.266	0.064	0.492	3.260	46.542	0.000	3.791

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	94	253	161	222	520	0	99
N.S.	1	1.00	0.75	2.02	1.29	1.78	4.16	0.00	0.79
time (sec)	N/A	0.051	0.196	0.054	0.495	3.543	29.410	0.000	3.480

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	64	120	78	139	99	0	34
N.S.	1	1.00	0.90	1.69	1.10	1.96	1.39	0.00	0.48
time (sec)	N/A	0.024	0.036	0.000	0.487	3.222	2.139	0.000	1.630

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	117	859	0	659	0	0	1427
N.S.	1	1.00	0.87	6.41	0.00	4.92	0.00	0.00	10.65
time (sec)	N/A	0.142	0.254	0.078	0.000	3.956	0.000	0.000	2.156

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	146	1325	0	1001	0	0	1153
N.S.	1	1.00	0.88	7.98	0.00	6.03	0.00	0.00	6.95
time (sec)	N/A	0.150	0.328	0.084	0.000	3.406	0.000	0.000	2.311

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	237	237	205	1638	0	1445	0	945	1476
N.S.	1	1.00	0.86	6.91	0.00	6.10	0.00	3.99	6.23
time (sec)	N/A	0.242	0.440	0.092	0.000	3.128	0.000	1.816	3.439

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	95	535	166	233	386	0	107
N.S.	1	1.00	0.75	4.25	1.32	1.85	3.06	0.00	0.85
time (sec)	N/A	0.058	0.217	0.062	0.474	2.873	37.555	0.000	1.727

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	66	348	129	158	114	0	63
N.S.	1	1.00	0.90	4.77	1.77	2.16	1.56	0.00	0.86
time (sec)	N/A	0.036	0.155	0.043	0.495	2.500	24.776	0.000	1.621

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	51	174	109	115	82	89	88
N.S.	1	1.00	1.00	3.41	2.14	2.25	1.61	1.75	1.73
time (sec)	N/A	0.021	0.099	0.039	0.501	2.645	23.033	0.598	1.977

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	71	67	98	44	71	66
N.S.	1	1.00	1.00	1.65	1.56	2.28	1.02	1.65	1.53
time (sec)	N/A	0.013	0.015	0.000	0.557	3.218	1.166	1.134	1.441

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	104	229	0	542	0	0	1183
N.S.	1	1.00	0.96	2.12	0.00	5.02	0.00	0.00	10.95
time (sec)	N/A	0.065	0.242	0.050	0.000	3.528	0.000	0.000	1.982

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	149	1137	0	1163	0	501	2500
N.S.	1	1.00	0.87	6.61	0.00	6.76	0.00	2.91	14.53
time (sec)	N/A	0.143	0.546	0.079	0.000	3.388	0.000	1.880	3.536

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	250	250	215	2269	0	2307	0	901	2890
N.S.	1	1.00	0.86	9.08	0.00	9.23	0.00	3.60	11.56
time (sec)	N/A	0.252	1.245	0.098	0.000	4.359	0.000	2.227	5.479

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	113	969	200	336	0	0	172
N.S.	1	1.00	0.86	7.34	1.52	2.55	0.00	0.00	1.30
time (sec)	N/A	0.069	0.210	0.087	0.576	2.030	0.000	0.000	1.908

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	93	89	789	164	272	0	186	120
N.S.	1	0.99	0.95	8.39	1.74	2.89	0.00	1.98	1.28
time (sec)	N/A	0.051	0.209	0.066	0.493	2.580	0.000	1.484	1.834

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	70	387	144	210	224	148	71
N.S.	1	1.00	0.92	5.09	1.89	2.76	2.95	1.95	0.93
time (sec)	N/A	0.032	0.138	0.062	0.490	2.163	20.955	1.603	2.438

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	57	198	85	156	71	118	34
N.S.	1	1.00	0.95	3.30	1.42	2.60	1.18	1.97	0.57
time (sec)	N/A	0.018	0.041	0.034	0.541	2.100	1.732	1.721	1.868

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	144	964	0	1075	0	0	3000
N.S.	1	1.00	0.98	6.56	0.00	7.31	0.00	0.00	20.41
time (sec)	N/A	0.128	0.461	0.074	0.000	2.545	0.000	0.000	2.684

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	224	224	215	3119	0	2321	0	0	2500
N.S.	1	1.00	0.96	13.92	0.00	10.36	0.00	0.00	11.16
time (sec)	N/A	0.214	0.778	0.106	0.000	4.160	0.000	0.000	6.202

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	320	320	298	5158	0	4093	0	0	2500
N.S.	1	1.00	0.93	16.12	0.00	12.79	0.00	0.00	7.81
time (sec)	N/A	0.343	1.258	0.135	0.000	4.814	0.000	0.000	9.488

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	134	1150	228	483	0	436	194
N.S.	1	1.00	0.94	8.04	1.59	3.38	0.00	3.05	1.36
time (sec)	N/A	0.102	0.234	0.098	0.506	2.478	0.000	2.022	2.050

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	121	112	588	190	407	0	370	144
N.S.	1	0.99	0.92	4.82	1.56	3.34	0.00	3.03	1.18
time (sec)	N/A	0.061	0.195	0.077	0.483	2.378	0.000	3.406	2.219

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	91	541	170	331	1479	266	87
N.S.	1	1.00	0.88	5.25	1.65	3.21	14.36	2.58	0.84
time (sec)	N/A	0.041	0.154	0.068	0.503	2.333	49.747	2.006	2.910

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	72	271	101	225	774	173	34
N.S.	1	1.00	0.91	3.43	1.28	2.85	9.80	2.19	0.43
time (sec)	N/A	0.025	0.029	0.058	0.497	2.476	3.748	1.983	1.722

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	200	1767	0	1990	0	0	2500
N.S.	1	1.00	1.00	8.79	0.00	9.90	0.00	0.00	12.44
time (sec)	N/A	0.200	0.511	0.105	0.000	7.514	0.000	0.000	4.622

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	287	287	305	4644	0	3887	0	0	2500
N.S.	1	1.00	1.06	16.18	0.00	13.54	0.00	0.00	8.71
time (sec)	N/A	0.360	1.113	0.130	0.000	7.731	0.000	0.000	8.729

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	409	409	404	7300	0	6171	0	1354	2500
N.S.	1	1.00	0.99	17.85	0.00	15.09	0.00	3.31	6.11
time (sec)	N/A	0.479	1.660	0.228	0.000	14.146	0.000	1.029	8.227

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	176	218	0	890	0	0	2500
N.S.	1	1.00	1.43	1.77	0.00	7.24	0.00	0.00	20.33
time (sec)	N/A	0.108	0.353	0.047	0.000	4.408	0.000	0.000	22.224

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	179	155	0	247	0	0	478
N.S.	1	1.00	2.21	1.91	0.00	3.05	0.00	0.00	5.90
time (sec)	N/A	0.066	0.395	0.034	0.000	3.151	0.000	0.000	6.583

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	126	280	0	319	0	0	-1
N.S.	1	1.00	1.03	2.30	0.00	2.61	0.00	0.00	-0.01
time (sec)	N/A	0.167	0.311	0.041	0.000	2.759	0.000	0.000	0.000

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	206	0	0	0	0	0	-1
N.S.	1	1.00	2.15	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.050	0.303	0.036	0.000	0.000	0.000	0.000	0.000

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	40	34	33	98	82	33	32
N.S.	1	1.00	1.03	0.87	0.85	2.51	2.10	0.85	0.82
time (sec)	N/A	0.018	0.032	0.037	0.476	2.144	0.295	0.567	0.069

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	233	233	205	277	0	0	0	0	-1
N.S.	1	1.00	0.88	1.19	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.149	1.127	0.086	0.000	0.000	0.000	0.000	0.000

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	232	232	86	94	0	0	0	0	-1
N.S.	1	1.00	0.37	0.41	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.150	0.841	0.043	0.000	0.000	0.000	0.000	0.000

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	262	262	191	185	0	0	0	0	-1
N.S.	1	1.00	0.73	0.71	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.191	2.061	0.065	0.000	0.000	0.000	0.000	0.000

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	104	0	0	0	0	0	-1
N.S.	1	1.00	1.32	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.058	0.104	0.037	0.000	0.000	0.000	0.000	0.000

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	129	110	128	390	71	133	123
N.S.	1	1.00	0.89	0.76	0.88	2.69	0.49	0.92	0.85
time (sec)	N/A	0.075	0.060	0.023	0.502	2.317	0.321	1.528	0.274

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	51	48	47	48	82	49	49
N.S.	1	1.00	1.04	0.98	0.96	0.98	1.67	1.00	1.00
time (sec)	N/A	0.035	0.032	0.249	0.280	2.544	0.267	1.170	0.074

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	21	20	20	24	20	20
N.S.	1	1.00	1.00	0.81	0.77	0.77	0.92	0.77	0.77
time (sec)	N/A	0.011	0.014	0.245	0.264	2.089	0.143	1.430	0.034

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	24	23	23	27	24	13
N.S.	1	1.00	1.00	1.41	1.35	1.35	1.59	1.41	0.76
time (sec)	N/A	0.010	0.016	0.263	0.277	2.637	0.188	1.367	1.458

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	104	66	71	71	102	71	90
N.S.	1	1.00	1.00	0.63	0.68	0.68	0.98	0.68	0.87
time (sec)	N/A	0.059	0.111	56.063	0.480	2.845	1.854	1.218	1.497

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	28	23	22	22	26	23	22
N.S.	1	1.00	0.93	0.77	0.73	0.73	0.87	0.77	0.73
time (sec)	N/A	0.013	0.015	0.245	0.272	2.149	0.054	1.296	0.041

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	79	0	0	0	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.023	0.243	0.003	0.000	0.000	0.000	0.000	0.000

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	76	0	0	0	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.018	0.078	0.004	0.000	0.000	0.000	0.000	0.000

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	72	0	0	0	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.019	0.045	0.130	0.000	0.000	0.000	0.000	0.000

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	123	128	186	527	2744	740	131
N.S.	1	1.00	0.93	0.97	1.41	3.99	20.79	5.61	0.99
time (sec)	N/A	0.080	0.493	0.255	0.298	2.605	0.817	1.808	1.638

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	92	96	140	319	1540	450	99
N.S.	1	1.00	0.93	0.97	1.41	3.22	15.56	4.55	1.00
time (sec)	N/A	0.052	0.143	0.250	0.297	2.632	1.210	0.911	1.557

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	65	68	94	175	726	232	71
N.S.	1	1.00	0.93	0.97	1.34	2.50	10.37	3.31	1.01
time (sec)	N/A	0.033	0.097	0.232	0.291	3.448	0.295	0.713	1.535

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	37	39	48	69	236	83	38
N.S.	1	1.00	0.92	0.98	1.20	1.72	5.90	2.08	0.95
time (sec)	N/A	0.015	0.077	0.021	0.289	3.370	0.179	0.829	1.483

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	40	0	0	0	73	0	-1
N.S.	1	1.00	0.93	0.00	0.00	0.00	1.70	0.00	-0.02
time (sec)	N/A	0.012	0.084	0.007	0.000	0.000	1.143	0.000	0.000

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	56	0	0	0	592	0	-1
N.S.	1	1.00	0.77	0.00	0.00	0.00	8.11	0.00	-0.01
time (sec)	N/A	0.021	0.054	0.010	0.000	0.000	3.495	0.000	0.000

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	58	0	0	0	0	0	-1
N.S.	1	1.00	0.74	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.021	0.061	0.017	0.000	0.000	0.000	0.000	0.000

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	58	0	0	0	18149	0	-1
N.S.	1	1.00	0.74	0.00	0.00	0.00	232.68	0.00	-0.01
time (sec)	N/A	0.021	0.069	0.026	0.000	0.000	190.753	0.000	0.000

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	149	154	242	624	3380	947	157
N.S.	1	1.00	0.94	0.97	1.53	3.95	21.39	5.99	0.99
time (sec)	N/A	0.089	0.574	0.260	0.287	2.184	20.849	0.566	1.705

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	105	109	168	369	1765	539	108
N.S.	1	1.00	0.94	0.97	1.50	3.29	15.76	4.81	0.96
time (sec)	N/A	0.057	0.585	0.263	0.293	3.234	12.358	0.655	1.566

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	65	68	94	175	726	232	71
N.S.	1	1.00	0.93	0.97	1.34	2.50	10.37	3.31	1.01
time (sec)	N/A	0.034	0.096	0.239	0.294	3.378	0.300	1.579	1.531

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	82	0	0	0	170	0	-1
N.S.	1	1.00	0.98	0.00	0.00	0.00	2.02	0.00	-0.01
time (sec)	N/A	0.061	0.144	0.012	0.000	0.000	1.936	0.000	0.000

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	95	0	0	0	0	0	-1
N.S.	1	1.00	0.83	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.062	0.171	0.010	0.000	0.000	0.000	0.000	0.000

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	133	0	0	0	0	0	-1
N.S.	1	1.00	0.83	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.106	0.137	0.016	0.000	0.000	0.000	0.000	0.000

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	310	310	133	0	0	0	369	0	-1
N.S.	1	1.00	0.43	0.00	0.00	0.00	1.19	0.00	-0.00
time (sec)	N/A	0.342	2.343	0.018	0.000	0.000	4.810	0.000	0.000

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	173	104	0	0	0	269	0	-1
N.S.	1	1.00	0.60	0.00	0.00	0.00	1.55	0.00	-0.01
time (sec)	N/A	0.183	0.984	0.013	0.000	0.000	2.797	0.000	0.000

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	75	0	0	0	170	0	-1
N.S.	1	1.00	0.89	0.00	0.00	0.00	2.02	0.00	-0.01
time (sec)	N/A	0.065	0.311	0.010	0.000	0.000	1.972	0.000	0.000

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	40	0	0	0	73	0	-1
N.S.	1	1.00	0.95	0.00	0.00	0.00	1.74	0.00	-0.02
time (sec)	N/A	0.011	0.086	0.008	0.000	0.000	1.992	0.000	0.000

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	64	0	0	0	0	0	-1
N.S.	1	1.00	0.89	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.019	0.062	0.108	0.000	0.000	0.000	0.000	0.000

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	121	0	0	0	0	0	-1
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.098	0.135	0.020	0.000	0.000	0.000	0.000	0.000

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	210	210	0	0	0	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.231	0.186	0.037	0.000	0.000	0.000	0.000	0.000

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	341	341	217	0	0	0	0	0	-1
N.S.	1	1.00	0.64	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.381	6.096	0.019	0.000	0.000	0.000	0.000	0.000

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	2050	0	0	0	0	0	-1
N.S.	1	1.00	10.25	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.180	3.351	0.014	0.000	0.000	0.000	0.000	0.000

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	666	0	0	0	0	0	-1
N.S.	1	1.00	5.79	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.064	1.211	0.011	0.000	0.000	0.000	0.000	0.000

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	56	0	0	0	592	0	-1
N.S.	1	1.00	0.78	0.00	0.00	0.00	8.22	0.00	-0.01
time (sec)	N/A	0.020	0.056	0.013	0.000	0.000	3.470	0.000	0.000

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	108	0	0	0	0	0	-1
N.S.	1	1.00	0.89	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.098	0.147	0.020	0.000	0.000	0.000	0.000	0.000

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	193	147	0	0	0	0	0	-1
N.S.	1	1.00	0.76	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.197	0.194	0.026	0.000	0.000	0.000	0.000	0.000

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	299	299	233	0	0	0	0	0	-1
N.S.	1	1.00	0.78	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.382	0.283	0.043	0.000	0.000	0.000	0.000	0.000

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	190	0	0	0	0	0	-1
N.S.	1	1.00	2.35	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.044	0.295	0.122	0.000	0.000	0.000	0.000	0.000

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	402	401	168	0	0	0	199	0	-1
N.S.	1	1.00	0.42	0.00	0.00	0.00	0.50	0.00	-0.00
time (sec)	N/A	0.412	5.222	0.072	0.000	0.000	54.422	0.000	0.000

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	197	140	0	0	0	143	0	-1
N.S.	1	0.98	0.69	0.00	0.00	0.00	0.71	0.00	-0.00
time (sec)	N/A	0.181	5.154	0.069	0.000	0.000	25.052	0.000	0.000

Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	89	94	0	0	0	87	0	-1
N.S.	1	0.91	0.96	0.00	0.00	0.00	0.89	0.00	-0.01
time (sec)	N/A	0.034	0.052	0.080	0.000	0.000	3.825	0.000	0.000

Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	46	0	0	0	37	0	47
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.80	0.00	1.02
time (sec)	N/A	0.009	0.005	0.002	0.000	0.000	0.799	0.000	2.299

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	180	0	0	0	0	0	-1
N.S.	1	1.00	3.05	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.021	0.211	0.145	0.000	0.000	0.000	0.000	0.000

Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	180	0	0	0	0	0	-1
N.S.	1	1.00	3.05	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.020	0.231	0.107	0.000	0.000	0.000	0.000	0.000

Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	180	0	0	0	0	0	-1
N.S.	1	1.00	3.05	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.020	0.323	0.097	0.000	0.000	0.000	0.000	0.000

Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	94	0	0	0	0	0	-1
N.S.	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.018	0.109	0.129	0.000	0.000	0.000	0.000	0.000

Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	178	218	0	0	478	0	0	-1
N.S.	1	1.00	1.22	0.00	0.00	2.69	0.00	0.00	-0.01
time (sec)	N/A	0.062	0.137	0.069	0.000	3.414	0.000	0.000	0.000

Problem 322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	113	0	0	231	0	0	-1
N.S.	1	1.00	0.97	0.00	0.00	1.99	0.00	0.00	-0.01
time (sec)	N/A	0.025	0.092	0.086	0.000	2.469	0.000	0.000	0.000

Problem 323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	82	0	0	85	0	0	-1
N.S.	1	1.00	1.41	0.00	0.00	1.47	0.00	0.00	-0.02
time (sec)	N/A	0.009	0.110	0.076	0.000	3.006	0.000	0.000	0.000

Problem 324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	53	0	31	207	0	75
N.S.	1	1.00	1.00	2.94	0.00	1.72	11.50	0.00	4.17
time (sec)	N/A	0.002	0.033	0.325	0.000	3.084	11.382	0.000	1.756

Problem 325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	52	0	0	0	0	0	-1
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.010	0.044	0.128	0.000	0.000	0.000	0.000	0.000

Problem 326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	53	0	0	0	0	0	-1
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.010	0.057	0.100	0.000	0.000	0.000	0.000	0.000

Problem 327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	55	0	0	0	0	0	-1
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.010	0.054	0.101	0.000	0.000	0.000	0.000	0.000

Problem 328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	193	179	1414	0	0	0	0	0	-1
N.S.	1	0.93	7.33	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.060	37.640	0.124	0.000	0.000	0.000	0.000	0.000

Problem 329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	55	0	0	108	0	0	-1
N.S.	1	1.00	0.96	0.00	0.00	1.89	0.00	0.00	-0.02
time (sec)	N/A	0.018	0.384	0.155	0.000	2.848	0.000	0.000	0.000

Problem 330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	327	327	136	0	0	400	0	0	-1
N.S.	1	1.00	0.42	0.00	0.00	1.22	0.00	0.00	-0.00
time (sec)	N/A	0.128	0.312	0.069	0.000	2.199	0.000	0.000	0.000

Problem 331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	94	0	0	173	0	0	-1
N.S.	1	1.00	0.74	0.00	0.00	1.36	0.00	0.00	-0.01
time (sec)	N/A	0.044	0.127	0.078	0.000	2.468	0.000	0.000	0.000

Problem 332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F(-2)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	55	0	0	68	0	0	64
N.S.	1	1.00	1.10	0.00	0.00	1.36	0.00	0.00	1.28
time (sec)	N/A	0.010	0.040	0.073	0.000	3.364	0.000	0.000	1.763

Problem 333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	153	0	0	0	0	0	-1
N.S.	1	1.00	1.61	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.025	6.005	0.144	0.000	0.000	0.000	0.000	0.000

Problem 334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	1070	0	0	0	0	0	-1
N.S.	1	1.00	8.43	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.036	36.734	0.101	0.000	0.000	0.000	0.000	0.000

Problem 335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	1241	0	0	0	0	0	-1
N.S.	1	1.00	9.47	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.037	40.118	0.108	0.000	0.000	0.000	0.000	0.000

Problem 336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F(-1)	F	F	F	F(-2)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.040	180.008	0.099	0.000	0.000	0.000	0.000	0.000

Problem 337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	164	97	104	178	114	0	621	152
N.S.	1	1.08	0.64	0.68	1.17	0.75	0.00	4.09	1.00
time (sec)	N/A	0.080	0.172	0.283	0.331	3.720	0.000	0.668	1.761

Problem 338	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	118	75	80	124	90	0	495	118
N.S.	1	1.08	0.69	0.73	1.14	0.83	0.00	4.54	1.08
time (sec)	N/A	0.055	0.134	0.312	0.273	3.647	0.000	0.761	1.736

Problem 339	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	72	49	56	70	66	0	361	83
N.S.	1	1.07	0.73	0.84	1.04	0.99	0.00	5.39	1.24
time (sec)	N/A	0.030	0.100	0.274	0.265	2.648	0.000	0.743	1.641

Problem 340	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	75	174	52	80	0	78	248
N.S.	1	1.00	0.94	2.18	0.65	1.00	0.00	0.98	3.10
time (sec)	N/A	0.051	0.137	0.283	0.509	1.885	0.000	0.628	3.598

Problem 341	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	114	74	182	98	85	0	157	584
N.S.	1	1.19	0.77	1.90	1.02	0.89	0.00	1.64	6.08
time (sec)	N/A	0.057	0.175	0.325	0.485	2.391	0.000	0.638	6.890

Problem 342	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	164	99	226	162	100	0	324	1004
N.S.	1	1.36	0.82	1.87	1.34	0.83	0.00	2.68	8.30
time (sec)	N/A	0.066	0.173	0.305	0.519	2.644	0.000	0.640	15.557

Problem 343	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	208	208	142	298	246	138	0	558	2314
N.S.	1	1.00	0.68	1.43	1.18	0.66	0.00	2.68	11.12
time (sec)	N/A	0.092	0.298	0.292	0.279	3.865	0.000	0.671	39.151

Problem 344	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	116	240	192	112	0	432	1681
N.S.	1	1.00	0.73	1.51	1.21	0.70	0.00	2.72	10.57
time (sec)	N/A	0.077	0.241	0.291	0.267	3.438	0.000	0.629	42.568

Problem 345	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	92	182	137	88	0	288	734
N.S.	1	1.00	0.81	1.60	1.20	0.77	0.00	2.53	6.44
time (sec)	N/A	0.030	0.176	0.288	0.262	3.130	0.000	0.715	17.425

Problem 346	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	74	153	105	83	0	110	243
N.S.	1	1.00	0.71	1.47	1.01	0.80	0.00	1.06	2.34
time (sec)	N/A	0.059	0.196	0.308	0.503	2.848	0.000	0.583	3.486

Problem 347	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	81	153	75	100	0	171	236
N.S.	1	1.00	0.96	1.82	0.89	1.19	0.00	2.04	2.81
time (sec)	N/A	0.051	0.163	0.298	0.561	2.347	0.000	0.630	3.438

Problem 348	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F(-1)	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	96	191	153	96	0	172	1154
N.S.	1	1.00	0.77	1.53	1.22	0.77	0.00	1.38	9.23
time (sec)	N/A	0.058	0.212	0.288	0.283	3.282	0.000	0.654	32.625

Problem 349	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	61	57	95	55	216	108	108
N.S.	1	1.00	0.59	0.55	0.92	0.53	2.10	1.05	1.05
time (sec)	N/A	0.078	0.097	0.299	0.269	2.712	42.074	0.764	2.440

Problem 350	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F(-1)	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	77	147	113	77	0	121	720
N.S.	1	1.00	0.89	1.69	1.30	0.89	0.00	1.39	8.28
time (sec)	N/A	0.098	0.162	0.276	0.305	2.890	0.000	0.952	22.496

Problem 351	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	43	38	54	37	202	59	66
N.S.	1	1.00	0.66	0.58	0.83	0.57	3.11	0.91	1.02
time (sec)	N/A	0.057	0.073	0.271	0.286	2.847	27.303	0.687	2.359

Problem 352	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F(-1)	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	58	103	74	55	0	69	293
N.S.	1	1.00	1.23	2.19	1.57	1.17	0.00	1.47	6.23
time (sec)	N/A	0.030	0.104	0.302	0.280	2.968	0.000	0.686	12.685

Problem 353	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	45	62	29	48	162	45	77
N.S.	1	1.00	0.98	1.35	0.63	1.04	3.52	0.98	1.67
time (sec)	N/A	0.067	0.075	0.271	0.604	4.058	38.994	0.612	3.865

Problem 354	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	C	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	48	77	44	56	148	58	61
N.S.	1	1.00	1.45	2.33	1.33	1.70	4.48	1.76	1.85
time (sec)	N/A	0.036	0.087	0.287	0.501	3.324	38.912	0.682	2.593

Problem 355	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	55	84	45	57	0	114	297
N.S.	1	1.00	0.92	1.40	0.75	0.95	0.00	1.90	4.95
time (sec)	N/A	0.046	0.096	0.309	0.561	3.090	0.000	0.606	8.668

Problem 356	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	42	41	54	52	146	116	53
N.S.	1	1.00	0.68	0.66	0.87	0.84	2.35	1.87	0.85
time (sec)	N/A	0.039	0.086	0.279	0.511	3.353	26.803	0.635	2.440

Problem 357	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	78	125	85	78	0	268	650
N.S.	1	1.00	0.79	1.26	0.86	0.79	0.00	2.71	6.57
time (sec)	N/A	0.051	0.176	0.295	0.490	3.487	0.000	0.804	21.455

Problem 358	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	119	240	196	115	0	203	1682
N.S.	1	1.00	0.73	1.46	1.20	0.70	0.00	1.24	10.26
time (sec)	N/A	0.078	0.248	0.331	0.280	3.424	0.000	0.653	42.656

Problem 359	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	74	68	124	66	240	124	130
N.S.	1	1.00	0.63	0.58	1.05	0.56	2.03	1.05	1.10
time (sec)	N/A	0.057	0.120	0.280	0.316	3.322	30.638	0.670	2.700

Problem 360	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	92	182	142	90	0	140	1048
N.S.	1	1.00	0.78	1.54	1.20	0.76	0.00	1.19	8.88
time (sec)	N/A	0.066	0.180	0.292	0.310	3.384	0.000	0.646	25.513

Problem 361	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	48	43	69	42	223	65	76
N.S.	1	1.00	0.67	0.60	0.96	0.58	3.10	0.90	1.06
time (sec)	N/A	0.030	0.085	0.304	0.329	2.844	19.782	0.609	2.663

Problem 362	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	68	124	89	63	0	79	417
N.S.	1	1.00	1.00	1.82	1.31	0.93	0.00	1.16	6.13
time (sec)	N/A	0.022	0.115	0.289	0.311	2.722	0.000	0.562	10.800

Problem 363	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	54	108	37	61	178	55	108
N.S.	1	1.00	0.96	1.93	0.66	1.09	3.18	0.98	1.93
time (sec)	N/A	0.047	0.091	0.296	0.486	2.657	28.478	0.624	3.967

Problem 364	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	57	97	55	68	165	66	77
N.S.	1	1.00	1.00	1.70	0.96	1.19	2.89	1.16	1.35
time (sec)	N/A	0.048	0.101	0.301	0.504	2.176	27.986	0.666	2.945

Problem 365	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	70	158	60	73	0	141	457
N.S.	1	1.00	0.92	2.08	0.79	0.96	0.00	1.86	6.01
time (sec)	N/A	0.047	0.113	0.303	0.488	2.409	0.000	0.597	7.500

Problem 366	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	54	53	75	67	170	137	79
N.S.	1	1.00	0.72	0.71	1.00	0.89	2.27	1.83	1.05
time (sec)	N/A	0.045	0.103	0.293	0.482	2.606	32.116	0.574	2.765

Problem 367	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	101	227	114	100	0	325	1005
N.S.	1	1.00	0.82	1.85	0.93	0.81	0.00	2.64	8.17
time (sec)	N/A	0.061	0.176	0.298	0.531	2.351	0.000	0.583	19.135

Problem 368	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	137	324	196	190	0	214	-1
N.S.	1	1.00	0.85	2.01	1.22	1.18	0.00	1.33	-0.01
time (sec)	N/A	0.082	0.248	0.320	0.307	3.143	0.000	0.600	0.000

Problem 369	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	72	76	123	80	0	200	90
N.S.	1	1.00	0.63	0.66	1.07	0.70	0.00	1.74	0.78
time (sec)	N/A	0.062	0.121	0.326	0.274	3.328	0.000	0.572	2.801

Problem 370	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	108	263	138	159	0	147	-1
N.S.	1	1.00	0.71	1.73	0.91	1.05	0.00	0.97	-0.01
time (sec)	N/A	0.073	0.183	0.324	0.302	3.713	0.000	0.603	0.000

Problem 371	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	45	50	69	56	0	152	67
N.S.	1	1.00	0.59	0.66	0.91	0.74	0.00	2.00	0.88
time (sec)	N/A	0.037	0.100	0.293	0.279	2.997	0.000	0.553	2.747

Problem 372	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	71	166	76	129	0	113	-1
N.S.	1	1.00	1.13	2.63	1.21	2.05	0.00	1.79	-0.02
time (sec)	N/A	0.022	0.174	0.302	0.290	3.522	0.000	0.577	0.000

Problem 373	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	70	194	58	101	0	115	-1
N.S.	1	1.00	1.08	2.98	0.89	1.55	0.00	1.77	-0.02
time (sec)	N/A	0.053	0.147	0.296	0.530	3.129	0.000	0.593	0.000

Problem 374	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	51	60	71	103	0	219	73
N.S.	1	1.00	0.76	0.90	1.06	1.54	0.00	3.27	1.09
time (sec)	N/A	0.050	0.106	0.298	0.497	4.109	0.000	0.880	2.866

Problem 375	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	93	324	104	138	0	211	-1
N.S.	1	1.00	0.79	2.77	0.89	1.18	0.00	1.80	-0.01
time (sec)	N/A	0.066	0.202	0.337	0.505	4.139	0.000	0.712	0.000

Problem 376	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	77	85	125	132	0	242	104
N.S.	1	1.00	0.65	0.71	1.05	1.11	0.00	2.03	0.87
time (sec)	N/A	0.067	0.135	0.296	0.546	3.948	0.000	0.873	2.897

Problem 377	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	122	395	162	165	0	402	-1
N.S.	1	1.00	0.73	2.38	0.98	0.99	0.00	2.42	-0.01
time (sec)	N/A	0.085	0.278	0.344	0.499	3.787	0.000	1.036	0.000

Problem 378	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	40	53	23	39	148	40	72
N.S.	1	1.00	1.00	1.32	0.58	0.98	3.70	1.00	1.80
time (sec)	N/A	0.036	0.063	0.280	0.489	3.190	26.876	0.683	3.652

Problem 379	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F(-2)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	296	66	79	65	0	0	96
N.S.	1	1.00	5.58	1.25	1.49	1.23	0.00	0.00	1.81
time (sec)	N/A	0.057	10.327	0.292	0.371	2.718	0.000	0.000	3.274

Problem 380	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	44	0	0	69	0	0	-1
N.S.	1	1.00	1.22	0.00	0.00	1.92	0.00	0.00	-0.03
time (sec)	N/A	0.013	7.180	0.090	0.000	1.897	0.000	0.000	0.000

Problem 381	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	75	0	0	50	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.67	0.00	0.00	-0.01
time (sec)	N/A	0.035	10.024	0.091	0.000	2.083	0.000	0.000	0.000

Problem 382	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-2)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.063	0.210	0.373	0.000	0.000	0.000	0.000	0.000

Problem 383	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	87	0	0	0	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.042	0.076	0.352	0.000	0.000	0.000	0.000	0.000

Problem 384	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	258	0	0	0	0	0	-1
N.S.	1	1.00	3.39	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.039	0.319	0.164	0.000	0.000	0.000	0.000	0.000

Problem 385	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	104	103	0	0	180	0	0	-1
N.S.	1	1.08	1.07	0.00	0.00	1.88	0.00	0.00	-0.01
time (sec)	N/A	0.088	0.902	0.391	0.000	3.270	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [385] had the largest ratio of [76]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	2	1	1.00	17	0.059
2	A	2	1	1.00	17	0.059
3	A	2	1	1.00	17	0.059
4	A	2	1	1.00	15	0.067
5	A	7	7	1.00	17	0.412
6	A	7	7	1.00	17	0.412
7	A	8	8	1.00	17	0.471
8	A	2	1	1.00	19	0.053
9	A	2	1	1.00	19	0.053
10	A	2	1	1.00	17	0.059
11	A	8	7	1.00	19	0.368
12	A	9	8	1.00	19	0.421
13	A	8	8	1.00	19	0.421
14	A	8	7	1.00	19	0.368
15	A	8	7	1.00	19	0.368
16	A	8	7	1.00	19	0.368
17	A	7	7	1.00	17	0.412
18	A	13	7	1.00	19	0.368
19	A	14	8	1.00	19	0.421
20	A	9	8	1.00	19	0.421
21	A	9	8	1.00	19	0.421
22	A	9	8	1.00	19	0.421
23	A	9	8	1.00	19	0.421
24	A	7	7	1.00	17	0.412
25	A	14	8	1.00	19	0.421

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
26	A	15	9	1.00	19	0.474
27	A	3	3	1.00	20	0.150
28	A	2	2	1.00	20	0.100
29	A	2	2	1.00	20	0.100
30	A	2	2	1.00	20	0.100
31	A	3	3	1.00	20	0.150
32	A	4	3	1.00	20	0.150
33	A	5	3	1.00	20	0.150
34	A	22	14	1.00	22	0.636
35	A	21	13	1.00	22	0.591
36	A	14	8	1.00	22	0.364
37	A	17	11	1.00	22	0.500
38	A	21	13	1.00	22	0.591
39	A	22	14	1.00	22	0.636
40	A	4	4	1.00	22	0.182
41	A	3	3	1.00	22	0.136
42	A	3	3	1.00	22	0.136
43	A	3	3	1.00	22	0.136
44	A	3	2	1.00	22	0.091
45	A	4	3	1.00	22	0.136
46	A	5	4	1.00	22	0.182
47	A	6	4	1.00	22	0.182
48	A	4	4	1.00	22	0.182
49	A	4	4	1.00	22	0.182
50	A	4	4	1.00	22	0.182
51	A	4	4	1.00	22	0.182
52	A	4	4	1.00	22	0.182
53	A	4	4	1.00	22	0.182
54	A	4	4	1.00	22	0.182
55	A	4	4	1.00	22	0.182
56	A	4	3	1.00	19	0.158
57	A	3	3	1.00	19	0.158
58	A	2	2	1.00	19	0.105
59	A	2	2	1.00	19	0.105
60	A	2	2	1.00	19	0.105

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
61	A	3	3	1.00	19	0.158
62	A	4	3	1.00	19	0.158
63	A	5	3	1.00	19	0.158
64	A	3	3	1.00	19	0.158
65	A	3	3	1.00	19	0.158
66	A	3	3	1.00	19	0.158
67	A	3	3	1.00	19	0.158
68	A	3	3	1.00	19	0.158
69	A	3	3	1.00	19	0.158
70	A	5	4	1.00	21	0.190
71	A	4	4	1.00	21	0.190
72	A	3	3	1.00	21	0.143
73	A	3	3	1.00	21	0.143
74	A	3	3	1.00	21	0.143
75	A	3	2	1.00	21	0.095
76	A	4	3	1.00	21	0.143
77	A	5	4	1.00	21	0.190
78	A	6	4	1.00	21	0.190
79	A	4	4	1.00	21	0.190
80	A	4	4	1.00	21	0.190
81	A	4	4	1.00	21	0.190
82	A	4	4	1.00	21	0.190
83	A	4	4	1.00	21	0.190
84	A	4	4	1.00	21	0.190
85	A	4	2	1.00	21	0.095
86	A	5	5	1.00	21	0.238
87	A	4	4	1.00	21	0.190
88	A	3	3	1.00	21	0.143
89	A	1	1	1.00	21	0.048
90	A	2	2	1.00	21	0.095
91	A	4	4	1.00	21	0.190
92	A	5	4	1.00	21	0.190
93	A	2	2	1.00	21	0.095
94	A	2	2	1.00	21	0.095
95	A	2	2	1.00	21	0.095

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
96	A	2	2	1.00	21	0.095
97	A	2	2	1.00	21	0.095
98	A	5	5	1.00	21	0.238
99	A	4	4	1.00	21	0.190
100	A	2	2	1.00	21	0.095
101	A	2	2	1.00	21	0.095
102	A	4	4	1.00	21	0.190
103	A	5	4	1.00	21	0.190
104	A	2	2	1.00	21	0.095
105	A	2	2	1.00	21	0.095
106	A	2	2	1.00	21	0.095
107	A	2	2	1.00	21	0.095
108	A	2	2	1.00	21	0.095
109	A	7	6	1.00	21	0.286
110	A	6	6	1.00	21	0.286
111	A	5	5	1.00	21	0.238
112	A	3	2	1.00	21	0.095
113	A	3	3	1.00	21	0.143
114	A	4	4	1.00	21	0.190
115	A	5	4	1.00	21	0.190
116	A	6	4	1.00	21	0.190
117	A	2	2	1.00	21	0.095
118	A	2	2	1.00	21	0.095
119	A	2	2	1.00	21	0.095
120	A	2	2	1.00	21	0.095
121	A	2	2	1.00	21	0.095
122	A	3	2	1.00	23	0.087
123	A	3	2	1.00	23	0.087
124	A	2	2	1.00	23	0.087
125	A	2	2	1.00	23	0.087
126	A	1	1	1.00	23	0.043
127	A	1	1	1.00	23	0.043
128	A	1	1	1.00	23	0.043
129	A	2	2	1.00	23	0.087
130	A	2	2	1.00	23	0.087

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
131	A	3	2	1.00	23	0.087
132	A	3	2	1.00	23	0.087
133	A	3	2	1.00	19	0.105
134	A	4	4	1.05	19	0.210
135	A	3	3	1.01	17	0.176
136	A	2	2	1.00	19	0.105
137	A	2	2	1.00	19	0.105
138	A	5	5	1.00	19	0.263
139	A	4	4	1.00	19	0.210
140	A	3	3	0.91	17	0.176
141	A	2	2	1.00	9	0.222
142	A	2	2	1.00	19	0.105
143	A	2	2	1.00	19	0.105
144	A	2	2	1.00	19	0.105
145	A	1	1	1.00	50	0.020
146	A	2	1	1.00	17	0.059
147	A	2	1	1.00	17	0.059
148	A	2	1	1.00	17	0.059
149	A	2	1	1.00	15	0.067
150	A	10	7	1.00	17	0.412
151	A	10	7	1.00	17	0.412
152	A	11	8	1.00	17	0.471
153	A	2	1	1.00	19	0.053
154	A	2	1	1.00	19	0.053
155	A	2	1	1.00	19	0.053
156	A	2	1	1.00	17	0.059
157	A	11	7	1.00	19	0.368
158	A	12	8	1.00	19	0.421
159	A	11	8	1.00	19	0.421
160	A	11	7	1.00	19	0.368
161	A	11	7	1.00	19	0.368
162	A	11	7	1.00	19	0.368
163	A	10	7	1.00	17	0.412
164	A	19	7	1.00	19	0.368
165	A	20	8	1.00	19	0.421

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
166	A	12	8	1.00	19	0.421
167	A	12	8	1.00	19	0.421
168	A	12	8	1.00	19	0.421
169	A	12	8	1.00	19	0.421
170	A	10	7	1.00	17	0.412
171	A	20	8	1.00	19	0.421
172	A	21	9	1.00	19	0.474
173	A	10	8	1.00	23	0.348
174	A	9	7	1.00	23	0.304
175	A	8	6	1.00	23	0.261
176	A	5	3	1.00	23	0.130
177	A	9	7	1.00	23	0.304
178	A	10	8	1.00	23	0.348
179	A	10	6	1.00	21	0.286
180	A	9	5	1.00	21	0.238
181	A	7	4	1.00	21	0.190
182	A	10	6	1.00	21	0.286
183	A	11	7	1.00	21	0.333
184	A	11	8	1.00	23	0.348
185	A	10	8	1.00	23	0.348
186	A	9	7	1.00	23	0.304
187	A	9	7	1.00	23	0.304
188	A	9	7	1.00	23	0.304
189	A	10	8	1.00	23	0.348
190	A	11	8	1.00	23	0.348
191	A	4	4	1.00	25	0.160
192	A	1	1	1.00	25	0.040
193	A	10	9	1.00	21	0.429
194	A	9	8	1.00	21	0.381
195	A	4	4	1.00	21	0.190
196	A	5	5	1.00	21	0.238
197	A	7	7	1.00	21	0.333
198	A	8	7	1.00	21	0.333
199	A	11	10	1.00	21	0.476
200	A	10	9	1.00	21	0.429

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
201	A	4	3	1.00	21	0.143
202	A	9	8	1.00	21	0.381
203	A	10	9	1.00	21	0.429
204	A	11	10	1.00	21	0.476
205	A	11	10	1.00	21	0.476
206	A	10	9	1.00	21	0.429
207	A	5	5	1.00	21	0.238
208	A	5	5	1.00	21	0.238
209	A	7	7	1.00	21	0.333
210	A	8	7	1.00	21	0.333
211	A	11	10	1.00	21	0.476
212	A	10	9	1.00	21	0.429
213	A	10	9	1.00	21	0.429
214	A	10	9	1.00	21	0.429
215	A	11	10	1.00	21	0.476
216	A	4	4	1.00	17	0.235
217	A	4	4	1.00	26	0.154
218	A	3	2	1.00	19	0.105
219	A	4	4	1.00	19	0.210
220	A	3	3	0.91	17	0.176
221	A	2	2	1.00	19	0.105
222	A	2	2	1.00	19	0.105
223	A	7	7	1.00	21	0.333
224	A	6	6	1.00	21	0.286
225	A	6	6	1.00	21	0.286
226	A	5	5	1.00	19	0.263
227	A	4	4	1.00	11	0.364
228	A	7	6	1.00	21	0.286
229	A	8	7	1.00	21	0.333
230	A	9	7	1.00	21	0.333
231	A	7	7	1.00	21	0.333
232	A	7	6	1.00	21	0.286
233	A	6	5	1.00	19	0.263
234	A	5	5	1.00	11	0.454
235	A	7	6	1.00	21	0.286

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
236	A	8	7	1.00	21	0.333
237	A	9	7	1.00	21	0.333
238	A	8	7	1.00	21	0.333
239	A	8	6	1.00	21	0.286
240	A	7	5	1.00	19	0.263
241	A	6	5	1.00	11	0.454
242	A	8	7	1.00	21	0.333
243	A	8	7	1.00	21	0.333
244	A	9	8	1.00	21	0.381
245	A	5	5	1.00	21	0.238
246	A	5	5	1.00	21	0.238
247	A	4	4	1.00	19	0.210
248	A	4	4	1.00	11	0.364
249	A	7	6	1.00	21	0.286
250	A	8	7	1.00	21	0.333
251	A	9	7	1.00	21	0.333
252	A	5	5	1.00	21	0.238
253	A	5	5	0.99	21	0.238
254	A	5	5	1.00	19	0.263
255	A	5	5	1.00	11	0.454
256	A	8	7	1.00	21	0.333
257	A	9	8	1.00	21	0.381
258	A	10	8	1.00	21	0.381
259	A	5	5	1.00	21	0.238
260	A	6	6	0.99	21	0.286
261	A	6	5	1.00	19	0.263
262	A	6	5	1.00	11	0.454
263	A	9	7	1.00	21	0.333
264	A	10	8	1.00	21	0.381
265	A	11	8	1.00	21	0.381
266	A	8	8	1.00	23	0.348
267	A	4	4	1.00	23	0.174
268	A	5	5	1.00	23	0.217
269	A	3	3	1.00	19	0.158
270	A	3	3	1.00	17	0.176

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
271	A	6	6	1.00	23	0.261
272	A	7	7	1.00	23	0.304
273	A	7	7	1.00	23	0.304
274	A	4	3	1.00	19	0.158
275	A	8	8	1.00	17	0.471
276	A	3	2	1.00	21	0.095
277	A	3	2	1.00	17	0.118
278	A	4	4	1.00	17	0.235
279	A	9	9	1.00	17	0.529
280	A	4	3	1.00	17	0.176
281	A	3	3	1.00	24	0.125
282	A	3	3	1.00	24	0.125
283	A	3	3	1.00	20	0.150
284	A	2	1	1.00	17	0.059
285	A	2	1	1.00	17	0.059
286	A	2	1	1.00	17	0.059
287	A	2	1	1.00	15	0.067
288	A	2	2	1.00	17	0.118
289	A	2	2	1.00	17	0.118
290	A	2	2	1.00	17	0.118
291	A	2	2	1.00	17	0.118
292	A	2	1	1.00	19	0.053
293	A	2	1	1.00	19	0.053
294	A	2	1	1.00	17	0.059
295	A	3	3	1.00	19	0.158
296	A	3	3	1.00	19	0.158
297	A	3	3	1.00	19	0.158
298	A	5	4	1.00	19	0.210
299	A	4	4	1.00	19	0.210
300	A	3	3	1.00	19	0.158
301	A	2	2	1.00	17	0.118
302	A	3	2	1.00	19	0.105
303	A	4	3	1.00	19	0.158
304	A	5	4	1.00	19	0.210
305	A	5	4	1.00	19	0.210

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
306	A	4	4	1.00	19	0.210
307	A	3	3	1.00	19	0.158
308	A	2	2	1.00	17	0.118
309	A	4	3	1.00	19	0.158
310	A	5	4	1.00	19	0.210
311	A	6	4	1.00	19	0.210
312	A	3	2	1.00	19	0.105
313	A	5	5	1.00	19	0.263
314	A	4	4	0.98	19	0.210
315	A	3	3	0.91	17	0.176
316	A	2	2	1.00	9	0.222
317	A	2	2	1.00	19	0.105
318	A	2	2	1.00	19	0.105
319	A	2	2	1.00	19	0.105
320	A	1	1	1.00	28	0.036
321	A	4	2	1.00	25	0.080
322	A	3	2	1.00	25	0.080
323	A	2	2	1.00	23	0.087
324	A	1	1	1.00	15	0.067
325	A	1	1	1.00	23	0.043
326	A	1	1	1.00	25	0.040
327	A	1	1	1.00	25	0.040
328	A	2	2	0.93	28	0.071
329	A	1	1	1.00	69	0.014
330	A	5	3	1.00	25	0.120
331	A	3	3	1.00	23	0.130
332	A	2	2	1.00	15	0.133
333	A	2	2	1.00	25	0.080
334	A	2	2	1.00	23	0.087
335	A	2	2	1.00	25	0.080
336	A	2	2	1.00	25	0.080
337	A	6	4	1.08	31	0.129
338	A	4	4	1.08	31	0.129
339	A	2	2	1.07	29	0.069
340	A	5	5	1.00	31	0.161

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
341	A	5	5	1.19	31	0.161
342	A	5	4	1.36	31	0.129
343	A	9	8	1.00	31	0.258
344	A	7	7	1.00	31	0.226
345	A	5	5	1.00	28	0.179
346	A	5	5	1.00	31	0.161
347	A	6	6	1.00	31	0.194
348	A	5	5	1.00	29	0.172
349	A	4	4	1.00	29	0.138
350	A	3	3	1.00	29	0.103
351	A	2	2	1.00	27	0.074
352	A	2	2	1.00	26	0.077
353	A	3	3	1.00	29	0.103
354	A	2	2	1.00	29	0.069
355	A	3	3	1.00	29	0.103
356	A	2	2	1.00	29	0.069
357	A	5	5	1.00	29	0.172
358	A	8	7	1.00	31	0.226
359	A	4	4	1.00	31	0.129
360	A	6	6	1.00	31	0.194
361	A	2	2	1.00	29	0.069
362	A	4	4	1.00	28	0.143
363	A	3	3	1.00	31	0.097
364	A	4	4	1.00	31	0.129
365	A	3	3	1.00	31	0.097
366	A	2	2	1.00	31	0.065
367	A	5	5	1.00	31	0.161
368	A	8	8	1.00	31	0.258
369	A	4	4	1.00	31	0.129
370	A	7	7	1.00	31	0.226
371	A	2	2	1.00	29	0.069
372	A	4	4	1.00	28	0.143
373	A	3	3	1.00	31	0.097
374	A	2	2	1.00	31	0.065
375	A	5	5	1.00	31	0.161

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
376	A	4	4	1.00	31	0.129
377	A	7	7	1.00	31	0.226
378	A	3	3	1.00	31	0.097
379	A	1	1	1.00	57	0.018
380	A	3	3	1.00	32	0.094
381	A	4	4	1.00	41	0.098
382	A	4	3	1.00	31	0.097
383	A	4	4	1.00	35	0.114
384	A	3	3	1.00	31	0.097
385	A	2	2	1.08	76	0.026

Chapter 3

Listing of integrals

Local contents

3.1 $\int (a + bx^3) (c + dx^3)^4 \, dx$

Optimal. Leaf size=94

$$ac^4x+\frac{1}{4}c^3(bc+4ad)x^4+\frac{2}{7}c^2d(2bc+3ad)x^7+\frac{1}{5}cd^2(3bc+2ad)x^{10}+\frac{1}{13}d^3(4bc+ad)x^{13}+\frac{1}{16}bd^4x^{16}$$

```
[Out] a*c^4*x+1/4*c^3*(4*a*d+b*c)*x^4+2/7*c^2*d*(3*a*d+2*b*c)*x^7+1/5*c*d^2*(2*a*
d+3*b*c)*x^10+1/13*d^3*(a*d+4*b*c)*x^13+1/16*b*d^4*x^16
```

Rubi [A]
time = 0.05, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {380}

$$\frac{1}{4}c^3x^4(4ad+bc)+\frac{2}{7}c^2dx^7(3ad+2bc)+\frac{1}{13}d^3x^{13}(ad+4bc)+\frac{1}{5}cd^2x^{10}(2ad+3bc)+ac^4x+\frac{1}{16}bd^4x^{16}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)*(c + d*x^3)^4,x]
```

```
[Out] a*c^4*x + (c^3*(b*c + 4*a*d)*x^4)/4 + (2*c^2*d*(2*b*c + 3*a*d)*x^7)/7 + (c*
d^2*(3*b*c + 2*a*d)*x^10)/5 + (d^3*(4*b*c + a*d)*x^13)/13 + (b*d^4*x^16)/16
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int (a+bx^3)\left(c+dx^3\right)^4 \, dx &= \int \left(ac^4+c^3(bc+4ad)x^3+2c^2d(2bc+3ad)x^6+2cd^2(3bc+2ad)x^9+d^3(4bc\right.\\ &= ac^4x+\frac{1}{4}c^3(bc+4ad)x^4+\frac{2}{7}c^2d(2bc+3ad)x^7+\frac{1}{5}cd^2(3bc+2ad)x^{10}+\frac{1}{13}d^3(\end{aligned}$$

Mathematica [A]
time = 0.02, size = 94, normalized size = 1.00

$$ac^4x+\frac{1}{4}c^3(bc+4ad)x^4+\frac{2}{7}c^2d(2bc+3ad)x^7+\frac{1}{5}cd^2(3bc+2ad)x^{10}+\frac{1}{13}d^3(4bc+ad)x^{13}+\frac{1}{16}bd^4x^{16}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)*(c + d*x^3)^4,x]
```

[Out] a*c^4*x + (c^3*(b*c + 4*a*d)*x^4)/4 + (2*c^2*d*(2*b*c + 3*a*d)*x^7)/7 + (c*d^2*(3*b*c + 2*a*d)*x^10)/5 + (d^3*(4*b*c + a*d)*x^13)/13 + (b*d^4*x^16)/16

Maple [A]
time = 0.30, size = 97, normalized size = 1.03

method	result
norman	$a c^4 x + \left(a c^3 d + \frac{1}{4} b c^4\right) x^4 + \left(\frac{6}{7} a c^2 d^2 + \frac{4}{7} b c^3 d\right) x^7 + \left(\frac{2}{5} ac d^3 + \frac{3}{5} d^2 b c^2\right) x^{10} + \left(\frac{1}{13} a d^4 + \frac{4}{13} bc d^3\right) x^{13}$
default	$\frac{b d^4 x^{16}}{16} + \frac{\left(a d^4 + 4bc d^3\right) x^{13}}{13} + \frac{\left(4ac d^3 + 6d^2 b c^2\right) x^{10}}{10} + \frac{\left(6a c^2 d^2 + 4b c^3 d\right) x^7}{7} + \frac{\left(4a c^3 d + b c^4\right) x^4}{4} + a c^4 x$
gosper	$a c^4 x + x^4 a c^3 d + \frac{1}{4} x^4 b c^4 + \frac{6}{7} x^7 a c^2 d^2 + \frac{4}{7} x^7 b c^3 d + \frac{2}{5} x^{10} ac d^3 + \frac{3}{5} x^{10} d^2 b c^2 + \frac{1}{13} x^{13} a d^4 + \frac{4}{13} x^{13} bc d$
risch	$a c^4 x + x^4 a c^3 d + \frac{1}{4} x^4 b c^4 + \frac{6}{7} x^7 a c^2 d^2 + \frac{4}{7} x^7 b c^3 d + \frac{2}{5} x^{10} ac d^3 + \frac{3}{5} x^{10} d^2 b c^2 + \frac{1}{13} x^{13} a d^4 + \frac{4}{13} x^{13} bc d$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^3+a)*(d*x^3+c)^4,x,method=_RETURNVERBOSE)

[Out] 1/16*b*d^4*x^16+1/13*(a*d^4+4*b*c*d^3)*x^13+1/10*(4*a*c*d^3+6*b*c^2*d^2)*x^10+1/7*(6*a*c^2*d^2+4*b*c^3*d)*x^7+1/4*(4*a*c^3*d+b*c^4)*x^4+a*c^4*x

Maxima [A]
time = 0.28, size = 96, normalized size = 1.02

$$\frac{1}{16}\, bd^4 x^{16} + \frac{1}{13}\left(4\, bcd^3 + ad^4\right) x^{13} + \frac{1}{5}\left(3\, bc^2 d^2 + 2\, acd^3\right) x^{10} + \frac{2}{7}\left(2\, bc^3 d + 3\, ac^2 d^2\right) x^7 + ac^4 x + \frac{1}{4}\left(bc^4 + 4\, ac^3 d\right) x^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)*(d*x^3+c)^4,x, algorithm="maxima")

[Out] 1/16*b*d^4*x^16 + 1/13*(4*b*c*d^3 + a*d^4)*x^13 + 1/5*(3*b*c^2*d^2 + 2*a*c*d^3)*x^10 + 2/7*(2*b*c^3*d + 3*a*c^2*d^2)*x^7 + a*c^4*x + 1/4*(b*c^4 + 4*a*c^3*d)*x^4

Fricas [A]
time = 3.45, size = 96, normalized size = 1.02

$$\frac{1}{16}\, bd^4 x^{16} + \frac{1}{13}\left(4\, bcd^3 + ad^4\right) x^{13} + \frac{1}{5}\left(3\, bc^2 d^2 + 2\, acd^3\right) x^{10} + \frac{2}{7}\left(2\, bc^3 d + 3\, ac^2 d^2\right) x^7 + ac^4 x + \frac{1}{4}\left(bc^4 + 4\, ac^3 d\right) x^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)*(d*x^3+c)^4,x, algorithm="fricas")

[Out] 1/16*b*d^4*x^16 + 1/13*(4*b*c*d^3 + a*d^4)*x^13 + 1/5*(3*b*c^2*d^2 + 2*a*c*d^3)*x^10 + 2/7*(2*b*c^3*d + 3*a*c^2*d^2)*x^7 + a*c^4*x + 1/4*(b*c^4 + 4*a*c^3*d)*x^4

Sympy [A]
time = 0.02, size = 104, normalized size = 1.11

$$ac^4 x + \frac{bd^4 x^{16}}{16} + x^{13}\left(\frac{ad^4}{13} + \frac{4bcd^3}{13}\right) + x^{10}\cdot\left(\frac{2acd^3}{5} + \frac{3bc^2 d^2}{5}\right) + x^7\cdot\left(\frac{6ac^2 d^2}{7} + \frac{4bc^3 d}{7}\right) + x^4\left(ac^3 d + \frac{bc^4}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)*(d*x**3+c)**4,x)
```

```
[Out] a*c**4*x + b*d**4*x**16/16 + x**13*(a*d**4/13 + 4*b*c*d**3/13) + x**10*(2*a
*c*d**3/5 + 3*b*c**2*d**2/5) + x**7*(6*a*c**2*d**2/7 + 4*b*c**3*d/7) + x**4
*(a*c**3*d + b*c**4/4)
```

Giac [A]
time = 1.04, size = 97, normalized size = 1.03

$$\frac{1}{16}\,bd^4x^{16}+\frac{4}{13}\,bcd^3x^{13}+\frac{1}{13}\,ad^4x^{13}+\frac{3}{5}\,bc^2d^2x^{10}+\frac{2}{5}\,acd^3x^{10}+\frac{4}{7}\,bc^3dx^7+\frac{6}{7}\,ac^2d^2x^7+\frac{1}{4}\,bc^4x^4+ac^3dx^4+ac^4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(d*x^3+c)^4,x, algorithm="giac")
```

```
[Out] 1/16*b*d^4*x^16 + 4/13*b*c*d^3*x^13 + 1/13*a*d^4*x^13 + 3/5*b*c^2*d^2*x^10
+ 2/5*a*c*d^3*x^10 + 4/7*b*c^3*d*x^7 + 6/7*a*c^2*d^2*x^7 + 1/4*b*c^4*x^4 +
a*c^3*d*x^4 + a*c^4*x
```

Mupad [B]
time = 0.05, size = 87, normalized size = 0.93

$$x^4\left(\frac{b\,c^4}{4}+a\,d\,c^3\right)+x^{13}\left(\frac{a\,d^4}{13}+\frac{4\,b\,c\,d^3}{13}\right)+\frac{b\,d^4\,x^{16}}{16}+a\,c^4\,x+\frac{2\,c^2\,d\,x^7\,\left(3\,a\,d+2\,b\,c\right)}{7}+\frac{c\,d^2\,x^{10}\,\left(2\,a\,d+3\,b\,c\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)*(c + d*x^3)^4,x)
```

```
[Out] x^4*((b*c^4)/4 + a*c^3*d) + x^13*((a*d^4)/13 + (4*b*c*d^3)/13) + (b*d^4*x^1
6)/16 + a*c^4*x + (2*c^2*d*x^7*(3*a*d + 2*b*c))/7 + (c*d^2*x^10*(2*a*d + 3*
b*c))/5
```

3.2 $\int (a+bx^3)(c+dx^3)^3 \, dx$

Optimal. Leaf size=70

$$ac^3x+\frac{1}{4}c^2(bc+3ad)x^4+\frac{3}{7}cd(bc+ad)x^7+\frac{1}{10}d^2(3bc+ad)x^{10}+\frac{1}{13}bd^3x^{13}$$

```
[Out] a*c^3*x+1/4*c^2*(3*a*d+b*c)*x^4+3/7*c*d*(a*d+b*c)*x^7+1/10*d^2*(a*d+3*b*c)*
x^10+1/13*b*d^3*x^13
```

Rubi [A]
time = 0.03, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {380}

$$\frac{1}{4}c^2x^4(3ad+bc)+\frac{1}{10}d^2x^{10}(ad+3bc)+\frac{3}{7}cdx^7(ad+bc)+ac^3x+\frac{1}{13}bd^3x^{13}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)*(c + d*x^3)^3,x]
```

```
[Out] a*c^3*x + (c^2*(b*c + 3*a*d)*x^4)/4 + (3*c*d*(b*c + a*d)*x^7)/7 + (d^2*(3*b
*c + a*d)*x^10)/10 + (b*d^3*x^13)/13
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int (a+bx^3)(c+dx^3)^3 \, dx &= \int \left(ac^3+c^2(bc+3ad)x^3+3cd(bc+ad)x^6+d^2(3bc+ad)x^9+bd^3x^{12}\right) dx\\ &= ac^3x+\frac{1}{4}c^2(bc+3ad)x^4+\frac{3}{7}cd(bc+ad)x^7+\frac{1}{10}d^2(3bc+ad)x^{10}+\frac{1}{13}bd^3x^{13}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 70, normalized size = 1.00

$$ac^3x+\frac{1}{4}c^2(bc+3ad)x^4+\frac{3}{7}cd(bc+ad)x^7+\frac{1}{10}d^2(3bc+ad)x^{10}+\frac{1}{13}bd^3x^{13}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)*(c + d*x^3)^3,x]
```

```
[Out] a*c^3*x + (c^2*(b*c + 3*a*d)*x^4)/4 + (3*c*d*(b*c + a*d)*x^7)/7 + (d^2*(3*b
*c + a*d)*x^10)/10 + (b*d^3*x^13)/13
```

Maple [A]
time = 0.28, size = 73, normalized size = 1.04

method	result	size
norman	$\frac{b\,d^{3} x^{13}}{13}+\left(\frac{1}{10} a\,d^{3}+\frac{3}{10} b c\,d^{2}\right) x^{10}+\left(\frac{3}{7} a c\,d^{2}+\frac{3}{7} b\,c^{2} d\right) x^{7}+\left(\frac{3}{4} a\,c^{2} d+\frac{1}{4} b\,c^{3}\right) x^{4}+a\,c^{3} x$	72
default	$\frac{b\,d^{3} x^{13}}{13}+\frac{\left(a\,d^{3}+3 b c\,d^{2}\right) x^{10}}{10}+\frac{\left(3 a c\,d^{2}+3 b\,c^{2} d\right) x^{7}}{7}+\frac{\left(3 a\,c^{2} d+b\,c^{3}\right) x^{4}}{4}+a\,c^{3} x$	73
gosper	$\frac{1}{13} b\,d^{3} x^{13}+\frac{1}{10} x^{10} a\,d^{3}+\frac{3}{10} x^{10} b c\,d^{2}+\frac{3}{7} x^{7} a c\,d^{2}+\frac{3}{7} x^{7} b\,c^{2} d+\frac{3}{4} x^{4} a\,c^{2} d+\frac{1}{4} x^{4} b\,c^{3}+a\,c^{3} x$	75
risch	$\frac{1}{13} b\,d^{3} x^{13}+\frac{1}{10} x^{10} a\,d^{3}+\frac{3}{10} x^{10} b c\,d^{2}+\frac{3}{7} x^{7} a c\,d^{2}+\frac{3}{7} x^{7} b\,c^{2} d+\frac{3}{4} x^{4} a\,c^{2} d+\frac{1}{4} x^{4} b\,c^{3}+a\,c^{3} x$	75

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)*(d*x^3+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/13*b*d^3*x^13+1/10*(a*d^3+3*b*c*d^2)*x^10+1/7*(3*a*c*d^2+3*b*c^2*d)*x^7+1
/4*(3*a*c^2*d+b*c^3)*x^4+a*c^3*x
```

Maxima [A]
time = 0.31, size = 70, normalized size = 1.00

$$\frac{1}{13}\,bd^{3}x^{13}+\frac{1}{10}\,\left(3\,bcd^{2}+ad^{3}\right)x^{10}+\frac{3}{7}\,\left(bc^{2}d+acd^{2}\right)x^{7}+ac^{3}x+\frac{1}{4}\,\left(bc^{3}+3\,ac^{2}d\right)x^{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(d*x^3+c)^3,x, algorithm="maxima")
```

```
[Out] 1/13*b*d^3*x^13 + 1/10*(3*b*c*d^2 + a*d^3)*x^10 + 3/7*(b*c^2*d + a*c*d^2)*x
^7 + a*c^3*x + 1/4*(b*c^3 + 3*a*c^2*d)*x^4
```

Fricas [A]
time = 2.74, size = 70, normalized size = 1.00

$$\frac{1}{13}\,bd^{3}x^{13}+\frac{1}{10}\,\left(3\,bcd^{2}+ad^{3}\right)x^{10}+\frac{3}{7}\,\left(bc^{2}d+acd^{2}\right)x^{7}+ac^{3}x+\frac{1}{4}\,\left(bc^{3}+3\,ac^{2}d\right)x^{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(d*x^3+c)^3,x, algorithm="fricas")
```

```
[Out] 1/13*b*d^3*x^13 + 1/10*(3*b*c*d^2 + a*d^3)*x^10 + 3/7*(b*c^2*d + a*c*d^2)*x
^7 + a*c^3*x + 1/4*(b*c^3 + 3*a*c^2*d)*x^4
```

Sympy [A]
time = 0.01, size = 80, normalized size = 1.14

$$ac^{3}x+\frac{bd^{3}x^{13}}{13}+x^{10}\left(\frac{ad^{3}}{10}+\frac{3bcd^{2}}{10}\right)+x^{7}\cdot\left(\frac{3acd^{2}}{7}+\frac{3bc^{2}d}{7}\right)+x^{4}\cdot\left(\frac{3ac^{2}d}{4}+\frac{bc^{3}}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)*(d*x**3+c)**3,x)
```

```
[Out] a*c**3*x + b*d**3*x**13/13 + x**10*(a*d**3/10 + 3*b*c*d**2/10) + x**7*(3*a*
c*d**2/7 + 3*b*c**2*d/7) + x**4*(3*a*c**2*d/4 + b*c**3/4)
```

Giac [A]
time = 0.75, size = 74, normalized size = 1.06

$$\frac{1}{13}\,bd^3x^{13}+\frac{3}{10}\,bcd^2x^{10}+\frac{1}{10}\,ad^3x^{10}+\frac{3}{7}\,bc^2dx^7+\frac{3}{7}\,acd^2x^7+\frac{1}{4}\,bc^3x^4+\frac{3}{4}\,ac^2dx^4+ac^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(d*x^3+c)^3,x, algorithm="giac")
```

```
[Out] 1/13*b*d^3*x^13 + 3/10*b*c*d^2*x^10 + 1/10*a*d^3*x^10 + 3/7*b*c^2*d*x^7 + 3
/7*a*c*d^2*x^7 + 1/4*b*c^3*x^4 + 3/4*a*c^2*d*x^4 + a*c^3*x
```

Mupad [B]
time = 0.03, size = 66, normalized size = 0.94

$$x^4\left(\frac{b\,c^3}{4}+\frac{3\,a\,d\,c^2}{4}\right)+x^{10}\left(\frac{a\,d^3}{10}+\frac{3\,b\,c\,d^2}{10}\right)+\frac{b\,d^3\,x^{13}}{13}+a\,c^3\,x+\frac{3\,c\,d\,x^7\,\left(a\,d+b\,c\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)*(c + d*x^3)^3,x)
```

```
[Out] x^4*((b*c^3)/4 + (3*a*c^2*d)/4) + x^10*((a*d^3)/10 + (3*b*c*d^2)/10) + (b*d
^3*x^13)/13 + a*c^3*x + (3*c*d*x^7*(a*d + b*c))/7
```

3.3 $\int (a+bx^3)(c+dx^3)^2\,dx$

Optimal. Leaf size=50

$$ac^2x+\frac{1}{4}c(bc+2ad)x^4+\frac{1}{7}d(2bc+ad)x^7+\frac{1}{10}bd^2x^{10}$$

```
[Out] a*c^2*x+1/4*c*(2*a*d+b*c)*x^4+1/7*d*(a*d+2*b*c)*x^7+1/10*b*d^2*x^10
```

Rubi [A]
time = 0.02, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = $\{380\}$

$$\frac{1}{7}dx^7(ad+2bc)+\frac{1}{4}cx^4(2ad+bc)+ac^2x+\frac{1}{10}bd^2x^{10}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)*(c + d*x^3)^2,x]
```

```
[Out] a*c^2*x + (c*(b*c + 2*a*d)*x^4)/4 + (d*(2*b*c + a*d)*x^7)/7 + (b*d^2*x^10)/
10
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int (a+bx^3)(c+dx^3)^2\,dx &= \int \left(ac^2+c(bc+2ad)x^3+d(2bc+ad)x^6+bd^2x^9\right)dx\\ &= ac^2x+\frac{1}{4}c(bc+2ad)x^4+\frac{1}{7}d(2bc+ad)x^7+\frac{1}{10}bd^2x^{10}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 50, normalized size = 1.00

$$ac^2x+\frac{1}{4}c(bc+2ad)x^4+\frac{1}{7}d(2bc+ad)x^7+\frac{1}{10}bd^2x^{10}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)*(c + d*x^3)^2,x]
```

```
[Out] a*c^2*x + (c*(b*c + 2*a*d)*x^4)/4 + (d*(2*b*c + a*d)*x^7)/7 + (b*d^2*x^10)/
10
```

Maple [A]
time = 0.31, size = 49, normalized size = 0.98

method	result	size
default	$\frac{b\,d^{2} x^{10}}{10}+\frac{\left(a\,d^{2}+2 b c d \right) x^{7}}{7}+\frac{\left(2 a c d +b\,c^{2}\right) x^{4}}{4}+a\,c^{2} x$	49
norman	$\frac{b\,d^{2} x^{10}}{10}+\left(\frac{1}{7} a\,d^{2}+\frac{2}{7} b c d \right) x^{7}+\left(\frac{1}{2} a c d +\frac{1}{4} b\,c^{2}\right) x^{4}+a\,c^{2} x$	49
gosper	$\frac{1}{10} b\,d^{2} x^{10}+\frac{1}{7} x^{7} a\,d^{2}+\frac{2}{7} x^{7} b c d +\frac{1}{2} x^{4} a c d +\frac{1}{4} x^{4} b\,c^{2}+a\,c^{2} x$	51
risch	$\frac{1}{10} b\,d^{2} x^{10}+\frac{1}{7} x^{7} a\,d^{2}+\frac{2}{7} x^{7} b c d +\frac{1}{2} x^{4} a c d +\frac{1}{4} x^{4} b\,c^{2}+a\,c^{2} x$	51

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)*(d*x^3+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/10*b*d^2*x^10+1/7*(a*d^2+2*b*c*d)*x^7+1/4*(2*a*c*d+b*c^2)*x^4+a*c^2*x
```

Maxima [A]
time = 0.26, size = 48, normalized size = 0.96

$$\frac{1}{10}\,bd^2x^{10}+\frac{1}{7}\,\left(2\,bcd+ad^2\right)x^7+\frac{1}{4}\,\left(bc^2+2\,acd\right)x^4+ac^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(d*x^3+c)^2,x, algorithm="maxima")
```

```
[Out] 1/10*b*d^2*x^10 + 1/7*(2*b*c*d + a*d^2)*x^7 + 1/4*(b*c^2 + 2*a*c*d)*x^4 + a
*c^2*x
```

Fricas [A]
time = 2.65, size = 48, normalized size = 0.96

$$\frac{1}{10}\,bd^2x^{10}+\frac{1}{7}\,\left(2\,bcd+ad^2\right)x^7+\frac{1}{4}\,\left(bc^2+2\,acd\right)x^4+ac^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(d*x^3+c)^2,x, algorithm="fricas")
```

```
[Out] 1/10*b*d^2*x^10 + 1/7*(2*b*c*d + a*d^2)*x^7 + 1/4*(b*c^2 + 2*a*c*d)*x^4 + a
*c^2*x
```

Sympy [A]
time = 0.01, size = 51, normalized size = 1.02

$$ac^2x+\frac{bd^2x^{10}}{10}+x^7\left(\frac{ad^2}{7}+\frac{2bcd}{7}\right)+x^4\left(\frac{acd}{2}+\frac{bc^2}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)*(d*x**3+c)**2,x)
```

```
[Out] a*c**2*x + b*d**2*x**10/10 + x**7*(a*d**2/7 + 2*b*c*d/7) + x**4*(a*c*d/2 + b*c**2/4)
```

Giac [A]
time = 1.53, size = 50, normalized size = 1.00

$$\frac{1}{10}\,bd^2x^{10}+\frac{2}{7}\,bcdx^7+\frac{1}{7}\,ad^2x^7+\frac{1}{4}\,bc^2x^4+\frac{1}{2}\,acdx^4+ac^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(d*x^3+c)^2,x, algorithm="giac")
```

```
[Out] 1/10*b*d^2*x^10 + 2/7*b*c*d*x^7 + 1/7*a*d^2*x^7 + 1/4*b*c^2*x^4 + 1/2*a*c*d
*x^4 + a*c^2*x
```

Mupad [B]
time = 0.05, size = 48, normalized size = 0.96

$$x^4\left(\frac{b\,c^2}{4}+\frac{a\,d\,c}{2}\right)+x^7\left(\frac{a\,d^2}{7}+\frac{2\,b\,c\,d}{7}\right)+\frac{b\,d^2\,x^{10}}{10}+a\,c^2\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)*(c + d*x^3)^2,x)
```

```
[Out] x^4*((b*c^2)/4 + (a*c*d)/2) + x^7*((a*d^2)/7 + (2*b*c*d)/7) + (b*d^2*x^10)/
10 + a*c^2*x
```

3.4 $\int (a+bx^3)(c+dx^3)\,dx$

Optimal. Leaf size=28

$$acx+\frac{1}{4}(bc+ad)x^4+\frac{1}{7}bdx^7$$

```
[Out] a*c*x+1/4*(a*d+b*c)*x^4+1/7*b*d*x^7
```

Rubi [A]
time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {380}

$$\frac{1}{4}x^4(ad+bc)+acx+\frac{1}{7}bdx^7$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)*(c + d*x^3),x]
```

```
[Out] a*c*x + ((b*c + a*d)*x^4)/4 + (b*d*x^7)/7
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int (a+bx^3)(c+dx^3)\,dx &= \int (ac+(bc+ad)x^3+bdx^6)\,dx\\ &= acx+\frac{1}{4}(bc+ad)x^4+\frac{1}{7}bdx^7\end{aligned}$$

Mathematica [A]
time = 0.00, size = 28, normalized size = 1.00

$$acx+\frac{1}{4}(bc+ad)x^4+\frac{1}{7}bdx^7$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)*(c + d*x^3),x]
```

```
[Out] a*c*x + ((b*c + a*d)*x^4)/4 + (b*d*x^7)/7
```

Maple [A]
time = 0.13, size = 25, normalized size = 0.89

method	result	size
default	$acx + \frac{(ad+bc)x^4}{4} + \frac{bd\,x^7}{7}$	25
norman	$\frac{bd\,x^7}{7} + \left(\frac{ad}{4} + \frac{bc}{4}\right)x^4 + acx$	26
gosper	$\frac{1}{7}bd\,x^7 + \frac{1}{4}ad\,x^4 + \frac{1}{4}x^4bc + acx$	27
risch	$\frac{1}{7}bd\,x^7 + \frac{1}{4}ad\,x^4 + \frac{1}{4}x^4bc + acx$	27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)*(d*x^3+c),x,method=_RETURNVERBOSE)
[Out] a*c*x+1/4*(a*d+b*c)*x^4+1/7*b*d*x^7
```

Maxima [A]
time = 0.29, size = 24, normalized size = 0.86

$$\frac{1}{7}\,bdx^7 + \frac{1}{4}\,(bc + ad)x^4 + acx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(d*x^3+c),x, algorithm="maxima")
[Out] 1/7*b*d*x^7 + 1/4*(b*c + a*d)*x^4 + a*c*x
```

Fricas [A]
time = 3.35, size = 24, normalized size = 0.86

$$\frac{1}{7}\,bdx^7 + \frac{1}{4}\,(bc + ad)x^4 + acx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(d*x^3+c),x, algorithm="fricas")
[Out] 1/7*b*d*x^7 + 1/4*(b*c + a*d)*x^4 + a*c*x
```

Sympy [A]
time = 0.01, size = 26, normalized size = 0.93

$$acx + \frac{bdx^7}{7} + x^4\left(\frac{ad}{4} + \frac{bc}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)*(d*x**3+c),x)
```

```
[Out] a*c*x + b*d*x**7/7 + x**4*(a*d/4 + b*c/4)
```

Giac [**A**]
time = 1.17, size = 26, normalized size = 0.93

$$\frac{1}{7}\,bdx^7+\frac{1}{4}\,bcx^4+\frac{1}{4}\,adx^4+acx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(d*x^3+c),x, algorithm="giac")
```

```
[Out] 1/7*b*d*x^7 + 1/4*b*c*x^4 + 1/4*a*d*x^4 + a*c*x
```

Mupad [**B**]
time = 0.04, size = 25, normalized size = 0.89

$$\frac{b\,d\,x^7}{7}+\left(\frac{a\,d}{4}+\frac{b\,c}{4}\right)\,x^4+a\,c\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)*(c + d*x^3),x)
```

```
[Out] x^4*((a*d)/4 + (b*c)/4) + a*c*x + (b*d*x^7)/7
```

3.5 $\int \frac{a+bx^3}{c+dx^3}\,dx$

Optimal. Leaf size=144

$$\frac{bx}{d}+\frac{(bc-ad)\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}\,x}{\sqrt{3}\,\sqrt[3]{c}}\right)}{\sqrt{3}\,c^{2/3}d^{4/3}}-\frac{(bc-ad)\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{3c^{2/3}d^{4/3}}+\frac{(bc-ad)\log\left(c^{2/3}-\sqrt[3]{c}\,\sqrt[3]{d}\,x+d^{2/3}x\right.}{6c^{2/3}d^{4/3}}$$

```
[Out] b*x/d-1/3*(-a*d+b*c)*ln(c^(1/3)+d^(1/3)*x)/c^(2/3)/d^(4/3)+1/6*(-a*d+b*c)*l
n(c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/c^(2/3)/d^(4/3)+1/3*(-a*d+b*c)*arc
tan(1/3*(c^(1/3)-2*d^(1/3)*x)/c^(1/3)*3^(1/2))/c^(2/3)/d^(4/3)*3^(1/2)
```

Rubi [A]

time = 0.07, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {396, 206, 31, 648, 631, 210, 642}

$$\frac{(bc-ad)\text{ArcTan}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}\,x}{\sqrt{3}\,\sqrt[3]{c}}\right)}{\sqrt{3}\,c^{2/3}d^{4/3}}+\frac{(bc-ad)\log\left(c^{2/3}-\sqrt[3]{c}\,\sqrt[3]{d}\,x+d^{2/3}x^2\right)}{6c^{2/3}d^{4/3}}-\frac{(bc-ad)\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{3c^{2/3}d^{4/3}}+\frac{bx}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)/(c + d*x^3),x]
```

```
[Out] (b*x)/d + ((b*c - a*d)*ArcTan[(c^(1/3) - 2*d^(1/3)*x)/(Sqrt[3]*c^(1/3))])/(
Sqrt[3]*c^(2/3)*d^(4/3)) - ((b*c - a*d)*Log[c^(1/3) + d^(1/3)*x])/(3*c^(2/3
)*d^(4/3)) + ((b*c - a*d)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/(
6*c^(2/3)*d^(4/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+bx^3}{c+dx^3}\,dx &= \frac{bx}{d}-\frac{(bc-ad)\int \frac{1}{c+dx^3}\,dx}{d}\\
&= \frac{bx}{d}-\frac{(bc-ad)\int \frac{1}{\sqrt[3]{c}+\sqrt[3]{d}\,x}\,dx}{3c^{2/3}d}-\frac{(bc-ad)\int \frac{2\sqrt[3]{c}-\sqrt[3]{d}\,x}{c^{2/3}-\sqrt[3]{c}\,\sqrt[3]{d}\,x+d^{2/3}x^2}\,dx}{3c^{2/3}d}\\
&= \frac{bx}{d}-\frac{(bc-ad)\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{3c^{2/3}d^{4/3}}+\frac{(bc-ad)\int \frac{-\sqrt[3]{c}\,\sqrt[3]{d}+2d^{2/3}x}{c^{2/3}-\sqrt[3]{c}\,\sqrt[3]{d}\,x+d^{2/3}x^2}\,dx}{6c^{2/3}d^{4/3}}-\frac{(bc-ad)\int \frac{}{c^{2/3}-\cdots}}{2\cdots}\\
&= \frac{bx}{d}-\frac{(bc-ad)\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{3c^{2/3}d^{4/3}}+\frac{(bc-ad)\log\left(c^{2/3}-\sqrt[3]{c}\,\sqrt[3]{d}\,x+d^{2/3}x^2\right)}{6c^{2/3}d^{4/3}}-\frac{(bc-ad)\cdots}{\cdots}\\
&= \frac{bx}{d}+\frac{(bc-ad)\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}\,x}{\sqrt{3}\,\sqrt[3]{c}}\right)}{\sqrt{3}\,c^{2/3}d^{4/3}}-\frac{(bc-ad)\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{3c^{2/3}d^{4/3}}+\frac{(bc-ad)\log\left(c^{2/3}-\cdots\right.}{6c^{2/\cdots}}
\end{aligned}$$

Mathematica [A]
time = 0.06, size = 128, normalized size = 0.89

$$\frac{6bc^{2/3}\sqrt[3]{d}\,x+2\sqrt{3}\,(bc-ad)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\,x}{\sqrt[3]{c}}}{\sqrt{3}}\right)-2(bc-ad)\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)+(bc-ad)\log\left(c^{2/3}-\sqrt[3]{c}\,\sqrt[3]{d}\,x+d^{2/3}x^2\right)}{6c^{2/3}d^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)/(c + d*x^3),x]
```

```
[Out] (6*b*c^(2/3)*d^(1/3)*x + 2*Sqrt[3]*(b*c - a*d)*ArcTan[(1 - (2*d^(1/3)*x)/c^
(1/3))/Sqrt[3]] - 2*(b*c - a*d)*Log[c^(1/3) + d^(1/3)*x] + (b*c - a*d)*Log[
c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/(6*c^(2/3)*d^(4/3))
```

Maple [A]
time = 0.26, size = 110, normalized size = 0.76

method	result	size
risch	$\frac{bx}{d}+\frac{\sum_{\_R=\mathrm{RootOf}\left(d\_Z^3+c\right)}\frac{(ad-bc)\ln\left(x-\_R\right)}{\_R^2}}{3d^2}$	42
default	$\frac{bx}{d}+\frac{\left(\frac{\ln\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3d\left(\frac{c}{d}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{c}{d}\right)^{\frac{1}{3}}x+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6d\left(\frac{c}{d}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{c}{d}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3d\left(\frac{c}{d}\right)^{\frac{2}{3}}}\right)(ad-bc)}{d}$	110

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

```
[Out] b*x/d+(1/3/d/(c/d)^(2/3)*ln(x+(c/d)^(1/3))-1/6/d/(c/d)^(2/3)*ln(x^2-(c/d)^(
1/3)*x+(c/d)^(2/3))+1/3/d/(c/d)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(c/d)^(
1/3)*x-1)))*(a*d-b*c)/d
```

Maxima [A]
time = 0.49, size = 128, normalized size = 0.89

$$\frac{bx}{d}-\frac{\sqrt{3}\,(bc-ad)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{3\,d^2\left(\frac{c}{d}\right)^{\frac{2}{3}}}+\frac{(bc-ad)\log\left(x^2-x\left(\frac{c}{d}\right)^{\frac{1}{3}}+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\,d^2\left(\frac{c}{d}\right)^{\frac{2}{3}}}-\frac{(bc-ad)\log\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\,d^2\left(\frac{c}{d}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] b*x/d - 1/3*sqrt(3)*(b*c - a*d)*arctan(1/3*sqrt(3)*(2*x - (c/d)^(1/3))/(c/d
)^(1/3))/(d^2*(c/d)^(2/3)) + 1/6*(b*c - a*d)*log(x^2 - x*(c/d)^(1/3) + (c/d
)^(2/3))/(d^2*(c/d)^(2/3)) - 1/3*(b*c - a*d)*log(x + (c/d)^(1/3))/(d^2*(c/d
)^(2/3))
```

Fricas [A]
time = 3.09, size = 369, normalized size = 2.56

$$\left[\frac{6\,bc^2dx-3\,\sqrt{\frac{1}{3}}\,(bc^2d-acd^2)\sqrt{-\frac{(c^2d)^{\frac{1}{3}}}{d}}\,\log\left(\frac{2\,cdx^3-3\,(c^2d)^{\frac{1}{3}}cx-c^2+3\,\sqrt{\frac{1}{3}}\left(2\,cdx^2+(c^2d)^{\frac{2}{3}}x-(c^2d)^{\frac{1}{3}}c\right)\sqrt{-\frac{(c^2d)^{\frac{1}{3}}}{d}}}{dx^3+c}\right)+(c^2d)^{\frac{2}{3}}\,(bc-ad)\log\left(cdx^2-(c^2d)^{\frac{2}{3}}\,x+(c^2d)^{\frac{1}{3}}\,c\right)-2\,(c^2d)^{\frac{2}{3}}\,(bc-ad)\log\left(cdx+(c^2d)^{\frac{2}{3}}\right)}{6\,c^2d^2},\ \frac{6\,bc^2dx-6\,\sqrt{\frac{1}{3}}\,(bc^2d-acd^2)\sqrt{\frac{(c^2d)^{\frac{1}{3}}}{d}}\,\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(2\,(c^2d)^{\frac{2}{3}}x-(c^2d)^{\frac{1}{3}}c\right)\sqrt{\frac{(c^2d)^{\frac{1}{3}}}{d}}}{c^2}\right)+(c^2d)^{\frac{2}{3}}\,(bc-ad)\log\left(cdx^2-(c^2d)^{\frac{2}{3}}\,x+(c^2d)^{\frac{1}{3}}\,c\right)-2\,(c^2d)^{\frac{2}{3}}\,(bc-ad)\log\left(cdx+(c^2d)^{\frac{2}{3}}\right)}{6\,c^2d^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] [1/6*(6*b*c^2*d*x - 3*sqrt(1/3)*(b*c^2*d - a*c*d^2)*sqrt(-(c^2*d)^(1/3)/d)*
log((2*c*d*x^3 - 3*(c^2*d)^(1/3)*c*x - c^2 + 3*sqrt(1/3)*(2*c*d*x^2 + (c^2*
d)^(2/3)*x - (c^2*d)^(1/3)*c)*sqrt(-(c^2*d)^(1/3)/d))/(d*x^3 + c)) + (c^2*d
)^(2/3)*(b*c - a*d)*log(c*d*x^2 - (c^2*d)^(2/3)*x + (c^2*d)^(1/3)*c) - 2*(c
^2*d)^(2/3)*(b*c - a*d)*log(c*d*x + (c^2*d)^(2/3)))/(c^2*d^2), 1/6*(6*b*c^2
*d*x - 6*sqrt(1/3)*(b*c^2*d - a*c*d^2)*sqrt((c^2*d)^(1/3)/d)*arctan(sqrt(1/
3)*(2*(c^2*d)^(2/3)*x - (c^2*d)^(1/3)*c)*sqrt((c^2*d)^(1/3)/d)/c^2) + (c^2*
d)^(2/3)*(b*c - a*d)*log(c*d*x^2 - (c^2*d)^(2/3)*x + (c^2*d)^(1/3)*c) - 2*(
c^2*d)^(2/3)*(b*c - a*d)*log(c*d*x + (c^2*d)^(2/3)))/(c^2*d^2)]
```

Sympy [A]
time = 0.23, size = 71, normalized size = 0.49

$$\frac{bx}{d}+\operatorname{RootSum}\left(27t^3c^2d^4-a^3d^3+3a^2bcd^2-3ab^2c^2d+b^3c^3,\left(t\mapsto t\log\left(\frac{3tcd}{ad-bc}+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)/(d*x**3+c),x)
```

```
[Out] b*x/d + RootSum(27*_t**3*c**2*d**4 - a**3*d**3 + 3*a**2*b*c*d**2 - 3*a*b**2
*c**2*d + b**3*c**3, Lambda(_t, _t*log(3*_t*c*d/(a*d - b*c) + x)))
```

Giac [A]
time = 0.88, size = 133, normalized size = 0.92

$$\frac{\sqrt{3}\,(bc-ad)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{3\,(-cd^2)^{\frac{2}{3}}}+\frac{(bc-ad)\log\left(x^2+x\left(-\frac{c}{d}\right)^{\frac{1}{3}}+\left(-\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\,(-cd^2)^{\frac{2}{3}}}+\frac{bx}{d}+\frac{(bc-ad)\left(-\frac{c}{d}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right|\right)}{3\,cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*(b*c - a*d)*arctan(1/3*sqrt(3)*(2*x + (-c/d)^(1/3))/(-c/d)^(1/3
))/(-c*d^2)^(2/3) + 1/6*(b*c - a*d)*log(x^2 + x*(-c/d)^(1/3) + (-c/d)^(2/3)
)/(-c*d^2)^(2/3) + b*x/d + 1/3*(b*c - a*d)*(-c/d)^(1/3)*log(abs(x - (-c/d)^
(1/3)))/(c*d)
```

Mupad [B]
time = 1.38, size = 123, normalized size = 0.85

$$\frac{b\,x}{d}+\frac{\ln\left(d^{1/3}\,x+c^{1/3}\right)\,\left(a\,d-b\,c\right)}{3\,c^{2/3}\,d^{4/3}}-\frac{\ln\left(c^{1/3}-2\,d^{1/3}\,x+\sqrt{3}\,c^{1/3}\,1{}\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(a\,d-b\,c\right)}{3\,c^{2/3}\,d^{4/3}}+\frac{\ln\left(2\,d^{1/3}\,x-c^{1/3}+\sqrt{3}\,c^{1/3}\,1{}\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(a\,d-b\,c\right)}{3\,c^{2/3}\,d^{4/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)/(c + d*x^3),x)
```

```
[Out] (b*x)/d + (log(d^(1/3)*x + c^(1/3))*(a*d - b*c))/(3*c^(2/3)*d^(4/3)) - (log
(3^(1/2)*c^(1/3)*1i - 2*d^(1/3)*x + c^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(a*d -
b*c))/(3*c^(2/3)*d^(4/3)) + (log(3^(1/2)*c^(1/3)*1i + 2*d^(1/3)*x - c^(1/3)
)*((3^(1/2)*1i)/2 - 1/2)*(a*d - b*c))/(3*c^(2/3)*d^(4/3))
```

3.6 $\int \frac{a+bx^3}{\left(c+dx^3\right)^2}\,dx$

Optimal. Leaf size=169

$$-\frac{(bc-ad)x}{3cd\left(c+dx^3\right)}-\frac{(bc+2ad)\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}\,x}{\sqrt{3}\,\sqrt[3]{c}}\right)}{3\sqrt{3}\,c^{5/3}d^{4/3}}+\frac{(bc+2ad)\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{9c^{5/3}d^{4/3}}-\frac{(bc+2ad)\log\left(c^{2/3}-\sqrt[3]{c}\right.}{18c^{5/3}d^{4/}}$$

```
[Out] -1/3*(-a*d+b*c)*x/c/d/(d*x^3+c)+1/9*(2*a*d+b*c)*ln(c^(1/3)+d^(1/3)*x)/c^(5/
3)/d^(4/3)-1/18*(2*a*d+b*c)*ln(c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/c^(5/
3)/d^(4/3)-1/9*(2*a*d+b*c)*arctan(1/3*(c^(1/3)-2*d^(1/3)*x)/c^(1/3)*3^(1/2)
)/c^(5/3)/d^(4/3)*3^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {393, 206, 31, 648, 631, 210, 642}

$$-\frac{(2ad+bc)\text{ArcTan}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}\,x}{\sqrt{3}\,\sqrt[3]{c}}\right)}{3\sqrt{3}\,c^{5/3}d^{4/3}}-\frac{(2ad+bc)\log\left(c^{2/3}-\sqrt[3]{c}\,\sqrt[3]{d}\,x+d^{2/3}x^2\right)}{18c^{5/3}d^{4/3}}+\frac{(2ad+bc)\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{9c^{5/3}d^{4/3}}-\frac{x(bc-ad)}{3cd\left(c+dx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)/(c + d*x^3)^2,x]
```

```
[Out] -1/3*((b*c - a*d)*x)/(c*d*(c + d*x^3)) - ((b*c + 2*a*d)*ArcTan[(c^(1/3) - 2
*d^(1/3)*x)/(Sqrt[3]*c^(1/3))])/(3*Sqrt[3]*c^(5/3)*d^(4/3)) + ((b*c + 2*a*d
)*Log[c^(1/3) + d^(1/3)*x]])/(9*c^(5/3)*d^(4/3)) - ((b*c + 2*a*d)*Log[c^(2/3
) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2]])/(18*c^(5/3)*d^(4/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{a+bx^3}{(c+dx^3)^2}\,dx = -\frac{(bc-ad)x}{3cd\,(c+dx^3)} + \frac{(bc+2ad)\int \frac{1}{c+dx^3}\,dx}{3cd}$$

$$= -\frac{(bc-ad)x}{3cd\,(c+dx^3)} + \frac{(bc+2ad)\int \frac{1}{\sqrt[3]{c}+\sqrt[3]{d}\,x}\,dx}{9c^{5/3}d} + \frac{(bc+2ad)\int \frac{2\sqrt[3]{c}-\sqrt[3]{d}\,x}{c^{2/3}-\sqrt[3]{c}\,\sqrt[3]{d}\,x+d^{2/3}x^2}\,dx}{9c^{5/3}d}$$

$$= -\frac{(bc-ad)x}{3cd\,(c+dx^3)} + \frac{(bc+2ad)\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{9c^{5/3}d^{4/3}} - \frac{(bc+2ad)\int \frac{-\sqrt[3]{c}\,\sqrt[3]{d}+2d^{2/3}x}{c^{2/3}-\sqrt[3]{c}\,\sqrt[3]{d}\,x+d^{2/3}x^2}\,dx}{18c^{5/3}d^{4/3}} +$$

$$= -\frac{(bc-ad)x}{3cd\,(c+dx^3)} + \frac{(bc+2ad)\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{9c^{5/3}d^{4/3}} - \frac{(bc+2ad)\log\left(c^{2/3}-\sqrt[3]{c}\,\sqrt[3]{d}\,x+d^{2/}\right.}{18c^{5/3}d^{4/3}}$$

$$= -\frac{(bc-ad)x}{3cd\,(c+dx^3)} - \frac{(bc+2ad)\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}\,x}{\sqrt{3}\,\sqrt[3]{c}}\right)}{3\sqrt{3}\,c^{5/3}d^{4/3}} + \frac{(bc+2ad)\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{9c^{5/3}d^{4/3}} - \frac{(b}{}$$

Mathematica [A]
time = 0.07, size = 145, normalized size = 0.86

$$\frac{-\frac{6c^{2/3}\sqrt[3]{d}\,(bc-ad)x}{c+dx^3} - 2\sqrt{3}\,(bc+2ad)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\,x}{\sqrt[3]{c}}}{\sqrt{3}}\right) + 2(bc+2ad)\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right) - (bc+2ad)\log\left(c^{2/3}-\sqrt[3]{c}\,\sqrt[3]{d}\,x+d^{2/3}x^2\right)}{18c^{5/3}d^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)/(c + d*x^3)^2,x]
```

```
[Out] ((-6*c^(2/3)*d^(1/3)*(b*c - a*d)*x)/(c + d*x^3) - 2*Sqrt[3]*(b*c + 2*a*d)*A
rcTan[(1 - (2*d^(1/3)*x)/c^(1/3))/Sqrt[3]] + 2*(b*c + 2*a*d)*Log[c^(1/3) +
d^(1/3)*x] - (b*c + 2*a*d)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/
(18*c^(5/3)*d^(4/3))
```

Maple [A]
time = 0.25, size = 134, normalized size = 0.79

method	result	size
risch	$\frac{(ad-bc)x}{3cd(d\,x^3+c)} + \frac{\sum_{\_R=\mathrm{RootOf}(d\,\_Z^3+c)} \frac{(2ad+bc)\ln(x-\_R)}{\_R^2}}{9c\,d^2}$	65

default	$\frac{(ad-bc)x}{3cd(d\,x^3+c)}+\frac{(2ad+bc)\left(\frac{\ln\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3d\left(\frac{c}{d}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{c}{d}\right)^{\frac{1}{3}}x+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6d\left(\frac{c}{d}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{c}{d}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3d\left(\frac{c}{d}\right)^{\frac{2}{3}}}\right)}{3cd}$	134

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)/(d*x^3+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(a*d-b*c)/c/d*x/(d*x^3+c)+1/3*(2*a*d+b*c)/c/d*(1/3/d/(c/d)^(2/3)*ln(x+(
c/d)^(1/3))-1/6/d/(c/d)^(2/3)*ln(x^2-(c/d)^(1/3)*x+(c/d)^(2/3))+1/3/d/(c/d)
^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(c/d)^(1/3)*x-1)))
```

<u>Maxima</u> [A]
time = 0.50, size = 158, normalized size = 0.93

$$-\frac{(bc-ad)x}{3\left(cd^2x^3+c^2d\right)}+\frac{\sqrt{3}\,(bc+2\,ad)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{9\,cd^2\left(\frac{c}{d}\right)^{\frac{2}{3}}}-\frac{(bc+2\,ad)\log\left(x^2-x\left(\frac{c}{d}\right)^{\frac{1}{3}}+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{18\,cd^2\left(\frac{c}{d}\right)^{\frac{2}{3}}}+\frac{(bc+2\,ad)\log\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{9\,cd^2\left(\frac{c}{d}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)/(d*x^3+c)^2,x, algorithm="maxima")
```

```
[Out] -1/3*(b*c - a*d)*x/(c*d^2*x^3 + c^2*d) + 1/9*sqrt(3)*(b*c + 2*a*d)*arctan(1
/3*sqrt(3)*(2*x - (c/d)^(1/3))/(c/d)^(1/3))/(c*d^2*(c/d)^(2/3)) - 1/18*(b*c
 + 2*a*d)*log(x^2 - x*(c/d)^(1/3) + (c/d)^(2/3))/(c*d^2*(c/d)^(2/3)) + 1/9*
(b*c + 2*a*d)*log(x + (c/d)^(1/3))/(c*d^2*(c/d)^(2/3))
```

<u>Fricas</u> [A]
time = 2.73, size = 537, normalized size = 3.18

$$\left[\frac{3\sqrt{\frac{1}{3}}\left(bc^3d+2\,ac^2d^2+\left(bc^2d^2+2\,acd^3\right)x^3\right)\sqrt{-\frac{\left(c^2d\right)^{\frac{1}{3}}}{d}}\log\left(\frac{2\,cdx^3-3\left(c^2d\right)^{\frac{1}{3}}cx-c^2+3\sqrt{\frac{1}{3}}\left(2\,cdx^2+\left(c^2d\right)^{\frac{2}{3}}x-\left(c^2d\right)^{\frac{1}{3}}c\right)\sqrt{-\frac{\left(c^2d\right)^{\frac{1}{3}}}{d}}}{dx^3+c}\right)-\left(\left(bcd+2\,ad^2\right)x^3+bc^2+2\,acd\right)\left(c^2d\right)^{\frac{2}{3}}\log\left(cdx^2-\left(c^2d\right)^{\frac{2}{3}}x+\left(c^2d\right)^{\frac{1}{3}}c\right)+2\left(\left(bcd+2\,ad^2\right)x^3+bc^2+2\,acd\right)\left(c^2d\right)^{\frac{2}{3}}\log\left(cdx+\left(c^2d\right)^{\frac{2}{3}}\right)-6\left(bc^3d-ac^2d^2\right)x}{18\left(c^3d^3x^3+c^4d^2\right)},\ \frac{6\sqrt{\frac{1}{3}}\left(bc^3d+2\,ac^2d^2+\left(bc^2d^2+2\,acd^3\right)x^3\right)\sqrt{\frac{\left(c^2d\right)^{\frac{1}{3}}}{d}}\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(2\left(c^2d\right)^{\frac{2}{3}}x-\left(c^2d\right)^{\frac{1}{3}}c\right)\sqrt{\frac{\left(c^2d\right)^{\frac{1}{3}}}{d}}}{c^2}\right)-\left(\left(bcd+2\,ad^2\right)x^3+bc^2+2\,acd\right)\left(c^2d\right)^{\frac{2}{3}}\log\left(cdx^2-\left(c^2d\right)^{\frac{2}{3}}x+\left(c^2d\right)^{\frac{1}{3}}c\right)+2\left(\left(bcd+2\,ad^2\right)x^3+bc^2+2\,acd\right)\left(c^2d\right)^{\frac{2}{3}}\log\left(cdx+\left(c^2d\right)^{\frac{2}{3}}\right)-6\left(bc^3d-ac^2d^2\right)x}{18\left(c^3d^3x^3+c^4d^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)/(d*x^3+c)^2,x, algorithm="fricas")
```

```
[Out] [1/18*(3*sqrt(1/3)*(b*c^3*d + 2*a*c^2*d^2 + (b*c^2*d^2 + 2*a*c*d^3)*x^3)*sq
rt(-(c^2*d)^(1/3)/d)*log((2*c*d*x^3 - 3*(c^2*d)^(1/3)*c*x - c^2 + 3*sqrt(1/
3)*(2*c*d*x^2 + (c^2*d)^(2/3)*x - (c^2*d)^(1/3)*c)*sqrt(-(c^2*d)^(1/3)/d))/
(d*x^3 + c)) - ((b*c*d + 2*a*d^2)*x^3 + b*c^2 + 2*a*c*d)*(c^2*d)^(2/3)*log(
```

```
c*d*x^2 - (c^2*d)^(2/3)*x + (c^2*d)^(1/3)*c) + 2*((b*c*d + 2*a*d^2)*x^3 + b
*c^2 + 2*a*c*d)*(c^2*d)^(2/3)*log(c*d*x + (c^2*d)^(2/3)) - 6*(b*c^3*d - a*c
^2*d^2)*x)/(c^3*d^3*x^3 + c^4*d^2), 1/18*(6*sqrt(1/3)*(b*c^3*d + 2*a*c^2*d^
2 + (b*c^2*d^2 + 2*a*c*d^3)*x^3)*sqrt((c^2*d)^(1/3)/d)*arctan(sqrt(1/3)*(2*
(c^2*d)^(2/3)*x - (c^2*d)^(1/3)*c)*sqrt((c^2*d)^(1/3)/d)/c^2) - ((b*c*d + 2
*a*d^2)*x^3 + b*c^2 + 2*a*c*d)*(c^2*d)^(2/3)*log(c*d*x^2 - (c^2*d)^(2/3)*x
+ (c^2*d)^(1/3)*c) + 2*((b*c*d + 2*a*d^2)*x^3 + b*c^2 + 2*a*c*d)*(c^2*d)^(2
/3)*log(c*d*x + (c^2*d)^(2/3)) - 6*(b*c^3*d - a*c^2*d^2)*x)/(c^3*d^3*x^3 +
c^4*d^2)]
```

Sympy [A]
time = 0.33, size = 97, normalized size = 0.57

$$\frac{x(ad-bc)}{3c^2d+3cd^2x^3}+\operatorname{RootSum}\left(729t^3c^5d^4-8a^3d^3-12a^2bcd^2-6ab^2c^2d-b^3c^3,\left(t\mapsto t\log\left(\frac{9tc^2d}{2ad+bc}+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)/(d*x**3+c)**2,x)
```

```
[Out] x*(a*d - b*c)/(3*c**2*d + 3*c*d**2*x**3) + RootSum(729*_t**3*c**5*d**4 - 8*
a**3*d**3 - 12*a**2*b*c*d**2 - 6*a*b**2*c**2*d - b**3*c**3, Lambda(_t, _t*l
og(9*_t*c**2*d/(2*a*d + b*c) + x)))
```

Giac [A]
time = 0.91, size = 160, normalized size = 0.95

$$-\frac{\sqrt{3}\,(bc+2\,ad)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{9\left(-cd^2\right)^{\frac{2}{3}}c}-\frac{(bc+2\,ad)\log\left(x^2+x\left(-\frac{c}{d}\right)^{\frac{1}{3}}+\left(-\frac{c}{d}\right)^{\frac{2}{3}}\right)}{18\left(-cd^2\right)^{\frac{2}{3}}c}-\frac{(bc+2\,ad)\left(-\frac{c}{d}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right|\right)}{9\,c^2d}-\frac{bcx-adx}{3\,(dx^3+c)cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)/(d*x^3+c)^2,x, algorithm="giac")
```

```
[Out] -1/9*sqrt(3)*(b*c + 2*a*d)*arctan(1/3*sqrt(3)*(2*x + (-c/d)^(1/3))/(-c/d)^(
1/3))/((-c*d^2)^(2/3)*c) - 1/18*(b*c + 2*a*d)*log(x^2 + x*(-c/d)^(1/3) + (-
c/d)^(2/3))/((-c*d^2)^(2/3)*c) - 1/9*(b*c + 2*a*d)*(-c/d)^(1/3)*log(abs(x -
 (-c/d)^(1/3)))/(c^2*d) - 1/3*(b*c*x - a*d*x)/((d*x^3 + c)*c*d)
```

Mupad [B]
time = 1.40, size = 143, normalized size = 0.85

$$\frac{\ln\left(d^{1/3}\,x+c^{1/3}\right)\,\left(2\,a\,d+b\,c\right)}{9\,c^{5/3}\,d^{4/3}}-\frac{\ln\left(c^{1/3}-2\,d^{1/3}\,x+\sqrt{3}\,c^{1/3}\,1{}\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(2\,a\,d+b\,c\right)}{9\,c^{5/3}\,d^{4/3}}+\frac{\ln\left(2\,d^{1/3}\,x-c^{1/3}+\sqrt{3}\,c^{1/3}\,1{}\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(2\,a\,d+b\,c\right)}{9\,c^{5/3}\,d^{4/3}}+\frac{x\,\left(a\,d-b\,c\right)}{3\,c\,d\,\left(d\,x^3+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)/(c + d*x^3)^2,x)
```

```
[Out] (log(d^(1/3)*x + c^(1/3))*(2*a*d + b*c))/(9*c^(5/3)*d^(4/3)) - (log(3^(1/2)
*c^(1/3)*1i - 2*d^(1/3)*x + c^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(2*a*d + b*c))/
(9*c^(5/3)*d^(4/3)) + (log(3^(1/2)*c^(1/3)*1i + 2*d^(1/3)*x - c^(1/3))*((3^
(1/2)*1i)/2 - 1/2)*(2*a*d + b*c))/(9*c^(5/3)*d^(4/3)) + (x*(a*d - b*c))/(3*
c*d*(c + d*x^3))
```

3.7 $\int \frac{a+bx^3}{\left(c+dx^3\right)^3}\,dx$

Optimal. Leaf size=197

$$-\frac{(bc-ad)x}{6cd\left(c+dx^3\right)^2}+\frac{(bc+5ad)x}{18c^2d\left(c+dx^3\right)}-\frac{(bc+5ad)\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}\,x}{\sqrt{3}\,\sqrt[3]{c}}\right)}{9\sqrt{3}\,c^{8/3}d^{4/3}}+\frac{(bc+5ad)\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{27c^{8/3}d^{4/3}}-(bc+5$$

```
[Out] -1/6*(-a*d+b*c)*x/c/d/(d*x^3+c)^2+1/18*(5*a*d+b*c)*x/c^2/d/(d*x^3+c)+1/27*(
5*a*d+b*c)*ln(c^(1/3)+d^(1/3)*x)/c^(8/3)/d^(4/3)-1/54*(5*a*d+b*c)*ln(c^(2/3
)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/c^(8/3)/d^(4/3)-1/27*(5*a*d+b*c)*arctan(1/
3*(c^(1/3)-2*d^(1/3)*x)/c^(1/3)*3^(1/2))/c^(8/3)/d^(4/3)*3^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.471$, Rules used = {393, 205, 206, 31, 648, 631, 210, 642}

$$-\frac{(5ad+bc)\text{ArcTan}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}\,x}{\sqrt{3}\,\sqrt[3]{c}}\right)}{9\sqrt{3}\,c^{8/3}d^{4/3}}-\frac{(5ad+bc)\log\left(c^{2/3}-\sqrt[3]{c}\,\sqrt[3]{d}\,x+d^{2/3}x^2\right)}{54c^{8/3}d^{4/3}}+\frac{(5ad+bc)\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{27c^{8/3}d^{4/3}}+\frac{x(5ad+bc)}{18c^2d\left(c+dx^3\right)}-\frac{x(bc-ad)}{6cd\left(c+dx^3\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)/(c + d*x^3)^3,x]
```

```
[Out] -1/6*((b*c - a*d)*x)/(c*d*(c + d*x^3)^2) + ((b*c + 5*a*d)*x)/(18*c^2*d*(c +
 d*x^3)) - ((b*c + 5*a*d)*ArcTan[(c^(1/3) - 2*d^(1/3)*x)/(Sqrt[3]*c^(1/3))]
)/(9*Sqrt[3]*c^(8/3)*d^(4/3)) + ((b*c + 5*a*d)*Log[c^(1/3) + d^(1/3)*x])/(2
7*c^(8/3)*d^(4/3)) - ((b*c + 5*a*d)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/
3)*x^2])/(54*c^(8/3)*d^(4/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{a+bx^3}{(c+dx^3)^3}\,dx = -\frac{(bc-ad)x}{6cd\,(c+dx^3)^2} + \frac{(bc+5ad)\int \frac{1}{(c+dx^3)^2}\,dx}{6cd}$$

$$= -\frac{(bc-ad)x}{6cd\,(c+dx^3)^2} + \frac{(bc+5ad)x}{18c^2d\,(c+dx^3)} + \frac{(bc+5ad)\int \frac{1}{c+dx^3}\,dx}{9c^2d}$$

$$= -\frac{(bc-ad)x}{6cd\,(c+dx^3)^2} + \frac{(bc+5ad)x}{18c^2d\,(c+dx^3)} + \frac{(bc+5ad)\int \frac{1}{\sqrt[3]{c}+\sqrt[3]{d}\,x}\,dx}{27c^{8/3}d} + \frac{(bc+5ad)\int \frac{2\sqrt[3]{c}}{c^{2/3}-\sqrt[3]{c}}}{27c^{8/3}d}$$

$$= -\frac{(bc-ad)x}{6cd\,(c+dx^3)^2} + \frac{(bc+5ad)x}{18c^2d\,(c+dx^3)} + \frac{(bc+5ad)\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{27c^{8/3}d^{4/3}} - \frac{(bc+5ad)\int \frac{-\sqrt[3]{}}{c^{2/3}-}}{54c^{8/}}$$

$$= -\frac{(bc-ad)x}{6cd\,(c+dx^3)^2} + \frac{(bc+5ad)x}{18c^2d\,(c+dx^3)} + \frac{(bc+5ad)\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{27c^{8/3}d^{4/3}} - \frac{(bc+5ad)\log\left(c^{2/}\right.}{54}$$

$$= -\frac{(bc-ad)x}{6cd\,(c+dx^3)^2} + \frac{(bc+5ad)x}{18c^2d\,(c+dx^3)} - \frac{(bc+5ad)\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}\,x}{\sqrt{3}\,\sqrt[3]{c}}\right)}{9\sqrt{3}\,c^{8/3}d^{4/3}} + \frac{(bc+5ad)\log\left(\right.}{27c^{8/3}}$$

Mathematica [A]
time = 0.10, size = 175, normalized size = 0.89

$$\frac{-\frac{9c^{5/3}\sqrt[3]{d}\,(bc-ad)x}{(c+dx^3)^2} + \frac{3c^{2/3}\sqrt[3]{d}\,(bc+5ad)x}{c+dx^3} - 2\sqrt{3}\,(bc+5ad)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\,x}{\sqrt[3]{c}}}{\sqrt{3}}\right) + 2(bc+5ad)\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right) - (bc+5ad)\log\left(c^{2/3}-\sqrt[3]{c}\,\sqrt[3]{d}\,x+d^{2/3}x^2\right)}{54c^{8/3}d^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)/(c + d*x^3)^3,x]
```

```
[Out] ((-9*c^(5/3)*d^(1/3)*(b*c - a*d)*x)/(c + d*x^3)^2 + (3*c^(2/3)*d^(1/3)*(b*c
 + 5*a*d)*x)/(c + d*x^3) - 2*Sqrt[3]*(b*c + 5*a*d)*ArcTan[(1 - (2*d^(1/3)*x
)/c^(1/3))/Sqrt[3]] + 2*(b*c + 5*a*d)*Log[c^(1/3) + d^(1/3)*x] - (b*c + 5*a
*d)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/(54*c^(8/3)*d^(4/3))
```

Maple [A]
time = 0.26, size = 153, normalized size = 0.78

method	result	size
risch	$\frac{\frac{(5ad+bc)x^4}{18c^2}+\frac{(4ad-bc)x}{9cd}}{(d\,x^3+c)^2} + \frac{\sum_{\_R=\mathrm{RootOf}\left(d\_Z^3+c\right)} \frac{(5ad+bc)\ln\left(x-\_R\right)}{\_R^2}}{27c^2d^2}$	84

default	$\frac{\frac{(5ad+bc)x^4}{18c^2}+\frac{(4ad-bc)x}{9cd}}{(dx^3+c)^2}+\frac{(5ad+bc)\left(\frac{\ln\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3d\left(\frac{c}{d}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{c}{d}\right)^{\frac{1}{3}}x+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6d\left(\frac{c}{d}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{c}{d}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3d\left(\frac{c}{d}\right)^{\frac{2}{3}}}\right)}{9c^2d}$	153

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)/(d*x^3+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] (1/18*(5*a*d+b*c)/c^2*x^4+1/9*(4*a*d-b*c)/c/d*x)/(d*x^3+c)^2+1/9*(5*a*d+b*c
)/c^2/d*(1/3/d/(c/d)^(2/3)*ln(x+(c/d)^(1/3))-1/6/d/(c/d)^(2/3)*ln(x^2-(c/d)
^(1/3)*x+(c/d)^(2/3))+1/3/d/(c/d)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(c/d)
^(1/3)*x-1)))
```

Maxima [A]
time = 0.48, size = 192, normalized size = 0.97

$$\frac{(bcd+5\,ad^2)x^4-2\,(bc^2-4\,acd)x}{18\,(c^2d^3x^6+2\,c^3d^2x^3+c^4d)}+\frac{\sqrt{3}\,(bc+5\,ad)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{27\,c^2d^2\left(\frac{c}{d}\right)^{\frac{2}{3}}}-\frac{(bc+5\,ad)\log\left(x^2-x\left(\frac{c}{d}\right)^{\frac{1}{3}}+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{54\,c^2d^2\left(\frac{c}{d}\right)^{\frac{2}{3}}}+\frac{(bc+5\,ad)\log\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{27\,c^2d^2\left(\frac{c}{d}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)/(d*x^3+c)^3,x, algorithm="maxima")
```

```
[Out] 1/18*((b*c*d + 5*a*d^2)*x^4 - 2*(b*c^2 - 4*a*c*d)*x)/(c^2*d^3*x^6 + 2*c^3*d
^2*x^3 + c^4*d) + 1/27*sqrt(3)*(b*c + 5*a*d)*arctan(1/3*sqrt(3)*(2*x - (c/d
)^(1/3))/(c/d)^(1/3))/(c^2*d^2*(c/d)^(2/3)) - 1/54*(b*c + 5*a*d)*log(x^2 -
x*(c/d)^(1/3) + (c/d)^(2/3))/(c^2*d^2*(c/d)^(2/3)) + 1/27*(b*c + 5*a*d)*log
(x + (c/d)^(1/3))/(c^2*d^2*(c/d)^(2/3))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 351 vs. $2(156) = 312$.
time = 2.54, size = 743, normalized size = 3.77

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)/(d*x^3+c)^3,x, algorithm="fricas")
```

```
[Out] [1/54*(3*(b*c^3*d^2 + 5*a*c^2*d^3)*x^4 + 3*sqrt(1/3)*((b*c^2*d^3 + 5*a*c*d^
4)*x^6 + b*c^4*d + 5*a*c^3*d^2 + 2*(b*c^3*d^2 + 5*a*c^2*d^3)*x^3)*sqrt(-(c^
```

```
2*d)^(1/3)/d)*log((2*c*d*x^3 - 3*(c^2*d)^(1/3)*c*x - c^2 + 3*sqrt(1/3)*(2*c
*d*x^2 + (c^2*d)^(2/3)*x - (c^2*d)^(1/3)*c)*sqrt(-(c^2*d)^(1/3)/d))/(d*x^3
+ c)) - ((b*c*d^2 + 5*a*d^3)*x^6 + b*c^3 + 5*a*c^2*d + 2*(b*c^2*d + 5*a*c*d
^2)*x^3)*(c^2*d)^(2/3)*log(c*d*x^2 - (c^2*d)^(2/3)*x + (c^2*d)^(1/3)*c) + 2
*((b*c*d^2 + 5*a*d^3)*x^6 + b*c^3 + 5*a*c^2*d + 2*(b*c^2*d + 5*a*c*d^2)*x^3
)*(c^2*d)^(2/3)*log(c*d*x + (c^2*d)^(2/3)) - 6*(b*c^4*d - 4*a*c^3*d^2)*x)/(
c^4*d^4*x^6 + 2*c^5*d^3*x^3 + c^6*d^2), 1/54*(3*(b*c^3*d^2 + 5*a*c^2*d^3)*x
^4 + 6*sqrt(1/3)*((b*c^2*d^3 + 5*a*c*d^4)*x^6 + b*c^4*d + 5*a*c^3*d^2 + 2*(
b*c^3*d^2 + 5*a*c^2*d^3)*x^3)*sqrt((c^2*d)^(1/3)/d)*arctan(sqrt(1/3)*(2*(c^
2*d)^(2/3)*x - (c^2*d)^(1/3)*c)*sqrt((c^2*d)^(1/3)/d)/c^2) - ((b*c*d^2 + 5*
a*d^3)*x^6 + b*c^3 + 5*a*c^2*d + 2*(b*c^2*d + 5*a*c*d^2)*x^3)*(c^2*d)^(2/3)
*log(c*d*x^2 - (c^2*d)^(2/3)*x + (c^2*d)^(1/3)*c) + 2*((b*c*d^2 + 5*a*d^3)*
x^6 + b*c^3 + 5*a*c^2*d + 2*(b*c^2*d + 5*a*c*d^2)*x^3)*(c^2*d)^(2/3)*log(c*
d*x + (c^2*d)^(2/3)) - 6*(b*c^4*d - 4*a*c^3*d^2)*x)/(c^4*d^4*x^6 + 2*c^5*d^
3*x^3 + c^6*d^2)]
```

Sympy [A]
time = 0.46, size = 133, normalized size = 0.68

$$\frac{x^4\cdot(5ad^2+bcd)+x(8acd-2bc^2)}{18c^4d+36c^3d^2x^3+18c^2d^3x^6}+\operatorname{RootSum}\left(19683t^3c^8d^4-125a^3d^3-75a^2bcd^2-15ab^2c^2d-b^3c^3,\left(t\mapsto t\log\left(\frac{27tc^3d}{5ad+bc}+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)/(d*x**3+c)**3,x)
```

```
[Out] (x**4*(5*a*d**2 + b*c*d) + x*(8*a*c*d - 2*b*c**2))/(18*c**4*d + 36*c**3*d**
2*x**3 + 18*c**2*d**3*x**6) + RootSum(19683*_t**3*c**8*d**4 - 125*a**3*d**3
 - 75*a**2*b*c*d**2 - 15*a*b**2*c**2*d - b**3*c**3, Lambda(_t, _t*log(27*_t
*c**3*d/(5*a*d + b*c) + x)))
```

Giac [A]
time = 0.84, size = 180, normalized size = 0.91

$$-\frac{\sqrt{3}\,(bc+5\,ad)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{27\,(-cd^2)^{\frac{2}{3}}\,c^2}-\frac{(bc+5\,ad)\log\left(x^2+x\left(-\frac{c}{d}\right)^{\frac{1}{3}}+\left(-\frac{c}{d}\right)^{\frac{2}{3}}\right)}{54\,(-cd^2)^{\frac{2}{3}}\,c^2}-\frac{(bc+5\,ad)\left(-\frac{c}{d}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right|\right)}{27\,c^3d}+\frac{bcdx^4+5\,ad^2x^4-2\,bc^2x+8\,acdx}{18\,(dx^3+c)^2c^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)/(d*x^3+c)^3,x, algorithm="giac")
```

```
[Out] -1/27*sqrt(3)*(b*c + 5*a*d)*arctan(1/3*sqrt(3)*(2*x + (-c/d)^(1/3))/(-c/d)^
(1/3))/((-c*d^2)^(2/3)*c^2) - 1/54*(b*c + 5*a*d)*log(x^2 + x*(-c/d)^(1/3) +
 (-c/d)^(2/3))/((-c*d^2)^(2/3)*c^2) - 1/27*(b*c + 5*a*d)*(-c/d)^(1/3)*log(a
bs(x - (-c/d)^(1/3)))/(c^3*d) + 1/18*(b*c*d*x^4 + 5*a*d^2*x^4 - 2*b*c^2*x +
 8*a*c*d*x)/((d*x^3 + c)^2*c^2*d)
```

Mupad [B]
time = 1.40, size = 173, normalized size = 0.88

$$\frac{\frac{x^4\,(5\,a\,d+b\,c)}{18\,c^2}+\frac{x\,(4\,a\,d-b\,c)}{9\,c\,d}}{c^2+2\,c\,d\,x^3+d^2\,x^6}+\frac{\ln\left(d^{1/3}\,x+c^{1/3}\right)\,(5\,a\,d+b\,c)}{27\,c^{8/3}\,d^{4/3}}-\frac{\ln\left(c^{1/3}-2\,d^{1/3}\,x+\sqrt{3}\,c^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(5\,a\,d+b\,c)}{27\,c^{8/3}\,d^{4/3}}+\frac{\ln\left(2\,d^{1/3}\,x-c^{1/3}+\sqrt{3}\,c^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(5\,a\,d+b\,c)}{27\,c^{8/3}\,d^{4/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)/(c + d*x^3)^3,x)
```

```
[Out] ((x^4*(5*a*d + b*c))/(18*c^2) + (x*(4*a*d - b*c))/(9*c*d))/(c^2 + d^2*x^6 +
 2*c*d*x^3) + (log(d^(1/3)*x + c^(1/3))*(5*a*d + b*c))/(27*c^(8/3)*d^(4/3))
 - (log(3^(1/2)*c^(1/3)*1i - 2*d^(1/3)*x + c^(1/3))*((3^(1/2)*1i)/2 + 1/2)*
(5*a*d + b*c))/(27*c^(8/3)*d^(4/3)) + (log(3^(1/2)*c^(1/3)*1i + 2*d^(1/3)*x
 - c^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(5*a*d + b*c))/(27*c^(8/3)*d^(4/3))
```

3.8 $\int \left(a+bx^3\right)^2 \left(c+dx^3\right)^3 \, dx$

Optimal. Leaf size=122

$$a^2c^3x+\frac{1}{4}ac^2(2bc+3ad)x^4+\frac{1}{7}c\left(b^2c^2+6abcd+3a^2d^2\right)x^7+\frac{1}{10}d\left(3b^2c^2+6abcd+a^2d^2\right)x^{10}+\frac{1}{13}bd^2(3bc+2ad)x^{13}$$

```
[Out] a^2*c^3*x+1/4*a*c^2*(3*a*d+2*b*c)*x^4+1/7*c*(3*a^2*d^2+6*a*b*c*d+b^2*c^2)*x
^7+1/10*d*(a^2*d^2+6*a*b*c*d+3*b^2*c^2)*x^10+1/13*b*d^2*(2*a*d+3*b*c)*x^13+
1/16*b^2*d^3*x^16
```

Rubi [A]
time = 0.05, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = $\{380\}$

$$\frac{1}{10}dx^{10}\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{1}{7}cx^7\left(3a^2d^2+6abcd+b^2c^2\right)+a^2c^3x+\frac{1}{4}ac^2x^4(3ad+2bc)+\frac{1}{13}bd^2x^{13}(2ad+3bc)+\frac{1}{16}b^2d^3x^{16}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^2*(c + d*x^3)^3,x]
```

```
[Out] a^2*c^3*x + (a*c^2*(2*b*c + 3*a*d)*x^4)/4 + (c*(b^2*c^2 + 6*a*b*c*d + 3*a^2
*d^2)*x^7)/7 + (d*(3*b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^10)/10 + (b*d^2*(3*b*
c + 2*a*d)*x^13)/13 + (b^2*d^3*x^16)/16
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int \left(a+bx^3\right)^2 \left(c+dx^3\right)^3 \, dx = \int \left(a^2c^3+ac^2(2bc+3ad)x^3+c\left(b^2c^2+6abcd+3a^2d^2\right)x^6+d\left(3b^2c^2+6abcd\right.\right.$$

$$= a^2c^3x+\frac{1}{4}ac^2(2bc+3ad)x^4+\frac{1}{7}c\left(b^2c^2+6abcd+3a^2d^2\right)x^7+\frac{1}{10}d\left(3b^2c^2+6ab\right.$$

Mathematica [A]
time = 0.01, size = 122, normalized size = 1.00

$$a^2c^3x+\frac{1}{4}ac^2(2bc+3ad)x^4+\frac{1}{7}c\left(b^2c^2+6abcd+3a^2d^2\right)x^7+\frac{1}{10}d\left(3b^2c^2+6abcd+a^2d^2\right)x^{10}+\frac{1}{13}bd^2(3bc+2ad)x^{13}+\frac{1}{16}b^2d^3x^{16}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^2*(c + d*x^3)^3,x]
```

```
[Out] a^2*c^3*x + (a*c^2*(2*b*c + 3*a*d)*x^4)/4 + (c*(b^2*c^2 + 6*a*b*c*d + 3*a^2
*d^2)*x^7)/7 + (d*(3*b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^10)/10 + (b*d^2*(3*b*
c + 2*a*d)*x^13)/13 + (b^2*d^3*x^16)/16
```

Maple [A]
time = 0.30, size = 125, normalized size = 1.02

method	result
norman	$a^2c^3x+\left(\frac{3}{4}a^2c^2d+\frac{1}{2}ab\,c^3\right)x^4+\left(\frac{3}{7}a^2c\,d^2+\frac{6}{7}ab\,c^2d+\frac{1}{7}b^2c^3\right)x^7+\left(\frac{1}{10}a^2d^3+\frac{3}{5}abc\,d^2+\frac{3}{10}b^2c^2d\right)x$
default	$\frac{b^2d^3x^{16}}{16}+\frac{(2ab\,d^3+3b^2c\,d^2)x^{13}}{13}+\frac{(a^2d^3+6abc\,d^2+3b^2c^2d)x^{10}}{10}+\frac{(3a^2c\,d^2+6ab\,c^2d+b^2c^3)x^7}{7}+\frac{(3a^2c^2d+2ab\,c^3)x^4}{4}+a^2c^3$
gosper	$a^2c^3x+\frac{3}{4}x^4a^2c^2d+\frac{1}{2}x^4ab\,c^3+\frac{3}{7}x^7a^2c\,d^2+\frac{6}{7}x^7ab\,c^2d+\frac{1}{7}x^7b^2c^3+\frac{1}{10}x^{10}a^2d^3+\frac{3}{5}x^{10}abc\,d^2+\frac{3}{10}x$
risch	$a^2c^3x+\frac{3}{4}x^4a^2c^2d+\frac{1}{2}x^4ab\,c^3+\frac{3}{7}x^7a^2c\,d^2+\frac{6}{7}x^7ab\,c^2d+\frac{1}{7}x^7b^2c^3+\frac{1}{10}x^{10}a^2d^3+\frac{3}{5}x^{10}abc\,d^2+\frac{3}{10}x$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^2*(d*x^3+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/16*b^2*d^3*x^16+1/13*(2*a*b*d^3+3*b^2*c*d^2)*x^13+1/10*(a^2*d^3+6*a*b*c*d
^2+3*b^2*c^2*d)*x^10+1/7*(3*a^2*c*d^2+6*a*b*c^2*d+b^2*c^3)*x^7+1/4*(3*a^2*c
^2*d+2*a*b*c^3)*x^4+a^2*c^3*x
```

Maxima [A]
time = 0.30, size = 124, normalized size = 1.02

$$\frac{1}{16}b^2d^3x^{16}+\frac{1}{13}\left(3\,b^2cd^2+2\,abd^3\right)x^{13}+\frac{1}{10}\left(3\,b^2c^2d+6\,abcd^2+a^2d^3\right)x^{10}+\frac{1}{7}\left(b^2c^3+6\,abc^2d+3\,a^2cd^2\right)x^7+a^2c^3x+\frac{1}{4}\left(2\,abc^3+3\,a^2c^2d\right)x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(d*x^3+c)^3,x, algorithm="maxima")
```

```
[Out] 1/16*b^2*d^3*x^16 + 1/13*(3*b^2*c*d^2 + 2*a*b*d^3)*x^13 + 1/10*(3*b^2*c^2*d
 + 6*a*b*c*d^2 + a^2*d^3)*x^10 + 1/7*(b^2*c^3 + 6*a*b*c^2*d + 3*a^2*c*d^2)*
x^7 + a^2*c^3*x + 1/4*(2*a*b*c^3 + 3*a^2*c^2*d)*x^4
```

Fricas [A]
time = 2.33, size = 124, normalized size = 1.02

$$\frac{1}{16}b^2d^3x^{16}+\frac{1}{13}\left(3\,b^2cd^2+2\,abd^3\right)x^{13}+\frac{1}{10}\left(3\,b^2c^2d+6\,abcd^2+a^2d^3\right)x^{10}+\frac{1}{7}\left(b^2c^3+6\,abc^2d+3\,a^2cd^2\right)x^7+a^2c^3x+\frac{1}{4}\left(2\,abc^3+3\,a^2c^2d\right)x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(d*x^3+c)^3,x, algorithm="fricas")
```

```
[Out] 1/16*b^2*d^3*x^16 + 1/13*(3*b^2*c*d^2 + 2*a*b*d^3)*x^13 + 1/10*(3*b^2*c^2*d
 + 6*a*b*c*d^2 + a^2*d^3)*x^10 + 1/7*(b^2*c^3 + 6*a*b*c^2*d + 3*a^2*c*d^2)*
x^7 + a^2*c^3*x + 1/4*(2*a*b*c^3 + 3*a^2*c^2*d)*x^4
```

Sympy [A]
time = 0.02, size = 139, normalized size = 1.14

$$a^2c^3x+\frac{b^2d^3x^{16}}{16}+x^{13}\cdot\left(\frac{2abd^3}{13}+\frac{3b^2cd^2}{13}\right)+x^{10}\left(\frac{a^2d^3}{10}+\frac{3abcd^2}{5}+\frac{3b^2c^2d}{10}\right)+x^7\cdot\left(\frac{3a^2cd^2}{7}+\frac{6abc^2d}{7}+\frac{b^2c^3}{7}\right)+x^4\cdot\left(\frac{3a^2c^2d}{4}+\frac{abc^3}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**2*(d*x**3+c)**3,x)
```

```
[Out] a**2*c**3*x + b**2*d**3*x**16/16 + x**13*(2*a*b*d**3/13 + 3*b**2*c*d**2/13)
 + x**10*(a**2*d**3/10 + 3*a*b*c*d**2/5 + 3*b**2*c**2*d/10) + x**7*(3*a**2*
c*d**2/7 + 6*a*b*c**2*d/7 + b**2*c**3/7) + x**4*(3*a**2*c**2*d/4 + a*b*c**3
/2)
```

Giac [A]
time = 0.70, size = 132, normalized size = 1.08

$$\frac{1}{16}b^2d^3x^{16}+\frac{3}{13}b^2cd^2x^{13}+\frac{2}{13}abd^3x^{13}+\frac{3}{10}b^2c^2dx^{10}+\frac{3}{5}abcd^2x^{10}+\frac{1}{10}a^2d^3x^{10}+\frac{1}{7}b^2c^3x^7+\frac{6}{7}abc^2dx^7+\frac{3}{7}a^2cd^2x^7+\frac{1}{2}abc^3x^4+\frac{3}{4}a^2c^2dx^4+a^2c^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(d*x^3+c)^3,x, algorithm="giac")
```

```
[Out] 1/16*b^2*d^3*x^16 + 3/13*b^2*c*d^2*x^13 + 2/13*a*b*d^3*x^13 + 3/10*b^2*c^2*
d*x^10 + 3/5*a*b*c*d^2*x^10 + 1/10*a^2*d^3*x^10 + 1/7*b^2*c^3*x^7 + 6/7*a*b
*c^2*d*x^7 + 3/7*a^2*c*d^2*x^7 + 1/2*a*b*c^3*x^4 + 3/4*a^2*c^2*d*x^4 + a^2*
c^3*x
```

Mupad [B]
time = 1.20, size = 116, normalized size = 0.95

$$x^7\left(\frac{3\,a^2\,c\,d^2}{7}+\frac{6\,a\,b\,c^2\,d}{7}+\frac{b^2\,c^3}{7}\right)+x^{10}\left(\frac{a^2\,d^3}{10}+\frac{3\,a\,b\,c\,d^2}{5}+\frac{3\,b^2\,c^2\,d}{10}\right)+a^2\,c^3\,x+\frac{b^2\,d^3\,x^{16}}{16}+\frac{a\,c^2\,x^4\,\left(3\,a\,d+2\,b\,c\right)}{4}+\frac{b\,d^2\,x^{13}\,\left(2\,a\,d+3\,b\,c\right)}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^2*(c + d*x^3)^3,x)
```

```
[Out] x^7*((b^2*c^3)/7 + (3*a^2*c*d^2)/7 + (6*a*b*c^2*d)/7) + x^10*((a^2*d^3)/10
+ (3*b^2*c^2*d)/10 + (3*a*b*c*d^2)/5) + a^2*c^3*x + (b^2*d^3*x^16)/16 + (a*
c^2*x^4*(3*a*d + 2*b*c))/4 + (b*d^2*x^13*(2*a*d + 3*b*c))/13
```

3.9 $\int (a+bx^3)^2 (c+dx^3)^2 \, dx$

Optimal. Leaf size=82

$$a^2c^2x+\frac{1}{2}ac(bc+ad)x^4+\frac{1}{7}\left(b^2c^2+4abcd+a^2d^2\right)x^7+\frac{1}{5}bd(bc+ad)x^{10}+\frac{1}{13}b^2d^2x^{13}$$

```
[Out] a^2*c^2*x+1/2*a*c*(a*d+b*c)*x^4+1/7*(a^2*d^2+4*a*b*c*d+b^2*c^2)*x^7+1/5*b*d
*(a*d+b*c)*x^10+1/13*b^2*d^2*x^13
```

Rubi [A]
time = 0.03, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = $\{380\}$

$$\frac{1}{7}x^7\left(a^2d^2+4abcd+b^2c^2\right)+a^2c^2x+\frac{1}{5}bdx^{10}(ad+bc)+\frac{1}{2}acx^4(ad+bc)+\frac{1}{13}b^2d^2x^{13}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^2*(c + d*x^3)^2,x]
```

```
[Out] a^2*c^2*x + (a*c*(b*c + a*d)*x^4)/2 + ((b^2*c^2 + 4*a*b*c*d + a^2*d^2)*x^7)
/7 + (b*d*(b*c + a*d)*x^10)/5 + (b^2*d^2*x^13)/13
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int \left(a+bx^3\right)^2\left(c+dx^3\right)^2 \, dx = \int \left(a^2c^2+2ac(bc+ad)x^3+\left(b^2c^2+4abcd+a^2d^2\right)x^6+2bd(bc+ad)x^9+b^2\right.$$

$$=a^2c^2x+\frac{1}{2}ac(bc+ad)x^4+\frac{1}{7}\left(b^2c^2+4abcd+a^2d^2\right)x^7+\frac{1}{5}bd(bc+ad)x^{10}+$$

Mathematica [A]
time = 0.01, size = 82, normalized size = 1.00

$$a^2c^2x+\frac{1}{2}ac(bc+ad)x^4+\frac{1}{7}\left(b^2c^2+4abcd+a^2d^2\right)x^7+\frac{1}{5}bd(bc+ad)x^{10}+\frac{1}{13}b^2d^2x^{13}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^2*(c + d*x^3)^2,x]
```

```
[Out] a^2*c^2*x + (a*c*(b*c + a*d)*x^4)/2 + ((b^2*c^2 + 4*a*b*c*d + a^2*d^2)*x^7)
/7 + (b*d*(b*c + a*d)*x^10)/5 + (b^2*d^2*x^13)/13
```

Maple [A]
time = 0.32, size = 87, normalized size = 1.06

method	result
norman	$\frac{b^2d^2x^{13}}{13}+\left(\frac{1}{5}ab\,d^2+\frac{1}{5}b^2cd\right)x^{10}+\left(\frac{1}{7}a^2d^2+\frac{4}{7}abcd+\frac{1}{7}b^2c^2\right)x^7+\left(\frac{1}{2}a^2cd+\frac{1}{2}ab\,c^2\right)x^4+a^2c^2x$
default	$\frac{b^2d^2x^{13}}{13}+\frac{(2ab\,d^2+2b^2cd)x^{10}}{10}+\frac{(a^2d^2+4abcd+b^2c^2)x^7}{7}+\frac{(2a^2cd+2ab\,c^2)x^4}{4}+a^2c^2x$
gosper	$\frac{1}{13}b^2d^2x^{13}+\frac{1}{5}x^{10}ab\,d^2+\frac{1}{5}x^{10}b^2cd+\frac{1}{7}x^7a^2d^2+\frac{4}{7}x^7abcd+\frac{1}{7}x^7b^2c^2+\frac{1}{2}x^4a^2cd+\frac{1}{2}x^4ab\,c^2+a^2c^2x$
risch	$\frac{1}{13}b^2d^2x^{13}+\frac{1}{5}x^{10}ab\,d^2+\frac{1}{5}x^{10}b^2cd+\frac{1}{7}x^7a^2d^2+\frac{4}{7}x^7abcd+\frac{1}{7}x^7b^2c^2+\frac{1}{2}x^4a^2cd+\frac{1}{2}x^4ab\,c^2+a^2c^2x$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^2*(d*x^3+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/13*b^2*d^2*x^13+1/10*(2*a*b*d^2+2*b^2*c*d)*x^10+1/7*(a^2*d^2+4*a*b*c*d+b^
2*c^2)*x^7+1/4*(2*a^2*c*d+2*a*b*c^2)*x^4+a^2*c^2*x
```

Maxima [A]
time = 0.26, size = 82, normalized size = 1.00

$$\frac{1}{13}\,b^2d^2x^{13}+\frac{1}{5}\,(b^2cd+abd^2)x^{10}+\frac{1}{7}\,(b^2c^2+4\,abcd+a^2d^2)x^7+a^2c^2x+\frac{1}{2}\,(abc^2+a^2cd)x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(d*x^3+c)^2,x, algorithm="maxima")
```

```
[Out] 1/13*b^2*d^2*x^13 + 1/5*(b^2*c*d + a*b*d^2)*x^10 + 1/7*(b^2*c^2 + 4*a*b*c*d
 + a^2*d^2)*x^7 + a^2*c^2*x + 1/2*(a*b*c^2 + a^2*c*d)*x^4
```

Fricas [A]
time = 2.79, size = 82, normalized size = 1.00

$$\frac{1}{13}\,b^2d^2x^{13}+\frac{1}{5}\,(b^2cd+abd^2)x^{10}+\frac{1}{7}\,(b^2c^2+4\,abcd+a^2d^2)x^7+a^2c^2x+\frac{1}{2}\,(abc^2+a^2cd)x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(d*x^3+c)^2,x, algorithm="fricas")
```

```
[Out] 1/13*b^2*d^2*x^13 + 1/5*(b^2*c*d + a*b*d^2)*x^10 + 1/7*(b^2*c^2 + 4*a*b*c*d
 + a^2*d^2)*x^7 + a^2*c^2*x + 1/2*(a*b*c^2 + a^2*c*d)*x^4
```

Sympy [A]
time = 0.01, size = 90, normalized size = 1.10

$$a^2c^2x+\frac{b^2d^2x^{13}}{13}+x^{10}\left(\frac{abd^2}{5}+\frac{b^2cd}{5}\right)+x^7\left(\frac{a^2d^2}{7}+\frac{4abcd}{7}+\frac{b^2c^2}{7}\right)+x^4\left(\frac{a^2cd}{2}+\frac{abc^2}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**2*(d*x**3+c)**2,x)
```

```
[Out] a**2*c**2*x + b**2*d**2*x**13/13 + x**10*(a*b*d**2/5 + b**2*c*d/5) + x**7*(
a**2*d**2/7 + 4*a*b*c*d/7 + b**2*c**2/7) + x**4*(a**2*c*d/2 + a*b*c**2/2)
```

Giac [A]
time = 0.85, size = 91, normalized size = 1.11

$$\frac{1}{13}\,b^{2}d^{2}x^{13}+\frac{1}{5}\,b^{2}cdx^{10}+\frac{1}{5}\,abd^{2}x^{10}+\frac{1}{7}\,b^{2}c^{2}x^{7}+\frac{4}{7}\,abcdx^{7}+\frac{1}{7}\,a^{2}d^{2}x^{7}+\frac{1}{2}\,abc^{2}x^{4}+\frac{1}{2}\,a^{2}cdx^{4}+a^{2}c^{2}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(d*x^3+c)^2,x, algorithm="giac")
```

```
[Out] 1/13*b^2*d^2*x^13 + 1/5*b^2*c*d*x^10 + 1/5*a*b*d^2*x^10 + 1/7*b^2*c^2*x^7 +
 4/7*a*b*c*d*x^7 + 1/7*a^2*d^2*x^7 + 1/2*a*b*c^2*x^4 + 1/2*a^2*c*d*x^4 + a^
2*c^2*x
```

Mupad [B]
time = 0.04, size = 75, normalized size = 0.91

$$x^7\,\left(\frac{a^2\,d^2}{7}+\frac{4\,a\,b\,c\,d}{7}+\frac{b^2\,c^2}{7}\right)+a^2\,c^2\,x+\frac{b^2\,d^2\,x^{13}}{13}+\frac{a\,c\,x^4\,\left(a\,d+b\,c\right)}{2}+\frac{b\,d\,x^{10}\,\left(a\,d+b\,c\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^2*(c + d*x^3)^2,x)
```

```
[Out] x^7*((a^2*d^2)/7 + (b^2*c^2)/7 + (4*a*b*c*d)/7) + a^2*c^2*x + (b^2*d^2*x^13
)/13 + (a*c*x^4*(a*d + b*c))/2 + (b*d*x^10*(a*d + b*c))/5
```

3.10 $\int (a+bx^3)^2 (c+dx^3)\,dx$

Optimal. Leaf size=50

$$a^2cx+\frac{1}{4}a(2bc+ad)x^4+\frac{1}{7}b(bc+2ad)x^7+\frac{1}{10}b^2dx^{10}$$

```
[Out] a^2*c*x+1/4*a*(a*d+2*b*c)*x^4+1/7*b*(2*a*d+b*c)*x^7+1/10*b^2*d*x^10
```

Rubi [A]
time = 0.02, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {380}

$$a^2cx+\frac{1}{7}bx^7(2ad+bc)+\frac{1}{4}ax^4(ad+2bc)+\frac{1}{10}b^2dx^{10}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^2*(c + d*x^3),x]
```

```
[Out] a^2*c*x + (a*(2*b*c + a*d)*x^4)/4 + (b*(b*c + 2*a*d)*x^7)/7 + (b^2*d*x^10)/
10
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int (a+bx^3)^2 (c+dx^3)\,dx &= \int \left(a^2c+a(2bc+ad)x^3+b(bc+2ad)x^6+b^2dx^9\right)dx\\ &= a^2cx+\frac{1}{4}a(2bc+ad)x^4+\frac{1}{7}b(bc+2ad)x^7+\frac{1}{10}b^2dx^{10}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 50, normalized size = 1.00

$$a^2cx+\frac{1}{4}a(2bc+ad)x^4+\frac{1}{7}b(bc+2ad)x^7+\frac{1}{10}b^2dx^{10}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^2*(c + d*x^3),x]
```

```
[Out] a^2*c*x + (a*(2*b*c + a*d)*x^4)/4 + (b*(b*c + 2*a*d)*x^7)/7 + (b^2*d*x^10)/
10
```

Maple [A]
time = 0.30, size = 49, normalized size = 0.98

method	result	size
default	$\frac{b^2 d x^{10}}{10}+\frac{(2abd+b^2c)x^7}{7}+\frac{(a^2d+2abc)x^4}{4}+a^2cx$	49
norman	$\frac{b^2 d x^{10}}{10}+\left(\frac{2}{7}abd+\frac{1}{7}b^2c\right)x^7+\left(\frac{1}{4}a^2d+\frac{1}{2}abc\right)x^4+a^2cx$	49
gosper	$\frac{1}{10}b^2 d x^{10}+\frac{2}{7}x^7abd+\frac{1}{7}x^7b^2c+\frac{1}{4}x^4a^2d+\frac{1}{2}x^4abc+a^2cx$	51
risch	$\frac{1}{10}b^2 d x^{10}+\frac{2}{7}x^7abd+\frac{1}{7}x^7b^2c+\frac{1}{4}x^4a^2d+\frac{1}{2}x^4abc+a^2cx$	51

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^2*(d*x^3+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/10*b^2*d*x^10+1/7*(2*a*b*d+b^2*c)*x^7+1/4*(a^2*d+2*a*b*c)*x^4+a^2*c*x
```

Maxima [A]
time = 0.28, size = 48, normalized size = 0.96

$$\frac{1}{10}\,b^2dx^{10}+\frac{1}{7}\,\left(b^2c+2\,abd\right)x^7+\frac{1}{4}\,\left(2\,abc+a^2d\right)x^4+a^2cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(d*x^3+c),x, algorithm="maxima")
```

```
[Out] 1/10*b^2*d*x^10 + 1/7*(b^2*c + 2*a*b*d)*x^7 + 1/4*(2*a*b*c + a^2*d)*x^4 + a
^2*c*x
```

Fricas [A]
time = 1.78, size = 48, normalized size = 0.96

$$\frac{1}{10}\,b^2dx^{10}+\frac{1}{7}\,\left(b^2c+2\,abd\right)x^7+\frac{1}{4}\,\left(2\,abc+a^2d\right)x^4+a^2cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(d*x^3+c),x, algorithm="fricas")
```

```
[Out] 1/10*b^2*d*x^10 + 1/7*(b^2*c + 2*a*b*d)*x^7 + 1/4*(2*a*b*c + a^2*d)*x^4 + a
^2*c*x
```

Sympy [A]
time = 0.01, size = 51, normalized size = 1.02

$$a^2cx+\frac{b^2dx^{10}}{10}+x^7\cdot\left(\frac{2abd}{7}+\frac{b^2c}{7}\right)+x^4\left(\frac{a^2d}{4}+\frac{abc}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**2*(d*x**3+c),x)
```

```
[Out] a**2*c*x + b**2*d*x**10/10 + x**7*(2*a*b*d/7 + b**2*c/7) + x**4*(a**2*d/4 +
 a*b*c/2)
```

Giac [A]
time = 0.66, size = 50, normalized size = 1.00

$$\frac{1}{10}\,b^2dx^{10}+\frac{1}{7}\,b^2cx^7+\frac{2}{7}\,abdx^7+\frac{1}{2}\,abcx^4+\frac{1}{4}\,a^2dx^4+a^2cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(d*x^3+c),x, algorithm="giac")
```

```
[Out] 1/10*b^2*d*x^10 + 1/7*b^2*c*x^7 + 2/7*a*b*d*x^7 + 1/2*a*b*c*x^4 + 1/4*a^2*d
*x^4 + a^2*c*x
```

Mupad [B]
time = 0.04, size = 48, normalized size = 0.96

$$x^4\left(\frac{d\,a^2}{4}+\frac{b\,c\,a}{2}\right)+x^7\left(\frac{c\,b^2}{7}+\frac{2\,a\,d\,b}{7}\right)+\frac{b^2\,d\,x^{10}}{10}+a^2\,c\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^2*(c + d*x^3),x)
```

```
[Out] x^4*((a^2*d)/4 + (a*b*c)/2) + x^7*((b^2*c)/7 + (2*a*b*d)/7) + (b^2*d*x^10)/
10 + a^2*c*x
```

3.11 $\int \frac{(a+bx^3)^2}{c+dx^3}\,dx$

Optimal. Leaf size=173

$$-\frac{b(bc-2ad)x}{d^2}+\frac{b^2x^4}{4d}-\frac{(bc-ad)^2\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}\,x}{\sqrt{3}\,\sqrt[3]{c}}\right)}{\sqrt{3}\,c^{2/3}d^{7/3}}+\frac{(bc-ad)^2\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{3c^{2/3}d^{7/3}}-\frac{(bc-ad)^2\log\left(c^{2/\cdots}\right.}{6\cdots}$$

```
[Out] -b*(-2*a*d+b*c)*x/d^2+1/4*b^2*x^4/d+1/3*(-a*d+b*c)^2*ln(c^(1/3)+d^(1/3)*x)/
c^(2/3)/d^(7/3)-1/6*(-a*d+b*c)^2*ln(c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/
c^(2/3)/d^(7/3)-1/3*(-a*d+b*c)^2*arctan(1/3*(c^(1/3)-2*d^(1/3)*x)/c^(1/3)*3
^(1/2))/c^(2/3)/d^(7/3)*3^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {398, 206, 31, 648, 631, 210, 642}

$$-\frac{(bc-ad)^2\text{ArcTan}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}\,x}{\sqrt{3}\,\sqrt[3]{c}}\right)}{\sqrt{3}\,c^{2/3}d^{7/3}}-\frac{(bc-ad)^2\log\left(c^{2/3}-\sqrt[3]{c}\,\sqrt[3]{d}\,x+d^{2/3}x^2\right)}{6c^{2/3}d^{7/3}}+\frac{(bc-ad)^2\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{3c^{2/3}d^{7/3}}-\frac{bx(bc-2ad)}{d^2}+\frac{b^2x^4}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^2/(c + d*x^3),x]
```

```
[Out] -((b*(b*c - 2*a*d)*x)/d^2) + (b^2*x^4)/(4*d) - ((b*c - a*d)^2*ArcTan[(c^(1/
3) - 2*d^(1/3)*x)/(Sqrt[3]*c^(1/3))])/(Sqrt[3]*c^(2/3)*d^(7/3)) + ((b*c - a
*d)^2*Log[c^(1/3) + d^(1/3)*x])/(3*c^(2/3)*d^(7/3)) - ((b*c - a*d)^2*Log[c^
(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/(6*c^(2/3)*d^(7/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{(a+bx^3)^2}{c+dx^3}\,dx = \int \left(-\frac{b(bc-2ad)}{d^2} + \frac{b^2x^3}{d} + \frac{b^2c^2-2abcd+a^2d^2}{d^2(c+dx^3)}\right)dx$$
$$= -\frac{b(bc-2ad)x}{d^2} + \frac{b^2x^4}{4d} + \frac{(bc-ad)^2\int\frac{1}{c+dx^3}\,dx}{d^2}$$
$$= -\frac{b(bc-2ad)x}{d^2} + \frac{b^2x^4}{4d} + \frac{(bc-ad)^2\int\frac{1}{\sqrt[3]{c}+\sqrt[3]{d}\,x}\,dx}{3c^{2/3}d^2} + \frac{(bc-ad)^2\int\frac{2\sqrt[3]{c}-\sqrt[3]{d}\,x}{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}\,x+d^{2/3}x^2}}{3c^{2/3}d^2}$$
$$= -\frac{b(bc-2ad)x}{d^2} + \frac{b^2x^4}{4d} + \frac{(bc-ad)^2\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{3c^{2/3}d^{7/3}} - \frac{(bc-ad)^2\int\frac{-\sqrt[3]{c}\sqrt[3]{d}+2d^{2/3}}{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}\,x+d^2}}{6c^{2/3}d^{7/3}}$$
$$= -\frac{b(bc-2ad)x}{d^2} + \frac{b^2x^4}{4d} + \frac{(bc-ad)^2\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{3c^{2/3}d^{7/3}} - \frac{(bc-ad)^2\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}\right.}{6c^{2/3}d^{7/3}}$$
$$= -\frac{b(bc-2ad)x}{d^2} + \frac{b^2x^4}{4d} - \frac{(bc-ad)^2\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}\,x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}\,c^{2/3}d^{7/3}} + \frac{(bc-ad)^2\log\left(\sqrt[3]{c}+\sqrt[3]{d}\right.}{3c^{2/3}d^{7/3}}$$

Mathematica [A]
time = 0.06, size = 167, normalized size = 0.97

$$\frac{-12bc^{2/3}\sqrt[3]{d}\,(bc-2ad)x+3b^2c^{2/3}d^{4/3}x^4+4\sqrt{3}\,(bc-ad)^2\tan^{-1}\left(\frac{-\sqrt[3]{c}+2\sqrt[3]{d}\,x}{\sqrt{3}\sqrt[3]{c}}\right)+4(bc-ad)^2\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)-2(bc-ad)^2\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}\,x+d^{2/3}x^2\right)}{12c^{2/3}d^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^2/(c + d*x^3),x]
```

```
[Out] (-12*b*c^(2/3)*d^(1/3)*(b*c - 2*a*d)*x + 3*b^2*c^(2/3)*d^(4/3)*x^4 + 4*Sqrt
[3]*(b*c - a*d)^2*ArcTan[(-c^(1/3) + 2*d^(1/3)*x)/(Sqrt[3]*c^(1/3))] + 4*(b
*c - a*d)^2*Log[c^(1/3) + d^(1/3)*x] - 2*(b*c - a*d)^2*Log[c^(2/3) - c^(1/3
)*d^(1/3)*x + d^(2/3)*x^2])/(12*c^(2/3)*d^(7/3))
```

Maple [A]
time = 0.27, size = 140, normalized size = 0.81

method	result
risch	$\frac{b^2x^4}{4d}+\frac{2bax}{d}-\frac{b^2cx}{d^2}+\frac{\sum_{\_R=\mathrm{RootOf}\left(d\_Z^3+c\right)}\frac{\left(a^2d^2-2abcd+b^2c^2\right)\ln\left(x-\_R\right)}{\_R^2}}{3d^3}$

default	$\frac{b\left(\frac{1}{4}bd\,x^{4}+2adx-bcx\right)}{d^{2}}+\frac{\left(\frac{\ln\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3d\left(\frac{c}{d}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{c}{d}\right)^{\frac{1}{3}}x+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6d\left(\frac{c}{d}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,\left(\frac{2x}{\left(\frac{c}{d}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3d\left(\frac{c}{d}\right)^{\frac{2}{3}}}\right)\left(a^{2}d^{2}-2abcd+b^{2}c^{2}\right)}{d^{2}}$	14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^2/(d*x^3+c),x,method=_RETURNVERBOSE)
```

```
[Out] b/d^2*(1/4*b*d*x^4+2*a*d*x-b*c*x)+(1/3/d/(c/d)^(2/3)*ln(x+(c/d)^(1/3))-1/6/
d/(c/d)^(2/3)*ln(x^2-(c/d)^(1/3)*x+(c/d)^(2/3))+1/3/d/(c/d)^(2/3)*3^(1/2)*a
rctan(1/3*3^(1/2)*(2/(c/d)^(1/3)*x-1)))*(a^2*d^2-2*a*b*c*d+b^2*c^2)/d^2
```

Maxima [A]
time = 0.51, size = 189, normalized size = 1.09

$$\frac{b^2dx^4-4\left(b^2c-2\,abd\right)x}{4\,d^2}+\frac{\sqrt{3}\left(b^2c^2-2\,abcd+a^2d^2\right)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{3\,d^3\left(\frac{c}{d}\right)^{\frac{2}{3}}}-\frac{\left(b^2c^2-2\,abcd+a^2d^2\right)\log\left(x^2-x\left(\frac{c}{d}\right)^{\frac{1}{3}}+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\,d^3\left(\frac{c}{d}\right)^{\frac{2}{3}}}+\frac{\left(b^2c^2-2\,abcd+a^2d^2\right)\log\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\,d^3\left(\frac{c}{d}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] 1/4*(b^2*d*x^4 - 4*(b^2*c - 2*a*b*d)*x)/d^2 + 1/3*sqrt(3)*(b^2*c^2 - 2*a*b*
c*d + a^2*d^2)*arctan(1/3*sqrt(3)*(2*x - (c/d)^(1/3))/(c/d)^(1/3))/(d^3*(c/
d)^(2/3)) - 1/6*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(x^2 - x*(c/d)^(1/3) + (
c/d)^(2/3))/(d^3*(c/d)^(2/3)) + 1/3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(x +
 (c/d)^(1/3))/(d^3*(c/d)^(2/3))
```

Fricas [A]
time = 4.26, size = 505, normalized size = 2.92

$$\left[\frac{3\,b^2c^2d^2x^4+6\sqrt{\frac{1}{3}}\left(b^2c^3d-2\,abc^2d^2+a^2cd^3\right)\sqrt{-\frac{\left(c^2d\right)^{\frac{1}{3}}}{d}}\log\left(\frac{2\,cdx^3-3\left(c^2d\right)^{\frac{1}{3}}cx-c^2+3\sqrt{\frac{1}{3}}\left(2\,cdx^2+\left(c^2d\right)^{\frac{2}{3}}x-\left(c^2d\right)^{\frac{1}{3}}c\right)\sqrt{-\frac{\left(c^2d\right)^{\frac{1}{3}}}{d}}}{dx^3+c}\right)-2\left(b^2c^2-2\,abcd+a^2d^2\right)\left(c^2d\right)^{\frac{2}{3}}\log\left(cdx^2-\left(c^2d\right)^{\frac{2}{3}}x+\left(c^2d\right)^{\frac{1}{3}}c\right)+4\left(b^2c^2-2\,abcd+a^2d^2\right)\left(c^2d\right)^{\frac{2}{3}}\log\left(cdx+\left(c^2d\right)^{\frac{2}{3}}\right)-12\left(b^2c^3d-2\,abc^2d^2\right)x}{12\,c^2d^3},\ \frac{3\,b^2c^2d^2x^4+12\sqrt{\frac{1}{3}}\left(b^2c^3d-2\,abc^2d^2+a^2cd^3\right)\sqrt{\frac{\left(c^2d\right)^{\frac{1}{3}}}{d}}\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(2\left(c^2d\right)^{\frac{2}{3}}x-\left(c^2d\right)^{\frac{1}{3}}c\right)\sqrt{\frac{\left(c^2d\right)^{\frac{1}{3}}}{d}}}{c^2}\right)-2\left(b^2c^2-2\,abcd+a^2d^2\right)\left(c^2d\right)^{\frac{2}{3}}\log\left(cdx^2-\left(c^2d\right)^{\frac{2}{3}}x+\left(c^2d\right)^{\frac{1}{3}}c\right)+4\left(b^2c^2-2\,abcd+a^2d^2\right)\left(c^2d\right)^{\frac{2}{3}}\log\left(cdx+\left(c^2d\right)^{\frac{2}{3}}\right)-12\left(b^2c^3d-2\,abc^2d^2\right)x}{12\,c^2d^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] [1/12*(3*b^2*c^2*d^2*x^4 + 6*sqrt(1/3)*(b^2*c^3*d - 2*a*b*c^2*d^2 + a^2*c*d
^3)*sqrt(-(c^2*d)^(1/3)/d)*log((2*c*d*x^3 - 3*(c^2*d)^(1/3)*c*x - c^2 + 3*s
qrt(1/3)*(2*c*d*x^2 + (c^2*d)^(2/3)*x - (c^2*d)^(1/3)*c)*sqrt(-(c^2*d)^(1/3
)/d))/(d*x^3 + c)) - 2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(c^2*d)^(2/3)*log(c*
```

```
d*x^2 - (c^2*d)^(2/3)*x + (c^2*d)^(1/3)*c) + 4*(b^2*c^2 - 2*a*b*c*d + a^2*d
^2)*(c^2*d)^(2/3)*log(c*d*x + (c^2*d)^(2/3)) - 12*(b^2*c^3*d - 2*a*b*c^2*d^
2)*x)/(c^2*d^3), 1/12*(3*b^2*c^2*d^2*x^4 + 12*sqrt(1/3)*(b^2*c^3*d - 2*a*b*
c^2*d^2 + a^2*c*d^3)*sqrt((c^2*d)^(1/3)/d)*arctan(sqrt(1/3)*(2*(c^2*d)^(2/3
)*x - (c^2*d)^(1/3)*c)*sqrt((c^2*d)^(1/3)/d)/c^2) - 2*(b^2*c^2 - 2*a*b*c*d
+ a^2*d^2)*(c^2*d)^(2/3)*log(c*d*x^2 - (c^2*d)^(2/3)*x + (c^2*d)^(1/3)*c) +
 4*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(c^2*d)^(2/3)*log(c*d*x + (c^2*d)^(2/3))
 - 12*(b^2*c^3*d - 2*a*b*c^2*d^2)*x)/(c^2*d^3)]
```

Sympy [A]
time = 0.36, size = 156, normalized size = 0.90

$$\frac{b^2x^4}{4d} + x\left(\frac{2ab}{d} - \frac{b^2c}{d^2}\right) + \operatorname{RootSum}\left(27t^3c^2d^7 - a^6d^6 + 6a^5bcd^5 - 15a^4b^2c^2d^4 + 20a^3b^3c^3d^3 - 15a^2b^4c^4d^2 + 6ab^5c^5d - b^6c^6, \left(t \mapsto t\log\left(\frac{3tcd^2}{a^2d^2 - 2abcd + b^2c^2} + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**2/(d*x**3+c),x)
```

```
[Out] b**2*x**4/(4*d) + x*(2*a*b/d - b**2*c/d**2) + RootSum(27*_t**3*c**2*d**7 -
a**6*d**6 + 6*a**5*b*c*d**5 - 15*a**4*b**2*c**2*d**4 + 20*a**3*b**3*c**3*d*
*3 - 15*a**2*b**4*c**4*d**2 + 6*a*b**5*c**5*d - b**6*c**6, Lambda(_t, _t*lo
g(3*_t*c*d**2/(a**2*d**2 - 2*a*b*c*d + b**2*c**2) + x)))
```

Giac [A]
time = 0.94, size = 211, normalized size = 1.22

$$-\frac{\sqrt{3}\,(b^2c^2 - 2\,abcd + a^2d^2)\arctan\left(\frac{\sqrt{3}\left(2\,x + \left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{3\left(-cd^2\right)^{\frac{2}{3}}d} - \frac{(b^2c^2 - 2\,abcd + a^2d^2)\log\left(x^2 + x\left(-\frac{c}{d}\right)^{\frac{1}{3}} + \left(-\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\left(-cd^2\right)^{\frac{2}{3}}d} - \frac{(b^2c^2d^2 - 2\,abcd^3 + a^2d^4)\left(-\frac{c}{d}\right)^{\frac{1}{3}}\log\left(\left|x - \left(-\frac{c}{d}\right)^{\frac{1}{3}}\right|\right)}{3\,cd^4} + \frac{b^2d^3x^4 - 4\,b^2cd^2x + 8\,abd^3x}{4\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2/(d*x^3+c),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*arctan(1/3*sqrt(3)*(2*x + (-c/
d)^(1/3))/(-c/d)^(1/3))/((-c*d^2)^(2/3)*d) - 1/6*(b^2*c^2 - 2*a*b*c*d + a^2
*d^2)*log(x^2 + x*(-c/d)^(1/3) + (-c/d)^(2/3))/((-c*d^2)^(2/3)*d) - 1/3*(b^
2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(-c/d)^(1/3)*log(abs(x - (-c/d)^(1/3)))/
(c*d^4) + 1/4*(b^2*d^3*x^4 - 4*b^2*c*d^2*x + 8*a*b*d^3*x)/d^4
```

Mupad [B]
time = 1.39, size = 152, normalized size = 0.88

$$\frac{b^2\,x^4}{4\,d} - x\left(\frac{b^2\,c}{d^2} - \frac{2\,a\,b}{d}\right) + \frac{\ln\left(d^{1/3}\,x + c^{1/3}\right)\,\left(a\,d - b\,c\right)^2}{3\,c^{2/3}\,d^{7/3}} + \frac{\ln\left(2\,d^{1/3}\,x - c^{1/3} + \sqrt{3}\,c^{1/3}\,1{}\mathrm{i}\right)\,\left(-\frac{1}{6} + \frac{\sqrt{3}\,1{}\mathrm{i}}{6}\right)\,\left(a\,d - b\,c\right)^2}{c^{2/3}\,d^{7/3}} - \frac{\ln\left(c^{1/3} - 2\,d^{1/3}\,x + \sqrt{3}\,c^{1/3}\,1{}\mathrm{i}\right)\,\left(\frac{1}{2} + \frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(a\,d - b\,c\right)^2}{3\,c^{2/3}\,d^{7/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^2/(c + d*x^3),x)
```

```
[Out] (b^2*x^4)/(4*d) - x*((b^2*c)/d^2 - (2*a*b)/d) + (log(d^(1/3)*x + c^(1/3))*(
a*d - b*c)^2)/(3*c^(2/3)*d^(7/3)) + (log(3^(1/2)*c^(1/3)*1i + 2*d^(1/3)*x -
 c^(1/3))*((3^(1/2)*1i)/6 - 1/6)*(a*d - b*c)^2)/(c^(2/3)*d^(7/3)) - (log(3^
(1/2)*c^(1/3)*1i - 2*d^(1/3)*x + c^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(a*d - b*c
)^2)/(3*c^(2/3)*d^(7/3))
```

3.12 $\int \frac{(a+bx^3)^2}{(c+dx^3)^2}\, dx$

Optimal. Leaf size=203

$$\frac{b^2x}{d^2}+\frac{(bc-ad)^2x}{3cd^2\left(c+dx^3\right)}+\frac{2(bc-ad)(2bc+ad)\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}\,x}{\sqrt{3}\sqrt[3]{c}}\right)}{3\sqrt{3}\,c^{5/3}d^{7/3}}-\frac{2(bc-ad)(2bc+ad)\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{9c^{5/3}d^{7/3}}$$

```
[Out] b^2*x/d^2+1/3*(-a*d+b*c)^2*x/c/d^2/(d*x^3+c)-2/9*(-a*d+b*c)*(a*d+2*b*c)*ln(
c^(1/3)+d^(1/3)*x)/c^(5/3)/d^(7/3)+1/9*(-a*d+b*c)*(a*d+2*b*c)*ln(c^(2/3)-c^
(1/3)*d^(1/3)*x+d^(2/3)*x^2)/c^(5/3)/d^(7/3)+2/9*(-a*d+b*c)*(a*d+2*b*c)*arc
tan(1/3*(c^(1/3)-2*d^(1/3)*x)/c^(1/3)*3^(1/2))/c^(5/3)/d^(7/3)*3^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {398, 393, 206, 31, 648, 631, 210, 642}

$$\frac{2(bc-ad)(ad+2bc)\text{ArcTan}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}\,x}{\sqrt{3}\sqrt[3]{c}}\right)}{3\sqrt{3}\,c^{5/3}d^{7/3}}+\frac{(bc-ad)(ad+2bc)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}\,x+d^{2/3}x^2\right)}{9c^{5/3}d^{7/3}}-\frac{2(bc-ad)(ad+2bc)\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{9c^{5/3}d^{7/3}}+\frac{x(bc-ad)^2}{3cd^2\left(c+dx^3\right)}+\frac{b^2x}{d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^2/(c + d*x^3)^2,x]
```

```
[Out] (b^2*x)/d^2 + ((b*c - a*d)^2*x)/(3*c*d^2*(c + d*x^3)) + (2*(b*c - a*d)*(2*b
*c + a*d)*ArcTan[(c^(1/3) - 2*d^(1/3)*x)/(Sqrt[3]*c^(1/3))])/(3*Sqrt[3]*c^(
5/3)*d^(7/3)) - (2*(b*c - a*d)*(2*b*c + a*d)*Log[c^(1/3) + d^(1/3)*x])/(9*c
^(5/3)*d^(7/3)) + ((b*c - a*d)*(2*b*c + a*d)*Log[c^(2/3) - c^(1/3)*d^(1/3)*
x + d^(2/3)*x^2])/(9*c^(5/3)*d^(7/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{(a+bx^3)^2}{(c+dx^3)^2}\,dx = \int \left(\frac{b^2}{d^2} - \frac{b^2c^2-a^2d^2+2bd(bc-ad)x^3}{d^2(c+dx^3)^2}\right)dx$$

$$= \frac{b^2x}{d^2} - \frac{\int \frac{b^2c^2-a^2d^2+2bd(bc-ad)x^3}{(c+dx^3)^2}\,dx}{d^2}$$

$$= \frac{b^2x}{d^2} + \frac{(bc-ad)^2x}{3cd^2(c+dx^3)} - \frac{(2(bc-ad)(2bc+ad))\int \frac{1}{c+dx^3}\,dx}{3cd^2}$$

$$= \frac{b^2x}{d^2} + \frac{(bc-ad)^2x}{3cd^2(c+dx^3)} - \frac{(2(bc-ad)(2bc+ad))\int \frac{1}{\sqrt[3]{c}+\sqrt[3]{d}\,x}\,dx}{9c^{5/3}d^2} - \frac{(2(bc-ad)(2bc+ad)}{9}$$

$$= \frac{b^2x}{d^2} + \frac{(bc-ad)^2x}{3cd^2(c+dx^3)} - \frac{2(bc-ad)(2bc+ad)\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{9c^{5/3}d^{7/3}} + \frac{((bc-ad)(2bc+ad)}{9}$$

$$= \frac{b^2x}{d^2} + \frac{(bc-ad)^2x}{3cd^2(c+dx^3)} - \frac{2(bc-ad)(2bc+ad)\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{9c^{5/3}d^{7/3}} + \frac{(bc-ad)(2bc+ad)}{}$$

$$= \frac{b^2x}{d^2} + \frac{(bc-ad)^2x}{3cd^2(c+dx^3)} + \frac{2(bc-ad)(2bc+ad)\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}\,x}{\sqrt{3}\sqrt[3]{c}}\right)}{3\sqrt{3}\,c^{5/3}d^{7/3}} - \frac{2(bc-ad)(2bc+}{9c}$$

Mathematica [A]
time = 0.14, size = 210, normalized size = 1.03

$$\frac{9b^2\sqrt[3]{d}\,x + \frac{3\sqrt[3]{d}\,(bc-ad)^2x}{c(c+dx^3)} + \frac{2\sqrt{3}\,(2b^2c^2-abcd-a^2d^2)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\,x}{\sqrt[3]{c}}}{\sqrt{3}}\right)}{c^{5/3}} - \frac{2(2b^2c^2-abcd-a^2d^2)\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{c^{5/3}} + \frac{(2b^2c^2-abcd-a^2d^2)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}\,x+d^{2/3}x^2\right)}{c^{5/3}}}{9d^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^2/(c + d*x^3)^2,x]
```

```
[Out] (9*b^2*d^(1/3)*x + (3*d^(1/3)*(b*c - a*d)^2*x)/(c*(c + d*x^3)) + (2*Sqrt[3]
*(2*b^2*c^2 - a*b*c*d - a^2*d^2)*ArcTan[(1 - (2*d^(1/3)*x)/c^(1/3))/Sqrt[3]
])/c^(5/3) - (2*(2*b^2*c^2 - a*b*c*d - a^2*d^2)*Log[c^(1/3) + d^(1/3)*x])/c
^(5/3) + ((2*b^2*c^2 - a*b*c*d - a^2*d^2)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x +
 d^(2/3)*x^2])/c^(5/3))/(9*d^(7/3))
```

Maple [A]
time = 0.27, size = 167, normalized size = 0.82

method	result
risch	$\frac{b^{2} x}{d^{2}}+\frac{\left(a^{2} d^{2}-2 a b c d+b^{2} c^{2}\right) x}{3 c\, d^{2} \left(d\, x^{3}+c\right)}+\frac{2 \left(\sum_{\_R =\mathrm{RootOf}\left(d \_Z^{3}+c \right)}\frac{\left(a^{2} d^{2}+a b c d-2 b^{2} c^{2}\right) \ln \left(x -\_R \right)}{\_R^{2}}\right)}{9 d^{3} c}$
default	$\frac{b^{2} x}{d^{2}}+\frac{\frac{\left(a^{2} d^{2}-2 a b c d+b^{2} c^{2}\right) x}{3 c \left(d\, x^{3}+c \right)}+\frac{2 \left(a^{2} d^{2}+a b c d-2 b^{2} c^{2}\right) \left(\frac{\ln \left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3 d \left(\frac{c}{d}\right)^{\frac{2}{3}}}-\frac{\ln \left(x^{2}-\left(\frac{c}{d}\right)^{\frac{1}{3}} x+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6 d \left(\frac{c}{d}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\, \arctan \left(\frac{\sqrt{3}\, \left(\frac{2 x}{\left(\frac{c}{d}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3 d \left(\frac{c}{d}\right)^{\frac{2}{3}}}\right)}{3 c}}{d^{2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^2/(d*x^3+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] b^2*x/d^2+1/d^2*(1/3*(a^2*d^2-2*a*b*c*d+b^2*c^2)/c*x/(d*x^3+c)+2/3*(a^2*d^2
+a*b*c*d-2*b^2*c^2)/c*(1/3/d/(c/d)^(2/3)*ln(x+(c/d)^(1/3))-1/6/d/(c/d)^(2/3
)*ln(x^2-(c/d)^(1/3)*x+(c/d)^(2/3))+1/3/d/(c/d)^(2/3)*3^(1/2)*arctan(1/3*3^
(1/2)*(2/(c/d)^(1/3)*x-1))))
```

Maxima [A]
time = 0.49, size = 226, normalized size = 1.11

$$\frac{(b^2c^2-2\,abcd+a^2d^2)x}{3\,(cd^3x^3+c^2d^2)}+\frac{b^2x}{d^2}-\frac{2\,\sqrt{3}\,(2\,b^2c^2-abcd-a^2d^2)\arctan\left(\frac{\sqrt{3}\,\left(2\,x-\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{9\,cd^3\left(\frac{c}{d}\right)^{\frac{2}{3}}}+\frac{(2\,b^2c^2-abcd-a^2d^2)\log\left(x^2-x\left(\frac{c}{d}\right)^{\frac{1}{3}}+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{9\,cd^3\left(\frac{c}{d}\right)^{\frac{2}{3}}}-\frac{2\,(2\,b^2c^2-abcd-a^2d^2)\log\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{9\,cd^3\left(\frac{c}{d}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2/(d*x^3+c)^2,x, algorithm="maxima")
```

```
[Out] 1/3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x/(c*d^3*x^3 + c^2*d^2) + b^2*x/d^2 - 2
/9*sqrt(3)*(2*b^2*c^2 - a*b*c*d - a^2*d^2)*arctan(1/3*sqrt(3)*(2*x - (c/d)^
(1/3))/(c/d)^(1/3))/(c*d^3*(c/d)^(2/3)) + 1/9*(2*b^2*c^2 - a*b*c*d - a^2*d^
2)*log(x^2 - x*(c/d)^(1/3) + (c/d)^(2/3))/(c*d^3*(c/d)^(2/3)) - 2/9*(2*b^2*
c^2 - a*b*c*d - a^2*d^2)*log(x + (c/d)^(1/3))/(c*d^3*(c/d)^(2/3))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 365 vs. $2(164) = 328$.
time = 2.98, size = 771, normalized size = 3.80

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2/(d*x^3+c)^2,x, algorithm="fricas")
```

```
[Out] [1/9*(9*b^2*c^3*d^2*x^4 - 3*sqrt(1/3)*(2*b^2*c^4*d - a*b*c^3*d^2 - a^2*c^2*
d^3 + (2*b^2*c^3*d^2 - a*b*c^2*d^3 - a^2*c*d^4)*x^3)*sqrt(-(c^2*d)^(1/3)/d)
*log((2*c*d*x^3 - 3*(c^2*d)^(1/3)*c*x - c^2 + 3*sqrt(1/3)*(2*c*d*x^2 + (c^2
*d)^(2/3)*x - (c^2*d)^(1/3)*c)*sqrt(-(c^2*d)^(1/3)/d))/(d*x^3 + c)) + (2*b^
2*c^3 - a*b*c^2*d - a^2*c*d^2 + (2*b^2*c^2*d - a*b*c*d^2 - a^2*d^3)*x^3)*(c
^2*d)^(2/3)*log(c*d*x^2 - (c^2*d)^(2/3)*x + (c^2*d)^(1/3)*c) - 2*(2*b^2*c^3
 - a*b*c^2*d - a^2*c*d^2 + (2*b^2*c^2*d - a*b*c*d^2 - a^2*d^3)*x^3)*(c^2*d)
^(2/3)*log(c*d*x + (c^2*d)^(2/3)) + 3*(4*b^2*c^4*d - 2*a*b*c^3*d^2 + a^2*c^
2*d^3)*x)/(c^3*d^4*x^3 + c^4*d^3), 1/9*(9*b^2*c^3*d^2*x^4 - 6*sqrt(1/3)*(2*
b^2*c^4*d - a*b*c^3*d^2 - a^2*c^2*d^3 + (2*b^2*c^3*d^2 - a*b*c^2*d^3 - a^2*
c*d^4)*x^3)*sqrt((c^2*d)^(1/3)/d)*arctan(sqrt(1/3)*(2*(c^2*d)^(2/3)*x - (c^
2*d)^(1/3)*c)*sqrt((c^2*d)^(1/3)/d)/c^2) + (2*b^2*c^3 - a*b*c^2*d - a^2*c*d
^2 + (2*b^2*c^2*d - a*b*c*d^2 - a^2*d^3)*x^3)*(c^2*d)^(2/3)*log(c*d*x^2 - (
c^2*d)^(2/3)*x + (c^2*d)^(1/3)*c) - 2*(2*b^2*c^3 - a*b*c^2*d - a^2*c*d^2 +
(2*b^2*c^2*d - a*b*c*d^2 - a^2*d^3)*x^3)*(c^2*d)^(2/3)*log(c*d*x + (c^2*d)^
(2/3)) + 3*(4*b^2*c^4*d - 2*a*b*c^3*d^2 + a^2*c^2*d^3)*x)/(c^3*d^4*x^3 + c^
4*d^3)]
```

Sympy [A]
time = 0.79, size = 189, normalized size = 0.93

$$\frac{b^2x}{d^2} + \frac{x(a^2d^2 - 2abcd + b^2c^2)}{3c^2d^2 + 3cd^3x^3} + \operatorname{RootSum}\left(729t^3c^5d^7 - 8a^6d^6 - 24a^5bcd^5 + 24a^4b^2c^2d^4 + 88a^3b^3c^3d^3 - 48a^2b^4c^4d^2 - 96ab^5c^5d + 64b^6c^6, \left(t \mapsto t\log\left(\frac{9tc^2d^2}{2a^2d^2 + 2abcd - 4b^2c^2} + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**2/(d*x**3+c)**2,x)
```

```
[Out] b**2*x/d**2 + x*(a**2*d**2 - 2*a*b*c*d + b**2*c**2)/(3*c**2*d**2 + 3*c*d**3
*x**3) + RootSum(729*_t**3*c**5*d**7 - 8*a**6*d**6 - 24*a**5*b*c*d**5 + 24*
a**4*b**2*c**2*d**4 + 88*a**3*b**3*c**3*d**3 - 48*a**2*b**4*c**4*d**2 - 96*
a*b**5*c**5*d + 64*b**6*c**6, Lambda(_t, _t*log(9*_t*c**2*d**2/(2*a**2*d**2
 + 2*a*b*c*d - 4*b**2*c**2) + x)))
```

Giac [A]
time = 0.71, size = 233, normalized size = 1.15

$$\frac{b^2x}{d^2} + \frac{2\sqrt{3}\,(2\,b^2c^2 - abcd - a^2d^2)\arctan\left(\frac{\sqrt{3}\left(2\,x + \left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{9\,(-cd^2)^{\frac{2}{3}}\,cd} + \frac{(2\,b^2c^2 - abcd - a^2d^2)\log\left(x^2 + x\left(-\frac{c}{d}\right)^{\frac{1}{3}} + \left(-\frac{c}{d}\right)^{\frac{2}{3}}\right)}{9\,(-cd^2)^{\frac{2}{3}}\,cd} + \frac{2\,(2\,b^2c^2 - abcd - a^2d^2)\left(-\frac{c}{d}\right)^{\frac{1}{3}}\log\left(\left|x - \left(-\frac{c}{d}\right)^{\frac{1}{3}}\right|\right)}{9\,c^2d^2} + \frac{b^2c^2x - 2\,abcdx + a^2d^2x}{3\,(dx^3 + c)cd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2/(d*x^3+c)^2,x, algorithm="giac")
```

```
[Out] b^2*x/d^2 + 2/9*sqrt(3)*(2*b^2*c^2 - a*b*c*d - a^2*d^2)*arctan(1/3*sqrt(3)*
(2*x + (-c/d)^(1/3))/(-c/d)^(1/3))/((-c*d^2)^(2/3)*c*d) + 1/9*(2*b^2*c^2 -
```

```
a*b*c*d - a^2*d^2)*log(x^2 + x*(-c/d)^(1/3) + (-c/d)^(2/3))/((-c*d^2)^(2/3)
*c*d) + 2/9*(2*b^2*c^2 - a*b*c*d - a^2*d^2)*(-c/d)^(1/3)*log(abs(x - (-c/d)
^(1/3)))/(c^2*d^2) + 1/3*(b^2*c^2*x - 2*a*b*c*d*x + a^2*d^2*x)/((d*x^3 + c)
*c*d^2)
```

Mupad [B]
time = 1.41, size = 191, normalized size = 0.94

$$\frac{b^2\,x}{d^2}+\frac{x\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}{3\,c\,\left(d^3\,x^3+c\,d^2\right)}+\frac{2\,\ln\left(d^{1/3}\,x+c^{1/3}\right)\,\left(a\,d-b\,c\right)\,\left(a\,d+2\,b\,c\right)}{9\,c^{5/3}\,d^{7/3}}+\frac{2\,\ln\left(2\,d^{1/3}\,x-c^{1/3}+\sqrt{3}\,c^{1/3}\,1{}\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(a\,d-b\,c\right)\,\left(a\,d+2\,b\,c\right)}{9\,c^{5/3}\,d^{7/3}}-\frac{2\,\ln\left(c^{1/3}-2\,d^{1/3}\,x+\sqrt{3}\,c^{1/3}\,1{}\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(a\,d-b\,c\right)\,\left(a\,d+2\,b\,c\right)}{9\,c^{5/3}\,d^{7/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^2/(c + d*x^3)^2,x)
```

```
[Out] (b^2*x)/d^2 + (x*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))/(3*c*(c*d^2 + d^3*x^3)) +
 (2*log(d^(1/3)*x + c^(1/3))*(a*d - b*c)*(a*d + 2*b*c))/(9*c^(5/3)*d^(7/3))
 + (2*log(3^(1/2)*c^(1/3)*1i + 2*d^(1/3)*x - c^(1/3))*((3^(1/2)*1i)/2 - 1/2
)*(a*d - b*c)*(a*d + 2*b*c))/(9*c^(5/3)*d^(7/3)) - (2*log(3^(1/2)*c^(1/3)*1
i - 2*d^(1/3)*x + c^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(a*d - b*c)*(a*d + 2*b*c)
)/(9*c^(5/3)*d^(7/3))
```

3.13 $\int \frac{(a+bx^3)^2}{(c+dx^3)^3} \, dx$

Optimal. Leaf size=258

$$-\frac{(bc-ad)x(a+bx^3)}{6cd\,(c+dx^3)^2}-\frac{(bc-ad)(4bc+5ad)x}{18c^2d^2\,(c+dx^3)}-\frac{(2b^2c^2+2abcd+5a^2d^2)\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}\,x}{\sqrt{3}\,\sqrt[3]{c}}\right)}{9\sqrt{3}\,c^{8/3}d^{7/3}}+\frac{(2b^2c^2+2a$$

```
[Out] -1/6*(-a*d+b*c)*x*(b*x^3+a)/c/d/(d*x^3+c)^2-1/18*(-a*d+b*c)*(5*a*d+4*b*c)*x
/c^2/d^2/(d*x^3+c)+1/27*(5*a^2*d^2+2*a*b*c*d+2*b^2*c^2)*ln(c^(1/3)+d^(1/3)*
x)/c^(8/3)/d^(7/3)-1/54*(5*a^2*d^2+2*a*b*c*d+2*b^2*c^2)*ln(c^(2/3)-c^(1/3)*
d^(1/3)*x+d^(2/3)*x^2)/c^(8/3)/d^(7/3)-1/27*(5*a^2*d^2+2*a*b*c*d+2*b^2*c^2)
*arctan(1/3*(c^(1/3)-2*d^(1/3)*x)/c^(1/3)*3^(1/2))/c^(8/3)/d^(7/3)*3^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 258, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {424, 393, 206, 31, 648, 631, 210, 642}

$$-\frac{(5a^2d^2+2abcd+2b^2c^2)\,\text{ArcTan}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}\,x}{\sqrt{3}\,\sqrt[3]{c}}\right)}{9\sqrt{3}\,c^{8/3}d^{7/3}}-\frac{(5a^2d^2+2abcd+2b^2c^2)\log\left(c^{2/3}-\sqrt[3]{c}\,\sqrt[3]{d}\,x+d^{2/3}x^2\right)}{54c^{8/3}d^{7/3}}+\frac{(5a^2d^2+2abcd+2b^2c^2)\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{27c^{8/3}d^{7/3}}-\frac{x(bc-ad)(5ad+4bc)}{18c^2d^2\,(c+dx^3)}-\frac{x(a+bx^3)\,(bc-ad)}{6cd\,(c+dx^3)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^2/(c + d*x^3)^3,x]
```

```
[Out] -1/6*((b*c - a*d)*x*(a + b*x^3))/(c*d*(c + d*x^3)^2) - ((b*c - a*d)*(4*b*c
+ 5*a*d)*x)/(18*c^2*d^2*(c + d*x^3)) - ((2*b^2*c^2 + 2*a*b*c*d + 5*a^2*d^2)
*ArcTan[(c^(1/3) - 2*d^(1/3)*x)/(Sqrt[3]*c^(1/3))])/(9*Sqrt[3]*c^(8/3)*d^(7
/3)) + ((2*b^2*c^2 + 2*a*b*c*d + 5*a^2*d^2)*Log[c^(1/3) + d^(1/3)*x])/(27*c
^(8/3)*d^(7/3)) - ((2*b^2*c^2 + 2*a*b*c*d + 5*a^2*d^2)*Log[c^(2/3) - c^(1/3
)*d^(1/3)*x + d^(2/3)*x^2])/(54*c^(8/3)*d^(7/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{(a+bx^3)^2}{(c+dx^3)^3}\,dx = -\frac{(bc-ad)x(a+bx^3)}{6cd\,(c+dx^3)^2} + \frac{\int \frac{a(bc+5ad)+2b(2bc+ad)x^3}{(c+dx^3)^2}\,dx}{6cd}$$

$$= -\frac{(bc-ad)x(a+bx^3)}{6cd\,(c+dx^3)^2} - \frac{(bc-ad)(4bc+5ad)x}{18c^2d^2\,(c+dx^3)} + \frac{(2b^2c^2+2abcd+5a^2d^2)\int \frac{1}{c+dx^3}\,dx}{9c^2d^2}$$

$$= -\frac{(bc-ad)x(a+bx^3)}{6cd\,(c+dx^3)^2} - \frac{(bc-ad)(4bc+5ad)x}{18c^2d^2\,(c+dx^3)} + \frac{(2b^2c^2+2abcd+5a^2d^2)\int \frac{1}{\sqrt[3]{c}+\sqrt[3]{d}\,x}}{27c^{8/3}d^2}$$

$$= -\frac{(bc-ad)x(a+bx^3)}{6cd\,(c+dx^3)^2} - \frac{(bc-ad)(4bc+5ad)x}{18c^2d^2\,(c+dx^3)} + \frac{(2b^2c^2+2abcd+5a^2d^2)\log\left(\sqrt[3]{c}+\right.}{27c^{8/3}d^{7/3}}$$

$$= -\frac{(bc-ad)x(a+bx^3)}{6cd\,(c+dx^3)^2} - \frac{(bc-ad)(4bc+5ad)x}{18c^2d^2\,(c+dx^3)} + \frac{(2b^2c^2+2abcd+5a^2d^2)\log\left(\sqrt[3]{c}+\right.}{27c^{8/3}d^{7/3}}$$

$$= -\frac{(bc-ad)x(a+bx^3)}{6cd\,(c+dx^3)^2} - \frac{(bc-ad)(4bc+5ad)x}{18c^2d^2\,(c+dx^3)} - \frac{(2b^2c^2+2abcd+5a^2d^2)\tan^{-1}\left(\frac{\sqrt[3]{c}}{\sqrt{3}}\right.}{9\sqrt{3}\,c^{8/3}d^{7/3}}$$

Mathematica [**A**]
time = 0.20, size = 234, normalized size = 0.91

$$\frac{-\frac{3c^{2/3}\sqrt[3]{d}\,x(2abcd(2c-dx^3)-a^2d^2(8c+5dx^3)+b^2c^2(4c+7dx^3))}{(c+dx^3)^2} - 2\sqrt{3}\,(2b^2c^2+2abcd+5a^2d^2)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\,x}{\sqrt[3]{c}}}{\sqrt{3}}\right) + 2(2b^2c^2+2abcd+5a^2d^2)\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right) - (2b^2c^2+2abcd+5a^2d^2)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}\,x+d^{2/3}x^2\right)}{54c^{8/3}d^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^2/(c + d*x^3)^3,x]
```

```
[Out] ((-3*c^(2/3)*d^(1/3)*x*(2*a*b*c*d*(2*c - d*x^3) - a^2*d^2*(8*c + 5*d*x^3) +
 b^2*c^2*(4*c + 7*d*x^3)))/(c + d*x^3)^2 - 2*Sqrt[3]*(2*b^2*c^2 + 2*a*b*c*d
 + 5*a^2*d^2)*ArcTan[(1 - (2*d^(1/3)*x)/c^(1/3))/Sqrt[3]] + 2*(2*b^2*c^2 +
2*a*b*c*d + 5*a^2*d^2)*Log[c^(1/3) + d^(1/3)*x] - (2*b^2*c^2 + 2*a*b*c*d +
5*a^2*d^2)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/(54*c^(8/3)*d^(7
/3))
```

Maple [**A**]
time = 0.27, size = 200, normalized size = 0.78

method	result

risch	$\frac{\frac{(5a^2d^2+2abcd-7b^2c^2)x^4}{18c^2d}+\frac{2(2a^2d^2-abcd-b^2c^2)x}{9d^2c}}{(d\,x^3+c)^2}+\frac{\sum_{\_R=\mathrm{RootOf}(d\_Z^3+c)}\frac{(5a^2d^2+2abcd+2b^2c^2)\ln(x-\_R)}{\_R^2}}{27c^2d^3}$
default	$\frac{\frac{(5a^2d^2+2abcd-7b^2c^2)x^4}{18c^2d}+\frac{2(2a^2d^2-abcd-b^2c^2)x}{9d^2c}}{(d\,x^3+c)^2}+\frac{(5a^2d^2+2abcd+2b^2c^2)\left(\frac{\ln\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3d\left(\frac{c}{d}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{c}{d}\right)^{\frac{1}{3}}x+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6d\left(\frac{c}{d}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan(\ldots}{3\ldots}\right.}{9c^2d^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^2/(d*x^3+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] (1/18*(5*a^2*d^2+2*a*b*c*d-7*b^2*c^2)/c^2/d*x^4+2/9*(2*a^2*d^2-a*b*c*d-b^2*
c^2)/d^2/c*x)/(d*x^3+c)^2+1/9*(5*a^2*d^2+2*a*b*c*d+2*b^2*c^2)/c^2/d^2*(1/3/
d/(c/d)^(2/3)*ln(x+(c/d)^(1/3))-1/6/d/(c/d)^(2/3)*ln(x^2-(c/d)^(1/3)*x+(c/d
)^(2/3))+1/3/d/(c/d)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(c/d)^(1/3)*x-1)))
```

Maxima [A]
time = 0.52, size = 267, normalized size = 1.03

$$-\frac{(7\,b^2c^2d-2\,abcd^2-5\,a^2d^3)x^4+4\,(b^2c^3+abc^2d-2\,a^2cd^2)x}{18\,(c^2d^4x^6+2\,c^3d^3x^3+c^4d^2)}+\frac{\sqrt{3}\,(2\,b^2c^2+2\,abcd+5\,a^2d^2)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{27\,c^2d^3\left(\frac{c}{d}\right)^{\frac{2}{3}}}-\frac{(2\,b^2c^2+2\,abcd+5\,a^2d^2)\log\left(x^2-x\left(\frac{c}{d}\right)^{\frac{1}{3}}+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{54\,c^2d^3\left(\frac{c}{d}\right)^{\frac{2}{3}}}+\frac{(2\,b^2c^2+2\,abcd+5\,a^2d^2)\log\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{27\,c^2d^3\left(\frac{c}{d}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2/(d*x^3+c)^3,x, algorithm="maxima")
```

```
[Out] -1/18*((7*b^2*c^2*d - 2*a*b*c*d^2 - 5*a^2*d^3)*x^4 + 4*(b^2*c^3 + a*b*c^2*d
 - 2*a^2*c*d^2)*x)/(c^2*d^4*x^6 + 2*c^3*d^3*x^3 + c^4*d^2) + 1/27*sqrt(3)*(
2*b^2*c^2 + 2*a*b*c*d + 5*a^2*d^2)*arctan(1/3*sqrt(3)*(2*x - (c/d)^(1/3))/(
c/d)^(1/3))/(c^2*d^3*(c/d)^(2/3)) - 1/54*(2*b^2*c^2 + 2*a*b*c*d + 5*a^2*d^2
)*log(x^2 - x*(c/d)^(1/3) + (c/d)^(2/3))/(c^2*d^3*(c/d)^(2/3)) + 1/27*(2*b^
2*c^2 + 2*a*b*c*d + 5*a^2*d^2)*log(x + (c/d)^(1/3))/(c^2*d^3*(c/d)^(2/3))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 513 vs. $2(217) = 434$.
time = 3.45, size = 1067, normalized size = 4.14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2/(d*x^3+c)^3,x, algorithm="fricas")
```

```
[Out] [-1/54*(3*(7*b^2*c^4*d^2 - 2*a*b*c^3*d^3 - 5*a^2*c^2*d^4)*x^4 - 3*sqrt(1/3)
*(2*b^2*c^5*d + 2*a*b*c^4*d^2 + 5*a^2*c^3*d^3 + (2*b^2*c^3*d^3 + 2*a*b*c^2*
d^4 + 5*a^2*c*d^5)*x^6 + 2*(2*b^2*c^4*d^2 + 2*a*b*c^3*d^3 + 5*a^2*c^2*d^4)*
x^3)*sqrt(-(c^2*d)^(1/3)/d)*log((2*c*d*x^3 - 3*(c^2*d)^(1/3)*c*x - c^2 + 3*
sqrt(1/3)*(2*c*d*x^2 + (c^2*d)^(2/3)*x - (c^2*d)^(1/3)*c)*sqrt(-(c^2*d)^(1/
3)/d))/(d*x^3 + c)) + ((2*b^2*c^2*d^2 + 2*a*b*c*d^3 + 5*a^2*d^4)*x^6 + 2*b^
2*c^4 + 2*a*b*c^3*d + 5*a^2*c^2*d^2 + 2*(2*b^2*c^3*d + 2*a*b*c^2*d^2 + 5*a^
2*c*d^3)*x^3)*(c^2*d)^(2/3)*log(c*d*x^2 - (c^2*d)^(2/3)*x + (c^2*d)^(1/3)*c
) - 2*((2*b^2*c^2*d^2 + 2*a*b*c*d^3 + 5*a^2*d^4)*x^6 + 2*b^2*c^4 + 2*a*b*c^
3*d + 5*a^2*c^2*d^2 + 2*(2*b^2*c^3*d + 2*a*b*c^2*d^2 + 5*a^2*c*d^3)*x^3)*(c
^2*d)^(2/3)*log(c*d*x + (c^2*d)^(2/3)) + 12*(b^2*c^5*d + a*b*c^4*d^2 - 2*a^
2*c^3*d^3)*x)/(c^4*d^5*x^6 + 2*c^5*d^4*x^3 + c^6*d^3), -1/54*(3*(7*b^2*c^4*
d^2 - 2*a*b*c^3*d^3 - 5*a^2*c^2*d^4)*x^4 - 6*sqrt(1/3)*(2*b^2*c^5*d + 2*a*b
*c^4*d^2 + 5*a^2*c^3*d^3 + (2*b^2*c^3*d^3 + 2*a*b*c^2*d^4 + 5*a^2*c*d^5)*x^
6 + 2*(2*b^2*c^4*d^2 + 2*a*b*c^3*d^3 + 5*a^2*c^2*d^4)*x^3)*sqrt((c^2*d)^(1/
3)/d)*arctan(sqrt(1/3)*(2*(c^2*d)^(2/3)*x - (c^2*d)^(1/3)*c)*sqrt((c^2*d)^(
1/3)/d)/c^2) + ((2*b^2*c^2*d^2 + 2*a*b*c*d^3 + 5*a^2*d^4)*x^6 + 2*b^2*c^4 +
 2*a*b*c^3*d + 5*a^2*c^2*d^2 + 2*(2*b^2*c^3*d + 2*a*b*c^2*d^2 + 5*a^2*c*d^3
)*x^3)*(c^2*d)^(2/3)*log(c*d*x^2 - (c^2*d)^(2/3)*x + (c^2*d)^(1/3)*c) - 2*(
(2*b^2*c^2*d^2 + 2*a*b*c*d^3 + 5*a^2*d^4)*x^6 + 2*b^2*c^4 + 2*a*b*c^3*d + 5
*a^2*c^2*d^2 + 2*(2*b^2*c^3*d + 2*a*b*c^2*d^2 + 5*a^2*c*d^3)*x^3)*(c^2*d)^(
2/3)*log(c*d*x + (c^2*d)^(2/3)) + 12*(b^2*c^5*d + a*b*c^4*d^2 - 2*a^2*c^3*d
^3)*x)/(c^4*d^5*x^6 + 2*c^5*d^4*x^3 + c^6*d^3)]
```

Sympy [A]
time = 1.18, size = 233, normalized size = 0.90

$$\frac{x^4\cdot\left(5a^2d^3+2abcd^2-7b^2c^2d\right)+x\left(8a^2cd^2-4abc^2d-4b^2c^3\right)}{18c^4d^2+36c^3d^3x^3+18c^2d^4x^6}+\operatorname{RootSum}\left(19683t^3c^8d^7-125a^6d^6-150a^5bcd^5-210a^4b^2c^2d^4-128a^3b^3c^3d^3-84a^2b^4c^4d^2-24ab^5c^5d-8b^6c^6,\left(t\mapsto t\log\left(\frac{27tc^3d^2}{5a^2d^2+2abcd+2b^2c^2}+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**2/(d*x**3+c)**3,x)
```

```
[Out] (x**4*(5*a**2*d**3 + 2*a*b*c*d**2 - 7*b**2*c**2*d) + x*(8*a**2*c*d**2 - 4*a
*b*c**2*d - 4*b**2*c**3))/(18*c**4*d**2 + 36*c**3*d**3*x**3 + 18*c**2*d**4*
x**6) + RootSum(19683*_t**3*c**8*d**7 - 125*a**6*d**6 - 150*a**5*b*c*d**5 -
 210*a**4*b**2*c**2*d**4 - 128*a**3*b**3*c**3*d**3 - 84*a**2*b**4*c**4*d**2
 - 24*a*b**5*c**5*d - 8*b**6*c**6, Lambda(_t, _t*log(27*_t*c**3*d**2/(5*a**
2*d**2 + 2*a*b*c*d + 2*b**2*c**2) + x)))
```

Giac [A]
time = 0.69, size = 264, normalized size = 1.02

$$-\frac{\sqrt{3}\,(2\,b^2c^2+2\,abcd+5\,a^2d^2)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{27\,(-cd^2)^{\frac{2}{3}}\,c^2d}-\frac{(2\,b^2c^2+2\,abcd+5\,a^2d^2)\log\left(x^2+x\left(-\frac{c}{d}\right)^{\frac{1}{3}}+\left(-\frac{c}{d}\right)^{\frac{2}{3}}\right)}{54\,(-cd^2)^{\frac{2}{3}}\,c^2d}-\frac{(2\,b^2c^2+2\,abcd+5\,a^2d^2)\left(-\frac{c}{d}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right|\right)}{27\,c^3d^2}-\frac{7\,b^2c^2dx^4-2\,abcd^2x^4-5\,a^2d^3x^4+4\,b^2c^3x+4\,abc^2dx-8\,a^2cd^2x}{18\,(dx^3+c)^2c^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2/(d*x^3+c)^3,x, algorithm="giac")
```

```
[Out] -1/27*sqrt(3)*(2*b^2*c^2 + 2*a*b*c*d + 5*a^2*d^2)*arctan(1/3*sqrt(3)*(2*x +
 (-c/d)^(1/3))/(-c/d)^(1/3))/((-c*d^2)^(2/3)*c^2*d) - 1/54*(2*b^2*c^2 + 2*a
*b*c*d + 5*a^2*d^2)*log(x^2 + x*(-c/d)^(1/3) + (-c/d)^(2/3))/((-c*d^2)^(2/3
)*c^2*d) - 1/27*(2*b^2*c^2 + 2*a*b*c*d + 5*a^2*d^2)*(-c/d)^(1/3)*log(abs(x
- (-c/d)^(1/3)))/(c^3*d^2) - 1/18*(7*b^2*c^2*d*x^4 - 2*a*b*c*d^2*x^4 - 5*a^
2*d^3*x^4 + 4*b^2*c^3*x + 4*a*b*c^2*d*x - 8*a^2*c*d^2*x)/((d*x^3 + c)^2*c^2
*d^2)
```

Mupad [B]

time = 1.43, size = 249, normalized size = 0.97

$$\frac{\ln\left(d^{1/3}\,x+c^{1/3}\right)\,\left(5\,a^2\,d^2+2\,a\,b\,c\,d+2\,b^2\,c^2\right)}{27\,c^{8/3}\,d^{7/3}}-\frac{\frac{2\,x\,\left(-2\,a^2\,d^2+a\,b\,c\,d+b^2\,c^2\right)}{9\,c\,d^2}-\frac{x^4\,\left(5\,a^2\,d^2+2\,a\,b\,c\,d-7\,b^2\,c^2\right)}{18\,c^2\,d}}{c^2+2\,c\,d\,x^3+d^2\,x^6}+\frac{\ln\left(2\,d^{1/3}\,x-c^{1/3}+\sqrt{3}\,c^{1/3}\,1{}\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(5\,a^2\,d^2+2\,a\,b\,c\,d+2\,b^2\,c^2\right)}{27\,c^{8/3}\,d^{7/3}}-\frac{\ln\left(c^{1/3}-2\,d^{1/3}\,x+\sqrt{3}\,c^{1/3}\,1{}\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(5\,a^2\,d^2+2\,a\,b\,c\,d+2\,b^2\,c^2\right)}{27\,c^{8/3}\,d^{7/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^2/(c + d*x^3)^3,x)
```

```
[Out] (log(d^(1/3)*x + c^(1/3))*(5*a^2*d^2 + 2*b^2*c^2 + 2*a*b*c*d))/(27*c^(8/3)*
d^(7/3)) - ((2*x*(b^2*c^2 - 2*a^2*d^2 + a*b*c*d))/(9*c*d^2) - (x^4*(5*a^2*d
^2 - 7*b^2*c^2 + 2*a*b*c*d))/(18*c^2*d))/(c^2 + d^2*x^6 + 2*c*d*x^3) + (log
(3^(1/2)*c^(1/3)*1i + 2*d^(1/3)*x - c^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(5*a^2*
d^2 + 2*b^2*c^2 + 2*a*b*c*d))/(27*c^(8/3)*d^(7/3)) - (log(3^(1/2)*c^(1/3)*1
i - 2*d^(1/3)*x + c^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(5*a^2*d^2 + 2*b^2*c^2 +
2*a*b*c*d))/(27*c^(8/3)*d^(7/3))
```

3.14 $\int \frac{(c+dx^3)^4}{a+bx^3}\,dx$

Optimal. Leaf size=252

$$\frac{d(2bc-ad)\left(2b^2c^2-2abcd+a^2d^2\right)x}{b^4}+\frac{d^2\left(6b^2c^2-4abcd+a^2d^2\right)x^4}{4b^3}+\frac{d^3(4bc-ad)x^7}{7b^2}+\frac{d^4x^{10}}{10b}-\frac{(bc-ad)^4\tan}{\sqrt{3}}$$

```
[Out] d*(-a*d+2*b*c)*(a^2*d^2-2*a*b*c*d+2*b^2*c^2)*x/b^4+1/4*d^2*(a^2*d^2-4*a*b*c
*d+6*b^2*c^2)*x^4/b^3+1/7*d^3*(-a*d+4*b*c)*x^7/b^2+1/10*d^4*x^10/b+1/3*(-a*
d+b*c)^4*ln(a^(1/3)+b^(1/3)*x)/a^(2/3)/b^(13/3)-1/6*(-a*d+b*c)^4*ln(a^(2/3)
-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(2/3)/b^(13/3)-1/3*(-a*d+b*c)^4*arctan(1/
3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(2/3)/b^(13/3)*3^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 252, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {398, 206, 31, 648, 631, 210, 642}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)(bc-ad)^4}{\sqrt{3}\,a^{2/3}b^{13/3}}-\frac{(bc-ad)^4\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6a^{2/3}b^{13/3}}+\frac{(bc-ad)^4\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{2/3}b^{13/3}}+\frac{dx(2bc-ad)\left(a^2d^2-2abcd+2b^2c^2\right)}{b^4}+\frac{d^2x^4\left(a^2d^2-4abcd+6b^2c^2\right)}{4b^3}+\frac{d^3x^7(4bc-ad)}{7b^2}+\frac{d^4x^{10}}{10b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)^4/(a + b*x^3),x]
```

```
[Out] (d*(2*b*c - a*d)*(2*b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x)/b^4 + (d^2*(6*b^2*c^2
 - 4*a*b*c*d + a^2*d^2)*x^4)/(4*b^3) + (d^3*(4*b*c - a*d)*x^7)/(7*b^2) + (d
^4*x^10)/(10*b) - ((b*c - a*d)^4*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^
(1/3))])/(Sqrt[3]*a^(2/3)*b^(13/3)) + ((b*c - a*d)^4*Log[a^(1/3) + b^(1/3)*
x])/(3*a^(2/3)*b^(13/3)) - ((b*c - a*d)^4*Log[a^(2/3) - a^(1/3)*b^(1/3)*x +
 b^(2/3)*x^2])/(6*a^(2/3)*b^(13/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{(c+dx^3)^4}{a+bx^3}\,dx = \int \left(\frac{d(2bc-ad)\left(2b^2c^2-2abcd+a^2d^2\right)}{b^4} + \frac{d^2\left(6b^2c^2-4abcd+a^2d^2\right)x^3}{b^3} + \frac{d^3(4bc-ad)x}{b^2}\right.$$

$$= \frac{d(2bc-ad)\left(2b^2c^2-2abcd+a^2d^2\right)x}{b^4} + \frac{d^2\left(6b^2c^2-4abcd+a^2d^2\right)x^4}{4b^3} + \frac{d^3(4bc-ad)x^7}{7b^2}$$

$$= \frac{d(2bc-ad)\left(2b^2c^2-2abcd+a^2d^2\right)x}{b^4} + \frac{d^2\left(6b^2c^2-4abcd+a^2d^2\right)x^4}{4b^3} + \frac{d^3(4bc-ad)x^7}{7b^2}$$

$$= \frac{d(2bc-ad)\left(2b^2c^2-2abcd+a^2d^2\right)x}{b^4} + \frac{d^2\left(6b^2c^2-4abcd+a^2d^2\right)x^4}{4b^3} + \frac{d^3(4bc-ad)x^7}{7b^2}$$

$$= \frac{d(2bc-ad)\left(2b^2c^2-2abcd+a^2d^2\right)x}{b^4} + \frac{d^2\left(6b^2c^2-4abcd+a^2d^2\right)x^4}{4b^3} + \frac{d^3(4bc-ad)x^7}{7b^2}$$

$$= \frac{d(2bc-ad)\left(2b^2c^2-2abcd+a^2d^2\right)x}{b^4} + \frac{d^2\left(6b^2c^2-4abcd+a^2d^2\right)x^4}{4b^3} + \frac{d^3(4bc-ad)x^7}{7b^2}$$

Mathematica [A]
time = 0.09, size = 253, normalized size = 1.00

$$\frac{420\sqrt[3]{b}\,d\left(4b^3c^3-6ab^2c^2d+4a^2bcd^2-a^3d^3\right)x+105b^{4/3}d^2\left(6b^2c^2-4abcd+a^2d^2\right)x^4+60b^{7/3}d^3(4bc-ad)x^7+42b^{10/3}d^4x^{10}+\frac{140\sqrt{3}(bc-ad)^4\tan^{-1}\left(\frac{-\sqrt[3]{a}+2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{a^{2/3}}+\frac{140(bc-ad)^4\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{a^{2/3}}-\frac{70(bc-ad)^4\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{a^{2/3}}}{420b^{13/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3)^4/(a + b*x^3),x]
```

```
[Out] (420*b^(1/3)*d*(4*b^3*c^3 - 6*a*b^2*c^2*d + 4*a^2*b*c*d^2 - a^3*d^3)*x + 10
5*b^(4/3)*d^2*(6*b^2*c^2 - 4*a*b*c*d + a^2*d^2)*x^4 + 60*b^(7/3)*d^3*(4*b*c
 - a*d)*x^7 + 42*b^(10/3)*d^4*x^10 + (140*Sqrt[3]*(b*c - a*d)^4*ArcTan[(-a^
(1/3) + 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/a^(2/3) + (140*(b*c - a*d)^4*Log[a
^(1/3) + b^(1/3)*x])/a^(2/3) - (70*(b*c - a*d)^4*Log[a^(2/3) - a^(1/3)*b^(1
/3)*x + b^(2/3)*x^2])/a^(2/3))/(420*b^(13/3))
```

Maple [A]
time = 0.33, size = 272, normalized size = 1.08

method	result
risch	$\frac{d^4x^{10}}{10b} - \frac{d^4x^7a}{7b^2} + \frac{4d^3x^7c}{7b} - \frac{d^3x^4ac}{b^2} + \frac{3d^2x^4c^2}{2b} + \frac{d^4x^4a^2}{4b^3} - \frac{d^4a^3x}{b^4} + \frac{4d^3a^2cx}{b^3} - \frac{6d^2ac^2x}{b^2} + \frac{4dc^3x}{b} + \frac{\sum_{\_R=\mathrm{RootOf}(b}}{}$

default	$-\frac{d\left(-\frac{d^{3}x^{10}b^{3}}{10}+\frac{\left((ad-2bc)b^{2}d^{2}-2b^{3}c\,d^{2}\right)x^{7}}{7}+\frac{\left(2(ad-2bc)b^{2}cd-bd\left(a^{2}d^{2}-2abcd+2b^{2}c^{2}\right)\right)x^{4}}{4}+(ad-2bc)\left(a^{2}d^{2}-2abcd+2b^{2}c^{2}\right)x\right)}{b^{4}}+\ldots$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^4/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] -d/b^4*(-1/10*d^3*x^10*b^3+1/7*((a*d-2*b*c)*b^2*d^2-2*b^3*c*d^2)*x^7+1/4*(2
*(a*d-2*b*c)*b^2*c*d-b*d*(a^2*d^2-2*a*b*c*d+2*b^2*c^2))*x^4+(a*d-2*b*c)*(a^
2*d^2-2*a*b*c*d+2*b^2*c^2)*x)+(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a
/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arcta
n(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))*(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^
2-4*a*b^3*c^3*d+b^4*c^4)/b^4
```

Maxima [A]
time = 0.57, size = 364, normalized size = 1.44

$$\frac{14\,b^{3}d^{4}x^{10}+20\,(4\,b^{3}cd^{3}-ab^{2}d^{4})x^{7}+35\,(6\,b^{3}c^{2}d^{2}-4\,ab^{2}cd^{3}+a^{2}bd^{4})x^{4}+140\,(4\,b^{3}c^{3}d-6\,ab^{2}c^{2}d^{2}+4\,a^{2}bcd^{3}-a^{3}d^{4})x}{140\,b^{4}}+\frac{\sqrt{3}\,(b^{4}c^{4}-4\,ab^{3}c^{3}d+6\,a^{2}b^{2}c^{2}d^{2}-4\,a^{3}bcd^{3}+a^{4}d^{4})\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b^{5}\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{(b^{4}c^{4}-4\,ab^{3}c^{3}d+6\,a^{2}b^{2}c^{2}d^{2}-4\,a^{3}bcd^{3}+a^{4}d^{4})\log\left(x^{2}-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b^{5}\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{(b^{4}c^{4}-4\,ab^{3}c^{3}d+6\,a^{2}b^{2}c^{2}d^{2}-4\,a^{3}bcd^{3}+a^{4}d^{4})\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b^{5}\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^4/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/140*(14*b^3*d^4*x^10 + 20*(4*b^3*c*d^3 - a*b^2*d^4)*x^7 + 35*(6*b^3*c^2*d
^2 - 4*a*b^2*c*d^3 + a^2*b*d^4)*x^4 + 140*(4*b^3*c^3*d - 6*a*b^2*c^2*d^2 +
4*a^2*b*c*d^3 - a^3*d^4)*x)/b^4 + 1/3*sqrt(3)*(b^4*c^4 - 4*a*b^3*c^3*d + 6*
a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*arctan(1/3*sqrt(3)*(2*x - (a/b)^
(1/3))/(a/b)^(1/3))/(b^5*(a/b)^(2/3)) - 1/6*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^
2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2
/3))/(b^5*(a/b)^(2/3)) + 1/3*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 -
 4*a^3*b*c*d^3 + a^4*d^4)*log(x + (a/b)^(1/3))/(b^5*(a/b)^(2/3))
```

Fricas [A]
time = 2.88, size = 873, normalized size = 3.46

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^4/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] [1/420*(42*a^2*b^4*d^4*x^10 + 60*(4*a^2*b^4*c*d^3 - a^3*b^3*d^4)*x^7 + 105*
(6*a^2*b^4*c^2*d^2 - 4*a^3*b^3*c*d^3 + a^4*b^2*d^4)*x^4 + 210*sqrt(1/3)*(a*
b^5*c^4 - 4*a^2*b^4*c^3*d + 6*a^3*b^3*c^2*d^2 - 4*a^4*b^2*c*d^3 + a^5*b*d^4
)*sqrt(-(a^2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*(a^2*b)^(1/3)*a*x - a^2 + 3*sqr
t(1/3)*(2*a*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt(-(a^2*b)^(1/3)/
b))/(b*x^3 + a)) - 70*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*
b*c*d^3 + a^4*d^4)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1
/3)*a) + 140*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 +
 a^4*d^4)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)) + 420*(4*a^2*b^4*c^3*d -
 6*a^3*b^3*c^2*d^2 + 4*a^4*b^2*c*d^3 - a^5*b*d^4)*x)/(a^2*b^5), 1/420*(42*a
^2*b^4*d^4*x^10 + 60*(4*a^2*b^4*c*d^3 - a^3*b^3*d^4)*x^7 + 105*(6*a^2*b^4*c
^2*d^2 - 4*a^3*b^3*c*d^3 + a^4*b^2*d^4)*x^4 + 420*sqrt(1/3)*(a*b^5*c^4 - 4*
a^2*b^4*c^3*d + 6*a^3*b^3*c^2*d^2 - 4*a^4*b^2*c*d^3 + a^5*b*d^4)*sqrt((a^2*
b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt((a^
2*b)^(1/3)/b)/a^2) - 70*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^
3*b*c*d^3 + a^4*d^4)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^
(1/3)*a) + 140*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3
 + a^4*d^4)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)) + 420*(4*a^2*b^4*c^3*d
 - 6*a^3*b^3*c^2*d^2 + 4*a^4*b^2*c*d^3 - a^5*b*d^4)*x)/(a^2*b^5)]
```

Sympy [**A**]
time = 0.97, size = 371, normalized size = 1.47

$$x^7\left(-\frac{ad^4}{7b^2}+\frac{4cd^3}{7b}\right)+x^4\left(\frac{a^2d^4}{4b^3}-\frac{acd^3}{b^2}+\frac{3c^2d^2}{2b}\right)+x\left(-\frac{a^3d^4}{b^4}+\frac{4a^2cd^3}{b^3}-\frac{6ac^2d^2}{b^2}+\frac{4c^3d}{b}\right)+\operatorname{RootSum}\left(27t^3a^2b^{13}-a^{12}d^{12}+12a^{11}bcd^{11}-66a^{10}b^2c^2d^{10}+220a^9b^3c^3d^9-495a^8b^4c^4d^8+792a^7b^5c^5d^7-924a^6b^6c^6d^6+792a^5b^7c^7d^5-495a^4b^8c^8d^4+220a^3b^9c^9d^3-66a^2b^{10}c^{10}d^2+12ab^{11}c^{11}d-b^{12}c^{12},\left(t\mapsto t\log\left(\frac{3tab^4}{a^4d^4-4a^3bcd^3+6a^2b^2c^2d^2-4ab^3c^3d+b^4c^4}+x\right)\right)\right)+\frac{d^4x^{10}}{10b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**4/(b*x**3+a),x)
```

```
[Out] x**7*(-a*d**4/(7*b**2) + 4*c*d**3/(7*b)) + x**4*(a**2*d**4/(4*b**3) - a*c*d
**3/b**2 + 3*c**2*d**2/(2*b)) + x*(-a**3*d**4/b**4 + 4*a**2*c*d**3/b**3 - 6
*a*c**2*d**2/b**2 + 4*c**3*d/b) + RootSum(27*_t**3*a**2*b**13 - a**12*d**12
 + 12*a**11*b*c*d**11 - 66*a**10*b**2*c**2*d**10 + 220*a**9*b**3*c**3*d**9
- 495*a**8*b**4*c**4*d**8 + 792*a**7*b**5*c**5*d**7 - 924*a**6*b**6*c**6*d*
*6 + 792*a**5*b**7*c**7*d**5 - 495*a**4*b**8*c**8*d**4 + 220*a**3*b**9*c**9
*d**3 - 66*a**2*b**10*c**10*d**2 + 12*a*b**11*c**11*d - b**12*c**12, Lambda
(_t, _t*log(3*_t*a*b**4/(a**4*d**4 - 4*a**3*b*c*d**3 + 6*a**2*b**2*c**2*d**
2 - 4*a*b**3*c**3*d + b**4*c**4) + x))) + d**4*x**10/(10*b)
```

Giac [**A**]
time = 0.71, size = 391, normalized size = 1.55

$$-\frac{\sqrt{3}\left(b^4c^4-4ab^3c^3d+6a^2b^2c^2d^2-4a^3bcd^3+a^4d^4\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\left(-ab^2\right)^{\frac{2}{3}}b^3}-\frac{\left(b^4c^4-4ab^3c^3d+6a^2b^2c^2d^2-4a^3bcd^3+a^4d^4\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(-ab^2\right)^{\frac{2}{3}}b^3}-\frac{\left(b^{10}c^4-4ab^9c^3d+6a^2b^8c^2d^2-4a^3b^7cd^3+a^4b^6d^4\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3ab^{10}}+\frac{14b^9d^4x^{10}+80b^9cd^3x^7-20ab^8d^4x^7+210b^9c^2d^2x^4-140ab^8cd^3x^4+35a^2b^7d^4x^4+560b^9c^3dx-840ab^8c^2d^2x+560a^2b^7cd^3x-140a^3b^6d^4x}{140b^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^4/(b*x^3+a),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 +
 a^4*d^4)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(
2/3)*b^3) - 1/6*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^
3 + a^4*d^4)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(2/3)*b^3)
- 1/3*(b^10*c^4 - 4*a*b^9*c^3*d + 6*a^2*b^8*c^2*d^2 - 4*a^3*b^7*c*d^3 + a^4
*b^6*d^4)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^10) + 1/140*(14*b^9*
d^4*x^10 + 80*b^9*c*d^3*x^7 - 20*a*b^8*d^4*x^7 + 210*b^9*c^2*d^2*x^4 - 140*
a*b^8*c*d^3*x^4 + 35*a^2*b^7*d^4*x^4 + 560*b^9*c^3*d*x - 840*a*b^8*c^2*d^2*
x + 560*a^2*b^7*c*d^3*x - 140*a^3*b^6*d^4*x)/b^10
```

Mupad [B]
time = 1.43, size = 250, normalized size = 0.99

$$x\left(\frac{4\,c^3\,d}{b}-\frac{a\left(\frac{a\left(\frac{a\,d^4}{b^2}-\frac{4\,c\,d^3}{b}\right)}{b}+\frac{6\,c^2\,d^2}{b}\right)}{b}\right)-x^7\left(\frac{a\,d^4}{7\,b^2}-\frac{4\,c\,d^3}{7\,b}\right)+x^4\left(\frac{a\left(\frac{a\,d^4}{b^2}-\frac{4\,c\,d^3}{b}\right)}{4\,b}+\frac{3\,c^2\,d^2}{2\,b}\right)+\frac{d^4\,x^{10}}{10\,b}+\frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,{\left(a\,d-b\,c\right)}^4}{3\,a^{2/3}\,b^{13/3}}+\frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(-\frac{1}{6}+\frac{\sqrt{3}\,1{}\mathrm{i}}{6}\right)\,{\left(a\,d-b\,c\right)}^4}{a^{2/3}\,b^{13/3}}-\frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,{\left(a\,d-b\,c\right)}^4}{3\,a^{2/3}\,b^{13/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^4/(a + b*x^3),x)
```

```
[Out] x*((4*c^3*d)/b - (a*((a*((a*d^4)/b^2 - (4*c*d^3)/b))/b + (6*c^2*d^2)/b))/b)
 - x^7*((a*d^4)/(7*b^2) - (4*c*d^3)/(7*b)) + x^4*((a*((a*d^4)/b^2 - (4*c*d^
3)/b))/(4*b) + (3*c^2*d^2)/(2*b)) + (d^4*x^10)/(10*b) + (log(b^(1/3)*x + a^
(1/3))*(a*d - b*c)^4)/(3*a^(2/3)*b^(13/3)) + (log(3^(1/2)*a^(1/3)*1i + 2*b^
(1/3)*x - a^(1/3))*((3^(1/2)*1i)/6 - 1/6)*(a*d - b*c)^4)/(a^(2/3)*b^(13/3))
 - (log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*
(a*d - b*c)^4)/(3*a^(2/3)*b^(13/3))
```

3.15 $\int \frac{(c+dx^3)^3}{a+bx^3} \, dx$

Optimal. Leaf size=208

$$\frac{d(3b^2c^2-3abcd+a^2d^2)\,x}{b^3}+\frac{d^2(3bc-ad)x^4}{4b^2}+\frac{d^3x^7}{7b}-\frac{(bc-ad)^3\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{2/3}b^{10/3}}+\frac{(bc-ad)^3\log\left(\sqrt[3]{a}\right.}{3a^{2/3}b^{10/3}}$$

```
[Out] d*(a^2*d^2-3*a*b*c*d+3*b^2*c^2)*x/b^3+1/4*d^2*(-a*d+3*b*c)*x^4/b^2+1/7*d^3*
x^7/b+1/3*(-a*d+b*c)^3*ln(a^(1/3)+b^(1/3)*x)/a^(2/3)/b^(10/3)-1/6*(-a*d+b*c
)^3*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(2/3)/b^(10/3)-1/3*(-a*d+b*
c)^3*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(2/3)/b^(10/3)*3^(
1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {398, 206, 31, 648, 631, 210, 642}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)(bc-ad)^3}{\sqrt{3}\,a^{2/3}b^{10/3}}-\frac{(bc-ad)^3\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6a^{2/3}b^{10/3}}+\frac{(bc-ad)^3\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{2/3}b^{10/3}}+\frac{dx(a^2d^2-3abcd+3b^2c^2)}{b^3}+\frac{d^2x^4(3bc-ad)}{4b^2}+\frac{d^3x^7}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)^3/(a + b*x^3),x]
```

```
[Out] (d*(3*b^2*c^2 - 3*a*b*c*d + a^2*d^2)*x)/b^3 + (d^2*(3*b*c - a*d)*x^4)/(4*b^
2) + (d^3*x^7)/(7*b) - ((b*c - a*d)^3*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[
3]*a^(1/3))])/(Sqrt[3]*a^(2/3)*b^(10/3)) + ((b*c - a*d)^3*Log[a^(1/3) + b^(
1/3)*x])/(3*a^(2/3)*b^(10/3)) - ((b*c - a*d)^3*Log[a^(2/3) - a^(1/3)*b^(1/3
)*x + b^(2/3)*x^2])/(6*a^(2/3)*b^(10/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{(c+dx^3)^3}{a+bx^3}\,dx = \int \left(\frac{d(3b^2c^2-3abcd+a^2d^2)}{b^3} + \frac{d^2(3bc-ad)x^3}{b^2} + \frac{d^3x^6}{b} + \frac{b^3c^3-3ab^2c^2d+3a^2bcd^2-a^3}{b^3(a+bx^3)}\right.$$

$$= \frac{d(3b^2c^2-3abcd+a^2d^2)\,x}{b^3} + \frac{d^2(3bc-ad)x^4}{4b^2} + \frac{d^3x^7}{7b} + \frac{(bc-ad)^3\int \frac{1}{a+bx^3}\,dx}{b^3}$$

$$= \frac{d(3b^2c^2-3abcd+a^2d^2)\,x}{b^3} + \frac{d^2(3bc-ad)x^4}{4b^2} + \frac{d^3x^7}{7b} + \frac{(bc-ad)^3\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx}{3a^{2/3}b^3} + \frac{(bc}{}$$

$$= \frac{d(3b^2c^2-3abcd+a^2d^2)\,x}{b^3} + \frac{d^2(3bc-ad)x^4}{4b^2} + \frac{d^3x^7}{7b} + \frac{(bc-ad)^3\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{2/3}b^{10/3}} -$$

$$= \frac{d(3b^2c^2-3abcd+a^2d^2)\,x}{b^3} + \frac{d^2(3bc-ad)x^4}{4b^2} + \frac{d^3x^7}{7b} + \frac{(bc-ad)^3\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{2/3}b^{10/3}} -$$

$$= \frac{d(3b^2c^2-3abcd+a^2d^2)\,x}{b^3} + \frac{d^2(3bc-ad)x^4}{4b^2} + \frac{d^3x^7}{7b} - \frac{(bc-ad)^3\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{2/3}b^{10/3}}$$

Mathematica [A]
time = 0.07, size = 203, normalized size = 0.98

$$\frac{84\sqrt[3]{b}\,d(3b^2c^2-3abcd+a^2d^2)\,x + 21b^{4/3}d^2(3bc-ad)x^4 + 12b^{7/3}d^3x^7 + \frac{28\sqrt{3}\,(bc-ad)^3\tan^{-1}\left(\frac{-\sqrt[3]{a}+2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{a^{2/3}} + \frac{28(bc-ad)^3\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{a^{2/3}} + \frac{14(-bc+ad)^3\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{a^{2/3}}}{84b^{10/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3)^3/(a + b*x^3),x]
```

```
[Out] (84*b^(1/3)*d*(3*b^2*c^2 - 3*a*b*c*d + a^2*d^2)*x + 21*b^(4/3)*d^2*(3*b*c -
 a*d)*x^4 + 12*b^(7/3)*d^3*x^7 + (28*Sqrt[3]*(b*c - a*d)^3*ArcTan[(-a^(1/3)
 + 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/a^(2/3) + (28*(b*c - a*d)^3*Log[a^(1/3)
 + b^(1/3)*x])/a^(2/3) + (14*(-(b*c) + a*d)^3*Log[a^(2/3) - a^(1/3)*b^(1/3)
*x + b^(2/3)*x^2])/a^(2/3))/(84*b^(10/3))
```

Maple [A]
time = 0.28, size = 193, normalized size = 0.93

method	result
risch	$\frac{d^3x^7}{7b} - \frac{d^3a\,x^4}{4b^2} + \frac{3d^2c\,x^4}{4b} + \frac{d^3a^2x}{b^3} - \frac{3d^2acx}{b^2} + \frac{3d\,c^2x}{b} + \frac{\sum_{\_R=\mathrm{RootOf}\left(b\_Z^3+a\right)} \frac{\left(-a^3d^3+3a^2bc\,d^2-3a\,b^2c^2d+b^3c^3\right)\ln\left(x-\_R\right)}{\_R^2}}{3b^4}$

default	$\frac{d\left(\frac{1}{7}b^{2}d^{2}x^{7}-\frac{1}{4}ab\,d^{2}x^{4}+\frac{3}{4}b^{2}cd\,x^{4}+a^{2}d^{2}x-3abcdx+3b^{2}c^{2}x\right)}{b^{3}}+\frac{\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)}\right.}{3}\right.}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right.}{b^{3}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^3/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] d/b^3*(1/7*b^2*d^2*x^7-1/4*a*b*d^2*x^4+3/4*b^2*c*d*x^4+a^2*d^2*x-3*a*b*c*d*
x+3*b^2*c^2*x)+(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*ln(x^
2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(
2/(a/b)^(1/3)*x-1)))*(-a^3*d^3+3*a^2*b*c*d^2-3*a*b^2*c^2*d+b^3*c^3)/b^3
```

Maxima [A]
time = 0.51, size = 273, normalized size = 1.31

$$\frac{4\,b^{2}d^{3}x^{7}+7\,(3\,b^{2}cd^{2}-abd^{3})x^{4}+28\,(3\,b^{2}c^{2}d-3\,abcd^{2}+a^{2}d^{3})x}{28\,b^{3}}+\frac{\sqrt{3}\,(b^{3}c^{3}-3\,ab^{2}c^{2}d+3\,a^{2}bcd^{2}-a^{3}d^{3})\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b^{4}\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{(b^{3}c^{3}-3\,ab^{2}c^{2}d+3\,a^{2}bcd^{2}-a^{3}d^{3})\log\left(x^{2}-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b^{4}\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{(b^{3}c^{3}-3\,ab^{2}c^{2}d+3\,a^{2}bcd^{2}-a^{3}d^{3})\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b^{4}\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^3/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/28*(4*b^2*d^3*x^7 + 7*(3*b^2*c*d^2 - a*b*d^3)*x^4 + 28*(3*b^2*c^2*d - 3*a
*b*c*d^2 + a^2*d^3)*x)/b^3 + 1/3*sqrt(3)*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b
*c*d^2 - a^3*d^3)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(b^4*
(a/b)^(2/3)) - 1/6*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(
x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^4*(a/b)^(2/3)) + 1/3*(b^3*c^3 - 3*a*b
^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(x + (a/b)^(1/3))/(b^4*(a/b)^(2/3))
```

Fricas [A]
time = 3.18, size = 700, normalized size = 3.37

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^3/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] [1/84*(12*a^2*b^3*d^3*x^7 + 21*(3*a^2*b^3*c*d^2 - a^3*b^2*d^3)*x^4 - 42*sqr
t(1/3)*(a*b^4*c^3 - 3*a^2*b^3*c^2*d + 3*a^3*b^2*c*d^2 - a^4*b*d^3)*sqrt((-a
^2*b)^(1/3)/b)*log((2*a*b*x^3 + 3*(-a^2*b)^(1/3)*a*x - a^2 - 3*sqrt(1/3)*(2
```

```
*a*b*x^2 + (-a^2*b)^(2/3)*x + (-a^2*b)^(1/3)*a)*sqrt((-a^2*b)^(1/3)/b))/(b*
x^3 + a)) - 14*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*(-a^2*b)
^(2/3)*log(a*b*x^2 - (-a^2*b)^(2/3)*x - (-a^2*b)^(1/3)*a) + 28*(b^3*c^3 - 3
*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*(-a^2*b)^(2/3)*log(a*b*x + (-a^2*b)
^(2/3)) + 84*(3*a^2*b^3*c^2*d - 3*a^3*b^2*c*d^2 + a^4*b*d^3)*x)/(a^2*b^4),
1/84*(12*a^2*b^3*d^3*x^7 + 21*(3*a^2*b^3*c*d^2 - a^3*b^2*d^3)*x^4 + 84*sqrt
(1/3)*(a*b^4*c^3 - 3*a^2*b^3*c^2*d + 3*a^3*b^2*c*d^2 - a^4*b*d^3)*sqrt(-(-a
^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(-a^2*b)^(2/3)*x + (-a^2*b)^(1/3)*a)*sqr
t(-(-a^2*b)^(1/3)/b)/a^2) - 14*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a
^3*d^3)*(-a^2*b)^(2/3)*log(a*b*x^2 - (-a^2*b)^(2/3)*x - (-a^2*b)^(1/3)*a) +
 28*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*(-a^2*b)^(2/3)*log(
a*b*x + (-a^2*b)^(2/3)) + 84*(3*a^2*b^3*c^2*d - 3*a^3*b^2*c*d^2 + a^4*b*d^3
)*x)/(a^2*b^4)]
```

Sympy [A]
time = 0.56, size = 257, normalized size = 1.24

$$x^{4}\left(-\frac{ad^{3}}{4b^{2}}+\frac{3cd^{2}}{4b}\right)+x\left(\frac{a^{2}d^{3}}{b^{3}}-\frac{3acd^{2}}{b^{2}}+\frac{3c^{2}d}{b}\right)+\operatorname{RootSum}\left(27t^{3}a^{2}b^{10}+a^{9}d^{9}-9a^{8}bcd^{8}+36a^{7}b^{2}c^{2}d^{7}-84a^{6}b^{3}c^{3}d^{6}+126a^{5}b^{4}c^{4}d^{5}-126a^{4}b^{5}c^{5}d^{4}+84a^{3}b^{6}c^{6}d^{3}-36a^{2}b^{7}c^{7}d^{2}+9ab^{8}c^{8}d-b^{9}c^{9},\left(t\mapsto t\log\left(-\frac{3tab^{3}}{a^{3}d^{3}-3a^{2}bcd^{2}+3ab^{2}c^{2}d-b^{3}c^{3}}+x\right)\right)\right)+\frac{d^{3}x^{7}}{7b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**3/(b*x**3+a),x)
```

```
[Out] x**4*(-a*d**3/(4*b**2) + 3*c*d**2/(4*b)) + x*(a**2*d**3/b**3 - 3*a*c*d**2/b
**2 + 3*c**2*d/b) + RootSum(27*_t**3*a**2*b**10 + a**9*d**9 - 9*a**8*b*c*d*
*8 + 36*a**7*b**2*c**2*d**7 - 84*a**6*b**3*c**3*d**6 + 126*a**5*b**4*c**4*d
**5 - 126*a**4*b**5*c**5*d**4 + 84*a**3*b**6*c**6*d**3 - 36*a**2*b**7*c**7*
d**2 + 9*a*b**8*c**8*d - b**9*c**9, Lambda(_t, _t*log(-3*_t*a*b**3/(a**3*d*
*3 - 3*a**2*b*c*d**2 + 3*a*b**2*c**2*d - b**3*c**3) + x))) + d**3*x**7/(7*b
)
```

Giac [A]
time = 0.65, size = 296, normalized size = 1.42

$$-\frac{\sqrt{3}\,(b^{3}c^{3}-3\,ab^{2}c^{2}d+3\,a^{2}bcd^{2}-a^{3}d^{3})\arctan\left(\frac{\sqrt{3}\,\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,(-ab^{2})^{\frac{2}{3}}\,b^{2}}-\frac{(b^{3}c^{3}-3\,ab^{2}c^{2}d+3\,a^{2}bcd^{2}-a^{3}d^{3})\log\left(x^{2}+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,(-ab^{2})^{\frac{2}{3}}\,b^{2}}-\frac{(b^{7}c^{3}-3\,ab^{6}c^{2}d+3\,a^{2}b^{5}cd^{2}-a^{3}b^{4}d^{3})\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,ab^{7}}+\frac{4\,b^{6}d^{3}x^{7}+21\,b^{6}cd^{2}x^{4}-7\,ab^{5}d^{3}x^{4}+84\,b^{6}c^{2}dx-84\,ab^{5}cd^{2}x+28\,a^{2}b^{4}d^{3}x}{28\,b^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^3/(b*x^3+a),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*arctan(1/3
*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*b^2) - 1/6*(b^3
*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(x^2 + x*(-a/b)^(1/3) +
(-a/b)^(2/3))/((-a*b^2)^(2/3)*b^2) - 1/3*(b^7*c^3 - 3*a*b^6*c^2*d + 3*a^2*b
^5*c*d^2 - a^3*b^4*d^3)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^7) + 1
/28*(4*b^6*d^3*x^7 + 21*b^6*c*d^2*x^4 - 7*a*b^5*d^3*x^4 + 84*b^6*c^2*d*x -
84*a*b^5*c*d^2*x + 28*a^2*b^4*d^3*x)/b^7
```

Mupad [B]
time = 1.40, size = 192, normalized size = 0.92

$$x\left(\frac{3\,c^2\,d}{b}+\frac{a\,\left(\frac{a\,d^3}{b^2}-\frac{3\,c\,d^2}{b}\right)}{b}\right)-x^4\,\left(\frac{a\,d^3}{4\,b^2}-\frac{3\,c\,d^2}{4\,b}\right)+\frac{d^3\,x^7}{7\,b}-\frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,{\left(a\,d-b\,c\right)}^3}{3\,a^{2/3}\,b^{10/3}}-\frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,{\left(a\,d-b\,c\right)}^3}{3\,a^{2/3}\,b^{10/3}}+\frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(\frac{1}{6}+\frac{\sqrt{3}\,1{}\mathrm{i}}{6}\right)\,{\left(a\,d-b\,c\right)}^3}{a^{2/3}\,b^{10/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^3/(a + b*x^3),x)
```

```
[Out] x*((3*c^2*d)/b + (a*((a*d^3)/b^2 - (3*c*d^2)/b))/b) - x^4*((a*d^3)/(4*b^2)
- (3*c*d^2)/(4*b)) + (d^3*x^7)/(7*b) - (log(b^(1/3)*x + a^(1/3))*(a*d - b*c
)^3)/(3*a^(2/3)*b^(10/3)) - (log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3)
)*((3^(1/2)*1i)/2 - 1/2)*(a*d - b*c)^3)/(3*a^(2/3)*b^(10/3)) + (log(3^(1/2)
*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/6 + 1/6)*(a*d - b*c)^3)/
(a^(2/3)*b^(10/3))
```

3.16 $\int \frac{(c+dx^3)^2}{a+bx^3}\,dx$

Optimal. Leaf size=173

$$\frac{d(2bc-ad)x}{b^2}+\frac{d^2x^4}{4b}-\frac{(bc-ad)^2\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{2/3}b^{7/3}}+\frac{(bc-ad)^2\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{2/3}b^{7/3}}-\frac{(bc-ad)^2\log\left(a^{2/3}\right.}{6a}$$

```
[Out] d*(-a*d+2*b*c)*x/b^2+1/4*d^2*x^4/b+1/3*(-a*d+b*c)^2*ln(a^(1/3)+b^(1/3)*x)/a
^(2/3)/b^(7/3)-1/6*(-a*d+b*c)^2*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a
^(2/3)/b^(7/3)-1/3*(-a*d+b*c)^2*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^
(1/2))/a^(2/3)/b^(7/3)*3^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {398, 206, 31, 648, 631, 210, 642}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)(bc-ad)^2}{\sqrt{3}\,a^{2/3}b^{7/3}}-\frac{(bc-ad)^2\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6a^{2/3}b^{7/3}}+\frac{(bc-ad)^2\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{2/3}b^{7/3}}+\frac{dx(2bc-ad)}{b^2}+\frac{d^2x^4}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)^2/(a + b*x^3),x]
```

```
[Out] (d*(2*b*c - a*d)*x)/b^2 + (d^2*x^4)/(4*b) - ((b*c - a*d)^2*ArcTan[(a^(1/3)
 - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(2/3)*b^(7/3)) + ((b*c - a*d)
^2*Log[a^(1/3) + b^(1/3)*x])/(3*a^(2/3)*b^(7/3)) - ((b*c - a*d)^2*Log[a^(2/
3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(2/3)*b^(7/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{(c+dx^3)^2}{a+bx^3}\,dx = \int \left(\frac{d(2bc-ad)}{b^2}+\frac{d^2x^3}{b}+\frac{b^2c^2-2abcd+a^2d^2}{b^2(a+bx^3)}\right)dx$$

$$= \frac{d(2bc-ad)x}{b^2}+\frac{d^2x^4}{4b}+\frac{(bc-ad)^2\int\frac{1}{a+bx^3}\,dx}{b^2}$$

$$= \frac{d(2bc-ad)x}{b^2}+\frac{d^2x^4}{4b}+\frac{(bc-ad)^2\int\frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx}{3a^{2/3}b^2}+\frac{(bc-ad)^2\int\frac{2\sqrt[3]{a}-\sqrt[3]{b}\,x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}\,d}{3a^{2/3}b^2}$$

$$= \frac{d(2bc-ad)x}{b^2}+\frac{d^2x^4}{4b}+\frac{(bc-ad)^2\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{2/3}b^{7/3}}-\frac{(bc-ad)^2\int\frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}}}{6a^{2/3}b^{7/3}}$$

$$= \frac{d(2bc-ad)x}{b^2}+\frac{d^2x^4}{4b}+\frac{(bc-ad)^2\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{2/3}b^{7/3}}-\frac{(bc-ad)^2\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\right.}{6a^{2/3}b^{7/3}}$$

$$= \frac{d(2bc-ad)x}{b^2}+\frac{d^2x^4}{4b}-\frac{(bc-ad)^2\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{2/3}b^{7/3}}+\frac{(bc-ad)^2\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right.}{3a^{2/3}b^{7/3}}$$

Mathematica [A]
time = 0.09, size = 167, normalized size = 0.97

$$\frac{-12a^{2/3}\sqrt[3]{b}\,d(-2bc+ad)x+3a^{2/3}b^{4/3}d^2x^4+4\sqrt{3}(bc-ad)^2\tan^{-1}\left(\frac{-\sqrt[3]{a}+2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)+4(bc-ad)^2\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)-2(bc-ad)^2\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{12a^{2/3}b^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3)^2/(a + b*x^3),x]
```

```
[Out] (-12*a^(2/3)*b^(1/3)*d*(-2*b*c + a*d)*x + 3*a^(2/3)*b^(4/3)*d^2*x^4 + 4*Sqr
t[3]*(b*c - a*d)^2*ArcTan[(-a^(1/3) + 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))] + 4*(
b*c - a*d)^2*Log[a^(1/3) + b^(1/3)*x] - 2*(b*c - a*d)^2*Log[a^(2/3) - a^(1/
3)*b^(1/3)*x + b^(2/3)*x^2])/(12*a^(2/3)*b^(7/3))
```

Maple [A]
time = 0.28, size = 140, normalized size = 0.81

method	result
risch	$\frac{d^2x^4}{4b}-\frac{d^2ax}{b^2}+\frac{2dcx}{b}+\frac{\sum_{\_R=\mathrm{RootOf}\left(b\_Z^3+a\right)}\frac{\left(a^2d^2-2abcd+b^2c^2\right)\ln\left(x-\_R\right)}{\_R^2}}{3b^3}$

default	$-\frac{d\left(-\frac{1}{4}bd\,x^{4}+adx-2bcx\right)}{b^{2}}+\frac{\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)\left(a^{2}d^{2}-2abcd+b^{2}c^{2}\right)}{b^{2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^2/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] -d/b^2*(-1/4*b*d*x^4+a*d*x-2*b*c*x)+(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/
6/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)
*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))*(a^2*d^2-2*a*b*c*d+b^2*c^2)/b^2
```

Maxima [A]
time = 0.49, size = 190, normalized size = 1.10

$$\frac{bd^{2}x^{4}+4\,(2\,bcd-ad^{2})x}{4\,b^{2}}+\frac{\sqrt{3}\,(b^{2}c^{2}-2\,abcd+a^{2}d^{2})\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b^{3}\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{(b^{2}c^{2}-2\,abcd+a^{2}d^{2})\log\left(x^{2}-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b^{3}\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{(b^{2}c^{2}-2\,abcd+a^{2}d^{2})\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b^{3}\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^2/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/4*(b*d^2*x^4 + 4*(2*b*c*d - a*d^2)*x)/b^2 + 1/3*sqrt(3)*(b^2*c^2 - 2*a*b*
c*d + a^2*d^2)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(b^3*(a/
b)^(2/3)) - 1/6*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(x^2 - x*(a/b)^(1/3) + (
a/b)^(2/3))/(b^3*(a/b)^(2/3)) + 1/3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(x +
 (a/b)^(1/3))/(b^3*(a/b)^(2/3))
```

Fricas [A]
time = 5.21, size = 507, normalized size = 2.93

$$\left[\frac{3\,a^{2}b^{2}d^{2}x^{4}+6\sqrt{\frac{1}{3}}\,(ab^{3}c^{2}-2\,a^{2}b^{2}cd+a^{3}bd^{2})\sqrt{-\frac{(a^{2}b)^{\frac{1}{3}}}{b}}\log\left(\frac{2\,abx^{3}-3\,(a^{2}b)^{\frac{1}{3}}ax-a^{2}+3\sqrt{\frac{1}{3}}\left(2\,abx^{2}+(a^{2}b)^{\frac{2}{3}}x-(a^{2}b)^{\frac{1}{3}}a\right)\sqrt{-\frac{(a^{2}b)^{\frac{1}{3}}}{b}}}{bx^{3}+a}\right)-2\,(b^{2}c^{2}-2\,abcd+a^{2}d^{2})(a^{2}b)^{\frac{2}{3}}\log\left(abx^{2}-(a^{2}b)^{\frac{2}{3}}x+(a^{2}b)^{\frac{1}{3}}a\right)+4\,(b^{2}c^{2}-2\,abcd+a^{2}d^{2})(a^{2}b)^{\frac{2}{3}}\log\left(abx+(a^{2}b)^{\frac{2}{3}}\right)+12\,(2\,a^{2}b^{2}cd-a^{3}bd^{2})x}{12\,a^{2}b^{3}},\frac{3\,a^{2}b^{2}d^{2}x^{4}+12\sqrt{\frac{1}{3}}\,(ab^{3}c^{2}-2\,a^{2}b^{2}cd+a^{3}bd^{2})\sqrt{\frac{(a^{2}b)^{\frac{1}{3}}}{b}}\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(2\,(a^{2}b)^{\frac{2}{3}}x-(a^{2}b)^{\frac{1}{3}}a\right)\sqrt{\frac{(a^{2}b)^{\frac{1}{3}}}{b}}}{a^{2}}\right)-2\,(b^{2}c^{2}-2\,abcd+a^{2}d^{2})(a^{2}b)^{\frac{2}{3}}\log\left(abx^{2}-(a^{2}b)^{\frac{2}{3}}x+(a^{2}b)^{\frac{1}{3}}a\right)+4\,(b^{2}c^{2}-2\,abcd+a^{2}d^{2})(a^{2}b)^{\frac{2}{3}}\log\left(abx+(a^{2}b)^{\frac{2}{3}}\right)+12\,(2\,a^{2}b^{2}cd-a^{3}bd^{2})x}{12\,a^{2}b^{3}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^2/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] [1/12*(3*a^2*b^2*d^2*x^4 + 6*sqrt(1/3)*(a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d
^2)*sqrt(-(a^2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*(a^2*b)^(1/3)*a*x - a^2 + 3*s
qrt(1/3)*(2*a*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt(-(a^2*b)^(1/3
)/b))/(b*x^3 + a)) - 2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(a^2*b)^(2/3)*log(a*
```

```
b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 4*(b^2*c^2 - 2*a*b*c*d + a^2*d
^2)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)) + 12*(2*a^2*b^2*c*d - a^3*b*d^
2)*x)/(a^2*b^3), 1/12*(3*a^2*b^2*d^2*x^4 + 12*sqrt(1/3)*(a*b^3*c^2 - 2*a^2*
b^2*c*d + a^3*b*d^2)*sqrt((a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(a^2*b)^(2/3
)*x - (a^2*b)^(1/3)*a)*sqrt((a^2*b)^(1/3)/b)/a^2) - 2*(b^2*c^2 - 2*a*b*c*d
+ a^2*d^2)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) +
 4*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3))
 + 12*(2*a^2*b^2*c*d - a^3*b*d^2)*x)/(a^2*b^3)]
```

Sympy [A]
time = 0.43, size = 156, normalized size = 0.90

$$x\left(-\frac{ad^2}{b^2}+\frac{2cd}{b}\right)+\operatorname{RootSum}\left(27t^3a^2b^7-a^6d^6+6a^5bcd^5-15a^4b^2c^2d^4+20a^3b^3c^3d^3-15a^2b^4c^4d^2+6ab^5c^5d-b^6c^6,\left(t\mapsto t\log\left(\frac{3tab^2}{a^2d^2-2abcd+b^2c^2}+x\right)\right)\right)+\frac{d^2x^4}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**2/(b*x**3+a),x)
```

```
[Out] x*(-a*d**2/b**2 + 2*c*d/b) + RootSum(27*_t**3*a**2*b**7 - a**6*d**6 + 6*a**
5*b*c*d**5 - 15*a**4*b**2*c**2*d**4 + 20*a**3*b**3*c**3*d**3 - 15*a**2*b**4
*c**4*d**2 + 6*a*b**5*c**5*d - b**6*c**6, Lambda(_t, _t*log(3*_t*a*b**2/(a*
*2*d**2 - 2*a*b*c*d + b**2*c**2) + x))) + d**2*x**4/(4*b)
```

Giac [A]
time = 0.69, size = 211, normalized size = 1.22

$$-\frac{\sqrt{3}\,(b^2c^2-2\,abcd+a^2d^2)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,(-ab^2)^{\frac{2}{3}}\,b}-\frac{(b^2c^2-2\,abcd+a^2d^2)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,(-ab^2)^{\frac{2}{3}}\,b}-\frac{(b^4c^2-2\,ab^3cd+a^2b^2d^2)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,ab^4}+\frac{b^3d^2x^4+8\,b^3cdx-4\,ab^2d^2x}{4\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^2/(b*x^3+a),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*arctan(1/3*sqrt(3)*(2*x + (-a/
b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*b) - 1/6*(b^2*c^2 - 2*a*b*c*d + a^2
*d^2)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(2/3)*b) - 1/3*(b^
4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/
(a*b^4) + 1/4*(b^3*d^2*x^4 + 8*b^3*c*d*x - 4*a*b^2*d^2*x)/b^4
```

Mupad [B]
time = 1.38, size = 152, normalized size = 0.88

$$\frac{d^2\,x^4}{4\,b}-x\left(\frac{a\,d^2}{b^2}-\frac{2\,c\,d}{b}\right)+\frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,{\left(a\,d-b\,c\right)}^2}{3\,a^{2/3}\,b^{7/3}}+\frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(-\frac{1}{6}+\frac{\sqrt{3}\,1{}\mathrm{i}}{6}\right)\,{\left(a\,d-b\,c\right)}^2}{a^{2/3}\,b^{7/3}}-\frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,{\left(a\,d-b\,c\right)}^2}{3\,a^{2/3}\,b^{7/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^2/(a + b*x^3),x)
```

```
[Out] (d^2*x^4)/(4*b) - x*((a*d^2)/b^2 - (2*c*d)/b) + (log(b^(1/3)*x + a^(1/3))*(
a*d - b*c)^2)/(3*a^(2/3)*b^(7/3)) + (log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x -
 a^(1/3))*((3^(1/2)*1i)/6 - 1/6)*(a*d - b*c)^2)/(a^(2/3)*b^(7/3)) - (log(3^
(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(a*d - b*c
)^2)/(3*a^(2/3)*b^(7/3))
```

3.17 $\int \frac{c+dx^3}{a+bx^3}\,dx$

Optimal. Leaf size=145

$$\frac{dx}{b}-\frac{(bc-ad)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{2/3}b^{4/3}}+\frac{(bc-ad)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{2/3}b^{4/3}}-\frac{(bc-ad)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6a^{2/3}b^{4/3}}$$

```
[Out] d*x/b+1/3*(-a*d+b*c)*ln(a^(1/3)+b^(1/3)*x)/a^(2/3)/b^(4/3)-1/6*(-a*d+b*c)*l
n(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(2/3)/b^(4/3)-1/3*(-a*d+b*c)*arc
tan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(2/3)/b^(4/3)*3^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {396, 206, 31, 648, 631, 210, 642}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)(bc-ad)}{\sqrt{3}\,a^{2/3}b^{4/3}}-\frac{(bc-ad)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6a^{2/3}b^{4/3}}+\frac{(bc-ad)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{2/3}b^{4/3}}+\frac{dx}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)/(a + b*x^3),x]
```

```
[Out] (d*x)/b - ((b*c - a*d)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(
Sqrt[3]*a^(2/3)*b^(4/3)) + ((b*c - a*d)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(2/3
)*b^(4/3)) - ((b*c - a*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(
6*a^(2/3)*b^(4/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{c+dx^3}{a+bx^3}\,dx = \frac{dx}{b} - \frac{(-bc+ad)\int \frac{1}{a+bx^3}\,dx}{b}$$

$$= \frac{dx}{b} + \frac{(bc-ad)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx}{3a^{2/3}b} + \frac{(bc-ad)\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}\,x}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx}{3a^{2/3}b}$$

$$= \frac{dx}{b} + \frac{(bc-ad)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{2/3}b^{4/3}} - \frac{(bc-ad)\int \frac{-\sqrt[3]{a}\,\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx}{6a^{2/3}b^{4/3}} + \frac{(bc-ad)\int \ldots}{\ldots}$$

$$= \frac{dx}{b} + \frac{(bc-ad)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{2/3}b^{4/3}} - \frac{(bc-ad)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6a^{2/3}b^{4/3}} + \frac{(bc-ad \ldots}{\ldots}$$

$$= \frac{dx}{b} - \frac{(bc-ad)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{2/3}b^{4/3}} + \frac{(bc-ad)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{2/3}b^{4/3}} - \frac{(bc-ad)\log\left(a^{2/3} \ldots\right.}{6a \ldots}$$

Mathematica [A]
time = 0.05, size = 129, normalized size = 0.89

$$\frac{6a^{2/3}\sqrt[3]{b}\,dx-2\sqrt{3}\,(bc-ad)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+2(bc-ad)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)-(bc-ad)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6a^{2/3}b^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3)/(a + b*x^3),x]
```

```
[Out] (6*a^(2/3)*b^(1/3)*d*x - 2*Sqrt[3]*(b*c - a*d)*ArcTan[(1 - (2*b^(1/3)*x)/a^
(1/3))/Sqrt[3]] + 2*(b*c - a*d)*Log[a^(1/3) + b^(1/3)*x] - (b*c - a*d)*Log[
a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(2/3)*b^(4/3))
```

Maple [A]
time = 0.26, size = 110, normalized size = 0.76

method	result	size
risch	$\frac{dx}{b}+\frac{\sum_{\_R=\mathrm{RootOf}\left(b\_Z^3+a\right)}\frac{(-ad+bc)\ln\left(x-\_R\right)}{\_R^2}}{3b^2}$	42
default	$\frac{dx}{b}+\frac{\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)(-ad+bc)}{b}$	110

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] d*x/b+(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*ln(x^2-(a/b)^(
1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(
1/3)*x-1)))*(-a*d+b*c)/b
```

Maxima [A]
time = 0.50, size = 128, normalized size = 0.88

$$\frac{dx}{b}+\frac{\sqrt{3}\,(bc-ad)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{(bc-ad)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{(bc-ad)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] d*x/b + 1/3*sqrt(3)*(b*c - a*d)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b
)^(1/3))/(b^2*(a/b)^(2/3)) - 1/6*(b*c - a*d)*log(x^2 - x*(a/b)^(1/3) + (a/b
)^(2/3))/(b^2*(a/b)^(2/3)) + 1/3*(b*c - a*d)*log(x + (a/b)^(1/3))/(b^2*(a/b
)^(2/3))
```

Fricas [A]
time = 1.95, size = 390, normalized size = 2.69

$$\left[\frac{6\,a^2bdx - 3\,\sqrt{\frac{1}{3}}\,(ab^2c - a^2bd)\sqrt{\frac{(-a^2b)^{\frac{1}{3}}}{b}}\log\left(\frac{2\,abx^3 + 3\,(-a^2b)^{\frac{1}{3}}ax - a^2 - 3\,\sqrt{\frac{1}{3}}\left(2\,abx^2 + (-a^2b)^{\frac{2}{3}}x + (-a^2b)^{\frac{1}{3}}a\right)\sqrt{\frac{(-a^2b)^{\frac{1}{3}}}{b}}}{bx^3 + a}\right) - (-a^2b)^{\frac{2}{3}}(bc - ad)\log\left(abx^2 - (-a^2b)^{\frac{2}{3}}x - (-a^2b)^{\frac{1}{3}}a\right) + 2\,(-a^2b)^{\frac{2}{3}}(bc - ad)\log\left(abx + (-a^2b)^{\frac{2}{3}}\right)}{6\,a^2b^2}, \frac{6\,a^2bdx + 6\,\sqrt{\frac{1}{3}}\,(ab^2c - a^2bd)\sqrt{-\frac{(-a^2b)^{\frac{1}{3}}}{b}}\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(2\,(-a^2b)^{\frac{2}{3}}x + (-a^2b)^{\frac{1}{3}}a\right)\sqrt{-\frac{(-a^2b)^{\frac{1}{3}}}{b}}}{a^2}\right) - (-a^2b)^{\frac{2}{3}}(bc - ad)\log\left(abx^2 - (-a^2b)^{\frac{2}{3}}x - (-a^2b)^{\frac{1}{3}}a\right) + 2\,(-a^2b)^{\frac{2}{3}}(bc - ad)\log\left(abx + (-a^2b)^{\frac{2}{3}}\right)}{6\,a^2b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] [1/6*(6*a^2*b*d*x - 3*sqrt(1/3)*(a*b^2*c - a^2*b*d)*sqrt((-a^2*b)^(1/3)/b)*
log((2*a*b*x^3 + 3*(-a^2*b)^(1/3)*a*x - a^2 - 3*sqrt(1/3)*(2*a*b*x^2 + (-a^
2*b)^(2/3)*x + (-a^2*b)^(1/3)*a)*sqrt((-a^2*b)^(1/3)/b))/(b*x^3 + a)) - (-a
^2*b)^(2/3)*(b*c - a*d)*log(a*b*x^2 - (-a^2*b)^(2/3)*x - (-a^2*b)^(1/3)*a)
+ 2*(-a^2*b)^(2/3)*(b*c - a*d)*log(a*b*x + (-a^2*b)^(2/3)))/(a^2*b^2), 1/6*
(6*a^2*b*d*x + 6*sqrt(1/3)*(a*b^2*c - a^2*b*d)*sqrt(-(-a^2*b)^(1/3)/b)*arct
an(sqrt(1/3)*(2*(-a^2*b)^(2/3)*x + (-a^2*b)^(1/3)*a)*sqrt(-(-a^2*b)^(1/3)/b
)/a^2) - (-a^2*b)^(2/3)*(b*c - a*d)*log(a*b*x^2 - (-a^2*b)^(2/3)*x - (-a^2*
b)^(1/3)*a) + 2*(-a^2*b)^(2/3)*(b*c - a*d)*log(a*b*x + (-a^2*b)^(2/3)))/(a^
2*b^2)]
```

Sympy [A]
time = 0.22, size = 71, normalized size = 0.49

$$\operatorname{RootSum}\left(27t^3a^2b^4 + a^3d^3 - 3a^2bcd^2 + 3ab^2c^2d - b^3c^3, \left(t \mapsto t\log\left(-\frac{3tab}{ad - bc} + x\right)\right)\right) + \frac{dx}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)/(b*x**3+a),x)
```

```
[Out] RootSum(27*_t**3*a**2*b**4 + a**3*d**3 - 3*a**2*b*c*d**2 + 3*a*b**2*c**2*d
- b**3*c**3, Lambda(_t, _t*log(-3*_t*a*b/(a*d - b*c) + x))) + d*x/b
```

Giac [A]
time = 0.62, size = 133, normalized size = 0.92

$$-\frac{\sqrt{3}\,(bc - ad)\arctan\left(\frac{\sqrt{3}\left(2\,x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,\left(-ab^2\right)^{\frac{2}{3}}} - \frac{(bc - ad)\log\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,\left(-ab^2\right)^{\frac{2}{3}}} + \frac{dx}{b} - \frac{(bc - ad)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*(b*c - a*d)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/
3))/(-a*b^2)^(2/3) - 1/6*(b*c - a*d)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3
))/(-a*b^2)^(2/3) + d*x/b - 1/3*(b*c - a*d)*(-a/b)^(1/3)*log(abs(x - (-a/b)
^(1/3)))/(a*b)
```

Mupad [B]
time = 1.38, size = 123, normalized size = 0.85

$$\frac{d\,x}{b}-\frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(a\,d-b\,c\right)}{3\,a^{2/3}\,b^{4/3}}+\frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(a\,d-b\,c\right)}{3\,a^{2/3}\,b^{4/3}}-\frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(a\,d-b\,c\right)}{3\,a^{2/3}\,b^{4/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)/(a + b*x^3),x)
```

```
[Out] (d*x)/b - (log(b^(1/3)*x + a^(1/3))*(a*d - b*c))/(3*a^(2/3)*b^(4/3)) + (log
(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(a*d -
b*c))/(3*a^(2/3)*b^(4/3)) - (log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3)
)*((3^(1/2)*1i)/2 - 1/2)*(a*d - b*c))/(3*a^(2/3)*b^(4/3))
```

3.18 $\int \frac{1}{(a+bx^3)(c+dx^3)}\,dx$

Optimal. Leaf size=288

$$-\frac{b^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{2/3}(bc-ad)}+\frac{d^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}\,x}{\sqrt{3}\,\sqrt[3]{c}}\right)}{\sqrt{3}\,c^{2/3}(bc-ad)}+\frac{b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{2/3}(bc-ad)}-\frac{d^{2/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{3c^{2/3}(bc-ad)}-$$

```
[Out] 1/3*b^(2/3)*ln(a^(1/3)+b^(1/3)*x)/a^(2/3)/(-a*d+b*c)-1/3*d^(2/3)*ln(c^(1/3)
+d^(1/3)*x)/c^(2/3)/(-a*d+b*c)-1/6*b^(2/3)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(
2/3)*x^2)/a^(2/3)/(-a*d+b*c)+1/6*d^(2/3)*ln(c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/
3)*x^2)/c^(2/3)/(-a*d+b*c)-1/3*b^(2/3)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(
1/3)*3^(1/2))/a^(2/3)/(-a*d+b*c)*3^(1/2)+1/3*d^(2/3)*arctan(1/3*(c^(1/3)-2*
d^(1/3)*x)/c^(1/3)*3^(1/2))/c^(2/3)/(-a*d+b*c)*3^(1/2)
```

Rubi [A]
time = 0.20, antiderivative size = 288, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {400, 206, 31, 648, 631, 210, 642}

$$-\frac{b^{2/3}\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{2/3}(bc-ad)}-\frac{b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6a^{2/3}(bc-ad)}+\frac{b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{2/3}(bc-ad)}+\frac{d^{2/3}\text{ArcTan}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}\,x}{\sqrt{3}\,\sqrt[3]{c}}\right)}{\sqrt{3}\,c^{2/3}(bc-ad)}+\frac{d^{2/3}\log\left(c^{2/3}-\sqrt[3]{c}\,\sqrt[3]{d}\,x+d^{2/3}x^2\right)}{6c^{2/3}(bc-ad)}-\frac{d^{2/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{3c^{2/3}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] -((b^(2/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(2
/3)*(b*c - a*d))) + (d^(2/3)*ArcTan[(c^(1/3) - 2*d^(1/3)*x)/(Sqrt[3]*c^(1/3
))])/(Sqrt[3]*c^(2/3)*(b*c - a*d)) + (b^(2/3)*Log[a^(1/3) + b^(1/3)*x])/(3*
a^(2/3)*(b*c - a*d)) - (d^(2/3)*Log[c^(1/3) + d^(1/3)*x])/(3*c^(2/3)*(b*c -
 a*d)) - (b^(2/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(2/3
)*(b*c - a*d)) + (d^(2/3)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/(
6*c^(2/3)*(b*c - a*d))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 400

```
Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[d/(b*c - a*d), Int[1/(c +
 d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{(a+bx^3)(c+dx^3)}\,dx = \frac{b\int \frac{1}{a+bx^3}\,dx}{bc-ad} - \frac{d\int \frac{1}{c+dx^3}\,dx}{bc-ad}$$

$$= \frac{b\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx}{3a^{2/3}(bc-ad)} + \frac{b\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}\,x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx}{3a^{2/3}(bc-ad)} - \frac{d\int \frac{1}{\sqrt[3]{c}+\sqrt[3]{d}\,x}\,dx}{3c^{2/3}(bc-ad)} - \frac{d\int \frac{\cdots}{c^{2/3}-\cdots}}{3\cdots}$$

$$= \frac{b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{2/3}(bc-ad)} - \frac{d^{2/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{3c^{2/3}(bc-ad)} - \frac{b^{2/3}\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}\cdots}{6a^{2/3}(bc-ad)}$$

$$= \frac{b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{2/3}(bc-ad)} - \frac{d^{2/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{3c^{2/3}(bc-ad)} - \frac{b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x\cdots\right.}{6a^{2/3}(bc-ad)}$$

$$= -\frac{b^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{2/3}(bc-ad)} + \frac{d^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}\,x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}\,c^{2/3}(bc-ad)} + \frac{b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\cdots\right.}{3a^{2/3}(bc-ad)}$$

Mathematica [A]
time = 0.08, size = 224, normalized size = 0.78

$$\frac{\frac{2\sqrt{3}\,b^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{2/3}} - \frac{2\sqrt{3}\,d^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\,x}{\sqrt[3]{c}}}{\sqrt{3}}\right)}{c^{2/3}} - \frac{2b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{a^{2/3}} + \frac{2d^{2/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{c^{2/3}} + \frac{b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{a^{2/3}} - \frac{d^{2/3}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}\,x+d^{2/3}x^2\right)}{c^{2/3}}}{-6bc+6ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] ((2*Sqrt[3]*b^(2/3)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(2/3) -
(2*Sqrt[3]*d^(2/3)*ArcTan[(1 - (2*d^(1/3)*x)/c^(1/3))/Sqrt[3]])/c^(2/3) - (
2*b^(2/3)*Log[a^(1/3) + b^(1/3)*x])/a^(2/3) + (2*d^(2/3)*Log[c^(1/3) + d^(1
/3)*x])/c^(2/3) + (b^(2/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/
a^(2/3) - (d^(2/3)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/c^(2/3))
/(-6*b*c + 6*a*d)
```

Maple [A]
time = 0.32, size = 207, normalized size = 0.72

method	result

default	$-\frac{\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)b}{ad-bc}+\frac{\left(\frac{\ln\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3d\left(\frac{c}{d}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{c}{d}\right)^{\frac{1}{3}}x+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6d\left(\frac{c}{d}\right)^{\frac{2}{3}}}+\cdots\right.}{ad-bc}$
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(a^5d^3-3a^4bcd^2+3a^3b^2c^2d-b^3a^2c^3\right)\_Z^3+b^2\right)}\_R\ln\left(\left(\left(-a^5d^5+3a^4bcd^4-2a^3b^2c^2d^3-2a^2b^3c^3d^2+3ab^4c^4d-b^5c^5\right)\_R^3\cdots\right.\right.\right.}{3}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b*x^3+a)/(d*x^3+c),x,method=_RETURNVERBOSE)

[Out] -(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))*b/(a*d-b*c)+(1/3/d/(c/d)^(2/3)*ln(x+(c/d)^(1/3))-1/6/d/(c/d)^(2/3)*ln(x^2-(c/d)^(1/3)*x+(c/d)^(2/3))+1/3/d/(c/d)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(c/d)^(1/3)*x-1)))*d/(a*d-b*c)

Maxima [A]
time = 0.53, size = 293, normalized size = 1.02

$$\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\left(bc\left(\frac{a}{b}\right)^{\frac{1}{3}}-ad\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{3\left(bc\left(\frac{c}{d}\right)^{\frac{1}{3}}-ad\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)\left(\frac{c}{d}\right)^{\frac{1}{3}}}-\frac{\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(bc\left(\frac{a}{b}\right)^{\frac{2}{3}}-ad\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}+\frac{\log\left(x^2-x\left(\frac{c}{d}\right)^{\frac{1}{3}}+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\left(bc\left(\frac{c}{d}\right)^{\frac{2}{3}}-ad\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}+\frac{\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(bc\left(\frac{a}{b}\right)^{\frac{2}{3}}-ad\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}-\frac{\log\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(bc\left(\frac{c}{d}\right)^{\frac{2}{3}}-ad\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x^3+a)/(d*x^3+c),x, algorithm="maxima")

[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/((b*c*(a/b)^(1/3) - a*d*(a/b)^(1/3))*(a/b)^(1/3)) - 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - (c/d)^(1/3))/(c/d)^(1/3))/((b*c*(c/d)^(1/3) - a*d*(c/d)^(1/3))*(c/d)^(1/3)) - 1/6*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b*c*(a/b)^(2/3) - a*d*(a/b)^(2/3)) + 1/6*log(x^2 - x*(c/d)^(1/3) + (c/d)^(2/3))/(b*c*(c/d)^(2/3) - a*d*(c/d)^(2/3)) + 1/3*log(x + (a/b)^(1/3))/(b*c*(a/b)^(2/3) - a*d*(a/b)^(2/3)) - 1/3*log(x + (c/d)^(1/3))/(b*c*(c/d)^(2/3) - a*d*(c/d)^(2/3))

Fricas [A]
time = 2.75, size = 254, normalized size = 0.88

$$-\frac{2\sqrt{3}\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}ax\left(-\frac{b^2}{a^2}\right)^{\frac{2}{3}}-\sqrt{3}b}{3b}\right)+2\sqrt{3}\left(\frac{d^2}{c^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}cx\left(\frac{d^2}{c^2}\right)^{\frac{2}{3}}-\sqrt{3}d}{3d}\right)-\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\log\left(b^2x^2+abx\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}+a^2\left(-\frac{b^2}{a^2}\right)^{\frac{2}{3}}\right)-\left(\frac{d^2}{c^2}\right)^{\frac{1}{3}}\log\left(d^2x^2-cdx\left(\frac{d^2}{c^2}\right)^{\frac{1}{3}}+c^2\left(\frac{d^2}{c^2}\right)^{\frac{2}{3}}\right)+2\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\log\left(bx-a\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\right)+2\left(\frac{d^2}{c^2}\right)^{\frac{1}{3}}\log\left(dx+c\left(\frac{d^2}{c^2}\right)^{\frac{1}{3}}\right)}{6\left(bc-ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] -1/6*(2*sqrt(3)*(-b^2/a^2)^(1/3)*arctan(1/3*(2*sqrt(3)*a*x*(-b^2/a^2)^(2/3)
 - sqrt(3)*b)/b) + 2*sqrt(3)*(d^2/c^2)^(1/3)*arctan(1/3*(2*sqrt(3)*c*x*(d^2
/c^2)^(2/3) - sqrt(3)*d)/d) - (-b^2/a^2)^(1/3)*log(b^2*x^2 + a*b*x*(-b^2/a^
2)^(1/3) + a^2*(-b^2/a^2)^(2/3)) - (d^2/c^2)^(1/3)*log(d^2*x^2 - c*d*x*(d^2
/c^2)^(1/3) + c^2*(d^2/c^2)^(2/3)) + 2*(-b^2/a^2)^(1/3)*log(b*x - a*(-b^2/a
^2)^(1/3)) + 2*(d^2/c^2)^(1/3)*log(d*x + c*(d^2/c^2)^(1/3)))/(b*c - a*d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3+a)/(d*x**3+c),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.83, size = 278, normalized size = 0.97

$$-\frac{b\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,(abc-a^2d)}+\frac{d\left(-\frac{c}{d}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right|\right)}{3\,(bc^2-acd)}+\frac{(-ab^2)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\,\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}\,abc-\sqrt{3}\,a^2d}-\frac{(-cd^2)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\,\left(2\,x+\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\,\left(-\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}\,bc^2-\sqrt{3}\,acd}+\frac{(-ab^2)^{\frac{1}{3}}\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,(abc-a^2d)}-\frac{(-cd^2)^{\frac{1}{3}}\log\left(x^2+x\left(-\frac{c}{d}\right)^{\frac{1}{3}}+\left(-\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\,(bc^2-acd)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] -1/3*b*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b*c - a^2*d) + 1/3*d*(-c/
d)^(1/3)*log(abs(x - (-c/d)^(1/3)))/(b*c^2 - a*c*d) + (-a*b^2)^(1/3)*arctan
(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(sqrt(3)*a*b*c - sqrt(3)*a^
2*d) - (-c*d^2)^(1/3)*arctan(1/3*sqrt(3)*(2*x + (-c/d)^(1/3))/(-c/d)^(1/3))
/(sqrt(3)*b*c^2 - sqrt(3)*a*c*d) + 1/6*(-a*b^2)^(1/3)*log(x^2 + x*(-a/b)^(1
/3) + (-a/b)^(2/3))/(a*b*c - a^2*d) - 1/6*(-c*d^2)^(1/3)*log(x^2 + x*(-c/d)
^(1/3) + (-c/d)^(2/3))/(b*c^2 - a*c*d)
```

Mupad [**B**]
time = 7.70, size = 1364, normalized size = 4.74

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^3)*(c + d*x^3)),x)
```

```
[Out] log(((-b^2/(a^2*(a*d - b*c)^3))^(1/3)*(9*a^2*b^4*d^6 + 9*b^6*c^2*d^4 - 18*a
*b^5*c*d^5 - 9*b^3*d^3*(x + a*c*(-b^2/(a^2*(a*d - b*c)^3))^(1/3))*(a*d + b*
```

```
c)*(a*d - b*c)^4*(-b^2/(a^2*(a*d - b*c)^3))^(2/3)))/3 - 6*b^5*d^5*x)*(-b^2/
(27*a^5*d^3 - 27*a^2*b^3*c^3 + 81*a^3*b^2*c^2*d - 81*a^4*b*c*d^2))^(1/3) +
log(((d^2/(c^2*(a*d - b*c)^3))^(1/3)*(9*a^2*b^4*d^6 + 9*b^6*c^2*d^4 - 18*a*
b^5*c*d^5 - 9*b^3*d^3*(x + a*c*(d^2/(c^2*(a*d - b*c)^3))^(1/3))*(a*d + b*c)
*(a*d - b*c)^4*(d^2/(c^2*(a*d - b*c)^3))^(2/3)))/3 - 6*b^5*d^5*x)*(-d^2/(27
*b^3*c^5 - 27*a^3*c^2*d^3 + 81*a^2*b*c^3*d^2 - 81*a*b^2*c^4*d))^(1/3) + (lo
g(6*b^5*d^5*x + ((3^(1/2)*1i - 1)*(-b^2/(a^2*(a*d - b*c)^3))^(1/3)*(((3^(1/
2)*1i - 1)^2*(81*b^3*d^3*x*(a*d + b*c)*(a*d - b*c)^4 + (81*a*b^3*c*d^3*(3^(
1/2)*1i - 1)*(a*d + b*c)*(a*d - b*c)^4*(-b^2/(a^2*(a*d - b*c)^3))^(1/3))/2)
*(-b^2/(a^2*(a*d - b*c)^3))^(2/3))/36 - 9*a^2*b^4*d^6 - 9*b^6*c^2*d^4 + 18*
a*b^5*c*d^5))/6)*(-b^2/(27*a^5*d^3 - 27*a^2*b^3*c^3 + 81*a^3*b^2*c^2*d - 81
*a^4*b*c*d^2))^(1/3)*(3^(1/2)*1i - 1))/2 - (log(6*b^5*d^5*x - ((3^(1/2)*1i
+ 1)*(-b^2/(a^2*(a*d - b*c)^3))^(1/3)*(((3^(1/2)*1i + 1)^2*(81*b^3*d^3*x*(a
*d + b*c)*(a*d - b*c)^4 - (81*a*b^3*c*d^3*(3^(1/2)*1i + 1)*(a*d + b*c)*(a*d
 - b*c)^4*(-b^2/(a^2*(a*d - b*c)^3))^(1/3))/2)*(-b^2/(a^2*(a*d - b*c)^3))^(
2/3))/36 - 9*a^2*b^4*d^6 - 9*b^6*c^2*d^4 + 18*a*b^5*c*d^5))/6)*(-b^2/(27*a^
5*d^3 - 27*a^2*b^3*c^3 + 81*a^3*b^2*c^2*d - 81*a^4*b*c*d^2))^(1/3)*(3^(1/2)
*1i + 1))/2 + (log(6*b^5*d^5*x + ((3^(1/2)*1i - 1)*(d^2/(c^2*(a*d - b*c)^3)
)^(1/3)*(((3^(1/2)*1i - 1)^2*(81*b^3*d^3*x*(a*d + b*c)*(a*d - b*c)^4 + (81*
a*b^3*c*d^3*(3^(1/2)*1i - 1)*(a*d + b*c)*(a*d - b*c)^4*(d^2/(c^2*(a*d - b*c
)^3))^(1/3))/2)*(d^2/(c^2*(a*d - b*c)^3))^(2/3))/36 - 9*a^2*b^4*d^6 - 9*b^6
*c^2*d^4 + 18*a*b^5*c*d^5))/6)*(-d^2/(27*b^3*c^5 - 27*a^3*c^2*d^3 + 81*a^2*
b*c^3*d^2 - 81*a*b^2*c^4*d))^(1/3)*(3^(1/2)*1i - 1))/2 - (log(6*b^5*d^5*x -
 ((3^(1/2)*1i + 1)*(d^2/(c^2*(a*d - b*c)^3))^(1/3)*(((3^(1/2)*1i + 1)^2*(81
*b^3*d^3*x*(a*d + b*c)*(a*d - b*c)^4 - (81*a*b^3*c*d^3*(3^(1/2)*1i + 1)*(a*
d + b*c)*(a*d - b*c)^4*(d^2/(c^2*(a*d - b*c)^3))^(1/3))/2)*(d^2/(c^2*(a*d -
 b*c)^3))^(2/3))/36 - 9*a^2*b^4*d^6 - 9*b^6*c^2*d^4 + 18*a*b^5*c*d^5))/6)*(
-d^2/(27*b^3*c^5 - 27*a^3*c^2*d^3 + 81*a^2*b*c^3*d^2 - 81*a*b^2*c^4*d))^(1/
3)*(3^(1/2)*1i + 1))/2
```

3.19 $\int \frac{1}{(a+bx^3)(c+dx^3)^2} \, dx$

Optimal. Leaf size=346

$$-\frac{dx}{3c(bc-ad)(c+dx^3)}-\frac{b^{5/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{2/3}(bc-ad)^2}+\frac{d^{2/3}(5bc-2ad)\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}\,x}{\sqrt{3}\,\sqrt[3]{c}}\right)}{3\sqrt{3}\,c^{5/3}(bc-ad)^2}+\frac{b^{5/3}\log\left(\sqrt[3]{a}+\right.}{3a^{2/3}(bc-}$$

```
[Out] -1/3*d*x/c/(-a*d+b*c)/(d*x^3+c)+1/3*b^(5/3)*ln(a^(1/3)+b^(1/3)*x)/a^(2/3)/(
-a*d+b*c)^2-1/9*d^(2/3)*(-2*a*d+5*b*c)*ln(c^(1/3)+d^(1/3)*x)/c^(5/3)/(-a*d+
b*c)^2-1/6*b^(5/3)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(2/3)/(-a*d+
b*c)^2+1/18*d^(2/3)*(-2*a*d+5*b*c)*ln(c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2
)/c^(5/3)/(-a*d+b*c)^2-1/3*b^(5/3)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)
*3^(1/2))/a^(2/3)/(-a*d+b*c)^2*3^(1/2)+1/9*d^(2/3)*(-2*a*d+5*b*c)*arctan(1/
3*(c^(1/3)-2*d^(1/3)*x)/c^(1/3)*3^(1/2))/c^(5/3)/(-a*d+b*c)^2*3^(1/2)
```

Rubi [A]
time = 0.21, antiderivative size = 346, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {425, 536, 206, 31, 648, 631, 210, 642}

$$-\frac{b^{5/3}\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{2/3}(bc-ad)^2}-\frac{b^{5/3}\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6a^{2/3}(bc-ad)^2}+\frac{b^{5/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{2/3}(bc-ad)^2}+\frac{d^{2/3}(5bc-2ad)\text{ArcTan}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}\,x}{\sqrt{3}\,\sqrt[3]{c}}\right)}{3\sqrt{3}\,c^{5/3}(bc-ad)^2}+\frac{d^{2/3}(5bc-2ad)\log\left(c^{2/3}-\sqrt[3]{c}\,\sqrt[3]{d}\,x+d^{2/3}x^2\right)}{18c^{5/3}(bc-ad)^2}-\frac{d^{2/3}(5bc-2ad)\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{9c^{5/3}(bc-ad)^2}-\frac{dx}{3c\left(c+dx^3\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^3)*(c + d*x^3)^2),x]
```

```
[Out] -1/3*(d*x)/(c*(b*c - a*d)*(c + d*x^3)) - (b^(5/3)*ArcTan[(a^(1/3) - 2*b^(1/
3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(2/3)*(b*c - a*d)^2) + (d^(2/3)*(5*b*c
 - 2*a*d)*ArcTan[(c^(1/3) - 2*d^(1/3)*x)/(Sqrt[3]*c^(1/3))])/(3*Sqrt[3]*c^(
5/3)*(b*c - a*d)^2) + (b^(5/3)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(2/3)*(b*c -
a*d)^2) - (d^(2/3)*(5*b*c - 2*a*d)*Log[c^(1/3) + d^(1/3)*x])/(9*c^(5/3)*(b*
c - a*d)^2) - (b^(5/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a
^(2/3)*(b*c - a*d)^2) + (d^(2/3)*(5*b*c - 2*a*d)*Log[c^(2/3) - c^(1/3)*d^(1
/3)*x + d^(2/3)*x^2])/(18*c^(5/3)*(b*c - a*d)^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
```

```
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{(a+bx^3)(c+dx^3)^2}\,dx = -\frac{dx}{3c(bc-ad)(c+dx^3)} + \frac{\int \frac{3bc-2ad-2bdx^3}{(a+bx^3)(c+dx^3)}\,dx}{3c(bc-ad)}$$

$$= -\frac{dx}{3c(bc-ad)(c+dx^3)} + \frac{b^2\int \frac{1}{a+bx^3}\,dx}{(bc-ad)^2} - \frac{(d(5bc-2ad))\int \frac{1}{c+dx^3}\,dx}{3c(bc-ad)^2}$$

$$= -\frac{dx}{3c(bc-ad)(c+dx^3)} + \frac{b^2\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx}{3a^{2/3}(bc-ad)^2} + \frac{b^2\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}\,x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx}{3a^{2/3}(bc-ad)^2} - $$

$$= -\frac{dx}{3c(bc-ad)(c+dx^3)} + \frac{b^{5/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{2/3}(bc-ad)^2} - \frac{d^{2/3}(5bc-2ad)\log\left(\sqrt[3]{c}\right.}{9c^{5/3}(bc-ad)^2}$$

$$= -\frac{dx}{3c(bc-ad)(c+dx^3)} + \frac{b^{5/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{2/3}(bc-ad)^2} - \frac{d^{2/3}(5bc-2ad)\log\left(\sqrt[3]{c}\right.}{9c^{5/3}(bc-ad)^2}$$

$$= -\frac{dx}{3c(bc-ad)(c+dx^3)} - \frac{b^{5/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{2/3}(bc-ad)^2} + \frac{d^{2/3}(5bc-2ad)\tan^{-1}\left(\right.}{3\sqrt{3}\,c^{5/3}(bc-}$$

Mathematica [A]
time = 0.14, size = 336, normalized size = 0.97

$$\frac{6a^{2/3}c^{2/3}d(-bc+ad)x-6\sqrt{3}\,b^{5/3}c^{5/3}(c+dx^3)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)-2\sqrt{3}\,a^{2/3}d^{2/3}(-5bc+2ad)(c+dx^3)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{3}}\right)+6b^{5/3}c^{5/3}(c+dx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)+2a^{2/3}d^{2/3}(-5bc+2ad)(c+dx^3)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)-3b^{5/3}c^{5/3}(c+dx^3)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)+a^{2/3}d^{2/3}(5bc-2ad)(c+dx^3)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{18a^{2/3}c^{5/3}(bc-ad)^2(c+dx^3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^3)*(c + d*x^3)^2),x]
```

```
[Out] (6*a^(2/3)*c^(2/3)*d*(-(b*c) + a*d)*x - 6*Sqrt[3]*b^(5/3)*c^(5/3)*(c + d*x^
3)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] - 2*Sqrt[3]*a^(2/3)*d^(2/3)*
(-5*b*c + 2*a*d)*(c + d*x^3)*ArcTan[(1 - (2*d^(1/3)*x)/c^(1/3))/Sqrt[3]] +
6*b^(5/3)*c^(5/3)*(c + d*x^3)*Log[a^(1/3) + b^(1/3)*x] + 2*a^(2/3)*d^(2/3)*
(-5*b*c + 2*a*d)*(c + d*x^3)*Log[c^(1/3) + d^(1/3)*x] - 3*b^(5/3)*c^(5/3)*(
c + d*x^3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] + a^(2/3)*d^(2/3)
*(5*b*c - 2*a*d)*(c + d*x^3)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2]
)/(18*a^(2/3)*c^(5/3)*(b*c - a*d)^2*(c + d*x^3))
```

Maple [A]
time = 0.36, size = 246, normalized size = 0.71

method	result

default	$\frac{\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)b^{2}}{(ad-bc)^{2}}+\frac{d\left(\frac{(ad-bc)x}{3c\left(dx^{3}+c\right)}+\frac{(2ad-5bc)\left(\frac{\ln\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3d\left(\frac{c}{d}\right)^{\frac{2}{3}}}-\frac{\ln(}{}\right.}{}\right.}{}$
risch	$\frac{dx}{3c(ad-bc)(dx^{3}+c)}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(a^{8}d^{6}-6a^{7}bc\,d^{5}+15a^{6}b^{2}c^{2}d^{4}-20a^{5}b^{3}c^{3}d^{3}+15a^{4}b^{4}c^{4}d^{2}-6a^{3}b^{5}c^{5}d+a^{2}b^{6}c^{6}\right)\_Z^{3}-b^{5}\right)}\_R\ln\left(\left(\left(-18\right.\right.\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^3+a)/(d*x^3+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x
+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x
-1)))*b^2/(a*d-b*c)^2+d/(a*d-b*c)^2*(1/3*(a*d-b*c)/c*x/(d*x^3+c)+1/3*(2*a*d
-5*b*c)/c*(1/3/d/(c/d)^(2/3)*ln(x+(c/d)^(1/3))-1/6/d/(c/d)^(2/3)*ln(x^2-(c/
d)^(1/3)*x+(c/d)^(2/3))+1/3/d/(c/d)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(c/
d)^(1/3)*x-1))))
```

Maxima [A]

time = 0.53, size = 489, normalized size = 1.41

$$\frac{\sqrt{3}\,b\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\left(b^{2}c^{2}\left(\frac{a}{b}\right)^{\frac{1}{3}}-2\,abcd\left(\frac{a}{b}\right)^{\frac{1}{3}}+a^{2}d^{2}\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{\sqrt{3}\,(5\,bc-2\,ad)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{9\left(b^{2}c^{3}\left(\frac{c}{d}\right)^{\frac{1}{3}}-2\,abc^{2}d\left(\frac{c}{d}\right)^{\frac{1}{3}}+a^{2}cd^{2}\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)\left(\frac{c}{d}\right)^{\frac{1}{3}}}-\frac{dx}{3\left(bc^{3}-ac^{2}d+(bc^{2}d-acd^{2})x^{3}\right)}-\frac{b\log\left(x^{2}-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(b^{2}c^{2}\left(\frac{a}{b}\right)^{\frac{2}{3}}-2\,abcd\left(\frac{a}{b}\right)^{\frac{2}{3}}+a^{2}d^{2}\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}+\frac{(5\,bc-2\,ad)\log\left(x^{2}-x\left(\frac{c}{d}\right)^{\frac{1}{3}}+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{18\left(b^{2}c^{3}\left(\frac{c}{d}\right)^{\frac{2}{3}}-2\,abc^{2}d\left(\frac{c}{d}\right)^{\frac{2}{3}}+a^{2}cd^{2}\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}+\frac{b\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(b^{2}c^{2}\left(\frac{a}{b}\right)^{\frac{2}{3}}-2\,abcd\left(\frac{a}{b}\right)^{\frac{2}{3}}+a^{2}d^{2}\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}-\frac{(5\,bc-2\,ad)\log\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{9\left(b^{2}c^{3}\left(\frac{c}{d}\right)^{\frac{2}{3}}-2\,abc^{2}d\left(\frac{c}{d}\right)^{\frac{2}{3}}+a^{2}cd^{2}\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)/(d*x^3+c)^2,x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*b*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/((b^2*c^2
*(a/b)^(1/3) - 2*a*b*c*d*(a/b)^(1/3) + a^2*d^2*(a/b)^(1/3))*(a/b)^(1/3)) -
1/9*sqrt(3)*(5*b*c - 2*a*d)*arctan(1/3*sqrt(3)*(2*x - (c/d)^(1/3))/(c/d)^(1
/3))/((b^2*c^3*(c/d)^(1/3) - 2*a*b*c^2*d*(c/d)^(1/3) + a^2*c*d^2*(c/d)^(1/3
```

```
))*(c/d)^(1/3)) - 1/3*d*x/(b*c^3 - a*c^2*d + (b*c^2*d - a*c*d^2)*x^3) - 1/6
*b*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^2*c^2*(a/b)^(2/3) - 2*a*b*c*d*
(a/b)^(2/3) + a^2*d^2*(a/b)^(2/3)) + 1/18*(5*b*c - 2*a*d)*log(x^2 - x*(c/d)
^(1/3) + (c/d)^(2/3))/(b^2*c^3*(c/d)^(2/3) - 2*a*b*c^2*d*(c/d)^(2/3) + a^2*
c*d^2*(c/d)^(2/3)) + 1/3*b*log(x + (a/b)^(1/3))/(b^2*c^2*(a/b)^(2/3) - 2*a*
b*c*d*(a/b)^(2/3) + a^2*d^2*(a/b)^(2/3)) - 1/9*(5*b*c - 2*a*d)*log(x + (c/d
)^(1/3))/(b^2*c^3*(c/d)^(2/3) - 2*a*b*c^2*d*(c/d)^(2/3) + a^2*c*d^2*(c/d)^(
2/3))
```

Fricas [**A**]
time = 18.56, size = 432, normalized size = 1.25

$$\frac{6\sqrt{3}(bcdx^3+bc^2)\left(\frac{b^2}{a^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}ax\left(\frac{b^2}{a^2}\right)^{\frac{2}{3}}-\sqrt{3}b}{3b}\right)-2\sqrt{3}((5bcd-2ad^2)x^3+5bc^2-2acd)\left(\frac{d^2}{c^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}cx\left(\frac{d^2}{c^2}\right)^{\frac{2}{3}}-\sqrt{3}d}{3d}\right)-3(bcdx^3+bc^2)\left(\frac{b^2}{a^2}\right)^{\frac{1}{3}}\log\left(b^2x^2-abx\left(\frac{b^2}{a^2}\right)^{\frac{1}{3}}+a^2\left(\frac{b^2}{a^2}\right)^{\frac{2}{3}}\right)+((5bcd-2ad^2)x^3+5bc^2-2acd)\left(\frac{d^2}{c^2}\right)^{\frac{1}{3}}\log\left(d^2x^2-cdx\left(\frac{d^2}{c^2}\right)^{\frac{1}{3}}+c^2\left(\frac{d^2}{c^2}\right)^{\frac{2}{3}}\right)+6(bcdx^3+bc^2)\left(\frac{b^2}{a^2}\right)^{\frac{1}{3}}\log\left(bx+a\left(\frac{b^2}{a^2}\right)^{\frac{1}{3}}\right)-2((5bcd-2ad^2)x^3+5bc^2-2acd)\left(\frac{d^2}{c^2}\right)^{\frac{1}{3}}\log\left(dx+c\left(\frac{d^2}{c^2}\right)^{\frac{1}{3}}\right)-6(bcd-ad^2)x}{18(b^2c^4-2abc^3d+a^2c^2d^2+(b^2c^3d-2abc^2d^2+a^2cd^3)x^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)/(d*x^3+c)^2,x, algorithm="fricas")
```

```
[Out] 1/18*(6*sqrt(3)*(b*c*d*x^3 + b*c^2)*(b^2/a^2)^(1/3)*arctan(1/3*(2*sqrt(3)*a
*x*(b^2/a^2)^(2/3) - sqrt(3)*b)/b) - 2*sqrt(3)*((5*b*c*d - 2*a*d^2)*x^3 + 5
*b*c^2 - 2*a*c*d)*(d^2/c^2)^(1/3)*arctan(1/3*(2*sqrt(3)*c*x*(d^2/c^2)^(2/3)
 - sqrt(3)*d)/d) - 3*(b*c*d*x^3 + b*c^2)*(b^2/a^2)^(1/3)*log(b^2*x^2 - a*b*
x*(b^2/a^2)^(1/3) + a^2*(b^2/a^2)^(2/3)) + ((5*b*c*d - 2*a*d^2)*x^3 + 5*b*c
^2 - 2*a*c*d)*(d^2/c^2)^(1/3)*log(d^2*x^2 - c*d*x*(d^2/c^2)^(1/3) + c^2*(d^
2/c^2)^(2/3)) + 6*(b*c*d*x^3 + b*c^2)*(b^2/a^2)^(1/3)*log(b*x + a*(b^2/a^2)
^(1/3)) - 2*((5*b*c*d - 2*a*d^2)*x^3 + 5*b*c^2 - 2*a*c*d)*(d^2/c^2)^(1/3)*l
og(d*x + c*(d^2/c^2)^(1/3)) - 6*(b*c*d - a*d^2)*x)/(b^2*c^4 - 2*a*b*c^3*d +
 a^2*c^2*d^2 + (b^2*c^3*d - 2*a*b*c^2*d^2 + a^2*c*d^3)*x^3)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3+a)/(d*x**3+c)**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.68, size = 443, normalized size = 1.28

$$-\frac{b^2\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3(ab^2c^2-2a^2bcd+a^3d^2)}+\frac{(-ab^2)^{\frac{1}{3}}b\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}ab^2c^2-2\sqrt{3}a^2bcd+\sqrt{3}a^3d^2}+\frac{(-ab^2)^{\frac{1}{3}}b\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6(ab^2c^2-2a^2bcd+a^3d^2)}+\frac{(5bcd-2ad^2)\left(-\frac{c}{d}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right|\right)}{9(b^2c^4-2abc^3d+a^2c^2d^2)}-\frac{\left(5(-cd^2)^{\frac{1}{3}}bc-2(-cd^2)^{\frac{1}{3}}ad\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{3\left(\sqrt{3}b^2c^4-2\sqrt{3}abc^3d+\sqrt{3}a^2c^2d^2\right)}-\frac{\left(5(-cd^2)^{\frac{1}{3}}bc-2(-cd^2)^{\frac{1}{3}}ad\right)\log\left(x^2+x\left(-\frac{c}{d}\right)^{\frac{1}{3}}+\left(-\frac{c}{d}\right)^{\frac{2}{3}}\right)}{18(b^2c^4-2abc^3d+a^2c^2d^2)}-\frac{dx}{3(dx^3+c)(bc^2-acd)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)/(d*x^3+c)^2,x, algorithm="giac")
```

```
[Out] -1/3*b^2*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^2*c^2 - 2*a^2*b*c*d +
 a^3*d^2) + (-a*b^2)^(1/3)*b*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)
^(1/3))/(sqrt(3)*a*b^2*c^2 - 2*sqrt(3)*a^2*b*c*d + sqrt(3)*a^3*d^2) + 1/6*(
-a*b^2)^(1/3)*b*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a*b^2*c^2 - 2*a^2
*b*c*d + a^3*d^2) + 1/9*(5*b*c*d - 2*a*d^2)*(-c/d)^(1/3)*log(abs(x - (-c/d)
^(1/3)))/(b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2) - 1/3*(5*(-c*d^2)^(1/3)*b*c
- 2*(-c*d^2)^(1/3)*a*d)*arctan(1/3*sqrt(3)*(2*x + (-c/d)^(1/3))/(-c/d)^(1/3
))/(sqrt(3)*b^2*c^4 - 2*sqrt(3)*a*b*c^3*d + sqrt(3)*a^2*c^2*d^2) - 1/18*(5*
(-c*d^2)^(1/3)*b*c - 2*(-c*d^2)^(1/3)*a*d)*log(x^2 + x*(-c/d)^(1/3) + (-c/d
)^(2/3))/(b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2) - 1/3*d*x/((d*x^3 + c)*(b*c^
2 - a*c*d))
```

Mupad [B]
time = 16.81, size = 2589, normalized size = 7.48

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^3)*(c + d*x^3)^2),x)
```

```
[Out] log((((((27*b^3*d^3*x*(a*d - b*c)^3*(3*b^2*c^2 - 2*a^2*d^2 + 3*a*b*c*d))/c
+ (27*a*b^3*c^4*d^3*(a*d + b*c)*(a*d - b*c)^5*((d^2*(2*a*d - 5*b*c)^3)/(c^5
*(a*d - b*c)^6))^(1/3))/(b*c^4 - a*c^3*d))*((d^2*(2*a*d - 5*b*c)^3)/(c^5*(a
*d - b*c)^6))^(2/3))/81 - (b^4*d^4*(8*a^3*d^3 - 27*b^3*c^3 + 98*a*b^2*c^2*d
 - 52*a^2*b*c*d^2))/(3*b*c^4 - 3*a*c^3*d))*((d^2*(2*a*d - 5*b*c)^3)/(c^5*(a
*d - b*c)^6))^(1/3))/9 + (2*b^6*d^5*x*(4*a^3*d^3 - 85*b^3*c^3 + 84*a*b^2*c^
2*d - 30*a^2*b*c*d^2))/(9*c^3*(a*d - b*c)^4))*((8*a^3*d^5 - 125*b^3*c^3*d^2
 + 150*a*b^2*c^2*d^3 - 60*a^2*b*c*d^4)/(729*b^6*c^11 + 729*a^6*c^5*d^6 - 43
74*a^5*b*c^6*d^5 + 10935*a^2*b^4*c^9*d^2 - 14580*a^3*b^3*c^8*d^3 + 10935*a^
4*b^2*c^7*d^4 - 4374*a*b^5*c^10*d))^(1/3) + log((((((27*b^3*d^3*x*(a*d - b*
c)^3*(3*b^2*c^2 - 2*a^2*d^2 + 3*a*b*c*d))/c + (81*a*b^3*c^4*d^3*(a*d + b*c)
*(a*d - b*c)^5*(b^5/(a^2*(a*d - b*c)^6))^(1/3))/(b*c^4 - a*c^3*d))*(b^5/(a^
2*(a*d - b*c)^6))^(2/3))/9 - (b^4*d^4*(8*a^3*d^3 - 27*b^3*c^3 + 98*a*b^2*c^
2*d - 52*a^2*b*c*d^2))/(3*b*c^4 - 3*a*c^3*d))*(b^5/(a^2*(a*d - b*c)^6))^(1/
3))/3 + (2*b^6*d^5*x*(4*a^3*d^3 - 85*b^3*c^3 + 84*a*b^2*c^2*d - 30*a^2*b*c*
d^2))/(9*c^3*(a*d - b*c)^4))*(b^5/(27*a^8*d^6 + 27*a^2*b^6*c^6 - 162*a^3*b^
5*c^5*d + 405*a^4*b^4*c^4*d^2 - 540*a^5*b^3*c^3*d^3 + 405*a^6*b^2*c^2*d^4 -
 162*a^7*b*c*d^5))^(1/3) + (log(((3^(1/2)*1i - 1)*(((3^(1/2)*1i - 1)^2*((27
*b^3*d^3*x*(a*d - b*c)^3*(3*b^2*c^2 - 2*a^2*d^2 + 3*a*b*c*d))/c + (27*a*b^3
*c^4*d^3*(3^(1/2)*1i - 1)*(a*d + b*c)*(a*d - b*c)^5*((d^2*(2*a*d - 5*b*c)^3
)/(c^5*(a*d - b*c)^6))^(1/3))/(2*(b*c^4 - a*c^3*d)))*((d^2*(2*a*d - 5*b*c)^
3)/(c^5*(a*d - b*c)^6))^(2/3))/324 - (b^4*d^4*(8*a^3*d^3 - 27*b^3*c^3 + 98*
a*b^2*c^2*d - 52*a^2*b*c*d^2))/(3*b*c^4 - 3*a*c^3*d))*((d^2*(2*a*d - 5*b*c)
^3)/(c^5*(a*d - b*c)^6))^(1/3))/18 + (2*b^6*d^5*x*(4*a^3*d^3 - 85*b^3*c^3 +
 84*a*b^2*c^2*d - 30*a^2*b*c*d^2))/(9*c^3*(a*d - b*c)^4))*(3^(1/2)*1i - 1)*
```

```
((8*a^3*d^5 - 125*b^3*c^3*d^2 + 150*a*b^2*c^2*d^3 - 60*a^2*b*c*d^4)/(729*b^
6*c^11 + 729*a^6*c^5*d^6 - 4374*a^5*b*c^6*d^5 + 10935*a^2*b^4*c^9*d^2 - 145
80*a^3*b^3*c^8*d^3 + 10935*a^4*b^2*c^7*d^4 - 4374*a*b^5*c^10*d))^(1/3))/2 -
 (log(((3^(1/2)*1i + 1)*(((3^(1/2)*1i + 1)^2*((27*b^3*d^3*x*(a*d - b*c)^3*(
3*b^2*c^2 - 2*a^2*d^2 + 3*a*b*c*d))/c - (27*a*b^3*c^4*d^3*(3^(1/2)*1i + 1)*
(a*d + b*c)*(a*d - b*c)^5*((d^2*(2*a*d - 5*b*c)^3)/(c^5*(a*d - b*c)^6))^(1/
3))/(2*(b*c^4 - a*c^3*d)))*((d^2*(2*a*d - 5*b*c)^3)/(c^5*(a*d - b*c)^6))^(2
/3))/324 - (b^4*d^4*(8*a^3*d^3 - 27*b^3*c^3 + 98*a*b^2*c^2*d - 52*a^2*b*c*d
^2))/(3*b*c^4 - 3*a*c^3*d))*((d^2*(2*a*d - 5*b*c)^3)/(c^5*(a*d - b*c)^6))^(
1/3))/18 - (2*b^6*d^5*x*(4*a^3*d^3 - 85*b^3*c^3 + 84*a*b^2*c^2*d - 30*a^2*b
*c*d^2))/(9*c^3*(a*d - b*c)^4))*(3^(1/2)*1i + 1)*((8*a^3*d^5 - 125*b^3*c^3*
d^2 + 150*a*b^2*c^2*d^3 - 60*a^2*b*c*d^4)/(729*b^6*c^11 + 729*a^6*c^5*d^6 -
 4374*a^5*b*c^6*d^5 + 10935*a^2*b^4*c^9*d^2 - 14580*a^3*b^3*c^8*d^3 + 10935
*a^4*b^2*c^7*d^4 - 4374*a*b^5*c^10*d))^(1/3))/2 + (log(((3^(1/2)*1i - 1)*((
(3^(1/2)*1i - 1)^2*((27*b^3*d^3*x*(a*d - b*c)^3*(3*b^2*c^2 - 2*a^2*d^2 + 3*
a*b*c*d))/c + (81*a*b^3*c^4*d^3*(3^(1/2)*1i - 1)*(a*d + b*c)*(a*d - b*c)^5*
(b^5/(a^2*(a*d - b*c)^6))^(1/3))/(2*(b*c^4 - a*c^3*d)))*(b^5/(a^2*(a*d - b*
c)^6))^(2/3))/36 - (b^4*d^4*(8*a^3*d^3 - 27*b^3*c^3 + 98*a*b^2*c^2*d - 52*a
^2*b*c*d^2))/(3*b*c^4 - 3*a*c^3*d))*(b^5/(a^2*(a*d - b*c)^6))^(1/3))/6 + (2
*b^6*d^5*x*(4*a^3*d^3 - 85*b^3*c^3 + 84*a*b^2*c^2*d - 30*a^2*b*c*d^2))/(9*c
^3*(a*d - b*c)^4))*(3^(1/2)*1i - 1)*(b^5/(27*a^8*d^6 + 27*a^2*b^6*c^6 - 162
*a^3*b^5*c^5*d + 405*a^4*b^4*c^4*d^2 - 540*a^5*b^3*c^3*d^3 + 405*a^6*b^2*c^
2*d^4 - 162*a^7*b*c*d^5))^(1/3))/2 - (log(((3^(1/2)*1i + 1)*(((3^(1/2)*1i +
 1)^2*((27*b^3*d^3*x*(a*d - b*c)^3*(3*b^2*c^2 - 2*a^2*d^2 + 3*a*b*c*d))/c -
 (81*a*b^3*c^4*d^3*(3^(1/2)*1i + 1)*(a*d + b*c)*(a*d - b*c)^5*(b^5/(a^2*(a*
d - b*c)^6))^(1/3))/(2*(b*c^4 - a*c^3*d)))*(b^5/(a^2*(a*d - b*c)^6))^(2/3))
/36 - (b^4*d^4*(8*a^3*d^3 - 27*b^3*c^3 + 98*a*b^2*c^2*d - 52*a^2*b*c*d^2))/
(3*b*c^4 - 3*a*c^3*d))*(b^5/(a^2*(a*d - b*c)^6))^(1/3))/6 - (2*b^6*d^5*x*(4
*a^3*d^3 - 85*b^3*c^3 + 84*a*b^2*c^2*d - 30*a^2*b*c*d^2))/(9*c^3*(a*d - b*c
)^4))*(3^(1/2)*1i + 1)*(b^5/(27*a^8*d^6 + 27*a^2*b^6*c^6 - 162*a^3*b^5*c^5*
d + 405*a^4*b^4*c^4*d^2 - 540*a^5*b^3*c^3*d^3 + 405*a^6*b^2*c^2*d^4 - 162*a
^7*b*c*d^5))^(1/3))/2 + (d*x)/(3*c*(c + d*x^3)*(a*d - b*c))
```

3.20 $\int \frac{(c+dx^3)^5}{(a+bx^3)^2}\,dx$

Optimal. Leaf size=320

$$\frac{d^2(10b^3c^3-20ab^2c^2d+15a^2bcd^2-4a^3d^3)\,x}{b^5}+\frac{d^3(10b^2c^2-10abcd+3a^2d^2)\,x^4}{4b^4}+\frac{d^4(5bc-2ad)x^7}{7b^3}+\frac{d^5x^{10}}{10b^2}+\frac{(b}{3ab}$$

```
[Out] d^2*(-4*a^3*d^3+15*a^2*b*c*d^2-20*a*b^2*c^2*d+10*b^3*c^3)*x/b^5+1/4*d^3*(3*
a^2*d^2-10*a*b*c*d+10*b^2*c^2)*x^4/b^4+1/7*d^4*(-2*a*d+5*b*c)*x^7/b^3+1/10*
d^5*x^10/b^2+1/3*(-a*d+b*c)^5*x/a/b^5/(b*x^3+a)+1/9*(-a*d+b*c)^4*(13*a*d+2*
b*c)*ln(a^(1/3)+b^(1/3)*x)/a^(5/3)/b^(16/3)-1/18*(-a*d+b*c)^4*(13*a*d+2*b*c
)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(5/3)/b^(16/3)-1/9*(-a*d+b*c)
^4*(13*a*d+2*b*c)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(5/3)
/b^(16/3)*3^(1/2)
```

Rubi [A]
time = 0.21, antiderivative size = 320, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {398, 393, 206, 31, 648, 631, 210, 642}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)(bc-ad)^4(13ad+2bc)}{3\sqrt{3}\,a^{5/3}b^{16/3}}-\frac{(bc-ad)^4(13ad+2bc)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{18a^{5/3}b^{16/3}}+\frac{(bc-ad)^4(13ad+2bc)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9a^{5/3}b^{16/3}}+\frac{d^3x^4(3a^2d^2-10abcd+10b^2c^2)}{4b^4}+\frac{d^2x(-4a^3d^3+15a^2bcd^2-20ab^2c^2d+10b^3c^3)}{b^5}+\frac{x(bc-ad)^5}{3ab^5\,(a+bx^3)}+\frac{d^4x^7(5bc-2ad)}{7b^3}+\frac{d^5x^{10}}{10b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)^5/(a + b*x^3)^2,x]
```

```
[Out] (d^2*(10*b^3*c^3 - 20*a*b^2*c^2*d + 15*a^2*b*c*d^2 - 4*a^3*d^3)*x)/b^5 + (d
^3*(10*b^2*c^2 - 10*a*b*c*d + 3*a^2*d^2)*x^4)/(4*b^4) + (d^4*(5*b*c - 2*a*d
)*x^7)/(7*b^3) + (d^5*x^10)/(10*b^2) + ((b*c - a*d)^5*x)/(3*a*b^5*(a + b*x^
3)) - ((b*c - a*d)^4*(2*b*c + 13*a*d)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[
3]*a^(1/3))])/(3*Sqrt[3]*a^(5/3)*b^(16/3)) + ((b*c - a*d)^4*(2*b*c + 13*a*d
)*Log[a^(1/3) + b^(1/3)*x])/(9*a^(5/3)*b^(16/3)) - ((b*c - a*d)^4*(2*b*c +
13*a*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^(5/3)*b^(16/3
))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
```

reeQ[{a, b}, x]

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{(c+dx^3)^5}{(a+bx^3)^2}\,dx = \int \left(\frac{d^2(10b^3c^3-20ab^2c^2d+15a^2bcd^2-4a^3d^3)}{b^5} + \frac{d^3(10b^2c^2-10abcd+3a^2d^2)\,x^3}{b^4} + \frac{d^4(5\cdots}{}\right.$$

$$= \frac{d^2(10b^3c^3-20ab^2c^2d+15a^2bcd^2-4a^3d^3)\,x}{b^5} + \frac{d^3(10b^2c^2-10abcd+3a^2d^2)\,x^4}{4b^4} + \frac{d^4(5bc\cdots}{7\cdots}$$

$$= \frac{d^2(10b^3c^3-20ab^2c^2d+15a^2bcd^2-4a^3d^3)\,x}{b^5} + \frac{d^3(10b^2c^2-10abcd+3a^2d^2)\,x^4}{4b^4} + \frac{d^4(5bc\cdots}{7\cdots}$$

$$= \frac{d^2(10b^3c^3-20ab^2c^2d+15a^2bcd^2-4a^3d^3)\,x}{b^5} + \frac{d^3(10b^2c^2-10abcd+3a^2d^2)\,x^4}{4b^4} + \frac{d^4(5bc\cdots}{7\cdots}$$

$$= \frac{d^2(10b^3c^3-20ab^2c^2d+15a^2bcd^2-4a^3d^3)\,x}{b^5} + \frac{d^3(10b^2c^2-10abcd+3a^2d^2)\,x^4}{4b^4} + \frac{d^4(5bc\cdots}{7\cdots}$$

$$= \frac{d^2(10b^3c^3-20ab^2c^2d+15a^2bcd^2-4a^3d^3)\,x}{b^5} + \frac{d^3(10b^2c^2-10abcd+3a^2d^2)\,x^4}{4b^4} + \frac{d^4(5bc\cdots}{7\cdots}$$

$$= \frac{d^2(10b^3c^3-20ab^2c^2d+15a^2bcd^2-4a^3d^3)\,x}{b^5} + \frac{d^3(10b^2c^2-10abcd+3a^2d^2)\,x^4}{4b^4} + \frac{d^4(5bc\cdots}{7\cdots}$$

Mathematica [A]
time = 0.18, size = 313, normalized size = 0.98

$$\frac{1260\sqrt[3]{b}\,d^2(10b^3c^3-20ab^2c^2d+15a^2bcd^2-4a^3d^3)\,x+315b^{4/3}d^3(10b^2c^2-10abcd+3a^2d^2)\,x^4+180b^{7/3}d^4(5bc-2ad)\,x^7+126b^{10/3}d^5x^{10}+\frac{420\sqrt[3]{b}\,(bc-ad)^5x}{a(a+bx^3)}+\frac{140\sqrt{3}\,(bc-ad)^4(2bc+13ad)\tan^{-1}\left(\frac{-\sqrt[3]{a}+2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{a^{5/3}}+\frac{140(bc-ad)^4(2bc+13ad)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{a^{5/3}}-\frac{70(bc-ad)^4(2bc+13ad)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{a^{5/3}}}{1260b^{16/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3)^5/(a + b*x^3)^2,x]
```

```
[Out] (1260*b^(1/3)*d^2*(10*b^3*c^3 - 20*a*b^2*c^2*d + 15*a^2*b*c*d^2 - 4*a^3*d^3
)*x + 315*b^(4/3)*d^3*(10*b^2*c^2 - 10*a*b*c*d + 3*a^2*d^2)*x^4 + 180*b^(7/
3)*d^4*(5*b*c - 2*a*d)*x^7 + 126*b^(10/3)*d^5*x^10 + (420*b^(1/3)*(b*c - a*
d)^5*x)/(a*(a + b*x^3)) + (140*Sqrt[3]*(b*c - a*d)^4*(2*b*c + 13*a*d)*ArcTa
n[(-a^(1/3) + 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/a^(5/3) + (140*(b*c - a*d)^4
*(2*b*c + 13*a*d)*Log[a^(1/3) + b^(1/3)*x])/a^(5/3) - (70*(b*c - a*d)^4*(2*
b*c + 13*a*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(5/3))/(126
0*b^(16/3))
```

Maple [A]
time = 0.29, size = 367, normalized size = 1.15

method	result
risch	$\frac{d^5x^{10}}{10b^2} - \frac{2d^5a\,x^7}{7b^3} + \frac{5d^4c\,x^7}{7b^2} + \frac{3d^5a^2x^4}{4b^4} - \frac{5d^4ac\,x^4}{2b^3} + \frac{5d^3c^2x^4}{2b^2} - \frac{4d^5a^3x}{b^5} + \frac{15d^4a^2cx}{b^4} - \frac{20d^3a\,c^2x}{b^3} + \frac{10d^2c^3x}{b^2} - \frac{(a^5}{}$
default	$-\frac{d^2\left(-\frac{1}{10}d^3x^{10}b^3+\frac{2}{7}a\,b^2d^3x^7-\frac{5}{7}b^3c\,d^2x^7-\frac{3}{4}a^2b\,d^3x^4+\frac{5}{2}a\,b^2c\,d^2x^4-\frac{5}{2}b^3c^2d\,x^4+4a^3d^3x-15a^2bc\,d^2x+20a\,b^2c^2dx-10b^3c^3x\right)}{b^5} + -$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^5/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -d^2/b^5*(-1/10*d^3*x^10*b^3+2/7*a*b^2*d^3*x^7-5/7*b^3*c*d^2*x^7-3/4*a^2*b*
d^3*x^4+5/2*a*b^2*c*d^2*x^4-5/2*b^3*c^2*d*x^4+4*a^3*d^3*x-15*a^2*b*c*d^2*x+
20*a*b^2*c^2*d*x-10*b^3*c^3*x)+1/b^5*(-1/3*(a^5*d^5-5*a^4*b*c*d^4+10*a^3*b^
2*c^2*d^3-10*a^2*b^3*c^3*d^2+5*a*b^4*c^4*d-b^5*c^5)/a*x/(b*x^3+a)+1/3*(13*a
^5*d^5-50*a^4*b*c*d^4+70*a^3*b^2*c^2*d^3-40*a^2*b^3*c^3*d^2+5*a*b^4*c^4*d+2
*b^5*c^5)/a*(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*ln(x^2-(
a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(
a/b)^(1/3)*x-1))))
```

Maxima [A]
time = 0.51, size = 509, normalized size = 1.59

$$\frac{(b^5c^5-5ab^4c^4d+10a^2b^3c^3d^2-10a^3b^2c^2d^3+5a^4bcd^4-a^5d^5)x}{3(ab^6x^3+a^2b^5)}+\frac{14b^3d^5x^{10}+20(5b^3cd^4-2ab^2d^5)x^7+35(10b^3c^2d^3-10ab^2cd^4+3a^2bd^5)x^4+140(10b^3c^3d^2-20ab^2c^2d^3+15a^2bcd^4-4a^3d^5)x}{140b^5}+\frac{\sqrt{3}(2b^5c^5+5ab^4c^4d-40a^2b^3c^3d^2+70a^3b^2c^2d^3-50a^4bcd^4+13a^5d^5)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9ab^6\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{(2b^5c^5+5ab^4c^4d-40a^2b^3c^3d^2+70a^3b^2c^2d^3-50a^4bcd^4+13a^5d^5)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18ab^6\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{(2b^5c^5+5ab^4c^4d-40a^2b^3c^3d^2+70a^3b^2c^2d^3-50a^4bcd^4+13a^5d^5)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9ab^6\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^5/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] 1/3*(b^5*c^5 - 5*a*b^4*c^4*d + 10*a^2*b^3*c^3*d^2 - 10*a^3*b^2*c^2*d^3 + 5*
a^4*b*c*d^4 - a^5*d^5)*x/(a*b^6*x^3 + a^2*b^5) + 1/140*(14*b^3*d^5*x^10 + 2
0*(5*b^3*c*d^4 - 2*a*b^2*d^5)*x^7 + 35*(10*b^3*c^2*d^3 - 10*a*b^2*c*d^4 + 3
*a^2*b*d^5)*x^4 + 140*(10*b^3*c^3*d^2 - 20*a*b^2*c^2*d^3 + 15*a^2*b*c*d^4 -
 4*a^3*d^5)*x)/b^5 + 1/9*sqrt(3)*(2*b^5*c^5 + 5*a*b^4*c^4*d - 40*a^2*b^3*c^
3*d^2 + 70*a^3*b^2*c^2*d^3 - 50*a^4*b*c*d^4 + 13*a^5*d^5)*arctan(1/3*sqrt(3
)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a*b^6*(a/b)^(2/3)) - 1/18*(2*b^5*c^5 +
5*a*b^4*c^4*d - 40*a^2*b^3*c^3*d^2 + 70*a^3*b^2*c^2*d^3 - 50*a^4*b*c*d^4 +
13*a^5*d^5)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a*b^6*(a/b)^(2/3)) + 1/
```

```
9*(2*b^5*c^5 + 5*a*b^4*c^4*d - 40*a^2*b^3*c^3*d^2 + 70*a^3*b^2*c^2*d^3 - 50
*a^4*b*c*d^4 + 13*a^5*d^5)*log(x + (a/b)^(1/3))/(a*b^6*(a/b)^(2/3))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 789 vs. $2(275) = 550$.
time = 3.38, size = 1619, normalized size = 5.06

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^5/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] [1/1260*(126*a^3*b^5*d^5*x^13 + 18*(50*a^3*b^5*c*d^4 - 13*a^4*b^4*d^5)*x^10
 + 45*(70*a^3*b^5*c^2*d^3 - 50*a^4*b^4*c*d^4 + 13*a^5*b^3*d^5)*x^7 + 315*(4
0*a^3*b^5*c^3*d^2 - 70*a^4*b^4*c^2*d^3 + 50*a^5*b^3*c*d^4 - 13*a^6*b^2*d^5)
*x^4 + 210*sqrt(1/3)*(2*a^2*b^6*c^5 + 5*a^3*b^5*c^4*d - 40*a^4*b^4*c^3*d^2
+ 70*a^5*b^3*c^2*d^3 - 50*a^6*b^2*c*d^4 + 13*a^7*b*d^5 + (2*a*b^7*c^5 + 5*a
^2*b^6*c^4*d - 40*a^3*b^5*c^3*d^2 + 70*a^4*b^4*c^2*d^3 - 50*a^5*b^3*c*d^4 +
 13*a^6*b^2*d^5)*x^3)*sqrt(-(a^2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*(a^2*b)^(1/
3)*a*x - a^2 + 3*sqrt(1/3)*(2*a*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*
sqrt(-(a^2*b)^(1/3)/b))/(b*x^3 + a)) - 70*(2*a*b^5*c^5 + 5*a^2*b^4*c^4*d -
40*a^3*b^3*c^3*d^2 + 70*a^4*b^2*c^2*d^3 - 50*a^5*b*c*d^4 + 13*a^6*d^5 + (2*
b^6*c^5 + 5*a*b^5*c^4*d - 40*a^2*b^4*c^3*d^2 + 70*a^3*b^3*c^2*d^3 - 50*a^4*
b^2*c*d^4 + 13*a^5*b*d^5)*x^3)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x
+ (a^2*b)^(1/3)*a) + 140*(2*a*b^5*c^5 + 5*a^2*b^4*c^4*d - 40*a^3*b^3*c^3*d^
2 + 70*a^4*b^2*c^2*d^3 - 50*a^5*b*c*d^4 + 13*a^6*d^5 + (2*b^6*c^5 + 5*a*b^5
*c^4*d - 40*a^2*b^4*c^3*d^2 + 70*a^3*b^3*c^2*d^3 - 50*a^4*b^2*c*d^4 + 13*a^
5*b*d^5)*x^3)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)) + 420*(a^2*b^6*c^5 -
 5*a^3*b^5*c^4*d + 40*a^4*b^4*c^3*d^2 - 70*a^5*b^3*c^2*d^3 + 50*a^6*b^2*c*d
^4 - 13*a^7*b*d^5)*x)/(a^3*b^7*x^3 + a^4*b^6), 1/1260*(126*a^3*b^5*d^5*x^13
 + 18*(50*a^3*b^5*c*d^4 - 13*a^4*b^4*d^5)*x^10 + 45*(70*a^3*b^5*c^2*d^3 - 5
0*a^4*b^4*c*d^4 + 13*a^5*b^3*d^5)*x^7 + 315*(40*a^3*b^5*c^3*d^2 - 70*a^4*b^
4*c^2*d^3 + 50*a^5*b^3*c*d^4 - 13*a^6*b^2*d^5)*x^4 + 420*sqrt(1/3)*(2*a^2*b
^6*c^5 + 5*a^3*b^5*c^4*d - 40*a^4*b^4*c^3*d^2 + 70*a^5*b^3*c^2*d^3 - 50*a^6
*b^2*c*d^4 + 13*a^7*b*d^5 + (2*a*b^7*c^5 + 5*a^2*b^6*c^4*d - 40*a^3*b^5*c^3
*d^2 + 70*a^4*b^4*c^2*d^3 - 50*a^5*b^3*c*d^4 + 13*a^6*b^2*d^5)*x^3)*sqrt((a
^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt(
(a^2*b)^(1/3)/b)/a^2) - 70*(2*a*b^5*c^5 + 5*a^2*b^4*c^4*d - 40*a^3*b^3*c^3*
d^2 + 70*a^4*b^2*c^2*d^3 - 50*a^5*b*c*d^4 + 13*a^6*d^5 + (2*b^6*c^5 + 5*a*b
^5*c^4*d - 40*a^2*b^4*c^3*d^2 + 70*a^3*b^3*c^2*d^3 - 50*a^4*b^2*c*d^4 + 13*
a^5*b*d^5)*x^3)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)
*a) + 140*(2*a*b^5*c^5 + 5*a^2*b^4*c^4*d - 40*a^3*b^3*c^3*d^2 + 70*a^4*b^2*
c^2*d^3 - 50*a^5*b*c*d^4 + 13*a^6*d^5 + (2*b^6*c^5 + 5*a*b^5*c^4*d - 40*a^2
*b^4*c^3*d^2 + 70*a^3*b^3*c^2*d^3 - 50*a^4*b^2*c*d^4 + 13*a^5*b*d^5)*x^3)*(
a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)) + 420*(a^2*b^6*c^5 - 5*a^3*b^5*c^4*
```

```
d + 40*a^4*b^4*c^3*d^2 - 70*a^5*b^3*c^2*d^3 + 50*a^6*b^2*c*d^4 - 13*a^7*b*d
^5)*x)/(a^3*b^7*x^3 + a^4*b^6)]
```

Sympy [A]
time = 128.22, size = 546, normalized size = 1.71

$$x^{7}\left(-\frac{2ad^{5}}{7b^{3}}+\frac{5cd^{4}}{7b^{2}}\right)+x^{4}\left(\frac{3a^{2}d^{5}}{4b^{4}}-\frac{5acd^{4}}{2b^{3}}+\frac{5c^{2}d^{3}}{2b^{2}}\right)+x\left(-\frac{4a^{3}d^{5}}{b^{5}}+\frac{15a^{2}cd^{4}}{b^{4}}-\frac{20ac^{2}d^{3}}{b^{3}}+\frac{10c^{3}d^{2}}{b^{2}}\right)+\frac{x\left(-a^{5}d^{5}+5a^{4}bcd^{4}-10a^{3}b^{2}c^{2}d^{3}+10a^{2}b^{3}c^{3}d^{2}-5ab^{4}c^{4}d+b^{5}c^{5}\right)}{3a^{2}b^{5}+3ab^{6}x^{3}}+\operatorname{RootSum}\left(729t^{3}a^{5}b^{16}-2197a^{15}d^{15}+25350a^{14}bcd^{14}-132990a^{13}b^{2}c^{2}d^{13}+418280a^{12}b^{3}c^{3}d^{12}-874635a^{11}b^{4}c^{4}d^{11}+1271886a^{10}b^{5}c^{5}d^{10}-1302400a^{9}b^{6}c^{6}d^{9}+922680a^{8}b^{7}c^{7}d^{8}-422235a^{7}b^{8}c^{8}d^{7}+97570a^{6}b^{9}c^{9}d^{6}+7194a^{5}b^{10}c^{10}d^{5}-10200a^{4}b^{11}c^{11}d^{4}+1435a^{3}b^{12}c^{12}d^{3}+330a^{2}b^{13}c^{13}d^{2}-60ab^{14}c^{14}d-8b^{15}c^{15},\left(t\mapsto t\log\left(\frac{9ta^{2}b^{5}}{13a^{5}d^{5}-50a^{4}bcd^{4}+70a^{3}b^{2}c^{2}d^{3}-40a^{2}b^{3}c^{3}d^{2}+5ab^{4}c^{4}d+2b^{5}c^{5}}+x\right)\right)\right)+\frac{d^{5}x^{10}}{10b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**5/(b*x**3+a)**2,x)
```

```
[Out] x**7*(-2*a*d**5/(7*b**3) + 5*c*d**4/(7*b**2)) + x**4*(3*a**2*d**5/(4*b**4)
- 5*a*c*d**4/(2*b**3) + 5*c**2*d**3/(2*b**2)) + x*(-4*a**3*d**5/b**5 + 15*a
**2*c*d**4/b**4 - 20*a*c**2*d**3/b**3 + 10*c**3*d**2/b**2) + x*(-a**5*d**5
+ 5*a**4*b*c*d**4 - 10*a**3*b**2*c**2*d**3 + 10*a**2*b**3*c**3*d**2 - 5*a*b
**4*c**4*d + b**5*c**5)/(3*a**2*b**5 + 3*a*b**6*x**3) + RootSum(729*_t**3*a
**5*b**16 - 2197*a**15*d**15 + 25350*a**14*b*c*d**14 - 132990*a**13*b**2*c*
*2*d**13 + 418280*a**12*b**3*c**3*d**12 - 874635*a**11*b**4*c**4*d**11 + 12
71886*a**10*b**5*c**5*d**10 - 1302400*a**9*b**6*c**6*d**9 + 922680*a**8*b**
7*c**7*d**8 - 422235*a**7*b**8*c**8*d**7 + 97570*a**6*b**9*c**9*d**6 + 7194
*a**5*b**10*c**10*d**5 - 10200*a**4*b**11*c**11*d**4 + 1435*a**3*b**12*c**1
2*d**3 + 330*a**2*b**13*c**13*d**2 - 60*a*b**14*c**14*d - 8*b**15*c**15, La
mbda(_t, _t*log(9*_t*a**2*b**5/(13*a**5*d**5 - 50*a**4*b*c*d**4 + 70*a**3*b
**2*c**2*d**3 - 40*a**2*b**3*c**3*d**2 + 5*a*b**4*c**4*d + 2*b**5*c**5) + x
))) + d**5*x**10/(10*b**2)
```

Giac [A]
time = 0.61, size = 529, normalized size = 1.65

$$-\frac{\sqrt{3}\left(2b^{5}c^{5}+5ab^{4}c^{4}d-40a^{2}b^{3}c^{3}d^{2}+70a^{3}b^{2}c^{2}d^{3}-50a^{4}bcd^{4}+13a^{5}d^{5}\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\left(-ab^{2}\right)^{\frac{2}{3}}ab^{4}}-\frac{\left(2b^{5}c^{5}+5ab^{4}c^{4}d-40a^{2}b^{3}c^{3}d^{2}+70a^{3}b^{2}c^{2}d^{3}-50a^{4}bcd^{4}+13a^{5}d^{5}\right)\log\left(x^{2}+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\left(-ab^{2}\right)^{\frac{2}{3}}ab^{4}}-\frac{\left(2b^{5}c^{5}+5ab^{4}c^{4}d-40a^{2}b^{3}c^{3}d^{2}+70a^{3}b^{2}c^{2}d^{3}-50a^{4}bcd^{4}+13a^{5}d^{5}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9a^{2}b^{5}}+\frac{b^{5}c^{5}x-5ab^{4}c^{4}dx+10a^{2}b^{3}c^{3}d^{2}x-10a^{3}b^{2}c^{2}d^{3}x+5a^{4}bcd^{4}x-a^{5}d^{5}x}{3\left(bx^{3}+a\right)ab^{5}}+\frac{14b^{18}d^{5}x^{10}+100b^{18}cd^{4}x^{7}-40ab^{17}d^{5}x^{7}+350b^{18}c^{2}d^{3}x^{4}-350ab^{17}cd^{4}x^{4}+105a^{2}b^{16}d^{5}x^{4}+1400b^{18}c^{3}d^{2}x-2800ab^{17}c^{2}d^{3}x+2100a^{2}b^{16}cd^{4}x-560a^{3}b^{15}d^{5}x}{140b^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^5/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] -1/9*sqrt(3)*(2*b^5*c^5 + 5*a*b^4*c^4*d - 40*a^2*b^3*c^3*d^2 + 70*a^3*b^2*c
^2*d^3 - 50*a^4*b*c*d^4 + 13*a^5*d^5)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3
))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*a*b^4) - 1/18*(2*b^5*c^5 + 5*a*b^4*c^4*d -
 40*a^2*b^3*c^3*d^2 + 70*a^3*b^2*c^2*d^3 - 50*a^4*b*c*d^4 + 13*a^5*d^5)*log
(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(2/3)*a*b^4) - 1/9*(2*b^5*c
^5 + 5*a*b^4*c^4*d - 40*a^2*b^3*c^3*d^2 + 70*a^3*b^2*c^2*d^3 - 50*a^4*b*c*d
^4 + 13*a^5*d^5)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^2*b^5) + 1/3*(b
^5*c^5*x - 5*a*b^4*c^4*d*x + 10*a^2*b^3*c^3*d^2*x - 10*a^3*b^2*c^2*d^3*x +
5*a^4*b*c*d^4*x - a^5*d^5*x)/((b*x^3 + a)*a*b^5) + 1/140*(14*b^18*d^5*x^10
+ 100*b^18*c*d^4*x^7 - 40*a*b^17*d^5*x^7 + 350*b^18*c^2*d^3*x^4 - 350*a*b^1
7*c*d^4*x^4 + 105*a^2*b^16*d^5*x^4 + 1400*b^18*c^3*d^2*x - 2800*a*b^17*c^2*
d^3*x + 2100*a^2*b^16*c*d^4*x - 560*a^3*b^15*d^5*x)/b^20
```

Mupad [B]
time = 0.39, size = 416, normalized size = 1.30

$$x\left(\frac{10\,c^3\,d^2}{b^2}-\frac{2\,a\,\left(\frac{2\,a\,\left(\frac{2\,a\,d^5}{b^3}-\frac{5\,c\,d^4}{b^2}\right)}{b}-\frac{a^2\,d^5}{b^4}+\frac{10\,c^2\,d^3}{b^2}\right)}{b}+\frac{a^2\,\left(\frac{2\,a\,d^5}{b^3}-\frac{5\,c\,d^4}{b^2}\right)}{b^2}\right)-x^7\,\left(\frac{2\,a\,d^5}{7\,b^3}-\frac{5\,c\,d^4}{7\,b^2}\right)+x^4\,\left(\frac{a\,\left(\frac{2\,a\,d^5}{b^3}-\frac{5\,c\,d^4}{b^2}\right)}{2\,b}-\frac{a^2\,d^5}{4\,b^4}+\frac{5\,c^2\,d^3}{2\,b^2}\right)+\frac{d^5\,x^{10}}{10\,b^2}-\frac{x\,\left(a^5\,d^5-5\,a^4\,b\,c\,d^4+10\,a^3\,b^2\,c^2\,d^3-10\,a^2\,b^3\,c^3\,d^2+5\,a\,b^4\,c^4\,d-b^5\,c^5\right)}{3\,a\,\left(b^6\,x^3+a\,b^5\right)}+\frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(a\,d-b\,c\right)^4\,\left(13\,a\,d+2\,b\,c\right)}{9\,a^{5/3}\,b^{16/3}}-\frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(a\,d-b\,c\right)^4\,\left(13\,a\,d+2\,b\,c\right)}{9\,a^{5/3}\,b^{16/3}}+\frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(a\,d-b\,c\right)^4\,\left(13\,a\,d+2\,b\,c\right)}{9\,a^{5/3}\,b^{16/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^5/(a + b*x^3)^2,x)
```

```
[Out] x*((10*c^3*d^2)/b^2 - (2*a*((2*a*((2*a*d^5)/b^3 - (5*c*d^4)/b^2))/b - (a^2*
d^5)/b^4 + (10*c^2*d^3)/b^2))/b + (a^2*((2*a*d^5)/b^3 - (5*c*d^4)/b^2))/b^2
) - x^7*((2*a*d^5)/(7*b^3) - (5*c*d^4)/(7*b^2)) + x^4*((a*((2*a*d^5)/b^3 -
(5*c*d^4)/b^2))/(2*b) - (a^2*d^5)/(4*b^4) + (5*c^2*d^3)/(2*b^2)) + (d^5*x^1
0)/(10*b^2) - (x*(a^5*d^5 - b^5*c^5 - 10*a^2*b^3*c^3*d^2 + 10*a^3*b^2*c^2*d
^3 + 5*a*b^4*c^4*d - 5*a^4*b*c*d^4))/(3*a*(a*b^5 + b^6*x^3)) + (log(b^(1/3)
*x + a^(1/3))*(a*d - b*c)^4*(13*a*d + 2*b*c))/(9*a^(5/3)*b^(16/3)) - (log(3
^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(a*d - b*
c)^4*(13*a*d + 2*b*c))/(9*a^(5/3)*b^(16/3)) + (log(3^(1/2)*a^(1/3)*1i + 2*b
^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(a*d - b*c)^4*(13*a*d + 2*b*c))/
(9*a^(5/3)*b^(16/3))
```

3.21 $\int \frac{(c+dx^3)^4}{(a+bx^3)^2} \, dx$

Optimal. Leaf size=267

$$\frac{d^2(6b^2c^2 - 8abcd + 3a^2d^2)\,x}{b^4} + \frac{d^3(2bc - ad)x^4}{2b^3} + \frac{d^4x^7}{7b^2} + \frac{(bc - ad)^4 x}{3ab^4\,(a + bx^3)} - \frac{2(bc - ad)^3(bc + 5ad)\tan^{-1}\left(\frac{\sqrt[3]{a} - 2}{\sqrt{3}}\right.}{3\sqrt{3}\,a^{5/3}b^{13/3}}$$

```
[Out] d^2*(3*a^2*d^2-8*a*b*c*d+6*b^2*c^2)*x/b^4+1/2*d^3*(-a*d+2*b*c)*x^4/b^3+1/7*
d^4*x^7/b^2+1/3*(-a*d+b*c)^4*x/a/b^4/(b*x^3+a)+2/9*(-a*d+b*c)^3*(5*a*d+b*c)
*ln(a^(1/3)+b^(1/3)*x)/a^(5/3)/b^(13/3)-1/9*(-a*d+b*c)^3*(5*a*d+b*c)*ln(a^(
2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(5/3)/b^(13/3)-2/9*(-a*d+b*c)^3*(5*a*
d+b*c)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(5/3)/b^(13/3)*3
^(1/2)
```

Rubi [A]
time = 0.17, antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {398, 393, 206, 31, 648, 631, 210, 642}

$$-\frac{2\text{ArcTan}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)(bc - ad)^3(5ad + bc)}{3\sqrt{3}\,a^{5/3}b^{13/3}} - \frac{(bc - ad)^3(5ad + bc)\log\left(a^{2/3} - \sqrt[3]{a}\,\sqrt[3]{b}\,x + b^{2/3}x^2\right)}{9a^{5/3}b^{13/3}} + \frac{2(bc - ad)^3(5ad + bc)\log\left(\sqrt[3]{a} + \sqrt[3]{b}\,x\right)}{9a^{5/3}b^{13/3}} + \frac{d^2x(3a^2d^2 - 8abcd + 6b^2c^2)}{b^4} + \frac{x(bc - ad)^4}{3ab^4\,(a + bx^3)} + \frac{d^3x^4(2bc - ad)}{2b^3} + \frac{d^4x^7}{7b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)^4/(a + b*x^3)^2,x]
```

```
[Out] (d^2*(6*b^2*c^2 - 8*a*b*c*d + 3*a^2*d^2)*x)/b^4 + (d^3*(2*b*c - a*d)*x^4)/(
2*b^3) + (d^4*x^7)/(7*b^2) + ((b*c - a*d)^4*x)/(3*a*b^4*(a + b*x^3)) - (2*(
b*c - a*d)^3*(b*c + 5*a*d)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))
])/(3*Sqrt[3]*a^(5/3)*b^(13/3)) + (2*(b*c - a*d)^3*(b*c + 5*a*d)*Log[a^(1/3
) + b^(1/3)*x])/(9*a^(5/3)*b^(13/3)) - ((b*c - a*d)^3*(b*c + 5*a*d)*Log[a^(
2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(9*a^(5/3)*b^(13/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{(c+dx^3)^4}{(a+bx^3)^2}\,dx = \int \left(\frac{d^2(6b^2c^2-8abcd+3a^2d^2)}{b^4} + \frac{2d^3(2bc-ad)x^3}{b^3} + \frac{d^4x^6}{b^2} + \frac{(bc-ad)^3(bc+3ad)+4bc}{b^4\,(a+bx^3)^2}\right.$$

$$= \frac{d^2(6b^2c^2-8abcd+3a^2d^2)\,x}{b^4} + \frac{d^3(2bc-ad)x^4}{2b^3} + \frac{d^4x^7}{7b^2} + \frac{\int \frac{(bc-ad)^3(bc+3ad)+4bd(bc-ad)^3x^3}{(a+bx^3)^2}\,d}{b^4}$$

$$= \frac{d^2(6b^2c^2-8abcd+3a^2d^2)\,x}{b^4} + \frac{d^3(2bc-ad)x^4}{2b^3} + \frac{d^4x^7}{7b^2} + \frac{(bc-ad)^4x}{3ab^4\,(a+bx^3)} + \frac{(2(bc-ad)^3}{}$$

$$= \frac{d^2(6b^2c^2-8abcd+3a^2d^2)\,x}{b^4} + \frac{d^3(2bc-ad)x^4}{2b^3} + \frac{d^4x^7}{7b^2} + \frac{(bc-ad)^4x}{3ab^4\,(a+bx^3)} + \frac{(2(bc-ad)^3}{}$$

$$= \frac{d^2(6b^2c^2-8abcd+3a^2d^2)\,x}{b^4} + \frac{d^3(2bc-ad)x^4}{2b^3} + \frac{d^4x^7}{7b^2} + \frac{(bc-ad)^4x}{3ab^4\,(a+bx^3)} + \frac{2(bc-ad)^3(}{}$$

$$= \frac{d^2(6b^2c^2-8abcd+3a^2d^2)\,x}{b^4} + \frac{d^3(2bc-ad)x^4}{2b^3} + \frac{d^4x^7}{7b^2} + \frac{(bc-ad)^4x}{3ab^4\,(a+bx^3)} + \frac{2(bc-ad)^3(}{}$$

$$= \frac{d^2(6b^2c^2-8abcd+3a^2d^2)\,x}{b^4} + \frac{d^3(2bc-ad)x^4}{2b^3} + \frac{d^4x^7}{7b^2} + \frac{(bc-ad)^4x}{3ab^4\,(a+bx^3)} - \frac{2(bc-ad)^3(}{}$$

Mathematica [A]
time = 0.15, size = 260, normalized size = 0.97

$$\frac{126\sqrt[3]{b}\,d^2(6b^2c^2-8abcd+3a^2d^2)\,x+63b^{4/3}d^3(2bc-ad)x^4+18b^{7/3}d^4x^7+\frac{42\sqrt[3]{b}\,(bc-ad)^4x}{a(a+bx^3)}+\frac{28\sqrt{3}\,(bc-ad)^3(bc+5ad)\tan^{-1}\left(\frac{-\sqrt[3]{a}+2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{a^{5/3}}+\frac{28(bc-ad)^3(bc+5ad)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{a^{5/3}}+\frac{14(-bc+ad)^3(bc+5ad)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{a^{5/3}}}{126b^{13/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3)^4/(a + b*x^3)^2,x]
```

```
[Out] (126*b^(1/3)*d^2*(6*b^2*c^2 - 8*a*b*c*d + 3*a^2*d^2)*x + 63*b^(4/3)*d^3*(2*
b*c - a*d)*x^4 + 18*b^(7/3)*d^4*x^7 + (42*b^(1/3)*(b*c - a*d)^4*x)/(a*(a +
b*x^3)) + (28*Sqrt[3]*(b*c - a*d)^3*(b*c + 5*a*d)*ArcTan[(-a^(1/3) + 2*b^(1
/3)*x)/(Sqrt[3]*a^(1/3))])/a^(5/3) + (28*(b*c - a*d)^3*(b*c + 5*a*d)*Log[a^
(1/3) + b^(1/3)*x])/a^(5/3) + (14*(-(b*c) + a*d)^3*(b*c + 5*a*d)*Log[a^(2/3
) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(5/3))/(126*b^(13/3))
```

Maple [A]
time = 0.28, size = 281, normalized size = 1.05

method	result

risch	$\frac{d^4x^7}{7b^2} - \frac{d^4a\,x^4}{2b^3} + \frac{d^3c\,x^4}{b^2} + \frac{3d^4a^2x}{b^4} - \frac{8d^3acx}{b^3} + \frac{6d^2c^2x}{b^2} + \frac{(a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+b^4c^4)x}{3a\,b^4(b\,x^3+a)} - \frac{2\left(\sum_{\_R=\text{RootOf}} \cdots\right.}{\cdots}$
default	$\frac{d^2\left(\frac{1}{7}b^2d^2x^7-\frac{1}{2}ab\,d^2x^4+b^2cd\,x^4+3a^2d^2x-8abcdx+6b^2c^2x\right)}{b^4} - \frac{-\frac{\left(a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+b^4c^4\right)x}{3a\left(b\,x^3+a\right)}+\frac{2\left(5a^4d^4-14a^3bc\,d^3+12\cdots\right.}{\cdots}}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^4/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] d^2/b^4*(1/7*b^2*d^2*x^7-1/2*a*b*d^2*x^4+b^2*c*d*x^4+3*a^2*d^2*x-8*a*b*c*d*
x+6*b^2*c^2*x)-1/b^4*(-1/3*(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3
*c^3*d+b^4*c^4)/a*x/(b*x^3+a)+2/3*(5*a^4*d^4-14*a^3*b*c*d^3+12*a^2*b^2*c^2*
d^2-2*a*b^3*c^3*d-b^4*c^4)/a*(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/
b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan
(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))))
```

Maxima [A]
time = 0.51, size = 397, normalized size = 1.49

$$\frac{(b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4)x}{3\,(ab^5x^3+a^2b^4)}+\frac{2\,b^2d^4x^7+7\,(2\,b^2cd^3-abd^4)x^4+14\,(6\,b^2c^2d^2-8\,abcd^3+3\,a^2d^4)x}{14\,b^4}+\frac{2\,\sqrt{3}\,(b^4c^4+2\,ab^3c^3d-12\,a^2b^2c^2d^2+14\,a^3bcd^3-5\,a^4d^4)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,ab^5\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{(b^4c^4+2\,ab^3c^3d-12\,a^2b^2c^2d^2+14\,a^3bcd^3-5\,a^4d^4)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\,ab^5\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{2\,(b^4c^4+2\,ab^3c^3d-12\,a^2b^2c^2d^2+14\,a^3bcd^3-5\,a^4d^4)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,ab^5\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^4/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] 1/3*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)
*x/(a*b^5*x^3 + a^2*b^4) + 1/14*(2*b^2*d^4*x^7 + 7*(2*b^2*c*d^3 - a*b*d^4)*
x^4 + 14*(6*b^2*c^2*d^2 - 8*a*b*c*d^3 + 3*a^2*d^4)*x)/b^4 + 2/9*sqrt(3)*(b^
4*c^4 + 2*a*b^3*c^3*d - 12*a^2*b^2*c^2*d^2 + 14*a^3*b*c*d^3 - 5*a^4*d^4)*ar
ctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a*b^5*(a/b)^(2/3)) - 1/9
*(b^4*c^4 + 2*a*b^3*c^3*d - 12*a^2*b^2*c^2*d^2 + 14*a^3*b*c*d^3 - 5*a^4*d^4
)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a*b^5*(a/b)^(2/3)) + 2/9*(b^4*c^4
 + 2*a*b^3*c^3*d - 12*a^2*b^2*c^2*d^2 + 14*a^3*b*c*d^3 - 5*a^4*d^4)*log(x +
 (a/b)^(1/3))/(a*b^5*(a/b)^(2/3))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 639 vs. $2(224) = 448$.

time = 3.61, size = 1316, normalized size = 4.93

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^4/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] [1/126*(18*a^3*b^4*d^4*x^10 + 9*(14*a^3*b^4*c*d^3 - 5*a^4*b^3*d^4)*x^7 + 63
*(12*a^3*b^4*c^2*d^2 - 14*a^4*b^3*c*d^3 + 5*a^5*b^2*d^4)*x^4 - 42*sqrt(1/3)
*(a^2*b^5*c^4 + 2*a^3*b^4*c^3*d - 12*a^4*b^3*c^2*d^2 + 14*a^5*b^2*c*d^3 - 5
*a^6*b*d^4 + (a*b^6*c^4 + 2*a^2*b^5*c^3*d - 12*a^3*b^4*c^2*d^2 + 14*a^4*b^3
*c*d^3 - 5*a^5*b^2*d^4)*x^3)*sqrt((-a^2*b)^(1/3)/b)*log((2*a*b*x^3 + 3*(-a^
2*b)^(1/3)*a*x - a^2 - 3*sqrt(1/3)*(2*a*b*x^2 + (-a^2*b)^(2/3)*x + (-a^2*b)
^(1/3)*a)*sqrt((-a^2*b)^(1/3)/b))/(b*x^3 + a)) - 14*(a*b^4*c^4 + 2*a^2*b^3*
c^3*d - 12*a^3*b^2*c^2*d^2 + 14*a^4*b*c*d^3 - 5*a^5*d^4 + (b^5*c^4 + 2*a*b^
4*c^3*d - 12*a^2*b^3*c^2*d^2 + 14*a^3*b^2*c*d^3 - 5*a^4*b*d^4)*x^3)*(-a^2*b
)^(2/3)*log(a*b*x^2 - (-a^2*b)^(2/3)*x - (-a^2*b)^(1/3)*a) + 28*(a*b^4*c^4
+ 2*a^2*b^3*c^3*d - 12*a^3*b^2*c^2*d^2 + 14*a^4*b*c*d^3 - 5*a^5*d^4 + (b^5*
c^4 + 2*a*b^4*c^3*d - 12*a^2*b^3*c^2*d^2 + 14*a^3*b^2*c*d^3 - 5*a^4*b*d^4)*
x^3)*(-a^2*b)^(2/3)*log(a*b*x + (-a^2*b)^(2/3)) + 42*(a^2*b^5*c^4 - 4*a^3*b
^4*c^3*d + 24*a^4*b^3*c^2*d^2 - 28*a^5*b^2*c*d^3 + 10*a^6*b*d^4)*x)/(a^3*b^
6*x^3 + a^4*b^5), 1/126*(18*a^3*b^4*d^4*x^10 + 9*(14*a^3*b^4*c*d^3 - 5*a^4*
b^3*d^4)*x^7 + 63*(12*a^3*b^4*c^2*d^2 - 14*a^4*b^3*c*d^3 + 5*a^5*b^2*d^4)*x
^4 + 84*sqrt(1/3)*(a^2*b^5*c^4 + 2*a^3*b^4*c^3*d - 12*a^4*b^3*c^2*d^2 + 14*
a^5*b^2*c*d^3 - 5*a^6*b*d^4 + (a*b^6*c^4 + 2*a^2*b^5*c^3*d - 12*a^3*b^4*c^2
*d^2 + 14*a^4*b^3*c*d^3 - 5*a^5*b^2*d^4)*x^3)*sqrt(-(-a^2*b)^(1/3)/b)*arcta
n(sqrt(1/3)*(2*(-a^2*b)^(2/3)*x + (-a^2*b)^(1/3)*a)*sqrt(-(-a^2*b)^(1/3)/b)
/a^2) - 14*(a*b^4*c^4 + 2*a^2*b^3*c^3*d - 12*a^3*b^2*c^2*d^2 + 14*a^4*b*c*d
^3 - 5*a^5*d^4 + (b^5*c^4 + 2*a*b^4*c^3*d - 12*a^2*b^3*c^2*d^2 + 14*a^3*b^2
*c*d^3 - 5*a^4*b*d^4)*x^3)*(-a^2*b)^(2/3)*log(a*b*x^2 - (-a^2*b)^(2/3)*x -
(-a^2*b)^(1/3)*a) + 28*(a*b^4*c^4 + 2*a^2*b^3*c^3*d - 12*a^3*b^2*c^2*d^2 +
14*a^4*b*c*d^3 - 5*a^5*d^4 + (b^5*c^4 + 2*a*b^4*c^3*d - 12*a^2*b^3*c^2*d^2
+ 14*a^3*b^2*c*d^3 - 5*a^4*b*d^4)*x^3)*(-a^2*b)^(2/3)*log(a*b*x + (-a^2*b)^
(2/3)) + 42*(a^2*b^5*c^4 - 4*a^3*b^4*c^3*d + 24*a^4*b^3*c^2*d^2 - 28*a^5*b^
2*c*d^3 + 10*a^6*b*d^4)*x)/(a^3*b^6*x^3 + a^4*b^5)]
```

Sympy [A]
time = 11.87, size = 405, normalized size = 1.52

$$x^4\left(-\frac{ad^4}{2b^3}+\frac{cd^3}{b^2}\right)+x\left(\frac{3a^2d^4}{b^4}-\frac{8acd^3}{b^3}+\frac{6c^2d^2}{b^2}\right)+\frac{x(a^4d^4-4a^3bcd^3+6a^2b^2c^2d^2-4ab^3c^3d+b^4c^4)}{3a^2b^4+3ab^5x^3}+\operatorname{RootSum}\left(729t^3a^5b^{13}+1000a^{12}d^{12}-8400a^{11}bcd^{11}+30720a^{10}b^2c^2d^{10}-63472a^9b^3c^3d^9+79848a^8b^4c^4d^8-60192a^7b^5c^5d^7+22848a^6b^6c^6d^6+288a^5b^7c^7d^5-3528a^4b^8c^8d^4+752a^3b^9c^9d^3+192a^2b^{10}c^{10}d^2-48ab^{11}c^{11}d-8b^{12}c^{12},\left(t\mapsto t\log\left(-\frac{9ta^2b^4}{10a^4d^4-28a^3bcd^3+24a^2b^2c^2d^2-4ab^3c^3d-2b^4c^4}+x\right)\right)\right)+\frac{d^4x^7}{7b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**4/(b*x**3+a)**2,x)
```

```
[Out] x**4*(-a*d**4/(2*b**3) + c*d**3/b**2) + x*(3*a**2*d**4/b**4 - 8*a*c*d**3/b*
*3 + 6*c**2*d**2/b**2) + x*(a**4*d**4 - 4*a**3*b*c*d**3 + 6*a**2*b**2*c**2*
```

```
d**2 - 4*a*b**3*c**3*d + b**4*c**4)/(3*a**2*b**4 + 3*a*b**5*x**3) + RootSum
(729*_t**3*a**5*b**13 + 1000*a**12*d**12 - 8400*a**11*b*c*d**11 + 30720*a**
10*b**2*c**2*d**10 - 63472*a**9*b**3*c**3*d**9 + 79848*a**8*b**4*c**4*d**8
- 60192*a**7*b**5*c**5*d**7 + 22848*a**6*b**6*c**6*d**6 + 288*a**5*b**7*c**
7*d**5 - 3528*a**4*b**8*c**8*d**4 + 752*a**3*b**9*c**9*d**3 + 192*a**2*b**1
0*c**10*d**2 - 48*a*b**11*c**11*d - 8*b**12*c**12, Lambda(_t, _t*log(-9*_t*
a**2*b**4/(10*a**4*d**4 - 28*a**3*b*c*d**3 + 24*a**2*b**2*c**2*d**2 - 4*a*b
**3*c**3*d - 2*b**4*c**4) + x))) + d**4*x**7/(7*b**2)
```

Giac [A]
time = 0.74, size = 412, normalized size = 1.54

$$-\frac{2\sqrt{3}\,(b^4c^4+2\,ab^3c^3d-12\,a^2b^2c^2d^2+14\,a^3bcd^3-5\,a^4d^4)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,(-ab^2)^{\frac{2}{3}}\,ab^3}-\frac{(b^4c^4+2\,ab^3c^3d-12\,a^2b^2c^2d^2+14\,a^3bcd^3-5\,a^4d^4)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\,(-ab^2)^{\frac{2}{3}}\,ab^3}-\frac{2\,(b^4c^4+2\,ab^3c^3d-12\,a^2b^2c^2d^2+14\,a^3bcd^3-5\,a^4d^4)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^2b^4}+\frac{b^4c^4x-4\,ab^3c^3dx+6\,a^2b^2c^2d^2x-4\,a^3bcd^3x+a^4d^4x}{3\,(bx^3+a)ab^4}+\frac{2\,b^{12}d^4x^7+14\,b^{12}cd^3x^4-7\,ab^{11}d^4x^4+84\,b^{12}c^2d^2x-112\,ab^{11}cd^3x+42\,a^2b^{10}d^4x}{14\,b^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^4/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] -2/9*sqrt(3)*(b^4*c^4 + 2*a*b^3*c^3*d - 12*a^2*b^2*c^2*d^2 + 14*a^3*b*c*d^3
 - 5*a^4*d^4)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^
2)^(2/3)*a*b^3) - 1/9*(b^4*c^4 + 2*a*b^3*c^3*d - 12*a^2*b^2*c^2*d^2 + 14*a^
3*b*c*d^3 - 5*a^4*d^4)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(
2/3)*a*b^3) - 2/9*(b^4*c^4 + 2*a*b^3*c^3*d - 12*a^2*b^2*c^2*d^2 + 14*a^3*b*
c*d^3 - 5*a^4*d^4)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^2*b^4) + 1/3*
(b^4*c^4*x - 4*a*b^3*c^3*d*x + 6*a^2*b^2*c^2*d^2*x - 4*a^3*b*c*d^3*x + a^4*
d^4*x)/((b*x^3 + a)*a*b^4) + 1/14*(2*b^12*d^4*x^7 + 14*b^12*c*d^3*x^4 - 7*a
*b^11*d^4*x^4 + 84*b^12*c^2*d^2*x - 112*a*b^11*c*d^3*x + 42*a^2*b^10*d^4*x)
/b^14
```

Mupad [B]
time = 1.49, size = 302, normalized size = 1.13

$$x\left(\frac{2\,a\left(\frac{2\,a\,d^4}{b^3}-\frac{4\,c\,d^3}{b^2}\right)}{b}-\frac{a^2\,d^4}{b^4}+\frac{6\,c^2\,d^2}{b^2}\right)-x^4\left(\frac{a\,d^4}{2\,b^3}-\frac{c\,d^3}{b^2}\right)+\frac{d^4\,x^7}{7\,b^2}+\frac{x\,(a^4\,d^4-4\,a^3\,b\,c\,d^3+6\,a^2\,b^2\,c^2\,d^2-4\,a\,b^3\,c^3\,d+b^4\,c^4)}{3\,a\,(b^5\,x^3+a\,b^4)}-\frac{2\,\ln\left(b^{1/3}\,x+a^{1/3}\right)\,(a\,d-b\,c)^3\,(5\,a\,d+b\,c)}{9\,a^{5/3}\,b^{13/3}}+\frac{2\,\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(a\,d-b\,c)^3\,(5\,a\,d+b\,c)}{9\,a^{5/3}\,b^{13/3}}-\frac{2\,\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(a\,d-b\,c)^3\,(5\,a\,d+b\,c)}{9\,a^{5/3}\,b^{13/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^4/(a + b*x^3)^2,x)
```

```
[Out] x*((2*a*((2*a*d^4)/b^3 - (4*c*d^3)/b^2))/b - (a^2*d^4)/b^4 + (6*c^2*d^2)/b^
2) - x^4*((a*d^4)/(2*b^3) - (c*d^3)/b^2) + (d^4*x^7)/(7*b^2) + (x*(a^4*d^4
+ b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3))/(3*a*(a*b^4
 + b^5*x^3)) - (2*log(b^(1/3)*x + a^(1/3))*(a*d - b*c)^3*(5*a*d + b*c))/(9*
a^(5/3)*b^(13/3)) + (2*log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^
(1/2)*1i)/2 + 1/2)*(a*d - b*c)^3*(5*a*d + b*c))/(9*a^(5/3)*b^(13/3)) - (2*l
og(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(a*d
- b*c)^3*(5*a*d + b*c))/(9*a^(5/3)*b^(13/3))
```

3.22 $\int \frac{(c+dx^3)^3}{(a+bx^3)^2}\,dx$

Optimal. Leaf size=234

$$\frac{d^2(3bc-2ad)x}{b^3}+\frac{d^3x^4}{4b^2}+\frac{(bc-ad)^3x}{3ab^3(a+bx^3)}-\frac{(bc-ad)^2(2bc+7ad)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{5/3}b^{10/3}}+\frac{(bc-ad)^2(2bc+7ad}{9a^{5/3}}$$

```
[Out] d^2*(-2*a*d+3*b*c)*x/b^3+1/4*d^3*x^4/b^2+1/3*(-a*d+b*c)^3*x/a/b^3/(b*x^3+a)
+1/9*(-a*d+b*c)^2*(7*a*d+2*b*c)*ln(a^(1/3)+b^(1/3)*x)/a^(5/3)/b^(10/3)-1/18
*(-a*d+b*c)^2*(7*a*d+2*b*c)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(5/
3)/b^(10/3)-1/9*(-a*d+b*c)^2*(7*a*d+2*b*c)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)
/a^(1/3)*3^(1/2))/a^(5/3)/b^(10/3)*3^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {398, 393, 206, 31, 648, 631, 210, 642}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)(bc-ad)^2(7ad+2bc)}{3\sqrt{3}\,a^{5/3}b^{10/3}}-\frac{(bc-ad)^2(7ad+2bc)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{18a^{5/3}b^{10/3}}+\frac{(bc-ad)^2(7ad+2bc)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9a^{5/3}b^{10/3}}+\frac{d^2x(3bc-2ad)}{b^3}+\frac{x(bc-ad)^3}{3ab^3(a+bx^3)}+\frac{d^3x^4}{4b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)^3/(a + b*x^3)^2,x]
```

```
[Out] (d^2*(3*b*c - 2*a*d)*x)/b^3 + (d^3*x^4)/(4*b^2) + ((b*c - a*d)^3*x)/(3*a*b^
3*(a + b*x^3)) - ((b*c - a*d)^2*(2*b*c + 7*a*d)*ArcTan[(a^(1/3) - 2*b^(1/3)
*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(5/3)*b^(10/3)) + ((b*c - a*d)^2*(2*b*
c + 7*a*d)*Log[a^(1/3) + b^(1/3)*x])/(9*a^(5/3)*b^(10/3)) - ((b*c - a*d)^2*
(2*b*c + 7*a*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^(5/3)
*b^(10/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{(c+dx^3)^3}{(a+bx^3)^2}\,dx = \int \left(\frac{d^2(3bc-2ad)}{b^3}+\frac{d^3x^3}{b^2}+\frac{(bc-ad)^2(bc+2ad)+3bd(bc-ad)^2x^3}{b^3\,(a+bx^3)^2}\right)dx$$

$$= \frac{d^2(3bc-2ad)x}{b^3}+\frac{d^3x^4}{4b^2}+\frac{\int \frac{(bc-ad)^2(bc+2ad)+3bd(bc-ad)^2x^3}{(a+bx^3)^2}\,dx}{b^3}$$

$$= \frac{d^2(3bc-2ad)x}{b^3}+\frac{d^3x^4}{4b^2}+\frac{(bc-ad)^3x}{3ab^3\,(a+bx^3)}+\frac{((bc-ad)^2(2bc+7ad))\int \frac{1}{a+bx^3}\,dx}{3ab^3}$$

$$= \frac{d^2(3bc-2ad)x}{b^3}+\frac{d^3x^4}{4b^2}+\frac{(bc-ad)^3x}{3ab^3\,(a+bx^3)}+\frac{((bc-ad)^2(2bc+7ad))\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx}{9a^{5/3}b^3}+ \cdots$$

$$= \frac{d^2(3bc-2ad)x}{b^3}+\frac{d^3x^4}{4b^2}+\frac{(bc-ad)^3x}{3ab^3\,(a+bx^3)}+\frac{(bc-ad)^2(2bc+7ad)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9a^{5/3}b^{10/3}}- \cdots$$

$$= \frac{d^2(3bc-2ad)x}{b^3}+\frac{d^3x^4}{4b^2}+\frac{(bc-ad)^3x}{3ab^3\,(a+bx^3)}+\frac{(bc-ad)^2(2bc+7ad)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9a^{5/3}b^{10/3}}- \cdots$$

$$= \frac{d^2(3bc-2ad)x}{b^3}+\frac{d^3x^4}{4b^2}+\frac{(bc-ad)^3x}{3ab^3\,(a+bx^3)}-\frac{(bc-ad)^2(2bc+7ad)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{5/3}b^{10/3}} \cdots$$

Mathematica [A]
time = 0.11, size = 227, normalized size = 0.97

$$\frac{36\sqrt[3]{b}\,d^2(3bc-2ad)x+9b^{4/3}d^3x^4+\frac{12\sqrt[3]{b}\,(bc-ad)^3x}{a(a+bx^3)}+\frac{4\sqrt{3}\,(bc-ad)^2(2bc+7ad)\tan^{-1}\left(\frac{-\sqrt[3]{a}+2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{a^{5/3}}+\frac{4(bc-ad)^2(2bc+7ad)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{a^{5/3}}-\frac{2(bc-ad)^2(2bc+7ad)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{a^{5/3}}}{36b^{10/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3)^3/(a + b*x^3)^2,x]
```

```
[Out] (36*b^(1/3)*d^2*(3*b*c - 2*a*d)*x + 9*b^(4/3)*d^3*x^4 + (12*b^(1/3)*(b*c -
a*d)^3*x)/(a*(a + b*x^3)) + (4*Sqrt[3]*(b*c - a*d)^2*(2*b*c + 7*a*d)*ArcTan
[(-a^(1/3) + 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/a^(5/3) + (4*(b*c - a*d)^2*(2
*b*c + 7*a*d)*Log[a^(1/3) + b^(1/3)*x])/a^(5/3) - (2*(b*c - a*d)^2*(2*b*c +
 7*a*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(5/3))/(36*b^(10/
3))
```

Maple [A]
time = 0.28, size = 216, normalized size = 0.92

method	result

risch	$\frac{d^3x^4}{4b^2}-\frac{2d^3ax}{b^3}+\frac{3d^2cx}{b^2}-\frac{(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3)x}{3a\,b^3(b\,x^3+a)}+\frac{\sum_{\_R=\mathrm{RootOf}(b\_Z^3+a)}\frac{(7a^3d^3-12a^2bc\,d^2+3a\,b^2c^2d+2b^3c^3)\ln(x-\dots}{\_R^2}}{9b^4a}$
default	$-\frac{d^2\left(-\frac{1}{4}bd\,x^4+2adx-3bcx\right)}{b^3}+\frac{-\frac{(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3)x}{3a(b\,x^3+a)}+\frac{(7a^3d^3-12a^2bc\,d^2+3a\,b^2c^2d+2b^3c^3)\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)\dots\right)}{6b\left(\dots\right.}\dots\right.}{3a}}{b^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^3/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -d^2/b^3*(-1/4*b*d*x^4+2*a*d*x-3*b*c*x)+1/b^3*(-1/3*(a^3*d^3-3*a^2*b*c*d^2+
3*a*b^2*c^2*d-b^3*c^3)/a*x/(b*x^3+a)+1/3*(7*a^3*d^3-12*a^2*b*c*d^2+3*a*b^2*
c^2*d+2*b^3*c^3)/a*(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*l
n(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/
2)*(2/(a/b)^(1/3)*x-1))))
```

Maxima [A]
time = 0.50, size = 306, normalized size = 1.31

$$\frac{(b^3c^3-3ab^2c^2d+3a^2bcd^2-a^3d^3)x}{3(ab^4x^3+a^2b^3)}+\frac{bd^3x^4+4(3bcd^2-2ad^3)x}{4b^3}+\frac{\sqrt{3}(2b^3c^3+3ab^2c^2d-12a^2bcd^2+7a^3d^3)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9ab^4\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{(2b^3c^3+3ab^2c^2d-12a^2bcd^2+7a^3d^3)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18ab^4\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{(2b^3c^3+3ab^2c^2d-12a^2bcd^2+7a^3d^3)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9ab^4\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^3/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] 1/3*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*x/(a*b^4*x^3 + a^2*
b^3) + 1/4*(b*d^3*x^4 + 4*(3*b*c*d^2 - 2*a*d^3)*x)/b^3 + 1/9*sqrt(3)*(2*b^3
*c^3 + 3*a*b^2*c^2*d - 12*a^2*b*c*d^2 + 7*a^3*d^3)*arctan(1/3*sqrt(3)*(2*x
- (a/b)^(1/3))/(a/b)^(1/3))/(a*b^4*(a/b)^(2/3)) - 1/18*(2*b^3*c^3 + 3*a*b^2
*c^2*d - 12*a^2*b*c*d^2 + 7*a^3*d^3)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))
/(a*b^4*(a/b)^(2/3)) + 1/9*(2*b^3*c^3 + 3*a*b^2*c^2*d - 12*a^2*b*c*d^2 + 7*
a^3*d^3)*log(x + (a/b)^(1/3))/(a*b^4*(a/b)^(2/3))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 493 vs. $2(193) = 386$.
time = 2.59, size = 1027, normalized size = 4.39

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^3/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] [1/36*(9*a^3*b^3*d^3*x^7 + 9*(12*a^3*b^3*c*d^2 - 7*a^4*b^2*d^3)*x^4 + 6*sqr
t(1/3)*(2*a^2*b^4*c^3 + 3*a^3*b^3*c^2*d - 12*a^4*b^2*c*d^2 + 7*a^5*b*d^3 +
(2*a*b^5*c^3 + 3*a^2*b^4*c^2*d - 12*a^3*b^3*c*d^2 + 7*a^4*b^2*d^3)*x^3)*sqr
t(-(a^2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*(a^2*b)^(1/3)*a*x - a^2 + 3*sqrt(1/3
)*(2*a*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt(-(a^2*b)^(1/3)/b))/(
b*x^3 + a)) - 2*(2*a*b^3*c^3 + 3*a^2*b^2*c^2*d - 12*a^3*b*c*d^2 + 7*a^4*d^3
 + (2*b^4*c^3 + 3*a*b^3*c^2*d - 12*a^2*b^2*c*d^2 + 7*a^3*b*d^3)*x^3)*(a^2*b
)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 4*(2*a*b^3*c^3 +
 3*a^2*b^2*c^2*d - 12*a^3*b*c*d^2 + 7*a^4*d^3 + (2*b^4*c^3 + 3*a*b^3*c^2*d
- 12*a^2*b^2*c*d^2 + 7*a^3*b*d^3)*x^3)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2
/3)) + 12*(a^2*b^4*c^3 - 3*a^3*b^3*c^2*d + 12*a^4*b^2*c*d^2 - 7*a^5*b*d^3)*
x)/(a^3*b^5*x^3 + a^4*b^4), 1/36*(9*a^3*b^3*d^3*x^7 + 9*(12*a^3*b^3*c*d^2 -
 7*a^4*b^2*d^3)*x^4 + 12*sqrt(1/3)*(2*a^2*b^4*c^3 + 3*a^3*b^3*c^2*d - 12*a^
4*b^2*c*d^2 + 7*a^5*b*d^3 + (2*a*b^5*c^3 + 3*a^2*b^4*c^2*d - 12*a^3*b^3*c*d
^2 + 7*a^4*b^2*d^3)*x^3)*sqrt((a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(a^2*b)^
(2/3)*x - (a^2*b)^(1/3)*a)*sqrt((a^2*b)^(1/3)/b)/a^2) - 2*(2*a*b^3*c^3 + 3*
a^2*b^2*c^2*d - 12*a^3*b*c*d^2 + 7*a^4*d^3 + (2*b^4*c^3 + 3*a*b^3*c^2*d - 1
2*a^2*b^2*c*d^2 + 7*a^3*b*d^3)*x^3)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/
3)*x + (a^2*b)^(1/3)*a) + 4*(2*a*b^3*c^3 + 3*a^2*b^2*c^2*d - 12*a^3*b*c*d^2
 + 7*a^4*d^3 + (2*b^4*c^3 + 3*a*b^3*c^2*d - 12*a^2*b^2*c*d^2 + 7*a^3*b*d^3)
*x^3)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)) + 12*(a^2*b^4*c^3 - 3*a^3*b^
3*c^2*d + 12*a^4*b^2*c*d^2 - 7*a^5*b*d^3)*x)/(a^3*b^5*x^3 + a^4*b^4)]
```

Sympy [A]
time = 1.47, size = 291, normalized size = 1.24

$$x\left(-\frac{2ad^3}{b^3}+\frac{3cd^2}{b^2}\right)+\frac{x(-a^3d^3+3a^2bcd^2-3ab^2c^2d+b^3c^3)}{3a^2b^3+3ab^4x^3}+\operatorname{RootSum}\left(729t^3a^5b^{10}-343a^9d^9+1764a^8bcd^8-3465a^7b^2c^2d^7+2946a^6b^3c^3d^6-477a^5b^4c^4d^5-792a^4b^5c^5d^4+321a^3b^6c^6d^3+90a^2b^7c^7d^2-36ab^8c^8d-8b^9c^9,\left(t\mapsto t\log\left(\frac{9ta^2b^3}{7a^3d^3-12a^2bcd^2+3ab^2c^2d+2b^3c^3}+x\right)\right)\right)+\frac{d^3x^4}{4b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**3/(b*x**3+a)**2,x)
```

```
[Out] x*(-2*a*d**3/b**3 + 3*c*d**2/b**2) + x*(-a**3*d**3 + 3*a**2*b*c*d**2 - 3*a*
b**2*c**2*d + b**3*c**3)/(3*a**2*b**3 + 3*a*b**4*x**3) + RootSum(729*_t**3*
a**5*b**10 - 343*a**9*d**9 + 1764*a**8*b*c*d**8 - 3465*a**7*b**2*c**2*d**7
+ 2946*a**6*b**3*c**3*d**6 - 477*a**5*b**4*c**4*d**5 - 792*a**4*b**5*c**5*d
**4 + 321*a**3*b**6*c**6*d**3 + 90*a**2*b**7*c**7*d**2 - 36*a*b**8*c**8*d -
 8*b**9*c**9, Lambda(_t, _t*log(9*_t*a**2*b**3/(7*a**3*d**3 - 12*a**2*b*c*d
**2 + 3*a*b**2*c**2*d + 2*b**3*c**3) + x))) + d**3*x**4/(4*b**2)
```

Giac [A]
time = 0.74, size = 319, normalized size = 1.36

$$-\frac{\sqrt{3}\,(2\,b^3c^3+3\,ab^2c^2d-12\,a^2bcd^2+7\,a^3d^3)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,(-ab^2)^{\frac{2}{3}}\,ab^2}-\frac{(2\,b^3c^3+3\,ab^2c^2d-12\,a^2bcd^2+7\,a^3d^3)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,(-ab^2)^{\frac{2}{3}}\,ab^2}-\frac{(2\,b^3c^3+3\,ab^2c^2d-12\,a^2bcd^2+7\,a^3d^3)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^2b^3}+\frac{b^3c^3x-3\,ab^2c^2dx+3\,a^2bcd^2x-a^3d^3x}{3\,(bx^3+a)ab^3}+\frac{b^6d^3x^4+12\,b^6cd^2x-8\,ab^5d^3x}{4\,b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^3/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] -1/9*sqrt(3)*(2*b^3*c^3 + 3*a*b^2*c^2*d - 12*a^2*b*c*d^2 + 7*a^3*d^3)*arcta
n(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*a*b^2) - 1
/18*(2*b^3*c^3 + 3*a*b^2*c^2*d - 12*a^2*b*c*d^2 + 7*a^3*d^3)*log(x^2 + x*(-
a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(2/3)*a*b^2) - 1/9*(2*b^3*c^3 + 3*a*b^
2*c^2*d - 12*a^2*b*c*d^2 + 7*a^3*d^3)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)
))/(a^2*b^3) + 1/3*(b^3*c^3*x - 3*a*b^2*c^2*d*x + 3*a^2*b*c*d^2*x - a^3*d^3
*x)/((b*x^3 + a)*a*b^3) + 1/4*(b^6*d^3*x^4 + 12*b^6*c*d^2*x - 8*a*b^5*d^3*x
)/b^8
```

Mupad [B]
time = 0.30, size = 240, normalized size = 1.03

$$\frac{d^3\,x^4}{4\,b^2}-x\,\left(\frac{2\,a\,d^3}{b^3}-\frac{3\,c\,d^2}{b^2}\right)-\frac{x\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3\right)}{3\,a\,\left(b^4\,x^3+a\,b^3\right)}+\frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,{\left(a\,d-b\,c\right)}^2\,\left(7\,a\,d+2\,b\,c\right)}{9\,a^{5/3}\,b^{10/3}}-\frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,{\left(a\,d-b\,c\right)}^2\,\left(7\,a\,d+2\,b\,c\right)}{9\,a^{5/3}\,b^{10/3}}+\frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,{\left(a\,d-b\,c\right)}^2\,\left(7\,a\,d+2\,b\,c\right)}{9\,a^{5/3}\,b^{10/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^3/(a + b*x^3)^2,x)
```

```
[Out] (d^3*x^4)/(4*b^2) - x*((2*a*d^3)/b^3 - (3*c*d^2)/b^2) - (x*(a^3*d^3 - b^3*c
^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))/(3*a*(a*b^3 + b^4*x^3)) + (log(b^(1/3)
*x + a^(1/3))*(a*d - b*c)^2*(7*a*d + 2*b*c))/(9*a^(5/3)*b^(10/3)) - (log(3^
(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(a*d - b*c
)^2*(7*a*d + 2*b*c))/(9*a^(5/3)*b^(10/3)) + (log(3^(1/2)*a^(1/3)*1i + 2*b^(
1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(a*d - b*c)^2*(7*a*d + 2*b*c))/(9*
a^(5/3)*b^(10/3))
```

3.23 $\int \frac{(c+dx^3)^2}{(a+bx^3)^2} dx$

Optimal. Leaf size=203

$$\frac{d^2x}{b^2}+\frac{(bc-ad)^2x}{3ab^2(a+bx^3)}-\frac{2(bc-ad)(bc+2ad)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{5/3}b^{7/3}}+\frac{2(bc-ad)(bc+2ad)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9a^{5/3}b^{7/3}}$$

```
[Out] d^2*x/b^2+1/3*(-a*d+b*c)^2*x/a/b^2/(b*x^3+a)+2/9*(-a*d+b*c)*(2*a*d+b*c)*ln(
a^(1/3)+b^(1/3)*x)/a^(5/3)/b^(7/3)-1/9*(-a*d+b*c)*(2*a*d+b*c)*ln(a^(2/3)-a^
(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(5/3)/b^(7/3)-2/9*(-a*d+b*c)*(2*a*d+b*c)*arc
tan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(5/3)/b^(7/3)*3^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$,
Rules used = {398, 393, 206, 31, 648, 631, 210, 642}

$$-\frac{2\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)(bc-ad)(2ad+bc)}{3\sqrt{3}\,a^{5/3}b^{7/3}}-\frac{(bc-ad)(2ad+bc)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{9a^{5/3}b^{7/3}}+\frac{2(bc-ad)(2ad+bc)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9a^{5/3}b^{7/3}}+\frac{x(bc-ad)^2}{3ab^2(a+bx^3)}+\frac{d^2x}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)^2/(a + b*x^3)^2,x]
```

```
[Out] (d^2*x)/b^2 + ((b*c - a*d)^2*x)/(3*a*b^2*(a + b*x^3)) - (2*(b*c - a*d)*(b*c
 + 2*a*d)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(
5/3)*b^(7/3)) + (2*(b*c - a*d)*(b*c + 2*a*d)*Log[a^(1/3) + b^(1/3)*x])/(9*a
^(5/3)*b^(7/3)) - ((b*c - a*d)*(b*c + 2*a*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*
x + b^(2/3)*x^2])/(9*a^(5/3)*b^(7/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^2}\,dx = \int \left(\frac{d^2}{b^2}+\frac{b^2c^2-a^2d^2+2bd(bc-ad)x^3}{b^2(a+bx^3)^2}\right)dx$$

$$= \frac{d^2x}{b^2}+\frac{\int \frac{b^2c^2-a^2d^2+2bd(bc-ad)x^3}{(a+bx^3)^2}\,dx}{b^2}$$

$$= \frac{d^2x}{b^2}+\frac{(bc-ad)^2x}{3ab^2(a+bx^3)}+\frac{(2(bc-ad)(bc+2ad))\int \frac{1}{a+bx^3}\,dx}{3ab^2}$$

$$= \frac{d^2x}{b^2}+\frac{(bc-ad)^2x}{3ab^2(a+bx^3)}+\frac{(2(bc-ad)(bc+2ad))\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx}{9a^{5/3}b^2}+\frac{(2(bc-ad)(bc+2ad}{9}$$

$$= \frac{d^2x}{b^2}+\frac{(bc-ad)^2x}{3ab^2(a+bx^3)}+\frac{2(bc-ad)(bc+2ad)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9a^{5/3}b^{7/3}}-\frac{((bc-ad)(bc+2ad}{9}$$

$$= \frac{d^2x}{b^2}+\frac{(bc-ad)^2x}{3ab^2(a+bx^3)}+\frac{2(bc-ad)(bc+2ad)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9a^{5/3}b^{7/3}}-\frac{(bc-ad)(bc+2ad)}{}$$

$$= \frac{d^2x}{b^2}+\frac{(bc-ad)^2x}{3ab^2(a+bx^3)}-\frac{2(bc-ad)(bc+2ad)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{5/3}b^{7/3}}+\frac{2(bc-ad)(bc+2}{9a}$$

Mathematica [A]
time = 0.15, size = 205, normalized size = 1.01

$$\frac{9\sqrt[3]{b}\,d^2x+\frac{3\sqrt[3]{b}\,(bc-ad)^2x}{a(a+bx^3)}-\frac{2\sqrt{3}\,(b^2c^2+abcd-2a^2d^2)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{5/3}}+\frac{2(b^2c^2+abcd-2a^2d^2)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{a^{5/3}}-\frac{(b^2c^2+abcd-2a^2d^2)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{a^{5/3}}}{9b^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3)^2/(a + b*x^3)^2,x]
```

```
[Out] (9*b^(1/3)*d^2*x + (3*b^(1/3)*(b*c - a*d)^2*x)/(a*(a + b*x^3)) - (2*Sqrt[3]
*(b^2*c^2 + a*b*c*d - 2*a^2*d^2)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]
])/a^(5/3) + (2*(b^2*c^2 + a*b*c*d - 2*a^2*d^2)*Log[a^(1/3) + b^(1/3)*x])/a
^(5/3) - ((b^2*c^2 + a*b*c*d - 2*a^2*d^2)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x +
 b^(2/3)*x^2])/a^(5/3))/(9*b^(7/3))
```

Maple [A]
time = 0.28, size = 170, normalized size = 0.84

method	result
risch	$\frac{d^2x}{b^2}+\frac{(a^2d^2-2abcd+b^2c^2)x}{3ab^2(bx^3+a)}-\frac{2\left(\sum_{\_R=\mathrm{RootOf}(b\_Z^3+a)}\frac{(2a^2d^2-abcd-b^2c^2)\ln(x-\_R)}{\_R^2}\right)}{9b^3a}$
default	$\frac{d^2x}{b^2}-\frac{-\frac{(a^2d^2-2abcd+b^2c^2)x}{3a(bx^3+a)}+\frac{2(2a^2d^2-abcd-b^2c^2)\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)}{3a}}{b^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^2/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] d^2*x/b^2-1/b^2*(-1/3*(a^2*d^2-2*a*b*c*d+b^2*c^2)/a*x/(b*x^3+a)+2/3*(2*a^2*
d^2-a*b*c*d-b^2*c^2)/a*(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/b)^(2/
3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3
^(1/2)*(2/(a/b)^(1/3)*x-1))))
```

Maxima [A]

time = 0.49, size = 220, normalized size = 1.08

$$\frac{(b^2c^2-2\,abcd+a^2d^2)x}{3\,(ab^3x^3+a^2b^2)}+\frac{d^2x}{b^2}+\frac{2\,\sqrt{3}\,(b^2c^2+abcd-2\,a^2d^2)\arctan\left(\frac{\sqrt{3}\,\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,ab^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{(b^2c^2+abcd-2\,a^2d^2)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\,ab^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{2\,(b^2c^2+abcd-2\,a^2d^2)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,ab^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^2/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] 1/3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x/(a*b^3*x^3 + a^2*b^2) + d^2*x/b^2 + 2
/9*sqrt(3)*(b^2*c^2 + a*b*c*d - 2*a^2*d^2)*arctan(1/3*sqrt(3)*(2*x - (a/b)^
(1/3))/(a/b)^(1/3))/(a*b^3*(a/b)^(2/3)) - 1/9*(b^2*c^2 + a*b*c*d - 2*a^2*d^
2)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a*b^3*(a/b)^(2/3)) + 2/9*(b^2*c^
2 + a*b*c*d - 2*a^2*d^2)*log(x + (a/b)^(1/3))/(a*b^3*(a/b)^(2/3))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 365 vs. $2(164) = 328$.

time = 2.15, size = 768, normalized size = 3.78

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^2/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] [1/9*(9*a^3*b^2*d^2*x^4 - 3*sqrt(1/3)*(a^2*b^3*c^2 + a^3*b^2*c*d - 2*a^4*b*
d^2 + (a*b^4*c^2 + a^2*b^3*c*d - 2*a^3*b^2*d^2)*x^3)*sqrt((-a^2*b)^(1/3)/b)
*log((2*a*b*x^3 + 3*(-a^2*b)^(1/3)*a*x - a^2 - 3*sqrt(1/3)*(2*a*b*x^2 + (-a
^2*b)^(2/3)*x + (-a^2*b)^(1/3)*a)*sqrt((-a^2*b)^(1/3)/b))/(b*x^3 + a)) - (a
*b^2*c^2 + a^2*b*c*d - 2*a^3*d^2 + (b^3*c^2 + a*b^2*c*d - 2*a^2*b*d^2)*x^3)
*(-a^2*b)^(2/3)*log(a*b*x^2 - (-a^2*b)^(2/3)*x - (-a^2*b)^(1/3)*a) + 2*(a*b
^2*c^2 + a^2*b*c*d - 2*a^3*d^2 + (b^3*c^2 + a*b^2*c*d - 2*a^2*b*d^2)*x^3)*(
-a^2*b)^(2/3)*log(a*b*x + (-a^2*b)^(2/3)) + 3*(a^2*b^3*c^2 - 2*a^3*b^2*c*d
+ 4*a^4*b*d^2)*x)/(a^3*b^4*x^3 + a^4*b^3), 1/9*(9*a^3*b^2*d^2*x^4 + 6*sqrt(
1/3)*(a^2*b^3*c^2 + a^3*b^2*c*d - 2*a^4*b*d^2 + (a*b^4*c^2 + a^2*b^3*c*d -
2*a^3*b^2*d^2)*x^3)*sqrt(-(-a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(-a^2*b)^(2
/3)*x + (-a^2*b)^(1/3)*a)*sqrt(-(-a^2*b)^(1/3)/b)/a^2) - (a*b^2*c^2 + a^2*b
*c*d - 2*a^3*d^2 + (b^3*c^2 + a*b^2*c*d - 2*a^2*b*d^2)*x^3)*(-a^2*b)^(2/3)*
log(a*b*x^2 - (-a^2*b)^(2/3)*x - (-a^2*b)^(1/3)*a) + 2*(a*b^2*c^2 + a^2*b*c
*d - 2*a^3*d^2 + (b^3*c^2 + a*b^2*c*d - 2*a^2*b*d^2)*x^3)*(-a^2*b)^(2/3)*lo
g(a*b*x + (-a^2*b)^(2/3)) + 3*(a^2*b^3*c^2 - 2*a^3*b^2*c*d + 4*a^4*b*d^2)*x
)/(a^3*b^4*x^3 + a^4*b^3)]
```

Sympy [A]
time = 0.70, size = 189, normalized size = 0.93

$$\frac{x(a^2d^2-2abcd+b^2c^2)}{3a^2b^2+3ab^3x^3}+\operatorname{RootSum}\left(729t^3a^5b^7+64a^6d^6-96a^5bcd^5-48a^4b^2c^2d^4+88a^3b^3c^3d^3+24a^2b^4c^4d^2-24ab^5c^5d-8b^6c^6,\left(t\mapsto t\log\left(-\frac{9ta^2b^2}{4a^2d^2-2abcd-2b^2c^2}+x\right)\right)\right)+\frac{d^2x}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**2/(b*x**3+a)**2,x)
```

```
[Out] x*(a**2*d**2 - 2*a*b*c*d + b**2*c**2)/(3*a**2*b**2 + 3*a*b**3*x**3) + RootS
um(729*_t**3*a**5*b**7 + 64*a**6*d**6 - 96*a**5*b*c*d**5 - 48*a**4*b**2*c**
2*d**4 + 88*a**3*b**3*c**3*d**3 + 24*a**2*b**4*c**4*d**2 - 24*a*b**5*c**5*d
 - 8*b**6*c**6, Lambda(_t, _t*log(-9*_t*a**2*b**2/(4*a**2*d**2 - 2*a*b*c*d
- 2*b**2*c**2) + x))) + d**2*x/b**2
```

Giac [A]
time = 0.67, size = 227, normalized size = 1.12

$$\frac{d^2x}{b^2}-\frac{2\sqrt{3}\,(b^2c^2+abcd-2\,a^2d^2)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,(-ab^2)^{\frac{2}{3}}\,ab}-\frac{(b^2c^2+abcd-2\,a^2d^2)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\,(-ab^2)^{\frac{2}{3}}\,ab}-\frac{2\,(b^2c^2+abcd-2\,a^2d^2)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^2b^2}+\frac{b^2c^2x-2\,abcdx+a^2d^2x}{3\,(bx^3+a)ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^2/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] d^2*x/b^2 - 2/9*sqrt(3)*(b^2*c^2 + a*b*c*d - 2*a^2*d^2)*arctan(1/3*sqrt(3)*
(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*a*b) - 1/9*(b^2*c^2 + a*
```

```
b*c*d - 2*a^2*d^2)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(2/3)
*a*b) - 2/9*(b^2*c^2 + a*b*c*d - 2*a^2*d^2)*(-a/b)^(1/3)*log(abs(x - (-a/b)
^(1/3)))/(a^2*b^2) + 1/3*(b^2*c^2*x - 2*a*b*c*d*x + a^2*d^2*x)/((b*x^3 + a)
*a*b^2)
```

Mupad [B]
time = 1.47, size = 191, normalized size = 0.94

$$\frac{d^2\,x}{b^2}+\frac{x\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}{3\,a\,\left(b^3\,x^3+a\,b^2\right)}-\frac{2\,\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(a\,d-b\,c\right)\,\left(2\,a\,d+b\,c\right)}{9\,a^{5/3}\,b^{7/3}}-\frac{2\,\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(a\,d-b\,c\right)\,\left(2\,a\,d+b\,c\right)}{9\,a^{5/3}\,b^{7/3}}+\frac{2\,\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(a\,d-b\,c\right)\,\left(2\,a\,d+b\,c\right)}{9\,a^{5/3}\,b^{7/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^2/(a + b*x^3)^2,x)
```

```
[Out] (d^2*x)/b^2 + (x*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))/(3*a*(a*b^2 + b^3*x^3)) -
 (2*log(b^(1/3)*x + a^(1/3))*(a*d - b*c)*(2*a*d + b*c))/(9*a^(5/3)*b^(7/3))
 - (2*log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 - 1/2
)*(a*d - b*c)*(2*a*d + b*c))/(9*a^(5/3)*b^(7/3)) + (2*log(3^(1/2)*a^(1/3)*1
i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(a*d - b*c)*(2*a*d + b*c)
)/(9*a^(5/3)*b^(7/3))
```

3.24 $\int \frac{c+dx^3}{(a+bx^3)^2}\,dx$

Optimal. Leaf size=169

$$\frac{(bc-ad)x}{3ab\,(a+bx^3)}-\frac{(2bc+ad)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{5/3}b^{4/3}}+\frac{(2bc+ad)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9a^{5/3}b^{4/3}}-\frac{(2bc+ad)\log\left(a^{2/3}-\sqrt[3]{a}\right.}{18a^{5/3}b^{4/3}}$$

```
[Out] 1/3*(-a*d+b*c)*x/a/b/(b*x^3+a)+1/9*(a*d+2*b*c)*ln(a^(1/3)+b^(1/3)*x)/a^(5/3
)/b^(4/3)-1/18*(a*d+2*b*c)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(5/3
)/b^(4/3)-1/9*(a*d+2*b*c)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))
/a^(5/3)/b^(4/3)*3^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {393, 206, 31, 648, 631, 210, 642}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)(ad+2bc)}{3\sqrt{3}\,a^{5/3}b^{4/3}}-\frac{(ad+2bc)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{18a^{5/3}b^{4/3}}+\frac{(ad+2bc)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9a^{5/3}b^{4/3}}+\frac{x(bc-ad)}{3ab\,(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)/(a + b*x^3)^2,x]
```

```
[Out] ((b*c - a*d)*x)/(3*a*b*(a + b*x^3)) - ((2*b*c + a*d)*ArcTan[(a^(1/3) - 2*b^
(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(5/3)*b^(4/3)) + ((2*b*c + a*d)*L
og[a^(1/3) + b^(1/3)*x])/(9*a^(5/3)*b^(4/3)) - ((2*b*c + a*d)*Log[a^(2/3) -
 a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^(5/3)*b^(4/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{c+dx^3}{(a+bx^3)^2}\,dx = \frac{(bc-ad)x}{3ab(a+bx^3)} + \frac{(2bc+ad)\int \frac{1}{a+bx^3}\,dx}{3ab}$$

$$= \frac{(bc-ad)x}{3ab(a+bx^3)} + \frac{(2bc+ad)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx}{9a^{5/3}b} + \frac{(2bc+ad)\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}\,x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx}{9a^{5/3}b}$$

$$= \frac{(bc-ad)x}{3ab(a+bx^3)} + \frac{(2bc+ad)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9a^{5/3}b^{4/3}} - \frac{(2bc+ad)\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx}{18a^{5/3}b^{4/3}} +$$

$$= \frac{(bc-ad)x}{3ab(a+bx^3)} + \frac{(2bc+ad)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9a^{5/3}b^{4/3}} - \frac{(2bc+ad)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}\right.}{18a^{5/3}b^{4/3}}$$

$$= \frac{(bc-ad)x}{3ab(a+bx^3)} - \frac{(2bc+ad)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{5/3}b^{4/3}} + \frac{(2bc+ad)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9a^{5/3}b^{4/3}} - \frac{(2b}{}$$

Mathematica [A]
time = 0.07, size = 145, normalized size = 0.86

$$\frac{-\frac{6a^{2/3}\sqrt[3]{b}(-bc+ad)x}{a+bx^3} - 2\sqrt{3}(2bc+ad)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right) + 2(2bc+ad)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right) - (2bc+ad)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{18a^{5/3}b^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3)/(a + b*x^3)^2,x]
```

```
[Out] ((-6*a^(2/3)*b^(1/3)*(-(b*c) + a*d)*x)/(a + b*x^3) - 2*Sqrt[3]*(2*b*c + a*d
)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 2*(2*b*c + a*d)*Log[a^(1/3)
 + b^(1/3)*x] - (2*b*c + a*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2
])/(18*a^(5/3)*b^(4/3))
```

Maple [A]
time = 0.25, size = 134, normalized size = 0.79

method	result	size
risch	$-\frac{(ad-bc)x}{3ab(bx^3+a)} + \frac{\sum_{\_R=\mathrm{RootOf}(b\_Z^3+a)} \frac{(ad+2bc)\ln(x-\_R)}{\_R^2}}{9a\,b^2}$	65

default	$-\frac{(ad-bc)x}{3ab(b\,x^3+a)}+\frac{(ad+2bc)\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)}{3ab}$	134

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*(a*d-b*c)/a/b*x/(b*x^3+a)+1/3*(a*d+2*b*c)/a/b*(1/3/b/(a/b)^(2/3)*ln(x+
(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b
)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))
```

Maxima [A]
time = 0.49, size = 158, normalized size = 0.93

$$\frac{(bc-ad)x}{3\left(ab^2x^3+a^2b\right)}+\frac{\sqrt{3}\,(2\,bc+ad)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,ab^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{(2\,bc+ad)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,ab^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{(2\,bc+ad)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,ab^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] 1/3*(b*c - a*d)*x/(a*b^2*x^3 + a^2*b) + 1/9*sqrt(3)*(2*b*c + a*d)*arctan(1/
3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a*b^2*(a/b)^(2/3)) - 1/18*(2*b*
c + a*d)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a*b^2*(a/b)^(2/3)) + 1/9*(
2*b*c + a*d)*log(x + (a/b)^(1/3))/(a*b^2*(a/b)^(2/3))
```

Fricas [A]
time = 2.41, size = 537, normalized size = 3.18

$$\left[\frac{3\sqrt{\frac{1}{3}}\,(2\,a^2b^2c+a^3bd+(2\,ab^3c+a^2b^2d)x^3)\sqrt{-\frac{(a^2b)^{\frac{1}{3}}}{b}}\log\left(\frac{2\,abx^3-3\,(a^2b)^{\frac{1}{3}}ax-a^2+3\sqrt{\frac{1}{3}}\left(2\,abx^2+(a^2b)^{\frac{2}{3}}x-(a^2b)^{\frac{1}{3}}a\right)\sqrt{-\frac{(a^2b)^{\frac{1}{3}}}{b}}}{bx^3+a}\right)-((2\,b^2c+abd)x^3+2\,abc+a^2d)(a^2b)^{\frac{2}{3}}\log\left(abx^2-(a^2b)^{\frac{2}{3}}x+(a^2b)^{\frac{1}{3}}a\right)+2\,((2\,b^2c+abd)x^3+2\,abc+a^2d)(a^2b)^{\frac{2}{3}}\log\left(abx+(a^2b)^{\frac{2}{3}}\right)+6\,(a^2b^2c-a^3bd)x}{18\,(a^3b^3x^3+a^4b^2)},\ \frac{6\sqrt{\frac{1}{3}}\,(2\,a^2b^2c+a^3bd+(2\,ab^3c+a^2b^2d)x^3)\sqrt{\frac{(a^2b)^{\frac{1}{3}}}{b}}\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(2\,(a^2b)^{\frac{2}{3}}x-(a^2b)^{\frac{1}{3}}a\right)\sqrt{\frac{(a^2b)^{\frac{1}{3}}}{b}}}{a^2}\right)-((2\,b^2c+abd)x^3+2\,abc+a^2d)(a^2b)^{\frac{2}{3}}\log\left(abx^2-(a^2b)^{\frac{2}{3}}x+(a^2b)^{\frac{1}{3}}a\right)+2\,((2\,b^2c+abd)x^3+2\,abc+a^2d)(a^2b)^{\frac{2}{3}}\log\left(abx+(a^2b)^{\frac{2}{3}}\right)+6\,(a^2b^2c-a^3bd)x}{18\,(a^3b^3x^3+a^4b^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] [1/18*(3*sqrt(1/3)*(2*a^2*b^2*c + a^3*b*d + (2*a*b^3*c + a^2*b^2*d)*x^3)*sq
rt(-(a^2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*(a^2*b)^(1/3)*a*x - a^2 + 3*sqrt(1/
3)*(2*a*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt(-(a^2*b)^(1/3)/b))/
(b*x^3 + a)) - ((2*b^2*c + a*b*d)*x^3 + 2*a*b*c + a^2*d)*(a^2*b)^(2/3)*log(
```

```
a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 2*((2*b^2*c + a*b*d)*x^3 + 2
*a*b*c + a^2*d)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)) + 6*(a^2*b^2*c - a
^3*b*d)*x)/(a^3*b^3*x^3 + a^4*b^2), 1/18*(6*sqrt(1/3)*(2*a^2*b^2*c + a^3*b*
d + (2*a*b^3*c + a^2*b^2*d)*x^3)*sqrt((a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*
(a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt((a^2*b)^(1/3)/b)/a^2) - ((2*b^2*c +
 a*b*d)*x^3 + 2*a*b*c + a^2*d)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x
+ (a^2*b)^(1/3)*a) + 2*((2*b^2*c + a*b*d)*x^3 + 2*a*b*c + a^2*d)*(a^2*b)^(2
/3)*log(a*b*x + (a^2*b)^(2/3)) + 6*(a^2*b^2*c - a^3*b*d)*x)/(a^3*b^3*x^3 +
a^4*b^2)]
```

Sympy [A]
time = 0.33, size = 97, normalized size = 0.57

$$\frac{x(-ad+bc)}{3a^2b+3ab^2x^3}+\operatorname{RootSum}\left(729t^3a^5b^4-a^3d^3-6a^2bcd^2-12ab^2c^2d-8b^3c^3,\left(t\mapsto t\log\left(\frac{9ta^2b}{ad+2bc}+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)/(b*x**3+a)**2,x)
```

```
[Out] x*(-a*d + b*c)/(3*a**2*b + 3*a*b**2*x**3) + RootSum(729*_t**3*a**5*b**4 - a
**3*d**3 - 6*a**2*b*c*d**2 - 12*a*b**2*c**2*d - 8*b**3*c**3, Lambda(_t, _t*
log(9*_t*a**2*b/(a*d + 2*b*c) + x)))
```

Giac [A]
time = 0.65, size = 160, normalized size = 0.95

$$-\frac{\sqrt{3}\,(2\,bc+ad)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,\left(-ab^2\right)^{\frac{2}{3}}\,a}-\frac{(2\,bc+ad)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,\left(-ab^2\right)^{\frac{2}{3}}\,a}-\frac{(2\,bc+ad)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^2b}+\frac{bcx-adx}{3\,(bx^3+a)ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] -1/9*sqrt(3)*(2*b*c + a*d)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(
1/3))/((-a*b^2)^(2/3)*a) - 1/18*(2*b*c + a*d)*log(x^2 + x*(-a/b)^(1/3) + (-
a/b)^(2/3))/((-a*b^2)^(2/3)*a) - 1/9*(2*b*c + a*d)*(-a/b)^(1/3)*log(abs(x -
 (-a/b)^(1/3)))/(a^2*b) + 1/3*(b*c*x - a*d*x)/((b*x^3 + a)*a*b)
```

Mupad [B]
time = 1.43, size = 143, normalized size = 0.85

$$\frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(a\,d+2\,b\,c\right)}{9\,a^{5/3}\,b^{4/3}}-\frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(a\,d+2\,b\,c\right)}{9\,a^{5/3}\,b^{4/3}}+\frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(a\,d+2\,b\,c\right)}{9\,a^{5/3}\,b^{4/3}}-\frac{x\,\left(a\,d-b\,c\right)}{3\,a\,b\,\left(b\,x^3+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)/(a + b*x^3)^2,x)
```

```
[Out] (log(b^(1/3)*x + a^(1/3))*(a*d + 2*b*c))/(9*a^(5/3)*b^(4/3)) - (log(3^(1/2)
*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(a*d + 2*b*c))/
(9*a^(5/3)*b^(4/3)) + (log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^
(1/2)*1i)/2 - 1/2)*(a*d + 2*b*c))/(9*a^(5/3)*b^(4/3)) - (x*(a*d - b*c))/(3*
a*b*(a + b*x^3))
```

3.25 $\int \frac{1}{(a+bx^3)^2 (c+dx^3)} \, dx$

Optimal. Leaf size=346

$$\frac{bx}{3a(bc-ad)(a+bx^3)} - \frac{b^{2/3}(2bc-5ad)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}(bc-ad)^2} - \frac{d^{5/3}\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}c^{2/3}(bc-ad)^2} + \frac{b^{2/3}(2bc-5ad)\,\mathrm{l}}{9a^{5/3}(b}$$

```
[Out] 1/3*b*x/a/(-a*d+b*c)/(b*x^3+a)+1/9*b^(2/3)*(-5*a*d+2*b*c)*ln(a^(1/3)+b^(1/3
)*x)/a^(5/3)/(-a*d+b*c)^2+1/3*d^(5/3)*ln(c^(1/3)+d^(1/3)*x)/c^(2/3)/(-a*d+b
*c)^2-1/18*b^(2/3)*(-5*a*d+2*b*c)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)
/a^(5/3)/(-a*d+b*c)^2-1/6*d^(5/3)*ln(c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)
/c^(2/3)/(-a*d+b*c)^2-1/9*b^(2/3)*(-5*a*d+2*b*c)*arctan(1/3*(a^(1/3)-2*b^(1
/3)*x)/a^(1/3)*3^(1/2))/a^(5/3)/(-a*d+b*c)^2*3^(1/2)-1/3*d^(5/3)*arctan(1/3
*(c^(1/3)-2*d^(1/3)*x)/c^(1/3)*3^(1/2))/c^(2/3)/(-a*d+b*c)^2*3^(1/2)
```

Rubi [A]
time = 0.18, antiderivative size = 346, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {425, 536, 206, 31, 648, 631, 210, 642}

$$-\frac{b^{2/3}\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)(2bc-5ad)}{3\sqrt{3}a^{5/3}(bc-ad)^2} - \frac{b^{2/3}(2bc-5ad)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{5/3}(bc-ad)^2} + \frac{b^{2/3}(2bc-5ad)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}(bc-ad)^2} - \frac{d^{5/3}\text{ArcTan}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}c^{2/3}(bc-ad)^2} - \frac{d^{5/3}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6c^{2/3}(bc-ad)^2} + \frac{d^{5/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}(bc-ad)^2} + \frac{bx}{3a(a+bx^3)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^3)^2*(c + d*x^3)),x]
```

```
[Out] (b*x)/(3*a*(b*c - a*d)*(a + b*x^3)) - (b^(2/3)*(2*b*c - 5*a*d)*ArcTan[(a^(1
/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(5/3)*(b*c - a*d)^2) -
(d^(5/3)*ArcTan[(c^(1/3) - 2*d^(1/3)*x)/(Sqrt[3]*c^(1/3))])/(Sqrt[3]*c^(2/3
)*(b*c - a*d)^2) + (b^(2/3)*(2*b*c - 5*a*d)*Log[a^(1/3) + b^(1/3)*x])/(9*a^
(5/3)*(b*c - a*d)^2) + (d^(5/3)*Log[c^(1/3) + d^(1/3)*x])/(3*c^(2/3)*(b*c -
 a*d)^2) - (b^(2/3)*(2*b*c - 5*a*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/
3)*x^2])/(18*a^(5/3)*(b*c - a*d)^2) - (d^(5/3)*Log[c^(2/3) - c^(1/3)*d^(1/3
)*x + d^(2/3)*x^2])/(6*c^(2/3)*(b*c - a*d)^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
```

```
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{(a+bx^3)^2(c+dx^3)}\,dx = \frac{bx}{3a(bc-ad)(a+bx^3)} - \frac{\int \frac{-2bc+3ad-2bdx^3}{(a+bx^3)(c+dx^3)}\,dx}{3a(bc-ad)}$$

$$= \frac{bx}{3a(bc-ad)(a+bx^3)} + \frac{d^2\int \frac{1}{c+dx^3}\,dx}{(bc-ad)^2} + \frac{(b(2bc-5ad))\int \frac{1}{a+bx^3}\,dx}{3a(bc-ad)^2}$$

$$= \frac{bx}{3a(bc-ad)(a+bx^3)} + \frac{d^2\int \frac{1}{\sqrt[3]{c}+\sqrt[3]{d}\,x}\,dx}{3c^{2/3}(bc-ad)^2} + \frac{d^2\int \frac{2\sqrt[3]{c}-\sqrt[3]{d}\,x}{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}\,x+d^{2/3}x^2}\,dx}{3c^{2/3}(bc-ad)^2} + \cdots$$

$$= \frac{bx}{3a(bc-ad)(a+bx^3)} + \frac{b^{2/3}(2bc-5ad)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9a^{5/3}(bc-ad)^2} + \frac{d^{5/3}\log\left(\sqrt[3]{c}+\cdots\right.}{3c^{2/3}(bc-\cdots}$$

$$= \frac{bx}{3a(bc-ad)(a+bx^3)} + \frac{b^{2/3}(2bc-5ad)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9a^{5/3}(bc-ad)^2} + \frac{d^{5/3}\log\left(\sqrt[3]{c}+\cdots\right.}{3c^{2/3}(bc-\cdots}$$

$$= \frac{bx}{3a(bc-ad)(a+bx^3)} - \frac{b^{2/3}(2bc-5ad)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{5/3}(bc-ad)^2} - \frac{d^{5/3}\tan^{-1}\left(\cdots\right.}{\sqrt{3}\,c^{2/3}(\cdots}$$

Mathematica [A]
time = 0.15, size = 337, normalized size = 0.97

$$\frac{6a^{2/3}bc^{2/3}(bc-ad)x - 2\sqrt{3}\,b^{2/3}c^{2/3}(2bc-5ad)(a+bx^3)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right) - 6\sqrt{3}\,a^{5/3}d^{5/3}(a+bx^3)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\,x}{\sqrt[3]{c}}}{\sqrt{3}}\right) + 2b^{2/3}c^{2/3}(2bc-5ad)(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right) + 6a^{5/3}d^{5/3}(a+bx^3)\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right) - b^{2/3}c^{2/3}(2bc-5ad)(a+bx^3)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right) - 3a^{5/3}d^{5/3}(a+bx^3)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}\,x+d^{2/3}x^2\right)}{18a^{5/3}c^{2/3}(bc-ad)^2(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^3)^2*(c + d*x^3)),x]
```

```
[Out] (6*a^(2/3)*b*c^(2/3)*(b*c - a*d)*x - 2*Sqrt[3]*b^(2/3)*c^(2/3)*(2*b*c - 5*a
*d)*(a + b*x^3)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] - 6*Sqrt[3]*a^(
5/3)*d^(5/3)*(a + b*x^3)*ArcTan[(1 - (2*d^(1/3)*x)/c^(1/3))/Sqrt[3]] + 2*b^
(2/3)*c^(2/3)*(2*b*c - 5*a*d)*(a + b*x^3)*Log[a^(1/3) + b^(1/3)*x] + 6*a^(5
/3)*d^(5/3)*(a + b*x^3)*Log[c^(1/3) + d^(1/3)*x] - b^(2/3)*c^(2/3)*(2*b*c -
 5*a*d)*(a + b*x^3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] - 3*a^(5
/3)*d^(5/3)*(a + b*x^3)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/(18
*a^(5/3)*c^(2/3)*(b*c - a*d)^2*(a + b*x^3))
```

Maple [A]
time = 0.34, size = 247, normalized size = 0.71

method	result

default	$-\frac{b\left(\frac{(ad-bc)x}{3a\left(b\,x^{3}+a\right)}+\frac{(5ad-2bc)\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)}{3a}\right)}{(ad-bc)^{2}}+\frac{\left(\frac{\ln\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3d\left(\frac{c}{d}\right)^{\frac{2}{3}}}-\cdots\right.}{\cdots}$
risch	$-\frac{bx}{3a(ad-bc)(b\,x^{3}+a)}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(c^{2}d^{6}a^{6}-6a^{5}b\,c^{3}d^{5}+15a^{4}b^{2}c^{4}d^{4}-20a^{3}b^{3}c^{5}d^{3}+15a^{2}b^{4}c^{6}d^{2}-6a\,b^{5}c^{7}d+b^{6}c^{8}\right)\_Z^{3}-d^{5}\right)}\_R\ln\left(\left(\left(\cdots\right.\right.\right.\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^3+a)^2/(d*x^3+c),x,method=_RETURNVERBOSE)
```

```
[Out] -b/(a*d-b*c)^2*(1/3*(a*d-b*c)/a*x/(b*x^3+a)+1/3*(5*a*d-2*b*c)/a*(1/3/b/(a/b
)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3
))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))))+(1/3
/d/(c/d)^(2/3)*ln(x+(c/d)^(1/3))-1/6/d/(c/d)^(2/3)*ln(x^2-(c/d)^(1/3)*x+(c/
d)^(2/3))+1/3/d/(c/d)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(c/d)^(1/3)*x-1))
)*d^2/(a*d-b*c)^2
```

Maxima [A]
time = 0.51, size = 489, normalized size = 1.41

$$\frac{\sqrt{3}\,(2\,bc-5\,ad)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\left(ab^{2}c^{2}\left(\frac{a}{b}\right)^{\frac{1}{3}}-2\,a^{2}bcd\left(\frac{a}{b}\right)^{\frac{1}{3}}+a^{3}d^{2}\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\,d\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{3\left(b^{2}c^{2}\left(\frac{c}{d}\right)^{\frac{1}{3}}-2\,abcd\left(\frac{c}{d}\right)^{\frac{1}{3}}+a^{2}d^{2}\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)\left(\frac{c}{d}\right)^{\frac{1}{3}}}+\frac{bx}{3\left(a^{2}bc-a^{3}d+(ab^{2}c-a^{2}bd)x^{3}\right)}-\frac{(2\,bc-5\,ad)\log\left(x^{2}-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\left(ab^{2}c^{2}\left(\frac{a}{b}\right)^{\frac{2}{3}}-2\,a^{2}bcd\left(\frac{a}{b}\right)^{\frac{2}{3}}+a^{3}d^{2}\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}-\frac{d\log\left(x^{2}-x\left(\frac{c}{d}\right)^{\frac{1}{3}}+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\left(b^{2}c^{2}\left(\frac{c}{d}\right)^{\frac{2}{3}}-2\,abcd\left(\frac{c}{d}\right)^{\frac{2}{3}}+a^{2}d^{2}\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}+\frac{(2\,bc-5\,ad)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(ab^{2}c^{2}\left(\frac{a}{b}\right)^{\frac{2}{3}}-2\,a^{2}bcd\left(\frac{a}{b}\right)^{\frac{2}{3}}+a^{3}d^{2}\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}+\frac{d\log\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(b^{2}c^{2}\left(\frac{c}{d}\right)^{\frac{2}{3}}-2\,abcd\left(\frac{c}{d}\right)^{\frac{2}{3}}+a^{2}d^{2}\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^2/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] 1/9*sqrt(3)*(2*b*c - 5*a*d)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1
/3))/((a*b^2*c^2*(a/b)^(1/3) - 2*a^2*b*c*d*(a/b)^(1/3) + a^3*d^2*(a/b)^(1/3
))*(a/b)^(1/3)) + 1/3*sqrt(3)*d*arctan(1/3*sqrt(3)*(2*x - (c/d)^(1/3))/(c/d
)^(1/3))/((b^2*c^2*(c/d)^(1/3) - 2*a*b*c*d*(c/d)^(1/3) + a^2*d^2*(c/d)^(1/3
```

```
))*(c/d)^(1/3)) + 1/3*b*x/(a^2*b*c - a^3*d + (a*b^2*c - a^2*b*d)*x^3) - 1/1
8*(2*b*c - 5*a*d)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a*b^2*c^2*(a/b)^(
2/3) - 2*a^2*b*c*d*(a/b)^(2/3) + a^3*d^2*(a/b)^(2/3)) - 1/6*d*log(x^2 - x*(
c/d)^(1/3) + (c/d)^(2/3))/(b^2*c^2*(c/d)^(2/3) - 2*a*b*c*d*(c/d)^(2/3) + a^
2*d^2*(c/d)^(2/3)) + 1/9*(2*b*c - 5*a*d)*log(x + (a/b)^(1/3))/(a*b^2*c^2*(a
/b)^(2/3) - 2*a^2*b*c*d*(a/b)^(2/3) + a^3*d^2*(a/b)^(2/3)) + 1/3*d*log(x +
(c/d)^(1/3))/(b^2*c^2*(c/d)^(2/3) - 2*a*b*c*d*(c/d)^(2/3) + a^2*d^2*(c/d)^(
2/3))
```

Fricas [**A**]
time = 22.04, size = 440, normalized size = 1.27

$$-\frac{2\sqrt{3}\left((2b^2c-5abd)x^3+2abc-5a^2d\right)\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}ax\left(-\frac{b^2}{a^2}\right)^{\frac{2}{3}}-\sqrt{3}b}{3b}\right)-6\sqrt{3}\left(abdx^3+a^2d\right)\left(\frac{d^2}{c^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}cx\left(\frac{d^2}{c^2}\right)^{\frac{2}{3}}-\sqrt{3}d}{3d}\right)-\left((2b^2c-5abd)x^3+2abc-5a^2d\right)\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\log\left(b^2x^2+abx\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}+a^2\left(-\frac{b^2}{a^2}\right)^{\frac{2}{3}}\right)+3\left(abdx^3+a^2d\right)\left(\frac{d^2}{c^2}\right)^{\frac{1}{3}}\log\left(d^2x^2-cdx\left(\frac{d^2}{c^2}\right)^{\frac{1}{3}}+c^2\left(\frac{d^2}{c^2}\right)^{\frac{2}{3}}\right)+2\left((2b^2c-5abd)x^3+2abc-5a^2d\right)\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\log\left(bx-a\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\right)-6\left(abdx^3+a^2d\right)\left(\frac{d^2}{c^2}\right)^{\frac{1}{3}}\log\left(dx+c\left(\frac{d^2}{c^2}\right)^{\frac{1}{3}}\right)-6\left(b^2c-abd\right)x}{18\left(a^2b^2c^2-2a^3bcd+a^4d^2+\left(ab^3c^2-2a^2b^2cd+a^3bd^2\right)x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^2/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] -1/18*(2*sqrt(3)*((2*b^2*c - 5*a*b*d)*x^3 + 2*a*b*c - 5*a^2*d)*(-b^2/a^2)^(
1/3)*arctan(1/3*(2*sqrt(3)*a*x*(-b^2/a^2)^(2/3) - sqrt(3)*b)/b) - 6*sqrt(3)
*(a*b*d*x^3 + a^2*d)*(d^2/c^2)^(1/3)*arctan(1/3*(2*sqrt(3)*c*x*(d^2/c^2)^(2
/3) - sqrt(3)*d)/d) - ((2*b^2*c - 5*a*b*d)*x^3 + 2*a*b*c - 5*a^2*d)*(-b^2/a
^2)^(1/3)*log(b^2*x^2 + a*b*x*(-b^2/a^2)^(1/3) + a^2*(-b^2/a^2)^(2/3)) + 3*
(a*b*d*x^3 + a^2*d)*(d^2/c^2)^(1/3)*log(d^2*x^2 - c*d*x*(d^2/c^2)^(1/3) + c
^2*(d^2/c^2)^(2/3)) + 2*((2*b^2*c - 5*a*b*d)*x^3 + 2*a*b*c - 5*a^2*d)*(-b^2
/a^2)^(1/3)*log(b*x - a*(-b^2/a^2)^(1/3)) - 6*(a*b*d*x^3 + a^2*d)*(d^2/c^2)
^(1/3)*log(d*x + c*(d^2/c^2)^(1/3)) - 6*(b^2*c - a*b*d)*x)/(a^2*b^2*c^2 - 2
*a^3*b*c*d + a^4*d^2 + (a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*x^3)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3+a)**2/(d*x**3+c),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.88, size = 443, normalized size = 1.28

$$-\frac{d^2\left(-\frac{c}{d}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right|\right)}{3\left(b^2c^3-2abc^2d+a^2cd^2\right)}+\frac{\left(-cd^2\right)^{\frac{1}{3}}d\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}b^2c^3-2\sqrt{3}abc^2d+\sqrt{3}a^2cd^2}+\frac{\left(-cd^2\right)^{\frac{1}{3}}d\log\left(x^2+x\left(-\frac{c}{d}\right)^{\frac{1}{3}}+\left(-\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\left(b^2c^3-2abc^2d+a^2cd^2\right)}-\frac{\left(2b^2c-5abd\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\left(a^2b^2c^2-2a^3bcd+a^4d^2\right)}+\frac{\left(2\left(-ab^2\right)^{\frac{1}{3}}bc-5\left(-ab^2\right)^{\frac{1}{3}}ad\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\left(\sqrt{3}a^2b^2c^2-2\sqrt{3}a^3bcd+\sqrt{3}a^4d^2\right)}+\frac{\left(2\left(-ab^2\right)^{\frac{1}{3}}bc-5\left(-ab^2\right)^{\frac{1}{3}}ad\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\left(a^2b^2c^2-2a^3bcd+a^4d^2\right)}+\frac{bx}{3\left(bx^3+a\right)\left(abc-a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^2/(d*x^3+c),x, algorithm="giac")
```

```
[Out] -1/3*d^2*(-c/d)^(1/3)*log(abs(x - (-c/d)^(1/3)))/(b^2*c^3 - 2*a*b*c^2*d + a
^2*c*d^2) + (-c*d^2)^(1/3)*d*arctan(1/3*sqrt(3)*(2*x + (-c/d)^(1/3))/(-c/d)
^(1/3))/(sqrt(3)*b^2*c^3 - 2*sqrt(3)*a*b*c^2*d + sqrt(3)*a^2*c*d^2) + 1/6*(
-c*d^2)^(1/3)*d*log(x^2 + x*(-c/d)^(1/3) + (-c/d)^(2/3))/(b^2*c^3 - 2*a*b*c
^2*d + a^2*c*d^2) - 1/9*(2*b^2*c - 5*a*b*d)*(-a/b)^(1/3)*log(abs(x - (-a/b)
^(1/3)))/(a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2) + 1/3*(2*(-a*b^2)^(1/3)*b*c
- 5*(-a*b^2)^(1/3)*a*d)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3
))/(sqrt(3)*a^2*b^2*c^2 - 2*sqrt(3)*a^3*b*c*d + sqrt(3)*a^4*d^2) + 1/18*(2*
(-a*b^2)^(1/3)*b*c - 5*(-a*b^2)^(1/3)*a*d)*log(x^2 + x*(-a/b)^(1/3) + (-a/b
)^(2/3))/(a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2) + 1/3*b*x/((b*x^3 + a)*(a*b*
c - a^2*d))
```

Mupad [B]
time = 15.93, size = 2492, normalized size = 7.20

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^3)^2*(c + d*x^3)),x)
```

```
[Out] log((((((27*b^3*d^3*x*(a*d - b*c)^3*(3*a^2*d^2 - 2*b^2*c^2 + 3*a*b*c*d))/a
+ 27*a*b^3*c*d^3*(a*d + b*c)*(a*d - b*c)^4*(-(b^2*(5*a*d - 2*b*c)^3)/(a^5*(
a*d - b*c)^6))^(1/3))*(-(b^2*(5*a*d - 2*b*c)^3)/(a^5*(a*d - b*c)^6))^(2/3))
/81 - (b^4*d^4*(27*a^3*d^3 - 8*b^3*c^3 + 52*a*b^2*c^2*d - 98*a^2*b*c*d^2))/
(3*a^4*d - 3*a^3*b*c))*(-(b^2*(5*a*d - 2*b*c)^3)/(a^5*(a*d - b*c)^6))^(1/3)
)/9 + (2*b^5*d^6*x*(85*a^3*d^3 - 4*b^3*c^3 + 30*a*b^2*c^2*d - 84*a^2*b*c*d^
2))/(9*a^3*(a*d - b*c)^4))*((8*b^5*c^3 - 125*a^3*b^2*d^3 + 150*a^2*b^3*c*d^
2 - 60*a*b^4*c^2*d)/(729*a^11*d^6 + 729*a^5*b^6*c^6 - 4374*a^6*b^5*c^5*d +
10935*a^7*b^4*c^4*d^2 - 14580*a^8*b^3*c^3*d^3 + 10935*a^9*b^2*c^2*d^4 - 437
4*a^10*b*c*d^5))^(1/3) + log((((((27*b^3*d^3*x*(a*d - b*c)^3*(3*a^2*d^2 - 2
*b^2*c^2 + 3*a*b*c*d))/a + 81*a*b^3*c*d^3*(a*d + b*c)*(a*d - b*c)^4*(d^5/(c
^2*(a*d - b*c)^6))^(1/3))*(d^5/(c^2*(a*d - b*c)^6))^(2/3))/9 - (b^4*d^4*(27
*a^3*d^3 - 8*b^3*c^3 + 52*a*b^2*c^2*d - 98*a^2*b*c*d^2))/(3*a^4*d - 3*a^3*b
*c))*(d^5/(c^2*(a*d - b*c)^6))^(1/3))/3 + (2*b^5*d^6*x*(85*a^3*d^3 - 4*b^3*
c^3 + 30*a*b^2*c^2*d - 84*a^2*b*c*d^2))/(9*a^3*(a*d - b*c)^4))*(d^5/(27*b^6
*c^8 + 27*a^6*c^2*d^6 - 162*a^5*b*c^3*d^5 + 405*a^2*b^4*c^6*d^2 - 540*a^3*b
^3*c^5*d^3 + 405*a^4*b^2*c^4*d^4 - 162*a*b^5*c^7*d))^(1/3) + (log(((3^(1/2)
*1i - 1)*(((3^(1/2)*1i - 1)^2*((27*b^3*d^3*x*(a*d - b*c)^3*(3*a^2*d^2 - 2*b
^2*c^2 + 3*a*b*c*d))/a + (27*a*b^3*c*d^3*(3^(1/2)*1i - 1)*(a*d + b*c)*(a*d
- b*c)^4*(-(b^2*(5*a*d - 2*b*c)^3)/(a^5*(a*d - b*c)^6))^(1/3))/2)*(-(b^2*(5
*a*d - 2*b*c)^3)/(a^5*(a*d - b*c)^6))^(2/3))/324 - (b^4*d^4*(27*a^3*d^3 - 8
*b^3*c^3 + 52*a*b^2*c^2*d - 98*a^2*b*c*d^2))/(3*a^4*d - 3*a^3*b*c))*(-(b^2*
(5*a*d - 2*b*c)^3)/(a^5*(a*d - b*c)^6))^(1/3))/18 + (2*b^5*d^6*x*(85*a^3*d^
3 - 4*b^3*c^3 + 30*a*b^2*c^2*d - 84*a^2*b*c*d^2))/(9*a^3*(a*d - b*c)^4))*(3
^(1/2)*1i - 1)*((8*b^5*c^3 - 125*a^3*b^2*d^3 + 150*a^2*b^3*c*d^2 - 60*a*b^4
```

```
*c^2*d)/(729*a^11*d^6 + 729*a^5*b^6*c^6 - 4374*a^6*b^5*c^5*d + 10935*a^7*b^
4*c^4*d^2 - 14580*a^8*b^3*c^3*d^3 + 10935*a^9*b^2*c^2*d^4 - 4374*a^10*b*c*d
^5))^(1/3))/2 - (log(((3^(1/2)*1i + 1)*(((3^(1/2)*1i + 1)^2*((27*b^3*d^3*x*
(a*d - b*c)^3*(3*a^2*d^2 - 2*b^2*c^2 + 3*a*b*c*d))/a - (27*a*b^3*c*d^3*(3^(
1/2)*1i + 1)*(a*d + b*c)*(a*d - b*c)^4*(-(b^2*(5*a*d - 2*b*c)^3)/(a^5*(a*d
- b*c)^6))^(1/3))/2)*(-(b^2*(5*a*d - 2*b*c)^3)/(a^5*(a*d - b*c)^6))^(2/3))/
324 - (b^4*d^4*(27*a^3*d^3 - 8*b^3*c^3 + 52*a*b^2*c^2*d - 98*a^2*b*c*d^2))/
(3*a^4*d - 3*a^3*b*c))*(-(b^2*(5*a*d - 2*b*c)^3)/(a^5*(a*d - b*c)^6))^(1/3)
)/18 - (2*b^5*d^6*x*(85*a^3*d^3 - 4*b^3*c^3 + 30*a*b^2*c^2*d - 84*a^2*b*c*d
^2))/(9*a^3*(a*d - b*c)^4))*(3^(1/2)*1i + 1)*((8*b^5*c^3 - 125*a^3*b^2*d^3
+ 150*a^2*b^3*c*d^2 - 60*a*b^4*c^2*d)/(729*a^11*d^6 + 729*a^5*b^6*c^6 - 437
4*a^6*b^5*c^5*d + 10935*a^7*b^4*c^4*d^2 - 14580*a^8*b^3*c^3*d^3 + 10935*a^9
*b^2*c^2*d^4 - 4374*a^10*b*c*d^5))^(1/3))/2 + (log(((3^(1/2)*1i - 1)*(((3^(
1/2)*1i - 1)^2*((27*b^3*d^3*x*(a*d - b*c)^3*(3*a^2*d^2 - 2*b^2*c^2 + 3*a*b*
c*d))/a + (81*a*b^3*c*d^3*(3^(1/2)*1i - 1)*(a*d + b*c)*(a*d - b*c)^4*(d^5/(
c^2*(a*d - b*c)^6))^(1/3))/2)*(d^5/(c^2*(a*d - b*c)^6))^(2/3))/36 - (b^4*d^
4*(27*a^3*d^3 - 8*b^3*c^3 + 52*a*b^2*c^2*d - 98*a^2*b*c*d^2))/(3*a^4*d - 3*
a^3*b*c))*(d^5/(c^2*(a*d - b*c)^6))^(1/3))/6 + (2*b^5*d^6*x*(85*a^3*d^3 - 4
*b^3*c^3 + 30*a*b^2*c^2*d - 84*a^2*b*c*d^2))/(9*a^3*(a*d - b*c)^4))*(3^(1/2
)*1i - 1)*(d^5/(27*b^6*c^8 + 27*a^6*c^2*d^6 - 162*a^5*b*c^3*d^5 + 405*a^2*b
^4*c^6*d^2 - 540*a^3*b^3*c^5*d^3 + 405*a^4*b^2*c^4*d^4 - 162*a*b^5*c^7*d))^
(1/3))/2 - (log(((3^(1/2)*1i + 1)*(((3^(1/2)*1i + 1)^2*((27*b^3*d^3*x*(a*d
- b*c)^3*(3*a^2*d^2 - 2*b^2*c^2 + 3*a*b*c*d))/a - (81*a*b^3*c*d^3*(3^(1/2)*
1i + 1)*(a*d + b*c)*(a*d - b*c)^4*(d^5/(c^2*(a*d - b*c)^6))^(1/3))/2)*(d^5/
(c^2*(a*d - b*c)^6))^(2/3))/36 - (b^4*d^4*(27*a^3*d^3 - 8*b^3*c^3 + 52*a*b^
2*c^2*d - 98*a^2*b*c*d^2))/(3*a^4*d - 3*a^3*b*c))*(d^5/(c^2*(a*d - b*c)^6))
^(1/3))/6 - (2*b^5*d^6*x*(85*a^3*d^3 - 4*b^3*c^3 + 30*a*b^2*c^2*d - 84*a^2*
b*c*d^2))/(9*a^3*(a*d - b*c)^4))*(3^(1/2)*1i + 1)*(d^5/(27*b^6*c^8 + 27*a^6
*c^2*d^6 - 162*a^5*b*c^3*d^5 + 405*a^2*b^4*c^6*d^2 - 540*a^3*b^3*c^5*d^3 +
405*a^4*b^2*c^4*d^4 - 162*a*b^5*c^7*d))^(1/3))/2 - (b*x)/(3*a*(a + b*x^3)*(
a*d - b*c))
```

3.26 $\int \frac{1}{(a+bx^3)^2(c+dx^3)^2}\,dx$

Optimal. Leaf size=419

$$\frac{d(bc+ad)x}{3ac(bc-ad)^2(c+dx^3)}+\frac{bx}{3a(bc-ad)(a+bx^3)(c+dx^3)}-\frac{2b^{5/3}(bc-4ad)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{5/3}(bc-ad)^3}-\frac{2d^{5/3}(4bc-}{3}$$

```
[Out] 1/3*d*(a*d+b*c)*x/a/c/(-a*d+b*c)^2/(d*x^3+c)+1/3*b*x/a/(-a*d+b*c)/(b*x^3+a)
/(d*x^3+c)+2/9*b^(5/3)*(-4*a*d+b*c)*ln(a^(1/3)+b^(1/3)*x)/a^(5/3)/(-a*d+b*c
)^3+2/9*d^(5/3)*(-a*d+4*b*c)*ln(c^(1/3)+d^(1/3)*x)/c^(5/3)/(-a*d+b*c)^3-1/9
*b^(5/3)*(-4*a*d+b*c)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(5/3)/(-a
*d+b*c)^3-1/9*d^(5/3)*(-a*d+4*b*c)*ln(c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2
)/c^(5/3)/(-a*d+b*c)^3-2/9*b^(5/3)*(-4*a*d+b*c)*arctan(1/3*(a^(1/3)-2*b^(1/
3)*x)/a^(1/3)*3^(1/2))/a^(5/3)/(-a*d+b*c)^3*3^(1/2)-2/9*d^(5/3)*(-a*d+4*b*c
)*arctan(1/3*(c^(1/3)-2*d^(1/3)*x)/c^(1/3)*3^(1/2))/c^(5/3)/(-a*d+b*c)^3*3^
(1/2)
```

Rubi [A]

time = 0.35, antiderivative size = 419, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {425, 541, 536, 206, 31, 648, 631, 210, 642}

$$-\frac{2b^{5/3}\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)(bc-4ad)}{3\sqrt{3}\,a^{5/3}(bc-ad)^3}-\frac{b^{5/3}(bc-4ad)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{9a^{5/3}(bc-ad)^3}+\frac{2b^{5/3}(bc-4ad)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9a^{5/3}(bc-ad)^3}-\frac{2d^{5/3}(4bc-ad)\text{ArcTan}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}\,x}{\sqrt{3}\sqrt[3]{c}}\right)}{3\sqrt{3}\,c^{5/3}(bc-ad)^3}-\frac{d^{5/3}(4bc-ad)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}\,x+d^{2/3}x^2\right)}{9c^{5/3}(bc-ad)^3}+\frac{2d^{5/3}(4bc-ad)\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{9c^{5/3}(bc-ad)^3}+\frac{bx}{3a\left(a+bx^3\right)\left(c+dx^3\right)(bc-ad)}+\frac{dx(ad+bc)}{3ac\left(c+dx^3\right)(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^3)^2*(c + d*x^3)^2),x]
```

```
[Out] (d*(b*c + a*d)*x)/(3*a*c*(b*c - a*d)^2*(c + d*x^3)) + (b*x)/(3*a*(b*c - a*d
)*(a + b*x^3)*(c + d*x^3)) - (2*b^(5/3)*(b*c - 4*a*d)*ArcTan[(a^(1/3) - 2*b
^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(5/3)*(b*c - a*d)^3) - (2*d^(5/3
)*(4*b*c - a*d)*ArcTan[(c^(1/3) - 2*d^(1/3)*x)/(Sqrt[3]*c^(1/3))])/(3*Sqrt[
3]*c^(5/3)*(b*c - a*d)^3) + (2*b^(5/3)*(b*c - 4*a*d)*Log[a^(1/3) + b^(1/3)*
x])/(9*a^(5/3)*(b*c - a*d)^3) + (2*d^(5/3)*(4*b*c - a*d)*Log[c^(1/3) + d^(1
/3)*x])/(9*c^(5/3)*(b*c - a*d)^3) - (b^(5/3)*(b*c - 4*a*d)*Log[a^(2/3) - a^
(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(9*a^(5/3)*(b*c - a*d)^3) - (d^(5/3)*(4*b*c
 - a*d)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/(9*c^(5/3)*(b*c - a
*d)^3)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{(a+bx^3)^2(c+dx^3)^2}\,dx = \frac{bx}{3a(bc-ad)(a+bx^3)(c+dx^3)} - \frac{\int \frac{-2bc+3ad-5bdx^3}{(a+bx^3)(c+dx^3)^2}\,dx}{3a(bc-ad)}$$

$$= \frac{d(bc+ad)x}{3ac(bc-ad)^2(c+dx^3)} + \frac{bx}{3a(bc-ad)(a+bx^3)(c+dx^3)} - \frac{\int \frac{-6(b^2c^2-3abcd+a^2 \cdots}{(a+bx^3)(\cdots}}{9ac(bc \cdots}$$

$$= \frac{d(bc+ad)x}{3ac(bc-ad)^2(c+dx^3)} + \frac{bx}{3a(bc-ad)(a+bx^3)(c+dx^3)} + \frac{(2b^2(bc-4ad))\int \cdots}{3a(bc-a \cdots}$$

$$= \frac{d(bc+ad)x}{3ac(bc-ad)^2(c+dx^3)} + \frac{bx}{3a(bc-ad)(a+bx^3)(c+dx^3)} + \frac{(2b^2(bc-4ad))\int \cdots}{9a^{5/3}(bc \cdots}$$

$$= \frac{d(bc+ad)x}{3ac(bc-ad)^2(c+dx^3)} + \frac{bx}{3a(bc-ad)(a+bx^3)(c+dx^3)} + \frac{2b^{5/3}(bc-4ad)\,\mathrm{lo} \cdots}{9a^{5/3}(bc \cdots}$$

$$= \frac{d(bc+ad)x}{3ac(bc-ad)^2(c+dx^3)} + \frac{bx}{3a(bc-ad)(a+bx^3)(c+dx^3)} + \frac{2b^{5/3}(bc-4ad)\,\mathrm{lo} \cdots}{9a^{5/3}(bc \cdots}$$

$$= \frac{d(bc+ad)x}{3ac(bc-ad)^2(c+dx^3)} + \frac{bx}{3a(bc-ad)(a+bx^3)(c+dx^3)} - \frac{2b^{5/3}(bc-4ad)\,\mathrm{ta} \cdots}{3\sqrt{3}\,a^{5/ \cdots}$$

Mathematica [A]
time = 0.45, size = 381, normalized size = 0.91

$$\frac{1}{9}\left(\frac{3b^2x}{a(bc-ad)^2(a+bx^3)} + \frac{3d^2x}{c(bc-ad)^2(c+dx^3)} + \frac{2\sqrt{3}\,b^{5/3}(bc-4ad)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{5/3}(-bc+ad)^3} + \frac{2\sqrt{3}\,d^{5/3}(-4bc+ad)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\,x}{\sqrt[3]{c}}}{\sqrt{3}}\right)}{c^{5/3}(bc-ad)^3} + \frac{2b^{5/3}(-bc+4ad)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{a^{5/3}(-bc+ad)^3} + \frac{2d^{5/3}(4bc-ad)\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{c^{5/3}(bc-ad)^3} + \frac{b^{5/3}(bc-4ad)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{a^{5/3}(-bc+ad)^3} + \frac{d^{5/3}(-4bc+ad)\log\left(c^{2/3}-\sqrt[3]{c}\,\sqrt[3]{d}\,x+d^{2/3}x^2\right)}{c^{5/3}(bc-ad)^3}\right)$$

Antiderivative was successfully verified.

[In] Integrate[1/((a + b*x^3)^2*(c + d*x^3)^2),x]

[Out] ((3*b^2*x)/(a*(b*c - a*d)^2*(a + b*x^3)) + (3*d^2*x)/(c*(b*c - a*d)^2*(c + d*x^3)) + (2*Sqrt[3]*b^(5/3)*(b*c - 4*a*d)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/(a^(5/3)*(-(b*c) + a*d)^3) + (2*Sqrt[3]*d^(5/3)*(-4*b*c + a*d)*ArcTan[(1 - (2*d^(1/3)*x)/c^(1/3))/Sqrt[3]])/(c^(5/3)*(b*c - a*d)^3) + (2*b^(5/3)*(-(b*c) + 4*a*d)*Log[a^(1/3) + b^(1/3)*x])/(a^(5/3)*(-(b*c) + a*d)^3) + (2*d^(5/3)*(4*b*c - a*d)*Log[c^(1/3) + d^(1/3)*x])/(c^(5/3)*(b*c - a*d)^3) + (b^(5/3)*(b*c - 4*a*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(a^(5/3)*(-(b*c) + a*d)^3) + (d^(5/3)*(-4*b*c + a*d)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/(c^(5/3)*(b*c - a*d)^3))/9

Maple [A]
time = 0.41, size = 285, normalized size = 0.68

method	result
default	$\frac{b^2\left(\frac{(ad-bc)x}{3a\left(b\,x^3+a\right)}+\frac{2(4ad-bc)\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)}{3a}\right)}{(ad-bc)^3}+$ $d^2\left(\frac{(ad-bc)x}{3c\left(d\,x^3+c\right)}+\right.$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b*x^3+a)^2/(d*x^3+c)^2,x,method=_RETURNVERBOSE)

[Out] b^2/(a*d-b*c)^3*(1/3*(a*d-b*c)/a*x/(b*x^3+a)+2/3*(4*a*d-b*c)/a*(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))))+d^2/(a*d-b*c)^3*(1/3*(a*d-b*c)/c*x/(d*x^3+c)+2/3*(a*d-4*b*c)/c*(1/3/d/(c/d)^(2/3)*ln(x+(c/d)^(1/3))-1/6/d/(c/d)^(2/3)*ln(x^2-(c/d)^(1/3)*x+(c/d)^(2/3))+1/3/d/(c/d)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(c/d)^(1/3)*x-1))))

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 784 vs. $2(341) = 682$.

time = 0.50, size = 784, normalized size = 1.87

$$\frac{2\sqrt{3}\,(b^2c-4\,abd)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\left(ab^3c^3\left(\frac{a}{b}\right)^{\frac{1}{3}}-3\,a^2b^2c^2d\left(\frac{a}{b}\right)^{\frac{1}{3}}+3\,a^3bcd^2\left(\frac{a}{b}\right)^{\frac{1}{3}}-a^4d^3\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{2\sqrt{3}\,(4\,bcd-ad^2)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{9\left(b^3c^4\left(\frac{c}{d}\right)^{\frac{1}{3}}-3\,ab^2c^3d\left(\frac{c}{d}\right)^{\frac{1}{3}}+3\,a^2bc^2d^2\left(\frac{c}{d}\right)^{\frac{1}{3}}-a^3cd^3\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)\left(\frac{c}{d}\right)^{\frac{1}{3}}}-\frac{(b^2c-4\,abd)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\left(ab^3c^3\left(\frac{a}{b}\right)^{\frac{2}{3}}-3\,a^2b^2c^2d\left(\frac{a}{b}\right)^{\frac{2}{3}}+3\,a^3bcd^2\left(\frac{a}{b}\right)^{\frac{2}{3}}-a^4d^3\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}-\frac{(4\,bcd-ad^2)\log\left(x^2-x\left(\frac{c}{d}\right)^{\frac{1}{3}}+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{9\left(b^3c^4\left(\frac{c}{d}\right)^{\frac{2}{3}}-3\,ab^2c^3d\left(\frac{c}{d}\right)^{\frac{2}{3}}+3\,a^2bc^2d^2\left(\frac{c}{d}\right)^{\frac{2}{3}}-a^3cd^3\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}+\frac{2\,(b^2c-4\,abd)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(ab^3c^3\left(\frac{a}{b}\right)^{\frac{2}{3}}-3\,a^2b^2c^2d\left(\frac{a}{b}\right)^{\frac{2}{3}}+3\,a^3bcd^2\left(\frac{a}{b}\right)^{\frac{2}{3}}-a^4d^3\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}+\frac{2\,(4\,bcd-ad^2)\log\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{9\left(b^3c^4\left(\frac{c}{d}\right)^{\frac{2}{3}}-3\,ab^2c^3d\left(\frac{c}{d}\right)^{\frac{2}{3}}+3\,a^2bc^2d^2\left(\frac{c}{d}\right)^{\frac{2}{3}}-a^3cd^3\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}+\frac{(b^2cd+abd^2)x^4+(b^2c^2+a^2d^2)x}{3\left(a^2b^2c^4-2\,a^3bc^3d+a^4c^2d^2+(ab^3c^3d-2\,a^2b^2c^2d^2+a^3bcd^3)x^6+(ab^3c^4-a^2b^2c^3d-a^3bc^2d^2+a^4cd^3)x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^2/(d*x^3+c)^2,x, algorithm="maxima")
```

```
[Out] 2/9*sqrt(3)*(b^2*c - 4*a*b*d)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^
(1/3))/((a*b^3*c^3*(a/b)^(1/3) - 3*a^2*b^2*c^2*d*(a/b)^(1/3) + 3*a^3*b*c*d^
2*(a/b)^(1/3) - a^4*d^3*(a/b)^(1/3))*(a/b)^(1/3)) + 2/9*sqrt(3)*(4*b*c*d -
a*d^2)*arctan(1/3*sqrt(3)*(2*x - (c/d)^(1/3))/(c/d)^(1/3))/((b^3*c^4*(c/d)^
(1/3) - 3*a*b^2*c^3*d*(c/d)^(1/3) + 3*a^2*b*c^2*d^2*(c/d)^(1/3) - a^3*c*d^3
*(c/d)^(1/3))*(c/d)^(1/3)) - 1/9*(b^2*c - 4*a*b*d)*log(x^2 - x*(a/b)^(1/3)
+ (a/b)^(2/3))/(a*b^3*c^3*(a/b)^(2/3) - 3*a^2*b^2*c^2*d*(a/b)^(2/3) + 3*a^3
*b*c*d^2*(a/b)^(2/3) - a^4*d^3*(a/b)^(2/3)) - 1/9*(4*b*c*d - a*d^2)*log(x^2
 - x*(c/d)^(1/3) + (c/d)^(2/3))/(b^3*c^4*(c/d)^(2/3) - 3*a*b^2*c^3*d*(c/d)^
(2/3) + 3*a^2*b*c^2*d^2*(c/d)^(2/3) - a^3*c*d^3*(c/d)^(2/3)) + 2/9*(b^2*c -
 4*a*b*d)*log(x + (a/b)^(1/3))/(a*b^3*c^3*(a/b)^(2/3) - 3*a^2*b^2*c^2*d*(a/
b)^(2/3) + 3*a^3*b*c*d^2*(a/b)^(2/3) - a^4*d^3*(a/b)^(2/3)) + 2/9*(4*b*c*d
- a*d^2)*log(x + (c/d)^(1/3))/(b^3*c^4*(c/d)^(2/3) - 3*a*b^2*c^3*d*(c/d)^(2
/3) + 3*a^2*b*c^2*d^2*(c/d)^(2/3) - a^3*c*d^3*(c/d)^(2/3)) + 1/3*((b^2*c*d
+ a*b*d^2)*x^4 + (b^2*c^2 + a^2*d^2)*x)/(a^2*b^2*c^4 - 2*a^3*b*c^3*d + a^4*
c^2*d^2 + (a*b^3*c^3*d - 2*a^2*b^2*c^2*d^2 + a^3*b*c*d^3)*x^6 + (a*b^3*c^4
- a^2*b^2*c^3*d - a^3*b*c^2*d^2 + a^4*c*d^3)*x^3)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 897 vs. $2(341) = 682$.
time = 254.69, size = 897, normalized size = 2.14

$$\frac{3(b^3c^2d-a^2bd^3)x^4+2\sqrt{3}\left((b^3c^2d-4\,ab^2cd^2)x^6+ab^2c^3-4\,a^2bc^2d+(b^3c^3-3\,ab^2c^2d-4\,a^2bcd^2)x^3\right)\left(\frac{b^2}{a^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}\,ax\left(\frac{b^2}{a^2}\right)^{\frac{2}{3}}-\sqrt{3}\,b}{3b}\right)+2\sqrt{3}\left((4\,ab^2cd^2-a^2bd^3)x^6+4\,a^2bc^2d-a^3cd^2+(4\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)x^3\right)\left(\frac{d^2}{c^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}\,cx\left(\frac{d^2}{c^2}\right)^{\frac{2}{3}}-\sqrt{3}\,d}{3d}\right)-\left((b^3c^2d-4\,ab^2cd^2)x^6+ab^2c^3-4\,a^2bc^2d+(b^3c^3-3\,ab^2c^2d-4\,a^2bcd^2)x^3\right)\left(\frac{b^2}{a^2}\right)^{\frac{1}{3}}\log\left(b^2x^2-abx\left(\frac{b^2}{a^2}\right)^{\frac{1}{3}}+a^2\left(\frac{b^2}{a^2}\right)^{\frac{2}{3}}\right)-\left((4\,ab^2cd^2-a^2bd^3)x^6+4\,a^2bc^2d-a^3cd^2+(4\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)x^3\right)\left(\frac{d^2}{c^2}\right)^{\frac{1}{3}}\log\left(d^2x^2-cdx\left(\frac{d^2}{c^2}\right)^{\frac{1}{3}}+c^2\left(\frac{d^2}{c^2}\right)^{\frac{2}{3}}\right)+2\left((b^3c^2d-4\,ab^2cd^2)x^6+ab^2c^3-4\,a^2bc^2d+(b^3c^3-3\,ab^2c^2d-4\,a^2bcd^2)x^3\right)\left(\frac{b^2}{a^2}\right)^{\frac{1}{3}}\log\left(bx+a\left(\frac{b^2}{a^2}\right)^{\frac{1}{3}}\right)+2\left((4\,ab^2cd^2-a^2bd^3)x^6+4\,a^2bc^2d-a^3cd^2+(4\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)x^3\right)\left(\frac{d^2}{c^2}\right)^{\frac{1}{3}}\log\left(dx+c\left(\frac{d^2}{c^2}\right)^{\frac{1}{3}}\right)+3(b^3c^3-ab^2c^2d+a^2bcd^2-a^3d^3)x}{9\left(a^2b^3c^5-3\,a^3b^2c^4d+3\,a^4bc^3d^2-a^5c^2d^3+(ab^4c^4d-3\,a^2b^3c^3d^2+3\,a^3b^2c^2d^3-a^4bcd^4)x^6+(ab^4c^5-2\,a^2b^3c^4d+2\,a^4bc^2d^3-a^5cd^4)x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^2/(d*x^3+c)^2,x, algorithm="fricas")
```

```
[Out] 1/9*(3*(b^3*c^2*d - a^2*b*d^3)*x^4 + 2*sqrt(3)*((b^3*c^2*d - 4*a*b^2*c*d^2)
*x^6 + a*b^2*c^3 - 4*a^2*b*c^2*d + (b^3*c^3 - 3*a*b^2*c^2*d - 4*a^2*b*c*d^2
)*x^3)*(b^2/a^2)^(1/3)*arctan(1/3*(2*sqrt(3)*a*x*(b^2/a^2)^(2/3) - sqrt(3)*
b)/b) + 2*sqrt(3)*((4*a*b^2*c*d^2 - a^2*b*d^3)*x^6 + 4*a^2*b*c^2*d - a^3*c*
d^2 + (4*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*x^3)*(d^2/c^2)^(1/3)*arctan
(1/3*(2*sqrt(3)*c*x*(d^2/c^2)^(2/3) - sqrt(3)*d)/d) - ((b^3*c^2*d - 4*a*b^2
*c*d^2)*x^6 + a*b^2*c^3 - 4*a^2*b*c^2*d + (b^3*c^3 - 3*a*b^2*c^2*d - 4*a^2*
b*c*d^2)*x^3)*(b^2/a^2)^(1/3)*log(b^2*x^2 - a*b*x*(b^2/a^2)^(1/3) + a^2*(b^
2/a^2)^(2/3)) - ((4*a*b^2*c*d^2 - a^2*b*d^3)*x^6 + 4*a^2*b*c^2*d - a^3*c*d^
2 + (4*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*x^3)*(d^2/c^2)^(1/3)*log(d^2*
x^2 - c*d*x*(d^2/c^2)^(1/3) + c^2*(d^2/c^2)^(2/3)) + 2*((b^3*c^2*d - 4*a*b^
```

```
2*c*d^2)*x^6 + a*b^2*c^3 - 4*a^2*b*c^2*d + (b^3*c^3 - 3*a*b^2*c^2*d - 4*a^2
*b*c*d^2)*x^3)*(b^2/a^2)^(1/3)*log(b*x + a*(b^2/a^2)^(1/3)) + 2*((4*a*b^2*c
*d^2 - a^2*b*d^3)*x^6 + 4*a^2*b*c^2*d - a^3*c*d^2 + (4*a*b^2*c^2*d + 3*a^2*
b*c*d^2 - a^3*d^3)*x^3)*(d^2/c^2)^(1/3)*log(d*x + c*(d^2/c^2)^(1/3)) + 3*(b
^3*c^3 - a*b^2*c^2*d + a^2*b*c*d^2 - a^3*d^3)*x)/(a^2*b^3*c^5 - 3*a^3*b^2*c
^4*d + 3*a^4*b*c^3*d^2 - a^5*c^2*d^3 + (a*b^4*c^4*d - 3*a^2*b^3*c^3*d^2 + 3
*a^3*b^2*c^2*d^3 - a^4*b*c*d^4)*x^6 + (a*b^4*c^5 - 2*a^2*b^3*c^4*d + 2*a^4*
b*c^2*d^3 - a^5*c*d^4)*x^3)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3+a)**2/(d*x**3+c)**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.79, size = 664, normalized size = 1.58

$$-\frac{2\,(b^3c-4\,ab^2d)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,(a^2b^3c^3-3\,a^3b^2c^2d+3\,a^4bcd^2-a^5d^3)}-\frac{2\,(4\,bcd^2-ad^3)\left(-\frac{c}{d}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right|\right)}{9\,(b^3c^5-3\,ab^2c^4d+3\,a^2bc^3d^2-a^3c^2d^3)}+\frac{2\left((-ab^2)^{\frac{1}{3}}\,b^2c-4\,(-ab^2)^{\frac{1}{3}}\,abd\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\left(\sqrt{3}\,a^2b^3c^3-3\,\sqrt{3}\,a^3b^2c^2d+3\,\sqrt{3}\,a^4bcd^2-\sqrt{3}\,a^5d^3\right)}+\frac{2\left(4\,(-cd^2)^{\frac{1}{3}}\,bcd-(-cd^2)^{\frac{1}{3}}\,ad^2\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{3\left(\sqrt{3}\,b^3c^5-3\,\sqrt{3}\,ab^2c^4d+3\,\sqrt{3}\,a^2bc^3d^2-\sqrt{3}\,a^3c^2d^3\right)}+\frac{\left((-ab^2)^{\frac{1}{3}}\,b^2c-4\,(-ab^2)^{\frac{1}{3}}\,abd\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\,(a^2b^3c^3-3\,a^3b^2c^2d+3\,a^4bcd^2-a^5d^3)}+\frac{\left(4\,(-cd^2)^{\frac{1}{3}}\,bcd-(-cd^2)^{\frac{1}{3}}\,ad^2\right)\log\left(x^2+x\left(-\frac{c}{d}\right)^{\frac{1}{3}}+\left(-\frac{c}{d}\right)^{\frac{2}{3}}\right)}{9\,(b^3c^5-3\,ab^2c^4d+3\,a^2bc^3d^2-a^3c^2d^3)}+\frac{b^2cdx^4+abd^2x^4+b^2c^2x+a^2d^2x}{3\,(bdx^6+bcx^3+adx^3+ac)(ab^2c^3-2\,a^2bc^2d+a^3cd^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^2/(d*x^3+c)^2,x, algorithm="giac")
```

```
[Out] -2/9*(b^3*c - 4*a*b^2*d)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^2*b^3*c
^3 - 3*a^3*b^2*c^2*d + 3*a^4*b*c*d^2 - a^5*d^3) - 2/9*(4*b*c*d^2 - a*d^3)*(
-c/d)^(1/3)*log(abs(x - (-c/d)^(1/3)))/(b^3*c^5 - 3*a*b^2*c^4*d + 3*a^2*b*c
^3*d^2 - a^3*c^2*d^3) + 2/3*((-a*b^2)^(1/3)*b^2*c - 4*(-a*b^2)^(1/3)*a*b*d)
*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(sqrt(3)*a^2*b^3*c^3
 - 3*sqrt(3)*a^3*b^2*c^2*d + 3*sqrt(3)*a^4*b*c*d^2 - sqrt(3)*a^5*d^3) + 2/3
*(4*(-c*d^2)^(1/3)*b*c*d - (-c*d^2)^(1/3)*a*d^2)*arctan(1/3*sqrt(3)*(2*x +
(-c/d)^(1/3))/(-c/d)^(1/3))/(sqrt(3)*b^3*c^5 - 3*sqrt(3)*a*b^2*c^4*d + 3*sq
rt(3)*a^2*b*c^3*d^2 - sqrt(3)*a^3*c^2*d^3) + 1/9*((-a*b^2)^(1/3)*b^2*c - 4*
(-a*b^2)^(1/3)*a*b*d)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^2*b^3*c^3
 - 3*a^3*b^2*c^2*d + 3*a^4*b*c*d^2 - a^5*d^3) + 1/9*(4*(-c*d^2)^(1/3)*b*c*d
 - (-c*d^2)^(1/3)*a*d^2)*log(x^2 + x*(-c/d)^(1/3) + (-c/d)^(2/3))/(b^3*c^5
- 3*a*b^2*c^4*d + 3*a^2*b*c^3*d^2 - a^3*c^2*d^3) + 1/3*(b^2*c*d*x^4 + a*b*d
^2*x^4 + b^2*c^2*x + a^2*d^2*x)/((b*d*x^6 + b*c*x^3 + a*d*x^3 + a*c)*(a*b^2
*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2))
```

Mupad [**B**]
time = 24.31, size = 2500, normalized size = 5.97

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^3)^2*(c + d*x^3)^2),x)
```

```
[Out] ((x*(a^2*d^2 + b^2*c^2))/(3*a*c*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) + (b*d*x^4
*(a*d + b*c))/(3*a*c*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)))/(a*c + x^3*(a*d + b*
c) + b*d*x^6) + log((2*((4*((54*b^3*d^3*x*(a*d - b*c)^2*(a^3*d^3 + b^3*c^3
- 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))/(a*c) + 54*a*b^3*c*d^3*(a*d + b*c)*(a*d -
 b*c)^4*((b^5*(4*a*d - b*c)^3)/(a^5*(a*d - b*c)^9))^(1/3))*((b^5*(4*a*d - b
*c)^3)/(a^5*(a*d - b*c)^9))^(2/3))/81 - (8*b^4*d^4*(a^6*d^6 + b^6*c^6 + 37*
a^2*b^4*c^4*d^2 - 27*a^3*b^3*c^3*d^3 + 37*a^4*b^2*c^2*d^4 - 11*a*b^5*c^5*d
- 11*a^5*b*c*d^5))/(3*a^3*c^3*(a*d - b*c)^4))*((b^5*(4*a*d - b*c)^3)/(a^5*(
a*d - b*c)^9))^(1/3))/9 - (16*b^6*d^6*x*(4*a^6*d^6 + 4*b^6*c^6 + 268*a^2*b^
4*c^4*d^2 - 608*a^3*b^3*c^3*d^3 + 268*a^4*b^2*c^2*d^4 - 49*a*b^5*c^5*d - 49
*a^5*b*c*d^5))/(27*a^3*c^3*(a*d - b*c)^8))*(-(8*b^8*c^3 - 512*a^3*b^5*d^3 +
 384*a^2*b^6*c*d^2 - 96*a*b^7*c^2*d)/(729*a^14*d^9 - 729*a^5*b^9*c^9 + 6561
*a^6*b^8*c^8*d - 26244*a^7*b^7*c^7*d^2 + 61236*a^8*b^6*c^6*d^3 - 91854*a^9*
b^5*c^5*d^4 + 91854*a^10*b^4*c^4*d^5 - 61236*a^11*b^3*c^3*d^6 + 26244*a^12*
b^2*c^2*d^7 - 6561*a^13*b*c*d^8))^(1/3) + log((2*((4*((54*b^3*d^3*x*(a*d -
b*c)^2*(a^3*d^3 + b^3*c^3 - 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))/(a*c) + 54*a*b^
3*c*d^3*(a*d + b*c)*(a*d - b*c)^4*((d^5*(a*d - 4*b*c)^3)/(c^5*(a*d - b*c)^9
))^(1/3))*((d^5*(a*d - 4*b*c)^3)/(c^5*(a*d - b*c)^9))^(2/3))/81 - (8*b^4*d^
4*(a^6*d^6 + b^6*c^6 + 37*a^2*b^4*c^4*d^2 - 27*a^3*b^3*c^3*d^3 + 37*a^4*b^2
*c^2*d^4 - 11*a*b^5*c^5*d - 11*a^5*b*c*d^5))/(3*a^3*c^3*(a*d - b*c)^4))*((d
^5*(a*d - 4*b*c)^3)/(c^5*(a*d - b*c)^9))^(1/3))/9 - (16*b^6*d^6*x*(4*a^6*d^
6 + 4*b^6*c^6 + 268*a^2*b^4*c^4*d^2 - 608*a^3*b^3*c^3*d^3 + 268*a^4*b^2*c^2
*d^4 - 49*a*b^5*c^5*d - 49*a^5*b*c*d^5))/(27*a^3*c^3*(a*d - b*c)^8))*(-(8*a
^3*d^8 - 512*b^3*c^3*d^5 + 384*a*b^2*c^2*d^6 - 96*a^2*b*c*d^7)/(729*b^9*c^1
4 - 729*a^9*c^5*d^9 + 6561*a^8*b*c^6*d^8 + 26244*a^2*b^7*c^12*d^2 - 61236*a
^3*b^6*c^11*d^3 + 91854*a^4*b^5*c^10*d^4 - 91854*a^5*b^4*c^9*d^5 + 61236*a^
6*b^3*c^8*d^6 - 26244*a^7*b^2*c^7*d^7 - 6561*a*b^8*c^13*d))^(1/3) + (log(((
3^(1/2)*1i - 1)*(((3^(1/2)*1i - 1)^2*((54*b^3*d^3*x*(a*d - b*c)^2*(a^3*d^3
+ b^3*c^3 - 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))/(a*c) + 27*a*b^3*c*d^3*(3^(1/2)
*1i - 1)*(a*d + b*c)*(a*d - b*c)^4*((b^5*(4*a*d - b*c)^3)/(a^5*(a*d - b*c)^
9))^(1/3))*((b^5*(4*a*d - b*c)^3)/(a^5*(a*d - b*c)^9))^(2/3))/81 - (8*b^4*d
^4*(a^6*d^6 + b^6*c^6 + 37*a^2*b^4*c^4*d^2 - 27*a^3*b^3*c^3*d^3 + 37*a^4*b^
2*c^2*d^4 - 11*a*b^5*c^5*d - 11*a^5*b*c*d^5))/(3*a^3*c^3*(a*d - b*c)^4))*((
b^5*(4*a*d - b*c)^3)/(a^5*(a*d - b*c)^9))^(1/3))/9 - (16*b^6*d^6*x*(4*a^6*d
^6 + 4*b^6*c^6 + 268*a^2*b^4*c^4*d^2 - 608*a^3*b^3*c^3*d^3 + 268*a^4*b^2*c^
2*d^4 - 49*a*b^5*c^5*d - 49*a^5*b*c*d^5))/(27*a^3*c^3*(a*d - b*c)^8))*(3^(1
/2)*1i - 1)*(-(8*b^8*c^3 - 512*a^3*b^5*d^3 + 384*a^2*b^6*c*d^2 - 96*a*b^7*c
^2*d)/(729*a^14*d^9 - 729*a^5*b^9*c^9 + 6561*a^6*b^8*c^8*d - 26244*a^7*b^7*
c^7*d^2 + 61236*a^8*b^6*c^6*d^3 - 91854*a^9*b^5*c^5*d^4 + 91854*a^10*b^4*c^
4*d^5 - 61236*a^11*b^3*c^3*d^6 + 26244*a^12*b^2*c^2*d^7 - 6561*a^13*b*c*d^8
))^(1/3))/2 - (log(((3^(1/2)*1i + 1)*(((3^(1/2)*1i + 1)^2*((54*b^3*d^3*x*(a
```

```
*d - b*c)^2*(a^3*d^3 + b^3*c^3 - 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))/(a*c) - 27
*a*b^3*c*d^3*(3^(1/2)*1i + 1)*(a*d + b*c)*(a*d - b*c)^4*((b^5*(4*a*d - b*c)
^3)/(a^5*(a*d - b*c)^9))^(1/3))*((b^5*(4*a*d - b*c)^3)/(a^5*(a*d - b*c)^9))
^(2/3))/81 - (8*b^4*d^4*(a^6*d^6 + b^6*c^6 + 37*a^2*b^4*c^4*d^2 - 27*a^3*b^
3*c^3*d^3 + 37*a^4*b^2*c^2*d^4 - 11*a*b^5*c^5*d - 11*a^5*b*c*d^5))/(3*a^3*c
^3*(a*d - b*c)^4))*((b^5*(4*a*d - b*c)^3)/(a^5*(a*d - b*c)^9))^(1/3))/9 + (
16*b^6*d^6*x*(4*a^6*d^6 + 4*b^6*c^6 + 268*a^2*b^4*c^4*d^2 - 608*a^3*b^3*c^3
*d^3 + 268*a^4*b^2*c^2*d^4 - 49*a*b^5*c^5*d - 49*a^5*b*c*d^5))/(27*a^3*c^3*
(a*d - b*c)^8))*(3^(1/2)*1i + 1)*(-(8*b^8*c^3 - 512*a^3*b^5*d^3 + 384*a^2*b
^6*c*d^2 - 96*a*b^7*c^2*d)/(729*a^14*d^9 - 729*a^5*b^9*c^9 + 6561*a^6*b^8*c
^8*d - 26244*a^7*b^7*c^7*d^2 + 61236*a^8*b^6*c^6*d^3 - 91854*a^9*b^5*c^5*d^
4 + 91854*a^10*b^4*c^4*d^5 - 61236*a^11*b^3*c^3*d^6 + 26244*a^12*b^2*c^2*d^
7 - 6561*a^13*b*c*d^8))^(1/3))/2 + (log(((3^(1/2)*1i - 1)*(((3^(1/2)*1i - 1
)^2*((54*b^3*d^3*x*(a*d - b*c)^2*(a^3*d^3 + b^3*c^3 - 3*a*b^2*c^2*d - 3*a^2
*b*c*d^2))/(a*c) + 27*a*b^3*c*d^3*(3^(1/2)*1i - 1)*(a*d + b*c)*(a*d - b*c)^
4*((d^5*(a*d - 4*b*c)^3)/(c^5*(a*d - b*c)^9))^(1/3))*((d^5*(a*d - 4*b*c)^3)
/(c^5*(a*d - b*c)^9))^(2/3))/81 - (8*b^4*d^4*(a^6*d^6 + b^6*c^6 + 37*a^2*b^
4*c^4*d^2 - 27*a^3*b^3*c^3*d^3 + 37*a^4*b^2*c^2*d^4 - 11*a*b^5*c^5*d - 11*a
^5*b*c*d^5))/(3*a^3*c^3*(a*d - b*c)^4))*((d^5*(a*d - 4*b*c)^3)/(c^5*(a*d -
b*c)^9))^(1/3))/9 - (16*b^6*d^6*x*(4*a^6*d^6 + 4*b^6*c^6 + 268*a^2*b^4*c^4*
d^2 - 608*a^3*b^3*c^3*d^3 + 268*a^4*b^2*c^2*d^4 - 49*a*b^5*c^5*d - 49*a^5*b
*c*d^5))/(27*a^3*c^3*(a*d - b*c)^8))*(3^(1/2)*1i - 1)*(-(8*a^3*d^8 - 512*b^
3*c^3*d^5 + 384*a*b^2*c^2*d^6 - 96*a^2*b*c*d^7)/(729*b^9*c^14 - 729*a^9*c^5
*d^9 + 6561*a^8*b*c^6*d^8 + 26244*a^2*b^7*c^12*d^2 - 61236*a^3*b^6*c^11*d^3
 + 91854*a^4*b^5*c^10*d^4 - 91854*a^5*b^4*c^9*d^5 + 61236*a^6*b^3*c^8*d^6 -
 26244*a^7*b^2*c^7*d^7 - 6561*a*b^8*c^13*d))^(1...
```

3.27 $\int (a-bx^3)(a+bx^3)^{2/3}\,dx$

Optimal. Leaf size=112

$$\frac{7}{18}ax\left(a+bx^3\right)^{2/3}-\frac{1}{6}x\left(a+bx^3\right)^{5/3}+\frac{7a^2\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{9\sqrt{3}\sqrt[3]{b}}-\frac{7a^2\log\left(-\sqrt[3]{b}\,x+\sqrt[3]{a+bx^3}\right)}{18\sqrt[3]{b}}$$

```
[Out] 7/18*a*x*(b*x^3+a)^(2/3)-1/6*x*(b*x^3+a)^(5/3)-7/18*a^2*ln(-b^(1/3)*x+(b*x^
3+a)^(1/3))/b^(1/3)+7/27*a^2*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(
1/2))/b^(1/3)*3^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {396, 201, 245}

$$\frac{7a^2\text{ArcTan}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{9\sqrt{3}\sqrt[3]{b}}-\frac{7a^2\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}\,x\right)}{18\sqrt[3]{b}}+\frac{7}{18}ax\left(a+bx^3\right)^{2/3}-\frac{1}{6}x\left(a+bx^3\right)^{5/3}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^3)*(a + b*x^3)^(2/3),x]
```

```
[Out] (7*a*x*(a + b*x^3)^(2/3))/18 - (x*(a + b*x^3)^(5/3))/6 + (7*a^2*ArcTan[(1 +
 (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(9*Sqrt[3]*b^(1/3)) - (7*a^2*Lo
g[-(b^(1/3)*x) + (a + b*x^3)^(1/3)])/(18*b^(1/3))
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \left(a-bx^3\right)\left(a+bx^3\right)^{2/3}\,dx &= -\frac{1}{6}x\left(a+bx^3\right)^{5/3}+\frac{1}{6}(7a)\int \left(a+bx^3\right)^{2/3}\,dx\\
&= \frac{7}{18}ax\left(a+bx^3\right)^{2/3}-\frac{1}{6}x\left(a+bx^3\right)^{5/3}+\frac{1}{9}\left(7a^2\right)\int \frac{1}{\sqrt[3]{a+bx^3}}\,dx\\
&= \frac{7}{18}ax\left(a+bx^3\right)^{2/3}-\frac{1}{6}x\left(a+bx^3\right)^{5/3}+\frac{7a^2\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{9\sqrt{3}\,\sqrt[3]{b}}-\frac{7a^2\,\mathrm{lo}}{}
\end{aligned}$$

Mathematica [A]
time = 0.34, size = 156, normalized size = 1.39

$$\frac{3\sqrt[3]{b}\left(a+bx^3\right)^{2/3}\left(4ax-3bx^4\right)+14\sqrt{3}\,a^2\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[3]{b}\,x}{\sqrt[3]{b}\,x+2\sqrt[3]{a+bx^3}}\right)-14a^2\log\left(-\sqrt[3]{b}\,x+\sqrt[3]{a+bx^3}\right)+7a^2\log\left(b^{2/3}x^2+\sqrt[3]{b}\,x\sqrt[3]{a+bx^3}+\left(a+bx^3\right)^{2/3}\right)}{54\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^3)*(a + b*x^3)^(2/3),x]
```

```
[Out] (3*b^(1/3)*(a + b*x^3)^(2/3)*(4*a*x - 3*b*x^4) + 14*Sqrt[3]*a^2*ArcTan[(Sqr
t[3]*b^(1/3)*x)/(b^(1/3)*x + 2*(a + b*x^3)^(1/3))] - 14*a^2*Log[-(b^(1/3)*x
) + (a + b*x^3)^(1/3)] + 7*a^2*Log[b^(2/3)*x^2 + b^(1/3)*x*(a + b*x^3)^(1/3
) + (a + b*x^3)^(2/3)])/(54*b^(1/3))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \left(-b\,x^3+a\right)\left(b\,x^3+a\right)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^3+a)*(b*x^3+a)^(2/3),x)
```

```
[Out] int((-b*x^3+a)*(b*x^3+a)^(2/3),x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 322 vs. $2(85) = 170$.
time = 0.50, size = 322, normalized size = 2.88

$$-\frac{1}{9}\left(\frac{2\sqrt{3}\,a\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\,(bx^3+a)^{\frac{1}{3}}}{x}\right)}{3\,b^{\frac{1}{3}}}\right)}{b^{\frac{1}{3}}}-\frac{a\log\left(b^{\frac{2}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{(bx^3+a)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{1}{3}}}+\frac{2\,a\log\left(-b^{\frac{1}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}}{x}\right)}{b^{\frac{1}{3}}}+\frac{3\,(bx^3+a)^{\frac{2}{3}}a}{\left(b-\frac{bx^3+a}{x^3}\right)x^2}\right)a-\frac{1}{54}\left(\frac{2\sqrt{3}\,a^2\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\,(bx^3+a)^{\frac{1}{3}}}{x}\right)}{3\,b^{\frac{1}{3}}}\right)}{b^{\frac{4}{3}}}-\frac{a^2\log\left(b^{\frac{2}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{(bx^3+a)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{4}{3}}}+\frac{2\,a^2\log\left(-b^{\frac{1}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}}{x}\right)}{b^{\frac{4}{3}}}+\frac{3\left(\frac{(bx^3+a)^{\frac{2}{3}}a^2b}{x^2}+\frac{2\,(bx^3+a)^{\frac{5}{3}}a^2}{x^5}\right)}{b^3-\frac{2\,(bx^3+a)b^2}{x^3}+\frac{(bx^3+a)^2b}{x^6}}\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)*(b*x^3+a)^(2/3),x, algorithm="maxima")
```

```
[Out] -1/9*(2*sqrt(3)*a*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1
/3))/b^(1/3) - a*log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2
/3)/x^2)/b^(1/3) + 2*a*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(1/3) + 3*(b*x
^3 + a)^(2/3)*a/((b - (b*x^3 + a)/x^3)*x^2))*a - 1/54*(2*sqrt(3)*a^2*arctan
(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1/3))/b^(4/3) - a^2*log(b
^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2/3)/x^2)/b^(4/3) + 2*a
^2*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(4/3) + 3*((b*x^3 + a)^(2/3)*a^2*b
/x^2 + 2*(b*x^3 + a)^(5/3)*a^2/x^5)/(b^3 - 2*(b*x^3 + a)*b^2/x^3 + (b*x^3 +
 a)^2*b/x^6))*b
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 175 vs. $2(85) = 170$.
time = 4.43, size = 399, normalized size = 3.56

$$\left[\frac{21\sqrt{\frac{1}{3}}\,a^2b\sqrt{\frac{(-b)^{\frac{1}{3}}}{b}}\log\left(3bx^3-3\,(bx^3+a)^{\frac{1}{3}}(-b)^{\frac{2}{3}}x^2-3\sqrt{\frac{1}{3}}\left((-b)^{\frac{1}{3}}bx^3-(bx^3+a)^{\frac{1}{3}}bx^2+2\,(bx^3+a)^{\frac{2}{3}}(-b)^{\frac{2}{3}}x\right)\sqrt{\frac{(-b)^{\frac{1}{3}}}{b}}+2a\right)-14\,a^2(-b)^{\frac{2}{3}}\log\left(\frac{(-b)^{\frac{1}{3}}x+(bx^3+a)^{\frac{1}{3}}}{x}\right)+7\,a^2(-b)^{\frac{2}{3}}\log\left(\frac{(-b)^{\frac{2}{3}}x^2-(bx^3+a)^{\frac{1}{3}}(-b)^{\frac{1}{3}}x+(bx^3+a)^{\frac{2}{3}}}{x^2}\right)-3\,(3b^2x^4-4abx)(bx^3+a)^{\frac{2}{3}}}{54\,b},\ -\frac{42\sqrt{\frac{1}{3}}\,a^2b\sqrt{-\frac{(-b)^{\frac{1}{3}}}{b}}\arctan\left(-\frac{\sqrt{\frac{1}{3}}\left((-b)^{\frac{1}{3}}x-2\,(bx^3+a)^{\frac{1}{3}}\right)\sqrt{-\frac{(-b)^{\frac{1}{3}}}{b}}}{x}\right)+14\,a^2(-b)^{\frac{2}{3}}\log\left(\frac{(-b)^{\frac{1}{3}}x+(bx^3+a)^{\frac{1}{3}}}{x}\right)-7\,a^2(-b)^{\frac{2}{3}}\log\left(\frac{(-b)^{\frac{2}{3}}x^2-(bx^3+a)^{\frac{1}{3}}(-b)^{\frac{1}{3}}x+(bx^3+a)^{\frac{2}{3}}}{x^2}\right)+3\,(3b^2x^4-4abx)(bx^3+a)^{\frac{2}{3}}}{54\,b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)*(b*x^3+a)^(2/3),x, algorithm="fricas")
```

```
[Out] [1/54*(21*sqrt(1/3)*a^2*b*sqrt((-b)^(1/3)/b)*log(3*b*x^3 - 3*(b*x^3 + a)^(1
/3)*(-b)^(2/3)*x^2 - 3*sqrt(1/3)*((-b)^(1/3)*b*x^3 - (b*x^3 + a)^(1/3)*b*x^
2 + 2*(b*x^3 + a)^(2/3)*(-b)^(2/3)*x)*sqrt((-b)^(1/3)/b) + 2*a) - 14*a^2*(-
b)^(2/3)*log(((-b)^(1/3)*x + (b*x^3 + a)^(1/3))/x) + 7*a^2*(-b)^(2/3)*log((
(-b)^(2/3)*x^2 - (b*x^3 + a)^(1/3)*(-b)^(1/3)*x + (b*x^3 + a)^(2/3))/x^2) -
 3*(3*b^2*x^4 - 4*a*b*x)*(b*x^3 + a)^(2/3))/b, -1/54*(42*sqrt(1/3)*a^2*b*sq
rt(-(-b)^(1/3)/b)*arctan(-sqrt(1/3)*((-b)^(1/3)*x - 2*(b*x^3 + a)^(1/3))*sq
rt(-(-b)^(1/3)/b)/x) + 14*a^2*(-b)^(2/3)*log(((-b)^(1/3)*x + (b*x^3 + a)^(1
/3))/x) - 7*a^2*(-b)^(2/3)*log(((-b)^(2/3)*x^2 - (b*x^3 + a)^(1/3)*(-b)^(1/
3)*x + (b*x^3 + a)^(2/3))/x^2) + 3*(3*b^2*x^4 - 4*a*b*x)*(b*x^3 + a)^(2/3))
/b]
```

Sympy [C] Result contains complex when optimal does not.

time = 2.26, size = 80, normalized size = 0.71

$$\frac{a^{\frac{5}{3}}x\Gamma\left(\frac{1}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{2}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)}-\frac{a^{\frac{2}{3}}bx^{4}\Gamma\left(\frac{4}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{2}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**3+a)*(b*x**3+a)**(2/3),x)
```

```
[Out] a**(5/3)*x*gamma(1/3)*hyper((-2/3, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a)/
(3*gamma(4/3)) - a**(2/3)*b*x**4*gamma(4/3)*hyper((-2/3, 4/3), (7/3,), b*x*
*3*exp_polar(I*pi)/a)/(3*gamma(7/3))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)*(b*x^3+a)^(2/3),x, algorithm="giac")
```

```
[Out] integrate(-(b*x^3 + a)^(2/3)*(b*x^3 - a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(b\,x^3+a\right)^{2/3}\,\left(a-b\,x^3\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(2/3)*(a - b*x^3),x)
```

```
[Out] int((a + b*x^3)^(2/3)*(a - b*x^3), x)
```

3.28 $\int \frac{a-bx^3}{\sqrt[3]{a+bx^3}} dx$

Optimal. Leaf size=91

$$-\frac{1}{3}x\left(a+bx^3\right)^{2/3}+\frac{4a\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}\,\sqrt[3]{b}}-\frac{2a\log\left(-\sqrt[3]{b}\,x+\sqrt[3]{a+bx^3}\right)}{3\sqrt[3]{b}}$$

```
[Out] -1/3*x*(b*x^3+a)^(2/3)-2/3*a*ln(-b^(1/3)*x+(b*x^3+a)^(1/3))/b^(1/3)+4/9*a*a
rctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/b^(1/3)*3^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {396, 245}

$$\frac{4a\text{ArcTan}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{3\sqrt{3}\,\sqrt[3]{b}}-\frac{1}{3}x\left(a+bx^3\right)^{2/3}-\frac{2a\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}\,x\right)}{3\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^3)/(a + b*x^3)^(1/3),x]
```

```
[Out] -1/3*(x*(a + b*x^3)^(2/3)) + (4*a*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/
3))/Sqrt[3]])/(3*Sqrt[3]*b^(1/3)) - (2*a*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/
3)])/(3*b^(1/3))
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rubi steps

$$\int \frac{a-bx^3}{\sqrt[3]{a+bx^3}}\,dx = -\frac{1}{3}x\left(a+bx^3\right)^{2/3}+\frac{1}{3}(4a)\int \frac{1}{\sqrt[3]{a+bx^3}}\,dx$$

$$= -\frac{1}{3}x\left(a+bx^3\right)^{2/3}+\frac{4a\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}\,\sqrt[3]{b}}-\frac{2a\log\left(-\sqrt[3]{b}\,x+\sqrt[3]{a+bx^3}\right)}{3\sqrt[3]{b}}$$

Mathematica [A]
time = 0.28, size = 140, normalized size = 1.54

$$\frac{-3\sqrt[3]{b}\,x(a+bx^3)^{2/3}+4\sqrt{3}\,a\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[3]{b}\,x}{\sqrt[3]{b}\,x+2\sqrt[3]{a+bx^3}}\right)-4a\log\left(-\sqrt[3]{b}\,x+\sqrt[3]{a+bx^3}\right)+2a\log\left(b^{2/3}x^2+\sqrt[3]{b}\,x\sqrt[3]{a+bx^3}+(a+bx^3)^{2/3}\right)}{9\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^3)/(a + b*x^3)^(1/3),x]
```

```
[Out] (-3*b^(1/3)*x*(a + b*x^3)^(2/3) + 4*Sqrt[3]*a*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b
^(1/3)*x + 2*(a + b*x^3)^(1/3))] - 4*a*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)
] + 2*a*Log[b^(2/3)*x^2 + b^(1/3)*x*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)])
/(9*b^(1/3))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{-b\,x^3+a}{\left(b\,x^3+a\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^3+a)/(b*x^3+a)^(1/3),x)
```

```
[Out] int((-b*x^3+a)/(b*x^3+a)^(1/3),x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 244 vs. 2(68) = 136.
time = 0.51, size = 244, normalized size = 2.68

$$-\frac{1}{6}\left(\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)}{3b^{\frac{1}{3}}}\right)}{b^{\frac{1}{3}}}-\frac{\log\left(b^{\frac{2}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{(bx^3+a)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{1}{3}}}+\frac{2\log\left(-b^{\frac{1}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}}{x}\right)}{b^{\frac{1}{3}}}\right)a-\frac{1}{18}\left(\frac{2\sqrt{3}\,a\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)}{3b^{\frac{1}{3}}}\right)}{b^{\frac{4}{3}}}-\frac{a\log\left(b^{\frac{2}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{(bx^3+a)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{4}{3}}}+\frac{2a\log\left(-b^{\frac{1}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}}{x}\right)}{b^{\frac{4}{3}}}-\frac{6\left(bx^3+a\right)^{\frac{2}{3}}a}{\left(b^2-\frac{(bx^3+a)b}{x^3}\right)x^2}\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)/(b*x^3+a)^(1/3),x, algorithm="maxima")
```

```
[Out] -1/6*(2*sqrt(3)*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1/3
))/b^(1/3) - log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2/3)/
x^2)/b^(1/3) + 2*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(1/3))*a - 1/18*(2*s
qrt(3)*a*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1/3))/b^(4
/3) - a*log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2/3)/x^2)/
b^(4/3) + 2*a*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(4/3) - 6*(b*x^3 + a)^(
2/3)*a/((b^2 - (b*x^3 + a)*b/x^3)*x^2))*b
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 157 vs. $2(68) = 136$.
time = 2.25, size = 363, normalized size = 3.99

$$\left[\frac{6\sqrt{\frac{1}{3}}\,ab\sqrt{\frac{(-b)^{\frac{1}{3}}}{b}}\log\left(3\,bx^3 - 3\,(bx^3+a)^{\frac{1}{3}}(-b)^{\frac{2}{3}}x^2 - 3\sqrt{\frac{1}{3}}\left((-b)^{\frac{1}{3}}bx^3 - (bx^3+a)^{\frac{1}{3}}bx^2 + 2\,(bx^3+a)^{\frac{2}{3}}(-b)^{\frac{2}{3}}x\right)\sqrt{\frac{(-b)^{\frac{1}{3}}}{b}} + 2\,a\right) - 3\,(bx^3+a)^{\frac{2}{3}}bx - 4\,a(-b)^{\frac{2}{3}}\log\left(\frac{(-b)^{\frac{1}{3}}x+(bx^3+a)^{\frac{1}{3}}}{x}\right) + 2\,a(-b)^{\frac{2}{3}}\log\left(\frac{(-b)^{\frac{2}{3}}x^2-(bx^3+a)^{\frac{1}{3}}(-b)^{\frac{1}{3}}x+(bx^3+a)^{\frac{2}{3}}}{x^2}\right)}{9\,b}, -\frac{12\sqrt{\frac{1}{3}}\,ab\sqrt{-\frac{(-b)^{\frac{1}{3}}}{b}}\arctan\left(-\frac{\sqrt{\frac{1}{3}}\left((-b)^{\frac{1}{3}}x-2\,(bx^3+a)^{\frac{1}{3}}\right)\sqrt{-\frac{(-b)^{\frac{1}{3}}}{b}}}{x}\right) + 3\,(bx^3+a)^{\frac{2}{3}}bx + 4\,a(-b)^{\frac{2}{3}}\log\left(\frac{(-b)^{\frac{1}{3}}x+(bx^3+a)^{\frac{1}{3}}}{x}\right) - 2\,a(-b)^{\frac{2}{3}}\log\left(\frac{(-b)^{\frac{2}{3}}x^2-(bx^3+a)^{\frac{1}{3}}(-b)^{\frac{1}{3}}x+(bx^3+a)^{\frac{2}{3}}}{x^2}\right)}{9\,b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)/(b*x^3+a)^(1/3),x, algorithm="fricas")
```

```
[Out] [1/9*(6*sqrt(1/3)*a*b*sqrt((-b)^(1/3)/b)*log(3*b*x^3 - 3*(b*x^3 + a)^(1/3)*
(-b)^(2/3)*x^2 - 3*sqrt(1/3)*((-b)^(1/3)*b*x^3 - (b*x^3 + a)^(1/3)*b*x^2 +
2*(b*x^3 + a)^(2/3)*(-b)^(2/3)*x)*sqrt((-b)^(1/3)/b) + 2*a) - 3*(b*x^3 + a)
^(2/3)*b*x - 4*a*(-b)^(2/3)*log(((-b)^(1/3)*x + (b*x^3 + a)^(1/3))/x) + 2*a
*(-b)^(2/3)*log(((-b)^(2/3)*x^2 - (b*x^3 + a)^(1/3)*(-b)^(1/3)*x + (b*x^3 +
 a)^(2/3))/x^2))/b, -1/9*(12*sqrt(1/3)*a*b*sqrt(-(-b)^(1/3)/b)*arctan(-sqrt
(1/3)*((-b)^(1/3)*x - 2*(b*x^3 + a)^(1/3))*sqrt(-(-b)^(1/3)/b)/x) + 3*(b*x^
3 + a)^(2/3)*b*x + 4*a*(-b)^(2/3)*log(((-b)^(1/3)*x + (b*x^3 + a)^(1/3))/x)
 - 2*a*(-b)^(2/3)*log(((-b)^(2/3)*x^2 - (b*x^3 + a)^(1/3)*(-b)^(1/3)*x + (b
*x^3 + a)^(2/3))/x^2))/b]
```

Sympy [**C**] Result contains complex when optimal does not.
time = 1.41, size = 76, normalized size = 0.84

$$\frac{a^{\frac{2}{3}}x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{3},\frac{1}{3}\\\frac{4}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)} - \frac{bx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{3},\frac{4}{3}\\\frac{7}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt[3]{a}\,\Gamma\left(\frac{7}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**3+a)/(b*x**3+a)**(1/3),x)
```

```
[Out] a**(2/3)*x*gamma(1/3)*hyper((1/3, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a)/(
3*gamma(4/3)) - b*x**4*gamma(4/3)*hyper((1/3, 4/3), (7/3,), b*x**3*exp_pola
r(I*pi)/a)/(3*a**(1/3)*gamma(7/3))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)/(b*x^3+a)^(1/3),x, algorithm="giac")
[Out] integrate(-(b*x^3 - a)/(b*x^3 + a)^(1/3), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a-b\,x^3}{{\left(b\,x^3+a\right)}^{1/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x^3)/(a + b*x^3)^(1/3),x)
[Out] int((a - b*x^3)/(a + b*x^3)^(1/3), x)
```

3.29 $\int \frac{a-bx^3}{\left(a+bx^3\right)^{4/3}} dx$

Optimal. Leaf size=85

$$\frac{2x}{\sqrt[3]{a+bx^3}}-\frac{\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,\sqrt[3]{b}}+\frac{\log\left(-\sqrt[3]{b}\,x+\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{b}}$$

```
[Out] 2*x/(b*x^3+a)^(1/3)+1/2*ln(-b^(1/3)*x+(b*x^3+a)^(1/3))/b^(1/3)-1/3*arctan(1
/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/b^(1/3)*3^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {393, 245}

$$-\frac{\text{ArcTan}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,\sqrt[3]{b}}+\frac{2x}{\sqrt[3]{a+bx^3}}+\frac{\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}\,x\right)}{2\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^3)/(a + b*x^3)^(4/3),x]
```

```
[Out] (2*x)/(a + b*x^3)^(1/3) - ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt
[3]]/(Sqrt[3]*b^(1/3)) + Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)]/(2*b^(1/3))
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rubi steps

$$\int \frac{a-bx^3}{(a+bx^3)^{4/3}}\,dx = \frac{2x}{\sqrt[3]{a+bx^3}} - \int \frac{1}{\sqrt[3]{a+bx^3}}\,dx$$

$$= \frac{2x}{\sqrt[3]{a+bx^3}} - \frac{\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,\sqrt[3]{b}} + \frac{\log\left(-\sqrt[3]{b}\,x+\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{b}}$$

Mathematica [**A**]
time = 0.24, size = 142, normalized size = 1.67

$$\frac{2x}{\sqrt[3]{a+bx^3}} - \frac{\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[3]{b}\,x}{\sqrt[3]{b}\,x+2\sqrt[3]{a+bx^3}}\right)}{\sqrt{3}\,\sqrt[3]{b}} + \frac{\log\left(-\sqrt[3]{b}\,x+\sqrt[3]{a+bx^3}\right)}{3\sqrt[3]{b}} - \frac{\log\left(b^{2/3}x^2+\sqrt[3]{b}\,x\sqrt[3]{a+bx^3}+(a+bx^3)^{2/3}\right)}{6\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^3)/(a + b*x^3)^(4/3),x]
```

```
[Out] (2*x)/(a + b*x^3)^(1/3) - ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2*(a + b*
x^3)^(1/3))]/(Sqrt[3]*b^(1/3)) + Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)]/(3*b
^(1/3)) - Log[b^(2/3)*x^2 + b^(1/3)*x*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)
]/(6*b^(1/3))
```

Maple [**F**]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{-b\,x^3+a}{(b\,x^3+a)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^3+a)/(b*x^3+a)^(4/3),x)
```

```
[Out] int((-b*x^3+a)/(b*x^3+a)^(4/3),x)
```

Maxima [**A**]
time = 0.52, size = 130, normalized size = 1.53

$$\frac{1}{6}\,b\left(\frac{2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\,(bx^3+a)^{\frac{1}{3}}}{x}\right)}{3\,b^{\frac{1}{3}}}\right)}{b^{\frac{4}{3}}} + \frac{6\,x}{(bx^3+a)^{\frac{1}{3}}b} - \frac{\log\left(b^{\frac{2}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{(bx^3+a)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{4}{3}}} + \frac{2\,\log\left(-b^{\frac{1}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}}{x}\right)}{b^{\frac{4}{3}}}\right) + \frac{x}{(bx^3+a)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)/(b*x^3+a)^(4/3),x, algorithm="maxima")
```

```
[Out] 1/6*b*(2*sqrt(3)*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1/
3))/b^(4/3) + 6*x/((b*x^3 + a)^(1/3)*b) - log(b^(2/3) + (b*x^3 + a)^(1/3)*b
^(1/3)/x + (b*x^3 + a)^(2/3)/x^2)/b^(4/3) + 2*log(-b^(1/3) + (b*x^3 + a)^(1
/3)/x)/b^(4/3)) + x/(b*x^3 + a)^(1/3)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 159 vs. $2(66) = 132$.
time = 4.35, size = 372, normalized size = 4.38

$$\left[\frac{3\sqrt{\frac{1}{3}}\left(b^{2}x^{3}+ab\right)\sqrt{-\frac{1}{b^{\frac{2}{3}}}}\log\left(3bx^{3}-3\left(bx^{3}+a\right)^{\frac{1}{3}}b^{\frac{2}{3}}x^{2}-3\sqrt{\frac{1}{3}}\left(b^{\frac{4}{3}}x^{3}+\left(bx^{3}+a\right)^{\frac{1}{3}}bx^{2}-2\left(bx^{3}+a\right)^{\frac{2}{3}}b^{\frac{2}{3}}x\right)\sqrt{-\frac{1}{b^{\frac{2}{3}}}}+2a\right)+12\left(bx^{3}+a\right)^{\frac{2}{3}}bx+2\left(bx^{3}+a\right)b^{\frac{2}{3}}\log\left(-\frac{b^{\frac{1}{3}}x-\left(bx^{3}+a\right)^{\frac{1}{3}}}{x}\right)-\left(bx^{3}+a\right)b^{\frac{2}{3}}\log\left(\frac{b^{\frac{2}{3}}x^{2}+\left(bx^{3}+a\right)^{\frac{1}{3}}b^{\frac{1}{3}}x+\left(bx^{3}+a\right)^{\frac{2}{3}}}{x^{2}}\right)}{6\left(b^{2}x^{3}+ab\right)}, \frac{12\left(bx^{3}+a\right)^{\frac{2}{3}}bx+2\left(bx^{3}+a\right)b^{\frac{2}{3}}\log\left(-\frac{b^{\frac{1}{3}}x-\left(bx^{3}+a\right)^{\frac{1}{3}}}{x}\right)-\left(bx^{3}+a\right)b^{\frac{2}{3}}\log\left(\frac{b^{\frac{2}{3}}x^{2}+\left(bx^{3}+a\right)^{\frac{1}{3}}b^{\frac{1}{3}}x+\left(bx^{3}+a\right)^{\frac{2}{3}}}{x^{2}}\right)+\frac{6\sqrt{\frac{1}{3}}\left(b^{2}x^{3}+ab\right)\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(b^{\frac{1}{3}}x+2\left(bx^{3}+a\right)^{\frac{1}{3}}\right)}{b^{\frac{1}{3}}x}\right)}{b^{\frac{1}{3}}}}{6\left(b^{2}x^{3}+ab\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)/(b*x^3+a)^(4/3),x, algorithm="fricas")
```

```
[Out] [1/6*(3*sqrt(1/3)*(b^2*x^3 + a*b)*sqrt(-1/b^(2/3))*log(3*b*x^3 - 3*(b*x^3 +
 a)^(1/3)*b^(2/3)*x^2 - 3*sqrt(1/3)*(b^(4/3)*x^3 + (b*x^3 + a)^(1/3)*b*x^2
- 2*(b*x^3 + a)^(2/3)*b^(2/3)*x)*sqrt(-1/b^(2/3)) + 2*a) + 12*(b*x^3 + a)^(
2/3)*b*x + 2*(b*x^3 + a)*b^(2/3)*log(-(b^(1/3)*x - (b*x^3 + a)^(1/3))/x) -
(b*x^3 + a)*b^(2/3)*log((b^(2/3)*x^2 + (b*x^3 + a)^(1/3)*b^(1/3)*x + (b*x^3
 + a)^(2/3))/x^2))/(b^2*x^3 + a*b), 1/6*(12*(b*x^3 + a)^(2/3)*b*x + 2*(b*x^
3 + a)*b^(2/3)*log(-(b^(1/3)*x - (b*x^3 + a)^(1/3))/x) - (b*x^3 + a)*b^(2/3
)*log((b^(2/3)*x^2 + (b*x^3 + a)^(1/3)*b^(1/3)*x + (b*x^3 + a)^(2/3))/x^2)
+ 6*sqrt(1/3)*(b^2*x^3 + a*b)*arctan(sqrt(1/3)*(b^(1/3)*x + 2*(b*x^3 + a)^(
1/3))/(b^(1/3)*x))/b^(1/3))/(b^2*x^3 + a*b)]
```

Sympy [C] Result contains complex when optimal does not.
time = 3.39, size = 70, normalized size = 0.82

$$\frac{x\Gamma\left(\frac{1}{3}\right)}{3\sqrt[3]{a}\sqrt[3]{1+\frac{bx^{3}}{a}}\,\Gamma\left(\frac{4}{3}\right)}-\frac{bx^{4}\Gamma\left(\frac{4}{3}\right){}_{2}F_{1}\left(\begin{matrix}\frac{4}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3a^{\frac{4}{3}}\Gamma\left(\frac{7}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**3+a)/(b*x**3+a)**(4/3),x)
```

```
[Out] x*gamma(1/3)/(3*a**(1/3)*(1 + b*x**3/a)**(1/3)*gamma(4/3)) - b*x**4*gamma(4
/3)*hyper((4/3, 4/3), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(4/3)*gamma(7
/3))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)/(b*x^3+a)^(4/3),x, algorithm="giac")
[Out] integrate(-(b*x^3 - a)/(b*x^3 + a)^(4/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a-b\,x^3}{{\left(b\,x^3+a\right)}^{4/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x^3)/(a + b*x^3)^(4/3),x)
[Out] int((a - b*x^3)/(a + b*x^3)^(4/3), x)
```

3.30 $\int \frac{a-bx^3}{(a+bx^3)^{7/3}} \, dx$

Optimal. Leaf size=47

$$\frac{x(a-bx^3)}{4a\left(a+bx^3\right)^{4/3}}+\frac{3x}{4a\sqrt[3]{a+bx^3}}$$

```
[Out] 1/4*x*(-b*x^3+a)/a/(b*x^3+a)^(4/3)+3/4*x/a/(b*x^3+a)^(1/3)
```

Rubi [A]
time = 0.01, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {386, 197}

$$\frac{3x}{4a\sqrt[3]{a+bx^3}}+\frac{x(a-bx^3)}{4a\left(a+bx^3\right)^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^3)/(a + b*x^3)^(7/3),x]
```

```
[Out] (x*(a - b*x^3))/(4*a*(a + b*x^3)^(4/3)) + (3*x)/(4*a*(a + b*x^3)^(1/3))
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 386

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] - Dist[
c*(q/(a*(p + 1))), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1,
0] && GtQ[q, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{a-bx^3}{\left(a+bx^3\right)^{7/3}}\,dx &= \frac{x(a-bx^3)}{4a\left(a+bx^3\right)^{4/3}}+\frac{3}{4}\int \frac{1}{\left(a+bx^3\right)^{4/3}}\,dx\\ &= \frac{x(a-bx^3)}{4a\left(a+bx^3\right)^{4/3}}+\frac{3x}{4a\sqrt[3]{a+bx^3}}\end{aligned}$$

Mathematica [A]
time = 0.16, size = 28, normalized size = 0.60

$$\frac{2ax+bx^4}{2a\left(a+bx^3\right)^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^3)/(a + b*x^3)^(7/3),x]
```

```
[Out] (2*a*x + b*x^4)/(2*a*(a + b*x^3)^(4/3))
```

Maple [A]
time = 0.26, size = 25, normalized size = 0.53

method	result	size
gosper	$\frac{x\left(b\,x^{3}+2a\right)}{2\left(b\,x^{3}+a\right)^{\frac{4}{3}}a}$	25
trager	$\frac{x\left(b\,x^{3}+2a\right)}{2\left(b\,x^{3}+a\right)^{\frac{4}{3}}a}$	25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^3+a)/(b*x^3+a)^(7/3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x*(b*x^3+2*a)/(b*x^3+a)^(4/3)/a
```

Maxima [A]
time = 0.28, size = 50, normalized size = 1.06

$$-\frac{\left(b-\frac{4\left(bx^3+a\right)}{x^3}\right)x^4}{4\left(bx^3+a\right)^{\frac{4}{3}}a}-\frac{bx^4}{4\left(bx^3+a\right)^{\frac{4}{3}}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)/(b*x^3+a)^(7/3),x, algorithm="maxima")
```

```
[Out] -1/4*(b - 4*(b*x^3 + a)/x^3)*x^4/((b*x^3 + a)^(4/3)*a) - 1/4*b*x^4/((b*x^3
+ a)^(4/3)*a)
```

Fricas [A]
time = 3.75, size = 44, normalized size = 0.94

$$\frac{\left(bx^4+2\,ax\right)\left(bx^3+a\right)^{\frac{2}{3}}}{2\left(ab^2x^6+2\,a^2bx^3+a^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)/(b*x^3+a)^(7/3),x, algorithm="fricas")
```

```
[Out] 1/2*(b*x^4 + 2*a*x)*(b*x^3 + a)^(2/3)/(a*b^2*x^6 + 2*a^2*b*x^3 + a^3)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 190 vs. 2(37) = 74.
time = 23.67, size = 190, normalized size = 4.04

$$a\left(\frac{4ax\Gamma\left(\frac{1}{3}\right)}{9a^{\frac{10}{3}}\sqrt[3]{1+\frac{bx^3}{a}}\,\Gamma\left(\frac{7}{3}\right)+9a^{\frac{7}{3}}bx^3\sqrt[3]{1+\frac{bx^3}{a}}\,\Gamma\left(\frac{7}{3}\right)}+\frac{3bx^4\Gamma\left(\frac{1}{3}\right)}{9a^{\frac{10}{3}}\sqrt[3]{1+\frac{bx^3}{a}}\,\Gamma\left(\frac{7}{3}\right)+9a^{\frac{7}{3}}bx^3\sqrt[3]{1+\frac{bx^3}{a}}\,\Gamma\left(\frac{7}{3}\right)}\right)-\frac{bx^4\Gamma\left(\frac{4}{3}\right)}{3a^{\frac{7}{3}}\sqrt[3]{1+\frac{bx^3}{a}}\,\Gamma\left(\frac{7}{3}\right)+3a^{\frac{4}{3}}bx^3\sqrt[3]{1+\frac{bx^3}{a}}\,\Gamma\left(\frac{7}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**3+a)/(b*x**3+a)**(7/3),x)
```

```
[Out] a*(4*a*x*gamma(1/3)/(9*a**(10/3)*(1 + b*x**3/a)**(1/3)*gamma(7/3) + 9*a**(7
/3)*b*x**3*(1 + b*x**3/a)**(1/3)*gamma(7/3)) + 3*b*x**4*gamma(1/3)/(9*a**(1
0/3)*(1 + b*x**3/a)**(1/3)*gamma(7/3) + 9*a**(7/3)*b*x**3*(1 + b*x**3/a)**(
1/3)*gamma(7/3))) - b*x**4*gamma(4/3)/(3*a**(7/3)*(1 + b*x**3/a)**(1/3)*gam
ma(7/3) + 3*a**(4/3)*b*x**3*(1 + b*x**3/a)**(1/3)*gamma(7/3))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)/(b*x^3+a)^(7/3),x, algorithm="giac")
```

```
[Out] integrate(-(b*x^3 - a)/(b*x^3 + a)^(7/3), x)
```

Mupad [B]
time = 1.35, size = 27, normalized size = 0.57

$$\frac{x\,\left(b\,x^3+a\right)+a\,x}{2\,a\,{\left(b\,x^3+a\right)}^{4/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x^3)/(a + b*x^3)^(7/3),x)
```

```
[Out] (x*(a + b*x^3) + a*x)/(2*a*(a + b*x^3)^(4/3))
```

3.31 $\int \frac{a-bx^3}{\left(a+bx^3\right)^{10/3}} \, dx$

Optimal. Leaf size=55

$$\frac{2x}{7\left(a+bx^3\right)^{7/3}}+\frac{5x}{28a\left(a+bx^3\right)^{4/3}}+\frac{15x}{28a^2\sqrt[3]{a+bx^3}}$$

```
[Out] 2/7*x/(b*x^3+a)^(7/3)+5/28*x/a/(b*x^3+a)^(4/3)+15/28*x/a^2/(b*x^3+a)^(1/3)
```

Rubi [A]
time = 0.01, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {393, 198, 197}

$$\frac{15x}{28a^2\sqrt[3]{a+bx^3}}+\frac{5x}{28a\left(a+bx^3\right)^{4/3}}+\frac{2x}{7\left(a+bx^3\right)^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^3)/(a + b*x^3)^(10/3),x]
```

```
[Out] (2*x)/(7*(a + b*x^3)^(7/3)) + (5*x)/(28*a*(a + b*x^3)^(4/3)) + (15*x)/(28*a
^2*(a + b*x^3)^(1/3))
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rubi steps

$$\int \frac{a-bx^3}{(a+bx^3)^{10/3}}\,dx = \frac{2x}{7(a+bx^3)^{7/3}} + \frac{5}{7}\int \frac{1}{(a+bx^3)^{7/3}}\,dx$$
$$= \frac{2x}{7(a+bx^3)^{7/3}} + \frac{5x}{28a(a+bx^3)^{4/3}} + \frac{15\int \frac{1}{(a+bx^3)^{4/3}}\,dx}{28a}$$
$$= \frac{2x}{7(a+bx^3)^{7/3}} + \frac{5x}{28a(a+bx^3)^{4/3}} + \frac{15x}{28a^2\sqrt[3]{a+bx^3}}$$

Mathematica [A]
time = 0.21, size = 40, normalized size = 0.73

$$\frac{28a^2x + 35abx^4 + 15b^2x^7}{28a^2(a+bx^3)^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^3)/(a + b*x^3)^(10/3),x]
[Out] (28*a^2*x + 35*a*b*x^4 + 15*b^2*x^7)/(28*a^2*(a + b*x^3)^(7/3))
```

Maple [A]
time = 0.28, size = 37, normalized size = 0.67

method	result	size
gosper	$\frac{x(15b^2x^6+35ab\,x^3+28a^2)}{28(b\,x^3+a)^{\frac{7}{3}}a^2}$	37
trager	$\frac{x(15b^2x^6+35ab\,x^3+28a^2)}{28(b\,x^3+a)^{\frac{7}{3}}a^2}$	37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^3+a)/(b*x^3+a)^(10/3),x,method=_RETURNVERBOSE)
[Out] 1/28*x*(15*b^2*x^6+35*a*b*x^3+28*a^2)/(b*x^3+a)^(7/3)/a^2
```

Maxima [A]
time = 0.29, size = 85, normalized size = 1.55

$$\frac{\left(4\,b - \frac{7\,(bx^3+a)}{x^3}\right)bx^7}{28\,(bx^3+a)^{\frac{7}{3}}a^2} + \frac{\left(2\,b^2 - \frac{7\,(bx^3+a)b}{x^3} + \frac{14\,(bx^3+a)^2}{x^6}\right)x^7}{14\,(bx^3+a)^{\frac{7}{3}}a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)/(b*x^3+a)^(10/3),x, algorithm="maxima")
```

```
[Out] 1/28*(4*b - 7*(b*x^3 + a)/x^3)*b*x^7/((b*x^3 + a)^(7/3)*a^2) + 1/14*(2*b^2
- 7*(b*x^3 + a)*b/x^3 + 14*(b*x^3 + a)^2/x^6)*x^7/((b*x^3 + a)^(7/3)*a^2)
```

Fricas [A]
time = 3.00, size = 69, normalized size = 1.25

$$\frac{(15\,b^2x^7+35\,abx^4+28\,a^2x)(bx^3+a)^{\frac{2}{3}}}{28\,(a^2b^3x^9+3\,a^3b^2x^6+3\,a^4bx^3+a^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)/(b*x^3+a)^(10/3),x, algorithm="fricas")
```

```
[Out] 1/28*(15*b^2*x^7 + 35*a*b*x^4 + 28*a^2*x)*(b*x^3 + a)^(2/3)/(a^2*b^3*x^9 +
3*a^3*b^2*x^6 + 3*a^4*b*x^3 + a^5)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 709 vs. $2(49) = 98$.
time = 117.23, size = 709, normalized size = 12.89

$$a\left(\frac{28a^5x\Gamma\left(\frac{1}{3}\right)}{27a^{\frac{25}{3}}\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{10}{3}\right)+81a^{\frac{22}{3}}bx^3\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{10}{3}\right)+81a^{\frac{19}{3}}b^2x^6\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{10}{3}\right)+27a^{\frac{16}{3}}b^3x^9\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{10}{3}\right)}+\frac{70a^4bx^4\Gamma\left(\frac{1}{3}\right)}{27a^{\frac{25}{3}}\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{10}{3}\right)+81a^{\frac{22}{3}}bx^3\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{10}{3}\right)+81a^{\frac{19}{3}}b^2x^6\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{10}{3}\right)+27a^{\frac{16}{3}}b^3x^9\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{10}{3}\right)}+\frac{60a^3b^2x^7\Gamma\left(\frac{1}{3}\right)}{27a^{\frac{25}{3}}\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{10}{3}\right)+81a^{\frac{22}{3}}bx^3\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{10}{3}\right)+81a^{\frac{19}{3}}b^2x^6\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{10}{3}\right)+27a^{\frac{16}{3}}b^3x^9\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{10}{3}\right)}+\frac{18a^2b^3x^{10}\Gamma\left(\frac{1}{3}\right)}{27a^{\frac{25}{3}}\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{10}{3}\right)+81a^{\frac{22}{3}}bx^3\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{10}{3}\right)+81a^{\frac{19}{3}}b^2x^6\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{10}{3}\right)+27a^{\frac{16}{3}}b^3x^9\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{10}{3}\right)}\right)-b\left(\frac{7ax^4\Gamma\left(\frac{4}{3}\right)}{9a^{\frac{13}{3}}\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{10}{3}\right)+18a^{\frac{10}{3}}bx^3\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{10}{3}\right)+9a^{\frac{7}{3}}b^2x^6\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{10}{3}\right)}+\frac{3bx^7\Gamma\left(\frac{4}{3}\right)}{9a^{\frac{13}{3}}\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{10}{3}\right)+18a^{\frac{10}{3}}bx^3\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{10}{3}\right)+9a^{\frac{7}{3}}b^2x^6\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{10}{3}\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**3+a)/(b*x**3+a)**(10/3),x)
```

```
[Out] a*(28*a**5*x*gamma(1/3)/(27*a**(25/3)*(1 + b*x**3/a)**(1/3)*gamma(10/3) + 8
1*a**(22/3)*b*x**3*(1 + b*x**3/a)**(1/3)*gamma(10/3) + 81*a**(19/3)*b**2*x*
*6*(1 + b*x**3/a)**(1/3)*gamma(10/3) + 27*a**(16/3)*b**3*x**9*(1 + b*x**3/a
)**(1/3)*gamma(10/3)) + 70*a**4*b*x**4*gamma(1/3)/(27*a**(25/3)*(1 + b*x**3
/a)**(1/3)*gamma(10/3) + 81*a**(22/3)*b*x**3*(1 + b*x**3/a)**(1/3)*gamma(10
/3) + 81*a**(19/3)*b**2*x**6*(1 + b*x**3/a)**(1/3)*gamma(10/3) + 27*a**(16/
3)*b**3*x**9*(1 + b*x**3/a)**(1/3)*gamma(10/3)) + 60*a**3*b**2*x**7*gamma(1
/3)/(27*a**(25/3)*(1 + b*x**3/a)**(1/3)*gamma(10/3) + 81*a**(22/3)*b*x**3*(
1 + b*x**3/a)**(1/3)*gamma(10/3) + 81*a**(19/3)*b**2*x**6*(1 + b*x**3/a)**(
1/3)*gamma(10/3) + 27*a**(16/3)*b**3*x**9*(1 + b*x**3/a)**(1/3)*gamma(10/3)
) + 18*a**2*b**3*x**10*gamma(1/3)/(27*a**(25/3)*(1 + b*x**3/a)**(1/3)*gamma
(10/3) + 81*a**(22/3)*b*x**3*(1 + b*x**3/a)**(1/3)*gamma(10/3) + 81*a**(19/
3)*b**2*x**6*(1 + b*x**3/a)**(1/3)*gamma(10/3) + 27*a**(16/3)*b**3*x**9*(1
+ b*x**3/a)**(1/3)*gamma(10/3))) - b*(7*a*x**4*gamma(4/3)/(9*a**(13/3)*(1 +
 b*x**3/a)**(1/3)*gamma(10/3) + 18*a**(10/3)*b*x**3*(1 + b*x**3/a)**(1/3)*g
amma(10/3) + 9*a**(7/3)*b**2*x**6*(1 + b*x**3/a)**(1/3)*gamma(10/3)) + 3*b*
x**7*gamma(4/3)/(9*a**(13/3)*(1 + b*x**3/a)**(1/3)*gamma(10/3) + 18*a**(10/
3)*b*x**3*(1 + b*x**3/a)**(1/3)*gamma(10/3) + 9*a**(7/3)*b**2*x**6*(1 + b*x
**3/a)**(1/3)*gamma(10/3)))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)/(b*x^3+a)^(10/3),x, algorithm="giac")
```

```
[Out] integrate(-(b*x^3 - a)/(b*x^3 + a)^(10/3), x)
```

Mupad [B]
time = 1.42, size = 44, normalized size = 0.80

$$\frac{15\,x\,{\left(b\,x^3+a\right)}^2+8\,a^2\,x+5\,a\,x\,\left(b\,x^3+a\right)}{28\,a^2\,{\left(b\,x^3+a\right)}^{7/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x^3)/(a + b*x^3)^(10/3),x)
```

```
[Out] (15*x*(a + b*x^3)^2 + 8*a^2*x + 5*a*x*(a + b*x^3))/(28*a^2*(a + b*x^3)^(7/3
))
```

3.32 $\int \frac{a-bx^3}{\left(a+bx^3\right)^{13/3}}\,dx$

Optimal. Leaf size=74

$$\frac{x}{5\left(a+bx^3\right)^{10/3}}+\frac{4x}{35a\left(a+bx^3\right)^{7/3}}+\frac{6x}{35a^2\left(a+bx^3\right)^{4/3}}+\frac{18x}{35a^3\sqrt[3]{a+bx^3}}$$

```
[Out] 1/5*x/(b*x^3+a)^(10/3)+4/35*x/a/(b*x^3+a)^(7/3)+6/35*x/a^2/(b*x^3+a)^(4/3)+
18/35*x/a^3/(b*x^3+a)^(1/3)
```

Rubi [A]

time = 0.01, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {393, 198, 197}

$$\frac{18x}{35a^3\sqrt[3]{a+bx^3}}+\frac{6x}{35a^2\left(a+bx^3\right)^{4/3}}+\frac{4x}{35a\left(a+bx^3\right)^{7/3}}+\frac{x}{5\left(a+bx^3\right)^{10/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^3)/(a + b*x^3)^(13/3),x]
```

```
[Out] x/(5*(a + b*x^3)^(10/3)) + (4*x)/(35*a*(a + b*x^3)^(7/3)) + (6*x)/(35*a^2*(
a + b*x^3)^(4/3)) + (18*x)/(35*a^3*(a + b*x^3)^(1/3))
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a-bx^3}{(a+bx^3)^{13/3}}\,dx &= \frac{x}{5(a+bx^3)^{10/3}}+\frac{4}{5}\int \frac{1}{(a+bx^3)^{10/3}}\,dx \\ &= \frac{x}{5(a+bx^3)^{10/3}}+\frac{4x}{35a(a+bx^3)^{7/3}}+\frac{24\int \frac{1}{(a+bx^3)^{7/3}}\,dx}{35a} \\ &= \frac{x}{5(a+bx^3)^{10/3}}+\frac{4x}{35a(a+bx^3)^{7/3}}+\frac{6x}{35a^2(a+bx^3)^{4/3}}+\frac{18\int \frac{1}{(a+bx^3)^{4/3}}\,dx}{35a^2} \\ &= \frac{x}{5(a+bx^3)^{10/3}}+\frac{4x}{35a(a+bx^3)^{7/3}}+\frac{6x}{35a^2(a+bx^3)^{4/3}}+\frac{18x}{35a^3\sqrt[3]{a+bx^3}}\end{aligned}$$

Mathematica [A]
time = 0.30, size = 51, normalized size = 0.69

$$\frac{35a^3x+70a^2bx^4+60ab^2x^7+18b^3x^{10}}{35a^3(a+bx^3)^{10/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^3)/(a + b*x^3)^(13/3),x]
```

```
[Out] (35*a^3*x + 70*a^2*b*x^4 + 60*a*b^2*x^7 + 18*b^3*x^10)/(35*a^3*(a + b*x^3)^
(10/3))
```

Maple [A]
time = 0.27, size = 48, normalized size = 0.65

method	result	size
gosper	$\frac{x\left(18b^3x^9+60a\,b^2x^6+70a^2b\,x^3+35a^3\right)}{35\left(b\,x^3+a\right)^{\frac{10}{3}}a^3}$	48
trager	$\frac{x\left(18b^3x^9+60a\,b^2x^6+70a^2b\,x^3+35a^3\right)}{35\left(b\,x^3+a\right)^{\frac{10}{3}}a^3}$	48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^3+a)/(b*x^3+a)^(13/3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/35*x*(18*b^3*x^9+60*a*b^2*x^6+70*a^2*b*x^3+35*a^3)/(b*x^3+a)^(10/3)/a^3
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 119 vs. 2(58) = 116.
time = 0.28, size = 119, normalized size = 1.61

$$-\frac{\left(14\,b^2-\frac{40\,(bx^3+a)b}{x^3}+\frac{35\,(bx^3+a)^2}{x^6}\right)bx^{10}}{140\,(bx^3+a)^{\frac{10}{3}}a^3}-\frac{\left(14\,b^3-\frac{60\,(bx^3+a)b^2}{x^3}+\frac{105\,(bx^3+a)^2b}{x^6}-\frac{140\,(bx^3+a)^3}{x^9}\right)x^{10}}{140\,(bx^3+a)^{\frac{10}{3}}a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)/(b*x^3+a)^(13/3),x, algorithm="maxima")
```

```
[Out] -1/140*(14*b^2 - 40*(b*x^3 + a)*b/x^3 + 35*(b*x^3 + a)^2/x^6)*b*x^10/((b*x^
3 + a)^(10/3)*a^3) - 1/140*(14*b^3 - 60*(b*x^3 + a)*b^2/x^3 + 105*(b*x^3 +
a)^2*b/x^6 - 140*(b*x^3 + a)^3/x^9)*x^10/((b*x^3 + a)^(10/3)*a^3)
```

Fricas [A]
time = 3.30, size = 91, normalized size = 1.23

$$\frac{(18\,b^3x^{10}+60\,ab^2x^7+70\,a^2bx^4+35\,a^3x)(bx^3+a)^{\frac{2}{3}}}{35\,(a^3b^4x^{12}+4\,a^4b^3x^9+6\,a^5b^2x^6+4\,a^6bx^3+a^7)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)/(b*x^3+a)^(13/3),x, algorithm="fricas")
```

```
[Out] 1/35*(18*b^3*x^10 + 60*a*b^2*x^7 + 70*a^2*b*x^4 + 35*a^3*x)*(b*x^3 + a)^(2/
3)/(a^3*b^4*x^12 + 4*a^4*b^3*x^9 + 6*a^5*b^2*x^6 + 4*a^6*b*x^3 + a^7)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**3+a)/(b*x**3+a)**(13/3),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)/(b*x^3+a)^(13/3),x, algorithm="giac")
```

```
[Out] integrate(-(b*x^3 - a)/(b*x^3 + a)^(13/3), x)
```

Mupad [B]
time = 1.39, size = 58, normalized size = 0.78

$$\frac{x}{5\,{\left(b\,x^3+a\right)}^{10/3}}+\frac{18\,x}{35\,a^3\,{\left(b\,x^3+a\right)}^{1/3}}+\frac{6\,x}{35\,a^2\,{\left(b\,x^3+a\right)}^{4/3}}+\frac{4\,x}{35\,a\,{\left(b\,x^3+a\right)}^{7/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x^3)/(a + b*x^3)^(13/3),x)
```

```
[Out] x/(5*(a + b*x^3)^(10/3)) + (18*x)/(35*a^3*(a + b*x^3)^(1/3)) + (6*x)/(35*a^
2*(a + b*x^3)^(4/3)) + (4*x)/(35*a*(a + b*x^3)^(7/3))
```

3.33 $\int \frac{a-bx^3}{\left(a+bx^3\right)^{16/3}}\,dx$

Optimal. Leaf size=93

$$\frac{2x}{13\left(a+bx^3\right)^{13/3}}+\frac{11x}{130a\left(a+bx^3\right)^{10/3}}+\frac{99x}{910a^2\left(a+bx^3\right)^{7/3}}+\frac{297x}{1820a^3\left(a+bx^3\right)^{4/3}}+\frac{891x}{1820a^4\sqrt[3]{a+bx^3}}$$

```
[Out] 2/13*x/(b*x^3+a)^(13/3)+11/130*x/a/(b*x^3+a)^(10/3)+99/910*x/a^2/(b*x^3+a)^
(7/3)+297/1820*x/a^3/(b*x^3+a)^(4/3)+891/1820*x/a^4/(b*x^3+a)^(1/3)
```

Rubi [A]
time = 0.02, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {393, 198, 197}

$$\frac{891x}{1820a^4\sqrt[3]{a+bx^3}}+\frac{297x}{1820a^3\left(a+bx^3\right)^{4/3}}+\frac{99x}{910a^2\left(a+bx^3\right)^{7/3}}+\frac{11x}{130a\left(a+bx^3\right)^{10/3}}+\frac{2x}{13\left(a+bx^3\right)^{13/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^3)/(a + b*x^3)^(16/3),x]
```

```
[Out] (2*x)/(13*(a + b*x^3)^(13/3)) + (11*x)/(130*a*(a + b*x^3)^(10/3)) + (99*x)/
(910*a^2*(a + b*x^3)^(7/3)) + (297*x)/(1820*a^3*(a + b*x^3)^(4/3)) + (891*x
)/(1820*a^4*(a + b*x^3)^(1/3))
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rubi steps

$$\int \frac{a-bx^3}{(a+bx^3)^{16/3}}\,dx = \frac{2x}{13(a+bx^3)^{13/3}} + \frac{11}{13}\int \frac{1}{(a+bx^3)^{13/3}}\,dx$$
$$= \frac{2x}{13(a+bx^3)^{13/3}} + \frac{11x}{130a(a+bx^3)^{10/3}} + \frac{99\int \frac{1}{(a+bx^3)^{10/3}}\,dx}{130a}$$
$$= \frac{2x}{13(a+bx^3)^{13/3}} + \frac{11x}{130a(a+bx^3)^{10/3}} + \frac{99x}{910a^2(a+bx^3)^{7/3}} + \frac{297\int \frac{1}{(a+bx^3)^{7/3}}\,dx}{455a^2}$$
$$= \frac{2x}{13(a+bx^3)^{13/3}} + \frac{11x}{130a(a+bx^3)^{10/3}} + \frac{99x}{910a^2(a+bx^3)^{7/3}} + \frac{297x}{1820a^3(a+bx^3)^{4/3}} + \cdots$$
$$= \frac{2x}{13(a+bx^3)^{13/3}} + \frac{11x}{130a(a+bx^3)^{10/3}} + \frac{99x}{910a^2(a+bx^3)^{7/3}} + \frac{297x}{1820a^3(a+bx^3)^{4/3}} + \cdots$$

Mathematica [A]
time = 0.44, size = 62, normalized size = 0.67

$$\frac{x(1820a^4+5005a^3bx^3+6435a^2b^2x^6+3861ab^3x^9+891b^4x^{12})}{1820a^4(a+bx^3)^{13/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^3)/(a + b*x^3)^(16/3),x]
```

```
[Out] (x*(1820*a^4 + 5005*a^3*b*x^3 + 6435*a^2*b^2*x^6 + 3861*a*b^3*x^9 + 891*b^4
*x^12))/(1820*a^4*(a + b*x^3)^(13/3))
```

Maple [A]
time = 0.27, size = 59, normalized size = 0.63

method	result	size
gosper	$\frac{x(891b^4x^{12}+3861a\,b^3x^9+6435a^2b^2x^6+5005a^3b\,x^3+1820a^4)}{1820(b\,x^3+a)^{\frac{13}{3}}\,a^4}$	59
trager	$\frac{x(891b^4x^{12}+3861a\,b^3x^9+6435a^2b^2x^6+5005a^3b\,x^3+1820a^4)}{1820(b\,x^3+a)^{\frac{13}{3}}\,a^4}$	59

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^3+a)/(b*x^3+a)^(16/3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/1820*x*(891*b^4*x^12+3861*a*b^3*x^9+6435*a^2*b^2*x^6+5005*a^3*b*x^3+1820*
a^4)/(b*x^3+a)^(13/3)/a^4
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 153 vs. $2(73) = 146$.
time = 0.27, size = 153, normalized size = 1.65

$$\frac{\left(140\,b^3-\frac{546\,(bx^3+a)b^2}{x^3}+\frac{780\,(bx^3+a)^2b}{x^6}-\frac{455\,(bx^3+a)^3}{x^9}\right)bx^{13}}{1820\,(bx^3+a)^{\frac{13}{3}}a^4}+\frac{\left(35\,b^4-\frac{182\,(bx^3+a)b^3}{x^3}+\frac{390\,(bx^3+a)^2b^2}{x^6}-\frac{455\,(bx^3+a)^3b}{x^9}+\frac{455\,(bx^3+a)^4}{x^{12}}\right)x^{13}}{455\,(bx^3+a)^{\frac{13}{3}}a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)/(b*x^3+a)^(16/3),x, algorithm="maxima")
```

```
[Out] 1/1820*(140*b^3 - 546*(b*x^3 + a)*b^2/x^3 + 780*(b*x^3 + a)^2*b/x^6 - 455*(
b*x^3 + a)^3/x^9)*b*x^13/((b*x^3 + a)^(13/3)*a^4) + 1/455*(35*b^4 - 182*(b*
x^3 + a)*b^3/x^3 + 390*(b*x^3 + a)^2*b^2/x^6 - 455*(b*x^3 + a)^3*b/x^9 + 45
5*(b*x^3 + a)^4/x^12)*x^13/((b*x^3 + a)^(13/3)*a^4)
```

Fricas [A]
time = 1.95, size = 113, normalized size = 1.22

$$\frac{(891\,b^4x^{13}+3861\,ab^3x^{10}+6435\,a^2b^2x^7+5005\,a^3bx^4+1820\,a^4x)(bx^3+a)^{\frac{2}{3}}}{1820\,(a^4b^5x^{15}+5\,a^5b^4x^{12}+10\,a^6b^3x^9+10\,a^7b^2x^6+5\,a^8bx^3+a^9)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)/(b*x^3+a)^(16/3),x, algorithm="fricas")
```

```
[Out] 1/1820*(891*b^4*x^13 + 3861*a*b^3*x^10 + 6435*a^2*b^2*x^7 + 5005*a^3*b*x^4
+ 1820*a^4*x)*(b*x^3 + a)^(2/3)/(a^4*b^5*x^15 + 5*a^5*b^4*x^12 + 10*a^6*b^3
*x^9 + 10*a^7*b^2*x^6 + 5*a^8*b*x^3 + a^9)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**3+a)/(b*x**3+a)**(16/3),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)/(b*x^3+a)^(16/3),x, algorithm="giac")
```

```
[Out] integrate(-(b*x^3 - a)/(b*x^3 + a)^(16/3), x)
```

Mupad [B]
time = 1.37, size = 73, normalized size = 0.78

$$\frac{2\,x}{13\,{\left(b\,x^3+a\right)}^{13/3}}+\frac{891\,x}{1820\,a^4\,{\left(b\,x^3+a\right)}^{1/3}}+\frac{297\,x}{1820\,a^3\,{\left(b\,x^3+a\right)}^{4/3}}+\frac{99\,x}{910\,a^2\,{\left(b\,x^3+a\right)}^{7/3}}+\frac{11\,x}{130\,a\,{\left(b\,x^3+a\right)}^{10/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x^3)/(a + b*x^3)^(16/3),x)
```

```
[Out] (2*x)/(13*(a + b*x^3)^(13/3)) + (891*x)/(1820*a^4*(a + b*x^3)^(1/3)) + (297
*x)/(1820*a^3*(a + b*x^3)^(4/3)) + (99*x)/(910*a^2*(a + b*x^3)^(7/3)) + (11
*x)/(130*a*(a + b*x^3)^(10/3))
```

3.34 $\int \frac{(a+bx^3)^{7/3}}{a-bx^3}\,dx$

Optimal. Leaf size=483

$$-\frac{7}{5}ax\sqrt[3]{a+bx^3}-\frac{1}{5}x\left(a+bx^3\right)^{4/3}-\frac{4\sqrt[3]{2}\,a^{5/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,\sqrt[3]{b}}-\frac{2\sqrt[3]{2}\,a^{5/3}\tan^{-1}\left(\frac{1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\cdots\right)}{\sqrt[3]{a+b\cdots}}}{\sqrt{3}}\right)}{\sqrt{3}\,\sqrt[3]{b}}$$

```
[Out] -7/5*a*x*(b*x^3+a)^(1/3)-1/5*x*(b*x^3+a)^(4/3)-7/5*a^2*x*(1+b*x^3/a)^(2/3)*
hypergeom([1/3, 2/3],[4/3],-b*x^3/a)/(b*x^3+a)^(2/3)-2/3*2^(1/3)*a^(5/3)*ln
(2^(2/3)+(-a^(1/3)-b^(1/3)*x)/(b*x^3+a)^(1/3))/b^(1/3)+2/3*2^(1/3)*a^(5/3)*
ln(1+2^(2/3)*(a^(1/3)+b^(1/3)*x)^2/(b*x^3+a)^(2/3)-2^(1/3)*(a^(1/3)+b^(1/3)
*x)/(b*x^3+a)^(1/3))/b^(1/3)-4/3*2^(1/3)*a^(5/3)*ln(1+2^(1/3)*(a^(1/3)+b^(1
/3)*x)/(b*x^3+a)^(1/3))/b^(1/3)+1/3*2^(1/3)*a^(5/3)*ln(2*2^(1/3)+(a^(1/3)+b
^(1/3)*x)^2/(b*x^3+a)^(2/3)+2^(2/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))/b^
(1/3)-4/3*2^(1/3)*a^(5/3)*arctan(1/3*(1-2*2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^
3+a)^(1/3))*3^(1/2))/b^(1/3)*3^(1/2)-2/3*2^(1/3)*a^(5/3)*arctan(1/3*(1+2^(1
/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*3^(1/2))/b^(1/3)*3^(1/2)
```

Rubi [A]
time = 0.40, antiderivative size = 483, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 14, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.636$, Rules used = {427, 542, 544, 252, 251, 421, 420, 493, 298, 31, 648, 631, 210, 642}

$$-\frac{4\sqrt[3]{2}\,a^{5/3}\text{ArcTan}\left(\frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,\sqrt[3]{b}}-\frac{2\sqrt[3]{2}\,a^{5/3}\text{ArcTan}\left(\frac{\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,\sqrt[3]{b}}-\frac{2\sqrt[3]{2}\,a^{5/3}\log\left(2^{2/3}-\frac{\sqrt[3]{a}+\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{b}}+\frac{2\sqrt[3]{2}\,a^{5/3}\log\left(\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}{(a+bx^3)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{3\sqrt[3]{b}}-\frac{4\sqrt[3]{2}\,a^{5/3}\log\left(\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{3\sqrt[3]{b}}+\frac{\sqrt[3]{2}\,a^{5/3}\log\left(\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}{(a+bx^3)^{2/3}}+\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{a+bx^3}}+2\sqrt[3]{2}\right)}{3\sqrt[3]{b}}-\frac{7a^2x\left(\frac{bx^3}{a}+1\right)^{2/3}{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{5\left(a+bx^3\right)^{2/3}}-\frac{7}{5}ax\sqrt[3]{a+bx^3}-\frac{1}{5}x\left(a+bx^3\right)^{4/3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(7/3)/(a - b*x^3),x]
```

```
[Out] (-7*a*x*(a + b*x^3)^(1/3))/5 - (x*(a + b*x^3)^(4/3))/5 - (4*2^(1/3)*a^(5/3)
*ArcTan[(1 - (2*2^(1/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3))/Sqrt[3]])
/(Sqrt[3]*b^(1/3)) - (2*2^(1/3)*a^(5/3)*ArcTan[(1 + (2^(1/3)*(a^(1/3) + b^(
1/3)*x))/(a + b*x^3)^(1/3))/Sqrt[3]])/(Sqrt[3]*b^(1/3)) - (7*a^2*x*(1 + (b*
x^3)/a)^(2/3)*Hypergeometric2F1[1/3, 2/3, 4/3, -((b*x^3)/a)])/(5*(a + b*x^3
)^(2/3)) - (2*2^(1/3)*a^(5/3)*Log[2^(2/3) - (a^(1/3) + b^(1/3)*x)/(a + b*x^
3)^(1/3)])/(3*b^(1/3)) + (2*2^(1/3)*a^(5/3)*Log[1 + (2^(2/3)*(a^(1/3) + b^(
1/3)*x)^2)/(a + b*x^3)^(2/3) - (2^(1/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^
(1/3)])/(3*b^(1/3)) - (4*2^(1/3)*a^(5/3)*Log[1 + (2^(1/3)*(a^(1/3) + b^(1/3
)*x))/(a + b*x^3)^(1/3)])/(3*b^(1/3)) + (2^(1/3)*a^(5/3)*Log[2*2^(1/3) + (a
^(1/3) + b^(1/3)*x)^2/(a + b*x^3)^(2/3) + (2^(2/3)*(a^(1/3) + b^(1/3)*x))/(
a + b*x^3)^(1/3)])/(3*b^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 420

```
Int[((a_) + (b_.)*(x_)^3)^(1/3)/((c_) + (d_.)*(x_)^3), x_Symbol] :> With[{q
 = Rt[b/a, 3]}, Dist[9*(a/(c*q)), Subst[Int[x/((4 - a*x^3)*(1 + 2*a*x^3)),
x], x, (1 + q*x)/(a + b*x^3)^(1/3)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b
*c - a*d, 0] && EqQ[b*c + a*d, 0]
```

Rule 421

```
Int[1/(((a_) + (b_.)*(x_)^3)^(2/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^3)^(2/3), x], x] - Dist[d/(b*c - a*d), Int[
(a + b*x^3)^(1/3)/(c + d*x^3), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0] && EqQ[b*c + a*d, 0]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 493

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Dist[b/(b*c - a*d), Int[(e*x)^m/(a + b*x^n), x], x] - Dist[d/
(b*c - a*d), Int[(e*x)^m/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, m}, x
] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 544

```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Dist[f/d, Int[(a + b*x^n)^p, x], x] + Dist[(d*e -
c*f)/d, Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, p
, n}, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{(a+bx^3)^{7/3}}{a-bx^3}\,dx = \frac{\left(a^2\sqrt[3]{a+bx^3}\right)\int \frac{\left(1+\frac{bx^3}{a}\right)^{7/3}}{a-bx^3}\,dx}{\sqrt[3]{1+\frac{bx^3}{a}}}$$

$$= \frac{ax\sqrt[3]{a+bx^3}\,F_1\left(\frac{1}{3};1,-\frac{7}{3};\frac{4}{3};\frac{bx^3}{a},-\frac{bx^3}{a}\right)}{\sqrt[3]{1+\frac{bx^3}{a}}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.
time = 10.22, size = 232, normalized size = 0.48

$$\frac{27abx^4\left(1+\frac{bx^3}{a}\right)^{2/3}F_1\left(\frac{4}{3};\frac{2}{3},1;\frac{7}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)+4x\left(-8a^2-9abx^3-b^2x^6+\frac{52a^4F_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)}{(a-bx^3)\left(4aF_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)+bx^3\left(3F_1\left(\frac{4}{3};\frac{2}{3},2;\frac{7}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)-2F_1\left(\frac{4}{3};\frac{5}{3},1;\frac{7}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)\right)\right)}\right)}{20\,(a+bx^3)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(7/3)/(a - b*x^3),x]
```

```
[Out] (27*a*b*x^4*(1 + (b*x^3)/a)^(2/3)*AppellF1[4/3, 2/3, 1, 7/3, -((b*x^3)/a),
(b*x^3)/a] + 4*x*(-8*a^2 - 9*a*b*x^3 - b^2*x^6 + (52*a^4*AppellF1[1/3, 2/3,
 1, 4/3, -((b*x^3)/a), (b*x^3)/a])/((a - b*x^3)*(4*a*AppellF1[1/3, 2/3, 1,
4/3, -((b*x^3)/a), (b*x^3)/a] + b*x^3*(3*AppellF1[4/3, 2/3, 2, 7/3, -((b*x^
3)/a), (b*x^3)/a] - 2*AppellF1[4/3, 5/3, 1, 7/3, -((b*x^3)/a), (b*x^3)/a]))
)))/(20*(a + b*x^3)^(2/3))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(b\,x^3+a)^{\frac{7}{3}}}{-b\,x^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(7/3)/(-b*x^3+a),x)
```

```
[Out] int((b*x^3+a)^(7/3)/(-b*x^3+a),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(7/3)/(-b*x^3+a),x, algorithm="maxima")
```

```
[Out] -integrate((b*x^3 + a)^(7/3)/(b*x^3 - a), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(7/3)/(-b*x^3+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{a^2\sqrt[3]{a+bx^3}}{-a+bx^3}\,dx-\int\frac{b^2x^6\sqrt[3]{a+bx^3}}{-a+bx^3}\,dx-\int\frac{2abx^3\sqrt[3]{a+bx^3}}{-a+bx^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(7/3)/(-b*x**3+a),x)
```

```
[Out] -Integral(a**2*(a + b*x**3)**(1/3)/(-a + b*x**3), x) - Integral(b**2*x**6*(
a + b*x**3)**(1/3)/(-a + b*x**3), x) - Integral(2*a*b*x**3*(a + b*x**3)**(1
/3)/(-a + b*x**3), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(7/3)/(-b*x^3+a),x, algorithm="giac")
```

```
[Out] integrate(-(b*x^3 + a)^(7/3)/(b*x^3 - a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(b\,x^3+a\right)^{7/3}}{a-b\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(7/3)/(a - b*x^3),x)
[Out] int((a + b*x^3)^(7/3)/(a - b*x^3), x)
```

3.35 $\int \frac{(a+bx^3)^{4/3}}{a-bx^3}\,dx$

Optimal. Leaf size=464

$$-\frac{1}{2}x\sqrt[3]{a+bx^3}-\frac{2\sqrt[3]{2}\,a^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,\sqrt[3]{b}}-\frac{\sqrt[3]{2}\,a^{2/3}\tan^{-1}\left(\frac{1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,\sqrt[3]{b}}-\frac{ax\left(1+\right.}{}$$

```
[Out] -1/2*x*(b*x^3+a)^(1/3)-1/2*a*x*(1+b*x^3/a)^(2/3)*hypergeom([1/3, 2/3],[4/3]
,-b*x^3/a)/(b*x^3+a)^(2/3)-1/3*2^(1/3)*a^(2/3)*ln(2^(2/3)+(-a^(1/3)-b^(1/3)
*x)/(b*x^3+a)^(1/3))/b^(1/3)+1/3*2^(1/3)*a^(2/3)*ln(1+2^(2/3)*(a^(1/3)+b^(1
/3)*x)^2/(b*x^3+a)^(2/3)-2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))/b^(1/
3)-2/3*2^(1/3)*a^(2/3)*ln(1+2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))/b^
(1/3)+1/6*a^(2/3)*ln(2*2^(1/3)+(a^(1/3)+b^(1/3)*x)^2/(b*x^3+a)^(2/3)+2^(2/3
)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*2^(1/3)/b^(1/3)-2/3*2^(1/3)*a^(2/3)*
arctan(1/3*(1-2*2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*3^(1/2))/b^(1/
3)*3^(1/2)-1/3*2^(1/3)*a^(2/3)*arctan(1/3*(1+2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b
*x^3+a)^(1/3))*3^(1/2))/b^(1/3)*3^(1/2)
```

Rubi [A]

time = 0.29, antiderivative size = 464, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 13, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.591$, Rules used = {427, 544, 252, 251, 421, 420, 493, 298, 31, 648, 631, 210, 642}

$$-\frac{2\sqrt[3]{2}\,a^{2/3}\text{ArcTan}\left(\frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,\sqrt[3]{b}}-\frac{\sqrt[3]{2}\,a^{2/3}\text{ArcTan}\left(\frac{\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,\sqrt[3]{b}}-\frac{\sqrt[3]{2}\,a^{2/3}\log\left(2^{2/3}-\frac{\sqrt[3]{a}+\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{b}}+\frac{\sqrt[3]{2}\,a^{2/3}\log\left(\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}{(a+bx^3)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{3\sqrt[3]{b}}-\frac{2\sqrt[3]{2}\,a^{2/3}\log\left(\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{3\sqrt[3]{b}}+\frac{a^{2/3}\log\left(\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}{(a+bx^3)^{2/3}}+\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{a+bx^3}}+2\sqrt[3]{2}\right)}{3\,2^{2/3}\sqrt[3]{b}}-\frac{ax\left(\frac{bx^3}{a}+1\right)^{2/3}{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{2\left(a+bx^3\right)^{2/3}}-\frac{1}{2}x\sqrt[3]{a+bx^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(4/3)/(a - b*x^3),x]
```

```
[Out] -1/2*(x*(a + b*x^3)^(1/3)) - (2*2^(1/3)*a^(2/3)*ArcTan[(1 - (2*2^(1/3)*(a^(
1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3))/Sqrt[3]])/(Sqrt[3]*b^(1/3)) - (2^(1/3
)*a^(2/3)*ArcTan[(1 + (2^(1/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3))/Sq
rt[3]])/(Sqrt[3]*b^(1/3)) - (a*x*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[1/
3, 2/3, 4/3, -((b*x^3)/a)])/(2*(a + b*x^3)^(2/3)) - (2^(1/3)*a^(2/3)*Log[2^
(2/3) - (a^(1/3) + b^(1/3)*x)/(a + b*x^3)^(1/3)])/(3*b^(1/3)) + (2^(1/3)*a^
(2/3)*Log[1 + (2^(2/3)*(a^(1/3) + b^(1/3)*x)^2)/(a + b*x^3)^(2/3) - (2^(1/3
)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3)])/(3*b^(1/3)) - (2*2^(1/3)*a^(2/
3)*Log[1 + (2^(1/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3)])/(3*b^(1/3))
+ (a^(2/3)*Log[2*2^(1/3) + (a^(1/3) + b^(1/3)*x)^2/(a + b*x^3)^(2/3) + (2^(
2/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3)])/(3*2^(2/3)*b^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 420

```
Int[((a_) + (b_.)*(x_)^3)^(1/3)/((c_) + (d_.)*(x_)^3), x_Symbol] :> With[{q
 = Rt[b/a, 3]}, Dist[9*(a/(c*q)), Subst[Int[x/((4 - a*x^3)*(1 + 2*a*x^3)),
x], x, (1 + q*x)/(a + b*x^3)^(1/3)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b
*c - a*d, 0] && EqQ[b*c + a*d, 0]
```

Rule 421

```
Int[1/(((a_) + (b_.)*(x_)^3)^(2/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^3)^(2/3), x], x] - Dist[d/(b*c - a*d), Int[
(a + b*x^3)^(1/3)/(c + d*x^3), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0] && EqQ[b*c + a*d, 0]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 493

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Dist[b/(b*c - a*d), Int[(e*x)^m/(a + b*x^n), x], x] - Dist[d/
(b*c - a*d), Int[(e*x)^m/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, m}, x
] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
```

Rule 544

```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Dist[f/d, Int[(a + b*x^n)^p, x], x] + Dist[(d*e -
c*f)/d, Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, p
, n}, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{(a+bx^3)^{4/3}}{a-bx^3}\,dx = \frac{\left(a\sqrt[3]{a+bx^3}\right)\int \frac{\left(1+\frac{bx^3}{a}\right)^{4/3}}{a-bx^3}\,dx}{\sqrt[3]{1+\frac{bx^3}{a}}}$$
$$= \frac{x\sqrt[3]{a+bx^3}\,F_1\left(\frac{1}{3};1,-\frac{4}{3};\frac{4}{3};\frac{bx^3}{a},-\frac{bx^3}{a}\right)}{\sqrt[3]{1+\frac{bx^3}{a}}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.
time = 10.11, size = 217, normalized size = 0.47

$$\frac{x\left(-4(a+bx^3)+5bx^3\left(1+\frac{bx^3}{a}\right)^{2/3}F_1\left(\frac{4}{3};\frac{2}{3},1;\frac{7}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)+\frac{48a^3F_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)}{(a-bx^3)\left(4aF_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)+bx^3\left(3F_1\left(\frac{4}{3};\frac{2}{3},2;\frac{7}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)-2F_1\left(\frac{4}{3};\frac{5}{3},1;\frac{7}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)\right)\right)}\right)}{8\,(a+bx^3)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(4/3)/(a - b*x^3),x]
```

```
[Out] (x*(-4*(a + b*x^3) + 5*b*x^3*(1 + (b*x^3)/a)^(2/3)*AppellF1[4/3, 2/3, 1, 7/
3, -((b*x^3)/a), (b*x^3)/a] + (48*a^3*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/
a), (b*x^3)/a])/((a - b*x^3)*(4*a*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a),
(b*x^3)/a] + b*x^3*(3*AppellF1[4/3, 2/3, 2, 7/3, -((b*x^3)/a), (b*x^3)/a] -
 2*AppellF1[4/3, 5/3, 1, 7/3, -((b*x^3)/a), (b*x^3)/a])))))/(8*(a + b*x^3)^
(2/3))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\,x^3+a)^{\frac{4}{3}}}{-b\,x^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(4/3)/(-b*x^3+a),x)
```

```
[Out] int((b*x^3+a)^(4/3)/(-b*x^3+a),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(4/3)/(-b*x^3+a),x, algorithm="maxima")
```

```
[Out] -integrate((b*x^3 + a)^(4/3)/(b*x^3 - a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(4/3)/(-b*x^3+a),x, algorithm="fricas")
```

```
[Out] integral(-(b*x^3 + a)^(4/3)/(b*x^3 - a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{a\sqrt[3]{a+bx^3}}{-a+bx^3}\,dx-\int \frac{bx^3\sqrt[3]{a+bx^3}}{-a+bx^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(4/3)/(-b*x**3+a),x)
```

```
[Out] -Integral(a*(a + b*x**3)**(1/3)/(-a + b*x**3), x) - Integral(b*x**3*(a + b*
x**3)**(1/3)/(-a + b*x**3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(4/3)/(-b*x^3+a),x, algorithm="giac")
```

```
[Out] integrate(-(b*x^3 + a)^(4/3)/(b*x^3 - a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(b\,x^3+a\right)}^{4/3}}{a-b\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(4/3)/(a - b*x^3),x)
```

```
[Out] int((a + b*x^3)^(4/3)/(a - b*x^3), x)
```

3.36 $\int \frac{\sqrt[3]{a+bx^3}}{a-bx^3}\,dx$

Optimal. Leaf size=398

$$-\frac{\sqrt[3]{2}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{a}\sqrt[3]{b}}-\frac{\tan^{-1}\left(\frac{1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(2^{2/3}-\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)}{3\,2^{2/3}\sqrt[3]{a}\sqrt[3]{b}}+\frac{\log\left(1+\right.}{}$$

```
[Out] -1/6*ln(2^(2/3)+(-a^(1/3)-b^(1/3)*x)/(b*x^3+a)^(1/3))*2^(1/3)/a^(1/3)/b^(1/
3)+1/6*ln(1+2^(2/3)*(a^(1/3)+b^(1/3)*x)^2/(b*x^3+a)^(2/3)-2^(1/3)*(a^(1/3)+
b^(1/3)*x)/(b*x^3+a)^(1/3))*2^(1/3)/a^(1/3)/b^(1/3)-1/3*2^(1/3)*ln(1+2^(1/3
)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))/a^(1/3)/b^(1/3)+1/12*ln(2*2^(1/3)+(a
^(1/3)+b^(1/3)*x)^2/(b*x^3+a)^(2/3)+2^(2/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(
1/3))*2^(1/3)/a^(1/3)/b^(1/3)-1/3*2^(1/3)*arctan(1/3*(1-2*2^(1/3)*(a^(1/3)+
b^(1/3)*x)/(b*x^3+a)^(1/3))*3^(1/2))/a^(1/3)/b^(1/3)*3^(1/2)-1/6*arctan(1/3
*(1+2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*3^(1/2))*2^(1/3)/a^(1/3)/b
^(1/3)*3^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 398, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {420, 493, 298, 31, 648, 631, 210, 642}

$$-\frac{\sqrt[3]{2}\,\text{ArcTan}\left(\frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{a}\sqrt[3]{b}}-\frac{\text{ArcTan}\left(\frac{\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(2^{2/3}-\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)}{3\,2^{2/3}\sqrt[3]{a}\sqrt[3]{b}}+\frac{\log\left(\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{(a+bx^3)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{3\,2^{2/3}\sqrt[3]{a}\sqrt[3]{b}}-\frac{\sqrt[3]{2}\log\left(\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{3\sqrt[3]{a}\sqrt[3]{b}}+\frac{\log\left(\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{(a+bx^3)^{2/3}}+\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+2\sqrt[3]{2}\right)}{6\,2^{2/3}\sqrt[3]{a}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(1/3)/(a - b*x^3),x]
```

```
[Out] -((2^(1/3)*ArcTan[(1 - (2*2^(1/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3))
/Sqrt[3]])/(Sqrt[3]*a^(1/3)*b^(1/3))) - ArcTan[(1 + (2^(1/3)*(a^(1/3) + b^(
1/3)*x))/(a + b*x^3)^(1/3))/Sqrt[3]]/(2^(2/3)*Sqrt[3]*a^(1/3)*b^(1/3)) - Lo
g[2^(2/3) - (a^(1/3) + b^(1/3)*x)/(a + b*x^3)^(1/3)]/(3*2^(2/3)*a^(1/3)*b^(
1/3)) + Log[1 + (2^(2/3)*(a^(1/3) + b^(1/3)*x)^2)/(a + b*x^3)^(2/3) - (2^(1
/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3)]/(3*2^(2/3)*a^(1/3)*b^(1/3)) -
 (2^(1/3)*Log[1 + (2^(1/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3)])/(3*a^
(1/3)*b^(1/3)) + Log[2*2^(1/3) + (a^(1/3) + b^(1/3)*x)^2/(a + b*x^3)^(2/3)
+ (2^(2/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3)]/(6*2^(2/3)*a^(1/3)*b^(
1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 420

```
Int[((a_) + (b_.)*(x_)^3)^(1/3)/((c_) + (d_.)*(x_)^3), x_Symbol] :> With[{q
 = Rt[b/a, 3]}, Dist[9*(a/(c*q)), Subst[Int[x/((4 - a*x^3)*(1 + 2*a*x^3)),
x], x, (1 + q*x)/(a + b*x^3)^(1/3)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b
*c - a*d, 0] && EqQ[b*c + a*d, 0]
```

Rule 493

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Dist[b/(b*c - a*d), Int[(e*x)^m/(a + b*x^n), x], x] - Dist[d/
(b*c - a*d), Int[(e*x)^m/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, m}, x
] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt[3]{a+bx^3}}{a-bx^3}\,dx &= \frac{\sqrt[3]{a+bx^3}\int \frac{\sqrt[3]{1+\frac{bx^3}{a}}}{a-bx^3}\,dx}{\sqrt[3]{1+\frac{bx^3}{a}}}\\ &= \frac{x\sqrt[3]{a+bx^3}\,F_1\left(\frac{1}{3};1,-\frac{1}{3};\frac{4}{3};\frac{bx^3}{a},-\frac{bx^3}{a}\right)}{a\sqrt[3]{1+\frac{bx^3}{a}}}\end{aligned}$$

Mathematica [A]
time = 2.58, size = 428, normalized size = 1.08

$$\frac{4\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{a+bx^3}}{-2\sqrt[3]{2}\sqrt[3]{a}-2\sqrt[3]{2}\sqrt[3]{b}x+\sqrt[3]{a+bx^3}}\right)+2\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{a+bx^3}}{\sqrt[3]{2}\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{b}x+\sqrt[3]{a+bx^3}}\right)-4\log\left(\sqrt[3]{2}\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)-2\log\left(-\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{b}x+2\sqrt[3]{a+bx^3}\right)+\log\left(2^{2/3}a^{2/3}+2^{2/3}b^{2/3}x^2+2\sqrt[3]{2}\sqrt[3]{b}x\sqrt[3]{a+bx^3}+4(a+bx^3)^{2/3}+2\sqrt[3]{2}\sqrt[3]{a}\left(\sqrt[3]{2}\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)\right)+2\log\left(2^{2/3}a^{2/3}+2^{2/3}b^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{b}x\sqrt[3]{a+bx^3}+(a+bx^3)^{2/3}+\sqrt[3]{a}\left(2\,2^{2/3}\sqrt[3]{b}x-\sqrt[3]{2}\sqrt[3]{a+bx^3}\right)\right)}{6\,2^{2/3}\sqrt[3]{a}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^(1/3)/(a - b*x^3),x]
```

```
[Out] (4*Sqrt[3]*ArcTan[(Sqrt[3]*(a + b*x^3)^(1/3))/(-2*2^(1/3)*a^(1/3) - 2*2^(1/
3)*b^(1/3)*x + (a + b*x^3)^(1/3))] + 2*Sqrt[3]*ArcTan[(Sqrt[3]*(a + b*x^3)^
(1/3))/(2^(1/3)*a^(1/3) + 2^(1/3)*b^(1/3)*x + (a + b*x^3)^(1/3))] - 4*Log[2
^(1/3)*a^(1/3) + 2^(1/3)*b^(1/3)*x + (a + b*x^3)^(1/3)] - 2*Log[-(2^(1/3)*a
^(1/3)) - 2^(1/3)*b^(1/3)*x + 2*(a + b*x^3)^(1/3)] + Log[2^(2/3)*a^(2/3) +
2^(2/3)*b^(2/3)*x^2 + 2*2^(1/3)*b^(1/3)*x*(a + b*x^3)^(1/3) + 4*(a + b*x^3)
^(2/3) + 2*2^(1/3)*a^(1/3)*(2^(1/3)*b^(1/3)*x + (a + b*x^3)^(1/3))] + 2*Log
[2^(2/3)*a^(2/3) + 2^(2/3)*b^(2/3)*x^2 - 2^(1/3)*b^(1/3)*x*(a + b*x^3)^(1/3
) + (a + b*x^3)^(2/3) + a^(1/3)*(2*2^(2/3)*b^(1/3)*x - 2^(1/3)*(a + b*x^3)^
(1/3))])/(6*2^(2/3)*a^(1/3)*b^(1/3))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^3+a\right)^{\frac{1}{3}}}{-b\,x^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(1/3)/(-b*x^3+a),x)
```

```
[Out] int((b*x^3+a)^(1/3)/(-b*x^3+a),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/(-b*x^3+a),x, algorithm="maxima")
```

```
[Out] -integrate((b*x^3 + a)^(1/3)/(b*x^3 - a), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 644 vs. 2(284) = 568.
time = 25.42, size = 644, normalized size = 1.62

$$-\frac{1}{18}\sqrt{3}\,2^{\frac{1}{3}}\left(-\frac{1}{ab}\right)^{\frac{1}{3}}\arctan\left(\frac{6\sqrt{3}\,2^{\frac{2}{3}}(ab^6x^{16}+33\,a^2b^5x^{13}+110\,a^3b^4x^{10}+110\,a^4b^3x^7+33\,a^5b^2x^4+a^6bx)(bx^3+a)^{\frac{1}{3}}\left(-\frac{1}{ab}\right)^{\frac{2}{3}}+24\sqrt{3}\,2^{\frac{1}{3}}(ab^5x^{14}+2\,a^2b^4x^{11}-6\,a^3b^3x^8+2\,a^4b^2x^5+a^5bx^2)(bx^3+a)^{\frac{2}{3}}\left(-\frac{1}{ab}\right)^{\frac{1}{3}}-\sqrt{3}\,(b^6x^{18}-42\,ab^5x^{15}-417\,a^2b^4x^{12}-812\,a^3b^3x^9-417\,a^4b^2x^6-42\,a^5bx^3+a^6)}{3\,(b^6x^{18}+102\,ab^5x^{15}+447\,a^2b^4x^{12}+628\,a^3b^3x^9+447\,a^4b^2x^6+102\,a^5bx^3+a^6)}\right)-\frac{1}{36}\cdot 2^{\frac{1}{3}}\left(-\frac{1}{ab}\right)^{\frac{1}{3}}\log\left(\frac{12\cdot 2^{\frac{2}{3}}(ab^3x^8+4\,a^2b^2x^5+a^3bx^2)(bx^3+a)^{\frac{2}{3}}\left(-\frac{1}{ab}\right)^{\frac{2}{3}}-2^{\frac{1}{3}}(b^4x^{12}+32\,ab^3x^9+78\,a^2b^2x^6+32\,a^3bx^3+a^4)\left(-\frac{1}{ab}\right)^{\frac{1}{3}}+6\,(b^3x^{10}+11\,ab^2x^7+11\,a^2bx^4+a^3x)(bx^3+a)^{\frac{1}{3}}}{b^4x^{12}-4\,ab^3x^9+6\,a^2b^2x^6-4\,a^3bx^3+a^4}\right)+\frac{1}{18}\cdot 2^{\frac{1}{3}}\left(-\frac{1}{ab}\right)^{\frac{1}{3}}\log\left(-\frac{12\,(bx^3+a)^{\frac{2}{3}}x^2+2^{\frac{2}{3}}(b^2x^6-2\,abx^3+a^2)\left(-\frac{1}{ab}\right)^{\frac{2}{3}}+6\cdot 2^{\frac{1}{3}}(bx^4+ax)(bx^3+a)^{\frac{1}{3}}\left(-\frac{1}{ab}\right)^{\frac{1}{3}}}{b^2x^6-2\,abx^3+a^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/(-b*x^3+a),x, algorithm="fricas")
```

```
[Out] -1/18*sqrt(3)*2^(1/3)*(-1/(a*b))^(1/3)*arctan(1/3*(6*sqrt(3)*2^(2/3)*(a*b^6
*x^16 + 33*a^2*b^5*x^13 + 110*a^3*b^4*x^10 + 110*a^4*b^3*x^7 + 33*a^5*b^2*x
^4 + a^6*b*x)*(b*x^3 + a)^(1/3)*(-1/(a*b))^(2/3) + 24*sqrt(3)*2^(1/3)*(a*b^
5*x^14 + 2*a^2*b^4*x^11 - 6*a^3*b^3*x^8 + 2*a^4*b^2*x^5 + a^5*b*x^2)*(b*x^3
 + a)^(2/3)*(-1/(a*b))^(1/3) - sqrt(3)*(b^6*x^18 - 42*a*b^5*x^15 - 417*a^2*
b^4*x^12 - 812*a^3*b^3*x^9 - 417*a^4*b^2*x^6 - 42*a^5*b*x^3 + a^6))/(b^6*x^
18 + 102*a*b^5*x^15 + 447*a^2*b^4*x^12 + 628*a^3*b^3*x^9 + 447*a^4*b^2*x^6
+ 102*a^5*b*x^3 + a^6)) - 1/36*2^(1/3)*(-1/(a*b))^(1/3)*log((12*2^(2/3)*(a*
b^3*x^8 + 4*a^2*b^2*x^5 + a^3*b*x^2)*(b*x^3 + a)^(2/3)*(-1/(a*b))^(2/3) - 2
^(1/3)*(b^4*x^12 + 32*a*b^3*x^9 + 78*a^2*b^2*x^6 + 32*a^3*b*x^3 + a^4)*(-1/
(a*b))^(1/3) + 6*(b^3*x^10 + 11*a*b^2*x^7 + 11*a^2*b*x^4 + a^3*x)*(b*x^3 +
a)^(1/3))/(b^4*x^12 - 4*a*b^3*x^9 + 6*a^2*b^2*x^6 - 4*a^3*b*x^3 + a^4)) + 1
/18*2^(1/3)*(-1/(a*b))^(1/3)*log(-(12*(b*x^3 + a)^(2/3)*x^2 + 2^(2/3)*(b^2*
x^6 - 2*a*b*x^3 + a^2)*(-1/(a*b))^(2/3) + 6*2^(1/3)*(b*x^4 + a*x)*(b*x^3 +
a)^(1/3)*(-1/(a*b))^(1/3))/(b^2*x^6 - 2*a*b*x^3 + a^2))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt[3]{a+bx^3}}{-a+bx^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(1/3)/(-b*x**3+a),x)
```

```
[Out] -Integral((a + b*x**3)**(1/3)/(-a + b*x**3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/(-b*x^3+a),x, algorithm="giac")
[Out] integrate(-(b*x^3 + a)^(1/3)/(b*x^3 - a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(b\,x^3+a\right)}^{1/3}}{a-b\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(1/3)/(a - b*x^3),x)
[Out] int((a + b*x^3)^(1/3)/(a - b*x^3), x)
```

3.37 $\int \frac{1}{(a-bx^3)(a+bx^3)^{2/3}}\,dx$

Optimal. Leaf size=452

$$-\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}\,a^{4/3}\sqrt[3]{b}}-\frac{\tan^{-1}\left(\frac{1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{2\,2^{2/3}\sqrt{3}\,a^{4/3}\sqrt[3]{b}}+\frac{x\left(1+\frac{bx^3}{a}\right)^{2/3}{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{2a\left(a+bx^3\right)^{2/3}}-\frac{\log\left(2\ldots\right.}{6\ldots}$$

```
[Out] 1/2*x*(1+b*x^3/a)^(2/3)*hypergeom([1/3, 2/3],[4/3],-b*x^3/a)/a/(b*x^3+a)^(2
/3)-1/12*ln(2^(2/3)+(-a^(1/3)-b^(1/3)*x)/(b*x^3+a)^(1/3))*2^(1/3)/a^(4/3)/b
^(1/3)+1/12*ln(1+2^(2/3)*(a^(1/3)+b^(1/3)*x)^2/(b*x^3+a)^(2/3)-2^(1/3)*(a^(
1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*2^(1/3)/a^(4/3)/b^(1/3)-1/6*ln(1+2^(1/3)*(
a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*2^(1/3)/a^(4/3)/b^(1/3)+1/24*ln(2*2^(1/
3)+(a^(1/3)+b^(1/3)*x)^2/(b*x^3+a)^(2/3)+2^(2/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3
+a)^(1/3))*2^(1/3)/a^(4/3)/b^(1/3)-1/6*arctan(1/3*(1-2*2^(1/3)*(a^(1/3)+b^(
1/3)*x)/(b*x^3+a)^(1/3))*3^(1/2))*2^(1/3)/a^(4/3)/b^(1/3)*3^(1/2)-1/12*arct
an(1/3*(1+2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*3^(1/2))*2^(1/3)/a^(
4/3)/b^(1/3)*3^(1/2)
```

Rubi [A]
time = 0.20, antiderivative size = 452, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 11, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {421, 252, 251, 420, 493, 298, 31, 648, 631, 210, 642}

$$-\frac{\text{ArcTan}\left(\frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}\,a^{4/3}\sqrt[3]{b}}-\frac{\text{ArcTan}\left(\frac{\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{2\,2^{2/3}\sqrt{3}\,a^{4/3}\sqrt[3]{b}}-\frac{\log\left(2^{2/3}-\frac{\sqrt[3]{a}+\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}\right)}{6\,2^{2/3}a^{4/3}\sqrt[3]{b}}+\frac{\log\left(\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}{(a+bx^3)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{6\,2^{2/3}a^{4/3}\sqrt[3]{b}}-\frac{\log\left(\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{3\,2^{2/3}a^{4/3}\sqrt[3]{b}}+\frac{\log\left(\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}{(a+bx^3)^{2/3}}+\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{a+bx^3}}+2\sqrt[3]{2}\right)}{12\,2^{2/3}a^{4/3}\sqrt[3]{b}}+\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{2a\left(a+bx^3\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - b*x^3)*(a + b*x^3)^(2/3)),x]
```

```
[Out] -(ArcTan[(1 - (2*2^(1/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3))/Sqrt[3]]
/(2^(2/3)*Sqrt[3]*a^(4/3)*b^(1/3))) - ArcTan[(1 + (2^(1/3)*(a^(1/3) + b^(1/
3)*x))/(a + b*x^3)^(1/3))/Sqrt[3]]/(2*2^(2/3)*Sqrt[3]*a^(4/3)*b^(1/3)) + (x
*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[1/3, 2/3, 4/3, -((b*x^3)/a)])/(2*a
*(a + b*x^3)^(2/3)) - Log[2^(2/3) - (a^(1/3) + b^(1/3)*x)/(a + b*x^3)^(1/3)
]/(6*2^(2/3)*a^(4/3)*b^(1/3)) + Log[1 + (2^(2/3)*(a^(1/3) + b^(1/3)*x)^2)/(
a + b*x^3)^(2/3) - (2^(1/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3)]/(6*2^
(2/3)*a^(4/3)*b^(1/3)) - Log[1 + (2^(1/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3
)^(1/3)]/(3*2^(2/3)*a^(4/3)*b^(1/3)) + Log[2*2^(1/3) + (a^(1/3) + b^(1/3)*x
)^2/(a + b*x^3)^(2/3) + (2^(2/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3)]/
(12*2^(2/3)*a^(4/3)*b^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 420

```
Int[((a_) + (b_.)*(x_)^3)^(1/3)/((c_) + (d_.)*(x_)^3), x_Symbol] :> With[{q
 = Rt[b/a, 3]}, Dist[9*(a/(c*q)), Subst[Int[x/((4 - a*x^3)*(1 + 2*a*x^3)),
x], x, (1 + q*x)/(a + b*x^3)^(1/3)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b
*c - a*d, 0] && EqQ[b*c + a*d, 0]
```

Rule 421

```
Int[1/(((a_) + (b_.)*(x_)^3)^(2/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^3)^(2/3), x], x] - Dist[d/(b*c - a*d), Int[
(a + b*x^3)^(1/3)/(c + d*x^3), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0] && EqQ[b*c + a*d, 0]
```

Rule 493

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Dist[b/(b*c - a*d), Int[(e*x)^m/(a + b*x^n), x], x] - Dist[d/
(b*c - a*d), Int[(e*x)^m/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, m}, x
] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a-bx^3)\,(a+bx^3)^{2/3}}\,dx &= \frac{\left(1+\frac{bx^3}{a}\right)^{2/3}\int \frac{1}{(a-bx^3)\left(1+\frac{bx^3}{a}\right)^{2/3}}\,dx}{(a+bx^3)^{2/3}}\\ &= \frac{x\left(1+\frac{bx^3}{a}\right)^{2/3} F_1\left(\frac{1}{3};1,\frac{2}{3};\frac{4}{3};\frac{bx^3}{a},-\frac{bx^3}{a}\right)}{a\,(a+bx^3)^{2/3}}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.
time = 10.05, size = 153, normalized size = 0.34

$$\frac{4axF_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)}{(a-bx^3)\,(a+bx^3)^{2/3}\left(4aF_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)+bx^3\left(3F_1\left(\frac{4}{3};\frac{2}{3},2;\frac{7}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)-2F_1\left(\frac{4}{3};\frac{5}{3},1;\frac{7}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a - b*x^3)*(a + b*x^3)^(2/3)),x]
```

```
[Out] (4*a*x*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), (b*x^3)/a])/((a - b*x^3)*(a
 + b*x^3)^(2/3)*(4*a*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), (b*x^3)/a] +
b*x^3*(3*AppellF1[4/3, 2/3, 2, 7/3, -((b*x^3)/a), (b*x^3)/a] - 2*AppellF1[4
/3, 5/3, 1, 7/3, -((b*x^3)/a), (b*x^3)/a])))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-b\,x^{3}+a\right)\left(b\,x^{3}+a\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x^3+a)/(b*x^3+a)^(2/3),x)
```

```
[Out] int(1/(-b*x^3+a)/(b*x^3+a)^(2/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^3+a)/(b*x^3+a)^(2/3),x, algorithm="maxima")
```

```
[Out] -integrate(1/((b*x^3 + a)^(2/3)*(b*x^3 - a)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^3+a)/(b*x^3+a)^(2/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{-a\left(a+bx^{3}\right)^{\frac{2}{3}}+bx^{3}\left(a+bx^{3}\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x**3+a)/(b*x**3+a)**(2/3),x)
[Out] -Integral(1/(-a*(a + b*x**3)**(2/3) + b*x**3*(a + b*x**3)**(2/3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^3+a)/(b*x^3+a)^(2/3),x, algorithm="giac")
[Out] integrate(-1/((b*x^3 + a)^(2/3)*(b*x^3 - a)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(b\,x^3+a\right)}^{2/3}\,\left(a-b\,x^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^3)^(2/3)*(a - b*x^3)),x)
[Out] int(1/((a + b*x^3)^(2/3)*(a - b*x^3)), x)
```

3.38 $\int \frac{1}{(a-bx^3)(a+bx^3)^{5/3}}\,dx$

Optimal. Leaf size=473

$$\frac{x}{4a^2\left(a+bx^3\right)^{2/3}}-\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{2\;2^{2/3}\sqrt{3}\,a^{7/3}\sqrt[3]{b}}-\frac{\tan^{-1}\left(\frac{1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{4\;2^{2/3}\sqrt{3}\,a^{7/3}\sqrt[3]{b}}+\frac{x\left(1+\frac{bx^3}{a}\right)^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\right.}{2a^2\left(a+bx^3\right)^{2/3}}$$

```
[Out] 1/4*x/a^2/(b*x^3+a)^(2/3)+1/2*x*(1+b*x^3/a)^(2/3)*hypergeom([1/3, 2/3],[4/3
],-b*x^3/a)/a^2/(b*x^3+a)^(2/3)-1/24*ln(2^(2/3)+(-a^(1/3)-b^(1/3)*x)/(b*x^3
+a)^(1/3))*2^(1/3)/a^(7/3)/b^(1/3)+1/24*ln(1+2^(2/3)*(a^(1/3)+b^(1/3)*x)^2/
(b*x^3+a)^(2/3)-2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*2^(1/3)/a^(7/3
)/b^(1/3)-1/12*ln(1+2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*2^(1/3)/a^
(7/3)/b^(1/3)+1/48*ln(2*2^(1/3)+(a^(1/3)+b^(1/3)*x)^2/(b*x^3+a)^(2/3)+2^(2/
3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*2^(1/3)/a^(7/3)/b^(1/3)-1/12*arctan
(1/3*(1-2*2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*3^(1/2))*2^(1/3)/a^(
7/3)/b^(1/3)*3^(1/2)-1/24*arctan(1/3*(1+2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+
a)^(1/3))*3^(1/2))*2^(1/3)/a^(7/3)/b^(1/3)*3^(1/2)
```

Rubi [A]
time = 0.28, antiderivative size = 473, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 13, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.591$, Rules used = {425, 544, 252, 251, 421, 420, 493, 298, 31, 648, 631, 210, 642}

$$-\frac{\text{ArcTan}\left(\frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{2\;2^{2/3}\sqrt{3}\,a^{7/3}\sqrt[3]{b}}-\frac{\text{ArcTan}\left(\frac{\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{4\;2^{2/3}\sqrt{3}\,a^{7/3}\sqrt[3]{b}}-\frac{\log\left(2^{2/3}-\frac{\sqrt[3]{a}+\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}\right)}{12\;2^{2/3}a^{7/3}\sqrt[3]{b}}+\frac{\log\left(\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}{(a+bx^3)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{12\;2^{2/3}a^{7/3}\sqrt[3]{b}}-\frac{\log\left(\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{6\;2^{2/3}a^{7/3}\sqrt[3]{b}}+\frac{\log\left(\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}{(a+bx^3)^{2/3}}+\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{a+bx^3}}+2\sqrt[3]{2}\right)}{24\;2^{2/3}a^{7/3}\sqrt[3]{b}}+\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{2a^2\left(a+bx^3\right)^{2/3}}+\frac{x}{4a^2\left(a+bx^3\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - b*x^3)*(a + b*x^3)^(5/3)),x]
```

```
[Out] x/(4*a^2*(a + b*x^3)^(2/3)) - ArcTan[(1 - (2*2^(1/3)*(a^(1/3) + b^(1/3)*x))
/(a + b*x^3)^(1/3))/Sqrt[3]]/(2*2^(2/3)*Sqrt[3]*a^(7/3)*b^(1/3)) - ArcTan[(
1 + (2^(1/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3))/Sqrt[3]]/(4*2^(2/3)*
Sqrt[3]*a^(7/3)*b^(1/3)) + (x*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[1/3,
2/3, 4/3, -((b*x^3)/a)])/(2*a^2*(a + b*x^3)^(2/3)) - Log[2^(2/3) - (a^(1/3)
 + b^(1/3)*x)/(a + b*x^3)^(1/3)]/(12*2^(2/3)*a^(7/3)*b^(1/3)) + Log[1 + (2^
(2/3)*(a^(1/3) + b^(1/3)*x)^2)/(a + b*x^3)^(2/3) - (2^(1/3)*(a^(1/3) + b^(1
/3)*x))/(a + b*x^3)^(1/3)]/(12*2^(2/3)*a^(7/3)*b^(1/3)) - Log[1 + (2^(1/3)*
(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3)]/(6*2^(2/3)*a^(7/3)*b^(1/3)) + Log
[2*2^(1/3) + (a^(1/3) + b^(1/3)*x)^2/(a + b*x^3)^(2/3) + (2^(2/3)*(a^(1/3)
+ b^(1/3)*x))/(a + b*x^3)^(1/3)]/(24*2^(2/3)*a^(7/3)*b^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 420

```
Int[((a_) + (b_.)*(x_)^3)^(1/3)/((c_) + (d_.)*(x_)^3), x_Symbol] :> With[{q
 = Rt[b/a, 3]}, Dist[9*(a/(c*q)), Subst[Int[x/((4 - a*x^3)*(1 + 2*a*x^3)),
x], x, (1 + q*x)/(a + b*x^3)^(1/3)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b
*c - a*d, 0] && EqQ[b*c + a*d, 0]
```

Rule 421

```
Int[1/(((a_) + (b_.)*(x_)^3)^(2/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^3)^(2/3), x], x] - Dist[d/(b*c - a*d), Int[
(a + b*x^3)^(1/3)/(c + d*x^3), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0] && EqQ[b*c + a*d, 0]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 493

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Dist[b/(b*c - a*d), Int[(e*x)^m/(a + b*x^n), x], x] - Dist[d/
(b*c - a*d), Int[(e*x)^m/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, m}, x
] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
```

Rule 544

```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Dist[f/d, Int[(a + b*x^n)^p, x], x] + Dist[(d*e -
c*f)/d, Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, p
, n}, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{(a-bx^3)(a+bx^3)^{5/3}}\,dx = \frac{\left(1+\frac{bx^3}{a}\right)^{2/3}\int \frac{1}{(a-bx^3)\left(1+\frac{bx^3}{a}\right)^{5/3}}\,dx}{a(a+bx^3)^{2/3}}$$
$$= \frac{x\left(1+\frac{bx^3}{a}\right)^{2/3} F_1\left(\frac{1}{3};1,\frac{5}{3};\frac{4}{3};\frac{bx^3}{a},-\frac{bx^3}{a}\right)}{a^2(a+bx^3)^{2/3}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.
time = 10.10, size = 213, normalized size = 0.45

$$\frac{x\left(\frac{4}{a^2}-\frac{bx^3\left(1+\frac{bx^3}{a}\right)^{2/3}F_1\left(\frac{4}{3};\frac{2}{3},1;\frac{7}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)}{a^3}+\frac{48F_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)}{(a-bx^3)\left(4aF_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)+bx^3\left(3F_1\left(\frac{4}{3};\frac{2}{3},2;\frac{7}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)-2F_1\left(\frac{4}{3};\frac{5}{3},1;\frac{7}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)\right)\right)}\right)}{16(a+bx^3)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a - b*x^3)*(a + b*x^3)^(5/3)),x]
```

```
[Out] (x*(4/a^2 - (b*x^3*(1 + (b*x^3)/a)^(2/3)*AppellF1[4/3, 2/3, 1, 7/3, -((b*x^
3)/a), (b*x^3)/a])/a^3 + (48*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), (b*x^
3)/a])/((a - b*x^3)*(4*a*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), (b*x^3)/a
] + b*x^3*(3*AppellF1[4/3, 2/3, 2, 7/3, -((b*x^3)/a), (b*x^3)/a] - 2*Appell
F1[4/3, 5/3, 1, 7/3, -((b*x^3)/a), (b*x^3)/a])))))/(16*(a + b*x^3)^(2/3))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-b\,x^{3}+a\right)\left(b\,x^{3}+a\right)^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x^3+a)/(b*x^3+a)^(5/3),x)
```

```
[Out] int(1/(-b*x^3+a)/(b*x^3+a)^(5/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^3+a)/(b*x^3+a)^(5/3),x, algorithm="maxima")
```

```
[Out] -integrate(1/((b*x^3 + a)^(5/3)*(b*x^3 - a)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^3+a)/(b*x^3+a)^(5/3),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{-a^2\left(a+bx^3\right)^{\frac{2}{3}}+b^2x^6\left(a+bx^3\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x**3+a)/(b*x**3+a)**(5/3),x)
[Out] -Integral(1/(-a**2*(a + b*x**3)**(2/3) + b**2*x**6*(a + b*x**3)**(2/3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^3+a)/(b*x^3+a)^(5/3),x, algorithm="giac")
[Out] integrate(-1/((b*x^3 + a)^(5/3)*(b*x^3 - a)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(b\,x^3+a\right)}^{5/3}\,\left(a-b\,x^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^3)^(5/3)*(a - b*x^3)),x)
[Out] int(1/((a + b*x^3)^(5/3)*(a - b*x^3)), x)
```

3.39 $\int \frac{1}{(a-bx^3)(a+bx^3)^{8/3}}\,dx$

Optimal. Leaf size=492

$$\frac{x}{10a^2\left(a+bx^3\right)^{5/3}}+\frac{13x}{40a^3\left(a+bx^3\right)^{2/3}}-\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{4\,2^{2/3}\sqrt{3}\,a^{10/3}\sqrt[3]{b}}-\frac{\tan^{-1}\left(\frac{1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{8\,2^{2/3}\sqrt{3}\,a^{10/3}\sqrt[3]{b}}+\frac{9x\left(1\right.}{}$$

```
[Out] 1/10*x/a^2/(b*x^3+a)^(5/3)+13/40*x/a^3/(b*x^3+a)^(2/3)+9/20*x*(1+b*x^3/a)^(
2/3)*hypergeom([1/3, 2/3],[4/3],-b*x^3/a)/a^3/(b*x^3+a)^(2/3)-1/48*ln(2^(2/
3)+(-a^(1/3)-b^(1/3)*x)/(b*x^3+a)^(1/3))*2^(1/3)/a^(10/3)/b^(1/3)+1/48*ln(1
+2^(2/3)*(a^(1/3)+b^(1/3)*x)^2/(b*x^3+a)^(2/3)-2^(1/3)*(a^(1/3)+b^(1/3)*x)/
(b*x^3+a)^(1/3))*2^(1/3)/a^(10/3)/b^(1/3)-1/24*ln(1+2^(1/3)*(a^(1/3)+b^(1/3
)*x)/(b*x^3+a)^(1/3))*2^(1/3)/a^(10/3)/b^(1/3)+1/96*ln(2*2^(1/3)+(a^(1/3)+b
^(1/3)*x)^2/(b*x^3+a)^(2/3)+2^(2/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*2^
(1/3)/a^(10/3)/b^(1/3)-1/24*arctan(1/3*(1-2*2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*
x^3+a)^(1/3))*3^(1/2))*2^(1/3)/a^(10/3)/b^(1/3)*3^(1/2)-1/48*arctan(1/3*(1+
2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*3^(1/2))*2^(1/3)/a^(10/3)/b^(1
/3)*3^(1/2)
```

Rubi [A]

time = 0.31, antiderivative size = 492, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 14, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.636$, Rules used = {425, 541, 544, 252, 251, 421, 420, 493, 298, 31, 648, 631, 210, 642}

$$-\frac{\text{ArcTan}\left(\frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{4\,2^{2/3}\sqrt{3}\,a^{10/3}\sqrt[3]{b}}-\frac{\text{ArcTan}\left(\frac{\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{8\,2^{2/3}\sqrt{3}\,a^{10/3}\sqrt[3]{b}}-\frac{\log\left(2^{2/3}-\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)}{24\,2^{2/3}a^{10/3}\sqrt[3]{b}}+\frac{\log\left(\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{(a+bx^3)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{24\,2^{2/3}a^{10/3}\sqrt[3]{b}}-\frac{\log\left(\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{12\,2^{2/3}a^{10/3}\sqrt[3]{b}}+\frac{\log\left(\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{(a+bx^3)^{2/3}}+\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+2\sqrt[3]{2}\right)}{48\,2^{2/3}a^{10/3}\sqrt[3]{b}}+\frac{9x\left(\frac{bx^3}{a}+1\right)^{2/3}{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{20a^3\left(a+bx^3\right)^{2/3}}+\frac{13x}{40a^3\left(a+bx^3\right)^{2/3}}+\frac{x}{10a^2\left(a+bx^3\right)^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - b*x^3)*(a + b*x^3)^(8/3)),x]
```

```
[Out] x/(10*a^2*(a + b*x^3)^(5/3)) + (13*x)/(40*a^3*(a + b*x^3)^(2/3)) - ArcTan[(
1 - (2*2^(1/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3))/Sqrt[3]]/(4*2^(2/3
)*Sqrt[3]*a^(10/3)*b^(1/3)) - ArcTan[(1 + (2^(1/3)*(a^(1/3) + b^(1/3)*x))/(
a + b*x^3)^(1/3))/Sqrt[3]]/(8*2^(2/3)*Sqrt[3]*a^(10/3)*b^(1/3)) + (9*x*(1 +
 (b*x^3)/a)^(2/3)*Hypergeometric2F1[1/3, 2/3, 4/3, -((b*x^3)/a)])/(20*a^3*(
a + b*x^3)^(2/3)) - Log[2^(2/3) - (a^(1/3) + b^(1/3)*x)/(a + b*x^3)^(1/3)]/
(24*2^(2/3)*a^(10/3)*b^(1/3)) + Log[1 + (2^(2/3)*(a^(1/3) + b^(1/3)*x)^2)/(
a + b*x^3)^(2/3) - (2^(1/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3)]/(24*2
^(2/3)*a^(10/3)*b^(1/3)) - Log[1 + (2^(1/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x
^3)^(1/3)]/(12*2^(2/3)*a^(10/3)*b^(1/3)) + Log[2*2^(1/3) + (a^(1/3) + b^(1/
```

```
3)*x)^2/(a + b*x^3)^(2/3) + (2^(2/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/
3)]/(48*2^(2/3)*a^(10/3)*b^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 420

```
Int[((a_) + (b_.)*(x_)^3)^(1/3)/((c_) + (d_.)*(x_)^3), x_Symbol] :> With[{q
 = Rt[b/a, 3]}, Dist[9*(a/(c*q)), Subst[Int[x/((4 - a*x^3)*(1 + 2*a*x^3)),
x], x, (1 + q*x)/(a + b*x^3)^(1/3)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b
*c - a*d, 0] && EqQ[b*c + a*d, 0]
```

Rule 421

```
Int[1/(((a_) + (b_.)*(x_)^3)^(2/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^3)^(2/3), x], x] - Dist[d/(b*c - a*d), Int[
(a + b*x^3)^(1/3)/(c + d*x^3), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
```

```
- a*d, 0] && EqQ[b*c + a*d, 0]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 493

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Dist[b/(b*c - a*d), Int[(e*x)^m/(a + b*x^n), x], x] - Dist[d/
(b*c - a*d), Int[(e*x)^m/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, m}, x
] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 544

```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Dist[f/d, Int[(a + b*x^n)^p, x], x] + Dist[(d*e -
c*f)/d, Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, p
, n}, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{(a-bx^3)(a+bx^3)^{8/3}}\,dx = \frac{\left(1+\frac{bx^3}{a}\right)^{2/3}\int \frac{1}{(a-bx^3)\left(1+\frac{bx^3}{a}\right)^{8/3}}\,dx}{a^2(a+bx^3)^{2/3}}$$
$$= \frac{x\left(1+\frac{bx^3}{a}\right)^{2/3} F_1\left(\frac{1}{3};1,\frac{8}{3};\frac{4}{3};\frac{bx^3}{a},-\frac{bx^3}{a}\right)}{a^3(a+bx^3)^{2/3}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.
time = 10.14, size = 240, normalized size = 0.49

$$\frac{x\left(16a^2+52a(a+bx^3)-13bx^3(a+bx^3)\left(1+\frac{bx^3}{a}\right)^{2/3}F_1\left(\frac{4}{3};\frac{2}{3},1;\frac{7}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)+\frac{368a^3(a+bx^3)F_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)}{(a-bx^3)\left(4aF_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)+bx^3\left(3F_1\left(\frac{4}{3};\frac{2}{3},2;\frac{7}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)-2F_1\left(\frac{4}{3};\frac{5}{3},1;\frac{7}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)\right)\right)}\right)}{160a^4(a+bx^3)^{5/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a - b*x^3)*(a + b*x^3)^(8/3)),x]
```

```
[Out] (x*(16*a^2 + 52*a*(a + b*x^3) - 13*b*x^3*(a + b*x^3)*(1 + (b*x^3)/a)^(2/3)*
AppellF1[4/3, 2/3, 1, 7/3, -((b*x^3)/a), (b*x^3)/a] + (368*a^3*(a + b*x^3)*
AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), (b*x^3)/a])/((a - b*x^3)*(4*a*Appe
llF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), (b*x^3)/a] + b*x^3*(3*AppellF1[4/3, 2/
3, 2, 7/3, -((b*x^3)/a), (b*x^3)/a] - 2*AppellF1[4/3, 5/3, 1, 7/3, -((b*x^3
)/a), (b*x^3)/a])))))/(160*a^4*(a + b*x^3)^(5/3))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{(-b\,x^3+a)\,(b\,x^3+a)^{\frac{8}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x^3+a)/(b*x^3+a)^(8/3),x)
[Out] int(1/(-b*x^3+a)/(b*x^3+a)^(8/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^3+a)/(b*x^3+a)^(8/3),x, algorithm="maxima")
[Out] -integrate(1/((b*x^3 + a)^(8/3)*(b*x^3 - a)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^3+a)/(b*x^3+a)^(8/3),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{-a^{3}\left(a+bx^{3}\right)^{\frac{2}{3}}-a^{2}bx^{3}\left(a+bx^{3}\right)^{\frac{2}{3}}+ab^{2}x^{6}\left(a+bx^{3}\right)^{\frac{2}{3}}+b^{3}x^{9}\left(a+bx^{3}\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x**3+a)/(b*x**3+a)**(8/3),x)
[Out] -Integral(1/(-a**3*(a + b*x**3)**(2/3) - a**2*b*x**3*(a + b*x**3)**(2/3) +
a*b**2*x**6*(a + b*x**3)**(2/3) + b**3*x**9*(a + b*x**3)**(2/3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^3+a)/(b*x^3+a)^(8/3),x, algorithm="giac")
[Out] integrate(-1/((b*x^3 + a)^(8/3)*(b*x^3 - a)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(b\,x^3+a\right)^{8/3}\,\left(a-b\,x^3\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^3)^(8/3)*(a - b*x^3)),x)
[Out] int(1/((a + b*x^3)^(8/3)*(a - b*x^3)), x)
```

3.40 $\int (a-bx^3)^2 (a+bx^3)^{2/3}\, dx$

Optimal. Leaf size=139

$$\frac{38}{81}a^2x\left(a+bx^3\right)^{2/3}-\frac{8}{27}ax\left(a+bx^3\right)^{5/3}-\frac{1}{9}x\left(a-bx^3\right)\left(a+bx^3\right)^{5/3}+\frac{76a^3\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{81\sqrt{3}\sqrt[3]{b}}-\frac{38a^3\log\left(\cdots\right.}{}$$

[Out] 38/81*a^2*x*(b*x^3+a)^(2/3)-8/27*a*x*(b*x^3+a)^(5/3)-1/9*x*(-b*x^3+a)*(b*x^3+a)^(5/3)-38/81*a^3*ln(-b^(1/3)*x+(b*x^3+a)^(1/3))/b^(1/3)+76/243*a^3*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/b^(1/3)*3^(1/2)

Rubi [A]
time = 0.04, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$,
Rules used = {427, 396, 201, 245}

$$\frac{76a^3\text{ArcTan}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{81\sqrt{3}\sqrt[3]{b}}-\frac{38a^3\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}\,x\right)}{81\sqrt[3]{b}}+\frac{38}{81}a^2x\left(a+bx^3\right)^{2/3}-\frac{8}{27}ax\left(a+bx^3\right)^{5/3}-\frac{1}{9}x\left(a-bx^3\right)\left(a+bx^3\right)^{5/3}$$

Antiderivative was successfully verified.

[In] Int[(a - b*x^3)^2*(a + b*x^3)^(2/3),x]

[Out] (38*a^2*x*(a + b*x^3)^(2/3))/81 - (8*a*x*(a + b*x^3)^(5/3))/27 - (x*(a - b*x^3)*(a + b*x^3)^(5/3))/9 + (76*a^3*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(81*Sqrt[3]*b^(1/3)) - (38*a^3*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)])/(81*b^(1/3))

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rubi steps

$$\int \left(a-bx^3\right)^2 \left(a+bx^3\right)^{2/3} \, dx = -\frac{1}{9}x\left(a-bx^3\right)\left(a+bx^3\right)^{5/3} + \frac{\int \left(a+bx^3\right)^{2/3} \left(10a^2b-16ab^2x^3\right) \, dx}{9b}$$

$$= -\frac{8}{27}ax\left(a+bx^3\right)^{5/3} - \frac{1}{9}x\left(a-bx^3\right)\left(a+bx^3\right)^{5/3} + \frac{1}{27}\left(38a^2\right)\int \left(a+bx^3\right)^{2/}$$

$$= \frac{38}{81}a^2x\left(a+bx^3\right)^{2/3} - \frac{8}{27}ax\left(a+bx^3\right)^{5/3} - \frac{1}{9}x\left(a-bx^3\right)\left(a+bx^3\right)^{5/3} + \frac{1}{81}($$

$$76a$$

$$= \frac{38}{81}a^2x\left(a+bx^3\right)^{2/3} - \frac{8}{27}ax\left(a+bx^3\right)^{5/3} - \frac{1}{9}x\left(a-bx^3\right)\left(a+bx^3\right)^{5/3} + \text{—}$$

Mathematica [A]
time = 0.47, size = 167, normalized size = 1.20

$$\frac{3\sqrt[3]{b}\left(a+bx^3\right)^{2/3}\left(5a^2x-24abx^4+9b^2x^7\right)+76\sqrt{3}\,a^3\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{b}\,x}{\sqrt[3]{b}\,x+2\sqrt[3]{a+bx^3}}\right)-76a^3\log\left(-\sqrt[3]{b}\,x+\sqrt[3]{a+bx^3}\right)+38a^3\log\left(b^{2/3}x^2+\sqrt[3]{b}\,x\sqrt[3]{a+bx^3}+\left(a+bx^3\right)^{2/3}\right)}{243\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^3)^2*(a + b*x^3)^(2/3),x]
```

```
[Out] (3*b^(1/3)*(a + b*x^3)^(2/3)*(5*a^2*x - 24*a*b*x^4 + 9*b^2*x^7) + 76*Sqrt[3
]*a^3*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2*(a + b*x^3)^(1/3))] - 76*a^
3*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)] + 38*a^3*Log[b^(2/3)*x^2 + b^(1/3)*
x*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)])/(243*b^(1/3))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \left(-b\,x^{3}+a\right)^{2}\left(b\,x^{3}+a\right)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^3+a)^2*(b*x^3+a)^(2/3),x)
```

```
[Out] int((-b*x^3+a)^2*(b*x^3+a)^(2/3),x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 552 vs. 2(109) = 218.
time = 0.51, size = 552, normalized size = 3.97

$$-\frac{1}{9}\left(\frac{2\sqrt{3}\,a\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\,(bx^3+a)^{\frac{1}{3}}}{x}\right)}{3\,b^{\frac{1}{3}}}\right)}{b^{\frac{1}{3}}}-\frac{a\log\left(b^{\frac{2}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{(bx^3+a)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{1}{3}}}+\frac{2\,a\log\left(-b^{\frac{1}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}}{x}\right)}{b^{\frac{1}{3}}}+\frac{3\,(bx^3+a)^{\frac{2}{3}}a}{\left(b-\frac{bx^3+a}{x^3}\right)x^2}\right)a^2-\frac{1}{27}\left(\frac{2\sqrt{3}\,a^2\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\,(bx^3+a)^{\frac{1}{3}}}{x}\right)}{3\,b^{\frac{1}{3}}}\right)}{b^{\frac{4}{3}}}-\frac{a^2\log\left(b^{\frac{2}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{(bx^3+a)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{4}{3}}}+\frac{2\,a^2\log\left(-b^{\frac{1}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}}{x}\right)}{b^{\frac{4}{3}}}+\frac{3\left(\frac{(bx^3+a)^{\frac{2}{3}}a^2b}{x^2}+\frac{2\,(bx^3+a)^{\frac{5}{3}}a^2}{x^5}\right)}{b^3-\frac{2\,(bx^3+a)b^2}{x^3}+\frac{(bx^3+a)^2b}{x^6}}\right)ab-\frac{1}{243}\left(\frac{4\sqrt{3}\,a^3\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\,(bx^3+a)^{\frac{1}{3}}}{x}\right)}{3\,b^{\frac{1}{3}}}\right)}{b^{\frac{7}{3}}}-\frac{2\,a^3\log\left(b^{\frac{2}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{(bx^3+a)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{7}{3}}}+\frac{4\,a^3\log\left(-b^{\frac{1}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}}{x}\right)}{b^{\frac{7}{3}}}+\frac{3\left(\frac{2\,(bx^3+a)^{\frac{2}{3}}a^3b^2}{x^2}+\frac{11\,(bx^3+a)^{\frac{5}{3}}a^3b}{x^5}-\frac{4\,(bx^3+a)^{\frac{8}{3}}a^3}{x^8}\right)}{b^5-\frac{3\,(bx^3+a)b^4}{x^3}+\frac{3\,(bx^3+a)^2b^3}{x^6}-\frac{(bx^3+a)^3b^2}{x^9}}\right)b^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)^2*(b*x^3+a)^(2/3),x, algorithm="maxima")
```

```
[Out] -1/9*(2*sqrt(3)*a*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1
/3))/b^(1/3) - a*log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2
/3)/x^2)/b^(1/3) + 2*a*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(1/3) + 3*(b*x
^3 + a)^(2/3)*a/((b - (b*x^3 + a)/x^3)*x^2))*a^2 - 1/27*(2*sqrt(3)*a^2*arct
an(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1/3))/b^(4/3) - a^2*log
(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2/3)/x^2)/b^(4/3) + 2
*a^2*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(4/3) + 3*((b*x^3 + a)^(2/3)*a^2
*b/x^2 + 2*(b*x^3 + a)^(5/3)*a^2/x^5)/(b^3 - 2*(b*x^3 + a)*b^2/x^3 + (b*x^3
 + a)^2*b/x^6))*a*b - 1/243*(4*sqrt(3)*a^3*arctan(1/3*sqrt(3)*(b^(1/3) + 2*
(b*x^3 + a)^(1/3)/x)/b^(1/3))/b^(7/3) - 2*a^3*log(b^(2/3) + (b*x^3 + a)^(1/
3)*b^(1/3)/x + (b*x^3 + a)^(2/3)/x^2)/b^(7/3) + 4*a^3*log(-b^(1/3) + (b*x^3
 + a)^(1/3)/x)/b^(7/3) + 3*(2*(b*x^3 + a)^(2/3)*a^3*b^2/x^2 + 11*(b*x^3 + a
)^(5/3)*a^3*b/x^5 - 4*(b*x^3 + a)^(8/3)*a^3/x^8)/(b^5 - 3*(b*x^3 + a)*b^4/x
^3 + 3*(b*x^3 + a)^2*b^3/x^6 - (b*x^3 + a)^3*b^2/x^9))*b^2
```

Fricas [A]
time = 4.42, size = 421, normalized size = 3.03

$$\left[\frac{114\sqrt{\frac{1}{3}}\,a^3b\sqrt{\frac{(-b)^{\frac{1}{3}}}{b}}\log\left(3\,bx^3-3\,(bx^3+a)^{\frac{1}{3}}(-b)^{\frac{2}{3}}x^2-3\sqrt{\frac{1}{3}}\left((-b)^{\frac{1}{3}}bx^3-(bx^3+a)^{\frac{1}{3}}bx^2+2\,(bx^3+a)^{\frac{2}{3}}(-b)^{\frac{2}{3}}x\right)\sqrt{\frac{(-b)^{\frac{1}{3}}}{b}}+2\,a\right)-76\,a^3(-b)^{\frac{2}{3}}\log\left(\frac{(-b)^{\frac{1}{3}}x+(bx^3+a)^{\frac{1}{3}}}{x}\right)+38\,a^3(-b)^{\frac{2}{3}}\log\left(\frac{(-b)^{\frac{2}{3}}x^2-(bx^3+a)^{\frac{1}{3}}(-b)^{\frac{1}{3}}x+(bx^3+a)^{\frac{2}{3}}}{x^2}\right)+3\,(9\,b^3x^7-24\,ab^2x^4+5\,a^2bx)(bx^3+a)^{\frac{2}{3}}}{243\,b},\ -\frac{228\sqrt{\frac{1}{3}}\,a^3b\sqrt{-\frac{(-b)^{\frac{1}{3}}}{b}}\arctan\left(-\frac{\sqrt{\frac{1}{3}}\left((-b)^{\frac{1}{3}}x-2\,(bx^3+a)^{\frac{1}{3}}\right)\sqrt{-\frac{(-b)^{\frac{1}{3}}}{b}}}{x}\right)+76\,a^3(-b)^{\frac{2}{3}}\log\left(\frac{(-b)^{\frac{1}{3}}x+(bx^3+a)^{\frac{1}{3}}}{x}\right)-38\,a^3(-b)^{\frac{2}{3}}\log\left(\frac{(-b)^{\frac{2}{3}}x^2-(bx^3+a)^{\frac{1}{3}}(-b)^{\frac{1}{3}}x+(bx^3+a)^{\frac{2}{3}}}{x^2}\right)-3\,(9\,b^3x^7-24\,ab^2x^4+5\,a^2bx)(bx^3+a)^{\frac{2}{3}}}{243\,b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)^2*(b*x^3+a)^(2/3),x, algorithm="fricas")
```

```
[Out] [1/243*(114*sqrt(1/3)*a^3*b*sqrt((-b)^(1/3)/b)*log(3*b*x^3 - 3*(b*x^3 + a)^
(1/3)*(-b)^(2/3)*x^2 - 3*sqrt(1/3)*((-b)^(1/3)*b*x^3 - (b*x^3 + a)^(1/3)*b*
```

```
x^2 + 2*(b*x^3 + a)^(2/3)*(-b)^(2/3)*x)*sqrt((-b)^(1/3)/b) + 2*a) - 76*a^3*
(-b)^(2/3)*log(((-b)^(1/3)*x + (b*x^3 + a)^(1/3))/x) + 38*a^3*(-b)^(2/3)*lo
g(((-b)^(2/3)*x^2 - (b*x^3 + a)^(1/3)*(-b)^(1/3)*x + (b*x^3 + a)^(2/3))/x^2
) + 3*(9*b^3*x^7 - 24*a*b^2*x^4 + 5*a^2*b*x)*(b*x^3 + a)^(2/3))/b, -1/243*(
228*sqrt(1/3)*a^3*b*sqrt(-(-b)^(1/3)/b)*arctan(-sqrt(1/3)*((-b)^(1/3)*x - 2
*(b*x^3 + a)^(1/3))*sqrt(-(-b)^(1/3)/b)/x) + 76*a^3*(-b)^(2/3)*log(((-b)^(1
/3)*x + (b*x^3 + a)^(1/3))/x) - 38*a^3*(-b)^(2/3)*log(((-b)^(2/3)*x^2 - (b*
x^3 + a)^(1/3)*(-b)^(1/3)*x + (b*x^3 + a)^(2/3))/x^2) - 3*(9*b^3*x^7 - 24*a
*b^2*x^4 + 5*a^2*b*x)*(b*x^3 + a)^(2/3))/b]
```

Sympy [C] Result contains complex when optimal does not.
time = 6.13, size = 126, normalized size = 0.91

$$\frac{a^{\frac{8}{3}}x\Gamma\left(\frac{1}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{2}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)}-\frac{2a^{\frac{5}{3}}bx^{4}\Gamma\left(\frac{4}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{2}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)}+\frac{a^{\frac{2}{3}}b^{2}x^{7}\Gamma\left(\frac{7}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{2}{3},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{10}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**3+a)**2*(b*x**3+a)**(2/3),x)
```

```
[Out] a**(8/3)*x*gamma(1/3)*hyper((-2/3, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a)/
(3*gamma(4/3)) - 2*a**(5/3)*b*x**4*gamma(4/3)*hyper((-2/3, 4/3), (7/3,), b*
x**3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + a**(2/3)*b**2*x**7*gamma(7/3)*hype
r((-2/3, 7/3), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(10/3))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)^2*(b*x^3+a)^(2/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(2/3)*(b*x^3 - a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(b\,x^3+a\right)}^{2/3}\,{\left(a-b\,x^3\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(2/3)*(a - b*x^3)^2,x)
```

```
[Out] int((a + b*x^3)^(2/3)*(a - b*x^3)^2, x)
```

3.41 $\int \frac{(a-bx^3)^2}{\sqrt[3]{a+bx^3}}\,dx$

Optimal. Leaf size=120

$$-\frac{13}{18}ax\left(a+bx^3\right)^{2/3}-\frac{1}{6}x\left(a-bx^3\right)\left(a+bx^3\right)^{2/3}+\frac{17a^2\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{9\sqrt{3}\,\sqrt[3]{b}}-\frac{17a^2\log\left(-\sqrt[3]{b}\,x+\sqrt[3]{a+bx^3}\right)}{18\sqrt[3]{b}}$$

```
[Out] -13/18*a*x*(b*x^3+a)^(2/3)-1/6*x*(-b*x^3+a)*(b*x^3+a)^(2/3)-17/18*a^2*ln(-b
^(1/3)*x+(b*x^3+a)^(1/3))/b^(1/3)+17/27*a^2*arctan(1/3*(1+2*b^(1/3)*x/(b*x^
3+a)^(1/3))*3^(1/2))/b^(1/3)*3^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {427, 396, 245}

$$\frac{17a^2\text{ArcTan}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{9\sqrt{3}\,\sqrt[3]{b}}-\frac{17a^2\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}\,x\right)}{18\sqrt[3]{b}}-\frac{13}{18}ax\left(a+bx^3\right)^{2/3}-\frac{1}{6}x\left(a-bx^3\right)\left(a+bx^3\right)^{2/3}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^3)^2/(a + b*x^3)^(1/3),x]
```

```
[Out] (-13*a*x*(a + b*x^3)^(2/3))/18 - (x*(a - b*x^3)*(a + b*x^3)^(2/3))/6 + (17*
a^2*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(9*Sqrt[3]*b^(1/
3)) - (17*a^2*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)])/(18*b^(1/3))
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rubi steps

$$\int \frac{(a-bx^3)^2}{\sqrt[3]{a+bx^3}}\,dx = -\frac{1}{6}x(a-bx^3)\,(a+bx^3)^{2/3} + \frac{\int \frac{7a^2b-13ab^2x^3}{\sqrt[3]{a+bx^3}}\,dx}{6b}$$
$$= -\frac{13}{18}ax(a+bx^3)^{2/3} - \frac{1}{6}x(a-bx^3)\,(a+bx^3)^{2/3} + \frac{1}{9}(17a^2)\int \frac{1}{\sqrt[3]{a+bx^3}}\,dx$$
$$= -\frac{13}{18}ax(a+bx^3)^{2/3} - \frac{1}{6}x(a-bx^3)\,(a+bx^3)^{2/3} + \frac{17a^2\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{9\sqrt{3}\,\sqrt[3]{b}} - \frac{17a^2}{}$$

Mathematica [A]
time = 0.37, size = 165, normalized size = 1.38

$$\frac{1}{18}(a+bx^3)^{2/3}(-16ax+3bx^4) + \frac{17a^2\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{b}\,x}{\sqrt[3]{b}\,x+2\sqrt[3]{a+bx^3}}\right)}{9\sqrt{3}\,\sqrt[3]{b}} - \frac{17a^2\log\left(-\sqrt[3]{b}\,x+\sqrt[3]{a+bx^3}\right)}{27\sqrt[3]{b}} + \frac{17a^2\log\left(b^{2/3}x^2+\sqrt[3]{b}\,x\sqrt[3]{a+bx^3}+(a+bx^3)^{2/3}\right)}{54\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^3)^2/(a + b*x^3)^(1/3),x]
```

```
[Out] ((a + b*x^3)^(2/3)*(-16*a*x + 3*b*x^4))/18 + (17*a^2*ArcTan[(Sqrt[3]*b^(1/3
)*x)/(b^(1/3)*x + 2*(a + b*x^3)^(1/3))])/(9*Sqrt[3]*b^(1/3)) - (17*a^2*Log[
-(b^(1/3)*x) + (a + b*x^3)^(1/3)])/(27*b^(1/3)) + (17*a^2*Log[b^(2/3)*x^2 +
 b^(1/3)*x*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)])/(54*b^(1/3))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(-b\,x^3+a)^2}{(b\,x^3+a)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^3+a)^2/(b*x^3+a)^(1/3),x)
```

```
[Out] int((-b*x^3+a)^2/(b*x^3+a)^(1/3),x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 436 vs. $2(94) = 188$.
time = 0.53, size = 436, normalized size = 3.63

$$-\frac{1}{6}\left(\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2(bx^3+a)^{\frac{1}{3}}}{x}\right)}{3b^{\frac{1}{3}}}\right)}{b^{\frac{1}{3}}}-\frac{\log\left(b^{\frac{2}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{(bx^3+a)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{1}{3}}}+\frac{2\log\left(-b^{\frac{1}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}}{x}\right)}{b^{\frac{1}{3}}}\right)a^2-\frac{1}{9}\left(\frac{2\sqrt{3}\,a\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2(bx^3+a)^{\frac{1}{3}}}{x}\right)}{3b^{\frac{1}{3}}}\right)}{b^{\frac{4}{3}}}-\frac{a\log\left(b^{\frac{2}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{(bx^3+a)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{4}{3}}}+\frac{2a\log\left(-b^{\frac{1}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}}{x}\right)}{b^{\frac{4}{3}}}-\frac{6(bx^3+a)^{\frac{2}{3}}a}{\left(b^2-\frac{(bx^3+a)b}{x^3}\right)x^2}\right)ab-\frac{1}{54}\left(\frac{4\sqrt{3}\,a^2\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2(bx^3+a)^{\frac{1}{3}}}{x}\right)}{3b^{\frac{1}{3}}}\right)}{b^{\frac{7}{3}}}-\frac{2a^2\log\left(b^{\frac{2}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{(bx^3+a)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{7}{3}}}+\frac{4a^2\log\left(-b^{\frac{1}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}}{x}\right)}{b^{\frac{7}{3}}}-\frac{3\left(\frac{7(bx^3+a)^{\frac{2}{3}}a^2b}{x^2}-\frac{4(bx^3+a)^{\frac{5}{3}}a^2}{x^5}\right)}{b^4-\frac{2(bx^3+a)b^3}{x^3}+\frac{(bx^3+a)^2b^2}{x^6}}\right)b^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)^2/(b*x^3+a)^(1/3),x, algorithm="maxima")
```

```
[Out] -1/6*(2*sqrt(3)*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1/3
))/b^(1/3) - log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2/3)/
x^2)/b^(1/3) + 2*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(1/3))*a^2 - 1/9*(2*
sqrt(3)*a*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1/3))/b^(
4/3) - a*log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2/3)/x^2)
/b^(4/3) + 2*a*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(4/3) - 6*(b*x^3 + a)^
(2/3)*a/((b^2 - (b*x^3 + a)*b/x^3)*x^2))*a*b - 1/54*(4*sqrt(3)*a^2*arctan(1
/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1/3))/b^(7/3) - 2*a^2*log(b
^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2/3)/x^2)/b^(7/3) + 4*a
^2*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(7/3) - 3*(7*(b*x^3 + a)^(2/3)*a^2
*b/x^2 - 4*(b*x^3 + a)^(5/3)*a^2/x^5)/(b^4 - 2*(b*x^3 + a)*b^3/x^3 + (b*x^3
 + a)^2*b^2/x^6))*b^2
```

Fricas [A]
time = 3.98, size = 399, normalized size = 3.32

$$\left[\frac{51\sqrt{\frac{1}{3}}\,a^2b\sqrt{\frac{(-b)^{\frac{1}{3}}}{b}}\log\left(3bx^3-3(bx^3+a)^{\frac{1}{3}}(-b)^{\frac{2}{3}}x^2-3\sqrt{\frac{1}{3}}\left((-b)^{\frac{1}{3}}bx^3-(bx^3+a)^{\frac{1}{3}}bx^2+2(bx^3+a)^{\frac{2}{3}}(-b)^{\frac{2}{3}}x\right)\sqrt{\frac{(-b)^{\frac{1}{3}}}{b}}+2a\right)-34a^2(-b)^{\frac{2}{3}}\log\left(\frac{(-b)^{\frac{1}{3}}x+(bx^3+a)^{\frac{1}{3}}}{x}\right)+17a^2(-b)^{\frac{2}{3}}\log\left(\frac{(-b)^{\frac{2}{3}}x^2-(bx^3+a)^{\frac{1}{3}}(-b)^{\frac{1}{3}}x+(bx^3+a)^{\frac{2}{3}}}{x^2}\right)+3(3b^2x^4-16abx)(bx^3+a)^{\frac{2}{3}}}{54b},\ -\frac{102\sqrt{\frac{1}{3}}\,a^2b\sqrt{-\frac{(-b)^{\frac{1}{3}}}{b}}\arctan\left(-\frac{\sqrt{\frac{1}{3}}\left((-b)^{\frac{1}{3}}x-2(bx^3+a)^{\frac{1}{3}}\right)\sqrt{-\frac{(-b)^{\frac{1}{3}}}{b}}}{x}\right)+34a^2(-b)^{\frac{2}{3}}\log\left(\frac{(-b)^{\frac{1}{3}}x+(bx^3+a)^{\frac{1}{3}}}{x}\right)-17a^2(-b)^{\frac{2}{3}}\log\left(\frac{(-b)^{\frac{2}{3}}x^2-(bx^3+a)^{\frac{1}{3}}(-b)^{\frac{1}{3}}x+(bx^3+a)^{\frac{2}{3}}}{x^2}\right)-3(3b^2x^4-16abx)(bx^3+a)^{\frac{2}{3}}}{54b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)^2/(b*x^3+a)^(1/3),x, algorithm="fricas")
```

```
[Out] [1/54*(51*sqrt(1/3)*a^2*b*sqrt((-b)^(1/3)/b)*log(3*b*x^3 - 3*(b*x^3 + a)^(1
/3)*(-b)^(2/3)*x^2 - 3*sqrt(1/3)*((-b)^(1/3)*b*x^3 - (b*x^3 + a)^(1/3)*b*x^
2 + 2*(b*x^3 + a)^(2/3)*(-b)^(2/3)*x)*sqrt((-b)^(1/3)/b) + 2*a) - 34*a^2*(-
b)^(2/3)*log(((-b)^(1/3)*x + (b*x^3 + a)^(1/3))/x) + 17*a^2*(-b)^(2/3)*log(
((-b)^(2/3)*x^2 - (b*x^3 + a)^(1/3)*(-b)^(1/3)*x + (b*x^3 + a)^(2/3))/x^2)
+ 3*(3*b^2*x^4 - 16*a*b*x)*(b*x^3 + a)^(2/3))/b, -1/54*(102*sqrt(1/3)*a^2*b
*sqrt(-(-b)^(1/3)/b)*arctan(-sqrt(1/3)*((-b)^(1/3)*x - 2*(b*x^3 + a)^(1/3))
*sqrt(-(-b)^(1/3)/b)/x) + 34*a^2*(-b)^(2/3)*log(((-b)^(1/3)*x + (b*x^3 + a)
^(1/3))/x) - 17*a^2*(-b)^(2/3)*log(((-b)^(2/3)*x^2 - (b*x^3 + a)^(1/3)*(-b)
^(1/3)*x + (b*x^3 + a)^(2/3))/x^2) - 3*(3*b^2*x^4 - 16*a*b*x)*(b*x^3 + a)^(
2/3))/b]
```

Sympy [C] Result contains complex when optimal does not.
time = 3.28, size = 121, normalized size = 1.01

$$\frac{a^{\frac{5}{3}}x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{3}\\\frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)}-\frac{2a^{\frac{2}{3}}bx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{4}{3}\\\frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)}+\frac{b^2x^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{7}{3}\\\frac{10}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt[3]{a}\,\Gamma\left(\frac{10}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**3+a)**2/(b*x**3+a)**(1/3),x)
```

```
[Out] a**(5/3)*x*gamma(1/3)*hyper((1/3, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a)/(
3*gamma(4/3)) - 2*a**(2/3)*b*x**4*gamma(4/3)*hyper((1/3, 4/3), (7/3,), b*x*
*3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + b**2*x**7*gamma(7/3)*hyper((1/3, 7/3
), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(1/3)*gamma(10/3))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)^2/(b*x^3+a)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 - a)^2/(b*x^3 + a)^(1/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a-b\,x^3\right)}^2}{{\left(b\,x^3+a\right)}^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x^3)^2/(a + b*x^3)^(1/3),x)
```

```
[Out] int((a - b*x^3)^2/(a + b*x^3)^(1/3), x)
```

3.42 $\int \frac{(a-bx^3)^2}{(a+bx^3)^{4/3}}\,dx$

Optimal. Leaf size=113

$$\frac{2x(a-bx^3)}{\sqrt[3]{a+bx^3}}+\frac{7}{3}x\left(a+bx^3\right)^{2/3}-\frac{10a\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}\,\sqrt[3]{b}}+\frac{5a\log\left(-\sqrt[3]{b}\,x+\sqrt[3]{a+bx^3}\right)}{3\sqrt[3]{b}}$$

```
[Out] 2*x*(-b*x^3+a)/(b*x^3+a)^(1/3)+7/3*x*(b*x^3+a)^(2/3)+5/3*a*ln(-b^(1/3)*x+(b
*x^3+a)^(1/3))/b^(1/3)-10/9*a*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^
(1/2))/b^(1/3)*3^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {424, 396, 245}

$$-\frac{10a\text{ArcTan}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{3\sqrt{3}\,\sqrt[3]{b}}+\frac{7}{3}x\left(a+bx^3\right)^{2/3}+\frac{2x(a-bx^3)}{\sqrt[3]{a+bx^3}}+\frac{5a\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}\,x\right)}{3\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^3)^2/(a + b*x^3)^(4/3),x]
```

```
[Out] (2*x*(a - b*x^3))/(a + b*x^3)^(1/3) + (7*x*(a + b*x^3)^(2/3))/3 - (10*a*Arc
Tan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(3*Sqrt[3]*b^(1/3)) + (
5*a*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)])/(3*b^(1/3))
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rubi steps

$$\begin{aligned}\int \frac{(a-bx^3)^2}{(a+bx^3)^{4/3}}\,dx &= \frac{2x(a-bx^3)}{\sqrt[3]{a+bx^3}}+\frac{\int \frac{-a^2b+7ab^2x^3}{\sqrt[3]{a+bx^3}}\,dx}{ab}\\ &= \frac{2x(a-bx^3)}{\sqrt[3]{a+bx^3}}+\frac{7}{3}x\left(a+bx^3\right)^{2/3}-\frac{1}{3}(10a)\int \frac{1}{\sqrt[3]{a+bx^3}}\,dx\\ &= \frac{2x(a-bx^3)}{\sqrt[3]{a+bx^3}}+\frac{7}{3}x\left(a+bx^3\right)^{2/3}-\frac{10a\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}\,\sqrt[3]{b}}+\frac{5a\log\left(-\sqrt[3]{b}\,x+\sqrt[3]{a}\right.}{3\sqrt[3]{b}}\end{aligned}$$

Mathematica [A]
time = 0.40, size = 154, normalized size = 1.36

$$\frac{1}{9}\left(\frac{3(13ax+bx^4)}{\sqrt[3]{a+bx^3}}-\frac{10\sqrt{3}\,a\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[3]{b}\,x}{\sqrt[3]{b}\,x+2\sqrt[3]{a+bx^3}}\right)}{\sqrt[3]{b}}+\frac{10a\log\left(-\sqrt[3]{b}\,x+\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{b}}-\frac{5a\log\left(b^{2/3}x^2+\sqrt[3]{b}\,x\sqrt[3]{a+bx^3}+(a+bx^3)^{2/3}\right)}{\sqrt[3]{b}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^3)^2/(a + b*x^3)^(4/3),x]
```

```
[Out] ((3*(13*a*x + b*x^4))/(a + b*x^3)^(1/3) - (10*Sqrt[3]*a*ArcTan[(Sqrt[3]*b^(
1/3)*x)/(b^(1/3)*x + 2*(a + b*x^3)^(1/3))])/b^(1/3) + (10*a*Log[-(b^(1/3)*x
) + (a + b*x^3)^(1/3)])/b^(1/3) - (5*a*Log[b^(2/3)*x^2 + b^(1/3)*x*(a + b*x
^3)^(1/3) + (a + b*x^3)^(2/3)])/b^(1/3))/9
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{\left(-b\,x^3+a\right)^2}{\left(b\,x^3+a\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^3+a)^2/(b*x^3+a)^(4/3),x)
[Out] int((-b*x^3+a)^2/(b*x^3+a)^(4/3),x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 296 vs. $2(89) = 178$.
time = 0.54, size = 296, normalized size = 2.62

$$\frac{1}{9}b^2\left(\frac{4\sqrt{3}\,a\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2(bx^3+a)^{\frac{1}{3}}}{x}\right)}{3b^{\frac{1}{3}}}\right)}{b^{\frac{7}{3}}}+\frac{3\left(3ab-\frac{4(bx^3+a)a}{x^3}\right)}{\frac{(bx^3+a)^{\frac{1}{3}}b^3}{x}-\frac{(bx^3+a)^{\frac{4}{3}}b^2}{x^4}}-\frac{2a\log\left(b^{\frac{2}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{(bx^3+a)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{7}{3}}}+\frac{4a\log\left(-b^{\frac{1}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}}{x}\right)}{b^{\frac{7}{3}}}\right)+\frac{1}{3}ab\left(\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2(bx^3+a)^{\frac{1}{3}}}{x}\right)}{3b^{\frac{1}{3}}}\right)}{b^{\frac{4}{3}}}+\frac{6x}{(bx^3+a)^{\frac{1}{3}}b}-\frac{\log\left(b^{\frac{2}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{(bx^3+a)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{4}{3}}}+\frac{2\log\left(-b^{\frac{1}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}}{x}\right)}{b^{\frac{4}{3}}}\right)+\frac{ax}{(bx^3+a)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)^2/(b*x^3+a)^(4/3),x, algorithm="maxima")
[Out] 1/9*b^2*(4*sqrt(3)*a*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b
^(1/3))/b^(7/3) + 3*(3*a*b - 4*(b*x^3 + a)*a/x^3)/((b*x^3 + a)^(1/3)*b^3/x
- (b*x^3 + a)^(4/3)*b^2/x^4) - 2*a*log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/
x + (b*x^3 + a)^(2/3)/x^2)/b^(7/3) + 4*a*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x
)/b^(7/3)) + 1/3*a*b*(2*sqrt(3)*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)
^(1/3)/x)/b^(1/3))/b^(4/3) + 6*x/((b*x^3 + a)^(1/3)*b) - log(b^(2/3) + (b*x
^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2/3)/x^2)/b^(4/3) + 2*log(-b^(1/3) +
 (b*x^3 + a)^(1/3)/x)/b^(4/3)) + a*x/(b*x^3 + a)^(1/3)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 179 vs. $2(89) = 178$.
time = 3.92, size = 412, normalized size = 3.65

$$\left[\frac{15\sqrt{\frac{1}{3}}\left(ab^2x^3+a^2b\right)\sqrt{-\frac{1}{b^{\frac{2}{3}}}}\log\left(3bx^3-3(bx^3+a)^{\frac{1}{3}}b^{\frac{2}{3}}x^2-3\sqrt{\frac{1}{3}}\left(b^{\frac{4}{3}}x^3+(bx^3+a)^{\frac{1}{3}}bx^2-2(bx^3+a)^{\frac{2}{3}}b^{\frac{2}{3}}x\right)\sqrt{-\frac{1}{b^{\frac{2}{3}}}}+2a\right)+10\left(abx^3+a^2\right)b^{\frac{2}{3}}\log\left(-\frac{b^{\frac{1}{3}}x-(bx^3+a)^{\frac{1}{3}}}{x}\right)-5\left(abx^3+a^2\right)b^{\frac{2}{3}}\log\left(\frac{b^{\frac{2}{3}}x^2+(bx^3+a)^{\frac{1}{3}}b^{\frac{1}{3}}x+(bx^3+a)^{\frac{2}{3}}}{x^2}\right)+3\left(b^2x^4+13abx\right)(bx^3+a)^{\frac{2}{3}}}{9\left(b^2x^3+ab\right)},\ \frac{10\left(abx^3+a^2\right)b^{\frac{2}{3}}\log\left(-\frac{b^{\frac{1}{3}}x-(bx^3+a)^{\frac{1}{3}}}{x}\right)-5\left(abx^3+a^2\right)b^{\frac{2}{3}}\log\left(\frac{b^{\frac{2}{3}}x^2+(bx^3+a)^{\frac{1}{3}}b^{\frac{1}{3}}x+(bx^3+a)^{\frac{2}{3}}}{x^2}\right)+\frac{30\sqrt{\frac{1}{3}}\left(ab^2x^3+a^2b\right)\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(b^{\frac{1}{3}}x+2(bx^3+a)^{\frac{1}{3}}\right)}{b^{\frac{1}{3}}x}\right)}{b^{\frac{1}{3}}}+3\left(b^2x^4+13abx\right)(bx^3+a)^{\frac{2}{3}}}{9\left(b^2x^3+ab\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)^2/(b*x^3+a)^(4/3),x, algorithm="fricas")
[Out] [1/9*(15*sqrt(1/3)*(a*b^2*x^3 + a^2*b)*sqrt(-1/b^(2/3))*log(3*b*x^3 - 3*(b*
x^3 + a)^(1/3)*b^(2/3)*x^2 - 3*sqrt(1/3)*(b^(4/3)*x^3 + (b*x^3 + a)^(1/3)*b
*x^2 - 2*(b*x^3 + a)^(2/3)*b^(2/3)*x)*sqrt(-1/b^(2/3)) + 2*a) + 10*(a*b*x^3
 + a^2)*b^(2/3)*log(-(b^(1/3)*x - (b*x^3 + a)^(1/3))/x) - 5*(a*b*x^3 + a^2)
*b^(2/3)*log((b^(2/3)*x^2 + (b*x^3 + a)^(1/3)*b^(1/3)*x + (b*x^3 + a)^(2/3)
)/x^2) + 3*(b^2*x^4 + 13*a*b*x)*(b*x^3 + a)^(2/3))/(b^2*x^3 + a*b), 1/9*(10
*(a*b*x^3 + a^2)*b^(2/3)*log(-(b^(1/3)*x - (b*x^3 + a)^(1/3))/x) - 5*(a*b*x
^3 + a^2)*b^(2/3)*log((b^(2/3)*x^2 + (b*x^3 + a)^(1/3)*b^(1/3)*x + (b*x^3 +
 a)^(2/3))/x^2) + 30*sqrt(1/3)*(a*b^2*x^3 + a^2*b)*arctan(sqrt(1/3)*(b^(1/3
)*x + 2*(b*x^3 + a)^(1/3))/(b^(1/3)*x))/b^(1/3) + 3*(b^2*x^4 + 13*a*b*x)*(b
*x^3 + a)^(2/3))/(b^2*x^3 + a*b)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-a+bx^3)^2}{(a+bx^3)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**3+a)**2/(b*x**3+a)**(4/3),x)
[Out] Integral((-a + b*x**3)**2/(a + b*x**3)**(4/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)^2/(b*x^3+a)^(4/3),x, algorithm="giac")
[Out] integrate((b*x^3 - a)^2/(b*x^3 + a)^(4/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a-b\,x^3)^2}{(b\,x^3+a)^{4/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x^3)^2/(a + b*x^3)^(4/3),x)
[Out] int((a - b*x^3)^2/(a + b*x^3)^(4/3), x)
```

3.43 $\int \frac{(a-bx^3)^2}{(a+bx^3)^{7/3}} \, dx$

Optimal. Leaf size=110

$$\frac{x(a-bx^3)}{2(a+bx^3)^{4/3}} - \frac{x}{2\sqrt[3]{a+bx^3}} + \frac{\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,\sqrt[3]{b}} - \frac{\log\left(-\sqrt[3]{b}\,x+\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{b}}$$

```
[Out] 1/2*x*(-b*x^3+a)/(b*x^3+a)^(4/3)-1/2*x/(b*x^3+a)^(1/3)-1/2*ln(-b^(1/3)*x+(b
*x^3+a)^(1/3))/b^(1/3)+1/3*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/
2))/b^(1/3)*3^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {424, 393, 245}

$$\frac{\text{ArcTan}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,\sqrt[3]{b}} - \frac{x}{2\sqrt[3]{a+bx^3}} + \frac{x(a-bx^3)}{2(a+bx^3)^{4/3}} - \frac{\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}\,x\right)}{2\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^3)^2/(a + b*x^3)^(7/3),x]
```

```
[Out] (x*(a - b*x^3))/(2*(a + b*x^3)^(4/3)) - x/(2*(a + b*x^3)^(1/3)) + ArcTan[(1
 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*b^(1/3)) - Log[-(b^(1
/3)*x) + (a + b*x^3)^(1/3)]/(2*b^(1/3))
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rubi steps

$$\begin{aligned}\int \frac{(a-bx^3)^2}{(a+bx^3)^{7/3}}\,dx &= \frac{x(a-bx^3)}{2\,(a+bx^3)^{4/3}}+\frac{\int \frac{2a^2b+4ab^2x^3}{(a+bx^3)^{4/3}}\,dx}{4ab}\\ &= \frac{x(a-bx^3)}{2\,(a+bx^3)^{4/3}}-\frac{x}{2\sqrt[3]{a+bx^3}}+\int \frac{1}{\sqrt[3]{a+bx^3}}\,dx\\ &= \frac{x(a-bx^3)}{2\,(a+bx^3)^{4/3}}-\frac{x}{2\sqrt[3]{a+bx^3}}+\frac{\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,\sqrt[3]{b}}-\frac{\log\left(-\sqrt[3]{b}\,x+\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{b}}\end{aligned}$$

Mathematica [A]
time = 0.31, size = 137, normalized size = 1.25

$$\frac{-\frac{6b^{4/3}x^4}{(a+bx^3)^{4/3}}+2\sqrt{3}\,\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[3]{b}\,x}{\sqrt[3]{b}\,x+2\sqrt[3]{a+bx^3}}\right)-2\log\left(-\sqrt[3]{b}\,x+\sqrt[3]{a+bx^3}\right)+\log\left(b^{2/3}x^2+\sqrt[3]{b}\,x\sqrt[3]{a+bx^3}+(a+bx^3)^{2/3}\right)}{6\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^3)^2/(a + b*x^3)^(7/3),x]
```

```
[Out] ((-6*b^(4/3)*x^4)/(a + b*x^3)^(4/3) + 2*Sqrt[3]*ArcTan[(Sqrt[3]*b^(1/3)*x)/
(b^(1/3)*x + 2*(a + b*x^3)^(1/3))] - 2*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)
] + Log[b^(2/3)*x^2 + b^(1/3)*x*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)])/(6*
b^(1/3))
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{(-b\,x^3+a)^2}{(b\,x^3+a)^{\frac{7}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^3+a)^2/(b*x^3+a)^(7/3),x)
```

```
[Out] int((-b*x^3+a)^2/(b*x^3+a)^(7/3),x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 180 vs. $2(87) = 174$.
time = 0.51, size = 180, normalized size = 1.64

$$-\frac{\left(b-\frac{4\,(bx^3+a)}{x^3}\right)x^4}{4\,(bx^3+a)^{\frac{4}{3}}}-\frac{bx^4}{2\,(bx^3+a)^{\frac{4}{3}}}-\frac{1}{12}\left(\frac{3\left(b+\frac{4\,(bx^3+a)}{x^3}\right)x^4}{(bx^3+a)^{\frac{4}{3}}b^2}+\frac{4\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\,(bx^3+a)^{\frac{1}{3}}}{x}\right)}{3\,b^{\frac{1}{3}}}\right)}{b^{\frac{7}{3}}}-\frac{2\,\log\left(b^{\frac{2}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{(bx^3+a)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{7}{3}}}+\frac{4\,\log\left(-b^{\frac{1}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}}{x}\right)}{b^{\frac{7}{3}}}\right)b^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)^2/(b*x^3+a)^(7/3),x, algorithm="maxima")
```

```
[Out] -1/4*(b - 4*(b*x^3 + a)/x^3)*x^4/(b*x^3 + a)^(4/3) - 1/2*b*x^4/(b*x^3 + a)^
(4/3) - 1/12*(3*(b + 4*(b*x^3 + a)/x^3)*x^4/((b*x^3 + a)^(4/3)*b^2) + 4*sqr
t(3)*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1/3))/b^(7/3)
- 2*log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2/3)/x^2)/b^(7
/3) + 4*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(7/3))*b^2
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 236 vs. $2(87) = 174$.
time = 2.56, size = 521, normalized size = 4.74

$$\left[-\frac{6\,(bx^3+a)^{\frac{2}{3}}b^2x^4-3\sqrt{\frac{1}{3}}\,(b^3x^6+2\,ab^2x^3+a^2b)\sqrt{\frac{(-b)^{\frac{1}{3}}}{b}}\log\left(3\,bx^3-3\,(bx^3+a)^{\frac{1}{3}}(-b)^{\frac{2}{3}}x^2-3\sqrt{\frac{1}{3}}\left((-b)^{\frac{1}{3}}bx^3-(bx^3+a)^{\frac{1}{3}}bx^2+2\,(bx^3+a)^{\frac{2}{3}}(-b)^{\frac{2}{3}}x\right)\sqrt{\frac{(-b)^{\frac{1}{3}}}{b}}+2\,a\right)+2\,(b^2x^6+2\,abx^3+a^2)(-b)^{\frac{2}{3}}\log\left(\frac{(-b)^{\frac{1}{3}}x+(bx^3+a)^{\frac{1}{3}}}{x}\right)-(b^2x^6+2\,abx^3+a^2)(-b)^{\frac{2}{3}}\log\left(\frac{(-b)^{\frac{2}{3}}x^2-(bx^3+a)^{\frac{1}{3}}(-b)^{\frac{1}{3}}x+(bx^3+a)^{\frac{2}{3}}}{x^2}\right)}{6\,(b^3x^6+2\,ab^2x^3+a^2b)},-\frac{6\,(bx^3+a)^{\frac{2}{3}}b^2x^4+6\sqrt{\frac{1}{3}}\,(b^3x^6+2\,ab^2x^3+a^2b)\sqrt{-\frac{(-b)^{\frac{1}{3}}}{b}}\arctan\left(-\frac{\sqrt{\frac{1}{3}}\left((-b)^{\frac{1}{3}}x-2\,(bx^3+a)^{\frac{1}{3}}\right)\sqrt{-\frac{(-b)^{\frac{1}{3}}}{b}}}{x}\right)+2\,(b^2x^6+2\,abx^3+a^2)(-b)^{\frac{2}{3}}\log\left(\frac{(-b)^{\frac{1}{3}}x+(bx^3+a)^{\frac{1}{3}}}{x}\right)-(b^2x^6+2\,abx^3+a^2)(-b)^{\frac{2}{3}}\log\left(\frac{(-b)^{\frac{2}{3}}x^2-(bx^3+a)^{\frac{1}{3}}(-b)^{\frac{1}{3}}x+(bx^3+a)^{\frac{2}{3}}}{x^2}\right)}{6\,(b^3x^6+2\,ab^2x^3+a^2b)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)^2/(b*x^3+a)^(7/3),x, algorithm="fricas")
```

```
[Out] [-1/6*(6*(b*x^3 + a)^(2/3)*b^2*x^4 - 3*sqrt(1/3)*(b^3*x^6 + 2*a*b^2*x^3 + a
^2*b)*sqrt((-b)^(1/3)/b)*log(3*b*x^3 - 3*(b*x^3 + a)^(1/3)*(-b)^(2/3)*x^2 -
 3*sqrt(1/3)*((-b)^(1/3)*b*x^3 - (b*x^3 + a)^(1/3)*b*x^2 + 2*(b*x^3 + a)^(2
/3)*(-b)^(2/3)*x)*sqrt((-b)^(1/3)/b) + 2*a) + 2*(b^2*x^6 + 2*a*b*x^3 + a^2)
*(-b)^(2/3)*log(((-b)^(1/3)*x + (b*x^3 + a)^(1/3))/x) - (b^2*x^6 + 2*a*b*x^
3 + a^2)*(-b)^(2/3)*log(((-b)^(2/3)*x^2 - (b*x^3 + a)^(1/3)*(-b)^(1/3)*x +
(b*x^3 + a)^(2/3))/x^2))/(b^3*x^6 + 2*a*b^2*x^3 + a^2*b), -1/6*(6*(b*x^3 +
a)^(2/3)*b^2*x^4 + 6*sqrt(1/3)*(b^3*x^6 + 2*a*b^2*x^3 + a^2*b)*sqrt(-(-b)^(
1/3)/b)*arctan(-sqrt(1/3)*((-b)^(1/3)*x - 2*(b*x^3 + a)^(1/3))*sqrt(-(-b)^(
1/3)/b)/x) + 2*(b^2*x^6 + 2*a*b*x^3 + a^2)*(-b)^(2/3)*log(((-b)^(1/3)*x + (
b*x^3 + a)^(1/3))/x) - (b^2*x^6 + 2*a*b*x^3 + a^2)*(-b)^(2/3)*log(((-b)^(2/
3)*x^2 - (b*x^3 + a)^(1/3)*(-b)^(1/3)*x + (b*x^3 + a)^(2/3))/x^2))/(b^3*x^6
 + 2*a*b^2*x^3 + a^2*b)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-a+bx^3)^2}{(a+bx^3)^{\frac{7}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**3+a)**2/(b*x**3+a)**(7/3),x)
[Out] Integral((-a + b*x**3)**2/(a + b*x**3)**(7/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)^2/(b*x^3+a)^(7/3),x, algorithm="giac")
[Out] integrate((b*x^3 - a)^2/(b*x^3 + a)^(7/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a-b\,x^3)^2}{(b\,x^3+a)^{7/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x^3)^2/(a + b*x^3)^(7/3),x)
[Out] int((a - b*x^3)^2/(a + b*x^3)^(7/3), x)
```

3.44 $\int \frac{(a-bx^3)^2}{(a+bx^3)^{10/3}}\,dx$

Optimal. Leaf size=76

$$\frac{x(a-bx^3)^2}{7a(a+bx^3)^{7/3}}+\frac{3x(a-bx^3)}{14a(a+bx^3)^{4/3}}+\frac{9x}{14a\sqrt[3]{a+bx^3}}$$

```
[Out] 1/7*x*(-b*x^3+a)^2/a/(b*x^3+a)^(7/3)+3/14*x*(-b*x^3+a)/a/(b*x^3+a)^(4/3)+9/
14*x/a/(b*x^3+a)^(1/3)
```

Rubi [A]
time = 0.02, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {386, 197}

$$\frac{x(a-bx^3)^2}{7a(a+bx^3)^{7/3}}+\frac{3x(a-bx^3)}{14a(a+bx^3)^{4/3}}+\frac{9x}{14a\sqrt[3]{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^3)^2/(a + b*x^3)^(10/3),x]
```

```
[Out] (x*(a - b*x^3)^2)/(7*a*(a + b*x^3)^(7/3)) + (3*x*(a - b*x^3))/(14*a*(a + b*
x^3)^(4/3)) + (9*x)/(14*a*(a + b*x^3)^(1/3))
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 386

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] - Dist[
c*(q/(a*(p + 1))), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1,
0] && GtQ[q, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(a-bx^3)^2}{(a+bx^3)^{10/3}}\,dx &= \frac{x(a-bx^3)^2}{7a\,(a+bx^3)^{7/3}}+\frac{6}{7}\int \frac{a-bx^3}{(a+bx^3)^{7/3}}\,dx\\ &= \frac{x(a-bx^3)^2}{7a\,(a+bx^3)^{7/3}}+\frac{3x(a-bx^3)}{14a\,(a+bx^3)^{4/3}}+\frac{9}{14}\int \frac{1}{(a+bx^3)^{4/3}}\,dx\\ &= \frac{x(a-bx^3)^2}{7a\,(a+bx^3)^{7/3}}+\frac{3x(a-bx^3)}{14a\,(a+bx^3)^{4/3}}+\frac{9x}{14a\sqrt[3]{a+bx^3}}\end{aligned}$$

Mathematica [A]
time = 0.25, size = 40, normalized size = 0.53

$$\frac{7a^2x+7abx^4+4b^2x^7}{7a\,(a+bx^3)^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^3)^2/(a + b*x^3)^(10/3),x]
[Out] (7*a^2*x + 7*a*b*x^4 + 4*b^2*x^7)/(7*a*(a + b*x^3)^(7/3))
```

Maple [A]
time = 0.30, size = 37, normalized size = 0.49

method	result	size
gosper	$\frac{x\left(4b^2x^6+7ab\,x^3+7a^2\right)}{7(b\,x^3+a)^{\frac{7}{3}}a}$	37
trager	$\frac{x\left(4b^2x^6+7ab\,x^3+7a^2\right)}{7(b\,x^3+a)^{\frac{7}{3}}a}$	37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^3+a)^2/(b*x^3+a)^(10/3),x,method=_RETURNVERBOSE)
[Out] 1/7*x*(4*b^2*x^6+7*a*b*x^3+7*a^2)/(b*x^3+a)^(7/3)/a
```

Maxima [A]
time = 0.29, size = 105, normalized size = 1.38

$$\frac{\left(4\,b-\frac{7\,(bx^3+a)}{x^3}\right)bx^7}{14\,(bx^3+a)^{\frac{7}{3}}a}+\frac{b^2x^7}{7\,(bx^3+a)^{\frac{7}{3}}a}+\frac{\left(2\,b^2-\frac{7\,(bx^3+a)b}{x^3}+\frac{14\,(bx^3+a)^2}{x^6}\right)x^7}{14\,(bx^3+a)^{\frac{7}{3}}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)^2/(b*x^3+a)^(10/3),x, algorithm="maxima")
```

```
[Out] 1/14*(4*b - 7*(b*x^3 + a)/x^3)*b*x^7/((b*x^3 + a)^(7/3)*a) + 1/7*b^2*x^7/((
b*x^3 + a)^(7/3)*a) + 1/14*(2*b^2 - 7*(b*x^3 + a)*b/x^3 + 14*(b*x^3 + a)^2/
x^6)*x^7/((b*x^3 + a)^(7/3)*a)
```

Fricas [A]
time = 2.84, size = 67, normalized size = 0.88

$$\frac{(4\,b^2x^7+7\,abx^4+7\,a^2x)(bx^3+a)^{\frac{2}{3}}}{7\,(ab^3x^9+3\,a^2b^2x^6+3\,a^3bx^3+a^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)^2/(b*x^3+a)^(10/3),x, algorithm="fricas")
[Out] 1/7*(4*b^2*x^7 + 7*a*b*x^4 + 7*a^2*x)*(b*x^3 + a)^(2/3)/(a*b^3*x^9 + 3*a^2*
b^2*x^6 + 3*a^3*b*x^3 + a^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(-a+bx^3)^2}{(a+bx^3)^{\frac{10}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**3+a)**2/(b*x**3+a)**(10/3),x)
[Out] Integral((-a + b*x**3)**2/(a + b*x**3)**(10/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)^2/(b*x^3+a)^(10/3),x, algorithm="giac")
[Out] integrate((b*x^3 - a)^2/(b*x^3 + a)^(10/3), x)
```

Mupad [B]
time = 1.43, size = 44, normalized size = 0.58

$$\frac{4\,x\,(b\,x^3+a)^2+4\,a^2\,x-a\,x\,(b\,x^3+a)}{7\,a\,(b\,x^3+a)^{7/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x^3)^2/(a + b*x^3)^(10/3),x)
[Out] (4*x*(a + b*x^3)^2 + 4*a^2*x - a*x*(a + b*x^3))/(7*a*(a + b*x^3)^(7/3))
```

3.45 $\int \frac{(a-bx^3)^2}{(a+bx^3)^{13/3}}\,dx$

Optimal. Leaf size=105

$$\frac{x(a-bx^3)^3}{20a^2(a+bx^3)^{10/3}}+\frac{19x(a-bx^3)^2}{140a^2(a+bx^3)^{7/3}}+\frac{57x(a-bx^3)}{280a^2(a+bx^3)^{4/3}}+\frac{171x}{280a^2\sqrt[3]{a+bx^3}}$$

```
[Out] 1/20*x*(-b*x^3+a)^3/a^2/(b*x^3+a)^(10/3)+19/140*x*(-b*x^3+a)^2/a^2/(b*x^3+a
)^(7/3)+57/280*x*(-b*x^3+a)/a^2/(b*x^3+a)^(4/3)+171/280*x/a^2/(b*x^3+a)^(1/
3)
```

Rubi [A]
time = 0.02, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {390, 386, 197}

$$\frac{x(a-bx^3)^3}{20a^2(a+bx^3)^{10/3}}+\frac{19x(a-bx^3)^2}{140a^2(a+bx^3)^{7/3}}+\frac{57x(a-bx^3)}{280a^2(a+bx^3)^{4/3}}+\frac{171x}{280a^2\sqrt[3]{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^3)^2/(a + b*x^3)^(13/3),x]
```

```
[Out] (x*(a - b*x^3)^3)/(20*a^2*(a + b*x^3)^(10/3)) + (19*x*(a - b*x^3)^2)/(140*a
^2*(a + b*x^3)^(7/3)) + (57*x*(a - b*x^3))/(280*a^2*(a + b*x^3)^(4/3)) + (1
71*x)/(280*a^2*(a + b*x^3)^(1/3))
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 386

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] - Dist[
c*(q/(a*(p + 1))), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1,
0] && GtQ[q, 0] && NeQ[p, -1]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)),
 Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q},
```

```
 x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !L
tQ[q, -1]) && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(a-bx^3)^2}{(a+bx^3)^{13/3}}\,dx &= \frac{x(a-bx^3)^3}{20a^2(a+bx^3)^{10/3}}+\frac{19\int \frac{(a-bx^3)^2}{(a+bx^3)^{10/3}}\,dx}{20a}\\ &= \frac{x(a-bx^3)^3}{20a^2(a+bx^3)^{10/3}}+\frac{19x(a-bx^3)^2}{140a^2(a+bx^3)^{7/3}}+\frac{57\int \frac{a-bx^3}{(a+bx^3)^{7/3}}\,dx}{70a}\\ &= \frac{x(a-bx^3)^3}{20a^2(a+bx^3)^{10/3}}+\frac{19x(a-bx^3)^2}{140a^2(a+bx^3)^{7/3}}+\frac{57x(a-bx^3)}{280a^2(a+bx^3)^{4/3}}+\frac{171\int \frac{1}{(a+bx^3)^{4/3}}\,dx}{280a}\\ &= \frac{x(a-bx^3)^3}{20a^2(a+bx^3)^{10/3}}+\frac{19x(a-bx^3)^2}{140a^2(a+bx^3)^{7/3}}+\frac{57x(a-bx^3)}{280a^2(a+bx^3)^{4/3}}+\frac{171x}{280a^2\sqrt[3]{a+bx^3}}\end{aligned}$$

Mathematica [**A**]
time = 0.38, size = 51, normalized size = 0.49

$$\frac{140a^3x+245a^2bx^4+230ab^2x^7+69b^3x^{10}}{140a^2(a+bx^3)^{10/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^3)^2/(a + b*x^3)^(13/3),x]
```

```
[Out] (140*a^3*x + 245*a^2*b*x^4 + 230*a*b^2*x^7 + 69*b^3*x^10)/(140*a^2*(a + b*x
^3)^(10/3))
```

Maple [**A**]
time = 0.29, size = 48, normalized size = 0.46

method	result	size
gosper	$\frac{x\left(69b^3x^9+230a\,b^2x^6+245a^2b\,x^3+140a^3\right)}{140\left(b\,x^3+a\right)^{\frac{10}{3}}a^2}$	48
trager	$\frac{x\left(69b^3x^9+230a\,b^2x^6+245a^2b\,x^3+140a^3\right)}{140\left(b\,x^3+a\right)^{\frac{10}{3}}a^2}$	48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^3+a)^2/(b*x^3+a)^(13/3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/140*x*(69*b^3*x^9+230*a*b^2*x^6+245*a^2*b*x^3+140*a^3)/(b*x^3+a)^(10/3)/a
^2
```

Maxima [A]
time = 0.31, size = 155, normalized size = 1.48

$$-\frac{\left(7\,b-\frac{10\,(bx^3+a)}{x^3}\right)b^2x^{10}}{70\,(bx^3+a)^{\frac{10}{3}}a^2}-\frac{\left(14\,b^2-\frac{40\,(bx^3+a)b}{x^3}+\frac{35\,(bx^3+a)^2}{x^6}\right)bx^{10}}{70\,(bx^3+a)^{\frac{10}{3}}a^2}-\frac{\left(14\,b^3-\frac{60\,(bx^3+a)b^2}{x^3}+\frac{105\,(bx^3+a)^2b}{x^6}-\frac{140\,(bx^3+a)^3}{x^9}\right)x^{10}}{140\,(bx^3+a)^{\frac{10}{3}}a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)^2/(b*x^3+a)^(13/3),x, algorithm="maxima")
```

```
[Out] -1/70*(7*b - 10*(b*x^3 + a)/x^3)*b^2*x^10/((b*x^3 + a)^(10/3)*a^2) - 1/70*(
14*b^2 - 40*(b*x^3 + a)*b/x^3 + 35*(b*x^3 + a)^2/x^6)*b*x^10/((b*x^3 + a)^(
10/3)*a^2) - 1/140*(14*b^3 - 60*(b*x^3 + a)*b^2/x^3 + 105*(b*x^3 + a)^2*b/x
^6 - 140*(b*x^3 + a)^3/x^9)*x^10/((b*x^3 + a)^(10/3)*a^2)
```

Fricas [A]
time = 4.21, size = 91, normalized size = 0.87

$$\frac{(69\,b^3x^{10}+230\,ab^2x^7+245\,a^2bx^4+140\,a^3x)(bx^3+a)^{\frac{2}{3}}}{140\,(a^2b^4x^{12}+4\,a^3b^3x^9+6\,a^4b^2x^6+4\,a^5bx^3+a^6)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)^2/(b*x^3+a)^(13/3),x, algorithm="fricas")
```

```
[Out] 1/140*(69*b^3*x^10 + 230*a*b^2*x^7 + 245*a^2*b*x^4 + 140*a^3*x)*(b*x^3 + a)
^(2/3)/(a^2*b^4*x^12 + 4*a^3*b^3*x^9 + 6*a^4*b^2*x^6 + 4*a^5*b*x^3 + a^6)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**3+a)**2/(b*x**3+a)**(13/3),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)^2/(b*x^3+a)^(13/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 - a)^2/(b*x^3 + a)^(13/3), x)
```

Mupad [B]
time = 1.39, size = 56, normalized size = 0.53

$$\frac{69\,x}{140\,a^2\,{\left(b\,x^3+a\right)}^{1/3}}-\frac{2\,x}{35\,{\left(b\,x^3+a\right)}^{7/3}}+\frac{23\,x}{140\,a\,{\left(b\,x^3+a\right)}^{4/3}}+\frac{2\,a\,x}{5\,{\left(b\,x^3+a\right)}^{10/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x^3)^2/(a + b*x^3)^(13/3),x)
[Out] (69*x)/(140*a^2*(a + b*x^3)^(1/3)) - (2*x)/(35*(a + b*x^3)^(7/3)) + (23*x)/
(140*a*(a + b*x^3)^(4/3)) + (2*a*x)/(5*(a + b*x^3)^(10/3))
```

3.46 $\int \frac{(a-bx^3)^2}{(a+bx^3)^{16/3}}\,dx$

Optimal. Leaf size=98

$$\frac{2x(a-bx^3)}{13\,(a+bx^3)^{13/3}}+\frac{8x}{65\,(a+bx^3)^{10/3}}+\frac{47x}{455a\,(a+bx^3)^{7/3}}+\frac{141x}{910a^2\,(a+bx^3)^{4/3}}+\frac{423x}{910a^3\sqrt[3]{a+bx^3}}$$

```
[Out] 2/13*x*(-b*x^3+a)/(b*x^3+a)^(13/3)+8/65*x/(b*x^3+a)^(10/3)+47/455*x/a/(b*x^
3+a)^(7/3)+141/910*x/a^2/(b*x^3+a)^(4/3)+423/910*x/a^3/(b*x^3+a)^(1/3)
```

Rubi [A]
time = 0.02, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {424, 393, 198, 197}

$$\frac{423x}{910a^3\sqrt[3]{a+bx^3}}+\frac{141x}{910a^2\,(a+bx^3)^{4/3}}+\frac{47x}{455a\,(a+bx^3)^{7/3}}+\frac{8x}{65\,(a+bx^3)^{10/3}}+\frac{2x(a-bx^3)}{13\,(a+bx^3)^{13/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^3)^2/(a + b*x^3)^(16/3),x]
```

```
[Out] (2*x*(a - b*x^3))/(13*(a + b*x^3)^(13/3)) + (8*x)/(65*(a + b*x^3)^(10/3)) +
 (47*x)/(455*a*(a + b*x^3)^(7/3)) + (141*x)/(910*a^2*(a + b*x^3)^(4/3)) + (
423*x)/(910*a^3*(a + b*x^3)^(1/3))
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a-bx^3)^2}{(a+bx^3)^{16/3}}\,dx &= \frac{2x(a-bx^3)}{13(a+bx^3)^{13/3}} + \frac{\int \frac{11a^2b-5ab^2x^3}{(a+bx^3)^{13/3}}\,dx}{13ab} \\
&= \frac{2x(a-bx^3)}{13(a+bx^3)^{13/3}} + \frac{8x}{65(a+bx^3)^{10/3}} + \frac{47}{65}\int \frac{1}{(a+bx^3)^{10/3}}\,dx \\
&= \frac{2x(a-bx^3)}{13(a+bx^3)^{13/3}} + \frac{8x}{65(a+bx^3)^{10/3}} + \frac{47x}{455a(a+bx^3)^{7/3}} + \frac{282\int \frac{1}{(a+bx^3)^{7/3}}\,dx}{455a} \\
&= \frac{2x(a-bx^3)}{13(a+bx^3)^{13/3}} + \frac{8x}{65(a+bx^3)^{10/3}} + \frac{47x}{455a(a+bx^3)^{7/3}} + \frac{141x}{910a^2(a+bx^3)^{4/3}} + \frac{423\int \cdots}{9\cdots} \\
&= \frac{2x(a-bx^3)}{13(a+bx^3)^{13/3}} + \frac{8x}{65(a+bx^3)^{10/3}} + \frac{47x}{455a(a+bx^3)^{7/3}} + \frac{141x}{910a^2(a+bx^3)^{4/3}} + \frac{4\cdots}{910a^3\sqrt{\cdots}}
\end{aligned}
$$

Mathematica [A]
time = 0.50, size = 62, normalized size = 0.63

$$\frac{910a^4x+2275a^3bx^4+3055a^2b^2x^7+1833ab^3x^{10}+423b^4x^{13}}{910a^3(a+bx^3)^{13/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^3)^2/(a + b*x^3)^(16/3),x]
```

```
[Out] (910*a^4*x + 2275*a^3*b*x^4 + 3055*a^2*b^2*x^7 + 1833*a*b^3*x^10 + 423*b^4*
x^13)/(910*a^3*(a + b*x^3)^(13/3))
```

Maple [A]
time = 0.27, size = 59, normalized size = 0.60

method	result	size
gosper	$\frac{x(423b^4x^{12}+1833a\,b^3x^9+3055a^2b^2x^6+2275a^3b\,x^3+910a^4)}{910(b\,x^3+a)^{\frac{13}{3}}\,a^3}$	59

trager	$\frac{x\left(423b^4x^{12}+1833a\,b^3x^9+3055a^2b^2x^6+2275a^3b\,x^3+910a^4\right)}{910(b\,x^3+a)^{\frac{13}{3}}a^3}$	59

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^3+a)^2/(b*x^3+a)^(16/3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/910*x*(423*b^4*x^12+1833*a*b^3*x^9+3055*a^2*b^2*x^6+2275*a^3*b*x^3+910*a^
4)/(b*x^3+a)^(13/3)/a^3
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 206 vs. 2(79) = 158.
time = 0.29, size = 206, normalized size = 2.10

$$\frac{\left(35\,b^2-\frac{91\,(bx^3+a)b}{x^3}+\frac{65\,(bx^3+a)^2}{x^6}\right)b^2x^{13}}{455\,(bx^3+a)^{\frac{13}{3}}a^3}+\frac{\left(140\,b^3-\frac{546\,(bx^3+a)b^2}{x^3}+\frac{780\,(bx^3+a)^2b}{x^6}-\frac{455\,(bx^3+a)^3}{x^9}\right)bx^{13}}{910\,(bx^3+a)^{\frac{13}{3}}a^3}+\frac{\left(35\,b^4-\frac{182\,(bx^3+a)b^3}{x^3}+\frac{390\,(bx^3+a)^2b^2}{x^6}-\frac{455\,(bx^3+a)^3b}{x^9}+\frac{455\,(bx^3+a)^4}{x^{12}}\right)x^{13}}{455\,(bx^3+a)^{\frac{13}{3}}a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)^2/(b*x^3+a)^(16/3),x, algorithm="maxima")
```

```
[Out] 1/455*(35*b^2 - 91*(b*x^3 + a)*b/x^3 + 65*(b*x^3 + a)^2/x^6)*b^2*x^13/((b*x
^3 + a)^(13/3)*a^3) + 1/910*(140*b^3 - 546*(b*x^3 + a)*b^2/x^3 + 780*(b*x^3
 + a)^2*b/x^6 - 455*(b*x^3 + a)^3/x^9)*b*x^13/((b*x^3 + a)^(13/3)*a^3) + 1/
455*(35*b^4 - 182*(b*x^3 + a)*b^3/x^3 + 390*(b*x^3 + a)^2*b^2/x^6 - 455*(b*
x^3 + a)^3*b/x^9 + 455*(b*x^3 + a)^4/x^12)*x^13/((b*x^3 + a)^(13/3)*a^3)
```

Fricas [A]
time = 2.44, size = 113, normalized size = 1.15

$$\frac{\left(423\,b^4x^{13}+1833\,ab^3x^{10}+3055\,a^2b^2x^7+2275\,a^3bx^4+910\,a^4x\right)(bx^3+a)^{\frac{2}{3}}}{910\,\left(a^3b^5x^{15}+5\,a^4b^4x^{12}+10\,a^5b^3x^9+10\,a^6b^2x^6+5\,a^7bx^3+a^8\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)^2/(b*x^3+a)^(16/3),x, algorithm="fricas")
```

```
[Out] 1/910*(423*b^4*x^13 + 1833*a*b^3*x^10 + 3055*a^2*b^2*x^7 + 2275*a^3*b*x^4 +
 910*a^4*x)*(b*x^3 + a)^(2/3)/(a^3*b^5*x^15 + 5*a^4*b^4*x^12 + 10*a^5*b^3*x
^9 + 10*a^6*b^2*x^6 + 5*a^7*b*x^3 + a^8)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**3+a)**2/(b*x**3+a)**(16/3),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

$$\text{could not integrate}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)^2/(b*x^3+a)^(16/3),x, algorithm="giac")
[Out] integrate((b*x^3 - a)^2/(b*x^3 + a)^(16/3), x)
```

Mupad [B]
time = 1.44, size = 71, normalized size = 0.72

$$\frac{423\,x}{910\,a^3\,{\left(b\,x^3+a\right)}^{1/3}}-\frac{2\,x}{65\,{\left(b\,x^3+a\right)}^{10/3}}+\frac{141\,x}{910\,a^2\,{\left(b\,x^3+a\right)}^{4/3}}+\frac{47\,x}{455\,a\,{\left(b\,x^3+a\right)}^{7/3}}+\frac{4\,a\,x}{13\,{\left(b\,x^3+a\right)}^{13/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x^3)^2/(a + b*x^3)^(16/3),x)
[Out] (423*x)/(910*a^3*(a + b*x^3)^(1/3)) - (2*x)/(65*(a + b*x^3)^(10/3)) + (141*
x)/(910*a^2*(a + b*x^3)^(4/3)) + (47*x)/(455*a*(a + b*x^3)^(7/3)) + (4*a*x)
/(13*(a + b*x^3)^(13/3))
```

3.47 $\int \frac{(a-bx^3)^2}{(a+bx^3)^{19/3}} \, dx$

Optimal. Leaf size=117

$$\frac{x(a-bx^3)}{8(a+bx^3)^{16/3}}+\frac{11x}{104(a+bx^3)^{13/3}}+\frac{x}{13a(a+bx^3)^{10/3}}+\frac{9x}{91a^2(a+bx^3)^{7/3}}+\frac{27x}{182a^3(a+bx^3)^{4/3}}+\frac{81x}{182a^4\sqrt[3]{a+b}}$$

```
[Out] 1/8*x*(-b*x^3+a)/(b*x^3+a)^(16/3)+11/104*x/(b*x^3+a)^(13/3)+1/13*x/a/(b*x^3
+a)^(10/3)+9/91*x/a^2/(b*x^3+a)^(7/3)+27/182*x/a^3/(b*x^3+a)^(4/3)+81/182*x
/a^4/(b*x^3+a)^(1/3)
```

Rubi [A]
time = 0.03, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {424, 393, 198, 197}

$$\frac{81x}{182a^4\sqrt[3]{a+bx^3}}+\frac{27x}{182a^3(a+bx^3)^{4/3}}+\frac{9x}{91a^2(a+bx^3)^{7/3}}+\frac{x}{13a(a+bx^3)^{10/3}}+\frac{11x}{104(a+bx^3)^{13/3}}+\frac{x(a-bx^3)}{8(a+bx^3)^{16/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^3)^2/(a + b*x^3)^(19/3),x]
```

```
[Out] (x*(a - b*x^3))/(8*(a + b*x^3)^(16/3)) + (11*x)/(104*(a + b*x^3)^(13/3)) +
x/(13*a*(a + b*x^3)^(10/3)) + (9*x)/(91*a^2*(a + b*x^3)^(7/3)) + (27*x)/(18
2*a^3*(a + b*x^3)^(4/3)) + (81*x)/(182*a^4*(a + b*x^3)^(1/3))
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a-bx^3)^2}{(a+bx^3)^{19/3}}\,dx &= \frac{x(a-bx^3)}{8\,(a+bx^3)^{16/3}} + \frac{\int \frac{14a^2b-8ab^2x^3}{(a+bx^3)^{16/3}}\,dx}{16ab} \\
&= \frac{x(a-bx^3)}{8\,(a+bx^3)^{16/3}} + \frac{11x}{104\,(a+bx^3)^{13/3}} + \frac{10}{13}\int \frac{1}{(a+bx^3)^{13/3}}\,dx \\
&= \frac{x(a-bx^3)}{8\,(a+bx^3)^{16/3}} + \frac{11x}{104\,(a+bx^3)^{13/3}} + \frac{x}{13a\,(a+bx^3)^{10/3}} + \frac{9\int \frac{1}{(a+bx^3)^{10/3}}\,dx}{13a} \\
&= \frac{x(a-bx^3)}{8\,(a+bx^3)^{16/3}} + \frac{11x}{104\,(a+bx^3)^{13/3}} + \frac{x}{13a\,(a+bx^3)^{10/3}} + \frac{9x}{91a^2\,(a+bx^3)^{7/3}} + \frac{54\int \frac{1}{(a+\cdots}}{91\cdots} \\
&= \frac{x(a-bx^3)}{8\,(a+bx^3)^{16/3}} + \frac{11x}{104\,(a+bx^3)^{13/3}} + \frac{x}{13a\,(a+bx^3)^{10/3}} + \frac{9x}{91a^2\,(a+bx^3)^{7/3}} + \frac{2\cdots}{182a^3\,(a\cdots} \\
&= \frac{x(a-bx^3)}{8\,(a+bx^3)^{16/3}} + \frac{11x}{104\,(a+bx^3)^{13/3}} + \frac{x}{13a\,(a+bx^3)^{10/3}} + \frac{9x}{91a^2\,(a+bx^3)^{7/3}} + \frac{2\cdots}{182a^3\,(a\cdots}
\end{aligned}$$

Mathematica [A]
time = 0.76, size = 73, normalized size = 0.62

$$\frac{364a^5x+1183a^4bx^4+2080a^3b^2x^7+1872a^2b^3x^{10}+864ab^4x^{13}+162b^5x^{16}}{364a^4\,(a+bx^3)^{16/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^3)^2/(a + b*x^3)^(19/3),x]
```

```
[Out] (364*a^5*x + 1183*a^4*b*x^4 + 2080*a^3*b^2*x^7 + 1872*a^2*b^3*x^10 + 864*a*
b^4*x^13 + 162*b^5*x^16)/(364*a^4*(a + b*x^3)^(16/3))
```

Maple [A]
time = 0.28, size = 70, normalized size = 0.60

method	result	size
gosper	$\frac{x\left(162b^{5}x^{15}+864a\,b^{4}x^{12}+1872a^{2}b^{3}x^{9}+2080a^{3}b^{2}x^{6}+1183a^{4}b\,x^{3}+364a^{5}\right)}{364\left(b\,x^{3}+a\right)^{\frac{16}{3}}a^{4}}$	70
trager	$\frac{x\left(162b^{5}x^{15}+864a\,b^{4}x^{12}+1872a^{2}b^{3}x^{9}+2080a^{3}b^{2}x^{6}+1183a^{4}b\,x^{3}+364a^{5}\right)}{364\left(b\,x^{3}+a\right)^{\frac{16}{3}}a^{4}}$	70

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^3+a)^2/(b*x^3+a)^(19/3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/364*x*(162*b^5*x^15+864*a*b^4*x^12+1872*a^2*b^3*x^9+2080*a^3*b^2*x^6+1183
*a^4*b*x^3+364*a^5)/(b*x^3+a)^(16/3)/a^4
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 257 vs. $2(94) = 188$.
time = 0.31, size = 257, normalized size = 2.20

$$-\frac{\left(455\,b^{3}-\frac{1680\,(bx^{3}+a)b^{2}}{x^{3}}+\frac{2184\,(bx^{3}+a)^{2}b}{x^{6}}-\frac{1040\,(bx^{3}+a)^{3}}{x^{9}}\right)b^{2}x^{16}}{7280\,(bx^{3}+a)^{\frac{16}{3}}a^{4}}-\frac{\left(455\,b^{4}-\frac{2240\,(bx^{3}+a)b^{3}}{x^{3}}+\frac{4368\,(bx^{3}+a)^{2}b^{2}}{x^{6}}-\frac{4160\,(bx^{3}+a)^{3}b}{x^{9}}+\frac{1820\,(bx^{3}+a)^{4}}{x^{12}}\right)bx^{16}}{3640\,(bx^{3}+a)^{\frac{16}{3}}a^{4}}-\frac{\left(91\,b^{5}-\frac{560\,(bx^{3}+a)b^{4}}{x^{3}}+\frac{1456\,(bx^{3}+a)^{2}b^{3}}{x^{6}}-\frac{2080\,(bx^{3}+a)^{3}b^{2}}{x^{9}}+\frac{1820\,(bx^{3}+a)^{4}b}{x^{12}}-\frac{1456\,(bx^{3}+a)^{5}}{x^{15}}\right)x^{16}}{1456\,(bx^{3}+a)^{\frac{16}{3}}a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)^2/(b*x^3+a)^(19/3),x, algorithm="maxima")
```

```
[Out] -1/7280*(455*b^3 - 1680*(b*x^3 + a)*b^2/x^3 + 2184*(b*x^3 + a)^2*b/x^6 - 10
40*(b*x^3 + a)^3/x^9)*b^2*x^16/((b*x^3 + a)^(16/3)*a^4) - 1/3640*(455*b^4 -
 2240*(b*x^3 + a)*b^3/x^3 + 4368*(b*x^3 + a)^2*b^2/x^6 - 4160*(b*x^3 + a)^3
*b/x^9 + 1820*(b*x^3 + a)^4/x^12)*b*x^16/((b*x^3 + a)^(16/3)*a^4) - 1/1456*
(91*b^5 - 560*(b*x^3 + a)*b^4/x^3 + 1456*(b*x^3 + a)^2*b^3/x^6 - 2080*(b*x^
3 + a)^3*b^2/x^9 + 1820*(b*x^3 + a)^4*b/x^12 - 1456*(b*x^3 + a)^5/x^15)*x^1
6/((b*x^3 + a)^(16/3)*a^4)
```

Fricas [A]
time = 3.02, size = 135, normalized size = 1.15

$$\frac{\left(162\,b^{5}x^{16}+864\,ab^{4}x^{13}+1872\,a^{2}b^{3}x^{10}+2080\,a^{3}b^{2}x^{7}+1183\,a^{4}bx^{4}+364\,a^{5}x\right)\left(bx^{3}+a\right)^{\frac{2}{3}}}{364\,\left(a^{4}b^{6}x^{18}+6\,a^{5}b^{5}x^{15}+15\,a^{6}b^{4}x^{12}+20\,a^{7}b^{3}x^{9}+15\,a^{8}b^{2}x^{6}+6\,a^{9}bx^{3}+a^{10}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)^2/(b*x^3+a)^(19/3),x, algorithm="fricas")
```

```
[Out] 1/364*(162*b^5*x^16 + 864*a*b^4*x^13 + 1872*a^2*b^3*x^10 + 2080*a^3*b^2*x^7
 + 1183*a^4*b*x^4 + 364*a^5*x)*(b*x^3 + a)^(2/3)/(a^4*b^6*x^18 + 6*a^5*b^5*
x^15 + 15*a^6*b^4*x^12 + 20*a^7*b^3*x^9 + 15*a^8*b^2*x^6 + 6*a^9*b*x^3 + a^
10)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**3+a)**2/(b*x**3+a)**(19/3),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)^2/(b*x^3+a)^(19/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 - a)^2/(b*x^3 + a)^(19/3), x)
```

Mupad [B]
time = 1.43, size = 86, normalized size = 0.74

$$\frac{81\,x}{182\,a^4\,{\left(b\,x^3+a\right)}^{1/3}}-\frac{x}{52\,{\left(b\,x^3+a\right)}^{13/3}}+\frac{27\,x}{182\,a^3\,{\left(b\,x^3+a\right)}^{4/3}}+\frac{9\,x}{91\,a^2\,{\left(b\,x^3+a\right)}^{7/3}}+\frac{x}{13\,a\,{\left(b\,x^3+a\right)}^{10/3}}+\frac{a\,x}{4\,{\left(b\,x^3+a\right)}^{16/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x^3)^2/(a + b*x^3)^(19/3),x)
```

```
[Out] (81*x)/(182*a^4*(a + b*x^3)^(1/3)) - x/(52*(a + b*x^3)^(13/3)) + (27*x)/(18
2*a^3*(a + b*x^3)^(4/3)) + (9*x)/(91*a^2*(a + b*x^3)^(7/3)) + x/(13*a*(a +
b*x^3)^(10/3)) + (a*x)/(4*(a + b*x^3)^(16/3))
```

3.48 $\int (a-bx^3)^2 (a+bx^3)^{4/3}\,dx$

Optimal. Leaf size=94

$$-\frac{9}{44}ax\left(a+bx^3\right)^{7/3}-\frac{1}{11}x\left(a-bx^3\right)\left(a+bx^3\right)^{7/3}+\frac{57a^3x\sqrt[3]{a+bx^3}\;{}_2F_1\left(-\frac{4}{3},\frac{1}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{44\sqrt[3]{1+\frac{bx^3}{a}}}$$

```
[Out] -9/44*a*x*(b*x^3+a)^(7/3)-1/11*x*(-b*x^3+a)*(b*x^3+a)^(7/3)+57/44*a^3*x*(b*
x^3+a)^(1/3)*hypergeom([-4/3, 1/3],[4/3],-b*x^3/a)/(1+b*x^3/a)^(1/3)
```

Rubi [A]
time = 0.02, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {427, 396, 252, 251}

$$\frac{57a^3x\sqrt[3]{a+bx^3}\;{}_2F_1\left(-\frac{4}{3},\frac{1}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{44\sqrt[3]{\frac{bx^3}{a}+1}}-\frac{9}{44}ax\left(a+bx^3\right)^{7/3}-\frac{1}{11}x\left(a-bx^3\right)\left(a+bx^3\right)^{7/3}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^3)^2*(a + b*x^3)^(4/3),x]
```

```
[Out] (-9*a*x*(a + b*x^3)^(7/3))/44 - (x*(a - b*x^3)*(a + b*x^3)^(7/3))/11 + (57*
a^3*x*(a + b*x^3)^(1/3)*Hypergeometric2F1[-4/3, 1/3, 4/3, -((b*x^3)/a)])/(4
4*(1 + (b*x^3)/a)^(1/3))
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
```

```
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rubi steps

$$
\begin{aligned}
\int \left(a-bx^3\right)^2 \left(a+bx^3\right)^{4/3}\,dx &= -\frac{1}{11}x\left(a-bx^3\right)\left(a+bx^3\right)^{7/3}+\frac{\int \left(a+bx^3\right)^{4/3}\left(12a^2b-18ab^2x^3\right)\,dx}{11b}\\
&= -\frac{9}{44}ax\left(a+bx^3\right)^{7/3}-\frac{1}{11}x\left(a-bx^3\right)\left(a+bx^3\right)^{7/3}+\frac{1}{44}\left(57a^2\right)\int \left(a+bx^3\right)^{4/\ldots}\\
&= -\frac{9}{44}ax\left(a+bx^3\right)^{7/3}-\frac{1}{11}x\left(a-bx^3\right)\left(a+bx^3\right)^{7/3}+\frac{\left(57a^3\sqrt[3]{a+bx^3}\right)\int \left(1\ldots\right.}{44\sqrt[3]{1+\frac{bx^3}{a}}}\\
&= -\frac{9}{44}ax\left(a+bx^3\right)^{7/3}-\frac{1}{11}x\left(a-bx^3\right)\left(a+bx^3\right)^{7/3}+\frac{57a^3x\sqrt[3]{a+bx^3}\,{}_2F_1\left(-\ldots\right.}{44\sqrt[3]{1+\frac{bx^3}{a}}}
\end{aligned}
$$

Mathematica [A]
time = 7.52, size = 97, normalized size = 1.03

$$
\frac{x\left(106a^4+53a^3bx^3-78a^2b^2x^6-5ab^3x^9+20b^4x^{12}+114a^4\left(1+\frac{bx^3}{a}\right)^{2/3}{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)\right)}{220\left(a+bx^3\right)^{2/3}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^3)^2*(a + b*x^3)^(4/3),x]
```

```
[Out] (x*(106*a^4 + 53*a^3*b*x^3 - 78*a^2*b^2*x^6 - 5*a*b^3*x^9 + 20*b^4*x^12 + 1
14*a^4*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[1/3, 2/3, 4/3, -((b*x^3)/a)]
))/(220*(a + b*x^3)^(2/3))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \left(-b\,x^{3}+a\right)^{2} \left(b\,x^{3}+a\right)^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^3+a)^2*(b*x^3+a)^(4/3),x)
[Out] int((-b*x^3+a)^2*(b*x^3+a)^(4/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)^2*(b*x^3+a)^(4/3),x, algorithm="maxima")
[Out] integrate((b*x^3 + a)^(4/3)*(b*x^3 - a)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)^2*(b*x^3+a)^(4/3),x, algorithm="fricas")
[Out] integral((b^3*x^9 - a*b^2*x^6 - a^2*b*x^3 + a^3)*(b*x^3 + a)^(1/3), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 2.72, size = 168, normalized size = 1.79

$$\frac{a^{\frac{10}{3}}x\Gamma\left(\frac{1}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)}-\frac{a^{\frac{7}{3}}bx^{4}\Gamma\left(\frac{4}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)}-\frac{a^{\frac{4}{3}}b^{2}x^{7}\Gamma\left(\frac{7}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{3},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{10}{3}\right)}+\frac{\sqrt[3]{a}\,b^{3}x^{10}\Gamma\left(\frac{10}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{3},\frac{10}{3}\\ \frac{13}{3}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{13}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**3+a)**2*(b*x**3+a)**(4/3),x)
[Out] a**(10/3)*x*gamma(1/3)*hyper((-1/3, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a)
/(3*gamma(4/3)) - a**(7/3)*b*x**4*gamma(4/3)*hyper((-1/3, 4/3), (7/3,), b*x
**3*exp_polar(I*pi)/a)/(3*gamma(7/3)) - a**(4/3)*b**2*x**7*gamma(7/3)*hyper
((-1/3, 7/3), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(10/3)) + a**(1/3)
*b**3*x**10*gamma(10/3)*hyper((-1/3, 10/3), (13/3,), b*x**3*exp_polar(I*pi)
/a)/(3*gamma(13/3))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)^2*(b*x^3+a)^(4/3),x, algorithm="giac")
[Out] integrate((b*x^3 + a)^(4/3)*(b*x^3 - a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(b\,x^3+a\right)}^{4/3}\,{\left(a-b\,x^3\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(4/3)*(a - b*x^3)^2,x)
[Out] int((a + b*x^3)^(4/3)*(a - b*x^3)^2, x)
```

3.49 $\int (a-bx^3)^2 \sqrt[3]{a+bx^3}\, dx$

Optimal. Leaf size=94

$$-\frac{3}{8}ax\left(a+bx^3\right)^{4/3}-\frac{1}{8}x\left(a-bx^3\right)\left(a+bx^3\right)^{4/3}+\frac{3a^2x\sqrt[3]{a+bx^3}\ {}_2F_1\left(-\frac{1}{3},\frac{1}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{2\sqrt[3]{1+\frac{bx^3}{a}}}$$

```
[Out] -3/8*a*x*(b*x^3+a)^(4/3)-1/8*x*(-b*x^3+a)*(b*x^3+a)^(4/3)+3/2*a^2*x*(b*x^3+
a)^(1/3)*hypergeom([-1/3, 1/3],[4/3],-b*x^3/a)/(1+b*x^3/a)^(1/3)
```

Rubi [A]
time = 0.02, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {427, 396, 252, 251}

$$\frac{3a^2x\sqrt[3]{a+bx^3}\ {}_2F_1\left(-\frac{1}{3},\frac{1}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{2\sqrt[3]{\frac{bx^3}{a}+1}}-\frac{3}{8}ax\left(a+bx^3\right)^{4/3}-\frac{1}{8}x\left(a-bx^3\right)\left(a+bx^3\right)^{4/3}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^3)^2*(a + b*x^3)^(1/3),x]
```

```
[Out] (-3*a*x*(a + b*x^3)^(4/3))/8 - (x*(a - b*x^3)*(a + b*x^3)^(4/3))/8 + (3*a^2
*x*(a + b*x^3)^(1/3)*Hypergeometric2F1[-1/3, 1/3, 4/3, -((b*x^3)/a)])/(2*(1
 + (b*x^3)/a)^(1/3))
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
```

```
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rubi steps

$$\begin{aligned}
\int \left(a-bx^3\right)^2 \sqrt[3]{a+bx^3}\,dx &= -\frac{1}{8}x\left(a-bx^3\right)\left(a+bx^3\right)^{4/3}+\frac{\int \sqrt[3]{a+bx^3}\left(9a^2b-15ab^2x^3\right)dx}{8b}\\
&= -\frac{3}{8}ax\left(a+bx^3\right)^{4/3}-\frac{1}{8}x\left(a-bx^3\right)\left(a+bx^3\right)^{4/3}+\frac{1}{2}\left(3a^2\right)\int \sqrt[3]{a+bx^3}\,dx\\
&= -\frac{3}{8}ax\left(a+bx^3\right)^{4/3}-\frac{1}{8}x\left(a-bx^3\right)\left(a+bx^3\right)^{4/3}+\frac{\left(3a^2\sqrt[3]{a+bx^3}\right)\int \sqrt[3]{1+\frac{bx^3}{a}}}{2\sqrt[3]{1+\frac{bx^3}{a}}}\\
&= -\frac{3}{8}ax\left(a+bx^3\right)^{4/3}-\frac{1}{8}x\left(a-bx^3\right)\left(a+bx^3\right)^{4/3}+\frac{3a^2x\sqrt[3]{a+bx^3}\ {}_2F_1\left(-\frac{1}{3},\frac{1}{3};\frac{4}{3}\right.}{2\sqrt[3]{1+\frac{bx^3}{a}}}
\end{aligned}$$

Mathematica [A]
time = 5.84, size = 85, normalized size = 0.90

$$\frac{x\left(2a^3-a^2bx^3-2ab^2x^6+b^3x^9+6a^3\left(1+\frac{bx^3}{a}\right)^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)\right)}{8\left(a+bx^3\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^3)^2*(a + b*x^3)^(1/3),x]
```

```
[Out] (x*(2*a^3 - a^2*b*x^3 - 2*a*b^2*x^6 + b^3*x^9 + 6*a^3*(1 + (b*x^3)/a)^(2/3)
*Hypergeometric2F1[1/3, 2/3, 4/3, -((b*x^3)/a)]))/(8*(a + b*x^3)^(2/3))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \left(-b\,x^{3}+a\right)^{2} \left(b\,x^{3}+a\right)^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^3+a)^2*(b*x^3+a)^(1/3),x)
[Out] int((-b*x^3+a)^2*(b*x^3+a)^(1/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)^2*(b*x^3+a)^(1/3),x, algorithm="maxima")
[Out] integrate((b*x^3 + a)^(1/3)*(b*x^3 - a)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)^2*(b*x^3+a)^(1/3),x, algorithm="fricas")
[Out] integral((b^2*x^6 - 2*a*b*x^3 + a^2)*(b*x^3 + a)^(1/3), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.78, size = 126, normalized size = 1.34

$$\frac{a^{\frac{7}{3}}x\Gamma\left(\frac{1}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{3},\frac{1}{3}\\\frac{4}{3}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)}-\frac{2a^{\frac{4}{3}}bx^{4}\Gamma\left(\frac{4}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{3},\frac{4}{3}\\\frac{7}{3}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)}+\frac{\sqrt[3]{a}\,b^{2}x^{7}\Gamma\left(\frac{7}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{3},\frac{7}{3}\\\frac{10}{3}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{10}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**3+a)**2*(b*x**3+a)**(1/3),x)
[Out] a**(7/3)*x*gamma(1/3)*hyper((-1/3, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a)/
(3*gamma(4/3)) - 2*a**(4/3)*b*x**4*gamma(4/3)*hyper((-1/3, 4/3), (7/3,), b*
x**3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + a**(1/3)*b**2*x**7*gamma(7/3)*hype
r((-1/3, 7/3), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(10/3))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)^2*(b*x^3+a)^(1/3),x, algorithm="giac")
[Out] integrate((b*x^3 + a)^(1/3)*(b*x^3 - a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(b\,x^3+a\right)^{1/3}\,\left(a-b\,x^3\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(1/3)*(a - b*x^3)^2,x)
[Out] int((a + b*x^3)^(1/3)*(a - b*x^3)^2, x)
```

3.50 $\int \frac{(a-bx^3)^2}{(a+bx^3)^{2/3}} \, dx$

Optimal. Leaf size=94

$$-\frac{6}{5}ax\sqrt[3]{a+bx^3}-\frac{1}{5}x\left(a-bx^3\right)\sqrt[3]{a+bx^3}+\frac{12a^2x\left(1+\frac{bx^3}{a}\right)^{2/3}{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{5\left(a+bx^3\right)^{2/3}}$$

```
[Out] -6/5*a*x*(b*x^3+a)^(1/3)-1/5*x*(-b*x^3+a)*(b*x^3+a)^(1/3)+12/5*a^2*x*(1+b*x
^3/a)^(2/3)*hypergeom([1/3, 2/3],[4/3],-b*x^3/a)/(b*x^3+a)^(2/3)
```

Rubi [A]
time = 0.02, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {427, 396, 252, 251}

$$\frac{12a^2x\left(\frac{bx^3}{a}+1\right)^{2/3}{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{5\left(a+bx^3\right)^{2/3}}-\frac{6}{5}ax\sqrt[3]{a+bx^3}-\frac{1}{5}x\left(a-bx^3\right)\sqrt[3]{a+bx^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^3)^2/(a + b*x^3)^(2/3),x]
```

```
[Out] (-6*a*x*(a + b*x^3)^(1/3))/5 - (x*(a - b*x^3)*(a + b*x^3)^(1/3))/5 + (12*a^
2*x*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[1/3, 2/3, 4/3, -((b*x^3)/a)])/(
5*(a + b*x^3)^(2/3))
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
```

```
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a-bx^3)^2}{(a+bx^3)^{2/3}}\,dx &= -\frac{1}{5}x(a-bx^3)\sqrt[3]{a+bx^3}+\frac{\int \frac{6a^2b-12ab^2x^3}{(a+bx^3)^{2/3}}\,dx}{5b}\\
&= -\frac{6}{5}ax\sqrt[3]{a+bx^3}-\frac{1}{5}x(a-bx^3)\sqrt[3]{a+bx^3}+\frac{1}{5}(12a^2)\int \frac{1}{(a+bx^3)^{2/3}}\,dx\\
&= -\frac{6}{5}ax\sqrt[3]{a+bx^3}-\frac{1}{5}x(a-bx^3)\sqrt[3]{a+bx^3}+\frac{\left(12a^2\left(1+\frac{bx^3}{a}\right)^{2/3}\right)\int \frac{1}{\left(1+\frac{bx^3}{a}\right)^{2/3}}\,dx}{5(a+bx^3)^{2/3}}\\
&= -\frac{6}{5}ax\sqrt[3]{a+bx^3}-\frac{1}{5}x(a-bx^3)\sqrt[3]{a+bx^3}+\frac{12a^2x\left(1+\frac{bx^3}{a}\right)^{2/3}{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{5(a+bx^3)^{2/3}}
\end{aligned}
$$

Mathematica [A]
time = 10.04, size = 75, normalized size = 0.80

$$\frac{-7a^2x-6abx^4+b^2x^7+12a^2x\left(1+\frac{bx^3}{a}\right)^{2/3}{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{5(a+bx^3)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^3)^2/(a + b*x^3)^(2/3),x]
[Out] (-7*a^2*x - 6*a*b*x^4 + b^2*x^7 + 12*a^2*x*(1 + (b*x^3)/a)^(2/3)*Hypergeome
tric2F1[1/3, 2/3, 4/3, -((b*x^3)/a)])/(5*(a + b*x^3)^(2/3))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(-b\,x^3+a)^2}{(b\,x^3+a)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^3+a)^2/(b*x^3+a)^(2/3),x)
```

```
[Out] int((-b*x^3+a)^2/(b*x^3+a)^(2/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)^2/(b*x^3+a)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 - a)^2/(b*x^3 + a)^(2/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)^2/(b*x^3+a)^(2/3),x, algorithm="fricas")
```

```
[Out] integral((b^2*x^6 - 2*a*b*x^3 + a^2)/(b*x^3 + a)^(2/3), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.84, size = 121, normalized size = 1.29

$$\frac{a^{\frac{4}{3}}x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{2}{3}\\\frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)}-\frac{2\sqrt[3]{a}\,bx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{2}{3},\frac{4}{3}\\\frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)}+\frac{b^2x^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\begin{matrix}\frac{2}{3},\frac{7}{3}\\\frac{10}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{2}{3}}\Gamma\left(\frac{10}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**3+a)**2/(b*x**3+a)**(2/3),x)
```

```
[Out] a**(4/3)*x*gamma(1/3)*hyper((1/3, 2/3), (4/3,), b*x**3*exp_polar(I*pi)/a)/(
3*gamma(4/3)) - 2*a**(1/3)*b*x**4*gamma(4/3)*hyper((2/3, 4/3), (7/3,), b*x*
*3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + b**2*x**7*gamma(7/3)*hyper((2/3, 7/3
), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(2/3)*gamma(10/3))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)^2/(b*x^3+a)^(2/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 - a)^2/(b*x^3 + a)^(2/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a-b\,x^3\right)^2}{\left(b\,x^3+a\right)^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x^3)^2/(a + b*x^3)^(2/3),x)
```

```
[Out] int((a - b*x^3)^2/(a + b*x^3)^(2/3), x)
```

3.51 $\int \frac{(a-bx^3)^2}{(a+bx^3)^{5/3}}\,dx$

Optimal. Leaf size=74

$$\frac{x(a-bx^3)}{(a+bx^3)^{2/3}}+\frac{3bx^4\left(1+\frac{bx^3}{a}\right)^{2/3}\,{}_2F_1\left(\frac{2}{3},\frac{4}{3};\frac{7}{3};-\frac{bx^3}{a}\right)}{4\,(a+bx^3)^{2/3}}$$

```
[Out] x*(-b*x^3+a)/(b*x^3+a)^(2/3)+3/4*b*x^4*(1+b*x^3/a)^(2/3)*hypergeom([2/3, 4/
3],[7/3],-b*x^3/a)/(b*x^3+a)^(2/3)
```

Rubi [A]
time = 0.03, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {424, 12, 372, 371}

$$\frac{3bx^4\left(\frac{bx^3}{a}+1\right)^{2/3}\,{}_2F_1\left(\frac{2}{3},\frac{4}{3};\frac{7}{3};-\frac{bx^3}{a}\right)}{4\,(a+bx^3)^{2/3}}+\frac{x(a-bx^3)}{(a+bx^3)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^3)^2/(a + b*x^3)^(5/3),x]
```

```
[Out] (x*(a - b*x^3))/(a + b*x^3)^(2/3) + (3*b*x^4*(1 + (b*x^3)/a)^(2/3)*Hypergeo
metric2F1[2/3, 4/3, 7/3, -((b*x^3)/a)])/(4*(a + b*x^3)^(2/3))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rubi steps

$$\begin{aligned}\int \frac{(a-bx^3)^2}{(a+bx^3)^{5/3}}\,dx &= \frac{x(a-bx^3)}{(a+bx^3)^{2/3}}+\frac{\int \frac{6ab^2x^3}{(a+bx^3)^{2/3}}\,dx}{2ab}\\ &= \frac{x(a-bx^3)}{(a+bx^3)^{2/3}}+(3b)\int \frac{x^3}{(a+bx^3)^{2/3}}\,dx\\ &= \frac{x(a-bx^3)}{(a+bx^3)^{2/3}}+\frac{\left(3b\left(1+\frac{bx^3}{a}\right)^{2/3}\right)\int \frac{x^3}{\left(1+\frac{bx^3}{a}\right)^{2/3}}\,dx}{(a+bx^3)^{2/3}}\\ &= \frac{x(a-bx^3)}{(a+bx^3)^{2/3}}+\frac{3bx^4\left(1+\frac{bx^3}{a}\right)^{2/3}\,{}_2F_1\left(\frac{2}{3},\frac{4}{3};\frac{7}{3};-\frac{bx^3}{a}\right)}{4\,(a+bx^3)^{2/3}}\end{aligned}$$

Mathematica [A]
time = 10.03, size = 62, normalized size = 0.84

$$\frac{5ax+bx^4-3ax\left(1+\frac{bx^3}{a}\right)^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{2\,(a+bx^3)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^3)^2/(a + b*x^3)^(5/3),x]
[Out] (5*a*x + b*x^4 - 3*a*x*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[1/3, 2/3, 4/
3, -((b*x^3)/a)])/(2*(a + b*x^3)^(2/3))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(-b\,x^3+a)^2}{(b\,x^3+a)^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^3+a)^2/(b*x^3+a)^(5/3),x)
[Out] int((-b*x^3+a)^2/(b*x^3+a)^(5/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)^2/(b*x^3+a)^(5/3),x, algorithm="maxima")
[Out] integrate((b*x^3 - a)^2/(b*x^3 + a)^(5/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)^2/(b*x^3+a)^(5/3),x, algorithm="fricas")
[Out] integral((b^2*x^6 - 2*a*b*x^3 + a^2)*(b*x^3 + a)^(1/3)/(b^2*x^6 + 2*a*b*x^3
 + a^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-a + bx^3\right)^2}{\left(a + bx^3\right)^{\frac{5}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**3+a)**2/(b*x**3+a)**(5/3),x)
[Out] Integral((-a + b*x**3)**2/(a + b*x**3)**(5/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)^2/(b*x^3+a)^(5/3),x, algorithm="giac")
[Out] integrate((b*x^3 - a)^2/(b*x^3 + a)^(5/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a-b\,x^3)^2}{(b\,x^3+a)^{5/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x^3)^2/(a + b*x^3)^(5/3),x)
[Out] int((a - b*x^3)^2/(a + b*x^3)^(5/3), x)
```

3.52 $\int \frac{(a-bx^3)^2}{(a+bx^3)^{8/3}}\,dx$

Optimal. Leaf size=74

$$\frac{2x(a-bx^3)}{5\,(a+bx^3)^{5/3}}+\frac{3x\left(1+\frac{bx^3}{a}\right)^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{5\,(a+bx^3)^{2/3}}$$

```
[Out] 2/5*x*(-b*x^3+a)/(b*x^3+a)^(5/3)+3/5*x*(1+b*x^3/a)^(2/3)*hypergeom([1/3, 2/
3],[4/3],-b*x^3/a)/(b*x^3+a)^(2/3)
```

Rubi [A]
time = 0.02, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {424, 21, 252, 251}

$$\frac{3x\left(\frac{bx^3}{a}+1\right)^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{5\,(a+bx^3)^{2/3}}+\frac{2x(a-bx^3)}{5\,(a+bx^3)^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^3)^2/(a + b*x^3)^(8/3),x]
```

```
[Out] (2*x*(a - b*x^3))/(5*(a + b*x^3)^(5/3)) + (3*x*(1 + (b*x^3)/a)^(2/3)*Hyperg
eometric2F1[1/3, 2/3, 4/3, -((b*x^3)/a)])/(5*(a + b*x^3)^(2/3))
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
```

```
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rubi steps

$$\begin{aligned}\int \frac{(a-bx^3)^2}{(a+bx^3)^{8/3}}\,dx &= \frac{2x(a-bx^3)}{5\,(a+bx^3)^{5/3}}+\frac{\int \frac{3a^2b+3ab^2x^3}{(a+bx^3)^{5/3}}\,dx}{5ab}\\ &= \frac{2x(a-bx^3)}{5\,(a+bx^3)^{5/3}}+\frac{3}{5}\int \frac{1}{(a+bx^3)^{2/3}}\,dx\\ &= \frac{2x(a-bx^3)}{5\,(a+bx^3)^{5/3}}+\frac{\left(3\left(1+\frac{bx^3}{a}\right)^{2/3}\right)\int \frac{1}{\left(1+\frac{bx^3}{a}\right)^{2/3}}\,dx}{5\,(a+bx^3)^{2/3}}\\ &= \frac{2x(a-bx^3)}{5\,(a+bx^3)^{5/3}}+\frac{3x\left(1+\frac{bx^3}{a}\right)^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{5\,(a+bx^3)^{2/3}}\end{aligned}$$

Mathematica [A]
time = 10.05, size = 70, normalized size = 0.95

$$\frac{2x(a-bx^3)+3x(a+bx^3)\left(1+\frac{bx^3}{a}\right)^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{5\,(a+bx^3)^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^3)^2/(a + b*x^3)^(8/3),x]
[Out] (2*x*(a - b*x^3) + 3*x*(a + b*x^3)*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[
1/3, 2/3, 4/3, -((b*x^3)/a)])/(5*(a + b*x^3)^(5/3))
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{\left(-b\,x^{3}+a\right)^{2}}{\left(b\,x^{3}+a\right)^{\frac{8}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^3+a)^2/(b*x^3+a)^(8/3),x)
```

```
[Out] int((-b*x^3+a)^2/(b*x^3+a)^(8/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)^2/(b*x^3+a)^(8/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 - a)^2/(b*x^3 + a)^(8/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)^2/(b*x^3+a)^(8/3),x, algorithm="fricas")
```

```
[Out] integral((b^2*x^6 - 2*a*b*x^3 + a^2)*(b*x^3 + a)^(1/3)/(b^3*x^9 + 3*a*b^2*x
^6 + 3*a^2*b*x^3 + a^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-a+bx^3)^2}{(a+bx^3)^{\frac{8}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**3+a)**2/(b*x**3+a)**(8/3),x)
```

```
[Out] Integral((-a + b*x**3)**2/(a + b*x**3)**(8/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)^2/(b*x^3+a)^(8/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 - a)^2/(b*x^3 + a)^(8/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a-b\,x^3\right)^2}{{\left(b\,x^3+a\right)}^{8/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x^3)^2/(a + b*x^3)^(8/3),x)
```

```
[Out] int((a - b*x^3)^2/(a + b*x^3)^(8/3), x)
```

3.53 $\int \frac{(a-bx^3)^2}{(a+bx^3)^{11/3}} \, dx$

Optimal. Leaf size=77

$$\frac{x(a-bx^3)}{4(a+bx^3)^{8/3}}+\frac{3x\left(1+\frac{bx^3}{a}\right)^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{8}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{4a(a+bx^3)^{2/3}}$$

```
[Out] 1/4*x*(-b*x^3+a)/(b*x^3+a)^(8/3)+3/4*x*(1+b*x^3/a)^(2/3)*hypergeom([1/3, 8/
3],[4/3],-b*x^3/a)/a/(b*x^3+a)^(2/3)
```

Rubi [A]
time = 0.02, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {424, 12, 252, 251}

$$\frac{3x\left(\frac{bx^3}{a}+1\right)^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{8}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{4a(a+bx^3)^{2/3}}+\frac{x(a-bx^3)}{4(a+bx^3)^{8/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^3)^2/(a + b*x^3)^(11/3),x]
```

```
[Out] (x*(a - b*x^3))/(4*(a + b*x^3)^(8/3)) + (3*x*(1 + (b*x^3)/a)^(2/3)*Hypergeo
metric2F1[1/3, 8/3, 4/3, -((b*x^3)/a)])/(4*a*(a + b*x^3)^(2/3))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rubi steps

$$\begin{aligned}\int \frac{(a-bx^3)^2}{(a+bx^3)^{11/3}}\,dx &= \frac{x(a-bx^3)}{4\,(a+bx^3)^{8/3}}+\frac{\int \frac{6a^2b}{(a+bx^3)^{8/3}}\,dx}{8ab}\\ &= \frac{x(a-bx^3)}{4\,(a+bx^3)^{8/3}}+\frac{1}{4}(3a)\int \frac{1}{(a+bx^3)^{8/3}}\,dx\\ &= \frac{x(a-bx^3)}{4\,(a+bx^3)^{8/3}}+\frac{\left(3\left(1+\frac{bx^3}{a}\right)^{2/3}\right)\int \frac{1}{\left(1+\frac{bx^3}{a}\right)^{8/3}}\,dx}{4a\,(a+bx^3)^{2/3}}\\ &= \frac{x(a-bx^3)}{4\,(a+bx^3)^{8/3}}+\frac{3x\left(1+\frac{bx^3}{a}\right)^{2/3}{}_2F_1\left(\frac{1}{3},\frac{8}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{4a\,(a+bx^3)^{2/3}}\end{aligned}$$

Mathematica [A]
time = 10.06, size = 85, normalized size = 1.10

$$\frac{7a^2x+5abx^4+3b^2x^7+3x(a+bx^3)^2\left(1+\frac{bx^3}{a}\right)^{2/3}{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{10a\,(a+bx^3)^{8/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^3)^2/(a + b*x^3)^(11/3),x]
```

```
[Out] (7*a^2*x + 5*a*b*x^4 + 3*b^2*x^7 + 3*x*(a + b*x^3)^2*(1 + (b*x^3)/a)^(2/3)*
Hypergeometric2F1[1/3, 2/3, 4/3, -((b*x^3)/a)])/(10*a*(a + b*x^3)^(8/3))
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{\left(-b\,x^3+a\right)^2}{\left(b\,x^3+a\right)^{\frac{11}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^3+a)^2/(b*x^3+a)^(11/3),x)
```

```
[Out] int((-b*x^3+a)^2/(b*x^3+a)^(11/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)^2/(b*x^3+a)^(11/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 - a)^2/(b*x^3 + a)^(11/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)^2/(b*x^3+a)^(11/3),x, algorithm="fricas")
```

```
[Out] integral((b^2*x^6 - 2*a*b*x^3 + a^2)*(b*x^3 + a)^(1/3)/(b^4*x^12 + 4*a*b^3*
x^9 + 6*a^2*b^2*x^6 + 4*a^3*b*x^3 + a^4), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**3+a)**2/(b*x**3+a)**(11/3),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)^2/(b*x^3+a)^(11/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 - a)^2/(b*x^3 + a)^(11/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a-b\,x^3)^2}{(b\,x^3+a)^{11/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x^3)^2/(a + b*x^3)^(11/3),x)
[Out] int((a - b*x^3)^2/(a + b*x^3)^(11/3), x)
```

3.54 $\int \frac{(a-bx^3)^2}{(a+bx^3)^{14/3}}\,dx$

Optimal. Leaf size=93

$$\frac{2x(a-bx^3)}{11\,(a+bx^3)^{11/3}}+\frac{3x}{22\,(a+bx^3)^{8/3}}+\frac{15x\left(1+\frac{bx^3}{a}\right)^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{8}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{22a^2\,(a+bx^3)^{2/3}}$$

```
[Out] 2/11*x*(-b*x^3+a)/(b*x^3+a)^(11/3)+3/22*x/(b*x^3+a)^(8/3)+15/22*x*(1+b*x^3/
a)^(2/3)*hypergeom([1/3, 8/3],[4/3],-b*x^3/a)/a^2/(b*x^3+a)^(2/3)
```

Rubi [A]
time = 0.02, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {424, 393, 252, 251}

$$\frac{15x\left(\frac{bx^3}{a}+1\right)^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{8}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{22a^2\,(a+bx^3)^{2/3}}+\frac{3x}{22\,(a+bx^3)^{8/3}}+\frac{2x(a-bx^3)}{11\,(a+bx^3)^{11/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^3)^2/(a + b*x^3)^(14/3),x]
```

```
[Out] (2*x*(a - b*x^3))/(11*(a + b*x^3)^(11/3)) + (3*x)/(22*(a + b*x^3)^(8/3)) +
(15*x*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[1/3, 8/3, 4/3, -((b*x^3)/a)])
/(22*a^2*(a + b*x^3)^(2/3))
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
```

```
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rubi steps

$$\begin{aligned}\int \frac{(a-bx^3)^2}{(a+bx^3)^{14/3}}\,dx &= \frac{2x(a-bx^3)}{11\,(a+bx^3)^{11/3}}+\frac{\int \frac{9a^2b-3ab^2x^3}{(a+bx^3)^{11/3}}\,dx}{11ab}\\ &= \frac{2x(a-bx^3)}{11\,(a+bx^3)^{11/3}}+\frac{3x}{22\,(a+bx^3)^{8/3}}+\frac{15}{22}\int \frac{1}{(a+bx^3)^{8/3}}\,dx\\ &= \frac{2x(a-bx^3)}{11\,(a+bx^3)^{11/3}}+\frac{3x}{22\,(a+bx^3)^{8/3}}+\frac{\left(15\left(1+\frac{bx^3}{a}\right)^{2/3}\right)\int \frac{1}{\left(1+\frac{bx^3}{a}\right)^{8/3}}\,dx}{22a^2\,(a+bx^3)^{2/3}}\\ &= \frac{2x(a-bx^3)}{11\,(a+bx^3)^{11/3}}+\frac{3x}{22\,(a+bx^3)^{8/3}}+\frac{15x\left(1+\frac{bx^3}{a}\right)^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{8}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{22a^2\,(a+bx^3)^{2/3}}\end{aligned}$$

Mathematica [A]
time = 10.08, size = 95, normalized size = 1.02

$$\frac{x\left(16a^3+23a^2bx^3+21ab^2x^6+6b^3x^9+6(a+bx^3)^3\left(1+\frac{bx^3}{a}\right)^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)\right)}{22a^2\,(a+bx^3)^{11/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^3)^2/(a + b*x^3)^(14/3),x]
```

```
[Out] (x*(16*a^3 + 23*a^2*b*x^3 + 21*a*b^2*x^6 + 6*b^3*x^9 + 6*(a + b*x^3)^3*(1 +
 (b*x^3)/a)^(2/3)*Hypergeometric2F1[1/3, 2/3, 4/3, -((b*x^3)/a)]))/(22*a^2*
(a + b*x^3)^(11/3))
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{\left(-b\,x^{3}+a\right)^{2}}{\left(b\,x^{3}+a\right)^{\frac{14}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^3+a)^2/(b*x^3+a)^(14/3),x)
[Out] int((-b*x^3+a)^2/(b*x^3+a)^(14/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)^2/(b*x^3+a)^(14/3),x, algorithm="maxima")
[Out] integrate((b*x^3 - a)^2/(b*x^3 + a)^(14/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)^2/(b*x^3+a)^(14/3),x, algorithm="fricas")
[Out] integral((b^2*x^6 - 2*a*b*x^3 + a^2)*(b*x^3 + a)^(1/3)/(b^5*x^15 + 5*a*b^4*
x^12 + 10*a^2*b^3*x^9 + 10*a^3*b^2*x^6 + 5*a^4*b*x^3 + a^5), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**3+a)**2/(b*x**3+a)**(14/3),x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)^2/(b*x^3+a)^(14/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 - a)^2/(b*x^3 + a)^(14/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a-b\,x^3\right)^2}{{\left(b\,x^3+a\right)}^{14/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x^3)^2/(a + b*x^3)^(14/3),x)
```

```
[Out] int((a - b*x^3)^2/(a + b*x^3)^(14/3), x)
```

3.55 $\int \frac{(a-bx^3)^2}{(a+bx^3)^{17/3}} \, dx$

Optimal. Leaf size=93

$$\frac{x(a-bx^3)}{7(a+bx^3)^{14/3}}+\frac{9x}{77(a+bx^3)^{11/3}}+\frac{57x\left(1+\frac{bx^3}{a}\right)^{2/3}{}_2F_1\left(\frac{1}{3},\frac{11}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{77a^3(a+bx^3)^{2/3}}$$

```
[Out] 1/7*x*(-b*x^3+a)/(b*x^3+a)^(14/3)+9/77*x/(b*x^3+a)^(11/3)+57/77*x*(1+b*x^3/
a)^(2/3)*hypergeom([1/3, 11/3],[4/3],-b*x^3/a)/a^3/(b*x^3+a)^(2/3)
```

Rubi [A]
time = 0.02, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {424, 393, 252, 251}

$$\frac{57x\left(\frac{bx^3}{a}+1\right)^{2/3}{}_2F_1\left(\frac{1}{3},\frac{11}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{77a^3(a+bx^3)^{2/3}}+\frac{9x}{77(a+bx^3)^{11/3}}+\frac{x(a-bx^3)}{7(a+bx^3)^{14/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^3)^2/(a + b*x^3)^(17/3),x]
```

```
[Out] (x*(a - b*x^3))/(7*(a + b*x^3)^(14/3)) + (9*x)/(77*(a + b*x^3)^(11/3)) + (5
7*x*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[1/3, 11/3, 4/3, -((b*x^3)/a)])/
(77*a^3*(a + b*x^3)^(2/3))
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
```

```
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a-bx^3)^2}{(a+bx^3)^{17/3}}\,dx &= \frac{x(a-bx^3)}{7\,(a+bx^3)^{14/3}}+\frac{\int \frac{12a^2b-6ab^2x^3}{(a+bx^3)^{14/3}}\,dx}{14ab}\\
&= \frac{x(a-bx^3)}{7\,(a+bx^3)^{14/3}}+\frac{9x}{77\,(a+bx^3)^{11/3}}+\frac{57}{77}\int \frac{1}{(a+bx^3)^{11/3}}\,dx\\
&= \frac{x(a-bx^3)}{7\,(a+bx^3)^{14/3}}+\frac{9x}{77\,(a+bx^3)^{11/3}}+\frac{\left(57\left(1+\frac{bx^3}{a}\right)^{2/3}\right)\int \frac{1}{\left(1+\frac{bx^3}{a}\right)^{11/3}}\,dx}{77a^3\,(a+bx^3)^{2/3}}\\
&= \frac{x(a-bx^3)}{7\,(a+bx^3)^{14/3}}+\frac{9x}{77\,(a+bx^3)^{11/3}}+\frac{57x\left(1+\frac{bx^3}{a}\right)^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{11}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{77a^3\,(a+bx^3)^{2/3}}
\end{aligned}$$

Mathematica [**A**]
time = 10.09, size = 106, normalized size = 1.14

$$\frac{x\left(2282a^4+4879a^3bx^3+6270a^2b^2x^6+3591ab^3x^9+798b^4x^{12}+798(a+bx^3)^4\left(1+\frac{bx^3}{a}\right)^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)\right)}{3080a^3\,(a+bx^3)^{14/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^3)^2/(a + b*x^3)^(17/3),x]
```

```
[Out] (x*(2282*a^4 + 4879*a^3*b*x^3 + 6270*a^2*b^2*x^6 + 3591*a*b^3*x^9 + 798*b^4
*x^12 + 798*(a + b*x^3)^4*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[1/3, 2/3,
 4/3, -((b*x^3)/a)]))/(3080*a^3*(a + b*x^3)^(14/3))
```

Maple [**F**]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{\left(-b\,x^{3}+a\right)^{2}}{\left(b\,x^{3}+a\right)^{\frac{17}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^3+a)^2/(b*x^3+a)^(17/3),x)
[Out] int((-b*x^3+a)^2/(b*x^3+a)^(17/3),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)^2/(b*x^3+a)^(17/3),x, algorithm="maxima")
[Out] integrate((b*x^3 - a)^2/(b*x^3 + a)^(17/3), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)^2/(b*x^3+a)^(17/3),x, algorithm="fricas")
[Out] integral((b^2*x^6 - 2*a*b*x^3 + a^2)*(b*x^3 + a)^(1/3)/(b^6*x^18 + 6*a*b^5*
x^15 + 15*a^2*b^4*x^12 + 20*a^3*b^3*x^9 + 15*a^4*b^2*x^6 + 6*a^5*b*x^3 + a^
6), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**3+a)**2/(b*x**3+a)**(17/3),x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)^2/(b*x^3+a)^(17/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 - a)^2/(b*x^3 + a)^(17/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a-b\,x^3)^2}{(b\,x^3+a)^{17/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x^3)^2/(a + b*x^3)^(17/3),x)
```

```
[Out] int((a - b*x^3)^2/(a + b*x^3)^(17/3), x)
```

3.56 $\int (a+bx^3)^{5/3} (c+dx^3)\, dx$

Optimal. Leaf size=174

$$\frac{5a(9bc-ad)x(a+bx^3)^{2/3}}{162b}+\frac{(9bc-ad)x(a+bx^3)^{5/3}}{54b}+\frac{dx(a+bx^3)^{8/3}}{9b}+\frac{5a^2(9bc-ad)\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{81\sqrt{3}\,b^{4/3}}$$

```
[Out] 5/162*a*(-a*d+9*b*c)*x*(b*x^3+a)^(2/3)/b+1/54*(-a*d+9*b*c)*x*(b*x^3+a)^(5/3
)/b+1/9*d*x*(b*x^3+a)^(8/3)/b-5/162*a^2*(-a*d+9*b*c)*ln(-b^(1/3)*x+(b*x^3+a
)^(1/3))/b^(4/3)+5/243*a^2*(-a*d+9*b*c)*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)
^(1/3))*3^(1/2))/b^(4/3)*3^(1/2)
```

Rubi [A]

time = 0.04, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {396, 201, 245}

$$\frac{5a^2\text{ArcTan}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)(9bc-ad)}{81\sqrt{3}\,b^{4/3}}-\frac{5a^2(9bc-ad)\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}\,x\right)}{162b^{4/3}}+\frac{x(a+bx^3)^{5/3}(9bc-ad)}{54b}+\frac{5ax(a+bx^3)^{2/3}(9bc-ad)}{162b}+\frac{dx(a+bx^3)^{8/3}}{9b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(5/3)*(c + d*x^3),x]
```

```
[Out] (5*a*(9*b*c - a*d)*x*(a + b*x^3)^(2/3))/(162*b) + ((9*b*c - a*d)*x*(a + b*x
^3)^(5/3))/(54*b) + (d*x*(a + b*x^3)^(8/3))/(9*b) + (5*a^2*(9*b*c - a*d)*Ar
cTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(81*Sqrt[3]*b^(4/3)) -
 (5*a^2*(9*b*c - a*d)*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)])/(162*b^(4/3))
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rubi steps

$$\int \left(a+bx^3\right)^{5/3}\left(c+dx^3\right)\,dx = \frac{dx\left(a+bx^3\right)^{8/3}}{9b} - \frac{(-9bc+ad)\int \left(a+bx^3\right)^{5/3}\,dx}{9b}$$

$$= \frac{(9bc-ad)x\left(a+bx^3\right)^{5/3}}{54b} + \frac{dx\left(a+bx^3\right)^{8/3}}{9b} + \frac{(5a(9bc-ad))\int \left(a+bx^3\right)^{2/3}\,dx}{54b}$$

$$= \frac{5a(9bc-ad)x\left(a+bx^3\right)^{2/3}}{162b} + \frac{(9bc-ad)x\left(a+bx^3\right)^{5/3}}{54b} + \frac{dx\left(a+bx^3\right)^{8/3}}{9b} + \frac{(5 \cdots}{\cdots}$$

$$= \frac{5a(9bc-ad)x\left(a+bx^3\right)^{2/3}}{162b} + \frac{(9bc-ad)x\left(a+bx^3\right)^{5/3}}{54b} + \frac{dx\left(a+bx^3\right)^{8/3}}{9b} + \frac{5a \cdots}{\cdots}$$

Mathematica [A]
time = 0.61, size = 208, normalized size = 1.20

$$\frac{3\sqrt[3]{b}\,x\left(a+bx^3\right)^{2/3}\left(10a^2d+9b^2x^3\left(3c+2dx^3\right)+ab\left(72c+33dx^3\right)\right)-10\sqrt{3}\,a^2(-9bc+ad)\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[3]{b}\,x}{\sqrt[3]{b}\,x+2\sqrt[3]{a+bx^3}}\right)+10a^2(-9bc+ad)\log\left(-\sqrt[3]{b}\,x+\sqrt[3]{a+bx^3}\right)-5a^2(-9bc+ad)\log\left(b^{2/3}x^2+\sqrt[3]{b}\,x\sqrt[3]{a+bx^3}+\left(a+bx^3\right)^{2/3}\right)}{486b^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^(5/3)*(c + d*x^3),x]
```

```
[Out] (3*b^(1/3)*x*(a + b*x^3)^(2/3)*(10*a^2*d + 9*b^2*x^3*(3*c + 2*d*x^3) + a*b*
(72*c + 33*d*x^3)) - 10*Sqrt[3]*a^2*(-9*b*c + a*d)*ArcTan[(Sqrt[3]*b^(1/3)*
x)/(b^(1/3)*x + 2*(a + b*x^3)^(1/3))] + 10*a^2*(-9*b*c + a*d)*Log[-(b^(1/3)
*x) + (a + b*x^3)^(1/3)] - 5*a^2*(-9*b*c + a*d)*Log[b^(2/3)*x^2 + b^(1/3)*x
*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)])/(486*b^(4/3))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\,x^{3}+a\right)^{\frac{5}{3}}\left(d\,x^{3}+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(5/3)*(d*x^3+c),x)
```

[Out] int((b*x^3+a)^(5/3)*(d*x^3+c),x)

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 406 vs. $2(143) = 286$.
time = 0.49, size = 406, normalized size = 2.33

$$-\frac{1}{54}\left(\frac{10\sqrt{3}a^2\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2(bx^3+a)^{\frac{1}{3}}}{x}\right)}{3b^{\frac{1}{3}}}\right)}{b^{\frac{1}{3}}}-\frac{5a^2\log\left(b^{\frac{2}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{(bx^3+a)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{1}{3}}}+\frac{10a^2\log\left(-b^{\frac{1}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}}{x}\right)}{b^{\frac{1}{3}}}+\frac{3\left(\frac{5(bx^3+a)^{\frac{2}{3}}a^2b}{x^2}-\frac{8(bx^3+a)^{\frac{5}{3}}a^2}{x^5}\right)}{b^2-\frac{2(bx^3+a)b}{x^3}+\frac{(bx^3+a)^2}{x^6}}\right)c+\frac{1}{486}\left(\frac{10\sqrt{3}a^3\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2(bx^3+a)^{\frac{1}{3}}}{x}\right)}{3b^{\frac{1}{3}}}\right)}{b^{\frac{4}{3}}}-\frac{5a^3\log\left(b^{\frac{2}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{(bx^3+a)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{4}{3}}}+\frac{10a^3\log\left(-b^{\frac{1}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}}{x}\right)}{b^{\frac{4}{3}}}+\frac{3\left(\frac{5(bx^3+a)^{\frac{2}{3}}a^3b^2}{x^2}-\frac{13(bx^3+a)^{\frac{5}{3}}a^3b}{x^5}-\frac{10(bx^3+a)^{\frac{8}{3}}a^3}{x^8}\right)}{b^4-\frac{3(bx^3+a)b^3}{x^3}+\frac{3(bx^3+a)^2b^2}{x^6}-\frac{(bx^3+a)^3b}{x^9}}\right)d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^(5/3)*(d*x^3+c),x, algorithm="maxima")

[Out] -1/54*(10*sqrt(3)*a^2*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1/3))/b^(1/3) - 5*a^2*log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2/3)/x^2)/b^(1/3) + 10*a^2*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(1/3) + 3*(5*(b*x^3 + a)^(2/3)*a^2*b/x^2 - 8*(b*x^3 + a)^(5/3)*a^2/x^5)/(b^2 - 2*(b*x^3 + a)*b/x^3 + (b*x^3 + a)^2/x^6))*c + 1/486*(10*sqrt(3)*a^3*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1/3))/b^(4/3) - 5*a^3*log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2/3)/x^2)/b^(4/3) + 10*a^3*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(4/3) + 3*(5*(b*x^3 + a)^(2/3)*a^3*b^2/x^2 - 13*(b*x^3 + a)^(5/3)*a^3*b/x^5 - 10*(b*x^3 + a)^(8/3)*a^3/x^8)/(b^4 - 3*(b*x^3 + a)*b^3/x^3 + 3*(b*x^3 + a)^2*b^2/x^6 - (b*x^3 + a)^3*b/x^9))*d

Fricas [A]
time = 3.55, size = 482, normalized size = 2.77

$$\left[-\frac{15\sqrt{\frac{1}{3}}(9a^2b^2c-a^3bd)\sqrt{-\frac{1}{b^{\frac{2}{3}}}}\log\left(3bx^3-3(bx^3+a)^{\frac{1}{3}}b^{\frac{2}{3}}x^2-3\sqrt{\frac{1}{3}}\left(b^{\frac{4}{3}}x^3+(bx^3+a)^{\frac{1}{3}}bx^2-2(bx^3+a)^{\frac{2}{3}}b^{\frac{2}{3}}x\right)\sqrt{-\frac{1}{b^{\frac{2}{3}}}}+2a\right)+10(9a^2bc-a^3d)b^{\frac{2}{3}}\log\left(-\frac{b^{\frac{1}{3}}x-(bx^3+a)^{\frac{1}{3}}}{x}\right)-5(9a^2bc-a^3d)b^{\frac{2}{3}}\log\left(\frac{b^{\frac{2}{3}}x^2+(bx^3+a)^{\frac{1}{3}}b^{\frac{1}{3}}x+(bx^3+a)^{\frac{2}{3}}}{x^2}\right)-3\left(18b^3dx^7+3(9b^3c+11ab^2d)x^4+2(36ab^2c+5a^2bd)x\right)(bx^3+a)^{\frac{2}{3}}}{486b^2},\ -\frac{10(9a^2bc-a^3d)b^{\frac{2}{3}}\log\left(-\frac{b^{\frac{1}{3}}x-(bx^3+a)^{\frac{1}{3}}}{x}\right)-5(9a^2bc-a^3d)b^{\frac{2}{3}}\log\left(\frac{b^{\frac{2}{3}}x^2+(bx^3+a)^{\frac{1}{3}}b^{\frac{1}{3}}x+(bx^3+a)^{\frac{2}{3}}}{x^2}\right)+\frac{30\sqrt{\frac{1}{3}}(9a^2b^2c-a^3bd)\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(b^{\frac{1}{3}}x+2(bx^3+a)^{\frac{1}{3}}\right)}{b^{\frac{1}{3}}x}\right)}{b^{\frac{1}{3}}}-3\left(18b^3dx^7+3(9b^3c+11ab^2d)x^4+2(36ab^2c+5a^2bd)x\right)(bx^3+a)^{\frac{2}{3}}}{486b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^(5/3)*(d*x^3+c),x, algorithm="fricas")

[Out] [-1/486*(15*sqrt(1/3)*(9*a^2*b^2*c - a^3*b*d)*sqrt(-1/b^(2/3))*log(3*b*x^3 - 3*(b*x^3 + a)^(1/3)*b^(2/3)*x^2 - 3*sqrt(1/3)*(b^(4/3)*x^3 + (b*x^3 + a)^(1/3)*b*x^2 - 2*(b*x^3 + a)^(2/3)*b^(2/3)*x)*sqrt(-1/b^(2/3)) + 2*a) + 10*(9*a^2*b*c - a^3*d)*b^(2/3)*log(-(b^(1/3)*x - (b*x^3 + a)^(1/3))/x) - 5*(9*a^2*b*c - a^3*d)*b^(2/3)*log((b^(2/3)*x^2 + (b*x^3 + a)^(1/3)*b^(1/3)*x + (b*x^3 + a)^(2/3))/x^2) - 3*(18*b^3*d*x^7 + 3*(9*b^3*c + 11*a*b^2*d)*x^4 + 2*(36*a*b^2*c + 5*a^2*b*d)*x)*(b*x^3 + a)^(2/3))/b^2, -1/486*(10*(9*a^2*b*c - a^3*d)*b^(2/3)*log(-(b^(1/3)*x - (b*x^3 + a)^(1/3))/x) - 5*(9*a^2*b*c - a^3*d)*b^(2/3)*log((b^(2/3)*x^2 + (b*x^3 + a)^(1/3)*b^(1/3)*x + (b*x^3 + a)^(2/3))/x^2) + 30*sqrt(1/3)*(9*a^2*b^2*c - a^3*b*d)*arctan(sqrt(1/3)*(b^(1/3)*x + 2*(b*x^3 + a)^(1/3))/(b^(1/3)*x))/b^(1/3) - 3*(18*b^3*d*x^7 + 3*(9*b^3*c + 11*a*b^2*d)*x^4 + 2*(36*a*b^2*c + 5*a^2*b*d)*x)*(b*x^3 + a)^(2/3))/b^2
]

Sympy [C] Result contains complex when optimal does not.
time = 7.07, size = 170, normalized size = 0.98

$$\frac{a^{\frac{5}{3}}cx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)}+\frac{a^{\frac{5}{3}}dx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)}+\frac{a^{\frac{2}{3}}bcx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)}+\frac{a^{\frac{2}{3}}bdx^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{10}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(5/3)*(d*x**3+c),x)
```

```
[Out] a**(5/3)*c*x*gamma(1/3)*hyper((-2/3, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a
)/(3*gamma(4/3)) + a**(5/3)*d*x**4*gamma(4/3)*hyper((-2/3, 4/3), (7/3,), b*
x**3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + a**(2/3)*b*c*x**4*gamma(4/3)*hyper
((-2/3, 4/3), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + a**(2/3)*b
*d*x**7*gamma(7/3)*hyper((-2/3, 7/3), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3
*gamma(10/3))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(5/3)*(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(5/3)*(d*x^3 + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(b\,x^3+a\right)}^{5/3}\,\left(d\,x^3+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(5/3)*(c + d*x^3),x)
```

```
[Out] int((a + b*x^3)^(5/3)*(c + d*x^3), x)
```

3.57 $\int (a+bx^3)^{2/3} (c+dx^3)\, dx$

Optimal. Leaf size=141

$$\frac{(6bc-ad)x(a+bx^3)^{2/3}}{18b}+\frac{dx(a+bx^3)^{5/3}}{6b}+\frac{a(6bc-ad)\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{9\sqrt{3}\,b^{4/3}}-\frac{a(6bc-ad)\log\left(-\sqrt[3]{b}\,x+\right.}{18b^{4/3}}$$

```
[Out] 1/18*(-a*d+6*b*c)*x*(b*x^3+a)^(2/3)/b+1/6*d*x*(b*x^3+a)^(5/3)/b-1/18*a*(-a*
d+6*b*c)*ln(-b^(1/3)*x+(b*x^3+a)^(1/3))/b^(4/3)+1/27*a*(-a*d+6*b*c)*arctan(
1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/b^(4/3)*3^(1/2)
```

Rubi [A]

time = 0.03, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {396, 201, 245}

$$\frac{a\text{ArcTan}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)(6bc-ad)}{9\sqrt{3}\,b^{4/3}}-\frac{a(6bc-ad)\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}\,x\right)}{18b^{4/3}}+\frac{x(a+bx^3)^{2/3}(6bc-ad)}{18b}+\frac{dx(a+bx^3)^{5/3}}{6b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(2/3)*(c + d*x^3),x]
```

```
[Out] ((6*b*c - a*d)*x*(a + b*x^3)^(2/3))/(18*b) + (d*x*(a + b*x^3)^(5/3))/(6*b)
+ (a*(6*b*c - a*d)*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(
9*Sqrt[3]*b^(4/3)) - (a*(6*b*c - a*d)*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)]
)/(18*b^(4/3))
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a+bx^3\right)^{2/3}\left(c+dx^3\right)\,dx &= \frac{dx\left(a+bx^3\right)^{5/3}}{6b}-\frac{(-6bc+ad)\int \left(a+bx^3\right)^{2/3}\,dx}{6b}\\ &= \frac{(6bc-ad)x\left(a+bx^3\right)^{2/3}}{18b}+\frac{dx\left(a+bx^3\right)^{5/3}}{6b}+\frac{(a(6bc-ad))\int \frac{1}{\sqrt[3]{a+bx^3}}\,dx}{9b}\\ &= \frac{(6bc-ad)x\left(a+bx^3\right)^{2/3}}{18b}+\frac{dx\left(a+bx^3\right)^{5/3}}{6b}+\frac{a(6bc-ad)\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right.}{9\sqrt{3}\,b^{4/3}}\end{aligned}$$

Mathematica [A]
time = 0.57, size = 180, normalized size = 1.28

$$\frac{3\sqrt[3]{b}\,x\left(a+bx^3\right)^{2/3}\left(6bc+2ad+3bdx^3\right)-2\sqrt{3}\,a(-6bc+ad)\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{b}\,x}{\sqrt[3]{b}\,x+2\sqrt[3]{a+bx^3}}\right)+2a(-6bc+ad)\log\left(-\sqrt[3]{b}\,x+\sqrt[3]{a+bx^3}\right)-a(-6bc+ad)\log\left(b^{2/3}x^2+\sqrt[3]{b}\,x\sqrt[3]{a+bx^3}+\left(a+bx^3\right)^{2/3}\right)}{54b^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^(2/3)*(c + d*x^3),x]
```

```
[Out] (3*b^(1/3)*x*(a + b*x^3)^(2/3)*(6*b*c + 2*a*d + 3*b*d*x^3) - 2*Sqrt[3]*a*(-
6*b*c + a*d)*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2*(a + b*x^3)^(1/3))]
+ 2*a*(-6*b*c + a*d)*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)] - a*(-6*b*c + a*
d)*Log[b^(2/3)*x^2 + b^(1/3)*x*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)])/(54*
b^(4/3))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \left(b\,x^3+a\right)^{\frac{2}{3}}\left(d\,x^3+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(2/3)*(d*x^3+c),x)
```

```
[Out] int((b*x^3+a)^(2/3)*(d*x^3+c),x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 322 vs. $2(114) = 228$.

time = 0.51, size = 322, normalized size = 2.28

$$-\frac{1}{9}\left(\frac{2\sqrt{3}a\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2(bx^3+a)^{\frac{1}{3}}}{x}\right)}{3b^{\frac{1}{3}}}\right)}{b^{\frac{1}{3}}}-\frac{a\log\left(b^{\frac{2}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{(bx^3+a)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{1}{3}}}+\frac{2a\log\left(-b^{\frac{1}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}}{x}\right)}{b^{\frac{1}{3}}}+\frac{3(bx^3+a)^{\frac{2}{3}}a}{\left(b-\frac{bx^3+a}{x^3}\right)x^2}\right)c+\frac{1}{54}\left(\frac{2\sqrt{3}a^2\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2(bx^3+a)^{\frac{1}{3}}}{x}\right)}{3b^{\frac{1}{3}}}\right)}{b^{\frac{4}{3}}}-\frac{a^2\log\left(b^{\frac{2}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{(bx^3+a)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{4}{3}}}+\frac{2a^2\log\left(-b^{\frac{1}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}}{x}\right)}{b^{\frac{4}{3}}}+\frac{3\left(\frac{(bx^3+a)^{\frac{2}{3}}a^2b}{x^2}+\frac{2(bx^3+a)^{\frac{5}{3}}a^2}{x^5}\right)}{b^3-\frac{2(bx^3+a)b^2}{x^3}+\frac{(bx^3+a)^2b}{x^6}}\right)d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)*(d*x^3+c),x, algorithm="maxima")
```

```
[Out] -1/9*(2*sqrt(3)*a*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1
/3))/b^(1/3) - a*log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2
/3)/x^2)/b^(1/3) + 2*a*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(1/3) + 3*(b*x
^3 + a)^(2/3)*a/((b - (b*x^3 + a)/x^3)*x^2))*c + 1/54*(2*sqrt(3)*a^2*arctan
(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1/3))/b^(4/3) - a^2*log(b
^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2/3)/x^2)/b^(4/3) + 2*a
^2*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(4/3) + 3*((b*x^3 + a)^(2/3)*a^2*b
/x^2 + 2*(b*x^3 + a)^(5/3)*a^2/x^5)/(b^3 - 2*(b*x^3 + a)*b^2/x^3 + (b*x^3 +
 a)^2*b/x^6))*d
```

Fricas [A]

time = 3.69, size = 424, normalized size = 3.01

$$\left[-\frac{3\sqrt{\frac{1}{3}}(6ab^2c-a^2bd)\sqrt{-\frac{1}{b^{\frac{2}{3}}}}\log\left(3bx^3-3(bx^3+a)^{\frac{1}{3}}b^{\frac{2}{3}}x^2-3\sqrt{\frac{1}{3}}\left(b^{\frac{4}{3}}x^3+(bx^3+a)^{\frac{1}{3}}bx^2-2(bx^3+a)^{\frac{2}{3}}b^{\frac{2}{3}}x\right)\sqrt{-\frac{1}{b^{\frac{2}{3}}}}+2a\right)+2(6abc-a^2d)b^{\frac{2}{3}}\log\left(-\frac{b^{\frac{1}{3}}x-(bx^3+a)^{\frac{1}{3}}}{x}\right)-(6abc-a^2d)b^{\frac{2}{3}}\log\left(\frac{b^{\frac{2}{3}}x^2+(bx^3+a)^{\frac{1}{3}}b^{\frac{1}{3}}x+(bx^3+a)^{\frac{2}{3}}}{x^2}\right)-3(3b^2dx^4+2(3b^2c+abd)x)(bx^3+a)^{\frac{2}{3}}}{54b^2},\ -\frac{2(6abc-a^2d)b^{\frac{2}{3}}\log\left(-\frac{b^{\frac{1}{3}}x-(bx^3+a)^{\frac{1}{3}}}{x}\right)-(6abc-a^2d)b^{\frac{2}{3}}\log\left(\frac{b^{\frac{2}{3}}x^2+(bx^3+a)^{\frac{1}{3}}b^{\frac{1}{3}}x+(bx^3+a)^{\frac{2}{3}}}{x^2}\right)+\frac{6\sqrt{\frac{1}{3}}(6ab^2c-a^2bd)\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(b^{\frac{1}{3}}x+2(bx^3+a)^{\frac{1}{3}}\right)}{b^{\frac{1}{3}}x}\right)}{b^{\frac{1}{3}}}-3(3b^2dx^4+2(3b^2c+abd)x)(bx^3+a)^{\frac{2}{3}}}{54b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)*(d*x^3+c),x, algorithm="fricas")
```

```
[Out] [-1/54*(3*sqrt(1/3)*(6*a*b^2*c - a^2*b*d)*sqrt(-1/b^(2/3))*log(3*b*x^3 - 3*
(b*x^3 + a)^(1/3)*b^(2/3)*x^2 - 3*sqrt(1/3)*(b^(4/3)*x^3 + (b*x^3 + a)^(1/3
)*b*x^2 - 2*(b*x^3 + a)^(2/3)*b^(2/3)*x)*sqrt(-1/b^(2/3)) + 2*a) + 2*(6*a*b
*c - a^2*d)*b^(2/3)*log(-(b^(1/3)*x - (b*x^3 + a)^(1/3))/x) - (6*a*b*c - a^
2*d)*b^(2/3)*log((b^(2/3)*x^2 + (b*x^3 + a)^(1/3)*b^(1/3)*x + (b*x^3 + a)^(
2/3))/x^2) - 3*(3*b^2*d*x^4 + 2*(3*b^2*c + a*b*d)*x)*(b*x^3 + a)^(2/3))/b^2
, -1/54*(2*(6*a*b*c - a^2*d)*b^(2/3)*log(-(b^(1/3)*x - (b*x^3 + a)^(1/3))/x
) - (6*a*b*c - a^2*d)*b^(2/3)*log((b^(2/3)*x^2 + (b*x^3 + a)^(1/3)*b^(1/3)*
x + (b*x^3 + a)^(2/3))/x^2) + 6*sqrt(1/3)*(6*a*b^2*c - a^2*b*d)*arctan(sqrt
(1/3)*(b^(1/3)*x + 2*(b*x^3 + a)^(1/3))/(b^(1/3)*x))/b^(1/3) - 3*(3*b^2*d*x
^4 + 2*(3*b^2*c + a*b*d)*x)*(b*x^3 + a)^(2/3))/b^2]
```

Sympy [C] Result contains complex when optimal does not.

time = 1.96, size = 82, normalized size = 0.58

$$\frac{a^{\frac{2}{3}}cx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)}+\frac{a^{\frac{2}{3}}dx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(2/3)*(d*x**3+c),x)
```

```
[Out] a**(2/3)*c*x*gamma(1/3)*hyper((-2/3, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a
)/(3*gamma(4/3)) + a**(2/3)*d*x**4*gamma(4/3)*hyper((-2/3, 4/3), (7/3,), b*
x**3*exp_polar(I*pi)/a)/(3*gamma(7/3))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)*(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(2/3)*(d*x^3 + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(b\,x^3+a\right)^{2/3}\,\left(d\,x^3+c\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(2/3)*(c + d*x^3),x)
```

```
[Out] int((a + b*x^3)^(2/3)*(c + d*x^3), x)
```

3.58 $\int \frac{c+dx^3}{\sqrt[3]{a+bx^3}}\,dx$

Optimal. Leaf size=111

$$\frac{dx(a+bx^3)^{2/3}}{3b}+\frac{(3bc-ad)\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}\,b^{4/3}}-\frac{(3bc-ad)\log\left(-\sqrt[3]{b}\,x+\sqrt[3]{a+bx^3}\right)}{6b^{4/3}}$$

```
[Out] 1/3*d*x*(b*x^3+a)^(2/3)/b-1/6*(-a*d+3*b*c)*ln(-b^(1/3)*x+(b*x^3+a)^(1/3))/b
^(4/3)+1/9*(-a*d+3*b*c)*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))
/b^(4/3)*3^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {396, 245}

$$\frac{\text{ArcTan}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)(3bc-ad)}{3\sqrt{3}\,b^{4/3}}-\frac{(3bc-ad)\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}\,x\right)}{6b^{4/3}}+\frac{dx(a+bx^3)^{2/3}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)/(a + b*x^3)^(1/3),x]
```

```
[Out] (d*x*(a + b*x^3)^(2/3))/(3*b) + ((3*b*c - a*d)*ArcTan[(1 + (2*b^(1/3)*x)/(a
 + b*x^3)^(1/3))/Sqrt[3]])/(3*Sqrt[3]*b^(4/3)) - ((3*b*c - a*d)*Log[-(b^(1/
3)*x) + (a + b*x^3)^(1/3)])/(6*b^(4/3))
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rubi steps

$$\int \frac{c+dx^3}{\sqrt[3]{a+bx^3}}\,dx = \frac{dx(a+bx^3)^{2/3}}{3b} - \frac{(-3bc+ad)\int \frac{1}{\sqrt[3]{a+bx^3}}\,dx}{3b}$$

$$= \frac{dx(a+bx^3)^{2/3}}{3b} + \frac{(3bc-ad)\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}\,b^{4/3}} - \frac{(3bc-ad)\log\left(-\sqrt[3]{b}\,x+\sqrt[3]{a+b\right.}{6b^{4/3}}$$

Mathematica [A]
time = 0.39, size = 163, normalized size = 1.47

$$\frac{6\sqrt[3]{b}\,dx(a+bx^3)^{2/3}+2\sqrt{3}\,(3bc-ad)\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[3]{b}\,x}{\sqrt[3]{b}\,x+2\sqrt[3]{a+bx^3}}\right)+2(-3bc+ad)\log\left(-\sqrt[3]{b}\,x+\sqrt[3]{a+bx^3}\right)+(3bc-ad)\log\left(b^{2/3}x^2+\sqrt[3]{b}\,x\sqrt[3]{a+bx^3}+(a+bx^3)^{2/3}\right)}{18b^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3)/(a + b*x^3)^(1/3),x]
```

```
[Out] (6*b^(1/3)*d*x*(a + b*x^3)^(2/3) + 2*Sqrt[3]*(3*b*c - a*d)*ArcTan[(Sqrt[3]*
b^(1/3)*x)/(b^(1/3)*x + 2*(a + b*x^3)^(1/3))] + 2*(-3*b*c + a*d)*Log[-(b^(1
/3)*x) + (a + b*x^3)^(1/3)] + (3*b*c - a*d)*Log[b^(2/3)*x^2 + b^(1/3)*x*(a
+ b*x^3)^(1/3) + (a + b*x^3)^(2/3)])/(18*b^(4/3))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d\,x^3+c}{(b\,x^3+a)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)/(b*x^3+a)^(1/3),x)
```

```
[Out] int((d*x^3+c)/(b*x^3+a)^(1/3),x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 244 vs. 2(88) = 176.
time = 0.52, size = 244, normalized size = 2.20

$$-\frac{1}{6}\left(\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2(bx^3+a)^{\frac{1}{3}}}{x}\right)}{3b^{\frac{1}{3}}}\right)}{b^{\frac{1}{3}}}-\frac{\log\left(b^{\frac{2}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{(bx^3+a)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{1}{3}}}+\frac{2\log\left(-b^{\frac{1}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}}{x}\right)}{b^{\frac{1}{3}}}\right)c+\frac{1}{18}\left(\frac{2\sqrt{3}\,a\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2(bx^3+a)^{\frac{1}{3}}}{x}\right)}{3b^{\frac{1}{3}}}\right)}{b^{\frac{4}{3}}}-\frac{a\log\left(b^{\frac{2}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{(bx^3+a)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{4}{3}}}+\frac{2a\log\left(-b^{\frac{1}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}}{x}\right)}{b^{\frac{4}{3}}}-\frac{6(bx^3+a)^{\frac{2}{3}}a}{\left(b^2-\frac{(bx^3+a)b}{x^3}\right)x^2}\right)d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)/(b*x^3+a)^(1/3),x, algorithm="maxima")
```

```
[Out] -1/6*(2*sqrt(3)*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1/3
))/b^(1/3) - log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2/3)/
x^2)/b^(1/3) + 2*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(1/3))*c + 1/18*(2*s
qrt(3)*a*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1/3))/b^(4
/3) - a*log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2/3)/x^2)/
b^(4/3) + 2*a*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(4/3) - 6*(b*x^3 + a)^(
2/3)*a/((b^2 - (b*x^3 + a)*b/x^3)*x^2))*d
```

Fricas [**A**]
time = 2.45, size = 362, normalized size = 3.26

$$\left[\frac{6\,(bx^3+a)^{\frac{2}{3}}bdx - 3\,\sqrt{\frac{1}{3}}\,(3\,b^2c-abd)\sqrt{-\frac{1}{b^{\frac{2}{3}}}}\,\log\left(3\,bx^3 - 3\,(bx^3+a)^{\frac{1}{3}}b^{\frac{2}{3}}x^2 - 3\,\sqrt{\frac{1}{3}}\left(b^{\frac{4}{3}}x^3 + (bx^3+a)^{\frac{1}{3}}bx^2 - 2\,(bx^3+a)^{\frac{2}{3}}b^{\frac{2}{3}}x\right)\sqrt{-\frac{1}{b^{\frac{2}{3}}}} + 2\,a\right) - 2\,(3\,bc-ad)b^{\frac{2}{3}}\log\left(-\frac{b^{\frac{1}{3}}x-(bx^3+a)^{\frac{1}{3}}}{x}\right) + (3\,bc-ad)b^{\frac{2}{3}}\log\left(\frac{b^{\frac{2}{3}}x^2+(bx^3+a)^{\frac{1}{3}}b^{\frac{1}{3}}x+(bx^3+a)^{\frac{2}{3}}}{x^2}\right)}{18\,b^2}, \frac{6\,(bx^3+a)^{\frac{2}{3}}bdx - 2\,(3\,bc-ad)b^{\frac{2}{3}}\log\left(-\frac{b^{\frac{1}{3}}x-(bx^3+a)^{\frac{1}{3}}}{x}\right) + (3\,bc-ad)b^{\frac{2}{3}}\log\left(\frac{b^{\frac{2}{3}}x^2+(bx^3+a)^{\frac{1}{3}}b^{\frac{1}{3}}x+(bx^3+a)^{\frac{2}{3}}}{x^2}\right) - \frac{6\,\sqrt{\frac{1}{3}}\,(3\,b^2c-abd)\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(b^{\frac{1}{3}}x+2\,(bx^3+a)^{\frac{1}{3}}\right)}{b^{\frac{1}{3}}x}\right)}{b^{\frac{1}{3}}}}{18\,b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)/(b*x^3+a)^(1/3),x, algorithm="fricas")
```

```
[Out] [1/18*(6*(b*x^3 + a)^(2/3)*b*d*x - 3*sqrt(1/3)*(3*b^2*c - a*b*d)*sqrt(-1/b^
(2/3))*log(3*b*x^3 - 3*(b*x^3 + a)^(1/3)*b^(2/3)*x^2 - 3*sqrt(1/3)*(b^(4/3)
*x^3 + (b*x^3 + a)^(1/3)*b*x^2 - 2*(b*x^3 + a)^(2/3)*b^(2/3)*x)*sqrt(-1/b^(
2/3)) + 2*a) - 2*(3*b*c - a*d)*b^(2/3)*log(-(b^(1/3)*x - (b*x^3 + a)^(1/3))
/x) + (3*b*c - a*d)*b^(2/3)*log((b^(2/3)*x^2 + (b*x^3 + a)^(1/3)*b^(1/3)*x
+ (b*x^3 + a)^(2/3))/x^2))/b^2, 1/18*(6*(b*x^3 + a)^(2/3)*b*d*x - 2*(3*b*c
- a*d)*b^(2/3)*log(-(b^(1/3)*x - (b*x^3 + a)^(1/3))/x) + (3*b*c - a*d)*b^(2
/3)*log((b^(2/3)*x^2 + (b*x^3 + a)^(1/3)*b^(1/3)*x + (b*x^3 + a)^(2/3))/x^2
) - 6*sqrt(1/3)*(3*b^2*c - a*b*d)*arctan(sqrt(1/3)*(b^(1/3)*x + 2*(b*x^3 +
a)^(1/3))/(b^(1/3)*x))/b^(1/3))/b^2]
```

Sympy [**C**] Result contains complex when optimal does not.
time = 1.29, size = 78, normalized size = 0.70

$$\frac{cx\Gamma\left(\frac{1}{3}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{3},\frac{1}{3}\\\frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt[3]{a}\,\Gamma\left(\frac{4}{3}\right)} + \frac{dx^4\Gamma\left(\frac{4}{3}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{3},\frac{4}{3}\\\frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt[3]{a}\,\Gamma\left(\frac{7}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)/(b*x**3+a)**(1/3),x)
```

```
[Out] c*x*gamma(1/3)*hyper((1/3, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(1
/3)*gamma(4/3)) + d*x**4*gamma(4/3)*hyper((1/3, 4/3), (7/3,), b*x**3*exp_po
lar(I*pi)/a)/(3*a**(1/3)*gamma(7/3))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)/(b*x^3+a)^(1/3),x, algorithm="giac")
[Out] integrate((d*x^3 + c)/(b*x^3 + a)^(1/3), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{d\,x^3+c}{{\left(b\,x^3+a\right)}^{1/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)/(a + b*x^3)^(1/3),x)
[Out] int((c + d*x^3)/(a + b*x^3)^(1/3), x)
```

3.59 $\int \frac{c+dx^3}{(a+bx^3)^{4/3}}\,dx$

Optimal. Leaf size=99

$$\frac{(bc-ad)x}{ab\sqrt[3]{a+bx^3}}+\frac{d\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,b^{4/3}}-\frac{d\log\left(-\sqrt[3]{b}\,x+\sqrt[3]{a+bx^3}\right)}{2b^{4/3}}$$

```
[Out] (-a*d+b*c)*x/a/b/(b*x^3+a)^(1/3)-1/2*d*ln(-b^(1/3)*x+(b*x^3+a)^(1/3))/b^(4/
3)+1/3*d*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/b^(4/3)*3^(1/2
)
```

Rubi [A]
time = 0.02, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {393, 245}

$$\frac{d\text{ArcTan}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,b^{4/3}}-\frac{d\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}\,x\right)}{2b^{4/3}}+\frac{x(bc-ad)}{ab\sqrt[3]{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)/(a + b*x^3)^(4/3),x]
```

```
[Out] ((b*c - a*d)*x)/(a*b*(a + b*x^3)^(1/3)) + (d*ArcTan[(1 + (2*b^(1/3)*x)/(a +
 b*x^3)^(1/3))/Sqrt[3]])/(Sqrt[3]*b^(4/3)) - (d*Log[-(b^(1/3)*x) + (a + b*x
^3)^(1/3)])/(2*b^(4/3))
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rubi steps

$$\int \frac{c+dx^3}{(a+bx^3)^{4/3}}\,dx = \frac{(bc-ad)x}{ab\sqrt[3]{a+bx^3}} + \frac{d\int \frac{1}{\sqrt[3]{a+bx^3}}\,dx}{b}$$

$$= \frac{(bc-ad)x}{ab\sqrt[3]{a+bx^3}} + \frac{d\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,b^{4/3}} - \frac{d\log\left(-\sqrt[3]{b}\,x+\sqrt[3]{a+bx^3}\right)}{2b^{4/3}}$$

Mathematica [A]
time = 0.33, size = 150, normalized size = 1.52

$$\frac{\frac{6\sqrt[3]{b}\,(bc-ad)x}{a\sqrt[3]{a+bx^3}} + 2\sqrt{3}\,d\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[3]{b}\,x}{\sqrt[3]{b}\,x+2\sqrt[3]{a+bx^3}}\right) - 2d\log\left(-\sqrt[3]{b}\,x+\sqrt[3]{a+bx^3}\right) + d\log\left(b^{2/3}x^2+\sqrt[3]{b}\,x\sqrt[3]{a+bx^3}+(a+bx^3)^{2/3}\right)}{6b^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3)/(a + b*x^3)^(4/3),x]
[Out] ((6*b^(1/3)*(b*c - a*d)*x)/(a*(a + b*x^3)^(1/3)) + 2*Sqrt[3]*d*ArcTan[(Sqrt
[3]*b^(1/3)*x)/(b^(1/3)*x + 2*(a + b*x^3)^(1/3))] - 2*d*Log[-(b^(1/3)*x) +
(a + b*x^3)^(1/3)] + d*Log[b^(2/3)*x^2 + b^(1/3)*x*(a + b*x^3)^(1/3) + (a +
 b*x^3)^(2/3)])/(6*b^(4/3))
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{d\,x^3+c}{(b\,x^3+a)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)/(b*x^3+a)^(4/3),x)
[Out] int((d*x^3+c)/(b*x^3+a)^(4/3),x)
```

Maxima [A]
time = 0.50, size = 134, normalized size = 1.35

$$-\frac{1}{6}\,d\left(\frac{2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\,(bx^3+a)^{\frac{1}{3}}}{x}\right)}{3\,b^{\frac{1}{3}}}\right)}{b^{\frac{4}{3}}} + \frac{6\,x}{(bx^3+a)^{\frac{1}{3}}b} - \frac{\log\left(b^{\frac{2}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{(bx^3+a)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{4}{3}}} + \frac{2\,\log\left(-b^{\frac{1}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}}{x}\right)}{b^{\frac{4}{3}}}\right) + \frac{cx}{(bx^3+a)^{\frac{1}{3}}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)/(b*x^3+a)^(4/3),x, algorithm="maxima")
```

```
[Out] -1/6*d*(2*sqrt(3)*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1
/3))/b^(4/3) + 6*x/((b*x^3 + a)^(1/3)*b) - log(b^(2/3) + (b*x^3 + a)^(1/3)*
b^(1/3)/x + (b*x^3 + a)^(2/3)/x^2)/b^(4/3) + 2*log(-b^(1/3) + (b*x^3 + a)^(
1/3)/x)/b^(4/3)) + c*x/((b*x^3 + a)^(1/3)*a)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 220 vs. $2(81) = 162$.
time = 3.93, size = 488, normalized size = 4.93

$$\left[\frac{3\sqrt{\frac{1}{3}}\left(ab^{2}dx^{3}+a^{2}bd\right)\sqrt{\frac{(-b)^{\frac{1}{3}}}{b}}\log\left(3bx^{3}-3\left(bx^{3}+a\right)^{\frac{1}{3}}(-b)^{\frac{2}{3}}x^{2}-3\sqrt{\frac{1}{3}}\left((-b)^{\frac{1}{3}}bx^{3}-\left(bx^{3}+a\right)^{\frac{1}{3}}bx^{2}+2\left(bx^{3}+a\right)^{\frac{2}{3}}(-b)^{\frac{2}{3}}x\right)\sqrt{\frac{(-b)^{\frac{1}{3}}}{b}}+2a\right)+6\left(bx^{3}+a\right)^{\frac{2}{3}}\left(b^{2}c-abd\right)x-2\left(abdx^{3}+a^{2}d\right)(-b)^{\frac{2}{3}}\log\left(\frac{(-b)^{\frac{1}{3}}x+\left(bx^{3}+a\right)^{\frac{1}{3}}}{x}\right)+\left(abdx^{3}+a^{2}d\right)(-b)^{\frac{2}{3}}\log\left(\frac{(-b)^{\frac{2}{3}}x^{2}-\left(bx^{3}+a\right)^{\frac{1}{3}}(-b)^{\frac{1}{3}}x+\left(bx^{3}+a\right)^{\frac{2}{3}}}{x^{2}}\right)}{6\left(ab^{3}x^{3}+a^{2}b^{2}\right)},-\frac{6\sqrt{\frac{1}{3}}\left(ab^{2}dx^{3}+a^{2}bd\right)\sqrt{-\frac{(-b)^{\frac{1}{3}}}{b}}\arctan\left(-\frac{\sqrt{\frac{1}{3}}\left((-b)^{\frac{1}{3}}x-2\left(bx^{3}+a\right)^{\frac{1}{3}}\right)\sqrt{-\frac{(-b)^{\frac{1}{3}}}{b}}}{x}\right)-6\left(bx^{3}+a\right)^{\frac{2}{3}}\left(b^{2}c-abd\right)x+2\left(abdx^{3}+a^{2}d\right)(-b)^{\frac{2}{3}}\log\left(\frac{(-b)^{\frac{1}{3}}x+\left(bx^{3}+a\right)^{\frac{1}{3}}}{x}\right)-\left(abdx^{3}+a^{2}d\right)(-b)^{\frac{2}{3}}\log\left(\frac{(-b)^{\frac{2}{3}}x^{2}-\left(bx^{3}+a\right)^{\frac{1}{3}}(-b)^{\frac{1}{3}}x+\left(bx^{3}+a\right)^{\frac{2}{3}}}{x^{2}}\right)}{6\left(ab^{3}x^{3}+a^{2}b^{2}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)/(b*x^3+a)^(4/3),x, algorithm="fricas")
```

```
[Out] [1/6*(3*sqrt(1/3)*(a*b^2*d*x^3 + a^2*b*d)*sqrt((-b)^(1/3)/b)*log(3*b*x^3 -
3*(b*x^3 + a)^(1/3)*(-b)^(2/3)*x^2 - 3*sqrt(1/3)*((-b)^(1/3)*b*x^3 - (b*x^3
 + a)^(1/3)*b*x^2 + 2*(b*x^3 + a)^(2/3)*(-b)^(2/3)*x)*sqrt((-b)^(1/3)/b) +
2*a) + 6*(b*x^3 + a)^(2/3)*(b^2*c - a*b*d)*x - 2*(a*b*d*x^3 + a^2*d)*(-b)^(
2/3)*log(((-b)^(1/3)*x + (b*x^3 + a)^(1/3))/x) + (a*b*d*x^3 + a^2*d)*(-b)^(
2/3)*log(((-b)^(2/3)*x^2 - (b*x^3 + a)^(1/3)*(-b)^(1/3)*x + (b*x^3 + a)^(2/
3))/x^2))/(a*b^3*x^3 + a^2*b^2), -1/6*(6*sqrt(1/3)*(a*b^2*d*x^3 + a^2*b*d)*
sqrt(-(-b)^(1/3)/b)*arctan(-sqrt(1/3)*((-b)^(1/3)*x - 2*(b*x^3 + a)^(1/3))*
sqrt(-(-b)^(1/3)/b)/x) - 6*(b*x^3 + a)^(2/3)*(b^2*c - a*b*d)*x + 2*(a*b*d*x
^3 + a^2*d)*(-b)^(2/3)*log(((-b)^(1/3)*x + (b*x^3 + a)^(1/3))/x) - (a*b*d*x
^3 + a^2*d)*(-b)^(2/3)*log(((-b)^(2/3)*x^2 - (b*x^3 + a)^(1/3)*(-b)^(1/3)*x
 + (b*x^3 + a)^(2/3))/x^2))/(a*b^3*x^3 + a^2*b^2)]
```

Sympy [C] Result contains complex when optimal does not.
time = 3.08, size = 71, normalized size = 0.72

$$\frac{cx\Gamma\left(\frac{1}{3}\right)}{3a^{\frac{4}{3}}\sqrt[3]{1+\frac{bx^{3}}{a}}\,\Gamma\left(\frac{4}{3}\right)}+\frac{dx^{4}\Gamma\left(\frac{4}{3}\right){}_{2}F_{1}\left(\begin{matrix}\frac{4}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3a^{\frac{4}{3}}\Gamma\left(\frac{7}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)/(b*x**3+a)**(4/3),x)
```

```
[Out] c*x*gamma(1/3)/(3*a**(4/3)*(1 + b*x**3/a)**(1/3)*gamma(4/3)) + d*x**4*gamma
(4/3)*hyper((4/3, 4/3), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(4/3)*gamma
(7/3))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)/(b*x^3+a)^(4/3),x, algorithm="giac")
```

```
[Out] integrate((d*x^3 + c)/(b*x^3 + a)^(4/3), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{d\,x^3+c}{\left(b\,x^3+a\right)^{4/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)/(a + b*x^3)^(4/3),x)
```

```
[Out] int((c + d*x^3)/(a + b*x^3)^(4/3), x)
```

3.60 $\int \frac{c+dx^3}{\left(a+bx^3\right)^{7/3}}\,dx$

Optimal. Leaf size=47

$$\frac{3cx}{4a^2\sqrt[3]{a+bx^3}}+\frac{x(c+dx^3)}{4a\left(a+bx^3\right)^{4/3}}$$

```
[Out] 3/4*c*x/a^2/(b*x^3+a)^(1/3)+1/4*x*(d*x^3+c)/a/(b*x^3+a)^(4/3)
```

Rubi [A]
time = 0.01, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {386, 197}

$$\frac{3cx}{4a^2\sqrt[3]{a+bx^3}}+\frac{x(c+dx^3)}{4a\left(a+bx^3\right)^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)/(a + b*x^3)^(7/3),x]
```

```
[Out] (3*c*x)/(4*a^2*(a + b*x^3)^(1/3)) + (x*(c + d*x^3))/(4*a*(a + b*x^3)^(4/3))
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 386

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] - Dist[
c*(q/(a*(p + 1))), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1,
0] && GtQ[q, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{c+dx^3}{\left(a+bx^3\right)^{7/3}}\,dx &= \frac{x(c+dx^3)}{4a\left(a+bx^3\right)^{4/3}}+\frac{(3c)\int \frac{1}{\left(a+bx^3\right)^{4/3}}\,dx}{4a}\\ &= \frac{3cx}{4a^2\sqrt[3]{a+bx^3}}+\frac{x(c+dx^3)}{4a\left(a+bx^3\right)^{4/3}}\end{aligned}$$

Mathematica [A]
time = 0.22, size = 37, normalized size = 0.79

$$\frac{x(4ac + 3bcx^3 + adx^3)}{4a^2 (a + bx^3)^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3)/(a + b*x^3)^(7/3),x]
```

```
[Out] (x*(4*a*c + 3*b*c*x^3 + a*d*x^3))/(4*a^2*(a + b*x^3)^(4/3))
```

Maple [A]
time = 0.24, size = 34, normalized size = 0.72

method	result	size
gosper	$\frac{x\left(a d\,x^{3}+3 b c\,x^{3}+4 a c\right)}{4\left(b\,x^{3}+a\right)^{\frac{4}{3}} a^{2}}$	34
trager	$\frac{x\left(a d\,x^{3}+3 b c\,x^{3}+4 a c\right)}{4\left(b\,x^{3}+a\right)^{\frac{4}{3}} a^{2}}$	34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)/(b*x^3+a)^(7/3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*x*(a*d*x^3+3*b*c*x^3+4*a*c)/(b*x^3+a)^(4/3)/a^2
```

Maxima [A]
time = 0.30, size = 51, normalized size = 1.09

$$-\frac{\left(b - \frac{4\,(bx^3+a)}{x^3}\right)cx^4}{4\,(bx^3+a)^{\frac{4}{3}}a^2} + \frac{dx^4}{4\,(bx^3+a)^{\frac{4}{3}}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)/(b*x^3+a)^(7/3),x, algorithm="maxima")
```

```
[Out] -1/4*(b - 4*(b*x^3 + a)/x^3)*c*x^4/((b*x^3 + a)^(4/3)*a^2) + 1/4*d*x^4/((b*
x^3 + a)^(4/3)*a)
```

Fricas [A]
time = 4.60, size = 54, normalized size = 1.15

$$\frac{((3\,bc + ad)x^4 + 4\,acx)(bx^3 + a)^{\frac{2}{3}}}{4\,(a^2b^2x^6 + 2\,a^3bx^3 + a^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)/(b*x^3+a)^(7/3),x, algorithm="fricas")
```

```
[Out] 1/4*((3*b*c + a*d)*x^4 + 4*a*c*x)*(b*x^3 + a)^(2/3)/(a^2*b^2*x^6 + 2*a^3*b*
x^3 + a^4)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 190 vs. $2(41) = 82$.
time = 21.87, size = 190, normalized size = 4.04

$$c\left(\frac{4ax\Gamma\left(\frac{1}{3}\right)}{9a^{\frac{10}{3}}\sqrt[3]{1+\frac{bx^3}{a}}\,\Gamma\left(\frac{7}{3}\right)+9a^{\frac{7}{3}}bx^3\sqrt[3]{1+\frac{bx^3}{a}}\,\Gamma\left(\frac{7}{3}\right)}+\frac{3bx^4\Gamma\left(\frac{1}{3}\right)}{9a^{\frac{10}{3}}\sqrt[3]{1+\frac{bx^3}{a}}\,\Gamma\left(\frac{7}{3}\right)+9a^{\frac{7}{3}}bx^3\sqrt[3]{1+\frac{bx^3}{a}}\,\Gamma\left(\frac{7}{3}\right)}\right)+\frac{dx^4\Gamma\left(\frac{4}{3}\right)}{3a^{\frac{7}{3}}\sqrt[3]{1+\frac{bx^3}{a}}\,\Gamma\left(\frac{7}{3}\right)+3a^{\frac{4}{3}}bx^3\sqrt[3]{1+\frac{bx^3}{a}}\,\Gamma\left(\frac{7}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)/(b*x**3+a)**(7/3),x)
```

```
[Out] c*(4*a*x*gamma(1/3)/(9*a**(10/3)*(1 + b*x**3/a)**(1/3)*gamma(7/3) + 9*a**(7
/3)*b*x**3*(1 + b*x**3/a)**(1/3)*gamma(7/3)) + 3*b*x**4*gamma(1/3)/(9*a**(1
0/3)*(1 + b*x**3/a)**(1/3)*gamma(7/3) + 9*a**(7/3)*b*x**3*(1 + b*x**3/a)**(
1/3)*gamma(7/3))) + d*x**4*gamma(4/3)/(3*a**(7/3)*(1 + b*x**3/a)**(1/3)*gam
ma(7/3) + 3*a**(4/3)*b*x**3*(1 + b*x**3/a)**(1/3)*gamma(7/3))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)/(b*x^3+a)^(7/3),x, algorithm="giac")
```

```
[Out] integrate((d*x^3 + c)/(b*x^3 + a)^(7/3), x)
```

Mupad [B]
time = 1.37, size = 33, normalized size = 0.70

$$\frac{4\,a\,c\,x+a\,d\,x^4+3\,b\,c\,x^4}{4\,a^2\,{\left(b\,x^3+a\right)}^{4/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)/(a + b*x^3)^(7/3),x)
```

```
[Out] (4*a*c*x + a*d*x^4 + 3*b*c*x^4)/(4*a^2*(a + b*x^3)^(4/3))
```

3.61 $\int \frac{c+dx^3}{\left(a+bx^3\right)^{10/3}}\,dx$

Optimal. Leaf size=91

$$\frac{(bc-ad)x}{7ab\left(a+bx^3\right)^{7/3}}+\frac{(6bc+ad)x}{28a^2b\left(a+bx^3\right)^{4/3}}+\frac{3(6bc+ad)x}{28a^3b\sqrt[3]{a+bx^3}}$$

```
[Out] 1/7*(-a*d+b*c)*x/a/b/(b*x^3+a)^(7/3)+1/28*(a*d+6*b*c)*x/a^2/b/(b*x^3+a)^(4/
3)+3/28*(a*d+6*b*c)*x/a^3/b/(b*x^3+a)^(1/3)
```

Rubi [A]
time = 0.02, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {393, 198, 197}

$$\frac{3x(ad+6bc)}{28a^3b\sqrt[3]{a+bx^3}}+\frac{x(ad+6bc)}{28a^2b\left(a+bx^3\right)^{4/3}}+\frac{x(bc-ad)}{7ab\left(a+bx^3\right)^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)/(a + b*x^3)^(10/3),x]
```

```
[Out] ((b*c - a*d)*x)/(7*a*b*(a + b*x^3)^(7/3)) + ((6*b*c + a*d)*x)/(28*a^2*b*(a
+ b*x^3)^(4/3)) + (3*(6*b*c + a*d)*x)/(28*a^3*b*(a + b*x^3)^(1/3))
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rubi steps

$$\int \frac{c+dx^3}{(a+bx^3)^{10/3}}\,dx = \frac{(bc-ad)x}{7ab\,(a+bx^3)^{7/3}} + \frac{(6bc+ad)\int \frac{1}{(a+bx^3)^{7/3}}\,dx}{7ab}$$
$$= \frac{(bc-ad)x}{7ab\,(a+bx^3)^{7/3}} + \frac{(6bc+ad)x}{28a^2b\,(a+bx^3)^{4/3}} + \frac{(3(6bc+ad))\int \frac{1}{(a+bx^3)^{4/3}}\,dx}{28a^2b}$$
$$= \frac{(bc-ad)x}{7ab\,(a+bx^3)^{7/3}} + \frac{(6bc+ad)x}{28a^2b\,(a+bx^3)^{4/3}} + \frac{3(6bc+ad)x}{28a^3b\sqrt[3]{a+bx^3}}$$

Mathematica [A]
time = 0.31, size = 60, normalized size = 0.66

$$\frac{28a^2cx+42abcx^4+7a^2dx^4+18b^2cx^7+3abdx^7}{28a^3\,(a+bx^3)^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3)/(a + b*x^3)^(10/3),x]
```

```
[Out] (28*a^2*c*x + 42*a*b*c*x^4 + 7*a^2*d*x^4 + 18*b^2*c*x^7 + 3*a*b*d*x^7)/(28*
a^3*(a + b*x^3)^(7/3))
```

Maple [A]
time = 0.25, size = 57, normalized size = 0.63

method	result	size
gosper	$\frac{x\left(3abd\,x^6+18b^2c\,x^6+7a^2d\,x^3+42abc\,x^3+28a^2c\right)}{28\left(b\,x^3+a\right)^{\frac{7}{3}}a^3}$	57
trager	$\frac{x\left(3abd\,x^6+18b^2c\,x^6+7a^2d\,x^3+42abc\,x^3+28a^2c\right)}{28\left(b\,x^3+a\right)^{\frac{7}{3}}a^3}$	57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)/(b*x^3+a)^(10/3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/28*x*(3*a*b*d*x^6+18*b^2*c*x^6+7*a^2*d*x^3+42*a*b*c*x^3+28*a^2*c)/(b*x^3+
a)^(7/3)/a^3
```

Maxima [A]
time = 0.27, size = 86, normalized size = 0.95

$$-\frac{\left(4\,b-\frac{7\,(bx^3+a)}{x^3}\right)dx^7}{28\,(bx^3+a)^{\frac{7}{3}}a^2}+\frac{\left(2\,b^2-\frac{7\,(bx^3+a)b}{x^3}+\frac{14\,(bx^3+a)^2}{x^6}\right)cx^7}{14\,(bx^3+a)^{\frac{7}{3}}a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)/(b*x^3+a)^(10/3),x, algorithm="maxima")
```

```
[Out] -1/28*(4*b - 7*(b*x^3 + a)/x^3)*d*x^7/((b*x^3 + a)^(7/3)*a^2) + 1/14*(2*b^2
 - 7*(b*x^3 + a)*b/x^3 + 14*(b*x^3 + a)^2/x^6)*c*x^7/((b*x^3 + a)^(7/3)*a^3
)
```

Fricas [A]
time = 6.96, size = 87, normalized size = 0.96

$$\frac{(3\,(6\,b^2c+abd)x^7+7\,(6\,abc+a^2d)x^4+28\,a^2cx)(bx^3+a)^{\frac{2}{3}}}{28\,(a^3b^3x^9+3\,a^4b^2x^6+3\,a^5bx^3+a^6)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)/(b*x^3+a)^(10/3),x, algorithm="fricas")
```

```
[Out] 1/28*(3*(6*b^2*c + a*b*d)*x^7 + 7*(6*a*b*c + a^2*d)*x^4 + 28*a^2*c*x)*(b*x^
3 + a)^(2/3)/(a^3*b^3*x^9 + 3*a^4*b^2*x^6 + 3*a^5*b*x^3 + a^6)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 709 vs. 2(83) = 166.
time = 111.05, size = 709, normalized size = 7.79

$$c\left(\frac{28a^5x\Gamma\left(\frac{1}{3}\right)}{27a^{\frac{25}{3}}\sqrt[3]{1+\frac{bx^3}{a}}\,\Gamma\left(\frac{10}{3}\right)+81a^{\frac{22}{3}}bx^3\sqrt[3]{1+\frac{bx^3}{a}}\,\Gamma\left(\frac{10}{3}\right)+81a^{\frac{19}{3}}b^2x^6\sqrt[3]{1+\frac{bx^3}{a}}\,\Gamma\left(\frac{10}{3}\right)+27a^{\frac{16}{3}}b^3x^9\sqrt[3]{1+\frac{bx^3}{a}}\,\Gamma\left(\frac{10}{3}\right)}+\frac{70a^4bx^4\Gamma\left(\frac{1}{3}\right)}{27a^{\frac{25}{3}}\sqrt[3]{1+\frac{bx^3}{a}}\,\Gamma\left(\frac{10}{3}\right)+81a^{\frac{22}{3}}bx^3\sqrt[3]{1+\frac{bx^3}{a}}\,\Gamma\left(\frac{10}{3}\right)+81a^{\frac{19}{3}}b^2x^6\sqrt[3]{1+\frac{bx^3}{a}}\,\Gamma\left(\frac{10}{3}\right)+27a^{\frac{16}{3}}b^3x^9\sqrt[3]{1+\frac{bx^3}{a}}\,\Gamma\left(\frac{10}{3}\right)}+\frac{60a^3b^2x^7\Gamma\left(\frac{1}{3}\right)}{27a^{\frac{25}{3}}\sqrt[3]{1+\frac{bx^3}{a}}\,\Gamma\left(\frac{10}{3}\right)+81a^{\frac{22}{3}}bx^3\sqrt[3]{1+\frac{bx^3}{a}}\,\Gamma\left(\frac{10}{3}\right)+81a^{\frac{19}{3}}b^2x^6\sqrt[3]{1+\frac{bx^3}{a}}\,\Gamma\left(\frac{10}{3}\right)+27a^{\frac{16}{3}}b^3x^9\sqrt[3]{1+\frac{bx^3}{a}}\,\Gamma\left(\frac{10}{3}\right)}+\frac{18a^2b^3x^{10}\Gamma\left(\frac{1}{3}\right)}{27a^{\frac{25}{3}}\sqrt[3]{1+\frac{bx^3}{a}}\,\Gamma\left(\frac{10}{3}\right)+81a^{\frac{22}{3}}bx^3\sqrt[3]{1+\frac{bx^3}{a}}\,\Gamma\left(\frac{10}{3}\right)+81a^{\frac{19}{3}}b^2x^6\sqrt[3]{1+\frac{bx^3}{a}}\,\Gamma\left(\frac{10}{3}\right)+27a^{\frac{16}{3}}b^3x^9\sqrt[3]{1+\frac{bx^3}{a}}\,\Gamma\left(\frac{10}{3}\right)}\right)+d\left(\frac{7ax^4\Gamma\left(\frac{4}{3}\right)}{9a^{\frac{13}{3}}\sqrt[3]{1+\frac{bx^3}{a}}\,\Gamma\left(\frac{10}{3}\right)+18a^{\frac{10}{3}}bx^3\sqrt[3]{1+\frac{bx^3}{a}}\,\Gamma\left(\frac{10}{3}\right)+9a^{\frac{7}{3}}b^2x^6\sqrt[3]{1+\frac{bx^3}{a}}\,\Gamma\left(\frac{10}{3}\right)}+\frac{3bx^7\Gamma\left(\frac{4}{3}\right)}{9a^{\frac{13}{3}}\sqrt[3]{1+\frac{bx^3}{a}}\,\Gamma\left(\frac{10}{3}\right)+18a^{\frac{10}{3}}bx^3\sqrt[3]{1+\frac{bx^3}{a}}\,\Gamma\left(\frac{10}{3}\right)+9a^{\frac{7}{3}}b^2x^6\sqrt[3]{1+\frac{bx^3}{a}}\,\Gamma\left(\frac{10}{3}\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)/(b*x**3+a)**(10/3),x)
```

```
[Out] c*(28*a**5*x*gamma(1/3)/(27*a**(25/3)*(1 + b*x**3/a)**(1/3)*gamma(10/3) + 8
1*a**(22/3)*b*x**3*(1 + b*x**3/a)**(1/3)*gamma(10/3) + 81*a**(19/3)*b**2*x*
*6*(1 + b*x**3/a)**(1/3)*gamma(10/3) + 27*a**(16/3)*b**3*x**9*(1 + b*x**3/a
)**(1/3)*gamma(10/3)) + 70*a**4*b*x**4*gamma(1/3)/(27*a**(25/3)*(1 + b*x**3
/a)**(1/3)*gamma(10/3) + 81*a**(22/3)*b*x**3*(1 + b*x**3/a)**(1/3)*gamma(10
/3) + 81*a**(19/3)*b**2*x**6*(1 + b*x**3/a)**(1/3)*gamma(10/3) + 27*a**(16/
3)*b**3*x**9*(1 + b*x**3/a)**(1/3)*gamma(10/3)) + 60*a**3*b**2*x**7*gamma(1
/3)/(27*a**(25/3)*(1 + b*x**3/a)**(1/3)*gamma(10/3) + 81*a**(22/3)*b*x**3*(
1 + b*x**3/a)**(1/3)*gamma(10/3) + 81*a**(19/3)*b**2*x**6*(1 + b*x**3/a)**(
1/3)*gamma(10/3) + 27*a**(16/3)*b**3*x**9*(1 + b*x**3/a)**(1/3)*gamma(10/3)
) + 18*a**2*b**3*x**10*gamma(1/3)/(27*a**(25/3)*(1 + b*x**3/a)**(1/3)*gamma
(10/3) + 81*a**(22/3)*b*x**3*(1 + b*x**3/a)**(1/3)*gamma(10/3) + 81*a**(19/
3)*b**2*x**6*(1 + b*x**3/a)**(1/3)*gamma(10/3) + 27*a**(16/3)*b**3*x**9*(1
+ b*x**3/a)**(1/3)*gamma(10/3))) + d*(7*a*x**4*gamma(4/3)/(9*a**(13/3)*(1 +
 b*x**3/a)**(1/3)*gamma(10/3) + 18*a**(10/3)*b*x**3*(1 + b*x**3/a)**(1/3)*g
amma(10/3) + 9*a**(7/3)*b**2*x**6*(1 + b*x**3/a)**(1/3)*gamma(10/3)) + 3*b*
x**7*gamma(4/3)/(9*a**(13/3)*(1 + b*x**3/a)**(1/3)*gamma(10/3) + 18*a**(10/
```

```
3)*b*x**3*(1 + b*x**3/a)**(1/3)*gamma(10/3) + 9*a**(7/3)*b**2*x**6*(1 + b*x
**3/a)**(1/3)*gamma(10/3)))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)/(b*x^3+a)^(10/3),x, algorithm="giac")
```

```
[Out] integrate((d*x^3 + c)/(b*x^3 + a)^(10/3), x)
```

Mupad [B]
time = 1.42, size = 87, normalized size = 0.96

$$\frac{3\,a\,d\,x\,{\left(b\,x^3+a\right)}^2-4\,a^3\,d\,x+18\,b\,c\,x\,{\left(b\,x^3+a\right)}^2+a^2\,d\,x\,\left(b\,x^3+a\right)+4\,a^2\,b\,c\,x+6\,a\,b\,c\,x\,\left(b\,x^3+a\right)}{28\,a^3\,b\,{\left(b\,x^3+a\right)}^{7/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)/(a + b*x^3)^(10/3),x)
```

```
[Out] (3*a*d*x*(a + b*x^3)^2 - 4*a^3*d*x + 18*b*c*x*(a + b*x^3)^2 + a^2*d*x*(a +
b*x^3) + 4*a^2*b*c*x + 6*a*b*c*x*(a + b*x^3))/(28*a^3*b*(a + b*x^3)^(7/3))
```

3.62 $\int \frac{c+dx^3}{(a+bx^3)^{13/3}}\,dx$

Optimal. Leaf size=121

$$\frac{(bc-ad)x}{10ab\left(a+bx^3\right)^{10/3}}+\frac{(9bc+ad)x}{70a^2b\left(a+bx^3\right)^{7/3}}+\frac{3(9bc+ad)x}{140a^3b\left(a+bx^3\right)^{4/3}}+\frac{9(9bc+ad)x}{140a^4b\sqrt[3]{a+bx^3}}$$

```
[Out] 1/10*(-a*d+b*c)*x/a/b/(b*x^3+a)^(10/3)+1/70*(a*d+9*b*c)*x/a^2/b/(b*x^3+a)^(
7/3)+3/140*(a*d+9*b*c)*x/a^3/b/(b*x^3+a)^(4/3)+9/140*(a*d+9*b*c)*x/a^4/b/(b
*x^3+a)^(1/3)
```

Rubi [A]
time = 0.03, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {393, 198, 197}

$$\frac{9x(ad+9bc)}{140a^4b\sqrt[3]{a+bx^3}}+\frac{3x(ad+9bc)}{140a^3b\left(a+bx^3\right)^{4/3}}+\frac{x(ad+9bc)}{70a^2b\left(a+bx^3\right)^{7/3}}+\frac{x(bc-ad)}{10ab\left(a+bx^3\right)^{10/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)/(a + b*x^3)^(13/3),x]
```

```
[Out] ((b*c - a*d)*x)/(10*a*b*(a + b*x^3)^(10/3)) + ((9*b*c + a*d)*x)/(70*a^2*b*(
a + b*x^3)^(7/3)) + (3*(9*b*c + a*d)*x)/(140*a^3*b*(a + b*x^3)^(4/3)) + (9*
(9*b*c + a*d)*x)/(140*a^4*b*(a + b*x^3)^(1/3))
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rubi steps

$$\int \frac{c+dx^3}{(a+bx^3)^{13/3}}\,dx = \frac{(bc-ad)x}{10ab\,(a+bx^3)^{10/3}} + \frac{(9bc+ad)\int \frac{1}{(a+bx^3)^{10/3}}\,dx}{10ab}$$
$$= \frac{(bc-ad)x}{10ab\,(a+bx^3)^{10/3}} + \frac{(9bc+ad)x}{70a^2b\,(a+bx^3)^{7/3}} + \frac{(3(9bc+ad))\int \frac{1}{(a+bx^3)^{7/3}}\,dx}{35a^2b}$$
$$= \frac{(bc-ad)x}{10ab\,(a+bx^3)^{10/3}} + \frac{(9bc+ad)x}{70a^2b\,(a+bx^3)^{7/3}} + \frac{3(9bc+ad)x}{140a^3b\,(a+bx^3)^{4/3}} + \frac{(9(9bc+ad))\int \frac{1}{(a+bx^3)^{4/3}}\,dx}{140a^3b}$$
$$= \frac{(bc-ad)x}{10ab\,(a+bx^3)^{10/3}} + \frac{(9bc+ad)x}{70a^2b\,(a+bx^3)^{7/3}} + \frac{3(9bc+ad)x}{140a^3b\,(a+bx^3)^{4/3}} + \frac{9(9bc+ad)x}{140a^4b\sqrt[3]{a+bx^3}}$$

Mathematica [A]
time = 0.41, size = 80, normalized size = 0.66

$$\frac{x(81b^3cx^9+35a^3(4c+dx^3)+9ab^2x^6(30c+dx^3)+15a^2bx^3(21c+2dx^3))}{140a^4\,(a+bx^3)^{10/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3)/(a + b*x^3)^(13/3),x]
```

```
[Out] (x*(81*b^3*c*x^9 + 35*a^3*(4*c + d*x^3) + 9*a*b^2*x^6*(30*c + d*x^3) + 15*a
^2*b*x^3*(21*c + 2*d*x^3)))/(140*a^4*(a + b*x^3)^(10/3))
```

Maple [A]
time = 0.24, size = 81, normalized size = 0.67

method	result	size
gosper	$\frac{x\left(9a\,b^2d\,x^9+81b^3c\,x^9+30a^2bd\,x^6+270a\,b^2c\,x^6+35a^3d\,x^3+315a^2bc\,x^3+140c\,a^3\right)}{140(b\,x^3+a)^{\frac{10}{3}}\,a^4}$	81
trager	$\frac{x\left(9a\,b^2d\,x^9+81b^3c\,x^9+30a^2bd\,x^6+270a\,b^2c\,x^6+35a^3d\,x^3+315a^2bc\,x^3+140c\,a^3\right)}{140(b\,x^3+a)^{\frac{10}{3}}\,a^4}$	81

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)/(b*x^3+a)^(13/3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/140*x*(9*a*b^2*d*x^9+81*b^3*c*x^9+30*a^2*b*d*x^6+270*a*b^2*c*x^6+35*a^3*d
*x^3+315*a^2*b*c*x^3+140*a^3*c)/(b*x^3+a)^(10/3)/a^4
```

Maxima [A]
time = 0.30, size = 120, normalized size = 0.99

$$\frac{\left(14\,b^2-\frac{40\,(bx^3+a)b}{x^3}+\frac{35\,(bx^3+a)^2}{x^6}\right)dx^{10}}{140\,(bx^3+a)^{\frac{10}{3}}\,a^3}-\frac{\left(14\,b^3-\frac{60\,(bx^3+a)b^2}{x^3}+\frac{105\,(bx^3+a)^2b}{x^6}-\frac{140\,(bx^3+a)^3}{x^9}\right)cx^{10}}{140\,(bx^3+a)^{\frac{10}{3}}\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)/(b*x^3+a)^(13/3),x, algorithm="maxima")
```

```
[Out] 1/140*(14*b^2 - 40*(b*x^3 + a)*b/x^3 + 35*(b*x^3 + a)^2/x^6)*d*x^10/((b*x^3
 + a)^(10/3)*a^3) - 1/140*(14*b^3 - 60*(b*x^3 + a)*b^2/x^3 + 105*(b*x^3 + a
)^2*b/x^6 - 140*(b*x^3 + a)^3/x^9)*c*x^10/((b*x^3 + a)^(10/3)*a^4)
```

Fricas [A]
time = 3.44, size = 121, normalized size = 1.00

$$\frac{\left(9\left(9\,b^{3}c+ab^{2}d\right)x^{10}+30\left(9\,ab^{2}c+a^{2}bd\right)x^{7}+140\,a^{3}cx+35\left(9\,a^{2}bc+a^{3}d\right)x^{4}\right)\left(bx^{3}+a\right)^{\frac{2}{3}}}{140\left(a^{4}b^{4}x^{12}+4\,a^{5}b^{3}x^{9}+6\,a^{6}b^{2}x^{6}+4\,a^{7}bx^{3}+a^{8}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)/(b*x^3+a)^(13/3),x, algorithm="fricas")
```

```
[Out] 1/140*(9*(9*b^3*c + a*b^2*d)*x^10 + 30*(9*a*b^2*c + a^2*b*d)*x^7 + 140*a^3*
c*x + 35*(9*a^2*b*c + a^3*d)*x^4)*(b*x^3 + a)^(2/3)/(a^4*b^4*x^12 + 4*a^5*b
^3*x^9 + 6*a^6*b^2*x^6 + 4*a^7*b*x^3 + a^8)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)/(b*x**3+a)**(13/3),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)/(b*x^3+a)^(13/3),x, algorithm="giac")
```

```
[Out] integrate((d*x^3 + c)/(b*x^3 + a)^(13/3), x)
```

Mupad [B]
time = 1.46, size = 105, normalized size = 0.87

$$\frac{x\left(\frac{c}{10\,a}-\frac{d}{10\,b}\right)}{\left(b\,x^{3}+a\right)^{10/3}}+\frac{x\left(a\,d+9\,b\,c\right)}{70\,a^{2}\,b\left(b\,x^{3}+a\right)^{7/3}}+\frac{x\left(3\,a\,d+27\,b\,c\right)}{140\,a^{3}\,b\left(b\,x^{3}+a\right)^{4/3}}+\frac{x\left(9\,a\,d+81\,b\,c\right)}{140\,a^{4}\,b\left(b\,x^{3}+a\right)^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)/(a + b*x^3)^(13/3),x)
```

```
[Out] (x*(c/(10*a) - d/(10*b)))/(a + b*x^3)^(10/3) + (x*(a*d + 9*b*c))/(70*a^2*b*
(a + b*x^3)^(7/3)) + (x*(3*a*d + 27*b*c))/(140*a^3*b*(a + b*x^3)^(4/3)) + (
x*(9*a*d + 81*b*c))/(140*a^4*b*(a + b*x^3)^(1/3))
```

3.63 $\int \frac{c+dx^3}{(a+bx^3)^{16/3}}\,dx$

Optimal. Leaf size=151

$$\frac{(bc-ad)x}{13ab\,(a+bx^3)^{13/3}}+\frac{(12bc+ad)x}{130a^2b\,(a+bx^3)^{10/3}}+\frac{9(12bc+ad)x}{910a^3b\,(a+bx^3)^{7/3}}+\frac{27(12bc+ad)x}{1820a^4b\,(a+bx^3)^{4/3}}+\frac{81(12bc+ad)x}{1820a^5b\sqrt[3]{a+bx^3}}$$

```
[Out] 1/13*(-a*d+b*c)*x/a/b/(b*x^3+a)^(13/3)+1/130*(a*d+12*b*c)*x/a^2/b/(b*x^3+a)
^(10/3)+9/910*(a*d+12*b*c)*x/a^3/b/(b*x^3+a)^(7/3)+27/1820*(a*d+12*b*c)*x/a
^4/b/(b*x^3+a)^(4/3)+81/1820*(a*d+12*b*c)*x/a^5/b/(b*x^3+a)^(1/3)
```

Rubi [A]
time = 0.03, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {393, 198, 197}

$$\frac{81x(ad+12bc)}{1820a^5b\sqrt[3]{a+bx^3}}+\frac{27x(ad+12bc)}{1820a^4b\,(a+bx^3)^{4/3}}+\frac{9x(ad+12bc)}{910a^3b\,(a+bx^3)^{7/3}}+\frac{x(ad+12bc)}{130a^2b\,(a+bx^3)^{10/3}}+\frac{x(bc-ad)}{13ab\,(a+bx^3)^{13/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)/(a + b*x^3)^(16/3),x]
```

```
[Out] ((b*c - a*d)*x)/(13*a*b*(a + b*x^3)^(13/3)) + ((12*b*c + a*d)*x)/(130*a^2*b
*(a + b*x^3)^(10/3)) + (9*(12*b*c + a*d)*x)/(910*a^3*b*(a + b*x^3)^(7/3)) +
 (27*(12*b*c + a*d)*x)/(1820*a^4*b*(a + b*x^3)^(4/3)) + (81*(12*b*c + a*d)*
x)/(1820*a^5*b*(a + b*x^3)^(1/3))
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
```

```
 + p, 0])
```

Rubi steps

$$\int \frac{c+dx^3}{(a+bx^3)^{16/3}}\,dx = \frac{(bc-ad)x}{13ab\,(a+bx^3)^{13/3}} + \frac{(12bc+ad)\int \frac{1}{(a+bx^3)^{13/3}}\,dx}{13ab}$$
$$= \frac{(bc-ad)x}{13ab\,(a+bx^3)^{13/3}} + \frac{(12bc+ad)x}{130a^2b\,(a+bx^3)^{10/3}} + \frac{(9(12bc+ad))\int \frac{1}{(a+bx^3)^{10/3}}\,dx}{130a^2b}$$
$$= \frac{(bc-ad)x}{13ab\,(a+bx^3)^{13/3}} + \frac{(12bc+ad)x}{130a^2b\,(a+bx^3)^{10/3}} + \frac{9(12bc+ad)x}{910a^3b\,(a+bx^3)^{7/3}} + \frac{(27(12bc+ad))\int}{455a^3b}$$
$$= \frac{(bc-ad)x}{13ab\,(a+bx^3)^{13/3}} + \frac{(12bc+ad)x}{130a^2b\,(a+bx^3)^{10/3}} + \frac{9(12bc+ad)x}{910a^3b\,(a+bx^3)^{7/3}} + \frac{27(12bc+ad)x}{1820a^4b\,(a+bx^3)^{4}}$$
$$= \frac{(bc-ad)x}{13ab\,(a+bx^3)^{13/3}} + \frac{(12bc+ad)x}{130a^2b\,(a+bx^3)^{10/3}} + \frac{9(12bc+ad)x}{910a^3b\,(a+bx^3)^{7/3}} + \frac{27(12bc+ad)x}{1820a^4b\,(a+bx^3)^{4}}$$

Mathematica [A]
time = 0.61, size = 100, normalized size = 0.66

$$\frac{x(972b^4cx^{12}+455a^4(4c+dx^3)+351a^2b^2x^6(20c+dx^3)+81ab^3x^9(52c+dx^3)+195a^3bx^3(28c+3dx^3))}{1820a^5\,(a+bx^3)^{13/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3)/(a + b*x^3)^(16/3),x]
[Out] (x*(972*b^4*c*x^12 + 455*a^4*(4*c + d*x^3) + 351*a^2*b^2*x^6*(20*c + d*x^3)
 + 81*a*b^3*x^9*(52*c + d*x^3) + 195*a^3*b*x^3*(28*c + 3*d*x^3)))/(1820*a^5
*(a + b*x^3)^(13/3))
```

Maple [A]
time = 0.27, size = 105, normalized size = 0.70

method	result	size
gosper	$\frac{x\left(81a\,b^3d\,x^{12}+972b^4c\,x^{12}+351a^2b^2d\,x^9+4212a\,b^3c\,x^9+585a^3bd\,x^6+7020a^2b^2c\,x^6+455a^4d\,x^3+5460a^3bc\,x^3+1820c\,a^4\right)}{1820(b\,x^3+a)^{\frac{13}{3}}a^5}$	105
trager	$\frac{x\left(81a\,b^3d\,x^{12}+972b^4c\,x^{12}+351a^2b^2d\,x^9+4212a\,b^3c\,x^9+585a^3bd\,x^6+7020a^2b^2c\,x^6+455a^4d\,x^3+5460a^3bc\,x^3+1820c\,a^4\right)}{1820(b\,x^3+a)^{\frac{13}{3}}a^5}$	105

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)/(b*x^3+a)^(16/3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/1820*x*(81*a*b^3*d*x^12+972*b^4*c*x^12+351*a^2*b^2*d*x^9+4212*a*b^3*c*x^9
+585*a^3*b*d*x^6+7020*a^2*b^2*c*x^6+455*a^4*d*x^3+5460*a^3*b*c*x^3+1820*a^4
*c)/(b*x^3+a)^(13/3)/a^5
```

Maxima [A]
time = 0.30, size = 154, normalized size = 1.02

$$-\frac{\left(140\,b^{3}-\frac{546\,(bx^{3}+a)b^{2}}{x^{3}}+\frac{780\,(bx^{3}+a)^{2}b}{x^{6}}-\frac{455\,(bx^{3}+a)^{3}}{x^{9}}\right)dx^{13}}{1820\,(bx^{3}+a)^{\frac{13}{3}}a^{4}}+\frac{\left(35\,b^{4}-\frac{182\,(bx^{3}+a)b^{3}}{x^{3}}+\frac{390\,(bx^{3}+a)^{2}b^{2}}{x^{6}}-\frac{455\,(bx^{3}+a)^{3}b}{x^{9}}+\frac{455\,(bx^{3}+a)^{4}}{x^{12}}\right)cx^{13}}{455\,(bx^{3}+a)^{\frac{13}{3}}a^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)/(b*x^3+a)^(16/3),x, algorithm="maxima")
```

```
[Out] -1/1820*(140*b^3 - 546*(b*x^3 + a)*b^2/x^3 + 780*(b*x^3 + a)^2*b/x^6 - 455*
(b*x^3 + a)^3/x^9)*d*x^13/((b*x^3 + a)^(13/3)*a^4) + 1/455*(35*b^4 - 182*(b
*x^3 + a)*b^3/x^3 + 390*(b*x^3 + a)^2*b^2/x^6 - 455*(b*x^3 + a)^3*b/x^9 + 4
55*(b*x^3 + a)^4/x^12)*c*x^13/((b*x^3 + a)^(13/3)*a^5)
```

Fricas [A]
time = 6.31, size = 155, normalized size = 1.03

$$\frac{(81\,(12\,b^{4}c+ab^{3}d)x^{13}+351\,(12\,ab^{3}c+a^{2}b^{2}d)x^{10}+585\,(12\,a^{2}b^{2}c+a^{3}bd)x^{7}+1820\,a^{4}cx+455\,(12\,a^{3}bc+a^{4}d)x^{4})(bx^{3}+a)^{\frac{2}{3}}}{1820\,(a^{5}b^{5}x^{15}+5\,a^{6}b^{4}x^{12}+10\,a^{7}b^{3}x^{9}+10\,a^{8}b^{2}x^{6}+5\,a^{9}bx^{3}+a^{10})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)/(b*x^3+a)^(16/3),x, algorithm="fricas")
```

```
[Out] 1/1820*(81*(12*b^4*c + a*b^3*d)*x^13 + 351*(12*a*b^3*c + a^2*b^2*d)*x^10 +
585*(12*a^2*b^2*c + a^3*b*d)*x^7 + 1820*a^4*c*x + 455*(12*a^3*b*c + a^4*d)*
x^4)*(b*x^3 + a)^(2/3)/(a^5*b^5*x^15 + 5*a^6*b^4*x^12 + 10*a^7*b^3*x^9 + 10
*a^8*b^2*x^6 + 5*a^9*b*x^3 + a^10)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)/(b*x**3+a)**(16/3),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)/(b*x^3+a)^(16/3),x, algorithm="giac")
```

```
[Out] integrate((d*x^3 + c)/(b*x^3 + a)^(16/3), x)
```

Mupad [B]
time = 1.45, size = 132, normalized size = 0.87

$$\frac{x\left(\frac{c}{13a}-\frac{d}{13b}\right)}{(b x^3+a)^{13/3}}+\frac{x(a d+12 b c)}{130 a^2 b (b x^3+a)^{10/3}}+\frac{x(9 a d+108 b c)}{910 a^3 b (b x^3+a)^{7/3}}+\frac{x(27 a d+324 b c)}{1820 a^4 b (b x^3+a)^{4/3}}+\frac{x(81 a d+972 b c)}{1820 a^5 b (b x^3+a)^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)/(a + b*x^3)^(16/3),x)
```

```
[Out] (x*(c/(13*a) - d/(13*b)))/(a + b*x^3)^(13/3) + (x*(a*d + 12*b*c))/(130*a^2*
b*(a + b*x^3)^(10/3)) + (x*(9*a*d + 108*b*c))/(910*a^3*b*(a + b*x^3)^(7/3))
 + (x*(27*a*d + 324*b*c))/(1820*a^4*b*(a + b*x^3)^(4/3)) + (x*(81*a*d + 972
*b*c))/(1820*a^5*b*(a + b*x^3)^(1/3))
```

3.64 $\int (a+bx^3)^{7/3} (c+dx^3)\, dx$

Optimal. Leaf size=85

$$\frac{dx(a+bx^3)^{10/3}}{11b}+\frac{a^2(11bc-ad)x\sqrt[3]{a+bx^3}\ {}_2F_1\left(-\frac{7}{3},\frac{1}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{11b\sqrt[3]{1+\frac{bx^3}{a}}}$$

```
[Out] 1/11*d*x*(b*x^3+a)^(10/3)/b+1/11*a^2*(-a*d+11*b*c)*x*(b*x^3+a)^(1/3)*hyperg
eom([-7/3, 1/3],[4/3],-b*x^3/a)/b/(1+b*x^3/a)^(1/3)
```

Rubi [A]
time = 0.02, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {396, 252, 251}

$$\frac{a^2x\sqrt[3]{a+bx^3}\,(11bc-ad)\,{}_2F_1\left(-\frac{7}{3},\frac{1}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{11b\sqrt[3]{\frac{bx^3}{a}+1}}+\frac{dx(a+bx^3)^{10/3}}{11b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(7/3)*(c + d*x^3),x]
```

```
[Out] (d*x*(a + b*x^3)^(10/3))/(11*b) + (a^2*(11*b*c - a*d)*x*(a + b*x^3)^(1/3)*H
ypergeometric2F1[-7/3, 1/3, 4/3, -((b*x^3)/a)])/(11*b*(1 + (b*x^3)/a)^(1/3)
)
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
```

```
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rubi steps

$$
\begin{aligned}
\int \left(a+bx^3\right)^{7/3}\left(c+dx^3\right)\,dx &= \frac{dx(a+bx^3)^{10/3}}{11b}-\frac{(-11bc+ad)\int \left(a+bx^3\right)^{7/3}\,dx}{11b}\\
&= \frac{dx(a+bx^3)^{10/3}}{11b}-\frac{\left(a^2(-11bc+ad)\sqrt[3]{a+bx^3}\right)\int \left(1+\frac{bx^3}{a}\right)^{7/3}\,dx}{11b\sqrt[3]{1+\frac{bx^3}{a}}}\\
&= \frac{dx(a+bx^3)^{10/3}}{11b}+\frac{a^2(11bc-ad)x\sqrt[3]{a+bx^3}\;{}_2F_1\left(-\frac{7}{3},\frac{1}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{11b\sqrt[3]{1+\frac{bx^3}{a}}}
\end{aligned}
$$

Mathematica [A]
time = 8.74, size = 77, normalized size = 0.91

$$
\frac{x\sqrt[3]{a+bx^3}\left(d(a+bx^3)^3-\frac{a^2(-11bc+ad)\,{}_2F_1\left(-\frac{7}{3},\frac{1}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{\sqrt[3]{1+\frac{bx^3}{a}}}\right)}{11b}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^(7/3)*(c + d*x^3),x]
```

```
[Out] (x*(a + b*x^3)^(1/3)*(d*(a + b*x^3)^3 - (a^2*(-11*b*c + a*d)*Hypergeometric
2F1[-7/3, 1/3, 4/3, -((b*x^3)/a)])/(1 + (b*x^3)/a)^(1/3)))/(11*b)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$
\int \left(b\,x^{3}+a\right)^{\frac{7}{3}}\left(d\,x^{3}+c\right)dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(7/3)*(d*x^3+c),x)
```

```
[Out] int((b*x^3+a)^(7/3)*(d*x^3+c),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(7/3)*(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(7/3)*(d*x^3 + c), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(7/3)*(d*x^3+c),x, algorithm="fricas")
```

```
[Out] integral((b^2*d*x^9 + (b^2*c + 2*a*b*d)*x^6 + (2*a*b*c + a^2*d)*x^3 + a^2*c
)*(b*x^3 + a)^(1/3), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 3.61, size = 265, normalized size = 3.12

$$\frac{a^{\frac{7}{3}}cx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{1}{3}\\\frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)}+\frac{a^{\frac{7}{3}}dx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{4}{3}\\\frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)}+\frac{2a^{\frac{4}{3}}bcx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{4}{3}\\\frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)}+\frac{2a^{\frac{4}{3}}bdx^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{7}{3}\\\frac{10}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{10}{3}\right)}+\frac{\sqrt[3]{a}b^2cx^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{7}{3}\\\frac{10}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{10}{3}\right)}+\frac{\sqrt[3]{a}b^2dx^{10}\Gamma\left(\frac{10}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{10}{3}\\\frac{13}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{13}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(7/3)*(d*x**3+c),x)
```

```
[Out] a**(7/3)*c*x*gamma(1/3)*hyper((-1/3, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a
)/(3*gamma(4/3)) + a**(7/3)*d*x**4*gamma(4/3)*hyper((-1/3, 4/3), (7/3,), b*
x**3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + 2*a**(4/3)*b*c*x**4*gamma(4/3)*hyp
er((-1/3, 4/3), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + 2*a**(4/
3)*b*d*x**7*gamma(7/3)*hyper((-1/3, 7/3), (10/3,), b*x**3*exp_polar(I*pi)/a
)/(3*gamma(10/3)) + a**(1/3)*b**2*c*x**7*gamma(7/3)*hyper((-1/3, 7/3), (10/
3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(10/3)) + a**(1/3)*b**2*d*x**10*gamm
a(10/3)*hyper((-1/3, 10/3), (13/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(13/
3))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(7/3)*(d*x^3+c),x, algorithm="giac")
[Out] integrate((b*x^3 + a)^(7/3)*(d*x^3 + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(b\,x^3+a\right)^{7/3}\,\left(d\,x^3+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(7/3)*(c + d*x^3),x)
[Out] int((a + b*x^3)^(7/3)*(c + d*x^3), x)
```

3.65 $\int (a+bx^3)^{4/3} (c+dx^3)\, dx$

Optimal. Leaf size=83

$$\frac{dx(a+bx^3)^{7/3}}{8b}+\frac{a(8bc-ad)x\sqrt[3]{a+bx^3}\ {}_2F_1\left(-\frac{4}{3},\frac{1}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{8b\sqrt[3]{1+\frac{bx^3}{a}}}$$

```
[Out] 1/8*d*x*(b*x^3+a)^(7/3)/b+1/8*a*(-a*d+8*b*c)*x*(b*x^3+a)^(1/3)*hypergeom([-
4/3, 1/3],[4/3],-b*x^3/a)/b/(1+b*x^3/a)^(1/3)
```

Rubi [A]
time = 0.02, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {396, 252, 251}

$$\frac{ax\sqrt[3]{a+bx^3}\,(8bc-ad)\,{}_2F_1\left(-\frac{4}{3},\frac{1}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{8b\sqrt[3]{\frac{bx^3}{a}+1}}+\frac{dx(a+bx^3)^{7/3}}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(4/3)*(c + d*x^3),x]
```

```
[Out] (d*x*(a + b*x^3)^(7/3))/(8*b) + (a*(8*b*c - a*d)*x*(a + b*x^3)^(1/3)*Hyperg
eometric2F1[-4/3, 1/3, 4/3, -((b*x^3)/a)])/(8*b*(1 + (b*x^3)/a)^(1/3))
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
```

```
c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a+bx^3\right)^{4/3}\left(c+dx^3\right)\,dx &= \frac{dx(a+bx^3)^{7/3}}{8b}-\frac{(-8bc+ad)\int (a+bx^3)^{4/3}\,dx}{8b}\\ &= \frac{dx(a+bx^3)^{7/3}}{8b}-\frac{\left(a(-8bc+ad)\sqrt[3]{a+bx^3}\right)\int \left(1+\frac{bx^3}{a}\right)^{4/3}\,dx}{8b\sqrt[3]{1+\frac{bx^3}{a}}}\\ &= \frac{dx(a+bx^3)^{7/3}}{8b}+\frac{a(8bc-ad)x\sqrt[3]{a+bx^3}\;{}_2F_1\left(-\frac{4}{3},\frac{1}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{8b\sqrt[3]{1+\frac{bx^3}{a}}}\end{aligned}$$

Mathematica [A]
time = 7.49, size = 75, normalized size = 0.90

$$\frac{x\sqrt[3]{a+bx^3}\left(d(a+bx^3)^2-\frac{a(-8bc+ad)\,{}_2F_1\left(-\frac{4}{3},\frac{1}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{\sqrt[3]{1+\frac{bx^3}{a}}}\right)}{8b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^(4/3)*(c + d*x^3),x]
[Out] (x*(a + b*x^3)^(1/3)*(d*(a + b*x^3)^2 - (a*(-8*b*c + a*d)*Hypergeometric2F1
[-4/3, 1/3, 4/3, -((b*x^3)/a)])/(1 + (b*x^3)/a)^(1/3)))/(8*b)
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\,x^{3}+a\right)^{\frac{4}{3}}\left(d\,x^{3}+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(4/3)*(d*x^3+c),x)
[Out] int((b*x^3+a)^(4/3)*(d*x^3+c),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(4/3)*(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(4/3)*(d*x^3 + c), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(4/3)*(d*x^3+c),x, algorithm="fricas")
```

```
[Out] integral((b*d*x^6 + (b*c + a*d)*x^3 + a*c)*(b*x^3 + a)^(1/3), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 2.23, size = 170, normalized size = 2.05

$$\frac{a^{\frac{4}{3}}cx\Gamma\left(\frac{1}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)}+\frac{a^{\frac{4}{3}}dx^{4}\Gamma\left(\frac{4}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)}+\frac{\sqrt[3]{a}\,bcx^{4}\Gamma\left(\frac{4}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)}+\frac{\sqrt[3]{a}\,bdx^{7}\Gamma\left(\frac{7}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{3},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{10}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(4/3)*(d*x**3+c),x)
```

```
[Out] a**(4/3)*c*x*gamma(1/3)*hyper((-1/3, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a
)/(3*gamma(4/3)) + a**(4/3)*d*x**4*gamma(4/3)*hyper((-1/3, 4/3), (7/3,), b*
x**3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + a**(1/3)*b*c*x**4*gamma(4/3)*hyper
((-1/3, 4/3), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + a**(1/3)*b
*d*x**7*gamma(7/3)*hyper((-1/3, 7/3), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3
*gamma(10/3))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(4/3)*(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(4/3)*(d*x^3 + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(b\,x^3+a\right)}^{4/3}\,\left(d\,x^3+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(4/3)*(c + d*x^3),x)
```

```
[Out] int((a + b*x^3)^(4/3)*(c + d*x^3), x)
```

3.66 $\int \sqrt[3]{a+bx^3}\,(c+dx^3)\,dx$

Optimal. Leaf size=82

$$\frac{dx(a+bx^3)^{4/3}}{5b}+\frac{(5bc-ad)x\sqrt[3]{a+bx^3}\;{}_2F_1\left(-\frac{1}{3},\frac{1}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{5b\sqrt[3]{1+\frac{bx^3}{a}}}$$

```
[Out] 1/5*d*x*(b*x^3+a)^(4/3)/b+1/5*(-a*d+5*b*c)*x*(b*x^3+a)^(1/3)*hypergeom([-1/
3, 1/3],[4/3],-b*x^3/a)/b/(1+b*x^3/a)^(1/3)
```

Rubi [A]
time = 0.02, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}=0.158$, Rules used = {396, 252, 251}

$$\frac{x\sqrt[3]{a+bx^3}\,(5bc-ad)\,{}_2F_1\left(-\frac{1}{3},\frac{1}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{5b\sqrt[3]{\frac{bx^3}{a}+1}}+\frac{dx(a+bx^3)^{4/3}}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(1/3)*(c + d*x^3),x]
```

```
[Out] (d*x*(a + b*x^3)^(4/3))/(5*b) + ((5*b*c - a*d)*x*(a + b*x^3)^(1/3)*Hypergeo
metric2F1[-1/3, 1/3, 4/3, -((b*x^3)/a)])/(5*b*(1 + (b*x^3)/a)^(1/3))
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
```

```
c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt[3]{a+bx^3}\left(c+dx^3\right)\,dx &= \frac{dx(a+bx^3)^{4/3}}{5b}-\frac{(-5bc+ad)\int \sqrt[3]{a+bx^3}\,dx}{5b}\\ &= \frac{dx(a+bx^3)^{4/3}}{5b}-\frac{\left((-5bc+ad)\sqrt[3]{a+bx^3}\right)\int \sqrt[3]{1+\frac{bx^3}{a}}\,dx}{5b\sqrt[3]{1+\frac{bx^3}{a}}}\\ &= \frac{dx(a+bx^3)^{4/3}}{5b}+\frac{(5bc-ad)x\sqrt[3]{a+bx^3}\;{}_2F_1\left(-\frac{1}{3},\frac{1}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{5b\sqrt[3]{1+\frac{bx^3}{a}}}\end{aligned}$$

Mathematica [A]
time = 5.61, size = 72, normalized size = 0.88

$$\frac{x\sqrt[3]{a+bx^3}\left(d(a+bx^3)+\frac{(5bc-ad)\,{}_2F_1\left(-\frac{1}{3},\frac{1}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{\sqrt[3]{1+\frac{bx^3}{a}}}\right)}{5b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^(1/3)*(c + d*x^3),x]
[Out] (x*(a + b*x^3)^(1/3)*(d*(a + b*x^3) + ((5*b*c - a*d)*Hypergeometric2F1[-1/3
, 1/3, 4/3, -((b*x^3)/a)])/(1 + (b*x^3)/a)^(1/3)))/(5*b)
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\,x^{3}+a\right)^{\frac{1}{3}}\left(d\,x^{3}+c\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(1/3)*(d*x^3+c),x)
[Out] int((b*x^3+a)^(1/3)*(d*x^3+c),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)*(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(1/3)*(d*x^3 + c), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)*(d*x^3+c),x, algorithm="fricas")
```

```
[Out] integral((b*x^3 + a)^(1/3)*(d*x^3 + c), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.41, size = 82, normalized size = 1.00

$$\frac{\sqrt[3]{a}\, cx\Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{3}, \frac{1}{3} \\ \frac{4}{3} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)} + \frac{\sqrt[3]{a}\, dx^4\Gamma\left(\frac{4}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{3}, \frac{4}{3} \\ \frac{7}{3} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(1/3)*(d*x**3+c),x)
```

```
[Out] a**(1/3)*c*x*gamma(1/3)*hyper((-1/3, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a
)/(3*gamma(4/3)) + a**(1/3)*d*x**4*gamma(4/3)*hyper((-1/3, 4/3), (7/3,), b*
x**3*exp_polar(I*pi)/a)/(3*gamma(7/3))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)*(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(1/3)*(d*x^3 + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(b\,x^3+a\right)}^{1/3}\,\left(d\,x^3+c\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(1/3)*(c + d*x^3),x)
```

```
[Out] int((a + b*x^3)^(1/3)*(c + d*x^3), x)
```

3.67 $\int \frac{c+dx^3}{(a+bx^3)^{2/3}}\,dx$

Optimal. Leaf size=82

$$\frac{dx\sqrt[3]{a+bx^3}}{2b}+\frac{(2bc-ad)x\left(1+\frac{bx^3}{a}\right)^{2/3}{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{2b\left(a+bx^3\right)^{2/3}}$$

```
[Out] 1/2*d*x*(b*x^3+a)^(1/3)/b+1/2*(-a*d+2*b*c)*x*(1+b*x^3/a)^(2/3)*hypergeom([1
/3, 2/3],[4/3],-b*x^3/a)/b/(b*x^3+a)^(2/3)
```

Rubi [A]
time = 0.02, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {396, 252, 251}

$$\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}(2bc-ad)\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{2b\left(a+bx^3\right)^{2/3}}+\frac{dx\sqrt[3]{a+bx^3}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)/(a + b*x^3)^(2/3),x]
```

```
[Out] (d*x*(a + b*x^3)^(1/3))/(2*b) + ((2*b*c - a*d)*x*(1 + (b*x^3)/a)^(2/3)*Hype
rgeometric2F1[1/3, 2/3, 4/3, -((b*x^3)/a)])/(2*b*(a + b*x^3)^(2/3))
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
```

```
c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rubi steps

$$\int \frac{c+dx^3}{(a+bx^3)^{2/3}}\,dx = \frac{dx\sqrt[3]{a+bx^3}}{2b} - \frac{(-2bc+ad)\int \frac{1}{(a+bx^3)^{2/3}}\,dx}{2b}$$
$$= \frac{dx\sqrt[3]{a+bx^3}}{2b} - \frac{\left((-2bc+ad)\left(1+\frac{bx^3}{a}\right)^{2/3}\right)\int \frac{1}{\left(1+\frac{bx^3}{a}\right)^{2/3}}\,dx}{2b\,(a+bx^3)^{2/3}}$$
$$= \frac{dx\sqrt[3]{a+bx^3}}{2b} + \frac{(2bc-ad)x\left(1+\frac{bx^3}{a}\right)^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{2b\,(a+bx^3)^{2/3}}$$

Mathematica [A]
time = 10.04, size = 73, normalized size = 0.89

$$\frac{dx(a+bx^3)+(2bc-ad)x\left(1+\frac{bx^3}{a}\right)^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{2b\,(a+bx^3)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3)/(a + b*x^3)^(2/3),x]
[Out] (d*x*(a + b*x^3) + (2*b*c - a*d)*x*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[
1/3, 2/3, 4/3, -((b*x^3)/a)])/(2*b*(a + b*x^3)^(2/3))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d\,x^3+c}{(b\,x^3+a)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)/(b*x^3+a)^(2/3),x)
[Out] int((d*x^3+c)/(b*x^3+a)^(2/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)/(b*x^3+a)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((d*x^3 + c)/(b*x^3 + a)^(2/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)/(b*x^3+a)^(2/3),x, algorithm="fricas")
```

```
[Out] integral((d*x^3 + c)/(b*x^3 + a)^(2/3), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.13, size = 78, normalized size = 0.95

$$\frac{cx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{2}{3}\\\frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{2}{3}}\Gamma\left(\frac{4}{3}\right)}+\frac{dx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{2}{3},\frac{4}{3}\\\frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{2}{3}}\Gamma\left(\frac{7}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)/(b*x**3+a)**(2/3),x)
```

```
[Out] c*x*gamma(1/3)*hyper((1/3, 2/3), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(2
/3)*gamma(4/3)) + d*x**4*gamma(4/3)*hyper((2/3, 4/3), (7/3,), b*x**3*exp_po
lar(I*pi)/a)/(3*a**(2/3)*gamma(7/3))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)/(b*x^3+a)^(2/3),x, algorithm="giac")
```

```
[Out] integrate((d*x^3 + c)/(b*x^3 + a)^(2/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{d\,x^3+c}{{\left(b\,x^3+a\right)}^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)/(a + b*x^3)^(2/3),x)
```

```
[Out] int((c + d*x^3)/(a + b*x^3)^(2/3), x)
```

3.68 $\int \frac{c+dx^3}{(a+bx^3)^{5/3}}\,dx$

Optimal. Leaf size=93

$$\frac{(bc-ad)x}{2ab\,(a+bx^3)^{2/3}}+\frac{(bc+ad)x\left(1+\frac{bx^3}{a}\right)^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{2ab\,(a+bx^3)^{2/3}}$$

```
[Out] 1/2*(-a*d+b*c)*x/a/b/(b*x^3+a)^(2/3)+1/2*(a*d+b*c)*x*(1+b*x^3/a)^(2/3)*hype
rgeom([1/3, 2/3],[4/3],-b*x^3/a)/a/b/(b*x^3+a)^(2/3)
```

Rubi [A]
time = 0.02, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {393, 252, 251}

$$\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}(ad+bc)\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{2ab\,(a+bx^3)^{2/3}}+\frac{x(bc-ad)}{2ab\,(a+bx^3)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)/(a + b*x^3)^(5/3),x]
```

```
[Out] ((b*c - a*d)*x)/(2*a*b*(a + b*x^3)^(2/3)) + ((b*c + a*d)*x*(1 + (b*x^3)/a)^
(2/3)*Hypergeometric2F1[1/3, 2/3, 4/3, -((b*x^3)/a)])/(2*a*b*(a + b*x^3)^(2
/3))
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
```

```
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{c+dx^3}{(a+bx^3)^{5/3}}\,dx &= \frac{(bc-ad)x}{2ab\,(a+bx^3)^{2/3}}+\frac{(bc+ad)\int \frac{1}{(a+bx^3)^{2/3}}\,dx}{2ab}\\ &= \frac{(bc-ad)x}{2ab\,(a+bx^3)^{2/3}}+\frac{\left((bc+ad)\left(1+\frac{bx^3}{a}\right)^{2/3}\right)\int \frac{1}{\left(1+\frac{bx^3}{a}\right)^{2/3}}\,dx}{2ab\,(a+bx^3)^{2/3}}\\ &= \frac{(bc-ad)x}{2ab\,(a+bx^3)^{2/3}}+\frac{(bc+ad)x\left(1+\frac{bx^3}{a}\right)^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{2ab\,(a+bx^3)^{2/3}}\end{aligned}$$

Mathematica [A]
time = 10.03, size = 66, normalized size = 0.71

$$\frac{-adx+(bc+ad)x\left(1+\frac{bx^3}{a}\right)^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{5}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{ab\,(a+bx^3)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3)/(a + b*x^3)^(5/3),x]
[Out] (-(a*d*x) + (b*c + a*d)*x*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[1/3, 5/3,
 4/3, -((b*x^3)/a)])/(a*b*(a + b*x^3)^(2/3))
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{d\,x^3+c}{(b\,x^3+a)^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)/(b*x^3+a)^(5/3),x)
[Out] int((d*x^3+c)/(b*x^3+a)^(5/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)/(b*x^3+a)^(5/3),x, algorithm="maxima")
[Out] integrate((d*x^3 + c)/(b*x^3 + a)^(5/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)/(b*x^3+a)^(5/3),x, algorithm="fricas")
[Out] integral((b*x^3 + a)^(1/3)*(d*x^3 + c)/(b^2*x^6 + 2*a*b*x^3 + a^2), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 4.04, size = 78, normalized size = 0.84

$$\frac{cx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{5}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{5}{3}}\Gamma\left(\frac{4}{3}\right)}+\frac{dx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{4}{3},\frac{5}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{5}{3}}\Gamma\left(\frac{7}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)/(b*x**3+a)**(5/3),x)
[Out] c*x*gamma(1/3)*hyper((1/3, 5/3), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(5
/3)*gamma(4/3)) + d*x**4*gamma(4/3)*hyper((4/3, 5/3), (7/3,), b*x**3*exp_po
lar(I*pi)/a)/(3*a**(5/3)*gamma(7/3))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)/(b*x^3+a)^(5/3),x, algorithm="giac")
[Out] integrate((d*x^3 + c)/(b*x^3 + a)^(5/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{d\,x^3+c}{{\left(b\,x^3+a\right)}^{5/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)/(a + b*x^3)^(5/3),x)
[Out] int((c + d*x^3)/(a + b*x^3)^(5/3), x)
```

3.69 $\int \frac{c+dx^3}{(a+bx^3)^{8/3}} \, dx$

Optimal. Leaf size=94

$$\frac{(bc-ad)x}{5ab\left(a+bx^3\right)^{5/3}}+\frac{(4bc+ad)x\left(1+\frac{bx^3}{a}\right)^{2/3} {}_2F_1\left(\frac{1}{3},\frac{5}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{5a^2b\left(a+bx^3\right)^{2/3}}$$

```
[Out] 1/5*(-a*d+b*c)*x/a/b/(b*x^3+a)^(5/3)+1/5*(a*d+4*b*c)*x*(1+b*x^3/a)^(2/3)*hy
pergeom([1/3, 5/3],[4/3],-b*x^3/a)/a^2/b/(b*x^3+a)^(2/3)
```

Rubi [A]
time = 0.02, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {393, 252, 251}

$$\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}(ad+4bc)\, {}_2F_1\left(\frac{1}{3},\frac{5}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{5a^2b\left(a+bx^3\right)^{2/3}}+\frac{x(bc-ad)}{5ab\left(a+bx^3\right)^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)/(a + b*x^3)^(8/3),x]
```

```
[Out] ((b*c - a*d)*x)/(5*a*b*(a + b*x^3)^(5/3)) + ((4*b*c + a*d)*x*(1 + (b*x^3)/a
)^(2/3)*Hypergeometric2F1[1/3, 5/3, 4/3, -((b*x^3)/a)])/(5*a^2*b*(a + b*x^3
)^(2/3))
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
```

```
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{c+dx^3}{\left(a+bx^3\right)^{8/3}}\,dx &= \frac{(bc-ad)x}{5ab\left(a+bx^3\right)^{5/3}}+\frac{(4bc+ad)\int \frac{1}{\left(a+bx^3\right)^{5/3}}\,dx}{5ab}\\ &= \frac{(bc-ad)x}{5ab\left(a+bx^3\right)^{5/3}}+\frac{\left((4bc+ad)\left(1+\frac{bx^3}{a}\right)^{2/3}\right)\int \frac{1}{\left(1+\frac{bx^3}{a}\right)^{5/3}}\,dx}{5a^2b\left(a+bx^3\right)^{2/3}}\\ &= \frac{(bc-ad)x}{5ab\left(a+bx^3\right)^{5/3}}+\frac{(4bc+ad)x\left(1+\frac{bx^3}{a}\right)^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{5}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{5a^2b\left(a+bx^3\right)^{2/3}}\end{aligned}$$

Mathematica [A]
time = 10.03, size = 75, normalized size = 0.80

$$\frac{x\left(-d+\frac{(4bc+ad)\left(a+bx^3\right)\left(1+\frac{bx^3}{a}\right)^{2/3}{}_2F_1\left(\frac{1}{3},\frac{8}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{a^2}\right)}{4b\left(a+bx^3\right)^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3)/(a + b*x^3)^(8/3),x]
[Out] (x*(-d + ((4*b*c + a*d)*(a + b*x^3)*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1
[1/3, 8/3, 4/3, -((b*x^3)/a)])/a^2))/(4*b*(a + b*x^3)^(5/3))
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{d\,x^3+c}{\left(b\,x^3+a\right)^{\frac{8}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)/(b*x^3+a)^(8/3),x)
[Out] int((d*x^3+c)/(b*x^3+a)^(8/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)/(b*x^3+a)^(8/3),x, algorithm="maxima")
```

```
[Out] integrate((d*x^3 + c)/(b*x^3 + a)^(8/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)/(b*x^3+a)^(8/3),x, algorithm="fricas")
```

```
[Out] integral((b*x^3 + a)^(1/3)*(d*x^3 + c)/(b^3*x^9 + 3*a*b^2*x^6 + 3*a^2*b*x^3
 + a^3), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 41.84, size = 78, normalized size = 0.83

$$\frac{cx\Gamma\left(\frac{1}{3}\right){}_{2}F_{1}\left(\left.\begin{matrix}\frac{1}{3},\frac{8}{3}\\\frac{4}{3}\end{matrix}\right|\frac{bx^{3}e^{i\pi}}{a}\right)}{3a^{\frac{8}{3}}\Gamma\left(\frac{4}{3}\right)}+\frac{dx^{4}\Gamma\left(\frac{4}{3}\right){}_{2}F_{1}\left(\left.\begin{matrix}\frac{4}{3},\frac{8}{3}\\\frac{7}{3}\end{matrix}\right|\frac{bx^{3}e^{i\pi}}{a}\right)}{3a^{\frac{8}{3}}\Gamma\left(\frac{7}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)/(b*x**3+a)**(8/3),x)
```

```
[Out] c*x*gamma(1/3)*hyper((1/3, 8/3), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(8
/3)*gamma(4/3)) + d*x**4*gamma(4/3)*hyper((4/3, 8/3), (7/3,), b*x**3*exp_po
lar(I*pi)/a)/(3*a**(8/3)*gamma(7/3))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)/(b*x^3+a)^(8/3),x, algorithm="giac")
```

```
[Out] integrate((d*x^3 + c)/(b*x^3 + a)^(8/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{d\,x^3+c}{{\left(b\,x^3+a\right)}^{8/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)/(a + b*x^3)^(8/3),x)
```

```
[Out] int((c + d*x^3)/(a + b*x^3)^(8/3), x)
```

3.70 $\int (a+bx^3)^{5/3} (c+dx^3)^2 \, dx$

Optimal. Leaf size=262

$$\frac{5a(27b^2c^2-6abcd+a^2d^2)\,x(a+bx^3)^{2/3}}{486b^2}+\frac{(27b^2c^2-6abcd+a^2d^2)\,x(a+bx^3)^{5/3}}{162b^2}+\frac{d(15bc-4ad)x(a+bx^3)}{108b^2}$$

```
[Out] 5/486*a*(a^2*d^2-6*a*b*c*d+27*b^2*c^2)*x*(b*x^3+a)^(2/3)/b^2+1/162*(a^2*d^2
-6*a*b*c*d+27*b^2*c^2)*x*(b*x^3+a)^(5/3)/b^2+1/108*d*(-4*a*d+15*b*c)*x*(b*x
^3+a)^(8/3)/b^2+1/12*d*x*(b*x^3+a)^(8/3)*(d*x^3+c)/b-5/486*a^2*(a^2*d^2-6*a
*b*c*d+27*b^2*c^2)*ln(-b^(1/3)*x+(b*x^3+a)^(1/3))/b^(7/3)+5/729*a^2*(a^2*d^
2-6*a*b*c*d+27*b^2*c^2)*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))
/b^(7/3)*3^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 262, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {427, 396, 201, 245}

$$\frac{5a^2\text{ArcTan}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)(a^2d^2-6abcd+27b^2c^2)}{243\sqrt{3}\,b^{7/3}}+\frac{x(a+bx^3)^{5/3}(a^2d^2-6abcd+27b^2c^2)}{162b^2}+\frac{5ax(a+bx^3)^{2/3}(a^2d^2-6abcd+27b^2c^2)}{486b^2}-\frac{5a^2(a^2d^2-6abcd+27b^2c^2)\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}\,x\right)}{486b^{7/3}}+\frac{dx(a+bx^3)^{8/3}(15bc-4ad)}{108b^2}+\frac{dx(a+bx^3)^{8/3}(c+dx^3)}{12b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(5/3)*(c + d*x^3)^2,x]
```

```
[Out] (5*a*(27*b^2*c^2 - 6*a*b*c*d + a^2*d^2)*x*(a + b*x^3)^(2/3))/(486*b^2) + ((
27*b^2*c^2 - 6*a*b*c*d + a^2*d^2)*x*(a + b*x^3)^(5/3))/(162*b^2) + (d*(15*b
*c - 4*a*d)*x*(a + b*x^3)^(8/3))/(108*b^2) + (d*x*(a + b*x^3)^(8/3)*(c + d*
x^3))/(12*b) + (5*a^2*(27*b^2*c^2 - 6*a*b*c*d + a^2*d^2)*ArcTan[(1 + (2*b^(
1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(243*Sqrt[3]*b^(7/3)) - (5*a^2*(27*b^2
*c^2 - 6*a*b*c*d + a^2*d^2)*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)])/(486*b^(
7/3))
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rubi steps

$$\int \left(a+bx^3\right)^{5/3}\left(c+dx^3\right)^2\,dx=\frac{dx\left(a+bx^3\right)^{8/3}\left(c+dx^3\right)}{12b}+\frac{\int \left(a+bx^3\right)^{5/3}\left(c(12bc-ad)+d(15bc-4ad)x^3\right)}{12b}$$

$$=\frac{d(15bc-4ad)x\left(a+bx^3\right)^{8/3}}{108b^2}+\frac{dx\left(a+bx^3\right)^{8/3}\left(c+dx^3\right)}{12b}+\frac{\left(27b^2c^2-6abcd+\right.}{}$$

$$=\frac{\left(27b^2c^2-6abcd+a^2d^2\right)x\left(a+bx^3\right)^{5/3}}{162b^2}+\frac{d(15bc-4ad)x\left(a+bx^3\right)^{8/3}}{108b^2}+\frac{dx(a}{}$$

$$=\frac{5a\left(27b^2c^2-6abcd+a^2d^2\right)x\left(a+bx^3\right)^{2/3}}{486b^2}+\frac{\left(27b^2c^2-6abcd+a^2d^2\right)x(a+bx}{162b^2}$$

$$=\frac{5a\left(27b^2c^2-6abcd+a^2d^2\right)x\left(a+bx^3\right)^{2/3}}{486b^2}+\frac{\left(27b^2c^2-6abcd+a^2d^2\right)x(a+bx}{162b^2}$$

Mathematica [A]

time = 1.00, size = 293, normalized size = 1.12

$$\frac{3\sqrt[3]{b}\,x\left(a+bx^3\right)^{2/3}\left(-20a^3d^2+15a^2bd\left(8c+dx^3\right)+27b^3x^3\left(6c^2+8cdx^3+3d^2x^6\right)+18ab^2\left(24c^2+22cdx^3+7d^2x^6\right)\right)+20\sqrt{3}\,a^2\left(27b^2c^2-6abcd+a^2d^2\right)\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{b}\,x}{\sqrt[3]{b}\,x+2\sqrt[3]{a+bx^3}}\right)-20a^2\left(27b^2c^2-6abcd+a^2d^2\right)\log\left(-\sqrt[3]{b}\,x+\sqrt[3]{a+bx^3}\right)+10a^2\left(27b^2c^2-6abcd+a^2d^2\right)\log\left(b^{2/3}x^2+\sqrt[3]{b}\,x\sqrt[3]{a+bx^3}+\left(a+bx^3\right)^{2/3}\right)}{2916b^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^(5/3)*(c + d*x^3)^2,x]
```

```
[Out] (3*b^(1/3)*x*(a + b*x^3)^(2/3)*(-20*a^3*d^2 + 15*a^2*b*d*(8*c + d*x^3) + 27
*b^3*x^3*(6*c^2 + 8*c*d*x^3 + 3*d^2*x^6) + 18*a*b^2*(24*c^2 + 22*c*d*x^3 +
7*d^2*x^6)) + 20*Sqrt[3]*a^2*(27*b^2*c^2 - 6*a*b*c*d + a^2*d^2)*ArcTan[(Sqr
t[3]*b^(1/3)*x)/(b^(1/3)*x + 2*(a + b*x^3)^(1/3))] - 20*a^2*(27*b^2*c^2 - 6
*a*b*c*d + a^2*d^2)*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)] + 10*a^2*(27*b^2*
c^2 - 6*a*b*c*d + a^2*d^2)*Log[b^(2/3)*x^2 + b^(1/3)*x*(a + b*x^3)^(1/3) +
(a + b*x^3)^(2/3)])/(2916*b^(7/3))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \left(b\,x^{3}+a\right)^{\frac{5}{3}} \left(d\,x^{3}+c\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(5/3)*(d*x^3+c)^2,x)
```

```
[Out] int((b*x^3+a)^(5/3)*(d*x^3+c)^2,x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 672 vs. $2(227) = 454$.
time = 0.50, size = 672, normalized size = 2.56

$$-\frac{1}{54}\left(\frac{10\sqrt{3}\,a^{2}\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\,(bx^{3}+a)^{\frac{1}{3}}}{x}\right)}{3\,b^{\frac{1}{3}}}\right)}{b^{\frac{1}{3}}}-\frac{5\,a^{2}\log\left(b^{\frac{2}{3}}+\frac{(bx^{3}+a)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{(bx^{3}+a)^{\frac{2}{3}}}{x^{2}}\right)}{b^{\frac{1}{3}}}+\frac{10\,a^{2}\log\left(-b^{\frac{1}{3}}+\frac{(bx^{3}+a)^{\frac{1}{3}}}{x}\right)}{b^{\frac{1}{3}}}+\frac{3\left(\frac{5\,(bx^{3}+a)^{\frac{2}{3}}a^{2}b}{x^{2}}-\frac{8\,(bx^{3}+a)^{\frac{5}{3}}a^{2}}{x^{5}}\right)}{b^{2}-\frac{2\,(bx^{3}+a)b}{x^{3}}+\frac{(bx^{3}+a)^{2}}{x^{6}}}\right)c^{2}+\frac{1}{243}\left(\frac{10\sqrt{3}\,a^{3}\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\,(bx^{3}+a)^{\frac{1}{3}}}{x}\right)}{3\,b^{\frac{1}{3}}}\right)}{b^{\frac{4}{3}}}-\frac{5\,a^{3}\log\left(b^{\frac{2}{3}}+\frac{(bx^{3}+a)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{(bx^{3}+a)^{\frac{2}{3}}}{x^{2}}\right)}{b^{\frac{4}{3}}}+\frac{10\,a^{3}\log\left(-b^{\frac{1}{3}}+\frac{(bx^{3}+a)^{\frac{1}{3}}}{x}\right)}{b^{\frac{4}{3}}}+\frac{3\left(\frac{5\,(bx^{3}+a)^{\frac{2}{3}}a^{3}b^{2}}{x^{2}}-\frac{13\,(bx^{3}+a)^{\frac{5}{3}}a^{3}b}{x^{5}}-\frac{10\,(bx^{3}+a)^{\frac{8}{3}}a^{3}}{x^{8}}\right)}{b^{4}-\frac{3\,(bx^{3}+a)b^{3}}{x^{3}}+\frac{3\,(bx^{3}+a)^{2}b^{2}}{x^{6}}-\frac{(bx^{3}+a)^{3}b}{x^{9}}}\right)cd-\frac{1}{2916}\left(\frac{20\sqrt{3}\,a^{4}\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\,(bx^{3}+a)^{\frac{1}{3}}}{x}\right)}{3\,b^{\frac{1}{3}}}\right)}{b^{\frac{7}{3}}}-\frac{10\,a^{4}\log\left(b^{\frac{2}{3}}+\frac{(bx^{3}+a)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{(bx^{3}+a)^{\frac{2}{3}}}{x^{2}}\right)}{b^{\frac{7}{3}}}+\frac{20\,a^{4}\log\left(-b^{\frac{1}{3}}+\frac{(bx^{3}+a)^{\frac{1}{3}}}{x}\right)}{b^{\frac{7}{3}}}+\frac{3\left(\frac{10\,(bx^{3}+a)^{\frac{2}{3}}a^{4}b^{3}}{x^{2}}-\frac{36\,(bx^{3}+a)^{\frac{5}{3}}a^{4}b^{2}}{x^{5}}-\frac{75\,(bx^{3}+a)^{\frac{8}{3}}a^{4}b}{x^{8}}+\frac{20\,(bx^{3}+a)^{\frac{11}{3}}a^{4}}{x^{11}}\right)}{b^{6}-\frac{4\,(bx^{3}+a)b^{5}}{x^{3}}+\frac{6\,(bx^{3}+a)^{2}b^{4}}{x^{6}}-\frac{4\,(bx^{3}+a)^{3}b^{3}}{x^{9}}+\frac{(bx^{3}+a)^{4}b^{2}}{x^{12}}}\right)d^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(5/3)*(d*x^3+c)^2,x, algorithm="maxima")
```

```
[Out] -1/54*(10*sqrt(3)*a^2*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/
b^(1/3))/b^(1/3) - 5*a^2*log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3
 + a)^(2/3)/x^2)/b^(1/3) + 10*a^2*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(1/
3) + 3*(5*(b*x^3 + a)^(2/3)*a^2*b/x^2 - 8*(b*x^3 + a)^(5/3)*a^2/x^5)/(b^2 -
 2*(b*x^3 + a)*b/x^3 + (b*x^3 + a)^2/x^6))*c^2 + 1/243*(10*sqrt(3)*a^3*arct
an(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1/3))/b^(4/3) - 5*a^3*l
og(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2/3)/x^2)/b^(4/3) +
 10*a^3*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(4/3) + 3*(5*(b*x^3 + a)^(2/3
)*a^3*b^2/x^2 - 13*(b*x^3 + a)^(5/3)*a^3*b/x^5 - 10*(b*x^3 + a)^(8/3)*a^3/x
^8)/(b^4 - 3*(b*x^3 + a)*b^3/x^3 + 3*(b*x^3 + a)^2*b^2/x^6 - (b*x^3 + a)^3*
b/x^9))*c*d - 1/2916*(20*sqrt(3)*a^4*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3
 + a)^(1/3)/x)/b^(1/3))/b^(7/3) - 10*a^4*log(b^(2/3) + (b*x^3 + a)^(1/3)*b^
(1/3)/x + (b*x^3 + a)^(2/3)/x^2)/b^(7/3) + 20*a^4*log(-b^(1/3) + (b*x^3 + a
)^(1/3)/x)/b^(7/3) + 3*(10*(b*x^3 + a)^(2/3)*a^4*b^3/x^2 - 36*(b*x^3 + a)^(
```

```
5/3)*a^4*b^2/x^5 - 75*(b*x^3 + a)^(8/3)*a^4*b/x^8 + 20*(b*x^3 + a)^(11/3)*a
^4/x^11)/(b^6 - 4*(b*x^3 + a)*b^5/x^3 + 6*(b*x^3 + a)^2*b^4/x^6 - 4*(b*x^3
+ a)^3*b^3/x^9 + (b*x^3 + a)^4*b^2/x^12))*d^2
```

Fricas [A]
time = 2.64, size = 717, normalized size = 2.74

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(5/3)*(d*x^3+c)^2,x, algorithm="fricas")
```

```
[Out] [1/2916*(30*sqrt(1/3)*(27*a^2*b^3*c^2 - 6*a^3*b^2*c*d + a^4*b*d^2)*sqrt((-b
)^(1/3)/b)*log(3*b*x^3 - 3*(b*x^3 + a)^(1/3)*(-b)^(2/3)*x^2 - 3*sqrt(1/3)*(
(-b)^(1/3)*b*x^3 - (b*x^3 + a)^(1/3)*b*x^2 + 2*(b*x^3 + a)^(2/3)*(-b)^(2/3)
*x)*sqrt((-b)^(1/3)/b) + 2*a) - 20*(27*a^2*b^2*c^2 - 6*a^3*b*c*d + a^4*d^2)
*(-b)^(2/3)*log(((-b)^(1/3)*x + (b*x^3 + a)^(1/3))/x) + 10*(27*a^2*b^2*c^2
- 6*a^3*b*c*d + a^4*d^2)*(-b)^(2/3)*log(((-b)^(2/3)*x^2 - (b*x^3 + a)^(1/3)
*(-b)^(1/3)*x + (b*x^3 + a)^(2/3))/x^2) + 3*(81*b^4*d^2*x^10 + 18*(12*b^4*c
*d + 7*a*b^3*d^2)*x^7 + 3*(54*b^4*c^2 + 132*a*b^3*c*d + 5*a^2*b^2*d^2)*x^4
+ 4*(108*a*b^3*c^2 + 30*a^2*b^2*c*d - 5*a^3*b*d^2)*x)*(b*x^3 + a)^(2/3))/b^
3, -1/2916*(60*sqrt(1/3)*(27*a^2*b^3*c^2 - 6*a^3*b^2*c*d + a^4*b*d^2)*sqrt(
-(-b)^(1/3)/b)*arctan(-sqrt(1/3)*((-b)^(1/3)*x - 2*(b*x^3 + a)^(1/3))*sqrt(
-(-b)^(1/3)/b)/x) + 20*(27*a^2*b^2*c^2 - 6*a^3*b*c*d + a^4*d^2)*(-b)^(2/3)*
log(((-b)^(1/3)*x + (b*x^3 + a)^(1/3))/x) - 10*(27*a^2*b^2*c^2 - 6*a^3*b*c*
d + a^4*d^2)*(-b)^(2/3)*log(((-b)^(2/3)*x^2 - (b*x^3 + a)^(1/3)*(-b)^(1/3)*
x + (b*x^3 + a)^(2/3))/x^2) - 3*(81*b^4*d^2*x^10 + 18*(12*b^4*c*d + 7*a*b^3
*d^2)*x^7 + 3*(54*b^4*c^2 + 132*a*b^3*c*d + 5*a^2*b^2*d^2)*x^4 + 4*(108*a*b
^3*c^2 + 30*a^2*b^2*c*d - 5*a^3*b*d^2)*x)*(b*x^3 + a)^(2/3))/b^3]
```

Sympy [C] Result contains complex when optimal does not.
time = 32.75, size = 270, normalized size = 1.03

$$\frac{a^{\frac{5}{3}}c^{2}x\Gamma\left(\frac{1}{3}\right){}_{2}F_{1}\left(\left.\begin{matrix}-\frac{2}{3},\frac{1}{3}\\\frac{4}{3}\end{matrix}\right|\frac{bx^{3}e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)}+\frac{2a^{\frac{5}{3}}cdx^{4}\Gamma\left(\frac{4}{3}\right){}_{2}F_{1}\left(\left.\begin{matrix}-\frac{2}{3},\frac{4}{3}\\\frac{7}{3}\end{matrix}\right|\frac{bx^{3}e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)}+\frac{a^{\frac{5}{3}}d^{2}x^{7}\Gamma\left(\frac{7}{3}\right){}_{2}F_{1}\left(\left.\begin{matrix}-\frac{2}{3},\frac{7}{3}\\\frac{10}{3}\end{matrix}\right|\frac{bx^{3}e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{10}{3}\right)}+\frac{a^{\frac{2}{3}}bc^{2}x^{4}\Gamma\left(\frac{4}{3}\right){}_{2}F_{1}\left(\left.\begin{matrix}-\frac{2}{3},\frac{4}{3}\\\frac{7}{3}\end{matrix}\right|\frac{bx^{3}e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)}+\frac{2a^{\frac{2}{3}}bcdx^{7}\Gamma\left(\frac{7}{3}\right){}_{2}F_{1}\left(\left.\begin{matrix}-\frac{2}{3},\frac{7}{3}\\\frac{10}{3}\end{matrix}\right|\frac{bx^{3}e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{10}{3}\right)}+\frac{a^{\frac{2}{3}}bd^{2}x^{10}\Gamma\left(\frac{10}{3}\right){}_{2}F_{1}\left(\left.\begin{matrix}-\frac{2}{3},\frac{10}{3}\\\frac{13}{3}\end{matrix}\right|\frac{bx^{3}e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{13}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(5/3)*(d*x**3+c)**2,x)
```

```
[Out] a**(5/3)*c**2*x*gamma(1/3)*hyper((-2/3, 1/3), (4/3,), b*x**3*exp_polar(I*pi
)/a)/(3*gamma(4/3)) + 2*a**(5/3)*c*d*x**4*gamma(4/3)*hyper((-2/3, 4/3), (7/
3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + a**(5/3)*d**2*x**7*gamma(7/
3)*hyper((-2/3, 7/3), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(10/3)) +
a**(2/3)*b*c**2*x**4*gamma(4/3)*hyper((-2/3, 4/3), (7/3,), b*x**3*exp_polar
(I*pi)/a)/(3*gamma(7/3)) + 2*a**(2/3)*b*c*d*x**7*gamma(7/3)*hyper((-2/3, 7/
3), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(10/3)) + a**(2/3)*b*d**2*x*
```

```
*10*gamma(10/3)*hyper((-2/3, 10/3), (13/3,), b*x**3*exp_polar(I*pi)/a)/(3*g
amma(13/3))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(5/3)*(d*x^3+c)^2,x, algorithm="giac")
[Out] integrate((b*x^3 + a)^(5/3)*(d*x^3 + c)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \left(b\,x^3+a\right)^{5/3}\,\left(d\,x^3+c\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(5/3)*(c + d*x^3)^2,x)
[Out] int((a + b*x^3)^(5/3)*(c + d*x^3)^2, x)
```

3.71 $\int (a+bx^3)^{2/3} (c+dx^3)^2 \, dx$

Optimal. Leaf size=219

$$\frac{(27b^2c^2-9abcd+2a^2d^2)\,x(a+bx^3)^{2/3}}{81b^2}+\frac{2d(3bc-ad)x(a+bx^3)^{5/3}}{27b^2}+\frac{dx(a+bx^3)^{5/3}(c+dx^3)}{9b}+\frac{2a(27b^2c^2-\cdots}{\cdots}$$

```
[Out] 1/81*(2*a^2*d^2-9*a*b*c*d+27*b^2*c^2)*x*(b*x^3+a)^(2/3)/b^2+2/27*d*(-a*d+3*
b*c)*x*(b*x^3+a)^(5/3)/b^2+1/9*d*x*(b*x^3+a)^(5/3)*(d*x^3+c)/b-1/81*a*(2*a^
2*d^2-9*a*b*c*d+27*b^2*c^2)*ln(-b^(1/3)*x+(b*x^3+a)^(1/3))/b^(7/3)+2/243*a*
(2*a^2*d^2-9*a*b*c*d+27*b^2*c^2)*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))
*3^(1/2))/b^(7/3)*3^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 219, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {427, 396, 201, 245}

$$\frac{2a\,\text{ArcTan}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)(2a^2d^2-9abcd+27b^2c^2)}{81\sqrt{3}\,b^{7/3}}+\frac{x(a+bx^3)^{2/3}(2a^2d^2-9abcd+27b^2c^2)}{81b^2}-\frac{a(2a^2d^2-9abcd+27b^2c^2)\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}\,x\right)}{81b^{7/3}}+\frac{2dx(a+bx^3)^{5/3}(3bc-ad)}{27b^2}+\frac{dx(a+bx^3)^{5/3}(c+dx^3)}{9b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(2/3)*(c + d*x^3)^2,x]
```

```
[Out] ((27*b^2*c^2 - 9*a*b*c*d + 2*a^2*d^2)*x*(a + b*x^3)^(2/3))/(81*b^2) + (2*d*
(3*b*c - a*d)*x*(a + b*x^3)^(5/3))/(27*b^2) + (d*x*(a + b*x^3)^(5/3)*(c + d
*x^3))/(9*b) + (2*a*(27*b^2*c^2 - 9*a*b*c*d + 2*a^2*d^2)*ArcTan[(1 + (2*b^(
1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(81*Sqrt[3]*b^(7/3)) - (a*(27*b^2*c^2
- 9*a*b*c*d + 2*a^2*d^2)*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)])/(81*b^(7/3)
)
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
```

```
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rubi steps

$$\int \left(a+bx^3\right)^{2/3}\left(c+dx^3\right)^2\,dx = \frac{dx(a+bx^3)^{5/3}\left(c+dx^3\right)}{9b}+\frac{\int \left(a+bx^3\right)^{2/3}\left(c(9bc-ad)+4d(3bc-ad)x^3\right)\,d}{9b}$$

$$= \frac{2d(3bc-ad)x(a+bx^3)^{5/3}}{27b^2}+\frac{dx(a+bx^3)^{5/3}\left(c+dx^3\right)}{9b}-\frac{(4ad(3bc-ad)-}{}$$

$$= \frac{\left(27b^2c^2-9abcd+2a^2d^2\right)x(a+bx^3)^{2/3}}{81b^2}+\frac{2d(3bc-ad)x(a+bx^3)^{5/3}}{27b^2}+\frac{dx}{}$$

$$= \frac{\left(27b^2c^2-9abcd+2a^2d^2\right)x(a+bx^3)^{2/3}}{81b^2}+\frac{2d(3bc-ad)x(a+bx^3)^{5/3}}{27b^2}+\frac{dx}{}$$

Mathematica [A]
time = 0.76, size = 256, normalized size = 1.17

$$\frac{3\sqrt[3]{b}\,x(a+bx^3)^{2/3}\left(-4a^2d^2+3abd(6c+dx^3)+9b^2(3c^2+3cdx^3+d^2x^6)\right)+2\sqrt{3}\,a(27b^2c^2-9abcd+2a^2d^2)\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{b}\,x}{\sqrt[3]{b}\,x+2\sqrt[3]{a+bx^3}}\right)-2a(27b^2c^2-9abcd+2a^2d^2)\log\left(-\sqrt[3]{b}\,x+\sqrt[3]{a+bx^3}\right)+a(27b^2c^2-9abcd+2a^2d^2)\log\left(b^{2/3}x^2+\sqrt[3]{b}\,x\sqrt[3]{a+bx^3}+(a+bx^3)^{2/3}\right)}{243b^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^(2/3)*(c + d*x^3)^2,x]
```

```
[Out] (3*b^(1/3)*x*(a + b*x^3)^(2/3)*(-4*a^2*d^2 + 3*a*b*d*(6*c + d*x^3) + 9*b^2*
(3*c^2 + 3*c*d*x^3 + d^2*x^6)) + 2*Sqrt[3]*a*(27*b^2*c^2 - 9*a*b*c*d + 2*a^
2*d^2)*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2*(a + b*x^3)^(1/3))] - 2*a*
(27*b^2*c^2 - 9*a*b*c*d + 2*a^2*d^2)*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)]
+ a*(27*b^2*c^2 - 9*a*b*c*d + 2*a^2*d^2)*Log[b^(2/3)*x^2 + b^(1/3)*x*(a + b
*x^3)^(1/3) + (a + b*x^3)^(2/3)])/(243*b^(7/3))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \left(b\,x^{3}+a\right)^{\frac{2}{3}} \left(d\,x^{3}+c\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(2/3)*(d*x^3+c)^2,x)
```

```
[Out] int((b*x^3+a)^(2/3)*(d*x^3+c)^2,x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 552 vs. $2(188) = 376$.
time = 0.55, size = 552, normalized size = 2.52

$$-\frac{1}{9}\left(\frac{2\sqrt{3}\,a\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\,(bx^{3}+a)^{\frac{1}{3}}}{x}\right)}{3\,b^{\frac{1}{3}}}\right)}{b^{\frac{1}{3}}}-\frac{a\log\left(b^{\frac{2}{3}}+\frac{(bx^{3}+a)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{(bx^{3}+a)^{\frac{2}{3}}}{x^{2}}\right)}{b^{\frac{1}{3}}}+\frac{2\,a\log\left(-b^{\frac{1}{3}}+\frac{(bx^{3}+a)^{\frac{1}{3}}}{x}\right)}{b^{\frac{1}{3}}}+\frac{3\,(bx^{3}+a)^{\frac{2}{3}}a}{\left(b-\frac{bx^{3}+a}{x^{3}}\right)x^{2}}\right)c^{2}+\frac{1}{27}\left(\frac{2\sqrt{3}\,a^{2}\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\,(bx^{3}+a)^{\frac{1}{3}}}{x}\right)}{3\,b^{\frac{1}{3}}}\right)}{b^{\frac{4}{3}}}-\frac{a^{2}\log\left(b^{\frac{2}{3}}+\frac{(bx^{3}+a)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{(bx^{3}+a)^{\frac{2}{3}}}{x^{2}}\right)}{b^{\frac{4}{3}}}+\frac{2\,a^{2}\log\left(-b^{\frac{1}{3}}+\frac{(bx^{3}+a)^{\frac{1}{3}}}{x}\right)}{b^{\frac{4}{3}}}+\frac{3\left(\frac{(bx^{3}+a)^{\frac{2}{3}}a^{2}b}{x^{2}}+\frac{2\,(bx^{3}+a)^{\frac{5}{3}}a^{2}}{x^{5}}\right)}{b^{3}-\frac{2\,(bx^{3}+a)b^{2}}{x^{3}}+\frac{(bx^{3}+a)^{2}b}{x^{6}}}\right)cd-\frac{1}{243}\left(\frac{4\sqrt{3}\,a^{3}\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\,(bx^{3}+a)^{\frac{1}{3}}}{x}\right)}{3\,b^{\frac{1}{3}}}\right)}{b^{\frac{7}{3}}}-\frac{2\,a^{3}\log\left(b^{\frac{2}{3}}+\frac{(bx^{3}+a)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{(bx^{3}+a)^{\frac{2}{3}}}{x^{2}}\right)}{b^{\frac{7}{3}}}+\frac{4\,a^{3}\log\left(-b^{\frac{1}{3}}+\frac{(bx^{3}+a)^{\frac{1}{3}}}{x}\right)}{b^{\frac{7}{3}}}+\frac{3\left(\frac{2\,(bx^{3}+a)^{\frac{2}{3}}a^{3}b^{2}}{x^{2}}+\frac{11\,(bx^{3}+a)^{\frac{5}{3}}a^{3}b}{x^{5}}-\frac{4\,(bx^{3}+a)^{\frac{8}{3}}a^{3}}{x^{8}}\right)}{b^{5}-\frac{3\,(bx^{3}+a)b^{4}}{x^{3}}+\frac{3\,(bx^{3}+a)^{2}b^{3}}{x^{6}}-\frac{(bx^{3}+a)^{3}b^{2}}{x^{9}}}\right)d^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)*(d*x^3+c)^2,x, algorithm="maxima")
```

```
[Out] -1/9*(2*sqrt(3)*a*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1
/3))/b^(1/3) - a*log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2
/3)/x^2)/b^(1/3) + 2*a*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(1/3) + 3*(b*x
^3 + a)^(2/3)*a/((b - (b*x^3 + a)/x^3)*x^2))*c^2 + 1/27*(2*sqrt(3)*a^2*arct
an(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1/3))/b^(4/3) - a^2*log
(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2/3)/x^2)/b^(4/3) + 2
*a^2*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(4/3) + 3*((b*x^3 + a)^(2/3)*a^2
*b/x^2 + 2*(b*x^3 + a)^(5/3)*a^2/x^5)/(b^3 - 2*(b*x^3 + a)*b^2/x^3 + (b*x^3
 + a)^2*b/x^6))*c*d - 1/243*(4*sqrt(3)*a^3*arctan(1/3*sqrt(3)*(b^(1/3) + 2*
(b*x^3 + a)^(1/3)/x)/b^(1/3))/b^(7/3) - 2*a^3*log(b^(2/3) + (b*x^3 + a)^(1/
3)*b^(1/3)/x + (b*x^3 + a)^(2/3)/x^2)/b^(7/3) + 4*a^3*log(-b^(1/3) + (b*x^3
 + a)^(1/3)/x)/b^(7/3) + 3*(2*(b*x^3 + a)^(2/3)*a^3*b^2/x^2 + 11*(b*x^3 + a
)^(5/3)*a^3*b/x^5 - 4*(b*x^3 + a)^(8/3)*a^3/x^8)/(b^5 - 3*(b*x^3 + a)*b^4/x
^3 + 3*(b*x^3 + a)^2*b^3/x^6 - (b*x^3 + a)^3*b^2/x^9))*d^2
```

Fricas [A]
time = 3.41, size = 634, normalized size = 2.89

$$\left[\frac{3\sqrt{\frac{1}{3}}\left(27\,ab^{3}c^{2}-9\,a^{2}b^{2}cd+2\,a^{3}bd^{2}\right)\sqrt{\frac{(-b)^{\frac{1}{3}}}{b}}\log\left(3\,bx^{3}-3\,(bx^{3}+a)^{\frac{1}{3}}(-b)^{\frac{2}{3}}x^{2}-3\sqrt{\frac{1}{3}}\left((-b)^{\frac{1}{3}}bx^{3}-(bx^{3}+a)^{\frac{1}{3}}bx^{2}+2\,(bx^{3}+a)^{\frac{2}{3}}(-b)^{\frac{2}{3}}x\right)\sqrt{\frac{(-b)^{\frac{1}{3}}}{b}}+2\,a\right)-2\left(27\,ab^{2}c^{2}-9\,a^{2}bcd+2\,a^{3}d^{2}\right)(-b)^{\frac{2}{3}}\log\left(\frac{(-b)^{\frac{1}{3}}x+(bx^{3}+a)^{\frac{1}{3}}}{x}\right)+\left(27\,ab^{2}c^{2}-9\,a^{2}bcd+2\,a^{3}d^{2}\right)(-b)^{\frac{2}{3}}\log\left(\frac{(-b)^{\frac{2}{3}}x^{2}-(bx^{3}+a)^{\frac{1}{3}}(-b)^{\frac{1}{3}}x+(bx^{3}+a)^{\frac{2}{3}}}{x^{2}}\right)+3\left(9\,b^{3}d^{2}x^{7}+3\left(9\,b^{3}cd+ab^{2}d^{2}\right)x^{4}+\left(27\,b^{3}c^{2}+18\,ab^{2}cd-4\,a^{2}bd^{2}\right)x\right)(bx^{3}+a)^{\frac{2}{3}}}{243\,b^{3}},\;-\frac{6\sqrt{\frac{1}{3}}\left(27\,ab^{3}c^{2}-9\,a^{2}b^{2}cd+2\,a^{3}bd^{2}\right)\sqrt{-\frac{(-b)^{\frac{1}{3}}}{b}}\arctan\left(-\frac{\sqrt{\frac{1}{3}}\left((-b)^{\frac{1}{3}}x-2\,(bx^{3}+a)^{\frac{1}{3}}\right)\sqrt{-\frac{(-b)^{\frac{1}{3}}}{b}}}{x}\right)+2\left(27\,ab^{2}c^{2}-9\,a^{2}bcd+2\,a^{3}d^{2}\right)(-b)^{\frac{2}{3}}\log\left(\frac{(-b)^{\frac{1}{3}}x+(bx^{3}+a)^{\frac{1}{3}}}{x}\right)-\left(27\,ab^{2}c^{2}-9\,a^{2}bcd+2\,a^{3}d^{2}\right)(-b)^{\frac{2}{3}}\log\left(\frac{(-b)^{\frac{2}{3}}x^{2}-(bx^{3}+a)^{\frac{1}{3}}(-b)^{\frac{1}{3}}x+(bx^{3}+a)^{\frac{2}{3}}}{x^{2}}\right)-3\left(9\,b^{3}d^{2}x^{7}+3\left(9\,b^{3}cd+ab^{2}d^{2}\right)x^{4}+\left(27\,b^{3}c^{2}+18\,ab^{2}cd-4\,a^{2}bd^{2}\right)x\right)(bx^{3}+a)^{\frac{2}{3}}}{243\,b^{3}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)*(d*x^3+c)^2,x, algorithm="fricas")
```

```
[Out] [1/243*(3*sqrt(1/3)*(27*a*b^3*c^2 - 9*a^2*b^2*c*d + 2*a^3*b*d^2)*sqrt((-b)^
(1/3)/b)*log(3*b*x^3 - 3*(b*x^3 + a)^(1/3)*(-b)^(2/3)*x^2 - 3*sqrt(1/3)*((-
b)^(1/3)*b*x^3 - (b*x^3 + a)^(1/3)*b*x^2 + 2*(b*x^3 + a)^(2/3)*(-b)^(2/3)*x
)*sqrt((-b)^(1/3)/b) + 2*a) - 2*(27*a*b^2*c^2 - 9*a^2*b*c*d + 2*a^3*d^2)*(-
b)^(2/3)*log(((-b)^(1/3)*x + (b*x^3 + a)^(1/3))/x) + (27*a*b^2*c^2 - 9*a^2*
b*c*d + 2*a^3*d^2)*(-b)^(2/3)*log(((-b)^(2/3)*x^2 - (b*x^3 + a)^(1/3)*(-b)^
(1/3)*x + (b*x^3 + a)^(2/3))/x^2) + 3*(9*b^3*d^2*x^7 + 3*(9*b^3*c*d + a*b^2
*d^2)*x^4 + (27*b^3*c^2 + 18*a*b^2*c*d - 4*a^2*b*d^2)*x)*(b*x^3 + a)^(2/3))
/b^3, -1/243*(6*sqrt(1/3)*(27*a*b^3*c^2 - 9*a^2*b^2*c*d + 2*a^3*b*d^2)*sqrt
(-(-b)^(1/3)/b)*arctan(-sqrt(1/3)*((-b)^(1/3)*x - 2*(b*x^3 + a)^(1/3))*sqrt
(-(-b)^(1/3)/b)/x) + 2*(27*a*b^2*c^2 - 9*a^2*b*c*d + 2*a^3*d^2)*(-b)^(2/3)*
log(((-b)^(1/3)*x + (b*x^3 + a)^(1/3))/x) - (27*a*b^2*c^2 - 9*a^2*b*c*d + 2
*a^3*d^2)*(-b)^(2/3)*log(((-b)^(2/3)*x^2 - (b*x^3 + a)^(1/3)*(-b)^(1/3)*x +
 (b*x^3 + a)^(2/3))/x^2) - 3*(9*b^3*d^2*x^7 + 3*(9*b^3*c*d + a*b^2*d^2)*x^4
 + (27*b^3*c^2 + 18*a*b^2*c*d - 4*a^2*b*d^2)*x)*(b*x^3 + a)^(2/3))/b^3]
```

Sympy [C] Result contains complex when optimal does not.
time = 5.81, size = 131, normalized size = 0.60

$$\frac{a^{\frac{2}{3}}c^2x\Gamma\left(\frac{1}{3}\right)\,{}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{1}{3}\\\frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)}+\frac{2a^{\frac{2}{3}}cdx^4\Gamma\left(\frac{4}{3}\right)\,{}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{4}{3}\\\frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)}+\frac{a^{\frac{2}{3}}d^2x^7\Gamma\left(\frac{7}{3}\right)\,{}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{7}{3}\\\frac{10}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{10}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(2/3)*(d*x**3+c)**2,x)
```

```
[Out] a**(2/3)*c**2*x*gamma(1/3)*hyper((-2/3, 1/3), (4/3,), b*x**3*exp_polar(I*pi
)/a)/(3*gamma(4/3)) + 2*a**(2/3)*c*d*x**4*gamma(4/3)*hyper((-2/3, 4/3), (7/
3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + a**(2/3)*d**2*x**7*gamma(7/
3)*hyper((-2/3, 7/3), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(10/3))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)*(d*x^3+c)^2,x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(2/3)*(d*x^3 + c)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \left(b\,x^3+a\right)^{2/3}\,\left(d\,x^3+c\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(2/3)*(c + d*x^3)^2,x)
```

```
[Out] int((a + b*x^3)^(2/3)*(c + d*x^3)^2, x)
```

3.72 $\int \frac{(c+dx^3)^2}{\sqrt[3]{a+bx^3}}\,dx$

Optimal. Leaf size=175

$$\frac{d(9bc-4ad)x(a+bx^3)^{2/3}}{18b^2}+\frac{dx(a+bx^3)^{2/3}(c+dx^3)}{6b}+\frac{(9b^2c^2-6abcd+2a^2d^2)\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{9\sqrt{3}\,b^{7/3}}-\frac{(9$$

```
[Out] 1/18*d*(-4*a*d+9*b*c)*x*(b*x^3+a)^(2/3)/b^2+1/6*d*x*(b*x^3+a)^(2/3)*(d*x^3+
c)/b-1/18*(2*a^2*d^2-6*a*b*c*d+9*b^2*c^2)*ln(-b^(1/3)*x+(b*x^3+a)^(1/3))/b^
(7/3)+1/27*(2*a^2*d^2-6*a*b*c*d+9*b^2*c^2)*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3
+a)^(1/3))*3^(1/2))/b^(7/3)*3^(1/2)
```

Rubi [A]

time = 0.07, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {427, 396, 245}

$$\frac{\text{ArcTan}\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)(2a^2d^2-6abcd+9b^2c^2)}{9\sqrt{3}\,b^{7/3}}-\frac{(2a^2d^2-6abcd+9b^2c^2)\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{18b^{7/3}}+\frac{dx(a+bx^3)^{2/3}(9bc-4ad)}{18b^2}+\frac{dx(a+bx^3)^{2/3}(c+dx^3)}{6b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)^2/(a + b*x^3)^(1/3),x]
```

```
[Out] (d*(9*b*c - 4*a*d)*x*(a + b*x^3)^(2/3))/(18*b^2) + (d*x*(a + b*x^3)^(2/3)*(
c + d*x^3))/(6*b) + ((9*b^2*c^2 - 6*a*b*c*d + 2*a^2*d^2)*ArcTan[(1 + (2*b^(
1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(9*Sqrt[3]*b^(7/3)) - ((9*b^2*c^2 - 6*
a*b*c*d + 2*a^2*d^2)*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)])/(18*b^(7/3))
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rubi steps

$$\int \frac{(c+dx^3)^2}{\sqrt[3]{a+bx^3}}\,dx = \frac{dx(a+bx^3)^{2/3}(c+dx^3)}{6b} + \frac{\int \frac{c(6bc-ad)+d(9bc-4ad)x^3}{\sqrt[3]{a+bx^3}}\,dx}{6b}$$

$$= \frac{d(9bc-4ad)x(a+bx^3)^{2/3}}{18b^2} + \frac{dx(a+bx^3)^{2/3}(c+dx^3)}{6b} + \frac{(9b^2c^2-6abcd+2a^2d^2)\int \frac{\ldots}{\sqrt[3]{a+\ldots}}}{9b^2}$$

$$= \frac{d(9bc-4ad)x(a+bx^3)^{2/3}}{18b^2} + \frac{dx(a+bx^3)^{2/3}(c+dx^3)}{6b} + \frac{(9b^2c^2-6abcd+2a^2d^2)\tan^{-1}(\ldots}{9\sqrt{3}\,b^{7/3}}$$

Mathematica [A]
time = 0.62, size = 223, normalized size = 1.27

$$\frac{3\sqrt[3]{b}\,dx(a+bx^3)^{2/3}(-4ad+3b(4c+dx^3))+2\sqrt{3}(9b^2c^2-6abcd+2a^2d^2)\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{b}\,x}{\sqrt[3]{b}\,x+2\sqrt[3]{a+bx^3}}\right)-2(9b^2c^2-6abcd+2a^2d^2)\log\left(-\sqrt[3]{b}\,x+\sqrt[3]{a+bx^3}\right)+(9b^2c^2-6abcd+2a^2d^2)\log\left(b^{2/3}x^2+\sqrt[3]{b}\,x\sqrt[3]{a+bx^3}+(a+bx^3)^{2/3}\right)}{54b^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3)^2/(a + b*x^3)^(1/3),x]
```

```
[Out] (3*b^(1/3)*d*x*(a + b*x^3)^(2/3)*(-4*a*d + 3*b*(4*c + d*x^3)) + 2*Sqrt[3]*(
9*b^2*c^2 - 6*a*b*c*d + 2*a^2*d^2)*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x +
2*(a + b*x^3)^(1/3))] - 2*(9*b^2*c^2 - 6*a*b*c*d + 2*a^2*d^2)*Log[-(b^(1/3)
*x) + (a + b*x^3)^(1/3)] + (9*b^2*c^2 - 6*a*b*c*d + 2*a^2*d^2)*Log[b^(2/3)*
x^2 + b^(1/3)*x*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)])/(54*b^(7/3))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(d\,x^3+c)^2}{(b\,x^3+a)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^2/(b*x^3+a)^(1/3),x)
```

```
[Out] int((d*x^3+c)^2/(b*x^3+a)^(1/3),x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 436 vs. $2(148) = 296$.
time = 0.53, size = 436, normalized size = 2.49

$$-\frac{1}{6}\left(\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2(bx^3+a)^{\frac{1}{3}}}{x}\right)}{3b^{\frac{1}{3}}}\right)}{b^{\frac{1}{3}}}-\frac{\log\left(b^{\frac{2}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{(bx^3+a)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{1}{3}}}+\frac{2\log\left(-b^{\frac{1}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}}{x}\right)}{b^{\frac{1}{3}}}\right)c^2+\frac{1}{9}\left(\frac{2\sqrt{3}a\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2(bx^3+a)^{\frac{1}{3}}}{x}\right)}{3b^{\frac{1}{3}}}\right)}{b^{\frac{4}{3}}}-\frac{a\log\left(b^{\frac{2}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{(bx^3+a)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{4}{3}}}+\frac{2a\log\left(-b^{\frac{1}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}}{x}\right)}{b^{\frac{4}{3}}}-\frac{6(bx^3+a)^{\frac{2}{3}}a}{\left(b^2-\frac{(bx^3+a)b}{x^3}\right)x^2}\right)cd-\frac{1}{54}\left(\frac{4\sqrt{3}a^2\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2(bx^3+a)^{\frac{1}{3}}}{x}\right)}{3b^{\frac{1}{3}}}\right)}{b^{\frac{7}{3}}}-\frac{2a^2\log\left(b^{\frac{2}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{(bx^3+a)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{7}{3}}}+\frac{4a^2\log\left(-b^{\frac{1}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}}{x}\right)}{b^{\frac{7}{3}}}-\frac{3\left(\frac{7(bx^3+a)^{\frac{2}{3}}a^2b}{x^2}-\frac{4(bx^3+a)^{\frac{5}{3}}a^2}{x^5}\right)}{b^4-\frac{2(bx^3+a)b^3}{x^3}+\frac{(bx^3+a)^2b^2}{x^6}}\right)d^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^2/(b*x^3+a)^(1/3),x, algorithm="maxima")
```

```
[Out] -1/6*(2*sqrt(3)*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1/3
))/b^(1/3) - log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2/3)/
x^2)/b^(1/3) + 2*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(1/3))*c^2 + 1/9*(2*
sqrt(3)*a*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1/3))/b^(
4/3) - a*log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2/3)/x^2)
/b^(4/3) + 2*a*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(4/3) - 6*(b*x^3 + a)^
(2/3)*a/((b^2 - (b*x^3 + a)*b/x^3)*x^2))*c*d - 1/54*(4*sqrt(3)*a^2*arctan(1
/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1/3))/b^(7/3) - 2*a^2*log(b
^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2/3)/x^2)/b^(7/3) + 4*a
^2*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(7/3) - 3*(7*(b*x^3 + a)^(2/3)*a^2
*b/x^2 - 4*(b*x^3 + a)^(5/3)*a^2/x^5)/(b^4 - 2*(b*x^3 + a)*b^3/x^3 + (b*x^3
 + a)^2*b^2/x^6))*d^2
```

Fricas [A]
time = 3.13, size = 554, normalized size = 3.17

$$\left[\frac{3\sqrt{\frac{1}{3}}(9b^3c^2-6ab^2cd+2a^2bd^2)\sqrt{\frac{(-b)^{\frac{1}{3}}}{b}}\log\left(3bx^3-3(bx^3+a)^{\frac{1}{3}}(-b)^{\frac{2}{3}}x^2-3\sqrt{\frac{1}{3}}\left((-b)^{\frac{1}{3}}bx^3-(bx^3+a)^{\frac{1}{3}}bx^2+2(bx^3+a)^{\frac{2}{3}}(-b)^{\frac{2}{3}}x\right)\sqrt{\frac{(-b)^{\frac{1}{3}}}{b}}+2a\right)-2(9b^2c^2-6abcd+2a^2d^2)(-b)^{\frac{2}{3}}\log\left(\frac{(-b)^{\frac{1}{3}}x+(bx^3+a)^{\frac{1}{3}}}{x}\right)+(9b^2c^2-6abcd+2a^2d^2)(-b)^{\frac{2}{3}}\log\left(\frac{(-b)^{\frac{2}{3}}x^2-(bx^3+a)^{\frac{1}{3}}(-b)^{\frac{1}{3}}x+(bx^3+a)^{\frac{2}{3}}}{x^2}\right)+3(3b^2d^2x^4+4(3b^2cd-abd^2)x)(bx^3+a)^{\frac{2}{3}}}{54b^3},-\frac{6\sqrt{\frac{1}{3}}(9b^3c^2-6ab^2cd+2a^2bd^2)\sqrt{-\frac{(-b)^{\frac{1}{3}}}{b}}\arctan\left(-\frac{\sqrt{\frac{1}{3}}\left((-b)^{\frac{1}{3}}x-2(bx^3+a)^{\frac{1}{3}}\right)\sqrt{-\frac{(-b)^{\frac{1}{3}}}{b}}}{x}\right)+2(9b^2c^2-6abcd+2a^2d^2)(-b)^{\frac{2}{3}}\log\left(\frac{(-b)^{\frac{1}{3}}x+(bx^3+a)^{\frac{1}{3}}}{x}\right)-(9b^2c^2-6abcd+2a^2d^2)(-b)^{\frac{2}{3}}\log\left(\frac{(-b)^{\frac{2}{3}}x^2-(bx^3+a)^{\frac{1}{3}}(-b)^{\frac{1}{3}}x+(bx^3+a)^{\frac{2}{3}}}{x^2}\right)-3(3b^2d^2x^4+4(3b^2cd-abd^2)x)(bx^3+a)^{\frac{2}{3}}}{54b^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^2/(b*x^3+a)^(1/3),x, algorithm="fricas")
```

```
[Out] [1/54*(3*sqrt(1/3)*(9*b^3*c^2 - 6*a*b^2*c*d + 2*a^2*b*d^2)*sqrt((-b)^(1/3)/
b)*log(3*b*x^3 - 3*(b*x^3 + a)^(1/3)*(-b)^(2/3)*x^2 - 3*sqrt(1/3)*((-b)^(1/
3)*b*x^3 - (b*x^3 + a)^(1/3)*b*x^2 + 2*(b*x^3 + a)^(2/3)*(-b)^(2/3)*x)*sqrt
((-b)^(1/3)/b) + 2*a) - 2*(9*b^2*c^2 - 6*a*b*c*d + 2*a^2*d^2)*(-b)^(2/3)*lo
g(((-b)^(1/3)*x + (b*x^3 + a)^(1/3))/x) + (9*b^2*c^2 - 6*a*b*c*d + 2*a^2*d^
2)*(-b)^(2/3)*log(((-b)^(2/3)*x^2 - (b*x^3 + a)^(1/3)*(-b)^(1/3)*x + (b*x^3
 + a)^(2/3))/x^2) + 3*(3*b^2*d^2*x^4 + 4*(3*b^2*c*d - a*b*d^2)*x)*(b*x^3 +
a)^(2/3))/b^3, -1/54*(6*sqrt(1/3)*(9*b^3*c^2 - 6*a*b^2*c*d + 2*a^2*b*d^2)*s
qrt(-(-b)^(1/3)/b)*arctan(-sqrt(1/3)*((-b)^(1/3)*x - 2*(b*x^3 + a)^(1/3))*s
```

```
qrt(-(-b)^(1/3)/b)/x) + 2*(9*b^2*c^2 - 6*a*b*c*d + 2*a^2*d^2)*(-b)^(2/3)*lo
g(((-b)^(1/3)*x + (b*x^3 + a)^(1/3))/x) - (9*b^2*c^2 - 6*a*b*c*d + 2*a^2*d^
2)*(-b)^(2/3)*log(((-b)^(2/3)*x^2 - (b*x^3 + a)^(1/3)*(-b)^(1/3)*x + (b*x^3
 + a)^(2/3))/x^2) - 3*(3*b^2*d^2*x^4 + 4*(3*b^2*c*d - a*b*d^2)*x)*(b*x^3 +
a)^(2/3))/b^3]
```

Sympy [C] Result contains complex when optimal does not.
time = 3.42, size = 126, normalized size = 0.72

$$\frac{c^2x\Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\begin{matrix}\frac{1}{3}, \frac{1}{3} \\ \frac{4}{3}\end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt[3]{a}\,\Gamma\left(\frac{4}{3}\right)} + \frac{2cdx^4\Gamma\left(\frac{4}{3}\right) {}_2F_1\left(\begin{matrix}\frac{1}{3}, \frac{4}{3} \\ \frac{7}{3}\end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt[3]{a}\,\Gamma\left(\frac{7}{3}\right)} + \frac{d^2x^7\Gamma\left(\frac{7}{3}\right) {}_2F_1\left(\begin{matrix}\frac{1}{3}, \frac{7}{3} \\ \frac{10}{3}\end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt[3]{a}\,\Gamma\left(\frac{10}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**2/(b*x**3+a)**(1/3),x)
```

```
[Out] c**2*x*gamma(1/3)*hyper((1/3, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*a*
*(1/3)*gamma(4/3)) + 2*c*d*x**4*gamma(4/3)*hyper((1/3, 4/3), (7/3,), b*x**3
*exp_polar(I*pi)/a)/(3*a**(1/3)*gamma(7/3)) + d**2*x**7*gamma(7/3)*hyper((1
/3, 7/3), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(1/3)*gamma(10/3))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^2/(b*x^3+a)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((d*x^3 + c)^2/(b*x^3 + a)^(1/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(d\,x^3+c\right)}^2}{{\left(b\,x^3+a\right)}^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^2/(a + b*x^3)^(1/3),x)
```

```
[Out] int((c + d*x^3)^2/(a + b*x^3)^(1/3), x)
```

3.73 $\int \frac{(c+dx^3)^2}{(a+bx^3)^{4/3}} \, dx$

Optimal. Leaf size=159

$$-\frac{d(3bc-4ad)x(a+bx^3)^{2/3}}{3ab^2}+\frac{(bc-ad)x(c+dx^3)}{ab\sqrt[3]{a+bx^3}}+\frac{2d(3bc-2ad)\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}\,b^{7/3}}-\frac{d(3bc-2ad)\,\mathrm{l}}{}$$

```
[Out] -1/3*d*(-4*a*d+3*b*c)*x*(b*x^3+a)^(2/3)/a/b^2+(-a*d+b*c)*x*(d*x^3+c)/a/b/(b
*x^3+a)^(1/3)-1/3*d*(-2*a*d+3*b*c)*ln(-b^(1/3)*x+(b*x^3+a)^(1/3))/b^(7/3)+2
/9*d*(-2*a*d+3*b*c)*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/b^(
7/3)*3^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {424, 396, 245}

$$\frac{2d\text{ArcTan}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)(3bc-2ad)}{3\sqrt{3}\,b^{7/3}}-\frac{d(3bc-2ad)\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}\,x\right)}{3b^{7/3}}-\frac{dx(a+bx^3)^{2/3}(3bc-4ad)}{3ab^2}+\frac{x(c+dx^3)(bc-ad)}{ab\sqrt[3]{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)^2/(a + b*x^3)^(4/3),x]
```

```
[Out] -1/3*(d*(3*b*c - 4*a*d)*x*(a + b*x^3)^(2/3))/(a*b^2) + ((b*c - a*d)*x*(c +
d*x^3))/(a*b*(a + b*x^3)^(1/3)) + (2*d*(3*b*c - 2*a*d)*ArcTan[(1 + (2*b^(1/
3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(3*Sqrt[3]*b^(7/3)) - (d*(3*b*c - 2*a*d)
*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)])/(3*b^(7/3))
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rubi steps

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{4/3}}\,dx = \frac{(bc-ad)x(c+dx^3)}{ab\sqrt[3]{a+bx^3}} + \frac{\int \frac{acd-d(3bc-4ad)x^3}{\sqrt[3]{a+bx^3}}\,dx}{ab}$$

$$= -\frac{d(3bc-4ad)x(a+bx^3)^{2/3}}{3ab^2} + \frac{(bc-ad)x(c+dx^3)}{ab\sqrt[3]{a+bx^3}} + \frac{(2d(3bc-2ad))\int \frac{1}{\sqrt[3]{a+bx^3}}\,dx}{3b^2}$$

$$= -\frac{d(3bc-4ad)x(a+bx^3)^{2/3}}{3ab^2} + \frac{(bc-ad)x(c+dx^3)}{ab\sqrt[3]{a+bx^3}} + \frac{2d(3bc-2ad)\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}\ldots}{\sqrt[3]{a+b\ldots}}}{\sqrt{3}}\ldots\right.}{3\sqrt{3}\,b^{7/3}}\ldots$$

Mathematica [A]
time = 0.76, size = 199, normalized size = 1.25

$$\frac{\frac{3\sqrt[3]{b}\,x(3b^2c^2+4a^2d^2+abd(-6c+dx^3))}{a\sqrt[3]{a+bx^3}} + 2\sqrt{3}\,d(3bc-2ad)\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[3]{b}\,x}{\sqrt[3]{b}\,x+2\sqrt[3]{a+bx^3}}\right) + 2d(-3bc+2ad)\log\left(-\sqrt[3]{b}\,x+\sqrt[3]{a+bx^3}\right) + d(3bc-2ad)\log\left(b^{2/3}x^2+\sqrt[3]{b}\,x\sqrt[3]{a+bx^3}+(a+bx^3)^{2/3}\right)}{9b^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3)^2/(a + b*x^3)^(4/3),x]
```

```
[Out] ((3*b^(1/3)*x*(3*b^2*c^2 + 4*a^2*d^2 + a*b*d*(-6*c + d*x^3)))/(a*(a + b*x^3
)^(1/3)) + 2*Sqrt[3]*d*(3*b*c - 2*a*d)*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*
x + 2*(a + b*x^3)^(1/3))] + 2*d*(-3*b*c + 2*a*d)*Log[-(b^(1/3)*x) + (a + b*
x^3)^(1/3)] + d*(3*b*c - 2*a*d)*Log[b^(2/3)*x^2 + b^(1/3)*x*(a + b*x^3)^(1/
3) + (a + b*x^3)^(2/3)])/(9*b^(7/3))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(d\,x^3+c)^2}{(b\,x^3+a)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^2/(b*x^3+a)^(4/3),x)
```

```
[Out] int((d*x^3+c)^2/(b*x^3+a)^(4/3),x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 301 vs. $2(134) = 268$.
time = 0.49, size = 301, normalized size = 1.89

$$\frac{1}{9}\,d^{2}\left(\frac{4\,\sqrt{3}\,a\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\,(bx^{3}+a)^{\frac{1}{3}}}{x}\right)}{3\,b^{\frac{1}{3}}}\right)}{b^{\frac{7}{3}}}+\frac{3\left(3\,ab-\frac{4\,(bx^{3}+a)a}{x^{3}}\right)}{\frac{(bx^{3}+a)^{\frac{1}{3}}b^{3}}{x}-\frac{(bx^{3}+a)^{\frac{4}{3}}b^{2}}{x^{4}}}-\frac{2\,a\log\left(b^{\frac{2}{3}}+\frac{(bx^{3}+a)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{(bx^{3}+a)^{\frac{2}{3}}}{x^{2}}\right)}{b^{\frac{7}{3}}}+\frac{4\,a\log\left(-b^{\frac{1}{3}}+\frac{(bx^{3}+a)^{\frac{1}{3}}}{x}\right)}{b^{\frac{7}{3}}}\right)-\frac{1}{3}\,cd\left(\frac{2\,\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\,(bx^{3}+a)^{\frac{1}{3}}}{x}\right)}{3\,b^{\frac{1}{3}}}\right)}{b^{\frac{4}{3}}}+\frac{6\,x}{(bx^{3}+a)^{\frac{1}{3}}b}-\frac{\log\left(b^{\frac{2}{3}}+\frac{(bx^{3}+a)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{(bx^{3}+a)^{\frac{2}{3}}}{x^{2}}\right)}{b^{\frac{4}{3}}}+\frac{2\,\log\left(-b^{\frac{1}{3}}+\frac{(bx^{3}+a)^{\frac{1}{3}}}{x}\right)}{b^{\frac{4}{3}}}\right)+\frac{c^{2}x}{(bx^{3}+a)^{\frac{1}{3}}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^2/(b*x^3+a)^(4/3),x, algorithm="maxima")
```

```
[Out] 1/9*d^2*(4*sqrt(3)*a*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b
^(1/3))/b^(7/3) + 3*(3*a*b - 4*(b*x^3 + a)*a/x^3)/((b*x^3 + a)^(1/3)*b^3/x
- (b*x^3 + a)^(4/3)*b^2/x^4) - 2*a*log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/
x + (b*x^3 + a)^(2/3)/x^2)/b^(7/3) + 4*a*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x
)/b^(7/3)) - 1/3*c*d*(2*sqrt(3)*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)
^(1/3)/x)/b^(1/3))/b^(4/3) + 6*x/((b*x^3 + a)^(1/3)*b) - log(b^(2/3) + (b*x
^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2/3)/x^2)/b^(4/3) + 2*log(-b^(1/3) +
 (b*x^3 + a)^(1/3)/x)/b^(4/3)) + c^2*x/((b*x^3 + a)^(1/3)*a)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 299 vs. $2(134) = 268$.
time = 5.78, size = 652, normalized size = 4.10

$$\left[-\frac{3\,\sqrt{\frac{1}{3}}\,(3\,a^{2}b^{2}cd-2\,a^{3}bd^{2}+(3\,ab^{3}cd-2\,a^{2}b^{2}d^{2})x^{3})\sqrt{-\frac{1}{b^{\frac{2}{3}}}}\log\left(3\,bx^{3}-3\,(bx^{3}+a)^{\frac{1}{3}}b^{\frac{2}{3}}x^{2}-3\,\sqrt{\frac{1}{3}}\left(b^{\frac{4}{3}}x^{3}+(bx^{3}+a)^{\frac{1}{3}}bx^{2}-2\,(bx^{3}+a)^{\frac{2}{3}}b^{\frac{2}{3}}x\right)\sqrt{-\frac{1}{b^{\frac{2}{3}}}}+2\,a\right)+2\,(3\,a^{2}bcd-2\,a^{3}d^{2}+(3\,ab^{2}cd-2\,a^{2}bd^{2})x^{3})b^{\frac{2}{3}}\log\left(-\frac{b^{\frac{1}{3}}x-(bx^{3}+a)^{\frac{1}{3}}}{x}\right)-(3\,a^{2}bcd-2\,a^{3}d^{2}+(3\,ab^{2}cd-2\,a^{2}bd^{2})x^{3})b^{\frac{2}{3}}\log\left(\frac{b^{\frac{2}{3}}x^{2}+(bx^{3}+a)^{\frac{1}{3}}b^{\frac{1}{3}}x+(bx^{3}+a)^{\frac{2}{3}}}{x^{2}}\right)-3\,(ab^{2}d^{2}x^{4}+(3\,b^{3}c^{2}-6\,ab^{2}cd+4\,a^{2}bd^{2})x)(bx^{3}+a)^{\frac{2}{3}}}{9\,(ab^{4}x^{3}+a^{2}b^{3})},-\frac{2\,(3\,a^{2}bcd-2\,a^{3}d^{2}+(3\,ab^{2}cd-2\,a^{2}bd^{2})x^{3})b^{\frac{2}{3}}\log\left(-\frac{b^{\frac{1}{3}}x-(bx^{3}+a)^{\frac{1}{3}}}{x}\right)-(3\,a^{2}bcd-2\,a^{3}d^{2}+(3\,ab^{2}cd-2\,a^{2}bd^{2})x^{3})b^{\frac{2}{3}}\log\left(\frac{b^{\frac{2}{3}}x^{2}+(bx^{3}+a)^{\frac{1}{3}}b^{\frac{1}{3}}x+(bx^{3}+a)^{\frac{2}{3}}}{x^{2}}\right)+\frac{6\,\sqrt{\frac{1}{3}}\,(3\,a^{2}b^{2}cd-2\,a^{3}bd^{2}+(3\,ab^{3}cd-2\,a^{2}b^{2}d^{2})x^{3})\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(b^{\frac{1}{3}}x+2\,(bx^{3}+a)^{\frac{1}{3}}\right)}{b^{\frac{1}{3}}x}\right)}{b^{\frac{1}{3}}}-3\,(ab^{2}d^{2}x^{4}+(3\,b^{3}c^{2}-6\,ab^{2}cd+4\,a^{2}bd^{2})x)(bx^{3}+a)^{\frac{2}{3}}}{9\,(ab^{4}x^{3}+a^{2}b^{3})}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^2/(b*x^3+a)^(4/3),x, algorithm="fricas")
```

```
[Out] [-1/9*(3*sqrt(1/3)*(3*a^2*b^2*c*d - 2*a^3*b*d^2 + (3*a*b^3*c*d - 2*a^2*b^2*
d^2)*x^3)*sqrt(-1/b^(2/3))*log(3*b*x^3 - 3*(b*x^3 + a)^(1/3)*b^(2/3)*x^2 -
3*sqrt(1/3)*(b^(4/3)*x^3 + (b*x^3 + a)^(1/3)*b*x^2 - 2*(b*x^3 + a)^(2/3)*b^
(2/3)*x)*sqrt(-1/b^(2/3)) + 2*a) + 2*(3*a^2*b*c*d - 2*a^3*d^2 + (3*a*b^2*c*
d - 2*a^2*b*d^2)*x^3)*b^(2/3)*log(-(b^(1/3)*x - (b*x^3 + a)^(1/3))/x) - (3*
a^2*b*c*d - 2*a^3*d^2 + (3*a*b^2*c*d - 2*a^2*b*d^2)*x^3)*b^(2/3)*log((b^(2/
3)*x^2 + (b*x^3 + a)^(1/3)*b^(1/3)*x + (b*x^3 + a)^(2/3))/x^2) - 3*(a*b^2*d
^2*x^4 + (3*b^3*c^2 - 6*a*b^2*c*d + 4*a^2*b*d^2)*x)*(b*x^3 + a)^(2/3))/(a*b
^4*x^3 + a^2*b^3), -1/9*(2*(3*a^2*b*c*d - 2*a^3*d^2 + (3*a*b^2*c*d - 2*a^2*
b*d^2)*x^3)*b^(2/3)*log(-(b^(1/3)*x - (b*x^3 + a)^(1/3))/x) - (3*a^2*b*c*d
- 2*a^3*d^2 + (3*a*b^2*c*d - 2*a^2*b*d^2)*x^3)*b^(2/3)*log((b^(2/3)*x^2 + (
b*x^3 + a)^(1/3)*b^(1/3)*x + (b*x^3 + a)^(2/3))/x^2) + 6*sqrt(1/3)*(3*a^2*b
^2*c*d - 2*a^3*b*d^2 + (3*a*b^3*c*d - 2*a^2*b^2*d^2)*x^3)*arctan(sqrt(1/3)*
```

```
(b^(1/3)*x + 2*(b*x^3 + a)^(1/3))/(b^(1/3)*x))/b^(1/3) - 3*(a*b^2*d^2*x^4 +
 (3*b^3*c^2 - 6*a*b^2*c*d + 4*a^2*b*d^2)*x)*(b*x^3 + a)^(2/3))/(a*b^4*x^3 +
 a^2*b^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c + dx^3)^2}{(a + bx^3)^{\frac{4}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**2/(b*x**3+a)**(4/3),x)
```

```
[Out] Integral((c + d*x**3)**2/(a + b*x**3)**(4/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^2/(b*x^3+a)^(4/3),x, algorithm="giac")
```

```
[Out] integrate((d*x^3 + c)^2/(b*x^3 + a)^(4/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(d\,x^3 + c)^2}{(b\,x^3 + a)^{4/3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^2/(a + b*x^3)^(4/3),x)
```

```
[Out] int((c + d*x^3)^2/(a + b*x^3)^(4/3), x)
```

3.74 $\int \frac{(c+dx^3)^2}{(a+bx^3)^{7/3}}\,dx$

Optimal. Leaf size=152

$$\frac{(bc-ad)(3bc+4ad)x}{4a^2b^2\sqrt[3]{a+bx^3}}+\frac{(bc-ad)x(c+dx^3)}{4ab\,(a+bx^3)^{4/3}}+\frac{d^2\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,b^{7/3}}-\frac{d^2\log\left(-\sqrt[3]{b}\,x+\sqrt[3]{a+bx^3}\right)}{2b^{7/3}}$$

```
[Out] 1/4*(-a*d+b*c)*(4*a*d+3*b*c)*x/a^2/b^2/(b*x^3+a)^(1/3)+1/4*(-a*d+b*c)*x*(d*
x^3+c)/a/b/(b*x^3+a)^(4/3)-1/2*d^2*ln(-b^(1/3)*x+(b*x^3+a)^(1/3))/b^(7/3)+1
/3*d^2*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/b^(7/3)*3^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {424, 393, 245}

$$\frac{x(bc-ad)(4ad+3bc)}{4a^2b^2\sqrt[3]{a+bx^3}}+\frac{d^2\text{ArcTan}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,b^{7/3}}-\frac{d^2\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}\,x\right)}{2b^{7/3}}+\frac{x(c+dx^3)(bc-ad)}{4ab\,(a+bx^3)^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)^2/(a + b*x^3)^(7/3),x]
```

```
[Out] ((b*c - a*d)*(3*b*c + 4*a*d)*x)/(4*a^2*b^2*(a + b*x^3)^(1/3)) + ((b*c - a*d
)*x*(c + d*x^3))/(4*a*b*(a + b*x^3)^(4/3)) + (d^2*ArcTan[(1 + (2*b^(1/3)*x)
/(a + b*x^3)^(1/3))/Sqrt[3]])/(Sqrt[3]*b^(7/3)) - (d^2*Log[-(b^(1/3)*x) + (
a + b*x^3)^(1/3)])/(2*b^(7/3))
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx^3)^2}{(a+bx^3)^{7/3}}\,dx &= \frac{(bc-ad)x(c+dx^3)}{4ab\,(a+bx^3)^{4/3}}+\frac{\int \frac{c(3bc+ad)+4ad^2x^3}{(a+bx^3)^{4/3}}\,dx}{4ab}\\ &= \frac{(bc-ad)(3bc+4ad)x}{4a^2b^2\sqrt[3]{a+bx^3}}+\frac{(bc-ad)x(c+dx^3)}{4ab\,(a+bx^3)^{4/3}}+\frac{d^2\int \frac{1}{\sqrt[3]{a+bx^3}}\,dx}{b^2}\\ &= \frac{(bc-ad)(3bc+4ad)x}{4a^2b^2\sqrt[3]{a+bx^3}}+\frac{(bc-ad)x(c+dx^3)}{4ab\,(a+bx^3)^{4/3}}+\frac{d^2\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,b^{7/3}}-\frac{d^2\log\left(-\right.}{}\end{aligned}$$

Mathematica [A]
time = 0.61, size = 198, normalized size = 1.30

$$\frac{\frac{3\sqrt[3]{b}\,(-4a^3d^2x+3b^3c^2x^4-5a^2bd^2x^4+2ab^2cx(2c+dx^3))}{a^2(a+bx^3)^{4/3}}+4\sqrt{3}\,d^2\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[3]{b}\,x}{\sqrt[3]{b}\,x+2\sqrt[3]{a+bx^3}}\right)-4d^2\log\left(-\sqrt[3]{b}\,x+\sqrt[3]{a+bx^3}\right)+2d^2\log\left(b^{2/3}x^2+\sqrt[3]{b}\,x\sqrt[3]{a+bx^3}+(a+bx^3)^{2/3}\right)}{12b^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3)^2/(a + b*x^3)^(7/3),x]
```

```
[Out] ((3*b^(1/3)*(-4*a^3*d^2*x + 3*b^3*c^2*x^4 - 5*a^2*b*d^2*x^4 + 2*a*b^2*c*x*(
2*c + d*x^3)))/(a^2*(a + b*x^3)^(4/3)) + 4*Sqrt[3]*d^2*ArcTan[(Sqrt[3]*b^(1
/3)*x)/(b^(1/3)*x + 2*(a + b*x^3)^(1/3))] - 4*d^2*Log[-(b^(1/3)*x) + (a + b
*x^3)^(1/3)] + 2*d^2*Log[b^(2/3)*x^2 + b^(1/3)*x*(a + b*x^3)^(1/3) + (a + b
*x^3)^(2/3)])/(12*b^(7/3))
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{(d\,x^3+c)^2}{(b\,x^3+a)^{\frac{7}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^2/(b*x^3+a)^(7/3),x)
```

```
[Out] int((d*x^3+c)^2/(b*x^3+a)^(7/3),x)
```

Maxima [A]
time = 0.57, size = 190, normalized size = 1.25

$$-\frac{\left(b-\frac{4\,(bx^3+a)}{x^3}\right)c^2x^4}{4\,(bx^3+a)^{\frac{4}{3}}a^2}+\frac{cdx^4}{2\,(bx^3+a)^{\frac{4}{3}}a}-\frac{1}{12}\left(\frac{3\left(b+\frac{4\,(bx^3+a)}{x^3}\right)x^4}{(bx^3+a)^{\frac{4}{3}}b^2}+\frac{4\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\,(bx^3+a)^{\frac{1}{3}}}{x}\right)}{3\,b^{\frac{1}{3}}}\right)}{b^{\frac{7}{3}}}-\frac{2\,\log\left(b^{\frac{2}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{(bx^3+a)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{7}{3}}}+\frac{4\,\log\left(-b^{\frac{1}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}}{x}\right)}{b^{\frac{7}{3}}}\right)d^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^2/(b*x^3+a)^(7/3),x, algorithm="maxima")
```

```
[Out] -1/4*(b - 4*(b*x^3 + a)/x^3)*c^2*x^4/((b*x^3 + a)^(4/3)*a^2) + 1/2*c*d*x^4/
((b*x^3 + a)^(4/3)*a) - 1/12*(3*(b + 4*(b*x^3 + a)/x^3)*x^4/((b*x^3 + a)^(4
/3)*b^2) + 4*sqrt(3)*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b
^(1/3))/b^(7/3) - 2*log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)
^(2/3)/x^2)/b^(7/3) + 4*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(7/3))*d^2
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 335 vs. $2(128) = 256$.
time = 3.24, size = 719, normalized size = 4.73

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^2/(b*x^3+a)^(7/3),x, algorithm="fricas")
```

```
[Out] [1/12*(6*sqrt(1/3)*(a^2*b^3*d^2*x^6 + 2*a^3*b^2*d^2*x^3 + a^4*b*d^2)*sqrt((
-b)^(1/3)/b)*log(3*b*x^3 - 3*(b*x^3 + a)^(1/3)*(-b)^(2/3)*x^2 - 3*sqrt(1/3)
*((-b)^(1/3)*b*x^3 - (b*x^3 + a)^(1/3)*b*x^2 + 2*(b*x^3 + a)^(2/3)*(-b)^(2/
3)*x)*sqrt((-b)^(1/3)/b) + 2*a) - 4*(a^2*b^2*d^2*x^6 + 2*a^3*b*d^2*x^3 + a^
4*d^2)*(-b)^(2/3)*log(((-b)^(1/3)*x + (b*x^3 + a)^(1/3))/x) + 2*(a^2*b^2*d^
2*x^6 + 2*a^3*b*d^2*x^3 + a^4*d^2)*(-b)^(2/3)*log(((-b)^(2/3)*x^2 - (b*x^3
+ a)^(1/3)*(-b)^(1/3)*x + (b*x^3 + a)^(2/3))/x^2) + 3*((3*b^4*c^2 + 2*a*b^3
*c*d - 5*a^2*b^2*d^2)*x^4 + 4*(a*b^3*c^2 - a^3*b*d^2)*x)*(b*x^3 + a)^(2/3))
/(a^2*b^5*x^6 + 2*a^3*b^4*x^3 + a^4*b^3), -1/12*(12*sqrt(1/3)*(a^2*b^3*d^2*
x^6 + 2*a^3*b^2*d^2*x^3 + a^4*b*d^2)*sqrt(-(-b)^(1/3)/b)*arctan(-sqrt(1/3)*
((-b)^(1/3)*x - 2*(b*x^3 + a)^(1/3))*sqrt(-(-b)^(1/3)/b)/x) + 4*(a^2*b^2*d^
2*x^6 + 2*a^3*b*d^2*x^3 + a^4*d^2)*(-b)^(2/3)*log(((-b)^(1/3)*x + (b*x^3 +
a)^(1/3))/x) - 2*(a^2*b^2*d^2*x^6 + 2*a^3*b*d^2*x^3 + a^4*d^2)*(-b)^(2/3)*l
og(((-b)^(2/3)*x^2 - (b*x^3 + a)^(1/3)*(-b)^(1/3)*x + (b*x^3 + a)^(2/3))/x^
```

```
2) - 3*((3*b^4*c^2 + 2*a*b^3*c*d - 5*a^2*b^2*d^2)*x^4 + 4*(a*b^3*c^2 - a^3*
b*d^2)*x)*(b*x^3 + a)^(2/3))/(a^2*b^5*x^6 + 2*a^3*b^4*x^3 + a^4*b^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c + dx^3)^2}{(a + bx^3)^{\frac{7}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**2/(b*x**3+a)**(7/3),x)
[Out] Integral((c + d*x**3)**2/(a + b*x**3)**(7/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^2/(b*x^3+a)^(7/3),x, algorithm="giac")
[Out] integrate((d*x^3 + c)^2/(b*x^3 + a)^(7/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(d\,x^3 + c)^2}{(b\,x^3 + a)^{7/3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^2/(a + b*x^3)^(7/3),x)
[Out] int((c + d*x^3)^2/(a + b*x^3)^(7/3), x)
```

3.75 $\int \frac{(c+dx^3)^2}{(a+bx^3)^{10/3}} \, dx$

Optimal. Leaf size=78

$$\frac{9c^2x}{14a^3\sqrt[3]{a+bx^3}}+\frac{3cx(c+dx^3)}{14a^2\left(a+bx^3\right)^{4/3}}+\frac{x(c+dx^3)^2}{7a\left(a+bx^3\right)^{7/3}}$$

```
[Out] 9/14*c^2*x/a^3/(b*x^3+a)^(1/3)+3/14*c*x*(d*x^3+c)/a^2/(b*x^3+a)^(4/3)+1/7*x
*(d*x^3+c)^2/a/(b*x^3+a)^(7/3)
```

Rubi [A]
time = 0.02, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {386, 197}

$$\frac{9c^2x}{14a^3\sqrt[3]{a+bx^3}}+\frac{3cx(c+dx^3)}{14a^2\left(a+bx^3\right)^{4/3}}+\frac{x(c+dx^3)^2}{7a\left(a+bx^3\right)^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)^2/(a + b*x^3)^(10/3),x]
```

```
[Out] (9*c^2*x)/(14*a^3*(a + b*x^3)^(1/3)) + (3*c*x*(c + d*x^3))/(14*a^2*(a + b*x
^3)^(4/3)) + (x*(c + d*x^3)^2)/(7*a*(a + b*x^3)^(7/3))
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 386

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] - Dist[
c*(q/(a*(p + 1))), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1,
0] && GtQ[q, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{10/3}}\,dx = \frac{x(c+dx^3)^2}{7a\,(a+bx^3)^{7/3}} + \frac{(6c)\int \frac{c+dx^3}{(a+bx^3)^{7/3}}\,dx}{7a}$$
$$= \frac{3cx(c+dx^3)}{14a^2\,(a+bx^3)^{4/3}} + \frac{x(c+dx^3)^2}{7a\,(a+bx^3)^{7/3}} + \frac{(9c^2)\int \frac{1}{(a+bx^3)^{4/3}}\,dx}{14a^2}$$
$$= \frac{9c^2x}{14a^3\sqrt[3]{a+bx^3}} + \frac{3cx(c+dx^3)}{14a^2\,(a+bx^3)^{4/3}} + \frac{x(c+dx^3)^2}{7a\,(a+bx^3)^{7/3}}$$

Mathematica [A]
time = 0.42, size = 73, normalized size = 0.94

$$\frac{9b^2c^2x^7 + 3abcx^4(7c+dx^3) + a^2(14c^2x + 7cdx^4 + 2d^2x^7)}{14a^3\,(a+bx^3)^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3)^2/(a + b*x^3)^(10/3),x]
```

```
[Out] (9*b^2*c^2*x^7 + 3*a*b*c*x^4*(7*c + d*x^3) + a^2*(14*c^2*x + 7*c*d*x^4 + 2*
d^2*x^7))/(14*a^3*(a + b*x^3)^(7/3))
```

Maple [A]
time = 0.26, size = 76, normalized size = 0.97

method	result	size
gosper	$\frac{x\left(2a^2d^2x^6+3abcd\,x^6+9b^2c^2x^6+7a^2cd\,x^3+21ab\,c^2x^3+14a^2c^2\right)}{14\left(b\,x^3+a\right)^{\frac{7}{3}}a^3}$	76
trager	$\frac{x\left(2a^2d^2x^6+3abcd\,x^6+9b^2c^2x^6+7a^2cd\,x^3+21ab\,c^2x^3+14a^2c^2\right)}{14\left(b\,x^3+a\right)^{\frac{7}{3}}a^3}$	76

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^2/(b*x^3+a)^(10/3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/14*x*(2*a^2*d^2*x^6+3*a*b*c*d*x^6+9*b^2*c^2*x^6+7*a^2*c*d*x^3+21*a*b*c^2*
x^3+14*a^2*c^2)/(b*x^3+a)^(7/3)/a^3
```

Maxima [A]
time = 0.28, size = 109, normalized size = 1.40

$$-\frac{\left(4\,b - \frac{7\,(bx^3+a)}{x^3}\right)cdx^7}{14\,(bx^3+a)^{\frac{7}{3}}a^2} + \frac{d^2x^7}{7\,(bx^3+a)^{\frac{7}{3}}a} + \frac{\left(2\,b^2 - \frac{7\,(bx^3+a)b}{x^3} + \frac{14\,(bx^3+a)^2}{x^6}\right)c^2x^7}{14\,(bx^3+a)^{\frac{7}{3}}a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^2/(b*x^3+a)^(10/3),x, algorithm="maxima")
```

```
[Out] -1/14*(4*b - 7*(b*x^3 + a)/x^3)*c*d*x^7/((b*x^3 + a)^(7/3)*a^2) + 1/7*d^2*x
^7/((b*x^3 + a)^(7/3)*a) + 1/14*(2*b^2 - 7*(b*x^3 + a)*b/x^3 + 14*(b*x^3 +
a)^2/x^6)*c^2*x^7/((b*x^3 + a)^(7/3)*a^3)
```

Fricas [A]
time = 3.73, size = 103, normalized size = 1.32

$$\frac{\left((9\,b^2c^2+3\,abcd+2\,a^2d^2)x^7+14\,a^2c^2x+7\,(3\,abc^2+a^2cd)x^4\right)(bx^3+a)^{\frac{2}{3}}}{14\,(a^3b^3x^9+3\,a^4b^2x^6+3\,a^5bx^3+a^6)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^2/(b*x^3+a)^(10/3),x, algorithm="fricas")
```

```
[Out] 1/14*((9*b^2*c^2 + 3*a*b*c*d + 2*a^2*d^2)*x^7 + 14*a^2*c^2*x + 7*(3*a*b*c^2
 + a^2*c*d)*x^4)*(b*x^3 + a)^(2/3)/(a^3*b^3*x^9 + 3*a^4*b^2*x^6 + 3*a^5*b*x
^3 + a^6)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c+dx^3\right)^2}{\left(a+bx^3\right)^{\frac{10}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**2/(b*x**3+a)**(10/3),x)
```

```
[Out] Integral((c + d*x**3)**2/(a + b*x**3)**(10/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^2/(b*x^3+a)^(10/3),x, algorithm="giac")
```

```
[Out] integrate((d*x^3 + c)^2/(b*x^3 + a)^(10/3), x)
```

Mupad [B]
time = 1.43, size = 148, normalized size = 1.90

$$\frac{2\,a^4\,d^2\,x+2\,a^2\,d^2\,x\,{\left(b\,x^3+a\right)}^2+9\,b^2\,c^2\,x\,{\left(b\,x^3+a\right)}^2+2\,a^2\,b^2\,c^2\,x-4\,a^3\,d^2\,x\,\left(b\,x^3+a\right)+3\,a\,b^2\,c^2\,x\,\left(b\,x^3+a\right)-4\,a^3\,b\,c\,d\,x+3\,a\,b\,c\,d\,x\,{\left(b\,x^3+a\right)}^2+a^2\,b\,c\,d\,x\,\left(b\,x^3+a\right)}{14\,a^3\,b^2\,{\left(b\,x^3+a\right)}^{7/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^2/(a + b*x^3)^(10/3),x)
```

```
[Out] (2*a^4*d^2*x + 2*a^2*d^2*x*(a + b*x^3)^2 + 9*b^2*c^2*x*(a + b*x^3)^2 + 2*a^
2*b^2*c^2*x - 4*a^3*d^2*x*(a + b*x^3) + 3*a*b^2*c^2*x*(a + b*x^3) - 4*a^3*b
*c*d*x + 3*a*b*c*d*x*(a + b*x^3)^2 + a^2*b*c*d*x*(a + b*x^3))/(14*a^3*b^2*(
a + b*x^3)^(7/3))
```

3.76 $\int \frac{(c+dx^3)^2}{(a+bx^3)^{13/3}}\,dx$

Optimal. Leaf size=174

$$\frac{9c^2(9bc-10ad)x}{140a^4(bc-ad)\sqrt[3]{a+bx^3}}+\frac{3c(9bc-10ad)x(c+dx^3)}{140a^3(bc-ad)\,(a+bx^3)^{4/3}}+\frac{(9bc-10ad)x(c+dx^3)^2}{70a^2(bc-ad)\,(a+bx^3)^{7/3}}+\frac{bx(c+dx^3)^3}{10a(bc-ad)\,(a+bx^3)^{10/3}}$$

```
[Out] 9/140*c^2*(-10*a*d+9*b*c)*x/a^4/(-a*d+b*c)/(b*x^3+a)^(1/3)+3/140*c*(-10*a*d
+9*b*c)*x*(d*x^3+c)/a^3/(-a*d+b*c)/(b*x^3+a)^(4/3)+1/70*(-10*a*d+9*b*c)*x*(
d*x^3+c)^2/a^2/(-a*d+b*c)/(b*x^3+a)^(7/3)+1/10*b*x*(d*x^3+c)^3/a/(-a*d+b*c)
/(b*x^3+a)^(10/3)
```

Rubi [A]
time = 0.05, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {390, 386, 197}

$$\frac{9c^2x(9bc-10ad)}{140a^4\sqrt[3]{a+bx^3}\,(bc-ad)}+\frac{3cx(c+dx^3)\,(9bc-10ad)}{140a^3\,(a+bx^3)^{4/3}\,(bc-ad)}+\frac{x(c+dx^3)^2\,(9bc-10ad)}{70a^2\,(a+bx^3)^{7/3}\,(bc-ad)}+\frac{bx(c+dx^3)^3}{10a\,(a+bx^3)^{10/3}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)^2/(a + b*x^3)^(13/3),x]
```

```
[Out] (9*c^2*(9*b*c - 10*a*d)*x)/(140*a^4*(b*c - a*d)*(a + b*x^3)^(1/3)) + (3*c*(
9*b*c - 10*a*d)*x*(c + d*x^3))/(140*a^3*(b*c - a*d)*(a + b*x^3)^(4/3)) + ((
9*b*c - 10*a*d)*x*(c + d*x^3)^2)/(70*a^2*(b*c - a*d)*(a + b*x^3)^(7/3)) + (
b*x*(c + d*x^3)^3)/(10*a*(b*c - a*d)*(a + b*x^3)^(10/3))
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 386

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] - Dist[
c*(q/(a*(p + 1))), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1,
0] && GtQ[q, 0] && NeQ[p, -1]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
```

```
 a*d))), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)),
 Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q},
 x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !L
tQ[q, -1]) && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx^3)^2}{(a+bx^3)^{13/3}}\,dx &= \frac{bx(c+dx^3)^3}{10a(bc-ad)\,(a+bx^3)^{10/3}}+\frac{(9bc-10ad)\int \frac{(c+dx^3)^2}{(a+bx^3)^{10/3}}\,dx}{10a(bc-ad)}\\ &= \frac{(9bc-10ad)x(c+dx^3)^2}{70a^2(bc-ad)\,(a+bx^3)^{7/3}}+\frac{bx(c+dx^3)^3}{10a(bc-ad)\,(a+bx^3)^{10/3}}+\frac{(3c(9bc-10ad))\int \frac{c+dx^3}{(a+bx^3)^{7/3}}}{35a^2(bc-ad)}\\ &= \frac{3c(9bc-10ad)x(c+dx^3)}{140a^3(bc-ad)\,(a+bx^3)^{4/3}}+\frac{(9bc-10ad)x(c+dx^3)^2}{70a^2(bc-ad)\,(a+bx^3)^{7/3}}+\frac{bx(c+dx^3)^3}{10a(bc-ad)\,(a+bx^3)^{10/3}}\\ &= \frac{9c^2(9bc-10ad)x}{140a^4(bc-ad)\sqrt[3]{a+bx^3}}+\frac{3c(9bc-10ad)x(c+dx^3)}{140a^3(bc-ad)\,(a+bx^3)^{4/3}}+\frac{(9bc-10ad)x(c+dx^3)^2}{70a^2(bc-ad)\,(a+bx^3)^{7/3}}\end{aligned}$$

Mathematica [A]
time = 0.58, size = 106, normalized size = 0.61

$$\frac{x(81b^3c^2x^9+18ab^2cx^6(15c+dx^3)+10a^3(14c^2+7cdx^3+2d^2x^6)+3a^2bx^3(105c^2+20cdx^3+2d^2x^6))}{140a^4\,(a+bx^3)^{10/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3)^2/(a + b*x^3)^(13/3),x]
[Out] (x*(81*b^3*c^2*x^9 + 18*a*b^2*c*x^6*(15*c + d*x^3) + 10*a^3*(14*c^2 + 7*c*d
*x^3 + 2*d^2*x^6) + 3*a^2*b*x^3*(105*c^2 + 20*c*d*x^3 + 2*d^2*x^6)))/(140*a
^4*(a + b*x^3)^(10/3))
```

Maple [A]
time = 0.26, size = 115, normalized size = 0.66

method	result	size
gosper	$\frac{x\left(6a^2b\,d^2x^9+18a\,b^2cd\,x^9+81b^3c^2x^9+20a^3d^2x^6+60a^2bcd\,x^6+270a\,b^2c^2x^6+70a^3cd\,x^3+315a^2b\,c^2x^3+140a^3c^2\right)}{140(b\,x^3+a)^{\frac{10}{3}}a^4}$	115
trager	$\frac{x\left(6a^2b\,d^2x^9+18a\,b^2cd\,x^9+81b^3c^2x^9+20a^3d^2x^6+60a^2bcd\,x^6+270a\,b^2c^2x^6+70a^3cd\,x^3+315a^2b\,c^2x^3+140a^3c^2\right)}{140(b\,x^3+a)^{\frac{10}{3}}a^4}$	115

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^2/(b*x^3+a)^(13/3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/140*x*(6*a^2*b*d^2*x^9+18*a*b^2*c*d*x^9+81*b^3*c^2*x^9+20*a^3*d^2*x^6+60*
a^2*b*c*d*x^6+270*a*b^2*c^2*x^6+70*a^3*c*d*x^3+315*a^2*b*c^2*x^3+140*a^3*c^
2)/(b*x^3+a)^(10/3)/a^4
```

Maxima [A]
time = 0.28, size = 159, normalized size = 0.91

$$-\frac{\left(7\,b-\frac{10\,(bx^3+a)}{x^3}\right)d^2x^{10}}{70\,(bx^3+a)^{\frac{10}{3}}\,a^2}+\frac{\left(14\,b^2-\frac{40\,(bx^3+a)b}{x^3}+\frac{35\,(bx^3+a)^2}{x^6}\right)cdx^{10}}{70\,(bx^3+a)^{\frac{10}{3}}\,a^3}-\frac{\left(14\,b^3-\frac{60\,(bx^3+a)b^2}{x^3}+\frac{105\,(bx^3+a)^2b}{x^6}-\frac{140\,(bx^3+a)^3}{x^9}\right)c^2x^{10}}{140\,(bx^3+a)^{\frac{10}{3}}\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^2/(b*x^3+a)^(13/3),x, algorithm="maxima")
```

```
[Out] -1/70*(7*b - 10*(b*x^3 + a)/x^3)*d^2*x^10/((b*x^3 + a)^(10/3)*a^2) + 1/70*(
14*b^2 - 40*(b*x^3 + a)*b/x^3 + 35*(b*x^3 + a)^2/x^6)*c*d*x^10/((b*x^3 + a)
^(10/3)*a^3) - 1/140*(14*b^3 - 60*(b*x^3 + a)*b^2/x^3 + 105*(b*x^3 + a)^2*b
/x^6 - 140*(b*x^3 + a)^3/x^9)*c^2*x^10/((b*x^3 + a)^(10/3)*a^4)
```

Fricas [A]
time = 2.82, size = 152, normalized size = 0.87

$$\frac{(3\,(27\,b^3c^2+6\,ab^2cd+2\,a^2bd^2)x^{10}+10\,(27\,ab^2c^2+6\,a^2bcd+2\,a^3d^2)x^7+140\,a^3c^2x+35\,(9\,a^2bc^2+2\,a^3cd)x^4)(bx^3+a)^{\frac{2}{3}}}{140\,(a^4b^4x^{12}+4\,a^5b^3x^9+6\,a^6b^2x^6+4\,a^7bx^3+a^8)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^2/(b*x^3+a)^(13/3),x, algorithm="fricas")
```

```
[Out] 1/140*(3*(27*b^3*c^2 + 6*a*b^2*c*d + 2*a^2*b*d^2)*x^10 + 10*(27*a*b^2*c^2 +
 6*a^2*b*c*d + 2*a^3*d^2)*x^7 + 140*a^3*c^2*x + 35*(9*a^2*b*c^2 + 2*a^3*c*d
)*x^4)*(b*x^3 + a)^(2/3)/(a^4*b^4*x^12 + 4*a^5*b^3*x^9 + 6*a^6*b^2*x^6 + 4*
a^7*b*x^3 + a^8)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**2/(b*x**3+a)**(13/3),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^2/(b*x^3+a)^(13/3),x, algorithm="giac")
```

```
[Out] integrate((d*x^3 + c)^2/(b*x^3 + a)^(13/3), x)
```

Mupad [B]
time = 1.45, size = 176, normalized size = 1.01

$$\frac{x\left(\frac{c^2}{10\,a}+\frac{a\left(\frac{d^2}{10\,b}-\frac{c\,d}{5\,a}\right)}{b}\right)}{\left(b\,x^3+a\right)^{10/3}}-\frac{x\left(\frac{d^2}{7\,b^2}-\frac{-a^2\,d^2+2\,a\,b\,c\,d+9\,b^2\,c^2}{70\,a^2\,b^2}\right)}{\left(b\,x^3+a\right)^{7/3}}+\frac{x\,\left(2\,a^2\,d^2+6\,a\,b\,c\,d+27\,b^2\,c^2\right)}{140\,a^3\,b^2\,\left(b\,x^3+a\right)^{4/3}}+\frac{x\,\left(6\,a^2\,d^2+18\,a\,b\,c\,d+81\,b^2\,c^2\right)}{140\,a^4\,b^2\,\left(b\,x^3+a\right)^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^2/(a + b*x^3)^(13/3),x)
```

```
[Out] (x*(c^2/(10*a) + (a*(d^2/(10*b) - (c*d)/(5*a)))/b))/(a + b*x^3)^(10/3) - (x
*(d^2/(7*b^2) - (9*b^2*c^2 - a^2*d^2 + 2*a*b*c*d)/(70*a^2*b^2)))/(a + b*x^3
)^(7/3) + (x*(2*a^2*d^2 + 27*b^2*c^2 + 6*a*b*c*d))/(140*a^3*b^2*(a + b*x^3)
^(4/3)) + (x*(6*a^2*d^2 + 81*b^2*c^2 + 18*a*b*c*d))/(140*a^4*b^2*(a + b*x^3
)^(1/3))
```

3.77 $\int \frac{\left(c+dx^3\right)^2}{\left(a+bx^3\right)^{16/3}}\,dx$

Optimal. Leaf size=211

$$\frac{2(bc-ad)(3bc+ad)x}{65a^2b^2\left(a+bx^3\right)^{10/3}}+\frac{\left(54b^2c^2+9abcd+2a^2d^2\right)x}{455a^3b^2\left(a+bx^3\right)^{7/3}}+\frac{3\left(54b^2c^2+9abcd+2a^2d^2\right)x}{910a^4b^2\left(a+bx^3\right)^{4/3}}+\frac{9\left(54b^2c^2+9abcd+2a^2\right.}{910a^5b^2\sqrt[3]{a+bx^3}}$$

```
[Out] 2/65*(-a*d+b*c)*(a*d+3*b*c)*x/a^2/b^2/(b*x^3+a)^(10/3)+1/455*(2*a^2*d^2+9*a
*b*c*d+54*b^2*c^2)*x/a^3/b^2/(b*x^3+a)^(7/3)+3/910*(2*a^2*d^2+9*a*b*c*d+54*
b^2*c^2)*x/a^4/b^2/(b*x^3+a)^(4/3)+9/910*(2*a^2*d^2+9*a*b*c*d+54*b^2*c^2)*x
/a^5/b^2/(b*x^3+a)^(1/3)+1/13*(-a*d+b*c)*x*(d*x^3+c)/a/b/(b*x^3+a)^(13/3)
```

Rubi [A]
time = 0.09, antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {424, 393, 198, 197}

$$\frac{2x(bc-ad)(ad+3bc)}{65a^2b^2\left(a+bx^3\right)^{10/3}}+\frac{9x\left(2a^2d^2+9abcd+54b^2c^2\right)}{910a^5b^2\sqrt[3]{a+bx^3}}+\frac{3x\left(2a^2d^2+9abcd+54b^2c^2\right)}{910a^4b^2\left(a+bx^3\right)^{4/3}}+\frac{x\left(2a^2d^2+9abcd+54b^2c^2\right)}{455a^3b^2\left(a+bx^3\right)^{7/3}}+\frac{x\left(c+dx^3\right)(bc-ad)}{13ab\left(a+bx^3\right)^{13/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)^2/(a + b*x^3)^(16/3),x]
```

```
[Out] (2*(b*c - a*d)*(3*b*c + a*d)*x)/(65*a^2*b^2*(a + b*x^3)^(10/3)) + ((54*b^2*
c^2 + 9*a*b*c*d + 2*a^2*d^2)*x)/(455*a^3*b^2*(a + b*x^3)^(7/3)) + (3*(54*b^
2*c^2 + 9*a*b*c*d + 2*a^2*d^2)*x)/(910*a^4*b^2*(a + b*x^3)^(4/3)) + (9*(54*
b^2*c^2 + 9*a*b*c*d + 2*a^2*d^2)*x)/(910*a^5*b^2*(a + b*x^3)^(1/3)) + ((b*c
 - a*d)*x*(c + d*x^3))/(13*a*b*(a + b*x^3)^(13/3))
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
```

```
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(c+dx^3)^2}{(a+bx^3)^{16/3}}\,dx &= \frac{(bc-ad)x(c+dx^3)}{13ab\,(a+bx^3)^{13/3}} + \frac{\int \frac{c(12bc+ad)+d(9bc+4ad)x^3}{(a+bx^3)^{13/3}}\,dx}{13ab} \\
&= \frac{2(bc-ad)(3bc+ad)x}{65a^2b^2\,(a+bx^3)^{10/3}} + \frac{(bc-ad)x(c+dx^3)}{13ab\,(a+bx^3)^{13/3}} + \frac{(54b^2c^2+9abcd+2a^2d^2)\int \frac{1}{(a+bx^3)^{10/3}}}{65a^2b^2} \\
&= \frac{2(bc-ad)(3bc+ad)x}{65a^2b^2\,(a+bx^3)^{10/3}} + \frac{(54b^2c^2+9abcd+2a^2d^2)\,x}{455a^3b^2\,(a+bx^3)^{7/3}} + \frac{(bc-ad)x(c+dx^3)}{13ab\,(a+bx^3)^{13/3}} + \frac{(6(54b^2}{} \\
&= \frac{2(bc-ad)(3bc+ad)x}{65a^2b^2\,(a+bx^3)^{10/3}} + \frac{(54b^2c^2+9abcd+2a^2d^2)\,x}{455a^3b^2\,(a+bx^3)^{7/3}} + \frac{3(54b^2c^2+9abcd+2a^2d^2)\,x}{910a^4b^2\,(a+bx^3)^{4/3}} + \\
&= \frac{2(bc-ad)(3bc+ad)x}{65a^2b^2\,(a+bx^3)^{10/3}} + \frac{(54b^2c^2+9abcd+2a^2d^2)\,x}{455a^3b^2\,(a+bx^3)^{7/3}} + \frac{3(54b^2c^2+9abcd+2a^2d^2)\,x}{910a^4b^2\,(a+bx^3)^{4/3}} +
\end{aligned}
$$

Mathematica [A]
time = 0.82, size = 138, normalized size = 0.65

$$
\frac{x(486b^4c^2x^{12}+81ab^3cx^9(26c+dx^3)+65a^4(14c^2+7cdx^3+2d^2x^6)+39a^3bx^3(70c^2+15cdx^3+2d^2x^6)+9a^2b^2x^6(390c^2+39cdx^3+2d^2x^6))}{910a^5\,(a+bx^3)^{13/3}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3)^2/(a + b*x^3)^(16/3),x]
```

```
[Out] (x*(486*b^4*c^2*x^12 + 81*a*b^3*c*x^9*(26*c + d*x^3) + 65*a^4*(14*c^2 + 7*c
*d*x^3 + 2*d^2*x^6) + 39*a^3*b*x^3*(70*c^2 + 15*c*d*x^3 + 2*d^2*x^6) + 9*a^
2*b^2*x^6*(390*c^2 + 39*c*d*x^3 + 2*d^2*x^6)))/(910*a^5*(a + b*x^3)^(13/3))
```

Maple [A]
time = 0.28, size = 156, normalized size = 0.74

method	result
gosper	$\frac{x\left(18a^2b^2d^2x^{12}+81a\,b^3cd\,x^{12}+486b^4c^2x^{12}+78a^3b\,d^2x^9+351a^2b^2cd\,x^9+2106a\,b^3c^2x^9+130a^4d^2x^6+585a^3bcd\,x^6+3510a^2b^2c^2x^6+45\right.}{910(b\,x^3+a)^{\frac{13}{3}}a^5}$
trager	$\frac{x\left(18a^2b^2d^2x^{12}+81a\,b^3cd\,x^{12}+486b^4c^2x^{12}+78a^3b\,d^2x^9+351a^2b^2cd\,x^9+2106a\,b^3c^2x^9+130a^4d^2x^6+585a^3bcd\,x^6+3510a^2b^2c^2x^6+45\right.}{910(b\,x^3+a)^{\frac{13}{3}}a^5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^2/(b*x^3+a)^(16/3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/910*x*(18*a^2*b^2*d^2*x^12+81*a*b^3*c*d*x^12+486*b^4*c^2*x^12+78*a^3*b*d^
2*x^9+351*a^2*b^2*c*d*x^9+2106*a*b^3*c^2*x^9+130*a^4*d^2*x^6+585*a^3*b*c*d*
x^6+3510*a^2*b^2*c^2*x^6+455*a^4*c*d*x^3+2730*a^3*b*c^2*x^3+910*a^4*c^2)/(b
*x^3+a)^(13/3)/a^5
```

Maxima [A]
time = 0.29, size = 210, normalized size = 1.00

$$\frac{\left(35\,b^2-\frac{91\,(bx^3+a)b}{x^3}+\frac{65\,(bx^3+a)^2}{x^6}\right)d^2x^{13}}{455\,(bx^3+a)^{\frac{13}{3}}a^3}-\frac{\left(140\,b^3-\frac{546\,(bx^3+a)b^2}{x^3}+\frac{780\,(bx^3+a)^2b}{x^6}-\frac{455\,(bx^3+a)^3}{x^9}\right)cdx^{13}}{910\,(bx^3+a)^{\frac{13}{3}}a^4}+\frac{\left(35\,b^4-\frac{182\,(bx^3+a)b^3}{x^3}+\frac{390\,(bx^3+a)^2b^2}{x^6}-\frac{455\,(bx^3+a)^3b}{x^9}+\frac{455\,(bx^3+a)^4}{x^{12}}\right)c^2x^{13}}{455\,(bx^3+a)^{\frac{13}{3}}a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^2/(b*x^3+a)^(16/3),x, algorithm="maxima")
```

```
[Out] 1/455*(35*b^2 - 91*(b*x^3 + a)*b/x^3 + 65*(b*x^3 + a)^2/x^6)*d^2*x^13/((b*x
^3 + a)^(13/3)*a^3) - 1/910*(140*b^3 - 546*(b*x^3 + a)*b^2/x^3 + 780*(b*x^3
 + a)^2*b/x^6 - 455*(b*x^3 + a)^3/x^9)*c*d*x^13/((b*x^3 + a)^(13/3)*a^4) +
1/455*(35*b^4 - 182*(b*x^3 + a)*b^3/x^3 + 390*(b*x^3 + a)^2*b^2/x^6 - 455*(
b*x^3 + a)^3*b/x^9 + 455*(b*x^3 + a)^4/x^12)*c^2*x^13/((b*x^3 + a)^(13/3)*a
^5)
```

Fricas [A]
time = 2.79, size = 200, normalized size = 0.95

$$\frac{(9\,(54\,b^4c^2+9\,ab^3cd+2\,a^2b^2d^2)x^{13}+39\,(54\,ab^3c^2+9\,a^2b^2cd+2\,a^3bd^2)x^{10}+65\,(54\,a^2b^2c^2+9\,a^3bcd+2\,a^4d^2)x^7+910\,a^4c^2x+455\,(6\,a^3bc^2+a^4cd)x^4)(bx^3+a)^{\frac{2}{3}}}{910\,(a^5b^5x^{15}+5\,a^6b^4x^{12}+10\,a^7b^3x^9+10\,a^8b^2x^6+5\,a^9bx^3+a^{10})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^2/(b*x^3+a)^(16/3),x, algorithm="fricas")
```

```
[Out] 1/910*(9*(54*b^4*c^2 + 9*a*b^3*c*d + 2*a^2*b^2*d^2)*x^13 + 39*(54*a*b^3*c^2
 + 9*a^2*b^2*c*d + 2*a^3*b*d^2)*x^10 + 65*(54*a^2*b^2*c^2 + 9*a^3*b*c*d + 2
*a^4*d^2)*x^7 + 910*a^4*c^2*x + 455*(6*a^3*b*c^2 + a^4*c*d)*x^4)*(b*x^3 + a
)^(2/3)/(a^5*b^5*x^15 + 5*a^6*b^4*x^12 + 10*a^7*b^3*x^9 + 10*a^8*b^2*x^6 +
5*a^9*b*x^3 + a^10)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**2/(b*x**3+a)**(16/3),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^2/(b*x^3+a)^(16/3),x, algorithm="giac")
```

```
[Out] integrate((d*x^3 + c)^2/(b*x^3 + a)^(16/3), x)
```

Mupad [**B**]
time = 1.43, size = 217, normalized size = 1.03

$$\frac{x\left(\frac{c^2}{13\,a}+\frac{a\left(\frac{d^2}{13\,b}-\frac{2\,c\,d}{13\,a}\right)}{b}\right)}{{\left(b\,x^3+a\right)}^{13/3}}-\frac{x\left(\frac{d^2}{10\,b^2}-\frac{-a^2\,d^2+2\,a\,b\,c\,d+12\,b^2\,c^2}{130\,a^2\,b^2}\right)}{{\left(b\,x^3+a\right)}^{10/3}}+\frac{x\,\left(2\,a^2\,d^2+9\,a\,b\,c\,d+54\,b^2\,c^2\right)}{455\,a^3\,b^2\,{\left(b\,x^3+a\right)}^{7/3}}+\frac{x\,\left(6\,a^2\,d^2+27\,a\,b\,c\,d+162\,b^2\,c^2\right)}{910\,a^4\,b^2\,{\left(b\,x^3+a\right)}^{4/3}}+\frac{x\,\left(18\,a^2\,d^2+81\,a\,b\,c\,d+486\,b^2\,c^2\right)}{910\,a^5\,b^2\,{\left(b\,x^3+a\right)}^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^2/(a + b*x^3)^(16/3),x)
```

```
[Out] (x*(c^2/(13*a) + (a*(d^2/(13*b) - (2*c*d)/(13*a)))/b))/(a + b*x^3)^(13/3) -
 (x*(d^2/(10*b^2) - (12*b^2*c^2 - a^2*d^2 + 2*a*b*c*d)/(130*a^2*b^2)))/(a +
 b*x^3)^(10/3) + (x*(2*a^2*d^2 + 54*b^2*c^2 + 9*a*b*c*d))/(455*a^3*b^2*(a +
 b*x^3)^(7/3)) + (x*(6*a^2*d^2 + 162*b^2*c^2 + 27*a*b*c*d))/(910*a^4*b^2*(a
 + b*x^3)^(4/3)) + (x*(18*a^2*d^2 + 486*b^2*c^2 + 81*a*b*c*d))/(910*a^5*b^2
*(a + b*x^3)^(1/3))
```

3.78 $\int \frac{(c+dx^3)^2}{(a+bx^3)^{19/3}} \, dx$

Optimal. Leaf size=253

$$\frac{(bc-ad)(15bc+4ad)x}{208a^2b^2(a+bx^3)^{13/3}}+\frac{(45b^2c^2+6abcd+a^2d^2)\,x}{520a^3b^2(a+bx^3)^{10/3}}+\frac{9(45b^2c^2+6abcd+a^2d^2)\,x}{3640a^4b^2(a+bx^3)^{7/3}}+\frac{27(45b^2c^2+6abcd+a^2d}{7280a^5b^2(a+bx^3)^{4/3}}$$

```
[Out] 1/208*(-a*d+b*c)*(4*a*d+15*b*c)*x/a^2/b^2/(b*x^3+a)^(13/3)+1/520*(a^2*d^2+6
*a*b*c*d+45*b^2*c^2)*x/a^3/b^2/(b*x^3+a)^(10/3)+9/3640*(a^2*d^2+6*a*b*c*d+4
5*b^2*c^2)*x/a^4/b^2/(b*x^3+a)^(7/3)+27/7280*(a^2*d^2+6*a*b*c*d+45*b^2*c^2)
*x/a^5/b^2/(b*x^3+a)^(4/3)+81/7280*(a^2*d^2+6*a*b*c*d+45*b^2*c^2)*x/a^6/b^2
/(b*x^3+a)^(1/3)+1/16*(-a*d+b*c)*x*(d*x^3+c)/a/b/(b*x^3+a)^(16/3)
```

Rubi [A]
time = 0.14, antiderivative size = 253, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {424, 393, 198, 197}

$$\frac{x(bc-ad)(4ad+15bc)}{208a^2b^2(a+bx^3)^{13/3}}+\frac{81x(a^2d^2+6abcd+45b^2c^2)}{7280a^6b^2\sqrt[3]{a+bx^3}}+\frac{27x(a^2d^2+6abcd+45b^2c^2)}{7280a^5b^2(a+bx^3)^{4/3}}+\frac{9x(a^2d^2+6abcd+45b^2c^2)}{3640a^4b^2(a+bx^3)^{7/3}}+\frac{x(a^2d^2+6abcd+45b^2c^2)}{520a^3b^2(a+bx^3)^{10/3}}+\frac{x(c+dx^3)(bc-ad)}{16ab(a+bx^3)^{16/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)^2/(a + b*x^3)^(19/3),x]
```

```
[Out] ((b*c - a*d)*(15*b*c + 4*a*d)*x)/(208*a^2*b^2*(a + b*x^3)^(13/3)) + ((45*b^
2*c^2 + 6*a*b*c*d + a^2*d^2)*x)/(520*a^3*b^2*(a + b*x^3)^(10/3)) + (9*(45*b
^2*c^2 + 6*a*b*c*d + a^2*d^2)*x)/(3640*a^4*b^2*(a + b*x^3)^(7/3)) + (27*(45
*b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x)/(7280*a^5*b^2*(a + b*x^3)^(4/3)) + (81*(
45*b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x)/(7280*a^6*b^2*(a + b*x^3)^(1/3)) + ((b
*c - a*d)*x*(c + d*x^3))/(16*a*b*(a + b*x^3)^(16/3))
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
```

```
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(c+dx^3)^2}{(a+bx^3)^{19/3}}\,dx &= \frac{(bc-ad)x(c+dx^3)}{16ab\,(a+bx^3)^{16/3}} + \frac{\int \frac{c(15bc+ad)+4d(3bc+ad)x^3}{(a+bx^3)^{16/3}}\,dx}{16ab} \\
&= \frac{(bc-ad)(15bc+4ad)x}{208a^2b^2\,(a+bx^3)^{13/3}} + \frac{(bc-ad)x(c+dx^3)}{16ab\,(a+bx^3)^{16/3}} + \frac{(45b^2c^2+6abcd+a^2d^2)\int \frac{1}{(a+bx^3)^{13/3}}}{52a^2b^2} \\
&= \frac{(bc-ad)(15bc+4ad)x}{208a^2b^2\,(a+bx^3)^{13/3}} + \frac{(45b^2c^2+6abcd+a^2d^2)\,x}{520a^3b^2\,(a+bx^3)^{10/3}} + \frac{(bc-ad)x(c+dx^3)}{16ab\,(a+bx^3)^{16/3}} + \frac{(9(45b^2}{} \\
&= \frac{(bc-ad)(15bc+4ad)x}{208a^2b^2\,(a+bx^3)^{13/3}} + \frac{(45b^2c^2+6abcd+a^2d^2)\,x}{520a^3b^2\,(a+bx^3)^{10/3}} + \frac{9(45b^2c^2+6abcd+a^2d^2)\,x}{3640a^4b^2\,(a+bx^3)^{7/3}} + \\
&= \frac{(bc-ad)(15bc+4ad)x}{208a^2b^2\,(a+bx^3)^{13/3}} + \frac{(45b^2c^2+6abcd+a^2d^2)\,x}{520a^3b^2\,(a+bx^3)^{10/3}} + \frac{9(45b^2c^2+6abcd+a^2d^2)\,x}{3640a^4b^2\,(a+bx^3)^{7/3}} + \\
&= \frac{(bc-ad)(15bc+4ad)x}{208a^2b^2\,(a+bx^3)^{13/3}} + \frac{(45b^2c^2+6abcd+a^2d^2)\,x}{520a^3b^2\,(a+bx^3)^{10/3}} + \frac{9(45b^2c^2+6abcd+a^2d^2)\,x}{3640a^4b^2\,(a+bx^3)^{7/3}} +
\end{aligned}$$

Mathematica [A]
time = 1.26, size = 169, normalized size = 0.67

$$\frac{x(3645b^5c^2x^{15}+486ab^4cx^{12}(40c+dx^3)+81a^2b^3x^9(520c^2+32cdx^3+d^2x^6)+520a^5(14c^2+7cdx^3+2d^2x^6)+144a^3b^2x^6(325c^2+39cdx^3+3d^2x^6)+156a^4bx^3(175c^2+40cdx^3+6d^2x^6))}{7280a^6\,(a+bx^3)^{16/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3)^2/(a + b*x^3)^(19/3),x]
```

```
[Out] (x*(3645*b^5*c^2*x^15 + 486*a*b^4*c*x^12*(40*c + d*x^3) + 81*a^2*b^3*x^9*(5
20*c^2 + 32*c*d*x^3 + d^2*x^6) + 520*a^5*(14*c^2 + 7*c*d*x^3 + 2*d^2*x^6) +
```

```
144*a^3*b^2*x^6*(325*c^2 + 39*c*d*x^3 + 3*d^2*x^6) + 156*a^4*b*x^3*(175*c^
2 + 40*c*d*x^3 + 6*d^2*x^6)))/(7280*a^6*(a + b*x^3)^(16/3))
```

Maple [A]
time = 0.26, size = 197, normalized size = 0.78

method	result
gosper	$\frac{x\left(81 a^{2} b^{3} d^{2} x^{15}+486 a\,b^{4} c d\,x^{15}+3645 b^{5} c^{2} x^{15}+432 a^{3} b^{2} d^{2} x^{12}+2592 a^{2} b^{3} c d\,x^{12}+19440 a\,b^{4} c^{2} x^{12}+936 a^{4} b\,d^{2} x^{9}+5616 a^{3} b^{2} c d\,x^{9}+4212}{7280 \left(b\,x^{3}+a \right)^{\frac{16}{3}} a^{6}}$
trager	$\frac{x\left(81 a^{2} b^{3} d^{2} x^{15}+486 a\,b^{4} c d\,x^{15}+3645 b^{5} c^{2} x^{15}+432 a^{3} b^{2} d^{2} x^{12}+2592 a^{2} b^{3} c d\,x^{12}+19440 a\,b^{4} c^{2} x^{12}+936 a^{4} b\,d^{2} x^{9}+5616 a^{3} b^{2} c d\,x^{9}+4212}{7280 \left(b\,x^{3}+a \right)^{\frac{16}{3}} a^{6}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^2/(b*x^3+a)^(19/3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/7280*x*(81*a^2*b^3*d^2*x^15+486*a*b^4*c*d*x^15+3645*b^5*c^2*x^15+432*a^3*
b^2*d^2*x^12+2592*a^2*b^3*c*d*x^12+19440*a*b^4*c^2*x^12+936*a^4*b*d^2*x^9+5
616*a^3*b^2*c*d*x^9+42120*a^2*b^3*c^2*x^9+1040*a^5*d^2*x^6+6240*a^4*b*c*d*x
^6+46800*a^3*b^2*c^2*x^6+3640*a^5*c*d*x^3+27300*a^4*b*c^2*x^3+7280*a^5*c^2)
/(b*x^3+a)^(16/3)/a^6
```

Maxima [A]
time = 0.28, size = 261, normalized size = 1.03

$$-\frac{\left(455\,b^{3}-\frac{1680\,(bx^{3}+a)b^{2}}{x^{3}}+\frac{2184\,(bx^{3}+a)^{2}b}{x^{6}}-\frac{1040\,(bx^{3}+a)^{3}}{x^{9}}\right)d^{2}x^{16}}{7280\,(bx^{3}+a)^{\frac{16}{3}}a^{4}}+\frac{\left(455\,b^{4}-\frac{2240\,(bx^{3}+a)b^{3}}{x^{3}}+\frac{4368\,(bx^{3}+a)^{2}b^{2}}{x^{6}}-\frac{4160\,(bx^{3}+a)^{3}b}{x^{9}}+\frac{1820\,(bx^{3}+a)^{4}}{x^{12}}\right)cdx^{16}}{3640\,(bx^{3}+a)^{\frac{16}{3}}a^{5}}-\frac{\left(91\,b^{5}-\frac{560\,(bx^{3}+a)b^{4}}{x^{3}}+\frac{1456\,(bx^{3}+a)^{2}b^{3}}{x^{6}}-\frac{2080\,(bx^{3}+a)^{3}b^{2}}{x^{9}}+\frac{1820\,(bx^{3}+a)^{4}b}{x^{12}}-\frac{1456\,(bx^{3}+a)^{5}}{x^{15}}\right)c^{2}x^{16}}{1456\,(bx^{3}+a)^{\frac{16}{3}}a^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^2/(b*x^3+a)^(19/3),x, algorithm="maxima")
```

```
[Out] -1/7280*(455*b^3 - 1680*(b*x^3 + a)*b^2/x^3 + 2184*(b*x^3 + a)^2*b/x^6 - 10
40*(b*x^3 + a)^3/x^9)*d^2*x^16/((b*x^3 + a)^(16/3)*a^4) + 1/3640*(455*b^4 -
 2240*(b*x^3 + a)*b^3/x^3 + 4368*(b*x^3 + a)^2*b^2/x^6 - 4160*(b*x^3 + a)^3
*b/x^9 + 1820*(b*x^3 + a)^4/x^12)*c*d*x^16/((b*x^3 + a)^(16/3)*a^5) - 1/145
6*(91*b^5 - 560*(b*x^3 + a)*b^4/x^3 + 1456*(b*x^3 + a)^2*b^3/x^6 - 2080*(b*
x^3 + a)^3*b^2/x^9 + 1820*(b*x^3 + a)^4*b/x^12 - 1456*(b*x^3 + a)^5/x^15)*c
^2*x^16/((b*x^3 + a)^(16/3)*a^6)
```

Fricas [A]
time = 3.91, size = 246, normalized size = 0.97

$$\frac{(81\,(45\,b^{5}c^{2}+6\,ab^{4}cd+a^{2}b^{3}d^{2})x^{16}+432\,(45\,ab^{4}c^{2}+6\,a^{2}b^{3}cd+a^{3}b^{2}d^{2})x^{13}+936\,(45\,a^{2}b^{3}c^{2}+6\,a^{3}b^{2}cd+a^{4}bd^{2})x^{10}+7280\,a^{5}c^{2}x+1040\,(45\,a^{3}b^{2}c^{2}+6\,a^{4}bcd+a^{5}d^{2})x^{7}+1820\,(15\,a^{4}bc^{2}+2\,a^{5}cd)x^{4})(bx^{3}+a)^{\frac{2}{3}}}{7280\,(a^{6}b^{6}x^{18}+6\,a^{7}b^{5}x^{15}+15\,a^{8}b^{4}x^{12}+20\,a^{9}b^{3}x^{9}+15\,a^{10}b^{2}x^{6}+6\,a^{11}bx^{3}+a^{12})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^2/(b*x^3+a)^(19/3),x, algorithm="fricas")
```

```
[Out] 1/7280*(81*(45*b^5*c^2 + 6*a*b^4*c*d + a^2*b^3*d^2)*x^16 + 432*(45*a*b^4*c^
2 + 6*a^2*b^3*c*d + a^3*b^2*d^2)*x^13 + 936*(45*a^2*b^3*c^2 + 6*a^3*b^2*c*d
 + a^4*b*d^2)*x^10 + 7280*a^5*c^2*x + 1040*(45*a^3*b^2*c^2 + 6*a^4*b*c*d +
a^5*d^2)*x^7 + 1820*(15*a^4*b*c^2 + 2*a^5*c*d)*x^4)*(b*x^3 + a)^(2/3)/(a^6*
b^6*x^18 + 6*a^7*b^5*x^15 + 15*a^8*b^4*x^12 + 20*a^9*b^3*x^9 + 15*a^10*b^2*
x^6 + 6*a^11*b*x^3 + a^12)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**2/(b*x**3+a)**(19/3),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^2/(b*x^3+a)^(19/3),x, algorithm="giac")
```

```
[Out] integrate((d*x^3 + c)^2/(b*x^3 + a)^(19/3), x)
```

Mupad [B]
time = 1.48, size = 257, normalized size = 1.02

$$\frac{x\left(\frac{c^2}{16\,a}+\frac{a\left(\frac{d^2}{16\,b}-\frac{c\,d}{8\,a}\right)}{b}\right)}{{\left(b\,x^3+a\right)}^{16/3}}-\frac{x\left(\frac{d^2}{13\,b^2}-\frac{-a^2\,d^2+2\,a\,b\,c\,d+15\,b^2\,c^2}{208\,a^2\,b^2}\right)}{{\left(b\,x^3+a\right)}^{13/3}}+\frac{x\,\left(a^2\,d^2+6\,a\,b\,c\,d+45\,b^2\,c^2\right)}{520\,a^3\,b^2\,{\left(b\,x^3+a\right)}^{10/3}}+\frac{x\,\left(9\,a^2\,d^2+54\,a\,b\,c\,d+405\,b^2\,c^2\right)}{3640\,a^4\,b^2\,{\left(b\,x^3+a\right)}^{7/3}}+\frac{x\,\left(27\,a^2\,d^2+162\,a\,b\,c\,d+1215\,b^2\,c^2\right)}{7280\,a^5\,b^2\,{\left(b\,x^3+a\right)}^{4/3}}+\frac{x\,\left(81\,a^2\,d^2+486\,a\,b\,c\,d+3645\,b^2\,c^2\right)}{7280\,a^6\,b^2\,{\left(b\,x^3+a\right)}^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^2/(a + b*x^3)^(19/3),x)
```

```
[Out] (x*(c^2/(16*a) + (a*(d^2/(16*b) - (c*d)/(8*a)))/b))/(a + b*x^3)^(16/3) - (x
*(d^2/(13*b^2) - (15*b^2*c^2 - a^2*d^2 + 2*a*b*c*d)/(208*a^2*b^2)))/(a + b*
x^3)^(13/3) + (x*(a^2*d^2 + 45*b^2*c^2 + 6*a*b*c*d))/(520*a^3*b^2*(a + b*x^
3)^(10/3)) + (x*(9*a^2*d^2 + 405*b^2*c^2 + 54*a*b*c*d))/(3640*a^4*b^2*(a +
b*x^3)^(7/3)) + (x*(27*a^2*d^2 + 1215*b^2*c^2 + 162*a*b*c*d))/(7280*a^5*b^2
*(a + b*x^3)^(4/3)) + (x*(81*a^2*d^2 + 3645*b^2*c^2 + 486*a*b*c*d))/(7280*a
^6*b^2*(a + b*x^3)^(1/3))
```

3.79 $\int (a+bx^3)^{7/3} (c+dx^3)^2 \, dx$

Optimal. Leaf size=135

$$\frac{d(17bc-4ad)x(a+bx^3)^{10/3}}{154b^2}+\frac{dx(a+bx^3)^{10/3}(c+dx^3)}{14b}+\frac{a^2(77b^2c^2-14abcd+2a^2d^2)\,x\sqrt[3]{a+bx^3}\;{}_2F_1\left(-\frac{7}{3}\right.}{77b^2\sqrt[3]{1+\frac{bx^3}{a}}}$$

```
[Out] 1/154*d*(-4*a*d+17*b*c)*x*(b*x^3+a)^(10/3)/b^2+1/14*d*x*(b*x^3+a)^(10/3)*(d
*x^3+c)/b+1/77*a^2*(2*a^2*d^2-14*a*b*c*d+77*b^2*c^2)*x*(b*x^3+a)^(1/3)*hype
rgeom([-7/3, 1/3],[4/3],-b*x^3/a)/b^2/(1+b*x^3/a)^(1/3)
```

Rubi [A]
time = 0.05, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {427, 396, 252, 251}

$$\frac{a^2x\sqrt[3]{a+bx^3}\,(2a^2d^2-14abcd+77b^2c^2)\;{}_2F_1\left(-\frac{7}{3},\frac{1}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{77b^2\sqrt[3]{\frac{bx^3}{a}+1}}+\frac{dx(a+bx^3)^{10/3}(17bc-4ad)}{154b^2}+\frac{dx(a+bx^3)^{10/3}(c+dx^3)}{14b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(7/3)*(c + d*x^3)^2,x]
```

```
[Out] (d*(17*b*c - 4*a*d)*x*(a + b*x^3)^(10/3))/(154*b^2) + (d*x*(a + b*x^3)^(10/
3)*(c + d*x^3))/(14*b) + (a^2*(77*b^2*c^2 - 14*a*b*c*d + 2*a^2*d^2)*x*(a +
b*x^3)^(1/3)*Hypergeometric2F1[-7/3, 1/3, 4/3, -((b*x^3)/a)])/(77*b^2*(1 +
(b*x^3)/a)^(1/3))
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
```

```
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rubi steps

$$\int \left(a+bx^3\right)^{7/3}\left(c+dx^3\right)^2\,dx = \frac{dx(a+bx^3)^{10/3}\left(c+dx^3\right)}{14b} + \frac{\int \left(a+bx^3\right)^{7/3}\left(c(14bc-ad)+d(17bc-4ad)x^3\right)}{14b}$$

$$= \frac{d(17bc-4ad)x(a+bx^3)^{10/3}}{154b^2} + \frac{dx(a+bx^3)^{10/3}\left(c+dx^3\right)}{14b} - \frac{(ad(17bc-4ad)}{}$$

$$= \frac{d(17bc-4ad)x(a+bx^3)^{10/3}}{154b^2} + \frac{dx(a+bx^3)^{10/3}\left(c+dx^3\right)}{14b} - \frac{\left(a^2(ad(17bc-\right.}{}$$

$$= \frac{d(17bc-4ad)x(a+bx^3)^{10/3}}{154b^2} + \frac{dx(a+bx^3)^{10/3}\left(c+dx^3\right)}{14b} + \frac{a^2(77b^2c^2-14a}{}$$

Mathematica [A]
time = 10.96, size = 177, normalized size = 1.31

$$\frac{ax\sqrt[3]{a+bx^3}\left(20a(14c^2+7cdx^3+2d^2x^6)\Gamma\left(-\frac{7}{3}\right)\,{}_2F_1\left(-\frac{7}{3},\frac{1}{3};\frac{10}{3};-\frac{bx^3}{a}\right)-3bx^3(11c^2+16cdx^3+5d^2x^6)\Gamma\left(-\frac{4}{3}\right)\,{}_2F_1\left(-\frac{4}{3},\frac{4}{3};\frac{13}{3};-\frac{bx^3}{a}\right)-9bx^3(c+dx^3)^2\Gamma\left(-\frac{4}{3}\right)\,{}_3F_2\left(-\frac{4}{3},\frac{4}{3},2;1,\frac{13}{3};-\frac{bx^3}{a}\right)\right)}{280\sqrt[3]{1+\frac{bx^3}{a}}\,\Gamma\left(-\frac{7}{3}\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(7/3)*(c + d*x^3)^2,x]
```

```
[Out] (a*x*(a + b*x^3)^(1/3)*(20*a*(14*c^2 + 7*c*d*x^3 + 2*d^2*x^6)*Gamma[-7/3]*H
ypergeometric2F1[-7/3, 1/3, 10/3, -((b*x^3)/a)] - 3*b*x^3*(11*c^2 + 16*c*d*
x^3 + 5*d^2*x^6)*Gamma[-4/3]*Hypergeometric2F1[-4/3, 4/3, 13/3, -((b*x^3)/a
)] - 9*b*x^3*(c + d*x^3)^2*Gamma[-4/3]*HypergeometricPFQ[{-4/3, 4/3, 2}, {1
, 13/3}, -((b*x^3)/a)]))/(280*(1 + (b*x^3)/a)^(1/3)*Gamma[-7/3])
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \left(b\,x^{3}+a\right)^{\frac{7}{3}} \left(d\,x^{3}+c\right)^{2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(7/3)*(d*x^3+c)^2,x)
```

```
[Out] int((b*x^3+a)^(7/3)*(d*x^3+c)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(7/3)*(d*x^3+c)^2,x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(7/3)*(d*x^3 + c)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(7/3)*(d*x^3+c)^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*d^2*x^12 + 2*(b^2*c*d + a*b*d^2)*x^9 + (b^2*c^2 + 4*a*b*c*d +
 a^2*d^2)*x^6 + a^2*c^2 + 2*(a*b*c^2 + a^2*c*d)*x^3)*(b*x^3 + a)^(1/3), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 5.12, size = 418, normalized size = 3.10

$$\frac{a^{\frac{7}{3}}c^{2}x\Gamma\left(\frac{1}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)}+\frac{2a^{\frac{7}{3}}cdx^{4}\Gamma\left(\frac{4}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)}+\frac{a^{\frac{7}{3}}d^{2}x^{7}\Gamma\left(\frac{7}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{3},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{10}{3}\right)}+\frac{2a^{\frac{4}{3}}bc^{2}x^{4}\Gamma\left(\frac{4}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)}+\frac{4a^{\frac{4}{3}}bcdx^{7}\Gamma\left(\frac{7}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{3},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{10}{3}\right)}+\frac{2a^{\frac{4}{3}}bd^{2}x^{10}\Gamma\left(\frac{10}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{3},\frac{10}{3}\\ \frac{13}{3}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{13}{3}\right)}+\frac{\sqrt[3]{a}\,b^{2}c^{2}x^{7}\Gamma\left(\frac{7}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{3},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{10}{3}\right)}+\frac{2\sqrt[3]{a}\,b^{2}cdx^{10}\Gamma\left(\frac{10}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{3},\frac{10}{3}\\ \frac{13}{3}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{13}{3}\right)}+\frac{\sqrt[3]{a}\,b^{2}d^{2}x^{13}\Gamma\left(\frac{13}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{3},\frac{13}{3}\\ \frac{16}{3}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{16}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(7/3)*(d*x**3+c)**2,x)
```

```
[Out] a**(7/3)*c**2*x*gamma(1/3)*hyper((-1/3, 1/3), (4/3,), b*x**3*exp_polar(I*pi
)/a)/(3*gamma(4/3)) + 2*a**(7/3)*c*d*x**4*gamma(4/3)*hyper((-1/3, 4/3), (7/
3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + a**(7/3)*d**2*x**7*gamma(7/
3)*hyper((-1/3, 7/3), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(10/3)) +
2*a**(4/3)*b*c**2*x**4*gamma(4/3)*hyper((-1/3, 4/3), (7/3,), b*x**3*exp_pol
ar(I*pi)/a)/(3*gamma(7/3)) + 4*a**(4/3)*b*c*d*x**7*gamma(7/3)*hyper((-1/3,
```

```
7/3), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(10/3)) + 2*a**(4/3)*b*d**
2*x**10*gamma(10/3)*hyper((-1/3, 10/3), (13/3,), b*x**3*exp_polar(I*pi)/a)/
(3*gamma(13/3)) + a**(1/3)*b**2*c**2*x**7*gamma(7/3)*hyper((-1/3, 7/3), (10
/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(10/3)) + 2*a**(1/3)*b**2*c*d*x**10
*gamma(10/3)*hyper((-1/3, 10/3), (13/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamm
a(13/3)) + a**(1/3)*b**2*d**2*x**13*gamma(13/3)*hyper((-1/3, 13/3), (16/3,)
, b*x**3*exp_polar(I*pi)/a)/(3*gamma(16/3))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(7/3)*(d*x^3+c)^2,x, algorithm="giac")
[Out] integrate((b*x^3 + a)^(7/3)*(d*x^3 + c)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(b\,x^3+a\right)}^{7/3}\,{\left(d\,x^3+c\right)}^2 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(7/3)*(c + d*x^3)^2,x)
[Out] int((a + b*x^3)^(7/3)*(c + d*x^3)^2, x)
```

3.80 $\int (a+bx^3)^{4/3} (c+dx^3)^2 \, dx$

Optimal. Leaf size=133

$$\frac{d(7bc-2ad)x(a+bx^3)^{7/3}}{44b^2}+\frac{dx(a+bx^3)^{7/3}(c+dx^3)}{11b}+\frac{a(44b^2c^2-11abcd+2a^2d^2)\,x\sqrt[3]{a+bx^3}\;{}_2F_1\left(-\frac{4}{3},\frac{1}{3};\right.}{44b^2\sqrt[3]{1+\frac{bx^3}{a}}}$$

```
[Out] 1/44*d*(-2*a*d+7*b*c)*x*(b*x^3+a)^(7/3)/b^2+1/11*d*x*(b*x^3+a)^(7/3)*(d*x^3
+c)/b+1/44*a*(2*a^2*d^2-11*a*b*c*d+44*b^2*c^2)*x*(b*x^3+a)^(1/3)*hypergeom(
[-4/3, 1/3],[4/3],-b*x^3/a)/b^2/(1+b*x^3/a)^(1/3)
```

Rubi [A]
time = 0.05, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {427, 396, 252, 251}

$$\frac{ax\sqrt[3]{a+bx^3}\,(2a^2d^2-11abcd+44b^2c^2)\;{}_2F_1\left(-\frac{4}{3},\frac{1}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{44b^2\sqrt[3]{\frac{bx^3}{a}+1}}+\frac{dx(a+bx^3)^{7/3}(7bc-2ad)}{44b^2}+\frac{dx(a+bx^3)^{7/3}(c+dx^3)}{11b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(4/3)*(c + d*x^3)^2,x]
```

```
[Out] (d*(7*b*c - 2*a*d)*x*(a + b*x^3)^(7/3))/(44*b^2) + (d*x*(a + b*x^3)^(7/3)*(
c + d*x^3))/(11*b) + (a*(44*b^2*c^2 - 11*a*b*c*d + 2*a^2*d^2)*x*(a + b*x^3)
^(1/3)*Hypergeometric2F1[-4/3, 1/3, 4/3, -((b*x^3)/a)])/(44*b^2*(1 + (b*x^3
)/a)^(1/3))
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rubi steps

$$\int \left(a+bx^3\right)^{4/3} \left(c+dx^3\right)^2 \, dx = \frac{dx(a+bx^3)^{7/3}\left(c+dx^3\right)}{11b} + \frac{\int \left(a+bx^3\right)^{4/3} \left(c(11bc-ad)+2d(7bc-2ad)x^3\right)}{11b}$$

$$= \frac{d(7bc-2ad)x(a+bx^3)^{7/3}}{44b^2} + \frac{dx(a+bx^3)^{7/3}\left(c+dx^3\right)}{11b} - \frac{(2ad(7bc-2ad)-}{}$$

$$= \frac{d(7bc-2ad)x(a+bx^3)^{7/3}}{44b^2} + \frac{dx(a+bx^3)^{7/3}\left(c+dx^3\right)}{11b} - \frac{\Big(a(2ad(7bc-2ad)}{}$$

$$= \frac{d(7bc-2ad)x(a+bx^3)^{7/3}}{44b^2} + \frac{dx(a+bx^3)^{7/3}\left(c+dx^3\right)}{11b} + \frac{a(44b^2c^2-11abcd}{}$$

Mathematica [A]
time = 10.37, size = 176, normalized size = 1.32

$$\frac{x\sqrt[3]{a+bx^3}\left(20a(14c^2+7cdx^3+2d^2x^6)\Gamma\left(-\frac{4}{3}\right){}_2F_1\left(-\frac{4}{3},\frac{1}{3};\frac{10}{3};-\frac{bx^3}{a}\right)-3bx^3(11c^2+16cdx^3+5d^2x^6)\Gamma\left(-\frac{1}{3}\right){}_2F_1\left(-\frac{1}{3},\frac{4}{3};\frac{13}{3};-\frac{bx^3}{a}\right)-9bx^3(c+dx^3)^2\Gamma\left(-\frac{1}{3}\right){}_3F_2\left(-\frac{1}{3},\frac{4}{3},2;1,\frac{13}{3};-\frac{bx^3}{a}\right)\right)}{280\sqrt[3]{1+\frac{bx^3}{a}}\,\Gamma\left(-\frac{4}{3}\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(4/3)*(c + d*x^3)^2,x]
```

```
[Out] (x*(a + b*x^3)^(1/3)*(20*a*(14*c^2 + 7*c*d*x^3 + 2*d^2*x^6)*Gamma[-4/3]*Hyp
ergeometric2F1[-4/3, 1/3, 10/3, -((b*x^3)/a)] - 3*b*x^3*(11*c^2 + 16*c*d*x^
3 + 5*d^2*x^6)*Gamma[-1/3]*Hypergeometric2F1[-1/3, 4/3, 13/3, -((b*x^3)/a)]
```

```
 - 9*b*x^3*(c + d*x^3)^2*Gamma[-1/3]*HypergeometricPFQ[{-1/3, 4/3, 2}, {1,
13/3}, -((b*x^3)/a)]))/(280*(1 + (b*x^3)/a)^(1/3)*Gamma[-4/3])
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \left(b\,x^{3}+a\right)^{\frac{4}{3}} \left(d\,x^{3}+c\right)^{2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(4/3)*(d*x^3+c)^2,x)
```

```
[Out] int((b*x^3+a)^(4/3)*(d*x^3+c)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(4/3)*(d*x^3+c)^2,x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(4/3)*(d*x^3 + c)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(4/3)*(d*x^3+c)^2,x, algorithm="fricas")
```

```
[Out] integral((b*d^2*x^9 + (2*b*c*d + a*d^2)*x^6 + (b*c^2 + 2*a*c*d)*x^3 + a*c^2
)*(b*x^3 + a)^(1/3), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 3.33, size = 270, normalized size = 2.03

$$\frac{a^{\frac{4}{3}} c^{2} x \Gamma\left(\frac{1}{3}\right) {}_{2}F_{1}\left(\begin{matrix} -\frac{1}{3}, \frac{1}{3} \\ \frac{4}{3} \end{matrix}\middle| {\frac{b x^{3} e^{i \pi}}{a}} \right)}{3 \Gamma\left(\frac{4}{3}\right)} + \frac{2 a^{\frac{4}{3}} c d x^{4} \Gamma\left(\frac{4}{3}\right) {}_{2}F_{1}\left(\begin{matrix} -\frac{1}{3}, \frac{4}{3} \\ \frac{7}{3} \end{matrix}\middle| {\frac{b x^{3} e^{i \pi}}{a}} \right)}{3 \Gamma\left(\frac{7}{3}\right)} + \frac{a^{\frac{4}{3}} d^{2} x^{7} \Gamma\left(\frac{7}{3}\right) {}_{2}F_{1}\left(\begin{matrix} -\frac{1}{3}, \frac{7}{3} \\ \frac{10}{3} \end{matrix}\middle| {\frac{b x^{3} e^{i \pi}}{a}} \right)}{3 \Gamma\left(\frac{10}{3}\right)} + \frac{\sqrt[3]{a} b c^{2} x^{4} \Gamma\left(\frac{4}{3}\right) {}_{2}F_{1}\left(\begin{matrix} -\frac{1}{3}, \frac{4}{3} \\ \frac{7}{3} \end{matrix}\middle| {\frac{b x^{3} e^{i \pi}}{a}} \right)}{3 \Gamma\left(\frac{7}{3}\right)} + \frac{2 \sqrt[3]{a} b c d x^{7} \Gamma\left(\frac{7}{3}\right) {}_{2}F_{1}\left(\begin{matrix} -\frac{1}{3}, \frac{7}{3} \\ \frac{10}{3} \end{matrix}\middle| {\frac{b x^{3} e^{i \pi}}{a}} \right)}{3 \Gamma\left(\frac{10}{3}\right)} + \frac{\sqrt[3]{a} b d^{2} x^{10} \Gamma\left(\frac{10}{3}\right) {}_{2}F_{1}\left(\begin{matrix} -\frac{1}{3}, \frac{10}{3} \\ \frac{13}{3} \end{matrix}\middle| {\frac{b x^{3} e^{i \pi}}{a}} \right)}{3 \Gamma\left(\frac{13}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(4/3)*(d*x**3+c)**2,x)
```

```
[Out] a**(4/3)*c**2*x*gamma(1/3)*hyper((-1/3, 1/3), (4/3,), b*x**3*exp_polar(I*pi
)/a)/(3*gamma(4/3)) + 2*a**(4/3)*c*d*x**4*gamma(4/3)*hyper((-1/3, 4/3), (7/
3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + a**(4/3)*d**2*x**7*gamma(7/
```

```
3)*hyper((-1/3, 7/3), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(10/3)) +
a**(1/3)*b*c**2*x**4*gamma(4/3)*hyper((-1/3, 4/3), (7/3,), b*x**3*exp_polar
(I*pi)/a)/(3*gamma(7/3)) + 2*a**(1/3)*b*c*d*x**7*gamma(7/3)*hyper((-1/3, 7/
3), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(10/3)) + a**(1/3)*b*d**2*x*
*10*gamma(10/3)*hyper((-1/3, 10/3), (13/3,), b*x**3*exp_polar(I*pi)/a)/(3*g
amma(13/3))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(4/3)*(d*x^3+c)^2,x, algorithm="giac")
[Out] integrate((b*x^3 + a)^(4/3)*(d*x^3 + c)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(b\,x^3+a\right)}^{4/3}\,{\left(d\,x^3+c\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(4/3)*(c + d*x^3)^2,x)
[Out] int((a + b*x^3)^(4/3)*(c + d*x^3)^2, x)
```

3.81 $\int \sqrt[3]{a+bx^3}\,(c+dx^3)^2\,dx$

Optimal. Leaf size=131

$$\frac{d(11bc-4ad)x(a+bx^3)^{4/3}}{40b^2}+\frac{dx(a+bx^3)^{4/3}(c+dx^3)}{8b}+\frac{(10b^2c^2-4abcd+a^2d^2)\,x\sqrt[3]{a+bx^3}\,{}_2F_1\left(-\frac{1}{3},\frac{1}{3};\frac{4}{3};-\right.}{10b^2\sqrt[3]{1+\frac{bx^3}{a}}}$$

```
[Out] 1/40*d*(-4*a*d+11*b*c)*x*(b*x^3+a)^(4/3)/b^2+1/8*d*x*(b*x^3+a)^(4/3)*(d*x^3
+c)/b+1/10*(a^2*d^2-4*a*b*c*d+10*b^2*c^2)*x*(b*x^3+a)^(1/3)*hypergeom([-1/3
, 1/3],[4/3],-b*x^3/a)/b^2/(1+b*x^3/a)^(1/3)
```

Rubi [A]
time = 0.05, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {427, 396, 252, 251}

$$\frac{x\sqrt[3]{a+bx^3}\,(a^2d^2-4abcd+10b^2c^2)\,{}_2F_1\left(-\frac{1}{3},\frac{1}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{10b^2\sqrt[3]{\frac{bx^3}{a}+1}}+\frac{dx(a+bx^3)^{4/3}(11bc-4ad)}{40b^2}+\frac{dx(a+bx^3)^{4/3}(c+dx^3)}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(1/3)*(c + d*x^3)^2,x]
```

```
[Out] (d*(11*b*c - 4*a*d)*x*(a + b*x^3)^(4/3))/(40*b^2) + (d*x*(a + b*x^3)^(4/3)*
(c + d*x^3))/(8*b) + ((10*b^2*c^2 - 4*a*b*c*d + a^2*d^2)*x*(a + b*x^3)^(1/3
)*Hypergeometric2F1[-1/3, 1/3, 4/3, -((b*x^3)/a)])/(10*b^2*(1 + (b*x^3)/a)^
(1/3))
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
```

```
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rubi steps

$$\int \sqrt[3]{a+bx^3}\left(c+dx^3\right)^2\,dx = \frac{dx(a+bx^3)^{4/3}\left(c+dx^3\right)}{8b} + \frac{\int \sqrt[3]{a+bx^3}\left(c(8bc-ad)+d(11bc-4ad)x^3\right)\,dx}{8b}$$

$$= \frac{d(11bc-4ad)x(a+bx^3)^{4/3}}{40b^2} + \frac{dx(a+bx^3)^{4/3}\left(c+dx^3\right)}{8b} + \frac{(10b^2c^2-4abcd+a}{10}$$

$$= \frac{d(11bc-4ad)x(a+bx^3)^{4/3}}{40b^2} + \frac{dx(a+bx^3)^{4/3}\left(c+dx^3\right)}{8b} + \frac{\left((10b^2c^2-4abcd+\right.}{1}$$

$$= \frac{d(11bc-4ad)x(a+bx^3)^{4/3}}{40b^2} + \frac{dx(a+bx^3)^{4/3}\left(c+dx^3\right)}{8b} + \frac{(10b^2c^2-4abcd+a}{}$$

Mathematica [A]
time = 7.69, size = 179, normalized size = 1.37

$$\frac{x\sqrt[3]{a+bx^3}\left(20a(14c^2+7cdx^3+2d^2x^6)\Gamma\left(-\frac{1}{3}\right){}_2F_1\left(-\frac{1}{3},\frac{1}{3};\frac{10}{3};-\frac{bx^3}{a}\right)-3bx^3(11c^2+16cdx^3+5d^2x^6)\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\frac{2}{3},\frac{4}{3};\frac{13}{3};-\frac{bx^3}{a}\right)-9bx^3(c+dx^3)^2\Gamma\left(\frac{2}{3}\right){}_3F_2\left(\frac{2}{3},\frac{4}{3},2;1,\frac{13}{3};-\frac{bx^3}{a}\right)\right)}{280a\sqrt[3]{1+\frac{bx^3}{a}}\,\Gamma\left(-\frac{1}{3}\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(1/3)*(c + d*x^3)^2,x]
```

```
[Out] (x*(a + b*x^3)^(1/3)*(20*a*(14*c^2 + 7*c*d*x^3 + 2*d^2*x^6)*Gamma[-1/3]*Hyp
ergeometric2F1[-1/3, 1/3, 10/3, -((b*x^3)/a)] - 3*b*x^3*(11*c^2 + 16*c*d*x^
3 + 5*d^2*x^6)*Gamma[2/3]*Hypergeometric2F1[2/3, 4/3, 13/3, -((b*x^3)/a)] -
 9*b*x^3*(c + d*x^3)^2*Gamma[2/3]*HypergeometricPFQ[{2/3, 4/3, 2}, {1, 13/3
}, -((b*x^3)/a)]))/(280*a*(1 + (b*x^3)/a)^(1/3)*Gamma[-1/3])
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \left(b\,x^{3}+a\right)^{\frac{1}{3}} \left(d\,x^{3}+c\right)^{2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(1/3)*(d*x^3+c)^2,x)
```

```
[Out] int((b*x^3+a)^(1/3)*(d*x^3+c)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)*(d*x^3+c)^2,x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(1/3)*(d*x^3 + c)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)*(d*x^3+c)^2,x, algorithm="fricas")
```

```
[Out] integral((d^2*x^6 + 2*c*d*x^3 + c^2)*(b*x^3 + a)^(1/3), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.81, size = 131, normalized size = 1.00

$$\frac{\sqrt[3]{a}\, c^{2} x \Gamma\left(\frac{1}{3}\right) {}_{2}F_{1}\left(\begin{matrix} -\frac{1}{3}, \frac{1}{3} \\ \frac{4}{3} \end{matrix}\middle| {\frac{b x^{3} e^{i \pi}}{a}} \right)}{3 \Gamma\left(\frac{4}{3}\right)} + \frac{2 \sqrt[3]{a}\, c d x^{4} \Gamma\left(\frac{4}{3}\right) {}_{2}F_{1}\left(\begin{matrix} -\frac{1}{3}, \frac{4}{3} \\ \frac{7}{3} \end{matrix}\middle| {\frac{b x^{3} e^{i \pi}}{a}} \right)}{3 \Gamma\left(\frac{7}{3}\right)} + \frac{\sqrt[3]{a}\, d^{2} x^{7} \Gamma\left(\frac{7}{3}\right) {}_{2}F_{1}\left(\begin{matrix} -\frac{1}{3}, \frac{7}{3} \\ \frac{10}{3} \end{matrix}\middle| {\frac{b x^{3} e^{i \pi}}{a}} \right)}{3 \Gamma\left(\frac{10}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(1/3)*(d*x**3+c)**2,x)
```

```
[Out] a**(1/3)*c**2*x*gamma(1/3)*hyper((-1/3, 1/3), (4/3,), b*x**3*exp_polar(I*pi
)/a)/(3*gamma(4/3)) + 2*a**(1/3)*c*d*x**4*gamma(4/3)*hyper((-1/3, 4/3), (7/
3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + a**(1/3)*d**2*x**7*gamma(7/
3)*hyper((-1/3, 7/3), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(10/3))
```

<u>**Giac**</u> [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)*(d*x^3+c)^2,x, algorithm="giac")
[Out] integrate((b*x^3 + a)^(1/3)*(d*x^3 + c)^2, x)
```

<u>**Mupad**</u> [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(b\,x^3+a\right)^{1/3}\,\left(d\,x^3+c\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(1/3)*(c + d*x^3)^2,x)
[Out] int((a + b*x^3)^(1/3)*(c + d*x^3)^2, x)
```

3.82 $\int \frac{(c+dx^3)^2}{(a+bx^3)^{2/3}}\,dx$

Optimal. Leaf size=132

$$\frac{2d(2bc-ad)x\sqrt[3]{a+bx^3}}{5b^2}+\frac{dx\sqrt[3]{a+bx^3}\,(c+dx^3)}{5b}+\frac{(5b^2c^2-5abcd+2a^2d^2)\,x\left(1+\frac{bx^3}{a}\right)^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{5b^2\,(a+bx^3)^{2/3}}$$

```
[Out] 2/5*d*(-a*d+2*b*c)*x*(b*x^3+a)^(1/3)/b^2+1/5*d*x*(b*x^3+a)^(1/3)*(d*x^3+c)/
b+1/5*(2*a^2*d^2-5*a*b*c*d+5*b^2*c^2)*x*(1+b*x^3/a)^(2/3)*hypergeom([1/3, 2
/3],[4/3],-b*x^3/a)/b^2/(b*x^3+a)^(2/3)
```

Rubi [A]

time = 0.05, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {427, 396, 252, 251}

$$\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}(2a^2d^2-5abcd+5b^2c^2)\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{5b^2\,(a+bx^3)^{2/3}}+\frac{2dx\sqrt[3]{a+bx^3}\,(2bc-ad)}{5b^2}+\frac{dx\sqrt[3]{a+bx^3}\,(c+dx^3)}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)^2/(a + b*x^3)^(2/3),x]
```

```
[Out] (2*d*(2*b*c - a*d)*x*(a + b*x^3)^(1/3))/(5*b^2) + (d*x*(a + b*x^3)^(1/3)*(c
 + d*x^3))/(5*b) + ((5*b^2*c^2 - 5*a*b*c*d + 2*a^2*d^2)*x*(1 + (b*x^3)/a)^(
2/3)*Hypergeometric2F1[1/3, 2/3, 4/3, -((b*x^3)/a)])/(5*b^2*(a + b*x^3)^(2/
3))
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
```

```
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rubi steps

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{2/3}}\,dx = \frac{dx\sqrt[3]{a+bx^3}\,(c+dx^3)}{5b} + \frac{\int \frac{c(5bc-ad)+4d(2bc-ad)x^3}{(a+bx^3)^{2/3}}\,dx}{5b}$$

$$= \frac{2d(2bc-ad)x\sqrt[3]{a+bx^3}}{5b^2} + \frac{dx\sqrt[3]{a+bx^3}\,(c+dx^3)}{5b} - \frac{(4ad(2bc-ad)-2bc(5bc-ad))\ \int}{10b^2}$$

$$= \frac{2d(2bc-ad)x\sqrt[3]{a+bx^3}}{5b^2} + \frac{dx\sqrt[3]{a+bx^3}\,(c+dx^3)}{5b} - \frac{\Big((4ad(2bc-ad)-2bc(5bc-ad)}{10b^2\,(a+}$$

$$= \frac{2d(2bc-ad)x\sqrt[3]{a+bx^3}}{5b^2} + \frac{dx\sqrt[3]{a+bx^3}\,(c+dx^3)}{5b} + \frac{(5b^2c^2-5abcd+2a^2d^2)\,x\Big(1+\frac{bx^3}{a}}{5b^2\,(a+bx^3)}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 304 vs. 2(132) = 264.
time = 12.95, size = 304, normalized size = 2.30

$$-\frac{x\left(1+\frac{bx^3}{a}\right)^{2/3}\Gamma\left(\frac{4}{3}\right)\left(-3920ac^2\Gamma\left(\frac{1}{3}\right)-1960acdx^3\Gamma\left(\frac{1}{3}\right)-560ad^2x^6\Gamma\left(\frac{1}{3}\right)+3780ac^2\Gamma\left(\frac{10}{3}\right)+1890acdx^3\Gamma\left(\frac{10}{3}\right)+540ad^2x^6\Gamma\left(\frac{10}{3}\right)-270a(14c^2+7cdx^3+2d^2x^6)\Gamma\left(\frac{10}{3}\right)\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{10}{3};-\frac{bx^3}{a}\right)+297bc^2x^3\Gamma\left(\frac{10}{3}\right)\,{}_2F_1\left(\frac{4}{3},\frac{5}{3};\frac{13}{3};-\frac{bx^3}{a}\right)+432bcdx^6\Gamma\left(\frac{10}{3}\right)\,{}_2F_1\left(\frac{4}{3},\frac{5}{3};\frac{13}{3};-\frac{bx^3}{a}\right)+135bd^2x^9\Gamma\left(\frac{10}{3}\right)\,{}_2F_1\left(\frac{4}{3},\frac{5}{3};\frac{13}{3};-\frac{bx^3}{a}\right)+81bx^3(c+dx^3)^2\Gamma\left(\frac{10}{3}\right)\,{}_3F_2\left(\frac{4}{3},\frac{5}{3},2;1,\frac{13}{3};-\frac{bx^3}{a}\right)\right)}{1260a\,(a+bx^3)^{2/3}\,\Gamma\left(\frac{1}{3}\right)\Gamma\left(\frac{10}{3}\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x^3)^2/(a + b*x^3)^(2/3),x]
```

```
[Out] -1/1260*(x*(1 + (b*x^3)/a)^(2/3)*Gamma[4/3]*(-3920*a*c^2*Gamma[1/3] - 1960*
a*c*d*x^3*Gamma[1/3] - 560*a*d^2*x^6*Gamma[1/3] + 3780*a*c^2*Gamma[10/3] +
1890*a*c*d*x^3*Gamma[10/3] + 540*a*d^2*x^6*Gamma[10/3] - 270*a*(14*c^2 + 7*
c*d*x^3 + 2*d^2*x^6)*Gamma[10/3]*Hypergeometric2F1[1/3, 2/3, 10/3, -((b*x^3
)/a)] + 297*b*c^2*x^3*Gamma[10/3]*Hypergeometric2F1[4/3, 5/3, 13/3, -((b*x^
```

```
3)/a)] + 432*b*c*d*x^6*Gamma[10/3]*Hypergeometric2F1[4/3, 5/3, 13/3, -((b*x
^3)/a)] + 135*b*d^2*x^9*Gamma[10/3]*Hypergeometric2F1[4/3, 5/3, 13/3, -((b*
x^3)/a)] + 81*b*x^3*(c + d*x^3)^2*Gamma[10/3]*HypergeometricPFQ[{4/3, 5/3,
2}, {1, 13/3}, -((b*x^3)/a)]))/(a*(a + b*x^3)^(2/3)*Gamma[1/3]*Gamma[10/3])
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{\left(d\,x^{3}+c\right)^{2}}{\left(b\,x^{3}+a\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^2/(b*x^3+a)^(2/3),x)
[Out] int((d*x^3+c)^2/(b*x^3+a)^(2/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^2/(b*x^3+a)^(2/3),x, algorithm="maxima")
[Out] integrate((d*x^3 + c)^2/(b*x^3 + a)^(2/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^2/(b*x^3+a)^(2/3),x, algorithm="fricas")
[Out] integral((d^2*x^6 + 2*c*d*x^3 + c^2)/(b*x^3 + a)^(2/3), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.98, size = 126, normalized size = 0.95

$$\frac{c^{2}x\Gamma\left(\frac{1}{3}\right){}_{2}F_{1}\left(\left.\begin{matrix}\frac{1}{3},\frac{2}{3}\\\frac{4}{3}\end{matrix}\right|\frac{bx^{3}e^{i\pi}}{a}\right)}{3a^{\frac{2}{3}}\Gamma\left(\frac{4}{3}\right)}+\frac{2cdx^{4}\Gamma\left(\frac{4}{3}\right){}_{2}F_{1}\left(\left.\begin{matrix}\frac{2}{3},\frac{4}{3}\\\frac{7}{3}\end{matrix}\right|\frac{bx^{3}e^{i\pi}}{a}\right)}{3a^{\frac{2}{3}}\Gamma\left(\frac{7}{3}\right)}+\frac{d^{2}x^{7}\Gamma\left(\frac{7}{3}\right){}_{2}F_{1}\left(\left.\begin{matrix}\frac{2}{3},\frac{7}{3}\\\frac{10}{3}\end{matrix}\right|\frac{bx^{3}e^{i\pi}}{a}\right)}{3a^{\frac{2}{3}}\Gamma\left(\frac{10}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**2/(b*x**3+a)**(2/3),x)
```

```
[Out] c**2*x*gamma(1/3)*hyper((1/3, 2/3), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*a*
*(2/3)*gamma(4/3)) + 2*c*d*x**4*gamma(4/3)*hyper((2/3, 4/3), (7/3,), b*x**3
*exp_polar(I*pi)/a)/(3*a**(2/3)*gamma(7/3)) + d**2*x**7*gamma(7/3)*hyper((2
/3, 7/3), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(2/3)*gamma(10/3))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^2/(b*x^3+a)^(2/3),x, algorithm="giac")
```

```
[Out] integrate((d*x^3 + c)^2/(b*x^3 + a)^(2/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(d\,x^3+c\right)}^2}{{\left(b\,x^3+a\right)}^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^2/(a + b*x^3)^(2/3),x)
```

```
[Out] int((c + d*x^3)^2/(a + b*x^3)^(2/3), x)
```

3.83 $\int \frac{\left(c+dx^3\right)^2}{\left(a+bx^3\right)^{5/3}}\,dx$

Optimal. Leaf size=146

$$-\frac{d(bc-2ad)x\sqrt[3]{a+bx^3}}{2ab^2}+\frac{(bc-ad)x(c+dx^3)}{2ab\left(a+bx^3\right)^{2/3}}+\frac{\left(b^2c^2+2abcd-2a^2d^2\right)x\left(1+\frac{bx^3}{a}\right)^{2/3}{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{2ab^2\left(a+bx^3\right)^{2/3}}$$

```
[Out] -1/2*d*(-2*a*d+b*c)*x*(b*x^3+a)^(1/3)/a/b^2+1/2*(-a*d+b*c)*x*(d*x^3+c)/a/b/
(b*x^3+a)^(2/3)+1/2*(-2*a^2*d^2+2*a*b*c*d+b^2*c^2)*x*(1+b*x^3/a)^(2/3)*hype
rgeom([1/3, 2/3],[4/3],-b*x^3/a)/a/b^2/(b*x^3+a)^(2/3)
```

Rubi [A]
time = 0.07, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {424, 396, 252, 251}

$$\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}\left(-2a^2d^2+2abcd+b^2c^2\right){}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{2ab^2\left(a+bx^3\right)^{2/3}}-\frac{dx\sqrt[3]{a+bx^3}\,(bc-2ad)}{2ab^2}+\frac{x\left(c+dx^3\right)(bc-ad)}{2ab\left(a+bx^3\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)^2/(a + b*x^3)^(5/3),x]
```

```
[Out] -1/2*(d*(b*c - 2*a*d)*x*(a + b*x^3)^(1/3))/(a*b^2) + ((b*c - a*d)*x*(c + d*
x^3))/(2*a*b*(a + b*x^3)^(2/3)) + ((b^2*c^2 + 2*a*b*c*d - 2*a^2*d^2)*x*(1 +
 (b*x^3)/a)^(2/3)*Hypergeometric2F1[1/3, 2/3, 4/3, -((b*x^3)/a)])/(2*a*b^2*
(a + b*x^3)^(2/3))
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
```

```
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rubi steps

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{5/3}}\,dx = \frac{(bc-ad)x(c+dx^3)}{2ab\,(a+bx^3)^{2/3}} + \frac{\int \frac{c(bc+ad)-2d(bc-2ad)x^3}{(a+bx^3)^{2/3}}\,dx}{2ab}$$

$$= -\frac{d(bc-2ad)x\sqrt[3]{a+bx^3}}{2ab^2} + \frac{(bc-ad)x(c+dx^3)}{2ab\,(a+bx^3)^{2/3}} + -\frac{(-2ad(bc-2ad)-2bc(bc+ad))}{4ab^2}$$

$$= -\frac{d(bc-2ad)x\sqrt[3]{a+bx^3}}{2ab^2} + \frac{(bc-ad)x(c+dx^3)}{2ab\,(a+bx^3)^{2/3}} + -\frac{\Big((-2ad(bc-2ad)-2bc(bc+ad}{4ab^2\,(a+}$$

$$= -\frac{d(bc-2ad)x\sqrt[3]{a+bx^3}}{2ab^2} + \frac{(bc-ad)x(c+dx^3)}{2ab\,(a+bx^3)^{2/3}} + \frac{(b^2c^2+2abcd-2a^2d^2)\,x\left(1+\frac{bx^3}{a}\right)^{2}}{2ab^2\,(a+bx^3)^{2/}}$$

Mathematica [A]
time = 11.93, size = 171, normalized size = 1.17

$$\frac{x\left(1+\frac{bx^3}{a}\right)^{2/3}\Gamma\left(\frac{2}{3}\right)\left(4a(14c^2+7cdx^3+2d^2x^6)\,{}_2F_1\left(\frac{1}{3},\frac{5}{3};\frac{10}{3};-\frac{bx^3}{a}\right)-bx^3(11c^2+16cdx^3+5d^2x^6)\,{}_2F_1\left(\frac{4}{3},\frac{8}{3};\frac{13}{3};-\frac{bx^3}{a}\right)-3bx^3(c+dx^3)^2\,{}_3F_2\left(\frac{4}{3},2,\frac{8}{3};1,\frac{13}{3};-\frac{bx^3}{a}\right)\right)}{84a^2\,(a+bx^3)^{2/3}\,\Gamma\left(\frac{5}{3}\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x^3)^2/(a + b*x^3)^(5/3),x]
```

```
[Out] (x*(1 + (b*x^3)/a)^(2/3)*Gamma[2/3]*(4*a*(14*c^2 + 7*c*d*x^3 + 2*d^2*x^6)*H
ypergeometric2F1[1/3, 5/3, 10/3, -((b*x^3)/a)] - b*x^3*(11*c^2 + 16*c*d*x^3
 + 5*d^2*x^6)*Hypergeometric2F1[4/3, 8/3, 13/3, -((b*x^3)/a)] - 3*b*x^3*(c
+ d*x^3)^2*HypergeometricPFQ[{4/3, 2, 8/3}, {1, 13/3}, -((b*x^3)/a)]))/(84*
a^2*(a + b*x^3)^(2/3)*Gamma[5/3])
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{\left(d\,x^{3}+c\right)^{2}}{\left(b\,x^{3}+a\right)^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^2/(b*x^3+a)^(5/3),x)
[Out] int((d*x^3+c)^2/(b*x^3+a)^(5/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^2/(b*x^3+a)^(5/3),x, algorithm="maxima")
[Out] integrate((d*x^3 + c)^2/(b*x^3 + a)^(5/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^2/(b*x^3+a)^(5/3),x, algorithm="fricas")
[Out] integral((d^2*x^6 + 2*c*d*x^3 + c^2)*(b*x^3 + a)^(1/3)/(b^2*x^6 + 2*a*b*x^3
 + a^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c + dx^{3}\right)^{2}}{\left(a + bx^{3}\right)^{\frac{5}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**2/(b*x**3+a)**(5/3),x)
[Out] Integral((c + d*x**3)**2/(a + b*x**3)**(5/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^2/(b*x^3+a)^(5/3),x, algorithm="giac")
```

```
[Out] integrate((d*x^3 + c)^2/(b*x^3 + a)^(5/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(d\,x^3+c)^2}{(b\,x^3+a)^{5/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^2/(a + b*x^3)^(5/3),x)
```

```
[Out] int((c + d*x^3)^2/(a + b*x^3)^(5/3), x)
```

3.84 $\int \frac{(c+dx^3)^2}{(a+bx^3)^{8/3}} dx$

Optimal. Leaf size=147

$$\frac{2\left(\frac{c^2}{a^2}-\frac{d^2}{b^2}\right)x}{5\left(a+bx^3\right)^{2/3}}+\frac{(bc-ad)x(c+dx^3)}{5ab\left(a+bx^3\right)^{5/3}}+\frac{\left(2b^2c^2+abcd+2a^2d^2\right)x\left(1+\frac{bx^3}{a}\right)^{2/3}\ {}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{5a^2b^2\left(a+bx^3\right)^{2/3}}$$

```
[Out] 2/5*(c^2/a^2-d^2/b^2)*x/(b*x^3+a)^(2/3)+1/5*(-a*d+b*c)*x*(d*x^3+c)/a/b/(b*x
^3+a)^(5/3)+1/5*(2*a^2*d^2+a*b*c*d+2*b^2*c^2)*x*(1+b*x^3/a)^(2/3)*hypergeom
([1/3, 2/3],[4/3],-b*x^3/a)/a^2/b^2/(b*x^3+a)^(2/3)
```

Rubi [A]
time = 0.06, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {424, 393, 252, 251}

$$\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}\left(2a^2d^2+abcd+2b^2c^2\right)\ {}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{5a^2b^2\left(a+bx^3\right)^{2/3}}+\frac{2x\left(\frac{c^2}{a^2}-\frac{d^2}{b^2}\right)}{5\left(a+bx^3\right)^{2/3}}+\frac{x(c+dx^3)(bc-ad)}{5ab\left(a+bx^3\right)^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)^2/(a + b*x^3)^(8/3),x]
```

```
[Out] (2*(c^2/a^2 - d^2/b^2)*x)/(5*(a + b*x^3)^(2/3)) + ((b*c - a*d)*x*(c + d*x^3
))/(5*a*b*(a + b*x^3)^(5/3)) + ((2*b^2*c^2 + a*b*c*d + 2*a^2*d^2)*x*(1 + (b
*x^3)/a)^(2/3)*Hypergeometric2F1[1/3, 2/3, 4/3, -((b*x^3)/a)])/(5*a^2*b^2*(
a + b*x^3)^(2/3))
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rubi steps

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{8/3}}\,dx = \frac{(bc-ad)x(c+dx^3)}{5ab\,(a+bx^3)^{5/3}} + \frac{\int \frac{c(4bc+ad)+d(bc+4ad)x^3}{(a+bx^3)^{5/3}}\,dx}{5ab}$$

$$= \frac{2\left(\frac{c^2}{a^2}-\frac{d^2}{b^2}\right)x}{5\,(a+bx^3)^{2/3}} + \frac{(bc-ad)x(c+dx^3)}{5ab\,(a+bx^3)^{5/3}} + \frac{(2b^2c^2+abcd+2a^2d^2)\int \frac{1}{(a+bx^3)^{2/3}}\,dx}{5a^2b^2}$$

$$= \frac{2\left(\frac{c^2}{a^2}-\frac{d^2}{b^2}\right)x}{5\,(a+bx^3)^{2/3}} + \frac{(bc-ad)x(c+dx^3)}{5ab\,(a+bx^3)^{5/3}} + \frac{\left((2b^2c^2+abcd+2a^2d^2)\left(1+\frac{bx^3}{a}\right)^{2/3}\right)\int \frac{1}{\left(1+\frac{bx}{a}\right.}}{5a^2b^2\,(a+bx^3)^{2/3}}$$

$$= \frac{2\left(\frac{c^2}{a^2}-\frac{d^2}{b^2}\right)x}{5\,(a+bx^3)^{2/3}} + \frac{(bc-ad)x(c+dx^3)}{5ab\,(a+bx^3)^{5/3}} + \frac{(2b^2c^2+abcd+2a^2d^2)\,x\left(1+\frac{bx^3}{a}\right)^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\right.}{5a^2b^2\,(a+bx^3)^{2/3}}$$

Mathematica [A]
time = 13.13, size = 171, normalized size = 1.16

$$\frac{x\left(1+\frac{bx^3}{a}\right)^{2/3}\Gamma\left(\frac{2}{3}\right)\left(5a(14c^2+7cdx^3+2d^2x^6)\,{}_2F_1\left(\frac{1}{3},\frac{8}{3};\frac{10}{3};-\frac{bx^3}{a}\right)-2bx^3(11c^2+16cdx^3+5d^2x^6)\,{}_2F_1\left(\frac{4}{3},\frac{11}{3};\frac{13}{3};-\frac{bx^3}{a}\right)-6bx^3(c+dx^3)^2\,{}_3F_2\left(\frac{4}{3},2,\frac{11}{3};1,\frac{13}{3};-\frac{bx^3}{a}\right)\right)}{63a^3\,(a+bx^3)^{2/3}\,\Gamma\left(\frac{8}{3}\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x^3)^2/(a + b*x^3)^(8/3),x]
```

```
[Out] (x*(1 + (b*x^3)/a)^(2/3)*Gamma[2/3]*(5*a*(14*c^2 + 7*c*d*x^3 + 2*d^2*x^6)*H
ypergeometric2F1[1/3, 8/3, 10/3, -((b*x^3)/a)] - 2*b*x^3*(11*c^2 + 16*c*d*x
^3 + 5*d^2*x^6)*Hypergeometric2F1[4/3, 11/3, 13/3, -((b*x^3)/a)] - 6*b*x^3*
```

```
(c + d*x^3)^2*HypergeometricPFQ[{4/3, 2, 11/3}, {1, 13/3}, -((b*x^3)/a)]))/
(63*a^3*(a + b*x^3)^(2/3)*Gamma[8/3])
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{\left(d\,x^{3}+c\right)^{2}}{\left(b\,x^{3}+a\right)^{\frac{8}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^2/(b*x^3+a)^(8/3),x)
```

```
[Out] int((d*x^3+c)^2/(b*x^3+a)^(8/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^2/(b*x^3+a)^(8/3),x, algorithm="maxima")
```

```
[Out] integrate((d*x^3 + c)^2/(b*x^3 + a)^(8/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^2/(b*x^3+a)^(8/3),x, algorithm="fricas")
```

```
[Out] integral((d^2*x^6 + 2*c*d*x^3 + c^2)*(b*x^3 + a)^(1/3)/(b^3*x^9 + 3*a*b^2*x
^6 + 3*a^2*b*x^3 + a^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c + dx^{3}\right)^{2}}{\left(a + bx^{3}\right)^{\frac{8}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**2/(b*x**3+a)**(8/3),x)
```

```
[Out] Integral((c + d*x**3)**2/(a + b*x**3)**(8/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^2/(b*x^3+a)^(8/3),x, algorithm="giac")
```

```
[Out] integrate((d*x^3 + c)^2/(b*x^3 + a)^(8/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(d\,x^3+c\right)^2}{\left(b\,x^3+a\right)^{8/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^2/(a + b*x^3)^(8/3),x)
```

```
[Out] int((c + d*x^3)^2/(a + b*x^3)^(8/3), x)
```

3.85 $\int \frac{(a+bx^3)^3}{(c+dx^3)^{13/3}}\,dx$

Optimal. Leaf size=109

$$\frac{x(a+bx^3)^3}{10c\,(c+dx^3)^{10/3}}+\frac{9ax(a+bx^3)^2}{70c^2\,(c+dx^3)^{7/3}}+\frac{27a^2x(a+bx^3)}{140c^3\,(c+dx^3)^{4/3}}+\frac{81a^3x}{140c^4\sqrt[3]{c+dx^3}}$$

```
[Out] 1/10*x*(b*x^3+a)^3/c/(d*x^3+c)^(10/3)+9/70*a*x*(b*x^3+a)^2/c^2/(d*x^3+c)^(7
/3)+27/140*a^2*x*(b*x^3+a)/c^3/(d*x^3+c)^(4/3)+81/140*a^3*x/c^4/(d*x^3+c)^(
1/3)
```

Rubi [A]
time = 0.03, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {386, 197}

$$\frac{81a^3x}{140c^4\sqrt[3]{c+dx^3}}+\frac{27a^2x(a+bx^3)}{140c^3\,(c+dx^3)^{4/3}}+\frac{9ax(a+bx^3)^2}{70c^2\,(c+dx^3)^{7/3}}+\frac{x(a+bx^3)^3}{10c\,(c+dx^3)^{10/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^3/(c + d*x^3)^(13/3),x]
```

```
[Out] (x*(a + b*x^3)^3)/(10*c*(c + d*x^3)^(10/3)) + (9*a*x*(a + b*x^3)^2)/(70*c^2
*(c + d*x^3)^(7/3)) + (27*a^2*x*(a + b*x^3))/(140*c^3*(c + d*x^3)^(4/3)) +
(81*a^3*x)/(140*c^4*(c + d*x^3)^(1/3))
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 386

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] - Dist[
c*(q/(a*(p + 1))), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1,
0] && GtQ[q, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{(a+bx^3)^3}{(c+dx^3)^{13/3}}\,dx = \frac{x(a+bx^3)^3}{10c\,(c+dx^3)^{10/3}} + \frac{(9a)\int \frac{(a+bx^3)^2}{(c+dx^3)^{10/3}}\,dx}{10c}$$
$$= \frac{x(a+bx^3)^3}{10c\,(c+dx^3)^{10/3}} + \frac{9ax(a+bx^3)^2}{70c^2\,(c+dx^3)^{7/3}} + \frac{(27a^2)\int \frac{a+bx^3}{(c+dx^3)^{7/3}}\,dx}{35c^2}$$
$$= \frac{x(a+bx^3)^3}{10c\,(c+dx^3)^{10/3}} + \frac{9ax(a+bx^3)^2}{70c^2\,(c+dx^3)^{7/3}} + \frac{27a^2x(a+bx^3)}{140c^3\,(c+dx^3)^{4/3}} + \frac{(81a^3)\int \frac{1}{(c+dx^3)^{4/3}}\,dx}{140c^3}$$
$$= \frac{x(a+bx^3)^3}{10c\,(c+dx^3)^{10/3}} + \frac{9ax(a+bx^3)^2}{70c^2\,(c+dx^3)^{7/3}} + \frac{27a^2x(a+bx^3)}{140c^3\,(c+dx^3)^{4/3}} + \frac{81a^3x}{140c^4\sqrt[3]{c+dx^3}}$$

Mathematica [A]
time = 0.73, size = 120, normalized size = 1.10

$$\frac{x(14b^3c^3x^9+6ab^2c^2x^6(10c+3dx^3)+3a^2bcx^3(35c^2+30cdx^3+9d^2x^6)+a^3(140c^3+315c^2dx^3+270cd^2x^6+81d^3x^9))}{140c^4\,(c+dx^3)^{10/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^3/(c + d*x^3)^(13/3),x]
```

```
[Out] (x*(14*b^3*c^3*x^9 + 6*a*b^2*c^2*x^6*(10*c + 3*d*x^3) + 3*a^2*b*c*x^3*(35*c
^2 + 30*c*d*x^3 + 9*d^2*x^6) + a^3*(140*c^3 + 315*c^2*d*x^3 + 270*c*d^2*x^6
 + 81*d^3*x^9)))/(140*c^4*(c + d*x^3)^(10/3))
```

Maple [A]
time = 0.26, size = 134, normalized size = 1.23

method	result
gosper	$\frac{x\left(81a^3d^3x^9+27a^2bc\,d^2x^9+18a\,b^2c^2d\,x^9+14b^3c^3x^9+270a^3c\,d^2x^6+90a^2b\,c^2d\,x^6+60a\,b^2c^3x^6+315a^3c^2d\,x^3+105a^2b\,c^3x^3+140a^3c^3\right)}{140(d\,x^3+c)^{\frac{10}{3}}c^4}$
trager	$\frac{x\left(81a^3d^3x^9+27a^2bc\,d^2x^9+18a\,b^2c^2d\,x^9+14b^3c^3x^9+270a^3c\,d^2x^6+90a^2b\,c^2d\,x^6+60a\,b^2c^3x^6+315a^3c^2d\,x^3+105a^2b\,c^3x^3+140a^3c^3\right)}{140(d\,x^3+c)^{\frac{10}{3}}c^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^3/(d*x^3+c)^(13/3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/140*x*(81*a^3*d^3*x^9+27*a^2*b*c*d^2*x^9+18*a*b^2*c^2*d*x^9+14*b^3*c^3*x^
9+270*a^3*c*d^2*x^6+90*a^2*b*c^2*d*x^6+60*a*b^2*c^3*x^6+315*a^3*c^2*d*x^3+1
05*a^2*b*c^3*x^3+140*a^3*c^3)/(d*x^3+c)^(10/3)/c^4
```

Maxima [A]
time = 0.30, size = 182, normalized size = 1.67

$$\frac{b^3x^{10}}{10\,(dx^3+c)^{\frac{10}{3}}c} - \frac{3\,ab^2\left(7\,d-\frac{10\,(dx^3+c)}{x^3}\right)x^{10}}{70\,(dx^3+c)^{\frac{10}{3}}c^2} + \frac{3\left(14\,d^2-\frac{40\,(dx^3+c)d}{x^3}+\frac{35\,(dx^3+c)^2}{x^6}\right)a^2bx^{10}}{140\,(dx^3+c)^{\frac{10}{3}}c^3} - \frac{\left(14\,d^3-\frac{60\,(dx^3+c)d^2}{x^3}+\frac{105\,(dx^3+c)^2d}{x^6}-\frac{140\,(dx^3+c)^3}{x^9}\right)a^3x^{10}}{140\,(dx^3+c)^{\frac{10}{3}}c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3/(d*x^3+c)^(13/3),x, algorithm="maxima")
```

```
[Out] 1/10*b^3*x^10/((d*x^3 + c)^(10/3)*c) - 3/70*a*b^2*(7*d - 10*(d*x^3 + c)/x^3
)*x^10/((d*x^3 + c)^(10/3)*c^2) + 3/140*(14*d^2 - 40*(d*x^3 + c)*d/x^3 + 35
*(d*x^3 + c)^2/x^6)*a^2*b*x^10/((d*x^3 + c)^(10/3)*c^3) - 1/140*(14*d^3 - 6
0*(d*x^3 + c)*d^2/x^3 + 105*(d*x^3 + c)^2*d/x^6 - 140*(d*x^3 + c)^3/x^9)*a^
3*x^10/((d*x^3 + c)^(10/3)*c^4)
```

Fricas [A]
time = 2.65, size = 166, normalized size = 1.52

$$\frac{((14\,b^3c^3 + 18\,ab^2c^2d + 27\,a^2bcd^2 + 81\,a^3d^3)x^{10} + 30\,(2\,ab^2c^3 + 3\,a^2bc^2d + 9\,a^3cd^2)x^7 + 140\,a^3c^3x + 105\,(a^2bc^3 + 3\,a^3c^2d)x^4)(dx^3 + c)^{\frac{2}{3}}}{140\,(c^4d^4x^{12} + 4\,c^5d^3x^9 + 6\,c^6d^2x^6 + 4\,c^7dx^3 + c^8)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3/(d*x^3+c)^(13/3),x, algorithm="fricas")
```

```
[Out] 1/140*((14*b^3*c^3 + 18*a*b^2*c^2*d + 27*a^2*b*c*d^2 + 81*a^3*d^3)*x^10 + 3
0*(2*a*b^2*c^3 + 3*a^2*b*c^2*d + 9*a^3*c*d^2)*x^7 + 140*a^3*c^3*x + 105*(a^
2*b*c^3 + 3*a^3*c^2*d)*x^4)*(d*x^3 + c)^(2/3)/(c^4*d^4*x^12 + 4*c^5*d^3*x^9
 + 6*c^6*d^2*x^6 + 4*c^7*d*x^3 + c^8)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**3/(d*x**3+c)**(13/3),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3/(d*x^3+c)^(13/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^3/(d*x^3 + c)^(13/3), x)
```

Mupad [B]
time = 1.56, size = 271, normalized size = 2.49

$$\frac{x\,\left(\frac{a^3}{10\,c}-\frac{c\,\left(\frac{c\,\left(\frac{b^3}{10\,d}-\frac{3\,a\,b^2}{10\,c}\right)}{d}+\frac{3\,a^2\,b}{10\,c}\right)}{d}\right)}{{\left(d\,x^3+c\right)}^{10/3}}-\frac{x\,\left(\frac{b^3}{4\,d^3}-\frac{27\,a^3\,d^3+9\,a^2\,b\,c\,d^2+6\,a\,b^2\,c^2\,d-7\,b^3\,c^3}{140\,c^3\,d^3}\right)}{{\left(d\,x^3+c\right)}^{4/3}}+\frac{x\,\left(\frac{c\,\left(\frac{b^3}{7\,d^2}-\frac{b^2\,\left(3\,a\,d-b\,c\right)}{7\,c\,d^2}\right)}{d}+\frac{9\,a^3\,d^3+3\,a^2\,b\,c\,d^2-3\,a\,b^2\,c^2\,d+b^3\,c^3}{70\,c^2\,d^3}\right)}{{\left(d\,x^3+c\right)}^{7/3}}+\frac{x\,\left(81\,a^3\,d^3+27\,a^2\,b\,c\,d^2+18\,a\,b^2\,c^2\,d+14\,b^3\,c^3\right)}{140\,c^4\,d^3\,{\left(d\,x^3+c\right)}^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^3/(c + d*x^3)^(13/3),x)
```

```
[Out] (x*(a^3/(10*c) - (c*((c*(b^3/(10*d) - (3*a*b^2)/(10*c)))/d + (3*a^2*b)/(10*
c)))/d))/(c + d*x^3)^(10/3) - (x*(b^3/(4*d^3) - (27*a^3*d^3 - 7*b^3*c^3 + 6
*a*b^2*c^2*d + 9*a^2*b*c*d^2)/(140*c^3*d^3)))/(c + d*x^3)^(4/3) + (x*((c*(b
^3/(7*d^2) - (b^2*(3*a*d - b*c))/(7*c*d^2)))/d + (9*a^3*d^3 + b^3*c^3 - 3*a
*b^2*c^2*d + 3*a^2*b*c*d^2)/(70*c^2*d^3)))/(c + d*x^3)^(7/3) + (x*(81*a^3*d
^3 + 14*b^3*c^3 + 18*a*b^2*c^2*d + 27*a^2*b*c*d^2))/(140*c^4*d^3*(c + d*x^3
)^(1/3))
```

3.86 $\int \frac{(a+bx^3)^{8/3}}{c+dx^3}\,dx$

Optimal. Leaf size=331

$$-\frac{b(6bc-11ad)x(a+bx^3)^{2/3}}{18d^2}+\frac{bx(a+bx^3)^{5/3}}{6d}+\frac{b^{2/3}(9b^2c^2-24abcd+20a^2d^2)\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{9\sqrt{3}\,d^3}-\frac{(bc}{}$$

```
[Out] -1/18*b*(-11*a*d+6*b*c)*x*(b*x^3+a)^(2/3)/d^2+1/6*b*x*(b*x^3+a)^(5/3)/d-1/6
*(-a*d+b*c)^(8/3)*ln(d*x^3+c)/c^(2/3)/d^3+1/2*(-a*d+b*c)^(8/3)*ln((-a*d+b*c
)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/c^(2/3)/d^3-1/18*b^(2/3)*(20*a^2*d^2-24*
a*b*c*d+9*b^2*c^2)*ln(-b^(1/3)*x+(b*x^3+a)^(1/3))/d^3+1/27*b^(2/3)*(20*a^2*
d^2-24*a*b*c*d+9*b^2*c^2)*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2
))/d^3*3^(1/2)-1/3*(-a*d+b*c)^(8/3)*arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*x/c^(1
/3)/(b*x^3+a)^(1/3))*3^(1/2))/c^(2/3)/d^3*3^(1/2)
```

Rubi [A]
time = 0.25, antiderivative size = 331, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {427, 542, 544, 245, 384}

$$\frac{b^{2/3}\text{ArcTan}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)(20a^2d^2-24abcd+9b^2c^2)}{9\sqrt{3}\,d^3}-\frac{b^{2/3}(20a^2d^2-24abcd+9b^2c^2)\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}\,x\right)}{18d^3}-\frac{(bc-ad)^{8/3}\text{ArcTan}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\,\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,c^{2/3}d^3}-\frac{(bc-ad)^{8/3}\log(c+dx^3)}{6c^{2/3}d^3}+\frac{(bc-ad)^{8/3}\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{2/3}d^3}-\frac{bx(a+bx^3)^{2/3}(6bc-11ad)}{18d^2}+\frac{bx(a+bx^3)^{5/3}}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(8/3)/(c + d*x^3),x]
```

```
[Out] -1/18*(b*(6*b*c - 11*a*d)*x*(a + b*x^3)^(2/3))/d^2 + (b*x*(a + b*x^3)^(5/3)
)/(6*d) + (b^(2/3)*(9*b^2*c^2 - 24*a*b*c*d + 20*a^2*d^2)*ArcTan[(1 + (2*b^(
1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(9*Sqrt[3]*d^3) - ((b*c - a*d)^(8/3)*A
rcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[3]])/(
Sqrt[3]*c^(2/3)*d^3) - ((b*c - a*d)^(8/3)*Log[c + d*x^3])/(6*c^(2/3)*d^3) +
 ((b*c - a*d)^(8/3)*Log[((b*c - a*d)^(1/3)*x)/c^(1/3) - (a + b*x^3)^(1/3)])
/(2*c^(2/3)*d^3) - (b^(2/3)*(9*b^2*c^2 - 24*a*b*c*d + 20*a^2*d^2)*Log[-(b^(
1/3)*x) + (a + b*x^3)^(1/3)])/(18*d^3)
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 384

```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 544

```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Dist[f/d, Int[(a + b*x^n)^p, x], x] + Dist[(d*e -
c*f)/d, Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, p
, n}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^3)^{8/3}}{c+dx^3}\,dx &= \frac{\left(a^2(a+bx^3)^{2/3}\right)\int \frac{\left(1+\frac{bx^3}{a}\right)^{8/3}}{c+dx^3}\,dx}{\left(1+\frac{bx^3}{a}\right)^{2/3}}\\ &= \frac{a^2x(a+bx^3)^{2/3}\,F_1\left(\frac{1}{3};-\frac{8}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c\left(1+\frac{bx^3}{a}\right)^{2/3}}\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.

time = 7.97, size = 525, normalized size = 1.59

$$\frac{6bdx(a+bx^3)^{2/3}(-6bc+14ad+3bdx^3)+4\sqrt{3}b^{2/3}(9b^2c^2-24abcd+20a^2d^2)\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2\sqrt[3]{a+bx^3}}\right)+\frac{18\sqrt{-6+6i\sqrt{3}}(bc-ad)^{8/3}\tan^{-1}\left(\frac{3\sqrt[3]{bc-ad}x}{\sqrt{3}\sqrt[3]{bc-ad}x-\left(3i+\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{a+bx^3}}\right)}{c^{2/3}}-4b^{2/3}(9b^2c^2-24abcd+20a^2d^2)\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)-\frac{18i\left(-i+\sqrt{3}\right)(bc-ad)^{8/3}\log\left(2\sqrt[3]{bc-ad}x+\left(1+i\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{a+bx^3}\right)}{c^{2/3}}+2b^{2/3}(9b^2c^2-24abcd+20a^2d^2)\log\left(b^{2/3}x^2+\sqrt[3]{b}x\sqrt[3]{a+bx^3}+(a+bx^3)^{2/3}\right)+\frac{9\left(1+i\sqrt{3}\right)(bc-ad)^{8/3}\log\left(2(bc-ad)^{2/3}x^2+\left(-1-i\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{bc-ad}x\sqrt[3]{a+bx^3}+i\left(i+\sqrt{3}\right)c^{2/3}(a+bx^3)^{2/3}\right)}{c^{2/3}}}{108d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^(8/3)/(c + d*x^3),x]
```

```
[Out] (6*b*d*x*(a + b*x^3)^(2/3)*(-6*b*c + 14*a*d + 3*b*d*x^3) + 4*Sqrt[3]*b^(2/3
)*(9*b^2*c^2 - 24*a*b*c*d + 20*a^2*d^2)*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)
*x + 2*(a + b*x^3)^(1/3))] + (18*Sqrt[-6 + (6*I)*Sqrt[3]]*(b*c - a*d)^(8/3)
*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(b*c - a*d)^(1/3)*x - (3*I + Sqrt[
3])*c^(1/3)*(a + b*x^3)^(1/3))])/c^(2/3) - 4*b^(2/3)*(9*b^2*c^2 - 24*a*b*c*
d + 20*a^2*d^2)*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)] - ((18*I)*(-I + Sqrt[
3])*(b*c - a*d)^(8/3)*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*(
a + b*x^3)^(1/3)])/c^(2/3) + 2*b^(2/3)*(9*b^2*c^2 - 24*a*b*c*d + 20*a^2*d^2
)*Log[b^(2/3)*x^2 + b^(1/3)*x*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)] + (9*(
1 + I*Sqrt[3])*(b*c - a*d)^(8/3)*Log[2*(b*c - a*d)^(2/3)*x^2 + (-1 - I*Sqrt
[3])*c^(1/3)*(b*c - a*d)^(1/3)*x*(a + b*x^3)^(1/3) + I*(I + Sqrt[3])*c^(2/3
)*(a + b*x^3)^(2/3)])/c^(2/3))/(108*d^3)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^{3}+a\right)^{\frac{8}{3}}}{d\,x^{3}+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(8/3)/(d*x^3+c),x)
```

```
[Out] int((b*x^3+a)^(8/3)/(d*x^3+c),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(8/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(8/3)/(d*x^3 + c), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 643 vs. 2(273) = 546.
time = 32.03, size = 643, normalized size = 1.94

$$-\frac{18\sqrt{3}(b^2c^2-2abcd+a^2d^2)\left(\frac{b^2c^2-2abcd+a^2d^2}{c^2}\right)^{\frac{1}{3}}\arctan\left(-\frac{\sqrt{3}(bc-ad)x+2\sqrt{3}(bx^3+a)^{\frac{1}{3}}c\left(\frac{b^2c^2-2abcd+a^2d^2}{c^2}\right)^{\frac{2}{3}}}{3(bc-ad)x}\right)+2\sqrt{3}(9b^2c^2-24abcd+20a^2d^2)(-b^2)^{\frac{1}{3}}\arctan\left(-\frac{\sqrt{3}bx-2\sqrt{3}(bx^3+a)^{\frac{1}{3}}(-b^2)^{\frac{1}{3}}}{3bx}\right)-18(b^2c^2-2abcd+a^2d^2)\left(\frac{b^2c^2-2abcd+a^2d^2}{c^2}\right)^{\frac{1}{3}}\log\left(\frac{cx\left(\frac{b^2c^2-2abcd+a^2d^2}{c^2}\right)^{\frac{2}{3}}-(bx^3+a)^{\frac{1}{3}}(bc-ad)}{x}\right)-2(9b^2c^2-24abcd+20a^2d^2)(-b^2)^{\frac{1}{3}}\log\left(-\frac{(-b^2)^{\frac{2}{3}}x-(bx^3+a)^{\frac{1}{3}}b}{x}\right)+(9b^2c^2-24abcd+20a^2d^2)(-b^2)^{\frac{1}{3}}\log\left(-\frac{(-b^2)^{\frac{1}{3}}bx^2-(bx^3+a)^{\frac{1}{3}}(-b^2)^{\frac{2}{3}}x-(bx^3+a)^{\frac{2}{3}}b}{x^2}\right)+9(b^2c^2-2abcd+a^2d^2)\left(\frac{b^2c^2-2abcd+a^2d^2}{c^2}\right)^{\frac{1}{3}}\log\left(-\frac{(bc-ad)x^2\left(\frac{b^2c^2-2abcd+a^2d^2}{c^2}\right)^{\frac{1}{3}}+(bx^3+a)^{\frac{1}{3}}cx\left(\frac{b^2c^2-2abcd+a^2d^2}{c^2}\right)^{\frac{2}{3}}+(bx^3+a)^{\frac{2}{3}}(bc-ad)}{x^2}\right)-3(3b^2d^2x^4-2(3b^2cd-7abd^2)x)(bx^3+a)^{\frac{2}{3}}}{54d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(8/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] -1/54*(18*sqrt(3)*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*((b^2*c^2 - 2*a*b*c*d + a
^2*d^2)/c^2)^(1/3)*arctan(-1/3*(sqrt(3)*(b*c - a*d)*x + 2*sqrt(3)*(b*x^3 +
a)^(1/3)*c*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(1/3))/((b*c - a*d)*x)) +
2*sqrt(3)*(9*b^2*c^2 - 24*a*b*c*d + 20*a^2*d^2)*(-b^2)^(1/3)*arctan(-1/3*(s
qrt(3)*b*x - 2*sqrt(3)*(b*x^3 + a)^(1/3)*(-b^2)^(1/3))/(b*x)) - 18*(b^2*c^2
 - 2*a*b*c*d + a^2*d^2)*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(1/3)*log((c*
x*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(2/3) - (b*x^3 + a)^(1/3)*(b*c - a*
d))/x) - 2*(9*b^2*c^2 - 24*a*b*c*d + 20*a^2*d^2)*(-b^2)^(1/3)*log(-((-b^2)^
(2/3)*x - (b*x^3 + a)^(1/3)*b)/x) + (9*b^2*c^2 - 24*a*b*c*d + 20*a^2*d^2)*(
-b^2)^(1/3)*log(-((-b^2)^(1/3)*b*x^2 - (b*x^3 + a)^(1/3)*(-b^2)^(2/3)*x - (
b*x^3 + a)^(2/3)*b)/x^2) + 9*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*((b^2*c^2 - 2*
a*b*c*d + a^2*d^2)/c^2)^(1/3)*log(-((b*c - a*d)*x^2*((b^2*c^2 - 2*a*b*c*d +
 a^2*d^2)/c^2)^(1/3) + (b*x^3 + a)^(1/3)*c*x*((b^2*c^2 - 2*a*b*c*d + a^2*d^
2)/c^2)^(2/3) + (b*x^3 + a)^(2/3)*(b*c - a*d))/x^2) - 3*(3*b^2*d^2*x^4 - 2*
(3*b^2*c*d - 7*a*b*d^2)*x)*(b*x^3 + a)^(2/3))/d^3
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + bx^{3}\right)^{\frac{8}{3}}}{c + dx^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(8/3)/(d*x**3+c),x)
```

```
[Out] Integral((a + b*x**3)**(8/3)/(c + d*x**3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(8/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(8/3)/(d*x^3 + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(b\,x^3+a\right)}^{8/3}}{d\,x^3+c} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(8/3)/(c + d*x^3),x)
```

```
[Out] int((a + b*x^3)^(8/3)/(c + d*x^3), x)
```

3.87 $\int \frac{(a+bx^3)^{5/3}}{c+dx^3}\,dx$

Optimal. Leaf size=273

$$\frac{bx(a+bx^3)^{2/3}}{3d}-\frac{b^{2/3}(3bc-5ad)\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}\,d^2}+\frac{(bc-ad)^{5/3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc-ad}\,x}{\sqrt[3]{c}\,\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,c^{2/3}d^2}+\frac{(bc-}{}$$

```
[Out] 1/3*b*x*(b*x^3+a)^(2/3)/d+1/6*(-a*d+b*c)^(5/3)*ln(d*x^3+c)/c^(2/3)/d^2-1/2*
(-a*d+b*c)^(5/3)*ln((-a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/c^(2/3)/d^2
+1/6*b^(2/3)*(-5*a*d+3*b*c)*ln(-b^(1/3)*x+(b*x^3+a)^(1/3))/d^2-1/9*b^(2/3)*
(-5*a*d+3*b*c)*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/d^2*3^(1
/2)+1/3*(-a*d+b*c)^(5/3)*arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+
a)^(1/3))*3^(1/2))/c^(2/3)/d^2*3^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 273, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {427, 544, 245, 384}

$$-\frac{b^{2/3}\text{ArcTan}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)(3bc-5ad)}{3\sqrt{3}\,d^2}+\frac{(bc-ad)^{5/3}\text{ArcTan}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\,\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,c^{2/3}d^2}+\frac{b^{2/3}(3bc-5ad)\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}\,x\right)}{6d^2}+\frac{(bc-ad)^{5/3}\log\left(c+dx^3\right)}{6c^{2/3}d^2}-\frac{(bc-ad)^{5/3}\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{2/3}d^2}+\frac{bx(a+bx^3)^{2/3}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(5/3)/(c + d*x^3),x]
```

```
[Out] (b*x*(a + b*x^3)^(2/3))/(3*d) - (b^(2/3)*(3*b*c - 5*a*d)*ArcTan[(1 + (2*b^(
1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(3*Sqrt[3]*d^2) + ((b*c - a*d)^(5/3)*A
rcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[3]])/(
Sqrt[3]*c^(2/3)*d^2) + ((b*c - a*d)^(5/3)*Log[c + d*x^3])/(6*c^(2/3)*d^2) -
 ((b*c - a*d)^(5/3)*Log[((b*c - a*d)^(1/3)*x)/c^(1/3) - (a + b*x^3)^(1/3)])
/(2*c^(2/3)*d^2) + (b^(2/3)*(3*b*c - 5*a*d)*Log[-(b^(1/3)*x) + (a + b*x^3)^
(1/3)])/(6*d^2)
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 384

```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
```

```
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 544

```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Dist[f/d, Int[(a + b*x^n)^p, x], x] + Dist[(d*e -
c*f)/d, Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, p
, n}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^3)^{5/3}}{c+dx^3}\,dx &= \frac{\left(a(a+bx^3)^{2/3}\right)\int \frac{\left(1+\frac{bx^3}{a}\right)^{5/3}}{c+dx^3}\,dx}{\left(1+\frac{bx^3}{a}\right)^{2/3}}\\ &= \frac{ax(a+bx^3)^{2/3}\,F_1\left(\frac{1}{3};-\frac{5}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c\left(1+\frac{bx^3}{a}\right)^{2/3}}\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 4.47, size = 467, normalized size = 1.71

$$\frac{12bdx(a+bx^3)^{2/3}-4\sqrt{3}\,b^{2/3}(3bc-5ad)\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{b}\,x}{\sqrt[3]{b}\,x+2\sqrt[3]{a+bx^3}}\right)-\frac{6\sqrt{-6+6i\sqrt{3}}\,(bc-ad)^{5/3}\tan^{-1}\left(\frac{3\sqrt[3]{bc-ad}\,x}{\sqrt{3}\sqrt[3]{bc-ad}\,x-\left(3i+\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{a+bx^3}}\right)}{c^{2/3}}+4b^{2/3}(3bc-5ad)\log\left(-\sqrt[3]{b}\,x+\sqrt[3]{a+bx^3}\right)+\frac{6\left(1+i\sqrt{3}\right)(bc-ad)^{5/3}\log\left(2\sqrt[3]{bc-ad}\,x+\left(1+i\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{a+bx^3}\right)}{c^{2/3}}-2b^{2/3}(3bc-5ad)\log\left(b^{2/3}x^2+\sqrt[3]{b}\,x\sqrt[3]{a+bx^3}+(a+bx^3)^{2/3}\right)-\frac{3i\left(-i+\sqrt{3}\right)(bc-ad)^{5/3}\log\left(2(bc-ad)^{2/3}x^2+\left(-1-i\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{bc-ad}\,x\sqrt[3]{a+bx^3}+i\left(i+\sqrt{3}\right)c^{2/3}(a+bx^3)^{2/3}\right)}{c^{2/3}}}{36d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^(5/3)/(c + d*x^3),x]
```

```
[Out] (12*b*d*x*(a + b*x^3)^(2/3) - 4*Sqrt[3]*b^(2/3)*(3*b*c - 5*a*d)*ArcTan[(Sqr
t[3]*b^(1/3)*x)/(b^(1/3)*x + 2*(a + b*x^3)^(1/3))] - (6*Sqrt[-6 + (6*I)*Sqr
t[3]]*(b*c - a*d)^(5/3)*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(b*c - a*d)
```

```
^(1/3)*x - (3*I + Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3))])/c^(2/3) + 4*b^(2/3)
*(3*b*c - 5*a*d)*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)] + (6*(1 + I*Sqrt[3])
*(b*c - a*d)^(5/3)*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*(a +
 b*x^3)^(1/3)])/c^(2/3) - 2*b^(2/3)*(3*b*c - 5*a*d)*Log[b^(2/3)*x^2 + b^(1/
3)*x*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)] - ((3*I)*(-I + Sqrt[3])*(b*c -
a*d)^(5/3)*Log[2*(b*c - a*d)^(2/3)*x^2 + (-1 - I*Sqrt[3])*c^(1/3)*(b*c - a*
d)^(1/3)*x*(a + b*x^3)^(1/3) + I*(I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)])/
c^(2/3))/(36*d^2)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^{3}+a\right)^{\frac{5}{3}}}{d\,x^{3}+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(5/3)/(d*x^3+c),x)
```

```
[Out] int((b*x^3+a)^(5/3)/(d*x^3+c),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(5/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(5/3)/(d*x^3 + c), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 535 vs. $2(220) = 440$.
time = 6.33, size = 535, normalized size = 1.96

$$\frac{6\,(bx^3+a)^{\frac{2}{3}}bdx + 6\sqrt{3}\,(bc-ad)\left(\frac{b^2c^2-2\,abcd+a^2d^2}{c^2}\right)^{\frac{1}{3}}\arctan\left(-\frac{\sqrt{3}\,(bc-ad)x+2\sqrt{3}\,(bx^3+a)^{\frac{1}{3}}c\left(\frac{b^2c^2-2\,abcd+a^2d^2}{c^2}\right)^{\frac{1}{3}}}{3\,(bc-ad)x}\right) + 2\sqrt{3}\,(-b^2)^{\frac{1}{3}}\,(3\,bc-5\,ad)\arctan\left(-\frac{\sqrt{3}\,bx-2\sqrt{3}\,(bx^3+a)^{\frac{1}{3}}(-b^2)^{\frac{1}{3}}}{3\,bx}\right) - 6\,(bc-ad)\left(\frac{b^2c^2-2\,abcd+a^2d^2}{c^2}\right)^{\frac{1}{3}}\log\left(\frac{cx\left(\frac{b^2c^2-2\,abcd+a^2d^2}{c^2}\right)^{\frac{2}{3}}-(bx^3+a)^{\frac{1}{3}}(bc-ad)}{x}\right) - 2\,(-b^2)^{\frac{1}{3}}\,(3\,bc-5\,ad)\log\left(-\frac{(-b^2)^{\frac{2}{3}}x-(bx^3+a)^{\frac{1}{3}}b}{x}\right) + (-b^2)^{\frac{1}{3}}\,(3\,bc-5\,ad)\log\left(-\frac{(-b^2)^{\frac{1}{3}}bx^2-(bx^3+a)^{\frac{1}{3}}(-b^2)^{\frac{2}{3}}x-(bx^3+a)^{\frac{2}{3}}b}{x^2}\right) + 3\,(bc-ad)\left(\frac{b^2c^2-2\,abcd+a^2d^2}{c^2}\right)^{\frac{1}{3}}\log\left(-\frac{(bc-ad)x^2\left(\frac{b^2c^2-2\,abcd+a^2d^2}{c^2}\right)^{\frac{1}{3}}+(bx^3+a)^{\frac{1}{3}}cx\left(\frac{b^2c^2-2\,abcd+a^2d^2}{c^2}\right)^{\frac{2}{3}}+(bx^3+a)^{\frac{2}{3}}(bc-ad)}{x^2}\right)}{18\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(5/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] 1/18*(6*(b*x^3 + a)^(2/3)*b*d*x + 6*sqrt(3)*(b*c - a*d)*((b^2*c^2 - 2*a*b*c
*d + a^2*d^2)/c^2)^(1/3)*arctan(-1/3*(sqrt(3)*(b*c - a*d)*x + 2*sqrt(3)*(b*
x^3 + a)^(1/3)*c*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(1/3))/((b*c - a*d)*
x)) + 2*sqrt(3)*(-b^2)^(1/3)*(3*b*c - 5*a*d)*arctan(-1/3*(sqrt(3)*b*x - 2*s
qrt(3)*(b*x^3 + a)^(1/3)*(-b^2)^(1/3))/(b*x)) - 6*(b*c - a*d)*((b^2*c^2 - 2
*a*b*c*d + a^2*d^2)/c^2)^(1/3)*log((c*x*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^
2)^(2/3) - (b*x^3 + a)^(1/3)*(b*c - a*d))/x) - 2*(-b^2)^(1/3)*(3*b*c - 5*a*
```

```
d)*log(-((-b^2)^(2/3)*x - (b*x^3 + a)^(1/3)*b)/x) + (-b^2)^(1/3)*(3*b*c - 5
*a*d)*log(-((-b^2)^(1/3)*b*x^2 - (b*x^3 + a)^(1/3)*(-b^2)^(2/3)*x - (b*x^3
+ a)^(2/3)*b)/x^2) + 3*(b*c - a*d)*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(1
/3)*log(-((b*c - a*d)*x^2*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(1/3) + (b*
x^3 + a)^(1/3)*c*x*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(2/3) + (b*x^3 + a
)^(2/3)*(b*c - a*d))/x^2))/d^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx^3)^{\frac{5}{3}}}{c+dx^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(5/3)/(d*x**3+c),x)
[Out] Integral((a + b*x**3)**(5/3)/(c + d*x**3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(5/3)/(d*x^3+c),x, algorithm="giac")
[Out] integrate((b*x^3 + a)^(5/3)/(d*x^3 + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(b\,x^3+a\right)}^{5/3}}{d\,x^3+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(5/3)/(c + d*x^3),x)
[Out] int((a + b*x^3)^(5/3)/(c + d*x^3), x)
```

3.88 $\int \frac{\left(a+bx^3\right)^{2/3}}{c+dx^3}\,dx$

Optimal. Leaf size=233

$$\frac{b^{2/3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,d}-\frac{(bc-ad)^{2/3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc-ad}\,x}{\sqrt[3]{c}\,\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,c^{2/3}d}-\frac{(bc-ad)^{2/3}\log\left(c+dx^3\right)}{6c^{2/3}d}+\frac{(bc-a\ldots}{}$$

```
[Out] -1/6*(-a*d+b*c)^(2/3)*ln(d*x^3+c)/c^(2/3)/d+1/2*(-a*d+b*c)^(2/3)*ln((-a*d+b
*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/c^(2/3)/d-1/2*b^(2/3)*ln(-b^(1/3)*x+(b
*x^3+a)^(1/3))/d+1/3*b^(2/3)*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(
1/2))/d*3^(1/2)-1/3*(-a*d+b*c)^(2/3)*arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*x/c^(
1/3)/(b*x^3+a)^(1/3))*3^(1/2))/c^(2/3)/d*3^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 233, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {399, 245, 384}

$$\frac{b^{2/3}\text{ArcTan}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,d}-\frac{(bc-ad)^{2/3}\text{ArcTan}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\,\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,c^{2/3}d}-\frac{b^{2/3}\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}\,x\right)}{2d}-\frac{(bc-ad)^{2/3}\log\left(c+dx^3\right)}{6c^{2/3}d}+\frac{(bc-ad)^{2/3}\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{2/3}d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(2/3)/(c + d*x^3),x]
```

```
[Out] (b^(2/3)*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(Sqrt[3]*d)
 - ((b*c - a*d)^(2/3)*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a + b*x
^3)^(1/3)))/Sqrt[3]])/(Sqrt[3]*c^(2/3)*d) - ((b*c - a*d)^(2/3)*Log[c + d*x^
3])/(6*c^(2/3)*d) + ((b*c - a*d)^(2/3)*Log[((b*c - a*d)^(1/3)*x)/c^(1/3) -
(a + b*x^3)^(1/3)])/(2*c^(2/3)*d) - (b^(2/3)*Log[-(b^(1/3)*x) + (a + b*x^3)
^(1/3)])/(2*d)
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 384

```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
```

```
+ Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0]
```

Rule 399

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Di
st[b/d, Int[(a + b*x^n)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^
n)^(p - 1)/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*
d, 0] && EqQ[n*(p - 1) + 1, 0] && IntegerQ[n]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a+bx^3)^{2/3}}{c+dx^3}\,dx &= \frac{(a+bx^3)^{2/3}\int \frac{\left(1+\frac{bx^3}{a}\right)^{2/3}}{c+dx^3}\,dx}{\left(1+\frac{bx^3}{a}\right)^{2/3}} \\
&= \frac{x(a+bx^3)^{2/3}\,F_1\left(\frac{1}{3};-\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c\left(1+\frac{bx^3}{a}\right)^{2/3}}
\end{aligned}
$$

Mathematica [C] Result contains complex when optimal does not.
time = 2.80, size = 423, normalized size = 1.82

$$
\frac{4\sqrt{3}\,b^{2/3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{b}\,x}{\sqrt[3]{b}\,x+2\sqrt[3]{a+bx^3}}\right)+\frac{2\sqrt{-6+6i\sqrt{3}}\,(bc-ad)^{2/3}\tan^{-1}\left(\frac{3\sqrt[3]{bc-ad}\,x}{\sqrt{3}\sqrt[3]{bc-ad}\,x-\left(3i+\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{a+bx^3}}\right)}{c^{2/3}}-4b^{2/3}\log\left(-\sqrt[3]{b}\,x+\sqrt[3]{a+bx^3}\right)-\frac{2i\left(-i+\sqrt{3}\right)(bc-ad)^{2/3}\log\left(2\sqrt[3]{bc-ad}\,x+\left(1+i\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{a+bx^3}\right)}{c^{2/3}}+2b^{2/3}\log\left(b^{2/3}x^2+\sqrt[3]{b}\,x\sqrt[3]{a+bx^3}+(a+bx^3)^{2/3}\right)+\frac{\left(1+i\sqrt{3}\right)(bc-ad)^{2/3}\log\left(2(bc-ad)^{2/3}x^2+\left(-1-i\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{bc-ad}\,x\sqrt[3]{a+bx^3}+i\left(i+\sqrt{3}\right)c^{2/3}(a+bx^3)^{2/3}\right)}{c^{2/3}}}{12d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^(2/3)/(c + d*x^3),x]
```

```
[Out] (4*Sqrt[3]*b^(2/3)*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2*(a + b*x^3)^(1
/3))] + (2*Sqrt[-6 + (6*I)*Sqrt[3]]*(b*c - a*d)^(2/3)*ArcTan[(3*(b*c - a*d)
^(1/3)*x)/(Sqrt[3]*(b*c - a*d)^(1/3)*x - (3*I + Sqrt[3])*c^(1/3)*(a + b*x^3
)^(1/3))])/c^(2/3) - 4*b^(2/3)*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)] - ((2*
I)*(-I + Sqrt[3])*(b*c - a*d)^(2/3)*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt
[3])*c^(1/3)*(a + b*x^3)^(1/3)])/c^(2/3) + 2*b^(2/3)*Log[b^(2/3)*x^2 + b^(1
/3)*x*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)] + ((1 + I*Sqrt[3])*(b*c - a*d)
^(2/3)*Log[2*(b*c - a*d)^(2/3)*x^2 + (-1 - I*Sqrt[3])*c^(1/3)*(b*c - a*d)^(
1/3)*x*(a + b*x^3)^(1/3) + I*(I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)])/c^(2
/3))/(12*d)
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$
\int \frac{(b\,x^3+a)^{\frac{2}{3}}}{d\,x^3+c}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(2/3)/(d*x^3+c),x)
```

```
[Out] int((b*x^3+a)^(2/3)/(d*x^3+c),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(2/3)/(d*x^3 + c), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 469 vs. $2(186) = 372$.
time = 2.91, size = 469, normalized size = 2.01

$$-\frac{2\sqrt{3}\left(\frac{b^2c^2-2abcd+a^2d^2}{c^2}\right)^{\frac{1}{3}}\arctan\left(-\frac{\sqrt{3}(bc-ad)x+2\sqrt{3}(bx^3+a)^{\frac{1}{3}}c\left(\frac{b^2c^2-2abcd+a^2d^2}{c^2}\right)^{\frac{1}{3}}}{3(bc-ad)x}\right)+2\sqrt{3}(-b^2)^{\frac{1}{3}}\arctan\left(-\frac{\sqrt{3}bx-2\sqrt{3}(bx^3+a)^{\frac{1}{3}}(-b^2)^{\frac{1}{3}}}{3bx}\right)-2\left(\frac{b^2c^2-2abcd+a^2d^2}{c^2}\right)^{\frac{1}{3}}\log\left(\frac{cx\left(\frac{b^2c^2-2abcd+a^2d^2}{c^2}\right)^{\frac{2}{3}}-(bx^3+a)^{\frac{1}{3}}(bc-ad)}{x}\right)-2(-b^2)^{\frac{1}{3}}\log\left(-\frac{(-b^2)^{\frac{2}{3}}x-(bx^3+a)^{\frac{1}{3}}b}{x}\right)+(-b^2)^{\frac{1}{3}}\log\left(-\frac{(-b^2)^{\frac{1}{3}}bx^2-(bx^3+a)^{\frac{1}{3}}(-b^2)^{\frac{2}{3}}x-(bx^3+a)^{\frac{2}{3}}b}{x^2}\right)+\left(\frac{b^2c^2-2abcd+a^2d^2}{c^2}\right)^{\frac{1}{3}}\log\left(-\frac{(bc-ad)x^2\left(\frac{b^2c^2-2abcd+a^2d^2}{c^2}\right)^{\frac{1}{3}}+(bx^3+a)^{\frac{1}{3}}cx\left(\frac{b^2c^2-2abcd+a^2d^2}{c^2}\right)^{\frac{2}{3}}+(bx^3+a)^{\frac{2}{3}}(bc-ad)}{x^2}\right)}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] -1/6*(2*sqrt(3)*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(1/3)*arctan(-1/3*(sq
rt(3)*(b*c - a*d)*x + 2*sqrt(3)*(b*x^3 + a)^(1/3)*c*((b^2*c^2 - 2*a*b*c*d +
 a^2*d^2)/c^2)^(1/3))/((b*c - a*d)*x)) + 2*sqrt(3)*(-b^2)^(1/3)*arctan(-1/3
*(sqrt(3)*b*x - 2*sqrt(3)*(b*x^3 + a)^(1/3)*(-b^2)^(1/3))/(b*x)) - 2*((b^2*
c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(1/3)*log((c*x*((b^2*c^2 - 2*a*b*c*d + a^2*
d^2)/c^2)^(2/3) - (b*x^3 + a)^(1/3)*(b*c - a*d))/x) - 2*(-b^2)^(1/3)*log(-(
(-b^2)^(2/3)*x - (b*x^3 + a)^(1/3)*b)/x) + (-b^2)^(1/3)*log(-((-b^2)^(1/3)*
b*x^2 - (b*x^3 + a)^(1/3)*(-b^2)^(2/3)*x - (b*x^3 + a)^(2/3)*b)/x^2) + ((b^
2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(1/3)*log(-((b*c - a*d)*x^2*((b^2*c^2 - 2
*a*b*c*d + a^2*d^2)/c^2)^(1/3) + (b*x^3 + a)^(1/3)*c*x*((b^2*c^2 - 2*a*b*c*
d + a^2*d^2)/c^2)^(2/3) + (b*x^3 + a)^(2/3)*(b*c - a*d))/x^2))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx^3)^{\frac{2}{3}}}{c+dx^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(2/3)/(d*x**3+c),x)
```

```
[Out] Integral((a + b*x**3)**(2/3)/(c + d*x**3), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
[Out] integrate((b*x^3 + a)^(2/3)/(d*x^3 + c), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(b\,x^3+a\right)^{2/3}}{d\,x^3+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(2/3)/(c + d*x^3),x)
[Out] int((a + b*x^3)^(2/3)/(c + d*x^3), x)
```

3.89 $\int \frac{1}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$

Optimal. Leaf size=148

$$\frac{\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc-ad}\,x}{\sqrt[3]{c}\,\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,c^{2/3}\sqrt[3]{bc-ad}}+\frac{\log\left(c+dx^3\right)}{6c^{2/3}\sqrt[3]{bc-ad}}-\frac{\log\left(\frac{\sqrt[3]{bc-ad}\,x}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{2/3}\sqrt[3]{bc-ad}}$$

```
[Out] 1/6*ln(d*x^3+c)/c^(2/3)/(-a*d+b*c)^(1/3)-1/2*ln((-a*d+b*c)^(1/3)*x/c^(1/3)-
(b*x^3+a)^(1/3))/c^(2/3)/(-a*d+b*c)^(1/3)+1/3*arctan(1/3*(1+2*(-a*d+b*c)^(1
/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))/c^(2/3)/(-a*d+b*c)^(1/3)*3^(1/2)
```

Rubi [A]

time = 0.02, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {384}

$$\frac{\text{ArcTan}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\,\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,c^{2/3}\sqrt[3]{bc-ad}}+\frac{\log\left(c+dx^3\right)}{6c^{2/3}\sqrt[3]{bc-ad}}-\frac{\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{2/3}\sqrt[3]{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[3]]/(
Sqrt[3]*c^(2/3)*(b*c - a*d)^(1/3)) + Log[c + d*x^3]/(6*c^(2/3)*(b*c - a*d)^
(1/3)) - Log[((b*c - a*d)^(1/3)*x)/c^(1/3) - (a + b*x^3)^(1/3)]/(2*c^(2/3)*
(b*c - a*d)^(1/3))
```

Rule 384

```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \text{Subst}\left(\int \frac{1}{c-(bc-ad)x^3}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)$$

$$= \frac{\text{Subst}\left(\int \frac{1}{\sqrt[3]{c}-\sqrt[3]{bc-ad}\,x}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{3c^{2/3}} + \frac{\text{Subst}\left(\int \frac{2\sqrt[3]{c}+\sqrt[3]{bc-\cdots}}{c^{2/3}+\sqrt[3]{c}\,\sqrt[3]{bc-ad}\,x+\cdots}\cdots\right)}{3c^{2\cdots}}$$

$$= -\frac{\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}\,x}{\sqrt[3]{a+bx^3}}\right)}{3c^{2/3}\sqrt[3]{bc-ad}} + \frac{\text{Subst}\left(\int \frac{1}{c^{2/3}+\sqrt[3]{c}\,\sqrt[3]{bc-ad}\,x+(bc-ad)^{2/3}x^2}\,dx, x, \cdots\right)}{2\sqrt[3]{c}}$$

$$= -\frac{\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}\,x}{\sqrt[3]{a+bx^3}}\right)}{3c^{2/3}\sqrt[3]{bc-ad}} + \frac{\log\left(c^{2/3}+\frac{(bc-ad)^{2/3}x^2}{(a+bx^3)^{2/3}}+\frac{\sqrt[3]{c}\,\sqrt[3]{bc-ad}\,x}{\sqrt[3]{a+bx^3}}\right)}{6c^{2/3}\sqrt[3]{bc-ad}} - \cdots$$

$$= \frac{\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc-ad}\,x}{\sqrt[3]{c}\,\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,c^{2/3}\sqrt[3]{bc-ad}} - \frac{\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}\,x}{\sqrt[3]{a+bx^3}}\right)}{3c^{2/3}\sqrt[3]{bc-ad}} + \frac{\log\left(c^{2/3}+\frac{(bc-ad)^{2\cdots}}{(a+bx^3)\cdots}\cdots\right)}{6c^{2/3}\cdots}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.27, size = 255, normalized size = 1.72

$$\frac{-2\sqrt{-6+6i\sqrt{3}}\,\tan^{-1}\left(\frac{3\sqrt[3]{bc-ad}\,x}{\sqrt{3}\,\sqrt[3]{bc-ad}\,x-\left(3i+\sqrt{3}\right)\sqrt[3]{c}\,\sqrt[3]{a+bx^3}}\right)+\left(1+i\sqrt{3}\right)\left(2\log\left(2\sqrt[3]{bc-ad}\,x+\left(1+i\sqrt{3}\right)\sqrt[3]{c}\,\sqrt[3]{a+bx^3}\right)-\log\left(2(bc-ad)^{2/3}x^2+\left(-1-i\sqrt{3}\right)\sqrt[3]{c}\,\sqrt[3]{bc-ad}\,x\sqrt[3]{a+bx^3}+i\left(i+\sqrt{3}\right)c^{2/3}(a+bx^3)^{2/3}\right)\right)}{12c^{2/3}\sqrt[3]{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] (-2*Sqrt[-6 + (6*I)*Sqrt[3]]*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(b*c -
 a*d)^(1/3)*x - (3*I + Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3))] + (1 + I*Sqrt[3
])*(2*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3)
] - Log[2*(b*c - a*d)^(2/3)*x^2 + (-1 - I*Sqrt[3])*c^(1/3)*(b*c - a*d)^(1/3
)*x*(a + b*x^3)^(1/3) + I*(I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)]]))/(12*c^
(2/3)*(b*c - a*d)^(1/3))
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\,x^3+a)^{\frac{1}{3}}\,(d\,x^3+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

```
[Out] int(1/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
[Out] integrate(1/((b*x^3 + a)^(1/3)*(d*x^3 + c)), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{a + bx^3}\,(c + dx^3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3+a)**(1/3)/(d*x**3+c),x)
[Out] Integral(1/((a + b*x**3)**(1/3)*(c + d*x**3)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
[Out] integrate(1/((b*x^3 + a)^(1/3)*(d*x^3 + c)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(b\,x^3+a\right)}^{1/3}\,\left(d\,x^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^3)^(1/3)*(c + d*x^3)),x)
[Out] int(1/((a + b*x^3)^(1/3)*(c + d*x^3)), x)
```

3.90 $\int \frac{1}{(a+bx^3)^{4/3}(c+dx^3)}\,dx$

Optimal. Leaf size=179

$$\frac{bx}{a(bc-ad)\sqrt[3]{a+bx^3}}-\frac{d\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc-ad}\,x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,c^{2/3}(bc-ad)^{4/3}}-\frac{d\log\left(c+dx^3\right)}{6c^{2/3}(bc-ad)^{4/3}}+\frac{d\log\left(\frac{\sqrt[3]{bc-ad}\,x}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{2/3}(bc-ad)^{4/3}}$$

```
[Out] b*x/a/(-a*d+b*c)/(b*x^3+a)^(1/3)-1/6*d*ln(d*x^3+c)/c^(2/3)/(-a*d+b*c)^(4/3)
+1/2*d*ln((-a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/c^(2/3)/(-a*d+b*c)^(4
/3)-1/3*d*arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/
2))/c^(2/3)/(-a*d+b*c)^(4/3)*3^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {390, 384}

$$-\frac{d\text{ArcTan}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,c^{2/3}(bc-ad)^{4/3}}-\frac{d\log\left(c+dx^3\right)}{6c^{2/3}(bc-ad)^{4/3}}+\frac{d\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{2/3}(bc-ad)^{4/3}}+\frac{bx}{a\sqrt[3]{a+bx^3}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] (b*x)/(a*(b*c - a*d)*(a + b*x^3)^(1/3)) - (d*ArcTan[(1 + (2*(b*c - a*d)^(1/
3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[3]])/(Sqrt[3]*c^(2/3)*(b*c - a*d)^(
4/3)) - (d*Log[c + d*x^3])/(6*c^(2/3)*(b*c - a*d)^(4/3)) + (d*Log[((b*c - a
*d)^(1/3)*x)/c^(1/3) - (a + b*x^3)^(1/3)])/(2*c^(2/3)*(b*c - a*d)^(4/3))
```

Rule 384

```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
```

```
 a*d))), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)),
 Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q},
 x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !L
tQ[q, -1]) && NeQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx &= \frac{bx}{a(bc-ad)\sqrt[3]{a+bx^3}}-\frac{d\int \frac{1}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx}{bc-ad}\\
&= \frac{bx}{a(bc-ad)\sqrt[3]{a+bx^3}}-\frac{d\text{Subst}\left(\int \frac{1}{c-(bc-ad)x^3}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{bc-ad}\\
&= \frac{bx}{a(bc-ad)\sqrt[3]{a+bx^3}}-\frac{d\text{Subst}\left(\int \frac{1}{\sqrt[3]{c}-\sqrt[3]{bc-ad}\,x}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{3c^{2/3}(bc-ad)}-\frac{dS}{}\\
&= \frac{bx}{a(bc-ad)\sqrt[3]{a+bx^3}}+\frac{d\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}\,x}{\sqrt[3]{a+bx^3}}\right)}{3c^{2/3}(bc-ad)^{4/3}}-\frac{d\text{Subst}\left(\int \frac{\sqrt[3]{c}\sqrt[3]{bc}}{c^{2/3}+\sqrt[3]{c}\sqrt[3]{}}\right.}{}\\
&= \frac{bx}{a(bc-ad)\sqrt[3]{a+bx^3}}+\frac{d\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}\,x}{\sqrt[3]{a+bx^3}}\right)}{3c^{2/3}(bc-ad)^{4/3}}-\frac{d\log\left(c^{2/3}+\frac{(bc-ad)^{2/}}{(a+bx^3)^{2}}\right.}{6c^{2/3}(bc}\\
&= \frac{bx}{a(bc-ad)\sqrt[3]{a+bx^3}}-\frac{d\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc-ad}\,x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,c^{2/3}(bc-ad)^{4/3}}+\frac{d\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-}}{\sqrt[3]{a+}}\right.}{3c^{2/3}(bc-ad)}
\end{aligned}
$$

Mathematica [C] Result contains complex when optimal does not.
time = 2.09, size = 328, normalized size = 1.83

$$
\frac{1}{12}\left(\frac{12bx}{(abc-a^2d)\sqrt[3]{a+bx^3}}+\frac{2\sqrt{-6+6i\sqrt{3}}\,d\tan^{-1}\left(\frac{3\sqrt[3]{bc-ad}\,x}{\sqrt{3}\sqrt[3]{bc-ad}\,x-\left(3i+\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{a+bx^3}}\right)}{c^{2/3}(bc-ad)^{4/3}}-\frac{2i\left(-i+\sqrt{3}\right)d\log\left(2\sqrt[3]{bc-ad}\,x+\left(1+i\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{a+bx^3}\right)}{c^{2/3}(bc-ad)^{4/3}}+\frac{\left(d+i\sqrt{3}\,d\right)\log\left(2(bc-ad)^{2/3}x^2+\left(-1-i\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{bc-ad}\,x\sqrt[3]{a+bx^3}+i\left(i+\sqrt{3}\right)c^{2/3}(a+bx^3)^{2/3}\right)}{c^{2/3}(bc-ad)^{4/3}}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] ((12*b*x)/((a*b*c - a^2*d)*(a + b*x^3)^(1/3)) + (2*Sqrt[-6 + (6*I)*Sqrt[3]]
*d*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(b*c - a*d)^(1/3)*x - (3*I + Sqr
t[3])*c^(1/3)*(a + b*x^3)^(1/3))])/(c^(2/3)*(b*c - a*d)^(4/3)) - ((2*I)*(-I
```

```
 + Sqrt[3])*d*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*(a + b*x^
3)^(1/3)])/(c^(2/3)*(b*c - a*d)^(4/3)) + ((d + I*Sqrt[3]*d)*Log[2*(b*c - a*
d)^(2/3)*x^2 + (-1 - I*Sqrt[3])*c^(1/3)*(b*c - a*d)^(1/3)*x*(a + b*x^3)^(1/
3) + I*(I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)])/(c^(2/3)*(b*c - a*d)^(4/3)
))/12
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^{3}+a\right)^{\frac{4}{3}} \left(d\,x^{3}+c\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^3+a)^(4/3)/(d*x^3+c),x)
[Out] int(1/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
[Out] integrate(1/((b*x^3 + a)^(4/3)*(d*x^3 + c)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + b x^{3}\right)^{\frac{4}{3}} \left(c + d x^{3}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3+a)**(4/3)/(d*x**3+c),x)
```

```
[Out] Integral(1/((a + b*x**3)**(4/3)*(c + d*x**3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
[Out] integrate(1/((b*x^3 + a)^(4/3)*(d*x^3 + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(b\,x^3+a\right)}^{4/3}\,\left(d\,x^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^3)^(4/3)*(c + d*x^3)),x)
[Out] int(1/((a + b*x^3)^(4/3)*(c + d*x^3)), x)
```

3.91 $\int \frac{1}{\left(a+bx^3\right)^{7/3}\left(c+dx^3\right)}\,dx$

Optimal. Leaf size=226

$$\frac{bx}{4a(bc-ad)\left(a+bx^3\right)^{4/3}}+\frac{b(3bc-7ad)x}{4a^2(bc-ad)^2\sqrt[3]{a+bx^3}}+\frac{d^2\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc-ad}\,x}{\sqrt[3]{c}\,\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,c^{2/3}(bc-ad)^{7/3}}+\frac{d^2\log\left(c+dx^3\right)}{6c^{2/3}(bc-ad)^{7/3}}-\ldots$$

```
[Out] 1/4*b*x/a/(-a*d+b*c)/(b*x^3+a)^(4/3)+1/4*b*(-7*a*d+3*b*c)*x/a^2/(-a*d+b*c)^
2/(b*x^3+a)^(1/3)+1/6*d^2*ln(d*x^3+c)/c^(2/3)/(-a*d+b*c)^(7/3)-1/2*d^2*ln((
-a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/c^(2/3)/(-a*d+b*c)^(7/3)+1/3*d^2
*arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))/c^(2/
3)/(-a*d+b*c)^(7/3)*3^(1/2)
```

Rubi [A]

time = 0.14, antiderivative size = 226, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {425, 541, 12, 384}

$$\frac{bx(3bc-7ad)}{4a^2\sqrt[3]{a+bx^3}\,(bc-ad)^2}+\frac{d^2\text{ArcTan}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\,\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,c^{2/3}(bc-ad)^{7/3}}+\frac{d^2\log\left(c+dx^3\right)}{6c^{2/3}(bc-ad)^{7/3}}-\frac{d^2\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{2/3}(bc-ad)^{7/3}}+\frac{bx}{4a\left(a+bx^3\right)^{4/3}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^3)^(7/3)*(c + d*x^3)),x]
```

```
[Out] (b*x)/(4*a*(b*c - a*d)*(a + b*x^3)^(4/3)) + (b*(3*b*c - 7*a*d)*x)/(4*a^2*(b
*c - a*d)^2*(a + b*x^3)^(1/3)) + (d^2*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(
c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[3]])/(Sqrt[3]*c^(2/3)*(b*c - a*d)^(7/3)) +
 (d^2*Log[c + d*x^3])/(6*c^(2/3)*(b*c - a*d)^(7/3)) - (d^2*Log[((b*c - a*d)
^(1/3)*x)/c^(1/3) - (a + b*x^3)^(1/3)])/(2*c^(2/3)*(b*c - a*d)^(7/3))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 384

```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
```

```
 a*d, 0]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{1}{\left(a+bx^3\right)^{7/3}\left(c+dx^3\right)}\,dx = \frac{\sqrt[3]{1+\frac{bx^3}{a}}\int \frac{1}{\left(1+\frac{bx^3}{a}\right)^{7/3}(c+dx^3)}\,dx}{a^2\sqrt[3]{a+bx^3}}$$

$$= -\frac{70c^4(bc-ad)x^3\left(a+bx^3\right)^2+105c^3d(bc-ad)x^6\left(a+bx^3\right)^2+45c^2d^2(bc-a}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 2.75, size = 364, normalized size = 1.61

$$\frac{1}{12}\left(\frac{3bx\left(-8a^2d+3b^2cx^3+ab\left(4c-7dx^3\right)\right)}{a^2(bc-ad)^2\left(a+bx^3\right)^{4/3}}-\frac{2\sqrt{-6+6i\sqrt{3}}\,d^2\tan^{-1}\left(\frac{3\sqrt[3]{bc-ad}\,x}{\sqrt{3}\sqrt[3]{bc-ad}\,x-\left(3i+\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{a+bx^3}}\right)}{c^{2/3}(bc-ad)^{7/3}}+\frac{2\left(1+i\sqrt{3}\right)d^2\log\left(2\sqrt[3]{bc-ad}\,x+\left(1+i\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{a+bx^3}\right)}{c^{2/3}(bc-ad)^{7/3}}-\frac{i\left(-i+\sqrt{3}\right)d^2\log\left(2(bc-ad)^{2/3}x^2+\left(-1-i\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{bc-ad}\,x\sqrt[3]{a+bx^3}+i\left(i+\sqrt{3}\right)c^{2/3}\left(a+bx^3\right)^{2/3}\right)}{c^{2/3}(bc-ad)^{7/3}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^3)^(7/3)*(c + d*x^3)),x]
```

```
[Out] ((3*b*x*(-8*a^2*d + 3*b^2*c*x^3 + a*b*(4*c - 7*d*x^3)))/(a^2*(b*c - a*d)^2*
(a + b*x^3)^(4/3)) - (2*Sqrt[-6 + (6*I)*Sqrt[3]]*d^2*ArcTan[(3*(b*c - a*d)^
(1/3)*x)/(Sqrt[3]*(b*c - a*d)^(1/3)*x - (3*I + Sqrt[3])*c^(1/3)*(a + b*x^3)
```

```
^(1/3))])/(c^(2/3)*(b*c - a*d)^(7/3)) + (2*(1 + I*Sqrt[3])*d^2*Log[2*(b*c -
 a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3)])/(c^(2/3)*(b*c -
 a*d)^(7/3)) - (I*(-I + Sqrt[3])*d^2*Log[2*(b*c - a*d)^(2/3)*x^2 + (-1 - I*
Sqrt[3])*c^(1/3)*(b*c - a*d)^(1/3)*x*(a + b*x^3)^(1/3) + I*(I + Sqrt[3])*c^
(2/3)*(a + b*x^3)^(2/3)])/(c^(2/3)*(b*c - a*d)^(7/3)))/12
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^{3}+a\right)^{\frac{7}{3}}\left(d\,x^{3}+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^3+a)^(7/3)/(d*x^3+c),x)
[Out] int(1/(b*x^3+a)^(7/3)/(d*x^3+c),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(7/3)/(d*x^3+c),x, algorithm="maxima")
[Out] integrate(1/((b*x^3 + a)^(7/3)*(d*x^3 + c)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(7/3)/(d*x^3+c),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+bx^{3}\right)^{\frac{7}{3}}\left(c+dx^{3}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3+a)**(7/3)/(d*x**3+c),x)
```

```
[Out] Integral(1/((a + b*x**3)**(7/3)*(c + d*x**3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(7/3)/(d*x^3+c),x, algorithm="giac")
[Out] integrate(1/((b*x^3 + a)^(7/3)*(d*x^3 + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(b\,x^3+a\right)}^{7/3}\,\left(d\,x^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^3)^(7/3)*(c + d*x^3)),x)
[Out] int(1/((a + b*x^3)^(7/3)*(c + d*x^3)), x)
```

3.92 $\int \frac{1}{\left(a+bx^3\right)^{10/3}\left(c+dx^3\right)}\,dx$

Optimal. Leaf size=280

$$\frac{bx}{7a(bc-ad)\left(a+bx^3\right)^{7/3}}+\frac{b(6bc-13ad)x}{28a^2(bc-ad)^2\left(a+bx^3\right)^{4/3}}+\frac{b\left(18b^2c^2-57abcd+67a^2d^2\right)x}{28a^3(bc-ad)^3\sqrt[3]{a+bx^3}}-\frac{d^3\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc}}{\sqrt[3]{c}\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,c^{2/3}(bc-a}$$

```
[Out] 1/7*b*x/a/(-a*d+b*c)/(b*x^3+a)^(7/3)+1/28*b*(-13*a*d+6*b*c)*x/a^2/(-a*d+b*c
)^2/(b*x^3+a)^(4/3)+1/28*b*(67*a^2*d^2-57*a*b*c*d+18*b^2*c^2)*x/a^3/(-a*d+b
*c)^3/(b*x^3+a)^(1/3)-1/6*d^3*ln(d*x^3+c)/c^(2/3)/(-a*d+b*c)^(10/3)+1/2*d^3
*ln((-a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/c^(2/3)/(-a*d+b*c)^(10/3)-1
/3*d^3*arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))
/c^(2/3)/(-a*d+b*c)^(10/3)*3^(1/2)
```

Rubi [A]

time = 0.21, antiderivative size = 280, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {425, 541, 12, 384}

$$\frac{bx(6bc-13ad)}{28a^2\left(a+bx^3\right)^{4/3}(bc-ad)^2}+\frac{bx\left(67a^2d^2-57abcd+18b^2c^2\right)}{28a^3\sqrt[3]{a+bx^3}\,(bc-ad)^3}-\frac{d^3\text{ArcTan}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,c^{2/3}(bc-ad)^{10/3}}-\frac{d^3\log\left(c+dx^3\right)}{6c^{2/3}(bc-ad)^{10/3}}+\frac{d^3\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{2/3}(bc-ad)^{10/3}}+\frac{bx}{7a\left(a+bx^3\right)^{7/3}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^3)^(10/3)*(c + d*x^3)),x]
```

```
[Out] (b*x)/(7*a*(b*c - a*d)*(a + b*x^3)^(7/3)) + (b*(6*b*c - 13*a*d)*x)/(28*a^2*
(b*c - a*d)^2*(a + b*x^3)^(4/3)) + (b*(18*b^2*c^2 - 57*a*b*c*d + 67*a^2*d^2
)*x)/(28*a^3*(b*c - a*d)^3*(a + b*x^3)^(1/3)) - (d^3*ArcTan[(1 + (2*(b*c -
a*d)^(1/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[3]])/(Sqrt[3]*c^(2/3)*(b*c
- a*d)^(10/3)) - (d^3*Log[c + d*x^3])/(6*c^(2/3)*(b*c - a*d)^(10/3)) + (d^3
*Log[((b*c - a*d)^(1/3)*x)/c^(1/3) - (a + b*x^3)^(1/3)])/(2*c^(2/3)*(b*c -
a*d)^(10/3))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 384

```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
```

```
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{1}{(a+bx^3)^{10/3}(c+dx^3)}\,dx = \frac{\sqrt[3]{1+\frac{bx^3}{a}}\int \frac{1}{\left(1+\frac{bx^3}{a}\right)^{10/3}(c+dx^3)}\,dx}{a^3\sqrt[3]{a+bx^3}}$$

$$= -\frac{7280c^5(bc-ad)^2x^6(a+bx^3)^2 + 16380c^4d(bc-ad)^2x^9(a+bx^3)^2 + 14040c}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 5.29, size = 420, normalized size = 1.50

$$\frac{1}{84}\left(-\frac{3bx(84a^4d^2+18b^4c^2x^6+3ab^3cx^3(14c-19dx^3)+21a^3bd(-4c+7dx^3)+a^2b^2(28c^2-133cdx^3+67d^2x^6))}{a^3(-bc+ad)^3(a+bx^3)^{7/3}}+\frac{14\sqrt{-6+6i\sqrt{3}}\,d^3\tan^{-1}\left(\frac{3\sqrt[3]{bc-ad}\,x}{\sqrt{3}\sqrt[3]{bc-ad}\,x-\left(3i+\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{a+bx^3}}\right)}{c^{2/3}(bc-ad)^{10/3}}-\frac{14i\left(-i+\sqrt{3}\right)d^3\log\left(2\sqrt[3]{bc-ad}\,x+\left(1+i\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{a+bx^3}\right)}{c^{2/3}(bc-ad)^{10/3}}+\frac{7\left(1+i\sqrt{3}\right)d^3\log\left(2(bc-ad)^{2/3}x^2+\left(-1-i\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{bc-ad}\,x\sqrt[3]{a+bx^3}+i\left(i+\sqrt{3}\right)c^{2/3}(a+bx^3)^{2/3}\right)}{c^{2/3}(bc-ad)^{10/3}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^3)^(10/3)*(c + d*x^3)),x]
```

```
[Out] ((-3*b*x*(84*a^4*d^2 + 18*b^4*c^2*x^6 + 3*a*b^3*c*x^3*(14*c - 19*d*x^3) + 2
1*a^3*b*d*(-4*c + 7*d*x^3) + a^2*b^2*(28*c^2 - 133*c*d*x^3 + 67*d^2*x^6)))/
```

```
(a^3*(-(b*c) + a*d)^3*(a + b*x^3)^(7/3)) + (14*Sqrt[-6 + (6*I)*Sqrt[3]]*d^3
*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(b*c - a*d)^(1/3)*x - (3*I + Sqrt[
3])*c^(1/3)*(a + b*x^3)^(1/3))]])/(c^(2/3)*(b*c - a*d)^(10/3)) - ((14*I)*(-I
 + Sqrt[3])*d^3*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*(a + b*
x^3)^(1/3)]])/(c^(2/3)*(b*c - a*d)^(10/3)) + (7*(1 + I*Sqrt[3])*d^3*Log[2*(b
*c - a*d)^(2/3)*x^2 + (-1 - I*Sqrt[3])*c^(1/3)*(b*c - a*d)^(1/3)*x*(a + b*x
^3)^(1/3) + I*(I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)]])/(c^(2/3)*(b*c - a*d
)^(10/3)))/84
```

Maple [**F**]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^{3}+a\right)^{\frac{10}{3}} \left(d\,x^{3}+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^3+a)^(10/3)/(d*x^3+c),x)
```

```
[Out] int(1/(b*x^3+a)^(10/3)/(d*x^3+c),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(10/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)^(10/3)*(d*x^3 + c)), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(10/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+bx^{3}\right)^{\frac{10}{3}} \left(c+dx^{3}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3+a)**(10/3)/(d*x**3+c),x)
[Out] Integral(1/((a + b*x**3)**(10/3)*(c + d*x**3)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(10/3)/(d*x^3+c),x, algorithm="giac")
[Out] integrate(1/((b*x^3 + a)^(10/3)*(d*x^3 + c)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(b\,x^3+a\right)}^{10/3}\,\left(d\,x^3+c\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^3)^(10/3)*(c + d*x^3)),x)
[Out] int(1/((a + b*x^3)^(10/3)*(c + d*x^3)), x)
```

3.93 $\int \frac{\left(a+bx^3\right)^{4/3}}{c+dx^3}\,dx$

Optimal. Leaf size=60

$$\frac{ax\sqrt[3]{a+bx^3}\,F_1\left(\frac{1}{3};-\frac{4}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c\sqrt[3]{1+\frac{bx^3}{a}}}$$

```
[Out] a*x*(b*x^3+a)^(1/3)*AppellF1(1/3,-4/3,1,4/3,-b*x^3/a,-d*x^3/c)/c/(1+b*x^3/a
)^(1/3)
```

Rubi [A]

time = 0.02, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {441, 440}

$$\frac{ax\sqrt[3]{a+bx^3}\,F_1\left(\frac{1}{3};-\frac{4}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c\sqrt[3]{\frac{bx^3}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(4/3)/(c + d*x^3),x]
```

```
[Out] (a*x*(a + b*x^3)^(1/3)*AppellF1[1/3, -4/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/
c)])/(c*(1 + (b*x^3)/a)^(1/3))
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{(a+bx^3)^{4/3}}{c+dx^3}\,dx = \frac{\left(a\sqrt[3]{a+bx^3}\right)\int \frac{\left(1+\frac{bx^3}{a}\right)^{4/3}}{c+dx^3}\,dx}{\sqrt[3]{1+\frac{bx^3}{a}}}$$
$$= \frac{ax\sqrt[3]{a+bx^3}\,F_1\left(\frac{1}{3};-\frac{4}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c\sqrt[3]{1+\frac{bx^3}{a}}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 346 vs. 2(60) = 120.
time = 10.32, size = 346, normalized size = 5.77

$$\frac{x\left(\frac{b(-2bc+3ad)x^3\left(1+\frac{bx^3}{a}\right)^{2/3}F_1\left(\frac{4}{3};\frac{2}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c}+\frac{4\left(-4ac(2a^2d+abdx^3+b^2x^3(c+dx^3))F_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+bx^3(a+bx^3)(c+dx^3)\left(3adF_1\left(\frac{4}{3};\frac{2}{3},2;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+2bcF_1\left(\frac{4}{3};\frac{5}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)\right)}{(c+dx^3)\left(-4acF_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+x^3\left(3adF_1\left(\frac{4}{3};\frac{2}{3},2;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+2bcF_1\left(\frac{4}{3};\frac{5}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)\right)}\right)}{8d\,(a+bx^3)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(4/3)/(c + d*x^3),x]
```

```
[Out] (x*((b*(-2*b*c + 3*a*d)*x^3*(1 + (b*x^3)/a)^(2/3)*AppellF1[4/3, 2/3, 1, 7/3
, -((b*x^3)/a), -((d*x^3)/c)])/c + (4*(-4*a*c*(2*a^2*d + a*b*d*x^3 + b^2*x^
3*(c + d*x^3))*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)] + b*x
^3*(a + b*x^3)*(c + d*x^3)*(3*a*d*AppellF1[4/3, 2/3, 2, 7/3, -((b*x^3)/a),
-((d*x^3)/c)] + 2*b*c*AppellF1[4/3, 5/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)
])))/((c + d*x^3)*(-4*a*c*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3
)/c)] + x^3*(3*a*d*AppellF1[4/3, 2/3, 2, 7/3, -((b*x^3)/a), -((d*x^3)/c)] +
 2*b*c*AppellF1[4/3, 5/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)])))))/(8*d*(a
+ b*x^3)^(2/3))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(b\,x^3+a)^{\frac{4}{3}}}{d\,x^3+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(4/3)/(d*x^3+c),x)
```

```
[Out] int((b*x^3+a)^(4/3)/(d*x^3+c),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(4/3)/(d*x^3 + c), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx^3)^{\frac{4}{3}}}{c+dx^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(4/3)/(d*x**3+c),x)
```

```
[Out] Integral((a + b*x**3)**(4/3)/(c + d*x**3), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(4/3)/(d*x^3 + c), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{{\left(b\,x^3+a\right)}^{4/3}}{d\,x^3+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(4/3)/(c + d*x^3),x)
```

```
[Out] int((a + b*x^3)^(4/3)/(c + d*x^3), x)
```

3.94 $\int \frac{\sqrt[3]{a+bx^3}}{c+dx^3} \, dx$

Optimal. Leaf size=59

$$\frac{x\sqrt[3]{a+bx^3}\,F_1\left(\frac{1}{3};-\frac{1}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c\sqrt[3]{1+\frac{bx^3}{a}}}$$

```
[Out] x*(b*x^3+a)^(1/3)*AppellF1(1/3,-1/3,1,4/3,-b*x^3/a,-d*x^3/c)/c/(1+b*x^3/a)^
(1/3)
```

Rubi [A]
time = 0.02, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {441, 440}

$$\frac{x\sqrt[3]{a+bx^3}\,F_1\left(\frac{1}{3};-\frac{1}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c\sqrt[3]{\frac{bx^3}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(1/3)/(c + d*x^3),x]
```

```
[Out] (x*(a + b*x^3)^(1/3)*AppellF1[1/3, -1/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)
])/(c*(1 + (b*x^3)/a)^(1/3))
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{\sqrt[3]{a+bx^3}}{c+dx^3}\,dx = \frac{\sqrt[3]{a+bx^3}\int \frac{\sqrt[3]{1+\frac{bx^3}{a}}}{c+dx^3}\,dx}{\sqrt[3]{1+\frac{bx^3}{a}}}$$
$$= \frac{x\sqrt[3]{a+bx^3}\,F_1\left(\frac{1}{3};-\frac{1}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c\sqrt[3]{1+\frac{bx^3}{a}}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 160 vs. 2(59) = 118.
time = 10.12, size = 160, normalized size = 2.71

$$\frac{4acx\sqrt[3]{a+bx^3}\,F_1\left(\frac{1}{3};-\frac{1}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{(c+dx^3)\left(4acF_1\left(\frac{1}{3};-\frac{1}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+x^3\left(-3adF_1\left(\frac{4}{3};-\frac{1}{3},2;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+bcF_1\left(\frac{4}{3};\frac{2}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(1/3)/(c + d*x^3),x]
```

```
[Out] (4*a*c*x*(a + b*x^3)^(1/3)*AppellF1[1/3, -1/3, 1, 4/3, -((b*x^3)/a), -((d*x
^3)/c)])/((c + d*x^3)*(4*a*c*AppellF1[1/3, -1/3, 1, 4/3, -((b*x^3)/a), -((d
*x^3)/c)] + x^3*(-3*a*d*AppellF1[4/3, -1/3, 2, 7/3, -((b*x^3)/a), -((d*x^3)
/c)] + b*c*AppellF1[4/3, 2/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)])))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(b\,x^3+a)^{\frac{1}{3}}}{d\,x^3+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(1/3)/(d*x^3+c),x)
```

```
[Out] int((b*x^3+a)^(1/3)/(d*x^3+c),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
[Out] integrate((b*x^3 + a)^(1/3)/(d*x^3 + c), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{a+bx^3}}{c+dx^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(1/3)/(d*x**3+c),x)
[Out] Integral((a + b*x**3)**(1/3)/(c + d*x**3), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
[Out] integrate((b*x^3 + a)^(1/3)/(d*x^3 + c), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{{\left(b\,x^3+a\right)}^{1/3}}{d\,x^3+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(1/3)/(c + d*x^3),x)
[Out] int((a + b*x^3)^(1/3)/(c + d*x^3), x)
```

3.95 $\int \frac{1}{(a+bx^3)^{2/3}(c+dx^3)} \, dx$

Optimal. Leaf size=59

$$\frac{x\left(1+\frac{bx^3}{a}\right)^{2/3} F_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c\,(a+bx^3)^{2/3}}$$

```
[Out] x*(1+b*x^3/a)^(2/3)*AppellF1(1/3,2/3,1,4/3,-b*x^3/a,-d*x^3/c)/c/(b*x^3+a)^(
2/3)
```

Rubi [A]
time = 0.02, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {441, 440}

$$\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3} F_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c\,(a+bx^3)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

```
[Out] (x*(1 + (b*x^3)/a)^(2/3)*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)
/c)])/(c*(a + b*x^3)^(2/3))
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{1}{(a+bx^3)^{2/3}(c+dx^3)}\,dx = \frac{\left(1+\frac{bx^3}{a}\right)^{2/3}\int \frac{1}{\left(1+\frac{bx^3}{a}\right)^{2/3}(c+dx^3)}\,dx}{(a+bx^3)^{2/3}}$$
$$= \frac{x\left(1+\frac{bx^3}{a}\right)^{2/3} F_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c\,(a+bx^3)^{2/3}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 161 vs. $2(59) = 118$.
time = 10.06, size = 161, normalized size = 2.73

$$-\frac{4acxF_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{(a+bx^3)^{2/3}(c+dx^3)\left(-4acF_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+x^3\left(3adF_1\left(\frac{4}{3};\frac{2}{3},2;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+2bcF_1\left(\frac{4}{3};\frac{5}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^3)^(2/3)*(c + d*x^3)),x]
[Out] (-4*a*c*x*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)])/((a + b*x
^3)^(2/3)*(c + d*x^3)*(-4*a*c*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d
*x^3)/c)] + x^3*(3*a*d*AppellF1[4/3, 2/3, 2, 7/3, -((b*x^3)/a), -((d*x^3)/c
)] + 2*b*c*AppellF1[4/3, 5/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)])))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\,x^3+a)^{\frac{2}{3}}\,(d\,x^3+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^3+a)^(2/3)/(d*x^3+c),x)
[Out] int(1/(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
[Out] integrate(1/((b*x^3 + a)^(2/3)*(d*x^3 + c)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx^3)^{\frac{2}{3}}(c+dx^3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3+a)**(2/3)/(d*x**3+c),x)
[Out] Integral(1/((a + b*x**3)**(2/3)*(c + d*x**3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
[Out] integrate(1/((b*x^3 + a)^(2/3)*(d*x^3 + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{{\left(b\,x^3+a\right)}^{2/3}\,\left(d\,x^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^3)^(2/3)*(c + d*x^3)),x)
[Out] int(1/((a + b*x^3)^(2/3)*(c + d*x^3)), x)
```

3.96 $\int \frac{1}{(a+bx^3)^{5/3}(c+dx^3)}\,dx$

Optimal. Leaf size=62

$$\frac{x\left(1+\frac{bx^3}{a}\right)^{2/3}F_1\left(\frac{1}{3};\frac{5}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{ac\left(a+bx^3\right)^{2/3}}$$

```
[Out] x*(1+b*x^3/a)^(2/3)*AppellF1(1/3,5/3,1,4/3,-b*x^3/a,-d*x^3/c)/a/c/(b*x^3+a)
^(2/3)
```

Rubi [A]
time = 0.02, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {441, 440}

$$\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}F_1\left(\frac{1}{3};\frac{5}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{ac\left(a+bx^3\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^3)^(5/3)*(c + d*x^3)),x]
```

```
[Out] (x*(1 + (b*x^3)/a)^(2/3)*AppellF1[1/3, 5/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)
/c)])/(a*c*(a + b*x^3)^(2/3))
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{1}{(a+bx^3)^{5/3}(c+dx^3)}\,dx = \frac{\left(1+\frac{bx^3}{a}\right)^{2/3}\int \frac{1}{\left(1+\frac{bx^3}{a}\right)^{5/3}(c+dx^3)}\,dx}{a(a+bx^3)^{2/3}}$$
$$= \frac{x\left(1+\frac{bx^3}{a}\right)^{2/3}F_1\left(\frac{1}{3};\frac{5}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{ac(a+bx^3)^{2/3}}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 332 vs. $2(62) = 124$.
time = 10.20, size = 332, normalized size = 5.35

$$\frac{x\left(-\frac{bdx^3\left(1+\frac{bx^3}{a}\right)^{2/3}F_1\left(\frac{4}{3};\frac{2}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c}+\frac{4\left(4ac(2ad-b(2c+dx^3))F_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+bx^3(c+dx^3)\left(3adF_1\left(\frac{4}{3};\frac{2}{3},2;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+2bcF_1\left(\frac{4}{3};\frac{5}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)\right)}{(c+dx^3)\left(4acF_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)-x^3\left(3adF_1\left(\frac{4}{3};\frac{2}{3},2;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+2bcF_1\left(\frac{4}{3};\frac{5}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)\right)}\right)}{8a(-bc+ad)(a+bx^3)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^3)^(5/3)*(c + d*x^3)),x]
```

```
[Out] (x*(-((b*d*x^3*(1 + (b*x^3)/a)^(2/3)*AppellF1[4/3, 2/3, 1, 7/3, -((b*x^3)/a
), -((d*x^3)/c)])/c) + (4*(4*a*c*(2*a*d - b*(2*c + d*x^3))*AppellF1[1/3, 2/
3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)] + b*x^3*(c + d*x^3)*(3*a*d*AppellF1[
4/3, 2/3, 2, 7/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*b*c*AppellF1[4/3, 5/3, 1,
 7/3, -((b*x^3)/a), -((d*x^3)/c)])))/((c + d*x^3)*(4*a*c*AppellF1[1/3, 2/3,
 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)] - x^3*(3*a*d*AppellF1[4/3, 2/3, 2, 7/3
, -((b*x^3)/a), -((d*x^3)/c)] + 2*b*c*AppellF1[4/3, 5/3, 1, 7/3, -((b*x^3)/
a), -((d*x^3)/c)])))))/(8*a*(-(b*c) + a*d)*(a + b*x^3)^(2/3))
```

Maple [**F**]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\,x^3+a)^{\frac{5}{3}}(d\,x^3+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^3+a)^(5/3)/(d*x^3+c),x)
```

```
[Out] int(1/(b*x^3+a)^(5/3)/(d*x^3+c),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(5/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)^(5/3)*(d*x^3 + c)), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(5/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx^3)^{\frac{5}{3}}\,(c+dx^3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3+a)**(5/3)/(d*x**3+c),x)
```

```
[Out] Integral(1/((a + b*x**3)**(5/3)*(c + d*x**3)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(5/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^3 + a)^(5/3)*(d*x^3 + c)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{{\left(b\,x^3+a\right)}^{5/3}\,\left(d\,x^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^3)^(5/3)*(c + d*x^3)),x)
```

```
[Out] int(1/((a + b*x^3)^(5/3)*(c + d*x^3)), x)
```

3.97 $\int \frac{1}{(a+bx^3)^{8/3}(c+dx^3)}\,dx$

Optimal. Leaf size=62

$$\frac{x\left(1+\frac{bx^3}{a}\right)^{2/3}F_1\left(\frac{1}{3};\frac{8}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2c\,(a+bx^3)^{2/3}}$$

```
[Out] x*(1+b*x^3/a)^(2/3)*AppellF1(1/3,8/3,1,4/3,-b*x^3/a,-d*x^3/c)/a^2/c/(b*x^3+
a)^(2/3)
```

Rubi [A]
time = 0.02, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {441, 440}

$$\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}F_1\left(\frac{1}{3};\frac{8}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2c\,(a+bx^3)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^3)^(8/3)*(c + d*x^3)),x]
```

```
[Out] (x*(1 + (b*x^3)/a)^(2/3)*AppellF1[1/3, 8/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)
/c)])/(a^2*c*(a + b*x^3)^(2/3))
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{1}{(a+bx^3)^{8/3}\,(c+dx^3)}\,dx=\frac{\left(1+\frac{bx^3}{a}\right)^{2/3}\int \frac{1}{\left(1+\frac{bx^3}{a}\right)^{8/3}(c+dx^3)}\,dx}{a^2\,(a+bx^3)^{2/3}}$$
$$=\frac{x\left(1+\frac{bx^3}{a}\right)^{2/3}F_1\left(\frac{1}{3};\frac{8}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2c\,(a+bx^3)^{2/3}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 429 vs. $2(62) = 124$.
time = 10.56, size = 429, normalized size = 6.92

$$-\frac{x\left(\frac{bd(-4bc+9ad)x^3\left(1+\frac{bx^3}{a}\right)^{2/3}F_1\left(\frac{4}{3};\frac{2}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c}+\frac{4\left(4ac\left(10a^3d^2+4b^3cx^3\left(2c+dx^3\right)-a^2bd\left(20c+dx^3\right)+ab^2\left(10c^2-12cdx^3-9d^2x^6\right)\right)F_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+bx^3\left(c+dx^3\right)\left(11a^2d-4b^2cx^3+ab\left(-6c+9dx^3\right)\right)\left(3adF_1\left(\frac{4}{3};\frac{2}{3},2;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+2bcF_1\left(\frac{4}{3};\frac{5}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)\right)}{(a+bx^3)(c+dx^3)\left(-4acF_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+x^3\left(3adF_1\left(\frac{4}{3};\frac{2}{3},2;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+2bcF_1\left(\frac{4}{3};\frac{5}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)\right)}\right)}{40a^2(bc-ad)^2\,(a+bx^3)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^3)^(8/3)*(c + d*x^3)),x]
```

```
[Out] -1/40*(x*((b*d*(-4*b*c + 9*a*d)*x^3*(1 + (b*x^3)/a)^(2/3)*AppellF1[4/3, 2/3
, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)])/c + (4*(4*a*c*(10*a^3*d^2 + 4*b^3*c*
x^3*(2*c + d*x^3) - a^2*b*d*(20*c + d*x^3) + a*b^2*(10*c^2 - 12*c*d*x^3 - 9
*d^2*x^6))*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)] + b*x^3*(
c + d*x^3)*(11*a^2*d - 4*b^2*c*x^3 + a*b*(-6*c + 9*d*x^3))*(3*a*d*AppellF1[
4/3, 2/3, 2, 7/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*b*c*AppellF1[4/3, 5/3, 1,
 7/3, -((b*x^3)/a), -((d*x^3)/c)])))/((a + b*x^3)*(c + d*x^3)*(-4*a*c*Appel
lF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)] + x^3*(3*a*d*AppellF1[4/3
, 2/3, 2, 7/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*b*c*AppellF1[4/3, 5/3, 1, 7/
3, -((b*x^3)/a), -((d*x^3)/c)])))))/(a^2*(b*c - a*d)^2*(a + b*x^3)^(2/3))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^3+a\right)^{\frac{8}{3}}\left(d\,x^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^3+a)^(8/3)/(d*x^3+c),x)
```

```
[Out] int(1/(b*x^3+a)^(8/3)/(d*x^3+c),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(8/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)^(8/3)*(d*x^3 + c)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(8/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx^3)^{\frac{8}{3}}\,(c+dx^3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3+a)**(8/3)/(d*x**3+c),x)
```

```
[Out] Integral(1/((a + b*x**3)**(8/3)*(c + d*x**3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(8/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^3 + a)^(8/3)*(d*x^3 + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{{\left(b\,x^3+a\right)}^{8/3}\,\left(d\,x^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^3)^(8/3)*(c + d*x^3)),x)
```

```
[Out] int(1/((a + b*x^3)^(8/3)*(c + d*x^3)), x)
```

3.98 $\int \frac{(a+bx^3)^{8/3}}{(c+dx^3)^2}\,dx$

Optimal. Leaf size=351

$$\frac{b(2bc-ad)x(a+bx^3)^{2/3}}{3cd^2}-\frac{(bc-ad)x(a+bx^3)^{5/3}}{3cd\,(c+dx^3)}-\frac{2b^{5/3}(3bc-4ad)\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}\,d^3}+\frac{2(bc-ad)^{5/3}}{}$$

```
[Out] 1/3*b*(-a*d+2*b*c)*x*(b*x^3+a)^(2/3)/c/d^2-1/3*(-a*d+b*c)*x*(b*x^3+a)^(5/3)
/c/d/(d*x^3+c)+1/9*(-a*d+b*c)^(5/3)*(a*d+3*b*c)*ln(d*x^3+c)/c^(5/3)/d^3-1/3
*(-a*d+b*c)^(5/3)*(a*d+3*b*c)*ln((-a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3)
)/c^(5/3)/d^3+1/3*b^(5/3)*(-4*a*d+3*b*c)*ln(-b^(1/3)*x+(b*x^3+a)^(1/3))/d^3
-2/9*b^(5/3)*(-4*a*d+3*b*c)*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1
/2))/d^3*3^(1/2)+2/9*(-a*d+b*c)^(5/3)*(a*d+3*b*c)*arctan(1/3*(1+2*(-a*d+b*c
)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))/c^(5/3)/d^3*3^(1/2)
```

Rubi [A]
time = 0.46, antiderivative size = 351, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {424, 542, 544, 245, 384}

$$-\frac{2b^{5/3}\text{ArcTan}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)(3bc-4ad)}{3\sqrt{3}\,d^3}+\frac{2(bc-ad)^{5/3}(ad+3bc)\text{ArcTan}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{3\sqrt{3}\,c^{5/3}d^3}+\frac{b^{5/3}(3bc-4ad)\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}\,x\right)}{3d^3}+\frac{(bc-ad)^{5/3}(ad+3bc)\log\left(c+dx^3\right)}{9c^{5/3}d^3}-\frac{(bc-ad)^{5/3}(ad+3bc)\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{3c^{5/3}d^3}+\frac{bx(a+bx^3)^{2/3}(2bc-ad)}{3cd^2}-\frac{x(a+bx^3)^{5/3}(bc-ad)}{3cd\,(c+dx^3)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(8/3)/(c + d*x^3)^2,x]
```

```
[Out] (b*(2*b*c - a*d)*x*(a + b*x^3)^(2/3))/(3*c*d^2) - ((b*c - a*d)*x*(a + b*x^3
)^(5/3))/(3*c*d*(c + d*x^3)) - (2*b^(5/3)*(3*b*c - 4*a*d)*ArcTan[(1 + (2*b^
(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(3*Sqrt[3]*d^3) + (2*(b*c - a*d)^(5/3
)*(3*b*c + a*d)*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a + b*x^3)^(1
/3)))/Sqrt[3]])/(3*Sqrt[3]*c^(5/3)*d^3) + ((b*c - a*d)^(5/3)*(3*b*c + a*d)*
Log[c + d*x^3])/(9*c^(5/3)*d^3) - ((b*c - a*d)^(5/3)*(3*b*c + a*d)*Log[((b*
c - a*d)^(1/3)*x)/c^(1/3) - (a + b*x^3)^(1/3)]])/(3*c^(5/3)*d^3) + (b^(5/3)*
(3*b*c - 4*a*d)*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)]])/(3*d^3)
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 384

```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 544

```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Dist[f/d, Int[(a + b*x^n)^p, x], x] + Dist[(d*e -
c*f)/d, Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, p
, n}, x]
```

Rubi steps

$$\int \frac{(a+bx^3)^{8/3}}{(c+dx^3)^2}\,dx = \frac{\left(a^2(a+bx^3)^{2/3}\right)\int \frac{\left(1+\frac{bx^3}{a}\right)^{8/3}}{(c+dx^3)^2}\,dx}{\left(1+\frac{bx^3}{a}\right)^{2/3}}$$

$$= \frac{a^2x(a+bx^3)^{2/3}\,F_1\left(\frac{1}{3};-\frac{8}{3},2;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^2\left(1+\frac{bx^3}{a}\right)^{2/3}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

time = 10.70, size = 698, normalized size = 1.99

$$\frac{1}{18}\left(\frac{6x(a+bx^3)^{2/3}\left(b^2+\frac{(bc-ad)^2}{c(c+dx^3)}\right)}{d^2}-\frac{9b^3x^4\sqrt[3]{1+\frac{bx^3}{a}}\,F_1\left(\frac{4}{3};\frac{1}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{d^2\sqrt[3]{a+bx^3}}+\frac{12ab^2x^4\sqrt[3]{1+\frac{bx^3}{a}}\,F_1\left(\frac{4}{3};\frac{1}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{cd\sqrt[3]{a+bx^3}}+\frac{2a^3\left(2\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc-ad}\,x}{\sqrt[3]{c}\sqrt[3]{b+ax^3}}}{\sqrt{3}}\right)-2\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}\,x}{\sqrt[3]{b+ax^3}}\right)+\log\left(c^{2/3}+\frac{(bc-ad)^{2/3}x^2}{(b+ax^3)^{2/3}}+\frac{\sqrt[3]{c}\sqrt[3]{bc-ad}\,x}{\sqrt[3]{b+ax^3}}\right)\right)}{c^{5/3}\sqrt[3]{bc-ad}}-\frac{2ab^2\sqrt[3]{c}\left(2\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc-ad}\,x}{\sqrt[3]{c}\sqrt[3]{b+ax^3}}}{\sqrt{3}}\right)-2\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}\,x}{\sqrt[3]{b+ax^3}}\right)+\log\left(c^{2/3}+\frac{(bc-ad)^{2/3}x^2}{(b+ax^3)^{2/3}}+\frac{\sqrt[3]{c}\sqrt[3]{bc-ad}\,x}{\sqrt[3]{b+ax^3}}\right)\right)}{d^2\sqrt[3]{bc-ad}}+\frac{2a^2b\left(2\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc-ad}\,x}{\sqrt[3]{c}\sqrt[3]{b+ax^3}}}{\sqrt{3}}\right)-2\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}\,x}{\sqrt[3]{b+ax^3}}\right)+\log\left(c^{2/3}+\frac{(bc-ad)^{2/3}x^2}{(b+ax^3)^{2/3}}+\frac{\sqrt[3]{c}\sqrt[3]{bc-ad}\,x}{\sqrt[3]{b+ax^3}}\right)\right)}{c^{2/3}d\sqrt[3]{bc-ad}}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(8/3)/(c + d*x^3)^2,x]
```

```
[Out] ((6*x*(a + b*x^3)^(2/3)*(b^2 + (b*c - a*d)^2/(c*(c + d*x^3))))/d^2 - (9*b^3
*x^4*(1 + (b*x^3)/a)^(1/3)*AppellF1[4/3, 1/3, 1, 7/3, -((b*x^3)/a), -((d*x^
3)/c)])/(d^2*(a + b*x^3)^(1/3)) + (12*a*b^2*x^4*(1 + (b*x^3)/a)^(1/3)*Appel
lF1[4/3, 1/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)])/(c*d*(a + b*x^3)^(1/3))
+ (2*a^3*(2*Sqrt[3]*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(b + a*x^3
)^(1/3)))/Sqrt[3]] - 2*Log[c^(1/3) - ((b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3
)] + Log[c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(b + a*x^3)^(2/3) + (c^(1/3)*(b*
c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)]))/(c^(5/3)*(b*c - a*d)^(1/3)) - (2*a*b
^2*c^(1/3)*(2*Sqrt[3]*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(b + a*x
^3)^(1/3)))/Sqrt[3]] - 2*Log[c^(1/3) - ((b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1
/3)] + Log[c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(b + a*x^3)^(2/3) + (c^(1/3)*(
b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)]))/(d^2*(b*c - a*d)^(1/3)) + (2*a^2*b
*(2*Sqrt[3]*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(b + a*x^3)^(1/3))
)/Sqrt[3]] - 2*Log[c^(1/3) - ((b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)] + Log
[c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(b + a*x^3)^(2/3) + (c^(1/3)*(b*c - a*d)
^(1/3)*x)/(b + a*x^3)^(1/3)]))/(c^(2/3)*d*(b*c - a*d)^(1/3)))/18
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^{3}+a\right)^{\frac{8}{3}}}{\left(d\,x^{3}+c\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(8/3)/(d*x^3+c)^2,x)
```

```
[Out] int((b*x^3+a)^(8/3)/(d*x^3+c)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(8/3)/(d*x^3+c)^2,x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(8/3)/(d*x^3 + c)^2, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 819 vs. $2(291) = 582$.
time = 11.01, size = 819, normalized size = 2.33

$$\frac{2\sqrt{3}\left(3b^2c^3-2abc^2d-a^2cd^2+\left(3b^2c^2d-2abcd^2-a^2d^3\right)x^3\right)\left(\frac{b^2c^2-2abcd+a^2d^2}{c^2}\right)^{\frac{1}{3}}\arctan\left(-\frac{\sqrt{3}(bc-ad)x+2\sqrt{3}(bx^3+a)^{\frac{1}{3}}c\left(\frac{b^2c^2-2abcd+a^2d^2}{c^2}\right)^{\frac{1}{3}}}{3(bc-ad)x}\right)+2\sqrt{3}\left(3b^2c^3-4abc^2d+\left(3b^2c^2d-4abcd^2\right)x^3\right)(-b^2)^{\frac{1}{3}}\arctan\left(-\frac{\sqrt{3}bx-2\sqrt{3}(bx^3+a)^{\frac{1}{3}}(-b^2)^{\frac{1}{3}}}{3bx}\right)-2\left(3b^2c^3-2abc^2d-a^2cd^2+\left(3b^2c^2d-2abcd^2-a^2d^3\right)x^3\right)\left(\frac{b^2c^2-2abcd+a^2d^2}{c^2}\right)^{\frac{1}{3}}\log\left(\frac{cx\left(\frac{b^2c^2-2abcd+a^2d^2}{c^2}\right)^{\frac{2}{3}}-(bx^3+a)^{\frac{1}{3}}(bc-ad)}{x}\right)-2\left(3b^2c^3-4abc^2d+\left(3b^2c^2d-4abcd^2\right)x^3\right)(-b^2)^{\frac{1}{3}}\log\left(-\frac{(-b^2)^{\frac{2}{3}}x-(bx^3+a)^{\frac{1}{3}}b}{x}\right)+\left(3b^2c^3-4abc^2d+\left(3b^2c^2d-4abcd^2\right)x^3\right)(-b^2)^{\frac{1}{3}}\log\left(-\frac{(-b^2)^{\frac{1}{3}}bx^2-(bx^3+a)^{\frac{1}{3}}(-b^2)^{\frac{2}{3}}x-(bx^3+a)^{\frac{2}{3}}b}{x^2}\right)+\left(3b^2c^3-2abc^2d-a^2cd^2+\left(3b^2c^2d-2abcd^2-a^2d^3\right)x^3\right)\left(\frac{b^2c^2-2abcd+a^2d^2}{c^2}\right)^{\frac{1}{3}}\log\left(-\frac{(bc-ad)x^2\left(\frac{b^2c^2-2abcd+a^2d^2}{c^2}\right)^{\frac{1}{3}}+(bx^3+a)^{\frac{1}{3}}cx\left(\frac{b^2c^2-2abcd+a^2d^2}{c^2}\right)^{\frac{2}{3}}+(bx^3+a)^{\frac{2}{3}}(bc-ad)}{x^2}\right)+3\left(b^2cd^2x^4+\left(2b^2c^2d-2abcd^2+a^2d^3\right)x\right)(bx^3+a)^{\frac{2}{3}}}{9\left(cd^4x^3+c^2d^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(8/3)/(d*x^3+c)^2,x, algorithm="fricas")
```

```
[Out] 1/9*(2*sqrt(3)*(3*b^2*c^3 - 2*a*b*c^2*d - a^2*c*d^2 + (3*b^2*c^2*d - 2*a*b*
c*d^2 - a^2*d^3)*x^3)*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(1/3)*arctan(-1
/3*(sqrt(3)*(b*c - a*d)*x + 2*sqrt(3)*(b*x^3 + a)^(1/3)*c*((b^2*c^2 - 2*a*b
*c*d + a^2*d^2)/c^2)^(1/3))/((b*c - a*d)*x)) + 2*sqrt(3)*(3*b^2*c^3 - 4*a*b
*c^2*d + (3*b^2*c^2*d - 4*a*b*c*d^2)*x^3)*(-b^2)^(1/3)*arctan(-1/3*(sqrt(3)
*b*x - 2*sqrt(3)*(b*x^3 + a)^(1/3)*(-b^2)^(1/3))/(b*x)) - 2*(3*b^2*c^3 - 2*
a*b*c^2*d - a^2*c*d^2 + (3*b^2*c^2*d - 2*a*b*c*d^2 - a^2*d^3)*x^3)*((b^2*c^
2 - 2*a*b*c*d + a^2*d^2)/c^2)^(1/3)*log((c*x*((b^2*c^2 - 2*a*b*c*d + a^2*d^
2)/c^2)^(2/3) - (b*x^3 + a)^(1/3)*(b*c - a*d))/x) - 2*(3*b^2*c^3 - 4*a*b*c^
2*d + (3*b^2*c^2*d - 4*a*b*c*d^2)*x^3)*(-b^2)^(1/3)*log(-((-b^2)^(2/3)*x -
(b*x^3 + a)^(1/3)*b)/x) + (3*b^2*c^3 - 4*a*b*c^2*d + (3*b^2*c^2*d - 4*a*b*c
*d^2)*x^3)*(-b^2)^(1/3)*log(-((-b^2)^(1/3)*b*x^2 - (b*x^3 + a)^(1/3)*(-b^2)
^(2/3)*x - (b*x^3 + a)^(2/3)*b)/x^2) + (3*b^2*c^3 - 2*a*b*c^2*d - a^2*c*d^2
 + (3*b^2*c^2*d - 2*a*b*c*d^2 - a^2*d^3)*x^3)*((b^2*c^2 - 2*a*b*c*d + a^2*d
^2)/c^2)^(1/3)*log(-((b*c - a*d)*x^2*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^
(1/3) + (b*x^3 + a)^(1/3)*c*x*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(2/3) +
 (b*x^3 + a)^(2/3)*(b*c - a*d))/x^2) + 3*(b^2*c*d^2*x^4 + (2*b^2*c^2*d - 2*
a*b*c*d^2 + a^2*d^3)*x)*(b*x^3 + a)^(2/3))/(c*d^4*x^3 + c^2*d^3)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(8/3)/(d*x**3+c)**2,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(8/3)/(d*x^3+c)^2,x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(8/3)/(d*x^3 + c)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(b\,x^3+a\right)^{8/3}}{\left(d\,x^3+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(8/3)/(c + d*x^3)^2,x)
[Out] int((a + b*x^3)^(8/3)/(c + d*x^3)^2, x)
```

3.99 $\int \frac{(a+bx^3)^{5/3}}{(c+dx^3)^2} \, dx$

Optimal. Leaf size=301

$$-\frac{(bc-ad)x(a+bx^3)^{2/3}}{3cd\,(c+dx^3)}+\frac{b^{5/3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,d^2}-\frac{(bc-ad)^{2/3}(3bc+2ad)\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc-ad}\,x}{\sqrt[3]{c}\,\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}\,c^{5/3}d^2}$$

```
[Out] -1/3*(-a*d+b*c)*x*(b*x^3+a)^(2/3)/c/d/(d*x^3+c)-1/18*(-a*d+b*c)^(2/3)*(2*a*
d+3*b*c)*ln(d*x^3+c)/c^(5/3)/d^2+1/6*(-a*d+b*c)^(2/3)*(2*a*d+3*b*c)*ln((-a*
d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/c^(5/3)/d^2-1/2*b^(5/3)*ln(-b^(1/3)
*x+(b*x^3+a)^(1/3))/d^2+1/3*b^(5/3)*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/
3))*3^(1/2))/d^2*3^(1/2)-1/9*(-a*d+b*c)^(2/3)*(2*a*d+3*b*c)*arctan(1/3*(1+2
*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))/c^(5/3)/d^2*3^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 301, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {424, 544, 245, 384}

$$\frac{b^{5/3}\text{ArcTan}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,d^2}-\frac{(bc-ad)^{2/3}(2ad+3bc)\text{ArcTan}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\,\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{3\sqrt{3}\,c^{5/3}d^2}-\frac{b^{5/3}\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}\,x\right)}{2d^2}-\frac{(bc-ad)^{2/3}(2ad+3bc)\log(c+dx^3)}{18c^{5/3}d^2}+\frac{(bc-ad)^{2/3}(2ad+3bc)\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{6c^{5/3}d^2}-\frac{x(a+bx^3)^{2/3}(bc-ad)}{3cd\,(c+dx^3)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(5/3)/(c + d*x^3)^2,x]
```

```
[Out] -1/3*((b*c - a*d)*x*(a + b*x^3)^(2/3))/(c*d*(c + d*x^3)) + (b^(5/3)*ArcTan[
(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(Sqrt[3]*d^2) - ((b*c - a*d
)^(2/3)*(3*b*c + 2*a*d)*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a + b
*x^3)^(1/3)))/Sqrt[3]])/(3*Sqrt[3]*c^(5/3)*d^2) - ((b*c - a*d)^(2/3)*(3*b*c
 + 2*a*d)*Log[c + d*x^3])/(18*c^(5/3)*d^2) + ((b*c - a*d)^(2/3)*(3*b*c + 2*
a*d)*Log[((b*c - a*d)^(1/3)*x)/c^(1/3) - (a + b*x^3)^(1/3)])/(6*c^(5/3)*d^2
) - (b^(5/3)*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)])/(2*d^2)
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 384

```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
```

```
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 544

```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Dist[f/d, Int[(a + b*x^n)^p, x], x] + Dist[(d*e -
c*f)/d, Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, p
, n}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^3)^{5/3}}{(c+dx^3)^2}\,dx &= \frac{\left(a(a+bx^3)^{2/3}\right)\int \frac{\left(1+\frac{bx^3}{a}\right)^{5/3}}{(c+dx^3)^2}\,dx}{\left(1+\frac{bx^3}{a}\right)^{2/3}}\\ &= \frac{ax(a+bx^3)^{2/3}\,F_1\left(\frac{1}{3};-\frac{5}{3},2;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^2\left(1+\frac{bx^3}{a}\right)^{2/3}}\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 5.34, size = 510, normalized size = 1.69

$$\frac{-\frac{12d(bc-ad)x(a+bx^3)^{2/3}}{c(c+dx^3)}+12\sqrt{3}\,b^{5/3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{b}\,x}{\sqrt[3]{b}\,x+2\sqrt[3]{a+bx^3}}\right)+\frac{2i\left(3i+\sqrt{3}\right)(3b^2c^2-abcd-2a^2d^2)\tanh^{-1}\left(\frac{i+\frac{\left(-i+\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}\,x}}{\sqrt{3}}\right)}{c^{5/3}\sqrt[3]{bc-ad}}-12b^{5/3}\log\left(-\sqrt[3]{b}\,x+\sqrt[3]{a+bx^3}\right)-\frac{2i\left(-i+\sqrt{3}\right)(3b^2c^2-abcd-2a^2d^2)\log\left(2\sqrt[3]{bc-ad}\,x+\left(1+i\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{a+bx^3}\right)}{c^{5/3}\sqrt[3]{bc-ad}}+6b^{5/3}\log\left(b^{2/3}x^2+\sqrt[3]{b}\,x\sqrt[3]{a+bx^3}+(a+bx^3)^{2/3}\right)+\frac{\left(1+i\sqrt{3}\right)(3b^2c^2-abcd-2a^2d^2)\log\left(2(bc-ad)^{2/3}x^2+\left(-1-i\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{bc-ad}\,x\sqrt[3]{a+bx^3}+i\left(i+\sqrt{3}\right)c^{2/3}(a+bx^3)^{2/3}\right)}{c^{5/3}\sqrt[3]{bc-ad}}}{36d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^(5/3)/(c + d*x^3)^2,x]
```

```
[Out] ((-12*d*(b*c - a*d)*x*(a + b*x^3)^(2/3))/(c*(c + d*x^3)) + 12*Sqrt[3]*b^(5/
3)*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2*(a + b*x^3)^(1/3))] + ((2*I)*(
3*I + Sqrt[3])*(3*b^2*c^2 - a*b*c*d - 2*a^2*d^2)*ArcTanh[(I + ((-I + Sqrt[3
])*c^(1/3)*(a + b*x^3)^(1/3))/((b*c - a*d)^(1/3)*x))/Sqrt[3]])/(c^(5/3)*(b*
```

```
c - a*d)^(1/3)) - 12*b^(5/3)*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)] - ((2*I)
*(-I + Sqrt[3])*(3*b^2*c^2 - a*b*c*d - 2*a^2*d^2)*Log[2*(b*c - a*d)^(1/3)*x
 + (1 + I*Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3)])/(c^(5/3)*(b*c - a*d)^(1/3))
+ 6*b^(5/3)*Log[b^(2/3)*x^2 + b^(1/3)*x*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/
3)] + ((1 + I*Sqrt[3])*(3*b^2*c^2 - a*b*c*d - 2*a^2*d^2)*Log[2*(b*c - a*d)^
(2/3)*x^2 + (-1 - I*Sqrt[3])*c^(1/3)*(b*c - a*d)^(1/3)*x*(a + b*x^3)^(1/3)
+ I*(I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)])/(c^(5/3)*(b*c - a*d)^(1/3)))/
(36*d^2)
```

Maple [**F**]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^{3}+a\right)^{\frac{5}{3}}}{\left(d\,x^{3}+c\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(5/3)/(d*x^3+c)^2,x)
```

```
[Out] int((b*x^3+a)^(5/3)/(d*x^3+c)^2,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(5/3)/(d*x^3+c)^2,x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(5/3)/(d*x^3 + c)^2, x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 631 vs. $2(248) = 496$.
time = 5.55, size = 631, normalized size = 2.10

$$-\frac{2\sqrt{3}\left((3bcd+2ad^2)x^3+3bc^2+2acd\right)\left(\frac{b^2c^2-2abcd+a^2d^2}{c^2}\right)^{\frac{1}{3}}\arctan\left(-\frac{\sqrt{3}(bc-ad)x+2\sqrt{3}(bx^3+a)^{\frac{1}{3}}c\left(\frac{b^2c^2-2abcd+a^2d^2}{c^2}\right)^{\frac{1}{3}}}{3(bc-ad)x}\right)+6\sqrt{3}(bcdx^3+bc^2)(-b^2)^{\frac{1}{3}}\arctan\left(-\frac{\sqrt{3}bx-2\sqrt{3}(bx^3+a)^{\frac{1}{3}}(-b^2)^{\frac{1}{3}}}{3bx}\right)+6(bx^3+a)^{\frac{2}{3}}(bcd-ad^2)x-2\left((3bcd+2ad^2)x^3+3bc^2+2acd\right)\left(\frac{b^2c^2-2abcd+a^2d^2}{c^2}\right)^{\frac{1}{3}}\log\left(\frac{cx\left(\frac{b^2c^2-2abcd+a^2d^2}{c^2}\right)^{\frac{2}{3}}-(bx^3+a)^{\frac{1}{3}}(bc-ad)}{x}\right)-6(bcdx^3+bc^2)(-b^2)^{\frac{1}{3}}\log\left(-\frac{(-b^2)^{\frac{2}{3}}x-(bx^3+a)^{\frac{1}{3}}b}{x}\right)+3(bcdx^3+bc^2)(-b^2)^{\frac{1}{3}}\log\left(-\frac{(-b^2)^{\frac{1}{3}}bx^2-(bx^3+a)^{\frac{1}{3}}(-b^2)^{\frac{2}{3}}x-(bx^3+a)^{\frac{2}{3}}b}{x^2}\right)+\left((3bcd+2ad^2)x^3+3bc^2+2acd\right)\left(\frac{b^2c^2-2abcd+a^2d^2}{c^2}\right)^{\frac{1}{3}}\log\left(-\frac{(bc-ad)x^2\left(\frac{b^2c^2-2abcd+a^2d^2}{c^2}\right)^{\frac{1}{3}}+(bx^3+a)^{\frac{1}{3}}cx\left(\frac{b^2c^2-2abcd+a^2d^2}{c^2}\right)^{\frac{2}{3}}+(bx^3+a)^{\frac{2}{3}}(bc-ad)}{x^2}\right)}{18\left(cd^3x^3+c^2d^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(5/3)/(d*x^3+c)^2,x, algorithm="fricas")
```

```
[Out] -1/18*(2*sqrt(3)*((3*b*c*d + 2*a*d^2)*x^3 + 3*b*c^2 + 2*a*c*d)*((b^2*c^2 -
2*a*b*c*d + a^2*d^2)/c^2)^(1/3)*arctan(-1/3*(sqrt(3)*(b*c - a*d)*x + 2*sqrt
(3)*(b*x^3 + a)^(1/3)*c*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(1/3))/((b*c
- a*d)*x)) + 6*sqrt(3)*(b*c*d*x^3 + b*c^2)*(-b^2)^(1/3)*arctan(-1/3*(sqrt(3
)*b*x - 2*sqrt(3)*(b*x^3 + a)^(1/3)*(-b^2)^(1/3))/(b*x)) + 6*(b*x^3 + a)^(2
/3)*(b*c*d - a*d^2)*x - 2*((3*b*c*d + 2*a*d^2)*x^3 + 3*b*c^2 + 2*a*c*d)*((b
^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(1/3)*log((c*x*((b^2*c^2 - 2*a*b*c*d + a
```

```
^2*d^2)/c^2)^(2/3) - (b*x^3 + a)^(1/3)*(b*c - a*d))/x) - 6*(b*c*d*x^3 + b*c
^2)*(-b^2)^(1/3)*log(-((-b^2)^(2/3)*x - (b*x^3 + a)^(1/3)*b)/x) + 3*(b*c*d*
x^3 + b*c^2)*(-b^2)^(1/3)*log(-((-b^2)^(1/3)*b*x^2 - (b*x^3 + a)^(1/3)*(-b^
2)^(2/3)*x - (b*x^3 + a)^(2/3)*b)/x^2) + ((3*b*c*d + 2*a*d^2)*x^3 + 3*b*c^2
 + 2*a*c*d)*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(1/3)*log(-((b*c - a*d)*x
^2*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(1/3) + (b*x^3 + a)^(1/3)*c*x*((b^
2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(2/3) + (b*x^3 + a)^(2/3)*(b*c - a*d))/x^
2))/(c*d^3*x^3 + c^2*d^2)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx^3)^{\frac{5}{3}}}{(c+dx^3)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(5/3)/(d*x**3+c)**2,x)
[Out] Integral((a + b*x**3)**(5/3)/(c + d*x**3)**2, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(5/3)/(d*x^3+c)^2,x, algorithm="giac")
[Out] integrate((b*x^3 + a)^(5/3)/(d*x^3 + c)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(b\,x^3+a\right)}^{5/3}}{{\left(d\,x^3+c\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(5/3)/(c + d*x^3)^2,x)
[Out] int((a + b*x^3)^(5/3)/(c + d*x^3)^2, x)
```

3.100 $\int \frac{(a+bx^3)^{2/3}}{(c+dx^3)^2}\,dx$

Optimal. Leaf size=182

$$\frac{x(a+bx^3)^{2/3}}{3c(c+dx^3)}+\frac{2a\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc-ad}\,x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}\,c^{5/3}\sqrt[3]{bc-ad}}+\frac{a\log(c+dx^3)}{9c^{5/3}\sqrt[3]{bc-ad}}-\frac{a\log\left(\frac{\sqrt[3]{bc-ad}\,x}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{3c^{5/3}\sqrt[3]{bc-ad}}$$

```
[Out] 1/3*x*(b*x^3+a)^(2/3)/c/(d*x^3+c)+1/9*a*ln(d*x^3+c)/c^(5/3)/(-a*d+b*c)^(1/3
)-1/3*a*ln((-a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/c^(5/3)/(-a*d+b*c)^(
1/3)+2/9*a*arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1
/2))/c^(5/3)/(-a*d+b*c)^(1/3)*3^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}=0.095$, Rules used = {386, 384}

$$\frac{2a\text{ArcTan}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{3\sqrt{3}\,c^{5/3}\sqrt[3]{bc-ad}}+\frac{a\log(c+dx^3)}{9c^{5/3}\sqrt[3]{bc-ad}}-\frac{a\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{3c^{5/3}\sqrt[3]{bc-ad}}+\frac{x(a+bx^3)^{2/3}}{3c(c+dx^3)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(2/3)/(c + d*x^3)^2,x]
```

```
[Out] (x*(a + b*x^3)^(2/3))/(3*c*(c + d*x^3)) + (2*a*ArcTan[(1 + (2*(b*c - a*d)^(
1/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[3]])/(3*Sqrt[3]*c^(5/3)*(b*c - a*
d)^(1/3)) + (a*Log[c + d*x^3])/(9*c^(5/3)*(b*c - a*d)^(1/3)) - (a*Log[((b*c
 - a*d)^(1/3)*x)/c^(1/3) - (a + b*x^3)^(1/3)])/(3*c^(5/3)*(b*c - a*d)^(1/3)
)
```

Rule 384

```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 386

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] - Dist[
c*(q/(a*(p + 1))), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1,
0] && GtQ[q, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{(a+bx^3)^{2/3}}{(c+dx^3)^2}\,dx = \frac{x(a+bx^3)^{2/3}}{3c\,(c+dx^3)} + \frac{(2a)\int \frac{1}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx}{3c}$$

$$= \frac{x(a+bx^3)^{2/3}}{3c\,(c+dx^3)} + \frac{(2a)\text{Subst}\left(\int \frac{1}{c-(bc-ad)x^3}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{3c}$$

$$= \frac{x(a+bx^3)^{2/3}}{3c\,(c+dx^3)} + \frac{(2a)\text{Subst}\left(\int \frac{1}{\sqrt[3]{c}-\sqrt[3]{bc-ad}\,x}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{9c^{5/3}} + \frac{(2a)\text{Subst}\left(\int \frac{}{c^{2/\cdots}}\right.}{} \cdots$$

$$= \frac{x(a+bx^3)^{2/3}}{3c\,(c+dx^3)} - \frac{2a\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}\,x}{\sqrt[3]{a+bx^3}}\right)}{9c^{5/3}\sqrt[3]{bc-ad}} + \frac{a\text{Subst}\left(\int \frac{1}{c^{2/3}+\sqrt[3]{c}\,\sqrt[3]{bc-ad}\,x+(bc-ad\cdots}\right.}{3c^{4/3}} \cdots$$

$$= \frac{x(a+bx^3)^{2/3}}{3c\,(c+dx^3)} - \frac{2a\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}\,x}{\sqrt[3]{a+bx^3}}\right)}{9c^{5/3}\sqrt[3]{bc-ad}} + \frac{a\log\left(c^{2/3}+\frac{(bc-ad)^{2/3}x^2}{(a+bx^3)^{2/3}}+\frac{\sqrt[3]{c}\,\sqrt[3]{bc-\cdots}}{\sqrt[3]{a+b\cdots}}\right.}{9c^{5/3}\sqrt[3]{bc-ad}} \cdots$$

$$= \frac{x(a+bx^3)^{2/3}}{3c\,(c+dx^3)} + \frac{2a\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc-ad}\,x}{\sqrt[3]{c}\,\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}\,c^{5/3}\sqrt[3]{bc-ad}} - \frac{2a\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}\,x}{\sqrt[3]{a+bx^3}}\right)}{9c^{5/3}\sqrt[3]{bc-ad}} + \frac{a\,\text{lo}\cdots}{} \cdots$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.74, size = 319, normalized size = 1.75

$$\frac{\frac{6c^{2/3}x(a+bx^3)^{2/3}}{c+dx^3} - \frac{2\sqrt{-6+6i\sqrt{3}}\,a\tan^{-1}\left(\frac{3\sqrt[3]{bc-ad}\,x}{\sqrt{3}\,\sqrt[3]{bc-ad}\,x-\left(3i+\sqrt{3}\right)\sqrt[3]{c}\,\sqrt[3]{a+bx^3}}\right)}{\sqrt[3]{bc-ad}} + \frac{2\left(a+i\sqrt{3}\,a\right)\log\left(2\sqrt[3]{bc-ad}\,x+\left(1+i\sqrt{3}\right)\sqrt[3]{c}\,\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{bc-ad}} - \frac{i\left(-i+\sqrt{3}\right)a\log\left(2(bc-ad)^{2/3}x^2+\left(-1-i\sqrt{3}\right)\sqrt[3]{c}\,\sqrt[3]{bc-ad}\,x\sqrt[3]{a+bx^3}+i\left(i+\sqrt{3}\right)c^{2/3}(a+bx^3)^{2/3}\right)}{\sqrt[3]{bc-ad}}}{18c^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^(2/3)/(c + d*x^3)^2,x]
```

```
[Out] ((6*c^(2/3)*x*(a + b*x^3)^(2/3))/(c + d*x^3) - (2*Sqrt[-6 + (6*I)*Sqrt[3]]*
a*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(b*c - a*d)^(1/3)*x - (3*I + Sqrt
[3])*c^(1/3)*(a + b*x^3)^(1/3))])/(b*c - a*d)^(1/3) + (2*(a + I*Sqrt[3]*a)*
```

```
Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3)])/(b*
c - a*d)^(1/3) - (I*(-I + Sqrt[3])*a*Log[2*(b*c - a*d)^(2/3)*x^2 + (-1 - I*
Sqrt[3])*c^(1/3)*(b*c - a*d)^(1/3)*x*(a + b*x^3)^(1/3) + I*(I + Sqrt[3])*c^
(2/3)*(a + b*x^3)^(2/3)])/(b*c - a*d)^(1/3))/(18*c^(5/3))
```

Maple [**F**]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^{3}+a\right)^{\frac{2}{3}}}{\left(d\,x^{3}+c\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(2/3)/(d*x^3+c)^2,x)
```

```
[Out] int((b*x^3+a)^(2/3)/(d*x^3+c)^2,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/(d*x^3+c)^2,x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(2/3)/(d*x^3 + c)^2, x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/(d*x^3+c)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^{3}\right)^{\frac{2}{3}}}{\left(c+dx^{3}\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(2/3)/(d*x**3+c)**2,x)
```

```
[Out] Integral((a + b*x**3)**(2/3)/(c + d*x**3)**2, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/(d*x^3+c)^2,x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(2/3)/(d*x^3 + c)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(b\,x^3+a)^{2/3}}{(d\,x^3+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(2/3)/(c + d*x^3)^2,x)
```

```
[Out] int((a + b*x^3)^(2/3)/(c + d*x^3)^2, x)
```

3.101 $\int \frac{1}{\sqrt[3]{a+bx^3}\,(c+dx^3)^2}\,dx$

Optimal. Leaf size=217

$$-\frac{dx(a+bx^3)^{2/3}}{3c(bc-ad)\,(c+dx^3)}+\frac{(3bc-2ad)\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc-ad}\,x}{\sqrt[3]{c}\,\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}\,c^{5/3}(bc-ad)^{4/3}}+\frac{(3bc-2ad)\log(c+dx^3)}{18c^{5/3}(bc-ad)^{4/3}}-\frac{(3bc-2ad)\log\ldots}{6\ldots}$$

```
[Out] -1/3*d*x*(b*x^3+a)^(2/3)/c/(-a*d+b*c)/(d*x^3+c)+1/18*(-2*a*d+3*b*c)*ln(d*x^
3+c)/c^(5/3)/(-a*d+b*c)^(4/3)-1/6*(-2*a*d+3*b*c)*ln((-a*d+b*c)^(1/3)*x/c^(1
/3)-(b*x^3+a)^(1/3))/c^(5/3)/(-a*d+b*c)^(4/3)+1/9*(-2*a*d+3*b*c)*arctan(1/3
*(1+2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))/c^(5/3)/(-a*d+b*
c)^(4/3)*3^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {390, 384}

$$\frac{(3bc-2ad)\text{ArcTan}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\,\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{3\sqrt{3}\,c^{5/3}(bc-ad)^{4/3}}+\frac{(3bc-2ad)\log(c+dx^3)}{18c^{5/3}(bc-ad)^{4/3}}-\frac{(3bc-2ad)\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{6c^{5/3}(bc-ad)^{4/3}}-\frac{dx(a+bx^3)^{2/3}}{3c\,(c+dx^3)\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^3)^(1/3)*(c + d*x^3)^2),x]
```

```
[Out] -1/3*(d*x*(a + b*x^3)^(2/3))/(c*(b*c - a*d)*(c + d*x^3)) + ((3*b*c - 2*a*d)
*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[3]])
/(3*Sqrt[3]*c^(5/3)*(b*c - a*d)^(4/3)) + ((3*b*c - 2*a*d)*Log[c + d*x^3])/(
18*c^(5/3)*(b*c - a*d)^(4/3)) - ((3*b*c - 2*a*d)*Log[((b*c - a*d)^(1/3)*x)/
c^(1/3) - (a + b*x^3)^(1/3)])/(6*c^(5/3)*(b*c - a*d)^(4/3))
```

Rule 384

```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
```

```
a*d))), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)),
 Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q},
 x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !L
tQ[q, -1]) && NeQ[p, -1]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{a+bx^3}\,(c+dx^3)^2}\,dx = -\frac{dx(a+bx^3)^{2/3}}{3c(bc-ad)(c+dx^3)} + \frac{(3bc-2ad)\int \frac{1}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx}{3c(bc-ad)}$$

$$= -\frac{dx(a+bx^3)^{2/3}}{3c(bc-ad)(c+dx^3)} + \frac{(3bc-2ad)\text{Subst}\left(\int \frac{1}{c-(bc-ad)x^3}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{3c(bc-ad)}$$

$$= -\frac{dx(a+bx^3)^{2/3}}{3c(bc-ad)(c+dx^3)} + \frac{(3bc-2ad)\text{Subst}\left(\int \frac{1}{\sqrt[3]{c}-\sqrt[3]{bc-ad}\,x}\,dx, x, \frac{x}{\sqrt[3]{a+\cdots}}\right.}{9c^{5/3}(bc-ad)}$$

$$= -\frac{dx(a+bx^3)^{2/3}}{3c(bc-ad)(c+dx^3)} - \frac{(3bc-2ad)\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}\,x}{\sqrt[3]{a+bx^3}}\right)}{9c^{5/3}(bc-ad)^{4/3}} + \frac{(3bc-2a\cdots}{}$$

$$= -\frac{dx(a+bx^3)^{2/3}}{3c(bc-ad)(c+dx^3)} - \frac{(3bc-2ad)\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}\,x}{\sqrt[3]{a+bx^3}}\right)}{9c^{5/3}(bc-ad)^{4/3}} + \frac{(3bc-2a\cdots}{}$$

$$= -\frac{dx(a+bx^3)^{2/3}}{3c(bc-ad)(c+dx^3)} + \frac{(3bc-2ad)\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc-ad}\,x}{\sqrt[3]{c}\,\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}\,c^{5/3}(bc-ad)^{4/3}} - \frac{(3bc-\cdots}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 2.13, size = 336, normalized size = 1.55

$$\frac{-12c^{2/3}d\sqrt[3]{bc-ad}\,x(a+bx^3)^{2/3}+2\left(3-i\sqrt{3}\right)(3bc-2ad)(c+dx^3)\tanh^{-1}\left(\frac{i+\frac{\left(-i+\sqrt{3}\right)\sqrt[3]{c}\,\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}\,x}}{\sqrt{3}}\right)+2\left(1+i\sqrt{3}\right)(3bc-2ad)(c+dx^3)\log\left(2\sqrt[3]{bc-ad}\,x+\left(1+i\sqrt{3}\right)\sqrt[3]{c}\,\sqrt[3]{a+bx^3}\right)-i\left(-i+\sqrt{3}\right)(3bc-2ad)(c+dx^3)\log\left(2(bc-ad)^{2/3}x^2+\left(-1-i\sqrt{3}\right)\sqrt[3]{c}\,\sqrt[3]{bc-ad}\,x\sqrt[3]{a+bx^3}+i\left(i+\sqrt{3}\right)c^{2/3}(a+bx^3)^{2/3}\right)}{36c^{5/3}(bc-ad)^{4/3}(c+dx^3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^3)^(1/3)*(c + d*x^3)^2),x]
```

```
[Out] (-12*c^(2/3)*d*(b*c - a*d)^(1/3)*x*(a + b*x^3)^(2/3) + 2*(3 - I*Sqrt[3])*(3
*b*c - 2*a*d)*(c + d*x^3)*ArcTanh[(I + ((-I + Sqrt[3])*c^(1/3)*(a + b*x^3)^
(1/3))/((b*c - a*d)^(1/3)*x))/Sqrt[3]] + 2*(1 + I*Sqrt[3])*(3*b*c - 2*a*d)*
(c + d*x^3)*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*(a + b*x^3)
```

```
^(1/3)] - I*(-I + Sqrt[3])*(3*b*c - 2*a*d)*(c + d*x^3)*Log[2*(b*c - a*d)^(2
/3)*x^2 + (-1 - I*Sqrt[3])*c^(1/3)*(b*c - a*d)^(1/3)*x*(a + b*x^3)^(1/3) +
I*(I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)])/(36*c^(5/3)*(b*c - a*d)^(4/3)*(
c + d*x^3))
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^{3}+a\right)^{\frac{1}{3}} \left(d\,x^{3}+c\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^3+a)^(1/3)/(d*x^3+c)^2,x)
```

```
[Out] int(1/(b*x^3+a)^(1/3)/(d*x^3+c)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(1/3)/(d*x^3+c)^2,x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)^(1/3)*(d*x^3 + c)^2), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(1/3)/(d*x^3+c)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{a + bx^{3}}\, \left(c + dx^{3}\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3+a)**(1/3)/(d*x**3+c)**2,x)
```

```
[Out] Integral(1/((a + b*x**3)**(1/3)*(c + d*x**3)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(1/3)/(d*x^3+c)^2,x, algorithm="giac")
[Out] integrate(1/((b*x^3 + a)^(1/3)*(d*x^3 + c)^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(b\,x^3+a\right)}^{1/3}\,{\left(d\,x^3+c\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^3)^(1/3)*(c + d*x^3)^2),x)
[Out] int(1/((a + b*x^3)^(1/3)*(c + d*x^3)^2), x)
```

3.102 $\int \frac{1}{(a+bx^3)^{4/3}(c+dx^3)^2}\,dx$

Optimal. Leaf size=261

$$\frac{b(3bc+ad)x}{3ac(bc-ad)^2\sqrt[3]{a+bx^3}}-\frac{dx}{3c(bc-ad)\sqrt[3]{a+bx^3}\,(c+dx^3)}-\frac{2d(3bc-ad)\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc-ad}\,x}{\sqrt[3]{c}\,\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}\,c^{5/3}(bc-ad)^{7/3}}-\frac{d(3bc}{9}$$

```
[Out] 1/3*b*(a*d+3*b*c)*x/a/c/(-a*d+b*c)^2/(b*x^3+a)^(1/3)-1/3*d*x/c/(-a*d+b*c)/(
b*x^3+a)^(1/3)/(d*x^3+c)-1/9*d*(-a*d+3*b*c)*ln(d*x^3+c)/c^(5/3)/(-a*d+b*c)^
(7/3)+1/3*d*(-a*d+3*b*c)*ln((-a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/c^(
5/3)/(-a*d+b*c)^(7/3)-2/9*d*(-a*d+3*b*c)*arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*x
/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))/c^(5/3)/(-a*d+b*c)^(7/3)*3^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 261, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {425, 541, 12, 384}

$$-\frac{2d(3bc-ad)\text{ArcTan}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\,\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{3\sqrt{3}\,c^{5/3}(bc-ad)^{7/3}}-\frac{d(3bc-ad)\log\left(c+dx^3\right)}{9c^{5/3}(bc-ad)^{7/3}}+\frac{d(3bc-ad)\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{3c^{5/3}(bc-ad)^{7/3}}+\frac{bx(ad+3bc)}{3ac\sqrt[3]{a+bx^3}\,(bc-ad)^2}-\frac{dx}{3c\sqrt[3]{a+bx^3}\,(c+dx^3)\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^3)^(4/3)*(c + d*x^3)^2),x]
```

```
[Out] (b*(3*b*c + a*d)*x)/(3*a*c*(b*c - a*d)^2*(a + b*x^3)^(1/3)) - (d*x)/(3*c*(b
*c - a*d)*(a + b*x^3)^(1/3)*(c + d*x^3)) - (2*d*(3*b*c - a*d)*ArcTan[(1 + (
2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[3]])/(3*Sqrt[3]*c^
(5/3)*(b*c - a*d)^(7/3)) - (d*(3*b*c - a*d)*Log[c + d*x^3])/(9*c^(5/3)*(b*c
 - a*d)^(7/3)) + (d*(3*b*c - a*d)*Log[((b*c - a*d)^(1/3)*x)/c^(1/3) - (a +
b*x^3)^(1/3)])/(3*c^(5/3)*(b*c - a*d)^(7/3))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 384

```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
```

```
a*d, 0]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{1}{\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)^2}\,dx = \frac{\sqrt[3]{1+\frac{bx^3}{a}}\int \frac{1}{\left(1+\frac{bx^3}{a}\right)^{4/3}\left(c+dx^3\right)^2}\,dx}{a\sqrt[3]{a+bx^3}}$$

$$= -\frac{c\left(a+bx^3\right)^{2/3}\left(6860+\frac{13720dx^3}{c}+\frac{6300d^2x^6}{c^2}-\frac{525(bc-ad)x^3}{c(a+bx^3)}-\frac{1890d(bc-ad)x^6}{c^2(a+bx^3)}-\right.}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 3.44, size = 370, normalized size = 1.42

$$\frac{\frac{6c^{2/3}x\left(a^2d^2+abd^2x^3+3b^2c\left(c+dx^3\right)\right)}{a(bc-ad)^2\sqrt[3]{a+bx^3}\left(c+dx^3\right)}+\frac{2i\left(3i+\sqrt{3}\right)d(3bc-ad)\tanh^{-1}\left(\frac{i+\frac{\left(-i+\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}\,x}}{\sqrt{3}}\right)}{(bc-ad)^{7/3}}+\frac{2\left(1+i\sqrt{3}\right)d(-3bc+ad)\log\left(2\sqrt[3]{bc-ad}\,x+\left(1+i\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{a+bx^3}\right)}{(bc-ad)^{7/3}}+\frac{\left(1+i\sqrt{3}\right)d(3bc-ad)\log\left(2(bc-ad)^{2/3}x^2+\left(-1-i\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{bc-ad}\,x\sqrt[3]{a+bx^3}+i\left(i+\sqrt{3}\right)c^{2/3}\left(a+bx^3\right)^{2/3}\right)}{(bc-ad)^{7/3}}}{18c^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^3)^(4/3)*(c + d*x^3)^2),x]
```

```
[Out] ((6*c^(2/3)*x*(a^2*d^2 + a*b*d^2*x^3 + 3*b^2*c*(c + d*x^3)))/(a*(b*c - a*d)
^2*(a + b*x^3)^(1/3)*(c + d*x^3)) + ((2*I)*(3*I + Sqrt[3])*d*(3*b*c - a*d)*
ArcTanh[(I + ((-I + Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3))/((b*c - a*d)^(1/3)*
x))/Sqrt[3]])/(b*c - a*d)^(7/3) + (2*(1 + I*Sqrt[3])*d*(-3*b*c + a*d)*Log[2
*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3)])/(b*c - a
*d)^(7/3) + ((1 + I*Sqrt[3])*d*(3*b*c - a*d)*Log[2*(b*c - a*d)^(2/3)*x^2 +
(-1 - I*Sqrt[3])*c^(1/3)*(b*c - a*d)^(1/3)*x*(a + b*x^3)^(1/3) + I*(I + Sqr
t[3])*c^(2/3)*(a + b*x^3)^(2/3)])/(b*c - a*d)^(7/3))/(18*c^(5/3))
```

Maple [**F**]
time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^{3}+a\right)^{\frac{4}{3}} \left(d\,x^{3}+c\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^3+a)^(4/3)/(d*x^3+c)^2,x)
```

```
[Out] int(1/(b*x^3+a)^(4/3)/(d*x^3+c)^2,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(4/3)/(d*x^3+c)^2,x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)^(4/3)*(d*x^3 + c)^2), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(4/3)/(d*x^3+c)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+bx^{3}\right)^{\frac{4}{3}} \left(c+dx^{3}\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3+a)**(4/3)/(d*x**3+c)**2,x)
[Out] Integral(1/((a + b*x**3)**(4/3)*(c + d*x**3)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(4/3)/(d*x^3+c)^2,x, algorithm="giac")
[Out] integrate(1/((b*x^3 + a)^(4/3)*(d*x^3 + c)^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(b\,x^3+a\right)}^{4/3}\,{\left(d\,x^3+c\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^3)^(4/3)*(c + d*x^3)^2),x)
[Out] int(1/((a + b*x^3)^(4/3)*(c + d*x^3)^2), x)
```

3.103 $\int \frac{1}{(a+bx^3)^{7/3}(c+dx^3)^2}\,dx$

Optimal. Leaf size=324

$$\frac{b(3bc+4ad)x}{12ac(bc-ad)^2\left(a+bx^3\right)^{4/3}}+\frac{b\left(9b^2c^2-33abcd-4a^2d^2\right)x}{12a^2c(bc-ad)^3\sqrt[3]{a+bx^3}}-\frac{dx}{3c(bc-ad)\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}+\frac{d^2(9bc-2ad}{3\sqrt{}}$$

[Out] 1/12*b*(4*a*d+3*b*c)*x/a/c/(-a*d+b*c)^2/(b*x^3+a)^(4/3)+1/12*b*(-4*a^2*d^2-33*a*b*c*d+9*b^2*c^2)*x/a^2/c/(-a*d+b*c)^3/(b*x^3+a)^(1/3)-1/3*d*x/c/(-a*d+b*c)/(b*x^3+a)^(4/3)/(d*x^3+c)+1/18*d^2*(-2*a*d+9*b*c)*ln(d*x^3+c)/c^(5/3)/(-a*d+b*c)^(10/3)-1/6*d^2*(-2*a*d+9*b*c)*ln((-a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/c^(5/3)/(-a*d+b*c)^(10/3)+1/9*d^2*(-2*a*d+9*b*c)*arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))/c^(5/3)/(-a*d+b*c)^(10/3)*3^(1/2)

Rubi [A]
time = 0.25, antiderivative size = 324, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {425, 541, 12, 384}

$$\frac{bx\left(-4a^2d^2-33abcd+9b^2c^2\right)}{12a^2c\sqrt[3]{a+bx^3}(bc-ad)^3}+\frac{d^2(9bc-2ad)\text{ArcTan}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{3\sqrt{3}c^{5/3}(bc-ad)^{10/3}}+\frac{d^2(9bc-2ad)\log\left(c+dx^3\right)}{18c^{5/3}(bc-ad)^{10/3}}-\frac{d^2(9bc-2ad)\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{6c^{5/3}(bc-ad)^{10/3}}-\frac{dx}{3c\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)(bc-ad)}+\frac{bx(4ad+3bc)}{12ac\left(a+bx^3\right)^{4/3}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^3)^(7/3)*(c + d*x^3)^2),x]
```

```
[Out] (b*(3*b*c + 4*a*d)*x)/(12*a*c*(b*c - a*d)^2*(a + b*x^3)^(4/3)) + (b*(9*b^2*
c^2 - 33*a*b*c*d - 4*a^2*d^2)*x)/(12*a^2*c*(b*c - a*d)^3*(a + b*x^3)^(1/3))
 - (d*x)/(3*c*(b*c - a*d)*(a + b*x^3)^(4/3)*(c + d*x^3)) + (d^2*(9*b*c - 2*
a*d)*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[
3]])/(3*Sqrt[3]*c^(5/3)*(b*c - a*d)^(10/3)) + (d^2*(9*b*c - 2*a*d)*Log[c +
d*x^3])/(18*c^(5/3)*(b*c - a*d)^(10/3)) - (d^2*(9*b*c - 2*a*d)*Log[((b*c -
a*d)^(1/3)*x)/c^(1/3) - (a + b*x^3)^(1/3)])/(6*c^(5/3)*(b*c - a*d)^(10/3))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 384

```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
```

```
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{1}{(a+bx^3)^{7/3}(c+dx^3)^2}\,dx = \frac{\sqrt[3]{1+\frac{bx^3}{a}}\int \frac{1}{\left(1+\frac{bx^3}{a}\right)^{7/3}(c+dx^3)^2}\,dx}{a^2\sqrt[3]{a+bx^3}}$$
$$= \frac{26130c^5(bc-ad)^2x^6(a+bx^3)^2+89505c^4d(bc-ad)^2x^9(a+bx^3)^2+84240c}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 6.31, size = 443, normalized size = 1.37

$$\frac{\frac{3c^{2/3}x\left(4a^4d^3+8a^3bd^3x^3-9b^4c^2x^3(c+dx^3)+4a^2b^2d(9c^2+9cdx^3+d^2x^6)+3ab^3c(-4c^2+7cdx^3+11d^2x^6)\right)}{a^2(-bc+ad)^3(a+bx^3)^{4/3}(c+dx^3)}+\frac{2i\left(3i+\sqrt{3}\right)d^2(-9bc+2ad)\tanh^{-1}\left(\frac{i+\frac{\left(-i+\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}\,x}}{\sqrt{3}}\right)}{(bc-ad)^{10/3}}+\frac{2\left(1+i\sqrt{3}\right)d^2(9bc-2ad)\log\left(2\sqrt[3]{bc-ad}\,x+\left(1+i\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{a+bx^3}\right)}{(bc-ad)^{10/3}}+\frac{\left(1+i\sqrt{3}\right)d^2(-9bc+2ad)\log\left(2(bc-ad)^{2/3}x^2+\left(-1-i\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{bc-ad}\,x\sqrt[3]{a+bx^3}+i\left(i+\sqrt{3}\right)c^{2/3}(a+bx^3)^{2/3}\right)}{(bc-ad)^{10/3}}}{36c^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^3)^(7/3)*(c + d*x^3)^2),x]
```

```
[Out] ((3*c^(2/3)*x*(4*a^4*d^3 + 8*a^3*b*d^3*x^3 - 9*b^4*c^2*x^3*(c + d*x^3) + 4*
a^2*b^2*d*(9*c^2 + 9*c*d*x^3 + d^2*x^6) + 3*a*b^3*c*(-4*c^2 + 7*c*d*x^3 + 1
1*d^2*x^6)))/(a^2*(-(b*c) + a*d)^3*(a + b*x^3)^(4/3)*(c + d*x^3)) + ((2*I)*
(3*I + Sqrt[3])*d^2*(-9*b*c + 2*a*d)*ArcTanh[(I + ((-I + Sqrt[3])*c^(1/3)*(
a + b*x^3)^(1/3))/((b*c - a*d)^(1/3)*x))/Sqrt[3]])/(b*c - a*d)^(10/3) + (2*
(1 + I*Sqrt[3])*d^2*(9*b*c - 2*a*d)*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt
[3])*c^(1/3)*(a + b*x^3)^(1/3)])/(b*c - a*d)^(10/3) + ((1 + I*Sqrt[3])*d^2*
(-9*b*c + 2*a*d)*Log[2*(b*c - a*d)^(2/3)*x^2 + (-1 - I*Sqrt[3])*c^(1/3)*(b*
c - a*d)^(1/3)*x*(a + b*x^3)^(1/3) + I*(I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2
/3)])/(b*c - a*d)^(10/3))/(36*c^(5/3))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^{3}+a\right)^{\frac{7}{3}} \left(d\,x^{3}+c\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^3+a)^(7/3)/(d*x^3+c)^2,x)
```

```
[Out] int(1/(b*x^3+a)^(7/3)/(d*x^3+c)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(7/3)/(d*x^3+c)^2,x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)^(7/3)*(d*x^3 + c)^2), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(7/3)/(d*x^3+c)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+bx^{3}\right)^{\frac{7}{3}} \left(c+dx^{3}\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3+a)**(7/3)/(d*x**3+c)**2,x)
```

```
[Out] Integral(1/((a + b*x**3)**(7/3)*(c + d*x**3)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(7/3)/(d*x^3+c)^2,x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^3 + a)^(7/3)*(d*x^3 + c)^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(b\,x^3+a\right)}^{7/3}\,{\left(d\,x^3+c\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^3)^(7/3)*(c + d*x^3)^2),x)
```

```
[Out] int(1/((a + b*x^3)^(7/3)*(c + d*x^3)^2), x)
```

3.104 $\int \frac{(a+bx^3)^{4/3}}{(c+dx^3)^2} dx$

Optimal. Leaf size=60

$$\frac{ax\sqrt[3]{a+bx^3}\, F_1\left(\frac{1}{3};-\frac{4}{3},2;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^2\sqrt[3]{1+\frac{bx^3}{a}}}$$

```
[Out] a*x*(b*x^3+a)^(1/3)*AppellF1(1/3,-4/3,2,4/3,-b*x^3/a,-d*x^3/c)/c^2/(1+b*x^3
/a)^(1/3)
```

Rubi [A]
time = 0.02, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {441, 440}

$$\frac{ax\sqrt[3]{a+bx^3}\, F_1\left(\frac{1}{3};-\frac{4}{3},2;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^2\sqrt[3]{\frac{bx^3}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(4/3)/(c + d*x^3)^2,x]
```

```
[Out] (a*x*(a + b*x^3)^(1/3)*AppellF1[1/3, -4/3, 2, 4/3, -((b*x^3)/a), -((d*x^3)/
c)])/(c^2*(1 + (b*x^3)/a)^(1/3))
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{(a+bx^3)^{4/3}}{(c+dx^3)^2}\,dx = \frac{\left(a\sqrt[3]{a+bx^3}\right)\int \frac{\left(1+\frac{bx^3}{a}\right)^{4/3}}{(c+dx^3)^2}\,dx}{\sqrt[3]{1+\frac{bx^3}{a}}} = \frac{ax\sqrt[3]{a+bx^3}\,F_1\left(\frac{1}{3};-\frac{4}{3},2;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^2\sqrt[3]{1+\frac{bx^3}{a}}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 341 vs. $2(60) = 120$.
time = 10.23, size = 341, normalized size = 5.68

$$\frac{x\left(b(2bc+ad)x^3\left(1+\frac{bx^3}{a}\right)^{2/3}F_1\left(\frac{4}{3};\frac{2}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+\frac{4c\left(-4ac(3a^2d-b^2cx^3+abdx^3)F_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+(-bc+ad)x^3(a+bx^3)\left(3adF_1\left(\frac{4}{3};\frac{2}{3},2;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+2bcF_1\left(\frac{4}{3};\frac{5}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)\right)}{(c+dx^3)\left(-4acF_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+x^3\left(3adF_1\left(\frac{4}{3};\frac{2}{3},2;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+2bcF_1\left(\frac{4}{3};\frac{5}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)\right)}\right)}{12c^2d\,(a+bx^3)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(4/3)/(c + d*x^3)^2,x]
```

```
[Out] (x*(b*(2*b*c + a*d)*x^3*(1 + (b*x^3)/a)^(2/3)*AppellF1[4/3, 2/3, 1, 7/3, -(
(b*x^3)/a), -((d*x^3)/c)] + (4*c*(-4*a*c*(3*a^2*d - b^2*c*x^3 + a*b*d*x^3)*
AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)] + (-(b*c) + a*d)*x^3
*(a + b*x^3)*(3*a*d*AppellF1[4/3, 2/3, 2, 7/3, -((b*x^3)/a), -((d*x^3)/c)]
+ 2*b*c*AppellF1[4/3, 5/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)])))/((c + d*x
^3)*(-4*a*c*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)] + x^3*(3
*a*d*AppellF1[4/3, 2/3, 2, 7/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*b*c*AppellF
1[4/3, 5/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)])))))/(12*c^2*d*(a + b*x^3)^
(2/3))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(b\,x^3+a)^{\frac{4}{3}}}{(d\,x^3+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(4/3)/(d*x^3+c)^2,x)
```

```
[Out] int((b*x^3+a)^(4/3)/(d*x^3+c)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(4/3)/(d*x^3+c)^2,x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(4/3)/(d*x^3 + c)^2, x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(4/3)/(d*x^3+c)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx^3)^{\frac{4}{3}}}{(c+dx^3)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(4/3)/(d*x**3+c)**2,x)
```

```
[Out] Integral((a + b*x**3)**(4/3)/(c + d*x**3)**2, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(4/3)/(d*x^3+c)^2,x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(4/3)/(d*x^3 + c)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(b\,x^3+a)^{4/3}}{(d\,x^3+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(4/3)/(c + d*x^3)^2,x)
```

```
[Out] int((a + b*x^3)^(4/3)/(c + d*x^3)^2, x)
```

3.105 $\int \frac{\sqrt[3]{a+bx^3}}{\left(c+dx^3\right)^2}\,dx$

Optimal. Leaf size=59

$$\frac{x\sqrt[3]{a+bx^3}\,F_1\left(\frac{1}{3};-\frac{1}{3},2;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^2\sqrt[3]{1+\frac{bx^3}{a}}}$$

```
[Out] x*(b*x^3+a)^(1/3)*AppellF1(1/3,-1/3,2,4/3,-b*x^3/a,-d*x^3/c)/c^2/(1+b*x^3/a
)^(1/3)
```

Rubi [A]
time = 0.02, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {441, 440}

$$\frac{x\sqrt[3]{a+bx^3}\,F_1\left(\frac{1}{3};-\frac{1}{3},2;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^2\sqrt[3]{\frac{bx^3}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(1/3)/(c + d*x^3)^2,x]
```

```
[Out] (x*(a + b*x^3)^(1/3)*AppellF1[1/3, -1/3, 2, 4/3, -((b*x^3)/a), -((d*x^3)/c)
])/(c^2*(1 + (b*x^3)/a)^(1/3))
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{\sqrt[3]{a+bx^3}}{(c+dx^3)^2}\,dx = \frac{\sqrt[3]{a+bx^3}\int \frac{\sqrt[3]{1+\frac{bx^3}{a}}}{(c+dx^3)^2}\,dx}{\sqrt[3]{1+\frac{bx^3}{a}}}$$
$$= \frac{x\sqrt[3]{a+bx^3}\,F_1\left(\frac{1}{3};-\frac{1}{3},2;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^2\sqrt[3]{1+\frac{bx^3}{a}}}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 232 vs. 2(59) = 118.
time = 10.16, size = 232, normalized size = 3.93

$$\frac{x\left(\frac{bx^3\left(1+\frac{bx^3}{a}\right)^{2/3}F_1\left(\frac{4}{3};\frac{2}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^2}+\frac{4\left(\frac{a+bx^3}{c}-\frac{8a^2F_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{-4acF_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+x^3\left(3adF_1\left(\frac{4}{3};\frac{2}{3},2;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+2bcF_1\left(\frac{4}{3};\frac{5}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)}\right)}{c+dx^3}\right)}{12\left(a+bx^3\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(1/3)/(c + d*x^3)^2,x]
```

```
[Out] (x*((b*x^3*(1 + (b*x^3)/a)^(2/3)*AppellF1[4/3, 2/3, 1, 7/3, -((b*x^3)/a), -
((d*x^3)/c)])/c^2 + (4*((a + b*x^3)/c - (8*a^2*AppellF1[1/3, 2/3, 1, 4/3, -
((b*x^3)/a), -((d*x^3)/c)])/(-4*a*c*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a)
, -((d*x^3)/c)] + x^3*(3*a*d*AppellF1[4/3, 2/3, 2, 7/3, -((b*x^3)/a), -((d*
x^3)/c)] + 2*b*c*AppellF1[4/3, 5/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)]))))
/(c + d*x^3)))/(12*(a + b*x^3)^(2/3))
```

Maple [**F**]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^3+a\right)^{\frac{1}{3}}}{\left(d\,x^3+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(1/3)/(d*x^3+c)^2,x)
```

```
[Out] int((b*x^3+a)^(1/3)/(d*x^3+c)^2,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/(d*x^3+c)^2,x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(1/3)/(d*x^3 + c)^2, x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/(d*x^3+c)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{a+bx^3}}{(c+dx^3)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(1/3)/(d*x**3+c)**2,x)
```

```
[Out] Integral((a + b*x**3)**(1/3)/(c + d*x**3)**2, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/(d*x^3+c)^2,x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(1/3)/(d*x^3 + c)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{{\left(b\,x^3+a\right)}^{1/3}}{{\left(d\,x^3+c\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(1/3)/(c + d*x^3)^2,x)
```

```
[Out] int((a + b*x^3)^(1/3)/(c + d*x^3)^2, x)
```

3.106 $\int \frac{1}{(a+bx^3)^{2/3}(c+dx^3)^2}\,dx$

Optimal. Leaf size=59

$$\frac{x\left(1+\frac{bx^3}{a}\right)^{2/3}F_1\left(\frac{1}{3};\frac{2}{3},2;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^2\left(a+bx^3\right)^{2/3}}$$

```
[Out] x*(1+b*x^3/a)^(2/3)*AppellF1(1/3,2/3,2,4/3,-b*x^3/a,-d*x^3/c)/c^2/(b*x^3+a)
^(2/3)
```

Rubi [A]
time = 0.02, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {441, 440}

$$\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}F_1\left(\frac{1}{3};\frac{2}{3},2;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^2\left(a+bx^3\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^3)^(2/3)*(c + d*x^3)^2),x]
```

```
[Out] (x*(1 + (b*x^3)/a)^(2/3)*AppellF1[1/3, 2/3, 2, 4/3, -((b*x^3)/a), -((d*x^3)
/c)])/(c^2*(a + b*x^3)^(2/3))
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{1}{(a+bx^3)^{2/3}(c+dx^3)^2}\,dx = \frac{\left(1+\frac{bx^3}{a}\right)^{2/3}\int \frac{1}{\left(1+\frac{bx^3}{a}\right)^{2/3}(c+dx^3)^2}\,dx}{(a+bx^3)^{2/3}}$$

$$= \frac{x\left(1+\frac{bx^3}{a}\right)^{2/3} F_1\left(\frac{1}{3};\frac{2}{3},2;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^2(a+bx^3)^{2/3}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 393 vs. 2(59) = 118.

time = 10.21, size = 393, normalized size = 6.66

$$\frac{4acxF_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\left(4c(-3bc+3ad+bdx^3)+bdx^3\left(1+\frac{bx^3}{a}\right)^{2/3}(c+dx^3)F_1\left(\frac{4}{3};\frac{2}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)-dx^4\left(4c(a+bx^3)+bx^3\left(1+\frac{bx^3}{a}\right)^{2/3}(c+dx^3)F_1\left(\frac{4}{3};\frac{2}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)\left(3adF_1\left(\frac{4}{3};\frac{2}{3},2;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+2bcF_1\left(\frac{4}{3};\frac{5}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)}{12c^2(bc-ad)(a+bx^3)^{2/3}(c+dx^3)\left(-4acF_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+x^3\left(3adF_1\left(\frac{4}{3};\frac{2}{3},2;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+2bcF_1\left(\frac{4}{3};\frac{5}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^3)^(2/3)*(c + d*x^3)^2),x]
```

```
[Out] (4*a*c*x*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)]*(4*c*(-3*b*
c + 3*a*d + b*d*x^3) + b*d*x^3*(1 + (b*x^3)/a)^(2/3)*(c + d*x^3)*AppellF1[4
/3, 2/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)]) - d*x^4*(4*c*(a + b*x^3) + b*
x^3*(1 + (b*x^3)/a)^(2/3)*(c + d*x^3)*AppellF1[4/3, 2/3, 1, 7/3, -((b*x^3)/
a), -((d*x^3)/c)])*(3*a*d*AppellF1[4/3, 2/3, 2, 7/3, -((b*x^3)/a), -((d*x^3
)/c)] + 2*b*c*AppellF1[4/3, 5/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)]))/(12*
c^2*(b*c - a*d)*(a + b*x^3)^(2/3)*(c + d*x^3)*(-4*a*c*AppellF1[1/3, 2/3, 1,
 4/3, -((b*x^3)/a), -((d*x^3)/c)] + x^3*(3*a*d*AppellF1[4/3, 2/3, 2, 7/3, -
((b*x^3)/a), -((d*x^3)/c)] + 2*b*c*AppellF1[4/3, 5/3, 1, 7/3, -((b*x^3)/a),
 -((d*x^3)/c)])))
```

Maple [F]

time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\,x^3+a)^{\frac{2}{3}}(d\,x^3+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^3+a)^(2/3)/(d*x^3+c)^2,x)
```

```
[Out] int(1/(b*x^3+a)^(2/3)/(d*x^3+c)^2,x)
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(2/3)/(d*x^3+c)^2,x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)^(2/3)*(d*x^3 + c)^2), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(2/3)/(d*x^3+c)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+bx^3\right)^{\frac{2}{3}}\left(c+dx^3\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3+a)**(2/3)/(d*x**3+c)**2,x)
```

```
[Out] Integral(1/((a + b*x**3)**(2/3)*(c + d*x**3)**2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(2/3)/(d*x^3+c)^2,x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^3 + a)^(2/3)*(d*x^3 + c)^2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{{\left(b\,x^3+a\right)}^{2/3}\,{\left(d\,x^3+c\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^3)^(2/3)*(c + d*x^3)^2),x)
```

```
[Out] int(1/((a + b*x^3)^(2/3)*(c + d*x^3)^2), x)
```

3.107 $\int \frac{1}{(a+bx^3)^{5/3}(c+dx^3)^2}\,dx$

Optimal. Leaf size=62

$$\frac{x\left(1+\frac{bx^3}{a}\right)^{2/3}F_1\left(\frac{1}{3};\frac{5}{3},2;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{ac^2\left(a+bx^3\right)^{2/3}}$$

```
[Out] x*(1+b*x^3/a)^(2/3)*AppellF1(1/3,5/3,2,4/3,-b*x^3/a,-d*x^3/c)/a/c^2/(b*x^3+
a)^(2/3)
```

Rubi [A]
time = 0.02, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = $\{441, 440\}$

$$\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}F_1\left(\frac{1}{3};\frac{5}{3},2;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{ac^2\left(a+bx^3\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^3)^(5/3)*(c + d*x^3)^2),x]
```

```
[Out] (x*(1 + (b*x^3)/a)^(2/3)*AppellF1[1/3, 5/3, 2, 4/3, -((b*x^3)/a), -((d*x^3)
/c)])/(a*c^2*(a + b*x^3)^(2/3))
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{1}{(a+bx^3)^{5/3}(c+dx^3)^2}\,dx = \frac{\left(1+\frac{bx^3}{a}\right)^{2/3}\int \frac{1}{\left(1+\frac{bx^3}{a}\right)^{5/3}(c+dx^3)^2}\,dx}{a(a+bx^3)^{2/3}}$$
$$= \frac{x\left(1+\frac{bx^3}{a}\right)^{2/3} F_1\left(\frac{1}{3};\frac{5}{3},2;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{ac^2(a+bx^3)^{2/3}}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 386 vs. 2(62) = 124.
time = 10.37, size = 386, normalized size = 6.23

$$\frac{x\left(bd(3bc+2ad)x^3\left(1+\frac{bx^3}{a}\right)^{2/3}F_1\left(\frac{4}{3};\frac{2}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+\frac{c\left(16ac(6a^2d^2+2abd(-6c+dx^3)+3b^2c(2c+dx^3))F_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)-4x^3(2a^2d^2+2abd^2x^3+3b^2c(c+dx^3))\left(3adF_1\left(\frac{4}{3};\frac{2}{3},2;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+2bcF_1\left(\frac{4}{3};\frac{5}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)\right)}{(c+dx^3)\left(4acF_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)-x^3\left(3adF_1\left(\frac{4}{3};\frac{2}{3},2;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+2bcF_1\left(\frac{4}{3};\frac{5}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)\right)}\right)}{24ac^2(bc-ad)^2(a+bx^3)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^3)^(5/3)*(c + d*x^3)^2),x]
```

```
[Out] (x*(b*d*(3*b*c + 2*a*d)*x^3*(1 + (b*x^3)/a)^(2/3)*AppellF1[4/3, 2/3, 1, 7/3
, -((b*x^3)/a), -((d*x^3)/c)] + (c*(16*a*c*(6*a^2*d^2 + 2*a*b*d*(-6*c + d*x
^3) + 3*b^2*c*(2*c + d*x^3))*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*
x^3)/c)] - 4*x^3*(2*a^2*d^2 + 2*a*b*d^2*x^3 + 3*b^2*c*(c + d*x^3))*(3*a*d*A
ppellF1[4/3, 2/3, 2, 7/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*b*c*AppellF1[4/3,
 5/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)])))/((c + d*x^3)*(4*a*c*AppellF1[1
/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)] - x^3*(3*a*d*AppellF1[4/3, 2/3
, 2, 7/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*b*c*AppellF1[4/3, 5/3, 1, 7/3, -(
(b*x^3)/a), -((d*x^3)/c)])))))/(24*a*c^2*(b*c - a*d)^2*(a + b*x^3)^(2/3))
```

Maple [**F**]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\,x^3+a)^{\frac{5}{3}}\,(d\,x^3+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^3+a)^(5/3)/(d*x^3+c)^2,x)
```

```
[Out] int(1/(b*x^3+a)^(5/3)/(d*x^3+c)^2,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(5/3)/(d*x^3+c)^2,x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)^(5/3)*(d*x^3 + c)^2), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(5/3)/(d*x^3+c)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx^3)^{\frac{5}{3}}\,(c+dx^3)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3+a)**(5/3)/(d*x**3+c)**2,x)
```

```
[Out] Integral(1/((a + b*x**3)**(5/3)*(c + d*x**3)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(5/3)/(d*x^3+c)^2,x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^3 + a)^(5/3)*(d*x^3 + c)^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{{\left(b\,x^3+a\right)}^{5/3}\,{\left(d\,x^3+c\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^3)^(5/3)*(c + d*x^3)^2),x)
```

```
[Out] int(1/((a + b*x^3)^(5/3)*(c + d*x^3)^2), x)
```

3.108 $\int \frac{1}{(a+bx^3)^{8/3}(c+dx^3)^2} dx$

Optimal. Leaf size=62

$$\frac{x\left(1+\frac{bx^3}{a}\right)^{2/3} F_1\left(\frac{1}{3};\frac{8}{3},2;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2c^2\left(a+bx^3\right)^{2/3}}$$

```
[Out] x*(1+b*x^3/a)^(2/3)*AppellF1(1/3,8/3,2,4/3,-b*x^3/a,-d*x^3/c)/a^2/c^2/(b*x^
3+a)^(2/3)
```

Rubi [A]

time = 0.02, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {441, 440}

$$\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3} F_1\left(\frac{1}{3};\frac{8}{3},2;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2c^2\left(a+bx^3\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^3)^(8/3)*(c + d*x^3)^2),x]
```

```
[Out] (x*(1 + (b*x^3)/a)^(2/3)*AppellF1[1/3, 8/3, 2, 4/3, -((b*x^3)/a), -((d*x^3)
/c)])/(a^2*c^2*(a + b*x^3)^(2/3))
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{1}{(a+bx^3)^{8/3}(c+dx^3)^2}\,dx = \frac{\left(1+\frac{bx^3}{a}\right)^{2/3}\int \frac{1}{\left(1+\frac{bx^3}{a}\right)^{8/3}(c+dx^3)^2}\,dx}{a^2\,(a+bx^3)^{2/3}}$$

$$= \frac{x\left(1+\frac{bx^3}{a}\right)^{2/3} F_1\left(\frac{1}{3};\frac{8}{3},2;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2c^2\,(a+bx^3)^{2/3}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 550 vs. 2(62) = 124.

time = 10.66, size = 550, normalized size = 8.87

$$\frac{\frac{bd(-6b^2c^2+21abcd+5a^2d^2)x^4\left(1+\frac{bx^3}{a}\right)^{2/3}F_1\left(\frac{4}{3};\frac{2}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{(-bc+ad)^3}+\frac{4c\left(-4acx(15a^4d^3-6b^4c^2x^3(2c+dx^3)+5a^3bd^2(-9c+4dx^3)+a^2b^2d(45c^2-21cdx^3+5d^2x^6)+3ab^3c(-5c^2+11cdx^3+7d^2x^6))F_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+x^4(5a^4d^3+10a^3bd^3x^3-6b^4c^2x^3(c+dx^3)+a^2b^2d(24c^2+24cdx^3+5d^2x^6)+3ab^3c(-3c^2+4cdx^3+7d^2x^6))\left(3adF_1\left(\frac{4}{3};\frac{2}{3},2;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+2bcF_1\left(\frac{4}{3};\frac{5}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)\right)}{(bc-ad)^3(a+bx^3)(c+dx^3)\left(4acF_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)-x^3\left(3adF_1\left(\frac{4}{3};\frac{2}{3},2;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+2bcF_1\left(\frac{4}{3};\frac{5}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)\right)}}{60a^2c^2\,(a+bx^3)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^3)^(8/3)*(c + d*x^3)^2),x]
```

```
[Out] ((b*d*(-6*b^2*c^2 + 21*a*b*c*d + 5*a^2*d^2)*x^4*(1 + (b*x^3)/a)^(2/3)*Appel
lF1[4/3, 2/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)])/(-(b*c) + a*d)^3 + (4*c*
(-4*a*c*x*(15*a^4*d^3 - 6*b^4*c^2*x^3*(2*c + d*x^3) + 5*a^3*b*d^2*(-9*c + 4
*d*x^3) + a^2*b^2*d*(45*c^2 - 21*c*d*x^3 + 5*d^2*x^6) + 3*a*b^3*c*(-5*c^2 +
 11*c*d*x^3 + 7*d^2*x^6))*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3
)/c)] + x^4*(5*a^4*d^3 + 10*a^3*b*d^3*x^3 - 6*b^4*c^2*x^3*(c + d*x^3) + a^2
*b^2*d*(24*c^2 + 24*c*d*x^3 + 5*d^2*x^6) + 3*a*b^3*c*(-3*c^2 + 4*c*d*x^3 +
7*d^2*x^6))*(3*a*d*AppellF1[4/3, 2/3, 2, 7/3, -((b*x^3)/a), -((d*x^3)/c)] +
 2*b*c*AppellF1[4/3, 5/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)])))/((b*c - a*
d)^3*(a + b*x^3)*(c + d*x^3)*(4*a*c*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a)
, -((d*x^3)/c)] - x^3*(3*a*d*AppellF1[4/3, 2/3, 2, 7/3, -((b*x^3)/a), -((d*
x^3)/c)] + 2*b*c*AppellF1[4/3, 5/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)]))))
/(60*a^2*c^2*(a + b*x^3)^(2/3))
```

Maple [F]

time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\,x^3+a)^{\frac{8}{3}}\,(d\,x^3+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^3+a)^(8/3)/(d*x^3+c)^2,x)
```

```
[Out] int(1/(b*x^3+a)^(8/3)/(d*x^3+c)^2,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(8/3)/(d*x^3+c)^2,x, algorithm="maxima")
[Out] integrate(1/((b*x^3 + a)^(8/3)*(d*x^3 + c)^2), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(8/3)/(d*x^3+c)^2,x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx^3)^{\frac{8}{3}}\,(c+dx^3)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3+a)**(8/3)/(d*x**3+c)**2,x)
[Out] Integral(1/((a + b*x**3)**(8/3)*(c + d*x**3)**2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(8/3)/(d*x^3+c)^2,x, algorithm="giac")
[Out] integrate(1/((b*x^3 + a)^(8/3)*(d*x^3 + c)^2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{{\left(b\,x^3+a\right)}^{8/3}\,{\left(d\,x^3+c\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^3)^(8/3)*(c + d*x^3)^2),x)
[Out] int(1/((a + b*x^3)^(8/3)*(c + d*x^3)^2), x)
```

3.109 $\int \frac{(a+bx^3)^{14/3}}{(c+dx^3)^3}\,dx$

Optimal. Leaf size=541

$$-\frac{b(2bc-ad)\left(18b^2c^2-18abcd-5a^2d^2\right)x\left(a+bx^3\right)^{2/3}}{18c^2d^4}+\frac{b\left(18b^2c^2-10abcd-5a^2d^2\right)x\left(a+bx^3\right)^{5/3}}{18c^2d^3}-\frac{(bc-ac}{6cc}$$

```
[Out] -1/18*b*(-a*d+2*b*c)*(-5*a^2*d^2-18*a*b*c*d+18*b^2*c^2)*x*(b*x^3+a)^(2/3)/c
^2/d^4+1/18*b*(-5*a^2*d^2-10*a*b*c*d+18*b^2*c^2)*x*(b*x^3+a)^(5/3)/c^2/d^3-
1/6*(-a*d+b*c)*x*(b*x^3+a)^(11/3)/c/d/(d*x^3+c)^2-1/18*(-a*d+b*c)*(5*a*d+12
*b*c)*x*(b*x^3+a)^(8/3)/c^2/d^2/(d*x^3+c)-1/54*(-a*d+b*c)^(8/3)*(5*a^2*d^2+
18*a*b*c*d+54*b^2*c^2)*ln(d*x^3+c)/c^(8/3)/d^5+1/18*(-a*d+b*c)^(8/3)*(5*a^2
*d^2+18*a*b*c*d+54*b^2*c^2)*ln((-a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/
c^(8/3)/d^5-1/18*b^(8/3)*(77*a^2*d^2-126*a*b*c*d+54*b^2*c^2)*ln(-b^(1/3)*x+
(b*x^3+a)^(1/3))/d^5+1/27*b^(8/3)*(77*a^2*d^2-126*a*b*c*d+54*b^2*c^2)*arcta
n(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/d^5*3^(1/2)-1/27*(-a*d+b*c)^
(8/3)*(5*a^2*d^2+18*a*b*c*d+54*b^2*c^2)*arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*x/
c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))/c^(8/3)/d^5*3^(1/2)
```

Rubi [A]
time = 0.50, antiderivative size = 541, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {424, 540, 542, 544, 245, 384}

$$-\frac{(bc-ad)^{8/3}\left(5a^2d^2+18abcd+54b^2c^2\right)\text{ArcTan}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{9\sqrt{3}c^{8/3}d^5}+\frac{b^{8/3}\text{ArcTan}\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)\left(77a^2d^2-126abcd+54b^2c^2\right)}{9\sqrt{3}d^5}-\frac{bx\left(a+bx^3\right)^{2/3}(2bc-ad)\left(-5a^2d^2-18abcd+18b^2c^2\right)}{18c^2d^4}+\frac{bx\left(a+bx^3\right)^{5/3}\left(-5a^2d^2-10abcd+18b^2c^2\right)}{18c^2d^3}-\frac{(bc-ad)^{8/3}\left(5a^2d^2+18abcd+54b^2c^2\right)\log\left(c+dx^3\right)}{54c^{8/3}d^5}+\frac{(bc-ad)^{8/3}\left(5a^2d^2+18abcd+54b^2c^2\right)\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{18c^{8/3}d^5}-\frac{b^{8/3}\left(77a^2d^2-126abcd+54b^2c^2\right)\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{18d^5}-\frac{x\left(a+bx^3\right)^{8/3}(bc-ad)(5ad+12bc)}{18c^2d^2\left(c+dx^3\right)}-\frac{x\left(a+bx^3\right)^{11/3}(bc-ad)}{6cd\left(c+dx^3\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(14/3)/(c + d*x^3)^3,x]
```

```
[Out] -1/18*(b*(2*b*c - a*d)*(18*b^2*c^2 - 18*a*b*c*d - 5*a^2*d^2)*x*(a + b*x^3)^
(2/3))/(c^2*d^4) + (b*(18*b^2*c^2 - 10*a*b*c*d - 5*a^2*d^2)*x*(a + b*x^3)^(
5/3))/(18*c^2*d^3) - ((b*c - a*d)*x*(a + b*x^3)^(11/3))/(6*c*d*(c + d*x^3)^
2) - ((b*c - a*d)*(12*b*c + 5*a*d)*x*(a + b*x^3)^(8/3))/(18*c^2*d^2*(c + d*
x^3)) + (b^(8/3)*(54*b^2*c^2 - 126*a*b*c*d + 77*a^2*d^2)*ArcTan[(1 + (2*b^(
1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(9*Sqrt[3]*d^5) - ((b*c - a*d)^(8/3)*(
54*b^2*c^2 + 18*a*b*c*d + 5*a^2*d^2)*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c
^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[3]])/(9*Sqrt[3]*c^(8/3)*d^5) - ((b*c - a*d)
^(8/3)*(54*b^2*c^2 + 18*a*b*c*d + 5*a^2*d^2)*Log[c + d*x^3])/(54*c^(8/3)*d^
5) + ((b*c - a*d)^(8/3)*(54*b^2*c^2 + 18*a*b*c*d + 5*a^2*d^2)*Log[((b*c - a
*d)^(1/3)*x)/c^(1/3) - (a + b*x^3)^(1/3)])/(18*c^(8/3)*d^5) - (b^(8/3)*(54*
```

```
b^2*c^2 - 126*a*b*c*d + 77*a^2*d^2)*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)])/
(18*d^5)
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 384

```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 540

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^q/(a*b*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(
p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(b*e*n*(p + 1) + b*e - a*f) + d*(b*e*n*(p
 + 1) + (b*e - a*f)*(n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f,
n}, x] && LtQ[p, -1] && GtQ[q, 0]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 544

```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Dist[f/d, Int[(a + b*x^n)^p, x], x] + Dist[(d*e -
```

```
c*f)/d, Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, p
, n}, x]
```

Rubi steps

$$\int \frac{(a+bx^3)^{14/3}}{(c+dx^3)^3}\,dx = \frac{\left(a^4(a+bx^3)^{2/3}\right)\int \frac{\left(1+\frac{bx^3}{a}\right)^{14/3}}{(c+dx^3)^3}\,dx}{\left(1+\frac{bx^3}{a}\right)^{2/3}}$$

$$= \frac{a^4x(a+bx^3)^{2/3}\,F_1\left(\frac{1}{3};-\frac{14}{3},3;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^3\left(1+\frac{bx^3}{a}\right)^{2/3}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.
time = 11.57, size = 1171, normalized size = 2.16

[illegible]

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(14/3)/(c + d*x^3)^3,x]
```

```
[Out] ((6*x*(a + b*x^3)^(2/3)*(-2*b^3*(9*b*c - 13*a*d) + 3*b^4*d*x^3 + (3*(b*c -
a*d)^4)/(c*(c + d*x^3)^2) - ((b*c - a*d)^3*(21*b*c + 5*a*d))/(c^2*(c + d*x^
3))))/d^4 + (162*b^5*c*x^4*(1 + (b*x^3)/a)^(1/3)*AppellF1[4/3, 1/3, 1, 7/3,
 -((b*x^3)/a), -((d*x^3)/c)])/(d^4*(a + b*x^3)^(1/3)) - (378*a*b^4*x^4*(1 +
 (b*x^3)/a)^(1/3)*AppellF1[4/3, 1/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)])/(
d^3*(a + b*x^3)^(1/3)) + (231*a^2*b^3*x^4*(1 + (b*x^3)/a)^(1/3)*AppellF1[4/
3, 1/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)])/(c*d^2*(a + b*x^3)^(1/3)) + (1
0*a^5*(2*Sqrt[3]*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(b + a*x^3)^(
1/3)))/Sqrt[3]] - 2*Log[c^(1/3) - ((b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)]
+ Log[c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(b + a*x^3)^(2/3) + (c^(1/3)*(b*c -
 a*d)^(1/3)*x)/(b + a*x^3)^(1/3)]))/(c^(8/3)*(b*c - a*d)^(1/3)) + (36*a*b^4
*c^(4/3)*(2*Sqrt[3]*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(b + a*x^3
)^(1/3)))/Sqrt[3]] - 2*Log[c^(1/3) - ((b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3
)] + Log[c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(b + a*x^3)^(2/3) + (c^(1/3)*(b*
c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)]))/(d^4*(b*c - a*d)^(1/3)) - (72*a^2*b^
3*c^(1/3)*(2*Sqrt[3]*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(b + a*x^
3)^(1/3)))/Sqrt[3]] - 2*Log[c^(1/3) - ((b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/
3)] + Log[c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(b + a*x^3)^(2/3) + (c^(1/3)*(b
*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)]))/(d^3*(b*c - a*d)^(1/3)) + (30*a^3*b
^2*(2*Sqrt[3]*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(b + a*x^3)^(1/3
```

```
)))/Sqrt[3]] - 2*Log[c^(1/3) - ((b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)] + L
og[c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(b + a*x^3)^(2/3) + (c^(1/3)*(b*c - a*
d)^(1/3)*x)/(b + a*x^3)^(1/3)]))/(c^(2/3)*d^2*(b*c - a*d)^(1/3)) + (6*a^4*b
*(2*Sqrt[3]*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(b + a*x^3)^(1/3))
)/Sqrt[3]] - 2*Log[c^(1/3) - ((b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)] + Log
[c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(b + a*x^3)^(2/3) + (c^(1/3)*(b*c - a*d)
^(1/3)*x)/(b + a*x^3)^(1/3)]))/(c^(5/3)*d*(b*c - a*d)^(1/3)))/108
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^{3}+a\right)^{\frac{14}{3}}}{\left(d\,x^{3}+c\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(14/3)/(d*x^3+c)^3,x)
[Out] int((b*x^3+a)^(14/3)/(d*x^3+c)^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(14/3)/(d*x^3+c)^3,x, algorithm="maxima")
[Out] integrate((b*x^3 + a)^(14/3)/(d*x^3 + c)^3, x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(14/3)/(d*x^3+c)^3,x, algorithm="fricas")
[Out] Timed out
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(14/3)/(d*x**3+c)**3,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(14/3)/(d*x^3+c)^3,x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(14/3)/(d*x^3 + c)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(b\,x^3+a)^{14/3}}{(d\,x^3+c)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(14/3)/(c + d*x^3)^3,x)
```

```
[Out] int((a + b*x^3)^(14/3)/(c + d*x^3)^3, x)
```

3.110 $\int \frac{(a+bx^3)^{11/3}}{(c+dx^3)^3}\,dx$

Optimal. Leaf size=458

$$\frac{b(18b^2c^2-7abcd-5a^2d^2)\,x(a+bx^3)^{2/3}}{18c^2d^3}-\frac{(bc-ad)x(a+bx^3)^{8/3}}{6cd\,(c+dx^3)^2}-\frac{(bc-ad)(9bc+5ad)x(a+bx^3)^{5/3}}{18c^2d^2\,(c+dx^3)}-\frac{b^{8/3}(9\cdots}{\cdots}$$

```
[Out] 1/18*b*(-5*a^2*d^2-7*a*b*c*d+18*b^2*c^2)*x*(b*x^3+a)^(2/3)/c^2/d^3-1/6*(-a*
d+b*c)*x*(b*x^3+a)^(8/3)/c/d/(d*x^3+c)^2-1/18*(-a*d+b*c)*(5*a*d+9*b*c)*x*(b
*x^3+a)^(5/3)/c^2/d^2/(d*x^3+c)+1/54*(-a*d+b*c)^(5/3)*(5*a^2*d^2+12*a*b*c*d
+27*b^2*c^2)*ln(d*x^3+c)/c^(8/3)/d^4-1/18*(-a*d+b*c)^(5/3)*(5*a^2*d^2+12*a*
b*c*d+27*b^2*c^2)*ln((-a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/c^(8/3)/d^
4+1/6*b^(8/3)*(-11*a*d+9*b*c)*ln(-b^(1/3)*x+(b*x^3+a)^(1/3))/d^4-1/9*b^(8/3
)*(-11*a*d+9*b*c)*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/d^4*3
^(1/2)+1/27*(-a*d+b*c)^(5/3)*(5*a^2*d^2+12*a*b*c*d+27*b^2*c^2)*arctan(1/3*(
1+2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))/c^(8/3)/d^4*3^(1/2
)
```

Rubi [A]
time = 0.37, antiderivative size = 458, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$,
Rules used = {424, 540, 542, 544, 245, 384}

$$\frac{(bc-ad)^{5/3}(5a^2d^2+12abcd+27b^2c^2)\,\text{ArcTan}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{9\sqrt{3}\,c^{8/3}d^4}+\frac{bx(a+bx^3)^{2/3}(-5a^2d^2-7abcd+18b^2c^2)}{18c^2d^3}+\frac{(bc-ad)^{5/3}(5a^2d^2+12abcd+27b^2c^2)\log(c+dx^3)}{54c^{8/3}d^4}-\frac{(bc-ad)^{5/3}(5a^2d^2+12abcd+27b^2c^2)\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{18c^{8/3}d^4}-\frac{b^{8/3}\text{ArcTan}\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)(9bc-11ad)}{3\sqrt{3}\,d^4}+\frac{b^{8/3}(9bc-11ad)\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}\,x\right)}{6d^4}-\frac{x(a+bx^3)^{5/3}(bc-ad)(5ad+9bc)}{18c^2d^2(c+dx^3)}-\frac{x(a+bx^3)^{8/3}(bc-ad)}{6cd(c+dx^3)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(11/3)/(c + d*x^3)^3,x]
```

```
[Out] (b*(18*b^2*c^2 - 7*a*b*c*d - 5*a^2*d^2)*x*(a + b*x^3)^(2/3))/(18*c^2*d^3) -
 ((b*c - a*d)*x*(a + b*x^3)^(8/3))/(6*c*d*(c + d*x^3)^2) - ((b*c - a*d)*(9*
b*c + 5*a*d)*x*(a + b*x^3)^(5/3))/(18*c^2*d^2*(c + d*x^3)) - (b^(8/3)*(9*b*
c - 11*a*d)*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(3*Sqrt[
3]*d^4) + ((b*c - a*d)^(5/3)*(27*b^2*c^2 + 12*a*b*c*d + 5*a^2*d^2)*ArcTan[(
1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[3]])/(9*Sqrt[
3]*c^(8/3)*d^4) + ((b*c - a*d)^(5/3)*(27*b^2*c^2 + 12*a*b*c*d + 5*a^2*d^2)*
Log[c + d*x^3])/(54*c^(8/3)*d^4) - ((b*c - a*d)^(5/3)*(27*b^2*c^2 + 12*a*b*
c*d + 5*a^2*d^2)*Log[((b*c - a*d)^(1/3)*x)/c^(1/3) - (a + b*x^3)^(1/3)])/(1
8*c^(8/3)*d^4) + (b^(8/3)*(9*b*c - 11*a*d)*Log[-(b^(1/3)*x) + (a + b*x^3)^(
1/3)])/(6*d^4)
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 384

```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 540

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^q/(a*b*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(
p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(b*e*n*(p + 1) + b*e - a*f) + d*(b*e*n*(p
 + 1) + (b*e - a*f)*(n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f,
n}, x] && LtQ[p, -1] && GtQ[q, 0]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 544

```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Dist[f/d, Int[(a + b*x^n)^p, x], x] + Dist[(d*e -
c*f)/d, Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, p
, n}, x]
```

Rubi steps

$$\int \frac{(a+bx^3)^{11/3}}{(c+dx^3)^3}\,dx = \frac{\left(a^3(a+bx^3)^{2/3}\right)\int \frac{\left(1+\frac{bx^3}{a}\right)^{11/3}}{(c+dx^3)^3}\,dx}{\left(1+\frac{bx^3}{a}\right)^{2/3}}$$
$$= \frac{a^3x(a+bx^3)^{2/3}\,F_1\left(\frac{1}{3};-\frac{11}{3},3;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^3\left(1+\frac{bx^3}{a}\right)^{2/3}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.
time = 11.33, size = 908, normalized size = 1.98

$$\frac{1}{108}\left(\frac{6x(a+bx^3)^{2/3}\left(6b^3-\frac{3(bc-ad)^3}{c(c+dx^3)^2}+\frac{5(bc-ad)^2(3bc+ad)}{c^2(c+dx^3)}\right)}{d^3}-\frac{81b^4x^4\sqrt[3]{1+\frac{bx^3}{a}}\,F_1\left(\frac{4}{3};\frac{1}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{d^3\sqrt[3]{a+bx^3}}+\frac{99ab^3x^4\sqrt[3]{1+\frac{bx^3}{a}}\,F_1\left(\frac{4}{3};\frac{1}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{cd^2\sqrt[3]{a+bx^3}}+\frac{10a^4\left(2\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc-ad}\,x}{\sqrt[3]{c}\sqrt[3]{b+ax^3}}}{\sqrt{3}}\right)-2\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}\,x}{\sqrt[3]{b+ax^3}}\right)+\log\left(c^{2/3}+\frac{(bc-ad)^{2/3}x^2}{(b+ax^3)^{2/3}}+\frac{\sqrt[3]{c}\sqrt[3]{bc-ad}\,x}{\sqrt[3]{b+ax^3}}\right)\right)}{c^{8/3}\sqrt[3]{bc-ad}}-\frac{18ab^3\sqrt[3]{c}\left(2\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc-ad}\,x}{\sqrt[3]{c}\sqrt[3]{b+ax^3}}}{\sqrt{3}}\right)-2\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}\,x}{\sqrt[3]{b+ax^3}}\right)+\log\left(c^{2/3}+\frac{(bc-ad)^{2/3}x^2}{(b+ax^3)^{2/3}}+\frac{\sqrt[3]{c}\sqrt[3]{bc-ad}\,x}{\sqrt[3]{b+ax^3}}\right)\right)}{d^3\sqrt[3]{bc-ad}}+\frac{16a^2b^2\left(2\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc-ad}\,x}{\sqrt[3]{c}\sqrt[3]{b+ax^3}}}{\sqrt{3}}\right)-2\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}\,x}{\sqrt[3]{b+ax^3}}\right)+\log\left(c^{2/3}+\frac{(bc-ad)^{2/3}x^2}{(b+ax^3)^{2/3}}+\frac{\sqrt[3]{c}\sqrt[3]{bc-ad}\,x}{\sqrt[3]{b+ax^3}}\right)\right)}{c^{2/3}d^2\sqrt[3]{bc-ad}}+\frac{4a^3b\left(2\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc-ad}\,x}{\sqrt[3]{c}\sqrt[3]{b+ax^3}}}{\sqrt{3}}\right)-2\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}\,x}{\sqrt[3]{b+ax^3}}\right)+\log\left(c^{2/3}+\frac{(bc-ad)^{2/3}x^2}{(b+ax^3)^{2/3}}+\frac{\sqrt[3]{c}\sqrt[3]{bc-ad}\,x}{\sqrt[3]{b+ax^3}}\right)\right)}{c^{5/3}d\sqrt[3]{bc-ad}}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(11/3)/(c + d*x^3)^3,x]
```

```
[Out] ((6*x*(a + b*x^3)^(2/3)*(6*b^3 - (3*(b*c - a*d)^3)/(c*(c + d*x^3)^2) + (5*(
b*c - a*d)^2*(3*b*c + a*d))/(c^2*(c + d*x^3))))/d^3 - (81*b^4*x^4*(1 + (b*x
^3)/a)^(1/3)*AppellF1[4/3, 1/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)])/(d^3*(
a + b*x^3)^(1/3)) + (99*a*b^3*x^4*(1 + (b*x^3)/a)^(1/3)*AppellF1[4/3, 1/3,
1, 7/3, -((b*x^3)/a), -((d*x^3)/c)])/(c*d^2*(a + b*x^3)^(1/3)) + (10*a^4*(2
*Sqrt[3]*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(b + a*x^3)^(1/3)))/S
qrt[3]] - 2*Log[c^(1/3) - ((b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)] + Log[c^
(2/3) + ((b*c - a*d)^(2/3)*x^2)/(b + a*x^3)^(2/3) + (c^(1/3)*(b*c - a*d)^(1
/3)*x)/(b + a*x^3)^(1/3)]))/(c^(8/3)*(b*c - a*d)^(1/3)) - (18*a*b^3*c^(1/3)
*(2*Sqrt[3]*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(b + a*x^3)^(1/3))
)/Sqrt[3]] - 2*Log[c^(1/3) - ((b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)] + Log
[c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(b + a*x^3)^(2/3) + (c^(1/3)*(b*c - a*d)
^(1/3)*x)/(b + a*x^3)^(1/3)]))/(d^3*(b*c - a*d)^(1/3)) + (16*a^2*b^2*(2*Sqr
t[3]*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(b + a*x^3)^(1/3)))/Sqrt[
3]] - 2*Log[c^(1/3) - ((b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)] + Log[c^(2/3
) + ((b*c - a*d)^(2/3)*x^2)/(b + a*x^3)^(2/3) + (c^(1/3)*(b*c - a*d)^(1/3)*
x)/(b + a*x^3)^(1/3)]))/(c^(2/3)*d^2*(b*c - a*d)^(1/3)) + (4*a^3*b*(2*Sqrt[
3]*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(b + a*x^3)^(1/3)))/Sqrt[3]
] - 2*Log[c^(1/3) - ((b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)] + Log[c^(2/3)
+ ((b*c - a*d)^(2/3)*x^2)/(b + a*x^3)^(2/3) + (c^(1/3)*(b*c - a*d)^(1/3)*x)
/(b + a*x^3)^(1/3)]))/(c^(5/3)*d*(b*c - a*d)^(1/3)))/108
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(b\,x^3+a)^{\frac{11}{3}}}{(d\,x^3+c)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^3+a)^(11/3)/(d*x^3+c)^3,x)

[Out] int((b*x^3+a)^(11/3)/(d*x^3+c)^3,x)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^(11/3)/(d*x^3+c)^3,x, algorithm="maxima")

[Out] integrate((b*x^3 + a)^(11/3)/(d*x^3 + c)^3, x)

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1246 vs. 2(394) = 788.
time = 59.21, size = 1246, normalized size = 2.72

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^(11/3)/(d*x^3+c)^3,x, algorithm="fricas")

[Out] 1/54*(2*sqrt(3)*(27*b^3*c^5 - 15*a*b^2*c^4*d - 7*a^2*b*c^3*d^2 - 5*a^3*c^2*d^3 + (27*b^3*c^3*d^2 - 15*a*b^2*c^2*d^3 - 7*a^2*b*c*d^4 - 5*a^3*d^5)*x^6 + 2*(27*b^3*c^4*d - 15*a*b^2*c^3*d^2 - 7*a^2*b*c^2*d^3 - 5*a^3*c*d^4)*x^3)*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(1/3)*arctan(-1/3*(sqrt(3)*(b*c - a*d)*x + 2*sqrt(3)*(b*x^3 + a)^(1/3)*c*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(1/3))/((b*c - a*d)*x)) + 6*sqrt(3)*(9*b^3*c^5 - 11*a*b^2*c^4*d + (9*b^3*c^3*d^2 - 11*a*b^2*c^2*d^3)*x^6 + 2*(9*b^3*c^4*d - 11*a*b^2*c^3*d^2)*x^3)*(-b^2)^(1/3)*arctan(-1/3*(sqrt(3)*b*x - 2*sqrt(3)*(b*x^3 + a)^(1/3)*(-b^2)^(1/3))/(b*x)) - 2*(27*b^3*c^5 - 15*a*b^2*c^4*d - 7*a^2*b*c^3*d^2 - 5*a^3*c^2*d^3 + (27*b^3*c^3*d^2 - 15*a*b^2*c^2*d^3 - 7*a^2*b*c*d^4 - 5*a^3*d^5)*x^6 + 2*(27*b^3*c^4*d - 15*a*b^2*c^3*d^2 - 7*a^2*b*c^2*d^3 - 5*a^3*c*d^4)*x^3)*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(1/3)*log((c*x*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(2/3) - (b*x^3 + a)^(1/3)*(b*c - a*d))/x) - 6*(9*b^3*c^5 - 11*a*b^2*c^4*d + (9*b^3*c^3*d^2 - 11*a*b^2*c^2*d^3)*x^6 + 2*(9*b^3*c^4*d - 11*a*b^2*c^3*d^2)*x^3)*(-b^2)^(1/3)*log(-((-b^2)^(2/3)*x - (b*x^3 + a)^(1/3)*b)/x) + 3*(9*b^3*c^5 - 11*a*b^2*c^4*d + (9*b^3*c^3*d^2 - 11*a*b^2*c^2*d^3)*x^6 + 2*(9*b^3*c^4*d - 11*a*b^2*c^3*d^2)*x^3)*(-b^2)^(1/3)*log(-((-b^2)^(1/3)*b*x^2 - (b*x^3 + a)^(1/3)*(-b^2)^(2/3)*x - (b*x^3 + a)^(2/3)*b)/x^2) + (27*b^3*c^5 - 15*a*b^2*c^4*d - 7*a^2*b*c^3*d^2 - 5*a^3*c^2*d^3 + (27*b^3*c^3*d^2 - 15*a*b^2*c^2*d^3 - 7*a^2*b*c*d^4 - 5*a^3*d^5)*x^6 + 2*(27*b^3*c^4*d - 15*a*b^2*c^3*d^2 - 7*a^2*b*c^2*d^3 - 5*a^3*c*d^4)*x^3)*((b^2*c^2 - 2*a*b*c*

```
d + a^2*d^2)/c^2)^(1/3)*log(-((b*c - a*d)*x^2*((b^2*c^2 - 2*a*b*c*d + a^2*d
^2)/c^2)^(1/3) + (b*x^3 + a)^(1/3)*c*x*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2
)^(2/3) + (b*x^3 + a)^(2/3)*(b*c - a*d))/x^2) + 3*(6*b^3*c^2*d^3*x^7 + (27*
b^3*c^3*d^2 - 25*a*b^2*c^2*d^3 + 5*a^2*b*c*d^4 + 5*a^3*d^5)*x^4 + 2*(9*b^3*
c^4*d - 8*a*b^2*c^3*d^2 - 2*a^2*b*c^2*d^3 + 4*a^3*c*d^4)*x)*(b*x^3 + a)^(2/
3))/(c^2*d^6*x^6 + 2*c^3*d^5*x^3 + c^4*d^4)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(11/3)/(d*x**3+c)**3,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(11/3)/(d*x^3+c)^3,x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(11/3)/(d*x^3 + c)^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(b\,x^3+a\right)}^{11/3}}{{\left(d\,x^3+c\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(11/3)/(c + d*x^3)^3,x)
```

```
[Out] int((a + b*x^3)^(11/3)/(c + d*x^3)^3, x)
```

3.111 $\int \frac{(a+bx^3)^{8/3}}{(c+dx^3)^3}\,dx$

Optimal. Leaf size=391

$$-\frac{(bc-ad)x(a+bx^3)^{5/3}}{6cd\,(c+dx^3)^2}-\frac{(bc-ad)(6bc+5ad)x(a+bx^3)^{2/3}}{18c^2d^2\,(c+dx^3)}+\frac{b^{8/3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,d^3}-\frac{(bc-ad)^{2/3}\,(9}{}$$

```
[Out] -1/6*(-a*d+b*c)*x*(b*x^3+a)^(5/3)/c/d/(d*x^3+c)^2-1/18*(-a*d+b*c)*(5*a*d+6*
b*c)*x*(b*x^3+a)^(2/3)/c^2/d^2/(d*x^3+c)-1/54*(-a*d+b*c)^(2/3)*(5*a^2*d^2+6
*a*b*c*d+9*b^2*c^2)*ln(d*x^3+c)/c^(8/3)/d^3+1/18*(-a*d+b*c)^(2/3)*(5*a^2*d^
2+6*a*b*c*d+9*b^2*c^2)*ln((-a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/c^(8/
3)/d^3-1/2*b^(8/3)*ln(-b^(1/3)*x+(b*x^3+a)^(1/3))/d^3+1/3*b^(8/3)*arctan(1/
3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/d^3*3^(1/2)-1/27*(-a*d+b*c)^(2/3
)*(5*a^2*d^2+6*a*b*c*d+9*b^2*c^2)*arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*x/c^(1/3
)/(b*x^3+a)^(1/3))*3^(1/2))/c^(8/3)/d^3*3^(1/2)
```

Rubi [A]
time = 0.27, antiderivative size = 391, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {424, 540, 544, 245, 384}

$$-\frac{(bc-ad)^{2/3}\left(5a^2d^2+6abcd+9b^2c^2\right)\text{ArcTan}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\,\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{9\sqrt{3}\,c^{8/3}d^3}-\frac{(bc-ad)^{2/3}\left(5a^2d^2+6abcd+9b^2c^2\right)\log\left(c+dx^3\right)}{54c^{8/3}d^3}+\frac{(bc-ad)^{2/3}\left(5a^2d^2+6abcd+9b^2c^2\right)\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{18c^{8/3}d^3}+\frac{b^{8/3}\text{ArcTan}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,d^3}-\frac{b^{8/3}\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}\,x\right)}{2d^3}-\frac{x(a+bx^3)^{2/3}(bc-ad)(5ad+6bc)}{18c^2d^2\,(c+dx^3)}-\frac{x(a+bx^3)^{5/3}(bc-ad)}{6cd\,(c+dx^3)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(8/3)/(c + d*x^3)^3,x]
```

```
[Out] -1/6*((b*c - a*d)*x*(a + b*x^3)^(5/3))/(c*d*(c + d*x^3)^2) - ((b*c - a*d)*(
6*b*c + 5*a*d)*x*(a + b*x^3)^(2/3))/(18*c^2*d^2*(c + d*x^3)) + (b^(8/3)*Arc
Tan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(Sqrt[3]*d^3) - ((b*c -
 a*d)^(2/3)*(9*b^2*c^2 + 6*a*b*c*d + 5*a^2*d^2)*ArcTan[(1 + (2*(b*c - a*d)^
(1/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[3]])/(9*Sqrt[3]*c^(8/3)*d^3) - (
(b*c - a*d)^(2/3)*(9*b^2*c^2 + 6*a*b*c*d + 5*a^2*d^2)*Log[c + d*x^3])/(54*c
^(8/3)*d^3) + ((b*c - a*d)^(2/3)*(9*b^2*c^2 + 6*a*b*c*d + 5*a^2*d^2)*Log[((
b*c - a*d)^(1/3)*x)/c^(1/3) - (a + b*x^3)^(1/3)])/(18*c^(8/3)*d^3) - (b^(8/
3)*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)])/(2*d^3)
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 384

```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 540

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^q/(a*b*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(
p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(b*e*n*(p + 1) + b*e - a*f) + d*(b*e*n*(p
 + 1) + (b*e - a*f)*(n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f,
n}, x] && LtQ[p, -1] && GtQ[q, 0]
```

Rule 544

```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Dist[f/d, Int[(a + b*x^n)^p, x], x] + Dist[(d*e -
c*f)/d, Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, p
, n}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^3)^{8/3}}{(c+dx^3)^3}\,dx &= \frac{\left(a^2(a+bx^3)^{2/3}\right)\int \frac{\left(1+\frac{bx^3}{a}\right)^{8/3}}{(c+dx^3)^3}\,dx}{\left(1+\frac{bx^3}{a}\right)^{2/3}}\\ &= \frac{a^2x(a+bx^3)^{2/3}\,F_1\left(\frac{1}{3};-\frac{8}{3},3;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^3\left(1+\frac{bx^3}{a}\right)^{2/3}}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

time = 10.71, size = 651, normalized size = 1.66

$$\frac{\frac{6c^{2/3}(-bc+ad)x(a+bx^3)^{2/3}(3bc(2c+3dx^3)+ad(8c+5dx^3))}{d^2(c+dx^3)^2}+\frac{27b^3c^{5/3}x^4\sqrt[3]{1+\frac{bx^3}{a}}\,F_1\left(\frac{4}{3};\frac{1}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{d^2\sqrt[3]{a+bx^3}}+\frac{10a^3\left(2\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc-ad}\,x}{\sqrt[3]{c}\sqrt[3]{b+ax^3}}}{\sqrt{3}}\right)-2\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}\,x}{\sqrt[3]{b+ax^3}}\right)+\log\left(c^{2/3}+\frac{(bc-ad)^{2/3}x^2}{(b+ax^3)^{2/3}}+\frac{\sqrt[3]{c}\sqrt[3]{bc-ad}\,x}{\sqrt[3]{b+ax^3}}\right)\right)}{\sqrt[3]{bc-ad}}+\frac{6ab^2c^2\left(2\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc-ad}\,x}{\sqrt[3]{c}\sqrt[3]{b+ax^3}}}{\sqrt{3}}\right)-2\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}\,x}{\sqrt[3]{b+ax^3}}\right)+\log\left(c^{2/3}+\frac{(bc-ad)^{2/3}x^2}{(b+ax^3)^{2/3}}+\frac{\sqrt[3]{c}\sqrt[3]{bc-ad}\,x}{\sqrt[3]{b+ax^3}}\right)\right)}{d^2\sqrt[3]{bc-ad}}+\frac{2a^2bc\left(2\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc-ad}\,x}{\sqrt[3]{c}\sqrt[3]{b+ax^3}}}{\sqrt{3}}\right)-2\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}\,x}{\sqrt[3]{b+ax^3}}\right)+\log\left(c^{2/3}+\frac{(bc-ad)^{2/3}x^2}{(b+ax^3)^{2/3}}+\frac{\sqrt[3]{c}\sqrt[3]{bc-ad}\,x}{\sqrt[3]{b+ax^3}}\right)\right)}{d\sqrt[3]{bc-ad}}}{108c^{8/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(8/3)/(c + d*x^3)^3,x]
```

```
[Out] ((6*c^(2/3)*(-(b*c) + a*d)*x*(a + b*x^3)^(2/3)*(3*b*c*(2*c + 3*d*x^3) + a*d
*(8*c + 5*d*x^3)))/(d^2*(c + d*x^3)^2) + (27*b^3*c^(5/3)*x^4*(1 + (b*x^3)/a
)^(1/3)*AppellF1[4/3, 1/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)])/(d^2*(a + b
*x^3)^(1/3)) + (10*a^3*(2*Sqrt[3]*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1
/3)*(b + a*x^3)^(1/3)))/Sqrt[3]] - 2*Log[c^(1/3) - ((b*c - a*d)^(1/3)*x)/(b
 + a*x^3)^(1/3)] + Log[c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(b + a*x^3)^(2/3)
+ (c^(1/3)*(b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)]))/(b*c - a*d)^(1/3) + (6
*a*b^2*c^2*(2*Sqrt[3]*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(b + a*x
^3)^(1/3)))/Sqrt[3]] - 2*Log[c^(1/3) - ((b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1
/3)] + Log[c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(b + a*x^3)^(2/3) + (c^(1/3)*(
b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)]))/(d^2*(b*c - a*d)^(1/3)) + (2*a^2*b
*c*(2*Sqrt[3]*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(b + a*x^3)^(1/3
)))/Sqrt[3]] - 2*Log[c^(1/3) - ((b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)] + L
og[c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(b + a*x^3)^(2/3) + (c^(1/3)*(b*c - a*
d)^(1/3)*x)/(b + a*x^3)^(1/3)]))/(d*(b*c - a*d)^(1/3)))/(108*c^(8/3))
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{(b\,x^3+a)^{\frac{8}{3}}}{(d\,x^3+c)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(8/3)/(d*x^3+c)^3,x)
```

```
[Out] int((b*x^3+a)^(8/3)/(d*x^3+c)^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(8/3)/(d*x^3+c)^3,x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(8/3)/(d*x^3 + c)^3, x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 954 vs. $2(334) = 668$.
time = 11.19, size = 954, normalized size = 2.44

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(8/3)/(d*x^3+c)^3,x, algorithm="fricas")
```

```
[Out] -1/54*(2*sqrt(3)*((9*b^2*c^2*d^2 + 6*a*b*c*d^3 + 5*a^2*d^4)*x^6 + 9*b^2*c^4
 + 6*a*b*c^3*d + 5*a^2*c^2*d^2 + 2*(9*b^2*c^3*d + 6*a*b*c^2*d^2 + 5*a^2*c*d
^3)*x^3)*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(1/3)*arctan(-1/3*(sqrt(3)*(
b*c - a*d)*x + 2*sqrt(3)*(b*x^3 + a)^(1/3)*c*((b^2*c^2 - 2*a*b*c*d + a^2*d^
2)/c^2)^(1/3))/((b*c - a*d)*x)) + 18*sqrt(3)*(b^2*c^2*d^2*x^6 + 2*b^2*c^3*d
*x^3 + b^2*c^4)*(-b^2)^(1/3)*arctan(-1/3*(sqrt(3)*b*x - 2*sqrt(3)*(b*x^3 +
a)^(1/3)*(-b^2)^(1/3))/(b*x)) - 2*((9*b^2*c^2*d^2 + 6*a*b*c*d^3 + 5*a^2*d^4
)*x^6 + 9*b^2*c^4 + 6*a*b*c^3*d + 5*a^2*c^2*d^2 + 2*(9*b^2*c^3*d + 6*a*b*c^
2*d^2 + 5*a^2*c*d^3)*x^3)*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(1/3)*log((
c*x*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(2/3) - (b*x^3 + a)^(1/3)*(b*c -
a*d))/x) - 18*(b^2*c^2*d^2*x^6 + 2*b^2*c^3*d*x^3 + b^2*c^4)*(-b^2)^(1/3)*lo
g(-((-b^2)^(2/3)*x - (b*x^3 + a)^(1/3)*b)/x) + 9*(b^2*c^2*d^2*x^6 + 2*b^2*c
^3*d*x^3 + b^2*c^4)*(-b^2)^(1/3)*log(-((-b^2)^(1/3)*b*x^2 - (b*x^3 + a)^(1/
3)*(-b^2)^(2/3)*x - (b*x^3 + a)^(2/3)*b)/x^2) + ((9*b^2*c^2*d^2 + 6*a*b*c*d
^3 + 5*a^2*d^4)*x^6 + 9*b^2*c^4 + 6*a*b*c^3*d + 5*a^2*c^2*d^2 + 2*(9*b^2*c^
3*d + 6*a*b*c^2*d^2 + 5*a^2*c*d^3)*x^3)*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^
2)^(1/3)*log(-((b*c - a*d)*x^2*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(1/3)
+ (b*x^3 + a)^(1/3)*c*x*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(2/3) + (b*x^
3 + a)^(2/3)*(b*c - a*d))/x^2) + 3*((9*b^2*c^2*d^2 - 4*a*b*c*d^3 - 5*a^2*d^
4)*x^4 + 2*(3*b^2*c^3*d + a*b*c^2*d^2 - 4*a^2*c*d^3)*x)*(b*x^3 + a)^(2/3))/
(c^2*d^5*x^6 + 2*c^3*d^4*x^3 + c^4*d^3)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(8/3)/(d*x**3+c)**3,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(8/3)/(d*x^3+c)^3,x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(8/3)/(d*x^3 + c)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(b\,x^3+a\right)}^{8/3}}{{\left(d\,x^3+c\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(8/3)/(c + d*x^3)^3,x)
```

```
[Out] int((a + b*x^3)^(8/3)/(c + d*x^3)^3, x)
```

3.112 $\int \frac{(a+bx^3)^{5/3}}{(c+dx^3)^3}\,dx$

Optimal. Leaf size=217

$$\frac{x(a+bx^3)^{5/3}}{6c\,(c+dx^3)^2}+\frac{5ax(a+bx^3)^{2/3}}{18c^2\,(c+dx^3)}+\frac{5a^2\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc-ad}\,x}{\sqrt[3]{c}\,\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{9\sqrt{3}\,c^{8/3}\sqrt[3]{bc-ad}}+\frac{5a^2\log\left(c+dx^3\right)}{54c^{8/3}\sqrt[3]{bc-ad}}-\frac{5a^2\log\left(\frac{\sqrt[3]{bc-ad}\,x}{\sqrt[3]{c}}-\right.}{18c^{8/3}\sqrt[3]{bc-}}$$

```
[Out] 1/6*x*(b*x^3+a)^(5/3)/c/(d*x^3+c)^2+5/18*a*x*(b*x^3+a)^(2/3)/c^2/(d*x^3+c)+
5/54*a^2*ln(d*x^3+c)/c^(8/3)/(-a*d+b*c)^(1/3)-5/18*a^2*ln((-a*d+b*c)^(1/3)*
x/c^(1/3)-(b*x^3+a)^(1/3))/c^(8/3)/(-a*d+b*c)^(1/3)+5/27*a^2*arctan(1/3*(1+
2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))/c^(8/3)/(-a*d+b*c)^(
1/3)*3^(1/2)
```

Rubi [A]

time = 0.06, antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {386, 384}

$$\frac{5a^2\text{ArcTan}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\,\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{9\sqrt{3}\,c^{8/3}\sqrt[3]{bc-ad}}+\frac{5a^2\log\left(c+dx^3\right)}{54c^{8/3}\sqrt[3]{bc-ad}}-\frac{5a^2\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{18c^{8/3}\sqrt[3]{bc-ad}}+\frac{5ax(a+bx^3)^{2/3}}{18c^2\,(c+dx^3)}+\frac{x(a+bx^3)^{5/3}}{6c\,(c+dx^3)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(5/3)/(c + d*x^3)^3,x]
```

```
[Out] (x*(a + b*x^3)^(5/3))/(6*c*(c + d*x^3)^2) + (5*a*x*(a + b*x^3)^(2/3))/(18*c
^2*(c + d*x^3)) + (5*a^2*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a +
b*x^3)^(1/3)))/Sqrt[3]])/(9*Sqrt[3]*c^(8/3)*(b*c - a*d)^(1/3)) + (5*a^2*Log
[c + d*x^3])/(54*c^(8/3)*(b*c - a*d)^(1/3)) - (5*a^2*Log[((b*c - a*d)^(1/3)
*x)/c^(1/3) - (a + b*x^3)^(1/3)])/(18*c^(8/3)*(b*c - a*d)^(1/3))
```

Rule 384

```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 386

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] - Dist[
c*(q/(a*(p + 1))), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1,
0] && GtQ[q, 0] && NeQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a+bx^3)^{5/3}}{(c+dx^3)^3}\,dx &= \frac{x(a+bx^3)^{5/3}}{6c\,(c+dx^3)^2}+\frac{(5a)\int \frac{(a+bx^3)^{2/3}}{(c+dx^3)^2}\,dx}{6c} \\
&= \frac{x(a+bx^3)^{5/3}}{6c\,(c+dx^3)^2}+\frac{5ax(a+bx^3)^{2/3}}{18c^2\,(c+dx^3)}+\frac{(5a^2)\int \frac{1}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx}{9c^2} \\
&= \frac{x(a+bx^3)^{5/3}}{6c\,(c+dx^3)^2}+\frac{5ax(a+bx^3)^{2/3}}{18c^2\,(c+dx^3)}+\frac{(5a^2)\,\text{Subst}\left(\int \frac{1}{c-(bc-ad)x^3}\,dx,x,\frac{x}{\sqrt[3]{a+bx^3}}\right)}{9c^2} \\
&= \frac{x(a+bx^3)^{5/3}}{6c\,(c+dx^3)^2}+\frac{5ax(a+bx^3)^{2/3}}{18c^2\,(c+dx^3)}+\frac{(5a^2)\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{c}-\sqrt[3]{bc-ad}\,x}\,dx,x,\frac{x}{\sqrt[3]{a+bx^3}}\right.}{27c^{8/3}} \\
&= \frac{x(a+bx^3)^{5/3}}{6c\,(c+dx^3)^2}+\frac{5ax(a+bx^3)^{2/3}}{18c^2\,(c+dx^3)}-\frac{5a^2\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}\,x}{\sqrt[3]{a+bx^3}}\right)}{27c^{8/3}\sqrt[3]{bc-ad}}+\frac{(5a^2)\,\text{Subst}\left(\int \frac{}{c^{2/}\cdots}\right.}{} \\
&= \frac{x(a+bx^3)^{5/3}}{6c\,(c+dx^3)^2}+\frac{5ax(a+bx^3)^{2/3}}{18c^2\,(c+dx^3)}-\frac{5a^2\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}\,x}{\sqrt[3]{a+bx^3}}\right)}{27c^{8/3}\sqrt[3]{bc-ad}}+\frac{5a^2\log\left(c^{2/3}+\frac{(b}{(}\cdots\right.}{54\cdots} \\
&= \frac{x(a+bx^3)^{5/3}}{6c\,(c+dx^3)^2}+\frac{5ax(a+bx^3)^{2/3}}{18c^2\,(c+dx^3)}+\frac{5a^2\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc-ad}\,x}{\sqrt[3]{c}\,\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{9\sqrt{3}\,c^{8/3}\sqrt[3]{bc-ad}}-\frac{5a^2\log\left(\sqrt[3]{c}-\cdots\right.}{27c^{8/3}\sqrt[3]{b\cdots}}
\end{aligned}
$$

Mathematica [C] Result contains complex when optimal does not.
time = 2.51, size = 344, normalized size = 1.59

$$
\frac{\frac{6c^{2/3}(a+bx^3)^{2/3}(8acx+3bcx^4+5adx^4)}{(c+dx^3)^2}-\frac{10\sqrt{-6+6i\sqrt{3}}\,a^2\tan^{-1}\left(\frac{3\sqrt[3]{bc-ad}\,x}{\sqrt{3}\,\sqrt[3]{bc-ad}\,x-\left(3i+\sqrt{3}\right)\sqrt[3]{c}\,\sqrt[3]{a+bx^3}}\right)}{\sqrt[3]{bc-ad}}+\frac{10\left(1+i\sqrt{3}\right)a^2\log\left(2\sqrt[3]{bc-ad}\,x+\left(1+i\sqrt{3}\right)\sqrt[3]{c}\,\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{bc-ad}}-\frac{5i\left(-i+\sqrt{3}\right)a^2\log\left(2(bc-ad)^{2/3}x^2+\left(-1-i\sqrt{3}\right)\sqrt[3]{c}\,\sqrt[3]{bc-ad}\,x\sqrt[3]{a+bx^3}+i\left(i+\sqrt{3}\right)c^{2/3}(a+bx^3)^{2/3}\right)}{\sqrt[3]{bc-ad}}}{108c^{8/3}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^(5/3)/(c + d*x^3)^3,x]
```

```
[Out] ((6*c^(2/3)*(a + b*x^3)^(2/3)*(8*a*c*x + 3*b*c*x^4 + 5*a*d*x^4))/(c + d*x^3
)^2 - (10*Sqrt[-6 + (6*I)*Sqrt[3]]*a^2*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt
[3]*(b*c - a*d)^(1/3)*x - (3*I + Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3))])/(b*c
 - a*d)^(1/3) + (10*(1 + I*Sqrt[3])*a^2*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*
Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3)])/(b*c - a*d)^(1/3) - ((5*I)*(-I + Sqrt[
3])*a^2*Log[2*(b*c - a*d)^(2/3)*x^2 + (-1 - I*Sqrt[3])*c^(1/3)*(b*c - a*d)^
(1/3)*x*(a + b*x^3)^(1/3) + I*(I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)])/(b*
c - a*d)^(1/3))/(108*c^(8/3))
```

Maple [**F**]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^{3}+a\right)^{\frac{5}{3}}}{\left(d\,x^{3}+c\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(5/3)/(d*x^3+c)^3,x)
```

```
[Out] int((b*x^3+a)^(5/3)/(d*x^3+c)^3,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(5/3)/(d*x^3+c)^3,x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(5/3)/(d*x^3 + c)^3, x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(5/3)/(d*x^3+c)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(5/3)/(d*x**3+c)**3,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(5/3)/(d*x^3+c)^3,x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(5/3)/(d*x^3 + c)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(b\,x^3+a\right)}^{5/3}}{{\left(d\,x^3+c\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(5/3)/(c + d*x^3)^3,x)
```

```
[Out] int((a + b*x^3)^(5/3)/(c + d*x^3)^3, x)
```

3.113 $\int \frac{(a+bx^3)^{2/3}}{(c+dx^3)^3} \, dx$

Optimal. Leaf size=267

$$-\frac{dx(a+bx^3)^{5/3}}{6c(bc-ad)(c+dx^3)^2}+\frac{(6bc-5ad)x(a+bx^3)^{2/3}}{18c^2(bc-ad)(c+dx^3)}+\frac{a(6bc-5ad)\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc-ad}\,x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{9\sqrt{3}\,c^{8/3}(bc-ad)^{4/3}}+\frac{a(6bc-5ad}{54c^{8/3}(b}$$

```
[Out] -1/6*d*x*(b*x^3+a)^(5/3)/c/(-a*d+b*c)/(d*x^3+c)^2+1/18*(-5*a*d+6*b*c)*x*(b*
x^3+a)^(2/3)/c^2/(-a*d+b*c)/(d*x^3+c)+1/54*a*(-5*a*d+6*b*c)*ln(d*x^3+c)/c^(
8/3)/(-a*d+b*c)^(4/3)-1/18*a*(-5*a*d+6*b*c)*ln((-a*d+b*c)^(1/3)*x/c^(1/3)-(
b*x^3+a)^(1/3))/c^(8/3)/(-a*d+b*c)^(4/3)+1/27*a*(-5*a*d+6*b*c)*arctan(1/3*(
1+2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))/c^(8/3)/(-a*d+b*c)
^(4/3)*3^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {390, 386, 384}

$$\frac{a(6bc-5ad)\text{ArcTan}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{9\sqrt{3}\,c^{8/3}(bc-ad)^{4/3}}+\frac{a(6bc-5ad)\log\left(c+dx^3\right)}{54c^{8/3}(bc-ad)^{4/3}}-\frac{a(6bc-5ad)\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{18c^{8/3}(bc-ad)^{4/3}}+\frac{x(a+bx^3)^{2/3}(6bc-5ad)}{18c^2\left(c+dx^3\right)(bc-ad)}-\frac{dx(a+bx^3)^{5/3}}{6c\left(c+dx^3\right)^2(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(2/3)/(c + d*x^3)^3,x]
```

```
[Out] -1/6*(d*x*(a + b*x^3)^(5/3))/(c*(b*c - a*d)*(c + d*x^3)^2) + ((6*b*c - 5*a*
d)*x*(a + b*x^3)^(2/3))/(18*c^2*(b*c - a*d)*(c + d*x^3)) + (a*(6*b*c - 5*a*
d)*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[3]
])/(9*Sqrt[3]*c^(8/3)*(b*c - a*d)^(4/3)) + (a*(6*b*c - 5*a*d)*Log[c + d*x^3
])/(54*c^(8/3)*(b*c - a*d)^(4/3)) - (a*(6*b*c - 5*a*d)*Log[((b*c - a*d)^(1/
3)*x)/c^(1/3) - (a + b*x^3)^(1/3)])/(18*c^(8/3)*(b*c - a*d)^(4/3))
```

Rule 384

```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 386

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] - Dist[
c*(q/(a*(p + 1))), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1,
0] && GtQ[q, 0] && NeQ[p, -1]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)),
 Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q},
 x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !L
tQ[q, -1]) && NeQ[p, -1]
```

Rubi steps

$$\int \frac{(a+bx^3)^{2/3}}{(c+dx^3)^3}\,dx = -\frac{dx(a+bx^3)^{5/3}}{6c(bc-ad)(c+dx^3)^2} + \frac{(6bc-5ad)\int \frac{(a+bx^3)^{2/3}}{(c+dx^3)^2}\,dx}{6c(bc-ad)}$$

$$= -\frac{dx(a+bx^3)^{5/3}}{6c(bc-ad)(c+dx^3)^2} + \frac{(6bc-5ad)x(a+bx^3)^{2/3}}{18c^2(bc-ad)(c+dx^3)} + \frac{(a(6bc-5ad))\int \frac{1}{\sqrt[3]{a+bx^3}\,(c+\ldots}}{9c^2(bc-ad)}$$

$$= -\frac{dx(a+bx^3)^{5/3}}{6c(bc-ad)(c+dx^3)^2} + \frac{(6bc-5ad)x(a+bx^3)^{2/3}}{18c^2(bc-ad)(c+dx^3)} + \frac{(a(6bc-5ad))\text{Subst}\left(\int \frac{1}{c-(bc-\ldots}\right.}{9c^2(bc-\ldots}$$

$$= -\frac{dx(a+bx^3)^{5/3}}{6c(bc-ad)(c+dx^3)^2} + \frac{(6bc-5ad)x(a+bx^3)^{2/3}}{18c^2(bc-ad)(c+dx^3)} + \frac{(a(6bc-5ad))\text{Subst}\left(\int \frac{\ldots}{\sqrt[3]{c}-\ldots}\right.}{27c^{8/3}(b\ldots}$$

$$= -\frac{dx(a+bx^3)^{5/3}}{6c(bc-ad)(c+dx^3)^2} + \frac{(6bc-5ad)x(a+bx^3)^{2/3}}{18c^2(bc-ad)(c+dx^3)} - \frac{a(6bc-5ad)\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc\ldots}}{\sqrt[3]{a\ldots}}\right.}{27c^{8/3}(bc-ad)^{4/3}}$$

$$= -\frac{dx(a+bx^3)^{5/3}}{6c(bc-ad)(c+dx^3)^2} + \frac{(6bc-5ad)x(a+bx^3)^{2/3}}{18c^2(bc-ad)(c+dx^3)} - \frac{a(6bc-5ad)\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc\ldots}}{\sqrt[3]{a\ldots}}\right.}{27c^{8/3}(bc-ad)^{4/3}}$$

$$= -\frac{dx(a+bx^3)^{5/3}}{6c(bc-ad)(c+dx^3)^2} + \frac{(6bc-5ad)x(a+bx^3)^{2/3}}{18c^2(bc-ad)(c+dx^3)} + \frac{a(6bc-5ad)\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc\ldots}}{\sqrt[3]{c}\sqrt[3]{\ldots}}}{\sqrt{\ldots}}\right.}{9\sqrt{3}\,c^{8/3}(bc-ad)^{4/\ldots}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 3.27, size = 366, normalized size = 1.37

$$\frac{\frac{6c^{2/3}x(a+bx^3)^{2/3}(3bc(2c+dx^3)-ad(8c+5dx^3))}{(bc-ad)(c+dx^3)^2}+\frac{2i\left(3i+\sqrt{3}\right)a(-6bc+5ad)\tanh^{-1}\left(\frac{i+\frac{\left(-i+\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}\,x}}{\sqrt{3}}\right)}{(bc-ad)^{4/3}}+\frac{2\left(1+i\sqrt{3}\right)a(6bc-5ad)\log\left(2\sqrt[3]{bc-ad}\,x+\left(1+i\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{a+bx^3}\right)}{(bc-ad)^{4/3}}+\frac{\left(1+i\sqrt{3}\right)a(-6bc+5ad)\log\left(2(bc-ad)^{2/3}x^2+\left(-1-i\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{bc-ad}\,x\sqrt[3]{a+bx^3}+i\left(i+\sqrt{3}\right)c^{2/3}(a+bx^3)^{2/3}\right)}{(bc-ad)^{4/3}}}{108c^{8/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^(2/3)/(c + d*x^3)^3,x]
```

```
[Out] ((6*c^(2/3)*x*(a + b*x^3)^(2/3)*(3*b*c*(2*c + d*x^3) - a*d*(8*c + 5*d*x^3))
)/((b*c - a*d)*(c + d*x^3)^2) + ((2*I)*(3*I + Sqrt[3])*a*(-6*b*c + 5*a*d)*A
rcTanh[(I + ((-I + Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3))/((b*c - a*d)^(1/3)*x
))/Sqrt[3]])/(b*c - a*d)^(4/3) + (2*(1 + I*Sqrt[3])*a*(6*b*c - 5*a*d)*Log[2
*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3)])/(b*c - a
*d)^(4/3) + ((1 + I*Sqrt[3])*a*(-6*b*c + 5*a*d)*Log[2*(b*c - a*d)^(2/3)*x^2
 + (-1 - I*Sqrt[3])*c^(1/3)*(b*c - a*d)^(1/3)*x*(a + b*x^3)^(1/3) + I*(I +
Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)])/(b*c - a*d)^(4/3))/(108*c^(8/3))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(b\,x^3+a)^{\frac{2}{3}}}{(d\,x^3+c)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(2/3)/(d*x^3+c)^3,x)
```

```
[Out] int((b*x^3+a)^(2/3)/(d*x^3+c)^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/(d*x^3+c)^3,x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(2/3)/(d*x^3 + c)^3, x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/(d*x^3+c)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(2/3)/(d*x**3+c)**3,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/(d*x^3+c)^3,x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(2/3)/(d*x^3 + c)^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(b\,x^3+a\right)}^{2/3}}{{\left(d\,x^3+c\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(2/3)/(c + d*x^3)^3,x)
```

```
[Out] int((a + b*x^3)^(2/3)/(c + d*x^3)^3, x)
```

3.114 $\int \frac{1}{\sqrt[3]{a+bx^3}\,(c+dx^3)^3}\,dx$

Optimal. Leaf size=307

$$-\frac{dx(a+bx^3)^{2/3}}{6c(bc-ad)(c+dx^3)^2}-\frac{d(9bc-5ad)x(a+bx^3)^{2/3}}{18c^2(bc-ad)^2(c+dx^3)}+\frac{(9b^2c^2-12abcd+5a^2d^2)\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc-ad}\,x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{9\sqrt{3}\,c^{8/3}(bc-ad)^{7/3}}-$$

```
[Out] -1/6*d*x*(b*x^3+a)^(2/3)/c/(-a*d+b*c)/(d*x^3+c)^2-1/18*d*(-5*a*d+9*b*c)*x*(
b*x^3+a)^(2/3)/c^2/(-a*d+b*c)^2/(d*x^3+c)+1/54*(5*a^2*d^2-12*a*b*c*d+9*b^2*
c^2)*ln(d*x^3+c)/c^(8/3)/(-a*d+b*c)^(7/3)-1/18*(5*a^2*d^2-12*a*b*c*d+9*b^2*
c^2)*ln((-a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/c^(8/3)/(-a*d+b*c)^(7/3
)+1/27*(5*a^2*d^2-12*a*b*c*d+9*b^2*c^2)*arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*x/
c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))/c^(8/3)/(-a*d+b*c)^(7/3)*3^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 307, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {425, 541, 12, 384}

$$\frac{(5a^2d^2-12abcd+9b^2c^2)\,\text{ArcTan}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{9\sqrt{3}\,c^{8/3}(bc-ad)^{7/3}}+\frac{(5a^2d^2-12abcd+9b^2c^2)\log(c+dx^3)}{54c^{8/3}(bc-ad)^{7/3}}-\frac{(5a^2d^2-12abcd+9b^2c^2)\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{18c^{8/3}(bc-ad)^{7/3}}-\frac{dx(a+bx^3)^{2/3}(9bc-5ad)}{18c^2(c+dx^3)(bc-ad)^2}-\frac{dx(a+bx^3)^{2/3}}{6c(c+dx^3)^2(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^3)^(1/3)*(c + d*x^3)^3),x]
```

```
[Out] -1/6*(d*x*(a + b*x^3)^(2/3))/(c*(b*c - a*d)*(c + d*x^3)^2) - (d*(9*b*c - 5*
a*d)*x*(a + b*x^3)^(2/3))/(18*c^2*(b*c - a*d)^2*(c + d*x^3)) + ((9*b^2*c^2
- 12*a*b*c*d + 5*a^2*d^2)*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a +
 b*x^3)^(1/3)))/Sqrt[3]])/(9*Sqrt[3]*c^(8/3)*(b*c - a*d)^(7/3)) + ((9*b^2*c
^2 - 12*a*b*c*d + 5*a^2*d^2)*Log[c + d*x^3])/(54*c^(8/3)*(b*c - a*d)^(7/3))
 - ((9*b^2*c^2 - 12*a*b*c*d + 5*a^2*d^2)*Log[((b*c - a*d)^(1/3)*x)/c^(1/3)
- (a + b*x^3)^(1/3)])/(18*c^(8/3)*(b*c - a*d)^(7/3))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 384

```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
```

```
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{a+bx^3}\,(c+dx^3)^3}\,dx = \frac{\sqrt[3]{1+\frac{bx^3}{a}}\int \frac{1}{\sqrt[3]{1+\frac{bx^3}{a}}\,(c+dx^3)^3}\,dx}{\sqrt[3]{a+bx^3}}$$

$$= -\frac{x\Big(cd(3b^2cx^3(4c+3dx^3)-a^2d(8c+5dx^3)+ab(12c^2+cdx^3-5d^2x^6))-2(}{18c^3(bc-ad)^2\sqrt[3]{a+bx^3}\,(}$$

Mathematica [C] Result contains complex when optimal does not.
time = 4.48, size = 407, normalized size = 1.33

$$\frac{\frac{6c^{2/3}dx(a+bx^3)^{2/3}(-3bc(4c+3dx^3)+ad(8c+5dx^3))}{(bc-ad)^2(c+dx^3)^2}+\frac{2\left(3-i\sqrt{3}\right)(9b^2c^2-12abcd+5a^2d^2)\tanh^{-1}\left(\frac{i+\frac{\left(-i+\sqrt{3}\right)\sqrt[3]{c}\,\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}\,x}}{\sqrt{3}}\right)}{(bc-ad)^{7/3}}+\frac{2\left(1+i\sqrt{3}\right)(9b^2c^2-12abcd+5a^2d^2)\log\left(2\sqrt[3]{bc-ad}\,x+\left(1+i\sqrt{3}\right)\sqrt[3]{c}\,\sqrt[3]{a+bx^3}\right)}{(bc-ad)^{7/3}}-\frac{i\left(-i+\sqrt{3}\right)(9b^2c^2-12abcd+5a^2d^2)\log\left(2(bc-ad)^{2/3}x^2+\left(-1-i\sqrt{3}\right)\sqrt[3]{c}\,\sqrt[3]{bc-ad}\,x\sqrt[3]{a+bx^3}+i\left(i+\sqrt{3}\right)c^{2/3}(a+bx^3)^{2/3}\right)}{(bc-ad)^{7/3}}}{108c^{8/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^3)^(1/3)*(c + d*x^3)^3),x]
```

```
[Out] ((6*c^(2/3)*d*x*(a + b*x^3)^(2/3)*(-3*b*c*(4*c + 3*d*x^3) + a*d*(8*c + 5*d*
x^3)))/((b*c - a*d)^2*(c + d*x^3)^2) + (2*(3 - I*Sqrt[3])*(9*b^2*c^2 - 12*a
*b*c*d + 5*a^2*d^2)*ArcTanh[(I + ((-I + Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3))
/((b*c - a*d)^(1/3)*x))/Sqrt[3]])/(b*c - a*d)^(7/3) + (2*(1 + I*Sqrt[3])*(9
*b^2*c^2 - 12*a*b*c*d + 5*a^2*d^2)*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[
3])*c^(1/3)*(a + b*x^3)^(1/3)])/(b*c - a*d)^(7/3) - (I*(-I + Sqrt[3])*(9*b^
2*c^2 - 12*a*b*c*d + 5*a^2*d^2)*Log[2*(b*c - a*d)^(2/3)*x^2 + (-1 - I*Sqrt[
3])*c^(1/3)*(b*c - a*d)^(1/3)*x*(a + b*x^3)^(1/3) + I*(I + Sqrt[3])*c^(2/3)
*(a + b*x^3)^(2/3)])/(b*c - a*d)^(7/3))/(108*c^(8/3))
```

Maple [**F**]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^{3}+a\right)^{\frac{1}{3}} \left(d\,x^{3}+c\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^3+a)^(1/3)/(d*x^3+c)^3,x)
```

```
[Out] int(1/(b*x^3+a)^(1/3)/(d*x^3+c)^3,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(1/3)/(d*x^3+c)^3,x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)^(1/3)*(d*x^3 + c)^3), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(1/3)/(d*x^3+c)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{a+bx^{3}}\,\left(c+dx^{3}\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3+a)**(1/3)/(d*x**3+c)**3,x)
```

```
[Out] Integral(1/((a + b*x**3)**(1/3)*(c + d*x**3)**3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(1/3)/(d*x^3+c)^3,x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^3 + a)^(1/3)*(d*x^3 + c)^3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(b\,x^3+a\right)}^{1/3}\,{\left(d\,x^3+c\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^3)^(1/3)*(c + d*x^3)^3),x)
```

```
[Out] int(1/((a + b*x^3)^(1/3)*(c + d*x^3)^3), x)
```

3.115 $\int \frac{1}{(a+bx^3)^{4/3}(c+dx^3)^3}\,dx$

Optimal. Leaf size=377

$$-\frac{dx}{6c(bc-ad)\sqrt[3]{a+bx^3}\,(c+dx^3)^2}+\frac{b(6bc+ad)x}{6ac(bc-ad)^2\sqrt[3]{a+bx^3}\,(c+dx^3)}+\frac{d(18b^2c^2+15abcd-5a^2d^2)\,x(a+b$$

```
[Out] -1/6*d*x/c/(-a*d+b*c)/(b*x^3+a)^(1/3)/(d*x^3+c)^2+1/6*b*(a*d+6*b*c)*x/a/c/(
-a*d+b*c)^2/(b*x^3+a)^(1/3)/(d*x^3+c)+1/18*d*(-5*a^2*d^2+15*a*b*c*d+18*b^2*
c^2)*x*(b*x^3+a)^(2/3)/a/c^2/(-a*d+b*c)^3/(d*x^3+c)-1/54*d*(5*a^2*d^2-18*a*
b*c*d+27*b^2*c^2)*ln(d*x^3+c)/c^(8/3)/(-a*d+b*c)^(10/3)+1/18*d*(5*a^2*d^2-1
8*a*b*c*d+27*b^2*c^2)*ln((-a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/c^(8/3
)/(-a*d+b*c)^(10/3)-1/27*d*(5*a^2*d^2-18*a*b*c*d+27*b^2*c^2)*arctan(1/3*(1+
2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))/c^(8/3)/(-a*d+b*c)^(
10/3)*3^(1/2)
```

Rubi [A]
time = 0.25, antiderivative size = 377, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {425, 541, 12, 384}

$$-\frac{d(5a^2d^2-18abcd+27b^2c^2)\,\text{ArcTan}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{9\sqrt{3}\,c^{8/3}(bc-ad)^{10/3}}+\frac{dx(a+bx^3)^{2/3}(-5a^2d^2+15abcd+18b^2c^2)}{18ac^2(c+dx^3)(bc-ad)^3}-\frac{d(5a^2d^2-18abcd+27b^2c^2)\log(c+dx^3)}{54c^{8/3}(bc-ad)^{10/3}}+\frac{d(5a^2d^2-18abcd+27b^2c^2)\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{18c^{8/3}(bc-ad)^{10/3}}+\frac{bx(ad+6bc)}{6ac\sqrt[3]{a+bx^3}\,(c+dx^3)(bc-ad)^2}-\frac{dx}{6c\sqrt[3]{a+bx^3}\,(c+dx^3)^2(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^3)^(4/3)*(c + d*x^3)^3),x]
```

```
[Out] -1/6*(d*x)/(c*(b*c - a*d)*(a + b*x^3)^(1/3)*(c + d*x^3)^2) + (b*(6*b*c + a*
d)*x)/(6*a*c*(b*c - a*d)^2*(a + b*x^3)^(1/3)*(c + d*x^3)) + (d*(18*b^2*c^2
+ 15*a*b*c*d - 5*a^2*d^2)*x*(a + b*x^3)^(2/3))/(18*a*c^2*(b*c - a*d)^3*(c +
 d*x^3)) - (d*(27*b^2*c^2 - 18*a*b*c*d + 5*a^2*d^2)*ArcTan[(1 + (2*(b*c - a
*d)^(1/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[3]])/(9*Sqrt[3]*c^(8/3)*(b*c
 - a*d)^(10/3)) - (d*(27*b^2*c^2 - 18*a*b*c*d + 5*a^2*d^2)*Log[c + d*x^3])/
(54*c^(8/3)*(b*c - a*d)^(10/3)) + (d*(27*b^2*c^2 - 18*a*b*c*d + 5*a^2*d^2)*
Log[((b*c - a*d)^(1/3)*x)/c^(1/3) - (a + b*x^3)^(1/3)])/(18*c^(8/3)*(b*c -
a*d)^(10/3))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 384

```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{1}{(a+bx^3)^{4/3}(c+dx^3)^3}\,dx = \frac{\sqrt[3]{1+\frac{bx^3}{a}}\int \frac{1}{\left(1+\frac{bx^3}{a}\right)^{4/3}(c+dx^3)^3}\,dx}{a\sqrt[3]{a+bx^3}}$$

$$= -\frac{65c^2(a+bx^3)^2\left(14000a^2c^5+21896abc^5x^3+48104a^2c^4dx^3+8391b^2c^5x^6\right.}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 7.99, size = 471, normalized size = 1.25

$$\frac{-\frac{6c^{2/3}x\left(18b^3c^2(c+dx^3)^2+3ab^2cd^2x^3(6c+5dx^3)-a^3d^3(8c+5dx^3)+a^2bd^2(18c^2+7cdx^3-5d^2x^6)\right)}{a(-bc+ad)^3\sqrt[3]{a+bx^3}\,(c+dx^3)^2}+\frac{2i\left(3i+\sqrt{3}\right)d(27b^2c^2-18abcd+5a^2d^2)\tanh^{-1}\left(\frac{i+\frac{\left(-i+\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}\,x}}{\sqrt{3}}\right)}{(bc-ad)^{10/3}}-\frac{2i\left(-i+\sqrt{3}\right)d(27b^2c^2-18abcd+5a^2d^2)\log\left(2\sqrt[3]{bc-ad}\,x+\left(1+i\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{a+bx^3}\right)}{(bc-ad)^{10/3}}+\frac{\left(1+i\sqrt{3}\right)d(27b^2c^2-18abcd+5a^2d^2)\log\left(2(bc-ad)^{2/3}x^2+\left(-1-i\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{bc-ad}\,x\sqrt[3]{a+bx^3}+i\left(i+\sqrt{3}\right)c^{2/3}(a+bx^3)^{2/3}\right)}{(bc-ad)^{10/3}}}{108c^{8/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^3)^(4/3)*(c + d*x^3)^3),x]
```

```
[Out] ((-6*c^(2/3)*x*(18*b^3*c^2*(c + d*x^3)^2 + 3*a*b^2*c*d^2*x^3*(6*c + 5*d*x^3
) - a^3*d^3*(8*c + 5*d*x^3) + a^2*b*d^2*(18*c^2 + 7*c*d*x^3 - 5*d^2*x^6)))/
(a*(-(b*c) + a*d)^3*(a + b*x^3)^(1/3)*(c + d*x^3)^2) + ((2*I)*(3*I + Sqrt[3
])*d*(27*b^2*c^2 - 18*a*b*c*d + 5*a^2*d^2)*ArcTanh[(I + ((-I + Sqrt[3])*c^(
1/3)*(a + b*x^3)^(1/3))/((b*c - a*d)^(1/3)*x))/Sqrt[3]])/(b*c - a*d)^(10/3)
 - ((2*I)*(-I + Sqrt[3])*d*(27*b^2*c^2 - 18*a*b*c*d + 5*a^2*d^2)*Log[2*(b*c
 - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3)])/(b*c - a*d)^(
10/3) + ((1 + I*Sqrt[3])*d*(27*b^2*c^2 - 18*a*b*c*d + 5*a^2*d^2)*Log[2*(b*c
 - a*d)^(2/3)*x^2 + (-1 - I*Sqrt[3])*c^(1/3)*(b*c - a*d)^(1/3)*x*(a + b*x^3
)^(1/3) + I*(I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)])/(b*c - a*d)^(10/3))/(
108*c^(8/3))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^{3}+a\right)^{\frac{4}{3}} \left(d\,x^{3}+c\right)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^3+a)^(4/3)/(d*x^3+c)^3,x)
```

```
[Out] int(1/(b*x^3+a)^(4/3)/(d*x^3+c)^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(4/3)/(d*x^3+c)^3,x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)^(4/3)*(d*x^3 + c)^3), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(4/3)/(d*x^3+c)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3+a)**(4/3)/(d*x**3+c)**3,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(4/3)/(d*x^3+c)^3,x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^3 + a)^(4/3)*(d*x^3 + c)^3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(b\,x^3+a\right)^{4/3}\,\left(d\,x^3+c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^3)^(4/3)*(c + d*x^3)^3),x)
```

```
[Out] int(1/((a + b*x^3)^(4/3)*(c + d*x^3)^3), x)
```

3.116 $\int \frac{1}{(a+bx^3)^{7/3}(c+dx^3)^3} dx$

Optimal. Leaf size=463

$$-\frac{dx}{6c(bc-ad)\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)^2}+\frac{b(3bc+2ad)x}{12ac(bc-ad)^2\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}+\frac{b(9b^2c^2-42abcd-2a^2d^2)x}{12a^2c(bc-ad)^3\sqrt[3]{a+bx^3}\left(c+\cdots\right.}$$

```
[Out] -1/6*d*x/c/(-a*d+b*c)/(b*x^3+a)^(4/3)/(d*x^3+c)^2+1/12*b*(2*a*d+3*b*c)*x/a/
c/(-a*d+b*c)^2/(b*x^3+a)^(4/3)/(d*x^3+c)+1/12*b*(-2*a^2*d^2-42*a*b*c*d+9*b^
2*c^2)*x/a^2/c/(-a*d+b*c)^3/(b*x^3+a)^(1/3)/(d*x^3+c)+1/36*d*(10*a^3*d^3-42
*a^2*b*c*d^2-135*a*b^2*c^2*d+27*b^3*c^3)*x*(b*x^3+a)^(2/3)/a^2/c^2/(-a*d+b*
c)^4/(d*x^3+c)+1/54*d^2*(5*a^2*d^2-24*a*b*c*d+54*b^2*c^2)*ln(d*x^3+c)/c^(8/
3)/(-a*d+b*c)^(13/3)-1/18*d^2*(5*a^2*d^2-24*a*b*c*d+54*b^2*c^2)*ln((-a*d+b*
c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/c^(8/3)/(-a*d+b*c)^(13/3)+1/27*d^2*(5*a
^2*d^2-24*a*b*c*d+54*b^2*c^2)*arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b
*x^3+a)^(1/3))*3^(1/2))/c^(8/3)/(-a*d+b*c)^(13/3)*3^(1/2)
```

Rubi [A]
time = 0.42, antiderivative size = 463, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {425, 541, 12, 384}

$$\frac{d^2(5a^2d^2-24abcd+54b^2c^2)\,\text{ArcTan}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{9\sqrt{3}\,c^{8/3}(bc-ad)^{13/3}}+\frac{bx(-2a^2d^2-42abcd+9b^2c^2)}{12a^2c\sqrt[3]{a+bx^3}\,(c+dx^3)\,(bc-ad)^3}+\frac{d^2(5a^2d^2-24abcd+54b^2c^2)\log\left(c+dx^3\right)}{54c^{8/3}(bc-ad)^{13/3}}-\frac{d^2(5a^2d^2-24abcd+54b^2c^2)\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{18c^{8/3}(bc-ad)^{13/3}}+\frac{dx(a+bx^3)^{2/3}\left(10a^3d^3-42a^2bcd^2-135ab^2c^2d+27b^3c^3\right)}{36a^2c^2\left(c+dx^3\right)(bc-ad)^4}-\frac{dx}{6c\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)^2(bc-ad)}+\frac{bx(2ad+3bc)}{12ac\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^3)^(7/3)*(c + d*x^3)^3),x]
```

```
[Out] -1/6*(d*x)/(c*(b*c - a*d)*(a + b*x^3)^(4/3)*(c + d*x^3)^2) + (b*(3*b*c + 2*
a*d)*x)/(12*a*c*(b*c - a*d)^2*(a + b*x^3)^(4/3)*(c + d*x^3)) + (b*(9*b^2*c^
2 - 42*a*b*c*d - 2*a^2*d^2)*x)/(12*a^2*c*(b*c - a*d)^3*(a + b*x^3)^(1/3)*(c
 + d*x^3)) + (d*(27*b^3*c^3 - 135*a*b^2*c^2*d - 42*a^2*b*c*d^2 + 10*a^3*d^3
)*x*(a + b*x^3)^(2/3))/(36*a^2*c^2*(b*c - a*d)^4*(c + d*x^3)) + (d^2*(54*b^
2*c^2 - 24*a*b*c*d + 5*a^2*d^2)*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3
)*(a + b*x^3)^(1/3)))/Sqrt[3]])/(9*Sqrt[3]*c^(8/3)*(b*c - a*d)^(13/3)) + (d
^2*(54*b^2*c^2 - 24*a*b*c*d + 5*a^2*d^2)*Log[c + d*x^3])/(54*c^(8/3)*(b*c -
 a*d)^(13/3)) - (d^2*(54*b^2*c^2 - 24*a*b*c*d + 5*a^2*d^2)*Log[((b*c - a*d)
^(1/3)*x)/c^(1/3) - (a + b*x^3)^(1/3)])/(18*c^(8/3)*(b*c - a*d)^(13/3))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 384

```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{1}{(a+bx^3)^{7/3}(c+dx^3)^3}\,dx = \frac{\sqrt[3]{1+\frac{bx^3}{a}}\int \frac{1}{\left(1+\frac{bx^3}{a}\right)^{7/3}(c+dx^3)^3}\,dx}{a^2\sqrt[3]{a+bx^3}}$$

$$= -\frac{522756c^6(bc-ad)^3x^9(a+bx^3)^2+1516320c^5d(bc-ad)^3x^{12}(a+bx^3)^2+2}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 15.10, size = 1991, normalized size = 4.30

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^3)^(7/3)*(c + d*x^3)^3),x]
```

```
[Out] (-522756*c^6*(b*c - a*d)^3*x^9*(a + b*x^3)^2 - 1516320*c^5*d*(b*c - a*d)^3*
x^12*(a + b*x^3)^2 - 2198664*c^4*d^2*(b*c - a*d)^3*x^15*(a + b*x^3)^2 - 141
5232*c^3*d^3*(b*c - a*d)^3*x^18*(a + b*x^3)^2 - 341172*c^2*d^4*(b*c - a*d)^
3*x^21*(a + b*x^3)^2 - 28042560*c^7*(b*c - a*d)^2*x^6*(a + b*x^3)^3 - 10760
2560*c^6*d*(b*c - a*d)^2*x^9*(a + b*x^3)^3 - 157697280*c^5*d^2*(b*c - a*d)^
2*x^12*(a + b*x^3)^3 - 101088000*c^4*d^3*(b*c - a*d)^2*x^15*(a + b*x^3)^3 -
 24261120*c^3*d^4*(b*c - a*d)^2*x^18*(a + b*x^3)^3 + 265470660*c^8*(b*c - a
*d)*x^3*(a + b*x^3)^4 + 1019636800*c^7*d*(b*c - a*d)*x^6*(a + b*x^3)^4 + 14
66086440*c^6*d^2*(b*c - a*d)*x^9*(a + b*x^3)^4 + 930252960*c^5*d^3*(b*c - a
*d)*x^12*(a + b*x^3)^4 + 221899860*c^4*d^4*(b*c - a*d)*x^15*(a + b*x^3)^4 -
 335877360*c^9*(a + b*x^3)^5 - 1279532800*c^8*d*x^3*(a + b*x^3)^5 - 1823334
240*c^7*d^2*x^6*(a + b*x^3)^5 - 1151579520*c^6*d^3*x^9*(a + b*x^3)^5 - 2739
39120*c^5*d^4*x^12*(a + b*x^3)^5 + 67420080*c^7*(b*c - a*d)^2*x^6*(a + b*x^
3)^3*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 25
9692160*c^6*d*(b*c - a*d)^2*x^9*(a + b*x^3)^3*Hypergeometric2F1[1/3, 1, 4/3
, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 377700960*c^5*d^2*(b*c - a*d)^2*x^12
*(a + b*x^3)^3*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x
^3))] + 241113600*c^4*d^3*(b*c - a*d)^2*x^15*(a + b*x^3)^3*Hypergeometric2F
1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 57723120*c^3*d^4*(b*c -
 a*d)^2*x^18*(a + b*x^3)^3*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)
/(c*(a + b*x^3))] - 349440000*c^8*(b*c - a*d)*x^3*(a + b*x^3)^4*Hypergeomet
ric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 1339520000*c^7*d*(
b*c - a*d)*x^6*(a + b*x^3)^4*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^
3)/(c*(a + b*x^3))] - 1921920000*c^6*d^2*(b*c - a*d)*x^9*(a + b*x^3)^4*Hype
rgeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 1218147840*
c^5*d^3*(b*c - a*d)*x^12*(a + b*x^3)^4*Hypergeometric2F1[1/3, 1, 4/3, ((b*c
 - a*d)*x^3)/(c*(a + b*x^3))] - 290384640*c^4*d^4*(b*c - a*d)*x^15*(a + b*x
^3)^4*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 3
35877360*c^9*(a + b*x^3)^5*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)
/(c*(a + b*x^3))] + 1279532800*c^8*d*x^3*(a + b*x^3)^5*Hypergeometric2F1[1/
3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 1823334240*c^7*d^2*x^6*(a +
 b*x^3)^5*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))]
 + 1151579520*c^6*d^3*x^9*(a + b*x^3)^5*Hypergeometric2F1[1/3, 1, 4/3, ((b*
c - a*d)*x^3)/(c*(a + b*x^3))] + 273939120*c^5*d^4*x^12*(a + b*x^3)^5*Hyper
geometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 57834*c^4*(b
*c - a*d)^5*x^15*HypergeometricPFQ[{2, 2, 2, 10/3}, {1, 1, 19/3}, ((b*c - a
*d)*x^3)/(c*(a + b*x^3))] + 224532*c^3*d*(b*c - a*d)^5*x^18*HypergeometricP
FQ[{2, 2, 2, 10/3}, {1, 1, 19/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 3265
92*c^2*d^2*(b*c - a*d)^5*x^21*HypergeometricPFQ[{2, 2, 2, 10/3}, {1, 1, 19/
3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 210924*c*d^3*(b*c - a*d)^5*x^24*Hy
pergeometricPFQ[{2, 2, 2, 10/3}, {1, 1, 19/3}, ((b*c - a*d)*x^3)/(c*(a + b*
x^3))] + 51030*d^4*(b*c - a*d)^5*x^27*HypergeometricPFQ[{2, 2, 2, 10/3}, {1
, 1, 19/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 5103*c^4*(b*c - a*d)^5*x^1
```

```
5*HypergeometricPFQ[{2, 2, 2, 2, 10/3}, {1, 1, 1, 19/3}, ((b*c - a*d)*x^3)/
(c*(a + b*x^3))] + 20412*c^3*d*(b*c - a*d)^5*x^18*HypergeometricPFQ[{2, 2,
2, 2, 10/3}, {1, 1, 1, 19/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 30618*c^
2*d^2*(b*c - a*d)^5*x^21*HypergeometricPFQ[{2, 2, 2, 2, 10/3}, {1, 1, 1, 19
/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 20412*c*d^3*(b*c - a*d)^5*x^24*Hy
pergeometricPFQ[{2, 2, 2, 2, 10/3}, {1, 1, 1, 19/3}, ((b*c - a*d)*x^3)/(c*(
a + b*x^3))] + 5103*d^4*(b*c - a*d)^5*x^27*HypergeometricPFQ[{2, 2, 2, 2, 1
0/3}, {1, 1, 1, 19/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))])/(524160*c^10*(b
- (a*d)/c)^4*x^11*(a + b*x^3)^(10/3)*(c + d*x^3)^2)
```

Maple [**F**]
time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^{3}+a\right)^{\frac{7}{3}} \left(d\,x^{3}+c\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^3+a)^(7/3)/(d*x^3+c)^3,x)
[Out] int(1/(b*x^3+a)^(7/3)/(d*x^3+c)^3,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(7/3)/(d*x^3+c)^3,x, algorithm="maxima")
[Out] integrate(1/((b*x^3 + a)^(7/3)*(d*x^3 + c)^3), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(7/3)/(d*x^3+c)^3,x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3+a)**(7/3)/(d*x**3+c)**3,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(7/3)/(d*x^3+c)^3,x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^3 + a)^(7/3)*(d*x^3 + c)^3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(b\,x^3+a\right)^{7/3}\,\left(d\,x^3+c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^3)^(7/3)*(c + d*x^3)^3),x)
```

```
[Out] int(1/((a + b*x^3)^(7/3)*(c + d*x^3)^3), x)
```

3.117 $\int \frac{(a+bx^3)^{4/3}}{(c+dx^3)^3}\,dx$

Optimal. Leaf size=60

$$\frac{ax\sqrt[3]{a+bx^3}\,F_1\left(\frac{1}{3};-\frac{4}{3},3;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^3\sqrt[3]{1+\frac{bx^3}{a}}}$$

```
[Out] a*x*(b*x^3+a)^(1/3)*AppellF1(1/3,-4/3,3,4/3,-b*x^3/a,-d*x^3/c)/c^3/(1+b*x^3
/a)^(1/3)
```

Rubi [A]
time = 0.02, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {441, 440}

$$\frac{ax\sqrt[3]{a+bx^3}\,F_1\left(\frac{1}{3};-\frac{4}{3},3;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^3\sqrt[3]{\frac{bx^3}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(4/3)/(c + d*x^3)^3,x]
```

```
[Out] (a*x*(a + b*x^3)^(1/3)*AppellF1[1/3, -4/3, 3, 4/3, -((b*x^3)/a), -((d*x^3)/
c)])/(c^3*(1 + (b*x^3)/a)^(1/3))
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{(a+bx^3)^{4/3}}{(c+dx^3)^3}\,dx = \frac{\left(a\sqrt[3]{a+bx^3}\right)\int \frac{\left(1+\frac{bx^3}{a}\right)^{4/3}}{(c+dx^3)^3}\,dx}{\sqrt[3]{1+\frac{bx^3}{a}}}$$
$$= \frac{ax\sqrt[3]{a+bx^3}\,F_1\left(\frac{1}{3};-\frac{4}{3},3;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^3\sqrt[3]{1+\frac{bx^3}{a}}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 316 vs. $2(60) = 120$.
time = 10.35, size = 316, normalized size = 5.27

$$\frac{x\left(b(2bc+5ad)x^3\left(1+\frac{bx^3}{a}\right)^{2/3}F_1\left(\frac{4}{3};\frac{2}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+\frac{4c\left(-abc^2+8a^2cd-b^2c^2x^3+10abcdx^3+5a^2d^2x^3+2b^2cdx^6+5abd^2x^6+\frac{4a^2c(bc+10ad)\left(c+dx^3\right)F_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4acF_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)-x^3\left(3adF_1\left(\frac{4}{3};\frac{2}{3},2;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+2bcF_1\left(\frac{4}{3};\frac{5}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)}\right)}{(c+dx^3)^2}\right)}{72c^3d\,(a+bx^3)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(4/3)/(c + d*x^3)^3,x]
```

```
[Out] (x*(b*(2*b*c + 5*a*d)*x^3*(1 + (b*x^3)/a)^(2/3)*AppellF1[4/3, 2/3, 1, 7/3,
-((b*x^3)/a), -((d*x^3)/c)] + (4*c*(-(a*b*c^2) + 8*a^2*c*d - b^2*c^2*x^3 +
10*a*b*c*d*x^3 + 5*a^2*d^2*x^3 + 2*b^2*c*d*x^6 + 5*a*b*d^2*x^6 + (4*a^2*c*(
b*c + 10*a*d)*(c + d*x^3)*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3
)/c)])/(4*a*c*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)] - x^3*
(3*a*d*AppellF1[4/3, 2/3, 2, 7/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*b*c*Appel
lF1[4/3, 5/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)]))))/(c + d*x^3)^2))/(72*c
^3*d*(a + b*x^3)^(2/3))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(b\,x^3+a)^{\frac{4}{3}}}{(d\,x^3+c)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(4/3)/(d*x^3+c)^3,x)
```

```
[Out] int((b*x^3+a)^(4/3)/(d*x^3+c)^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(4/3)/(d*x^3+c)^3,x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(4/3)/(d*x^3 + c)^3, x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(4/3)/(d*x^3+c)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(4/3)/(d*x**3+c)**3,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(4/3)/(d*x^3+c)^3,x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(4/3)/(d*x^3 + c)^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{{\left(b\,x^3+a\right)}^{4/3}}{{\left(d\,x^3+c\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(4/3)/(c + d*x^3)^3,x)
```

```
[Out] int((a + b*x^3)^(4/3)/(c + d*x^3)^3, x)
```

3.118 $\int \frac{\sqrt[3]{a+bx^3}}{(c+dx^3)^3}\,dx$

Optimal. Leaf size=59

$$\frac{x\sqrt[3]{a+bx^3}\,F_1\left(\frac{1}{3};-\frac{1}{3},3;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^3\sqrt[3]{1+\frac{bx^3}{a}}}$$

```
[Out] x*(b*x^3+a)^(1/3)*AppellF1(1/3,-1/3,3,4/3,-b*x^3/a,-d*x^3/c)/c^3/(1+b*x^3/a
)^(1/3)
```

Rubi [A]
time = 0.02, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {441, 440}

$$\frac{x\sqrt[3]{a+bx^3}\,F_1\left(\frac{1}{3};-\frac{1}{3},3;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^3\sqrt[3]{\frac{bx^3}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(1/3)/(c + d*x^3)^3,x]
```

```
[Out] (x*(a + b*x^3)^(1/3)*AppellF1[1/3, -1/3, 3, 4/3, -((b*x^3)/a), -((d*x^3)/c)
])/(c^3*(1 + (b*x^3)/a)^(1/3))
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{\sqrt[3]{a+bx^3}}{(c+dx^3)^3}\,dx = \frac{\sqrt[3]{a+bx^3}\int \frac{\sqrt[3]{1+\frac{bx^3}{a}}}{(c+dx^3)^3}\,dx}{\sqrt[3]{1+\frac{bx^3}{a}}}$$

$$= \frac{x\sqrt[3]{a+bx^3}\,F_1\left(\frac{1}{3};-\frac{1}{3},3;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^3\sqrt[3]{1+\frac{bx^3}{a}}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 431 vs. 2(59) = 118.

time = 10.40, size = 431, normalized size = 7.31

$$\frac{-b(-4bc+5ad)x^4\left(1+\frac{bx^3}{a}\right)^{2/3}F_1\left(\frac{4}{3};\frac{2}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+\frac{c\left(16acx\left(-b^2cx^3(7c+4dx^3)+3a^2d(6c+5dx^3)+ab(-18c^2-7cdx^3+5d^2x^6)\right)F_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)-4x^4\left(-b^2cx^3(7c+4dx^3)+a^2d(8c+5dx^3)+ab(-7c^2+4cdx^3+5d^2x^6)\right)\left(3adF_1\left(\frac{4}{3};\frac{2}{3},2;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+2bcF_1\left(\frac{4}{3};\frac{5}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)\right)}{(c+dx^3)^2\left(-4acF_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+x^3\left(3adF_1\left(\frac{4}{3};\frac{2}{3},2;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+2bcF_1\left(\frac{4}{3};\frac{5}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)\right)}}{72c^3(bc-ad)\,(a+bx^3)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(1/3)/(c + d*x^3)^3,x]
```

```
[Out] (-(b*(-4*b*c + 5*a*d)*x^4*(1 + (b*x^3)/a)^(2/3)*AppellF1[4/3, 2/3, 1, 7/3,
-((b*x^3)/a), -((d*x^3)/c)]) + (c*(16*a*c*x*(-(b^2*c*x^3*(7*c + 4*d*x^3)) +
 3*a^2*d*(6*c + 5*d*x^3) + a*b*(-18*c^2 - 7*c*d*x^3 + 5*d^2*x^6))*AppellF1[
1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)] - 4*x^4*(-(b^2*c*x^3*(7*c + 4
*d*x^3)) + a^2*d*(8*c + 5*d*x^3) + a*b*(-7*c^2 + 4*c*d*x^3 + 5*d^2*x^6))*(3
*a*d*AppellF1[4/3, 2/3, 2, 7/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*b*c*AppellF
1[4/3, 5/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)])))/((c + d*x^3)^2*(-4*a*c*A
ppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)] + x^3*(3*a*d*AppellF1
[4/3, 2/3, 2, 7/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*b*c*AppellF1[4/3, 5/3, 1
, 7/3, -((b*x^3)/a), -((d*x^3)/c)]))))/(72*c^3*(b*c - a*d)*(a + b*x^3)^(2/3
))
```

Maple [F]

time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^3+a\right)^{\frac{1}{3}}}{\left(d\,x^3+c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(1/3)/(d*x^3+c)^3,x)
```

```
[Out] int((b*x^3+a)^(1/3)/(d*x^3+c)^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/(d*x^3+c)^3,x, algorithm="maxima")
[Out] integrate((b*x^3 + a)^(1/3)/(d*x^3 + c)^3, x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/(d*x^3+c)^3,x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{a + bx^3}}{\left(c + dx^3\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(1/3)/(d*x**3+c)**3,x)
[Out] Integral((a + b*x**3)**(1/3)/(c + d*x**3)**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/(d*x^3+c)^3,x, algorithm="giac")
[Out] integrate((b*x^3 + a)^(1/3)/(d*x^3 + c)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(b\,x^3+a\right)^{1/3}}{\left(d\,x^3+c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(1/3)/(c + d*x^3)^3,x)
[Out] int((a + b*x^3)^(1/3)/(c + d*x^3)^3, x)
```

3.119 $\int \frac{1}{(a+bx^3)^{2/3}(c+dx^3)^3}\,dx$

Optimal. Leaf size=59

$$\frac{x\left(1+\frac{bx^3}{a}\right)^{2/3}F_1\left(\frac{1}{3};\frac{2}{3},3;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^3\left(a+bx^3\right)^{2/3}}$$

```
[Out] x*(1+b*x^3/a)^(2/3)*AppellF1(1/3,2/3,3,4/3,-b*x^3/a,-d*x^3/c)/c^3/(b*x^3+a)
^(2/3)
```

Rubi [A]
time = 0.02, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {441, 440}

$$\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}F_1\left(\frac{1}{3};\frac{2}{3},3;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^3\left(a+bx^3\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^3)^(2/3)*(c + d*x^3)^3),x]
```

```
[Out] (x*(1 + (b*x^3)/a)^(2/3)*AppellF1[1/3, 2/3, 3, 4/3, -((b*x^3)/a), -((d*x^3)
/c)])/(c^3*(a + b*x^3)^(2/3))
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{1}{(a+bx^3)^{2/3}(c+dx^3)^3}\,dx = \frac{\left(1+\frac{bx^3}{a}\right)^{2/3}\int \frac{1}{\left(1+\frac{bx^3}{a}\right)^{2/3}(c+dx^3)^3}\,dx}{(a+bx^3)^{2/3}}$$
$$= \frac{x\left(1+\frac{bx^3}{a}\right)^{2/3} F_1\left(\frac{1}{3};\frac{2}{3},3;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^3(a+bx^3)^{2/3}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 442 vs. 2(59) = 118.
time = 10.47, size = 442, normalized size = 7.49

$$\frac{x\left(5bd(-2bc+ad)x^3\left(1+\frac{bx^3}{a}\right)^{2/3}F_1\left(\frac{4}{3};\frac{2}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+\frac{4c\left(-4ac\left(3a^2d^2(6c+5dx^3)+b^2c(18c^2+5cdx^3-10d^2x^6)+abd(-36c^2-25cdx^3+5d^2x^6)\right)F_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+dx^3\left(a^2d(8c+5dx^3)-b^2cx^3(13c+10dx^3)+ab(-13c^2-2cdx^3+5d^2x^6)\right)\left(3adF_1\left(\frac{4}{3};\frac{2}{3},2;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+2bcF_1\left(\frac{4}{3};\frac{5}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)\right)}{(c+dx^3)^2\left(-4acF_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+x^3\left(3adF_1\left(\frac{4}{3};\frac{2}{3},2;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+2bcF_1\left(\frac{4}{3};\frac{5}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)\right)}\right)}{72c^3(bc-ad)^2(a+bx^3)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^3)^(2/3)*(c + d*x^3)^3),x]
```

```
[Out] (x*(5*b*d*(-2*b*c + a*d)*x^3*(1 + (b*x^3)/a)^(2/3)*AppellF1[4/3, 2/3, 1, 7/
3, -((b*x^3)/a), -((d*x^3)/c)] + (4*c*(-4*a*c*(3*a^2*d^2*(6*c + 5*d*x^3) +
b^2*c*(18*c^2 + 5*c*d*x^3 - 10*d^2*x^6) + a*b*d*(-36*c^2 - 25*c*d*x^3 + 5*d
^2*x^6))*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)] + d*x^3*(a^
2*d*(8*c + 5*d*x^3) - b^2*c*x^3*(13*c + 10*d*x^3) + a*b*(-13*c^2 - 2*c*d*x^
3 + 5*d^2*x^6))*(3*a*d*AppellF1[4/3, 2/3, 2, 7/3, -((b*x^3)/a), -((d*x^3)/c
)] + 2*b*c*AppellF1[4/3, 5/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)])))/((c +
d*x^3)^2*(-4*a*c*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)] + x
^3*(3*a*d*AppellF1[4/3, 2/3, 2, 7/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*b*c*Ap
pellF1[4/3, 5/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)]))))/(72*c^3*(b*c - a*
d)^2*(a + b*x^3)^(2/3))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\,x^3+a)^{\frac{2}{3}}(d\,x^3+c)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^3+a)^(2/3)/(d*x^3+c)^3,x)
```

```
[Out] int(1/(b*x^3+a)^(2/3)/(d*x^3+c)^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(2/3)/(d*x^3+c)^3,x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)^(2/3)*(d*x^3 + c)^3), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(2/3)/(d*x^3+c)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3+a)**(2/3)/(d*x**3+c)**3,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(2/3)/(d*x^3+c)^3,x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^3 + a)^(2/3)*(d*x^3 + c)^3), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{{\left(b\,x^3+a\right)}^{2/3}\,{\left(d\,x^3+c\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^3)^(2/3)*(c + d*x^3)^3),x)
```

```
[Out] int(1/((a + b*x^3)^(2/3)*(c + d*x^3)^3), x)
```

3.120 $\int \frac{1}{(a+bx^3)^{5/3}(c+dx^3)^3}\,dx$

Optimal. Leaf size=62

$$\frac{x\left(1+\frac{bx^3}{a}\right)^{2/3}F_1\left(\frac{1}{3};\frac{5}{3},3;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{ac^3\left(a+bx^3\right)^{2/3}}$$

```
[Out] x*(1+b*x^3/a)^(2/3)*AppellF1(1/3,5/3,3,4/3,-b*x^3/a,-d*x^3/c)/a/c^3/(b*x^3+
a)^(2/3)
```

Rubi [A]
time = 0.02, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {441, 440}

$$\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}F_1\left(\frac{1}{3};\frac{5}{3},3;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{ac^3\left(a+bx^3\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^3)^(5/3)*(c + d*x^3)^3),x]
```

```
[Out] (x*(1 + (b*x^3)/a)^(2/3)*AppellF1[1/3, 5/3, 3, 4/3, -((b*x^3)/a), -((d*x^3)
/c)])/(a*c^3*(a + b*x^3)^(2/3))
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{1}{(a+bx^3)^{5/3}(c+dx^3)^3}\,dx = \frac{\left(1+\frac{bx^3}{a}\right)^{2/3}\int \frac{1}{\left(1+\frac{bx^3}{a}\right)^{5/3}(c+dx^3)^3}\,dx}{a(a+bx^3)^{2/3}}$$
$$= \frac{x\left(1+\frac{bx^3}{a}\right)^{2/3} F_1\left(\frac{1}{3};\frac{5}{3},3;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{ac^3(a+bx^3)^{2/3}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 531 vs. 2(62) = 124.
time = 10.68, size = 531, normalized size = 8.56

$$\frac{\frac{bd(-9b^2c^2-16abcd+5a^2d^2)x^4\left(1+\frac{bx^3}{a}\right)^{2/3}F_1\left(\frac{4}{3};\frac{2}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{(-bc+ad)^3}-\frac{4c\left(4acx\left(3a^3d^3(6c+5dx^3)+ab^2cd(54c^2+35cdx^3-16d^2x^6)-9b^3c^2(2c^2+3cdx^3+d^2x^6)+a^2bd^2(-54c^2-43cdx^3+5d^2x^6)\right)F_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+x^4\left(9b^3c^2(c+dx^3)^2-a^3d^3(8c+5dx^3)+ab^2cd^2x^3(19c+16dx^3)+a^2bd^2(19c^2+8cdx^3-5d^2x^6)\right)\left(3adF_1\left(\frac{4}{3};\frac{2}{3},2;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+2bcF_1\left(\frac{4}{3};\frac{5}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)\right)}{(bc-ad)^3(c+dx^3)^2\left(4acF_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)-x^3\left(3adF_1\left(\frac{4}{3};\frac{2}{3},2;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+2bcF_1\left(\frac{4}{3};\frac{5}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)\right)}}{72ac^3(a+bx^3)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^3)^(5/3)*(c + d*x^3)^3),x]
```

```
[Out] ((b*d*(-9*b^2*c^2 - 16*a*b*c*d + 5*a^2*d^2)*x^4*(1 + (b*x^3)/a)^(2/3)*Appel
lF1[4/3, 2/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)])/(-(b*c) + a*d)^3 - (4*c*
(4*a*c*x*(3*a^3*d^3*(6*c + 5*d*x^3) + a*b^2*c*d*(54*c^2 + 35*c*d*x^3 - 16*d
^2*x^6) - 9*b^3*c^2*(2*c^2 + 3*c*d*x^3 + d^2*x^6) + a^2*b*d^2*(-54*c^2 - 43
*c*d*x^3 + 5*d^2*x^6))*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c
)] + x^4*(9*b^3*c^2*(c + d*x^3)^2 - a^3*d^3*(8*c + 5*d*x^3) + a*b^2*c*d^2*x
^3*(19*c + 16*d*x^3) + a^2*b*d^2*(19*c^2 + 8*c*d*x^3 - 5*d^2*x^6))*(3*a*d*A
ppellF1[4/3, 2/3, 2, 7/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*b*c*AppellF1[4/3,
 5/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)])))/((b*c - a*d)^3*(c + d*x^3)^2*(
4*a*c*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)] - x^3*(3*a*d*A
ppellF1[4/3, 2/3, 2, 7/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*b*c*AppellF1[4/3,
 5/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)]))))/(72*a*c^3*(a + b*x^3)^(2/3))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\,x^3+a)^{\frac{5}{3}}(d\,x^3+c)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^3+a)^(5/3)/(d*x^3+c)^3,x)
```

```
[Out] int(1/(b*x^3+a)^(5/3)/(d*x^3+c)^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(5/3)/(d*x^3+c)^3,x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)^(5/3)*(d*x^3 + c)^3), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(5/3)/(d*x^3+c)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3+a)**(5/3)/(d*x**3+c)**3,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(5/3)/(d*x^3+c)^3,x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^3 + a)^(5/3)*(d*x^3 + c)^3), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{{\left(b\,x^3+a\right)}^{5/3}\,{\left(d\,x^3+c\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^3)^(5/3)*(c + d*x^3)^3),x)
```

```
[Out] int(1/((a + b*x^3)^(5/3)*(c + d*x^3)^3), x)
```

3.121 $\int \frac{1}{\left(a+bx^3\right)^{8/3}\left(c+dx^3\right)^3}\,dx$

Optimal. Leaf size=62

$$\frac{x\left(1+\frac{bx^3}{a}\right)^{2/3}F_1\left(\frac{1}{3};\frac{8}{3},3;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2c^3\left(a+bx^3\right)^{2/3}}$$

```
[Out] x*(1+b*x^3/a)^(2/3)*AppellF1(1/3,8/3,3,4/3,-b*x^3/a,-d*x^3/c)/a^2/c^3/(b*x^
3+a)^(2/3)
```

Rubi [A]
time = 0.02, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {441, 440}

$$\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}F_1\left(\frac{1}{3};\frac{8}{3},3;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2c^3\left(a+bx^3\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^3)^(8/3)*(c + d*x^3)^3),x]
```

```
[Out] (x*(1 + (b*x^3)/a)^(2/3)*AppellF1[1/3, 8/3, 3, 4/3, -((b*x^3)/a), -((d*x^3)
/c)])/(a^2*c^3*(a + b*x^3)^(2/3))
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{1}{(a+bx^3)^{8/3}(c+dx^3)^3}\,dx = \frac{\left(1+\frac{bx^3}{a}\right)^{2/3}\int \frac{1}{\left(1+\frac{bx^3}{a}\right)^{8/3}(c+dx^3)^3}\,dx}{a^2(a+bx^3)^{2/3}}$$

$$= \frac{x\left(1+\frac{bx^3}{a}\right)^{2/3} F_1\left(\frac{1}{3};\frac{8}{3},3;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2c^3(a+bx^3)^{2/3}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 515 vs. 2(62) = 124.
time = 11.08, size = 515, normalized size = 8.31

$$\frac{x\left(bd(36b^3c^3-171ab^2c^2d-110a^2bcd^2+25a^3d^3)x^3\left(1+\frac{bx^3}{a}\right)^{2/3}F_1\left(\frac{4}{3};\frac{2}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+\frac{4c\left(\frac{36b^5c^3x^3(c+dx^3)^2+9ab^4c^2(6c-19dx^3)(c+dx^3)^2+5a^5d^4(8c+5dx^3)+5a^3b^2d^3x^3(-50c^2-36cdx^3+5d^2x^6)+5a^4bd^3(-25c^2-6cdx^3+10d^2x^6)-a^2b^3cd(189c^3+378c^2dx^3+314cd^2x^6+110d^3x^9)}{a+bx^3}+\frac{4ac(36b^4c^4-171ab^3c^3d+540a^2b^2c^2d^2-235a^3bcd^3+50a^4d^4)(c+dx^3)F_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4acF_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)-x^3\left(3adF_1\left(\frac{4}{3};\frac{2}{3},2;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+2bcF_1\left(\frac{4}{3};\frac{5}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)}\right)}{(c+dx^3)^2}\right)}{360a^2c^3(bc-ad)^4(a+bx^3)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^3)^(8/3)*(c + d*x^3)^3),x]
```

```
[Out] (x*(b*d*(36*b^3*c^3 - 171*a*b^2*c^2*d - 110*a^2*b*c*d^2 + 25*a^3*d^3)*x^3*(
1 + (b*x^3)/a)^(2/3)*AppellF1[4/3, 2/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)]
 + (4*c*((36*b^5*c^3*x^3*(c + d*x^3)^2 + 9*a*b^4*c^2*(6*c - 19*d*x^3)*(c +
d*x^3)^2 + 5*a^5*d^4*(8*c + 5*d*x^3) + 5*a^3*b^2*d^3*x^3*(-50*c^2 - 36*c*d*
x^3 + 5*d^2*x^6) + 5*a^4*b*d^3*(-25*c^2 - 6*c*d*x^3 + 10*d^2*x^6) - a^2*b^3
*c*d*(189*c^3 + 378*c^2*d*x^3 + 314*c*d^2*x^6 + 110*d^3*x^9))/(a + b*x^3) +
 (4*a*c*(36*b^4*c^4 - 171*a*b^3*c^3*d + 540*a^2*b^2*c^2*d^2 - 235*a^3*b*c*d
^3 + 50*a^4*d^4)*(c + d*x^3)*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*
x^3)/c)])/(4*a*c*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)] - x
^3*(3*a*d*AppellF1[4/3, 2/3, 2, 7/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*b*c*Ap
pellF1[4/3, 5/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)]))))/(c + d*x^3)^2))/(3
60*a^2*c^3*(b*c - a*d)^4*(a + b*x^3)^(2/3))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\,x^3+a)^{\frac{8}{3}}(d\,x^3+c)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^3+a)^(8/3)/(d*x^3+c)^3,x)
```

```
[Out] int(1/(b*x^3+a)^(8/3)/(d*x^3+c)^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(8/3)/(d*x^3+c)^3,x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)^(8/3)*(d*x^3 + c)^3), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(8/3)/(d*x^3+c)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3+a)**(8/3)/(d*x**3+c)**3,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(8/3)/(d*x^3+c)^3,x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^3 + a)^(8/3)*(d*x^3 + c)^3), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{{\left(b\,x^3+a\right)}^{8/3}\,{\left(d\,x^3+c\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^3)^(8/3)*(c + d*x^3)^3),x)
```

```
[Out] int(1/((a + b*x^3)^(8/3)*(c + d*x^3)^3), x)
```

3.122 $\int \frac{(a+bx^3)^{7/4}}{(c+dx^3)^{37/12}}\,dx$

Optimal. Leaf size=155

$$\frac{4x(a+bx^3)^{7/4}}{25c\,(c+dx^3)^{25/12}}+\frac{84ax(a+bx^3)^{3/4}}{325c^2\,(c+dx^3)^{13/12}}+\frac{189a^2x\sqrt[4]{\frac{c\,(a+bx^3)}{a\,(c+dx^3)}}\;{}_2F_1\left(\frac{1}{4},\frac{1}{3};\frac{4}{3};-\frac{(bc-ad)x^3}{a(c+dx^3)}\right)}{325c^3\sqrt[4]{a+bx^3}\;\sqrt[12]{c+dx^3}}$$

```
[Out] 4/25*x*(b*x^3+a)^(7/4)/c/(d*x^3+c)^(25/12)+84/325*a*x*(b*x^3+a)^(3/4)/c^2/(
d*x^3+c)^(13/12)+189/325*a^2*x*(c*(b*x^3+a)/a/(d*x^3+c))^(1/4)*hypergeom([1
/4, 1/3],[4/3],-(-a*d+b*c)*x^3/a/(d*x^3+c))/c^3/(b*x^3+a)^(1/4)/(d*x^3+c)^(
1/12)
```

Rubi [A]
time = 0.04, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {386, 388}

$$\frac{189a^2x\sqrt[4]{\frac{c\,(a+bx^3)}{a\,(c+dx^3)}}\;{}_2F_1\left(\frac{1}{4},\frac{1}{3};\frac{4}{3};-\frac{(bc-ad)x^3}{a(dx^3+c)}\right)}{325c^3\sqrt[4]{a+bx^3}\;\sqrt[12]{c+dx^3}}+\frac{84ax(a+bx^3)^{3/4}}{325c^2\,(c+dx^3)^{13/12}}+\frac{4x(a+bx^3)^{7/4}}{25c\,(c+dx^3)^{25/12}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(7/4)/(c + d*x^3)^(37/12),x]
```

```
[Out] (4*x*(a + b*x^3)^(7/4))/(25*c*(c + d*x^3)^(25/12)) + (84*a*x*(a + b*x^3)^(3
/4))/(325*c^2*(c + d*x^3)^(13/12)) + (189*a^2*x*((c*(a + b*x^3))/(a*(c + d*
x^3)))^(1/4)*Hypergeometric2F1[1/4, 1/3, 4/3, -(((b*c - a*d)*x^3)/(a*(c + d
*x^3)))])/(325*c^3*(a + b*x^3)^(1/4)*(c + d*x^3)^(1/12))
```

Rule 386

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] - Dist[
c*(q/(a*(p + 1))), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1,
0] && GtQ[q, 0] && NeQ[p, -1]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[x*((a + b*x^n)^p/(c*(c*((a + b*x^n)/(a*(c + d*x^n))))^p*(c + d*x^n)
^(1/n + p)))*Hypergeometric2F1[1/n, -p, 1 + 1/n, (-(b*c - a*d))*(x^n/(a*(c
```

```
+ d*x^n)))], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] &&
EqQ[n*(p + q + 1) + 1, 0]
```

Rubi steps

$$\int \frac{(a+bx^3)^{7/4}}{(c+dx^3)^{37/12}}\,dx = \frac{4x(a+bx^3)^{7/4}}{25c\,(c+dx^3)^{25/12}} + \frac{(21a)\int \frac{(a+bx^3)^{3/4}}{(c+dx^3)^{25/12}}\,dx}{25c}$$

$$= \frac{4x(a+bx^3)^{7/4}}{25c\,(c+dx^3)^{25/12}} + \frac{84ax(a+bx^3)^{3/4}}{325c^2\,(c+dx^3)^{13/12}} + \frac{(189a^2)\int \frac{1}{\sqrt[4]{a+bx^3}\,(c+dx^3)^{13/12}}\,dx}{325c^2}$$

$$= \frac{4x(a+bx^3)^{7/4}}{25c\,(c+dx^3)^{25/12}} + \frac{84ax(a+bx^3)^{3/4}}{325c^2\,(c+dx^3)^{13/12}} + \frac{189a^2x\sqrt[4]{\frac{c(a+bx^3)}{a(c+dx^3)}}\;{}_2F_1\left(\frac{1}{4},\frac{1}{3};\frac{4}{3};-\frac{(bc-}{a(c+}\right.}{325c^3\sqrt[4]{a+bx^3}\;\sqrt[12]{c+dx^3}}$$

Mathematica [A]
time = 5.80, size = 90, normalized size = 0.58

$$\frac{ax(a+bx^3)^{3/4}\;{}_2F_1\left(-\frac{7}{4},\frac{1}{3};\frac{4}{3};\frac{(-bc+ad)x^3}{a(c+dx^3)}\right)}{c^3\left(1+\frac{bx^3}{a}\right)^{3/4}\sqrt[12]{c+dx^3}\,\sqrt[4]{1+\frac{dx^3}{c}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(7/4)/(c + d*x^3)^(37/12),x]
```

```
[Out] (a*x*(a + b*x^3)^(3/4)*Hypergeometric2F1[-7/4, 1/3, 4/3, ((-(b*c) + a*d)*x^
3)/(a*(c + d*x^3))])/(c^3*(1 + (b*x^3)/a)^(3/4)*(c + d*x^3)^(1/12)*(1 + (d*
x^3)/c)^(1/4))
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{(b\,x^3+a)^{\frac{7}{4}}}{(d\,x^3+c)^{\frac{37}{12}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(7/4)/(d*x^3+c)^(37/12),x)
```

```
[Out] int((b*x^3+a)^(7/4)/(d*x^3+c)^(37/12),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(7/4)/(d*x^3+c)^(37/12),x, algorithm="maxima")
[Out] integrate((b*x^3 + a)^(7/4)/(d*x^3 + c)^(37/12), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(7/4)/(d*x^3+c)^(37/12),x, algorithm="fricas")
[Out] integral((b*x^3 + a)^(7/4)*(d*x^3 + c)^(11/12)/(d^4*x^12 + 4*c*d^3*x^9 + 6*
c^2*d^2*x^6 + 4*c^3*d*x^3 + c^4), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(7/4)/(d*x**3+c)**(37/12),x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(7/4)/(d*x^3+c)^(37/12),x, algorithm="giac")
[Out] integrate((b*x^3 + a)^(7/4)/(d*x^3 + c)^(37/12), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(b\,x^3+a\right)}^{7/4}}{{\left(d\,x^3+c\right)}^{37/12}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(7/4)/(c + d*x^3)^(37/12),x)
[Out] int((a + b*x^3)^(7/4)/(c + d*x^3)^(37/12), x)
```

3.123 $\int \frac{(a+bx^3)^{5/4}}{(c+dx^3)^{31/12}} \, dx$

Optimal. Leaf size=155

$$\frac{4x(a+bx^3)^{5/4}}{19c\,(c+dx^3)^{19/12}}+\frac{60ax\sqrt[4]{a+bx^3}}{133c^2\,(c+dx^3)^{7/12}}+\frac{45a^2x\left(\frac{c(a+bx^3)}{a(c+dx^3)}\right)^{3/4}(c+dx^3)^{5/12}\,{}_2F_1\left(\frac{1}{3},\frac{3}{4};\frac{4}{3};-\frac{(bc-ad)x^3}{a(c+dx^3)}\right)}{133c^3\,(a+bx^3)^{3/4}}$$

```
[Out] 4/19*x*(b*x^3+a)^(5/4)/c/(d*x^3+c)^(19/12)+60/133*a*x*(b*x^3+a)^(1/4)/c^2/(
d*x^3+c)^(7/12)+45/133*a^2*x*(c*(b*x^3+a)/a/(d*x^3+c))^(3/4)*(d*x^3+c)^(5/1
2)*hypergeom([1/3, 3/4],[4/3],-(-a*d+b*c)*x^3/a/(d*x^3+c))/c^3/(b*x^3+a)^(3
/4)
```

Rubi [A]

time = 0.04, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {386, 388}

$$\frac{45a^2x(c+dx^3)^{5/12}\left(\frac{c(a+bx^3)}{a(c+dx^3)}\right)^{3/4}\,{}_2F_1\left(\frac{1}{3},\frac{3}{4};\frac{4}{3};-\frac{(bc-ad)x^3}{a(dx^3+c)}\right)}{133c^3\,(a+bx^3)^{3/4}}+\frac{60ax\sqrt[4]{a+bx^3}}{133c^2\,(c+dx^3)^{7/12}}+\frac{4x(a+bx^3)^{5/4}}{19c\,(c+dx^3)^{19/12}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(5/4)/(c + d*x^3)^(31/12),x]
```

```
[Out] (4*x*(a + b*x^3)^(5/4))/(19*c*(c + d*x^3)^(19/12)) + (60*a*x*(a + b*x^3)^(1
/4))/(133*c^2*(c + d*x^3)^(7/12)) + (45*a^2*x*((c*(a + b*x^3))/(a*(c + d*x^
3)))^(3/4)*(c + d*x^3)^(5/12)*Hypergeometric2F1[1/3, 3/4, 4/3, -(((b*c - a*
d)*x^3)/(a*(c + d*x^3)))])/(133*c^3*(a + b*x^3)^(3/4))
```

Rule 386

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] - Dist[
c*(q/(a*(p + 1))), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1,
0] && GtQ[q, 0] && NeQ[p, -1]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[x*((a + b*x^n)^p/(c*(c*((a + b*x^n)/(a*(c + d*x^n))))^p*(c + d*x^n)
^(1/n + p)))*Hypergeometric2F1[1/n, -p, 1 + 1/n, (-(b*c - a*d))*(x^n/(a*(c
+ d*x^n)))], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] &&
EqQ[n*(p + q + 1) + 1, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a+bx^3)^{5/4}}{(c+dx^3)^{31/12}}\,dx &= \frac{4x(a+bx^3)^{5/4}}{19c\,(c+dx^3)^{19/12}} + \frac{(15a)\int \frac{\sqrt[4]{a+bx^3}}{(c+dx^3)^{19/12}}\,dx}{19c} \\
&= \frac{4x(a+bx^3)^{5/4}}{19c\,(c+dx^3)^{19/12}} + \frac{60ax\sqrt[4]{a+bx^3}}{133c^2\,(c+dx^3)^{7/12}} + \frac{(45a^2)\int \frac{1}{(a+bx^3)^{3/4}(c+dx^3)^{7/12}}\,dx}{133c^2} \\
&= \frac{4x(a+bx^3)^{5/4}}{19c\,(c+dx^3)^{19/12}} + \frac{60ax\sqrt[4]{a+bx^3}}{133c^2\,(c+dx^3)^{7/12}} + \frac{45a^2x\left(\frac{c(a+bx^3)}{a(c+dx^3)}\right)^{3/4}(c+dx^3)^{5/12}\,{}_2F_1\left(\frac{1}{3},\frac{3}{4};\right.}{133c^3\,(a+bx^3)^{3/4}}
\end{aligned}
$$

Mathematica [A]
time = 5.54, size = 90, normalized size = 0.58

$$
\frac{ax\sqrt[4]{a+bx^3}\,\sqrt[4]{1+\frac{dx^3}{c}}\,{}_2F_1\left(-\frac{5}{4},\frac{1}{3};\frac{4}{3};\frac{(-bc+ad)x^3}{a(c+dx^3)}\right)}{c^2\sqrt[4]{1+\frac{bx^3}{a}}\,(c+dx^3)^{7/12}}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(5/4)/(c + d*x^3)^(31/12),x]
```

```
[Out] (a*x*(a + b*x^3)^(1/4)*(1 + (d*x^3)/c)^(1/4)*Hypergeometric2F1[-5/4, 1/3, 4
/3, ((-(b*c) + a*d)*x^3)/(a*(c + d*x^3))])/(c^2*(1 + (b*x^3)/a)^(1/4)*(c +
d*x^3)^(7/12))
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$
\int \frac{(b\,x^3+a)^{\frac{5}{4}}}{(d\,x^3+c)^{\frac{31}{12}}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(5/4)/(d*x^3+c)^(31/12),x)
```

```
[Out] int((b*x^3+a)^(5/4)/(d*x^3+c)^(31/12),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(5/4)/(d*x^3+c)^(31/12),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(5/4)/(d*x^3 + c)^(31/12), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(5/4)/(d*x^3+c)^(31/12),x, algorithm="fricas")
```

```
[Out] integral((b*x^3 + a)^(5/4)*(d*x^3 + c)^(5/12)/(d^3*x^9 + 3*c*d^2*x^6 + 3*c^
2*d*x^3 + c^3), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(5/4)/(d*x**3+c)**(31/12),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(5/4)/(d*x^3+c)^(31/12),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(5/4)/(d*x^3 + c)^(31/12), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(b\,x^3+a\right)}^{5/4}}{{\left(d\,x^3+c\right)}^{31/12}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(5/4)/(c + d*x^3)^(31/12),x)
```

```
[Out] int((a + b*x^3)^(5/4)/(c + d*x^3)^(31/12), x)
```

3.124 $\int \frac{(a+bx^3)^{3/4}}{(c+dx^3)^{25/12}}\,dx$

Optimal. Leaf size=122

$$\frac{4x(a+bx^3)^{3/4}}{13c\,(c+dx^3)^{13/12}}+\frac{9ax\sqrt[4]{\frac{c\,(a+bx^3)}{a\,(c+dx^3)}}\;{}_2F_1\left(\frac{1}{4},\frac{1}{3};\frac{4}{3};-\frac{(bc-ad)x^3}{a(c+dx^3)}\right)}{13c^2\sqrt[4]{a+bx^3}\;\sqrt[12]{c+dx^3}}$$

```
[Out] 4/13*x*(b*x^3+a)^(3/4)/c/(d*x^3+c)^(13/12)+9/13*a*x*(c*(b*x^3+a)/a/(d*x^3+c
))^(1/4)*hypergeom([1/4, 1/3],[4/3],-(-a*d+b*c)*x^3/a/(d*x^3+c))/c^2/(b*x^3
+a)^(1/4)/(d*x^3+c)^(1/12)
```

Rubi [A]
time = 0.03, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {386, 388}

$$\frac{9ax\sqrt[4]{\frac{c\,(a+bx^3)}{a\,(c+dx^3)}}\;{}_2F_1\left(\frac{1}{4},\frac{1}{3};\frac{4}{3};-\frac{(bc-ad)x^3}{a(dx^3+c)}\right)}{13c^2\sqrt[4]{a+bx^3}\;\sqrt[12]{c+dx^3}}+\frac{4x(a+bx^3)^{3/4}}{13c\,(c+dx^3)^{13/12}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(3/4)/(c + d*x^3)^(25/12),x]
```

```
[Out] (4*x*(a + b*x^3)^(3/4))/(13*c*(c + d*x^3)^(13/12)) + (9*a*x*((c*(a + b*x^3)
)/(a*(c + d*x^3)))^(1/4)*Hypergeometric2F1[1/4, 1/3, 4/3, -(((b*c - a*d)*x^
3)/(a*(c + d*x^3)))])/(13*c^2*(a + b*x^3)^(1/4)*(c + d*x^3)^(1/12))
```

Rule 386

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] - Dist[
c*(q/(a*(p + 1))), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1,
0] && GtQ[q, 0] && NeQ[p, -1]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[x*((a + b*x^n)^p/(c*(c*((a + b*x^n)/(a*(c + d*x^n))))^p*(c + d*x^n)
^(1/n + p)))*Hypergeometric2F1[1/n, -p, 1 + 1/n, (-(b*c - a*d))*(x^n/(a*(c
+ d*x^n)))], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] &&
EqQ[n*(p + q + 1) + 1, 0]
```

Rubi steps

$$\int \frac{(a+bx^3)^{3/4}}{(c+dx^3)^{25/12}}\,dx = \frac{4x(a+bx^3)^{3/4}}{13c\,(c+dx^3)^{13/12}} + \frac{(9a)\int \frac{1}{\sqrt[4]{a+bx^3}\,(c+dx^3)^{13/12}}\,dx}{13c}$$

$$= \frac{4x(a+bx^3)^{3/4}}{13c\,(c+dx^3)^{13/12}} + \frac{9ax\sqrt[4]{\frac{c(a+bx^3)}{a(c+dx^3)}}\;{}_2F_1\left(\frac{1}{4},\frac{1}{3};\frac{4}{3};-\frac{(bc-ad)x^3}{a(c+dx^3)}\right)}{13c^2\sqrt[4]{a+bx^3}\,\sqrt[12]{c+dx^3}}$$

Mathematica [A]
time = 5.81, size = 89, normalized size = 0.73

$$\frac{x(a+bx^3)^{3/4}\;{}_2F_1\left(-\frac{3}{4},\frac{1}{3};\frac{4}{3};\frac{(-bc+ad)x^3}{a(c+dx^3)}\right)}{c^2\left(1+\frac{bx^3}{a}\right)^{3/4}\sqrt[12]{c+dx^3}\,\sqrt[4]{1+\frac{dx^3}{c}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(3/4)/(c + d*x^3)^(25/12),x]
```

```
[Out] (x*(a + b*x^3)^(3/4)*Hypergeometric2F1[-3/4, 1/3, 4/3, ((-(b*c) + a*d)*x^3)
/(a*(c + d*x^3))])/(c^2*(1 + (b*x^3)/a)^(3/4)*(c + d*x^3)^(1/12)*(1 + (d*x^
3)/c)^(1/4))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(b\,x^3+a)^{\frac{3}{4}}}{(d\,x^3+c)^{\frac{25}{12}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(3/4)/(d*x^3+c)^(25/12),x)
```

```
[Out] int((b*x^3+a)^(3/4)/(d*x^3+c)^(25/12),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/4)/(d*x^3+c)^(25/12),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(3/4)/(d*x^3 + c)^(25/12), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/4)/(d*x^3+c)^(25/12),x, algorithm="fricas")
[Out] integral((b*x^3 + a)^(3/4)*(d*x^3 + c)^(11/12)/(d^3*x^9 + 3*c*d^2*x^6 + 3*c
^2*d*x^3 + c^3), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(3/4)/(d*x**3+c)**(25/12),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 9881 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/4)/(d*x^3+c)^(25/12),x, algorithm="giac")
[Out] integrate((b*x^3 + a)^(3/4)/(d*x^3 + c)^(25/12), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(b\,x^3+a)^{3/4}}{(d\,x^3+c)^{25/12}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(3/4)/(c + d*x^3)^(25/12),x)
[Out] int((a + b*x^3)^(3/4)/(c + d*x^3)^(25/12), x)
```

3.125 $\int \frac{\sqrt[4]{a+bx^3}}{(c+dx^3)^{19/12}}\,dx$

Optimal. Leaf size=122

$$\frac{4x\sqrt[4]{a+bx^3}}{7c\left(c+dx^3\right)^{7/12}}+\frac{3ax\left(\frac{c(a+bx^3)}{a(c+dx^3)}\right)^{3/4}\left(c+dx^3\right)^{5/12}{}_2F_1\left(\frac{1}{3},\frac{3}{4};\frac{4}{3};-\frac{(bc-ad)x^3}{a(c+dx^3)}\right)}{7c^2\left(a+bx^3\right)^{3/4}}$$

```
[Out] 4/7*x*(b*x^3+a)^(1/4)/c/(d*x^3+c)^(7/12)+3/7*a*x*(c*(b*x^3+a)/a/(d*x^3+c))^
(3/4)*(d*x^3+c)^(5/12)*hypergeom([1/3, 3/4],[4/3],-(-a*d+b*c)*x^3/a/(d*x^3+
c))/c^2/(b*x^3+a)^(3/4)
```

Rubi [A]

time = 0.03, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {386, 388}

$$\frac{3ax(c+dx^3)^{5/12}\left(\frac{c(a+bx^3)}{a(c+dx^3)}\right)^{3/4}{}_2F_1\left(\frac{1}{3},\frac{3}{4};\frac{4}{3};-\frac{(bc-ad)x^3}{a(dx^3+c)}\right)}{7c^2\left(a+bx^3\right)^{3/4}}+\frac{4x\sqrt[4]{a+bx^3}}{7c\left(c+dx^3\right)^{7/12}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(1/4)/(c + d*x^3)^(19/12),x]
```

```
[Out] (4*x*(a + b*x^3)^(1/4))/(7*c*(c + d*x^3)^(7/12)) + (3*a*x*((c*(a + b*x^3))/
(a*(c + d*x^3)))^(3/4)*(c + d*x^3)^(5/12)*Hypergeometric2F1[1/3, 3/4, 4/3,
-(((b*c - a*d)*x^3)/(a*(c + d*x^3)))])/(7*c^2*(a + b*x^3)^(3/4))
```

Rule 386

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] - Dist[
c*(q/(a*(p + 1))), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1,
0] && GtQ[q, 0] && NeQ[p, -1]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[x*((a + b*x^n)^p/(c*(c*((a + b*x^n)/(a*(c + d*x^n))))^p*(c + d*x^n)
^(1/n + p)))*Hypergeometric2F1[1/n, -p, 1 + 1/n, (-(b*c - a*d))*(x^n/(a*(c
+ d*x^n)))], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] &&
EqQ[n*(p + q + 1) + 1, 0]
```

Rubi steps

$$\int \frac{\sqrt[4]{a+bx^3}}{(c+dx^3)^{19/12}}\,dx = \frac{4x\sqrt[4]{a+bx^3}}{7c\,(c+dx^3)^{7/12}} + \frac{(3a)\int \frac{1}{(a+bx^3)^{3/4}(c+dx^3)^{7/12}}\,dx}{7c}$$
$$= \frac{4x\sqrt[4]{a+bx^3}}{7c\,(c+dx^3)^{7/12}} + \frac{3ax\left(\frac{c(a+bx^3)}{a(c+dx^3)}\right)^{3/4}(c+dx^3)^{5/12}\,{}_2F_1\left(\frac{1}{3},\frac{3}{4};\frac{4}{3};-\frac{(bc-ad)x^3}{a(c+dx^3)}\right)}{7c^2\,(a+bx^3)^{3/4}}$$

Mathematica [A]
time = 3.84, size = 89, normalized size = 0.73

$$\frac{x\sqrt[4]{a+bx^3}\,\sqrt[4]{1+\frac{dx^3}{c}}\;{}_2F_1\left(-\frac{1}{4},\frac{1}{3};\frac{4}{3};\frac{(-bc+ad)x^3}{a(c+dx^3)}\right)}{c\sqrt[4]{1+\frac{bx^3}{a}}\;(c+dx^3)^{7/12}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(1/4)/(c + d*x^3)^(19/12),x]
[Out] (x*(a + b*x^3)^(1/4)*(1 + (d*x^3)/c)^(1/4)*Hypergeometric2F1[-1/4, 1/3, 4/3
, ((-(b*c) + a*d)*x^3)/(a*(c + d*x^3))])/(c*(1 + (b*x^3)/a)^(1/4)*(c + d*x^
3)^(7/12))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(b\,x^3+a)^{\frac{1}{4}}}{(d\,x^3+c)^{\frac{19}{12}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(1/4)/(d*x^3+c)^(19/12),x)
[Out] int((b*x^3+a)^(1/4)/(d*x^3+c)^(19/12),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/4)/(d*x^3+c)^(19/12),x, algorithm="maxima")
[Out] integrate((b*x^3 + a)^(1/4)/(d*x^3 + c)^(19/12), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/4)/(d*x^3+c)^(19/12),x, algorithm="fricas")
[Out] integral((b*x^3 + a)^(1/4)*(d*x^3 + c)^(5/12)/(d^2*x^6 + 2*c*d*x^3 + c^2),
x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(1/4)/(d*x**3+c)**(19/12),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3277 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/4)/(d*x^3+c)^(19/12),x, algorithm="giac")
[Out] integrate((b*x^3 + a)^(1/4)/(d*x^3 + c)^(19/12), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(b\,x^3+a\right)}^{1/4}}{{\left(d\,x^3+c\right)}^{19/12}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(1/4)/(c + d*x^3)^(19/12),x)
[Out] int((a + b*x^3)^(1/4)/(c + d*x^3)^(19/12), x)
```

3.126 $\int \frac{1}{\sqrt[4]{a+bx^3}\,(c+dx^3)^{13/12}}\,dx$

Optimal. Leaf size=87

$$\frac{x\sqrt[4]{\frac{c(a+bx^3)}{a(c+dx^3)}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{3};\frac{4}{3};-\frac{(bc-ad)x^3}{a(c+dx^3)}\right)}{c\sqrt[4]{a+bx^3}\,\sqrt[12]{c+dx^3}}$$

```
[Out] x*(c*(b*x^3+a)/a/(d*x^3+c))^(1/4)*hypergeom([1/4, 1/3],[4/3],-(-a*d+b*c)*x^
3/a/(d*x^3+c))/c/(b*x^3+a)^(1/4)/(d*x^3+c)^(1/12)
```

Rubi [A]
time = 0.01, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = $\{388\}$

$$\frac{x\sqrt[4]{\frac{c(a+bx^3)}{a(c+dx^3)}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{3};\frac{4}{3};-\frac{(bc-ad)x^3}{a(dx^3+c)}\right)}{c\sqrt[4]{a+bx^3}\,\sqrt[12]{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^3)^(1/4)*(c + d*x^3)^(13/12)),x]
```

```
[Out] (x*((c*(a + b*x^3))/(a*(c + d*x^3)))^(1/4)*Hypergeometric2F1[1/4, 1/3, 4/3,
 -(((b*c - a*d)*x^3)/(a*(c + d*x^3)))])/(c*(a + b*x^3)^(1/4)*(c + d*x^3)^(1
/12))
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[x*((a + b*x^n)^p/(c*(c*((a + b*x^n)/(a*(c + d*x^n))))^p*(c + d*x^n)
^(1/n + p)))*Hypergeometric2F1[1/n, -p, 1 + 1/n, (-(b*c - a*d))*(x^n/(a*(c
+ d*x^n)))], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] &&
EqQ[n*(p + q + 1) + 1, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt[4]{a+bx^3}\,(c+dx^3)^{13/12}}\,dx = \frac{x\sqrt[4]{\frac{c(a+bx^3)}{a(c+dx^3)}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{3};\frac{4}{3};-\frac{(bc-ad)x^3}{a(c+dx^3)}\right)}{c\sqrt[4]{a+bx^3}\,\sqrt[12]{c+dx^3}}$$

Mathematica [A]
time = 3.56, size = 86, normalized size = 0.99

$$\frac{x\sqrt[4]{1+\frac{bx^3}{a}}\left(1+\frac{dx^3}{c}\right)^{3/4}{}_2F_1\left(\frac{1}{4},\frac{1}{3};\frac{4}{3};\frac{(-bc+ad)x^3}{a(c+dx^3)}\right)}{\sqrt[4]{a+bx^3}\,(c+dx^3)^{13/12}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^3)^(1/4)*(c + d*x^3)^(13/12)),x]
```

```
[Out] (x*(1 + (b*x^3)/a)^(1/4)*(1 + (d*x^3)/c)^(3/4)*Hypergeometric2F1[1/4, 1/3,
4/3, ((-(b*c) + a*d)*x^3)/(a*(c + d*x^3))])/((a + b*x^3)^(1/4)*(c + d*x^3)^
(13/12))
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\,x^3+a)^{\frac{1}{4}}\,(d\,x^3+c)^{\frac{13}{12}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^3+a)^(1/4)/(d*x^3+c)^(13/12),x)
```

```
[Out] int(1/(b*x^3+a)^(1/4)/(d*x^3+c)^(13/12),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(1/4)/(d*x^3+c)^(13/12),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)^(1/4)*(d*x^3 + c)^(13/12)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(1/4)/(d*x^3+c)^(13/12),x, algorithm="fricas")
```

```
[Out] integral((b*x^3 + a)^(3/4)*(d*x^3 + c)^(11/12)/(b*d^2*x^9 + (2*b*c*d + a*d^
2)*x^6 + (b*c^2 + 2*a*c*d)*x^3 + a*c^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[4]{a+bx^3}\,(c+dx^3)^{\frac{13}{12}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3+a)**(1/4)/(d*x**3+c)**(13/12),x)
[Out] Integral(1/((a + b*x**3)**(1/4)*(c + d*x**3)**(13/12)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(1/4)/(d*x^3+c)^(13/12),x, algorithm="giac")
[Out] integrate(1/((b*x^3 + a)^(1/4)*(d*x^3 + c)^(13/12)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(b\,x^3+a\right)}^{1/4}\,{\left(d\,x^3+c\right)}^{13/12}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^3)^(1/4)*(c + d*x^3)^(13/12)),x)
[Out] int(1/((a + b*x^3)^(1/4)*(c + d*x^3)^(13/12)), x)
```

3.127 $\int \frac{1}{(a+bx^3)^{3/4}(c+dx^3)^{7/12}}\,dx$

Optimal. Leaf size=87

$$\frac{x\left(\frac{c(a+bx^3)}{a(c+dx^3)}\right)^{3/4}(c+dx^3)^{5/12}\,{}_2F_1\left(\frac{1}{3},\frac{3}{4};\frac{4}{3};-\frac{(bc-ad)x^3}{a(c+dx^3)}\right)}{c\,(a+bx^3)^{3/4}}$$

```
[Out] x*(c*(b*x^3+a)/a/(d*x^3+c))^(3/4)*(d*x^3+c)^(5/12)*hypergeom([1/3, 3/4],[4/
3],-(-a*d+b*c)*x^3/a/(d*x^3+c))/c/(b*x^3+a)^(3/4)
```

Rubi [A]
time = 0.01, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {388}

$$\frac{x(c+dx^3)^{5/12}\left(\frac{c(a+bx^3)}{a(c+dx^3)}\right)^{3/4}\,{}_2F_1\left(\frac{1}{3},\frac{3}{4};\frac{4}{3};-\frac{(bc-ad)x^3}{a(dx^3+c)}\right)}{c\,(a+bx^3)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^3)^(3/4)*(c + d*x^3)^(7/12)),x]
```

```
[Out] (x*((c*(a + b*x^3))/(a*(c + d*x^3)))^(3/4)*(c + d*x^3)^(5/12)*Hypergeometri
c2F1[1/3, 3/4, 4/3, -(((b*c - a*d)*x^3)/(a*(c + d*x^3)))])/(c*(a + b*x^3)^(
3/4))
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[x*((a + b*x^n)^p/(c*(c*((a + b*x^n)/(a*(c + d*x^n))))^p*(c + d*x^n)
^(1/n + p)))*Hypergeometric2F1[1/n, -p, 1 + 1/n, (-(b*c - a*d))*(x^n/(a*(c
+ d*x^n)))], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] &&
EqQ[n*(p + q + 1) + 1, 0]
```

Rubi steps

$$\int \frac{1}{(a+bx^3)^{3/4}(c+dx^3)^{7/12}}\,dx = \frac{x\left(\frac{c(a+bx^3)}{a(c+dx^3)}\right)^{3/4}(c+dx^3)^{5/12}\,{}_2F_1\left(\frac{1}{3},\frac{3}{4};\frac{4}{3};-\frac{(bc-ad)x^3}{a(c+dx^3)}\right)}{c\,(a+bx^3)^{3/4}}$$

Mathematica [A]
time = 5.64, size = 86, normalized size = 0.99

$$\frac{x\left(1+\frac{bx^3}{a}\right)^{3/4}\sqrt[4]{1+\frac{dx^3}{c}}\,{}_2F_1\left(\frac{1}{3},\frac{3}{4};\frac{4}{3};\frac{(-bc+ad)x^3}{a(c+dx^3)}\right)}{(a+bx^3)^{3/4}(c+dx^3)^{7/12}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^3)^(3/4)*(c + d*x^3)^(7/12)),x]
```

```
[Out] (x*(1 + (b*x^3)/a)^(3/4)*(1 + (d*x^3)/c)^(1/4)*Hypergeometric2F1[1/3, 3/4,
4/3, ((-(b*c) + a*d)*x^3)/(a*(c + d*x^3))])/((a + b*x^3)^(3/4)*(c + d*x^3)^
(7/12))
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^{3}+a\right)^{\frac{3}{4}} \left(d\,x^{3}+c\right)^{\frac{7}{12}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^3+a)^(3/4)/(d*x^3+c)^(7/12),x)
```

```
[Out] int(1/(b*x^3+a)^(3/4)/(d*x^3+c)^(7/12),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(3/4)/(d*x^3+c)^(7/12),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)^(3/4)*(d*x^3 + c)^(7/12)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(3/4)/(d*x^3+c)^(7/12),x, algorithm="fricas")
```

```
[Out] integral((b*x^3 + a)^(1/4)*(d*x^3 + c)^(5/12)/(b*d*x^6 + (b*c + a*d)*x^3 +
a*c), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + bx^{3}\right)^{\frac{3}{4}} \left(c + dx^{3}\right)^{\frac{7}{12}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3+a)**(3/4)/(d*x**3+c)**(7/12),x)
[Out] Integral(1/((a + b*x**3)**(3/4)*(c + d*x**3)**(7/12)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(3/4)/(d*x^3+c)^(7/12),x, algorithm="giac")
[Out] integrate(1/((b*x^3 + a)^(3/4)*(d*x^3 + c)^(7/12)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(b\,x^3+a\right)}^{3/4}\,{\left(d\,x^3+c\right)}^{7/12}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^3)^(3/4)*(c + d*x^3)^(7/12)),x)
[Out] int(1/((a + b*x^3)^(3/4)*(c + d*x^3)^(7/12)), x)
```

3.128 $\int \frac{1}{(a+bx^3)^{5/4}\sqrt[12]{c+dx^3}}\,dx$

Optimal. Leaf size=87

$$\frac{x\left(\frac{c(a+bx^3)}{a(c+dx^3)}\right)^{5/4}(c+dx^3)^{11/12}\,{}_2F_1\left(\frac{1}{3},\frac{5}{4};\frac{4}{3};-\frac{(bc-ad)x^3}{a(c+dx^3)}\right)}{c\,(a+bx^3)^{5/4}}$$

```
[Out] x*(c*(b*x^3+a)/a/(d*x^3+c))^(5/4)*(d*x^3+c)^(11/12)*hypergeom([1/3, 5/4],[4
/3],-(-a*d+b*c)*x^3/a/(d*x^3+c))/c/(b*x^3+a)^(5/4)
```

Rubi [A]

time = 0.01, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {388}

$$\frac{x(c+dx^3)^{11/12}\left(\frac{c(a+bx^3)}{a(c+dx^3)}\right)^{5/4}\,{}_2F_1\left(\frac{1}{3},\frac{5}{4};\frac{4}{3};-\frac{(bc-ad)x^3}{a(dx^3+c)}\right)}{c\,(a+bx^3)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^3)^(5/4)*(c + d*x^3)^(1/12)),x]
```

```
[Out] (x*((c*(a + b*x^3))/(a*(c + d*x^3)))^(5/4)*(c + d*x^3)^(11/12)*Hypergeometr
ic2F1[1/3, 5/4, 4/3, -(((b*c - a*d)*x^3)/(a*(c + d*x^3)))])/(c*(a + b*x^3)^
(5/4))
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[x*((a + b*x^n)^p/(c*(c*((a + b*x^n)/(a*(c + d*x^n))))^p*(c + d*x^n)
^(1/n + p)))*Hypergeometric2F1[1/n, -p, 1 + 1/n, (-(b*c - a*d))*(x^n/(a*(c
+ d*x^n)))], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] &&
EqQ[n*(p + q + 1) + 1, 0]
```

Rubi steps

$$\int \frac{1}{(a+bx^3)^{5/4}\sqrt[12]{c+dx^3}}\,dx = \frac{x\left(\frac{c(a+bx^3)}{a(c+dx^3)}\right)^{5/4}(c+dx^3)^{11/12}\,{}_2F_1\left(\frac{1}{3},\frac{5}{4};\frac{4}{3};-\frac{(bc-ad)x^3}{a(c+dx^3)}\right)}{c\,(a+bx^3)^{5/4}}$$

Mathematica [A]
time = 3.55, size = 89, normalized size = 1.02

$$\frac{x\sqrt[4]{1+\frac{bx^3}{a}}\ {}_2F_1\left(\frac{1}{3},\frac{5}{4};\frac{4}{3};\frac{(-bc+ad)x^3}{a(c+dx^3)}\right)}{a\sqrt[4]{a+bx^3}\ \sqrt[12]{c+dx^3}\ \sqrt[4]{1+\frac{dx^3}{c}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^3)^(5/4)*(c + d*x^3)^(1/12)),x]
```

```
[Out] (x*(1 + (b*x^3)/a)^(1/4)*Hypergeometric2F1[1/3, 5/4, 4/3, ((-(b*c) + a*d)*x
^3)/(a*(c + d*x^3))])/(a*(a + b*x^3)^(1/4)*(c + d*x^3)^(1/12)*(1 + (d*x^3)/
c)^(1/4))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^{3}+a\right)^{\frac{5}{4}} \left(d\,x^{3}+c\right)^{\frac{1}{12}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^3+a)^(5/4)/(d*x^3+c)^(1/12),x)
```

```
[Out] int(1/(b*x^3+a)^(5/4)/(d*x^3+c)^(1/12),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(5/4)/(d*x^3+c)^(1/12),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)^(5/4)*(d*x^3 + c)^(1/12)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(5/4)/(d*x^3+c)^(1/12),x, algorithm="fricas")
```

```
[Out] integral((b*x^3 + a)^(3/4)*(d*x^3 + c)^(11/12)/(b^2*d*x^9 + (b^2*c + 2*a*b*
d)*x^6 + (2*a*b*c + a^2*d)*x^3 + a^2*c), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx^3)^{\frac{5}{4}}\sqrt[12]{c+dx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3+a)**(5/4)/(d*x**3+c)**(1/12),x)
```

```
[Out] Integral(1/((a + b*x**3)**(5/4)*(c + d*x**3)**(1/12)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(5/4)/(d*x^3+c)^(1/12),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^3 + a)^(5/4)*(d*x^3 + c)^(1/12)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(b\,x^3+a\right)}^{5/4}\,{\left(d\,x^3+c\right)}^{1/12}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^3)^(5/4)*(c + d*x^3)^(1/12)),x)
```

```
[Out] int(1/((a + b*x^3)^(5/4)*(c + d*x^3)^(1/12)), x)
```

3.129 $\int \frac{(c+dx^3)^{5/12}}{(a+bx^3)^{7/4}} dx$

Optimal. Leaf size=121

$$\frac{4x(c+dx^3)^{5/12}}{9a(a+bx^3)^{3/4}}+\frac{5x\left(\frac{c(a+bx^3)}{a(c+dx^3)}\right)^{3/4}(c+dx^3)^{5/12}\,{}_2F_1\left(\frac{1}{3},\frac{3}{4};\frac{4}{3};-\frac{(bc-ad)x^3}{a(c+dx^3)}\right)}{9a(a+bx^3)^{3/4}}$$

```
[Out] 4/9*x*(d*x^3+c)^(5/12)/a/(b*x^3+a)^(3/4)+5/9*x*(c*(b*x^3+a)/a/(d*x^3+c))^(3
/4)*(d*x^3+c)^(5/12)*hypergeom([1/3, 3/4],[4/3],-(-a*d+b*c)*x^3/a/(d*x^3+c)
)/a/(b*x^3+a)^(3/4)
```

Rubi [A]

time = 0.03, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {386, 388}

$$\frac{5x(c+dx^3)^{5/12}\left(\frac{c(a+bx^3)}{a(c+dx^3)}\right)^{3/4}{}_2F_1\left(\frac{1}{3},\frac{3}{4};\frac{4}{3};-\frac{(bc-ad)x^3}{a(dx^3+c)}\right)}{9a(a+bx^3)^{3/4}}+\frac{4x(c+dx^3)^{5/12}}{9a(a+bx^3)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)^(5/12)/(a + b*x^3)^(7/4),x]
```

```
[Out] (4*x*(c + d*x^3)^(5/12))/(9*a*(a + b*x^3)^(3/4)) + (5*x*((c*(a + b*x^3))/(a
*(c + d*x^3)))^(3/4)*(c + d*x^3)^(5/12)*Hypergeometric2F1[1/3, 3/4, 4/3, -(
((b*c - a*d)*x^3)/(a*(c + d*x^3)))])/(9*a*(a + b*x^3)^(3/4))
```

Rule 386

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] - Dist[
c*(q/(a*(p + 1))), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1,
0] && GtQ[q, 0] && NeQ[p, -1]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[x*((a + b*x^n)^p/(c*(c*((a + b*x^n)/(a*(c + d*x^n))))^p*(c + d*x^n)
^(1/n + p)))*Hypergeometric2F1[1/n, -p, 1 + 1/n, (-(b*c - a*d))*(x^n/(a*(c
+ d*x^n)))], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] &&
EqQ[n*(p + q + 1) + 1, 0]
```

Rubi steps

$$\int \frac{(c+dx^3)^{5/12}}{(a+bx^3)^{7/4}}\,dx = \frac{4x(c+dx^3)^{5/12}}{9a\,(a+bx^3)^{3/4}} + \frac{(5c)\int \frac{1}{(a+bx^3)^{3/4}(c+dx^3)^{7/12}}\,dx}{9a}$$
$$= \frac{4x(c+dx^3)^{5/12}}{9a\,(a+bx^3)^{3/4}} + \frac{5x\left(\frac{c(a+bx^3)}{a(c+dx^3)}\right)^{3/4}(c+dx^3)^{5/12}\,{}_2F_1\left(\frac{1}{3},\frac{3}{4};\frac{4}{3};-\frac{(bc-ad)x^3}{a(c+dx^3)}\right)}{9a\,(a+bx^3)^{3/4}}$$

Mathematica [A]
time = 5.59, size = 89, normalized size = 0.74

$$\frac{x\left(1+\frac{bx^3}{a}\right)^{3/4}(c+dx^3)^{5/12}\,{}_2F_1\left(\frac{1}{3},\frac{7}{4};\frac{4}{3};\frac{(-bc+ad)x^3}{a(c+dx^3)}\right)}{a\,(a+bx^3)^{3/4}\left(1+\frac{dx^3}{c}\right)^{3/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x^3)^(5/12)/(a + b*x^3)^(7/4),x]
[Out] (x*(1 + (b*x^3)/a)^(3/4)*(c + d*x^3)^(5/12)*Hypergeometric2F1[1/3, 7/4, 4/3
, ((-(b*c) + a*d)*x^3)/(a*(c + d*x^3))])/(a*(a + b*x^3)^(3/4)*(1 + (d*x^3)/
c)^(3/4))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(d\,x^3+c)^{\frac{5}{12}}}{(b\,x^3+a)^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(5/12)/(b*x^3+a)^(7/4),x)
[Out] int((d*x^3+c)^(5/12)/(b*x^3+a)^(7/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(5/12)/(b*x^3+a)^(7/4),x, algorithm="maxima")
[Out] integrate((d*x^3 + c)^(5/12)/(b*x^3 + a)^(7/4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(5/12)/(b*x^3+a)^(7/4),x, algorithm="fricas")
[Out] integral((b*x^3 + a)^(1/4)*(d*x^3 + c)^(5/12)/(b^2*x^6 + 2*a*b*x^3 + a^2),
x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c + dx^3)^{\frac{5}{12}}}{(a + bx^3)^{\frac{7}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(5/12)/(b*x**3+a)**(7/4),x)
[Out] Integral((c + d*x**3)**(5/12)/(a + b*x**3)**(7/4), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(5/12)/(b*x^3+a)^(7/4),x, algorithm="giac")
[Out] integrate((d*x^3 + c)^(5/12)/(b*x^3 + a)^(7/4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(d\,x^3+c\right)}^{5/12}}{{\left(b\,x^3+a\right)}^{7/4}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^(5/12)/(a + b*x^3)^(7/4),x)
[Out] int((c + d*x^3)^(5/12)/(a + b*x^3)^(7/4), x)
```

3.130 $\int \frac{(c+dx^3)^{11/12}}{(a+bx^3)^{9/4}}\,dx$

Optimal. Leaf size=121

$$\frac{4x(c+dx^3)^{11/12}}{15a(a+bx^3)^{5/4}}+\frac{11x\left(\frac{c(a+bx^3)}{a(c+dx^3)}\right)^{5/4}(c+dx^3)^{11/12}\,{}_2F_1\left(\frac{1}{3},\frac{5}{4};\frac{4}{3};-\frac{(bc-ad)x^3}{a(c+dx^3)}\right)}{15a(a+bx^3)^{5/4}}$$

```
[Out] 4/15*x*(d*x^3+c)^(11/12)/a/(b*x^3+a)^(5/4)+11/15*x*(c*(b*x^3+a)/a/(d*x^3+c)
)^(5/4)*(d*x^3+c)^(11/12)*hypergeom([1/3, 5/4],[4/3],-(-a*d+b*c)*x^3/a/(d*x
^3+c))/a/(b*x^3+a)^(5/4)
```

Rubi [A]
time = 0.03, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}=0.087$, Rules used = {386, 388}

$$\frac{11x(c+dx^3)^{11/12}\left(\frac{c(a+bx^3)}{a(c+dx^3)}\right)^{5/4}\,{}_2F_1\left(\frac{1}{3},\frac{5}{4};\frac{4}{3};-\frac{(bc-ad)x^3}{a(dx^3+c)}\right)}{15a(a+bx^3)^{5/4}}+\frac{4x(c+dx^3)^{11/12}}{15a(a+bx^3)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)^(11/12)/(a + b*x^3)^(9/4),x]
```

```
[Out] (4*x*(c + d*x^3)^(11/12))/(15*a*(a + b*x^3)^(5/4)) + (11*x*((c*(a + b*x^3))
/(a*(c + d*x^3)))^(5/4)*(c + d*x^3)^(11/12)*Hypergeometric2F1[1/3, 5/4, 4/3
, -(((b*c - a*d)*x^3)/(a*(c + d*x^3)))])/(15*a*(a + b*x^3)^(5/4))
```

Rule 386

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] - Dist[
c*(q/(a*(p + 1))), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1,
0] && GtQ[q, 0] && NeQ[p, -1]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[x*((a + b*x^n)^p/(c*(c*((a + b*x^n)/(a*(c + d*x^n))))^p*(c + d*x^n)
^(1/n + p)))*Hypergeometric2F1[1/n, -p, 1 + 1/n, (-(b*c - a*d))*(x^n/(a*(c
+ d*x^n)))], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] &&
EqQ[n*(p + q + 1) + 1, 0]
```

Rubi steps

$$\int \frac{(c+dx^3)^{11/12}}{(a+bx^3)^{9/4}}\,dx = \frac{4x(c+dx^3)^{11/12}}{15a\,(a+bx^3)^{5/4}} + \frac{(11c)\int \frac{1}{(a+bx^3)^{5/4}\sqrt[12]{c+dx^3}}\,dx}{15a}$$
$$= \frac{4x(c+dx^3)^{11/12}}{15a\,(a+bx^3)^{5/4}} + \frac{11x\left(\frac{c(a+bx^3)}{a(c+dx^3)}\right)^{5/4}(c+dx^3)^{11/12}\,{}_2F_1\left(\frac{1}{3},\frac{5}{4};\frac{4}{3};-\frac{(bc-ad)x^3}{a(c+dx^3)}\right)}{15a\,(a+bx^3)^{5/4}}$$

Mathematica [A]
time = 5.74, size = 89, normalized size = 0.74

$$\frac{x\sqrt[4]{1+\frac{bx^3}{a}}\,(c+dx^3)^{11/12}\,{}_2F_1\left(\frac{1}{3},\frac{9}{4};\frac{4}{3};\frac{(-bc+ad)x^3}{a(c+dx^3)}\right)}{a^2\sqrt[4]{a+bx^3}\,\left(1+\frac{dx^3}{c}\right)^{5/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x^3)^(11/12)/(a + b*x^3)^(9/4),x]
[Out] (x*(1 + (b*x^3)/a)^(1/4)*(c + d*x^3)^(11/12)*Hypergeometric2F1[1/3, 9/4, 4/
3, ((-(b*c) + a*d)*x^3)/(a*(c + d*x^3))])/(a^2*(a + b*x^3)^(1/4)*(1 + (d*x^
3)/c)^(5/4))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(d\,x^3+c)^{\frac{11}{12}}}{(b\,x^3+a)^{\frac{9}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(11/12)/(b*x^3+a)^(9/4),x)
[Out] int((d*x^3+c)^(11/12)/(b*x^3+a)^(9/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(11/12)/(b*x^3+a)^(9/4),x, algorithm="maxima")
[Out] integrate((d*x^3 + c)^(11/12)/(b*x^3 + a)^(9/4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(11/12)/(b*x^3+a)^(9/4),x, algorithm="fricas")
[Out] integral((b*x^3 + a)^(3/4)*(d*x^3 + c)^(11/12)/(b^3*x^9 + 3*a*b^2*x^6 + 3*a
^2*b*x^3 + a^3), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(11/12)/(b*x**3+a)**(9/4),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 6546 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(11/12)/(b*x^3+a)^(9/4),x, algorithm="giac")
[Out] integrate((d*x^3 + c)^(11/12)/(b*x^3 + a)^(9/4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(d\,x^3+c\right)^{11/12}}{\left(b\,x^3+a\right)^{9/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^(11/12)/(a + b*x^3)^(9/4),x)
[Out] int((c + d*x^3)^(11/12)/(a + b*x^3)^(9/4), x)
```

3.131 $\int \frac{(c+dx^3)^{17/12}}{(a+bx^3)^{11/4}}\,dx$

Optimal. Leaf size=153

$$\frac{68cx(c+dx^3)^{5/12}}{189a^2(a+bx^3)^{3/4}}+\frac{4x(c+dx^3)^{17/12}}{21a(a+bx^3)^{7/4}}+\frac{85cx\left(\frac{c(a+bx^3)}{a(c+dx^3)}\right)^{3/4}(c+dx^3)^{5/12}\,{}_2F_1\left(\frac{1}{3},\frac{3}{4};\frac{4}{3};-\frac{(bc-ad)x^3}{a(c+dx^3)}\right)}{189a^2(a+bx^3)^{3/4}}$$

```
[Out] 68/189*c*x*(d*x^3+c)^(5/12)/a^2/(b*x^3+a)^(3/4)+4/21*x*(d*x^3+c)^(17/12)/a/
(b*x^3+a)^(7/4)+85/189*c*x*(c*(b*x^3+a)/a/(d*x^3+c))^(3/4)*(d*x^3+c)^(5/12)
*hypergeom([1/3, 3/4],[4/3],-(-a*d+b*c)*x^3/a/(d*x^3+c))/a^2/(b*x^3+a)^(3/4
)
```

Rubi [A]
time = 0.04, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {386, 388}

$$\frac{85cx(c+dx^3)^{5/12}\left(\frac{c(a+bx^3)}{a(c+dx^3)}\right)^{3/4}{}_2F_1\left(\frac{1}{3},\frac{3}{4};\frac{4}{3};-\frac{(bc-ad)x^3}{a(dx^3+c)}\right)}{189a^2(a+bx^3)^{3/4}}+\frac{68cx(c+dx^3)^{5/12}}{189a^2(a+bx^3)^{3/4}}+\frac{4x(c+dx^3)^{17/12}}{21a(a+bx^3)^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)^(17/12)/(a + b*x^3)^(11/4),x]
```

```
[Out] (68*c*x*(c + d*x^3)^(5/12))/(189*a^2*(a + b*x^3)^(3/4)) + (4*x*(c + d*x^3)^
(17/12))/(21*a*(a + b*x^3)^(7/4)) + (85*c*x*((c*(a + b*x^3))/(a*(c + d*x^3)
))^(3/4)*(c + d*x^3)^(5/12)*Hypergeometric2F1[1/3, 3/4, 4/3, -(((b*c - a*d)
*x^3)/(a*(c + d*x^3)))])/(189*a^2*(a + b*x^3)^(3/4))
```

Rule 386

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] - Dist[
c*(q/(a*(p + 1))), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1,
0] && GtQ[q, 0] && NeQ[p, -1]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[x*((a + b*x^n)^p/(c*(c*((a + b*x^n)/(a*(c + d*x^n))))^p*(c + d*x^n)
^(1/n + p)))*Hypergeometric2F1[1/n, -p, 1 + 1/n, (-(b*c - a*d))*(x^n/(a*(c
+ d*x^n)))], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] &&
EqQ[n*(p + q + 1) + 1, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(c+dx^3)^{17/12}}{(a+bx^3)^{11/4}}\,dx &= \frac{4x(c+dx^3)^{17/12}}{21a\,(a+bx^3)^{7/4}} + \frac{(17c)\int \frac{(c+dx^3)^{5/12}}{(a+bx^3)^{7/4}}\,dx}{21a} \\
&= \frac{68cx(c+dx^3)^{5/12}}{189a^2\,(a+bx^3)^{3/4}} + \frac{4x(c+dx^3)^{17/12}}{21a\,(a+bx^3)^{7/4}} + \frac{(85c^2)\int \frac{1}{(a+bx^3)^{3/4}(c+dx^3)^{7/12}}\,dx}{189a^2} \\
&= \frac{68cx(c+dx^3)^{5/12}}{189a^2\,(a+bx^3)^{3/4}} + \frac{4x(c+dx^3)^{17/12}}{21a\,(a+bx^3)^{7/4}} + \frac{85cx\left(\frac{c(a+bx^3)}{a(c+dx^3)}\right)^{3/4}(c+dx^3)^{5/12}\,{}_2F_1\left(\frac{1}{3},\frac{3}{4};\frac{4}{3};-\right.}{189a^2\,(a+bx^3)^{3/4}}
\end{aligned}
$$

Mathematica [A]
time = 5.85, size = 90, normalized size = 0.59

$$
\frac{cx\left(1+\frac{bx^3}{a}\right)^{3/4}(c+dx^3)^{5/12}\,{}_2F_1\left(\frac{1}{3},\frac{11}{4};\frac{4}{3};\frac{(-bc+ad)x^3}{a(c+dx^3)}\right)}{a^2\,(a+bx^3)^{3/4}\left(1+\frac{dx^3}{c}\right)^{3/4}}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x^3)^(17/12)/(a + b*x^3)^(11/4),x]
[Out] (c*x*(1 + (b*x^3)/a)^(3/4)*(c + d*x^3)^(5/12)*Hypergeometric2F1[1/3, 11/4,
4/3, ((-(b*c) + a*d)*x^3)/(a*(c + d*x^3))])/(a^2*(a + b*x^3)^(3/4)*(1 + (d*
x^3)/c)^(3/4))
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$
\int \frac{(d\,x^3+c)^{\frac{17}{12}}}{(b\,x^3+a)^{\frac{11}{4}}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(17/12)/(b*x^3+a)^(11/4),x)
[Out] int((d*x^3+c)^(17/12)/(b*x^3+a)^(11/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(17/12)/(b*x^3+a)^(11/4),x, algorithm="maxima")
```

```
[Out] integrate((d*x^3 + c)^(17/12)/(b*x^3 + a)^(11/4), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(17/12)/(b*x^3+a)^(11/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^3 + a)^(1/4)*(d*x^3 + c)^(17/12)/(b^3*x^9 + 3*a*b^2*x^6 + 3*a
^2*b*x^3 + a^3), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(17/12)/(b*x**3+a)**(11/4),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(17/12)/(b*x^3+a)^(11/4),x, algorithm="giac")
```

```
[Out] integrate((d*x^3 + c)^(17/12)/(b*x^3 + a)^(11/4), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(d\,x^3+c\right)}^{17/12}}{{\left(b\,x^3+a\right)}^{11/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^(17/12)/(a + b*x^3)^(11/4),x)
```

```
[Out] int((c + d*x^3)^(17/12)/(a + b*x^3)^(11/4), x)
```

3.132 $\int \frac{(c+dx^3)^{23/12}}{(a+bx^3)^{13/4}}\,dx$

Optimal. Leaf size=153

$$\frac{92cx(c+dx^3)^{11/12}}{405a^2(a+bx^3)^{5/4}}+\frac{4x(c+dx^3)^{23/12}}{27a(a+bx^3)^{9/4}}+\frac{253cx\left(\frac{c(a+bx^3)}{a(c+dx^3)}\right)^{5/4}(c+dx^3)^{11/12}\,{}_2F_1\left(\frac{1}{3},\frac{5}{4};\frac{4}{3};-\frac{(bc-ad)x^3}{a(c+dx^3)}\right)}{405a^2(a+bx^3)^{5/4}}$$

```
[Out] 92/405*c*x*(d*x^3+c)^(11/12)/a^2/(b*x^3+a)^(5/4)+4/27*x*(d*x^3+c)^(23/12)/a
/(b*x^3+a)^(9/4)+253/405*c*x*(c*(b*x^3+a)/a/(d*x^3+c))^(5/4)*(d*x^3+c)^(11/
12)*hypergeom([1/3, 5/4],[4/3],-(-a*d+b*c)*x^3/a/(d*x^3+c))/a^2/(b*x^3+a)^(
5/4)
```

Rubi [A]
time = 0.04, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {386, 388}

$$\frac{253cx(c+dx^3)^{11/12}\left(\frac{c(a+bx^3)}{a(c+dx^3)}\right)^{5/4}{}_2F_1\left(\frac{1}{3},\frac{5}{4};\frac{4}{3};-\frac{(bc-ad)x^3}{a(dx^3+c)}\right)}{405a^2(a+bx^3)^{5/4}}+\frac{92cx(c+dx^3)^{11/12}}{405a^2(a+bx^3)^{5/4}}+\frac{4x(c+dx^3)^{23/12}}{27a(a+bx^3)^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)^(23/12)/(a + b*x^3)^(13/4),x]
```

```
[Out] (92*c*x*(c + d*x^3)^(11/12))/(405*a^2*(a + b*x^3)^(5/4)) + (4*x*(c + d*x^3)
^(23/12))/(27*a*(a + b*x^3)^(9/4)) + (253*c*x*((c*(a + b*x^3))/(a*(c + d*x^
3)))^(5/4)*(c + d*x^3)^(11/12)*Hypergeometric2F1[1/3, 5/4, 4/3, -(((b*c - a
*d)*x^3)/(a*(c + d*x^3)))])/(405*a^2*(a + b*x^3)^(5/4))
```

Rule 386

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] - Dist[
c*(q/(a*(p + 1))), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1,
0] && GtQ[q, 0] && NeQ[p, -1]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[x*((a + b*x^n)^p/(c*(c*((a + b*x^n)/(a*(c + d*x^n))))^p*(c + d*x^n)
^(1/n + p)))*Hypergeometric2F1[1/n, -p, 1 + 1/n, (-(b*c - a*d))*(x^n/(a*(c
+ d*x^n)))], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] &&
EqQ[n*(p + q + 1) + 1, 0]
```

Rubi steps

$$\int \frac{(c+dx^3)^{23/12}}{(a+bx^3)^{13/4}}\,dx = \frac{4x(c+dx^3)^{23/12}}{27a\,(a+bx^3)^{9/4}} + \frac{(23c)\int \frac{(c+dx^3)^{11/12}}{(a+bx^3)^{9/4}}\,dx}{27a}$$
$$= \frac{92cx(c+dx^3)^{11/12}}{405a^2\,(a+bx^3)^{5/4}} + \frac{4x(c+dx^3)^{23/12}}{27a\,(a+bx^3)^{9/4}} + \frac{(253c^2)\int \frac{1}{(a+bx^3)^{5/4}\sqrt[12]{c+dx^3}}\,dx}{405a^2}$$
$$= \frac{92cx(c+dx^3)^{11/12}}{405a^2\,(a+bx^3)^{5/4}} + \frac{4x(c+dx^3)^{23/12}}{27a\,(a+bx^3)^{9/4}} + \frac{253cx\left(\frac{c(a+bx^3)}{a(c+dx^3)}\right)^{5/4}(c+dx^3)^{11/12}\,{}_2F_1\left(\frac{1}{3},\frac{5}{4}\right.}{405a^2\,(a+bx^3)^{5/4}}$$

Mathematica [A]
time = 5.82, size = 90, normalized size = 0.59

$$\frac{cx\sqrt[4]{1+\frac{bx^3}{a}}\,(c+dx^3)^{11/12}\,{}_2F_1\left(\frac{1}{3},\frac{13}{4};\frac{4}{3};\frac{(-bc+ad)x^3}{a(c+dx^3)}\right)}{a^3\sqrt[4]{a+bx^3}\,\left(1+\frac{dx^3}{c}\right)^{5/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x^3)^(23/12)/(a + b*x^3)^(13/4),x]
[Out] (c*x*(1 + (b*x^3)/a)^(1/4)*(c + d*x^3)^(11/12)*Hypergeometric2F1[1/3, 13/4,
 4/3, ((-(b*c) + a*d)*x^3)/(a*(c + d*x^3))])/(a^3*(a + b*x^3)^(1/4)*(1 + (d
*x^3)/c)^(5/4))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(d\,x^3+c)^{\frac{23}{12}}}{(b\,x^3+a)^{\frac{13}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(23/12)/(b*x^3+a)^(13/4),x)
[Out] int((d*x^3+c)^(23/12)/(b*x^3+a)^(13/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(23/12)/(b*x^3+a)^(13/4),x, algorithm="maxima")
```

```
[Out] integrate((d*x^3 + c)^(23/12)/(b*x^3 + a)^(13/4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(23/12)/(b*x^3+a)^(13/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^3 + a)^(3/4)*(d*x^3 + c)^(23/12)/(b^4*x^12 + 4*a*b^3*x^9 + 6*
a^2*b^2*x^6 + 4*a^3*b*x^3 + a^4), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(23/12)/(b*x**3+a)**(13/4),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(23/12)/(b*x^3+a)^(13/4),x, algorithm="giac")
```

```
[Out] integrate((d*x^3 + c)^(23/12)/(b*x^3 + a)^(13/4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(d\,x^3+c\right)^{23/12}}{\left(b\,x^3+a\right)^{13/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^(23/12)/(a + b*x^3)^(13/4),x)
```

```
[Out] int((c + d*x^3)^(23/12)/(a + b*x^3)^(13/4), x)
```

3.133 $\int (a+bx^3)^m (c+dx^3)^p \, dx$

Optimal. Leaf size=79

$$x(a+bx^3)^m \left(1+\frac{bx^3}{a}\right)^{-m} (c+dx^3)^p \left(1+\frac{dx^3}{c}\right)^{-p} F_1\left(\frac{1}{3}; -m, -p; \frac{4}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c}\right)$$

```
[Out] x*(b*x^3+a)^m*(d*x^3+c)^p*AppellF1(1/3,-m,-p,4/3,-b*x^3/a,-d*x^3/c)/((1+b*x
^3/a)^m)/((1+d*x^3/c)^p)
```

Rubi [A]
time = 0.03, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {441, 440}

$$x(a+bx^3)^m \left(\frac{bx^3}{a}+1\right)^{-m} (c+dx^3)^p \left(\frac{dx^3}{c}+1\right)^{-p} F_1\left(\frac{1}{3}; -m, -p; \frac{4}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^m*(c + d*x^3)^p,x]
```

```
[Out] (x*(a + b*x^3)^m*(c + d*x^3)^p*AppellF1[1/3, -m, -p, 4/3, -((b*x^3)/a), -((
d*x^3)/c)])/((1 + (b*x^3)/a)^m*(1 + (d*x^3)/c)^p)
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int (a+bx^3)^m (c+dx^3)^p \, dx = \left((a+bx^3)^m \left(1+\frac{bx^3}{a}\right)^{-m}\right) \int \left(1+\frac{bx^3}{a}\right)^m (c+dx^3)^p \, dx$$
$$= \left((a+bx^3)^m \left(1+\frac{bx^3}{a}\right)^{-m} (c+dx^3)^p \left(1+\frac{dx^3}{c}\right)^{-p}\right) \int \left(1+\frac{bx^3}{a}\right)^m \left(1+\right.$$
$$= x(a+bx^3)^m \left(1+\frac{bx^3}{a}\right)^{-m} (c+dx^3)^p \left(1+\frac{dx^3}{c}\right)^{-p} F_1\left(\frac{1}{3};-m,-p;\frac{4}{3};-\frac{bx^3}{a}\right.$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 172 vs. 2(79) = 158.
time = 0.22, size = 172, normalized size = 2.18

$$\frac{4acx(a+bx^3)^m (c+dx^3)^p F_1\left(\frac{1}{3};-m,-p;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4acF_1\left(\frac{1}{3};-m,-p;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+3x^3\left(bcmF_1\left(\frac{4}{3};1-m,-p;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+adpF_1\left(\frac{4}{3};-m,1-p;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^m*(c + d*x^3)^p,x]
```

```
[Out] (4*a*c*x*(a + b*x^3)^m*(c + d*x^3)^p*AppellF1[1/3, -m, -p, 4/3, -((b*x^3)/a
), -((d*x^3)/c)])/(4*a*c*AppellF1[1/3, -m, -p, 4/3, -((b*x^3)/a), -((d*x^3)
/c)] + 3*x^3*(b*c*m*AppellF1[4/3, 1 - m, -p, 7/3, -((b*x^3)/a), -((d*x^3)/c
)] + a*d*p*AppellF1[4/3, -m, 1 - p, 7/3, -((b*x^3)/a), -((d*x^3)/c)]))
```

Maple [F]
time = 0.10, size = 0, normalized size = 0.00

$$\int \left(b\,x^3+a\right)^m \left(d\,x^3+c\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^m*(d*x^3+c)^p,x)
```

```
[Out] int((b*x^3+a)^m*(d*x^3+c)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^m*(d*x^3+c)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^m*(d*x^3 + c)^p, x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^m*(d*x^3+c)^p,x, algorithm="fricas")
```

```
[Out] integral((b*x^3 + a)^m*(d*x^3 + c)^p, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**m*(d*x**3+c)**p,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^m*(d*x^3+c)^p,x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^m*(d*x^3 + c)^p, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(b\,x^3+a\right)}^m\,{\left(d\,x^3+c\right)}^p \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^m*(c + d*x^3)^p,x)
```

```
[Out] int((a + b*x^3)^m*(c + d*x^3)^p, x)
```

3.134 $\int (a+bx^3)^2 (c+dx^3)^q \, dx$

Optimal. Leaf size=167

$$-\frac{b(4bc-ad(10+3q))x(c+dx^3)^{1+q}}{d^2(4+3q)(7+3q)}+\frac{bx(a+bx^3)(c+dx^3)^{1+q}}{d(7+3q)}+\frac{(4b^2c^2-2abcd(7+3q)+a^2d^2(28+33q+9}{cd^2(4+3q}$$

```
[Out] -b*(4*b*c-a*d*(10+3*q))*x*(d*x^3+c)^(1+q)/d^2/(9*q^2+33*q+28)+b*x*(b*x^3+a)
*(d*x^3+c)^(1+q)/d/(7+3*q)+(4*b^2*c^2-2*a*b*c*d*(7+3*q)+a^2*d^2*(9*q^2+33*q
+28))*x*(d*x^3+c)^(1+q)*hypergeom([1, 4/3+q],[4/3],-d*x^3/c)/c/d^2/(9*q^2+3
3*q+28)
```

Rubi [A]
time = 0.09, antiderivative size = 176, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {427, 396, 252, 251}

$$\frac{x(c+dx^3)^q\left(\frac{dx^3}{c}+1\right)^{-q}(a^2d^2(9q^2+33q+28)-2abcd(3q+7)+4b^2c^2)\,{}_2F_1\left(\frac{1}{3},-q;\frac{4}{3};-\frac{dx^3}{c}\right)}{d^2(3q+4)(3q+7)}-\frac{bx(c+dx^3)^{q+1}(4bc-ad(3q+10))}{d^2(3q+4)(3q+7)}+\frac{bx(a+bx^3)(c+dx^3)^{q+1}}{d(3q+7)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^2*(c + d*x^3)^q,x]
```

```
[Out] -((b*(4*b*c - a*d*(10 + 3*q))*x*(c + d*x^3)^(1 + q))/(d^2*(4 + 3*q)*(7 + 3*
q))) + (b*x*(a + b*x^3)*(c + d*x^3)^(1 + q))/(d*(7 + 3*q)) + ((4*b^2*c^2 -
2*a*b*c*d*(7 + 3*q) + a^2*d^2*(28 + 33*q + 9*q^2))*x*(c + d*x^3)^q*Hypergeo
metric2F1[1/3, -q, 4/3, -((d*x^3)/c)])/(d^2*(4 + 3*q)*(7 + 3*q)*(1 + (d*x^3
)/c)^q)
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
```

```
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rubi steps

$$\int \left(a+bx^3\right)^2 \left(c+dx^3\right)^q \, dx = \frac{bx(a+bx^3)\left(c+dx^3\right)^{1+q}}{d(7+3q)} + \frac{\int \left(c+dx^3\right)^q \left(-a(bc-ad(7+3q)) - b(4bc-ad(\right.}{d(7+3q)}$$

$$= -\frac{b(4bc-ad(10+3q))x(c+dx^3)^{1+q}}{d^2(4+3q)(7+3q)} + \frac{bx(a+bx^3)\left(c+dx^3\right)^{1+q}}{d(7+3q)} + \frac{(4b^2c^2 -}{}$$

$$= -\frac{b(4bc-ad(10+3q))x(c+dx^3)^{1+q}}{d^2(4+3q)(7+3q)} + \frac{bx(a+bx^3)\left(c+dx^3\right)^{1+q}}{d(7+3q)} + \frac{\Big(\big((4b^2c^2}{}$$

$$= -\frac{b(4bc-ad(10+3q))x(c+dx^3)^{1+q}}{d^2(4+3q)(7+3q)} + \frac{bx(a+bx^3)\left(c+dx^3\right)^{1+q}}{d(7+3q)} + \frac{(4b^2c^2 -}{}$$

Mathematica [A]
time = 0.16, size = 106, normalized size = 0.63

$$\frac{1}{14}x\left(c+dx^3\right)^q\left(1+\frac{dx^3}{c}\right)^{-q}\left(14a^2\,{}_2F_1\left(\frac{1}{3},-q;\frac{4}{3};-\frac{dx^3}{c}\right)+bx^3\left(7a\,{}_2F_1\left(\frac{4}{3},-q;\frac{7}{3};-\frac{dx^3}{c}\right)+2bx^3\,{}_2F_1\left(\frac{7}{3},-q;\frac{10}{3};-\frac{dx^3}{c}\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^2*(c + d*x^3)^q,x]
```

```
[Out] (x*(c + d*x^3)^q*(14*a^2*Hypergeometric2F1[1/3, -q, 4/3, -((d*x^3)/c)] + b*
x^3*(7*a*Hypergeometric2F1[4/3, -q, 7/3, -((d*x^3)/c)] + 2*b*x^3*Hypergeome
tric2F1[7/3, -q, 10/3, -((d*x^3)/c)])))/(14*(1 + (d*x^3)/c)^q)
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \left(b\,x^3+a\right)^2 \left(d\,x^3+c\right)^q \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^2*(d*x^3+c)^q,x)
```

```
[Out] int((b*x^3+a)^2*(d*x^3+c)^q,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(d*x^3+c)^q,x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^2*(d*x^3 + c)^q, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(d*x^3+c)^q,x, algorithm="fricas")
```

```
[Out] integral((b^2*x^6 + 2*a*b*x^3 + a^2)*(d*x^3 + c)^q, x)
```

Sympy [C] Result contains complex when optimal does not.
time = 109.74, size = 121, normalized size = 0.72

$$\frac{a^2c^qx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},-q\\\frac{4}{3}\end{matrix}\middle|\frac{dx^3e^{i\pi}}{c}\right)}{3\Gamma\left(\frac{4}{3}\right)}+\frac{2abc^qx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{4}{3},-q\\\frac{7}{3}\end{matrix}\middle|\frac{dx^3e^{i\pi}}{c}\right)}{3\Gamma\left(\frac{7}{3}\right)}+\frac{b^2c^qx^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\begin{matrix}\frac{7}{3},-q\\\frac{10}{3}\end{matrix}\middle|\frac{dx^3e^{i\pi}}{c}\right)}{3\Gamma\left(\frac{10}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**2*(d*x**3+c)**q,x)
```

```
[Out] a**2*c**q*x*gamma(1/3)*hyper((1/3, -q), (4/3,), d*x**3*exp_polar(I*pi)/c)/(
3*gamma(4/3)) + 2*a*b*c**q*x**4*gamma(4/3)*hyper((4/3, -q), (7/3,), d*x**3*
exp_polar(I*pi)/c)/(3*gamma(7/3)) + b**2*c**q*x**7*gamma(7/3)*hyper((7/3, -
q), (10/3,), d*x**3*exp_polar(I*pi)/c)/(3*gamma(10/3))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(d*x^3+c)^q,x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^2*(d*x^3 + c)^q, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (b\,x^3+a)^2\,(d\,x^3+c)^q\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^2*(c + d*x^3)^q,x)
```

```
[Out] int((a + b*x^3)^2*(c + d*x^3)^q, x)
```

3.135 $\int (a+bx^3)(c+dx^3)^q \, dx$

Optimal. Leaf size=84

$$\frac{bx(c+dx^3)^{1+q}}{d(4+3q)} - \frac{(bc-ad(4+3q))x(c+dx^3)^{1+q} \, {}_2F_1\left(1,\frac{4}{3}+q;\frac{4}{3};-\frac{dx^3}{c}\right)}{cd(4+3q)}$$

```
[Out] b*x*(d*x^3+c)^(1+q)/d/(4+3*q)-(b*c-a*d*(4+3*q))*x*(d*x^3+c)^(1+q)*hypergeom
([1, 4/3+q],[4/3],-d*x^3/c)/c/d/(4+3*q)
```

Rubi [A]
time = 0.03, antiderivative size = 85, normalized size of antiderivative = 1.01, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {396, 252, 251}

$$x(c+dx^3)^q\left(\frac{dx^3}{c}+1\right)^{-q}\left(a-\frac{bc}{3dq+4d}\right){}_2F_1\left(\frac{1}{3},-q;\frac{4}{3};-\frac{dx^3}{c}\right)+\frac{bx(c+dx^3)^{q+1}}{d(3q+4)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)*(c + d*x^3)^q,x]
```

```
[Out] (b*x*(c + d*x^3)^(1 + q))/(d*(4 + 3*q)) + ((a - (b*c)/(4*d + 3*d*q))*x*(c +
 d*x^3)^q*Hypergeometric2F1[1/3, -q, 4/3, -((d*x^3)/c)])/(1 + (d*x^3)/c)^q
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rubi steps

$$\int (a+bx^3)(c+dx^3)^q\,dx = \frac{bx(c+dx^3)^{1+q}}{d(4+3q)} - \left(-a+\frac{bc}{4d+3dq}\right)\int (c+dx^3)^q\,dx$$
$$= \frac{bx(c+dx^3)^{1+q}}{d(4+3q)} - \left(\left(-a+\frac{bc}{4d+3dq}\right)(c+dx^3)^q\left(1+\frac{dx^3}{c}\right)^{-q}\right)\int \left(1+\frac{dx^3}{c}\right.$$
$$= \frac{bx(c+dx^3)^{1+q}}{d(4+3q)} + \left(a-\frac{bc}{4d+3dq}\right)x(c+dx^3)^q\left(1+\frac{dx^3}{c}\right)^{-q} {}_2F_1\left(\frac{1}{3},-q;\frac{4}{3};\right.$$

Mathematica [A]
time = 0.10, size = 90, normalized size = 1.07

$$\frac{x(c+dx^3)^q\left(1+\frac{dx^3}{c}\right)^{-q}\left(b(c+dx^3)\left(1+\frac{dx^3}{c}\right)^q+(-bc+ad(4+3q))\,{}_2F_1\left(\frac{1}{3},-q;\frac{4}{3};-\frac{dx^3}{c}\right)\right)}{d(4+3q)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)*(c + d*x^3)^q,x]
[Out] (x*(c + d*x^3)^q*(b*(c + d*x^3)*(1 + (d*x^3)/c)^q + (-(b*c) + a*d*(4 + 3*q)
)*Hypergeometric2F1[1/3, -q, 4/3, -((d*x^3)/c)]))/(d*(4 + 3*q)*(1 + (d*x^3)
/c)^q)
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \left(b\,x^3+a\right)\left(d\,x^3+c\right)^q\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)*(d*x^3+c)^q,x)
[Out] int((b*x^3+a)*(d*x^3+c)^q,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(d*x^3+c)^q,x, algorithm="maxima")
[Out] integrate((b*x^3 + a)*(d*x^3 + c)^q, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(d*x^3+c)^q,x, algorithm="fricas")
```

```
[Out] integral((b*x^3 + a)*(d*x^3 + c)^q, x)
```

Sympy [C] Result contains complex when optimal does not.
time = 40.41, size = 75, normalized size = 0.89

$$\frac{ac^q x\Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\begin{matrix} \frac{1}{3}, -q \\ \frac{4}{3} \end{matrix}\middle| \frac{dx^3 e^{i\pi}}{c}\right)}{3\Gamma\left(\frac{4}{3}\right)} + \frac{bc^q x^4\Gamma\left(\frac{4}{3}\right) {}_2F_1\left(\begin{matrix} \frac{4}{3}, -q \\ \frac{7}{3} \end{matrix}\middle| \frac{dx^3 e^{i\pi}}{c}\right)}{3\Gamma\left(\frac{7}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)*(d*x**3+c)**q,x)
```

```
[Out] a*c**q*x*gamma(1/3)*hyper((1/3, -q), (4/3,), d*x**3*exp_polar(I*pi)/c)/(3*g
amma(4/3)) + b*c**q*x**4*gamma(4/3)*hyper((4/3, -q), (7/3,), d*x**3*exp_pol
ar(I*pi)/c)/(3*gamma(7/3))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(d*x^3+c)^q,x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)*(d*x^3 + c)^q, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(b\,x^3+a\right)\,{\left(d\,x^3+c\right)}^q \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)*(c + d*x^3)^q,x)
```

```
[Out] int((a + b*x^3)*(c + d*x^3)^q, x)
```

3.136 $\int \frac{(c+dx^3)^q}{a+bx^3} \, dx$

Optimal. Leaf size=57

$$\frac{x(c+dx^3)^q \left(1+\frac{dx^3}{c}\right)^{-q} F_1\left(\frac{1}{3};1,-q;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a}$$

```
[Out] x*(d*x^3+c)^q*AppellF1(1/3,1,-q,4/3,-b*x^3/a,-d*x^3/c)/a/((1+d*x^3/c)^q)
```

Rubi [A]
time = 0.02, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {441, 440}

$$\frac{x(c+dx^3)^q \left(\frac{dx^3}{c}+1\right)^{-q} F_1\left(\frac{1}{3};1,-q;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)^q/(a + b*x^3),x]
[Out] (x*(c + d*x^3)^q*AppellF1[1/3, 1, -q, 4/3, -((b*x^3)/a), -((d*x^3)/c)])/(a*
(1 + (d*x^3)/c)^q)
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx^3)^q}{a+bx^3} \, dx &= \left((c+dx^3)^q \left(1+\frac{dx^3}{c}\right)^{-q}\right) \int \frac{\left(1+\frac{dx^3}{c}\right)^q}{a+bx^3} \, dx \\ &= \frac{x(c+dx^3)^q \left(1+\frac{dx^3}{c}\right)^{-q} F_1\left(\frac{1}{3};1,-q;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 162 vs. 2(57) = 114.
time = 0.25, size = 162, normalized size = 2.84

$$\frac{4acx(c+dx^3)^q F_1\left(\frac{1}{3};-q,1;\frac{4}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)}{(a+bx^3)\left(4acF_1\left(\frac{1}{3};-q,1;\frac{4}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+3x^3\left(adqF_1\left(\frac{4}{3};1-q,1;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)-bcF_1\left(\frac{4}{3};-q,2;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x^3)^q/(a + b*x^3),x]
```

```
[Out] (4*a*c*x*(c + d*x^3)^q*AppellF1[1/3, -q, 1, 4/3, -((d*x^3)/c), -((b*x^3)/a)
])/((a + b*x^3)*(4*a*c*AppellF1[1/3, -q, 1, 4/3, -((d*x^3)/c), -((b*x^3)/a)
] + 3*x^3*(a*d*q*AppellF1[4/3, 1 - q, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a)] -
 b*c*AppellF1[4/3, -q, 2, 7/3, -((d*x^3)/c), -((b*x^3)/a)])))
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{(d\,x^3+c)^q}{b\,x^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^q/(b*x^3+a),x)
```

```
[Out] int((d*x^3+c)^q/(b*x^3+a),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^q/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] integrate((d*x^3 + c)^q/(b*x^3 + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^q/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] integral((d*x^3 + c)^q/(b*x^3 + a), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**q/(b*x**3+a),x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^q/(b*x^3+a),x, algorithm="giac")
[Out] integrate((d*x^3 + c)^q/(b*x^3 + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(d\,x^3+c)^q}{b\,x^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^q/(a + b*x^3),x)
[Out] int((c + d*x^3)^q/(a + b*x^3), x)
```

3.137 $\int \frac{(c+dx^3)^q}{(a+bx^3)^2} \, dx$

Optimal. Leaf size=57

$$\frac{x(c+dx^3)^q \left(1+\frac{dx^3}{c}\right)^{-q} F_1\left(\frac{1}{3};2,-q;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2}$$

```
[Out] x*(d*x^3+c)^q*AppellF1(1/3,2,-q,4/3,-b*x^3/a,-d*x^3/c)/a^2/((1+d*x^3/c)^q)
```

Rubi [A]

time = 0.02, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {441, 440}

$$\frac{x(c+dx^3)^q \left(\frac{dx^3}{c}+1\right)^{-q} F_1\left(\frac{1}{3};2,-q;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)^q/(a + b*x^3)^2,x]
```

```
[Out] (x*(c + d*x^3)^q*AppellF1[1/3, 2, -q, 4/3, -((b*x^3)/a), -((d*x^3)/c)])/(a^
2*(1 + (d*x^3)/c)^q)
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(c+dx^3)^q}{(a+bx^3)^2} \, dx &= \left((c+dx^3)^q \left(1+\frac{dx^3}{c}\right)^{-q}\right) \int \frac{\left(1+\frac{dx^3}{c}\right)^q}{(a+bx^3)^2} \, dx \\
&= \frac{x(c+dx^3)^q \left(1+\frac{dx^3}{c}\right)^{-q} F_1\left(\frac{1}{3};2,-q;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2}
\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 162 vs. 2(57) = 114.
time = 0.33, size = 162, normalized size = 2.84

$$\frac{4acx(c+dx^3)^q F_1\left(\frac{1}{3};2,-q;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{(a+bx^3)^2\left(4acF_1\left(\frac{1}{3};2,-q;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+3x^3\left(adqF_1\left(\frac{4}{3};2,1-q;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)-2bcF_1\left(\frac{4}{3};3,-q;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x^3)^q/(a + b*x^3)^2,x]
```

```
[Out] (4*a*c*x*(c + d*x^3)^q*AppellF1[1/3, 2, -q, 4/3, -((b*x^3)/a), -((d*x^3)/c)
])/((a + b*x^3)^2*(4*a*c*AppellF1[1/3, 2, -q, 4/3, -((b*x^3)/a), -((d*x^3)/
c)] + 3*x^3*(a*d*q*AppellF1[4/3, 2, 1 - q, 7/3, -((b*x^3)/a), -((d*x^3)/c)]
 - 2*b*c*AppellF1[4/3, 3, -q, 7/3, -((b*x^3)/a), -((d*x^3)/c)])))
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{(d\,x^3+c)^q}{(b\,x^3+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^q/(b*x^3+a)^2,x)
```

```
[Out] int((d*x^3+c)^q/(b*x^3+a)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^q/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*x^3 + c)^q/(b*x^3 + a)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^q/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] integral((d*x^3 + c)^q/(b^2*x^6 + 2*a*b*x^3 + a^2), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**q/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^q/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x^3 + c)^q/(b*x^3 + a)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(d\,x^3+c)^q}{(b\,x^3+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^q/(a + b*x^3)^2,x)
```

```
[Out] int((c + d*x^3)^q/(a + b*x^3)^2, x)
```

3.138 $\int (a+bx^3)^m (c+dx^3)^3 \, dx$

Optimal. Leaf size=298

$$\frac{d(28a^2d^2-abcd(92+15m)+b^2c^2(118+60m+9m^2))\,x(a+bx^3)^{1+m}}{b^3(4+3m)(7+3m)(10+3m)}-\frac{d(7ad-bc(16+3m))x(a+bx^3)^{1+}}{b^2(7+3m)(10+3m)}$$

```
[Out] d*(28*a^2*d^2-a*b*c*d*(92+15*m)+b^2*c^2*(9*m^2+60*m+118))*x*(b*x^3+a)^(1+m)
/b^3/(10+3*m)/(9*m^2+33*m+28)-d*(7*a*d-b*c*(16+3*m))*x*(b*x^3+a)^(1+m)*(d*x
^3+c)/b^2/(9*m^2+51*m+70)+d*x*(b*x^3+a)^(1+m)*(d*x^3+c)^2/b/(10+3*m)-(28*a^
3*d^3-12*a^2*b*c*d^2*(10+3*m)+3*a*b^2*c^2*d*(9*m^2+51*m+70)-b^3*c^3*(27*m^3
+189*m^2+414*m+280))*x*(b*x^3+a)^m*hypergeom([1/3, -m],[4/3],-b*x^3/a)/b^3/
(10+3*m)/(9*m^2+33*m+28)/((1+b*x^3/a)^m)
```

Rubi [A]
time = 0.21, antiderivative size = 298, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {427, 542, 396, 252, 251}

$$\frac{dx(a+bx^3)^{m+1}(28a^2d^2-abcd(15m+92)+b^2c^2(9m^2+60m+118))}{b^3(3m+4)(3m+7)(3m+10)}-\frac{x(a+bx^3)^m\left(\frac{bx^3}{a}+1\right)^{-m}(28a^3d^3-12a^2bcd^2(3m+10)+3ab^2c^2d(9m^2+51m+70)-b^3c^3(27m^3+189m^2+414m+280))\,{}_2F_1\left(\frac{1}{3},-m;\frac{4}{3};-\frac{bx^3}{a}\right)}{b^3(3m+4)(3m+7)(3m+10)}-\frac{dx(c+dx^3)(a+bx^3)^{m+1}(7ad-bc(3m+16))}{b^2(3m+7)(3m+10)}+\frac{dx(c+dx^3)^2(a+bx^3)^{m+1}}{b(3m+10)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^m*(c + d*x^3)^3,x]
```

```
[Out] (d*(28*a^2*d^2 - a*b*c*d*(92 + 15*m) + b^2*c^2*(118 + 60*m + 9*m^2))*x*(a +
 b*x^3)^(1 + m))/(b^3*(4 + 3*m)*(7 + 3*m)*(10 + 3*m)) - (d*(7*a*d - b*c*(16
 + 3*m))*x*(a + b*x^3)^(1 + m)*(c + d*x^3))/(b^2*(7 + 3*m)*(10 + 3*m)) + (d
*x*(a + b*x^3)^(1 + m)*(c + d*x^3)^2)/(b*(10 + 3*m)) - ((28*a^3*d^3 - 12*a^
2*b*c*d^2*(10 + 3*m) + 3*a*b^2*c^2*d*(70 + 51*m + 9*m^2) - b^3*c^3*(280 + 4
14*m + 189*m^2 + 27*m^3))*x*(a + b*x^3)^m*Hypergeometric2F1[1/3, -m, 4/3, -
((b*x^3)/a)])/(b^3*(4 + 3*m)*(7 + 3*m)*(10 + 3*m)*(1 + (b*x^3)/a)^m)
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rubi steps

$$
\begin{aligned}
\int \left(a+bx^3\right)^m \left(c+dx^3\right)^3\,dx &= \frac{dx(a+bx^3)^{1+m}\left(c+dx^3\right)^2}{b(10+3m)} + \frac{\int \left(a+bx^3\right)^m \left(c+dx^3\right)\left(-c(ad-bc(10+3m))\right.}{b(10+3m)} \\
&= -\frac{d(7ad-bc(16+3m))x(a+bx^3)^{1+m}\left(c+dx^3\right)}{b^2(7+3m)(10+3m)} + \frac{dx(a+bx^3)^{1+m}\left(c+dx^3\right)^2}{b(10+3m)} \\
&= \frac{d\left(28a^2d^2-abcd(92+15m)+b^2c^2\left(118+60m+9m^2\right)\right)x(a+bx^3)^{1+m}}{b^3(4+3m)(7+3m)(10+3m)} - \frac{d(7}{} \\
&= \frac{d\left(28a^2d^2-abcd(92+15m)+b^2c^2\left(118+60m+9m^2\right)\right)x(a+bx^3)^{1+m}}{b^3(4+3m)(7+3m)(10+3m)} - \frac{d(7}{} \\
&= \frac{d\left(28a^2d^2-abcd(92+15m)+b^2c^2\left(118+60m+9m^2\right)\right)x(a+bx^3)^{1+m}}{b^3(4+3m)(7+3m)(10+3m)} - \frac{d(7}{}
\end{aligned}
$$

Mathematica [A]

time = 5.20, size = 137, normalized size = 0.46

$$\frac{1}{140}x\left(a+bx^3\right)^m\left(1+\frac{bx^3}{a}\right)^{-m}\left(140c^3\,{}_2F_1\left(\frac{1}{3},-m;\frac{4}{3};-\frac{bx^3}{a}\right)+dx^3\left(105c^2\,{}_2F_1\left(\frac{4}{3},-m;\frac{7}{3};-\frac{bx^3}{a}\right)+2dx^3\left(30c\,{}_2F_1\left(\frac{7}{3},-m;\frac{10}{3};-\frac{bx^3}{a}\right)+7dx^3\,{}_2F_1\left(\frac{10}{3},-m;\frac{13}{3};-\frac{bx^3}{a}\right)\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^m*(c + d*x^3)^3,x]
```

```
[Out] (x*(a + b*x^3)^m*(140*c^3*Hypergeometric2F1[1/3, -m, 4/3, -((b*x^3)/a)] + d
*x^3*(105*c^2*Hypergeometric2F1[4/3, -m, 7/3, -((b*x^3)/a)] + 2*d*x^3*(30*c
*Hypergeometric2F1[7/3, -m, 10/3, -((b*x^3)/a)] + 7*d*x^3*Hypergeometric2F1
[10/3, -m, 13/3, -((b*x^3)/a)]))))/(140*(1 + (b*x^3)/a)^m)
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \left(b\,x^3+a\right)^m \left(d\,x^3+c\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^m*(d*x^3+c)^3,x)
```

```
[Out] int((b*x^3+a)^m*(d*x^3+c)^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^m*(d*x^3+c)^3,x, algorithm="maxima")
```

```
[Out] integrate((d*x^3 + c)^3*(b*x^3 + a)^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^m*(d*x^3+c)^3,x, algorithm="fricas")
```

```
[Out] integral((d^3*x^9 + 3*c*d^2*x^6 + 3*c^2*d*x^3 + c^3)*(b*x^3 + a)^m, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**m*(d*x**3+c)**3,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^m*(d*x^3+c)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x^3 + c)^3*(b*x^3 + a)^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(b\,x^3+a\right)}^m\,{\left(d\,x^3+c\right)}^3 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^m*(c + d*x^3)^3,x)
```

```
[Out] int((a + b*x^3)^m*(c + d*x^3)^3, x)
```

3.139 $\int (a+bx^3)^m (c+dx^3)^2 \, dx$

Optimal. Leaf size=176

$$-\frac{d(4ad-bc(10+3m))x(a+bx^3)^{1+m}}{b^2(4+3m)(7+3m)}+\frac{dx(a+bx^3)^{1+m}(c+dx^3)}{b(7+3m)}+\frac{(4a^2d^2-2abcd(7+3m)+b^2c^2(28+33$$

```
[Out] -d*(4*a*d-b*c*(10+3*m))*x*(b*x^3+a)^(1+m)/b^2/(9*m^2+33*m+28)+d*x*(b*x^3+a)
^(1+m)*(d*x^3+c)/b/(7+3*m)+(4*a^2*d^2-2*a*b*c*d*(7+3*m)+b^2*c^2*(9*m^2+33*m
+28))*x*(b*x^3+a)^m*hypergeom([1/3, -m],[4/3],-b*x^3/a)/b^2/(9*m^2+33*m+28)
/((1+b*x^3/a)^m)
```

Rubi [A]
time = 0.09, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {427, 396, 252, 251}

$$\frac{x(a+bx^3)^m\left(\frac{bx^3}{a}+1\right)^{-m}(4a^2d^2-2abcd(3m+7)+b^2c^2(9m^2+33m+28))\,{}_2F_1\left(\frac{1}{3},-m;\frac{4}{3};-\frac{bx^3}{a}\right)}{b^2(3m+4)(3m+7)}-\frac{dx(a+bx^3)^{m+1}(4ad-bc(3m+10))}{b^2(3m+4)(3m+7)}+\frac{dx(c+dx^3)(a+bx^3)^{m+1}}{b(3m+7)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^m*(c + d*x^3)^2,x]
```

```
[Out] -((d*(4*a*d - b*c*(10 + 3*m))*x*(a + b*x^3)^(1 + m))/(b^2*(4 + 3*m)*(7 + 3*
m))) + (d*x*(a + b*x^3)^(1 + m)*(c + d*x^3))/(b*(7 + 3*m)) + ((4*a^2*d^2 -
2*a*b*c*d*(7 + 3*m) + b^2*c^2*(28 + 33*m + 9*m^2))*x*(a + b*x^3)^m*Hypergeo
metric2F1[1/3, -m, 4/3, -((b*x^3)/a)])/(b^2*(4 + 3*m)*(7 + 3*m)*(1 + (b*x^3
)/a)^m)
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
```

```
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rubi steps

$$\int \left(a+bx^3\right)^m \left(c+dx^3\right)^2\,dx = \frac{dx(a+bx^3)^{1+m}\left(c+dx^3\right)}{b(7+3m)} + \frac{\int \left(a+bx^3\right)^m \left(-c(ad-bc(7+3m))-d(4ad-b\right.}{b(7+3m)}$$

$$= -\frac{d(4ad-bc(10+3m))x(a+bx^3)^{1+m}}{b^2(4+3m)(7+3m)} + \frac{dx(a+bx^3)^{1+m}\left(c+dx^3\right)}{b(7+3m)} + \frac{(4a^2d^2}{}$$

$$= -\frac{d(4ad-bc(10+3m))x(a+bx^3)^{1+m}}{b^2(4+3m)(7+3m)} + \frac{dx(a+bx^3)^{1+m}\left(c+dx^3\right)}{b(7+3m)} + \frac{\left(\left(4a^2d\right.\right.}{}$$

$$= -\frac{d(4ad-bc(10+3m))x(a+bx^3)^{1+m}}{b^2(4+3m)(7+3m)} + \frac{dx(a+bx^3)^{1+m}\left(c+dx^3\right)}{b(7+3m)} + \frac{(4a^2d^2}{}$$

Mathematica [A]
time = 5.68, size = 106, normalized size = 0.60

$$\frac{1}{14}x\left(a+bx^3\right)^m\left(1+\frac{bx^3}{a}\right)^{-m}\left(14c^2\,{}_2F_1\left(\frac{1}{3},-m;\frac{4}{3};-\frac{bx^3}{a}\right)+dx^3\left(7c\,{}_2F_1\left(\frac{4}{3},-m;\frac{7}{3};-\frac{bx^3}{a}\right)+2dx^3\,{}_2F_1\left(\frac{7}{3},-m;\frac{10}{3};-\frac{bx^3}{a}\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^m*(c + d*x^3)^2,x]
```

```
[Out] (x*(a + b*x^3)^m*(14*c^2*Hypergeometric2F1[1/3, -m, 4/3, -((b*x^3)/a)] + d*
x^3*(7*c*Hypergeometric2F1[4/3, -m, 7/3, -((b*x^3)/a)] + 2*d*x^3*Hypergeome
tric2F1[7/3, -m, 10/3, -((b*x^3)/a)])))/(14*(1 + (b*x^3)/a)^m)
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \left(b\,x^3+a\right)^m \left(d\,x^3+c\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^m*(d*x^3+c)^2,x)
```

```
[Out] int((b*x^3+a)^m*(d*x^3+c)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^m*(d*x^3+c)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*x^3 + c)^2*(b*x^3 + a)^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^m*(d*x^3+c)^2,x, algorithm="fricas")
```

```
[Out] integral((d^2*x^6 + 2*c*d*x^3 + c^2)*(b*x^3 + a)^m, x)
```

Sympy [C] Result contains complex when optimal does not.
time = 105.62, size = 121, normalized size = 0.69

$$\frac{a^m c^2 x \Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\begin{matrix} \frac{1}{3}, -m \\ \frac{4}{3} \end{matrix}\middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)} + \frac{2a^m c d x^4 \Gamma\left(\frac{4}{3}\right) {}_2F_1\left(\begin{matrix} \frac{4}{3}, -m \\ \frac{7}{3} \end{matrix}\middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)} + \frac{a^m d^2 x^7 \Gamma\left(\frac{7}{3}\right) {}_2F_1\left(\begin{matrix} \frac{7}{3}, -m \\ \frac{10}{3} \end{matrix}\middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{10}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**m*(d*x**3+c)**2,x)
```

```
[Out] a**m*c**2*x*gamma(1/3)*hyper((1/3, -m), (4/3,), b*x**3*exp_polar(I*pi)/a)/(
3*gamma(4/3)) + 2*a**m*c*d*x**4*gamma(4/3)*hyper((4/3, -m), (7/3,), b*x**3*
exp_polar(I*pi)/a)/(3*gamma(7/3)) + a**m*d**2*x**7*gamma(7/3)*hyper((7/3, -
m), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(10/3))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^m*(d*x^3+c)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x^3 + c)^2*(b*x^3 + a)^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(b\,x^3+a\right)^m\,{\left(d\,x^3+c\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^m*(c + d*x^3)^2,x)
```

```
[Out] int((a + b*x^3)^m*(c + d*x^3)^2, x)
```

3.140 $\int (a+bx^3)^m (c+dx^3)\, dx$

Optimal. Leaf size=93

$$\frac{dx(a+bx^3)^{1+m}}{b(4+3m)} - \frac{(ad-bc(4+3m))x(a+bx^3)^m \left(1+\frac{bx^3}{a}\right)^{-m} {}_2F_1\left(\frac{1}{3},-m;\frac{4}{3};-\frac{bx^3}{a}\right)}{b(4+3m)}$$

```
[Out] d*x*(b*x^3+a)^(1+m)/b/(4+3*m)-(a*d-b*c*(4+3*m))*x*(b*x^3+a)^m*hypergeom([1/
3, -m],[4/3],-b*x^3/a)/b/(4+3*m)/((1+b*x^3/a)^m)
```

Rubi [A]

time = 0.03, antiderivative size = 85, normalized size of antiderivative = 0.91, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {396, 252, 251}

$$x(a+bx^3)^m \left(\frac{bx^3}{a}+1\right)^{-m} \left(c-\frac{ad}{3bm+4b}\right) {}_2F_1\left(\frac{1}{3},-m;\frac{4}{3};-\frac{bx^3}{a}\right) + \frac{dx(a+bx^3)^{m+1}}{b(3m+4)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^m*(c + d*x^3),x]
```

```
[Out] (d*x*(a + b*x^3)^(1 + m))/(b*(4 + 3*m)) + ((c - (a*d)/(4*b + 3*b*m))*x*(a +
 b*x^3)^m*Hypergeometric2F1[1/3, -m, 4/3, -((b*x^3)/a)])/(1 + (b*x^3)/a)^m
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rubi steps

$$\int \left(a+bx^3\right)^m \left(c+dx^3\right) dx = \frac{dx(a+bx^3)^{1+m}}{b(4+3m)} - \left(-c+\frac{ad}{4b+3bm}\right)\int \left(a+bx^3\right)^m dx$$
$$= \frac{dx(a+bx^3)^{1+m}}{b(4+3m)} - \left(\left(-c+\frac{ad}{4b+3bm}\right)\left(a+bx^3\right)^m \left(1+\frac{bx^3}{a}\right)^{-m}\right)\int \left(1+\right.$$
$$= \frac{dx(a+bx^3)^{1+m}}{b(4+3m)} + \left(c-\frac{ad}{4b+3bm}\right)x\left(a+bx^3\right)^m \left(1+\frac{bx^3}{a}\right)^{-m} {}_2F_1\left(\frac{1}{3},-m;\right.$$

Mathematica [A]
time = 0.11, size = 90, normalized size = 0.97

$$\frac{x(a+bx^3)^m \left(1+\frac{bx^3}{a}\right)^{-m} \left(d(a+bx^3)\left(1+\frac{bx^3}{a}\right)^m + (-ad+bc(4+3m))\,{}_2F_1\left(\frac{1}{3},-m;\frac{4}{3};-\frac{bx^3}{a}\right)\right)}{b(4+3m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^m*(c + d*x^3),x]
```

```
[Out] (x*(a + b*x^3)^m*(d*(a + b*x^3)*(1 + (b*x^3)/a)^m + (-(a*d) + b*c*(4 + 3*m)
)*Hypergeometric2F1[1/3, -m, 4/3, -((b*x^3)/a)]))/(b*(4 + 3*m)*(1 + (b*x^3)
/a)^m)
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \left(b\,x^3+a\right)^m \left(d\,x^3+c\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^m*(d*x^3+c),x)
```

```
[Out] int((b*x^3+a)^m*(d*x^3+c),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^m*(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate((d*x^3 + c)*(b*x^3 + a)^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^m*(d*x^3+c),x, algorithm="fricas")
```

```
[Out] integral((d*x^3 + c)*(b*x^3 + a)^m, x)
```

Sympy [C] Result contains complex when optimal does not.
time = 40.02, size = 75, normalized size = 0.81

$$\frac{a^{m}cx\Gamma\left(\frac{1}{3}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{3}, -m\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)}+\frac{a^{m}dx^{4}\Gamma\left(\frac{4}{3}\right){}_{2}F_{1}\left(\begin{matrix}\frac{4}{3}, -m\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**m*(d*x**3+c),x)
```

```
[Out] a**m*c*x*gamma(1/3)*hyper((1/3, -m), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*g
amma(4/3)) + a**m*d*x**4*gamma(4/3)*hyper((4/3, -m), (7/3,), b*x**3*exp_pol
ar(I*pi)/a)/(3*gamma(7/3))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^m*(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate((d*x^3 + c)*(b*x^3 + a)^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(b\,x^3+a\right)}^m\,\left(d\,x^3+c\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^m*(c + d*x^3),x)
```

```
[Out] int((a + b*x^3)^m*(c + d*x^3), x)
```

3.141 $\int (a+bx^3)^m \, dx$

Optimal. Leaf size=44

$$x\left(a+bx^3\right)^m\left(1+\frac{bx^3}{a}\right)^{-m}{}_2F_1\left(\frac{1}{3},-m;\frac{4}{3};-\frac{bx^3}{a}\right)$$

```
[Out] x*(b*x^3+a)^m*hypergeom([1/3, -m],[4/3],-b*x^3/a)/((1+b*x^3/a)^m)
```

Rubi [A]
time = 0.01, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {252, 251}

$$x\left(a+bx^3\right)^m\left(\frac{bx^3}{a}+1\right)^{-m}{}_2F_1\left(\frac{1}{3},-m;\frac{4}{3};-\frac{bx^3}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^m,x]
```

```
[Out] (x*(a + b*x^3)^m*Hypergeometric2F1[1/3, -m, 4/3, -((b*x^3)/a)])/(1 + (b*x^3
)/a)^m
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \left(a+bx^3\right)^m \, dx &= \left(\left(a+bx^3\right)^m\left(1+\frac{bx^3}{a}\right)^{-m}\right)\int \left(1+\frac{bx^3}{a}\right)^m \, dx\\ &= x\left(a+bx^3\right)^m\left(1+\frac{bx^3}{a}\right)^{-m}{}_2F_1\left(\frac{1}{3},-m;\frac{4}{3};-\frac{bx^3}{a}\right)\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.
time = 0.13, size = 196, normalized size = 4.45

$$\frac{2^{-m}\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\left(\frac{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}\,x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}\right)^{-m}\left(\frac{i\left(1+\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a}}\right)}{3i+\sqrt{3}}\right)^{-m}(a+bx^3)^m\,F_1\left(1+m;-m,-m;2+m;-\frac{i\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt{3}\,\sqrt[3]{a}},\frac{i+\sqrt{3}-\frac{2i\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{3i+\sqrt{3}}\right)}{\sqrt[3]{b}\,(1+m)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^m,x]
```

```
[Out] (((-1)^(2/3)*a^(1/3) + b^(1/3)*x)*(a + b*x^3)^m*AppellF1[1 + m, -m, -m, 2 +
 m, ((-I)*((-1)^(2/3)*a^(1/3) + b^(1/3)*x))/(Sqrt[3]*a^(1/3)), (I + Sqrt[3]
 - ((2*I)*b^(1/3)*x)/a^(1/3))/(3*I + Sqrt[3])])/(2^m*b^(1/3)*(1 + m)*((a^(1
/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3)))^m*((I*(1 + (b^(1/3)
*x)/a^(1/3)))/(3*I + Sqrt[3]))^m)
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \left(b\,x^{3}+a\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^m,x)
```

```
[Out] int((b*x^3+a)^m,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^m,x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^m,x, algorithm="fricas")
```

```
[Out] integral((b*x^3 + a)^m, x)
```

Sympy [C] Result contains complex when optimal does not.
time = 6.02, size = 34, normalized size = 0.77

$$\frac{a^m x\Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\begin{matrix} \frac{1}{3}, -m \\ \frac{4}{3} \end{matrix} \middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**m,x)
```

```
[Out] a**m*x*gamma(1/3)*hyper((1/3, -m), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*gam
ma(4/3))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^m,x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^m, x)
```

Mupad [B]
time = 1.35, size = 41, normalized size = 0.93

$$\frac{x\,\left(b\,x^3+a\right)^m\,{}_2\mathrm{F}_1\left(\frac{1}{3},-m;\ \frac{4}{3};\ -\frac{b\,x^3}{a}\right)}{\left(\frac{b\,x^3}{a}+1\right)^m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^m,x)
```

```
[Out] (x*(a + b*x^3)^m*hypergeom([1/3, -m], 4/3, -(b*x^3)/a))/((b*x^3)/a + 1)^m
```

3.142 $\int \frac{(a+bx^3)^m}{c+dx^3} dx$

Optimal. Leaf size=57

$$\frac{x(a+bx^3)^m \left(1+\frac{bx^3}{a}\right)^{-m} F_1\left(\frac{1}{3};-m,1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c}$$

```
[Out] x*(b*x^3+a)^m*AppellF1(1/3,-m,1,4/3,-b*x^3/a,-d*x^3/c)/c/((1+b*x^3/a)^m)
```

Rubi [A]
time = 0.02, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {441, 440}

$$\frac{x(a+bx^3)^m \left(\frac{bx^3}{a}+1\right)^{-m} F_1\left(\frac{1}{3};-m,1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^m/(c + d*x^3),x]
```

```
[Out] (x*(a + b*x^3)^m*AppellF1[1/3, -m, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)])/(c*
(1 + (b*x^3)/a)^m)
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^3)^m}{c+dx^3} dx &= \left((a+bx^3)^m \left(1+\frac{bx^3}{a}\right)^{-m}\right) \int \frac{\left(1+\frac{bx^3}{a}\right)^m}{c+dx^3} dx \\ &= \frac{x(a+bx^3)^m \left(1+\frac{bx^3}{a}\right)^{-m} F_1\left(\frac{1}{3};-m,1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 162 vs. 2(57) = 114.
time = 0.26, size = 162, normalized size = 2.84

$$-\frac{4acx(a+bx^3)^m F_1\left(\frac{1}{3};-m,1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{(c+dx^3)\left(-4acF_1\left(\frac{1}{3};-m,1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+3x^3\left(-bcmF_1\left(\frac{4}{3};1-m,1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+adF_1\left(\frac{4}{3};-m,2;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^m/(c + d*x^3),x]
[Out] (-4*a*c*x*(a + b*x^3)^m*AppellF1[1/3, -m, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c
)])/((c + d*x^3)*(-4*a*c*AppellF1[1/3, -m, 1, 4/3, -((b*x^3)/a), -((d*x^3)/
c)] + 3*x^3*(-(b*c*m*AppellF1[4/3, 1 - m, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c
)]) + a*d*AppellF1[4/3, -m, 2, 7/3, -((b*x^3)/a), -((d*x^3)/c)])))
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{(b\,x^3+a)^m}{d\,x^3+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^m/(d*x^3+c),x)
[Out] int((b*x^3+a)^m/(d*x^3+c),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^m/(d*x^3+c),x, algorithm="maxima")
[Out] integrate((b*x^3 + a)^m/(d*x^3 + c), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^m/(d*x^3+c),x, algorithm="fricas")
[Out] integral((b*x^3 + a)^m/(d*x^3 + c), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**m/(d*x**3+c),x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^m/(d*x^3+c),x, algorithm="giac")
[Out] integrate((b*x^3 + a)^m/(d*x^3 + c), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(b\,x^3+a)^m}{d\,x^3+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^m/(c + d*x^3),x)
[Out] int((a + b*x^3)^m/(c + d*x^3), x)
```

3.143 $\int \frac{(a+bx^3)^m}{(c+dx^3)^2} dx$

Optimal. Leaf size=57

$$\frac{x(a+bx^3)^m \left(1+\frac{bx^3}{a}\right)^{-m} F_1\left(\frac{1}{3};-m,2;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^2}$$

```
[Out] x*(b*x^3+a)^m*AppellF1(1/3,-m,2,4/3,-b*x^3/a,-d*x^3/c)/c^2/((1+b*x^3/a)^m)
```

Rubi [A]
time = 0.02, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {441, 440}

$$\frac{x(a+bx^3)^m \left(\frac{bx^3}{a}+1\right)^{-m} F_1\left(\frac{1}{3};-m,2;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^m/(c + d*x^3)^2,x]
```

```
[Out] (x*(a + b*x^3)^m*AppellF1[1/3, -m, 2, 4/3, -((b*x^3)/a), -((d*x^3)/c)])/(c^
2*(1 + (b*x^3)/a)^m)
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^3)^m}{(c+dx^3)^2} dx &= \left((a+bx^3)^m \left(1+\frac{bx^3}{a}\right)^{-m}\right) \int \frac{\left(1+\frac{bx^3}{a}\right)^m}{(c+dx^3)^2} dx \\
&= \frac{x(a+bx^3)^m \left(1+\frac{bx^3}{a}\right)^{-m} F_1\left(\frac{1}{3};-m,2;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^2}
\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 162 vs. 2(57) = 114.
time = 0.34, size = 162, normalized size = 2.84

$$-\frac{4acx(a+bx^3)^m F_1\left(\frac{1}{3};-m,2;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{(c+dx^3)^2\left(-4acF_1\left(\frac{1}{3};-m,2;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)-3x^3\left(bcmF_1\left(\frac{4}{3};1-m,2;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)-2adF_1\left(\frac{4}{3};-m,3;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^m/(c + d*x^3)^2,x]
```

```
[Out] (-4*a*c*x*(a + b*x^3)^m*AppellF1[1/3, -m, 2, 4/3, -((b*x^3)/a), -((d*x^3)/c
)])/((c + d*x^3)^2*(-4*a*c*AppellF1[1/3, -m, 2, 4/3, -((b*x^3)/a), -((d*x^3
)/c)] - 3*x^3*(b*c*m*AppellF1[4/3, 1 - m, 2, 7/3, -((b*x^3)/a), -((d*x^3)/c
)] - 2*a*d*AppellF1[4/3, -m, 3, 7/3, -((b*x^3)/a), -((d*x^3)/c)])))
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{(b\,x^3+a)^m}{(d\,x^3+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^m/(d*x^3+c)^2,x)
```

```
[Out] int((b*x^3+a)^m/(d*x^3+c)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^m/(d*x^3+c)^2,x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^m/(d*x^3 + c)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^m/(d*x^3+c)^2,x, algorithm="fricas")
```

```
[Out] integral((b*x^3 + a)^m/(d^2*x^6 + 2*c*d*x^3 + c^2), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**m/(d*x**3+c)**2,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^m/(d*x^3+c)^2,x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^m/(d*x^3 + c)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(b\,x^3+a)^m}{(d\,x^3+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^m/(c + d*x^3)^2,x)
```

```
[Out] int((a + b*x^3)^m/(c + d*x^3)^2, x)
```

3.144 $\int \frac{(a+bx^3)^m}{(c+dx^3)^3}\,dx$

Optimal. Leaf size=57

$$\frac{x(a+bx^3)^m\left(1+\frac{bx^3}{a}\right)^{-m}F_1\left(\frac{1}{3};-m,3;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^3}$$

```
[Out] x*(b*x^3+a)^m*AppellF1(1/3,-m,3,4/3,-b*x^3/a,-d*x^3/c)/c^3/((1+b*x^3/a)^m)
```

Rubi [A]
time = 0.02, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {441, 440}

$$\frac{x(a+bx^3)^m\left(\frac{bx^3}{a}+1\right)^{-m}F_1\left(\frac{1}{3};-m,3;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^m/(c + d*x^3)^3,x]
```

```
[Out] (x*(a + b*x^3)^m*AppellF1[1/3, -m, 3, 4/3, -((b*x^3)/a), -((d*x^3)/c)])/(c^
3*(1 + (b*x^3)/a)^m)
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^3)^m}{(c+dx^3)^3}\,dx &= \left((a+bx^3)^m\left(1+\frac{bx^3}{a}\right)^{-m}\right)\int \frac{\left(1+\frac{bx^3}{a}\right)^m}{(c+dx^3)^3}\,dx\\ &= \frac{x(a+bx^3)^m\left(1+\frac{bx^3}{a}\right)^{-m}F_1\left(\frac{1}{3};-m,3;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^3}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 162 vs. 2(57) = 114.
time = 0.47, size = 162, normalized size = 2.84

$$-\frac{4acx(a+bx^3)^m F_1\left(\frac{1}{3};-m,3;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{(c+dx^3)^3\left(-4acF_1\left(\frac{1}{3};-m,3;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)-3x^3\left(bcmF_1\left(\frac{4}{3};1-m,3;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)-3adF_1\left(\frac{4}{3};-m,4;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^m/(c + d*x^3)^3,x]
```

```
[Out] (-4*a*c*x*(a + b*x^3)^m*AppellF1[1/3, -m, 3, 4/3, -((b*x^3)/a), -((d*x^3)/c
)])/((c + d*x^3)^3*(-4*a*c*AppellF1[1/3, -m, 3, 4/3, -((b*x^3)/a), -((d*x^3
)/c)] - 3*x^3*(b*c*m*AppellF1[4/3, 1 - m, 3, 7/3, -((b*x^3)/a), -((d*x^3)/c
)] - 3*a*d*AppellF1[4/3, -m, 4, 7/3, -((b*x^3)/a), -((d*x^3)/c)])))
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{(b\,x^3+a)^m}{(d\,x^3+c)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^m/(d*x^3+c)^3,x)
```

```
[Out] int((b*x^3+a)^m/(d*x^3+c)^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^m/(d*x^3+c)^3,x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^m/(d*x^3 + c)^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^m/(d*x^3+c)^3,x, algorithm="fricas")
```

```
[Out] integral((b*x^3 + a)^m/(d^3*x^9 + 3*c*d^2*x^6 + 3*c^2*d*x^3 + c^3), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**m/(d*x**3+c)**3,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^m/(d*x^3+c)^3,x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^m/(d*x^3 + c)^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(b\,x^3+a)^m}{(d\,x^3+c)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^m/(c + d*x^3)^3,x)
```

```
[Out] int((a + b*x^3)^m/(c + d*x^3)^3, x)
```

3.145 $\int \left(a+bx^3\right)^{-1-\frac{bc}{3bc-3ad}} \left(c+dx^3\right)^{-1+\frac{ad}{3bc-3ad}} \, dx$

Optimal. Leaf size=53

$$\frac{x\left(a+bx^3\right)^{-\frac{bc}{3bc-3ad}} \left(c+dx^3\right)^{\frac{ad}{3bc-3ad}}}{ac}$$

```
[Out] x*(d*x^3+c)^(a*d/(-3*a*d+3*b*c))/a/c/((b*x^3+a)^(b*c/(-3*a*d+3*b*c)))
```

Rubi [A]
time = 0.01, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 50, $\frac{\text{number of rules}}{\text{integrand size}} = 0.020$, Rules used = $\{389\}$

$$\frac{x\left(a+bx^3\right)^{-\frac{bc}{3bc-3ad}} \left(c+dx^3\right)^{\frac{ad}{3bc-3ad}}}{ac}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(-1 - (b*c)/(3*b*c - 3*a*d))*(c + d*x^3)^(-1 + (a*d)/(3*b*c
 - 3*a*d)),x]
```

```
[Out] (x*(c + d*x^3)^((a*d)/(3*b*c - 3*a*d)))/(a*c*(a + b*x^3)^((b*c)/(3*b*c - 3*
a*d)))
```

Rule 389

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c)), x] /; FreeQ[{a,
b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] &&
EqQ[a*d*(p + 1) + b*c*(q + 1), 0]
```

Rubi steps

$$\int \left(a+bx^3\right)^{-1-\frac{bc}{3bc-3ad}} \left(c+dx^3\right)^{-1+\frac{ad}{3bc-3ad}} \, dx = \frac{x\left(a+bx^3\right)^{-\frac{bc}{3bc-3ad}} \left(c+dx^3\right)^{\frac{ad}{3bc-3ad}}}{ac}$$

Mathematica [A]
time = 0.39, size = 52, normalized size = 0.98

$$\frac{x\left(a+bx^3\right)^{\frac{bc}{-3bc+3ad}} \left(c+dx^3\right)^{\frac{ad}{3bc-3ad}}}{ac}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^(-1 - (b*c)/(3*b*c - 3*a*d))*(c + d*x^3)^(-1 + (a*d)/
(3*b*c - 3*a*d)),x]
```

```
[Out] (x*(a + b*x^3)^((b*c)/(-3*b*c + 3*a*d))*(c + d*x^3)^((a*d)/(3*b*c - 3*a*d))
)/(a*c)
```

Maple [A]
time = 0.33, size = 71, normalized size = 1.34

method	result	size
gosper	$\frac{\left(b\,x^{3}+a\right)^{1-\frac{3ad-4bc}{3(ad-bc)}}\left(d\,x^{3}+c\right)^{1-\frac{4ad-3bc}{3(ad-bc)}}x}{ac}$	71

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(-1-b*c/(-3*a*d+3*b*c))*(d*x^3+c)^(-1+a*d/(-3*a*d+3*b*c)),x,m
ethod=_RETURNVERBOSE)
```

```
[Out] (b*x^3+a)^(1-1/3*(3*a*d-4*b*c)/(a*d-b*c))*(d*x^3+c)^(1-1/3*(4*a*d-3*b*c)/(a
*d-b*c))/a/c*x
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(-1-b*c/(-3*a*d+3*b*c))*(d*x^3+c)^(-1+a*d/(-3*a*d+3*b*c
)),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(-1/3*b*c/(b*c - a*d) - 1)*(d*x^3 + c)^(1/3*a*d/(b*c
- a*d) - 1), x)
```

Fricas [A]
time = 2.39, size = 91, normalized size = 1.72

$$\frac{bdx^7 + (bc + ad)x^4 + acx}{(bx^3 + a)^{\frac{4\,bc-3\,ad}{3\,(bc-ad)}}(dx^3 + c)^{\frac{3\,bc-4\,ad}{3\,(bc-ad)}}ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(-1-b*c/(-3*a*d+3*b*c))*(d*x^3+c)^(-1+a*d/(-3*a*d+3*b*c
)),x, algorithm="fricas")
```

```
[Out] (b*d*x^7 + (b*c + a*d)*x^4 + a*c*x)/((b*x^3 + a)^(1/3*(4*b*c - 3*a*d)/(b*c
- a*d))*(d*x^3 + c)^(1/3*(3*b*c - 4*a*d)/(b*c - a*d))*a*c)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+bx^3\right)^{-\frac{bc}{-3ad+3bc}-1} \left(c+dx^3\right)^{\frac{ad}{-3ad+3bc}-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(-1-b*c/(-3*a*d+3*b*c))*(d*x**3+c)**(-1+a*d/(-3*a*d+3
*b*c)),x)
```

```
[Out] Integral((a + b*x**3)**(-b*c/(-3*a*d + 3*b*c) - 1)*(c + d*x**3)**(a*d/(-3*a
*d + 3*b*c) - 1), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(-1-b*c/(-3*a*d+3*b*c))*(d*x^3+c)^(-1+a*d/(-3*a*d+3*b*c
)),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(-1/3*b*c/(b*c - a*d) - 1)*(d*x^3 + c)^(1/3*a*d/(b*c
- a*d) - 1), x)
```

Mupad [B]
time = 1.90, size = 131, normalized size = 2.47

$$\frac{x\,{\left(b\,x^3+a\right)}^{\frac{b\,c}{3\,a\,d-3\,b\,c}-1}+\frac{x^4\,{\left(b\,x^3+a\right)}^{\frac{b\,c}{3\,a\,d-3\,b\,c}-1}\,\left(a\,d+b\,c\right)}{a\,c}+\frac{b\,d\,x^7\,{\left(b\,x^3+a\right)}^{\frac{b\,c}{3\,a\,d-3\,b\,c}-1}}{a\,c}}{{\left(d\,x^3+c\right)}^{\frac{a\,d}{3\,a\,d-3\,b\,c}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^((b*c)/(3*a*d - 3*b*c) - 1)/(c + d*x^3)^((a*d)/(3*a*d - 3*b
*c) + 1),x)
```

```
[Out] (x*(a + b*x^3)^((b*c)/(3*a*d - 3*b*c) - 1) + (x^4*(a + b*x^3)^((b*c)/(3*a*d
 - 3*b*c) - 1)*(a*d + b*c))/(a*c) + (b*d*x^7*(a + b*x^3)^((b*c)/(3*a*d - 3*
b*c) - 1))/(a*c))/(c + d*x^3)^((a*d)/(3*a*d - 3*b*c) + 1)
```

3.146 $\int (a+bx^4)(c+dx^4)^4\,dx$

Optimal. Leaf size=94

$$ac^4x+\frac{1}{5}c^3(bc+4ad)x^5+\frac{2}{9}c^2d(2bc+3ad)x^9+\frac{2}{13}cd^2(3bc+2ad)x^{13}+\frac{1}{17}d^3(4bc+ad)x^{17}+\frac{1}{21}bd^4x^{21}$$

```
[Out] a*c^4*x+1/5*c^3*(4*a*d+b*c)*x^5+2/9*c^2*d*(3*a*d+2*b*c)*x^9+2/13*c*d^2*(2*a
*d+3*b*c)*x^13+1/17*d^3*(a*d+4*b*c)*x^17+1/21*b*d^4*x^21
```

Rubi [A]
time = 0.05, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {380}

$$\frac{1}{5}c^3x^5(4ad+bc)+\frac{2}{9}c^2dx^9(3ad+2bc)+\frac{1}{17}d^3x^{17}(ad+4bc)+\frac{2}{13}cd^2x^{13}(2ad+3bc)+ac^4x+\frac{1}{21}bd^4x^{21}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)*(c + d*x^4)^4,x]
```

```
[Out] a*c^4*x + (c^3*(b*c + 4*a*d)*x^5)/5 + (2*c^2*d*(2*b*c + 3*a*d)*x^9)/9 + (2*
c*d^2*(3*b*c + 2*a*d)*x^13)/13 + (d^3*(4*b*c + a*d)*x^17)/17 + (b*d^4*x^21)
/21
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int (a+bx^4)(c+dx^4)^4\,dx &= \int \left(ac^4+c^3(bc+4ad)x^4+2c^2d(2bc+3ad)x^8+2cd^2(3bc+2ad)x^{12}+d^3(4bc\right.\\ &= ac^4x+\frac{1}{5}c^3(bc+4ad)x^5+\frac{2}{9}c^2d(2bc+3ad)x^9+\frac{2}{13}cd^2(3bc+2ad)x^{13}+\frac{1}{17}d^3(\end{aligned}$$

Mathematica [A]
time = 0.02, size = 94, normalized size = 1.00

$$ac^4x+\frac{1}{5}c^3(bc+4ad)x^5+\frac{2}{9}c^2d(2bc+3ad)x^9+\frac{2}{13}cd^2(3bc+2ad)x^{13}+\frac{1}{17}d^3(4bc+ad)x^{17}+\frac{1}{21}bd^4x^{21}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^4)*(c + d*x^4)^4,x]
```

```
[Out] a*c^4*x + (c^3*(b*c + 4*a*d)*x^5)/5 + (2*c^2*d*(2*b*c + 3*a*d)*x^9)/9 + (2*
c*d^2*(3*b*c + 2*a*d)*x^13)/13 + (d^3*(4*b*c + a*d)*x^17)/17 + (b*d^4*x^21)
/21
```

Maple [A]
time = 0.28, size = 97, normalized size = 1.03

method	result
norman	$a\,c^{4} x +\left(\frac{4}{5} a\,c^{3} d +\frac{1}{5} b\,c^{4}\right) x^{5}+\left(\frac{2}{3} a\,c^{2} d^{2}+\frac{4}{9} b\,c^{3} d \right) x^{9}+\left(\frac{4}{13} a c\,d^{3}+\frac{6}{13} d^{2} b\,c^{2}\right) x^{13}+\left(\frac{1}{17} a\,d^{4}+\frac{4}{17} b c\,d^{3}\right)$
default	$\frac{b\,d^{4} x^{21}}{21}+\frac{\left(a\,d^{4}+4 b c\,d^{3}\right) x^{17}}{17}+\frac{\left(4 a c\,d^{3}+6 d^{2} b\,c^{2}\right) x^{13}}{13}+\frac{\left(6 a\,c^{2} d^{2}+4 b\,c^{3} d \right) x^{9}}{9}+\frac{\left(4 a\,c^{3} d +b\,c^{4}\right) x^{5}}{5}+a\,c^{4} x$
gosper	$a\,c^{4} x +\frac{4}{5} x^{5} a\,c^{3} d +\frac{1}{5} x^{5} b\,c^{4}+\frac{2}{3} x^{9} a\,c^{2} d^{2}+\frac{4}{9} x^{9} b\,c^{3} d +\frac{4}{13} x^{13} a c\,d^{3}+\frac{6}{13} x^{13} d^{2} b\,c^{2}+\frac{1}{17} x^{17} a\,d^{4}+\frac{4}{17} x^{17}$
risch	$a\,c^{4} x +\frac{4}{5} x^{5} a\,c^{3} d +\frac{1}{5} x^{5} b\,c^{4}+\frac{2}{3} x^{9} a\,c^{2} d^{2}+\frac{4}{9} x^{9} b\,c^{3} d +\frac{4}{13} x^{13} a c\,d^{3}+\frac{6}{13} x^{13} d^{2} b\,c^{2}+\frac{1}{17} x^{17} a\,d^{4}+\frac{4}{17} x^{17}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)*(d*x^4+c)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/21*b*d^4*x^21+1/17*(a*d^4+4*b*c*d^3)*x^17+1/13*(4*a*c*d^3+6*b*c^2*d^2)*x^
13+1/9*(6*a*c^2*d^2+4*b*c^3*d)*x^9+1/5*(4*a*c^3*d+b*c^4)*x^5+a*c^4*x
```

Maxima [A]
time = 0.28, size = 96, normalized size = 1.02

$$\frac{1}{21}\,bd^4x^{21}+\frac{1}{17}\,(4\,bcd^3+ad^4)x^{17}+\frac{2}{13}\,(3\,bc^2d^2+2\,acd^3)x^{13}+\frac{2}{9}\,(2\,bc^3d+3\,ac^2d^2)x^9+ac^4x+\frac{1}{5}\,(bc^4+4\,ac^3d)x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)*(d*x^4+c)^4,x, algorithm="maxima")
```

```
[Out] 1/21*b*d^4*x^21 + 1/17*(4*b*c*d^3 + a*d^4)*x^17 + 2/13*(3*b*c^2*d^2 + 2*a*c
*d^3)*x^13 + 2/9*(2*b*c^3*d + 3*a*c^2*d^2)*x^9 + a*c^4*x + 1/5*(b*c^4 + 4*a
*c^3*d)*x^5
```

Fricas [A]
time = 2.96, size = 96, normalized size = 1.02

$$\frac{1}{21}\,bd^4x^{21}+\frac{1}{17}\,(4\,bcd^3+ad^4)x^{17}+\frac{2}{13}\,(3\,bc^2d^2+2\,acd^3)x^{13}+\frac{2}{9}\,(2\,bc^3d+3\,ac^2d^2)x^9+ac^4x+\frac{1}{5}\,(bc^4+4\,ac^3d)x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)*(d*x^4+c)^4,x, algorithm="fricas")
```

```
[Out] 1/21*b*d^4*x^21 + 1/17*(4*b*c*d^3 + a*d^4)*x^17 + 2/13*(3*b*c^2*d^2 + 2*a*c
*d^3)*x^13 + 2/9*(2*b*c^3*d + 3*a*c^2*d^2)*x^9 + a*c^4*x + 1/5*(b*c^4 + 4*a
*c^3*d)*x^5
```

Sympy [A]
time = 0.02, size = 107, normalized size = 1.14

$$ac^4x + \frac{bd^4x^{21}}{21} + x^{17}\left(\frac{ad^4}{17} + \frac{4bcd^3}{17}\right) + x^{13}\cdot\left(\frac{4acd^3}{13} + \frac{6bc^2d^2}{13}\right) + x^9\cdot\left(\frac{2ac^2d^2}{3} + \frac{4bc^3d}{9}\right) + x^5\cdot\left(\frac{4ac^3d}{5} + \frac{bc^4}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)*(d*x**4+c)**4,x)
```

```
[Out] a*c**4*x + b*d**4*x**21/21 + x**17*(a*d**4/17 + 4*b*c*d**3/17) + x**13*(4*a
*c*d**3/13 + 6*b*c**2*d**2/13) + x**9*(2*a*c**2*d**2/3 + 4*b*c**3*d/9) + x*
*5*(4*a*c**3*d/5 + b*c**4/5)
```

Giac [A]
time = 0.54, size = 98, normalized size = 1.04

$$\frac{1}{21}\,bd^4x^{21} + \frac{4}{17}\,bcd^3x^{17} + \frac{1}{17}\,ad^4x^{17} + \frac{6}{13}\,bc^2d^2x^{13} + \frac{4}{13}\,acd^3x^{13} + \frac{4}{9}\,bc^3dx^9 + \frac{2}{3}\,ac^2d^2x^9 + \frac{1}{5}\,bc^4x^5 + \frac{4}{5}\,ac^3dx^5 + ac^4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)*(d*x^4+c)^4,x, algorithm="giac")
```

```
[Out] 1/21*b*d^4*x^21 + 4/17*b*c*d^3*x^17 + 1/17*a*d^4*x^17 + 6/13*b*c^2*d^2*x^13
 + 4/13*a*c*d^3*x^13 + 4/9*b*c^3*d*x^9 + 2/3*a*c^2*d^2*x^9 + 1/5*b*c^4*x^5
+ 4/5*a*c^3*d*x^5 + a*c^4*x
```

Mupad [B]
time = 1.30, size = 88, normalized size = 0.94

$$x^5\left(\frac{b\,c^4}{5} + \frac{4\,a\,d\,c^3}{5}\right) + x^{17}\left(\frac{a\,d^4}{17} + \frac{4\,b\,c\,d^3}{17}\right) + \frac{b\,d^4\,x^{21}}{21} + a\,c^4\,x + \frac{2\,c^2\,d\,x^9\,(3\,a\,d + 2\,b\,c)}{9} + \frac{2\,c\,d^2\,x^{13}\,(2\,a\,d + 3\,b\,c)}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)*(c + d*x^4)^4,x)
```

```
[Out] x^5*((b*c^4)/5 + (4*a*c^3*d)/5) + x^17*((a*d^4)/17 + (4*b*c*d^3)/17) + (b*d
^4*x^21)/21 + a*c^4*x + (2*c^2*d*x^9*(3*a*d + 2*b*c))/9 + (2*c*d^2*x^13*(2*
a*d + 3*b*c))/13
```

3.147 $\int (a+bx^4)(c+dx^4)^3\,dx$

Optimal. Leaf size=70

$$ac^3x+\frac{1}{5}c^2(bc+3ad)x^5+\frac{1}{3}cd(bc+ad)x^9+\frac{1}{13}d^2(3bc+ad)x^{13}+\frac{1}{17}bd^3x^{17}$$

```
[Out] a*c^3*x+1/5*c^2*(3*a*d+b*c)*x^5+1/3*c*d*(a*d+b*c)*x^9+1/13*d^2*(a*d+3*b*c)*
x^13+1/17*b*d^3*x^17
```

Rubi [A]
time = 0.03, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {380}

$$\frac{1}{5}c^2x^5(3ad+bc)+\frac{1}{13}d^2x^{13}(ad+3bc)+\frac{1}{3}cdx^9(ad+bc)+ac^3x+\frac{1}{17}bd^3x^{17}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)*(c + d*x^4)^3,x]
```

```
[Out] a*c^3*x + (c^2*(b*c + 3*a*d)*x^5)/5 + (c*d*(b*c + a*d)*x^9)/3 + (d^2*(3*b*c
 + a*d)*x^13)/13 + (b*d^3*x^17)/17
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int (a+bx^4)(c+dx^4)^3\,dx &= \int \left(ac^3+c^2(bc+3ad)x^4+3cd(bc+ad)x^8+d^2(3bc+ad)x^{12}+bd^3x^{16}\right)dx\\ &= ac^3x+\frac{1}{5}c^2(bc+3ad)x^5+\frac{1}{3}cd(bc+ad)x^9+\frac{1}{13}d^2(3bc+ad)x^{13}+\frac{1}{17}bd^3x^{17}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 70, normalized size = 1.00

$$ac^3x+\frac{1}{5}c^2(bc+3ad)x^5+\frac{1}{3}cd(bc+ad)x^9+\frac{1}{13}d^2(3bc+ad)x^{13}+\frac{1}{17}bd^3x^{17}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^4)*(c + d*x^4)^3,x]
```

```
[Out] a*c^3*x + (c^2*(b*c + 3*a*d)*x^5)/5 + (c*d*(b*c + a*d)*x^9)/3 + (d^2*(3*b*c
 + a*d)*x^13)/13 + (b*d^3*x^17)/17
```

Maple [A]
time = 0.27, size = 73, normalized size = 1.04

method	result	size
norman	$a\,c^{3} x+\left(\frac{3}{5} a\,c^{2} d+\frac{1}{5} b\,c^{3}\right) x^{5}+\left(\frac{1}{3} a c\,d^{2}+\frac{1}{3} b\,c^{2} d\right) x^{9}+\left(\frac{1}{13} a\,d^{3}+\frac{3}{13} b c\,d^{2}\right) x^{13}+\frac{b\,d^{3} x^{17}}{17}$	72
default	$\frac{b\,d^{3} x^{17}}{17}+\frac{\left(a\,d^{3}+3 b c\,d^{2}\right) x^{13}}{13}+\frac{\left(3 a c\,d^{2}+3 b\,c^{2} d\right) x^{9}}{9}+\frac{\left(3 a\,c^{2} d+b\,c^{3}\right) x^{5}}{5}+a\,c^{3} x$	73
gosper	$a\,c^{3} x+\frac{3}{5} x^{5} a\,c^{2} d+\frac{1}{5} x^{5} b\,c^{3}+\frac{1}{3} x^{9} a c\,d^{2}+\frac{1}{3} x^{9} b\,c^{2} d+\frac{1}{13} x^{13} a\,d^{3}+\frac{3}{13} x^{13} b c\,d^{2}+\frac{1}{17} b\,d^{3} x^{17}$	75
risch	$a\,c^{3} x+\frac{3}{5} x^{5} a\,c^{2} d+\frac{1}{5} x^{5} b\,c^{3}+\frac{1}{3} x^{9} a c\,d^{2}+\frac{1}{3} x^{9} b\,c^{2} d+\frac{1}{13} x^{13} a\,d^{3}+\frac{3}{13} x^{13} b c\,d^{2}+\frac{1}{17} b\,d^{3} x^{17}$	75

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)*(d*x^4+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/17*b*d^3*x^17+1/13*(a*d^3+3*b*c*d^2)*x^13+1/9*(3*a*c*d^2+3*b*c^2*d)*x^9+1
/5*(3*a*c^2*d+b*c^3)*x^5+a*c^3*x
```

Maxima [A]
time = 0.29, size = 70, normalized size = 1.00

$$\frac{1}{17}\,bd^3x^{17}+\frac{1}{13}\,\left(3\,bcd^2+ad^3\right)x^{13}+\frac{1}{3}\,\left(bc^2d+acd^2\right)x^9+\frac{1}{5}\,\left(bc^3+3\,ac^2d\right)x^5+ac^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)*(d*x^4+c)^3,x, algorithm="maxima")
```

```
[Out] 1/17*b*d^3*x^17 + 1/13*(3*b*c*d^2 + a*d^3)*x^13 + 1/3*(b*c^2*d + a*c*d^2)*x
^9 + 1/5*(b*c^3 + 3*a*c^2*d)*x^5 + a*c^3*x
```

Fricas [A]
time = 3.61, size = 70, normalized size = 1.00

$$\frac{1}{17}\,bd^3x^{17}+\frac{1}{13}\,\left(3\,bcd^2+ad^3\right)x^{13}+\frac{1}{3}\,\left(bc^2d+acd^2\right)x^9+\frac{1}{5}\,\left(bc^3+3\,ac^2d\right)x^5+ac^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)*(d*x^4+c)^3,x, algorithm="fricas")
```

```
[Out] 1/17*b*d^3*x^17 + 1/13*(3*b*c*d^2 + a*d^3)*x^13 + 1/3*(b*c^2*d + a*c*d^2)*x
^9 + 1/5*(b*c^3 + 3*a*c^2*d)*x^5 + a*c^3*x
```

Sympy [A]
time = 0.02, size = 76, normalized size = 1.09

$$ac^3x+\frac{bd^3x^{17}}{17}+x^{13}\left(\frac{ad^3}{13}+\frac{3bcd^2}{13}\right)+x^9\left(\frac{acd^2}{3}+\frac{bc^2d}{3}\right)+x^5\cdot\left(\frac{3ac^2d}{5}+\frac{bc^3}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)*(d*x**4+c)**3,x)
```

```
[Out] a*c**3*x + b*d**3*x**17/17 + x**13*(a*d**3/13 + 3*b*c*d**2/13) + x**9*(a*c*
d**2/3 + b*c**2*d/3) + x**5*(3*a*c**2*d/5 + b*c**3/5)
```

Giac [A]
time = 0.54, size = 74, normalized size = 1.06

$$\frac{1}{17}\,bd^3x^{17}+\frac{3}{13}\,bcd^2x^{13}+\frac{1}{13}\,ad^3x^{13}+\frac{1}{3}\,bc^2dx^9+\frac{1}{3}\,acd^2x^9+\frac{1}{5}\,bc^3x^5+\frac{3}{5}\,ac^2dx^5+ac^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)*(d*x^4+c)^3,x, algorithm="giac")
```

```
[Out] 1/17*b*d^3*x^17 + 3/13*b*c*d^2*x^13 + 1/13*a*d^3*x^13 + 1/3*b*c^2*d*x^9 + 1
/3*a*c*d^2*x^9 + 1/5*b*c^3*x^5 + 3/5*a*c^2*d*x^5 + a*c^3*x
```

Mupad [B]
time = 1.24, size = 66, normalized size = 0.94

$$x^5\left(\frac{b\,c^3}{5}+\frac{3\,a\,d\,c^2}{5}\right)+x^{13}\left(\frac{a\,d^3}{13}+\frac{3\,b\,c\,d^2}{13}\right)+\frac{b\,d^3\,x^{17}}{17}+a\,c^3\,x+\frac{c\,d\,x^9\,\left(a\,d+b\,c\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)*(c + d*x^4)^3,x)
```

```
[Out] x^5*((b*c^3)/5 + (3*a*c^2*d)/5) + x^13*((a*d^3)/13 + (3*b*c*d^2)/13) + (b*d
^3*x^17)/17 + a*c^3*x + (c*d*x^9*(a*d + b*c))/3
```

3.148 $\int (a+bx^4)(c+dx^4)^2\,dx$

Optimal. Leaf size=50

$$ac^2x+\frac{1}{5}c(bc+2ad)x^5+\frac{1}{9}d(2bc+ad)x^9+\frac{1}{13}bd^2x^{13}$$

```
[Out] a*c^2*x+1/5*c*(2*a*d+b*c)*x^5+1/9*d*(a*d+2*b*c)*x^9+1/13*b*d^2*x^13
```

Rubi [A]
time = 0.02, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {380}

$$\frac{1}{9}dx^9(ad+2bc)+\frac{1}{5}cx^5(2ad+bc)+ac^2x+\frac{1}{13}bd^2x^{13}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)*(c + d*x^4)^2,x]
```

```
[Out] a*c^2*x + (c*(b*c + 2*a*d)*x^5)/5 + (d*(2*b*c + a*d)*x^9)/9 + (b*d^2*x^13)/
13
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int (a+bx^4)(c+dx^4)^2\,dx &= \int \left(ac^2+c(bc+2ad)x^4+d(2bc+ad)x^8+bd^2x^{12}\right)dx\\ &= ac^2x+\frac{1}{5}c(bc+2ad)x^5+\frac{1}{9}d(2bc+ad)x^9+\frac{1}{13}bd^2x^{13}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 50, normalized size = 1.00

$$ac^2x+\frac{1}{5}c(bc+2ad)x^5+\frac{1}{9}d(2bc+ad)x^9+\frac{1}{13}bd^2x^{13}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^4)*(c + d*x^4)^2,x]
```

```
[Out] a*c^2*x + (c*(b*c + 2*a*d)*x^5)/5 + (d*(2*b*c + a*d)*x^9)/9 + (b*d^2*x^13)/
13
```

Maple [A]
time = 0.27, size = 49, normalized size = 0.98

method	result	size
default	$\frac{b\,d^2x^{13}}{13}+\frac{(a\,d^2+2bcd)x^9}{9}+\frac{(2acd+b\,c^2)x^5}{5}+a\,c^2x$	49
norman	$\frac{b\,d^2x^{13}}{13}+\left(\frac{1}{9}a\,d^2+\frac{2}{9}bcd\right)x^9+\left(\frac{2}{5}acd+\frac{1}{5}b\,c^2\right)x^5+a\,c^2x$	49
gosper	$\frac{1}{13}b\,d^2x^{13}+\frac{1}{9}x^9a\,d^2+\frac{2}{9}x^9bcd+\frac{2}{5}x^5acd+\frac{1}{5}x^5b\,c^2+a\,c^2x$	51
risch	$\frac{1}{13}b\,d^2x^{13}+\frac{1}{9}x^9a\,d^2+\frac{2}{9}x^9bcd+\frac{2}{5}x^5acd+\frac{1}{5}x^5b\,c^2+a\,c^2x$	51

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)*(d*x^4+c)^2,x,method=_RETURNVERBOSE)
[Out] 1/13*b*d^2*x^13+1/9*(a*d^2+2*b*c*d)*x^9+1/5*(2*a*c*d+b*c^2)*x^5+a*c^2*x
```

Maxima [A]
time = 0.27, size = 48, normalized size = 0.96

$$\frac{1}{13}\,bd^2x^{13}+\frac{1}{9}\,\left(2\,bcd+ad^2\right)x^9+\frac{1}{5}\,\left(bc^2+2\,acd\right)x^5+ac^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)*(d*x^4+c)^2,x, algorithm="maxima")
[Out] 1/13*b*d^2*x^13 + 1/9*(2*b*c*d + a*d^2)*x^9 + 1/5*(b*c^2 + 2*a*c*d)*x^5 + a
*c^2*x
```

Fricas [A]
time = 3.43, size = 48, normalized size = 0.96

$$\frac{1}{13}\,bd^2x^{13}+\frac{1}{9}\,\left(2\,bcd+ad^2\right)x^9+\frac{1}{5}\,\left(bc^2+2\,acd\right)x^5+ac^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)*(d*x^4+c)^2,x, algorithm="fricas")
[Out] 1/13*b*d^2*x^13 + 1/9*(2*b*c*d + a*d^2)*x^9 + 1/5*(b*c^2 + 2*a*c*d)*x^5 + a
*c^2*x
```

Sympy [A]
time = 0.01, size = 53, normalized size = 1.06

$$ac^2x+\frac{bd^2x^{13}}{13}+x^9\left(\frac{ad^2}{9}+\frac{2bcd}{9}\right)+x^5\cdot\left(\frac{2acd}{5}+\frac{bc^2}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)*(d*x**4+c)**2,x)
```

```
[Out] a*c**2*x + b*d**2*x**13/13 + x**9*(a*d**2/9 + 2*b*c*d/9) + x**5*(2*a*c*d/5
+ b*c**2/5)
```

Giac [A]
time = 0.50, size = 50, normalized size = 1.00

$$\frac{1}{13}\,bd^2x^{13} + \frac{2}{9}\,bcdx^9 + \frac{1}{9}\,ad^2x^9 + \frac{1}{5}\,bc^2x^5 + \frac{2}{5}\,acdx^5 + ac^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)*(d*x^4+c)^2,x, algorithm="giac")
```

```
[Out] 1/13*b*d^2*x^13 + 2/9*b*c*d*x^9 + 1/9*a*d^2*x^9 + 1/5*b*c^2*x^5 + 2/5*a*c*d
*x^5 + a*c^2*x
```

Mupad [B]
time = 0.05, size = 48, normalized size = 0.96

$$x^5\left(\frac{b\,c^2}{5}+\frac{2\,a\,d\,c}{5}\right)+x^9\left(\frac{a\,d^2}{9}+\frac{2\,b\,c\,d}{9}\right)+\frac{b\,d^2\,x^{13}}{13}+a\,c^2\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)*(c + d*x^4)^2,x)
```

```
[Out] x^5*((b*c^2)/5 + (2*a*c*d)/5) + x^9*((a*d^2)/9 + (2*b*c*d)/9) + (b*d^2*x^13
)/13 + a*c^2*x
```

3.149 $\int (a+bx^4)(c+dx^4)\,dx$

Optimal. Leaf size=28

$$acx+\frac{1}{5}(bc+ad)x^5+\frac{1}{9}bdx^9$$

```
[Out] a*c*x+1/5*(a*d+b*c)*x^5+1/9*b*d*x^9
```

Rubi [A]
time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {380}

$$\frac{1}{5}x^5(ad+bc)+acx+\frac{1}{9}bdx^9$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)*(c + d*x^4),x]
```

```
[Out] a*c*x + ((b*c + a*d)*x^5)/5 + (b*d*x^9)/9
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int (a+bx^4)(c+dx^4)\,dx &= \int (ac+(bc+ad)x^4+bdx^8)\,dx\\ &= acx+\frac{1}{5}(bc+ad)x^5+\frac{1}{9}bdx^9\end{aligned}$$

Mathematica [A]
time = 0.00, size = 28, normalized size = 1.00

$$acx+\frac{1}{5}(bc+ad)x^5+\frac{1}{9}bdx^9$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^4)*(c + d*x^4),x]
```

```
[Out] a*c*x + ((b*c + a*d)*x^5)/5 + (b*d*x^9)/9
```

Maple [A]
time = 0.10, size = 25, normalized size = 0.89

method	result	size
default	$acx+\frac{(ad+bc)x^5}{5}+\frac{bd\,x^9}{9}$	25
norman	$\frac{bd\,x^9}{9}+\left(\frac{ad}{5}+\frac{bc}{5}\right)x^5+acx$	26
gosper	$\frac{1}{9}bd\,x^9+\frac{1}{5}x^5ad+\frac{1}{5}x^5bc+acx$	27
risch	$\frac{1}{9}bd\,x^9+\frac{1}{5}x^5ad+\frac{1}{5}x^5bc+acx$	27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)*(d*x^4+c),x,method=_RETURNVERBOSE)
[Out] a*c*x+1/5*(a*d+b*c)*x^5+1/9*b*d*x^9
```

Maxima [A]
time = 0.27, size = 24, normalized size = 0.86

$$\frac{1}{9}\,bdx^9+\frac{1}{5}\,(bc+ad)x^5+acx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)*(d*x^4+c),x, algorithm="maxima")
[Out] 1/9*b*d*x^9 + 1/5*(b*c + a*d)*x^5 + a*c*x
```

Fricas [A]
time = 3.55, size = 24, normalized size = 0.86

$$\frac{1}{9}\,bdx^9+\frac{1}{5}\,(bc+ad)x^5+acx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)*(d*x^4+c),x, algorithm="fricas")
[Out] 1/9*b*d*x^9 + 1/5*(b*c + a*d)*x^5 + a*c*x
```

Sympy [A]
time = 0.01, size = 26, normalized size = 0.93

$$acx+\frac{bdx^9}{9}+x^5\left(\frac{ad}{5}+\frac{bc}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)*(d*x**4+c),x)
```

```
[Out] a*c*x + b*d*x**9/9 + x**5*(a*d/5 + b*c/5)
```

Giac [A]
time = 1.13, size = 26, normalized size = 0.93

$$\frac{1}{9}\,bdx^9+\frac{1}{5}\,bcx^5+\frac{1}{5}\,adx^5+acx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)*(d*x^4+c),x, algorithm="giac")
```

```
[Out] 1/9*b*d*x^9 + 1/5*b*c*x^5 + 1/5*a*d*x^5 + a*c*x
```

Mupad [B]
time = 0.04, size = 25, normalized size = 0.89

$$\frac{b\,d\,x^9}{9}+\left(\frac{a\,d}{5}+\frac{b\,c}{5}\right)\,x^5+a\,c\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)*(c + d*x^4),x)
```

```
[Out] x^5*((a*d)/5 + (b*c)/5) + a*c*x + (b*d*x^9)/9
```

3.150 $\int \frac{a+bx^4}{c+dx^4} \, dx$

Optimal. Leaf size=223

$$\frac{bx}{d}+\frac{(bc-ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right)}{2\sqrt{2}\,c^{3/4}d^{5/4}}-\frac{(bc-ad)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right)}{2\sqrt{2}\,c^{3/4}d^{5/4}}+\frac{(bc-ad)\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{\ }\right.}{4\sqrt{2}\,c^{3/4}d^{5/4}}$$

```
[Out] b*x/d-1/4*(-a*d+b*c)*arctan(-1+d^(1/4)*x*2^(1/2)/c^(1/4))/c^(3/4)/d^(5/4)*2
^(1/2)-1/4*(-a*d+b*c)*arctan(1+d^(1/4)*x*2^(1/2)/c^(1/4))/c^(3/4)/d^(5/4)*2
^(1/2)+1/8*(-a*d+b*c)*ln(-c^(1/4)*d^(1/4)*x*2^(1/2)+c^(1/2)+x^2*d^(1/2))/c^
(3/4)/d^(5/4)*2^(1/2)-1/8*(-a*d+b*c)*ln(c^(1/4)*d^(1/4)*x*2^(1/2)+c^(1/2)+x
^2*d^(1/2))/c^(3/4)/d^(5/4)*2^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {396, 217, 1179, 642, 1176, 631, 210}

$$\frac{(bc-ad)\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right)}{2\sqrt{2}\,c^{3/4}d^{5/4}}-\frac{(bc-ad)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{d}\,x}{\sqrt[4]{c}}+1\right)}{2\sqrt{2}\,c^{3/4}d^{5/4}}+\frac{(bc-ad)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\,x+\sqrt{c}+\sqrt{d}\,x^2\right)}{4\sqrt{2}\,c^{3/4}d^{5/4}}-\frac{(bc-ad)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\,x+\sqrt{c}+\sqrt{d}\,x^2\right)}{4\sqrt{2}\,c^{3/4}d^{5/4}}+\frac{bx}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)/(c + d*x^4),x]
```

```
[Out] (b*x)/d + ((b*c - a*d)*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/(2*Sqrt[2]*
c^(3/4)*d^(5/4)) - ((b*c - a*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/(2
*Sqrt[2]*c^(3/4)*d^(5/4)) + ((b*c - a*d)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1
/4)*x + Sqrt[d]*x^2])/(4*Sqrt[2]*c^(3/4)*d^(5/4)) - ((b*c - a*d)*Log[Sqrt[c
] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(4*Sqrt[2]*c^(3/4)*d^(5/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{a+bx^4}{c+dx^4}\,dx = \frac{bx}{d} - \frac{(bc-ad)\int \frac{1}{c+dx^4}\,dx}{d}$$

$$= \frac{bx}{d} - \frac{(bc-ad)\int \frac{\sqrt{c}-\sqrt{d}\,x^2}{c+dx^4}\,dx}{2\sqrt{c}\,d} - \frac{(bc-ad)\int \frac{\sqrt{c}+\sqrt{d}\,x^2}{c+dx^4}\,dx}{2\sqrt{c}\,d}$$

$$= \frac{bx}{d} - \frac{(bc-ad)\int \frac{1}{\frac{\sqrt{c}}{\sqrt{d}}-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{d}}+x^2}\,dx}{4\sqrt{c}\,d^{3/2}} - \frac{(bc-ad)\int \frac{1}{\frac{\sqrt{c}}{\sqrt{d}}+\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{d}}+x^2}\,dx}{4\sqrt{c}\,d^{3/2}} + \frac{(bc-ad)\int}{}$$

$$= \frac{bx}{d} + \frac{(bc-ad)\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\,x+\sqrt{d}\,x^2\right)}{4\sqrt{2}\,c^{3/4}d^{5/4}} - \frac{(bc-ad)\log\left(\sqrt{c}+\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\,x\right.}{4\sqrt{2}\,c^{3/4}d^{5/4}}$$

$$= \frac{bx}{d} + \frac{(bc-ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right)}{2\sqrt{2}\,c^{3/4}d^{5/4}} - \frac{(bc-ad)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right)}{2\sqrt{2}\,c^{3/4}d^{5/4}} + \frac{(bc-ad)\log}{}$$

Mathematica [A]
time = 0.09, size = 196, normalized size = 0.88

$$\frac{8bc^{3/4}\sqrt[4]{d}\,x+2\sqrt{2}\,(bc-ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right)-2\sqrt{2}\,(bc-ad)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right)+\sqrt{2}\,(bc-ad)\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\,x+\sqrt{d}\,x^2\right)-\sqrt{2}\,(bc-ad)\log\left(\sqrt{c}+\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\,x+\sqrt{d}\,x^2\right)}{8c^{3/4}d^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^4)/(c + d*x^4),x]
```

```
[Out] (8*b*c^(3/4)*d^(1/4)*x + 2*Sqrt[2]*(b*c - a*d)*ArcTan[1 - (Sqrt[2]*d^(1/4)*
x)/c^(1/4)] - 2*Sqrt[2]*(b*c - a*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)]
 + Sqrt[2]*(b*c - a*d)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^
2] - Sqrt[2]*(b*c - a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*
x^2])/(8*c^(3/4)*d^(5/4))
```

Maple [A]
time = 0.25, size = 120, normalized size = 0.54

method	result	size
risch	$\frac{bx}{d}+\frac{\sum_{\_R=\mathrm{RootOf}\left(d\_Z^4+c\right)}\frac{(ad-bc)\ln\left(x-\_R\right)}{\_R^3}}{4d^2}$	42

default	$\frac{bx}{d}+\frac{(ad-bc)\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\left(\ln\left(\frac{x^{2}+\left(\frac{c}{d}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{c}{d}}}{x^{2}-\left(\frac{c}{d}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{c}{d}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)\right)}{8dc}$	120

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)/(d*x^4+c),x,method=_RETURNVERBOSE)
```

```
[Out] b*x/d+1/8*(a*d-b*c)/d*(c/d)^(1/4)/c*2^(1/2)*(ln((x^2+(c/d)^(1/4)*x*2^(1/2)+
(c/d)^(1/2))/(x^2-(c/d)^(1/4)*x*2^(1/2)+(c/d)^(1/2)))+2*arctan(2^(1/2)/(c/d
)^(1/4)*x+1)+2*arctan(2^(1/2)/(c/d)^(1/4)*x-1))
```

Maxima [A]
time = 0.50, size = 212, normalized size = 0.95

$$\frac{bx}{d}-\frac{\frac{2\sqrt{2}\,(bc-ad)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}\,x+\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\,\sqrt{d}}}\right)}{\sqrt{c}\,\sqrt{\sqrt{c}\,\sqrt{d}}}+\frac{2\sqrt{2}\,(bc-ad)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}\,x-\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\,\sqrt{d}}}\right)}{\sqrt{c}\,\sqrt{\sqrt{c}\,\sqrt{d}}}+\frac{\sqrt{2}\,(bc-ad)\log\left(\sqrt{d}\,x^{2}+\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}x+\sqrt{c}\right)}{c^{\frac{3}{4}}d^{\frac{1}{4}}}-\frac{\sqrt{2}\,(bc-ad)\log\left(\sqrt{d}\,x^{2}-\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}x+\sqrt{c}\right)}{c^{\frac{3}{4}}d^{\frac{1}{4}}}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)/(d*x^4+c),x, algorithm="maxima")
```

```
[Out] b*x/d - 1/8*(2*sqrt(2)*(b*c - a*d)*arctan(1/2*sqrt(2)*(2*sqrt(d)*x + sqrt(2
)*c^(1/4)*d^(1/4))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) +
 2*sqrt(2)*(b*c - a*d)*arctan(1/2*sqrt(2)*(2*sqrt(d)*x - sqrt(2)*c^(1/4)*d^
(1/4))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + sqrt(2)*(b*
c - a*d)*log(sqrt(d)*x^2 + sqrt(2)*c^(1/4)*d^(1/4)*x + sqrt(c))/(c^(3/4)*d^
(1/4)) - sqrt(2)*(b*c - a*d)*log(sqrt(d)*x^2 - sqrt(2)*c^(1/4)*d^(1/4)*x +
sqrt(c))/(c^(3/4)*d^(1/4)))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 639 vs. 2(158) = 316.
time = 3.36, size = 639, normalized size = 2.87

$$\frac{4\,d\left(-\frac{b^{4}c^{4}-4\,ab^{3}c^{3}d+6\,a^{2}b^{2}c^{2}d^{2}-4\,a^{3}bcd^{3}+a^{4}d^{4}}{c^{3}d^{5}}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{c^{2}d^{2}\sqrt{-\frac{b^{4}c^{4}-4\,ab^{3}c^{3}d+6\,a^{2}b^{2}c^{2}d^{2}-4\,a^{3}bcd^{3}+a^{4}d^{4}}{c^{3}d^{5}}}+(b^{2}c^{2}-2\,abcd+a^{2}d^{2})x^{2}}\;c^{2}d^{4}\left(-\frac{b^{4}c^{4}-4\,ab^{3}c^{3}d+6\,a^{2}b^{2}c^{2}d^{2}-4\,a^{3}bcd^{3}+a^{4}d^{4}}{c^{3}d^{5}}\right)^{\frac{3}{4}}+(bc^{3}d^{4}-ac^{2}d^{5})x\left(-\frac{b^{4}c^{4}-4\,ab^{3}c^{3}d+6\,a^{2}b^{2}c^{2}d^{2}-4\,a^{3}bcd^{3}+a^{4}d^{4}}{c^{3}d^{5}}\right)^{\frac{3}{4}}}{b^{4}c^{4}-4\,ab^{3}c^{3}d+6\,a^{2}b^{2}c^{2}d^{2}-4\,a^{3}bcd^{3}+a^{4}d^{4}}\right)+d\left(-\frac{b^{4}c^{4}-4\,ab^{3}c^{3}d+6\,a^{2}b^{2}c^{2}d^{2}-4\,a^{3}bcd^{3}+a^{4}d^{4}}{c^{3}d^{5}}\right)^{\frac{1}{4}}\log\left(cd\left(-\frac{b^{4}c^{4}-4\,ab^{3}c^{3}d+6\,a^{2}b^{2}c^{2}d^{2}-4\,a^{3}bcd^{3}+a^{4}d^{4}}{c^{3}d^{5}}\right)^{\frac{1}{4}}-(bc-ad)x\right)-d\left(-\frac{b^{4}c^{4}-4\,ab^{3}c^{3}d+6\,a^{2}b^{2}c^{2}d^{2}-4\,a^{3}bcd^{3}+a^{4}d^{4}}{c^{3}d^{5}}\right)^{\frac{1}{4}}\log\left(-cd\left(-\frac{b^{4}c^{4}-4\,ab^{3}c^{3}d+6\,a^{2}b^{2}c^{2}d^{2}-4\,a^{3}bcd^{3}+a^{4}d^{4}}{c^{3}d^{5}}\right)^{\frac{1}{4}}-(bc-ad)x\right)+4\,bx}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)/(d*x^4+c),x, algorithm="fricas")
```

```
[Out] 1/4*(4*d*(-(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a
^4*d^4)/(c^3*d^5))^(1/4)*arctan((sqrt(c^2*d^2*sqrt(-(b^4*c^4 - 4*a*b^3*c^3*
d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)/(c^3*d^5)) + (b^2*c^2 - 2*
a*b*c*d + a^2*d^2)*x^2)*c^2*d^4*(-(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*
d^2 - 4*a^3*b*c*d^3 + a^4*d^4)/(c^3*d^5))^(3/4) + (b*c^3*d^4 - a*c^2*d^5)*x
*(-(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)/
```

```
(c^3*d^5))^(3/4))/(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*
d^3 + a^4*d^4)) + d*(-(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*
b*c*d^3 + a^4*d^4)/(c^3*d^5))^(1/4)*log(c*d*(-(b^4*c^4 - 4*a*b^3*c^3*d + 6*
a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)/(c^3*d^5))^(1/4) - (b*c - a*d)*x
) - d*(-(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*
d^4)/(c^3*d^5))^(1/4)*log(-c*d*(-(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d
^2 - 4*a^3*b*c*d^3 + a^4*d^4)/(c^3*d^5))^(1/4) - (b*c - a*d)*x) + 4*b*x)/d
```

Sympy [A]
time = 0.29, size = 87, normalized size = 0.39

$$\frac{bx}{d} + \operatorname{RootSum}\left(256t^4c^3d^5 + a^4d^4 - 4a^3bcd^3 + 6a^2b^2c^2d^2 - 4ab^3c^3d + b^4c^4, \left(t \mapsto t\log\left(\frac{4tcd}{ad - bc} + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)/(d*x**4+c),x)
```

```
[Out] b*x/d + RootSum(256*_t**4*c**3*d**5 + a**4*d**4 - 4*a**3*b*c*d**3 + 6*a**2*
b**2*c**2*d**2 - 4*a*b**3*c**3*d + b**4*c**4, Lambda(_t, _t*log(4*_t*c*d/(a
*d - b*c) + x)))
```

Giac [A]
time = 1.54, size = 245, normalized size = 1.10

$$\frac{bx}{d} - \frac{\sqrt{2}\left(\left(cd^3\right)^{\frac{1}{4}}bc - \left(cd^3\right)^{\frac{1}{4}}ad\right)\arctan\left(\frac{\sqrt{2}\left(2x + \sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{4cd^2} - \frac{\sqrt{2}\left(\left(cd^3\right)^{\frac{1}{4}}bc - \left(cd^3\right)^{\frac{1}{4}}ad\right)\arctan\left(\frac{\sqrt{2}\left(2x - \sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{4cd^2} - \frac{\sqrt{2}\left(\left(cd^3\right)^{\frac{1}{4}}bc - \left(cd^3\right)^{\frac{1}{4}}ad\right)\log\left(x^2 + \sqrt{2}x\left(\frac{c}{d}\right)^{\frac{1}{4}} + \sqrt{\frac{c}{d}}\right)}{8cd^2} + \frac{\sqrt{2}\left(\left(cd^3\right)^{\frac{1}{4}}bc - \left(cd^3\right)^{\frac{1}{4}}ad\right)\log\left(x^2 - \sqrt{2}x\left(\frac{c}{d}\right)^{\frac{1}{4}} + \sqrt{\frac{c}{d}}\right)}{8cd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)/(d*x^4+c),x, algorithm="giac")
```

```
[Out] b*x/d - 1/4*sqrt(2)*((c*d^3)^(1/4)*b*c - (c*d^3)^(1/4)*a*d)*arctan(1/2*sqrt
(2)*(2*x + sqrt(2)*(c/d)^(1/4))/(c/d)^(1/4))/(c*d^2) - 1/4*sqrt(2)*((c*d^3)
^(1/4)*b*c - (c*d^3)^(1/4)*a*d)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(c/d)^(1/
4))/(c/d)^(1/4))/(c*d^2) - 1/8*sqrt(2)*((c*d^3)^(1/4)*b*c - (c*d^3)^(1/4)*a
*d)*log(x^2 + sqrt(2)*x*(c/d)^(1/4) + sqrt(c/d))/(c*d^2) + 1/8*sqrt(2)*((c*
d^3)^(1/4)*b*c - (c*d^3)^(1/4)*a*d)*log(x^2 - sqrt(2)*x*(c/d)^(1/4) + sqrt(
c/d))/(c*d^2)
```

Mupad [B]
time = 1.48, size = 720, normalized size = 3.23

$$\frac{bx}{d} - \frac{\operatorname{atan}\left(\frac{\frac{\left(x\left(4a^2d^3 - 8abcd^2 + 4b^2c^2d\right) - \frac{\left(16bc^2d^2 - 16acd^3\right)(ad - bc)}{4(-c)^{3/4}d^{5/4}}\right)(ad - bc)\,1i}{4(-c)^{3/4}d^{5/4}} + \frac{\left(x\left(4a^2d^3 - 8abcd^2 + 4b^2c^2d\right) + \frac{\left(16bc^2d^2 - 16acd^3\right)(ad - bc)}{4(-c)^{3/4}d^{5/4}}\right)(ad - bc)\,1i}{4(-c)^{3/4}d^{5/4}}}{\frac{\left(x\left(4a^2d^3 - 8abcd^2 + 4b^2c^2d\right) - \frac{\left(16bc^2d^2 - 16acd^3\right)(ad - bc)}{4(-c)^{3/4}d^{5/4}}\right)(ad - bc)}{4(-c)^{3/4}d^{5/4}} - \frac{\left(x\left(4a^2d^3 - 8abcd^2 + 4b^2c^2d\right) + \frac{\left(16bc^2d^2 - 16acd^3\right)(ad - bc)}{4(-c)^{3/4}d^{5/4}}\right)(ad - bc)}{4(-c)^{3/4}d^{5/4}}}\right)(ad - bc)\,1i}{2(-c)^{3/4}d^{5/4}} - \frac{\operatorname{atan}\left(\frac{\frac{\left(x\left(4a^2d^3 - 8abcd^2 + 4b^2c^2d\right) - \frac{\left(16bc^2d^2 - 16acd^3\right)(ad - bc)\,1i}{4(-c)^{3/4}d^{5/4}}\right)(ad - bc)}{4(-c)^{3/4}d^{5/4}} + \frac{\left(x\left(4a^2d^3 - 8abcd^2 + 4b^2c^2d\right) + \frac{\left(16bc^2d^2 - 16acd^3\right)(ad - bc)\,1i}{4(-c)^{3/4}d^{5/4}}\right)(ad - bc)}{4(-c)^{3/4}d^{5/4}}}{\frac{\left(x\left(4a^2d^3 - 8abcd^2 + 4b^2c^2d\right) - \frac{\left(16bc^2d^2 - 16acd^3\right)(ad - bc)\,1i}{4(-c)^{3/4}d^{5/4}}\right)(ad - bc)\,1i}{4(-c)^{3/4}d^{5/4}} - \frac{\left(x\left(4a^2d^3 - 8abcd^2 + 4b^2c^2d\right) + \frac{\left(16bc^2d^2 - 16acd^3\right)(ad - bc)\,1i}{4(-c)^{3/4}d^{5/4}}\right)(ad - bc)\,1i}{4(-c)^{3/4}d^{5/4}}}\right)(ad - bc)}{2(-c)^{3/4}d^{5/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)/(c + d*x^4),x)
```

```
[Out] (b*x)/d - (atan((((x*(4*a^2*d^3 + 4*b^2*c^2*d - 8*a*b*c*d^2) - ((16*b*c^2*d
^2 - 16*a*c*d^3)*(a*d - b*c))/(4*(-c)^(3/4)*d^(5/4)))*(a*d - b*c)*1i)/(4*(-
c)^(3/4)*d^(5/4)) + ((x*(4*a^2*d^3 + 4*b^2*c^2*d - 8*a*b*c*d^2) + ((16*b*c^
2*d^2 - 16*a*c*d^3)*(a*d - b*c))/(4*(-c)^(3/4)*d^(5/4)))*(a*d - b*c)*1i)/(4
*(-c)^(3/4)*d^(5/4)))/(((x*(4*a^2*d^3 + 4*b^2*c^2*d - 8*a*b*c*d^2) - ((16*b
*c^2*d^2 - 16*a*c*d^3)*(a*d - b*c))/(4*(-c)^(3/4)*d^(5/4)))*(a*d - b*c))/(4
*(-c)^(3/4)*d^(5/4)) - ((x*(4*a^2*d^3 + 4*b^2*c^2*d - 8*a*b*c*d^2) + ((16*b
*c^2*d^2 - 16*a*c*d^3)*(a*d - b*c))/(4*(-c)^(3/4)*d^(5/4)))*(a*d - b*c))/(4
*(-c)^(3/4)*d^(5/4))))*(a*d - b*c)*1i)/(2*(-c)^(3/4)*d^(5/4)) - (atan((((x*
(4*a^2*d^3 + 4*b^2*c^2*d - 8*a*b*c*d^2) - ((16*b*c^2*d^2 - 16*a*c*d^3)*(a*d
 - b*c)*1i)/(4*(-c)^(3/4)*d^(5/4)))*(a*d - b*c))/(4*(-c)^(3/4)*d^(5/4)) + (
(x*(4*a^2*d^3 + 4*b^2*c^2*d - 8*a*b*c*d^2) + ((16*b*c^2*d^2 - 16*a*c*d^3)*(
a*d - b*c)*1i)/(4*(-c)^(3/4)*d^(5/4)))*(a*d - b*c))/(4*(-c)^(3/4)*d^(5/4)))
/(((x*(4*a^2*d^3 + 4*b^2*c^2*d - 8*a*b*c*d^2) - ((16*b*c^2*d^2 - 16*a*c*d^3
)*(a*d - b*c)*1i)/(4*(-c)^(3/4)*d^(5/4)))*(a*d - b*c)*1i)/(4*(-c)^(3/4)*d^(
5/4)) - ((x*(4*a^2*d^3 + 4*b^2*c^2*d - 8*a*b*c*d^2) + ((16*b*c^2*d^2 - 16*a
*c*d^3)*(a*d - b*c)*1i)/(4*(-c)^(3/4)*d^(5/4)))*(a*d - b*c)*1i)/(4*(-c)^(3/
4)*d^(5/4))))*(a*d - b*c))/(2*(-c)^(3/4)*d^(5/4))
```

3.151 $\int \frac{a+bx^4}{\left(c+dx^4\right)^2}\,dx$

Optimal. Leaf size=245

$$-\frac{(bc-ad)x}{4cd\,(c+dx^4)}-\frac{(bc+3ad)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right)}{8\sqrt{2}\,c^{7/4}d^{5/4}}+\frac{(bc+3ad)\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right)}{8\sqrt{2}\,c^{7/4}d^{5/4}}-\frac{(bc+3ad)\log\left(\sqrt{\right.}}{}$$

```
[Out] -1/4*(-a*d+b*c)*x/c/d/(d*x^4+c)+1/16*(3*a*d+b*c)*arctan(-1+d^(1/4)*x*2^(1/2
)/c^(1/4))/c^(7/4)/d^(5/4)*2^(1/2)+1/16*(3*a*d+b*c)*arctan(1+d^(1/4)*x*2^(1
/2)/c^(1/4))/c^(7/4)/d^(5/4)*2^(1/2)-1/32*(3*a*d+b*c)*ln(-c^(1/4)*d^(1/4)*x
*2^(1/2)+c^(1/2)+x^2*d^(1/2))/c^(7/4)/d^(5/4)*2^(1/2)+1/32*(3*a*d+b*c)*ln(c
^(1/4)*d^(1/4)*x*2^(1/2)+c^(1/2)+x^2*d^(1/2))/c^(7/4)/d^(5/4)*2^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 245, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {393, 217, 1179, 642, 1176, 631, 210}

$$-\frac{(3ad+bc)\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right)}{8\sqrt{2}\,c^{7/4}d^{5/4}}+\frac{(3ad+bc)\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{d}\,x}{\sqrt[4]{c}}+1\right)}{8\sqrt{2}\,c^{7/4}d^{5/4}}-\frac{(3ad+bc)\log\left(-\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,x+\sqrt{c}+\sqrt{d}\,x^2\right)}{16\sqrt{2}\,c^{7/4}d^{5/4}}+\frac{(3ad+bc)\log\left(\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,x+\sqrt{c}+\sqrt{d}\,x^2\right)}{16\sqrt{2}\,c^{7/4}d^{5/4}}-\frac{x(bc-ad)}{4cd\,(c+dx^4)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)/(c + d*x^4)^2,x]
```

```
[Out] -1/4*((b*c - a*d)*x)/(c*d*(c + d*x^4)) - ((b*c + 3*a*d)*ArcTan[1 - (Sqrt[2]
*d^(1/4)*x)/c^(1/4)])/(8*Sqrt[2]*c^(7/4)*d^(5/4)) + ((b*c + 3*a*d)*ArcTan[1
 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/(8*Sqrt[2]*c^(7/4)*d^(5/4)) - ((b*c + 3*a*
d)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(16*Sqrt[2]*c^(7
/4)*d^(5/4)) + ((b*c + 3*a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqr
t[d]*x^2])/(16*Sqrt[2]*c^(7/4)*d^(5/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{a+bx^4}{(c+dx^4)^2}\,dx = -\frac{(bc-ad)x}{4cd\,(c+dx^4)} + \frac{(bc+3ad)\int \frac{1}{c+dx^4}\,dx}{4cd}$$

$$= -\frac{(bc-ad)x}{4cd\,(c+dx^4)} + \frac{(bc+3ad)\int \frac{\sqrt{c}-\sqrt{d}\,x^2}{c+dx^4}\,dx}{8c^{3/2}d} + \frac{(bc+3ad)\int \frac{\sqrt{c}+\sqrt{d}\,x^2}{c+dx^4}\,dx}{8c^{3/2}d}$$

$$= -\frac{(bc-ad)x}{4cd\,(c+dx^4)} + \frac{(bc+3ad)\int \frac{1}{\frac{\sqrt{c}}{\sqrt{d}}-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{d}}+x^2}\,dx}{16c^{3/2}d^{3/2}} + \frac{(bc+3ad)\int \frac{1}{\frac{\sqrt{c}}{\sqrt{d}}+\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{d}}+x^2}}{16c^{3/2}d^{3/2}}$$

$$= -\frac{(bc-ad)x}{4cd\,(c+dx^4)} - \frac{(bc+3ad)\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\,x+\sqrt{d}\,x^2\right)}{16\sqrt{2}\,c^{7/4}d^{5/4}} + \frac{(bc+3ad)\log\left(\sqrt{c}\right.}{16}$$

$$= -\frac{(bc-ad)x}{4cd\,(c+dx^4)} - \frac{(bc+3ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right)}{8\sqrt{2}\,c^{7/4}d^{5/4}} + \frac{(bc+3ad)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right.}{8\sqrt{2}\,c^{7/4}d^{5/4}}$$

Mathematica [A]
time = 0.12, size = 212, normalized size = 0.87

$$\frac{-\frac{8c^{3/4}\sqrt[4]{d}(bc-ad)x}{c+dx^4}-2\sqrt{2}(bc+3ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right)+2\sqrt{2}(bc+3ad)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right)-\sqrt{2}(bc+3ad)\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\,x+\sqrt{d}\,x^2\right)+\sqrt{2}(bc+3ad)\log\left(\sqrt{c}+\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\,x+\sqrt{d}\,x^2\right)}{32c^{7/4}d^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^4)/(c + d*x^4)^2,x]
```

```
[Out] ((-8*c^(3/4)*d^(1/4)*(b*c - a*d)*x)/(c + d*x^4) - 2*Sqrt[2]*(b*c + 3*a*d)*A
rcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)] + 2*Sqrt[2]*(b*c + 3*a*d)*ArcTan[1 +
 (Sqrt[2]*d^(1/4)*x)/c^(1/4)] - Sqrt[2]*(b*c + 3*a*d)*Log[Sqrt[c] - Sqrt[2]
*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2] + Sqrt[2]*(b*c + 3*a*d)*Log[Sqrt[c] + Sqr
t[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(32*c^(7/4)*d^(5/4))
```

Maple [A]
time = 0.25, size = 140, normalized size = 0.57

method	result	size
risch	$\frac{(ad-bc)x}{4cd(d\,x^4+c)}+\frac{\sum_{\_R=\mathrm{RootOf}\left(d\_Z^4+c\right)}\frac{(3ad+bc)\ln\left(x-\_R\right)}{\_R^3}}{16c\,d^2}$	65

default	$\frac{(ad-bc)x}{4cd(d\,x^4+c)}+\frac{(3ad+bc)\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{c}{d}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{c}{d}}}{x^2-\left(\frac{c}{d}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{c}{d}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)\right)}{32c^2d}$	140

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)/(d*x^4+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(a*d-b*c)/c/d*x/(d*x^4+c)+1/32*(3*a*d+b*c)/c^2/d*(c/d)^(1/4)*2^(1/2)*(l
n((x^2+(c/d)^(1/4)*x*2^(1/2)+(c/d)^(1/2))/(x^2-(c/d)^(1/4)*x*2^(1/2)+(c/d)^
(1/2)))+2*arctan(2^(1/2)/(c/d)^(1/4)*x+1)+2*arctan(2^(1/2)/(c/d)^(1/4)*x-1)
)
```

Maxima [A]
time = 0.49, size = 236, normalized size = 0.96

$$-\frac{(bc-ad)x}{4\,(cd^2x^4+c^2d)}+\frac{\frac{2\sqrt{2}\,(bc+3\,ad)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}\,x+\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\,\sqrt{d}}}\right)}{\sqrt{c}\,\sqrt{\sqrt{c}\,\sqrt{d}}}+\frac{2\sqrt{2}\,(bc+3\,ad)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}\,x-\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\,\sqrt{d}}}\right)}{\sqrt{c}\,\sqrt{\sqrt{c}\,\sqrt{d}}}+\frac{\sqrt{2}\,(bc+3\,ad)\log\left(\sqrt{d}\,x^2+\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}x+\sqrt{c}\right)}{c^{\frac{3}{4}}d^{\frac{1}{4}}}-\frac{\sqrt{2}\,(bc+3\,ad)\log\left(\sqrt{d}\,x^2-\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}x+\sqrt{c}\right)}{c^{\frac{3}{4}}d^{\frac{1}{4}}}}{32\,cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)/(d*x^4+c)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(b*c - a*d)*x/(c*d^2*x^4 + c^2*d) + 1/32*(2*sqrt(2)*(b*c + 3*a*d)*arct
an(1/2*sqrt(2)*(2*sqrt(d)*x + sqrt(2)*c^(1/4)*d^(1/4))/sqrt(sqrt(c)*sqrt(d)
))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + 2*sqrt(2)*(b*c + 3*a*d)*arctan(1/2*sqr
t(2)*(2*sqrt(d)*x - sqrt(2)*c^(1/4)*d^(1/4))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c
)*sqrt(sqrt(c)*sqrt(d))) + sqrt(2)*(b*c + 3*a*d)*log(sqrt(d)*x^2 + sqrt(2)*
c^(1/4)*d^(1/4)*x + sqrt(c))/(c^(3/4)*d^(1/4)) - sqrt(2)*(b*c + 3*a*d)*log(
sqrt(d)*x^2 - sqrt(2)*c^(1/4)*d^(1/4)*x + sqrt(c))/(c^(3/4)*d^(1/4)))/(c*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 711 vs. $2(178) = 356$.
time = 3.59, size = 711, normalized size = 2.90

$$\frac{4\,(cd^2x^4+c^2d)\left(-\frac{b^4c^4+12\,ab^3c^3d+54\,a^2b^2c^2d^2+108\,a^3bcd^3+81\,a^4d^4}{c^7d^5}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{c^4d^2\sqrt{-\frac{b^4c^4+12\,ab^3c^3d+54\,a^2b^2c^2d^2+108\,a^3bcd^3+81\,a^4d^4}{c^7d^5}}+(b^2c^2+6\,abcd+9\,a^2d^2)x^2}\,c^5d^4\left(-\frac{b^4c^4+12\,ab^3c^3d+54\,a^2b^2c^2d^2+108\,a^3bcd^3+81\,a^4d^4}{c^7d^5}\right)^{\frac{3}{4}}-(bc^6d^4+3\,ac^5d^5)x\left(-\frac{b^4c^4+12\,ab^3c^3d+54\,a^2b^2c^2d^2+108\,a^3bcd^3+81\,a^4d^4}{c^7d^5}\right)^{\frac{3}{4}}}{b^4c^4+12\,ab^3c^3d+54\,a^2b^2c^2d^2+108\,a^3bcd^3+81\,a^4d^4}\right)+(cd^2x^4+c^2d)\left(-\frac{b^4c^4+12\,ab^3c^3d+54\,a^2b^2c^2d^2+108\,a^3bcd^3+81\,a^4d^4}{c^7d^5}\right)^{\frac{1}{4}}\log\left(c^2d\left(-\frac{b^4c^4+12\,ab^3c^3d+54\,a^2b^2c^2d^2+108\,a^3bcd^3+81\,a^4d^4}{c^7d^5}\right)^{\frac{1}{4}}+(bc+3\,ad)x\right)-(cd^2x^4+c^2d)\left(-\frac{b^4c^4+12\,ab^3c^3d+54\,a^2b^2c^2d^2+108\,a^3bcd^3+81\,a^4d^4}{c^7d^5}\right)^{\frac{1}{4}}\log\left(-c^2d\left(-\frac{b^4c^4+12\,ab^3c^3d+54\,a^2b^2c^2d^2+108\,a^3bcd^3+81\,a^4d^4}{c^7d^5}\right)^{\frac{1}{4}}+(bc+3\,ad)x\right)-4\,(bc-ad)x}{16\,(cd^2x^4+c^2d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)/(d*x^4+c)^2,x, algorithm="fricas")
```

```
[Out] 1/16*(4*(c*d^2*x^4 + c^2*d)*(-(b^4*c^4 + 12*a*b^3*c^3*d + 54*a^2*b^2*c^2*d^
2 + 108*a^3*b*c*d^3 + 81*a^4*d^4)/(c^7*d^5))^(1/4)*arctan((sqrt(c^4*d^2*sqr
t(-(b^4*c^4 + 12*a*b^3*c^3*d + 54*a^2*b^2*c^2*d^2 + 108*a^3*b*c*d^3 + 81*a^
4*d^4)/(c^7*d^5)) + (b^2*c^2 + 6*a*b*c*d + 9*a^2*d^2)*x^2)*c^5*d^4*(-(b^4*c
^4 + 12*a*b^3*c^3*d + 54*a^2*b^2*c^2*d^2 + 108*a^3*b*c*d^3 + 81*a^4*d^4)/(c
^7*d^5))^(3/4) - (b*c^6*d^4 + 3*a*c^5*d^5)*x*(-(b^4*c^4 + 12*a*b^3*c^3*d +
```

```
54*a^2*b^2*c^2*d^2 + 108*a^3*b*c*d^3 + 81*a^4*d^4)/(c^7*d^5))^(3/4))/(b^4*c
^4 + 12*a*b^3*c^3*d + 54*a^2*b^2*c^2*d^2 + 108*a^3*b*c*d^3 + 81*a^4*d^4)) +
 (c*d^2*x^4 + c^2*d)*(-(b^4*c^4 + 12*a*b^3*c^3*d + 54*a^2*b^2*c^2*d^2 + 108
*a^3*b*c*d^3 + 81*a^4*d^4)/(c^7*d^5))^(1/4)*log(c^2*d*(-(b^4*c^4 + 12*a*b^3
*c^3*d + 54*a^2*b^2*c^2*d^2 + 108*a^3*b*c*d^3 + 81*a^4*d^4)/(c^7*d^5))^(1/4
) + (b*c + 3*a*d)*x) - (c*d^2*x^4 + c^2*d)*(-(b^4*c^4 + 12*a*b^3*c^3*d + 54
*a^2*b^2*c^2*d^2 + 108*a^3*b*c*d^3 + 81*a^4*d^4)/(c^7*d^5))^(1/4)*log(-c^2*
d*(-(b^4*c^4 + 12*a*b^3*c^3*d + 54*a^2*b^2*c^2*d^2 + 108*a^3*b*c*d^3 + 81*a
^4*d^4)/(c^7*d^5))^(1/4) + (b*c + 3*a*d)*x) - 4*(b*c - a*d)*x)/(c*d^2*x^4 +
 c^2*d)
```

Sympy [A]
time = 0.45, size = 112, normalized size = 0.46

$$\frac{x(ad-bc)}{4c^2d+4cd^2x^4}+\operatorname{RootSum}\left(65536t^4c^7d^5+81a^4d^4+108a^3bcd^3+54a^2b^2c^2d^2+12ab^3c^3d+b^4c^4,\left(t\mapsto t\log\left(\frac{16tc^2d}{3ad+bc}+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)/(d*x**4+c)**2,x)
```

```
[Out] x*(a*d - b*c)/(4*c**2*d + 4*c*d**2*x**4) + RootSum(65536*_t**4*c**7*d**5 +
81*a**4*d**4 + 108*a**3*b*c*d**3 + 54*a**2*b**2*c**2*d**2 + 12*a*b**3*c**3*
d + b**4*c**4, Lambda(_t, _t*log(16*_t*c**2*d/(3*a*d + b*c) + x)))
```

Giac [A]
time = 1.05, size = 266, normalized size = 1.09

$$\frac{\sqrt{2}\left(\left(cd^3\right)^{\frac{1}{4}}bc+3\left(cd^3\right)^{\frac{1}{4}}ad\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{16\,c^2d^2}+\frac{\sqrt{2}\left(\left(cd^3\right)^{\frac{1}{4}}bc+3\left(cd^3\right)^{\frac{1}{4}}ad\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{16\,c^2d^2}+\frac{\sqrt{2}\left(\left(cd^3\right)^{\frac{1}{4}}bc+3\left(cd^3\right)^{\frac{1}{4}}ad\right)\log\left(x^2+\sqrt{2}\,x\left(\frac{c}{d}\right)^{\frac{1}{4}}+\sqrt{\frac{c}{d}}\right)}{32\,c^2d^2}-\frac{\sqrt{2}\left(\left(cd^3\right)^{\frac{1}{4}}bc+3\left(cd^3\right)^{\frac{1}{4}}ad\right)\log\left(x^2-\sqrt{2}\,x\left(\frac{c}{d}\right)^{\frac{1}{4}}+\sqrt{\frac{c}{d}}\right)}{32\,c^2d^2}-\frac{bcx-adx}{4\left(dx^4+c\right)cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)/(d*x^4+c)^2,x, algorithm="giac")
```

```
[Out] 1/16*sqrt(2)*((c*d^3)^(1/4)*b*c + 3*(c*d^3)^(1/4)*a*d)*arctan(1/2*sqrt(2)*(
2*x + sqrt(2)*(c/d)^(1/4))/(c/d)^(1/4))/(c^2*d^2) + 1/16*sqrt(2)*((c*d^3)^(
1/4)*b*c + 3*(c*d^3)^(1/4)*a*d)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(c/d)^(1/
4))/(c/d)^(1/4))/(c^2*d^2) + 1/32*sqrt(2)*((c*d^3)^(1/4)*b*c + 3*(c*d^3)^(1
/4)*a*d)*log(x^2 + sqrt(2)*x*(c/d)^(1/4) + sqrt(c/d))/(c^2*d^2) - 1/32*sqrt
(2)*((c*d^3)^(1/4)*b*c + 3*(c*d^3)^(1/4)*a*d)*log(x^2 - sqrt(2)*x*(c/d)^(1/
4) + sqrt(c/d))/(c^2*d^2) - 1/4*(b*c*x - a*d*x)/((d*x^4 + c)*c*d)
```

Mupad [B]
time = 1.52, size = 740, normalized size = 3.02

$$\frac{\mathrm{atan}\left(\frac{\frac{\left(\frac{x\left(9a^2d^3+6abcd^2+b^2c^2d\right)}{4c^2}-\frac{(3ad+bc)\left(12ad^3+4bcd^2\right)1i}{16(-c)^{7/4}d^{5/4}}\right)(3ad+bc)}{16(-c)^{7/4}d^{5/4}}+\frac{\left(\frac{x\left(9a^2d^3+6abcd^2+b^2c^2d\right)}{4c^2}+\frac{(3ad+bc)\left(12ad^3+4bcd^2\right)1i}{16(-c)^{7/4}d^{5/4}}\right)(3ad+bc)}{16(-c)^{7/4}d^{5/4}}}{\frac{\left(\frac{x\left(9a^2d^3+6abcd^2+b^2c^2d\right)}{4c^2}-\frac{(3ad+bc)\left(12ad^3+4bcd^2\right)1i}{16(-c)^{7/4}d^{5/4}}\right)(3ad+bc)1i}{16(-c)^{7/4}d^{5/4}}-\frac{\left(\frac{x\left(9a^2d^3+6abcd^2+b^2c^2d\right)}{4c^2}+\frac{(3ad+bc)\left(12ad^3+4bcd^2\right)1i}{16(-c)^{7/4}d^{5/4}}\right)(3ad+bc)1i}{16(-c)^{7/4}d^{5/4}}}\right)(3ad+bc)}{8(-c)^{7/4}d^{5/4}}+\frac{x(ad-bc)}{4cd\,(dx^4+c)}+\frac{\mathrm{atan}\left(\frac{\frac{\left(\frac{x\left(9a^2d^3+6abcd^2+b^2c^2d\right)}{4c^2}-\frac{(3ad+bc)\left(12ad^3+4bcd^2\right)}{16(-c)^{7/4}d^{5/4}}\right)(3ad+bc)1i}{16(-c)^{7/4}d^{5/4}}+\frac{\left(\frac{x\left(9a^2d^3+6abcd^2+b^2c^2d\right)}{4c^2}+\frac{(3ad+bc)\left(12ad^3+4bcd^2\right)}{16(-c)^{7/4}d^{5/4}}\right)(3ad+bc)1i}{16(-c)^{7/4}d^{5/4}}}{\frac{\left(\frac{x\left(9a^2d^3+6abcd^2+b^2c^2d\right)}{4c^2}-\frac{(3ad+bc)\left(12ad^3+4bcd^2\right)}{16(-c)^{7/4}d^{5/4}}\right)(3ad+bc)}{16(-c)^{7/4}d^{5/4}}-\frac{\left(\frac{x\left(9a^2d^3+6abcd^2+b^2c^2d\right)}{4c^2}+\frac{(3ad+bc)\left(12ad^3+4bcd^2\right)}{16(-c)^{7/4}d^{5/4}}\right)(3ad+bc)}{16(-c)^{7/4}d^{5/4}}}\right)(3ad+bc)\,1i}{8(-c)^{7/4}d^{5/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)/(c + d*x^4)^2,x)
```

```
[Out] (atan(((((x*(9*a^2*d^3 + b^2*c^2*d + 6*a*b*c*d^2))/(4*c^2) - ((3*a*d + b*c)
*(12*a*d^3 + 4*b*c*d^2))/(16*(-c)^(7/4)*d^(5/4)))*(3*a*d + b*c)*1i)/(16*(-c
)^(7/4)*d^(5/4)) + (((x*(9*a^2*d^3 + b^2*c^2*d + 6*a*b*c*d^2))/(4*c^2) + ((
3*a*d + b*c)*(12*a*d^3 + 4*b*c*d^2))/(16*(-c)^(7/4)*d^(5/4)))*(3*a*d + b*c)
*1i)/(16*(-c)^(7/4)*d^(5/4)))/((((x*(9*a^2*d^3 + b^2*c^2*d + 6*a*b*c*d^2))/
(4*c^2) - ((3*a*d + b*c)*(12*a*d^3 + 4*b*c*d^2))/(16*(-c)^(7/4)*d^(5/4)))*(
3*a*d + b*c))/(16*(-c)^(7/4)*d^(5/4)) - (((x*(9*a^2*d^3 + b^2*c^2*d + 6*a*b
*c*d^2))/(4*c^2) + ((3*a*d + b*c)*(12*a*d^3 + 4*b*c*d^2))/(16*(-c)^(7/4)*d^
(5/4)))*(3*a*d + b*c))/(16*(-c)^(7/4)*d^(5/4))))*(3*a*d + b*c)*1i)/(8*(-c)^
(7/4)*d^(5/4)) + (atan(((((x*(9*a^2*d^3 + b^2*c^2*d + 6*a*b*c*d^2))/(4*c^2)
 - ((3*a*d + b*c)*(12*a*d^3 + 4*b*c*d^2)*1i)/(16*(-c)^(7/4)*d^(5/4)))*(3*a*
d + b*c))/(16*(-c)^(7/4)*d^(5/4)) + (((x*(9*a^2*d^3 + b^2*c^2*d + 6*a*b*c*d
^2))/(4*c^2) + ((3*a*d + b*c)*(12*a*d^3 + 4*b*c*d^2)*1i)/(16*(-c)^(7/4)*d^(
5/4)))*(3*a*d + b*c))/(16*(-c)^(7/4)*d^(5/4)))/((((x*(9*a^2*d^3 + b^2*c^2*d
 + 6*a*b*c*d^2))/(4*c^2) - ((3*a*d + b*c)*(12*a*d^3 + 4*b*c*d^2)*1i)/(16*(-
c)^(7/4)*d^(5/4)))*(3*a*d + b*c)*1i)/(16*(-c)^(7/4)*d^(5/4)) - (((x*(9*a^2*
d^3 + b^2*c^2*d + 6*a*b*c*d^2))/(4*c^2) + ((3*a*d + b*c)*(12*a*d^3 + 4*b*c*
d^2)*1i)/(16*(-c)^(7/4)*d^(5/4)))*(3*a*d + b*c)*1i)/(16*(-c)^(7/4)*d^(5/4))
))*(3*a*d + b*c))/(8*(-c)^(7/4)*d^(5/4)) + (x*(a*d - b*c))/(4*c*d*(c + d*x^
4))
```

3.152 $\int \frac{a+bx^4}{(c+dx^4)^3}\,dx$

Optimal. Leaf size=273

$$-\frac{(bc-ad)x}{8cd\left(c+dx^4\right)^2}+\frac{(bc+7ad)x}{32c^2d\left(c+dx^4\right)}-\frac{3(bc+7ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right)}{64\sqrt{2}\,c^{11/4}d^{5/4}}+\frac{3(bc+7ad)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right)}{64\sqrt{2}\,c^{11/4}d^{5/4}}$$

```
[Out] -1/8*(-a*d+b*c)*x/c/d/(d*x^4+c)^2+1/32*(7*a*d+b*c)*x/c^2/d/(d*x^4+c)+3/128*
(7*a*d+b*c)*arctan(-1+d^(1/4)*x*2^(1/2)/c^(1/4))/c^(11/4)/d^(5/4)*2^(1/2)+3
/128*(7*a*d+b*c)*arctan(1+d^(1/4)*x*2^(1/2)/c^(1/4))/c^(11/4)/d^(5/4)*2^(1/
2)-3/256*(7*a*d+b*c)*ln(-c^(1/4)*d^(1/4)*x*2^(1/2)+c^(1/2)+x^2*d^(1/2))/c^(
11/4)/d^(5/4)*2^(1/2)+3/256*(7*a*d+b*c)*ln(c^(1/4)*d^(1/4)*x*2^(1/2)+c^(1/2
)+x^2*d^(1/2))/c^(11/4)/d^(5/4)*2^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 273, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.471$, Rules used = {393, 205, 217, 1179, 642, 1176, 631, 210}

$$-\frac{3(7ad+bc)\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right)}{64\sqrt{2}\,c^{11/4}d^{5/4}}+\frac{3(7ad+bc)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{d}\,x}{\sqrt[4]{c}}+1\right)}{64\sqrt{2}\,c^{11/4}d^{5/4}}-\frac{3(7ad+bc)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\,x+\sqrt{c}+\sqrt{d}\,x^2\right)}{128\sqrt{2}\,c^{11/4}d^{5/4}}+\frac{3(7ad+bc)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\,x+\sqrt{c}+\sqrt{d}\,x^2\right)}{128\sqrt{2}\,c^{11/4}d^{5/4}}+\frac{x(7ad+bc)}{32c^2d\left(c+dx^4\right)}-\frac{x(bc-ad)}{8cd\left(c+dx^4\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)/(c + d*x^4)^3,x]
```

```
[Out] -1/8*((b*c - a*d)*x)/(c*d*(c + d*x^4)^2) + ((b*c + 7*a*d)*x)/(32*c^2*d*(c +
 d*x^4)) - (3*(b*c + 7*a*d)*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/(64*Sq
rt[2]*c^(11/4)*d^(5/4)) + (3*(b*c + 7*a*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c
^(1/4)])/(64*Sqrt[2]*c^(11/4)*d^(5/4)) - (3*(b*c + 7*a*d)*Log[Sqrt[c] - Sqr
t[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(128*Sqrt[2]*c^(11/4)*d^(5/4)) + (3*
(b*c + 7*a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(128*
Sqrt[2]*c^(11/4)*d^(5/4))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{a+bx^4}{(c+dx^4)^3}\,dx = -\frac{(bc-ad)x}{8cd\,(c+dx^4)^2} + \frac{(bc+7ad)\int \frac{1}{(c+dx^4)^2}\,dx}{8cd}$$

$$= -\frac{(bc-ad)x}{8cd\,(c+dx^4)^2} + \frac{(bc+7ad)x}{32c^2d\,(c+dx^4)} + \frac{(3(bc+7ad))\int \frac{1}{c+dx^4}\,dx}{32c^2d}$$

$$= -\frac{(bc-ad)x}{8cd\,(c+dx^4)^2} + \frac{(bc+7ad)x}{32c^2d\,(c+dx^4)} + \frac{(3(bc+7ad))\int \frac{\sqrt{c}-\sqrt{d}\,x^2}{c+dx^4}\,dx}{64c^{5/2}d} + \frac{(3(bc+7ad))\int}{64c^5}$$

$$= -\frac{(bc-ad)x}{8cd\,(c+dx^4)^2} + \frac{(bc+7ad)x}{32c^2d\,(c+dx^4)} + \frac{(3(bc+7ad))\int \frac{1}{\frac{\sqrt{c}}{\sqrt{d}}-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{d}}+x^2}\,dx}{128c^{5/2}d^{3/2}} + \frac{(3(bc+7}{}$$

$$= -\frac{(bc-ad)x}{8cd\,(c+dx^4)^2} + \frac{(bc+7ad)x}{32c^2d\,(c+dx^4)} - \frac{3(bc+7ad)\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\,x+\sqrt{d}\,x^2\right)}{128\sqrt{2}\,c^{11/4}d^{5/4}}$$

$$= -\frac{(bc-ad)x}{8cd\,(c+dx^4)^2} + \frac{(bc+7ad)x}{32c^2d\,(c+dx^4)} - \frac{3(bc+7ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right)}{64\sqrt{2}\,c^{11/4}d^{5/4}} + \frac{3(bc+7ad}{6}$$

Mathematica [A]
time = 0.14, size = 243, normalized size = 0.89

$$\frac{-\frac{32c^{7/4}\sqrt[4]{d}(bc-ad)x}{(c+dx^4)^2}+\frac{8c^{3/4}\sqrt[4]{d}(bc+7ad)x}{c+dx^4}-6\sqrt{2}(bc+7ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)+6\sqrt{2}(bc+7ad)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)-3\sqrt{2}(bc+7ad)\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{d}x^2\right)+3\sqrt{2}(bc+7ad)\log\left(\sqrt{c}+\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{d}x^2\right)}{256c^{11/4}d^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^4)/(c + d*x^4)^3,x]
```

```
[Out] ((-32*c^(7/4)*d^(1/4)*(b*c - a*d)*x)/(c + d*x^4)^2 + (8*c^(3/4)*d^(1/4)*(b*
c + 7*a*d)*x)/(c + d*x^4) - 6*Sqrt[2]*(b*c + 7*a*d)*ArcTan[1 - (Sqrt[2]*d^(
1/4)*x)/c^(1/4)] + 6*Sqrt[2]*(b*c + 7*a*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c
^(1/4)] - 3*Sqrt[2]*(b*c + 7*a*d)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x +
 Sqrt[d]*x^2] + 3*Sqrt[2]*(b*c + 7*a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/
4)*x + Sqrt[d]*x^2])/(256*c^(11/4)*d^(5/4))
```

Maple [A]
time = 0.24, size = 159, normalized size = 0.58

method	result

risch	$\frac{\frac{(7ad+bc)x^5}{32c^2}+\frac{(11ad-3bc)x}{32cd}}{(d\,x^4+c)^2}+\frac{3\left(\sum_{\_R=\mathrm{RootOf}\left(d\_Z^4+c\right)}\frac{(7ad+bc)\ln\left(x-\_R\right)}{\_R^3}\right)}{128c^2d^2}$
default	$\frac{\frac{(7ad+bc)x^5}{32c^2}+\frac{(11ad-3bc)x}{32cd}}{(d\,x^4+c)^2}+\frac{3(7ad+bc)\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{c}{d}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{c}{d}}}{x^2-\left(\frac{c}{d}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{c}{d}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right.\right.}{256c^3d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)/(d*x^4+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] (1/32*(7*a*d+b*c)/c^2*x^5+1/32*(11*a*d-3*b*c)/c/d*x)/(d*x^4+c)^2+3/256*(7*a
*d+b*c)/c^3/d*(c/d)^(1/4)*2^(1/2)*(ln((x^2+(c/d)^(1/4)*x*2^(1/2)+(c/d)^(1/2
))/(x^2-(c/d)^(1/4)*x*2^(1/2)+(c/d)^(1/2)))+2*arctan(2^(1/2)/(c/d)^(1/4)*x+
1)+2*arctan(2^(1/2)/(c/d)^(1/4)*x-1))
```

Maxima [A]
time = 0.50, size = 271, normalized size = 0.99

$$\frac{(bcd+7\,ad^2)x^5-(3\,bc^2-11\,acd)x}{32\,(c^2d^3x^8+2\,c^3d^2x^4+c^4d)}+\frac{3\left(\frac{2\sqrt{2}\,(bc+7\,ad)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}\,x+\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\,\sqrt{d}}}\right)}{\sqrt{c}\,\sqrt{\sqrt{c}\,\sqrt{d}}}+\frac{2\sqrt{2}\,(bc+7\,ad)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}\,x-\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\,\sqrt{d}}}\right)}{\sqrt{c}\,\sqrt{\sqrt{c}\,\sqrt{d}}}+\frac{\sqrt{2}\,(bc+7\,ad)\log\left(\sqrt{d}\,x^2+\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}x+\sqrt{c}\right)}{c^{\frac{3}{4}}d^{\frac{1}{4}}}-\frac{\sqrt{2}\,(bc+7\,ad)\log\left(\sqrt{d}\,x^2-\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}x+\sqrt{c}\right)}{c^{\frac{3}{4}}d^{\frac{1}{4}}}\right)}{256\,c^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)/(d*x^4+c)^3,x, algorithm="maxima")
```

```
[Out] 1/32*((b*c*d + 7*a*d^2)*x^5 - (3*b*c^2 - 11*a*c*d)*x)/(c^2*d^3*x^8 + 2*c^3*
d^2*x^4 + c^4*d) + 3/256*(2*sqrt(2)*(b*c + 7*a*d)*arctan(1/2*sqrt(2)*(2*sqr
t(d)*x + sqrt(2)*c^(1/4)*d^(1/4))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt
(c)*sqrt(d))) + 2*sqrt(2)*(b*c + 7*a*d)*arctan(1/2*sqrt(2)*(2*sqrt(d)*x - s
qrt(2)*c^(1/4)*d^(1/4))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d
))) + sqrt(2)*(b*c + 7*a*d)*log(sqrt(d)*x^2 + sqrt(2)*c^(1/4)*d^(1/4)*x + s
qrt(c))/(c^(3/4)*d^(1/4)) - sqrt(2)*(b*c + 7*a*d)*log(sqrt(d)*x^2 - sqrt(2)
*c^(1/4)*d^(1/4)*x + sqrt(c))/(c^(3/4)*d^(1/4)))/(c^2*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 787 vs. 2(204) = 408.
time = 3.02, size = 787, normalized size = 2.88

$$\frac{4\,(bcd+7\,ad^2)x^5+12\,(c^2d^3x^8+2\,c^3d^2x^4+c^4d)\left(-\frac{b^4c^4+28\,ab^3c^3d+294\,a^2b^2c^2d^2+1372\,a^3bcd^3+2401\,a^4d^4}{c^{11}d^5}\right)^{\frac{1}{4}}\arctan\left([illegible]\right)+3\,(c^2d^3x^8+2\,c^3d^2x^4+c^4d)\left(-\frac{b^4c^4+28\,ab^3c^3d+294\,a^2b^2c^2d^2+1372\,a^3bcd^3+2401\,a^4d^4}{c^{11}d^5}\right)^{\frac{1}{4}}\log\left(3\,c^3d\left([illegible]\right)^{\frac{3}{4}}+3\,(bc+7\,ad)x\right)-3\,(c^2d^3x^8+2\,c^3d^2x^4+c^4d)\left(-\frac{b^4c^4+28\,ab^3c^3d+294\,a^2b^2c^2d^2+1372\,a^3bcd^3+2401\,a^4d^4}{c^{11}d^5}\right)^{\frac{1}{4}}\log\left(-3\,c^3d\left([illegible]\right)^{\frac{3}{4}}+3\,(bc+7\,ad)x\right)-4\,(3\,bc^2-11\,acd)x}{128\,(c^2d^3x^8+2\,c^3d^2x^4+c^4d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)/(d*x^4+c)^3,x, algorithm="fricas")
```

```
[Out] 1/128*(4*(b*c*d + 7*a*d^2)*x^5 + 12*(c^2*d^3*x^8 + 2*c^3*d^2*x^4 + c^4*d)*(
-(b^4*c^4 + 28*a*b^3*c^3*d + 294*a^2*b^2*c^2*d^2 + 1372*a^3*b*c*d^3 + 2401*
a^4*d^4)/(c^11*d^5))^(1/4)*arctan((sqrt(c^6*d^2*sqrt(-(b^4*c^4 + 28*a*b^3*c
^3*d + 294*a^2*b^2*c^2*d^2 + 1372*a^3*b*c*d^3 + 2401*a^4*d^4)/(c^11*d^5)) +
 (b^2*c^2 + 14*a*b*c*d + 49*a^2*d^2)*x^2)*c^8*d^4*(-(b^4*c^4 + 28*a*b^3*c^3
*d + 294*a^2*b^2*c^2*d^2 + 1372*a^3*b*c*d^3 + 2401*a^4*d^4)/(c^11*d^5))^(3/
4) - (b*c^9*d^4 + 7*a*c^8*d^5)*x*(-(b^4*c^4 + 28*a*b^3*c^3*d + 294*a^2*b^2*
c^2*d^2 + 1372*a^3*b*c*d^3 + 2401*a^4*d^4)/(c^11*d^5))^(3/4))/(b^4*c^4 + 28
*a*b^3*c^3*d + 294*a^2*b^2*c^2*d^2 + 1372*a^3*b*c*d^3 + 2401*a^4*d^4)) + 3*
(c^2*d^3*x^8 + 2*c^3*d^2*x^4 + c^4*d)*(-(b^4*c^4 + 28*a*b^3*c^3*d + 294*a^2
*b^2*c^2*d^2 + 1372*a^3*b*c*d^3 + 2401*a^4*d^4)/(c^11*d^5))^(1/4)*log(3*c^3
*d*(-(b^4*c^4 + 28*a*b^3*c^3*d + 294*a^2*b^2*c^2*d^2 + 1372*a^3*b*c*d^3 + 2
401*a^4*d^4)/(c^11*d^5))^(1/4) + 3*(b*c + 7*a*d)*x) - 3*(c^2*d^3*x^8 + 2*c^
3*d^2*x^4 + c^4*d)*(-(b^4*c^4 + 28*a*b^3*c^3*d + 294*a^2*b^2*c^2*d^2 + 1372
*a^3*b*c*d^3 + 2401*a^4*d^4)/(c^11*d^5))^(1/4)*log(-3*c^3*d*(-(b^4*c^4 + 28
*a*b^3*c^3*d + 294*a^2*b^2*c^2*d^2 + 1372*a^3*b*c*d^3 + 2401*a^4*d^4)/(c^11
*d^5))^(1/4) + 3*(b*c + 7*a*d)*x) - 4*(3*b*c^2 - 11*a*c*d)*x)/(c^2*d^3*x^8
+ 2*c^3*d^2*x^4 + c^4*d)
```

Sympy [A]
time = 0.61, size = 151, normalized size = 0.55

$$\frac{x^5 \cdot (7ad^2 + bcd) + x(11acd - 3bc^2)}{32c^4d + 64c^3d^2x^4 + 32c^2d^3x^8} + \operatorname{RootSum}\left(268435456t^4c^{11}d^5 + 194481a^4d^4 + 111132a^3bcd^3 + 23814a^2b^2c^2d^2 + 2268ab^3c^3d + 81b^4c^4, \left(t \mapsto t\log\left(\frac{128tc^3d}{21ad + 3bc} + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)/(d*x**4+c)**3,x)
```

```
[Out] (x**5*(7*a*d**2 + b*c*d) + x*(11*a*c*d - 3*b*c**2))/(32*c**4*d + 64*c**3*d*
*2*x**4 + 32*c**2*d**3*x**8) + RootSum(268435456*_t**4*c**11*d**5 + 194481*
a**4*d**4 + 111132*a**3*b*c*d**3 + 23814*a**2*b**2*c**2*d**2 + 2268*a*b**3*
c**3*d + 81*b**4*c**4, Lambda(_t, _t*log(128*_t*c**3*d/(21*a*d + 3*b*c) + x
)))
```

Giac [A]
time = 1.11, size = 286, normalized size = 1.05

$$\frac{3\sqrt{2}\left(\left(cd^3\right)^{\frac{1}{4}}bc + 7\left(cd^3\right)^{\frac{1}{4}}ad\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{128\,c^3d^2} + \frac{3\sqrt{2}\left(\left(cd^3\right)^{\frac{1}{4}}bc + 7\left(cd^3\right)^{\frac{1}{4}}ad\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{128\,c^3d^2} + \frac{3\sqrt{2}\left(\left(cd^3\right)^{\frac{1}{4}}bc + 7\left(cd^3\right)^{\frac{1}{4}}ad\right)\log\left(x^2 + \sqrt{2}\,x\left(\frac{c}{d}\right)^{\frac{1}{4}} + \sqrt{\frac{c}{d}}\right)}{256\,c^3d^2} - \frac{3\sqrt{2}\left(\left(cd^3\right)^{\frac{1}{4}}bc + 7\left(cd^3\right)^{\frac{1}{4}}ad\right)\log\left(x^2 - \sqrt{2}\,x\left(\frac{c}{d}\right)^{\frac{1}{4}} + \sqrt{\frac{c}{d}}\right)}{256\,c^3d^2} + \frac{bcdx^5 + 7\,ad^2x^5 - 3\,bc^2x + 11\,acdx}{32\,(dx^4+c)^2c^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)/(d*x^4+c)^3,x, algorithm="giac")
```

```
[Out] 3/128*sqrt(2)*((c*d^3)^(1/4)*b*c + 7*(c*d^3)^(1/4)*a*d)*arctan(1/2*sqrt(2)*
(2*x + sqrt(2)*(c/d)^(1/4))/(c/d)^(1/4))/(c^3*d^2) + 3/128*sqrt(2)*((c*d^3)
^(1/4)*b*c + 7*(c*d^3)^(1/4)*a*d)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(c/d)^(
1/4))/(c/d)^(1/4))/(c^3*d^2) + 3/256*sqrt(2)*((c*d^3)^(1/4)*b*c + 7*(c*d^3)
```

^(1/4)*a*d)*log(x^2 + sqrt(2)*x*(c/d)^(1/4) + sqrt(c/d))/(c^3*d^2) - 3/256*
sqrt(2)*((c*d^3)^(1/4)*b*c + 7*(c*d^3)^(1/4)*a*d)*log(x^2 - sqrt(2)*x*(c/d)
^(1/4) + sqrt(c/d))/(c^3*d^2) + 1/32*(b*c*d*x^5 + 7*a*d^2*x^5 - 3*b*c^2*x +
11*a*c*d*x)/((d*x^4 + c)^2*c^2*d)

Mupad [B]
time = 1.58, size = 762, normalized size = 2.79

$$\frac{\frac{x^5\,(7\,a\,d+b\,c)}{32\,c^2}+\frac{x\,(11\,a\,d-3\,b\,c)}{32\,c\,d}}{c^2+2\,c\,d\,x^4+d^2\,x^8}-\frac{\mathrm{atan}\left(\frac{\frac{\left(\frac{9\,x\,\left(49\,a^2\,d^3+14\,a\,b\,c\,d^2+b^2\,c^2\,d\right)}{256\,c^4}-\frac{9\,(7\,a\,d+b\,c)\,\left(7\,a\,d^3+b\,c\,d^2\right)}{256\,(-c)^{15/4}\,d^{5/4}}\right)\,(7\,a\,d+b\,c)\,3\mathrm{i}}{128\,(-c)^{11/4}\,d^{5/4}}+\frac{\left(\frac{9\,x\,\left(49\,a^2\,d^3+14\,a\,b\,c\,d^2+b^2\,c^2\,d\right)}{256\,c^4}+\frac{9\,(7\,a\,d+b\,c)\,\left(7\,a\,d^3+b\,c\,d^2\right)}{256\,(-c)^{15/4}\,d^{5/4}}\right)\,(7\,a\,d+b\,c)\,3\mathrm{i}}{128\,(-c)^{11/4}\,d^{5/4}}}{\frac{3\,\left(\frac{9\,x\,\left(49\,a^2\,d^3+14\,a\,b\,c\,d^2+b^2\,c^2\,d\right)}{256\,c^4}-\frac{9\,(7\,a\,d+b\,c)\,\left(7\,a\,d^3+b\,c\,d^2\right)}{256\,(-c)^{15/4}\,d^{5/4}}\right)\,(7\,a\,d+b\,c)}{128\,(-c)^{11/4}\,d^{5/4}}-\frac{3\,\left(\frac{9\,x\,\left(49\,a^2\,d^3+14\,a\,b\,c\,d^2+b^2\,c^2\,d\right)}{256\,c^4}+\frac{9\,(7\,a\,d+b\,c)\,\left(7\,a\,d^3+b\,c\,d^2\right)}{256\,(-c)^{15/4}\,d^{5/4}}\right)\,(7\,a\,d+b\,c)}{128\,(-c)^{11/4}\,d^{5/4}}}\right)\,(7\,a\,d+b\,c)\,3\mathrm{i}}{64\,(-c)^{11/4}\,d^{5/4}}-\frac{3\,\mathrm{atan}\left(\frac{\frac{3\,\left(\frac{9\,x\,\left(49\,a^2\,d^3+14\,a\,b\,c\,d^2+b^2\,c^2\,d\right)}{256\,c^4}-\frac{(7\,a\,d+b\,c)\,\left(7\,a\,d^3+b\,c\,d^2\right)\,9\mathrm{i}}{256\,(-c)^{15/4}\,d^{5/4}}\right)\,(7\,a\,d+b\,c)}{128\,(-c)^{11/4}\,d^{5/4}}+\frac{3\,\left(\frac{9\,x\,\left(49\,a^2\,d^3+14\,a\,b\,c\,d^2+b^2\,c^2\,d\right)}{256\,c^4}+\frac{(7\,a\,d+b\,c)\,\left(7\,a\,d^3+b\,c\,d^2\right)\,9\mathrm{i}}{256\,(-c)^{15/4}\,d^{5/4}}\right)\,(7\,a\,d+b\,c)}{128\,(-c)^{11/4}\,d^{5/4}}}{\frac{\left(\frac{9\,x\,\left(49\,a^2\,d^3+14\,a\,b\,c\,d^2+b^2\,c^2\,d\right)}{256\,c^4}-\frac{(7\,a\,d+b\,c)\,\left(7\,a\,d^3+b\,c\,d^2\right)\,9\mathrm{i}}{256\,(-c)^{15/4}\,d^{5/4}}\right)\,(7\,a\,d+b\,c)\,3\mathrm{i}}{128\,(-c)^{11/4}\,d^{5/4}}-\frac{\left(\frac{9\,x\,\left(49\,a^2\,d^3+14\,a\,b\,c\,d^2+b^2\,c^2\,d\right)}{256\,c^4}+\frac{(7\,a\,d+b\,c)\,\left(7\,a\,d^3+b\,c\,d^2\right)\,9\mathrm{i}}{256\,(-c)^{15/4}\,d^{5/4}}\right)\,(7\,a\,d+b\,c)\,3\mathrm{i}}{128\,(-c)^{11/4}\,d^{5/4}}}\right)\,(7\,a\,d+b\,c)}{64\,(-c)^{11/4}\,d^{5/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*x^4)/(c + d*x^4)^3,x)

[Out] ((x^5*(7*a*d + b*c))/(32*c^2) + (x*(11*a*d - 3*b*c))/(32*c*d))/(c^2 + d^2*x
^8 + 2*c*d*x^4) - (atan(((((9*x*(49*a^2*d^3 + b^2*c^2*d + 14*a*b*c*d^2))/(2
56*c^4) - (9*(7*a*d + b*c)*(7*a*d^3 + b*c*d^2))/(256*(-c)^(15/4)*d^(5/4)))*
(7*a*d + b*c)*3i)/(128*(-c)^(11/4)*d^(5/4)) + (((9*x*(49*a^2*d^3 + b^2*c^2*
d + 14*a*b*c*d^2))/(256*c^4) + (9*(7*a*d + b*c)*(7*a*d^3 + b*c*d^2))/(256*(
-c)^(15/4)*d^(5/4)))*(7*a*d + b*c)*3i)/(128*(-c)^(11/4)*d^(5/4)))/((3*((9*x
*(49*a^2*d^3 + b^2*c^2*d + 14*a*b*c*d^2))/(256*c^4) - (9*(7*a*d + b*c)*(7*a
*d^3 + b*c*d^2))/(256*(-c)^(15/4)*d^(5/4)))*(7*a*d + b*c))/(128*(-c)^(11/4)
d^(5/4)) - (3((9*x*(49*a^2*d^3 + b^2*c^2*d + 14*a*b*c*d^2))/(256*c^4) + (
9*(7*a*d + b*c)*(7*a*d^3 + b*c*d^2))/(256*(-c)^(15/4)*d^(5/4)))*(7*a*d + b*
c))/(128*(-c)^(11/4)*d^(5/4))))*(7*a*d + b*c)*3i)/(64*(-c)^(11/4)*d^(5/4))
- (3*atan(((3*((9*x*(49*a^2*d^3 + b^2*c^2*d + 14*a*b*c*d^2))/(256*c^4) - ((
7*a*d + b*c)*(7*a*d^3 + b*c*d^2)*9i)/(256*(-c)^(15/4)*d^(5/4)))*(7*a*d + b*
c))/(128*(-c)^(11/4)*d^(5/4)) + (3*((9*x*(49*a^2*d^3 + b^2*c^2*d + 14*a*b*c
*d^2))/(256*c^4) + ((7*a*d + b*c)*(7*a*d^3 + b*c*d^2)*9i)/(256*(-c)^(15/4)*
d^(5/4)))*(7*a*d + b*c))/(128*(-c)^(11/4)*d^(5/4)))/((((9*x*(49*a^2*d^3 + b
^2*c^2*d + 14*a*b*c*d^2))/(256*c^4) - ((7*a*d + b*c)*(7*a*d^3 + b*c*d^2)*9i
)/(256*(-c)^(15/4)*d^(5/4)))*(7*a*d + b*c)*3i)/(128*(-c)^(11/4)*d^(5/4)) -
(((9*x*(49*a^2*d^3 + b^2*c^2*d + 14*a*b*c*d^2))/(256*c^4) + ((7*a*d + b*c)*
(7*a*d^3 + b*c*d^2)*9i)/(256*(-c)^(15/4)*d^(5/4)))*(7*a*d + b*c)*3i)/(128*(
-c)^(11/4)*d^(5/4))))*(7*a*d + b*c))/(64*(-c)^(11/4)*d^(5/4))

3.153 $\int (a+bx^4)^2 (c+dx^4)^4 \, dx$

Optimal. Leaf size=154

$$a^2c^4x+\frac{2}{5}ac^3(bc+2ad)x^5+\frac{1}{9}c^2\left(b^2c^2+8abcd+6a^2d^2\right)x^9+\frac{4}{13}cd\left(b^2c^2+3abcd+a^2d^2\right)x^{13}+\frac{1}{17}d^2\left(6b^2c^2+8abc\right.$$

```
[Out] a^2*c^4*x+2/5*a*c^3*(2*a*d+b*c)*x^5+1/9*c^2*(6*a^2*d^2+8*a*b*c*d+b^2*c^2)*x
^9+4/13*c*d*(a^2*d^2+3*a*b*c*d+b^2*c^2)*x^13+1/17*d^2*(a^2*d^2+8*a*b*c*d+6*
b^2*c^2)*x^17+2/21*b*d^3*(a*d+2*b*c)*x^21+1/25*b^2*d^4*x^25
```

Rubi [A]
time = 0.08, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {380}

$$\frac{1}{17}d^2x^{17}\left(a^2d^2+8abcd+6b^2c^2\right)+\frac{4}{13}cdx^{13}\left(a^2d^2+3abcd+b^2c^2\right)+\frac{1}{9}c^2x^9\left(6a^2d^2+8abcd+b^2c^2\right)+a^2c^4x+\frac{2}{5}ac^3x^5(2ad+bc)+\frac{2}{21}bd^3x^{21}(ad+2bc)+\frac{1}{25}b^2d^4x^{25}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)^2*(c + d*x^4)^4,x]
```

```
[Out] a^2*c^4*x + (2*a*c^3*(b*c + 2*a*d)*x^5)/5 + (c^2*(b^2*c^2 + 8*a*b*c*d + 6*a
^2*d^2)*x^9)/9 + (4*c*d*(b^2*c^2 + 3*a*b*c*d + a^2*d^2)*x^13)/13 + (d^2*(6*
b^2*c^2 + 8*a*b*c*d + a^2*d^2)*x^17)/17 + (2*b*d^3*(2*b*c + a*d)*x^21)/21 +
 (b^2*d^4*x^25)/25
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int \left(a+bx^4\right)^2 \left(c+dx^4\right)^4 \, dx = \int \left(a^2c^4+2ac^3(bc+2ad)x^4+c^2\left(b^2c^2+8abcd+6a^2d^2\right)x^8+4cd\left(b^2c^2+3ab\right.\right.$$
$$= a^2c^4x+\frac{2}{5}ac^3(bc+2ad)x^5+\frac{1}{9}c^2\left(b^2c^2+8abcd+6a^2d^2\right)x^9+\frac{4}{13}cd\left(b^2c^2+3a\right.$$

Mathematica [A]
time = 0.02, size = 154, normalized size = 1.00

$$a^2c^4x+\frac{2}{5}ac^3(bc+2ad)x^5+\frac{1}{9}c^2\left(b^2c^2+8abcd+6a^2d^2\right)x^9+\frac{4}{13}cd\left(b^2c^2+3abcd+a^2d^2\right)x^{13}+\frac{1}{17}d^2\left(6b^2c^2+8abcd+a^2d^2\right)x^{17}+\frac{2}{21}bd^3(2bc+ad)x^{21}+\frac{1}{25}b^2d^4x^{25}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^4)^2*(c + d*x^4)^4,x]
```

```
[Out] a^2*c^4*x + (2*a*c^3*(b*c + 2*a*d)*x^5)/5 + (c^2*(b^2*c^2 + 8*a*b*c*d + 6*a
^2*d^2)*x^9)/9 + (4*c*d*(b^2*c^2 + 3*a*b*c*d + a^2*d^2)*x^13)/13 + (d^2*(6*
b^2*c^2 + 8*a*b*c*d + a^2*d^2)*x^17)/17 + (2*b*d^3*(2*b*c + a*d)*x^21)/21 +
 (b^2*d^4*x^25)/25
```

Maple [A]
time = 0.30, size = 163, normalized size = 1.06

method	result
norman	$\frac{b^2d^4x^{25}}{25} + \left(\frac{1}{17}d^4a^2 + \frac{8}{17}abc\,d^3 + \frac{6}{17}b^2c^2d^2\right)x^{17} + \left(\frac{2}{21}ab\,d^4 + \frac{4}{21}b^2c\,d^3\right)x^{21} + \left(\frac{4}{13}a^2c\,d^3 + \frac{12}{13}ab\,c^2d^2 + \right.$
default	$\frac{b^2d^4x^{25}}{25} + \frac{(2ab\,d^4+4b^2c\,d^3)x^{21}}{21} + \frac{(d^4a^2+8abc\,d^3+6b^2c^2d^2)x^{17}}{17} + \frac{(4a^2c\,d^3+12ab\,c^2d^2+4b^2c^3d)x^{13}}{13} + \frac{(6a^2c^2d^2+8ab\,c^3d+b^2c^4}{9}$
gosper	$\frac{1}{25}b^2d^4x^{25} + \frac{1}{17}x^{17}d^4a^2 + \frac{8}{17}x^{17}abc\,d^3 + \frac{6}{17}x^{17}b^2c^2d^2 + \frac{2}{21}x^{21}ab\,d^4 + \frac{4}{21}x^{21}b^2c\,d^3 + \frac{4}{13}x^{13}a^2c\,d^3 + \frac{12}{13}x$
risch	$\frac{1}{25}b^2d^4x^{25} + \frac{1}{17}x^{17}d^4a^2 + \frac{8}{17}x^{17}abc\,d^3 + \frac{6}{17}x^{17}b^2c^2d^2 + \frac{2}{21}x^{21}ab\,d^4 + \frac{4}{21}x^{21}b^2c\,d^3 + \frac{4}{13}x^{13}a^2c\,d^3 + \frac{12}{13}x$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)^2*(d*x^4+c)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/25*b^2*d^4*x^25+1/21*(2*a*b*d^4+4*b^2*c*d^3)*x^21+1/17*(a^2*d^4+8*a*b*c*d
^3+6*b^2*c^2*d^2)*x^17+1/13*(4*a^2*c*d^3+12*a*b*c^2*d^2+4*b^2*c^3*d)*x^13+1
/9*(6*a^2*c^2*d^2+8*a*b*c^3*d+b^2*c^4)*x^9+1/5*(4*a^2*c^3*d+2*a*b*c^4)*x^5+
a^2*c^4*x
```

Maxima [A]
time = 0.28, size = 158, normalized size = 1.03

$$\frac{1}{25}\,b^2d^4x^{25} + \frac{2}{21}\,(2\,b^2cd^3 + abd^4)x^{21} + \frac{1}{17}\,(6\,b^2c^2d^2 + 8\,abcd^3 + a^2d^4)x^{17} + \frac{4}{13}\,(b^2c^3d + 3\,abc^2d^2 + a^2cd^3)x^{13} + \frac{1}{9}\,(b^2c^4 + 8\,abc^3d + 6\,a^2c^2d^2)x^9 + a^2c^4x + \frac{2}{5}\,(abc^4 + 2\,a^2c^3d)x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^2*(d*x^4+c)^4,x, algorithm="maxima")
```

```
[Out] 1/25*b^2*d^4*x^25 + 2/21*(2*b^2*c*d^3 + a*b*d^4)*x^21 + 1/17*(6*b^2*c^2*d^2
 + 8*a*b*c*d^3 + a^2*d^4)*x^17 + 4/13*(b^2*c^3*d + 3*a*b*c^2*d^2 + a^2*c*d^
3)*x^13 + 1/9*(b^2*c^4 + 8*a*b*c^3*d + 6*a^2*c^2*d^2)*x^9 + a^2*c^4*x + 2/5
*(a*b*c^4 + 2*a^2*c^3*d)*x^5
```

Fricas [A]
time = 2.85, size = 158, normalized size = 1.03

$$\frac{1}{25}\,b^2d^4x^{25} + \frac{2}{21}\,(2\,b^2cd^3 + abd^4)x^{21} + \frac{1}{17}\,(6\,b^2c^2d^2 + 8\,abcd^3 + a^2d^4)x^{17} + \frac{4}{13}\,(b^2c^3d + 3\,abc^2d^2 + a^2cd^3)x^{13} + \frac{1}{9}\,(b^2c^4 + 8\,abc^3d + 6\,a^2c^2d^2)x^9 + a^2c^4x + \frac{2}{5}\,(abc^4 + 2\,a^2c^3d)x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^2*(d*x^4+c)^4,x, algorithm="fricas")
```

```
[Out] 1/25*b^2*d^4*x^25 + 2/21*(2*b^2*c*d^3 + a*b*d^4)*x^21 + 1/17*(6*b^2*c^2*d^2
 + 8*a*b*c*d^3 + a^2*d^4)*x^17 + 4/13*(b^2*c^3*d + 3*a*b*c^2*d^2 + a^2*c*d^
3)*x^13 + 1/9*(b^2*c^4 + 8*a*b*c^3*d + 6*a^2*c^2*d^2)*x^9 + a^2*c^4*x + 2/5
*(a*b*c^4 + 2*a^2*c^3*d)*x^5
```

Sympy [A]
time = 0.02, size = 185, normalized size = 1.20

$$a^{2}c^{4}x + \frac{b^{2}d^{4}x^{25}}{25} + x^{21}\cdot\left(\frac{2abd^{4}}{21} + \frac{4b^{2}cd^{3}}{21}\right) + x^{17}\left(\frac{a^{2}d^{4}}{17} + \frac{8abcd^{3}}{17} + \frac{6b^{2}c^{2}d^{2}}{17}\right) + x^{13}\cdot\left(\frac{4a^{2}cd^{3}}{13} + \frac{12abc^{2}d^{2}}{13} + \frac{4b^{2}c^{3}d}{13}\right) + x^{9}\cdot\left(\frac{2a^{2}c^{2}d^{2}}{3} + \frac{8abc^{3}d}{9} + \frac{b^{2}c^{4}}{9}\right) + x^{5}\cdot\left(\frac{4a^{2}c^{3}d}{5} + \frac{2abc^{4}}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)**2*(d*x**4+c)**4,x)
```

```
[Out] a**2*c**4*x + b**2*d**4*x**25/25 + x**21*(2*a*b*d**4/21 + 4*b**2*c*d**3/21)
 + x**17*(a**2*d**4/17 + 8*a*b*c*d**3/17 + 6*b**2*c**2*d**2/17) + x**13*(4*
a**2*c*d**3/13 + 12*a*b*c**2*d**2/13 + 4*b**2*c**3*d/13) + x**9*(2*a**2*c**
2*d**2/3 + 8*a*b*c**3*d/9 + b**2*c**4/9) + x**5*(4*a**2*c**3*d/5 + 2*a*b*c*
*4/5)
```

Giac [A]
time = 1.01, size = 173, normalized size = 1.12

$$\frac{1}{25}\,b^{2}d^{4}x^{25} + \frac{4}{21}\,b^{2}cd^{3}x^{21} + \frac{2}{21}\,abd^{4}x^{21} + \frac{6}{17}\,b^{2}c^{2}d^{2}x^{17} + \frac{8}{17}\,abcd^{3}x^{17} + \frac{1}{17}\,a^{2}d^{4}x^{17} + \frac{4}{13}\,b^{2}c^{3}dx^{13} + \frac{12}{13}\,abc^{2}d^{2}x^{13} + \frac{4}{13}\,a^{2}cd^{3}x^{13} + \frac{1}{9}\,b^{2}c^{4}x^{9} + \frac{8}{9}\,abc^{3}dx^{9} + \frac{2}{3}\,a^{2}c^{2}d^{2}x^{9} + \frac{2}{5}\,abc^{4}x^{5} + \frac{4}{5}\,a^{2}c^{3}dx^{5} + a^{2}c^{4}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^2*(d*x^4+c)^4,x, algorithm="giac")
```

```
[Out] 1/25*b^2*d^4*x^25 + 4/21*b^2*c*d^3*x^21 + 2/21*a*b*d^4*x^21 + 6/17*b^2*c^2*
d^2*x^17 + 8/17*a*b*c*d^3*x^17 + 1/17*a^2*d^4*x^17 + 4/13*b^2*c^3*d*x^13 +
12/13*a*b*c^2*d^2*x^13 + 4/13*a^2*c*d^3*x^13 + 1/9*b^2*c^4*x^9 + 8/9*a*b*c^
3*d*x^9 + 2/3*a^2*c^2*d^2*x^9 + 2/5*a*b*c^4*x^5 + 4/5*a^2*c^3*d*x^5 + a^2*c
^4*x
```

Mupad [B]
time = 0.07, size = 146, normalized size = 0.95

$$x^{9}\left(\frac{2\,a^{2}\,c^{2}\,d^{2}}{3} + \frac{8\,a\,b\,c^{3}\,d}{9} + \frac{b^{2}\,c^{4}}{9}\right) + x^{17}\left(\frac{a^{2}\,d^{4}}{17} + \frac{8\,a\,b\,c\,d^{3}}{17} + \frac{6\,b^{2}\,c^{2}\,d^{2}}{17}\right) + a^{2}\,c^{4}\,x + \frac{b^{2}\,d^{4}\,x^{25}}{25} + \frac{2\,a\,c^{3}\,x^{5}\,(2\,a\,d + b\,c)}{5} + \frac{2\,b\,d^{3}\,x^{21}\,(a\,d + 2\,b\,c)}{21} + \frac{4\,c\,d\,x^{13}\,(a^{2}\,d^{2} + 3\,a\,b\,c\,d + b^{2}\,c^{2})}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)^2*(c + d*x^4)^4,x)
```

```
[Out] x^9*((b^2*c^4)/9 + (2*a^2*c^2*d^2)/3 + (8*a*b*c^3*d)/9) + x^17*((a^2*d^4)/1
7 + (6*b^2*c^2*d^2)/17 + (8*a*b*c*d^3)/17) + a^2*c^4*x + (b^2*d^4*x^25)/25
+ (2*a*c^3*x^5*(2*a*d + b*c))/5 + (2*b*d^3*x^21*(a*d + 2*b*c))/21 + (4*c*d*
x^13*(a^2*d^2 + b^2*c^2 + 3*a*b*c*d))/13
```

3.154 $\int (a+bx^4)^2 (c+dx^4)^3 \, dx$

Optimal. Leaf size=122

$$a^2c^3x+\frac{1}{5}ac^2(2bc+3ad)x^5+\frac{1}{9}c(b^2c^2+6abcd+3a^2d^2)\,x^9+\frac{1}{13}d(3b^2c^2+6abcd+a^2d^2)\,x^{13}+\frac{1}{17}bd^2(3bc+2ad)x^{17}$$

```
[Out] a^2*c^3*x+1/5*a*c^2*(3*a*d+2*b*c)*x^5+1/9*c*(3*a^2*d^2+6*a*b*c*d+b^2*c^2)*x
^9+1/13*d*(a^2*d^2+6*a*b*c*d+3*b^2*c^2)*x^13+1/17*b*d^2*(2*a*d+3*b*c)*x^17+
1/21*b^2*d^3*x^21
```

Rubi [A]
time = 0.06, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {380}

$$\frac{1}{13}dx^{13}(a^2d^2+6abcd+3b^2c^2)+\frac{1}{9}cx^9(3a^2d^2+6abcd+b^2c^2)+a^2c^3x+\frac{1}{5}ac^2x^5(3ad+2bc)+\frac{1}{17}bd^2x^{17}(2ad+3bc)+\frac{1}{21}b^2d^3x^{21}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)^2*(c + d*x^4)^3,x]
```

```
[Out] a^2*c^3*x + (a*c^2*(2*b*c + 3*a*d)*x^5)/5 + (c*(b^2*c^2 + 6*a*b*c*d + 3*a^2
*d^2)*x^9)/9 + (d*(3*b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^13)/13 + (b*d^2*(3*b*
c + 2*a*d)*x^17)/17 + (b^2*d^3*x^21)/21
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int (a+bx^4)^2 (c+dx^4)^3 \, dx = \int \left(a^2c^3+ac^2(2bc+3ad)x^4+c(b^2c^2+6abcd+3a^2d^2)\,x^8+d(3b^2c^2+6abcd\right.$$
$$= a^2c^3x+\frac{1}{5}ac^2(2bc+3ad)x^5+\frac{1}{9}c(b^2c^2+6abcd+3a^2d^2)\,x^9+\frac{1}{13}d(3b^2c^2+6ab$$

Mathematica [A]
time = 0.02, size = 122, normalized size = 1.00

$$a^2c^3x+\frac{1}{5}ac^2(2bc+3ad)x^5+\frac{1}{9}c(b^2c^2+6abcd+3a^2d^2)\,x^9+\frac{1}{13}d(3b^2c^2+6abcd+a^2d^2)\,x^{13}+\frac{1}{17}bd^2(3bc+2ad)x^{17}+\frac{1}{21}b^2d^3x^{21}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^4)^2*(c + d*x^4)^3,x]
```

```
[Out] a^2*c^3*x + (a*c^2*(2*b*c + 3*a*d)*x^5)/5 + (c*(b^2*c^2 + 6*a*b*c*d + 3*a^2
*d^2)*x^9)/9 + (d*(3*b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^13)/13 + (b*d^2*(3*b*
c + 2*a*d)*x^17)/17 + (b^2*d^3*x^21)/21
```

Maple [A]
time = 0.28, size = 125, normalized size = 1.02

method	result
norman	$a^2c^3x + \left(\frac{3}{5}a^2c^2d + \frac{2}{5}ab\,c^3\right)x^5 + \left(\frac{1}{3}a^2c\,d^2 + \frac{2}{3}ab\,c^2d + \frac{1}{9}b^2c^3\right)x^9 + \left(\frac{1}{13}a^2d^3 + \frac{6}{13}abc\,d^2 + \frac{3}{13}b^2c^2d\right)$
default	$\frac{b^2d^3x^{21}}{21} + \frac{(2ab\,d^3+3b^2c\,d^2)x^{17}}{17} + \frac{(a^2d^3+6abc\,d^2+3b^2c^2d)x^{13}}{13} + \frac{(3a^2c\,d^2+6ab\,c^2d+b^2c^3)x^9}{9} + \frac{(3a^2c^2d+2ab\,c^3)x^5}{5} + a^2c$
gosper	$a^2c^3x + \frac{3}{5}x^5a^2c^2d + \frac{2}{5}x^5ab\,c^3 + \frac{1}{3}x^9a^2c\,d^2 + \frac{2}{3}x^9ab\,c^2d + \frac{1}{9}x^9b^2c^3 + \frac{1}{13}x^{13}a^2d^3 + \frac{6}{13}x^{13}abc\,d^2 + \frac{3}{13}$
risch	$a^2c^3x + \frac{3}{5}x^5a^2c^2d + \frac{2}{5}x^5ab\,c^3 + \frac{1}{3}x^9a^2c\,d^2 + \frac{2}{3}x^9ab\,c^2d + \frac{1}{9}x^9b^2c^3 + \frac{1}{13}x^{13}a^2d^3 + \frac{6}{13}x^{13}abc\,d^2 + \frac{3}{13}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)^2*(d*x^4+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/21*b^2*d^3*x^21+1/17*(2*a*b*d^3+3*b^2*c*d^2)*x^17+1/13*(a^2*d^3+6*a*b*c*d
^2+3*b^2*c^2*d)*x^13+1/9*(3*a^2*c*d^2+6*a*b*c^2*d+b^2*c^3)*x^9+1/5*(3*a^2*c
^2*d+2*a*b*c^3)*x^5+a^2*c^3*x
```

Maxima [A]
time = 0.28, size = 124, normalized size = 1.02

$$\frac{1}{21}b^2d^3x^{21} + \frac{1}{17}\left(3b^2cd^2 + 2abd^3\right)x^{17} + \frac{1}{13}\left(3b^2c^2d + 6abcd^2 + a^2d^3\right)x^{13} + \frac{1}{9}\left(b^2c^3 + 6abc^2d + 3a^2cd^2\right)x^9 + a^2c^3x + \frac{1}{5}\left(2abc^3 + 3a^2c^2d\right)x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^2*(d*x^4+c)^3,x, algorithm="maxima")
```

```
[Out] 1/21*b^2*d^3*x^21 + 1/17*(3*b^2*c*d^2 + 2*a*b*d^3)*x^17 + 1/13*(3*b^2*c^2*d
 + 6*a*b*c*d^2 + a^2*d^3)*x^13 + 1/9*(b^2*c^3 + 6*a*b*c^2*d + 3*a^2*c*d^2)*
x^9 + a^2*c^3*x + 1/5*(2*a*b*c^3 + 3*a^2*c^2*d)*x^5
```

Fricas [A]
time = 4.28, size = 124, normalized size = 1.02

$$\frac{1}{21}b^2d^3x^{21} + \frac{1}{17}\left(3b^2cd^2 + 2abd^3\right)x^{17} + \frac{1}{13}\left(3b^2c^2d + 6abcd^2 + a^2d^3\right)x^{13} + \frac{1}{9}\left(b^2c^3 + 6abc^2d + 3a^2cd^2\right)x^9 + a^2c^3x + \frac{1}{5}\left(2abc^3 + 3a^2c^2d\right)x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^2*(d*x^4+c)^3,x, algorithm="fricas")
```

```
[Out] 1/21*b^2*d^3*x^21 + 1/17*(3*b^2*c*d^2 + 2*a*b*d^3)*x^17 + 1/13*(3*b^2*c^2*d
 + 6*a*b*c*d^2 + a^2*d^3)*x^13 + 1/9*(b^2*c^3 + 6*a*b*c^2*d + 3*a^2*c*d^2)*
x^9 + a^2*c^3*x + 1/5*(2*a*b*c^3 + 3*a^2*c^2*d)*x^5
```

Sympy [A]
time = 0.02, size = 139, normalized size = 1.14

$$a^2c^3x+\frac{b^2d^3x^{21}}{21}+x^{17}\cdot\left(\frac{2abd^3}{17}+\frac{3b^2cd^2}{17}\right)+x^{13}\left(\frac{a^2d^3}{13}+\frac{6abcd^2}{13}+\frac{3b^2c^2d}{13}\right)+x^9\left(\frac{a^2cd^2}{3}+\frac{2abc^2d}{3}+\frac{b^2c^3}{9}\right)+x^5\cdot\left(\frac{3a^2c^2d}{5}+\frac{2abc^3}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)**2*(d*x**4+c)**3,x)
```

```
[Out] a**2*c**3*x + b**2*d**3*x**21/21 + x**17*(2*a*b*d**3/17 + 3*b**2*c*d**2/17)
 + x**13*(a**2*d**3/13 + 6*a*b*c*d**2/13 + 3*b**2*c**2*d/13) + x**9*(a**2*c
*d**2/3 + 2*a*b*c**2*d/3 + b**2*c**3/9) + x**5*(3*a**2*c**2*d/5 + 2*a*b*c**
3/5)
```

Giac [A]
time = 0.61, size = 132, normalized size = 1.08

$$\frac{1}{21}b^2d^3x^{21}+\frac{3}{17}b^2cd^2x^{17}+\frac{2}{17}abd^3x^{17}+\frac{3}{13}b^2c^2dx^{13}+\frac{6}{13}abcd^2x^{13}+\frac{1}{13}a^2d^3x^{13}+\frac{1}{9}b^2c^3x^9+\frac{2}{3}abc^2dx^9+\frac{1}{3}a^2cd^2x^9+\frac{2}{5}abc^3x^5+\frac{3}{5}a^2c^2dx^5+a^2c^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^2*(d*x^4+c)^3,x, algorithm="giac")
```

```
[Out] 1/21*b^2*d^3*x^21 + 3/17*b^2*c*d^2*x^17 + 2/17*a*b*d^3*x^17 + 3/13*b^2*c^2*
d*x^13 + 6/13*a*b*c*d^2*x^13 + 1/13*a^2*d^3*x^13 + 1/9*b^2*c^3*x^9 + 2/3*a*
b*c^2*d*x^9 + 1/3*a^2*c*d^2*x^9 + 2/5*a*b*c^3*x^5 + 3/5*a^2*c^2*d*x^5 + a^2
*c^3*x
```

Mupad [B]
time = 1.30, size = 116, normalized size = 0.95

$$x^9\left(\frac{a^2\,c\,d^2}{3}+\frac{2\,a\,b\,c^2\,d}{3}+\frac{b^2\,c^3}{9}\right)+x^{13}\left(\frac{a^2\,d^3}{13}+\frac{6\,a\,b\,c\,d^2}{13}+\frac{3\,b^2\,c^2\,d}{13}\right)+a^2\,c^3\,x+\frac{b^2\,d^3\,x^{21}}{21}+\frac{a\,c^2\,x^5\,(3\,a\,d+2\,b\,c)}{5}+\frac{b\,d^2\,x^{17}\,(2\,a\,d+3\,b\,c)}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)^2*(c + d*x^4)^3,x)
```

```
[Out] x^9*((b^2*c^3)/9 + (a^2*c*d^2)/3 + (2*a*b*c^2*d)/3) + x^13*((a^2*d^3)/13 +
(3*b^2*c^2*d)/13 + (6*a*b*c*d^2)/13) + a^2*c^3*x + (b^2*d^3*x^21)/21 + (a*c
^2*x^5*(3*a*d + 2*b*c))/5 + (b*d^2*x^17*(2*a*d + 3*b*c))/17
```

3.155 $\int (a+bx^4)^2 (c+dx^4)^2 \, dx$

Optimal. Leaf size=82

$$a^2c^2x+\frac{2}{5}ac(bc+ad)x^5+\frac{1}{9}\left(b^2c^2+4abcd+a^2d^2\right)x^9+\frac{2}{13}bd(bc+ad)x^{13}+\frac{1}{17}b^2d^2x^{17}$$

```
[Out] a^2*c^2*x+2/5*a*c*(a*d+b*c)*x^5+1/9*(a^2*d^2+4*a*b*c*d+b^2*c^2)*x^9+2/13*b*
d*(a*d+b*c)*x^13+1/17*b^2*d^2*x^17
```

Rubi [A]
time = 0.04, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {380}

$$\frac{1}{9}x^9\left(a^2d^2+4abcd+b^2c^2\right)+a^2c^2x+\frac{2}{13}bdx^{13}(ad+bc)+\frac{2}{5}acx^5(ad+bc)+\frac{1}{17}b^2d^2x^{17}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)^2*(c + d*x^4)^2,x]
```

```
[Out] a^2*c^2*x + (2*a*c*(b*c + a*d)*x^5)/5 + ((b^2*c^2 + 4*a*b*c*d + a^2*d^2)*x^
9)/9 + (2*b*d*(b*c + a*d)*x^13)/13 + (b^2*d^2*x^17)/17
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a+bx^4\right)^2\left(c+dx^4\right)^2\,dx &= \int \left(a^2c^2+2ac(bc+ad)x^4+\left(b^2c^2+4abcd+a^2d^2\right)x^8+2bd(bc+ad)x^{12}+b^2\right.\\ &= a^2c^2x+\frac{2}{5}ac(bc+ad)x^5+\frac{1}{9}\left(b^2c^2+4abcd+a^2d^2\right)x^9+\frac{2}{13}bd(bc+ad)x^{13}+\end{aligned}$$

Mathematica [A]
time = 0.01, size = 82, normalized size = 1.00

$$a^2c^2x+\frac{2}{5}ac(bc+ad)x^5+\frac{1}{9}\left(b^2c^2+4abcd+a^2d^2\right)x^9+\frac{2}{13}bd(bc+ad)x^{13}+\frac{1}{17}b^2d^2x^{17}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^4)^2*(c + d*x^4)^2,x]
```

```
[Out] a^2*c^2*x + (2*a*c*(b*c + a*d)*x^5)/5 + ((b^2*c^2 + 4*a*b*c*d + a^2*d^2)*x^
9)/9 + (2*b*d*(b*c + a*d)*x^13)/13 + (b^2*d^2*x^17)/17
```

Maple [A]
time = 0.27, size = 87, normalized size = 1.06

method	result
norman	$a^2c^2x + \left(\frac{2}{5}a^2cd + \frac{2}{5}ab\,c^2\right)x^5 + \left(\frac{1}{9}a^2d^2 + \frac{4}{9}abcd + \frac{1}{9}b^2c^2\right)x^9 + \left(\frac{2}{13}ab\,d^2 + \frac{2}{13}b^2cd\right)x^{13} + \frac{b^2d^2x^{17}}{17}$
default	$\frac{b^2d^2x^{17}}{17} + \frac{(2ab\,d^2+2b^2cd)x^{13}}{13} + \frac{(a^2d^2+4abcd+b^2c^2)x^9}{9} + \frac{(2a^2cd+2ab\,c^2)x^5}{5} + a^2c^2x$
gosper	$a^2c^2x + \frac{2}{5}x^5a^2cd + \frac{2}{5}x^5ab\,c^2 + \frac{1}{9}x^9a^2d^2 + \frac{4}{9}x^9abcd + \frac{1}{9}x^9b^2c^2 + \frac{2}{13}x^{13}ab\,d^2 + \frac{2}{13}x^{13}b^2cd + \frac{1}{17}b^2d^2x^{17}$
risch	$a^2c^2x + \frac{2}{5}x^5a^2cd + \frac{2}{5}x^5ab\,c^2 + \frac{1}{9}x^9a^2d^2 + \frac{4}{9}x^9abcd + \frac{1}{9}x^9b^2c^2 + \frac{2}{13}x^{13}ab\,d^2 + \frac{2}{13}x^{13}b^2cd + \frac{1}{17}b^2d^2x^{17}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)^2*(d*x^4+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/17*b^2*d^2*x^17+1/13*(2*a*b*d^2+2*b^2*c*d)*x^13+1/9*(a^2*d^2+4*a*b*c*d+b^
2*c^2)*x^9+1/5*(2*a^2*c*d+2*a*b*c^2)*x^5+a^2*c^2*x
```

Maxima [A]
time = 0.29, size = 82, normalized size = 1.00

$$\frac{1}{17}b^2d^2x^{17} + \frac{2}{13}\left(b^2cd + abd^2\right)x^{13} + \frac{1}{9}\left(b^2c^2 + 4\,abcd + a^2d^2\right)x^9 + \frac{2}{5}\left(abc^2 + a^2cd\right)x^5 + a^2c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^2*(d*x^4+c)^2,x, algorithm="maxima")
```

```
[Out] 1/17*b^2*d^2*x^17 + 2/13*(b^2*c*d + a*b*d^2)*x^13 + 1/9*(b^2*c^2 + 4*a*b*c*
d + a^2*d^2)*x^9 + 2/5*(a*b*c^2 + a^2*c*d)*x^5 + a^2*c^2*x
```

Fricas [A]
time = 3.79, size = 82, normalized size = 1.00

$$\frac{1}{17}b^2d^2x^{17} + \frac{2}{13}\left(b^2cd + abd^2\right)x^{13} + \frac{1}{9}\left(b^2c^2 + 4\,abcd + a^2d^2\right)x^9 + \frac{2}{5}\left(abc^2 + a^2cd\right)x^5 + a^2c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^2*(d*x^4+c)^2,x, algorithm="fricas")
```

```
[Out] 1/17*b^2*d^2*x^17 + 2/13*(b^2*c*d + a*b*d^2)*x^13 + 1/9*(b^2*c^2 + 4*a*b*c*
d + a^2*d^2)*x^9 + 2/5*(a*b*c^2 + a^2*c*d)*x^5 + a^2*c^2*x
```

Sympy [A]
time = 0.02, size = 97, normalized size = 1.18

$$a^2c^2x + \frac{b^2d^2x^{17}}{17} + x^{13}\cdot\left(\frac{2abd^2}{13} + \frac{2b^2cd}{13}\right) + x^9\left(\frac{a^2d^2}{9} + \frac{4abcd}{9} + \frac{b^2c^2}{9}\right) + x^5\cdot\left(\frac{2a^2cd}{5} + \frac{2abc^2}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)**2*(d*x**4+c)**2,x)
```

```
[Out] a**2*c**2*x + b**2*d**2*x**17/17 + x**13*(2*a*b*d**2/13 + 2*b**2*c*d/13) +
x**9*(a**2*d**2/9 + 4*a*b*c*d/9 + b**2*c**2/9) + x**5*(2*a**2*c*d/5 + 2*a*b
*c**2/5)
```

Giac [A]
time = 0.82, size = 91, normalized size = 1.11

$$\frac{1}{17}\,b^2d^2x^{17}+\frac{2}{13}\,b^2cdx^{13}+\frac{2}{13}\,abd^2x^{13}+\frac{1}{9}\,b^2c^2x^9+\frac{4}{9}\,abcdx^9+\frac{1}{9}\,a^2d^2x^9+\frac{2}{5}\,abc^2x^5+\frac{2}{5}\,a^2cdx^5+a^2c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^2*(d*x^4+c)^2,x, algorithm="giac")
```

```
[Out] 1/17*b^2*d^2*x^17 + 2/13*b^2*c*d*x^13 + 2/13*a*b*d^2*x^13 + 1/9*b^2*c^2*x^9
 + 4/9*a*b*c*d*x^9 + 1/9*a^2*d^2*x^9 + 2/5*a*b*c^2*x^5 + 2/5*a^2*c*d*x^5 +
a^2*c^2*x
```

Mupad [B]
time = 0.05, size = 75, normalized size = 0.91

$$x^9\left(\frac{a^2\,d^2}{9}+\frac{4\,a\,b\,c\,d}{9}+\frac{b^2\,c^2}{9}\right)+a^2\,c^2\,x+\frac{b^2\,d^2\,x^{17}}{17}+\frac{2\,a\,c\,x^5\,\left(a\,d+b\,c\right)}{5}+\frac{2\,b\,d\,x^{13}\,\left(a\,d+b\,c\right)}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)^2*(c + d*x^4)^2,x)
```

```
[Out] x^9*((a^2*d^2)/9 + (b^2*c^2)/9 + (4*a*b*c*d)/9) + a^2*c^2*x + (b^2*d^2*x^17
)/17 + (2*a*c*x^5*(a*d + b*c))/5 + (2*b*d*x^13*(a*d + b*c))/13
```

3.156 $\int (a+bx^4)^2 (c+dx^4)\, dx$

Optimal. Leaf size=50

$$a^2cx+\frac{1}{5}a(2bc+ad)x^5+\frac{1}{9}b(bc+2ad)x^9+\frac{1}{13}b^2dx^{13}$$

```
[Out] a^2*c*x+1/5*a*(a*d+2*b*c)*x^5+1/9*b*(2*a*d+b*c)*x^9+1/13*b^2*d*x^13
```

Rubi [A]
time = 0.02, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {380}

$$a^2cx+\frac{1}{9}bx^9(2ad+bc)+\frac{1}{5}ax^5(ad+2bc)+\frac{1}{13}b^2dx^{13}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)^2*(c + d*x^4),x]
```

```
[Out] a^2*c*x + (a*(2*b*c + a*d)*x^5)/5 + (b*(b*c + 2*a*d)*x^9)/9 + (b^2*d*x^13)/
13
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int (a+bx^4)^2 (c+dx^4)\, dx &= \int \left(a^2c+a(2bc+ad)x^4+b(bc+2ad)x^8+b^2dx^{12}\right) dx\\ &= a^2cx+\frac{1}{5}a(2bc+ad)x^5+\frac{1}{9}b(bc+2ad)x^9+\frac{1}{13}b^2dx^{13}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 50, normalized size = 1.00

$$a^2cx+\frac{1}{5}a(2bc+ad)x^5+\frac{1}{9}b(bc+2ad)x^9+\frac{1}{13}b^2dx^{13}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^4)^2*(c + d*x^4),x]
```

[Out] a^2*c*x + (a*(2*b*c + a*d)*x^5)/5 + (b*(b*c + 2*a*d)*x^9)/9 + (b^2*d*x^13)/13

Maple [A]
time = 0.28, size = 49, normalized size = 0.98

method	result	size
default	$\frac{b^2 d\,x^{13}}{13}+\frac{(2abd+b^2c)x^9}{9}+\frac{(a^2d+2abc)x^5}{5}+a^2cx$	49
norman	$\frac{b^2 d\,x^{13}}{13}+\left(\frac{2}{9}abd+\frac{1}{9}b^2c\right)x^9+\left(\frac{1}{5}a^2d+\frac{2}{5}abc\right)x^5+a^2cx$	49
gosper	$\frac{1}{13}b^2d\,x^{13}+\frac{2}{9}x^9abd+\frac{1}{9}x^9b^2c+\frac{1}{5}x^5a^2d+\frac{2}{5}x^5abc+a^2cx$	51
risch	$\frac{1}{13}b^2d\,x^{13}+\frac{2}{9}x^9abd+\frac{1}{9}x^9b^2c+\frac{1}{5}x^5a^2d+\frac{2}{5}x^5abc+a^2cx$	51

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^4+a)^2*(d*x^4+c),x,method=_RETURNVERBOSE)

[Out] 1/13*b^2*d*x^13+1/9*(2*a*b*d+b^2*c)*x^9+1/5*(a^2*d+2*a*b*c)*x^5+a^2*c*x

Maxima [A]
time = 0.27, size = 48, normalized size = 0.96

$$\frac{1}{13}\,b^2dx^{13}+\frac{1}{9}\,\left(b^2c+2\,abd\right)x^9+\frac{1}{5}\,\left(2\,abc+a^2d\right)x^5+a^2cx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^4+a)^2*(d*x^4+c),x, algorithm="maxima")

[Out] 1/13*b^2*d*x^13 + 1/9*(b^2*c + 2*a*b*d)*x^9 + 1/5*(2*a*b*c + a^2*d)*x^5 + a^2*c*x

Fricas [A]
time = 3.86, size = 48, normalized size = 0.96

$$\frac{1}{13}\,b^2dx^{13}+\frac{1}{9}\,\left(b^2c+2\,abd\right)x^9+\frac{1}{5}\,\left(2\,abc+a^2d\right)x^5+a^2cx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^4+a)^2*(d*x^4+c),x, algorithm="fricas")

[Out] 1/13*b^2*d*x^13 + 1/9*(b^2*c + 2*a*b*d)*x^9 + 1/5*(2*a*b*c + a^2*d)*x^5 + a^2*c*x

Sympy [A]
time = 0.01, size = 53, normalized size = 1.06

$$a^2cx+\frac{b^2dx^{13}}{13}+x^9\cdot\left(\frac{2abd}{9}+\frac{b^2c}{9}\right)+x^5\left(\frac{a^2d}{5}+\frac{2abc}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)**2*(d*x**4+c),x)
```

```
[Out] a**2*c*x + b**2*d*x**13/13 + x**9*(2*a*b*d/9 + b**2*c/9) + x**5*(a**2*d/5 +
 2*a*b*c/5)
```

Giac [A]
time = 0.74, size = 50, normalized size = 1.00

$$\frac{1}{13}\,b^2dx^{13}+\frac{1}{9}\,b^2cx^9+\frac{2}{9}\,abdx^9+\frac{2}{5}\,abcx^5+\frac{1}{5}\,a^2dx^5+a^2cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^2*(d*x^4+c),x, algorithm="giac")
```

```
[Out] 1/13*b^2*d*x^13 + 1/9*b^2*c*x^9 + 2/9*a*b*d*x^9 + 2/5*a*b*c*x^5 + 1/5*a^2*d
*x^5 + a^2*c*x
```

Mupad [B]
time = 0.04, size = 48, normalized size = 0.96

$$x^5\left(\frac{d\,a^2}{5}+\frac{2\,b\,c\,a}{5}\right)+x^9\left(\frac{c\,b^2}{9}+\frac{2\,a\,d\,b}{9}\right)+\frac{b^2\,d\,x^{13}}{13}+a^2\,c\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)^2*(c + d*x^4),x)
```

```
[Out] x^5*((a^2*d)/5 + (2*a*b*c)/5) + x^9*((b^2*c)/9 + (2*a*b*d)/9) + (b^2*d*x^13
)/13 + a^2*c*x
```

3.157 $\int \frac{(a+bx^4)^2}{c+dx^4}\,dx$

Optimal. Leaf size=253

$$-\frac{b(bc-2ad)x}{d^2}+\frac{b^2x^5}{5d}-\frac{(bc-ad)^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right)}{2\sqrt{2}\,c^{3/4}d^{9/4}}+\frac{(bc-ad)^2\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right)}{2\sqrt{2}\,c^{3/4}d^{9/4}}-\frac{(bc-ad)^2\,\mathrm{l}}{}$$

```
[Out] -b*(-2*a*d+b*c)*x/d^2+1/5*b^2*x^5/d+1/4*(-a*d+b*c)^2*arctan(-1+d^(1/4)*x*2^
(1/2)/c^(1/4))/c^(3/4)/d^(9/4)*2^(1/2)+1/4*(-a*d+b*c)^2*arctan(1+d^(1/4)*x*
2^(1/2)/c^(1/4))/c^(3/4)/d^(9/4)*2^(1/2)-1/8*(-a*d+b*c)^2*ln(-c^(1/4)*d^(1/
4)*x*2^(1/2)+c^(1/2)+x^2*d^(1/2))/c^(3/4)/d^(9/4)*2^(1/2)+1/8*(-a*d+b*c)^2*
ln(c^(1/4)*d^(1/4)*x*2^(1/2)+c^(1/2)+x^2*d^(1/2))/c^(3/4)/d^(9/4)*2^(1/2)
```

Rubi [A]

time = 0.13, antiderivative size = 253, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {398, 217, 1179, 642, 1176, 631, 210}

$$-\frac{(bc-ad)^2\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right)}{2\sqrt{2}\,c^{3/4}d^{9/4}}+\frac{(bc-ad)^2\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{d}\,x}{\sqrt[4]{c}}+1\right)}{2\sqrt{2}\,c^{3/4}d^{9/4}}-\frac{(bc-ad)^2\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\,x+\sqrt{c}+\sqrt{d}\,x^2\right)}{4\sqrt{2}\,c^{3/4}d^{9/4}}+\frac{(bc-ad)^2\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\,x+\sqrt{c}+\sqrt{d}\,x^2\right)}{4\sqrt{2}\,c^{3/4}d^{9/4}}-\frac{bx(bc-2ad)}{d^2}+\frac{b^2x^5}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)^2/(c + d*x^4),x]
```

```
[Out] -((b*(b*c - 2*a*d)*x)/d^2) + (b^2*x^5)/(5*d) - ((b*c - a*d)^2*ArcTan[1 - (S
qrt[2]*d^(1/4)*x)/c^(1/4)])/(2*Sqrt[2]*c^(3/4)*d^(9/4)) + ((b*c - a*d)^2*Ar
cTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/(2*Sqrt[2]*c^(3/4)*d^(9/4)) - ((b*c
- a*d)^2*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(4*Sqrt[2]
*c^(3/4)*d^(9/4)) + ((b*c - a*d)^2*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x
+ Sqrt[d]*x^2])/(4*Sqrt[2]*c^(3/4)*d^(9/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(a+bx^4)^2}{c+dx^4}\,dx = \int \left(-\frac{b(bc-2ad)}{d^2}+\frac{b^2x^4}{d}+\frac{b^2c^2-2abcd+a^2d^2}{d^2\,(c+dx^4)}\right)dx$$

$$= -\frac{b(bc-2ad)x}{d^2}+\frac{b^2x^5}{5d}+\frac{(bc-ad)^2\int\frac{1}{c+dx^4}\,dx}{d^2}$$

$$= -\frac{b(bc-2ad)x}{d^2}+\frac{b^2x^5}{5d}+\frac{(bc-ad)^2\int\frac{\sqrt{c}-\sqrt{d}\,x^2}{c+dx^4}\,dx}{2\sqrt{c}\,d^2}+\frac{(bc-ad)^2\int\frac{\sqrt{c}+\sqrt{d}\,x^2}{c+dx^4}\,dx}{2\sqrt{c}\,d^2}$$

$$= -\frac{b(bc-2ad)x}{d^2}+\frac{b^2x^5}{5d}+\frac{(bc-ad)^2\int\frac{1}{\frac{\sqrt{c}}{\sqrt{d}}-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{d}}+x^2}\,dx}{4\sqrt{c}\,d^{5/2}}+\frac{(bc-ad)^2\int\frac{1}{\frac{\sqrt{c}}{\sqrt{d}}+\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{d}}+\cdots}}{4\sqrt{c}\,d^{5/2}}$$

$$= -\frac{b(bc-2ad)x}{d^2}+\frac{b^2x^5}{5d}-\frac{(bc-ad)^2\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\,x+\sqrt{d}\,x^2\right)}{4\sqrt{2}\,c^{3/4}d^{9/4}}+\frac{(bc-ad)^2\,\mathrm{l}\cdots}{\cdots}$$

$$= -\frac{b(bc-2ad)x}{d^2}+\frac{b^2x^5}{5d}-\frac{(bc-ad)^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right)}{2\sqrt{2}\,c^{3/4}d^{9/4}}+\frac{(bc-ad)^2\tan^{-1}\left(1+\cdots\right)}{2\sqrt{2}\,c^{3/4}d^{9/4}}$$

Mathematica [A]
time = 0.08, size = 231, normalized size = 0.91

$$\frac{-40bc^{3/4}\sqrt[4]{d}\,(bc-2ad)x+8b^2c^{3/4}d^{5/4}x^5-10\sqrt{2}\,(bc-ad)^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right)+10\sqrt{2}\,(bc-ad)^2\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right)-5\sqrt{2}\,(bc-ad)^2\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\,x+\sqrt{d}\,x^2\right)+5\sqrt{2}\,(bc-ad)^2\log\left(\sqrt{c}+\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\,x+\sqrt{d}\,x^2\right)}{40c^{3/4}d^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^4)^2/(c + d*x^4),x]
```

```
[Out] (-40*b*c^(3/4)*d^(1/4)*(b*c - 2*a*d)*x + 8*b^2*c^(3/4)*d^(5/4)*x^5 - 10*Sqr
t[2]*(b*c - a*d)^2*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)] + 10*Sqrt[2]*(b*
c - a*d)^2*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)] - 5*Sqrt[2]*(b*c - a*d)^
2*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2] + 5*Sqrt[2]*(b*c -
 a*d)^2*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(40*c^(3/4)
*d^(9/4))
```

Maple [A]
time = 0.25, size = 150, normalized size = 0.59

method	result

risch	$\frac{b^2x^5}{5d} + \frac{2bax}{d} - \frac{b^2cx}{d^2} + \frac{\sum_{\_R=\text{RootOf}(d\_Z^4+c)} \frac{(a^2d^2-2abcd+b^2c^2)\ln(x-\_R)}{\_R^3}}{4d^3}$
default	$\frac{b(\frac{1}{5}bd\,x^5+2adx-bcx)}{d^2} + \frac{(a^2d^2-2abcd+b^2c^2)(\frac{c}{d})^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+(\frac{c}{d})^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{c}{d}}}{x^2-(\frac{c}{d})^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{c}{d}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{(\frac{c}{d})^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}}{(\frac{c}{d})}\right.\right.}{8d^2c}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)^2/(d*x^4+c),x,method=_RETURNVERBOSE)
```

```
[Out] b/d^2*(1/5*b*d*x^5+2*a*d*x-b*c*x)+1/8*(a^2*d^2-2*a*b*c*d+b^2*c^2)/d^2*(c/d)
^(1/4)/c*2^(1/2)*(ln((x^2+(c/d)^(1/4)*x*2^(1/2)+(c/d)^(1/2))/(x^2-(c/d)^(1/
4)*x*2^(1/2)+(c/d)^(1/2)))+2*arctan(2^(1/2)/(c/d)^(1/4)*x+1)+2*arctan(2^(1/
2)/(c/d)^(1/4)*x-1))
```

Maxima [A]
time = 0.50, size = 286, normalized size = 1.13

$$\frac{b^2dx^5-5\,(b^2c-2\,abd)x}{5\,d^2}+\frac{\frac{2\sqrt{2}\,(b^2c^2-2\,abcd+a^2d^2)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}\,x+\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}+\frac{2\sqrt{2}\,(b^2c^2-2\,abcd+a^2d^2)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}\,x-\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}+\frac{\sqrt{2}\,(b^2c^2-2\,abcd+a^2d^2)\log\left(\sqrt{d}\,x^2+\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}x+\sqrt{c}\right)}{c^{\frac{3}{4}}d^{\frac{1}{4}}}-\frac{\sqrt{2}\,(b^2c^2-2\,abcd+a^2d^2)\log\left(\sqrt{d}\,x^2-\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}x+\sqrt{c}\right)}{c^{\frac{3}{4}}d^{\frac{1}{4}}}}{8\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^2/(d*x^4+c),x, algorithm="maxima")
```

```
[Out] 1/5*(b^2*d*x^5 - 5*(b^2*c - 2*a*b*d)*x)/d^2 + 1/8*(2*sqrt(2)*(b^2*c^2 - 2*a
*b*c*d + a^2*d^2)*arctan(1/2*sqrt(2)*(2*sqrt(d)*x + sqrt(2)*c^(1/4)*d^(1/4)
)/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + 2*sqrt(2)*(b^2*c
^2 - 2*a*b*c*d + a^2*d^2)*arctan(1/2*sqrt(2)*(2*sqrt(d)*x - sqrt(2)*c^(1/4)
*d^(1/4))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + sqrt(2)*
(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(sqrt(d)*x^2 + sqrt(2)*c^(1/4)*d^(1/4)*x
 + sqrt(c))/(c^(3/4)*d^(1/4)) - sqrt(2)*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log
(sqrt(d)*x^2 - sqrt(2)*c^(1/4)*d^(1/4)*x + sqrt(c))/(c^(3/4)*d^(1/4)))/d^2
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1239 vs. 2(186) = 372.
time = 3.67, size = 1239, normalized size = 4.90

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^2/(d*x^4+c),x, algorithm="fricas")
```

```
[Out] 1/20*(4*b^2*d*x^5 + 20*d^2*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2
- 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2
```

```
*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(c^3*d^9))^(1/4)*arctan((sqrt(c^2*d^4*s
qrt(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 7
0*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7
 + a^8*d^8)/(c^3*d^9)) + (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a
^3*b*c*d^3 + a^4*d^4)*x^2)*c^2*d^7*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*
c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28
*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(c^3*d^9))^(3/4) - (b^2*c^4*d^7
 - 2*a*b*c^3*d^8 + a^2*c^2*d^9)*x*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c
^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*
a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(c^3*d^9))^(3/4))/(b^8*c^8 - 8*a
*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 -
 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)) + 5*d^
2*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70
*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7
+ a^8*d^8)/(c^3*d^9))^(1/4)*log(c*d^2*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b
^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 +
 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(c^3*d^9))^(1/4) + (b^2*c^2
- 2*a*b*c*d + a^2*d^2)*x) - 5*d^2*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c
^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*
a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(c^3*d^9))^(1/4)*log(-c*d^2*(-(b
^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b
^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*
d^8)/(c^3*d^9))^(1/4) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x) - 20*(b^2*c - 2*
a*b*d)*x)/d^2
```

Sympy [A]
time = 0.69, size = 187, normalized size = 0.74

$$\frac{b^2x^5}{5d}+x\left(\frac{2ab}{d}-\frac{b^2c}{d^2}\right)+\operatorname{RootSum}\left(256t^4c^3d^9+a^8d^8-8a^7bcd^7+28a^6b^2c^2d^6-56a^5b^3c^3d^5+70a^4b^4c^4d^4-56a^3b^5c^5d^3+28a^2b^6c^6d^2-8ab^7c^7d+b^8c^8,\left(t\mapsto t\log\left(\frac{4tcd^2}{a^2d^2-2abcd+b^2c^2}+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)**2/(d*x**4+c),x)
[Out] b**2*x**5/(5*d) + x*(2*a*b/d - b**2*c/d**2) + RootSum(256*_t**4*c**3*d**9 +
 a**8*d**8 - 8*a**7*b*c*d**7 + 28*a**6*b**2*c**2*d**6 - 56*a**5*b**3*c**3*d
**5 + 70*a**4*b**4*c**4*d**4 - 56*a**3*b**5*c**5*d**3 + 28*a**2*b**6*c**6*d
**2 - 8*a*b**7*c**7*d + b**8*c**8, Lambda(_t, _t*log(4*_t*c*d**2/(a**2*d**2
 - 2*a*b*c*d + b**2*c**2) + x)))
```

Giac [A]
time = 0.66, size = 353, normalized size = 1.40

$$\frac{\sqrt{2}\left((cd^3)^{\frac{1}{4}}b^2c^2-2(cd^3)^{\frac{1}{4}}abcd+(cd^3)^{\frac{1}{4}}a^2d^2\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{4cd^3}+\frac{\sqrt{2}\left((cd^3)^{\frac{1}{4}}b^2c^2-2(cd^3)^{\frac{1}{4}}abcd+(cd^3)^{\frac{1}{4}}a^2d^2\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{4cd^3}+\frac{\sqrt{2}\left((cd^3)^{\frac{1}{4}}b^2c^2-2(cd^3)^{\frac{1}{4}}abcd+(cd^3)^{\frac{1}{4}}a^2d^2\right)\log\left(x^2+\sqrt{2}x\left(\frac{c}{d}\right)^{\frac{1}{4}}+\sqrt{\frac{c}{d}}\right)}{8cd^3}-\frac{\sqrt{2}\left((cd^3)^{\frac{1}{4}}b^2c^2-2(cd^3)^{\frac{1}{4}}abcd+(cd^3)^{\frac{1}{4}}a^2d^2\right)\log\left(x^2-\sqrt{2}x\left(\frac{c}{d}\right)^{\frac{1}{4}}+\sqrt{\frac{c}{d}}\right)}{8cd^3}+\frac{b^2d^4x^5-5b^2cd^3x+10abd^4x}{5d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^2/(d*x^4+c),x, algorithm="giac")
```

[Out] 1/4*sqrt(2)*((c*d^3)^(1/4)*b^2*c^2 - 2*(c*d^3)^(1/4)*a*b*c*d + (c*d^3)^(1/4
)*a^2*d^2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(c/d)^(1/4))/(c/d)^(1/4))/(c*d
^3) + 1/4*sqrt(2)*((c*d^3)^(1/4)*b^2*c^2 - 2*(c*d^3)^(1/4)*a*b*c*d + (c*d^3
)^(1/4)*a^2*d^2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(c/d)^(1/4))/(c/d)^(1/4)
)/(c*d^3) + 1/8*sqrt(2)*((c*d^3)^(1/4)*b^2*c^2 - 2*(c*d^3)^(1/4)*a*b*c*d +
(c*d^3)^(1/4)*a^2*d^2)*log(x^2 + sqrt(2)*x*(c/d)^(1/4) + sqrt(c/d))/(c*d^3)
 - 1/8*sqrt(2)*((c*d^3)^(1/4)*b^2*c^2 - 2*(c*d^3)^(1/4)*a*b*c*d + (c*d^3)^(
1/4)*a^2*d^2)*log(x^2 - sqrt(2)*x*(c/d)^(1/4) + sqrt(c/d))/(c*d^3) + 1/5*(b
^2*d^4*x^5 - 5*b^2*c*d^3*x + 10*a*b*d^4*x)/d^5

Mupad [B]

time = 1.48, size = 1081, normalized size = 4.27

$$\frac{b^2\,x^5}{5\,d}-x\,\left(\frac{b^2\,c}{d^2}-\frac{2\,a\,b}{d}\right)+\frac{\mathrm{atan}\left(\frac{\frac{{\left(a\,d-b\,c\right)}^2\,\left(\frac{x\,\left(a^4\,d^4-4\,a^3\,b\,c\,d^3+6\,a^2\,b^2\,c^2\,d^2-4\,a\,b^3\,c^3\,d+b^4\,c^4\right)}{d}-\frac{{\left(a\,d-b\,c\right)}^2\,\left(4\,a^2\,c\,d^3-8\,a\,b\,c^2\,d^2+4\,b^2\,c^3\,d\right)}{4\,{\left(-c\right)}^{3/4}\,d^{9/4}}\right)\,1{}\mathrm{i}}{{\left(-c\right)}^{3/4}\,d^{9/4}}+\frac{{\left(a\,d-b\,c\right)}^2\,\left(\frac{x\,\left(a^4\,d^4-4\,a^3\,b\,c\,d^3+6\,a^2\,b^2\,c^2\,d^2-4\,a\,b^3\,c^3\,d+b^4\,c^4\right)}{d}+\frac{{\left(a\,d-b\,c\right)}^2\,\left(4\,a^2\,c\,d^3-8\,a\,b\,c^2\,d^2+4\,b^2\,c^3\,d\right)}{4\,{\left(-c\right)}^{3/4}\,d^{9/4}}\right)\,1{}\mathrm{i}}{{\left(-c\right)}^{3/4}\,d^{9/4}}}{\frac{{\left(a\,d-b\,c\right)}^2\,\left(\frac{x\,\left(a^4\,d^4-4\,a^3\,b\,c\,d^3+6\,a^2\,b^2\,c^2\,d^2-4\,a\,b^3\,c^3\,d+b^4\,c^4\right)}{d}-\frac{{\left(a\,d-b\,c\right)}^2\,\left(4\,a^2\,c\,d^3-8\,a\,b\,c^2\,d^2+4\,b^2\,c^3\,d\right)}{4\,{\left(-c\right)}^{3/4}\,d^{9/4}}\right)}{{\left(-c\right)}^{3/4}\,d^{9/4}}-\frac{{\left(a\,d-b\,c\right)}^2\,\left(\frac{x\,\left(a^4\,d^4-4\,a^3\,b\,c\,d^3+6\,a^2\,b^2\,c^2\,d^2-4\,a\,b^3\,c^3\,d+b^4\,c^4\right)}{d}+\frac{{\left(a\,d-b\,c\right)}^2\,\left(4\,a^2\,c\,d^3-8\,a\,b\,c^2\,d^2+4\,b^2\,c^3\,d\right)}{4\,{\left(-c\right)}^{3/4}\,d^{9/4}}\right)}{{\left(-c\right)}^{3/4}\,d^{9/4}}}\right)\,{\left(a\,d-b\,c\right)}^2\,1{}\mathrm{i}}{2\,{\left(-c\right)}^{3/4}\,d^{9/4}}+\frac{\mathrm{atan}\left(\frac{\frac{{\left(a\,d-b\,c\right)}^2\,\left(\frac{x\,\left(a^4\,d^4-4\,a^3\,b\,c\,d^3+6\,a^2\,b^2\,c^2\,d^2-4\,a\,b^3\,c^3\,d+b^4\,c^4\right)}{d}-\frac{{\left(a\,d-b\,c\right)}^2\,\left(4\,a^2\,c\,d^3-8\,a\,b\,c^2\,d^2+4\,b^2\,c^3\,d\right)\,1{}\mathrm{i}}{4\,{\left(-c\right)}^{3/4}\,d^{9/4}}\right)}{{\left(-c\right)}^{3/4}\,d^{9/4}}+\frac{{\left(a\,d-b\,c\right)}^2\,\left(\frac{x\,\left(a^4\,d^4-4\,a^3\,b\,c\,d^3+6\,a^2\,b^2\,c^2\,d^2-4\,a\,b^3\,c^3\,d+b^4\,c^4\right)}{d}+\frac{{\left(a\,d-b\,c\right)}^2\,\left(4\,a^2\,c\,d^3-8\,a\,b\,c^2\,d^2+4\,b^2\,c^3\,d\right)\,1{}\mathrm{i}}{4\,{\left(-c\right)}^{3/4}\,d^{9/4}}\right)}{{\left(-c\right)}^{3/4}\,d^{9/4}}}{\frac{{\left(a\,d-b\,c\right)}^2\,\left(\frac{x\,\left(a^4\,d^4-4\,a^3\,b\,c\,d^3+6\,a^2\,b^2\,c^2\,d^2-4\,a\,b^3\,c^3\,d+b^4\,c^4\right)}{d}-\frac{{\left(a\,d-b\,c\right)}^2\,\left(4\,a^2\,c\,d^3-8\,a\,b\,c^2\,d^2+4\,b^2\,c^3\,d\right)\,1{}\mathrm{i}}{4\,{\left(-c\right)}^{3/4}\,d^{9/4}}\right)\,1{}\mathrm{i}}{{\left(-c\right)}^{3/4}\,d^{9/4}}-\frac{{\left(a\,d-b\,c\right)}^2\,\left(\frac{x\,\left(a^4\,d^4-4\,a^3\,b\,c\,d^3+6\,a^2\,b^2\,c^2\,d^2-4\,a\,b^3\,c^3\,d+b^4\,c^4\right)}{d}+\frac{{\left(a\,d-b\,c\right)}^2\,\left(4\,a^2\,c\,d^3-8\,a\,b\,c^2\,d^2+4\,b^2\,c^3\,d\right)\,1{}\mathrm{i}}{4\,{\left(-c\right)}^{3/4}\,d^{9/4}}\right)\,1{}\mathrm{i}}{{\left(-c\right)}^{3/4}\,d^{9/4}}}\right)\,{\left(a\,d-b\,c\right)}^2}{2\,{\left(-c\right)}^{3/4}\,d^{9/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*x^4)^2/(c + d*x^4),x)

[Out] (b^2*x^5)/(5*d) - x*((b^2*c)/d^2 - (2*a*b)/d) + (atan((((a*d - b*c)^2*((x*(
a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3))/d -
 ((a*d - b*c)^2*(4*a^2*c*d^3 + 4*b^2*c^3*d - 8*a*b*c^2*d^2))/(4*(-c)^(3/4)*
d^(9/4)))*1i)/((-c)^(3/4)*d^(9/4)) + ((a*d - b*c)^2*((x*(a^4*d^4 + b^4*c^4
+ 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3))/d + ((a*d - b*c)^2*(4
*a^2*c*d^3 + 4*b^2*c^3*d - 8*a*b*c^2*d^2))/(4*(-c)^(3/4)*d^(9/4)))*1i)/((-c
)^(3/4)*d^(9/4)))/(((a*d - b*c)^2*((x*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^
2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3))/d - ((a*d - b*c)^2*(4*a^2*c*d^3 + 4*b^2
*c^3*d - 8*a*b*c^2*d^2))/(4*(-c)^(3/4)*d^(9/4))))/((-c)^(3/4)*d^(9/4)) - ((
a*d - b*c)^2*((x*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4
*a^3*b*c*d^3))/d + ((a*d - b*c)^2*(4*a^2*c*d^3 + 4*b^2*c^3*d - 8*a*b*c^2*d^
2))/(4*(-c)^(3/4)*d^(9/4))))/((-c)^(3/4)*d^(9/4))))*(a*d - b*c)^2*1i)/(2*(-
c)^(3/4)*d^(9/4)) + (atan((((a*d - b*c)^2*((x*(a^4*d^4 + b^4*c^4 + 6*a^2*b^
2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3))/d - ((a*d - b*c)^2*(4*a^2*c*d^3
 + 4*b^2*c^3*d - 8*a*b*c^2*d^2)*1i)/(4*(-c)^(3/4)*d^(9/4))))/((-c)^(3/4)*d^
(9/4)) + ((a*d - b*c)^2*((x*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^
3*c^3*d - 4*a^3*b*c*d^3))/d + ((a*d - b*c)^2*(4*a^2*c*d^3 + 4*b^2*c^3*d - 8
*a*b*c^2*d^2)*1i)/(4*(-c)^(3/4)*d^(9/4))))/((-c)^(3/4)*d^(9/4)))/(((a*d - b
c)^2((x*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*
c*d^3))/d - ((a*d - b*c)^2*(4*a^2*c*d^3 + 4*b^2*c^3*d - 8*a*b*c^2*d^2)*1i)/
(4*(-c)^(3/4)*d^(9/4)))*1i)/((-c)^(3/4)*d^(9/4)) - ((a*d - b*c)^2*((x*(a^4*
d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3))/d + ((a
*d - b*c)^2*(4*a^2*c*d^3 + 4*b^2*c^3*d - 8*a*b*c^2*d^2)*1i)/(4*(-c)^(3/4)*d
^(9/4)))*1i)/((-c)^(3/4)*d^(9/4))))*(a*d - b*c)^2)/(2*(-c)^(3/4)*d^(9/4))

3.158 $\int \frac{(a+bx^4)^2}{(c+dx^4)^2} \, dx$

Optimal. Leaf size=291

$$\frac{b^2x}{d^2}+\frac{(bc-ad)^2x}{4cd^2\left(c+dx^4\right)}+\frac{(bc-ad)(5bc+3ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{8\sqrt{2}c^{7/4}d^{9/4}}-\frac{(bc-ad)(5bc+3ad)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{8\sqrt{2}c^{7/4}d^{9/4}}$$

```
[Out] b^2*x/d^2+1/4*(-a*d+b*c)^2*x/c/d^2/(d*x^4+c)-1/16*(-a*d+b*c)*(3*a*d+5*b*c)*
arctan(-1+d^(1/4)*x*2^(1/2)/c^(1/4))/c^(7/4)/d^(9/4)*2^(1/2)-1/16*(-a*d+b*c
)*(3*a*d+5*b*c)*arctan(1+d^(1/4)*x*2^(1/2)/c^(1/4))/c^(7/4)/d^(9/4)*2^(1/2)
+1/32*(-a*d+b*c)*(3*a*d+5*b*c)*ln(-c^(1/4)*d^(1/4)*x*2^(1/2)+c^(1/2)+x^2*d^
(1/2))/c^(7/4)/d^(9/4)*2^(1/2)-1/32*(-a*d+b*c)*(3*a*d+5*b*c)*ln(c^(1/4)*d^(
1/4)*x*2^(1/2)+c^(1/2)+x^2*d^(1/2))/c^(7/4)/d^(9/4)*2^(1/2)
```

Rubi [A]

time = 0.38, antiderivative size = 291, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {398, 393, 217, 1179, 642, 1176, 631, 210}

$$\frac{(bc-ad)(3ad+5bc)\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{8\sqrt{2}c^{7/4}d^{9/4}}-\frac{(bc-ad)(3ad+5bc)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{8\sqrt{2}c^{7/4}d^{9/4}}+\frac{(bc-ad)(3ad+5bc)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{16\sqrt{2}c^{7/4}d^{9/4}}-\frac{(bc-ad)(3ad+5bc)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{16\sqrt{2}c^{7/4}d^{9/4}}+\frac{x(bc-ad)^2}{4cd^2\left(c+dx^4\right)}+\frac{b^2x}{d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)^2/(c + d*x^4)^2,x]
```

```
[Out] (b^2*x)/d^2 + ((b*c - a*d)^2*x)/(4*c*d^2*(c + d*x^4)) + ((b*c - a*d)*(5*b*c
 + 3*a*d)*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/(8*Sqrt[2]*c^(7/4)*d^(9/
4)) - ((b*c - a*d)*(5*b*c + 3*a*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)])
/(8*Sqrt[2]*c^(7/4)*d^(9/4)) + ((b*c - a*d)*(5*b*c + 3*a*d)*Log[Sqrt[c] - S
qrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(16*Sqrt[2]*c^(7/4)*d^(9/4)) - ((b
*c - a*d)*(5*b*c + 3*a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]
*x^2])/(16*Sqrt[2]*c^(7/4)*d^(9/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
```

```
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(a+bx^4)^2}{(c+dx^4)^2}\,dx = \int \left(\frac{b^2}{d^2} - \frac{b^2c^2-a^2d^2+2bd(bc-ad)x^4}{d^2\,(c+dx^4)^2}\right)\,dx$$

$$= \frac{b^2x}{d^2} - \frac{\int \frac{b^2c^2-a^2d^2+2bd(bc-ad)x^4}{(c+dx^4)^2}\,dx}{d^2}$$

$$= \frac{b^2x}{d^2} + \frac{(bc-ad)^2x}{4cd^2\,(c+dx^4)} - \frac{((bc-ad)(5bc+3ad))\int \frac{1}{c+dx^4}\,dx}{4cd^2}$$

$$= \frac{b^2x}{d^2} + \frac{(bc-ad)^2x}{4cd^2\,(c+dx^4)} - \frac{((bc-ad)(5bc+3ad))\int \frac{\sqrt{c}-\sqrt{d}\,x^2}{c+dx^4}\,dx}{8c^{3/2}d^2} - \frac{((bc-ad)(5bc+3ac}{8c^{3/}}$$

$$= \frac{b^2x}{d^2} + \frac{(bc-ad)^2x}{4cd^2\,(c+dx^4)} - \frac{((bc-ad)(5bc+3ad))\int \frac{1}{\frac{\sqrt{c}}{\sqrt{d}}-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{d}}+x^2}\,dx}{16c^{3/2}d^{5/2}} - \frac{((bc-ad)(5b}{}$$

$$= \frac{b^2x}{d^2} + \frac{(bc-ad)^2x}{4cd^2\,(c+dx^4)} + \frac{(bc-ad)(5bc+3ad)\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\,x+\sqrt{d}\,x^2\right)}{16\sqrt{2}\,c^{7/4}d^{9/4}} - \frac{(bc}{}$$

$$= \frac{b^2x}{d^2} + \frac{(bc-ad)^2x}{4cd^2\,(c+dx^4)} + \frac{(bc-ad)(5bc+3ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right)}{8\sqrt{2}\,c^{7/4}d^{9/4}} - \frac{(bc-ad)(5bc+}{}$$

Mathematica [A]

time = 0.12, size = 298, normalized size = 1.02

$$\frac{32b^2\sqrt[4]{d}\,x + \frac{8\sqrt[4]{d}\,(bc-ad)^2x}{c(c+dx^4)} + \frac{2\sqrt{2}\,(5b^2c^2-2abcd-3a^2d^2)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right)}{c^{7/4}} - \frac{2\sqrt{2}\,(5b^2c^2-2abcd-3a^2d^2)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right)}{c^{7/4}} + \frac{\sqrt{2}\,(5b^2c^2-2abcd-3a^2d^2)\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\,x+\sqrt{d}\,x^2\right)}{c^{7/4}} - \frac{\sqrt{2}\,(5b^2c^2-2abcd-3a^2d^2)\log\left(\sqrt{c}+\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\,x+\sqrt{d}\,x^2\right)}{c^{7/4}}}{32d^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^4)^2/(c + d*x^4)^2,x]
```

```
[Out] (32*b^2*d^(1/4)*x + (8*d^(1/4)*(b*c - a*d)^2*x)/(c*(c + d*x^4)) + (2*Sqrt[2
]*(5*b^2*c^2 - 2*a*b*c*d - 3*a^2*d^2)*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4
)])/c^(7/4) - (2*Sqrt[2]*(5*b^2*c^2 - 2*a*b*c*d - 3*a^2*d^2)*ArcTan[1 + (Sq
rt[2]*d^(1/4)*x)/c^(1/4)])/c^(7/4) + (Sqrt[2]*(5*b^2*c^2 - 2*a*b*c*d - 3*a^
2*d^2)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/c^(7/4) - (S
qrt[2]*(5*b^2*c^2 - 2*a*b*c*d - 3*a^2*d^2)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^
(1/4)*x + Sqrt[d]*x^2])/c^(7/4))/(32*d^(9/4))
```

Maple [A]

time = 0.27, size = 175, normalized size = 0.60

method	result
risch	$\frac{b^2x}{d^2}+\frac{(a^2d^2-2abcd+b^2c^2)x}{4c\,d^2(d\,x^4+c)}+\frac{\sum_{\_R=\mathrm{RootOf}\left(d\_Z^4+c\right)}\frac{\left(3a^2d^2+2abcd-5b^2c^2\right)\ln\left(x-\_R\right)}{\_R^3}}{16d^3c}$
default	$\frac{b^2x}{d^2}+\frac{\frac{\left(a^2d^2-2abcd+b^2c^2\right)x}{4c\left(d\,x^4+c\right)}+\frac{\left(3a^2d^2+2abcd-5b^2c^2\right)\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{c}{d}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{c}{d}}}{x^2-\left(\frac{c}{d}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{c}{d}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}}{\left(\frac{c}{d}\right)}\right.\right.}{32c^2}}{d^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)^2/(d*x^4+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] b^2*x/d^2+1/d^2*(1/4*(a^2*d^2-2*a*b*c*d+b^2*c^2)/c*x/(d*x^4+c)+1/32*(3*a^2*
d^2+2*a*b*c*d-5*b^2*c^2)/c^2*(c/d)^(1/4)*2^(1/2)*(ln((x^2+(c/d)^(1/4)*x*2^(
1/2)+(c/d)^(1/2))/(x^2-(c/d)^(1/4)*x*2^(1/2)+(c/d)^(1/2)))+2*arctan(2^(1/2)
/(c/d)^(1/4)*x+1)+2*arctan(2^(1/2)/(c/d)^(1/4)*x-1)))
```

Maxima [A]
time = 0.48, size = 319, normalized size = 1.10

$$\frac{(b^2c^2-2\,abcd+a^2d^2)x}{4\,(cd^3x^4+c^2d^2)}+\frac{b^2x}{d^2}-\frac{\frac{2\sqrt{2}\,(5b^2c^2-2abcd-3a^2d^2)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}\,x+\sqrt{2}\,c^{\frac14}d^{\frac14}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}+\frac{2\sqrt{2}\,(5b^2c^2-2abcd-3a^2d^2)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}\,x-\sqrt{2}\,c^{\frac14}d^{\frac14}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}+\frac{\sqrt{2}\,(5b^2c^2-2abcd-3a^2d^2)\log\left(\sqrt{d}\,x^2+\sqrt{2}\,c^{\frac14}d^{\frac14}x+\sqrt{c}\right)}{c^{\frac34}d^{\frac14}}-\frac{\sqrt{2}\,(5b^2c^2-2abcd-3a^2d^2)\log\left(\sqrt{d}\,x^2-\sqrt{2}\,c^{\frac14}d^{\frac14}x+\sqrt{c}\right)}{c^{\frac34}d^{\frac14}}}{32\,cd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^2/(d*x^4+c)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x/(c*d^3*x^4 + c^2*d^2) + b^2*x/d^2 - 1
/32*(2*sqrt(2)*(5*b^2*c^2 - 2*a*b*c*d - 3*a^2*d^2)*arctan(1/2*sqrt(2)*(2*sq
rt(d)*x + sqrt(2)*c^(1/4)*d^(1/4))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqr
t(c)*sqrt(d))) + 2*sqrt(2)*(5*b^2*c^2 - 2*a*b*c*d - 3*a^2*d^2)*arctan(1/2*s
qrt(2)*(2*sqrt(d)*x - sqrt(2)*c^(1/4)*d^(1/4))/sqrt(sqrt(c)*sqrt(d)))/(sqrt
(c)*sqrt(sqrt(c)*sqrt(d))) + sqrt(2)*(5*b^2*c^2 - 2*a*b*c*d - 3*a^2*d^2)*lo
g(sqrt(d)*x^2 + sqrt(2)*c^(1/4)*d^(1/4)*x + sqrt(c))/(c^(3/4)*d^(1/4)) - sq
rt(2)*(5*b^2*c^2 - 2*a*b*c*d - 3*a^2*d^2)*log(sqrt(d)*x^2 - sqrt(2)*c^(1/4)
*d^(1/4)*x + sqrt(c))/(c^(3/4)*d^(1/4)))/(c*d^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1335 vs. $2(224) = 448$.
time = 3.25, size = 1335, normalized size = 4.59

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^2/(d*x^4+c)^2,x, algorithm="fricas")
```

```
[Out] 1/16*(16*b^2*c*d*x^5 + 4*(c*d^3*x^4 + c^2*d^2)*(-(625*b^8*c^8 - 1000*a*b^7*
c^7*d - 900*a^2*b^6*c^6*d^2 + 1640*a^3*b^5*c^5*d^3 + 646*a^4*b^4*c^4*d^4 -
984*a^5*b^3*c^3*d^5 - 324*a^6*b^2*c^2*d^6 + 216*a^7*b*c*d^7 + 81*a^8*d^8)/(
c^7*d^9))^(1/4)*arctan((sqrt(c^4*d^4*sqrt(-(625*b^8*c^8 - 1000*a*b^7*c^7*d
- 900*a^2*b^6*c^6*d^2 + 1640*a^3*b^5*c^5*d^3 + 646*a^4*b^4*c^4*d^4 - 984*a^
5*b^3*c^3*d^5 - 324*a^6*b^2*c^2*d^6 + 216*a^7*b*c*d^7 + 81*a^8*d^8)/(c^7*d^
9)) + (25*b^4*c^4 - 20*a*b^3*c^3*d - 26*a^2*b^2*c^2*d^2 + 12*a^3*b*c*d^3 +
9*a^4*d^4)*x^2)*c^5*d^7*(-(625*b^8*c^8 - 1000*a*b^7*c^7*d - 900*a^2*b^6*c^6
*d^2 + 1640*a^3*b^5*c^5*d^3 + 646*a^4*b^4*c^4*d^4 - 984*a^5*b^3*c^3*d^5 - 3
24*a^6*b^2*c^2*d^6 + 216*a^7*b*c*d^7 + 81*a^8*d^8)/(c^7*d^9))^(3/4) + (5*b^
2*c^7*d^7 - 2*a*b*c^6*d^8 - 3*a^2*c^5*d^9)*x*(-(625*b^8*c^8 - 1000*a*b^7*c^
7*d - 900*a^2*b^6*c^6*d^2 + 1640*a^3*b^5*c^5*d^3 + 646*a^4*b^4*c^4*d^4 - 98
4*a^5*b^3*c^3*d^5 - 324*a^6*b^2*c^2*d^6 + 216*a^7*b*c*d^7 + 81*a^8*d^8)/(c^
7*d^9))^(3/4))/(625*b^8*c^8 - 1000*a*b^7*c^7*d - 900*a^2*b^6*c^6*d^2 + 1640
*a^3*b^5*c^5*d^3 + 646*a^4*b^4*c^4*d^4 - 984*a^5*b^3*c^3*d^5 - 324*a^6*b^2*
c^2*d^6 + 216*a^7*b*c*d^7 + 81*a^8*d^8)) + (c*d^3*x^4 + c^2*d^2)*(-(625*b^8
*c^8 - 1000*a*b^7*c^7*d - 900*a^2*b^6*c^6*d^2 + 1640*a^3*b^5*c^5*d^3 + 646*
a^4*b^4*c^4*d^4 - 984*a^5*b^3*c^3*d^5 - 324*a^6*b^2*c^2*d^6 + 216*a^7*b*c*d
^7 + 81*a^8*d^8)/(c^7*d^9))^(1/4)*log(c^2*d^2*(-(625*b^8*c^8 - 1000*a*b^7*c
^7*d - 900*a^2*b^6*c^6*d^2 + 1640*a^3*b^5*c^5*d^3 + 646*a^4*b^4*c^4*d^4 - 9
84*a^5*b^3*c^3*d^5 - 324*a^6*b^2*c^2*d^6 + 216*a^7*b*c*d^7 + 81*a^8*d^8)/(c
^7*d^9))^(1/4) - (5*b^2*c^2 - 2*a*b*c*d - 3*a^2*d^2)*x) - (c*d^3*x^4 + c^2*
d^2)*(-(625*b^8*c^8 - 1000*a*b^7*c^7*d - 900*a^2*b^6*c^6*d^2 + 1640*a^3*b^5
*c^5*d^3 + 646*a^4*b^4*c^4*d^4 - 984*a^5*b^3*c^3*d^5 - 324*a^6*b^2*c^2*d^6
+ 216*a^7*b*c*d^7 + 81*a^8*d^8)/(c^7*d^9))^(1/4)*log(-c^2*d^2*(-(625*b^8*c^
8 - 1000*a*b^7*c^7*d - 900*a^2*b^6*c^6*d^2 + 1640*a^3*b^5*c^5*d^3 + 646*a^4
*b^4*c^4*d^4 - 984*a^5*b^3*c^3*d^5 - 324*a^6*b^2*c^2*d^6 + 216*a^7*b*c*d^7
+ 81*a^8*d^8)/(c^7*d^9))^(1/4) - (5*b^2*c^2 - 2*a*b*c*d - 3*a^2*d^2)*x) + 4
*(5*b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x)/(c*d^3*x^4 + c^2*d^2)
```

Sympy [A]
time = 2.01, size = 219, normalized size = 0.75

$$\frac{b^2x}{d^2} + \frac{x(a^2d^2 - 2abcd + b^2c^2)}{4c^2d^2 + 4cd^3x^4} + \mathrm{RootSum}\left(65536t^4c^7d^9 + 81a^8d^8 + 216a^7bcd^7 - 324a^6b^2c^2d^6 - 984a^5b^3c^3d^5 + 646a^4b^4c^4d^4 + 1640a^3b^5c^5d^3 - 900a^2b^6c^6d^2 - 1000ab^7c^7d + 625b^8c^8, \left(t \mapsto t\log\left(\frac{16tc^2d^2}{3a^2d^2 + 2abcd - 5b^2c^2} + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)**2/(d*x**4+c)**2,x)
```

```
[Out] b**2*x/d**2 + x*(a**2*d**2 - 2*a*b*c*d + b**2*c**2)/(4*c**2*d**2 + 4*c*d**3
*x**4) + RootSum(65536*_t**4*c**7*d**9 + 81*a**8*d**8 + 216*a**7*b*c*d**7 -
 324*a**6*b**2*c**2*d**6 - 984*a**5*b**3*c**3*d**5 + 646*a**4*b**4*c**4*d**
4 + 1640*a**3*b**5*c**5*d**3 - 900*a**2*b**6*c**6*d**2 - 1000*a*b**7*c**7*d
 + 625*b**8*c**8, Lambda(_t, _t*log(16*_t*c**2*d**2/(3*a**2*d**2 + 2*a*b*c*
d - 5*b**2*c**2) + x)))
```

Giac [A]

time = 0.51, size = 376, normalized size = 1.29

$$\frac{b^2x}{d^2}-\frac{\sqrt{2}\left(5\left(cd^3\right)^{\frac14}b^2c^2-2\left(cd^3\right)^{\frac14}abcd-3\left(cd^3\right)^{\frac14}a^2d^2\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{c}{d}\right)^{\frac14}\right)}{2\left(\frac{c}{d}\right)^{\frac14}}\right)}{16\,c^2d^3}-\frac{\sqrt{2}\left(5\left(cd^3\right)^{\frac14}b^2c^2-2\left(cd^3\right)^{\frac14}abcd-3\left(cd^3\right)^{\frac14}a^2d^2\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{c}{d}\right)^{\frac14}\right)}{2\left(\frac{c}{d}\right)^{\frac14}}\right)}{16\,c^2d^3}-\frac{\sqrt{2}\left(5\left(cd^3\right)^{\frac14}b^2c^2-2\left(cd^3\right)^{\frac14}abcd-3\left(cd^3\right)^{\frac14}a^2d^2\right)\log\left(x^2+\sqrt{2}\,x\left(\frac{c}{d}\right)^{\frac14}+\sqrt{\frac{c}{d}}\right)}{32\,c^2d^3}+\frac{\sqrt{2}\left(5\left(cd^3\right)^{\frac14}b^2c^2-2\left(cd^3\right)^{\frac14}abcd-3\left(cd^3\right)^{\frac14}a^2d^2\right)\log\left(x^2-\sqrt{2}\,x\left(\frac{c}{d}\right)^{\frac14}+\sqrt{\frac{c}{d}}\right)}{32\,c^2d^3}+\frac{b^2c^2x-2abcdx+a^2d^2x}{4\,(dx^4+c)cd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^2/(d*x^4+c)^2,x, algorithm="giac")
```

```
[Out] b^2*x/d^2 - 1/16*sqrt(2)*(5*(c*d^3)^(1/4)*b^2*c^2 - 2*(c*d^3)^(1/4)*a*b*c*d
 - 3*(c*d^3)^(1/4)*a^2*d^2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(c/d)^(1/4))/
(c/d)^(1/4))/(c^2*d^3) - 1/16*sqrt(2)*(5*(c*d^3)^(1/4)*b^2*c^2 - 2*(c*d^3)^
(1/4)*a*b*c*d - 3*(c*d^3)^(1/4)*a^2*d^2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*
(c/d)^(1/4))/(c/d)^(1/4))/(c^2*d^3) - 1/32*sqrt(2)*(5*(c*d^3)^(1/4)*b^2*c^2
 - 2*(c*d^3)^(1/4)*a*b*c*d - 3*(c*d^3)^(1/4)*a^2*d^2)*log(x^2 + sqrt(2)*x*(
c/d)^(1/4) + sqrt(c/d))/(c^2*d^3) + 1/32*sqrt(2)*(5*(c*d^3)^(1/4)*b^2*c^2 -
 2*(c*d^3)^(1/4)*a*b*c*d - 3*(c*d^3)^(1/4)*a^2*d^2)*log(x^2 - sqrt(2)*x*(c/
d)^(1/4) + sqrt(c/d))/(c^2*d^3) + 1/4*(b^2*c^2*x - 2*a*b*c*d*x + a^2*d^2*x)
/((d*x^4 + c)*c*d^2)
```

Mupad [B]
time = 1.54, size = 1254, normalized size = 4.31

$$\frac{b^2x}{d^2}+\frac{x\left(a^2d^2-2abcd+b^2c^2\right)}{4c\left(d^3x^4+cd^2\right)}+\frac{\mathrm{atan}\left(\frac{\frac{\left(\frac{x\left(9a^4d^4+12a^3bcd^3-26a^2b^2c^2d^2-20ab^3c^3d+25b^4c^4\right)}{4c^2d}-\frac{(ad-bc)(3ad+5bc)\left(12a^2d^3+8abcd^2-20b^2c^2d\right)}{16(-c)^{7/4}d^{9/4}}\right)(ad-bc)(3ad+5bc)\,1\mathrm{i}}{16(-c)^{7/4}d^{9/4}}+\frac{\left(\frac{x\left(9a^4d^4+12a^3bcd^3-26a^2b^2c^2d^2-20ab^3c^3d+25b^4c^4\right)}{4c^2d}+\frac{(ad-bc)(3ad+5bc)\left(12a^2d^3+8abcd^2-20b^2c^2d\right)}{16(-c)^{7/4}d^{9/4}}\right)(ad-bc)(3ad+5bc)\,1\mathrm{i}}{16(-c)^{7/4}d^{9/4}}}{\frac{\left(\frac{x\left(9a^4d^4+12a^3bcd^3-26a^2b^2c^2d^2-20ab^3c^3d+25b^4c^4\right)}{4c^2d}-\frac{(ad-bc)(3ad+5bc)\left(12a^2d^3+8abcd^2-20b^2c^2d\right)}{16(-c)^{7/4}d^{9/4}}\right)(ad-bc)(3ad+5bc)}{16(-c)^{7/4}d^{9/4}}-\frac{\left(\frac{x\left(9a^4d^4+12a^3bcd^3-26a^2b^2c^2d^2-20ab^3c^3d+25b^4c^4\right)}{4c^2d}+\frac{(ad-bc)(3ad+5bc)\left(12a^2d^3+8abcd^2-20b^2c^2d\right)}{16(-c)^{7/4}d^{9/4}}\right)(ad-bc)(3ad+5bc)}{16(-c)^{7/4}d^{9/4}}}\right)(ad-bc)(3ad+5bc)\,1\mathrm{i}}{8(-c)^{7/4}d^{9/4}}+\frac{\mathrm{atan}\left(\frac{\frac{\left(\frac{x\left(9a^4d^4+12a^3bcd^3-26a^2b^2c^2d^2-20ab^3c^3d+25b^4c^4\right)}{4c^2d}-\frac{(ad-bc)(3ad+5bc)\left(12a^2d^3+8abcd^2-20b^2c^2d\right)1\mathrm{i}}{16(-c)^{7/4}d^{9/4}}\right)(ad-bc)(3ad+5bc)}{16(-c)^{7/4}d^{9/4}}+\frac{\left(\frac{x\left(9a^4d^4+12a^3bcd^3-26a^2b^2c^2d^2-20ab^3c^3d+25b^4c^4\right)}{4c^2d}+\frac{(ad-bc)(3ad+5bc)\left(12a^2d^3+8abcd^2-20b^2c^2d\right)1\mathrm{i}}{16(-c)^{7/4}d^{9/4}}\right)(ad-bc)(3ad+5bc)}{16(-c)^{7/4}d^{9/4}}}{\frac{\left(\frac{x\left(9a^4d^4+12a^3bcd^3-26a^2b^2c^2d^2-20ab^3c^3d+25b^4c^4\right)}{4c^2d}-\frac{(ad-bc)(3ad+5bc)\left(12a^2d^3+8abcd^2-20b^2c^2d\right)1\mathrm{i}}{16(-c)^{7/4}d^{9/4}}\right)(ad-bc)(3ad+5bc)\,1\mathrm{i}}{16(-c)^{7/4}d^{9/4}}-\frac{\left(\frac{x\left(9a^4d^4+12a^3bcd^3-26a^2b^2c^2d^2-20ab^3c^3d+25b^4c^4\right)}{4c^2d}+\frac{(ad-bc)(3ad+5bc)\left(12a^2d^3+8abcd^2-20b^2c^2d\right)1\mathrm{i}}{16(-c)^{7/4}d^{9/4}}\right)(ad-bc)(3ad+5bc)\,1\mathrm{i}}{16(-c)^{7/4}d^{9/4}}}\right)(ad-bc)(3ad+5bc)}{8(-c)^{7/4}d^{9/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)^2/(c + d*x^4)^2,x)
```

```
[Out] (b^2*x)/d^2 + (x*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))/(4*c*(c*d^2 + d^3*x^4)) +
 (atan(((((x*(9*a^4*d^4 + 25*b^4*c^4 - 26*a^2*b^2*c^2*d^2 - 20*a*b^3*c^3*d
+ 12*a^3*b*c*d^3))/(4*c^2*d) - ((a*d - b*c)*(3*a*d + 5*b*c)*(12*a^2*d^3 - 2
0*b^2*c^2*d + 8*a*b*c*d^2))/(16*(-c)^(7/4)*d^(9/4)))*(a*d - b*c)*(3*a*d + 5
*b*c)*1i)/(16*(-c)^(7/4)*d^(9/4)) + (((x*(9*a^4*d^4 + 25*b^4*c^4 - 26*a^2*b
^2*c^2*d^2 - 20*a*b^3*c^3*d + 12*a^3*b*c*d^3))/(4*c^2*d) + ((a*d - b*c)*(3*
a*d + 5*b*c)*(12*a^2*d^3 - 20*b^2*c^2*d + 8*a*b*c*d^2))/(16*(-c)^(7/4)*d^(9
/4)))*(a*d - b*c)*(3*a*d + 5*b*c)*1i)/(16*(-c)^(7/4)*d^(9/4)))/((((x*(9*a^4
*d^4 + 25*b^4*c^4 - 26*a^2*b^2*c^2*d^2 - 20*a*b^3*c^3*d + 12*a^3*b*c*d^3))/
(4*c^2*d) - ((a*d - b*c)*(3*a*d + 5*b*c)*(12*a^2*d^3 - 20*b^2*c^2*d + 8*a*b
*c*d^2))/(16*(-c)^(7/4)*d^(9/4)))*(a*d - b*c)*(3*a*d + 5*b*c))/(16*(-c)^(7/
4)*d^(9/4)) - (((x*(9*a^4*d^4 + 25*b^4*c^4 - 26*a^2*b^2*c^2*d^2 - 20*a*b^3*
c^3*d + 12*a^3*b*c*d^3))/(4*c^2*d) + ((a*d - b*c)*(3*a*d + 5*b*c)*(12*a^2*d
^3 - 20*b^2*c^2*d + 8*a*b*c*d^2))/(16*(-c)^(7/4)*d^(9/4)))*(a*d - b*c)*(3*a
*d + 5*b*c))/(16*(-c)^(7/4)*d^(9/4))))*(a*d - b*c)*(3*a*d + 5*b*c)*1i)/(8*(
-c)^(7/4)*d^(9/4)) + (atan(((((x*(9*a^4*d^4 + 25*b^4*c^4 - 26*a^2*b^2*c^2*d
^2 - 20*a*b^3*c^3*d + 12*a^3*b*c*d^3))/(4*c^2*d) - ((a*d - b*c)*(3*a*d + 5*
b*c)*(12*a^2*d^3 - 20*b^2*c^2*d + 8*a*b*c*d^2)*1i)/(16*(-c)^(7/4)*d^(9/4)))
*(a*d - b*c)*(3*a*d + 5*b*c))/(16*(-c)^(7/4)*d^(9/4)) + (((x*(9*a^4*d^4 + 2
```

```
5*b^4*c^4 - 26*a^2*b^2*c^2*d^2 - 20*a*b^3*c^3*d + 12*a^3*b*c*d^3))/(4*c^2*d
) + ((a*d - b*c)*(3*a*d + 5*b*c)*(12*a^2*d^3 - 20*b^2*c^2*d + 8*a*b*c*d^2)*
1i)/(16*(-c)^(7/4)*d^(9/4)))*(a*d - b*c)*(3*a*d + 5*b*c))/(16*(-c)^(7/4)*d^
(9/4)))/((((x*(9*a^4*d^4 + 25*b^4*c^4 - 26*a^2*b^2*c^2*d^2 - 20*a*b^3*c^3*d
 + 12*a^3*b*c*d^3))/(4*c^2*d) - ((a*d - b*c)*(3*a*d + 5*b*c)*(12*a^2*d^3 -
20*b^2*c^2*d + 8*a*b*c*d^2)*1i)/(16*(-c)^(7/4)*d^(9/4)))*(a*d - b*c)*(3*a*d
 + 5*b*c)*1i)/(16*(-c)^(7/4)*d^(9/4)) - (((x*(9*a^4*d^4 + 25*b^4*c^4 - 26*a
^2*b^2*c^2*d^2 - 20*a*b^3*c^3*d + 12*a^3*b*c*d^3))/(4*c^2*d) + ((a*d - b*c)
*(3*a*d + 5*b*c)*(12*a^2*d^3 - 20*b^2*c^2*d + 8*a*b*c*d^2)*1i)/(16*(-c)^(7/
4)*d^(9/4)))*(a*d - b*c)*(3*a*d + 5*b*c)*1i)/(16*(-c)^(7/4)*d^(9/4))))*(a*d
 - b*c)*(3*a*d + 5*b*c))/(8*(-c)^(7/4)*d^(9/4))
```

3.159 $\int \frac{(a+bx^4)^2}{(c+dx^4)^3} \, dx$

Optimal. Leaf size=349

$$-\frac{(bc-ad)x(a+bx^4)}{8cd(c+dx^4)^2}-\frac{(bc-ad)(5bc+7ad)x}{32c^2d^2(c+dx^4)}-\frac{(5b^2c^2+6abcd+21a^2d^2)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{64\sqrt{2}c^{11/4}d^{9/4}}+\frac{(5b^2c^2+}{}$$

```
[Out] -1/8*(-a*d+b*c)*x*(b*x^4+a)/c/d/(d*x^4+c)^2-1/32*(-a*d+b*c)*(7*a*d+5*b*c)*x
/c^2/d^2/(d*x^4+c)+1/128*(21*a^2*d^2+6*a*b*c*d+5*b^2*c^2)*arctan(-1+d^(1/4)
*x*2^(1/2)/c^(1/4))/c^(11/4)/d^(9/4)*2^(1/2)+1/128*(21*a^2*d^2+6*a*b*c*d+5*
b^2*c^2)*arctan(1+d^(1/4)*x*2^(1/2)/c^(1/4))/c^(11/4)/d^(9/4)*2^(1/2)-1/256
*(21*a^2*d^2+6*a*b*c*d+5*b^2*c^2)*ln(-c^(1/4)*d^(1/4)*x*2^(1/2)+c^(1/2)+x^2
*d^(1/2))/c^(11/4)/d^(9/4)*2^(1/2)+1/256*(21*a^2*d^2+6*a*b*c*d+5*b^2*c^2)*l
n(c^(1/4)*d^(1/4)*x*2^(1/2)+c^(1/2)+x^2*d^(1/2))/c^(11/4)/d^(9/4)*2^(1/2)
```

Rubi [A]
time = 0.23, antiderivative size = 349, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {424, 393, 217, 1179, 642, 1176, 631, 210}

$$-\frac{(21a^2d^2+6abcd+5b^2c^2)\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{64\sqrt{2}c^{11/4}d^{9/4}}+\frac{(21a^2d^2+6abcd+5b^2c^2)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{64\sqrt{2}c^{11/4}d^{9/4}}-\frac{(21a^2d^2+6abcd+5b^2c^2)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{128\sqrt{2}c^{11/4}d^{9/4}}+\frac{(21a^2d^2+6abcd+5b^2c^2)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{128\sqrt{2}c^{11/4}d^{9/4}}-\frac{x(bc-ad)(7ad+5bc)}{32c^2d^2(c+dx^4)}-\frac{x(a+bx^4)(bc-ad)}{8cd(c+dx^4)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)^2/(c + d*x^4)^3,x]
```

```
[Out] -1/8*((b*c - a*d)*x*(a + b*x^4))/(c*d*(c + d*x^4)^2) - ((b*c - a*d)*(5*b*c
+ 7*a*d)*x)/(32*c^2*d^2*(c + d*x^4)) - ((5*b^2*c^2 + 6*a*b*c*d + 21*a^2*d^2
)*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/(64*Sqrt[2]*c^(11/4)*d^(9/4)) +
((5*b^2*c^2 + 6*a*b*c*d + 21*a^2*d^2)*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4
)])/(64*Sqrt[2]*c^(11/4)*d^(9/4)) - ((5*b^2*c^2 + 6*a*b*c*d + 21*a^2*d^2)*L
og[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(128*Sqrt[2]*c^(11/4
)*d^(9/4)) + ((5*b^2*c^2 + 6*a*b*c*d + 21*a^2*d^2)*Log[Sqrt[c] + Sqrt[2]*c^
(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(128*Sqrt[2]*c^(11/4)*d^(9/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
```

```
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
```

```
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(a+bx^4)^2}{(c+dx^4)^3}\,dx = -\frac{(bc-ad)x(a+bx^4)}{8cd\,(c+dx^4)^2} + \frac{\int \frac{a(bc+7ad)+b(5bc+3ad)x^4}{(c+dx^4)^2}\,dx}{8cd}$$

$$= -\frac{(bc-ad)x(a+bx^4)}{8cd\,(c+dx^4)^2} - \frac{(bc-ad)(5bc+7ad)x}{32c^2d^2\,(c+dx^4)} + \frac{(5b^2c^2+6abcd+21a^2d^2)\int \frac{1}{c+dx^4}\,dx}{32c^2d^2}$$

$$= -\frac{(bc-ad)x(a+bx^4)}{8cd\,(c+dx^4)^2} - \frac{(bc-ad)(5bc+7ad)x}{32c^2d^2\,(c+dx^4)} + \frac{(5b^2c^2+6abcd+21a^2d^2)\int \frac{\sqrt{c}-\sqrt{d}\,x}{c+dx^4}}{64c^{5/2}d^2}$$

$$= -\frac{(bc-ad)x(a+bx^4)}{8cd\,(c+dx^4)^2} - \frac{(bc-ad)(5bc+7ad)x}{32c^2d^2\,(c+dx^4)} + \frac{(5b^2c^2+6abcd+21a^2d^2)\int \frac{1}{\frac{\sqrt{c}}{\sqrt{d}}-\frac{\sqrt{2}}{\sqrt[4]{}}}}{128c^{5/2}d^{5/2}}$$

$$= -\frac{(bc-ad)x(a+bx^4)}{8cd\,(c+dx^4)^2} - \frac{(bc-ad)(5bc+7ad)x}{32c^2d^2\,(c+dx^4)} - \frac{(5b^2c^2+6abcd+21a^2d^2)\log\left(\sqrt{c}-\right.}{128\sqrt{2}\,c^{11/4}d^{9/}}$$

$$= -\frac{(bc-ad)x(a+bx^4)}{8cd\,(c+dx^4)^2} - \frac{(bc-ad)(5bc+7ad)x}{32c^2d^2\,(c+dx^4)} - \frac{(5b^2c^2+6abcd+21a^2d^2)\tan^{-1}\left(1-\right.}{64\sqrt{2}\,c^{11/4}d^{9/4}}$$

Mathematica [A]
time = 0.13, size = 319, normalized size = 0.91

$$\frac{\frac{32c^{7/4}\sqrt[4]{d}\,(bc-ad)^2x}{(c+dx^4)^2} - \frac{8c^{3/4}\sqrt[4]{d}\,(9b^2c^2-2abcd-7a^2d^2)x}{c+dx^4} - 2\sqrt{2}\,(5b^2c^2+6abcd+21a^2d^2)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right) + 2\sqrt{2}\,(5b^2c^2+6abcd+21a^2d^2)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right) - \sqrt{2}\,(5b^2c^2+6abcd+21a^2d^2)\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\,x+\sqrt{d}\,x^2\right) + \sqrt{2}\,(5b^2c^2+6abcd+21a^2d^2)\log\left(\sqrt{c}+\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\,x+\sqrt{d}\,x^2\right)}{256c^{11/4}d^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^4)^2/(c + d*x^4)^3,x]
```

```
[Out] ((32*c^(7/4)*d^(1/4)*(b*c - a*d)^2*x)/(c + d*x^4)^2 - (8*c^(3/4)*d^(1/4)*(9
*b^2*c^2 - 2*a*b*c*d - 7*a^2*d^2)*x)/(c + d*x^4) - 2*Sqrt[2]*(5*b^2*c^2 + 6
*a*b*c*d + 21*a^2*d^2)*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)] + 2*Sqrt[2]*
(5*b^2*c^2 + 6*a*b*c*d + 21*a^2*d^2)*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)
] - Sqrt[2]*(5*b^2*c^2 + 6*a*b*c*d + 21*a^2*d^2)*Log[Sqrt[c] - Sqrt[2]*c^(1
/4)*d^(1/4)*x + Sqrt[d]*x^2] + Sqrt[2]*(5*b^2*c^2 + 6*a*b*c*d + 21*a^2*d^2)
*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(256*c^(11/4)*d^(9
/4))
```

Maple [A]
time = 0.27, size = 206, normalized size = 0.59

method	result
risch	$\frac{\frac{\left(7a^2d^2+2abcd-9b^2c^2\right)x^5}{32c^2d}+\frac{\left(11a^2d^2-6abcd-5b^2c^2\right)x}{32d^2c}}{\left(d\,x^4+c\right)^2}+\frac{\sum_{\_R=\mathrm{RootOf}\left(d\,\_Z^4+c\right)}\frac{\left(21a^2d^2+6abcd+5b^2c^2\right)\ln\left(x-\_R\right)}{\_R^3}}{128c^2d^3}$
default	$\frac{\frac{\left(7a^2d^2+2abcd-9b^2c^2\right)x^5}{32c^2d}+\frac{\left(11a^2d^2-6abcd-5b^2c^2\right)x}{32d^2c}}{\left(d\,x^4+c\right)^2}+\frac{\left(21a^2d^2+6abcd+5b^2c^2\right)\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\left(\ln\left(\frac{x^2+\left(\frac{c}{d}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{c}{d}}}{x^2-\left(\frac{c}{d}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{c}{d}}}\right)+2\,\mathrm{arcta}\right.}{256c^3d^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)^2/(d*x^4+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] (1/32*(7*a^2*d^2+2*a*b*c*d-9*b^2*c^2)/c^2/d*x^5+1/32*(11*a^2*d^2-6*a*b*c*d-
5*b^2*c^2)/d^2/c*x)/(d*x^4+c)^2+1/256*(21*a^2*d^2+6*a*b*c*d+5*b^2*c^2)/c^3/
d^2*(c/d)^(1/4)*2^(1/2)*(ln((x^2+(c/d)^(1/4)*x*2^(1/2)+(c/d)^(1/2))/(x^2-(c
/d)^(1/4)*x*2^(1/2)+(c/d)^(1/2)))+2*arctan(2^(1/2)/(c/d)^(1/4)*x+1)+2*arcta
n(2^(1/2)/(c/d)^(1/4)*x-1))
```

Maxima [A]
time = 0.50, size = 361, normalized size = 1.03

$$-\frac{(9\,b^2c^2d-2\,abcd^2-7\,a^2d^3)x^5+(5\,b^2c^3+6\,abc^2d-11\,a^2cd^2)x}{32\,(c^2d^4x^8+2\,c^3d^3x^4+c^4d^2)}+\frac{\frac{2\,\sqrt{2}\,(5\,b^2c^2+6\,abcd+21\,a^2d^2)\arctan\left(\frac{\sqrt{2}\,\left(2\,\sqrt{d}\,x+\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\,\sqrt{\sqrt{c}\,\sqrt{d}}}\right)}{\sqrt{c}\,\sqrt{\sqrt{c}\,\sqrt{d}}}+\frac{2\,\sqrt{2}\,(5\,b^2c^2+6\,abcd+21\,a^2d^2)\arctan\left(\frac{\sqrt{2}\,\left(2\,\sqrt{d}\,x-\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\,\sqrt{\sqrt{c}\,\sqrt{d}}}\right)}{\sqrt{c}\,\sqrt{\sqrt{c}\,\sqrt{d}}}+\frac{\sqrt{2}\,(5\,b^2c^2+6\,abcd+21\,a^2d^2)\log\left(\sqrt{d}\,x^2+\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}x+\sqrt{c}\right)}{c^{\frac{3}{4}}d^{\frac{1}{4}}}-\frac{\sqrt{2}\,(5\,b^2c^2+6\,abcd+21\,a^2d^2)\log\left(\sqrt{d}\,x^2-\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}x+\sqrt{c}\right)}{c^{\frac{3}{4}}d^{\frac{1}{4}}}}{256\,c^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^2/(d*x^4+c)^3,x, algorithm="maxima")
```

```
[Out] -1/32*((9*b^2*c^2*d - 2*a*b*c*d^2 - 7*a^2*d^3)*x^5 + (5*b^2*c^3 + 6*a*b*c^2
*d - 11*a^2*c*d^2)*x)/(c^2*d^4*x^8 + 2*c^3*d^3*x^4 + c^4*d^2) + 1/256*(2*sq
rt(2)*(5*b^2*c^2 + 6*a*b*c*d + 21*a^2*d^2)*arctan(1/2*sqrt(2)*(2*sqrt(d)*x
+ sqrt(2)*c^(1/4)*d^(1/4))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqr
t(d))) + 2*sqrt(2)*(5*b^2*c^2 + 6*a*b*c*d + 21*a^2*d^2)*arctan(1/2*sqrt(2)*
(2*sqrt(d)*x - sqrt(2)*c^(1/4)*d^(1/4))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqr
t(sqrt(c)*sqrt(d))) + sqrt(2)*(5*b^2*c^2 + 6*a*b*c*d + 21*a^2*d^2)*log(sqrt
(d)*x^2 + sqrt(2)*c^(1/4)*d^(1/4)*x + sqrt(c))/(c^(3/4)*d^(1/4)) - sqrt(2)*
(5*b^2*c^2 + 6*a*b*c*d + 21*a^2*d^2)*log(sqrt(d)*x^2 - sqrt(2)*c^(1/4)*d^(1
/4)*x + sqrt(c))/(c^(3/4)*d^(1/4)))/(c^2*d^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1411 vs. 2(280) = 560.
time = 4.07, size = 1411, normalized size = 4.04

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^2/(d*x^4+c)^3,x, algorithm="fricas")
```

```
[Out] -1/128*(4*(9*b^2*c^2*d - 2*a*b*c*d^2 - 7*a^2*d^3)*x^5 - 4*(c^2*d^4*x^8 + 2*
c^3*d^3*x^4 + c^4*d^2)*(-(625*b^8*c^8 + 3000*a*b^7*c^7*d + 15900*a^2*b^6*c^
6*d^2 + 42120*a^3*b^5*c^5*d^3 + 112806*a^4*b^4*c^4*d^4 + 176904*a^5*b^3*c^3
*d^5 + 280476*a^6*b^2*c^2*d^6 + 222264*a^7*b*c*d^7 + 194481*a^8*d^8)/(c^11*
d^9))^(1/4)*arctan((sqrt(c^6*d^4*sqrt(-(625*b^8*c^8 + 3000*a*b^7*c^7*d + 15
900*a^2*b^6*c^6*d^2 + 42120*a^3*b^5*c^5*d^3 + 112806*a^4*b^4*c^4*d^4 + 1769
04*a^5*b^3*c^3*d^5 + 280476*a^6*b^2*c^2*d^6 + 222264*a^7*b*c*d^7 + 194481*a
^8*d^8)/(c^11*d^9)) + (25*b^4*c^4 + 60*a*b^3*c^3*d + 246*a^2*b^2*c^2*d^2 +
252*a^3*b*c*d^3 + 441*a^4*d^4)*x^2)*c^8*d^7*(-(625*b^8*c^8 + 3000*a*b^7*c^7
*d + 15900*a^2*b^6*c^6*d^2 + 42120*a^3*b^5*c^5*d^3 + 112806*a^4*b^4*c^4*d^4
 + 176904*a^5*b^3*c^3*d^5 + 280476*a^6*b^2*c^2*d^6 + 222264*a^7*b*c*d^7 + 1
94481*a^8*d^8)/(c^11*d^9))^(3/4) - (5*b^2*c^10*d^7 + 6*a*b*c^9*d^8 + 21*a^2
*c^8*d^9)*x*(-(625*b^8*c^8 + 3000*a*b^7*c^7*d + 15900*a^2*b^6*c^6*d^2 + 421
20*a^3*b^5*c^5*d^3 + 112806*a^4*b^4*c^4*d^4 + 176904*a^5*b^3*c^3*d^5 + 2804
76*a^6*b^2*c^2*d^6 + 222264*a^7*b*c*d^7 + 194481*a^8*d^8)/(c^11*d^9))^(3/4)
)/(625*b^8*c^8 + 3000*a*b^7*c^7*d + 15900*a^2*b^6*c^6*d^2 + 42120*a^3*b^5*c
^5*d^3 + 112806*a^4*b^4*c^4*d^4 + 176904*a^5*b^3*c^3*d^5 + 280476*a^6*b^2*c
^2*d^6 + 222264*a^7*b*c*d^7 + 194481*a^8*d^8)) - (c^2*d^4*x^8 + 2*c^3*d^3*x
^4 + c^4*d^2)*(-(625*b^8*c^8 + 3000*a*b^7*c^7*d + 15900*a^2*b^6*c^6*d^2 + 4
2120*a^3*b^5*c^5*d^3 + 112806*a^4*b^4*c^4*d^4 + 176904*a^5*b^3*c^3*d^5 + 28
0476*a^6*b^2*c^2*d^6 + 222264*a^7*b*c*d^7 + 194481*a^8*d^8)/(c^11*d^9))^(1/
4)*log(c^3*d^2*(-(625*b^8*c^8 + 3000*a*b^7*c^7*d + 15900*a^2*b^6*c^6*d^2 +
42120*a^3*b^5*c^5*d^3 + 112806*a^4*b^4*c^4*d^4 + 176904*a^5*b^3*c^3*d^5 + 2
80476*a^6*b^2*c^2*d^6 + 222264*a^7*b*c*d^7 + 194481*a^8*d^8)/(c^11*d^9))^(1
/4) + (5*b^2*c^2 + 6*a*b*c*d + 21*a^2*d^2)*x) + (c^2*d^4*x^8 + 2*c^3*d^3*x^
4 + c^4*d^2)*(-(625*b^8*c^8 + 3000*a*b^7*c^7*d + 15900*a^2*b^6*c^6*d^2 + 42
120*a^3*b^5*c^5*d^3 + 112806*a^4*b^4*c^4*d^4 + 176904*a^5*b^3*c^3*d^5 + 280
476*a^6*b^2*c^2*d^6 + 222264*a^7*b*c*d^7 + 194481*a^8*d^8)/(c^11*d^9))^(1/4
)*log(-c^3*d^2*(-(625*b^8*c^8 + 3000*a*b^7*c^7*d + 15900*a^2*b^6*c^6*d^2 +
42120*a^3*b^5*c^5*d^3 + 112806*a^4*b^4*c^4*d^4 + 176904*a^5*b^3*c^3*d^5 + 2
80476*a^6*b^2*c^2*d^6 + 222264*a^7*b*c*d^7 + 194481*a^8*d^8)/(c^11*d^9))^(1
/4) + (5*b^2*c^2 + 6*a*b*c*d + 21*a^2*d^2)*x) + 4*(5*b^2*c^3 + 6*a*b*c^2*d
- 11*a^2*c*d^2)*x)/(c^2*d^4*x^8 + 2*c^3*d^3*x^4 + c^4*d^2)
```

Sympy [A]
time = 85.51, size = 264, normalized size = 0.76

$$\frac{x^5\cdot(7a^2d^3+2abcd^2-9b^2c^2d)+x(11a^2cd^2-6abc^2d-5b^2c^3)}{32c^4d^2+64c^3d^3x^4+32c^2d^4x^8}+\operatorname{RootSum}\left(268435456t^4c^{11}d^9+194481a^8d^8+222264a^7bcd^7+280476a^6b^2c^2d^6+176904a^5b^3c^3d^5+112806a^4b^4c^4d^4+42120a^3b^5c^5d^3+15900a^2b^6c^6d^2+3000ab^7c^7d+625b^8c^8,\left(t\mapsto t\log\left(\frac{128tc^3d^2}{21a^2d^2+6abcd+5b^2c^2}+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)**2/(d*x**4+c)**3,x)
```

```
[Out] (x**5*(7*a**2*d**3 + 2*a*b*c*d**2 - 9*b**2*c**2*d) + x*(11*a**2*c*d**2 - 6*
a*b*c**2*d - 5*b**2*c**3))/(32*c**4*d**2 + 64*c**3*d**3*x**4 + 32*c**2*d**4
*x**8) + RootSum(268435456*_t**4*c**11*d**9 + 194481*a**8*d**8 + 222264*a**
7*b*c*d**7 + 280476*a**6*b**2*c**2*d**6 + 176904*a**5*b**3*c**3*d**5 + 1128
06*a**4*b**4*c**4*d**4 + 42120*a**3*b**5*c**5*d**3 + 15900*a**2*b**6*c**6*d
**2 + 3000*a*b**7*c**7*d + 625*b**8*c**8, Lambda(_t, _t*log(128*_t*c**3*d**
2/(21*a**2*d**2 + 6*a*b*c*d + 5*b**2*c**2) + x)))
```

Giac [A]
time = 0.50, size = 407, normalized size = 1.17

$$\frac{\sqrt{2}\left(5\left(cd^{3}\right)^{\frac{1}{4}}b^{2}c^{2}+6\left(cd^{3}\right)^{\frac{1}{4}}abcd+21\left(cd^{3}\right)^{\frac{1}{4}}a^{2}d^{2}\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{128\,c^{3}d^{3}}+\frac{\sqrt{2}\left(5\left(cd^{3}\right)^{\frac{1}{4}}b^{2}c^{2}+6\left(cd^{3}\right)^{\frac{1}{4}}abcd+21\left(cd^{3}\right)^{\frac{1}{4}}a^{2}d^{2}\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{128\,c^{3}d^{3}}+\frac{\sqrt{2}\left(5\left(cd^{3}\right)^{\frac{1}{4}}b^{2}c^{2}+6\left(cd^{3}\right)^{\frac{1}{4}}abcd+21\left(cd^{3}\right)^{\frac{1}{4}}a^{2}d^{2}\right)\log\left(x^{2}+\sqrt{2}\,x\left(\frac{c}{d}\right)^{\frac{1}{4}}+\sqrt{\frac{c}{d}}\right)}{256\,c^{3}d^{3}}-\frac{\sqrt{2}\left(5\left(cd^{3}\right)^{\frac{1}{4}}b^{2}c^{2}+6\left(cd^{3}\right)^{\frac{1}{4}}abcd+21\left(cd^{3}\right)^{\frac{1}{4}}a^{2}d^{2}\right)\log\left(x^{2}-\sqrt{2}\,x\left(\frac{c}{d}\right)^{\frac{1}{4}}+\sqrt{\frac{c}{d}}\right)}{256\,c^{3}d^{3}}-\frac{9\,b^{2}c^{2}dx^{5}-2\,abcd^{2}x^{5}-7\,a^{2}d^{3}x^{5}+5\,b^{2}c^{3}x+6\,abc^{2}dx-11\,a^{2}cd^{2}x}{32\left(dx^{4}+c\right)^{2}c^{2}d^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^2/(d*x^4+c)^3,x, algorithm="giac")
```

```
[Out] 1/128*sqrt(2)*(5*(c*d^3)^(1/4)*b^2*c^2 + 6*(c*d^3)^(1/4)*a*b*c*d + 21*(c*d^
3)^(1/4)*a^2*d^2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(c/d)^(1/4))/(c/d)^(1/4
))/(c^3*d^3) + 1/128*sqrt(2)*(5*(c*d^3)^(1/4)*b^2*c^2 + 6*(c*d^3)^(1/4)*a*b
*c*d + 21*(c*d^3)^(1/4)*a^2*d^2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(c/d)^(1
/4))/(c/d)^(1/4))/(c^3*d^3) + 1/256*sqrt(2)*(5*(c*d^3)^(1/4)*b^2*c^2 + 6*(c
*d^3)^(1/4)*a*b*c*d + 21*(c*d^3)^(1/4)*a^2*d^2)*log(x^2 + sqrt(2)*x*(c/d)^(
1/4) + sqrt(c/d))/(c^3*d^3) - 1/256*sqrt(2)*(5*(c*d^3)^(1/4)*b^2*c^2 + 6*(c
*d^3)^(1/4)*a*b*c*d + 21*(c*d^3)^(1/4)*a^2*d^2)*log(x^2 - sqrt(2)*x*(c/d)^(
1/4) + sqrt(c/d))/(c^3*d^3) - 1/32*(9*b^2*c^2*d*x^5 - 2*a*b*c*d^2*x^5 - 7*a
^2*d^3*x^5 + 5*b^2*c^3*x + 6*a*b*c^2*d*x - 11*a^2*c*d^2*x)/((d*x^4 + c)^2*c
^2*d^2)
```

Mupad [B]
time = 1.66, size = 1401, normalized size = 4.01

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)^2/(c + d*x^4)^3,x)
```

```
[Out] - ((x*(5*b^2*c^2 - 11*a^2*d^2 + 6*a*b*c*d))/(32*c*d^2) - (x^5*(7*a^2*d^2 -
9*b^2*c^2 + 2*a*b*c*d))/(32*c^2*d))/(c^2 + d^2*x^8 + 2*c*d*x^4) - (atan((((
((21*a^2*d^2 + 5*b^2*c^2 + 6*a*b*c*d)*(21*a^2*d^3 + 5*b^2*c^2*d + 6*a*b*c*d
^2))/(256*(-c)^(15/4)*d^(9/4)) - (x*(441*a^4*d^4 + 25*b^4*c^4 + 246*a^2*b^2
*c^2*d^2 + 60*a*b^3*c^3*d + 252*a^3*b*c*d^3))/(256*c^4*d))*(21*a^2*d^2 + 5*
b^2*c^2 + 6*a*b*c*d)*1i)/(128*(-c)^(11/4)*d^(9/4)) - ((((21*a^2*d^2 + 5*b^2
*c^2 + 6*a*b*c*d)*(21*a^2*d^3 + 5*b^2*c^2*d + 6*a*b*c*d^2))/(256*(-c)^(15/4
)*d^(9/4)) + (x*(441*a^4*d^4 + 25*b^4*c^4 + 246*a^2*b^2*c^2*d^2 + 60*a*b^3*
c^3*d + 252*a^3*b*c*d^3))/(256*c^4*d))*(21*a^2*d^2 + 5*b^2*c^2 + 6*a*b*c*d)
*1i)/(128*(-c)^(11/4)*d^(9/4)))/(((((21*a^2*d^2 + 5*b^2*c^2 + 6*a*b*c*d)*(2
```

```
1*a^2*d^3 + 5*b^2*c^2*d + 6*a*b*c*d^2))/(256*(-c)^(15/4)*d^(9/4)) - (x*(441
*a^4*d^4 + 25*b^4*c^4 + 246*a^2*b^2*c^2*d^2 + 60*a*b^3*c^3*d + 252*a^3*b*c*
d^3))/(256*c^4*d))*(21*a^2*d^2 + 5*b^2*c^2 + 6*a*b*c*d))/(128*(-c)^(11/4)*d
^(9/4)) + ((((21*a^2*d^2 + 5*b^2*c^2 + 6*a*b*c*d)*(21*a^2*d^3 + 5*b^2*c^2*d
 + 6*a*b*c*d^2))/(256*(-c)^(15/4)*d^(9/4)) + (x*(441*a^4*d^4 + 25*b^4*c^4 +
 246*a^2*b^2*c^2*d^2 + 60*a*b^3*c^3*d + 252*a^3*b*c*d^3))/(256*c^4*d))*(21*
a^2*d^2 + 5*b^2*c^2 + 6*a*b*c*d))/(128*(-c)^(11/4)*d^(9/4))))*(21*a^2*d^2 +
 5*b^2*c^2 + 6*a*b*c*d)*1i)/(64*(-c)^(11/4)*d^(9/4)) - (atan((((((21*a^2*d^
2 + 5*b^2*c^2 + 6*a*b*c*d)*(21*a^2*d^3 + 5*b^2*c^2*d + 6*a*b*c*d^2)*1i)/(25
6*(-c)^(15/4)*d^(9/4)) - (x*(441*a^4*d^4 + 25*b^4*c^4 + 246*a^2*b^2*c^2*d^2
 + 60*a*b^3*c^3*d + 252*a^3*b*c*d^3))/(256*c^4*d))*(21*a^2*d^2 + 5*b^2*c^2
+ 6*a*b*c*d))/(128*(-c)^(11/4)*d^(9/4)) - ((((21*a^2*d^2 + 5*b^2*c^2 + 6*a*
b*c*d)*(21*a^2*d^3 + 5*b^2*c^2*d + 6*a*b*c*d^2)*1i)/(256*(-c)^(15/4)*d^(9/4
)) + (x*(441*a^4*d^4 + 25*b^4*c^4 + 246*a^2*b^2*c^2*d^2 + 60*a*b^3*c^3*d +
252*a^3*b*c*d^3))/(256*c^4*d))*(21*a^2*d^2 + 5*b^2*c^2 + 6*a*b*c*d))/(128*(
-c)^(11/4)*d^(9/4)))/(((((21*a^2*d^2 + 5*b^2*c^2 + 6*a*b*c*d)*(21*a^2*d^3 +
 5*b^2*c^2*d + 6*a*b*c*d^2)*1i)/(256*(-c)^(15/4)*d^(9/4)) - (x*(441*a^4*d^4
 + 25*b^4*c^4 + 246*a^2*b^2*c^2*d^2 + 60*a*b^3*c^3*d + 252*a^3*b*c*d^3))/(2
56*c^4*d))*(21*a^2*d^2 + 5*b^2*c^2 + 6*a*b*c*d)*1i)/(128*(-c)^(11/4)*d^(9/4
)) + ((((21*a^2*d^2 + 5*b^2*c^2 + 6*a*b*c*d)*(21*a^2*d^3 + 5*b^2*c^2*d + 6*
a*b*c*d^2)*1i)/(256*(-c)^(15/4)*d^(9/4)) + (x*(441*a^4*d^4 + 25*b^4*c^4 + 2
46*a^2*b^2*c^2*d^2 + 60*a*b^3*c^3*d + 252*a^3*b*c*d^3))/(256*c^4*d))*(21*a^
2*d^2 + 5*b^2*c^2 + 6*a*b*c*d)*1i)/(128*(-c)^(11/4)*d^(9/4))))*(21*a^2*d^2
+ 5*b^2*c^2 + 6*a*b*c*d))/(64*(-c)^(11/4)*d^(9/4))
```

3.160 $\int \frac{(c+dx^4)^4}{a+bx^4}\,dx$

Optimal. Leaf size=332

$$\frac{d(2bc-ad)\left(2b^2c^2-2abcd+a^2d^2\right)x}{b^4}+\frac{d^2\left(6b^2c^2-4abcd+a^2d^2\right)x^5}{5b^3}+\frac{d^3(4bc-ad)x^9}{9b^2}+\frac{d^4x^{13}}{13b}-\frac{(bc-ad)^4\tan}{2\sqrt{}}$$

```
[Out] d*(-a*d+2*b*c)*(a^2*d^2-2*a*b*c*d+2*b^2*c^2)*x/b^4+1/5*d^2*(a^2*d^2-4*a*b*c
*d+6*b^2*c^2)*x^5/b^3+1/9*d^3*(-a*d+4*b*c)*x^9/b^2+1/13*d^4*x^13/b+1/4*(-a*
d+b*c)^4*arctan(-1+b^(1/4)*x*2^(1/2)/a^(1/4))/a^(3/4)/b^(17/4)*2^(1/2)+1/4*
(-a*d+b*c)^4*arctan(1+b^(1/4)*x*2^(1/2)/a^(1/4))/a^(3/4)/b^(17/4)*2^(1/2)-1
/8*(-a*d+b*c)^4*ln(-a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))/a^(3/4)/
b^(17/4)*2^(1/2)+1/8*(-a*d+b*c)^4*ln(a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*
b^(1/2))/a^(3/4)/b^(17/4)*2^(1/2)
```

Rubi [A]
time = 0.19, antiderivative size = 332, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {398, 217, 1179, 642, 1176, 631, 210}

$$-\frac{\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)(bc-ad)^4}{2\sqrt{2}\,a^{3/4}b^{17/4}}+\frac{\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}+1\right)(bc-ad)^4}{2\sqrt{2}\,a^{3/4}b^{17/4}}-\frac{(bc-ad)^4\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{4\sqrt{2}\,a^{3/4}b^{17/4}}+\frac{(bc-ad)^4\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{4\sqrt{2}\,a^{3/4}b^{17/4}}+\frac{dx(2bc-ad)\left(a^2d^2-2abcd+2b^2c^2\right)}{b^4}+\frac{d^2x^5\left(a^2d^2-4abcd+6b^2c^2\right)}{5b^3}+\frac{d^3x^9(4bc-ad)}{9b^2}+\frac{d^4x^{13}}{13b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^4)^4/(a + b*x^4),x]
```

```
[Out] (d*(2*b*c - a*d)*(2*b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x)/b^4 + (d^2*(6*b^2*c^2
 - 4*a*b*c*d + a^2*d^2)*x^5)/(5*b^3) + (d^3*(4*b*c - a*d)*x^9)/(9*b^2) + (d
^4*x^13)/(13*b) - ((b*c - a*d)^4*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(
2*Sqrt[2]*a^(3/4)*b^(17/4)) + ((b*c - a*d)^4*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)
/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(17/4)) - ((b*c - a*d)^4*Log[Sqrt[a] - Sqrt
[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(17/4)) + ((b*c
- a*d)^4*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]
*a^(3/4)*b^(17/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
```

```
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(c+dx^4)^4}{a+bx^4}\,dx = \int \left(\frac{d(2bc-ad)\,(2b^2c^2-2abcd+a^2d^2)}{b^4} + \frac{d^2(6b^2c^2-4abcd+a^2d^2)\,x^4}{b^3} + \frac{d^3(4bc-ad)x}{b^2}\right.$$

$$= \frac{d(2bc-ad)\,(2b^2c^2-2abcd+a^2d^2)\,x}{b^4} + \frac{d^2(6b^2c^2-4abcd+a^2d^2)\,x^5}{5b^3} + \frac{d^3(4bc-ad)x^9}{9b^2}$$

$$= \frac{d(2bc-ad)\,(2b^2c^2-2abcd+a^2d^2)\,x}{b^4} + \frac{d^2(6b^2c^2-4abcd+a^2d^2)\,x^5}{5b^3} + \frac{d^3(4bc-ad)x^9}{9b^2}$$

$$= \frac{d(2bc-ad)\,(2b^2c^2-2abcd+a^2d^2)\,x}{b^4} + \frac{d^2(6b^2c^2-4abcd+a^2d^2)\,x^5}{5b^3} + \frac{d^3(4bc-ad)x^9}{9b^2}$$

$$= \frac{d(2bc-ad)\,(2b^2c^2-2abcd+a^2d^2)\,x}{b^4} + \frac{d^2(6b^2c^2-4abcd+a^2d^2)\,x^5}{5b^3} + \frac{d^3(4bc-ad)x^9}{9b^2}$$

$$= \frac{d(2bc-ad)\,(2b^2c^2-2abcd+a^2d^2)\,x}{b^4} + \frac{d^2(6b^2c^2-4abcd+a^2d^2)\,x^5}{5b^3} + \frac{d^3(4bc-ad)x^9}{9b^2}$$

Mathematica [A]
time = 0.14, size = 322, normalized size = 0.97

$$\frac{4680\sqrt[4]{b}\,d(4b^3c^3-6ab^2c^2d+4a^2bcd^2-a^3d^3)\,x+936b^{5/4}d^2(6b^2c^2-4abcd+a^2d^2)\,x^5+520b^{9/4}d^3(4bc-ad)x^9+360b^{13/4}d^4x^{13}-\frac{1170\sqrt{2}\,(bc-ad)^4\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{a^{3/4}}+\frac{1170\sqrt{2}\,(bc-ad)^4\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{a^{3/4}}-\frac{585\sqrt{2}\,(bc-ad)^4\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{b}\,x^2\right)}{a^{3/4}}+\frac{585\sqrt{2}\,(bc-ad)^4\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{b}\,x^2\right)}{a^{3/4}}}{4680b^{17/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^4)^4/(a + b*x^4),x]
```

```
[Out] (4680*b^(1/4)*d*(4*b^3*c^3 - 6*a*b^2*c^2*d + 4*a^2*b*c*d^2 - a^3*d^3)*x + 9
36*b^(5/4)*d^2*(6*b^2*c^2 - 4*a*b*c*d + a^2*d^2)*x^5 + 520*b^(9/4)*d^3*(4*b
*c - a*d)*x^9 + 360*b^(13/4)*d^4*x^13 - (1170*Sqrt[2]*(b*c - a*d)^4*ArcTan[
1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/a^(3/4) + (1170*Sqrt[2]*(b*c - a*d)^4*Arc
Tan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/a^(3/4) - (585*Sqrt[2]*(b*c - a*d)^4*
Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/a^(3/4) + (585*Sqrt
[2]*(b*c - a*d)^4*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/a
^(3/4))/(4680*b^(17/4))
```

Maple [A]
time = 0.29, size = 282, normalized size = 0.85

method	result

risch	$\frac{d^4x^{13}}{13b} - \frac{d^4x^9a}{9b^2} + \frac{4d^3x^9c}{9b} - \frac{4d^3x^5ac}{5b^2} + \frac{6d^2x^5c^2}{5b} + \frac{d^4x^5a^2}{5b^3} - \frac{d^4a^3x}{b^4} + \frac{4d^3a^2cx}{b^3} - \frac{6d^2ac^2x}{b^2} + \frac{4dc^3x}{b} + \frac{\sum_{\_R=\mathrm{RootOf}(b\_}}{}$
default	$-\frac{d\left(-\frac{b^3d^3x^{13}}{13}+\frac{((ad-2bc)b^2d^2-2b^3cd^2)x^9}{9}+\frac{(2(ad-2bc)b^2cd-bd(a^2d^2-2abcd+2b^2c^2))x^5}{5}+(ad-2bc)(a^2d^2-2abcd+2b^2c^2)x\right)}{b^4} + \frac{(a^4d^4}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^4+c)^4/(b*x^4+a),x,method=_RETURNVERBOSE)
```

```
[Out] -d/b^4*(-1/13*b^3*d^3*x^13+1/9*((a*d-2*b*c)*b^2*d^2-2*b^3*c*d^2)*x^9+1/5*(2
*(a*d-2*b*c)*b^2*c*d-b*d*(a^2*d^2-2*a*b*c*d+2*b^2*c^2))*x^5+(a*d-2*b*c)*(a^
2*d^2-2*a*b*c*d+2*b^2*c^2)*x)+1/8*(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-
4*a*b^3*c^3*d+b^4*c^4)/b^4*(a/b)^(1/4)/a*2^(1/2)*(ln((x^2+(a/b)^(1/4)*x*2^(
1/2)+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)
/(a/b)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x-1))
```

Maxima [A]
time = 0.50, size = 489, normalized size = 1.47

$$\frac{45\,b^3d^4x^{13}+65\,(4\,b^3cd^3-ab^2d^4)x^9+117\,(6\,b^3c^2d^2-4\,ab^2cd^3+a^2bd^4)x^5+585\,(4\,b^3c^3d-6\,ab^2c^2d^2+4\,a^2bcd^3-a^3d^4)x}{585\,b^4}+\frac{\frac{2\sqrt{2}\,(b^4c^4-4ab^3c^3d+6a^2b^2c^2d^2-4a^3bcd^3+a^4d^4)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}\,x+\sqrt{2}\,a^{\frac14}b^{\frac14}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{2\sqrt{2}\,(b^4c^4-4ab^3c^3d+6a^2b^2c^2d^2-4a^3bcd^3+a^4d^4)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}\,x-\sqrt{2}\,a^{\frac14}b^{\frac14}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{\sqrt{2}\,(b^4c^4-4ab^3c^3d+6a^2b^2c^2d^2-4a^3bcd^3+a^4d^4)\log\left(\sqrt{b}\,x^2+\sqrt{2}\,a^{\frac14}b^{\frac14}x+\sqrt{a}\right)}{a^{\frac34}b^{\frac14}}-\frac{\sqrt{2}\,(b^4c^4-4ab^3c^3d+6a^2b^2c^2d^2-4a^3bcd^3+a^4d^4)\log\left(\sqrt{b}\,x^2-\sqrt{2}\,a^{\frac14}b^{\frac14}x+\sqrt{a}\right)}{a^{\frac34}b^{\frac14}}}{8\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^4/(b*x^4+a),x, algorithm="maxima")
```

```
[Out] 1/585*(45*b^3*d^4*x^13 + 65*(4*b^3*c*d^3 - a*b^2*d^4)*x^9 + 117*(6*b^3*c^2*
d^2 - 4*a*b^2*c*d^3 + a^2*b*d^4)*x^5 + 585*(4*b^3*c^3*d - 6*a*b^2*c^2*d^2 +
 4*a^2*b*c*d^3 - a^3*d^4)*x)/b^4 + 1/8*(2*sqrt(2)*(b^4*c^4 - 4*a*b^3*c^3*d
+ 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*arctan(1/2*sqrt(2)*(2*sqrt(b
)*x + sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)
*sqrt(b))) + 2*sqrt(2)*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3
*b*c*d^3 + a^4*d^4)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/
4))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*(b^4*c
^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*log(sqrt(
b)*x^2 + sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(1/4)) - sqrt(2)*(
b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*log(
sqrt(b)*x^2 - sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(1/4)))/b^4
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2477 vs. $2(261) = 522$.
time = 3.65, size = 2477, normalized size = 7.46

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^4/(b*x^4+a),x, algorithm="fricas")
```

```
[Out] 1/2340*(180*b^3*d^4*x^13 + 260*(4*b^3*c*d^3 - a*b^2*d^4)*x^9 + 468*(6*b^3*c
^2*d^2 - 4*a*b^2*c*d^3 + a^2*b*d^4)*x^5 + 2340*b^4*(-(b^16*c^16 - 16*a*b^15
*c^15*d + 120*a^2*b^14*c^14*d^2 - 560*a^3*b^13*c^13*d^3 + 1820*a^4*b^12*c^1
2*d^4 - 4368*a^5*b^11*c^11*d^5 + 8008*a^6*b^10*c^10*d^6 - 11440*a^7*b^9*c^9
*d^7 + 12870*a^8*b^8*c^8*d^8 - 11440*a^9*b^7*c^7*d^9 + 8008*a^10*b^6*c^6*d^
10 - 4368*a^11*b^5*c^5*d^11 + 1820*a^12*b^4*c^4*d^12 - 560*a^13*b^3*c^3*d^1
3 + 120*a^14*b^2*c^2*d^14 - 16*a^15*b*c*d^15 + a^16*d^16)/(a^3*b^17))^(1/4)
*arctan((sqrt(a^2*b^8*sqrt(-(b^16*c^16 - 16*a*b^15*c^15*d + 120*a^2*b^14*c^
14*d^2 - 560*a^3*b^13*c^13*d^3 + 1820*a^4*b^12*c^12*d^4 - 4368*a^5*b^11*c^1
1*d^5 + 8008*a^6*b^10*c^10*d^6 - 11440*a^7*b^9*c^9*d^7 + 12870*a^8*b^8*c^8*
d^8 - 11440*a^9*b^7*c^7*d^9 + 8008*a^10*b^6*c^6*d^10 - 4368*a^11*b^5*c^5*d^
11 + 1820*a^12*b^4*c^4*d^12 - 560*a^13*b^3*c^3*d^13 + 120*a^14*b^2*c^2*d^14
 - 16*a^15*b*c*d^15 + a^16*d^16)/(a^3*b^17)) + (b^8*c^8 - 8*a*b^7*c^7*d + 2
8*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^
3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)*x^2)*a^2*b^13*(-(b^16
*c^16 - 16*a*b^15*c^15*d + 120*a^2*b^14*c^14*d^2 - 560*a^3*b^13*c^13*d^3 +
1820*a^4*b^12*c^12*d^4 - 4368*a^5*b^11*c^11*d^5 + 8008*a^6*b^10*c^10*d^6 -
11440*a^7*b^9*c^9*d^7 + 12870*a^8*b^8*c^8*d^8 - 11440*a^9*b^7*c^7*d^9 + 800
8*a^10*b^6*c^6*d^10 - 4368*a^11*b^5*c^5*d^11 + 1820*a^12*b^4*c^4*d^12 - 560
*a^13*b^3*c^3*d^13 + 120*a^14*b^2*c^2*d^14 - 16*a^15*b*c*d^15 + a^16*d^16)/
(a^3*b^17))^(3/4) - (a^2*b^17*c^4 - 4*a^3*b^16*c^3*d + 6*a^4*b^15*c^2*d^2 -
 4*a^5*b^14*c*d^3 + a^6*b^13*d^4)*x*(-(b^16*c^16 - 16*a*b^15*c^15*d + 120*a
^2*b^14*c^14*d^2 - 560*a^3*b^13*c^13*d^3 + 1820*a^4*b^12*c^12*d^4 - 4368*a^
5*b^11*c^11*d^5 + 8008*a^6*b^10*c^10*d^6 - 11440*a^7*b^9*c^9*d^7 + 12870*a^
8*b^8*c^8*d^8 - 11440*a^9*b^7*c^7*d^9 + 8008*a^10*b^6*c^6*d^10 - 4368*a^11*
b^5*c^5*d^11 + 1820*a^12*b^4*c^4*d^12 - 560*a^13*b^3*c^3*d^13 + 120*a^14*b^
2*c^2*d^14 - 16*a^15*b*c*d^15 + a^16*d^16)/(a^3*b^17))^(3/4))/(b^16*c^16 -
16*a*b^15*c^15*d + 120*a^2*b^14*c^14*d^2 - 560*a^3*b^13*c^13*d^3 + 1820*a^4
*b^12*c^12*d^4 - 4368*a^5*b^11*c^11*d^5 + 8008*a^6*b^10*c^10*d^6 - 11440*a^
7*b^9*c^9*d^7 + 12870*a^8*b^8*c^8*d^8 - 11440*a^9*b^7*c^7*d^9 + 8008*a^10*b
^6*c^6*d^10 - 4368*a^11*b^5*c^5*d^11 + 1820*a^12*b^4*c^4*d^12 - 560*a^13*b^
3*c^3*d^13 + 120*a^14*b^2*c^2*d^14 - 16*a^15*b*c*d^15 + a^16*d^16)) + 585*b
^4*(-(b^16*c^16 - 16*a*b^15*c^15*d + 120*a^2*b^14*c^14*d^2 - 560*a^3*b^13*c
^13*d^3 + 1820*a^4*b^12*c^12*d^4 - 4368*a^5*b^11*c^11*d^5 + 8008*a^6*b^10*c
^10*d^6 - 11440*a^7*b^9*c^9*d^7 + 12870*a^8*b^8*c^8*d^8 - 11440*a^9*b^7*c^7
*d^9 + 8008*a^10*b^6*c^6*d^10 - 4368*a^11*b^5*c^5*d^11 + 1820*a^12*b^4*c^4*
d^12 - 560*a^13*b^3*c^3*d^13 + 120*a^14*b^2*c^2*d^14 - 16*a^15*b*c*d^15 + a
^16*d^16)/(a^3*b^17))^(1/4)*log(a*b^4*(-(b^16*c^16 - 16*a*b^15*c^15*d + 120
*a^2*b^14*c^14*d^2 - 560*a^3*b^13*c^13*d^3 + 1820*a^4*b^12*c^12*d^4 - 4368*
a^5*b^11*c^11*d^5 + 8008*a^6*b^10*c^10*d^6 - 11440*a^7*b^9*c^9*d^7 + 12870*
a^8*b^8*c^8*d^8 - 11440*a^9*b^7*c^7*d^9 + 8008*a^10*b^6*c^6*d^10 - 4368*a^1
```

```
1*b^5*c^5*d^11 + 1820*a^12*b^4*c^4*d^12 - 560*a^13*b^3*c^3*d^13 + 120*a^14*
b^2*c^2*d^14 - 16*a^15*b*c*d^15 + a^16*d^16)/(a^3*b^17))^(1/4) + (b^4*c^4 -
 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*x) - 585*b^4*
(-(b^16*c^16 - 16*a*b^15*c^15*d + 120*a^2*b^14*c^14*d^2 - 560*a^3*b^13*c^13
*d^3 + 1820*a^4*b^12*c^12*d^4 - 4368*a^5*b^11*c^11*d^5 + 8008*a^6*b^10*c^10
*d^6 - 11440*a^7*b^9*c^9*d^7 + 12870*a^8*b^8*c^8*d^8 - 11440*a^9*b^7*c^7*d^
9 + 8008*a^10*b^6*c^6*d^10 - 4368*a^11*b^5*c^5*d^11 + 1820*a^12*b^4*c^4*d^1
2 - 560*a^13*b^3*c^3*d^13 + 120*a^14*b^2*c^2*d^14 - 16*a^15*b*c*d^15 + a^16
*d^16)/(a^3*b^17))^(1/4)*log(-a*b^4*(-(b^16*c^16 - 16*a*b^15*c^15*d + 120*a
^2*b^14*c^14*d^2 - 560*a^3*b^13*c^13*d^3 + 1820*a^4*b^12*c^12*d^4 - 4368*a^
5*b^11*c^11*d^5 + 8008*a^6*b^10*c^10*d^6 - 11440*a^7*b^9*c^9*d^7 + 12870*a^
8*b^8*c^8*d^8 - 11440*a^9*b^7*c^7*d^9 + 8008*a^10*b^6*c^6*d^10 - 4368*a^11*
b^5*c^5*d^11 + 1820*a^12*b^4*c^4*d^12 - 560*a^13*b^3*c^3*d^13 + 120*a^14*b^
2*c^2*d^14 - 16*a^15*b*c*d^15 + a^16*d^16)/(a^3*b^17))^(1/4) + (b^4*c^4 - 4
*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*x) + 2340*(4*b^
3*c^3*d - 6*a*b^2*c^2*d^2 + 4*a^2*b*c*d^3 - a^3*d^4)*x)/b^4
```

Sympy [A]
time = 22.79, size = 435, normalized size = 1.31

$$x^9\left(-\frac{ad^4}{9b^2}+\frac{4cd^3}{9b}\right)+x^5\left(\frac{a^2d^4}{5b^3}-\frac{4acd^3}{5b^2}+\frac{6c^2d^2}{5b}\right)+x\left(-\frac{a^3d^4}{b^4}+\frac{4a^2cd^3}{b^3}-\frac{6ac^2d^2}{b^2}+\frac{4c^3d}{b}\right)+\operatorname{RootSum}\left(256t^4a^3b^{17}+a^{16}d^{16}-16a^{15}bcd^{15}+120a^{14}b^2c^2d^{14}-560a^{13}b^3c^3d^{13}+1820a^{12}b^4c^4d^{12}-4368a^{11}b^5c^5d^{11}+8008a^{10}b^6c^6d^{10}-11440a^9b^7c^7d^9+12870a^8b^8c^8d^8-11440a^7b^9c^9d^7+8008a^6b^{10}c^{10}d^6-4368a^5b^{11}c^{11}d^5+1820a^4b^{12}c^{12}d^4-560a^3b^{13}c^{13}d^3+120a^2b^{14}c^{14}d^2-16ab^{15}c^{15}d+b^{16}c^{16},\left(t\mapsto t\log\left(\frac{4tab^4}{a^4d^4-4a^3bcd^3+6a^2b^2c^2d^2-4ab^3c^3d+b^4c^4}+x\right)\right)\right)+\frac{d^4x^{13}}{13b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**4+c)**4/(b*x**4+a),x)
```

```
[Out] x**9*(-a*d**4/(9*b**2) + 4*c*d**3/(9*b)) + x**5*(a**2*d**4/(5*b**3) - 4*a*c
*d**3/(5*b**2) + 6*c**2*d**2/(5*b)) + x*(-a**3*d**4/b**4 + 4*a**2*c*d**3/b*
*3 - 6*a*c**2*d**2/b**2 + 4*c**3*d/b) + RootSum(256*_t**4*a**3*b**17 + a**1
6*d**16 - 16*a**15*b*c*d**15 + 120*a**14*b**2*c**2*d**14 - 560*a**13*b**3*c
**3*d**13 + 1820*a**12*b**4*c**4*d**12 - 4368*a**11*b**5*c**5*d**11 + 8008*
a**10*b**6*c**6*d**10 - 11440*a**9*b**7*c**7*d**9 + 12870*a**8*b**8*c**8*d*
*8 - 11440*a**7*b**9*c**9*d**7 + 8008*a**6*b**10*c**10*d**6 - 4368*a**5*b**
11*c**11*d**5 + 1820*a**4*b**12*c**12*d**4 - 560*a**3*b**13*c**13*d**3 + 12
0*a**2*b**14*c**14*d**2 - 16*a*b**15*c**15*d + b**16*c**16, Lambda(_t, _t*l
og(4*_t*a*b**4/(a**4*d**4 - 4*a**3*b*c*d**3 + 6*a**2*b**2*c**2*d**2 - 4*a*b
**3*c**3*d + b**4*c**4) + x))) + d**4*x**13/(13*b)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 617 vs. 2(261) = 522.
time = 0.62, size = 617, normalized size = 1.86

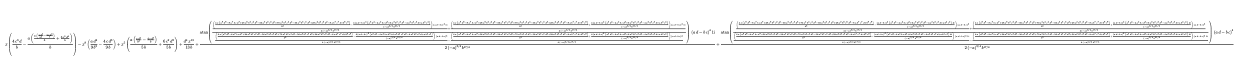
$$\frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}b^4c^4-4\left(ab^3\right)^{\frac{1}{4}}ab^3c^3d+6\left(ab^3\right)^{\frac{1}{4}}a^2b^2c^2d^2-4\left(ab^3\right)^{\frac{1}{4}}a^3bcd^3+\left(ab^3\right)^{\frac{1}{4}}a^4d^4\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4ab^5}+\frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}b^4c^4-4\left(ab^3\right)^{\frac{1}{4}}ab^3c^3d+6\left(ab^3\right)^{\frac{1}{4}}a^2b^2c^2d^2-4\left(ab^3\right)^{\frac{1}{4}}a^3bcd^3+\left(ab^3\right)^{\frac{1}{4}}a^4d^4\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4ab^5}+\frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}b^4c^4-4\left(ab^3\right)^{\frac{1}{4}}ab^3c^3d+6\left(ab^3\right)^{\frac{1}{4}}a^2b^2c^2d^2-4\left(ab^3\right)^{\frac{1}{4}}a^3bcd^3+\left(ab^3\right)^{\frac{1}{4}}a^4d^4\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{8ab^5}-\frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}b^4c^4-4\left(ab^3\right)^{\frac{1}{4}}ab^3c^3d+6\left(ab^3\right)^{\frac{1}{4}}a^2b^2c^2d^2-4\left(ab^3\right)^{\frac{1}{4}}a^3bcd^3+\left(ab^3\right)^{\frac{1}{4}}a^4d^4\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{8ab^5}+\frac{45b^{12}d^4x^{13}+260b^{12}cd^3x^9-65ab^{11}d^4x^9+702b^{12}c^2d^2x^5-468ab^{11}cd^3x^5+117a^2b^{10}d^4x^5+2340b^{12}c^3dx-3510ab^{11}c^2d^2x+2340a^2b^{10}cd^3x-585a^3b^9d^4x}{585b^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^4/(b*x^4+a),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*((a*b^3)^(1/4)*b^4*c^4 - 4*(a*b^3)^(1/4)*a*b^3*c^3*d + 6*(a*b^3
)^(1/4)*a^2*b^2*c^2*d^2 - 4*(a*b^3)^(1/4)*a^3*b*c*d^3 + (a*b^3)^(1/4)*a^4*d
^4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a*b^5) + 1
/4*sqrt(2)*((a*b^3)^(1/4)*b^4*c^4 - 4*(a*b^3)^(1/4)*a*b^3*c^3*d + 6*(a*b^3)
^(1/4)*a^2*b^2*c^2*d^2 - 4*(a*b^3)^(1/4)*a^3*b*c*d^3 + (a*b^3)^(1/4)*a^4*d^
4)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a*b^5) + 1/
8*sqrt(2)*((a*b^3)^(1/4)*b^4*c^4 - 4*(a*b^3)^(1/4)*a*b^3*c^3*d + 6*(a*b^3)^
(1/4)*a^2*b^2*c^2*d^2 - 4*(a*b^3)^(1/4)*a^3*b*c*d^3 + (a*b^3)^(1/4)*a^4*d^4
)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a*b^5) - 1/8*sqrt(2)*((a*b^
3)^(1/4)*b^4*c^4 - 4*(a*b^3)^(1/4)*a*b^3*c^3*d + 6*(a*b^3)^(1/4)*a^2*b^2*c^
2*d^2 - 4*(a*b^3)^(1/4)*a^3*b*c*d^3 + (a*b^3)^(1/4)*a^4*d^4)*log(x^2 - sqrt
(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a*b^5) + 1/585*(45*b^12*d^4*x^13 + 260*b^12
*c*d^3*x^9 - 65*a*b^11*d^4*x^9 + 702*b^12*c^2*d^2*x^5 - 468*a*b^11*c*d^3*x^
5 + 117*a^2*b^10*d^4*x^5 + 2340*b^12*c^3*d*x - 3510*a*b^11*c^2*d^2*x + 2340
*a^2*b^10*c*d^3*x - 585*a^3*b^9*d^4*x)/b^13
```

Mupad [B]
time = 1.51, size = 1822, normalized size = 5.49



Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^4)^4/(a + b*x^4),x)
```

```
[Out] x*((4*c^3*d)/b - (a*((a*((a*d^4)/b^2 - (4*c*d^3)/b))/b + (6*c^2*d^2)/b))/b)
 - x^9*((a*d^4)/(9*b^2) - (4*c*d^3)/(9*b)) + x^5*((a*((a*d^4)/b^2 - (4*c*d^
3)/b))/(5*b) + (6*c^2*d^2)/(5*b)) + (d^4*x^13)/(13*b) + (atan(((((4*x*(a^8*
d^8 + b^8*c^8 + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^
4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a*b^7*c^7*d - 8*a^7*b*c*d^7
))/b^5 - (4*(a*d - b*c)^4*(a^5*d^4 + a*b^4*c^4 - 4*a^2*b^3*c^3*d + 6*a^3*b^
2*c^2*d^2 - 4*a^4*b*c*d^3))/((-a)^(3/4)*b^(21/4)))*(a*d - b*c)^4*1i)/(4*(-a
)^(3/4)*b^(17/4)) + (((4*x*(a^8*d^8 + b^8*c^8 + 28*a^2*b^6*c^6*d^2 - 56*a^3
*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6
 - 8*a*b^7*c^7*d - 8*a^7*b*c*d^7))/b^5 + (4*(a*d - b*c)^4*(a^5*d^4 + a*b^4*
c^4 - 4*a^2*b^3*c^3*d + 6*a^3*b^2*c^2*d^2 - 4*a^4*b*c*d^3))/((-a)^(3/4)*b^(
21/4)))*(a*d - b*c)^4*1i)/(4*(-a)^(3/4)*b^(17/4)))/((((4*x*(a^8*d^8 + b^8*c
^8 + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*
b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a*b^7*c^7*d - 8*a^7*b*c*d^7))/b^5 - (4
*(a*d - b*c)^4*(a^5*d^4 + a*b^4*c^4 - 4*a^2*b^3*c^3*d + 6*a^3*b^2*c^2*d^2 -
 4*a^4*b*c*d^3))/((-a)^(3/4)*b^(21/4)))*(a*d - b*c)^4)/(4*(-a)^(3/4)*b^(17/
4)) - (((4*x*(a^8*d^8 + b^8*c^8 + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 +
 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a*b^7*c^7
*d - 8*a^7*b*c*d^7))/b^5 + (4*(a*d - b*c)^4*(a^5*d^4 + a*b^4*c^4 - 4*a^2*b^
3*c^3*d + 6*a^3*b^2*c^2*d^2 - 4*a^4*b*c*d^3))/((-a)^(3/4)*b^(21/4)))*(a*d -
 b*c)^4)/(4*(-a)^(3/4)*b^(17/4))))*(a*d - b*c)^4*1i)/(2*(-a)^(3/4)*b^(17/4)
```

```
) + (atan(((((4*x*(a^8*d^8 + b^8*c^8 + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*
d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a*b^
7*c^7*d - 8*a^7*b*c*d^7))/b^5 - ((a*d - b*c)^4*(a^5*d^4 + a*b^4*c^4 - 4*a^2
*b^3*c^3*d + 6*a^3*b^2*c^2*d^2 - 4*a^4*b*c*d^3)*4i)/((-a)^(3/4)*b^(21/4)))*
(a*d - b*c)^4)/(4*(-a)^(3/4)*b^(17/4)) + (((4*x*(a^8*d^8 + b^8*c^8 + 28*a^2
*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5
 + 28*a^6*b^2*c^2*d^6 - 8*a*b^7*c^7*d - 8*a^7*b*c*d^7))/b^5 + ((a*d - b*c)^
4*(a^5*d^4 + a*b^4*c^4 - 4*a^2*b^3*c^3*d + 6*a^3*b^2*c^2*d^2 - 4*a^4*b*c*d^
3)*4i)/((-a)^(3/4)*b^(21/4)))*(a*d - b*c)^4)/(4*(-a)^(3/4)*b^(17/4)))/((((4
*x*(a^8*d^8 + b^8*c^8 + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^
4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a*b^7*c^7*d - 8*a^7
*b*c*d^7))/b^5 - ((a*d - b*c)^4*(a^5*d^4 + a*b^4*c^4 - 4*a^2*b^3*c^3*d + 6*
a^3*b^2*c^2*d^2 - 4*a^4*b*c*d^3)*4i)/((-a)^(3/4)*b^(21/4)))*(a*d - b*c)^4*1
i)/(4*(-a)^(3/4)*b^(17/4)) - (((4*x*(a^8*d^8 + b^8*c^8 + 28*a^2*b^6*c^6*d^2
 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^
2*c^2*d^6 - 8*a*b^7*c^7*d - 8*a^7*b*c*d^7))/b^5 + ((a*d - b*c)^4*(a^5*d^4 +
 a*b^4*c^4 - 4*a^2*b^3*c^3*d + 6*a^3*b^2*c^2*d^2 - 4*a^4*b*c*d^3)*4i)/((-a)
^(3/4)*b^(21/4)))*(a*d - b*c)^4*1i)/(4*(-a)^(3/4)*b^(17/4))))*(a*d - b*c)^4
)/(2*(-a)^(3/4)*b^(17/4))
```

3.161 $\int \frac{(c+dx^4)^3}{a+bx^4} \, dx$

Optimal. Leaf size=288

$$\frac{d(3b^2c^2-3abcd+a^2d^2)\,x}{b^3}+\frac{d^2(3bc-ad)x^5}{5b^2}+\frac{d^3x^9}{9b}-\frac{(bc-ad)^3\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}b^{13/4}}+\frac{(bc-ad)^3\tan^{-1}\left(\right.}{2\sqrt{2}\,a^{3/4}}$$

```
[Out] d*(a^2*d^2-3*a*b*c*d+3*b^2*c^2)*x/b^3+1/5*d^2*(-a*d+3*b*c)*x^5/b^2+1/9*d^3*
x^9/b+1/4*(-a*d+b*c)^3*arctan(-1+b^(1/4)*x*2^(1/2)/a^(1/4))/a^(3/4)/b^(13/4
)*2^(1/2)+1/4*(-a*d+b*c)^3*arctan(1+b^(1/4)*x*2^(1/2)/a^(1/4))/a^(3/4)/b^(1
3/4)*2^(1/2)-1/8*(-a*d+b*c)^3*ln(-a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(
1/2))/a^(3/4)/b^(13/4)*2^(1/2)+1/8*(-a*d+b*c)^3*ln(a^(1/4)*b^(1/4)*x*2^(1/2
)+a^(1/2)+x^2*b^(1/2))/a^(3/4)/b^(13/4)*2^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 288, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {398, 217, 1179, 642, 1176, 631, 210}

$$-\frac{\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)(bc-ad)^3}{2\sqrt{2}\,a^{3/4}b^{13/4}}+\frac{\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a}}+1\right)(bc-ad)^3}{2\sqrt{2}\,a^{3/4}b^{13/4}}-\frac{(bc-ad)^3\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{4\sqrt{2}\,a^{3/4}b^{13/4}}+\frac{(bc-ad)^3\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{4\sqrt{2}\,a^{3/4}b^{13/4}}+\frac{dx(a^2d^2-3abcd+3b^2c^2)}{b^3}+\frac{d^2x^5(3bc-ad)}{5b^2}+\frac{d^3x^9}{9b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^4)^3/(a + b*x^4),x]
```

```
[Out] (d*(3*b^2*c^2 - 3*a*b*c*d + a^2*d^2)*x)/b^3 + (d^2*(3*b*c - a*d)*x^5)/(5*b^
2) + (d^3*x^9)/(9*b) - ((b*c - a*d)^3*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4
)])/(2*Sqrt[2]*a^(3/4)*b^(13/4)) + ((b*c - a*d)^3*ArcTan[1 + (Sqrt[2]*b^(1/
4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(13/4)) - ((b*c - a*d)^3*Log[Sqrt[a] -
 Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(13/4)) + (
(b*c - a*d)^3*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sq
rt[2]*a^(3/4)*b^(13/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
```

```
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(c+dx^4)^3}{a+bx^4}\,dx = \int \left(\frac{d(3b^2c^2-3abcd+a^2d^2)}{b^3} + \frac{d^2(3bc-ad)x^4}{b^2} + \frac{d^3x^8}{b} + \frac{b^3c^3-3ab^2c^2d+3a^2bcd^2-a}{b^3\,(a+bx^4)}\right.$$

$$= \frac{d(3b^2c^2-3abcd+a^2d^2)\,x}{b^3} + \frac{d^2(3bc-ad)x^5}{5b^2} + \frac{d^3x^9}{9b} + \frac{(bc-ad)^3\int \frac{1}{a+bx^4}\,dx}{b^3}$$

$$= \frac{d(3b^2c^2-3abcd+a^2d^2)\,x}{b^3} + \frac{d^2(3bc-ad)x^5}{5b^2} + \frac{d^3x^9}{9b} + \frac{(bc-ad)^3\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx}{2\sqrt{a}\,b^3} + \frac{(b}{}$$

$$= \frac{d(3b^2c^2-3abcd+a^2d^2)\,x}{b^3} + \frac{d^2(3bc-ad)x^5}{5b^2} + \frac{d^3x^9}{9b} + \frac{(bc-ad)^3\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}}{4\sqrt{a}\,b^{7/2}}$$

$$= \frac{d(3b^2c^2-3abcd+a^2d^2)\,x}{b^3} + \frac{d^2(3bc-ad)x^5}{5b^2} + \frac{d^3x^9}{9b} - \frac{(bc-ad)^3\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\right.}{4\sqrt{2}\,a^{3/4}b^{13/4}}$$

$$= \frac{d(3b^2c^2-3abcd+a^2d^2)\,x}{b^3} + \frac{d^2(3bc-ad)x^5}{5b^2} + \frac{d^3x^9}{9b} - \frac{(bc-ad)^3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right.}{2\sqrt{2}\,a^{3/4}b^{13/4}}$$

Mathematica [A]
time = 0.10, size = 271, normalized size = 0.94

$$\frac{360a^{3/4}\sqrt[4]{b}\,d(3b^2c^2-3abcd+a^2d^2)x-72a^{3/4}b^{5/4}d^2(-3bc+ad)x^5+40a^{3/4}b^{9/4}d^3x^9-90\sqrt{2}\,(bc-ad)^3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)+90\sqrt{2}\,(bc-ad)^3\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)-45\sqrt{2}\,(bc-ad)^3\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{b}\,x^2\right)+45\sqrt{2}\,(bc-ad)^3\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{b}\,x^2\right)}{360a^{3/4}b^{13/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^4)^3/(a + b*x^4),x]
```

```
[Out] (360*a^(3/4)*b^(1/4)*d*(3*b^2*c^2 - 3*a*b*c*d + a^2*d^2)*x - 72*a^(3/4)*b^(
5/4)*d^2*(-3*b*c + a*d)*x^5 + 40*a^(3/4)*b^(9/4)*d^3*x^9 - 90*Sqrt[2]*(b*c
- a*d)^3*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 90*Sqrt[2]*(b*c - a*d)^3
*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)] - 45*Sqrt[2]*(b*c - a*d)^3*Log[Sqr
t[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] + 45*Sqrt[2]*(b*c - a*d)^3*
Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(360*a^(3/4)*b^(13/
4))
```

Maple [A]
time = 0.25, size = 203, normalized size = 0.70

method	result

risch	$\frac{d^3x^9}{9b} - \frac{d^3a\,x^5}{5b^2} + \frac{3d^2c\,x^5}{5b} + \frac{d^3a^2x}{b^3} - \frac{3d^2acx}{b^2} + \frac{3d\,c^2x}{b} + \frac{\sum_{\_R=\mathrm{RootOf}\left(b\_Z^4+a\right)} \frac{\left(-a^3d^3+3a^2bc\,d^2-3a\,b^2c^2d+b^3c^3\right)\ln\left(x-\_R\right)}{\_R^3}}{4b^4}$
default	$\frac{d\left(\frac{1}{9}b^2d^2x^9-\frac{1}{5}ab\,d^2x^5+\frac{3}{5}b^2cd\,x^5+a^2d^2x-3abcdx+3b^2c^2x\right)}{b^3} + \frac{\left(-a^3d^3+3a^2bc\,d^2-3a\,b^2c^2d+b^3c^3\right)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}}\right.\right.}{8b^3a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^4+c)^3/(b*x^4+a),x,method=_RETURNVERBOSE)
```

```
[Out] d/b^3*(1/9*b^2*d^2*x^9-1/5*a*b*d^2*x^5+3/5*b^2*c*d*x^5+a^2*d^2*x-3*a*b*c*d*
x+3*b^2*c^2*x)+1/8*(-a^3*d^3+3*a^2*b*c*d^2-3*a*b^2*c^2*d+b^3*c^3)/b^3*(a/b)
^(1/4)/a*2^(1/2)*(ln((x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2))/(x^2-(a/b)^(1/
4)*x*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x+1)+2*arctan(2^(1/
2)/(a/b)^(1/4)*x-1))
```

Maxima [A]
time = 0.49, size = 385, normalized size = 1.34

$$\frac{5\,b^2d^3x^9+9\,(3\,b^2cd^2-abd^3)x^5+45\,(3\,b^2c^2d-3\,abcd^2+a^2d^3)x}{45\,b^3}+\frac{\frac{2\,\sqrt{2}\,(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)\arctan\left(\frac{\sqrt{2}\left(2\,\sqrt{b}\,x+\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}}+\frac{2\,\sqrt{2}\,(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)\arctan\left(\frac{\sqrt{2}\left(2\,\sqrt{b}\,x-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}}+\frac{\sqrt{2}\,(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)\log\left(\sqrt{b}\,x^2+\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}-\frac{\sqrt{2}\,(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)\log\left(\sqrt{b}\,x^2-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}}{8\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^3/(b*x^4+a),x, algorithm="maxima")
```

```
[Out] 1/45*(5*b^2*d^3*x^9 + 9*(3*b^2*c*d^2 - a*b*d^3)*x^5 + 45*(3*b^2*c^2*d - 3*a
*b*c*d^2 + a^2*d^3)*x)/b^3 + 1/8*(2*sqrt(2)*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^
2*b*c*d^2 - a^3*d^3)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1
/4))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqrt(2)*(b^
3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*arctan(1/2*sqrt(2)*(2*sqrt
(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(
a)*sqrt(b))) + sqrt(2)*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*
log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(1/4)) -
sqrt(2)*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(sqrt(b)*x^2
 - sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(1/4)))/b^3
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1855 vs. $2(219) = 438$.
time = 6.24, size = 1855, normalized size = 6.44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^3/(b*x^4+a),x, algorithm="fricas")
```

```
[Out] 1/180*(20*b^2*d^3*x^9 + 36*(3*b^2*c*d^2 - a*b*d^3)*x^5 - 180*b^3*(-(b^12*c^
12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^
4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5
*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 1
2*a^11*b*c*d^11 + a^12*d^12)/(a^3*b^13))^(1/4)*arctan((sqrt(a^2*b^6*sqrt(-(
b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 +
 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*
b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d
^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(a^3*b^13)) + (b^6*c^6 - 6*a*b^5*c^5*d
+ 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*
d^5 + a^6*d^6)*x^2)*a^2*b^10*(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*
c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5
+ 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9
*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(a^3*b^
13))^(3/4) + (a^2*b^13*c^3 - 3*a^3*b^12*c^2*d + 3*a^4*b^11*c*d^2 - a^5*b^10
*d^3)*x*(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^
9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6
 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^1
0*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(a^3*b^13))^(3/4))/(b^12*c^1
2 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4
*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*
d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12
*a^11*b*c*d^11 + a^12*d^12)) - 45*b^3*(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*
a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7
*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8
- 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12
)/(a^3*b^13))^(1/4)*log(a*b^3*(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10
*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5
 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^
9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(a^3*b
^13))^(1/4) - (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*x) + 45*b
^3*(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9
*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 79
2*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2
*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(a^3*b^13))^(1/4)*log(-a*b^3*(-(b
^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 +
495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b
^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^
10 - 12*a^11*b*c*d^11 + a^12*d^12)/(a^3*b^13))^(1/4) - (b^3*c^3 - 3*a*b^2*c
^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*x) + 180*(3*b^2*c^2*d - 3*a*b*c*d^2 + a^2*d
^3)*x)/b^3
```

Sympy [A]

time = 1.35, size = 303, normalized size = 1.05

$$x^5\left(-\frac{ad^3}{5b^2}+\frac{3cd^2}{5b}\right)+x\left(\frac{a^2d^3}{b^3}-\frac{3acd^2}{b^2}+\frac{3c^2d}{b}\right)+\operatorname{RootSum}\left(256t^4a^3b^{13}+a^{12}d^{12}-12a^{11}bcd^{11}+66a^{10}b^2c^2d^{10}-220a^9b^3c^3d^9+495a^8b^4c^4d^8-792a^7b^5c^5d^7+924a^6b^6c^6d^6-792a^5b^7c^7d^5+495a^4b^8c^8d^4-220a^3b^9c^9d^3+66a^2b^{10}c^{10}d^2-12ab^{11}c^{11}d+b^{12}c^{12},\left(t\mapsto t\log\left(-\frac{4tab^3}{a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3}+x\right)\right)\right)+\frac{d^3x^9}{9b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**4+c)**3/(b*x**4+a),x)
```

```
[Out] x**5*(-a*d**3/(5*b**2) + 3*c*d**2/(5*b)) + x*(a**2*d**3/b**3 - 3*a*c*d**2/b
**2 + 3*c**2*d/b) + RootSum(256*_t**4*a**3*b**13 + a**12*d**12 - 12*a**11*b
*c*d**11 + 66*a**10*b**2*c**2*d**10 - 220*a**9*b**3*c**3*d**9 + 495*a**8*b*
*4*c**4*d**8 - 792*a**7*b**5*c**5*d**7 + 924*a**6*b**6*c**6*d**6 - 792*a**5
*b**7*c**7*d**5 + 495*a**4*b**8*c**8*d**4 - 220*a**3*b**9*c**9*d**3 + 66*a*
*2*b**10*c**10*d**2 - 12*a*b**11*c**11*d + b**12*c**12, Lambda(_t, _t*log(-
4*_t*a*b**3/(a**3*d**3 - 3*a**2*b*c*d**2 + 3*a*b**2*c**2*d - b**3*c**3) + x
))) + d**3*x**9/(9*b)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 481 vs. 2(219) = 438.
time = 0.61, size = 481, normalized size = 1.67

$$\frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}b^3c^3-3\,(ab^3)^{\frac{1}{4}}ab^2c^2d+3\,(ab^3)^{\frac{1}{4}}a^2bcd^2-(ab^3)^{\frac{1}{4}}a^3d^3\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,ab^4}+\frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}b^3c^3-3\,(ab^3)^{\frac{1}{4}}ab^2c^2d+3\,(ab^3)^{\frac{1}{4}}a^2bcd^2-(ab^3)^{\frac{1}{4}}a^3d^3\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,ab^4}+\frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}b^3c^3-3\,(ab^3)^{\frac{1}{4}}ab^2c^2d+3\,(ab^3)^{\frac{1}{4}}a^2bcd^2-(ab^3)^{\frac{1}{4}}a^3d^3\right)\log\left(x^2+\sqrt{2}\,x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{8\,ab^4}-\frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}b^3c^3-3\,(ab^3)^{\frac{1}{4}}ab^2c^2d+3\,(ab^3)^{\frac{1}{4}}a^2bcd^2-(ab^3)^{\frac{1}{4}}a^3d^3\right)\log\left(x^2-\sqrt{2}\,x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{8\,ab^4}+\frac{5\,b^8d^3x^9+27\,b^8cd^2x^5-9\,ab^7d^3x^5+135\,b^8c^2dx-135\,ab^7cd^2x+45\,a^2b^6d^3x}{45\,b^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^3/(b*x^4+a),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*((a*b^3)^(1/4)*b^3*c^3 - 3*(a*b^3)^(1/4)*a*b^2*c^2*d + 3*(a*b^3
)^(1/4)*a^2*b*c*d^2 - (a*b^3)^(1/4)*a^3*d^3)*arctan(1/2*sqrt(2)*(2*x + sqrt
(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a*b^4) + 1/4*sqrt(2)*((a*b^3)^(1/4)*b^3*c^3
- 3*(a*b^3)^(1/4)*a*b^2*c^2*d + 3*(a*b^3)^(1/4)*a^2*b*c*d^2 - (a*b^3)^(1/4)
*a^3*d^3)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a*b^
4) + 1/8*sqrt(2)*((a*b^3)^(1/4)*b^3*c^3 - 3*(a*b^3)^(1/4)*a*b^2*c^2*d + 3*(
a*b^3)^(1/4)*a^2*b*c*d^2 - (a*b^3)^(1/4)*a^3*d^3)*log(x^2 + sqrt(2)*x*(a/b)
^(1/4) + sqrt(a/b))/(a*b^4) - 1/8*sqrt(2)*((a*b^3)^(1/4)*b^3*c^3 - 3*(a*b^3
)^(1/4)*a*b^2*c^2*d + 3*(a*b^3)^(1/4)*a^2*b*c*d^2 - (a*b^3)^(1/4)*a^3*d^3)*
log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a*b^4) + 1/45*(5*b^8*d^3*x^9
+ 27*b^8*c*d^2*x^5 - 9*a*b^7*d^3*x^5 + 135*b^8*c^2*d*x - 135*a*b^7*c*d^2*x
+ 45*a^2*b^6*d^3*x)/b^9
```

Mupad [B]
time = 1.49, size = 1433, normalized size = 4.98

$$x\left(\frac{3\,c^2\,d}{b}+\frac{a\left(\frac{a\,d^3}{b^2}-\frac{3\,c\,d^2}{b}\right)}{b}\right)-x^5\left(\frac{a\,d^3}{5\,b^2}-\frac{3\,c\,d^2}{5\,b}\right)+\frac{d^3\,x^9}{9\,b}-\frac{\mathrm{atan}\left(\frac{\frac{\left(\frac{x\left(a^6d^6-6a^5bcd^5+15a^4b^2c^2d^4-20a^3b^3c^3d^3+15a^2b^4c^4d^2-6ab^5c^5d+b^6c^6\right)}{b^3}-\frac{(ad-bc)^3\left(4a^4d^3-12a^3bcd^2+12a^2b^2c^2d-4ab^3c^3\right)1i}{4\,(-a)^{3/4}b^{13/4}}\right)(ad-bc)^3\,1i}{(-a)^{3/4}b^{13/4}}+\frac{\left(\frac{x\left(a^6d^6-6a^5bcd^5+15a^4b^2c^2d^4-20a^3b^3c^3d^3+15a^2b^4c^4d^2-6ab^5c^5d+b^6c^6\right)}{b^3}+\frac{(ad-bc)^3\left(4a^4d^3-12a^3bcd^2+12a^2b^2c^2d-4ab^3c^3\right)1i}{4\,(-a)^{3/4}b^{13/4}}\right)(ad-bc)^3\,1i}{(-a)^{3/4}b^{13/4}}}{\frac{\left(\frac{x\left(a^6d^6-6a^5bcd^5+15a^4b^2c^2d^4-20a^3b^3c^3d^3+15a^2b^4c^4d^2-6ab^5c^5d+b^6c^6\right)}{b^3}-\frac{(ad-bc)^3\left(4a^4d^3-12a^3bcd^2+12a^2b^2c^2d-4ab^3c^3\right)1i}{4\,(-a)^{3/4}b^{13/4}}\right)(ad-bc)^3}{(-a)^{3/4}b^{13/4}}-\frac{\left(\frac{x\left(a^6d^6-6a^5bcd^5+15a^4b^2c^2d^4-20a^3b^3c^3d^3+15a^2b^4c^4d^2-6ab^5c^5d+b^6c^6\right)}{b^3}+\frac{(ad-bc)^3\left(4a^4d^3-12a^3bcd^2+12a^2b^2c^2d-4ab^3c^3\right)1i}{4\,(-a)^{3/4}b^{13/4}}\right)(ad-bc)^3}{(-a)^{3/4}b^{13/4}}}\right)(a\,d-b\,c)^3\,1i}{2\,(-a)^{3/4}\,b^{13/4}}-\frac{\mathrm{atan}\left(\frac{\frac{\left(\frac{x\left(a^6d^6-6a^5bcd^5+15a^4b^2c^2d^4-20a^3b^3c^3d^3+15a^2b^4c^4d^2-6ab^5c^5d+b^6c^6\right)}{b^3}-\frac{(ad-bc)^3\left(4a^4d^3-12a^3bcd^2+12a^2b^2c^2d-4ab^3c^3\right)1i}{4\,(-a)^{3/4}b^{13/4}}\right)(ad-bc)^3}{(-a)^{3/4}b^{13/4}}+\frac{\left(\frac{x\left(a^6d^6-6a^5bcd^5+15a^4b^2c^2d^4-20a^3b^3c^3d^3+15a^2b^4c^4d^2-6ab^5c^5d+b^6c^6\right)}{b^3}+\frac{(ad-bc)^3\left(4a^4d^3-12a^3bcd^2+12a^2b^2c^2d-4ab^3c^3\right)1i}{4\,(-a)^{3/4}b^{13/4}}\right)(ad-bc)^3}{(-a)^{3/4}b^{13/4}}}{\frac{\left(\frac{x\left(a^6d^6-6a^5bcd^5+15a^4b^2c^2d^4-20a^3b^3c^3d^3+15a^2b^4c^4d^2-6ab^5c^5d+b^6c^6\right)}{b^3}-\frac{(ad-bc)^3\left(4a^4d^3-12a^3bcd^2+12a^2b^2c^2d-4ab^3c^3\right)1i}{4\,(-a)^{3/4}b^{13/4}}\right)(ad-bc)^3\,1i}{(-a)^{3/4}b^{13/4}}-\frac{\left(\frac{x\left(a^6d^6-6a^5bcd^5+15a^4b^2c^2d^4-20a^3b^3c^3d^3+15a^2b^4c^4d^2-6ab^5c^5d+b^6c^6\right)}{b^3}+\frac{(ad-bc)^3\left(4a^4d^3-12a^3bcd^2+12a^2b^2c^2d-4ab^3c^3\right)1i}{4\,(-a)^{3/4}b^{13/4}}\right)(ad-bc)^3\,1i}{(-a)^{3/4}b^{13/4}}}\right)(a\,d-b\,c)^3}{2\,(-a)^{3/4}\,b^{13/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^4)^3/(a + b*x^4),x)
```

```
[Out] x*((3*c^2*d)/b + (a*((a*d^3)/b^2 - (3*c*d^2)/b))/b) - x^5*((a*d^3)/(5*b^2)
- (3*c*d^2)/(5*b)) + (d^3*x^9)/(9*b) - (atan(((((x*(a^6*d^6 + b^6*c^6 + 15*
a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a*b^5*c^5*d -
```

6*a^5*b*c*d^5))/b^3 - ((a*d - b*c)^3*(4*a^4*d^3 - 4*a*b^3*c^3 + 12*a^2*b^2*c^2*d - 12*a^3*b*c*d^2))/(4*(-a)^(3/4)*b^(13/4)))*(a*d - b*c)^3*1i)/((-a)^(3/4)*b^(13/4)) + (((x*(a^6*d^6 + b^6*c^6 + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a*b^5*c^5*d - 6*a^5*b*c*d^5))/b^3 + ((a*d - b*c)^3*(4*a^4*d^3 - 4*a*b^3*c^3 + 12*a^2*b^2*c^2*d - 12*a^3*b*c*d^2))/(4*(-a)^(3/4)*b^(13/4)))*(a*d - b*c)^3*1i)/((-a)^(3/4)*b^(13/4)))/((((x*(a^6*d^6 + b^6*c^6 + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a*b^5*c^5*d - 6*a^5*b*c*d^5))/b^3 - ((a*d - b*c)^3*(4*a^4*d^3 - 4*a*b^3*c^3 + 12*a^2*b^2*c^2*d - 12*a^3*b*c*d^2))/(4*(-a)^(3/4)*b^(13/4)))*(a*d - b*c)^3)/((-a)^(3/4)*b^(13/4)) - (((x*(a^6*d^6 + b^6*c^6 + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a*b^5*c^5*d - 6*a^5*b*c*d^5))/b^3 + ((a*d - b*c)^3*(4*a^4*d^3 - 4*a*b^3*c^3 + 12*a^2*b^2*c^2*d - 12*a^3*b*c*d^2))/(4*(-a)^(3/4)*b^(13/4)))*(a*d - b*c)^3)/((-a)^(3/4)*b^(13/4)))*(a*d - b*c)^3*1i)/(2*(-a)^(3/4)*b^(13/4)) - (atan(((((x*(a^6*d^6 + b^6*c^6 + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a*b^5*c^5*d - 6*a^5*b*c*d^5))/b^3 - ((a*d - b*c)^3*(4*a^4*d^3 - 4*a*b^3*c^3 + 12*a^2*b^2*c^2*d - 12*a^3*b*c*d^2)*1i)/(4*(-a)^(3/4)*b^(13/4)))*(a*d - b*c)^3)/((-a)^(3/4)*b^(13/4)) + (((x*(a^6*d^6 + b^6*c^6 + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a*b^5*c^5*d - 6*a^5*b*c*d^5))/b^3 + ((a*d - b*c)^3*(4*a^4*d^3 - 4*a*b^3*c^3 + 12*a^2*b^2*c^2*d - 12*a^3*b*c*d^2)*1i)/(4*(-a)^(3/4)*b^(13/4)))*(a*d - b*c)^3)/((-a)^(3/4)*b^(13/4)))/((((x*(a^6*d^6 + b^6*c^6 + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a*b^5*c^5*d - 6*a^5*b*c*d^5))/b^3 - ((a*d - b*c)^3*(4*a^4*d^3 - 4*a*b^3*c^3 + 12*a^2*b^2*c^2*d - 12*a^3*b*c*d^2)*1i)/(4*(-a)^(3/4)*b^(13/4)))*(a*d - b*c)^3*1i)/((-a)^(3/4)*b^(13/4)) - (((x*(a^6*d^6 + b^6*c^6 + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a*b^5*c^5*d - 6*a^5*b*c*d^5))/b^3 + ((a*d - b*c)^3*(4*a^4*d^3 - 4*a*b^3*c^3 + 12*a^2*b^2*c^2*d - 12*a^3*b*c*d^2)*1i)/(4*(-a)^(3/4)*b^(13/4)))*(a*d - b*c)^3*1i)/((-a)^(3/4)*b^(13/4))))*(a*d - b*c)^3)/(2*(-a)^(3/4)*b^(13/4))

3.162 $\int \frac{(c+dx^4)^2}{a+bx^4}\,dx$

Optimal. Leaf size=253

$$\frac{d(2bc-ad)x}{b^2}+\frac{d^2x^5}{5b}-\frac{(bc-ad)^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}b^{9/4}}+\frac{(bc-ad)^2\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}b^{9/4}}-\frac{(bc-ad)^2\log}{}$$

```
[Out] d*(-a*d+2*b*c)*x/b^2+1/5*d^2*x^5/b+1/4*(-a*d+b*c)^2*arctan(-1+b^(1/4)*x*2^(
1/2)/a^(1/4))/a^(3/4)/b^(9/4)*2^(1/2)+1/4*(-a*d+b*c)^2*arctan(1+b^(1/4)*x*2
^(1/2)/a^(1/4))/a^(3/4)/b^(9/4)*2^(1/2)-1/8*(-a*d+b*c)^2*ln(-a^(1/4)*b^(1/4
)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))/a^(3/4)/b^(9/4)*2^(1/2)+1/8*(-a*d+b*c)^2*l
n(a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))/a^(3/4)/b^(9/4)*2^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 253, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {398, 217, 1179, 642, 1176, 631, 210}

$$-\frac{\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)(bc-ad)^2}{2\sqrt{2}\,a^{3/4}b^{9/4}}+\frac{\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}+1\right)(bc-ad)^2}{2\sqrt{2}\,a^{3/4}b^{9/4}}-\frac{(bc-ad)^2\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{4\sqrt{2}\,a^{3/4}b^{9/4}}+\frac{(bc-ad)^2\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{4\sqrt{2}\,a^{3/4}b^{9/4}}+\frac{dx(2bc-ad)}{b^2}+\frac{d^2x^5}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^4)^2/(a + b*x^4),x]
```

```
[Out] (d*(2*b*c - a*d)*x)/b^2 + (d^2*x^5)/(5*b) - ((b*c - a*d)^2*ArcTan[1 - (Sqrt
[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(9/4)) + ((b*c - a*d)^2*ArcTa
n[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(9/4)) - ((b*c - a
*d)^2*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^
(3/4)*b^(9/4)) + ((b*c - a*d)^2*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + S
qrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(9/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(c+dx^4)^2}{a+bx^4}\,dx = \int \left(\frac{d(2bc-ad)}{b^2} + \frac{d^2x^4}{b} + \frac{b^2c^2-2abcd+a^2d^2}{b^2(a+bx^4)}\right) dx$$

$$= \frac{d(2bc-ad)x}{b^2} + \frac{d^2x^5}{5b} + \frac{(bc-ad)^2 \int \frac{1}{a+bx^4}\,dx}{b^2}$$

$$= \frac{d(2bc-ad)x}{b^2} + \frac{d^2x^5}{5b} + \frac{(bc-ad)^2 \int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx}{2\sqrt{a}\,b^2} + \frac{(bc-ad)^2 \int \frac{\sqrt{a}+\sqrt{b}\,x^2}{a+bx^4}\,dx}{2\sqrt{a}\,b^2}$$

$$= \frac{d(2bc-ad)x}{b^2} + \frac{d^2x^5}{5b} + \frac{(bc-ad)^2 \int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}} - \frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}} + x^2}\,dx}{4\sqrt{a}\,b^{5/2}} + \frac{(bc-ad)^2 \int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}} + \frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}}}{4\sqrt{a}\,b^{5/2}}$$

$$= \frac{d(2bc-ad)x}{b^2} + \frac{d^2x^5}{5b} - \frac{(bc-ad)^2 \log\left(\sqrt{a} - \sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x + \sqrt{b}\,x^2\right)}{4\sqrt{2}\,a^{3/4}b^{9/4}} + \frac{(bc-ad)^2 \log}{}$$

$$= \frac{d(2bc-ad)x}{b^2} + \frac{d^2x^5}{5b} - \frac{(bc-ad)^2 \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}b^{9/4}} + \frac{(bc-ad)^2 \tan^{-1}\left(1 + \frac{\sqrt{2}}{\sqrt[4]{}}\right)}{2\sqrt{2}\,a^{3/4}b^{9/4}}$$

Mathematica [A]
time = 0.07, size = 231, normalized size = 0.91

$$\frac{-40a^{3/4}\sqrt[4]{b}\,d(-2bc+ad)x + 8a^{3/4}b^{5/4}d^2x^5 - 10\sqrt{2}\,(bc-ad)^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right) + 10\sqrt{2}\,(bc-ad)^2\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right) - 5\sqrt{2}\,(bc-ad)^2\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{b}\,x^2\right) + 5\sqrt{2}\,(bc-ad)^2\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{b}\,x^2\right)}{40a^{3/4}b^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^4)^2/(a + b*x^4),x]
```

```
[Out] (-40*a^(3/4)*b^(1/4)*d*(-2*b*c + a*d)*x + 8*a^(3/4)*b^(5/4)*d^2*x^5 - 10*Sq
rt[2]*(b*c - a*d)^2*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 10*Sqrt[2]*(b
*c - a*d)^2*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)] - 5*Sqrt[2]*(b*c - a*d)
^2*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] + 5*Sqrt[2]*(b*c
- a*d)^2*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(40*a^(3/4
)*b^(9/4))
```

Maple [A]
time = 0.26, size = 150, normalized size = 0.59

method	result

risch	$\frac{d^2x^5}{5b} - \frac{d^2ax}{b^2} + \frac{2dcx}{b} + \frac{\sum_{\_R=\mathrm{RootOf}(b\_Z^4+a)} \frac{(a^2d^2-2abcd+b^2c^2)\ln(x-\_R)}{\_R^3}}{4b^3}$
default	$-\frac{d(-\frac{1}{5}bd\,x^5+adx-2bcx)}{b^2} + \frac{(a^2d^2-2abcd+b^2c^2)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\right.}{8b^2a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^4+c)^2/(b*x^4+a),x,method=_RETURNVERBOSE)
```

```
[Out] -d/b^2*(-1/5*b*d*x^5+a*d*x-2*b*c*x)+1/8*(a^2*d^2-2*a*b*c*d+b^2*c^2)/b^2*(a/
b)^(1/4)/a*2^(1/2)*(ln((x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2))/(x^2-(a/b)^(
1/4)*x*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x+1)+2*arctan(2^(
1/2)/(a/b)^(1/4)*x-1))
```

Maxima [**A**]
time = 0.48, size = 287, normalized size = 1.13

$$\frac{bd^2x^5+5\,(2\,bcd-ad^2)x}{5\,b^2}+\frac{\frac{2\sqrt{2}\,(b^2c^2-2\,abcd+a^2d^2)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}\,x+\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{2\sqrt{2}\,(b^2c^2-2\,abcd+a^2d^2)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}\,x-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{\sqrt{2}\,(b^2c^2-2\,abcd+a^2d^2)\log\left(\sqrt{b}\,x^2+\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}-\frac{\sqrt{2}\,(b^2c^2-2\,abcd+a^2d^2)\log\left(\sqrt{b}\,x^2-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}}{8\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^2/(b*x^4+a),x, algorithm="maxima")
```

```
[Out] 1/5*(b*d^2*x^5 + 5*(2*b*c*d - a*d^2)*x)/b^2 + 1/8*(2*sqrt(2)*(b^2*c^2 - 2*a
*b*c*d + a^2*d^2)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4)
)/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqrt(2)*(b^2*c
^2 - 2*a*b*c*d + a^2*d^2)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)
*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*
(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1/4)*x
 + sqrt(a))/(a^(3/4)*b^(1/4)) - sqrt(2)*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log
(sqrt(b)*x^2 - sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(1/4)))/b^2
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1240 vs. 2(186) = 372.
time = 4.41, size = 1240, normalized size = 4.90

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^2/(b*x^4+a),x, algorithm="fricas")
```

```
[Out] 1/20*(4*b*d^2*x^5 + 20*b^2*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2
- 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2
```

```
*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(a^3*b^9))^(1/4)*arctan((sqrt(a^2*b^4*s
qrt(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 7
0*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7
 + a^8*d^8)/(a^3*b^9)) + (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a
^3*b*c*d^3 + a^4*d^4)*x^2)*a^2*b^7*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*
c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28
*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(a^3*b^9))^(3/4) - (a^2*b^9*c^2
 - 2*a^3*b^8*c*d + a^4*b^7*d^2)*x*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c
^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*
a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(a^3*b^9))^(3/4))/(b^8*c^8 - 8*a
*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 -
 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)) + 5*b^
2*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70
*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7
+ a^8*d^8)/(a^3*b^9))^(1/4)*log(a*b^2*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b
^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 +
 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(a^3*b^9))^(1/4) + (b^2*c^2
- 2*a*b*c*d + a^2*d^2)*x) - 5*b^2*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c
^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*
a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(a^3*b^9))^(1/4)*log(-a*b^2*(-(b
^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b
^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*
d^8)/(a^3*b^9))^(1/4) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x) + 20*(2*b*c*d -
a*d^2)*x)/b^2
```

Sympy [A]
time = 0.58, size = 187, normalized size = 0.74

$$x\left(-\frac{ad^2}{b^2}+\frac{2cd}{b}\right)+\operatorname{RootSum}\left(256t^4a^3b^9+a^8d^8-8a^7bcd^7+28a^6b^2c^2d^6-56a^5b^3c^3d^5+70a^4b^4c^4d^4-56a^3b^5c^5d^3+28a^2b^6c^6d^2-8ab^7c^7d+b^8c^8,\left(t\mapsto t\log\left(\frac{4tab^2}{a^2d^2-2abcd+b^2c^2}+x\right)\right)\right)+\frac{d^2x^5}{5b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**4+c)**2/(b*x**4+a),x)
```

```
[Out] x*(-a*d**2/b**2 + 2*c*d/b) + RootSum(256*_t**4*a**3*b**9 + a**8*d**8 - 8*a*
*7*b*c*d**7 + 28*a**6*b**2*c**2*d**6 - 56*a**5*b**3*c**3*d**5 + 70*a**4*b**
4*c**4*d**4 - 56*a**3*b**5*c**5*d**3 + 28*a**2*b**6*c**6*d**2 - 8*a*b**7*c*
*7*d + b**8*c**8, Lambda(_t, _t*log(4*_t*a*b**2/(a**2*d**2 - 2*a*b*c*d + b*
*2*c**2) + x))) + d**2*x**5/(5*b)
```

Giac [A]
time = 0.54, size = 353, normalized size = 1.40

$$\frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}b^2c^2-2\left(ab^3\right)^{\frac{1}{4}}abcd+\left(ab^3\right)^{\frac{1}{4}}a^2d^2\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4ab^3}+\frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}b^2c^2-2\left(ab^3\right)^{\frac{1}{4}}abcd+\left(ab^3\right)^{\frac{1}{4}}a^2d^2\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4ab^3}+\frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}b^2c^2-2\left(ab^3\right)^{\frac{1}{4}}abcd+\left(ab^3\right)^{\frac{1}{4}}a^2d^2\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{8ab^3}-\frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}b^2c^2-2\left(ab^3\right)^{\frac{1}{4}}abcd+\left(ab^3\right)^{\frac{1}{4}}a^2d^2\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{8ab^3}+\frac{b^4d^2x^5+10b^4cdx-5ab^3d^2x}{5b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^2/(b*x^4+a),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*((a*b^3)^(1/4)*b^2*c^2 - 2*(a*b^3)^(1/4)*a*b*c*d + (a*b^3)^(1/4
)*a^2*d^2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a*b
^3) + 1/4*sqrt(2)*((a*b^3)^(1/4)*b^2*c^2 - 2*(a*b^3)^(1/4)*a*b*c*d + (a*b^3
)^(1/4)*a^2*d^2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4)
)/(a*b^3) + 1/8*sqrt(2)*((a*b^3)^(1/4)*b^2*c^2 - 2*(a*b^3)^(1/4)*a*b*c*d +
(a*b^3)^(1/4)*a^2*d^2)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a*b^3)
 - 1/8*sqrt(2)*((a*b^3)^(1/4)*b^2*c^2 - 2*(a*b^3)^(1/4)*a*b*c*d + (a*b^3)^(
1/4)*a^2*d^2)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a*b^3) + 1/5*(b
^4*d^2*x^5 + 10*b^4*c*d*x - 5*a*b^3*d^2*x)/b^5
```

Mupad [B]

time = 1.47, size = 1081, normalized size = 4.27

$$\frac{d^2 x^5}{5 b}-x\left(\frac{a d^2}{b^2}-\frac{2 c d}{b}\right)+\frac{\mathrm{atan}\left(\frac{\frac{(a d-b c)^2\left(\frac{x\left(a^4 d^4-4 a^3 b c d^3+6 a^2 b^2 c^2 d^2-4 a b^3 c^3 d+b^4 c^4\right)}{b}-\frac{(a d-b c)^2\left(4 a^3 b d^2-8 a^2 b^2 c d+4 a b^3 c^2\right)}{4(-a)^{3/4} b^{9/4}}\right) 1i}{(-a)^{3/4} b^{9/4}}+\frac{(a d-b c)^2\left(\frac{x\left(a^4 d^4-4 a^3 b c d^3+6 a^2 b^2 c^2 d^2-4 a b^3 c^3 d+b^4 c^4\right)}{b}+\frac{(a d-b c)^2\left(4 a^3 b d^2-8 a^2 b^2 c d+4 a b^3 c^2\right)}{4(-a)^{3/4} b^{9/4}}\right) 1i}{(-a)^{3/4} b^{9/4}}}{\frac{(a d-b c)^2\left(\frac{x\left(a^4 d^4-4 a^3 b c d^3+6 a^2 b^2 c^2 d^2-4 a b^3 c^3 d+b^4 c^4\right)}{b}-\frac{(a d-b c)^2\left(4 a^3 b d^2-8 a^2 b^2 c d+4 a b^3 c^2\right)}{4(-a)^{3/4} b^{9/4}}\right)}{(-a)^{3/4} b^{9/4}}-\frac{(a d-b c)^2\left(\frac{x\left(a^4 d^4-4 a^3 b c d^3+6 a^2 b^2 c^2 d^2-4 a b^3 c^3 d+b^4 c^4\right)}{b}+\frac{(a d-b c)^2\left(4 a^3 b d^2-8 a^2 b^2 c d+4 a b^3 c^2\right)}{4(-a)^{3/4} b^{9/4}}\right)}{(-a)^{3/4} b^{9/4}}}\right)(a d-b c)^2\, 1i}{2(-a)^{3/4} b^{9/4}}+\frac{\mathrm{atan}\left(\frac{\frac{(a d-b c)^2\left(\frac{x\left(a^4 d^4-4 a^3 b c d^3+6 a^2 b^2 c^2 d^2-4 a b^3 c^3 d+b^4 c^4\right)}{b}-\frac{(a d-b c)^2\left(4 a^3 b d^2-8 a^2 b^2 c d+4 a b^3 c^2\right) 1i}{4(-a)^{3/4} b^{9/4}}\right)}{(-a)^{3/4} b^{9/4}}+\frac{(a d-b c)^2\left(\frac{x\left(a^4 d^4-4 a^3 b c d^3+6 a^2 b^2 c^2 d^2-4 a b^3 c^3 d+b^4 c^4\right)}{b}+\frac{(a d-b c)^2\left(4 a^3 b d^2-8 a^2 b^2 c d+4 a b^3 c^2\right) 1i}{4(-a)^{3/4} b^{9/4}}\right)}{(-a)^{3/4} b^{9/4}}}{\frac{(a d-b c)^2\left(\frac{x\left(a^4 d^4-4 a^3 b c d^3+6 a^2 b^2 c^2 d^2-4 a b^3 c^3 d+b^4 c^4\right)}{b}-\frac{(a d-b c)^2\left(4 a^3 b d^2-8 a^2 b^2 c d+4 a b^3 c^2\right) 1i}{4(-a)^{3/4} b^{9/4}}\right) 1i}{(-a)^{3/4} b^{9/4}}-\frac{(a d-b c)^2\left(\frac{x\left(a^4 d^4-4 a^3 b c d^3+6 a^2 b^2 c^2 d^2-4 a b^3 c^3 d+b^4 c^4\right)}{b}+\frac{(a d-b c)^2\left(4 a^3 b d^2-8 a^2 b^2 c d+4 a b^3 c^2\right) 1i}{4(-a)^{3/4} b^{9/4}}\right) 1i}{(-a)^{3/4} b^{9/4}}}\right)(a d-b c)^2}{2(-a)^{3/4} b^{9/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^4)^2/(a + b*x^4),x)
```

```
[Out] (d^2*x^5)/(5*b) - x*((a*d^2)/b^2 - (2*c*d)/b) + (atan((((a*d - b*c)^2*((x*(
a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3))/b -
 ((a*d - b*c)^2*(4*a*b^3*c^2 + 4*a^3*b*d^2 - 8*a^2*b^2*c*d))/(4*(-a)^(3/4)*
b^(9/4)))*1i)/((-a)^(3/4)*b^(9/4)) + ((a*d - b*c)^2*((x*(a^4*d^4 + b^4*c^4
+ 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3))/b + ((a*d - b*c)^2*(4
*a*b^3*c^2 + 4*a^3*b*d^2 - 8*a^2*b^2*c*d))/(4*(-a)^(3/4)*b^(9/4)))*1i)/((-a
)^(3/4)*b^(9/4)))/(((a*d - b*c)^2*((x*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^
2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3))/b - ((a*d - b*c)^2*(4*a*b^3*c^2 + 4*a^3
*b*d^2 - 8*a^2*b^2*c*d))/(4*(-a)^(3/4)*b^(9/4))))/((-a)^(3/4)*b^(9/4)) - ((
a*d - b*c)^2*((x*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4
*a^3*b*c*d^3))/b + ((a*d - b*c)^2*(4*a*b^3*c^2 + 4*a^3*b*d^2 - 8*a^2*b^2*c*
d))/(4*(-a)^(3/4)*b^(9/4))))/((-a)^(3/4)*b^(9/4))))*(a*d - b*c)^2*1i)/(2*(-
a)^(3/4)*b^(9/4)) + (atan((((a*d - b*c)^2*((x*(a^4*d^4 + b^4*c^4 + 6*a^2*b^
2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3))/b - ((a*d - b*c)^2*(4*a*b^3*c^2
 + 4*a^3*b*d^2 - 8*a^2*b^2*c*d)*1i)/(4*(-a)^(3/4)*b^(9/4))))/((-a)^(3/4)*b^
(9/4)) + ((a*d - b*c)^2*((x*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^
3*c^3*d - 4*a^3*b*c*d^3))/b + ((a*d - b*c)^2*(4*a*b^3*c^2 + 4*a^3*b*d^2 - 8
*a^2*b^2*c*d)*1i)/(4*(-a)^(3/4)*b^(9/4))))/((-a)^(3/4)*b^(9/4)))/(((a*d - b
*c)^2*((x*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*
c*d^3))/b - ((a*d - b*c)^2*(4*a*b^3*c^2 + 4*a^3*b*d^2 - 8*a^2*b^2*c*d)*1i)/
(4*(-a)^(3/4)*b^(9/4)))*1i)/((-a)^(3/4)*b^(9/4)) - ((a*d - b*c)^2*((x*(a^4*
d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3))/b + ((a
*d - b*c)^2*(4*a*b^3*c^2 + 4*a^3*b*d^2 - 8*a^2*b^2*c*d)*1i)/(4*(-a)^(3/4)*b
^(9/4)))*1i)/((-a)^(3/4)*b^(9/4))))*(a*d - b*c)^2)/(2*(-a)^(3/4)*b^(9/4))
```

3.163 $\int \frac{c+dx^4}{a+bx^4}\,dx$

Optimal. Leaf size=223

$$\frac{dx}{b}-\frac{(bc-ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}b^{5/4}}+\frac{(bc-ad)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}b^{5/4}}-\frac{(bc-ad)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\right.}{4\sqrt{2}\,a^{3/4}b^{5/4}}$$

```
[Out] d*x/b+1/4*(-a*d+b*c)*arctan(-1+b^(1/4)*x*2^(1/2)/a^(1/4))/a^(3/4)/b^(5/4)*2
^(1/2)+1/4*(-a*d+b*c)*arctan(1+b^(1/4)*x*2^(1/2)/a^(1/4))/a^(3/4)/b^(5/4)*2
^(1/2)-1/8*(-a*d+b*c)*ln(-a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))/a^
(3/4)/b^(5/4)*2^(1/2)+1/8*(-a*d+b*c)*ln(a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x
^2*b^(1/2))/a^(3/4)/b^(5/4)*2^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {396, 217, 1179, 642, 1176, 631, 210}

$$-\frac{\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)(bc-ad)}{2\sqrt{2}\,a^{3/4}b^{5/4}}+\frac{\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}+1\right)(bc-ad)}{2\sqrt{2}\,a^{3/4}b^{5/4}}-\frac{(bc-ad)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{4\sqrt{2}\,a^{3/4}b^{5/4}}+\frac{(bc-ad)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{4\sqrt{2}\,a^{3/4}b^{5/4}}+\frac{dx}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^4)/(a + b*x^4),x]
```

```
[Out] (d*x)/b - ((b*c - a*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*
a^(3/4)*b^(5/4)) + ((b*c - a*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2
*Sqrt[2]*a^(3/4)*b^(5/4)) - ((b*c - a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1
/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(5/4)) + ((b*c - a*d)*Log[Sqrt[a
] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(5/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{c+dx^4}{a+bx^4}\,dx = \frac{dx}{b} - \frac{(-bc+ad)\int \frac{1}{a+bx^4}\,dx}{b}$$

$$= \frac{dx}{b} + \frac{(bc-ad)\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx}{2\sqrt{a}\,b} + \frac{(bc-ad)\int \frac{\sqrt{a}+\sqrt{b}\,x^2}{a+bx^4}\,dx}{2\sqrt{a}\,b}$$

$$= \frac{dx}{b} + \frac{(bc-ad)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx}{4\sqrt{a}\,b^{3/2}} + \frac{(bc-ad)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx}{4\sqrt{a}\,b^{3/2}} - \frac{(bc-ad)\int}{4}$$

$$= \frac{dx}{b} - \frac{(bc-ad)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{b}\,x^2\right)}{4\sqrt{2}\,a^{3/4}b^{5/4}} + \frac{(bc-ad)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x\right.}{4\sqrt{2}\,a^{3/4}b^{5/4}}$$

$$= \frac{dx}{b} - \frac{(bc-ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}b^{5/4}} + \frac{(bc-ad)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}b^{5/4}} - \frac{(bc-ad)\log}{}$$

Mathematica [A]
time = 0.09, size = 196, normalized size = 0.88

$$\frac{8a^{3/4}\sqrt[4]{b}\,dx - 2\sqrt{2}\,(bc-ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right) + 2\sqrt{2}\,(bc-ad)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right) - \sqrt{2}\,(bc-ad)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{b}\,x^2\right) + \sqrt{2}\,(bc-ad)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{b}\,x^2\right)}{8a^{3/4}b^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^4)/(a + b*x^4),x]
```

```
[Out] (8*a^(3/4)*b^(1/4)*d*x - 2*Sqrt[2]*(b*c - a*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*
x)/a^(1/4)] + 2*Sqrt[2]*(b*c - a*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)]
 - Sqrt[2]*(b*c - a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^
2] + Sqrt[2]*(b*c - a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*
x^2])/(8*a^(3/4)*b^(5/4))
```

Maple [A]
time = 0.24, size = 120, normalized size = 0.54

method	result	size
risch	$\frac{dx}{b} + \frac{\sum_{\_R=\mathrm{RootOf}\left(b\_Z^4+a\right)} \frac{(-ad+bc)\ln\left(x-\_R\right)}{\_R^3}}{4b^2}$	42

default	$\frac{dx}{b}+\frac{(-ad+bc)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^{2}+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{8ba}$	120

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^4+c)/(b*x^4+a),x,method=_RETURNVERBOSE)
```

```
[Out] d*x/b+1/8*(-a*d+b*c)/b*(a/b)^(1/4)/a*2^(1/2)*(ln((x^2+(a/b)^(1/4)*x*2^(1/2)
+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/
b)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x-1))
```

Maxima [A]
time = 0.48, size = 212, normalized size = 0.95

$$\frac{dx}{b}+\frac{\frac{2\sqrt{2}\,(bc-ad)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}\,x+\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{2\sqrt{2}\,(bc-ad)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}\,x-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{\sqrt{2}\,(bc-ad)\log\left(\sqrt{b}\,x^{2}+\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}-\frac{\sqrt{2}\,(bc-ad)\log\left(\sqrt{b}\,x^{2}-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)/(b*x^4+a),x, algorithm="maxima")
```

```
[Out] d*x/b + 1/8*(2*sqrt(2)*(b*c - a*d)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2
)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) +
 2*sqrt(2)*(b*c - a*d)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^
(1/4))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*(b*
c - a*d)*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^
(1/4)) - sqrt(2)*(b*c - a*d)*log(sqrt(b)*x^2 - sqrt(2)*a^(1/4)*b^(1/4)*x +
sqrt(a))/(a^(3/4)*b^(1/4)))/b
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 639 vs. $2(158) = 316$.
time = 3.37, size = 639, normalized size = 2.87

$$-\frac{4\,b\left(-\frac{b^{4}c^{4}-4\,ab^{3}c^{3}d+6\,a^{2}b^{2}c^{2}d^{2}-4\,a^{3}bcd^{3}+a^{4}d^{4}}{a^{3}b^{5}}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{a^{2}b^{2}\sqrt{-\frac{b^{4}c^{4}-4\,ab^{3}c^{3}d+6\,a^{2}b^{2}c^{2}d^{2}-4\,a^{3}bcd^{3}+a^{4}d^{4}}{a^{3}b^{5}}}+(b^{2}c^{2}-2\,abcd+a^{2}d^{2})x^{2}}\,a^{2}b^{4}\left(-\frac{b^{4}c^{4}-4\,ab^{3}c^{3}d+6\,a^{2}b^{2}c^{2}d^{2}-4\,a^{3}bcd^{3}+a^{4}d^{4}}{a^{3}b^{5}}\right)^{\frac{3}{4}}+(a^{2}b^{5}c-a^{3}b^{4}d)x\left(-\frac{b^{4}c^{4}-4\,ab^{3}c^{3}d+6\,a^{2}b^{2}c^{2}d^{2}-4\,a^{3}bcd^{3}+a^{4}d^{4}}{a^{3}b^{5}}\right)^{\frac{3}{4}}}{b^{4}c^{4}-4\,ab^{3}c^{3}d+6\,a^{2}b^{2}c^{2}d^{2}-4\,a^{3}bcd^{3}+a^{4}d^{4}}\right)+b\left(-\frac{b^{4}c^{4}-4\,ab^{3}c^{3}d+6\,a^{2}b^{2}c^{2}d^{2}-4\,a^{3}bcd^{3}+a^{4}d^{4}}{a^{3}b^{5}}\right)^{\frac{1}{4}}\log\left(ab\left(-\frac{b^{4}c^{4}-4\,ab^{3}c^{3}d+6\,a^{2}b^{2}c^{2}d^{2}-4\,a^{3}bcd^{3}+a^{4}d^{4}}{a^{3}b^{5}}\right)^{\frac{1}{4}}-(bc-ad)x\right)-b\left(-\frac{b^{4}c^{4}-4\,ab^{3}c^{3}d+6\,a^{2}b^{2}c^{2}d^{2}-4\,a^{3}bcd^{3}+a^{4}d^{4}}{a^{3}b^{5}}\right)^{\frac{1}{4}}\log\left(-ab\left(-\frac{b^{4}c^{4}-4\,ab^{3}c^{3}d+6\,a^{2}b^{2}c^{2}d^{2}-4\,a^{3}bcd^{3}+a^{4}d^{4}}{a^{3}b^{5}}\right)^{\frac{1}{4}}-(bc-ad)x\right)-4\,dx}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)/(b*x^4+a),x, algorithm="fricas")
```

```
[Out] -1/4*(4*b*(-(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 +
a^4*d^4)/(a^3*b^5))^(1/4)*arctan((sqrt(a^2*b^2*sqrt(-(b^4*c^4 - 4*a*b^3*c^3
*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)/(a^3*b^5)) + (b^2*c^2 - 2
*a*b*c*d + a^2*d^2)*x^2)*a^2*b^4*(-(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2
*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)/(a^3*b^5))^(3/4) + (a^2*b^5*c - a^3*b^4*d)*
x*(-(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)
```

```
/(a^3*b^5))^(3/4))/(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c
*d^3 + a^4*d^4)) + b*(-(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3
*b*c*d^3 + a^4*d^4)/(a^3*b^5))^(1/4)*log(a*b*(-(b^4*c^4 - 4*a*b^3*c^3*d + 6
*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)/(a^3*b^5))^(1/4) - (b*c - a*d)*
x) - b*(-(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4
*d^4)/(a^3*b^5))^(1/4)*log(-a*b*(-(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*
d^2 - 4*a^3*b*c*d^3 + a^4*d^4)/(a^3*b^5))^(1/4) - (b*c - a*d)*x) - 4*d*x)/b
```

Sympy [A]
time = 0.31, size = 87, normalized size = 0.39

$$\operatorname{RootSum}\left(256t^4a^3b^5 + a^4d^4 - 4a^3bcd^3 + 6a^2b^2c^2d^2 - 4ab^3c^3d + b^4c^4, \left(t \mapsto t\log\left(-\frac{4tab}{ad-bc} + x\right)\right)\right) + \frac{dx}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**4+c)/(b*x**4+a),x)
```

```
[Out] RootSum(256*_t**4*a**3*b**5 + a**4*d**4 - 4*a**3*b*c*d**3 + 6*a**2*b**2*c**
2*d**2 - 4*a*b**3*c**3*d + b**4*c**4, Lambda(_t, _t*log(-4*_t*a*b/(a*d - b*
c) + x))) + d*x/b
```

Giac [A]
time = 0.61, size = 245, normalized size = 1.10

$$\frac{dx}{b} + \frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}bc - (ab^3)^{\frac{1}{4}}ad\right)\arctan\left(\frac{\sqrt{2}\left(2x + \sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4ab^2} + \frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}bc - (ab^3)^{\frac{1}{4}}ad\right)\arctan\left(\frac{\sqrt{2}\left(2x - \sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4ab^2} + \frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}bc - (ab^3)^{\frac{1}{4}}ad\right)\log\left(x^2 + \sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{b}}\right)}{8ab^2} - \frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}bc - (ab^3)^{\frac{1}{4}}ad\right)\log\left(x^2 - \sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{b}}\right)}{8ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)/(b*x^4+a),x, algorithm="giac")
```

```
[Out] d*x/b + 1/4*sqrt(2)*((a*b^3)^(1/4)*b*c - (a*b^3)^(1/4)*a*d)*arctan(1/2*sqrt
(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a*b^2) + 1/4*sqrt(2)*((a*b^3)
^(1/4)*b*c - (a*b^3)^(1/4)*a*d)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/
4))/(a/b)^(1/4))/(a*b^2) + 1/8*sqrt(2)*((a*b^3)^(1/4)*b*c - (a*b^3)^(1/4)*a
*d)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a*b^2) - 1/8*sqrt(2)*((a*
b^3)^(1/4)*b*c - (a*b^3)^(1/4)*a*d)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(
a/b))/(a*b^2)
```

Mupad [B]
time = 0.22, size = 720, normalized size = 3.23

$$\frac{d\,x}{b} - \frac{\operatorname{atan}\left(\frac{\frac{\left(x\,\left(4\,a^2\,b\,d^2-8\,a\,b^2\,c\,d+4\,b^3\,c^2\right)-\frac{\left(16\,a^2\,b^2\,d-16\,a\,b^3\,c\right)\,\left(a\,d-b\,c\right)}{4\,{\left(-a\right)}^{3/4}\,b^{5/4}}\right)\,\left(a\,d-b\,c\right)\,1{}\mathrm{i}}{4\,{\left(-a\right)}^{3/4}\,b^{5/4}}+\frac{\left(x\,\left(4\,a^2\,b\,d^2-8\,a\,b^2\,c\,d+4\,b^3\,c^2\right)+\frac{\left(16\,a^2\,b^2\,d-16\,a\,b^3\,c\right)\,\left(a\,d-b\,c\right)}{4\,{\left(-a\right)}^{3/4}\,b^{5/4}}\right)\,\left(a\,d-b\,c\right)\,1{}\mathrm{i}}{4\,{\left(-a\right)}^{3/4}\,b^{5/4}}}{\frac{\left(x\,\left(4\,a^2\,b\,d^2-8\,a\,b^2\,c\,d+4\,b^3\,c^2\right)-\frac{\left(16\,a^2\,b^2\,d-16\,a\,b^3\,c\right)\,\left(a\,d-b\,c\right)}{4\,{\left(-a\right)}^{3/4}\,b^{5/4}}\right)\,\left(a\,d-b\,c\right)}{4\,{\left(-a\right)}^{3/4}\,b^{5/4}}-\frac{\left(x\,\left(4\,a^2\,b\,d^2-8\,a\,b^2\,c\,d+4\,b^3\,c^2\right)+\frac{\left(16\,a^2\,b^2\,d-16\,a\,b^3\,c\right)\,\left(a\,d-b\,c\right)}{4\,{\left(-a\right)}^{3/4}\,b^{5/4}}\right)\,\left(a\,d-b\,c\right)}{4\,{\left(-a\right)}^{3/4}\,b^{5/4}}}\right)\,\left(a\,d-b\,c\right)\,1{}\mathrm{i}}{2\,{\left(-a\right)}^{3/4}\,b^{5/4}} - \frac{\operatorname{atan}\left(\frac{\frac{\left(x\,\left(4\,a^2\,b\,d^2-8\,a\,b^2\,c\,d+4\,b^3\,c^2\right)-\frac{\left(16\,a^2\,b^2\,d-16\,a\,b^3\,c\right)\,\left(a\,d-b\,c\right)\,1{}\mathrm{i}}{4\,{\left(-a\right)}^{3/4}\,b^{5/4}}\right)\,\left(a\,d-b\,c\right)}{4\,{\left(-a\right)}^{3/4}\,b^{5/4}}+\frac{\left(x\,\left(4\,a^2\,b\,d^2-8\,a\,b^2\,c\,d+4\,b^3\,c^2\right)+\frac{\left(16\,a^2\,b^2\,d-16\,a\,b^3\,c\right)\,\left(a\,d-b\,c\right)\,1{}\mathrm{i}}{4\,{\left(-a\right)}^{3/4}\,b^{5/4}}\right)\,\left(a\,d-b\,c\right)}{4\,{\left(-a\right)}^{3/4}\,b^{5/4}}}{\frac{\left(x\,\left(4\,a^2\,b\,d^2-8\,a\,b^2\,c\,d+4\,b^3\,c^2\right)-\frac{\left(16\,a^2\,b^2\,d-16\,a\,b^3\,c\right)\,\left(a\,d-b\,c\right)\,1{}\mathrm{i}}{4\,{\left(-a\right)}^{3/4}\,b^{5/4}}\right)\,\left(a\,d-b\,c\right)\,1{}\mathrm{i}}{4\,{\left(-a\right)}^{3/4}\,b^{5/4}}-\frac{\left(x\,\left(4\,a^2\,b\,d^2-8\,a\,b^2\,c\,d+4\,b^3\,c^2\right)+\frac{\left(16\,a^2\,b^2\,d-16\,a\,b^3\,c\right)\,\left(a\,d-b\,c\right)\,1{}\mathrm{i}}{4\,{\left(-a\right)}^{3/4}\,b^{5/4}}\right)\,\left(a\,d-b\,c\right)\,1{}\mathrm{i}}{4\,{\left(-a\right)}^{3/4}\,b^{5/4}}}\right)\,\left(a\,d-b\,c\right)}{2\,{\left(-a\right)}^{3/4}\,b^{5/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^4)/(a + b*x^4),x)
```

```
[Out] (d*x)/b - (atan((((x*(4*b^3*c^2 + 4*a^2*b*d^2 - 8*a*b^2*c*d) - ((16*a^2*b^2
*d - 16*a*b^3*c)*(a*d - b*c))/(4*(-a)^(3/4)*b^(5/4)))*(a*d - b*c)*1i)/(4*(-
a)^(3/4)*b^(5/4)) + ((x*(4*b^3*c^2 + 4*a^2*b*d^2 - 8*a*b^2*c*d) + ((16*a^2*
b^2*d - 16*a*b^3*c)*(a*d - b*c))/(4*(-a)^(3/4)*b^(5/4)))*(a*d - b*c)*1i)/(4
*(-a)^(3/4)*b^(5/4)))/(((x*(4*b^3*c^2 + 4*a^2*b*d^2 - 8*a*b^2*c*d) - ((16*a
^2*b^2*d - 16*a*b^3*c)*(a*d - b*c))/(4*(-a)^(3/4)*b^(5/4)))*(a*d - b*c))/(4
*(-a)^(3/4)*b^(5/4)) - ((x*(4*b^3*c^2 + 4*a^2*b*d^2 - 8*a*b^2*c*d) + ((16*a
^2*b^2*d - 16*a*b^3*c)*(a*d - b*c))/(4*(-a)^(3/4)*b^(5/4)))*(a*d - b*c))/(4
*(-a)^(3/4)*b^(5/4))))*(a*d - b*c)*1i)/(2*(-a)^(3/4)*b^(5/4)) - (atan((((x*
(4*b^3*c^2 + 4*a^2*b*d^2 - 8*a*b^2*c*d) - ((16*a^2*b^2*d - 16*a*b^3*c)*(a*d
 - b*c)*1i)/(4*(-a)^(3/4)*b^(5/4)))*(a*d - b*c))/(4*(-a)^(3/4)*b^(5/4)) + (
(x*(4*b^3*c^2 + 4*a^2*b*d^2 - 8*a*b^2*c*d) + ((16*a^2*b^2*d - 16*a*b^3*c)*(
a*d - b*c)*1i)/(4*(-a)^(3/4)*b^(5/4)))*(a*d - b*c))/(4*(-a)^(3/4)*b^(5/4)))
/(((x*(4*b^3*c^2 + 4*a^2*b*d^2 - 8*a*b^2*c*d) - ((16*a^2*b^2*d - 16*a*b^3*c
)*(a*d - b*c)*1i)/(4*(-a)^(3/4)*b^(5/4)))*(a*d - b*c)*1i)/(4*(-a)^(3/4)*b^(
5/4)) - ((x*(4*b^3*c^2 + 4*a^2*b*d^2 - 8*a*b^2*c*d) + ((16*a^2*b^2*d - 16*a
*b^3*c)*(a*d - b*c)*1i)/(4*(-a)^(3/4)*b^(5/4)))*(a*d - b*c)*1i)/(4*(-a)^(3/
4)*b^(5/4))))*(a*d - b*c))/(2*(-a)^(3/4)*b^(5/4))
```

3.164 $\int \frac{1}{(a+bx^4)(c+dx^4)}\,dx$

Optimal. Leaf size=449

$$-\frac{b^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}(bc-ad)}+\frac{b^{3/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}(bc-ad)}+\frac{d^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right)}{2\sqrt{2}\,c^{3/4}(bc-ad)}-\frac{d^{3/4}\tan^{-1}\left(1+\right.}{2\sqrt{2}\,c^{3/4}(bc}$$

```
[Out] 1/4*b^(3/4)*arctan(-1+b^(1/4)*x*2^(1/2)/a^(1/4))/a^(3/4)/(-a*d+b*c)*2^(1/2)
+1/4*b^(3/4)*arctan(1+b^(1/4)*x*2^(1/2)/a^(1/4))/a^(3/4)/(-a*d+b*c)*2^(1/2)
-1/4*d^(3/4)*arctan(-1+d^(1/4)*x*2^(1/2)/c^(1/4))/c^(3/4)/(-a*d+b*c)*2^(1/2
)-1/4*d^(3/4)*arctan(1+d^(1/4)*x*2^(1/2)/c^(1/4))/c^(3/4)/(-a*d+b*c)*2^(1/2
)-1/8*b^(3/4)*ln(-a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))/a^(3/4)/(-
a*d+b*c)*2^(1/2)+1/8*b^(3/4)*ln(a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/
2))/a^(3/4)/(-a*d+b*c)*2^(1/2)+1/8*d^(3/4)*ln(-c^(1/4)*d^(1/4)*x*2^(1/2)+c^
(1/2)+x^2*d^(1/2))/c^(3/4)/(-a*d+b*c)*2^(1/2)-1/8*d^(3/4)*ln(c^(1/4)*d^(1/4
)*x*2^(1/2)+c^(1/2)+x^2*d^(1/2))/c^(3/4)/(-a*d+b*c)*2^(1/2)
```

Rubi [A]
time = 0.18, antiderivative size = 449, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {400, 217, 1179, 642, 1176, 631, 210}

$$-\frac{b^{3/4}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}(bc-ad)}+\frac{b^{3/4}\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}\,a^{3/4}(bc-ad)}-\frac{b^{3/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{4\sqrt{2}\,a^{3/4}(bc-ad)}+\frac{b^{3/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{4\sqrt{2}\,a^{3/4}(bc-ad)}+\frac{d^{3/4}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right)}{2\sqrt{2}\,c^{3/4}(bc-ad)}-\frac{d^{3/4}\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{d}\,x}{\sqrt[4]{c}}+1\right)}{2\sqrt{2}\,c^{3/4}(bc-ad)}+\frac{d^{3/4}\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\,x+\sqrt{c}+\sqrt{d}\,x^2\right)}{4\sqrt{2}\,c^{3/4}(bc-ad)}-\frac{d^{3/4}\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\,x+\sqrt{c}+\sqrt{d}\,x^2\right)}{4\sqrt{2}\,c^{3/4}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] -1/2*(b^(3/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(Sqrt[2]*a^(3/4)*(b*
c - a*d)) + (b^(3/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^
(3/4)*(b*c - a*d)) + (d^(3/4)*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/(2*S
qrt[2]*c^(3/4)*(b*c - a*d)) - (d^(3/4)*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/
4)])/(2*Sqrt[2]*c^(3/4)*(b*c - a*d)) - (b^(3/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/
4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*(b*c - a*d)) + (b^(3/4)*Log
[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*(b*
c - a*d)) + (d^(3/4)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2]
)/(4*Sqrt[2]*c^(3/4)*(b*c - a*d)) - (d^(3/4)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*
d^(1/4)*x + Sqrt[d]*x^2])/(4*Sqrt[2]*c^(3/4)*(b*c - a*d))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 400

```
Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[d/(b*c - a*d), Int[1/(c +
 d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{(a+bx^4)(c+dx^4)}\,dx = \frac{b\int \frac{1}{a+bx^4}\,dx}{bc-ad} - \frac{d\int \frac{1}{c+dx^4}\,dx}{bc-ad}$$

$$= \frac{b\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx}{2\sqrt{a}\,(bc-ad)} + \frac{b\int \frac{\sqrt{a}+\sqrt{b}\,x^2}{a+bx^4}\,dx}{2\sqrt{a}\,(bc-ad)} - \frac{d\int \frac{\sqrt{c}-\sqrt{d}\,x^2}{c+dx^4}\,dx}{2\sqrt{c}\,(bc-ad)} - \frac{d\int \frac{\sqrt{c}+\sqrt{d}\,x^2}{c+dx^4}}{2\sqrt{c}\,(bc-ad}$$

$$= \frac{\sqrt{b}\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx}{4\sqrt{a}\,(bc-ad)} + \frac{\sqrt{b}\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx}{4\sqrt{a}\,(bc-ad)} - \frac{b^{3/4}\int \frac{\frac{\sqrt{2}\sqrt[4]{}}{\sqrt[4]{b}}}{-\frac{\sqrt{a}}{\sqrt{b}}-\sqrt{2}}}{4\sqrt{2}\,a^{3/4}(b}$$

$$= -\frac{b^{3/4}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{b}\,x^2\right)}{4\sqrt{2}\,a^{3/4}(bc-ad)} + \frac{b^{3/4}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{}\right.}{4\sqrt{2}\,a^{3/4}(bc-ad)}$$

$$= -\frac{b^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}(bc-ad)} + \frac{b^{3/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}(bc-ad)} + \frac{d^{3/4}\tan^{-1}\left(1-\right.}{2\sqrt{2}\,c^{3/4}(bc}$$

Mathematica [A]
time = 0.09, size = 340, normalized size = 0.76

$$\frac{-2b^{3/4}c^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)+2b^{3/4}c^{3/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)+2a^{3/4}d^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right)-2a^{3/4}d^{3/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right)-b^{3/4}c^{3/4}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{b}\,x^2\right)+b^{3/4}c^{3/4}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{b}\,x^2\right)+a^{3/4}d^{3/4}\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\,x+\sqrt{d}\,x^2\right)-a^{3/4}d^{3/4}\log\left(\sqrt{c}+\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\,x+\sqrt{d}\,x^2\right)}{4\sqrt{2}\,a^{3/4}c^{3/4}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] (-2*b^(3/4)*c^(3/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 2*b^(3/4)*c^(
3/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 2*a^(3/4)*d^(3/4)*ArcTan[1 -
 (Sqrt[2]*d^(1/4)*x)/c^(1/4)] - 2*a^(3/4)*d^(3/4)*ArcTan[1 + (Sqrt[2]*d^(1/
4)*x)/c^(1/4)] - b^(3/4)*c^(3/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x +
Sqrt[b]*x^2] + b^(3/4)*c^(3/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sq
rt[b]*x^2] + a^(3/4)*d^(3/4)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt
[d]*x^2] - a^(3/4)*d^(3/4)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d
]*x^2])/(4*Sqrt[2]*a^(3/4)*c^(3/4)*(b*c - a*d))
```

Maple [A]
time = 0.33, size = 226, normalized size = 0.50

method	result

default	$\frac{d\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\left(\ln\left(\frac{x^{2}+\left(\frac{c}{d}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{c}{d}}}{x^{2}-\left(\frac{c}{d}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{c}{d}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)\right)}{8(ad-bc)c}-\frac{b\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\left(\ln\left(\frac{x^{2}+\left(\frac{a}{b}\right)^{\frac{1}{4}}\cdots}{x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{4}}\cdots}\right.\right.}{}$
risch	$\left(\sum_{\_R=\mathrm{RootOf}\left(\left(a^{4}c^{3}d^{4}-4a^{3}b\,c^{4}d^{3}+6a^{2}b^{2}c^{5}d^{2}-4a\,b^{3}c^{6}d+b^{4}c^{7}\right)\_Z^{4}+d^{3}\right)}\_R\ln\left(\left(\left(-a^{7}d^{7}+4a^{6}bc\,d^{6}-6a^{5}b^{2}c^{2}d^{5}+3a^{4}b^{3}c^{3}d^{4}+3a^{3}b^{4}c^{4}\cdots\right.\right.\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^4+a)/(d*x^4+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*d/(a*d-b*c)*(c/d)^(1/4)/c*2^(1/2)*(ln((x^2+(c/d)^(1/4)*x*2^(1/2)+(c/d)^
(1/2))/(x^2-(c/d)^(1/4)*x*2^(1/2)+(c/d)^(1/2)))+2*arctan(2^(1/2)/(c/d)^(1/4
)*x+1)+2*arctan(2^(1/2)/(c/d)^(1/4)*x-1))-1/8*b/(a*d-b*c)*(a/b)^(1/4)/a*2^(
1/2)*(ln((x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*x*2^(1/2)
+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/b)^(1/
4)*x-1))
```

Maxima [A]
time = 0.49, size = 365, normalized size = 0.81

$$\frac{\frac{2\sqrt{2}\,b\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}\,x+\sqrt{2}\,a^{\frac14}b^{\frac14}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{2\sqrt{2}\,b\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}\,x-\sqrt{2}\,a^{\frac14}b^{\frac14}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{\sqrt{2}\,b^{\frac34}\log\left(\sqrt{b}\,x^{2}+\sqrt{2}\,a^{\frac14}b^{\frac14}x+\sqrt{a}\right)}{a^{\frac34}}-\frac{\sqrt{2}\,b^{\frac34}\log\left(\sqrt{b}\,x^{2}-\sqrt{2}\,a^{\frac14}b^{\frac14}x+\sqrt{a}\right)}{a^{\frac34}}}{8\,(bc-ad)}-\frac{\frac{2\sqrt{2}\,d\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}\,x+\sqrt{2}\,c^{\frac14}d^{\frac14}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}+\frac{2\sqrt{2}\,d\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}\,x-\sqrt{2}\,c^{\frac14}d^{\frac14}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}+\frac{\sqrt{2}\,d^{\frac34}\log\left(\sqrt{d}\,x^{2}+\sqrt{2}\,c^{\frac14}d^{\frac14}x+\sqrt{c}\right)}{c^{\frac34}}-\frac{\sqrt{2}\,d^{\frac34}\log\left(\sqrt{d}\,x^{2}-\sqrt{2}\,c^{\frac14}d^{\frac14}x+\sqrt{c}\right)}{c^{\frac34}}}{8\,(bc-ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")
```

```
[Out] 1/8*(2*sqrt(2)*b*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4))
/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqrt(2)*b*arcta
n(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b))
)/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*b^(3/4)*log(sqrt(b)*x^2 + sqrt(
2)*a^(1/4)*b^(1/4)*x + sqrt(a))/a^(3/4) - sqrt(2)*b^(3/4)*log(sqrt(b)*x^2 -
 sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/a^(3/4))/(b*c - a*d) - 1/8*(2*sqrt(2)
*d*arctan(1/2*sqrt(2)*(2*sqrt(d)*x + sqrt(2)*c^(1/4)*d^(1/4))/sqrt(sqrt(c)*
sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + 2*sqrt(2)*d*arctan(1/2*sqrt(2)*
(2*sqrt(d)*x - sqrt(2)*c^(1/4)*d^(1/4))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqr
t(sqrt(c)*sqrt(d))) + sqrt(2)*d^(3/4)*log(sqrt(d)*x^2 + sqrt(2)*c^(1/4)*d^(
1/4)*x + sqrt(c))/c^(3/4) - sqrt(2)*d^(3/4)*log(sqrt(d)*x^2 - sqrt(2)*c^(1/
4)*d^(1/4)*x + sqrt(c))/c^(3/4))/(b*c - a*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1354 vs. $2(319) = 638$.
time = 4.28, size = 1354, normalized size = 3.02

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")
```

```
[Out] -(-b^3/(a^3*b^4*c^4 - 4*a^4*b^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6*b*c*d^3 +
 a^7*d^4))^(1/4)*arctan(((a^2*b^4*c^3 - 3*a^3*b^3*c^2*d + 3*a^4*b^2*c*d^2 -
 a^5*b*d^3)*(-b^3/(a^3*b^4*c^4 - 4*a^4*b^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^
6*b*c*d^3 + a^7*d^4))^(3/4)*x - (a^2*b^3*c^3 - 3*a^3*b^2*c^2*d + 3*a^4*b*c*
d^2 - a^5*d^3)*sqrt(b^2*x^2 + (a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2)*sqrt(-b
^3/(a^3*b^4*c^4 - 4*a^4*b^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6*b*c*d^3 + a^7
*d^4)))*(-b^3/(a^3*b^4*c^4 - 4*a^4*b^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6*b*
c*d^3 + a^7*d^4))^(3/4))/b^3) + (-d^3/(b^4*c^7 - 4*a*b^3*c^6*d + 6*a^2*b^2*
c^5*d^2 - 4*a^3*b*c^4*d^3 + a^4*c^3*d^4))^(1/4)*arctan(((b^3*c^5*d - 3*a*b^
2*c^4*d^2 + 3*a^2*b*c^3*d^3 - a^3*c^2*d^4)*(-d^3/(b^4*c^7 - 4*a*b^3*c^6*d +
 6*a^2*b^2*c^5*d^2 - 4*a^3*b*c^4*d^3 + a^4*c^3*d^4))^(3/4)*x - (b^3*c^5 - 3
*a*b^2*c^4*d + 3*a^2*b*c^3*d^2 - a^3*c^2*d^3)*sqrt(d^2*x^2 + (b^2*c^4 - 2*a
*b*c^3*d + a^2*c^2*d^2)*sqrt(-d^3/(b^4*c^7 - 4*a*b^3*c^6*d + 6*a^2*b^2*c^5*
d^2 - 4*a^3*b*c^4*d^3 + a^4*c^3*d^4)))*(-d^3/(b^4*c^7 - 4*a*b^3*c^6*d + 6*a
^2*b^2*c^5*d^2 - 4*a^3*b*c^4*d^3 + a^4*c^3*d^4))^(3/4))/d^3) + 1/4*(-b^3/(a
^3*b^4*c^4 - 4*a^4*b^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6*b*c*d^3 + a^7*d^4)
)^(1/4)*log(b*x + (a*b*c - a^2*d)*(-b^3/(a^3*b^4*c^4 - 4*a^4*b^3*c^3*d + 6*
a^5*b^2*c^2*d^2 - 4*a^6*b*c*d^3 + a^7*d^4))^(1/4)) - 1/4*(-b^3/(a^3*b^4*c^4
 - 4*a^4*b^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6*b*c*d^3 + a^7*d^4))^(1/4)*lo
g(b*x - (a*b*c - a^2*d)*(-b^3/(a^3*b^4*c^4 - 4*a^4*b^3*c^3*d + 6*a^5*b^2*c^
2*d^2 - 4*a^6*b*c*d^3 + a^7*d^4))^(1/4)) - 1/4*(-d^3/(b^4*c^7 - 4*a*b^3*c^6
*d + 6*a^2*b^2*c^5*d^2 - 4*a^3*b*c^4*d^3 + a^4*c^3*d^4))^(1/4)*log(d*x + (b
*c^2 - a*c*d)*(-d^3/(b^4*c^7 - 4*a*b^3*c^6*d + 6*a^2*b^2*c^5*d^2 - 4*a^3*b*
c^4*d^3 + a^4*c^3*d^4))^(1/4)) + 1/4*(-d^3/(b^4*c^7 - 4*a*b^3*c^6*d + 6*a^2
*b^2*c^5*d^2 - 4*a^3*b*c^4*d^3 + a^4*c^3*d^4))^(1/4)*log(d*x - (b*c^2 - a*c
*d)*(-d^3/(b^4*c^7 - 4*a*b^3*c^6*d + 6*a^2*b^2*c^5*d^2 - 4*a^3*b*c^4*d^3 +
a^4*c^3*d^4))^(1/4))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**4+a)/(d*x**4+c),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.62, size = 437, normalized size = 0.97

$$\frac{\left(ab^{3}\right)^{\frac{1}{4}} \arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}\,abc-\sqrt{2}\,a^{2}d\right)}+\frac{\left(ab^{3}\right)^{\frac{1}{4}} \arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}\,abc-\sqrt{2}\,a^{2}d\right)}-\frac{\left(cd^{3}\right)^{\frac{1}{4}} \arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}\,bc^{2}-\sqrt{2}\,acd\right)}-\frac{\left(cd^{3}\right)^{\frac{1}{4}} \arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}\,bc^{2}-\sqrt{2}\,acd\right)}+\frac{\left(ab^{3}\right)^{\frac{1}{4}} \log\left(x^{2}+\sqrt{2}\,x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{4\left(\sqrt{2}\,abc-\sqrt{2}\,a^{2}d\right)}-\frac{\left(ab^{3}\right)^{\frac{1}{4}} \log\left(x^{2}-\sqrt{2}\,x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{4\left(\sqrt{2}\,abc-\sqrt{2}\,a^{2}d\right)}-\frac{\left(cd^{3}\right)^{\frac{1}{4}} \log\left(x^{2}+\sqrt{2}\,x\left(\frac{c}{d}\right)^{\frac{1}{4}}+\sqrt{\frac{c}{d}}\right)}{4\left(\sqrt{2}\,bc^{2}-\sqrt{2}\,acd\right)}+\frac{\left(cd^{3}\right)^{\frac{1}{4}} \log\left(x^{2}-\sqrt{2}\,x\left(\frac{c}{d}\right)^{\frac{1}{4}}+\sqrt{\frac{c}{d}}\right)}{4\left(\sqrt{2}\,bc^{2}-\sqrt{2}\,acd\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")

[Out] 1/2*(a*b^3)^(1/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(sqrt(2)*a*b*c - sqrt(2)*a^2*d) + 1/2*(a*b^3)^(1/4)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(sqrt(2)*a*b*c - sqrt(2)*a^2*d) - 1/2*(c*d^3)^(1/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(c/d)^(1/4))/(c/d)^(1/4))/(sqrt(2)*b*c^2 - sqrt(2)*a*c*d) - 1/2*(c*d^3)^(1/4)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(c/d)^(1/4))/(c/d)^(1/4))/(sqrt(2)*b*c^2 - sqrt(2)*a*c*d) + 1/4*(a*b^3)^(1/4)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(sqrt(2)*a*b*c - sqrt(2)*a^2*d) - 1/4*(a*b^3)^(1/4)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(sqrt(2)*a*b*c - sqrt(2)*a^2*d) - 1/4*(c*d^3)^(1/4)*log(x^2 + sqrt(2)*x*(c/d)^(1/4) + sqrt(c/d))/(sqrt(2)*b*c^2 - sqrt(2)*a*c*d) + 1/4*(c*d^3)^(1/4)*log(x^2 - sqrt(2)*x*(c/d)^(1/4) + sqrt(c/d))/(sqrt(2)*b*c^2 - sqrt(2)*a*c*d)

Mupad [B]
time = 2.76, size = 2500, normalized size = 5.57

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a + b*x^4)*(c + d*x^4)),x)

[Out] - atan((((-d^3/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d))^(1/4)*((-d^3/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d))^(3/4)*((-d^3/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d))^(1/4)*(4096*a*b^11*c^8*d^4 + 4096*a^8*b^4*c*d^11 - 20480*a^2*b^10*c^7*d^5 + 36864*a^3*b^9*c^6*d^6 - 20480*a^4*b^8*c^5*d^7 - 20480*a^5*b^7*c^4*d^8 + 36864*a^6*b^6*c^3*d^9 - 20480*a^7*b^5*c^2*d^10) + x*(1024*a^7*b^4*d^11 + 1024*b^11*c^7*d^4 - 4096*a*b^10*c^6*d^5 - 4096*a^6*b^5*c*d^10 + 6144*a^2*b^9*c^5*d^6 - 3072*a^3*b^8*c^4*d^7 - 3072*a^4*b^7*c^3*d^8 + 6144*a^5*b^6*c^2*d^9)) - 16*a^2*b^6*d^8 - 16*b^8*c^2*d^6 + 32*a*b^7*c*d^7) + 8*b^7*d^7*x)*(-d^3/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d))^(1/4)*1i - ((-d^3/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d))^(1/4)*((-d^3/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d))^(3/4)*((-d^3/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d))^(1/4)*(4096*a*b^11*c^8*d^4 + 4096*a^8*b^4*c*d^11 - 20480*a^2*b^10*c^7*d^5 + 36864*a^3*b^9*c^6*d^6 - 20480*a^4*b^8*c^5*d^7 - 20480*a^5*b^7*c^4*d^8 + 36864*a^6*b^6*c^3*d^9 - 20480*a^7*b^5*c^2*d^10) - x*(1024*a^7*b^4*d^11 + 1024*b^11*c^7*d^4 - 4096*a*b^10*c^6*d^5 - 4096*a^6*b^5*c*d^10 + 6144*a^2*b^9*c^5*d^6 - 3072*a^3*b^8*c^4*d^7 - 3072*a^4*b^7*c^3*d^8 + 6144*a^5*b^6*c

```
^2*d^9)) - 16*a^2*b^6*d^8 - 16*b^8*c^2*d^6 + 32*a*b^7*c*d^7) - 8*b^7*d^7*x)
*(-d^3/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a^2*b^2*c
^5*d^2 - 1024*a*b^3*c^6*d))^(1/4)*1i)/(((-d^3/(256*b^4*c^7 + 256*a^4*c^3*d^
4 - 1024*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d))^(1/4)*((
-d^3/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^5
*d^2 - 1024*a*b^3*c^6*d))^(3/4)*((-d^3/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 102
4*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d))^(1/4)*(4096*a*b
^11*c^8*d^4 + 4096*a^8*b^4*c*d^11 - 20480*a^2*b^10*c^7*d^5 + 36864*a^3*b^9*
c^6*d^6 - 20480*a^4*b^8*c^5*d^7 - 20480*a^5*b^7*c^4*d^8 + 36864*a^6*b^6*c^3
*d^9 - 20480*a^7*b^5*c^2*d^10) + x*(1024*a^7*b^4*d^11 + 1024*b^11*c^7*d^4 -
 4096*a*b^10*c^6*d^5 - 4096*a^6*b^5*c*d^10 + 6144*a^2*b^9*c^5*d^6 - 3072*a^
3*b^8*c^4*d^7 - 3072*a^4*b^7*c^3*d^8 + 6144*a^5*b^6*c^2*d^9)) - 16*a^2*b^6*
d^8 - 16*b^8*c^2*d^6 + 32*a*b^7*c*d^7) + 8*b^7*d^7*x)*(-d^3/(256*b^4*c^7 +
256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c^
6*d))^(1/4) + ((-d^3/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 +
1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d))^(1/4)*((-d^3/(256*b^4*c^7 + 256*a
^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d))
^(3/4)*((-d^3/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a^
2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d))^(1/4)*(4096*a*b^11*c^8*d^4 + 4096*a^8*b^
4*c*d^11 - 20480*a^2*b^10*c^7*d^5 + 36864*a^3*b^9*c^6*d^6 - 20480*a^4*b^8*c
^5*d^7 - 20480*a^5*b^7*c^4*d^8 + 36864*a^6*b^6*c^3*d^9 - 20480*a^7*b^5*c^2*
d^10) - x*(1024*a^7*b^4*d^11 + 1024*b^11*c^7*d^4 - 4096*a*b^10*c^6*d^5 - 40
96*a^6*b^5*c*d^10 + 6144*a^2*b^9*c^5*d^6 - 3072*a^3*b^8*c^4*d^7 - 3072*a^4*
b^7*c^3*d^8 + 6144*a^5*b^6*c^2*d^9)) - 16*a^2*b^6*d^8 - 16*b^8*c^2*d^6 + 32
*a*b^7*c*d^7) - 8*b^7*d^7*x)*(-d^3/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 1024*a^
3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d))^(1/4)))*(-d^3/(256*
b^4*c^7 + 256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 102
4*a*b^3*c^6*d))^(1/4)*2i - atan((((-b^3/(256*a^7*d^4 + 256*a^3*b^4*c^4 - 10
24*a^4*b^3*c^3*d + 1536*a^5*b^2*c^2*d^2 - 1024*a^6*b*c*d^3))^(1/4)*((-b^3/(
256*a^7*d^4 + 256*a^3*b^4*c^4 - 1024*a^4*b^3*c^3*d + 1536*a^5*b^2*c^2*d^2 -
 1024*a^6*b*c*d^3))^(3/4)*((-b^3/(256*a^7*d^4 + 256*a^3*b^4*c^4 - 1024*a^4*
b^3*c^3*d + 1536*a^5*b^2*c^2*d^2 - 1024*a^6*b*c*d^3))^(1/4)*(4096*a*b^11*c^
8*d^4 + 4096*a^8*b^4*c*d^11 - 20480*a^2*b^10*c^7*d^5 + 36864*a^3*b^9*c^6*d^
6 - 20480*a^4*b^8*c^5*d^7 - 20480*a^5*b^7*c^4*d^8 + 36864*a^6*b^6*c^3*d^9 -
 20480*a^7*b^5*c^2*d^10) + x*(1024*a^7*b^4*d^11 + 1024*b^11*c^7*d^4 - 4096*
a*b^10*c^6*d^5 - 4096*a^6*b^5*c*d^10 + 6144*a^2*b^9*c^5*d^6 - 3072*a^3*b^8*
c^4*d^7 - 3072*a^4*b^7*c^3*d^8 + 6144*a^5*b^6*c^2*d^9)) - 16*a^2*b^6*d^8 -
16*b^8*c^2*d^6 + 32*a*b^7*c*d^7) + 8*b^7*d^7*x)*(-b^3/(256*a^7*d^4 + 256*a^
3*b^4*c^4 - 1024*a^4*b^3*c^3*d + 1536*a^5*b^2*c^2*d^2 - 1024*a^6*b*c*d^3))^
(1/4)*1i - ((-b^3/(256*a^7*d^4 + 256*a^3*b^4*c^4 - 1024*a^4*b^3*c^3*d + 153
6*a^5*b^2*c^2*d^2 - 1024*a^6*b*c*d^3))^(1/4)*((-b^3/(256*a^7*d^4 + 256*a^3*
b^4*c^4 - 1024*a^4*b^3*c^3*d + 1536*a^5*b^2*c^2*d^2 - 1024*a^6*b*c*d^3))^(3
/4)*((-b^3/(256*a^7*d^4 + 256*a^3*b^4*c^4 - 1024*a^4*b^3*c^3*d + 1536*a^5*b
^2*c^2*d^2 - 1024*a^6*b*c*d^3))^(1/4)*(4096*a*b^11*c^8*d^4 + 4096*a^8*b^4*c
*d^11 - 20480*a^2*b^10*c^7*d^5 + 36864*a^3*b^9*c^6*d^6 - 20480*a^4*b^8*c^5*
```

```
d^7 - 20480*a^5*b^7*c^4*d^8 + 36864*a^6*b^6*c^3...
```

3.165 $\int \frac{1}{(a+bx^4)(c+dx^4)^2} \, dx$

Optimal. Leaf size=513

$$-\frac{dx}{4c(bc-ad)(c+dx^4)}-\frac{b^{7/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}(bc-ad)^2}+\frac{b^{7/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}(bc-ad)^2}+\frac{d^{3/4}(7bc-3ad)\tan^{-1}\left(1\right.}{8\sqrt{2}\,c^{7/4}(bc-}$$

```
[Out] -1/4*d*x/c/(-a*d+b*c)/(d*x^4+c)+1/4*b^(7/4)*arctan(-1+b^(1/4)*x*2^(1/2)/a^(
1/4))/a^(3/4)/(-a*d+b*c)^2*2^(1/2)+1/4*b^(7/4)*arctan(1+b^(1/4)*x*2^(1/2)/a
^(1/4))/a^(3/4)/(-a*d+b*c)^2*2^(1/2)-1/16*d^(3/4)*(-3*a*d+7*b*c)*arctan(-1+
d^(1/4)*x*2^(1/2)/c^(1/4))/c^(7/4)/(-a*d+b*c)^2*2^(1/2)-1/16*d^(3/4)*(-3*a*
d+7*b*c)*arctan(1+d^(1/4)*x*2^(1/2)/c^(1/4))/c^(7/4)/(-a*d+b*c)^2*2^(1/2)-1
/8*b^(7/4)*ln(-a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))/a^(3/4)/(-a*d
+b*c)^2*2^(1/2)+1/8*b^(7/4)*ln(a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2
))/a^(3/4)/(-a*d+b*c)^2*2^(1/2)+1/32*d^(3/4)*(-3*a*d+7*b*c)*ln(-c^(1/4)*d^(
1/4)*x*2^(1/2)+c^(1/2)+x^2*d^(1/2))/c^(7/4)/(-a*d+b*c)^2*2^(1/2)-1/32*d^(3/
4)*(-3*a*d+7*b*c)*ln(c^(1/4)*d^(1/4)*x*2^(1/2)+c^(1/2)+x^2*d^(1/2))/c^(7/4)
/(-a*d+b*c)^2*2^(1/2)
```

Rubi [A]
time = 0.29, antiderivative size = 513, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {425, 536, 217, 1179, 642, 1176, 631, 210}

$$-\frac{b^{7/4}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}(bc-ad)^2}+\frac{b^{7/4}\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}\,a^{3/4}(bc-ad)^2}-\frac{b^{7/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{4\sqrt{2}\,a^{3/4}(bc-ad)^2}+\frac{b^{7/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{4\sqrt{2}\,a^{3/4}(bc-ad)^2}+\frac{d^{3/4}(7bc-3ad)\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right)}{8\sqrt{2}\,c^{7/4}(bc-ad)^2}-\frac{d^{3/4}(7bc-3ad)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{d}\,x}{\sqrt[4]{c}}+1\right)}{8\sqrt{2}\,c^{7/4}(bc-ad)^2}+\frac{d^{3/4}(7bc-3ad)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\,x+\sqrt{c}+\sqrt{d}\,x^2\right)}{16\sqrt{2}\,c^{7/4}(bc-ad)^2}-\frac{d^{3/4}(7bc-3ad)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\,x+\sqrt{c}+\sqrt{d}\,x^2\right)}{16\sqrt{2}\,c^{7/4}(bc-ad)^2}-\frac{dx}{4c\left(c+dx^4\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^4)*(c + d*x^4)^2),x]
```

```
[Out] -1/4*(d*x)/(c*(b*c - a*d)*(c + d*x^4)) - (b^(7/4)*ArcTan[1 - (Sqrt[2]*b^(1/
4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(b*c - a*d)^2) + (b^(7/4)*ArcTan[1 + (Sq
rt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(b*c - a*d)^2) + (d^(3/4)*(7*
b*c - 3*a*d)*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/(8*Sqrt[2]*c^(7/4)*(b
*c - a*d)^2) - (d^(3/4)*(7*b*c - 3*a*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1
/4)])/(8*Sqrt[2]*c^(7/4)*(b*c - a*d)^2) - (b^(7/4)*Log[Sqrt[a] - Sqrt[2]*a^
(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*(b*c - a*d)^2) + (b^(7/4
)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4
)*(b*c - a*d)^2) + (d^(3/4)*(7*b*c - 3*a*d)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d
^(1/4)*x + Sqrt[d]*x^2])/(16*Sqrt[2]*c^(7/4)*(b*c - a*d)^2) - (d^(3/4)*(7*b
*c - 3*a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(16*Sqr
t[2]*c^(7/4)*(b*c - a*d)^2)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
```

```
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a+bx^4)(c+dx^4)^2}\,dx &= -\frac{dx}{4c(bc-ad)(c+dx^4)} + \frac{\int \frac{4bc-3ad-3bdx^4}{(a+bx^4)(c+dx^4)}\,dx}{4c(bc-ad)} \\
&= -\frac{dx}{4c(bc-ad)(c+dx^4)} + \frac{b^2\int \frac{1}{a+bx^4}\,dx}{(bc-ad)^2} - \frac{(d(7bc-3ad))\int \frac{1}{c+dx^4}\,dx}{4c(bc-ad)^2} \\
&= -\frac{dx}{4c(bc-ad)(c+dx^4)} + \frac{b^2\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx}{2\sqrt{a}\,(bc-ad)^2} + \frac{b^2\int \frac{\sqrt{a}+\sqrt{b}\,x^2}{a+bx^4}\,dx}{2\sqrt{a}\,(bc-ad)^2} - \frac{(d(7bc-
\\
&= -\frac{dx}{4c(bc-ad)(c+dx^4)} + \frac{b^{3/2}\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx}{4\sqrt{a}\,(bc-ad)^2} + \frac{b^{3/2}\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}}}{4\sqrt{a}\,(bc-ad)^2} \\
&= -\frac{dx}{4c(bc-ad)(c+dx^4)} - \frac{b^{7/4}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{b}\,x^2\right)}{4\sqrt{2}\,a^{3/4}(bc-ad)^2} + \frac{b^{7/4}\log\left(\right.}{} \\
&= -\frac{dx}{4c(bc-ad)(c+dx^4)} - \frac{b^{7/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}(bc-ad)^2} + \frac{b^{7/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{}}{\sqrt[4]{a}}\right.}{2\sqrt{2}\,a^{3/4}(bc-ad)}
\end{aligned}$$

Mathematica [A]
time = 0.20, size = 498, normalized size = 0.97

$$\frac{8a^{3/4}c^{3/4}d(-bc+ad)x - 8\sqrt{2}\,b^{7/4}c^{7/4}(c+dx^4)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right) + 8\sqrt{2}\,b^{7/4}c^{7/4}(c+dx^4)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right) - 2\sqrt{2}\,a^{3/4}d^{3/4}(-7bc+3ad)(c+dx^4)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right) + 2\sqrt{2}\,a^{3/4}d^{3/4}(-7bc+3ad)(c+dx^4)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right) - 4\sqrt{2}\,b^{7/4}c^{7/4}(c+dx^4)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{b}\,x^2\right) + 4\sqrt{2}\,b^{7/4}c^{7/4}(c+dx^4)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{b}\,x^2\right) + \sqrt{2}\,a^{3/4}d^{3/4}(7bc-3ad)(c+dx^4)\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\,x+\sqrt{d}\,x^2\right) + \sqrt{2}\,a^{3/4}d^{3/4}(-7bc+3ad)(c+dx^4)\log\left(\sqrt{c}+\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\,x+\sqrt{d}\,x^2\right)}{32a^{3/4}c^{7/4}(bc-ad)^2(c+dx^4)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^4)*(c + d*x^4)^2),x]
```

```
[Out] (8*a^(3/4)*c^(3/4)*d*(-(b*c) + a*d)*x - 8*Sqrt[2]*b^(7/4)*c^(7/4)*(c + d*x^
4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 8*Sqrt[2]*b^(7/4)*c^(7/4)*(c +
```

```
d*x^4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)] - 2*Sqrt[2]*a^(3/4)*d^(3/4)
*(-7*b*c + 3*a*d)*(c + d*x^4)*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)] + 2*S
qrt[2]*a^(3/4)*d^(3/4)*(-7*b*c + 3*a*d)*(c + d*x^4)*ArcTan[1 + (Sqrt[2]*d^(
1/4)*x)/c^(1/4)] - 4*Sqrt[2]*b^(7/4)*c^(7/4)*(c + d*x^4)*Log[Sqrt[a] - Sqrt
[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] + 4*Sqrt[2]*b^(7/4)*c^(7/4)*(c + d*x^4
)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] + Sqrt[2]*a^(3/4)*
d^(3/4)*(7*b*c - 3*a*d)*(c + d*x^4)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x
 + Sqrt[d]*x^2] + Sqrt[2]*a^(3/4)*d^(3/4)*(-7*b*c + 3*a*d)*(c + d*x^4)*Log[
Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(32*a^(3/4)*c^(7/4)*(b*
c - a*d)^2*(c + d*x^4))
```

Maple [A]
time = 0.40, size = 262, normalized size = 0.51

method	result
default	$\frac{d\left(\frac{(ad-bc)x}{4c\left(d\,x^4+c\right)}+\frac{(3ad-7bc)\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{c}{d}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{c}{d}}}{x^2-\left(\frac{c}{d}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{c}{d}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)\right)}{32c^2}\right)}{(ad-bc)^2}+\frac{b^2\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{}}{}$
risch	$\frac{dx}{4c(ad-bc)(d\,x^4+c)}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(a^{11}d^8-8a^{10}bc\,d^7+28a^9b^2c^2d^6-56a^8b^3c^3d^5+70a^7b^4c^4d^4-56a^6b^5c^5d^3+28a^5b^6c^6d^2-8a^4b^7c^7d+b^8a\right.\right.}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^4+a)/(d*x^4+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] d/(a*d-b*c)^2*(1/4*(a*d-b*c)/c*x/(d*x^4+c)+1/32*(3*a*d-7*b*c)/c^2*(c/d)^(1/
4)*2^(1/2)*(ln((x^2+(c/d)^(1/4)*x*2^(1/2)+(c/d)^(1/2))/(x^2-(c/d)^(1/4)*x*2
^(1/2)+(c/d)^(1/2)))+2*arctan(2^(1/2)/(c/d)^(1/4)*x+1)+2*arctan(2^(1/2)/(c/
d)^(1/4)*x-1)))+1/8*b^2/(a*d-b*c)^2*(a/b)^(1/4)/a*2^(1/2)*(ln((x^2+(a/b)^(1
/4)*x*2^(1/2)+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))+2*arcta
n(2^(1/2)/(a/b)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x-1))
```

Maxima [A]
time = 0.52, size = 481, normalized size = 0.94

$$-\frac{dx}{4\left(\left(bc^2d-acd^2\right)x^4+bc^3-ac^2d\right)}+\frac{\frac{2\sqrt{2}\,b^2\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}\,x+\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{2\sqrt{2}\,b^2\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}\,x-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{\sqrt{2}\,b^{\frac{7}{4}}\log\left(\sqrt{b}\,x^2+\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}}-\frac{\sqrt{2}\,b^{\frac{7}{4}}\log\left(\sqrt{b}\,x^2-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}}}{8\left(b^2c^2-2\,abcd+a^2d^2\right)}-\frac{\frac{2\sqrt{2}\left(7\,bcd-3\,ad^2\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}\,x+\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}+\frac{2\sqrt{2}\left(7\,bcd-3\,ad^2\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}\,x-\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}+\frac{\sqrt{2}\left(7\,bcd-3\,ad^2\right)\log\left(\sqrt{d}\,x^2+\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}x+\sqrt{c}\right)}{c^{\frac{3}{4}}d^{\frac{1}{4}}}-\frac{\sqrt{2}\left(7\,bcd-3\,ad^2\right)\log\left(\sqrt{d}\,x^2-\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}x+\sqrt{c}\right)}{c^{\frac{3}{4}}d^{\frac{1}{4}}}}{32\left(b^2c^3-2\,abc^2d+a^2cd^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)/(d*x^4+c)^2,x, algorithm="maxima")
```

```
[Out] -1/4*d*x/((b*c^2*d - a*c*d^2)*x^4 + b*c^3 - a*c^2*d) + 1/8*(2*sqrt(2)*b^2*a
rctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt
```

```
(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqrt(2)*b^2*arctan(1/2*sqrt(2)*(2
*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(
sqrt(a)*sqrt(b))) + sqrt(2)*b^(7/4)*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1/
4)*x + sqrt(a))/a^(3/4) - sqrt(2)*b^(7/4)*log(sqrt(b)*x^2 - sqrt(2)*a^(1/4)
*b^(1/4)*x + sqrt(a))/a^(3/4))/(b^2*c^2 - 2*a*b*c*d + a^2*d^2) - 1/32*(2*sq
rt(2)*(7*b*c*d - 3*a*d^2)*arctan(1/2*sqrt(2)*(2*sqrt(d)*x + sqrt(2)*c^(1/4)
*d^(1/4))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + 2*sqrt(2
)*(7*b*c*d - 3*a*d^2)*arctan(1/2*sqrt(2)*(2*sqrt(d)*x - sqrt(2)*c^(1/4)*d^(
1/4))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + sqrt(2)*(7*b
*c*d - 3*a*d^2)*log(sqrt(d)*x^2 + sqrt(2)*c^(1/4)*d^(1/4)*x + sqrt(c))/(c^(
3/4)*d^(1/4)) - sqrt(2)*(7*b*c*d - 3*a*d^2)*log(sqrt(d)*x^2 - sqrt(2)*c^(1/
4)*d^(1/4)*x + sqrt(c))/(c^(3/4)*d^(1/4)))/(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d
^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 3301 vs. $2(381) = 762$.
time = 75.67, size = 3301, normalized size = 6.43

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)/(d*x^4+c)^2,x, algorithm="fricas")
```

```
[Out] 1/16*(4*((b*c^2*d - a*c*d^2)*x^4 + b*c^3 - a*c^2*d)*(-(2401*b^4*c^4*d^3 - 4
116*a*b^3*c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6 + 81*a^4*d^7)/(b
^8*c^15 - 8*a*b^7*c^14*d + 28*a^2*b^6*c^13*d^2 - 56*a^3*b^5*c^12*d^3 + 70*a
^4*b^4*c^11*d^4 - 56*a^5*b^3*c^10*d^5 + 28*a^6*b^2*c^9*d^6 - 8*a^7*b*c^8*d^
7 + a^8*c^7*d^8))^(1/4)*arctan(((7*b^7*c^12*d - 45*a*b^6*c^11*d^2 + 123*a^2
*b^5*c^10*d^3 - 185*a^3*b^4*c^9*d^4 + 165*a^4*b^3*c^8*d^5 - 87*a^5*b^2*c^7*
d^6 + 25*a^6*b*c^6*d^7 - 3*a^7*c^5*d^8)*x*(-(2401*b^4*c^4*d^3 - 4116*a*b^3*
c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6 + 81*a^4*d^7)/(b^8*c^15 -
8*a*b^7*c^14*d + 28*a^2*b^6*c^13*d^2 - 56*a^3*b^5*c^12*d^3 + 70*a^4*b^4*c^1
1*d^4 - 56*a^5*b^3*c^10*d^5 + 28*a^6*b^2*c^9*d^6 - 8*a^7*b*c^8*d^7 + a^8*c^
7*d^8))^(3/4) + (b^6*c^11 - 6*a*b^5*c^10*d + 15*a^2*b^4*c^9*d^2 - 20*a^3*b^
3*c^8*d^3 + 15*a^4*b^2*c^7*d^4 - 6*a^5*b*c^6*d^5 + a^6*c^5*d^6)*sqrt((49*b^
2*c^2*d^2 - 42*a*b*c*d^3 + 9*a^2*d^4)*x^2 + (b^4*c^8 - 4*a*b^3*c^7*d + 6*a^
2*b^2*c^6*d^2 - 4*a^3*b*c^5*d^3 + a^4*c^4*d^4)*sqrt(-(2401*b^4*c^4*d^3 - 41
16*a*b^3*c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6 + 81*a^4*d^7)/(b^
8*c^15 - 8*a*b^7*c^14*d + 28*a^2*b^6*c^13*d^2 - 56*a^3*b^5*c^12*d^3 + 70*a^
4*b^4*c^11*d^4 - 56*a^5*b^3*c^10*d^5 + 28*a^6*b^2*c^9*d^6 - 8*a^7*b*c^8*d^7
 + a^8*c^7*d^8)))*(-(2401*b^4*c^4*d^3 - 4116*a*b^3*c^3*d^4 + 2646*a^2*b^2*c
^2*d^5 - 756*a^3*b*c*d^6 + 81*a^4*d^7)/(b^8*c^15 - 8*a*b^7*c^14*d + 28*a^2*
b^6*c^13*d^2 - 56*a^3*b^5*c^12*d^3 + 70*a^4*b^4*c^11*d^4 - 56*a^5*b^3*c^10*
d^5 + 28*a^6*b^2*c^9*d^6 - 8*a^7*b*c^8*d^7 + a^8*c^7*d^8))^(3/4))/(2401*b^4
*c^4*d^3 - 4116*a*b^3*c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6 + 81
```

```
*a^4*d^7)) + 16*(-b^7/(a^3*b^8*c^8 - 8*a^4*b^7*c^7*d + 28*a^5*b^6*c^6*d^2 -
 56*a^6*b^5*c^5*d^3 + 70*a^7*b^4*c^4*d^4 - 56*a^8*b^3*c^3*d^5 + 28*a^9*b^2*
c^2*d^6 - 8*a^10*b*c*d^7 + a^11*d^8))^(1/4)*((b*c^2*d - a*c*d^2)*x^4 + b*c^
3 - a*c^2*d)*arctan(-((a^2*b^8*c^6 - 6*a^3*b^7*c^5*d + 15*a^4*b^6*c^4*d^2 -
 20*a^5*b^5*c^3*d^3 + 15*a^6*b^4*c^2*d^4 - 6*a^7*b^3*c*d^5 + a^8*b^2*d^6)*(
-b^7/(a^3*b^8*c^8 - 8*a^4*b^7*c^7*d + 28*a^5*b^6*c^6*d^2 - 56*a^6*b^5*c^5*d
^3 + 70*a^7*b^4*c^4*d^4 - 56*a^8*b^3*c^3*d^5 + 28*a^9*b^2*c^2*d^6 - 8*a^10*
b*c*d^7 + a^11*d^8))^(3/4)*x - (a^2*b^6*c^6 - 6*a^3*b^5*c^5*d + 15*a^4*b^4*
c^4*d^2 - 20*a^5*b^3*c^3*d^3 + 15*a^6*b^2*c^2*d^4 - 6*a^7*b*c*d^5 + a^8*d^6
)*(-b^7/(a^3*b^8*c^8 - 8*a^4*b^7*c^7*d + 28*a^5*b^6*c^6*d^2 - 56*a^6*b^5*c^
5*d^3 + 70*a^7*b^4*c^4*d^4 - 56*a^8*b^3*c^3*d^5 + 28*a^9*b^2*c^2*d^6 - 8*a^
10*b*c*d^7 + a^11*d^8))^(3/4)*sqrt(b^4*x^2 + (a^2*b^4*c^4 - 4*a^3*b^3*c^3*d
 + 6*a^4*b^2*c^2*d^2 - 4*a^5*b*c*d^3 + a^6*d^4)*sqrt(-b^7/(a^3*b^8*c^8 - 8*
a^4*b^7*c^7*d + 28*a^5*b^6*c^6*d^2 - 56*a^6*b^5*c^5*d^3 + 70*a^7*b^4*c^4*d^
4 - 56*a^8*b^3*c^3*d^5 + 28*a^9*b^2*c^2*d^6 - 8*a^10*b*c*d^7 + a^11*d^8))))
/b^7) + 4*(-b^7/(a^3*b^8*c^8 - 8*a^4*b^7*c^7*d + 28*a^5*b^6*c^6*d^2 - 56*a^
6*b^5*c^5*d^3 + 70*a^7*b^4*c^4*d^4 - 56*a^8*b^3*c^3*d^5 + 28*a^9*b^2*c^2*d^
6 - 8*a^10*b*c*d^7 + a^11*d^8))^(1/4)*((b*c^2*d - a*c*d^2)*x^4 + b*c^3 - a*
c^2*d)*log(b^2*x + (-b^7/(a^3*b^8*c^8 - 8*a^4*b^7*c^7*d + 28*a^5*b^6*c^6*d^
2 - 56*a^6*b^5*c^5*d^3 + 70*a^7*b^4*c^4*d^4 - 56*a^8*b^3*c^3*d^5 + 28*a^9*b
^2*c^2*d^6 - 8*a^10*b*c*d^7 + a^11*d^8))^(1/4)*(a*b^2*c^2 - 2*a^2*b*c*d + a
^3*d^2)) - 4*(-b^7/(a^3*b^8*c^8 - 8*a^4*b^7*c^7*d + 28*a^5*b^6*c^6*d^2 - 56
*a^6*b^5*c^5*d^3 + 70*a^7*b^4*c^4*d^4 - 56*a^8*b^3*c^3*d^5 + 28*a^9*b^2*c^2
*d^6 - 8*a^10*b*c*d^7 + a^11*d^8))^(1/4)*((b*c^2*d - a*c*d^2)*x^4 + b*c^3 -
 a*c^2*d)*log(b^2*x - (-b^7/(a^3*b^8*c^8 - 8*a^4*b^7*c^7*d + 28*a^5*b^6*c^6
*d^2 - 56*a^6*b^5*c^5*d^3 + 70*a^7*b^4*c^4*d^4 - 56*a^8*b^3*c^3*d^5 + 28*a^
9*b^2*c^2*d^6 - 8*a^10*b*c*d^7 + a^11*d^8))^(1/4)*(a*b^2*c^2 - 2*a^2*b*c*d
+ a^3*d^2)) + ((b*c^2*d - a*c*d^2)*x^4 + b*c^3 - a*c^2*d)*(-(2401*b^4*c^4*d
^3 - 4116*a*b^3*c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6 + 81*a^4*d
^7)/(b^8*c^15 - 8*a*b^7*c^14*d + 28*a^2*b^6*c^13*d^2 - 56*a^3*b^5*c^12*d^3
+ 70*a^4*b^4*c^11*d^4 - 56*a^5*b^3*c^10*d^5 + 28*a^6*b^2*c^9*d^6 - 8*a^7*b*
c^8*d^7 + a^8*c^7*d^8))^(1/4)*log(-(7*b*c*d - 3*a*d^2)*x + (b^2*c^4 - 2*a*b
*c^3*d + a^2*c^2*d^2)*(-(2401*b^4*c^4*d^3 - 4116*a*b^3*c^3*d^4 + 2646*a^2*b
^2*c^2*d^5 - 756*a^3*b*c*d^6 + 81*a^4*d^7)/(b^8*c^15 - 8*a*b^7*c^14*d + 28*
a^2*b^6*c^13*d^2 - 56*a^3*b^5*c^12*d^3 + 70*a^4*b^4*c^11*d^4 - 56*a^5*b^3*c
^10*d^5 + 28*a^6*b^2*c^9*d^6 - 8*a^7*b*c^8*d^7 + a^8*c^7*d^8))^(1/4)) - ((b
*c^2*d - a*c*d^2)*x^4 + b*c^3 - a*c^2*d)*(-(2401*b^4*c^4*d^3 - 4116*a*b^3*c
^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6 + 81*a^4*d^7)/(b^8*c^15 - 8
*a*b^7*c^14*d + 28*a^2*b^6*c^13*d^2 - 56*a^3*b^5*c^12*d^3 + 70*a^4*b^4*c^11
*d^4 - 56*a^5*b^3*c^10*d^5 + 28*a^6*b^2*c^9*d^6 - 8*a^7*b*c^8*d^7 + a^8*c^7
*d^8))^(1/4)*log(-(7*b*c*d - 3*a*d^2)*x - (b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*
d^2)*(-(2401*b^4*c^4*d^3 - 4116*a*b^3*c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 756*
a^3*b*c*d^6 + 81*a^4*d^7)/(b^8*c^15 - 8*a*b^7*c^14*d + 28*a^2*b^6*c^13*d^2
- 56*a^3*b^5*c^12*d^3 + 70*a^4*b^4*c^11*d^4 - 56*a^5*b^3*c^10*d^5 + 28*a^6*
b^2*c^9*d^6 - 8*a^7*b*c^8*d^7 + a^8*c^7*d^8))^(...
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**4+a)/(d*x**4+c)**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.59, size = 667, normalized size = 1.30

$$\frac{(ab^3)^{\frac{1}{4}}\, b \arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}\, ab^2c^2-2\sqrt{2}\, a^2bcd+\sqrt{2}\, a^3d^2\right)}+\frac{(ab^3)^{\frac{1}{4}}\, b \arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}\, ab^2c^2-2\sqrt{2}\, a^2bcd+\sqrt{2}\, a^3d^2\right)}+\frac{(ab^3)^{\frac{1}{4}}\, b \log\left(x^2+\sqrt{2}\, x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{4\left(\sqrt{2}\, ab^2c^2-2\sqrt{2}\, a^2bcd+\sqrt{2}\, a^3d^2\right)}-\frac{(ab^3)^{\frac{1}{4}}\, b \log\left(x^2-\sqrt{2}\, x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{4\left(\sqrt{2}\, ab^2c^2-2\sqrt{2}\, a^2bcd+\sqrt{2}\, a^3d^2\right)}-\frac{\left(7\,(cd^3)^{\frac{1}{4}}\, bc-3\,(cd^3)^{\frac{1}{4}}\, ad\right) \arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{8\left(\sqrt{2}\, b^2c^4-2\sqrt{2}\, abc^3d+\sqrt{2}\, a^2c^2d^2\right)}-\frac{\left(7\,(cd^3)^{\frac{1}{4}}\, bc-3\,(cd^3)^{\frac{1}{4}}\, ad\right) \arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{8\left(\sqrt{2}\, b^2c^4-2\sqrt{2}\, abc^3d+\sqrt{2}\, a^2c^2d^2\right)}-\frac{\left(7\,(cd^3)^{\frac{1}{4}}\, bc-3\,(cd^3)^{\frac{1}{4}}\, ad\right) \log\left(x^2+\sqrt{2}\, x\left(\frac{c}{d}\right)^{\frac{1}{4}}+\sqrt{\frac{c}{d}}\right)}{16\left(\sqrt{2}\, b^2c^4-2\sqrt{2}\, abc^3d+\sqrt{2}\, a^2c^2d^2\right)}+\frac{\left(7\,(cd^3)^{\frac{1}{4}}\, bc-3\,(cd^3)^{\frac{1}{4}}\, ad\right) \log\left(x^2-\sqrt{2}\, x\left(\frac{c}{d}\right)^{\frac{1}{4}}+\sqrt{\frac{c}{d}}\right)}{16\left(\sqrt{2}\, b^2c^4-2\sqrt{2}\, abc^3d+\sqrt{2}\, a^2c^2d^2\right)}-\frac{dx}{4\,(dx^4+c)(bc^2-acd)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)/(d*x^4+c)^2,x, algorithm="giac")
```

```
[Out] 1/2*(a*b^3)^(1/4)*b*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1
/4))/(sqrt(2)*a*b^2*c^2 - 2*sqrt(2)*a^2*b*c*d + sqrt(2)*a^3*d^2) + 1/2*(a*b
^3)^(1/4)*b*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(sq
rt(2)*a*b^2*c^2 - 2*sqrt(2)*a^2*b*c*d + sqrt(2)*a^3*d^2) + 1/4*(a*b^3)^(1/4
)*b*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(sqrt(2)*a*b^2*c^2 - 2*sqr
t(2)*a^2*b*c*d + sqrt(2)*a^3*d^2) - 1/4*(a*b^3)^(1/4)*b*log(x^2 - sqrt(2)*x
*(a/b)^(1/4) + sqrt(a/b))/(sqrt(2)*a*b^2*c^2 - 2*sqrt(2)*a^2*b*c*d + sqrt(2
)*a^3*d^2) - 1/8*(7*(c*d^3)^(1/4)*b*c - 3*(c*d^3)^(1/4)*a*d)*arctan(1/2*sqr
t(2)*(2*x + sqrt(2)*(c/d)^(1/4))/(c/d)^(1/4))/(sqrt(2)*b^2*c^4 - 2*sqrt(2)*
a*b*c^3*d + sqrt(2)*a^2*c^2*d^2) - 1/8*(7*(c*d^3)^(1/4)*b*c - 3*(c*d^3)^(1/
4)*a*d)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(c/d)^(1/4))/(c/d)^(1/4))/(sqrt(2
)*b^2*c^4 - 2*sqrt(2)*a*b*c^3*d + sqrt(2)*a^2*c^2*d^2) - 1/16*(7*(c*d^3)^(1
/4)*b*c - 3*(c*d^3)^(1/4)*a*d)*log(x^2 + sqrt(2)*x*(c/d)^(1/4) + sqrt(c/d))
/(sqrt(2)*b^2*c^4 - 2*sqrt(2)*a*b*c^3*d + sqrt(2)*a^2*c^2*d^2) + 1/16*(7*(c
*d^3)^(1/4)*b*c - 3*(c*d^3)^(1/4)*a*d)*log(x^2 - sqrt(2)*x*(c/d)^(1/4) + sq
rt(c/d))/(sqrt(2)*b^2*c^4 - 2*sqrt(2)*a*b*c^3*d + sqrt(2)*a^2*c^2*d^2) - 1/
4*d*x/((d*x^4 + c)*(b*c^2 - a*c*d))
```

Mupad [B]
time = 4.00, size = 2500, normalized size = 4.87

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^4)*(c + d*x^4)^2),x)
```

```
[Out] 2*atan(((-(81*a^4*d^7 + 2401*b^4*c^4*d^3 - 4116*a*b^3*c^3*d^4 + 2646*a^2*b^
2*c^2*d^5 - 756*a^3*b*c*d^6)/(65536*b^8*c^15 + 65536*a^8*c^7*d^8 - 524288*a
```

^7*b*c^8*d^7 + 1835008*a^2*b^6*c^13*d^2 - 3670016*a^3*b^5*c^12*d^3 + 458752
0*a^4*b^4*c^11*d^4 - 3670016*a^5*b^3*c^10*d^5 + 1835008*a^6*b^2*c^9*d^6 - 5
24288*a*b^7*c^14*d))^(1/4)*((-(81*a^4*d^7 + 2401*b^4*c^4*d^3 - 4116*a*b^3*c
^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6)/(65536*b^8*c^15 + 65536*a^
8*c^7*d^8 - 524288*a^7*b*c^8*d^7 + 1835008*a^2*b^6*c^13*d^2 - 3670016*a^3*b
^5*c^12*d^3 + 4587520*a^4*b^4*c^11*d^4 - 3670016*a^5*b^3*c^10*d^5 + 1835008
*a^6*b^2*c^9*d^6 - 524288*a*b^7*c^14*d))^(1/4)*((((81*a^4*b^7*d^10)/16 + 28
*b^11*c^4*d^6 - (2145*a*b^10*c^3*d^7)/16 - (675*a^3*b^8*c*d^9)/16 + (1971*a
^2*b^9*c^2*d^8)/16)*1i)/(b^3*c^7 - a^3*c^4*d^3 + 3*a^2*b*c^5*d^2 - 3*a*b^2*
c^6*d) + (-(81*a^4*d^7 + 2401*b^4*c^4*d^3 - 4116*a*b^3*c^3*d^4 + 2646*a^2*b
^2*c^2*d^5 - 756*a^3*b*c*d^6)/(65536*b^8*c^15 + 65536*a^8*c^7*d^8 - 524288*
a^7*b*c^8*d^7 + 1835008*a^2*b^6*c^13*d^2 - 3670016*a^3*b^5*c^12*d^3 + 45875
20*a^4*b^4*c^11*d^4 - 3670016*a^5*b^3*c^10*d^5 + 1835008*a^6*b^2*c^9*d^6 -
524288*a*b^7*c^14*d))^(3/4)*(((-(81*a^4*d^7 + 2401*b^4*c^4*d^3 - 4116*a*b^3
*c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6)/(65536*b^8*c^15 + 65536*
a^8*c^7*d^8 - 524288*a^7*b*c^8*d^7 + 1835008*a^2*b^6*c^13*d^2 - 3670016*a^3
*b^5*c^12*d^3 + 4587520*a^4*b^4*c^11*d^4 - 3670016*a^5*b^3*c^10*d^5 + 18350
08*a^6*b^2*c^9*d^6 - 524288*a*b^7*c^14*d))^(1/4)*(28672*a^2*b^13*c^13*d^5 -
4096*a*b^14*c^14*d^4 - 78848*a^3*b^12*c^12*d^6 + 90112*a^4*b^11*c^11*d^7 +
28672*a^5*b^10*c^10*d^8 - 229376*a^6*b^9*c^9*d^9 + 329728*a^7*b^8*c^8*d^10
- 253952*a^8*b^7*c^7*d^11 + 114688*a^9*b^6*c^6*d^12 - 28672*a^10*b^5*c^5*d
^13 + 3072*a^11*b^4*c^4*d^14))/(b^3*c^7 - a^3*c^4*d^3 + 3*a^2*b*c^5*d^2 - 3
*a*b^2*c^6*d) - (x*(65536*b^17*c^15*d^4 - 524288*a*b^16*c^14*d^5 + 1835008*
a^2*b^15*c^13*d^6 - 3469312*a^3*b^14*c^12*d^7 + 2809856*a^4*b^13*c^11*d^8 +
3362816*a^5*b^12*c^10*d^9 - 14516224*a^6*b^11*c^9*d^10 + 24190976*a^7*b^10
*c^8*d^11 - 25280512*a^8*b^9*c^7*d^12 + 17833984*a^9*b^8*c^6*d^13 - 8486912
*a^10*b^7*c^5*d^14 + 2609152*a^11*b^6*c^4*d^15 - 466944*a^12*b^5*c^3*d^16 +
36864*a^13*b^4*c^2*d^17)*1i)/(64*(b^6*c^10 + a^6*c^4*d^6 - 6*a^5*b*c^5*d^5
+ 15*a^2*b^4*c^8*d^2 - 20*a^3*b^3*c^7*d^3 + 15*a^4*b^2*c^6*d^4 - 6*a*b^5*c
^9*d)))*1i) + (x*(81*a^4*b^9*d^11 + 3185*b^13*c^4*d^7 - 4788*a*b^12*c^3*d^8
- 756*a^3*b^10*c*d^10 + 2790*a^2*b^11*c^2*d^9))/(64*(b^6*c^10 + a^6*c^4*d^
6 - 6*a^5*b*c^5*d^5 + 15*a^2*b^4*c^8*d^2 - 20*a^3*b^3*c^7*d^3 + 15*a^4*b^2*
c^6*d^4 - 6*a*b^5*c^9*d))) - (-(81*a^4*d^7 + 2401*b^4*c^4*d^3 - 4116*a*b^3*
c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6)/(65536*b^8*c^15 + 65536*a
^8*c^7*d^8 - 524288*a^7*b*c^8*d^7 + 1835008*a^2*b^6*c^13*d^2 - 3670016*a^3*
b^5*c^12*d^3 + 4587520*a^4*b^4*c^11*d^4 - 3670016*a^5*b^3*c^10*d^5 + 183500
8*a^6*b^2*c^9*d^6 - 524288*a*b^7*c^14*d))^(1/4)*((-(81*a^4*d^7 + 2401*b^4*c
^4*d^3 - 4116*a*b^3*c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6)/(6553
6*b^8*c^15 + 65536*a^8*c^7*d^8 - 524288*a^7*b*c^8*d^7 + 1835008*a^2*b^6*c^1
3*d^2 - 3670016*a^3*b^5*c^12*d^3 + 4587520*a^4*b^4*c^11*d^4 - 3670016*a^5*b
^3*c^10*d^5 + 1835008*a^6*b^2*c^9*d^6 - 524288*a*b^7*c^14*d))^(1/4)*((((81*
a^4*b^7*d^10)/16 + 28*b^11*c^4*d^6 - (2145*a*b^10*c^3*d^7)/16 - (675*a^3*b^
8*c*d^9)/16 + (1971*a^2*b^9*c^2*d^8)/16)*1i)/(b^3*c^7 - a^3*c^4*d^3 + 3*a^2
*b*c^5*d^2 - 3*a*b^2*c^6*d) + (-(81*a^4*d^7 + 2401*b^4*c^4*d^3 - 4116*a*b^3
*c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6)/(65536*b^8*c^15 + 65536*

a^8*c^7*d^8 - 524288*a^7*b*c^8*d^7 + 1835008*a^2*b^6*c^13*d^2 - 3670016*a^3
*b^5*c^12*d^3 + 4587520*a^4*b^4*c^11*d^4 - 3670016*a^5*b^3*c^10*d^5 + 18350
08*a^6*b^2*c^9*d^6 - 524288*a*b^7*c^14*d))^(3/4)*(((-(81*a^4*d^7 + 2401*b^4
*c^4*d^3 - 4116*a*b^3*c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6)/(65
536*b^8*c^15 + 65536*a^8*c^7*d^8 - 524288*a^7*b*c^8*d^7 + 1835008*a^2*b^6*c
^13*d^2 - 3670016*a^3*b^5*c^12*d^3 + 4587520*a^4*b^4*c^11*d^4 - 3670016*a^5
*b^3*c^10*d^5 + 1835008*a^6*b^2*c^9*d^6 - 524288*a*b^7*c^14*d))^(1/4)*(2867
2*a^2*b^13*c^13*d^5 - 4096*a*b^14*c^14*d^4 - 78848*a^3*b^12*c^12*d^6 + 9011
2*a^4*b^11*c^11*d^7 + 28672*a^5*b^10*c^10*d^8 - 229376*a^6*b^9*c^9*d^9 + 32
9728*a^7*b^8*c^8*d^10 - 253952*a^8*b^7*c^7*d^11 + 114688*a^9*b^6*c^6*d^12 -
28672*a^10*b^5*c^5*d^13 + 3072*a^11*b^4*c^4*d^14))/(b^3*c^7 - a^3*c^4*d^3
+ 3*a^2*b*c^5*d^2 - 3*a*b^2*c^6*d) + (x*(65536*b^17*c^15*d^4 - 524288*a*b^1
6*c^14*d^5 + 1835008*a^2*b^15*c^13*d^6 - 3469312*a^3*b^14*c^12*d^7 + 280985
6*a^4*b^13*c^11*d^8 + 3362816*a^5*b^12*c^10*d^9 - 14516224*a^6*b^11*c^9*d^1
0 + 24190976*a^7*b^10*c^8*d^11 - 25280512*a^8*b^9*c^7*d^12 + 17833984*a^9*b
^8*c^6*d^13 - 8486912*a^10*b^7*c^5*d^14 + 2609152*a^11*b^6*c^4*d^15 - 46694
4*a^12*b^5*c^3*d^16 + 36864*a^13*b^4*c^2*d^17)*1i)/(64*(b^6*c^10 + a^6*c^4*
d^6 - 6*a^5*b*c^5*d^5 + 15*a^2*b^4*c^8*d^2 - 20*a^3*b^3*c^7*d^3 + 15*a^4*b^
2*c^6*d^4 - 6*a*b^5*c^9*d)))*1i) - (x*(81*a^4*b^9*d^11 + 3185*b^13*c^4*d^7
- 4788*a*b^12*c^3*d^8 - 756*a^3*b^10*c*d^10 + 2...

3.166 $\int \frac{(c+dx^4)^5}{(a+bx^4)^2} dx$

Optimal. Leaf size=407

$$\frac{d^2(10b^3c^3-20ab^2c^2d+15a^2bcd^2-4a^3d^3)\,x}{b^5}+\frac{d^3(10b^2c^2-10abcd+3a^2d^2)\,x^5}{5b^4}+\frac{d^4(5bc-2ad)x^9}{9b^3}+\frac{d^5x^{13}}{13b^2}+\frac{}{4}$$

```
[Out] d^2*(-4*a^3*d^3+15*a^2*b*c*d^2-20*a*b^2*c^2*d+10*b^3*c^3)*x/b^5+1/5*d^3*(3*
a^2*d^2-10*a*b*c*d+10*b^2*c^2)*x^5/b^4+1/9*d^4*(-2*a*d+5*b*c)*x^9/b^3+1/13*
d^5*x^13/b^2+1/4*(-a*d+b*c)^5*x/a/b^5/(b*x^4+a)+1/16*(-a*d+b*c)^4*(17*a*d+3
*b*c)*arctan(-1+b^(1/4)*x*2^(1/2)/a^(1/4))/a^(7/4)/b^(21/4)*2^(1/2)+1/16*(-
a*d+b*c)^4*(17*a*d+3*b*c)*arctan(1+b^(1/4)*x*2^(1/2)/a^(1/4))/a^(7/4)/b^(21
/4)*2^(1/2)-1/32*(-a*d+b*c)^4*(17*a*d+3*b*c)*ln(-a^(1/4)*b^(1/4)*x*2^(1/2)+
a^(1/2)+x^2*b^(1/2))/a^(7/4)/b^(21/4)*2^(1/2)+1/32*(-a*d+b*c)^4*(17*a*d+3*b
*c)*ln(a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))/a^(7/4)/b^(21/4)*2^(1
/2)
```

Rubi [A]
time = 0.27, antiderivative size = 407, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {398, 393, 217, 1179, 642, 1176, 631, 210}

$$-\frac{\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)(bc-ad)^4(17ad+3bc)}{8\sqrt{2}a^{7/4}b^{21/4}}+\frac{\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)(bc-ad)^4(17ad+3bc)}{8\sqrt{2}a^{7/4}b^{21/4}}-\frac{(bc-ad)^4(17ad+3bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{21/4}}+\frac{(bc-ad)^4(17ad+3bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{21/4}}+\frac{d^3x^5(3a^2d^2-10abcd+10b^2c^2)}{5b^4}+\frac{d^2x(-4a^3d^3+15a^2bcd^2-20ab^2c^2d+10b^3c^3)}{b^5}+\frac{x(bc-ad)^5}{4ab^5(a+bx^4)}+\frac{d^4x^9(5bc-2ad)}{9b^3}+\frac{d^5x^{13}}{13b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^4)^5/(a + b*x^4)^2,x]
```

```
[Out] (d^2*(10*b^3*c^3 - 20*a*b^2*c^2*d + 15*a^2*b*c*d^2 - 4*a^3*d^3)*x)/b^5 + (d
^3*(10*b^2*c^2 - 10*a*b*c*d + 3*a^2*d^2)*x^5)/(5*b^4) + (d^4*(5*b*c - 2*a*d
)*x^9)/(9*b^3) + (d^5*x^13)/(13*b^2) + ((b*c - a*d)^5*x)/(4*a*b^5*(a + b*x^
4)) - ((b*c - a*d)^4*(3*b*c + 17*a*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4
)])/(8*Sqrt[2]*a^(7/4)*b^(21/4)) + ((b*c - a*d)^4*(3*b*c + 17*a*d)*ArcTan[1
 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*b^(21/4)) - ((b*c - a*d
)^4*(3*b*c + 17*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2]
)/(16*Sqrt[2]*a^(7/4)*b^(21/4)) + ((b*c - a*d)^4*(3*b*c + 17*a*d)*Log[Sqrt[
a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)*b^(21/4)
)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
```

```
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(c+dx^4)^5}{(a+bx^4)^2}\,dx = \int \left(\frac{d^2(10b^3c^3-20ab^2c^2d+15a^2bcd^2-4a^3d^3)}{b^5}+\frac{d^3(10b^2c^2-10abcd+3a^2d^2)\,x^4}{b^4}+\frac{d^4(\right.$$

$$= \frac{d^2(10b^3c^3-20ab^2c^2d+15a^2bcd^2-4a^3d^3)\,x}{b^5}+\frac{d^3(10b^2c^2-10abcd+3a^2d^2)\,x^5}{5b^4}+\frac{d^4(5bc}{}$$

$$= \frac{d^2(10b^3c^3-20ab^2c^2d+15a^2bcd^2-4a^3d^3)\,x}{b^5}+\frac{d^3(10b^2c^2-10abcd+3a^2d^2)\,x^5}{5b^4}+\frac{d^4(5bc}{}$$

$$= \frac{d^2(10b^3c^3-20ab^2c^2d+15a^2bcd^2-4a^3d^3)\,x}{b^5}+\frac{d^3(10b^2c^2-10abcd+3a^2d^2)\,x^5}{5b^4}+\frac{d^4(5bc}{}$$

$$= \frac{d^2(10b^3c^3-20ab^2c^2d+15a^2bcd^2-4a^3d^3)\,x}{b^5}+\frac{d^3(10b^2c^2-10abcd+3a^2d^2)\,x^5}{5b^4}+\frac{d^4(5bc}{}$$

$$= \frac{d^2(10b^3c^3-20ab^2c^2d+15a^2bcd^2-4a^3d^3)\,x}{b^5}+\frac{d^3(10b^2c^2-10abcd+3a^2d^2)\,x^5}{5b^4}+\frac{d^4(5bc}{}$$

$$= \frac{d^2(10b^3c^3-20ab^2c^2d+15a^2bcd^2-4a^3d^3)\,x}{b^5}+\frac{d^3(10b^2c^2-10abcd+3a^2d^2)\,x^5}{5b^4}+\frac{d^4(5bc}{}$$

Mathematica [A]
time = 0.25, size = 391, normalized size = 0.96

$$\frac{18720\sqrt[4]{b}\,d^2(10b^3c^3-20ab^2c^2d+15a^2bcd^2-4a^3d^3)\,x+3744b^{5/4}d^3(10b^2c^2-10abcd+3a^2d^2)\,x^5+2080b^{9/4}d^4(5bc-2ad)x^9+1440b^{13/4}d^5x^{13}+\frac{4680\sqrt[4]{b}\,(bc-ad)^5x}{a(a+bx^4)}-\frac{1170\sqrt{2}\,(bc-ad)^4(3bc+17ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{a^{7/4}}+\frac{1170\sqrt{2}\,(bc-ad)^4(3bc+17ad)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{a^{7/4}}-\frac{585\sqrt{2}\,(bc-ad)^4(3bc+17ad)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{b}\,x^2\right)}{a^{7/4}}+\frac{585\sqrt{2}\,(bc-ad)^4(3bc+17ad)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{b}\,x^2\right)}{a^{7/4}}}{18720b^{21/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^4)^5/(a + b*x^4)^2,x]
```

```
[Out] (18720*b^(1/4)*d^2*(10*b^3*c^3 - 20*a*b^2*c^2*d + 15*a^2*b*c*d^2 - 4*a^3*d^
3)*x + 3744*b^(5/4)*d^3*(10*b^2*c^2 - 10*a*b*c*d + 3*a^2*d^2)*x^5 + 2080*b^
(9/4)*d^4*(5*b*c - 2*a*d)*x^9 + 1440*b^(13/4)*d^5*x^13 + (4680*b^(1/4)*(b*c
 - a*d)^5*x)/(a*(a + b*x^4)) - (1170*Sqrt[2]*(b*c - a*d)^4*(3*b*c + 17*a*d)
*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/a^(7/4) + (1170*Sqrt[2]*(b*c - a*
d)^4*(3*b*c + 17*a*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/a^(7/4) - (5
```

```
85*Sqrt[2]*(b*c - a*d)^4*(3*b*c + 17*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(
1/4)*x + Sqrt[b]*x^2])/a^(7/4) + (585*Sqrt[2]*(b*c - a*d)^4*(3*b*c + 17*a*d
)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/a^(7/4))/(18720*b
^(21/4))
```

Maple [A]
time = 0.28, size = 373, normalized size = 0.92

method	result
risch	$\frac{d^5x^{13}}{13b^2}-\frac{2d^5a\,x^9}{9b^3}+\frac{5d^4c\,x^9}{9b^2}+\frac{3d^5a^2x^5}{5b^4}-\frac{2d^4ac\,x^5}{b^3}+\frac{2d^3c^2x^5}{b^2}-\frac{4d^5a^3x}{b^5}+\frac{15d^4a^2cx}{b^4}-\frac{20d^3a\,c^2x}{b^3}+\frac{10d^2c^3x}{b^2}-\frac{(a^5d^5}{}$ …
default	$-\frac{d^2\left(-\frac{1}{13}b^3d^3x^{13}+\frac{2}{9}a\,b^2d^3x^9-\frac{5}{9}b^3c\,d^2x^9-\frac{3}{5}a^2b\,d^3x^5+2a\,b^2c\,d^2x^5-2b^3c^2d\,x^5+4a^3d^3x-15a^2bc\,d^2x+20a\,b^2c^2dx-10b^3c^3x\right)}{b^5}+\frac{-\frac{(a}{}}{}$ …

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^4+c)^5/(b*x^4+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -d^2/b^5*(-1/13*b^3*d^3*x^13+2/9*a*b^2*d^3*x^9-5/9*b^3*c*d^2*x^9-3/5*a^2*b*
d^3*x^5+2*a*b^2*c*d^2*x^5-2*b^3*c^2*d*x^5+4*a^3*d^3*x-15*a^2*b*c*d^2*x+20*a
*b^2*c^2*d*x-10*b^3*c^3*x)+1/b^5*(-1/4*(a^5*d^5-5*a^4*b*c*d^4+10*a^3*b^2*c^
2*d^3-10*a^2*b^3*c^3*d^2+5*a*b^4*c^4*d-b^5*c^5)/a*x/(b*x^4+a)+1/32*(17*a^5*
d^5-65*a^4*b*c*d^4+90*a^3*b^2*c^2*d^3-50*a^2*b^3*c^3*d^2+5*a*b^4*c^4*d+3*b^
5*c^5)/a^2*(a/b)^(1/4)*2^(1/2)*(ln((x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2))/
(x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x+1)+
2*arctan(2^(1/2)/(a/b)^(1/4)*x-1)))
```

Maxima [A]
time = 0.51, size = 644, normalized size = 1.58

$$\frac{(b^5c^5-5\,ab^4c^4d+10\,a^2b^3c^3d^2-10\,a^3b^2c^2d^3+5\,a^4bcd^4-a^5d^5)x}{4\,(ab^6x^4+a^2b^5)}+\frac{45\,b^3d^5x^{13}+65\,(5\,b^3cd^4-2\,ab^2d^5)x^9+117\,(10\,b^3c^2d^3-10\,ab^2cd^4+3\,a^2bd^5)x^5+585\,(10\,b^3c^3d^2-20\,ab^2c^2d^3+15\,a^2bcd^4-4\,a^3d^5)x}{585\,b^5}+\frac{\frac{2\sqrt{2}\,(3\,b^5c^5+5\,ab^4c^4d-50\,a^2b^3c^3d^2+90\,a^3b^2c^2d^3-65\,a^4bcd^4+17\,a^5d^5)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}\,x+\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{2\sqrt{2}\,(3\,b^5c^5+5\,ab^4c^4d-50\,a^2b^3c^3d^2+90\,a^3b^2c^2d^3-65\,a^4bcd^4+17\,a^5d^5)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}\,x-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{\sqrt{2}\,(3\,b^5c^5+5\,ab^4c^4d-50\,a^2b^3c^3d^2+90\,a^3b^2c^2d^3-65\,a^4bcd^4+17\,a^5d^5)\log\left(\sqrt{b}\,x^2+\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}-\frac{\sqrt{2}\,(3\,b^5c^5+5\,ab^4c^4d-50\,a^2b^3c^3d^2+90\,a^3b^2c^2d^3-65\,a^4bcd^4+17\,a^5d^5)\log\left(\sqrt{b}\,x^2-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}}{32\,ab^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^5/(b*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(b^5*c^5 - 5*a*b^4*c^4*d + 10*a^2*b^3*c^3*d^2 - 10*a^3*b^2*c^2*d^3 + 5*
a^4*b*c*d^4 - a^5*d^5)*x/(a*b^6*x^4 + a^2*b^5) + 1/585*(45*b^3*d^5*x^13 + 6
5*(5*b^3*c*d^4 - 2*a*b^2*d^5)*x^9 + 117*(10*b^3*c^2*d^3 - 10*a*b^2*c*d^4 +
3*a^2*b*d^5)*x^5 + 585*(10*b^3*c^3*d^2 - 20*a*b^2*c^2*d^3 + 15*a^2*b*c*d^4
- 4*a^3*d^5)*x)/b^5 + 1/32*(2*sqrt(2)*(3*b^5*c^5 + 5*a*b^4*c^4*d - 50*a^2*b
^3*c^3*d^2 + 90*a^3*b^2*c^2*d^3 - 65*a^4*b*c*d^4 + 17*a^5*d^5)*arctan(1/2*s
qrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(sqrt
(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqrt(2)*(3*b^5*c^5 + 5*a*b^4*c^4*d - 50*a^2*
```

```
b^3*c^3*d^2 + 90*a^3*b^2*c^2*d^3 - 65*a^4*b*c*d^4 + 17*a^5*d^5)*arctan(1/2*
sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(sqr
t(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*(3*b^5*c^5 + 5*a*b^4*c^4*d - 50*a^2*b
^3*c^3*d^2 + 90*a^3*b^2*c^2*d^3 - 65*a^4*b*c*d^4 + 17*a^5*d^5)*log(sqrt(b)*
x^2 + sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(1/4)) - sqrt(2)*(3*b
^5*c^5 + 5*a*b^4*c^4*d - 50*a^2*b^3*c^3*d^2 + 90*a^3*b^2*c^2*d^3 - 65*a^4*b
*c*d^4 + 17*a^5*d^5)*log(sqrt(b)*x^2 - sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))
/(a^(3/4)*b^(1/4)))/(a*b^5)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 3222 vs. $2(334) = 668$.
time = 3.55, size = 3222, normalized size = 7.92

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^5/(b*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] 1/9360*(720*a*b^4*d^5*x^17 + 80*(65*a*b^4*c*d^4 - 17*a^2*b^3*d^5)*x^13 + 20
8*(90*a*b^4*c^2*d^3 - 65*a^2*b^3*c*d^4 + 17*a^3*b^2*d^5)*x^9 + 1872*(50*a*b
^4*c^3*d^2 - 90*a^2*b^3*c^2*d^3 + 65*a^3*b^2*c*d^4 - 17*a^4*b*d^5)*x^5 + 23
40*(a*b^6*x^4 + a^2*b^5)*(-(81*b^20*c^20 + 540*a*b^19*c^19*d - 4050*a^2*b^1
8*c^18*d^2 - 15780*a^3*b^17*c^17*d^3 + 132205*a^4*b^16*c^16*d^4 - 13264*a^5
*b^15*c^15*d^5 - 1960920*a^6*b^14*c^14*d^6 + 6137200*a^7*b^13*c^13*d^7 - 50
0110*a^8*b^12*c^12*d^8 - 48530040*a^9*b^11*c^11*d^9 + 174873556*a^10*b^10*c
^10*d^10 - 360900280*a^11*b^9*c^9*d^11 + 517559250*a^12*b^8*c^8*d^12 - 5482
31440*a^13*b^7*c^7*d^13 + 438700840*a^14*b^6*c^6*d^14 - 266040144*a^15*b^5*
c^5*d^15 + 120836285*a^16*b^4*c^4*d^16 - 39944900*a^17*b^3*c^3*d^17 + 90948
30*a^18*b^2*c^2*d^18 - 1277380*a^19*b*c*d^19 + 83521*a^20*d^20)/(a^7*b^21))
^(1/4)*arctan((sqrt(a^4*b^10*sqrt(-(81*b^20*c^20 + 540*a*b^19*c^19*d - 4050
*a^2*b^18*c^18*d^2 - 15780*a^3*b^17*c^17*d^3 + 132205*a^4*b^16*c^16*d^4 - 1
3264*a^5*b^15*c^15*d^5 - 1960920*a^6*b^14*c^14*d^6 + 6137200*a^7*b^13*c^13*
d^7 - 500110*a^8*b^12*c^12*d^8 - 48530040*a^9*b^11*c^11*d^9 + 174873556*a^1
0*b^10*c^10*d^10 - 360900280*a^11*b^9*c^9*d^11 + 517559250*a^12*b^8*c^8*d^1
2 - 548231440*a^13*b^7*c^7*d^13 + 438700840*a^14*b^6*c^6*d^14 - 266040144*a
^15*b^5*c^5*d^15 + 120836285*a^16*b^4*c^4*d^16 - 39944900*a^17*b^3*c^3*d^17
 + 9094830*a^18*b^2*c^2*d^18 - 1277380*a^19*b*c*d^19 + 83521*a^20*d^20)/(a^
7*b^21)) + (9*b^10*c^10 + 30*a*b^9*c^9*d - 275*a^2*b^8*c^8*d^2 + 40*a^3*b^7
*c^7*d^3 + 3010*a^4*b^6*c^6*d^4 - 9548*a^5*b^5*c^5*d^5 + 14770*a^6*b^4*c^4*
d^6 - 13400*a^7*b^3*c^3*d^7 + 7285*a^8*b^2*c^2*d^8 - 2210*a^9*b*c*d^9 + 289
*a^10*d^10)*x^2)*a^5*b^16*(-(81*b^20*c^20 + 540*a*b^19*c^19*d - 4050*a^2*b^
18*c^18*d^2 - 15780*a^3*b^17*c^17*d^3 + 132205*a^4*b^16*c^16*d^4 - 13264*a^
5*b^15*c^15*d^5 - 1960920*a^6*b^14*c^14*d^6 + 6137200*a^7*b^13*c^13*d^7 - 5
00110*a^8*b^12*c^12*d^8 - 48530040*a^9*b^11*c^11*d^9 + 174873556*a^10*b^10*
c^10*d^10 - 360900280*a^11*b^9*c^9*d^11 + 517559250*a^12*b^8*c^8*d^12 - 548
```

```
231440*a^13*b^7*c^7*d^13 + 438700840*a^14*b^6*c^6*d^14 - 266040144*a^15*b^5
*c^5*d^15 + 120836285*a^16*b^4*c^4*d^16 - 39944900*a^17*b^3*c^3*d^17 + 9094
830*a^18*b^2*c^2*d^18 - 1277380*a^19*b*c*d^19 + 83521*a^20*d^20)/(a^7*b^21)
)^(3/4) - (3*a^5*b^21*c^5 + 5*a^6*b^20*c^4*d - 50*a^7*b^19*c^3*d^2 + 90*a^8
*b^18*c^2*d^3 - 65*a^9*b^17*c*d^4 + 17*a^10*b^16*d^5)*x*(-(81*b^20*c^20 + 5
40*a*b^19*c^19*d - 4050*a^2*b^18*c^18*d^2 - 15780*a^3*b^17*c^17*d^3 + 13220
5*a^4*b^16*c^16*d^4 - 13264*a^5*b^15*c^15*d^5 - 1960920*a^6*b^14*c^14*d^6 +
 6137200*a^7*b^13*c^13*d^7 - 500110*a^8*b^12*c^12*d^8 - 48530040*a^9*b^11*c
^11*d^9 + 174873556*a^10*b^10*c^10*d^10 - 360900280*a^11*b^9*c^9*d^11 + 517
559250*a^12*b^8*c^8*d^12 - 548231440*a^13*b^7*c^7*d^13 + 438700840*a^14*b^6
*c^6*d^14 - 266040144*a^15*b^5*c^5*d^15 + 120836285*a^16*b^4*c^4*d^16 - 399
44900*a^17*b^3*c^3*d^17 + 9094830*a^18*b^2*c^2*d^18 - 1277380*a^19*b*c*d^19
 + 83521*a^20*d^20)/(a^7*b^21))^(3/4))/(81*b^20*c^20 + 540*a*b^19*c^19*d -
4050*a^2*b^18*c^18*d^2 - 15780*a^3*b^17*c^17*d^3 + 132205*a^4*b^16*c^16*d^4
 - 13264*a^5*b^15*c^15*d^5 - 1960920*a^6*b^14*c^14*d^6 + 6137200*a^7*b^13*c
^13*d^7 - 500110*a^8*b^12*c^12*d^8 - 48530040*a^9*b^11*c^11*d^9 + 174873556
*a^10*b^10*c^10*d^10 - 360900280*a^11*b^9*c^9*d^11 + 517559250*a^12*b^8*c^8
*d^12 - 548231440*a^13*b^7*c^7*d^13 + 438700840*a^14*b^6*c^6*d^14 - 2660401
44*a^15*b^5*c^5*d^15 + 120836285*a^16*b^4*c^4*d^16 - 39944900*a^17*b^3*c^3*
d^17 + 9094830*a^18*b^2*c^2*d^18 - 1277380*a^19*b*c*d^19 + 83521*a^20*d^20)
) + 585*(a*b^6*x^4 + a^2*b^5)*(-(81*b^20*c^20 + 540*a*b^19*c^19*d - 4050*a^
2*b^18*c^18*d^2 - 15780*a^3*b^17*c^17*d^3 + 132205*a^4*b^16*c^16*d^4 - 1326
4*a^5*b^15*c^15*d^5 - 1960920*a^6*b^14*c^14*d^6 + 6137200*a^7*b^13*c^13*d^7
 - 500110*a^8*b^12*c^12*d^8 - 48530040*a^9*b^11*c^11*d^9 + 174873556*a^10*b
^10*c^10*d^10 - 360900280*a^11*b^9*c^9*d^11 + 517559250*a^12*b^8*c^8*d^12 -
 548231440*a^13*b^7*c^7*d^13 + 438700840*a^14*b^6*c^6*d^14 - 266040144*a^15
*b^5*c^5*d^15 + 120836285*a^16*b^4*c^4*d^16 - 39944900*a^17*b^3*c^3*d^17 +
9094830*a^18*b^2*c^2*d^18 - 1277380*a^19*b*c*d^19 + 83521*a^20*d^20)/(a^7*b
^21))^(1/4)*log(a^2*b^5*(-(81*b^20*c^20 + 540*a*b^19*c^19*d - 4050*a^2*b^18
*c^18*d^2 - 15780*a^3*b^17*c^17*d^3 + 132205*a^4*b^16*c^16*d^4 - 13264*a^5*
b^15*c^15*d^5 - 1960920*a^6*b^14*c^14*d^6 + 6137200*a^7*b^13*c^13*d^7 - 500
110*a^8*b^12*c^12*d^8 - 48530040*a^9*b^11*c^11*d^9 + 174873556*a^10*b^10*c^
10*d^10 - 360900280*a^11*b^9*c^9*d^11 + 517559250*a^12*b^8*c^8*d^12 - 54823
1440*a^13*b^7*c^7*d^13 + 438700840*a^14*b^6*c^6*d^14 - 266040144*a^15*b^5*c
^5*d^15 + 120836285*a^16*b^4*c^4*d^16 - 39944900*a^17*b^3*c^3*d^17 + 909483
0*a^18*b^2*c^2*d^18 - 1277380*a^19*b*c*d^19 + 83521*a^20*d^20)/(a^7*b^21))^
(1/4) + (3*b^5*c^5 + 5*a*b^4*c^4*d - 50*a^2*b^3*c^3*d^2 + 90*a^3*b^2*c^2*d^
3 - 65*a^4*b*c*d^4 + 17*a^5*d^5)*x) - 585*(a*b^6*x^4 + a^2*b^5)*(-(81*b^20*
c^20 + 540*a*b^19*c^19*d - 4050*a^2*b^18*c^18*d^2 - 15780*a^3*b^17*c^17*d^3
 + 132205*a^4*b^16*c^16*d^4 - 13264*a^5*b^15*c^...
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**4+c)**5/(b*x**4+a)**2,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 798 vs. 2(334) = 668.
time = 0.63, size = 798, normalized size = 1.96

$$\frac{\sqrt{2}\left(3\left(ab^{3}\right)^{\frac{1}{4}}b^{5}c^{5}+5\left(ab^{3}\right)^{\frac{1}{4}}ab^{4}c^{4}d-50\left(ab^{3}\right)^{\frac{1}{4}}a^{2}b^{3}c^{3}d^{2}+90\left(ab^{3}\right)^{\frac{1}{4}}a^{3}b^{2}c^{2}d^{3}-65\left(ab^{3}\right)^{\frac{1}{4}}a^{4}bcd^{4}+17\left(ab^{3}\right)^{\frac{1}{4}}a^{5}d^{5}\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16a^{2}b^{6}}+\frac{\sqrt{2}\left(3\left(ab^{3}\right)^{\frac{1}{4}}b^{5}c^{5}+5\left(ab^{3}\right)^{\frac{1}{4}}ab^{4}c^{4}d-50\left(ab^{3}\right)^{\frac{1}{4}}a^{2}b^{3}c^{3}d^{2}+90\left(ab^{3}\right)^{\frac{1}{4}}a^{3}b^{2}c^{2}d^{3}-65\left(ab^{3}\right)^{\frac{1}{4}}a^{4}bcd^{4}+17\left(ab^{3}\right)^{\frac{1}{4}}a^{5}d^{5}\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16a^{2}b^{6}}+\frac{\sqrt{2}\left(3\left(ab^{3}\right)^{\frac{1}{4}}b^{5}c^{5}+5\left(ab^{3}\right)^{\frac{1}{4}}ab^{4}c^{4}d-50\left(ab^{3}\right)^{\frac{1}{4}}a^{2}b^{3}c^{3}d^{2}+90\left(ab^{3}\right)^{\frac{1}{4}}a^{3}b^{2}c^{2}d^{3}-65\left(ab^{3}\right)^{\frac{1}{4}}a^{4}bcd^{4}+17\left(ab^{3}\right)^{\frac{1}{4}}a^{5}d^{5}\right)\log\left(x^{2}+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{32a^{2}b^{6}}-\frac{\sqrt{2}\left(3\left(ab^{3}\right)^{\frac{1}{4}}b^{5}c^{5}+5\left(ab^{3}\right)^{\frac{1}{4}}ab^{4}c^{4}d-50\left(ab^{3}\right)^{\frac{1}{4}}a^{2}b^{3}c^{3}d^{2}+90\left(ab^{3}\right)^{\frac{1}{4}}a^{3}b^{2}c^{2}d^{3}-65\left(ab^{3}\right)^{\frac{1}{4}}a^{4}bcd^{4}+17\left(ab^{3}\right)^{\frac{1}{4}}a^{5}d^{5}\right)\log\left(x^{2}-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{32a^{2}b^{6}}+\frac{b^{5}c^{5}x-5ab^{4}c^{4}dx+10a^{2}b^{3}c^{3}d^{2}x-10a^{3}b^{2}c^{2}d^{3}x+5a^{4}bcd^{4}x-a^{5}d^{5}x}{4\left(bx^{4}+a\right)ab^{5}}+\frac{45b^{24}d^{5}x^{13}+325b^{24}cd^{4}x^{9}-130ab^{23}d^{5}x^{9}+1170b^{24}c^{2}d^{3}x^{5}-1170ab^{23}cd^{4}x^{5}+351a^{2}b^{22}d^{5}x^{5}+5850b^{24}c^{3}d^{2}x-11700ab^{23}c^{2}d^{3}x+8775a^{2}b^{22}cd^{4}x-2340a^{3}b^{21}d^{5}x}{585b^{26}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^5/(b*x^4+a)^2,x, algorithm="giac")
```

```
[Out] 1/16*sqrt(2)*(3*(a*b^3)^(1/4)*b^5*c^5 + 5*(a*b^3)^(1/4)*a*b^4*c^4*d - 50*(a
*b^3)^(1/4)*a^2*b^3*c^3*d^2 + 90*(a*b^3)^(1/4)*a^3*b^2*c^2*d^3 - 65*(a*b^3)
^(1/4)*a^4*b*c*d^4 + 17*(a*b^3)^(1/4)*a^5*d^5)*arctan(1/2*sqrt(2)*(2*x + sq
rt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^2*b^6) + 1/16*sqrt(2)*(3*(a*b^3)^(1/4)*b
^5*c^5 + 5*(a*b^3)^(1/4)*a*b^4*c^4*d - 50*(a*b^3)^(1/4)*a^2*b^3*c^3*d^2 + 9
0*(a*b^3)^(1/4)*a^3*b^2*c^2*d^3 - 65*(a*b^3)^(1/4)*a^4*b*c*d^4 + 17*(a*b^3)
^(1/4)*a^5*d^5)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))
/(a^2*b^6) + 1/32*sqrt(2)*(3*(a*b^3)^(1/4)*b^5*c^5 + 5*(a*b^3)^(1/4)*a*b^4*
c^4*d - 50*(a*b^3)^(1/4)*a^2*b^3*c^3*d^2 + 90*(a*b^3)^(1/4)*a^3*b^2*c^2*d^3
 - 65*(a*b^3)^(1/4)*a^4*b*c*d^4 + 17*(a*b^3)^(1/4)*a^5*d^5)*log(x^2 + sqrt(
2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^2*b^6) - 1/32*sqrt(2)*(3*(a*b^3)^(1/4)*b^5
*c^5 + 5*(a*b^3)^(1/4)*a*b^4*c^4*d - 50*(a*b^3)^(1/4)*a^2*b^3*c^3*d^2 + 90*
(a*b^3)^(1/4)*a^3*b^2*c^2*d^3 - 65*(a*b^3)^(1/4)*a^4*b*c*d^4 + 17*(a*b^3)^(
1/4)*a^5*d^5)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^2*b^6) + 1/4*
(b^5*c^5*x - 5*a*b^4*c^4*d*x + 10*a^2*b^3*c^3*d^2*x - 10*a^3*b^2*c^2*d^3*x
+ 5*a^4*b*c*d^4*x - a^5*d^5*x)/((b*x^4 + a)*a*b^5) + 1/585*(45*b^24*d^5*x^1
3 + 325*b^24*c*d^4*x^9 - 130*a*b^23*d^5*x^9 + 1170*b^24*c^2*d^3*x^5 - 1170*
a*b^23*c*d^4*x^5 + 351*a^2*b^22*d^5*x^5 + 5850*b^24*c^3*d^2*x - 11700*a*b^2
3*c^2*d^3*x + 8775*a^2*b^22*c*d^4*x - 2340*a^3*b^21*d^5*x)/b^26
```

Mupad [B]
time = 1.71, size = 2490, normalized size = 6.12

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^4)^5/(a + b*x^4)^2,x)
```

```
[Out] x*((10*c^3*d^2)/b^2 - (2*a*((2*a*((2*a*d^5)/b^3 - (5*c*d^4)/b^2))/b - (a^2*
d^5)/b^4 + (10*c^2*d^3)/b^2))/b + (a^2*((2*a*d^5)/b^3 - (5*c*d^4)/b^2))/b^2
) - x^9*((2*a*d^5)/(9*b^3) - (5*c*d^4)/(9*b^2)) + x^5*((2*a*((2*a*d^5)/b^3
```

- (5*c*d^4)/b^2))/(5*b) - (a^2*d^5)/(5*b^4) + (2*c^2*d^3)/b^2) + (d^5*x^13)
/(13*b^2) - (x*(a^5*d^5 - b^5*c^5 - 10*a^2*b^3*c^3*d^2 + 10*a^3*b^2*c^2*d^3
+ 5*a*b^4*c^4*d - 5*a^4*b*c*d^4))/(4*a*(a*b^5 + b^6*x^4)) + (atan(((((x*(2
89*a^10*d^10 + 9*b^10*c^10 - 275*a^2*b^8*c^8*d^2 + 40*a^3*b^7*c^7*d^3 + 301
0*a^4*b^6*c^6*d^4 - 9548*a^5*b^5*c^5*d^5 + 14770*a^6*b^4*c^4*d^6 - 13400*a^
7*b^3*c^3*d^7 + 7285*a^8*b^2*c^2*d^8 + 30*a*b^9*c^9*d - 2210*a^9*b*c*d^9))/
(4*a^2*b^7) - ((a*d - b*c)^4*(17*a*d + 3*b*c)*(17*a^5*d^5 + 3*b^5*c^5 - 50*
a^2*b^3*c^3*d^2 + 90*a^3*b^2*c^2*d^3 + 5*a*b^4*c^4*d - 65*a^4*b*c*d^4))/(4*
(-a)^(7/4)*b^(29/4)))*(a*d - b*c)^4*(17*a*d + 3*b*c)*1i)/(16*(-a)^(7/4)*b^(
21/4)) + (((x*(289*a^10*d^10 + 9*b^10*c^10 - 275*a^2*b^8*c^8*d^2 + 40*a^3*b
^7*c^7*d^3 + 3010*a^4*b^6*c^6*d^4 - 9548*a^5*b^5*c^5*d^5 + 14770*a^6*b^4*c^
4*d^6 - 13400*a^7*b^3*c^3*d^7 + 7285*a^8*b^2*c^2*d^8 + 30*a*b^9*c^9*d - 221
0*a^9*b*c*d^9))/(4*a^2*b^7) + ((a*d - b*c)^4*(17*a*d + 3*b*c)*(17*a^5*d^5 +
3*b^5*c^5 - 50*a^2*b^3*c^3*d^2 + 90*a^3*b^2*c^2*d^3 + 5*a*b^4*c^4*d - 65*a
^4*b*c*d^4))/(4*(-a)^(7/4)*b^(29/4)))*(a*d - b*c)^4*(17*a*d + 3*b*c)*1i)/(1
6*(-a)^(7/4)*b^(21/4)))/((((x*(289*a^10*d^10 + 9*b^10*c^10 - 275*a^2*b^8*c^
8*d^2 + 40*a^3*b^7*c^7*d^3 + 3010*a^4*b^6*c^6*d^4 - 9548*a^5*b^5*c^5*d^5 +
14770*a^6*b^4*c^4*d^6 - 13400*a^7*b^3*c^3*d^7 + 7285*a^8*b^2*c^2*d^8 + 30*a
*b^9*c^9*d - 2210*a^9*b*c*d^9))/(4*a^2*b^7) - ((a*d - b*c)^4*(17*a*d + 3*b*
c)*(17*a^5*d^5 + 3*b^5*c^5 - 50*a^2*b^3*c^3*d^2 + 90*a^3*b^2*c^2*d^3 + 5*a*
b^4*c^4*d - 65*a^4*b*c*d^4))/(4*(-a)^(7/4)*b^(29/4)))*(a*d - b*c)^4*(17*a*d
+ 3*b*c))/(16*(-a)^(7/4)*b^(21/4)) - (((x*(289*a^10*d^10 + 9*b^10*c^10 - 2
75*a^2*b^8*c^8*d^2 + 40*a^3*b^7*c^7*d^3 + 3010*a^4*b^6*c^6*d^4 - 9548*a^5*b
^5*c^5*d^5 + 14770*a^6*b^4*c^4*d^6 - 13400*a^7*b^3*c^3*d^7 + 7285*a^8*b^2*c
^2*d^8 + 30*a*b^9*c^9*d - 2210*a^9*b*c*d^9))/(4*a^2*b^7) + ((a*d - b*c)^4*(
17*a*d + 3*b*c)*(17*a^5*d^5 + 3*b^5*c^5 - 50*a^2*b^3*c^3*d^2 + 90*a^3*b^2*c
^2*d^3 + 5*a*b^4*c^4*d - 65*a^4*b*c*d^4))/(4*(-a)^(7/4)*b^(29/4)))*(a*d - b
c)^4(17*a*d + 3*b*c))/(16*(-a)^(7/4)*b^(21/4))))*(a*d - b*c)^4*(17*a*d +
3*b*c)*1i)/(8*(-a)^(7/4)*b^(21/4)) + (atan(((((x*(289*a^10*d^10 + 9*b^10*c^
10 - 275*a^2*b^8*c^8*d^2 + 40*a^3*b^7*c^7*d^3 + 3010*a^4*b^6*c^6*d^4 - 9548
*a^5*b^5*c^5*d^5 + 14770*a^6*b^4*c^4*d^6 - 13400*a^7*b^3*c^3*d^7 + 7285*a^8
*b^2*c^2*d^8 + 30*a*b^9*c^9*d - 2210*a^9*b*c*d^9))/(4*a^2*b^7) - ((a*d - b*
c)^4*(17*a*d + 3*b*c)*(17*a^5*d^5 + 3*b^5*c^5 - 50*a^2*b^3*c^3*d^2 + 90*a^3
*b^2*c^2*d^3 + 5*a*b^4*c^4*d - 65*a^4*b*c*d^4)*1i)/(4*(-a)^(7/4)*b^(29/4)))
*(a*d - b*c)^4*(17*a*d + 3*b*c))/(16*(-a)^(7/4)*b^(21/4)) + (((x*(289*a^10*
d^10 + 9*b^10*c^10 - 275*a^2*b^8*c^8*d^2 + 40*a^3*b^7*c^7*d^3 + 3010*a^4*b^
6*c^6*d^4 - 9548*a^5*b^5*c^5*d^5 + 14770*a^6*b^4*c^4*d^6 - 13400*a^7*b^3*c^
3*d^7 + 7285*a^8*b^2*c^2*d^8 + 30*a*b^9*c^9*d - 2210*a^9*b*c*d^9))/(4*a^2*b
^7) + ((a*d - b*c)^4*(17*a*d + 3*b*c)*(17*a^5*d^5 + 3*b^5*c^5 - 50*a^2*b^3*
c^3*d^2 + 90*a^3*b^2*c^2*d^3 + 5*a*b^4*c^4*d - 65*a^4*b*c*d^4)*1i)/(4*(-a)^
(7/4)*b^(29/4)))*(a*d - b*c)^4*(17*a*d + 3*b*c))/(16*(-a)^(7/4)*b^(21/4)))/
((((x*(289*a^10*d^10 + 9*b^10*c^10 - 275*a^2*b^8*c^8*d^2 + 40*a^3*b^7*c^7*d
^3 + 3010*a^4*b^6*c^6*d^4 - 9548*a^5*b^5*c^5*d^5 + 14770*a^6*b^4*c^4*d^6 -
13400*a^7*b^3*c^3*d^7 + 7285*a^8*b^2*c^2*d^8 + 30*a*b^9*c^9*d - 2210*a^9*b*
c*d^9))/(4*a^2*b^7) - ((a*d - b*c)^4*(17*a*d + 3*b*c)*(17*a^5*d^5 + 3*b^5*c

```
^5 - 50*a^2*b^3*c^3*d^2 + 90*a^3*b^2*c^2*d^3 + 5*a*b^4*c^4*d - 65*a^4*b*c*d
^4)*1i)/(4*(-a)^(7/4)*b^(29/4)))*(a*d - b*c)^4*(17*a*d + 3*b*c)*1i)/(16*(-a
)^(7/4)*b^(21/4)) - (((x*(289*a^10*d^10 + 9*b^10*c^10 - 275*a^2*b^8*c^8*d^2
 + 40*a^3*b^7*c^7*d^3 + 3010*a^4*b^6*c^6*d^4 - 9548*a^5*b^5*c^5*d^5 + 14770
*a^6*b^4*c^4*d^6 - 13400*a^7*b^3*c^3*d^7 + 7285*a^8*b^2*c^2*d^8 + 30*a*b^9*
c^9*d - 2210*a^9*b*c*d^9))/(4*a^2*b^7) + ((a*d - b*c)^4*(17*a*d + 3*b*c)*(1
7*a^5*d^5 + 3*b^5*c^5 - 50*a^2*b^3*c^3*d^2 + 90*a^3*b^2*c^2*d^3 + 5*a*b^4*c
^4*d - 65*a^4*b*c*d^4)*1i)/(4*(-a)^(7/4)*b^(29/4)))*(a*d - b*c)^4*(17*a*d +
 3*b*c)*1i)/(16*(-a)^(7/4)*b^(21/4))))*(a*d - b*c)^4*(17*a*d + 3*b*c))/(8*(
-a)^(7/4)*b^(21/4))
```

3.167 $\int \frac{(c+dx^4)^4}{(a+bx^4)^2}\,dx$

Optimal. Leaf size=357

$$\frac{d^2(6b^2c^2-8abcd+3a^2d^2)\,x}{b^4}+\frac{2d^3(2bc-ad)x^5}{5b^3}+\frac{d^4x^9}{9b^2}+\frac{(bc-ad)^4x}{4ab^4\,(a+bx^4)}-\frac{(bc-ad)^3(3bc+13ad)\tan^{-1}\left(1-\right.}{8\sqrt{2}\,a^{7/4}b^{17/4}}$$

```
[Out] d^2*(3*a^2*d^2-8*a*b*c*d+6*b^2*c^2)*x/b^4+2/5*d^3*(-a*d+2*b*c)*x^5/b^3+1/9*
d^4*x^9/b^2+1/4*(-a*d+b*c)^4*x/a/b^4/(b*x^4+a)+1/16*(-a*d+b*c)^3*(13*a*d+3*
b*c)*arctan(-1+b^(1/4)*x*2^(1/2)/a^(1/4))/a^(7/4)/b^(17/4)*2^(1/2)+1/16*(-a
*d+b*c)^3*(13*a*d+3*b*c)*arctan(1+b^(1/4)*x*2^(1/2)/a^(1/4))/a^(7/4)/b^(17/
4)*2^(1/2)-1/32*(-a*d+b*c)^3*(13*a*d+3*b*c)*ln(-a^(1/4)*b^(1/4)*x*2^(1/2)+a
^(1/2)+x^2*b^(1/2))/a^(7/4)/b^(17/4)*2^(1/2)+1/32*(-a*d+b*c)^3*(13*a*d+3*b*
c)*ln(a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))/a^(7/4)/b^(17/4)*2^(1/
2)
```

Rubi [A]
time = 0.24, antiderivative size = 357, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$,
Rules used = {398, 393, 217, 1179, 642, 1176, 631, 210}

$$-\frac{\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)(bc-ad)^3(13ad+3bc)}{8\sqrt{2}\,a^{7/4}b^{17/4}}+\frac{\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}+1\right)(bc-ad)^3(13ad+3bc)}{8\sqrt{2}\,a^{7/4}b^{17/4}}-\frac{(bc-ad)^3(13ad+3bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{16\sqrt{2}\,a^{7/4}b^{17/4}}+\frac{(bc-ad)^3(13ad+3bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{16\sqrt{2}\,a^{7/4}b^{17/4}}+\frac{d^2x(3a^2d^2-8abcd+6b^2c^2)}{b^4}+\frac{x(bc-ad)^4}{4ab^4\,(a+bx^4)}+\frac{2d^3x^5(2bc-ad)}{5b^3}+\frac{d^4x^9}{9b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^4)^4/(a + b*x^4)^2,x]
```

```
[Out] (d^2*(6*b^2*c^2 - 8*a*b*c*d + 3*a^2*d^2)*x)/b^4 + (2*d^3*(2*b*c - a*d)*x^5)
/(5*b^3) + (d^4*x^9)/(9*b^2) + ((b*c - a*d)^4*x)/(4*a*b^4*(a + b*x^4)) - ((
b*c - a*d)^3*(3*b*c + 13*a*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(8*S
qrt[2]*a^(7/4)*b^(17/4)) + ((b*c - a*d)^3*(3*b*c + 13*a*d)*ArcTan[1 + (Sqrt
[2]*b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*b^(17/4)) - ((b*c - a*d)^3*(3*b
*c + 13*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sq
rt[2]*a^(7/4)*b^(17/4)) + ((b*c - a*d)^3*(3*b*c + 13*a*d)*Log[Sqrt[a] + Sqr
t[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)*b^(17/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
```

```
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(c+dx^4)^4}{(a+bx^4)^2}\,dx = \int \left(\frac{d^2(6b^2c^2-8abcd+3a^2d^2)}{b^4} + \frac{2d^3(2bc-ad)x^4}{b^3} + \frac{d^4x^8}{b^2} + \frac{(bc-ad)^3(bc+3ad)+4bd(}{b^4\,(a+bx^4)^2}\right.$$

$$= \frac{d^2(6b^2c^2-8abcd+3a^2d^2)\,x}{b^4} + \frac{2d^3(2bc-ad)x^5}{5b^3} + \frac{d^4x^9}{9b^2} + \frac{\int \frac{(bc-ad)^3(bc+3ad)+4bd(bc-ad)^3x^4}{(a+bx^4)^2}\,dx}{b^4}$$

$$= \frac{d^2(6b^2c^2-8abcd+3a^2d^2)\,x}{b^4} + \frac{2d^3(2bc-ad)x^5}{5b^3} + \frac{d^4x^9}{9b^2} + \frac{(bc-ad)^4x}{4ab^4\,(a+bx^4)} + \frac{((bc-ad)^3(3}{}$$

$$= \frac{d^2(6b^2c^2-8abcd+3a^2d^2)\,x}{b^4} + \frac{2d^3(2bc-ad)x^5}{5b^3} + \frac{d^4x^9}{9b^2} + \frac{(bc-ad)^4x}{4ab^4\,(a+bx^4)} + \frac{((bc-ad)^3(3}{}$$

$$= \frac{d^2(6b^2c^2-8abcd+3a^2d^2)\,x}{b^4} + \frac{2d^3(2bc-ad)x^5}{5b^3} + \frac{d^4x^9}{9b^2} + \frac{(bc-ad)^4x}{4ab^4\,(a+bx^4)} + \frac{((bc-ad)^3(3}{}$$

$$= \frac{d^2(6b^2c^2-8abcd+3a^2d^2)\,x}{b^4} + \frac{2d^3(2bc-ad)x^5}{5b^3} + \frac{d^4x^9}{9b^2} + \frac{(bc-ad)^4x}{4ab^4\,(a+bx^4)} - \frac{(bc-ad)^3(3b}{}$$

$$= \frac{d^2(6b^2c^2-8abcd+3a^2d^2)\,x}{b^4} + \frac{2d^3(2bc-ad)x^5}{5b^3} + \frac{d^4x^9}{9b^2} + \frac{(bc-ad)^4x}{4ab^4\,(a+bx^4)} - \frac{(bc-ad)^3(3b}{}$$

Mathematica [A]
time = 0.19, size = 341, normalized size = 0.96

$$\frac{1440\sqrt[4]{b}\,d^2(6b^2c^2-8abcd+3a^2d^2)\,x+576b^{5/4}d^3(2bc-ad)x^5+160b^{9/4}d^4x^9+\frac{360\sqrt[4]{b}\,(bc-ad)^4x}{a(a+bx^4)}+\frac{90\sqrt{2}\,(-bc+ad)^3(3bc+13ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{a^{7/4}}+\frac{90\sqrt{2}\,(bc-ad)^3(3bc+13ad)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{a^{7/4}}+\frac{45\sqrt{2}\,(-bc+ad)^3(3bc+13ad)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{b}\,x^2\right)}{a^{7/4}}+\frac{45\sqrt{2}\,(bc-ad)^3(3bc+13ad)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{b}\,x^2\right)}{a^{7/4}}}{1440b^{17/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^4)^4/(a + b*x^4)^2,x]
```

```
[Out] (1440*b^(1/4)*d^2*(6*b^2*c^2 - 8*a*b*c*d + 3*a^2*d^2)*x + 576*b^(5/4)*d^3*(
2*b*c - a*d)*x^5 + 160*b^(9/4)*d^4*x^9 + (360*b^(1/4)*(b*c - a*d)^4*x)/(a*(
a + b*x^4)) + (90*Sqrt[2]*(-(b*c) + a*d)^3*(3*b*c + 13*a*d)*ArcTan[1 - (Sqr
t[2]*b^(1/4)*x)/a^(1/4)])/a^(7/4) + (90*Sqrt[2]*(b*c - a*d)^3*(3*b*c + 13*a
*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/a^(7/4) + (45*Sqrt[2]*(-(b*c)
+ a*d)^3*(3*b*c + 13*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]
*x^2])/a^(7/4) + (45*Sqrt[2]*(b*c - a*d)^3*(3*b*c + 13*a*d)*Log[Sqrt[a] + S
qrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/a^(7/4))/(1440*b^(17/4))
```

Maple [A]
time = 0.27, size = 288, normalized size = 0.81

method	result
risch	$\frac{d^4x^9}{9b^2} - \frac{2d^4a\,x^5}{5b^3} + \frac{4d^3c\,x^5}{5b^2} + \frac{3d^4a^2x}{b^4} - \frac{8d^3acx}{b^3} + \frac{6d^2c^2x}{b^2} + \frac{(a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+b^4c^4)x}{4a\,b^4(b\,x^4+a)} - \frac{\sum_{\_R=\text{RootO}}}{}$
default	$\frac{d^2\left(\frac{1}{9}b^2d^2x^9-\frac{2}{5}ab\,d^2x^5+\frac{4}{5}b^2cd\,x^5+3a^2d^2x-8abcdx+6b^2c^2x\right)}{b^4} - \frac{-\frac{\left(a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+b^4c^4\right)x}{4a\left(b\,x^4+a\right)} + \frac{\left(13a^4d^4-36a^3bc\,d^3\right.}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^4+c)^4/(b*x^4+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] d^2/b^4*(1/9*b^2*d^2*x^9-2/5*a*b*d^2*x^5+4/5*b^2*c*d*x^5+3*a^2*d^2*x-8*a*b*
c*d*x+6*b^2*c^2*x)-1/b^4*(-1/4*(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a
*b^3*c^3*d+b^4*c^4)/a*x/(b*x^4+a)+1/32*(13*a^4*d^4-36*a^3*b*c*d^3+30*a^2*b^
2*c^2*d^2-4*a*b^3*c^3*d-3*b^4*c^4)/a^2*(a/b)^(1/4)*2^(1/2)*(ln((x^2+(a/b)^(
1/4)*x*2^(1/2)+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))+2*arct
an(2^(1/2)/(a/b)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x-1)))
```

Maxima [A]
time = 0.50, size = 521, normalized size = 1.46

$$\frac{(b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4)x}{4\,(ab^5x^4+a^2b^4)}+\frac{5\,b^2d^4x^9+18\,(2\,b^2cd^3-abd^4)x^5+45\,(6\,b^2c^2d^2-8\,abcd^3+3\,a^2d^4)x}{45\,b^4}+\frac{\frac{2\sqrt{2}\,(3\,b^4c^4+4\,ab^3c^3d-30\,a^2b^2c^2d^2+36\,a^3bcd^3-13\,a^4d^4)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}\,x+\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{2\sqrt{2}\,(3\,b^4c^4+4\,ab^3c^3d-30\,a^2b^2c^2d^2+36\,a^3bcd^3-13\,a^4d^4)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}\,x-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{\sqrt{2}\,(3\,b^4c^4+4\,ab^3c^3d-30\,a^2b^2c^2d^2+36\,a^3bcd^3-13\,a^4d^4)\log\left(\sqrt{b}\,x^2+\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}-\frac{\sqrt{2}\,(3\,b^4c^4+4\,ab^3c^3d-30\,a^2b^2c^2d^2+36\,a^3bcd^3-13\,a^4d^4)\log\left(\sqrt{b}\,x^2-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}}{32\,ab^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^4/(b*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)
*x/(a*b^5*x^4 + a^2*b^4) + 1/45*(5*b^2*d^4*x^9 + 18*(2*b^2*c*d^3 - a*b*d^4)
*x^5 + 45*(6*b^2*c^2*d^2 - 8*a*b*c*d^3 + 3*a^2*d^4)*x)/b^4 + 1/32*(2*sqrt(2
)*(3*b^4*c^4 + 4*a*b^3*c^3*d - 30*a^2*b^2*c^2*d^2 + 36*a^3*b*c*d^3 - 13*a^4
*d^4)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(
a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqrt(2)*(3*b^4*c^4 + 4*a*b
^3*c^3*d - 30*a^2*b^2*c^2*d^2 + 36*a^3*b*c*d^3 - 13*a^4*d^4)*arctan(1/2*sqr
t(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a
)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*(3*b^4*c^4 + 4*a*b^3*c^3*d - 30*a^2*b^2*
c^2*d^2 + 36*a^3*b*c*d^3 - 13*a^4*d^4)*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^
(1/4)*x + sqrt(a))/(a^(3/4)*b^(1/4)) - sqrt(2)*(3*b^4*c^4 + 4*a*b^3*c^3*d -
 30*a^2*b^2*c^2*d^2 + 36*a^3*b*c*d^3 - 13*a^4*d^4)*log(sqrt(b)*x^2 - sqrt(2
)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(1/4)))/(a*b^4)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2580 vs. $2(286) = 572$.

time = 3.33, size = 2580, normalized size = 7.23

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^4/(b*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] 1/720*(80*a*b^3*d^4*x^13 + 16*(36*a*b^3*c*d^3 - 13*a^2*b^2*d^4)*x^9 + 144*(
30*a*b^3*c^2*d^2 - 36*a^2*b^2*c*d^3 + 13*a^3*b*d^4)*x^5 - 180*(a*b^5*x^4 +
a^2*b^4)*(-(81*b^16*c^16 + 432*a*b^15*c^15*d - 2376*a^2*b^14*c^14*d^2 - 830
4*a^3*b^13*c^13*d^3 + 45724*a^4*b^12*c^12*d^4 + 20400*a^5*b^11*c^11*d^5 - 4
34808*a^6*b^10*c^10*d^6 + 772112*a^7*b^9*c^9*d^7 + 617958*a^8*b^8*c^8*d^8 -
 4810608*a^9*b^7*c^7*d^9 + 9723912*a^10*b^6*c^6*d^10 - 11486160*a^11*b^5*c^
5*d^11 + 8923164*a^12*b^4*c^4*d^12 - 4651504*a^13*b^3*c^3*d^13 + 1577784*a^
14*b^2*c^2*d^14 - 316368*a^15*b*c*d^15 + 28561*a^16*d^16)/(a^7*b^17))^(1/4)
*arctan((sqrt(a^4*b^8*sqrt(-(81*b^16*c^16 + 432*a*b^15*c^15*d - 2376*a^2*b^
14*c^14*d^2 - 8304*a^3*b^13*c^13*d^3 + 45724*a^4*b^12*c^12*d^4 + 20400*a^5*
b^11*c^11*d^5 - 434808*a^6*b^10*c^10*d^6 + 772112*a^7*b^9*c^9*d^7 + 617958*
a^8*b^8*c^8*d^8 - 4810608*a^9*b^7*c^7*d^9 + 9723912*a^10*b^6*c^6*d^10 - 114
86160*a^11*b^5*c^5*d^11 + 8923164*a^12*b^4*c^4*d^12 - 4651504*a^13*b^3*c^3*
d^13 + 1577784*a^14*b^2*c^2*d^14 - 316368*a^15*b*c*d^15 + 28561*a^16*d^16)/
(a^7*b^17)) + (9*b^8*c^8 + 24*a*b^7*c^7*d - 164*a^2*b^6*c^6*d^2 - 24*a^3*b^
5*c^5*d^3 + 1110*a^4*b^4*c^4*d^4 - 2264*a^5*b^3*c^3*d^5 + 2076*a^6*b^2*c^2*
d^6 - 936*a^7*b*c*d^7 + 169*a^8*d^8)*x^2)*a^5*b^13*(-(81*b^16*c^16 + 432*a*
b^15*c^15*d - 2376*a^2*b^14*c^14*d^2 - 8304*a^3*b^13*c^13*d^3 + 45724*a^4*b
^12*c^12*d^4 + 20400*a^5*b^11*c^11*d^5 - 434808*a^6*b^10*c^10*d^6 + 772112*
a^7*b^9*c^9*d^7 + 617958*a^8*b^8*c^8*d^8 - 4810608*a^9*b^7*c^7*d^9 + 972391
2*a^10*b^6*c^6*d^10 - 11486160*a^11*b^5*c^5*d^11 + 8923164*a^12*b^4*c^4*d^1
2 - 4651504*a^13*b^3*c^3*d^13 + 1577784*a^14*b^2*c^2*d^14 - 316368*a^15*b*c
*d^15 + 28561*a^16*d^16)/(a^7*b^17))^(3/4) + (3*a^5*b^17*c^4 + 4*a^6*b^16*c
^3*d - 30*a^7*b^15*c^2*d^2 + 36*a^8*b^14*c*d^3 - 13*a^9*b^13*d^4)*x*(-(81*b
^16*c^16 + 432*a*b^15*c^15*d - 2376*a^2*b^14*c^14*d^2 - 8304*a^3*b^13*c^13*
d^3 + 45724*a^4*b^12*c^12*d^4 + 20400*a^5*b^11*c^11*d^5 - 434808*a^6*b^10*c
^10*d^6 + 772112*a^7*b^9*c^9*d^7 + 617958*a^8*b^8*c^8*d^8 - 4810608*a^9*b^7
*c^7*d^9 + 9723912*a^10*b^6*c^6*d^10 - 11486160*a^11*b^5*c^5*d^11 + 8923164
*a^12*b^4*c^4*d^12 - 4651504*a^13*b^3*c^3*d^13 + 1577784*a^14*b^2*c^2*d^14
- 316368*a^15*b*c*d^15 + 28561*a^16*d^16)/(a^7*b^17))^(3/4))/(81*b^16*c^16
+ 432*a*b^15*c^15*d - 2376*a^2*b^14*c^14*d^2 - 8304*a^3*b^13*c^13*d^3 + 457
24*a^4*b^12*c^12*d^4 + 20400*a^5*b^11*c^11*d^5 - 434808*a^6*b^10*c^10*d^6 +
 772112*a^7*b^9*c^9*d^7 + 617958*a^8*b^8*c^8*d^8 - 4810608*a^9*b^7*c^7*d^9
+ 9723912*a^10*b^6*c^6*d^10 - 11486160*a^11*b^5*c^5*d^11 + 8923164*a^12*b^4
*c^4*d^12 - 4651504*a^13*b^3*c^3*d^13 + 1577784*a^14*b^2*c^2*d^14 - 316368*
a^15*b*c*d^15 + 28561*a^16*d^16)) - 45*(a*b^5*x^4 + a^2*b^4)*(-(81*b^16*c^1
6 + 432*a*b^15*c^15*d - 2376*a^2*b^14*c^14*d^2 - 8304*a^3*b^13*c^13*d^3 + 4
5724*a^4*b^12*c^12*d^4 + 20400*a^5*b^11*c^11*d^5 - 434808*a^6*b^10*c^10*d^6
```

```
 + 772112*a^7*b^9*c^9*d^7 + 617958*a^8*b^8*c^8*d^8 - 4810608*a^9*b^7*c^7*d^
9 + 9723912*a^10*b^6*c^6*d^10 - 11486160*a^11*b^5*c^5*d^11 + 8923164*a^12*b
^4*c^4*d^12 - 4651504*a^13*b^3*c^3*d^13 + 1577784*a^14*b^2*c^2*d^14 - 31636
8*a^15*b*c*d^15 + 28561*a^16*d^16)/(a^7*b^17))^(1/4)*log(a^2*b^4*(-(81*b^16
*c^16 + 432*a*b^15*c^15*d - 2376*a^2*b^14*c^14*d^2 - 8304*a^3*b^13*c^13*d^3
 + 45724*a^4*b^12*c^12*d^4 + 20400*a^5*b^11*c^11*d^5 - 434808*a^6*b^10*c^10
*d^6 + 772112*a^7*b^9*c^9*d^7 + 617958*a^8*b^8*c^8*d^8 - 4810608*a^9*b^7*c^
7*d^9 + 9723912*a^10*b^6*c^6*d^10 - 11486160*a^11*b^5*c^5*d^11 + 8923164*a^
12*b^4*c^4*d^12 - 4651504*a^13*b^3*c^3*d^13 + 1577784*a^14*b^2*c^2*d^14 - 3
16368*a^15*b*c*d^15 + 28561*a^16*d^16)/(a^7*b^17))^(1/4) - (3*b^4*c^4 + 4*a
*b^3*c^3*d - 30*a^2*b^2*c^2*d^2 + 36*a^3*b*c*d^3 - 13*a^4*d^4)*x) + 45*(a*b
^5*x^4 + a^2*b^4)*(-(81*b^16*c^16 + 432*a*b^15*c^15*d - 2376*a^2*b^14*c^14*
d^2 - 8304*a^3*b^13*c^13*d^3 + 45724*a^4*b^12*c^12*d^4 + 20400*a^5*b^11*c^1
1*d^5 - 434808*a^6*b^10*c^10*d^6 + 772112*a^7*b^9*c^9*d^7 + 617958*a^8*b^8*
c^8*d^8 - 4810608*a^9*b^7*c^7*d^9 + 9723912*a^10*b^6*c^6*d^10 - 11486160*a^
11*b^5*c^5*d^11 + 8923164*a^12*b^4*c^4*d^12 - 4651504*a^13*b^3*c^3*d^13 + 1
577784*a^14*b^2*c^2*d^14 - 316368*a^15*b*c*d^15 + 28561*a^16*d^16)/(a^7*b^1
7))^(1/4)*log(-a^2*b^4*(-(81*b^16*c^16 + 432*a*b^15*c^15*d - 2376*a^2*b^14*
c^14*d^2 - 8304*a^3*b^13*c^13*d^3 + 45724*a^4*b^12*c^12*d^4 + 20400*a^5*b^1
1*c^11*d^5 - 434808*a^6*b^10*c^10*d^6 + 772112*a^7*b^9*c^9*d^7 + 617958*a^8
*b^8*c^8*d^8 - 4810608*a^9*b^7*c^7*d^9 + 9723912*a^10*b^6*c^6*d^10 - 114861
60*a^11*b^5*c^5*d^11 + 8923164*a^12*b^4*c^4*d^12 - 4651504*a^13*b^3*c^3*d^1
3 + 1577784*a^14*b^2*c^2*d^14 - 316368*a^15*b*c*d^15 + 28561*a^16*d^16)/(a^
7*b^17))^(1/4) - (3*b^4*c^4 + 4*a*b^3*c^3*d - 30*a^2*b^2*c^2*d^2 + 36*a^3*b
*c*d^3 - 13*a^4*d^4)*x) + 180*(b^4*c^4 - 4*a*b^3*c^3*d + 30*a^2*b^2*c^2*d^2
 - 36*a^3*b*c*d^3 + 13*a^4*d^4)*x)/(a*b^5*x^4 + a^2*b^4)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**4+c)**4/(b*x**4+a)**2,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 642 vs. 2(286) = 572.
time = 0.65, size = 642, normalized size = 1.80

$$\frac{\sqrt{2}\left(3\,(ab^3)^{\frac{1}{4}}b^4c^4+4\,(ab^3)^{\frac{1}{4}}ab^3c^3d-30\,(ab^3)^{\frac{1}{4}}a^2b^2c^2d^2+36\,(ab^3)^{\frac{1}{4}}a^3bcd^3-13\,(ab^3)^{\frac{1}{4}}a^4d^4\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16\,a^2b^5}+\frac{\sqrt{2}\left(3\,(ab^3)^{\frac{1}{4}}b^4c^4+4\,(ab^3)^{\frac{1}{4}}ab^3c^3d-30\,(ab^3)^{\frac{1}{4}}a^2b^2c^2d^2+36\,(ab^3)^{\frac{1}{4}}a^3bcd^3-13\,(ab^3)^{\frac{1}{4}}a^4d^4\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16\,a^2b^5}+\frac{\sqrt{2}\left(3\,(ab^3)^{\frac{1}{4}}b^4c^4+4\,(ab^3)^{\frac{1}{4}}ab^3c^3d-30\,(ab^3)^{\frac{1}{4}}a^2b^2c^2d^2+36\,(ab^3)^{\frac{1}{4}}a^3bcd^3-13\,(ab^3)^{\frac{1}{4}}a^4d^4\right)\log\left(x^2+\sqrt{2}\,x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{32\,a^2b^5}-\frac{\sqrt{2}\left(3\,(ab^3)^{\frac{1}{4}}b^4c^4+4\,(ab^3)^{\frac{1}{4}}ab^3c^3d-30\,(ab^3)^{\frac{1}{4}}a^2b^2c^2d^2+36\,(ab^3)^{\frac{1}{4}}a^3bcd^3-13\,(ab^3)^{\frac{1}{4}}a^4d^4\right)\log\left(x^2-\sqrt{2}\,x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{32\,a^2b^5}+\frac{b^4c^4x-4\,ab^3c^3dx+6\,a^2b^2c^2d^2x-4\,a^3bcd^3x+a^4d^4x}{4\,(bx^4+a)ab^4}+\frac{5\,b^{16}d^4x^9+36\,b^{16}cd^3x^5-18\,ab^{15}d^4x^5+270\,b^{16}c^2d^2x-360\,ab^{15}cd^3x+135\,a^2b^{14}d^4x}{45\,b^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^4/(b*x^4+a)^2,x, algorithm="giac")
```

```
[Out] 1/16*sqrt(2)*(3*(a*b^3)^(1/4)*b^4*c^4 + 4*(a*b^3)^(1/4)*a*b^3*c^3*d - 30*(a
*b^3)^(1/4)*a^2*b^2*c^2*d^2 + 36*(a*b^3)^(1/4)*a^3*b*c*d^3 - 13*(a*b^3)^(1/
4)*a^4*d^4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^
2*b^5) + 1/16*sqrt(2)*(3*(a*b^3)^(1/4)*b^4*c^4 + 4*(a*b^3)^(1/4)*a*b^3*c^3*
d - 30*(a*b^3)^(1/4)*a^2*b^2*c^2*d^2 + 36*(a*b^3)^(1/4)*a^3*b*c*d^3 - 13*(a
*b^3)^(1/4)*a^4*d^4)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(
1/4))/(a^2*b^5) + 1/32*sqrt(2)*(3*(a*b^3)^(1/4)*b^4*c^4 + 4*(a*b^3)^(1/4)*a
*b^3*c^3*d - 30*(a*b^3)^(1/4)*a^2*b^2*c^2*d^2 + 36*(a*b^3)^(1/4)*a^3*b*c*d^
3 - 13*(a*b^3)^(1/4)*a^4*d^4)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/
(a^2*b^5) - 1/32*sqrt(2)*(3*(a*b^3)^(1/4)*b^4*c^4 + 4*(a*b^3)^(1/4)*a*b^3*c
^3*d - 30*(a*b^3)^(1/4)*a^2*b^2*c^2*d^2 + 36*(a*b^3)^(1/4)*a^3*b*c*d^3 - 13
*(a*b^3)^(1/4)*a^4*d^4)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^2*b
^5) + 1/4*(b^4*c^4*x - 4*a*b^3*c^3*d*x + 6*a^2*b^2*c^2*d^2*x - 4*a^3*b*c*d^
3*x + a^4*d^4*x)/((b*x^4 + a)*a*b^4) + 1/45*(5*b^16*d^4*x^9 + 36*b^16*c*d^3
*x^5 - 18*a*b^15*d^4*x^5 + 270*b^16*c^2*d^2*x - 360*a*b^15*c*d^3*x + 135*a^
2*b^14*d^4*x)/b^18
```

Mupad [B]
time = 0.30, size = 2043, normalized size = 5.72

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^4)^4/(a + b*x^4)^2,x)
```

```
[Out] x*((2*a*((2*a*d^4)/b^3 - (4*c*d^3)/b^2))/b - (a^2*d^4)/b^4 + (6*c^2*d^2)/b^
2) - x^5*((2*a*d^4)/(5*b^3) - (4*c*d^3)/(5*b^2)) + (d^4*x^9)/(9*b^2) + (x*(
a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3))/(4*
a*(a*b^4 + b^5*x^4)) + (atan(((((x*(169*a^8*d^8 + 9*b^8*c^8 - 164*a^2*b^6*c
^6*d^2 - 24*a^3*b^5*c^5*d^3 + 1110*a^4*b^4*c^4*d^4 - 2264*a^5*b^3*c^3*d^5 +
 2076*a^6*b^2*c^2*d^6 + 24*a*b^7*c^7*d - 936*a^7*b*c*d^7))/(4*a^2*b^5) - ((
a*d - b*c)^3*(13*a*d + 3*b*c)*(3*b^4*c^4 - 13*a^4*d^4 - 30*a^2*b^2*c^2*d^2
+ 4*a*b^3*c^3*d + 36*a^3*b*c*d^3))/(4*(-a)^(7/4)*b^(21/4)))*(a*d - b*c)^3*(
13*a*d + 3*b*c)*1i)/(16*(-a)^(7/4)*b^(17/4)) + (((x*(169*a^8*d^8 + 9*b^8*c^
8 - 164*a^2*b^6*c^6*d^2 - 24*a^3*b^5*c^5*d^3 + 1110*a^4*b^4*c^4*d^4 - 2264*
a^5*b^3*c^3*d^5 + 2076*a^6*b^2*c^2*d^6 + 24*a*b^7*c^7*d - 936*a^7*b*c*d^7))
/(4*a^2*b^5) + ((a*d - b*c)^3*(13*a*d + 3*b*c)*(3*b^4*c^4 - 13*a^4*d^4 - 30
*a^2*b^2*c^2*d^2 + 4*a*b^3*c^3*d + 36*a^3*b*c*d^3))/(4*(-a)^(7/4)*b^(21/4))
)*(a*d - b*c)^3*(13*a*d + 3*b*c)*1i)/(16*(-a)^(7/4)*b^(17/4)))/((((x*(169*a
^8*d^8 + 9*b^8*c^8 - 164*a^2*b^6*c^6*d^2 - 24*a^3*b^5*c^5*d^3 + 1110*a^4*b^
4*c^4*d^4 - 2264*a^5*b^3*c^3*d^5 + 2076*a^6*b^2*c^2*d^6 + 24*a*b^7*c^7*d -
936*a^7*b*c*d^7))/(4*a^2*b^5) - ((a*d - b*c)^3*(13*a*d + 3*b*c)*(3*b^4*c^4
- 13*a^4*d^4 - 30*a^2*b^2*c^2*d^2 + 4*a*b^3*c^3*d + 36*a^3*b*c*d^3))/(4*(-a
)^(7/4)*b^(21/4)))*(a*d - b*c)^3*(13*a*d + 3*b*c))/(16*(-a)^(7/4)*b^(17/4))
 - (((x*(169*a^8*d^8 + 9*b^8*c^8 - 164*a^2*b^6*c^6*d^2 - 24*a^3*b^5*c^5*d^3
```

```
 + 1110*a^4*b^4*c^4*d^4 - 2264*a^5*b^3*c^3*d^5 + 2076*a^6*b^2*c^2*d^6 + 24*
a*b^7*c^7*d - 936*a^7*b*c*d^7))/(4*a^2*b^5) + ((a*d - b*c)^3*(13*a*d + 3*b*
c)*(3*b^4*c^4 - 13*a^4*d^4 - 30*a^2*b^2*c^2*d^2 + 4*a*b^3*c^3*d + 36*a^3*b*
c*d^3))/(4*(-a)^(7/4)*b^(21/4)))*(a*d - b*c)^3*(13*a*d + 3*b*c))/(16*(-a)^(
7/4)*b^(17/4))))*(a*d - b*c)^3*(13*a*d + 3*b*c)*1i)/(8*(-a)^(7/4)*b^(17/4))
 + (atan(((((x*(169*a^8*d^8 + 9*b^8*c^8 - 164*a^2*b^6*c^6*d^2 - 24*a^3*b^5*
c^5*d^3 + 1110*a^4*b^4*c^4*d^4 - 2264*a^5*b^3*c^3*d^5 + 2076*a^6*b^2*c^2*d^
6 + 24*a*b^7*c^7*d - 936*a^7*b*c*d^7))/(4*a^2*b^5) - ((a*d - b*c)^3*(13*a*d
 + 3*b*c)*(3*b^4*c^4 - 13*a^4*d^4 - 30*a^2*b^2*c^2*d^2 + 4*a*b^3*c^3*d + 36
*a^3*b*c*d^3)*1i)/(4*(-a)^(7/4)*b^(21/4)))*(a*d - b*c)^3*(13*a*d + 3*b*c))/
(16*(-a)^(7/4)*b^(17/4)) + (((x*(169*a^8*d^8 + 9*b^8*c^8 - 164*a^2*b^6*c^6*
d^2 - 24*a^3*b^5*c^5*d^3 + 1110*a^4*b^4*c^4*d^4 - 2264*a^5*b^3*c^3*d^5 + 20
76*a^6*b^2*c^2*d^6 + 24*a*b^7*c^7*d - 936*a^7*b*c*d^7))/(4*a^2*b^5) + ((a*d
 - b*c)^3*(13*a*d + 3*b*c)*(3*b^4*c^4 - 13*a^4*d^4 - 30*a^2*b^2*c^2*d^2 + 4
*a*b^3*c^3*d + 36*a^3*b*c*d^3)*1i)/(4*(-a)^(7/4)*b^(21/4)))*(a*d - b*c)^3*(
13*a*d + 3*b*c))/(16*(-a)^(7/4)*b^(17/4)))/((((x*(169*a^8*d^8 + 9*b^8*c^8 -
 164*a^2*b^6*c^6*d^2 - 24*a^3*b^5*c^5*d^3 + 1110*a^4*b^4*c^4*d^4 - 2264*a^5
*b^3*c^3*d^5 + 2076*a^6*b^2*c^2*d^6 + 24*a*b^7*c^7*d - 936*a^7*b*c*d^7))/(4
*a^2*b^5) - ((a*d - b*c)^3*(13*a*d + 3*b*c)*(3*b^4*c^4 - 13*a^4*d^4 - 30*a^
2*b^2*c^2*d^2 + 4*a*b^3*c^3*d + 36*a^3*b*c*d^3)*1i)/(4*(-a)^(7/4)*b^(21/4))
)*(a*d - b*c)^3*(13*a*d + 3*b*c)*1i)/(16*(-a)^(7/4)*b^(17/4)) - (((x*(169*a
^8*d^8 + 9*b^8*c^8 - 164*a^2*b^6*c^6*d^2 - 24*a^3*b^5*c^5*d^3 + 1110*a^4*b^
4*c^4*d^4 - 2264*a^5*b^3*c^3*d^5 + 2076*a^6*b^2*c^2*d^6 + 24*a*b^7*c^7*d -
936*a^7*b*c*d^7))/(4*a^2*b^5) + ((a*d - b*c)^3*(13*a*d + 3*b*c)*(3*b^4*c^4
- 13*a^4*d^4 - 30*a^2*b^2*c^2*d^2 + 4*a*b^3*c^3*d + 36*a^3*b*c*d^3)*1i)/(4*
(-a)^(7/4)*b^(21/4)))*(a*d - b*c)^3*(13*a*d + 3*b*c)*1i)/(16*(-a)^(7/4)*b^(
17/4))))*(a*d - b*c)^3*(13*a*d + 3*b*c))/(8*(-a)^(7/4)*b^(17/4))
```

3.168 $\int \frac{(c+dx^4)^3}{(a+bx^4)^2} dx$

Optimal. Leaf size=317

$$\frac{d^2(3bc-2ad)x}{b^3}+\frac{d^3x^5}{5b^2}+\frac{(bc-ad)^3x}{4ab^3(a+bx^4)}-\frac{3(bc-ad)^2(bc+3ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}b^{13/4}}+\frac{3(bc-ad)^2(bc+3ad}{8\sqrt{2}}$$

```
[Out] d^2*(-2*a*d+3*b*c)*x/b^3+1/5*d^3*x^5/b^2+1/4*(-a*d+b*c)^3*x/a/b^3/(b*x^4+a)
+3/16*(-a*d+b*c)^2*(3*a*d+b*c)*arctan(-1+b^(1/4)*x*2^(1/2)/a^(1/4))/a^(7/4)
/b^(13/4)*2^(1/2)+3/16*(-a*d+b*c)^2*(3*a*d+b*c)*arctan(1+b^(1/4)*x*2^(1/2)/
a^(1/4))/a^(7/4)/b^(13/4)*2^(1/2)-3/32*(-a*d+b*c)^2*(3*a*d+b*c)*ln(-a^(1/4)
*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))/a^(7/4)/b^(13/4)*2^(1/2)+3/32*(-a*d
+b*c)^2*(3*a*d+b*c)*ln(a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))/a^(7/
4)/b^(13/4)*2^(1/2)
```

Rubi [A]

time = 0.21, antiderivative size = 317, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {398, 393, 217, 1179, 642, 1176, 631, 210}

$$-\frac{3\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)(bc-ad)^2(3ad+bc)}{8\sqrt{2}a^{7/4}b^{13/4}}+\frac{3\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)(bc-ad)^2(3ad+bc)}{8\sqrt{2}a^{7/4}b^{13/4}}-\frac{3(bc-ad)^2(3ad+bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{13/4}}+\frac{3(bc-ad)^2(3ad+bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{13/4}}+\frac{d^2x(3bc-2ad)}{b^3}+\frac{x(bc-ad)^3}{4ab^3(a+bx^4)}+\frac{d^3x^5}{5b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^4)^3/(a + b*x^4)^2,x]
```

```
[Out] (d^2*(3*b*c - 2*a*d)*x)/b^3 + (d^3*x^5)/(5*b^2) + ((b*c - a*d)^3*x)/(4*a*b^
3*(a + b*x^4)) - (3*(b*c - a*d)^2*(b*c + 3*a*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)
*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*b^(13/4)) + (3*(b*c - a*d)^2*(b*c + 3*a*d)
*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*b^(13/4)) - (3
*(b*c - a*d)^2*(b*c + 3*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt
[b]*x^2])/(16*Sqrt[2]*a^(7/4)*b^(13/4)) + (3*(b*c - a*d)^2*(b*c + 3*a*d)*Lo
g[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)*b
^(13/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
```

```
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{\left(c+dx^4\right)^3}{\left(a+bx^4\right)^2}\,dx = \int \left(\frac{d^2(3bc-2ad)}{b^3}+\frac{d^3x^4}{b^2}+\frac{(bc-ad)^2(bc+2ad)+3bd(bc-ad)^2x^4}{b^3\left(a+bx^4\right)^2}\right)dx$$

$$= \frac{d^2(3bc-2ad)x}{b^3}+\frac{d^3x^5}{5b^2}+\frac{\int \frac{(bc-ad)^2(bc+2ad)+3bd(bc-ad)^2x^4}{\left(a+bx^4\right)^2}\,dx}{b^3}$$

$$= \frac{d^2(3bc-2ad)x}{b^3}+\frac{d^3x^5}{5b^2}+\frac{(bc-ad)^3x}{4ab^3\left(a+bx^4\right)}+\frac{\left(3(bc-ad)^2(bc+3ad)\right)\int \frac{1}{a+bx^4}\,dx}{4ab^3}$$

$$= \frac{d^2(3bc-2ad)x}{b^3}+\frac{d^3x^5}{5b^2}+\frac{(bc-ad)^3x}{4ab^3\left(a+bx^4\right)}+\frac{\left(3(bc-ad)^2(bc+3ad)\right)\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx}{8a^{3/2}b^3}+\frac{(\ldots}{\ldots}$$

$$= \frac{d^2(3bc-2ad)x}{b^3}+\frac{d^3x^5}{5b^2}+\frac{(bc-ad)^3x}{4ab^3\left(a+bx^4\right)}+\frac{\left(3(bc-ad)^2(bc+3ad)\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\ldots}{16a^{3/2}b^{7/2}}$$

$$= \frac{d^2(3bc-2ad)x}{b^3}+\frac{d^3x^5}{5b^2}+\frac{(bc-ad)^3x}{4ab^3\left(a+bx^4\right)}-\frac{3(bc-ad)^2(bc+3ad)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{\ldots}\right.}{16\sqrt{2}\,a^{7/4}b^{13/4}}$$

$$= \frac{d^2(3bc-2ad)x}{b^3}+\frac{d^3x^5}{5b^2}+\frac{(bc-ad)^3x}{4ab^3\left(a+bx^4\right)}-\frac{3(bc-ad)^2(bc+3ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{8\sqrt{2}\,a^{7/4}b^{13/4}}\ldots$$

Mathematica [A]
time = 0.15, size = 301, normalized size = 0.95

$$\frac{160\sqrt[4]{b}\,d^2(3bc-2ad)x+32b^{5/4}d^3x^5+\frac{40\sqrt[4]{b}\,(bc-ad)^3x}{a(a+bx^4)}-\frac{30\sqrt{2}\,(bc-ad)^2(bc+3ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{a^{7/4}}+\frac{30\sqrt{2}\,(bc-ad)^2(bc+3ad)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{a^{7/4}}-\frac{15\sqrt{2}\,(bc-ad)^2(bc+3ad)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{b}\,x^2\right)}{a^{7/4}}+\frac{15\sqrt{2}\,(bc-ad)^2(bc+3ad)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{b}\,x^2\right)}{a^{7/4}}}{160b^{13/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^4)^3/(a + b*x^4)^2,x]
```

```
[Out] (160*b^(1/4)*d^2*(3*b*c - 2*a*d)*x + 32*b^(5/4)*d^3*x^5 + (40*b^(1/4)*(b*c
 - a*d)^3*x)/(a*(a + b*x^4)) - (30*Sqrt[2]*(b*c - a*d)^2*(b*c + 3*a*d)*ArcTa
n[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/a^(7/4) + (30*Sqrt[2]*(b*c - a*d)^2*(b*
c + 3*a*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/a^(7/4) - (15*Sqrt[2]*(
b*c - a*d)^2*(b*c + 3*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b
]*x^2])/a^(7/4) + (15*Sqrt[2]*(b*c - a*d)^2*(b*c + 3*a*d)*Log[Sqrt[a] + Sqr
t[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/a^(7/4))/(160*b^(13/4))
```

Maple [A]
time = 0.27, size = 220, normalized size = 0.69

method	result
risch	$\frac{d^3x^5}{5b^2}-\frac{2d^3ax}{b^3}+\frac{3d^2cx}{b^2}-\frac{(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3)x}{4a\,b^3(b\,x^4+a)}+\frac{3\left(\sum_{\_R=\mathrm{RootOf}(\_Z^4b+a)}\frac{(3a^3d^3-5a^2bc\,d^2+a\,b^2c^2d+b^3c^3)\ln(x}{\_R^3}\right.}{16b^4a}$
default	$-\frac{d^2(-\frac{1}{5}bd\,x^5+2adx-3bcx)}{b^3}+\frac{-\frac{(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3)x}{4a(b\,x^4+a)}+\frac{3(3a^3d^3-5a^2bc\,d^2+a\,b^2c^2d+b^3c^3)(\frac{a}{b})^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+(\frac{a}{b})^{\frac{1}{4}}x\sqrt{}}{x^2-(\frac{a}{b})^{\frac{1}{4}}x\sqrt{}}\right.\right.}{32a}}{b^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^4+c)^3/(b*x^4+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -d^2/b^3*(-1/5*b*d*x^5+2*a*d*x-3*b*c*x)+1/b^3*(-1/4*(a^3*d^3-3*a^2*b*c*d^2+
3*a*b^2*c^2*d-b^3*c^3)/a*x/(b*x^4+a)+3/32*(3*a^3*d^3-5*a^2*b*c*d^2+a*b^2*c^
2*d+b^3*c^3)/a^2*(a/b)^(1/4)*2^(1/2)*(ln((x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(
1/2))/(x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)
*x+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x-1)))
```

Maxima [A]
time = 0.51, size = 405, normalized size = 1.28

$$\frac{(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)x}{4\,(ab^4x^4+a^2b^3)}+\frac{bd^3x^5+5\,(3\,bcd^2-2\,ad^3)x}{5\,b^3}+\frac{3\left(\frac{2\sqrt{2}\,(b^3c^3+ab^2c^2d-5\,a^2bcd^2+3\,a^3d^3)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}\,x+\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{2\sqrt{2}\,(b^3c^3+ab^2c^2d-5\,a^2bcd^2+3\,a^3d^3)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}\,x-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{\sqrt{2}\,(b^3c^3+ab^2c^2d-5\,a^2bcd^2+3\,a^3d^3)\log\left(\sqrt{b}\,x^2+\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}-\frac{\sqrt{2}\,(b^3c^3+ab^2c^2d-5\,a^2bcd^2+3\,a^3d^3)\log\left(\sqrt{b}\,x^2-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}\right)}{32\,ab^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^3/(b*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*x/(a*b^4*x^4 + a^2*
b^3) + 1/5*(b*d^3*x^5 + 5*(3*b*c*d^2 - 2*a*d^3)*x)/b^3 + 3/32*(2*sqrt(2)*(b
^3*c^3 + a*b^2*c^2*d - 5*a^2*b*c*d^2 + 3*a^3*d^3)*arctan(1/2*sqrt(2)*(2*sqr
t(b)*x + sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt
(a)*sqrt(b))) + 2*sqrt(2)*(b^3*c^3 + a*b^2*c^2*d - 5*a^2*b*c*d^2 + 3*a^3*d^
3)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*
sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*(b^3*c^3 + a*b^2*c^2*d
- 5*a^2*b*c*d^2 + 3*a^3*d^3)*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1/4)*x +
sqrt(a))/(a^(3/4)*b^(1/4)) - sqrt(2)*(b^3*c^3 + a*b^2*c^2*d - 5*a^2*b*c*d^2
 + 3*a^3*d^3)*log(sqrt(b)*x^2 - sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/
4)*b^(1/4)))/(a*b^3)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1937 vs. 2(248) = 496.
time = 2.66, size = 1937, normalized size = 6.11

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^3/(b*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] 1/80*(16*a*b^2*d^3*x^9 + 48*(5*a*b^2*c*d^2 - 3*a^2*b*d^3)*x^5 + 60*(a*b^4*x
^4 + a^2*b^3)*(-(b^12*c^12 + 4*a*b^11*c^11*d - 14*a^2*b^10*c^10*d^2 - 44*a^
3*b^9*c^9*d^3 + 127*a^4*b^8*c^8*d^4 + 136*a^5*b^7*c^7*d^5 - 644*a^6*b^6*c^6
*d^6 + 328*a^7*b^5*c^5*d^7 + 1039*a^8*b^4*c^4*d^8 - 1932*a^9*b^3*c^3*d^9 +
1458*a^10*b^2*c^2*d^10 - 540*a^11*b*c*d^11 + 81*a^12*d^12)/(a^7*b^13))^(1/4
)*arctan((sqrt(a^4*b^6*sqrt(-(b^12*c^12 + 4*a*b^11*c^11*d - 14*a^2*b^10*c^1
0*d^2 - 44*a^3*b^9*c^9*d^3 + 127*a^4*b^8*c^8*d^4 + 136*a^5*b^7*c^7*d^5 - 64
4*a^6*b^6*c^6*d^6 + 328*a^7*b^5*c^5*d^7 + 1039*a^8*b^4*c^4*d^8 - 1932*a^9*b
^3*c^3*d^9 + 1458*a^10*b^2*c^2*d^10 - 540*a^11*b*c*d^11 + 81*a^12*d^12)/(a^
7*b^13)) + (b^6*c^6 + 2*a*b^5*c^5*d - 9*a^2*b^4*c^4*d^2 - 4*a^3*b^3*c^3*d^3
 + 31*a^4*b^2*c^2*d^4 - 30*a^5*b*c*d^5 + 9*a^6*d^6)*x^2)*a^5*b^10*(-(b^12*c
^12 + 4*a*b^11*c^11*d - 14*a^2*b^10*c^10*d^2 - 44*a^3*b^9*c^9*d^3 + 127*a^4
*b^8*c^8*d^4 + 136*a^5*b^7*c^7*d^5 - 644*a^6*b^6*c^6*d^6 + 328*a^7*b^5*c^5*
d^7 + 1039*a^8*b^4*c^4*d^8 - 1932*a^9*b^3*c^3*d^9 + 1458*a^10*b^2*c^2*d^10
- 540*a^11*b*c*d^11 + 81*a^12*d^12)/(a^7*b^13))^(3/4) - (a^5*b^13*c^3 + a^6
*b^12*c^2*d - 5*a^7*b^11*c*d^2 + 3*a^8*b^10*d^3)*x*(-(b^12*c^12 + 4*a*b^11*
c^11*d - 14*a^2*b^10*c^10*d^2 - 44*a^3*b^9*c^9*d^3 + 127*a^4*b^8*c^8*d^4 +
136*a^5*b^7*c^7*d^5 - 644*a^6*b^6*c^6*d^6 + 328*a^7*b^5*c^5*d^7 + 1039*a^8*
b^4*c^4*d^8 - 1932*a^9*b^3*c^3*d^9 + 1458*a^10*b^2*c^2*d^10 - 540*a^11*b*c*
d^11 + 81*a^12*d^12)/(a^7*b^13))^(3/4))/(b^12*c^12 + 4*a*b^11*c^11*d - 14*a
^2*b^10*c^10*d^2 - 44*a^3*b^9*c^9*d^3 + 127*a^4*b^8*c^8*d^4 + 136*a^5*b^7*c
^7*d^5 - 644*a^6*b^6*c^6*d^6 + 328*a^7*b^5*c^5*d^7 + 1039*a^8*b^4*c^4*d^8 -
 1932*a^9*b^3*c^3*d^9 + 1458*a^10*b^2*c^2*d^10 - 540*a^11*b*c*d^11 + 81*a^1
2*d^12)) + 15*(a*b^4*x^4 + a^2*b^3)*(-(b^12*c^12 + 4*a*b^11*c^11*d - 14*a^2
*b^10*c^10*d^2 - 44*a^3*b^9*c^9*d^3 + 127*a^4*b^8*c^8*d^4 + 136*a^5*b^7*c^7
*d^5 - 644*a^6*b^6*c^6*d^6 + 328*a^7*b^5*c^5*d^7 + 1039*a^8*b^4*c^4*d^8 - 1
932*a^9*b^3*c^3*d^9 + 1458*a^10*b^2*c^2*d^10 - 540*a^11*b*c*d^11 + 81*a^12*
d^12)/(a^7*b^13))^(1/4)*log(3*a^2*b^3*(-(b^12*c^12 + 4*a*b^11*c^11*d - 14*a
^2*b^10*c^10*d^2 - 44*a^3*b^9*c^9*d^3 + 127*a^4*b^8*c^8*d^4 + 136*a^5*b^7*c
^7*d^5 - 644*a^6*b^6*c^6*d^6 + 328*a^7*b^5*c^5*d^7 + 1039*a^8*b^4*c^4*d^8 -
 1932*a^9*b^3*c^3*d^9 + 1458*a^10*b^2*c^2*d^10 - 540*a^11*b*c*d^11 + 81*a^1
2*d^12)/(a^7*b^13))^(1/4) + 3*(b^3*c^3 + a*b^2*c^2*d - 5*a^2*b*c*d^2 + 3*a^
3*d^3)*x) - 15*(a*b^4*x^4 + a^2*b^3)*(-(b^12*c^12 + 4*a*b^11*c^11*d - 14*a^
2*b^10*c^10*d^2 - 44*a^3*b^9*c^9*d^3 + 127*a^4*b^8*c^8*d^4 + 136*a^5*b^7*c^
7*d^5 - 644*a^6*b^6*c^6*d^6 + 328*a^7*b^5*c^5*d^7 + 1039*a^8*b^4*c^4*d^8 -
1932*a^9*b^3*c^3*d^9 + 1458*a^10*b^2*c^2*d^10 - 540*a^11*b*c*d^11 + 81*a^12
*d^12)/(a^7*b^13))^(1/4)*log(-3*a^2*b^3*(-(b^12*c^12 + 4*a*b^11*c^11*d - 14
*a^2*b^10*c^10*d^2 - 44*a^3*b^9*c^9*d^3 + 127*a^4*b^8*c^8*d^4 + 136*a^5*b^7
*c^7*d^5 - 644*a^6*b^6*c^6*d^6 + 328*a^7*b^5*c^5*d^7 + 1039*a^8*b^4*c^4*d^8
 - 1932*a^9*b^3*c^3*d^9 + 1458*a^10*b^2*c^2*d^10 - 540*a^11*b*c*d^11 + 81*a
^12*d^12)/(a^7*b^13))^(1/4) + 3*(b^3*c^3 + a*b^2*c^2*d - 5*a^2*b*c*d^2 + 3*
```

```
a^3*d^3)*x) + 20*(b^3*c^3 - 3*a*b^2*c^2*d + 15*a^2*b*c*d^2 - 9*a^3*d^3)*x)/
(a*b^4*x^4 + a^2*b^3)
```

Sympy [A]
time = 82.86, size = 337, normalized size = 1.06

$$x\left(-\frac{2ad^3}{b^3}+\frac{3cd^2}{b^2}\right)+\frac{x(-a^3d^3+3a^2bcd^2-3ab^2c^2d+b^3c^3)}{4a^2b^3+4ab^4x^4}+\operatorname{RootSum}\left(65536t^4a^7b^{13}+6561a^{12}d^{12}-43740a^{11}bcd^{11}+118098a^{10}b^2c^2d^{10}-156492a^9b^3c^3d^9+84159a^8b^4c^4d^8+26568a^7b^5c^5d^7-52164a^6b^6c^6d^6+11016a^5b^7c^7d^5+10287a^4b^8c^8d^4-3564a^3b^9c^9d^3-1134a^2b^{10}c^{10}d^2+324ab^{11}c^{11}d+81b^{12}c^{12},\left(t\mapsto t\log\left(\frac{16ta^2b^3}{9a^3d^3-15a^2bcd^2+3ab^2c^2d+3b^3c^3}+x\right)\right)\right)+\frac{d^3x^5}{5b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**4+c)**3/(b*x**4+a)**2,x)
```

```
[Out] x*(-2*a*d**3/b**3 + 3*c*d**2/b**2) + x*(-a**3*d**3 + 3*a**2*b*c*d**2 - 3*a*
b**2*c**2*d + b**3*c**3)/(4*a**2*b**3 + 4*a*b**4*x**4) + RootSum(65536*_t**
4*a**7*b**13 + 6561*a**12*d**12 - 43740*a**11*b*c*d**11 + 118098*a**10*b**2
*c**2*d**10 - 156492*a**9*b**3*c**3*d**9 + 84159*a**8*b**4*c**4*d**8 + 2656
8*a**7*b**5*c**5*d**7 - 52164*a**6*b**6*c**6*d**6 + 11016*a**5*b**7*c**7*d*
*5 + 10287*a**4*b**8*c**8*d**4 - 3564*a**3*b**9*c**9*d**3 - 1134*a**2*b**10
*c**10*d**2 + 324*a*b**11*c**11*d + 81*b**12*c**12, Lambda(_t, _t*log(16*_t
*a**2*b**3/(9*a**3*d**3 - 15*a**2*b*c*d**2 + 3*a*b**2*c**2*d + 3*b**3*c**3)
 + x))) + d**3*x**5/(5*b**2)
```

Giac [A]
time = 0.64, size = 496, normalized size = 1.56

$$\frac{3\sqrt{2}\left((ab^3)^{\frac{1}{4}}b^3c^3+(ab^3)^{\frac{1}{4}}ab^2c^2d-5(ab^3)^{\frac{1}{4}}a^2bcd^2+3(ab^3)^{\frac{1}{4}}a^3d^3\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16a^2b^4}+\frac{3\sqrt{2}\left((ab^3)^{\frac{1}{4}}b^3c^3+(ab^3)^{\frac{1}{4}}ab^2c^2d-5(ab^3)^{\frac{1}{4}}a^2bcd^2+3(ab^3)^{\frac{1}{4}}a^3d^3\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16a^2b^4}+\frac{3\sqrt{2}\left((ab^3)^{\frac{1}{4}}b^3c^3+(ab^3)^{\frac{1}{4}}ab^2c^2d-5(ab^3)^{\frac{1}{4}}a^2bcd^2+3(ab^3)^{\frac{1}{4}}a^3d^3\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{32a^2b^4}-\frac{3\sqrt{2}\left((ab^3)^{\frac{1}{4}}b^3c^3+(ab^3)^{\frac{1}{4}}ab^2c^2d-5(ab^3)^{\frac{1}{4}}a^2bcd^2+3(ab^3)^{\frac{1}{4}}a^3d^3\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{32a^2b^4}+\frac{b^3c^3x-3ab^2c^2dx+3a^2bcd^2x-a^3d^3x}{4(bx^4+a)ab^3}+\frac{b^8d^3x^5+15b^8cd^2x-10ab^7d^3x}{5b^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^3/(b*x^4+a)^2,x, algorithm="giac")
```

```
[Out] 3/16*sqrt(2)*((a*b^3)^(1/4)*b^3*c^3 + (a*b^3)^(1/4)*a*b^2*c^2*d - 5*(a*b^3)
^(1/4)*a^2*b*c*d^2 + 3*(a*b^3)^(1/4)*a^3*d^3)*arctan(1/2*sqrt(2)*(2*x + sqr
t(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^2*b^4) + 3/16*sqrt(2)*((a*b^3)^(1/4)*b^3*
c^3 + (a*b^3)^(1/4)*a*b^2*c^2*d - 5*(a*b^3)^(1/4)*a^2*b*c*d^2 + 3*(a*b^3)^(
1/4)*a^3*d^3)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(
a^2*b^4) + 3/32*sqrt(2)*((a*b^3)^(1/4)*b^3*c^3 + (a*b^3)^(1/4)*a*b^2*c^2*d
 - 5*(a*b^3)^(1/4)*a^2*b*c*d^2 + 3*(a*b^3)^(1/4)*a^3*d^3)*log(x^2 + sqrt(2)*
x*(a/b)^(1/4) + sqrt(a/b))/(a^2*b^4) - 3/32*sqrt(2)*((a*b^3)^(1/4)*b^3*c^3
 + (a*b^3)^(1/4)*a*b^2*c^2*d - 5*(a*b^3)^(1/4)*a^2*b*c*d^2 + 3*(a*b^3)^(1/4)
*a^3*d^3)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^2*b^4) + 1/4*(b^3
*c^3*x - 3*a*b^2*c^2*d*x + 3*a^2*b*c*d^2*x - a^3*d^3*x)/((b*x^4 + a)*a*b^3)
 + 1/5*(b^8*d^3*x^5 + 15*b^8*c*d^2*x - 10*a*b^7*d^3*x)/b^10
```

Mupad [B]
time = 1.53, size = 1616, normalized size = 5.10

$$\frac{d^3x^5}{5b^2}-x\left(\frac{2ad^3}{b^3}-\frac{3cd^2}{b^2}\right)-\frac{x\left(a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3\right)}{4a\left(b^4x^4+ab^3\right)}+\ldots$$

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c + d*x^4)^3/(a + b*x^4)^2,x)

[Out] (d^3*x^5)/(5*b^2) - x*((2*a*d^3)/b^3 - (3*c*d^2)/b^2) - (x*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))/(4*a*(a*b^3 + b^4*x^4)) + (atan((((a*d - b*c)^2*(3*a*d + b*c)*((9*x*(9*a^6*d^6 + b^6*c^6 - 9*a^2*b^4*c^4*d^2 - 4*a^3*b^3*c^3*d^3 + 31*a^4*b^2*c^2*d^4 + 2*a*b^5*c^5*d - 30*a^5*b*c*d^5))/(4*a^2*b^3) - (3*(a*d - b*c)^2*(3*a*d + b*c)*(36*a^3*d^3 + 12*b^3*c^3 + 12*a*b^2*c^2*d - 60*a^2*b*c*d^2))/(16*(-a)^(7/4)*b^(13/4)))*3i)/(16*(-a)^(7/4)*b^(13/4)) + ((a*d - b*c)^2*(3*a*d + b*c)*((9*x*(9*a^6*d^6 + b^6*c^6 - 9*a^2*b^4*c^4*d^2 - 4*a^3*b^3*c^3*d^3 + 31*a^4*b^2*c^2*d^4 + 2*a*b^5*c^5*d - 30*a^5*b*c*d^5))/(4*a^2*b^3) + (3*(a*d - b*c)^2*(3*a*d + b*c)*(36*a^3*d^3 + 12*b^3*c^3 + 12*a*b^2*c^2*d - 60*a^2*b*c*d^2))/(16*(-a)^(7/4)*b^(13/4)))*3i)/(16*(-a)^(7/4)*b^(13/4)))/((3*(a*d - b*c)^2*(3*a*d + b*c)*((9*x*(9*a^6*d^6 + b^6*c^6 - 9*a^2*b^4*c^4*d^2 - 4*a^3*b^3*c^3*d^3 + 31*a^4*b^2*c^2*d^4 + 2*a*b^5*c^5*d - 30*a^5*b*c*d^5))/(4*a^2*b^3) - (3*(a*d - b*c)^2*(3*a*d + b*c)*(36*a^3*d^3 + 12*b^3*c^3 + 12*a*b^2*c^2*d - 60*a^2*b*c*d^2))/(16*(-a)^(7/4)*b^(13/4))))/(16*(-a)^(7/4)*b^(13/4)) - (3*(a*d - b*c)^2*(3*a*d + b*c)*((9*x*(9*a^6*d^6 + b^6*c^6 - 9*a^2*b^4*c^4*d^2 - 4*a^3*b^3*c^3*d^3 + 31*a^4*b^2*c^2*d^4 + 2*a*b^5*c^5*d - 30*a^5*b*c*d^5))/(4*a^2*b^3) + (3*(a*d - b*c)^2*(3*a*d + b*c)*(36*a^3*d^3 + 12*b^3*c^3 + 12*a*b^2*c^2*d - 60*a^2*b*c*d^2))/(16*(-a)^(7/4)*b^(13/4))))/(16*(-a)^(7/4)*b^(13/4))))*(a*d - b*c)^2*(3*a*d + b*c)*3i)/(8*(-a)^(7/4)*b^(13/4)) + (3*atan(((3*(a*d - b*c)^2*(3*a*d + b*c)*((9*x*(9*a^6*d^6 + b^6*c^6 - 9*a^2*b^4*c^4*d^2 - 4*a^3*b^3*c^3*d^3 + 31*a^4*b^2*c^2*d^4 + 2*a*b^5*c^5*d - 30*a^5*b*c*d^5))/(4*a^2*b^3) - ((a*d - b*c)^2*(3*a*d + b*c)*(36*a^3*d^3 + 12*b^3*c^3 + 12*a*b^2*c^2*d - 60*a^2*b*c*d^2)*3i)/(16*(-a)^(7/4)*b^(13/4))))/(16*(-a)^(7/4)*b^(13/4)) + (3*(a*d - b*c)^2*(3*a*d + b*c)*((9*x*(9*a^6*d^6 + b^6*c^6 - 9*a^2*b^4*c^4*d^2 - 4*a^3*b^3*c^3*d^3 + 31*a^4*b^2*c^2*d^4 + 2*a*b^5*c^5*d - 30*a^5*b*c*d^5))/(4*a^2*b^3) + ((a*d - b*c)^2*(3*a*d + b*c)*(36*a^3*d^3 + 12*b^3*c^3 + 12*a*b^2*c^2*d - 60*a^2*b*c*d^2)*3i)/(16*(-a)^(7/4)*b^(13/4))))/(16*(-a)^(7/4)*b^(13/4)))/(((a*d - b*c)^2*(3*a*d + b*c)*((9*x*(9*a^6*d^6 + b^6*c^6 - 9*a^2*b^4*c^4*d^2 - 4*a^3*b^3*c^3*d^3 + 31*a^4*b^2*c^2*d^4 + 2*a*b^5*c^5*d - 30*a^5*b*c*d^5))/(4*a^2*b^3) - ((a*d - b*c)^2*(3*a*d + b*c)*(36*a^3*d^3 + 12*b^3*c^3 + 12*a*b^2*c^2*d - 60*a^2*b*c*d^2)*3i)/(16*(-a)^(7/4)*b^(13/4)))*3i)/(16*(-a)^(7/4)*b^(13/4)) - ((a*d - b*c)^2*(3*a*d + b*c)*((9*x*(9*a^6*d^6 + b^6*c^6 - 9*a^2*b^4*c^4*d^2 - 4*a^3*b^3*c^3*d^3 + 31*a^4*b^2*c^2*d^4 + 2*a*b^5*c^5*d - 30*a^5*b*c*d^5))/(4*a^2*b^3) + ((a*d - b*c)^2*(3*a*d + b*c)*(36*a^3*d^3 + 12*b^3*c^3 + 12*a*b^2*c^2*d - 60*a^2*b*c*d^2)*3i)/(16*(-a)^(7/4)*b^(13/4)))*3i)/(16*(-a)^(7/4)*b^(13/4))))*(a*d - b*c)^2*(3*a*d + b*c))/(8*(-a)^(7/4)*b^(13/4))

3.169 $\int \frac{(c+dx^4)^2}{(a+bx^4)^2}\,dx$

Optimal. Leaf size=291

$$\frac{d^2x}{b^2}+\frac{(bc-ad)^2x}{4ab^2\,(a+bx^4)}-\frac{(bc-ad)(3bc+5ad)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{8\sqrt{2}\,a^{7/4}b^{9/4}}+\frac{(bc-ad)(3bc+5ad)\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{}}{\sqrt[4]{}}\right.}{8\sqrt{2}\,a^{7/4}b^{9/4}}$$

```
[Out] d^2*x/b^2+1/4*(-a*d+b*c)^2*x/a/b^2/(b*x^4+a)+1/16*(-a*d+b*c)*(5*a*d+3*b*c)*
arctan(-1+b^(1/4)*x*2^(1/2)/a^(1/4))/a^(7/4)/b^(9/4)*2^(1/2)+1/16*(-a*d+b*c
)*(5*a*d+3*b*c)*arctan(1+b^(1/4)*x*2^(1/2)/a^(1/4))/a^(7/4)/b^(9/4)*2^(1/2)
-1/32*(-a*d+b*c)*(5*a*d+3*b*c)*ln(-a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^
(1/2))/a^(7/4)/b^(9/4)*2^(1/2)+1/32*(-a*d+b*c)*(5*a*d+3*b*c)*ln(a^(1/4)*b^(
1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))/a^(7/4)/b^(9/4)*2^(1/2)
```

Rubi [A]
time = 0.25, antiderivative size = 291, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$,
Rules used = {398, 393, 217, 1179, 642, 1176, 631, 210}

$$-\frac{\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)(bc-ad)(5ad+3bc)}{8\sqrt{2}\,a^{7/4}b^{9/4}}+\frac{\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a}}+1\right)(bc-ad)(5ad+3bc)}{8\sqrt{2}\,a^{7/4}b^{9/4}}-\frac{(bc-ad)(5ad+3bc)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{16\sqrt{2}\,a^{7/4}b^{9/4}}+\frac{(bc-ad)(5ad+3bc)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{16\sqrt{2}\,a^{7/4}b^{9/4}}+\frac{x(bc-ad)^2}{4ab^2\,(a+bx^4)}+\frac{d^2x}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^4)^2/(a + b*x^4)^2,x]
```

```
[Out] (d^2*x)/b^2 + ((b*c - a*d)^2*x)/(4*a*b^2*(a + b*x^4)) - ((b*c - a*d)*(3*b*c
 + 5*a*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*b^(9/
4)) + ((b*c - a*d)*(3*b*c + 5*a*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])
/(8*Sqrt[2]*a^(7/4)*b^(9/4)) - ((b*c - a*d)*(3*b*c + 5*a*d)*Log[Sqrt[a] - S
qrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)*b^(9/4)) + ((b
*c - a*d)*(3*b*c + 5*a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]
*x^2])/(16*Sqrt[2]*a^(7/4)*b^(9/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
```

```
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(c+dx^4)^2}{(a+bx^4)^2}\,dx &= \int \left(\frac{d^2}{b^2}+\frac{b^2c^2-a^2d^2+2bd(bc-ad)x^4}{b^2(a+bx^4)^2}\right)dx \\
&= \frac{d^2x}{b^2}+\frac{\int \frac{b^2c^2-a^2d^2+2bd(bc-ad)x^4}{(a+bx^4)^2}\,dx}{b^2} \\
&= \frac{d^2x}{b^2}+\frac{(bc-ad)^2x}{4ab^2(a+bx^4)}+\frac{((bc-ad)(3bc+5ad))\int \frac{1}{a+bx^4}\,dx}{4ab^2} \\
&= \frac{d^2x}{b^2}+\frac{(bc-ad)^2x}{4ab^2(a+bx^4)}+\frac{((bc-ad)(3bc+5ad))\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx}{8a^{3/2}b^2}+\frac{((bc-ad)(3bc+5ad)}{8a^{3/}} \\
&= \frac{d^2x}{b^2}+\frac{(bc-ad)^2x}{4ab^2(a+bx^4)}+\frac{((bc-ad)(3bc+5ad))\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx}{16a^{3/2}b^{5/2}}+\frac{((bc-ad)(3b}{} \\
&= \frac{d^2x}{b^2}+\frac{(bc-ad)^2x}{4ab^2(a+bx^4)}-\frac{(bc-ad)(3bc+5ad)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{b}\,x^2\right)}{16\sqrt{2}\,a^{7/4}b^{9/4}}+\frac{(b}{} \\
&= \frac{d^2x}{b^2}+\frac{(bc-ad)^2x}{4ab^2(a+bx^4)}-\frac{(bc-ad)(3bc+5ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{8\sqrt{2}\,a^{7/4}b^{9/4}}+\frac{(bc-ad)(3bc+}{}
\end{aligned}
$$

Mathematica [A]
time = 0.13, size = 297, normalized size = 1.02

$$\frac{32\sqrt[4]{b}\,d^2x+\frac{8\sqrt[4]{b}\,(bc-ad)^2x}{a(a+bx^4)}+\frac{2\sqrt{2}\,(-3b^2c^2-2abcd+5a^2d^2)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{a^{7/4}}+\frac{2\sqrt{2}\,(3b^2c^2+2abcd-5a^2d^2)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{a^{7/4}}+\frac{\sqrt{2}\,(-3b^2c^2-2abcd+5a^2d^2)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{b}\,x^2\right)}{a^{7/4}}+\frac{\sqrt{2}\,(3b^2c^2+2abcd-5a^2d^2)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{b}\,x^2\right)}{a^{7/4}}}{32b^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^4)^2/(a + b*x^4)^2,x]
```

```
[Out] (32*b^(1/4)*d^2*x + (8*b^(1/4)*(b*c - a*d)^2*x)/(a*(a + b*x^4)) + (2*Sqrt[2
]*(-3*b^2*c^2 - 2*a*b*c*d + 5*a^2*d^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/
4)])/a^(7/4) + (2*Sqrt[2]*(3*b^2*c^2 + 2*a*b*c*d - 5*a^2*d^2)*ArcTan[1 + (S
qrt[2]*b^(1/4)*x)/a^(1/4)])/a^(7/4) + (Sqrt[2]*(-3*b^2*c^2 - 2*a*b*c*d + 5*
a^2*d^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/a^(7/4) +
(Sqrt[2]*(3*b^2*c^2 + 2*a*b*c*d - 5*a^2*d^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*
b^(1/4)*x + Sqrt[b]*x^2])/a^(7/4))/(32*b^(9/4))
```

Maple [A]
time = 0.26, size = 176, normalized size = 0.60

method	result
risch	$\frac{d^2x}{b^2}+\frac{(a^2d^2-2abcd+b^2c^2)x}{4a\,b^2(b\,x^4+a)}-\frac{\sum_{\_R=\mathrm{RootOf}(\_Z^4b+a)}\frac{(5a^2d^2-2abcd-3b^2c^2)\ln(x-\_R)}{\_R^3}}{16b^3a}$
default	$\frac{d^2x}{b^2}-\frac{-\frac{(a^2d^2-2abcd+b^2c^2)x}{4a(b\,x^4+a)}+\frac{(5a^2d^2-2abcd-3b^2c^2)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{32a^2}}{b^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^4+c)^2/(b*x^4+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] d^2*x/b^2-1/b^2*(-1/4*(a^2*d^2-2*a*b*c*d+b^2*c^2)/a*x/(b*x^4+a)+1/32*(5*a^2
*d^2-2*a*b*c*d-3*b^2*c^2)/a^2*(a/b)^(1/4)*2^(1/2)*(ln((x^2+(a/b)^(1/4)*x*2^
(1/2)+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2
)/(a/b)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x-1)))
```

Maxima [A]
time = 0.50, size = 319, normalized size = 1.10

$$\frac{(b^2c^2-2\,abcd+a^2d^2)x}{4\,(ab^3x^4+a^2b^2)}+\frac{d^2x}{b^2}+\frac{\frac{2\sqrt{2}\,(3b^2c^2+2abcd-5a^2d^2)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}\,x+\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{2\sqrt{2}\,(3b^2c^2+2abcd-5a^2d^2)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}\,x-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{\sqrt{2}\,(3b^2c^2+2abcd-5a^2d^2)\log\left(\sqrt{b}\,x^2+\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}-\frac{\sqrt{2}\,(3b^2c^2+2abcd-5a^2d^2)\log\left(\sqrt{b}\,x^2-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}}{32\,ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^2/(b*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x/(a*b^3*x^4 + a^2*b^2) + d^2*x/b^2 + 1
/32*(2*sqrt(2)*(3*b^2*c^2 + 2*a*b*c*d - 5*a^2*d^2)*arctan(1/2*sqrt(2)*(2*sq
rt(b)*x + sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqr
t(a)*sqrt(b))) + 2*sqrt(2)*(3*b^2*c^2 + 2*a*b*c*d - 5*a^2*d^2)*arctan(1/2*s
qrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(sqrt
(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*(3*b^2*c^2 + 2*a*b*c*d - 5*a^2*d^2)*lo
g(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(1/4)) - sq
rt(2)*(3*b^2*c^2 + 2*a*b*c*d - 5*a^2*d^2)*log(sqrt(b)*x^2 - sqrt(2)*a^(1/4)
*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(1/4)))/(a*b^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1335 vs. $2(224) = 448$.
time = 4.08, size = 1335, normalized size = 4.59

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^2/(b*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] 1/16*(16*a*b*d^2*x^5 - 4*(a*b^3*x^4 + a^2*b^2)*(-(81*b^8*c^8 + 216*a*b^7*c^
7*d - 324*a^2*b^6*c^6*d^2 - 984*a^3*b^5*c^5*d^3 + 646*a^4*b^4*c^4*d^4 + 164
0*a^5*b^3*c^3*d^5 - 900*a^6*b^2*c^2*d^6 - 1000*a^7*b*c*d^7 + 625*a^8*d^8)/(
a^7*b^9))^(1/4)*arctan((sqrt(a^4*b^4*sqrt(-(81*b^8*c^8 + 216*a*b^7*c^7*d -
324*a^2*b^6*c^6*d^2 - 984*a^3*b^5*c^5*d^3 + 646*a^4*b^4*c^4*d^4 + 1640*a^5*
b^3*c^3*d^5 - 900*a^6*b^2*c^2*d^6 - 1000*a^7*b*c*d^7 + 625*a^8*d^8)/(a^7*b^
9)) + (9*b^4*c^4 + 12*a*b^3*c^3*d - 26*a^2*b^2*c^2*d^2 - 20*a^3*b*c*d^3 + 2
5*a^4*d^4)*x^2)*a^5*b^7*(-(81*b^8*c^8 + 216*a*b^7*c^7*d - 324*a^2*b^6*c^6*d
^2 - 984*a^3*b^5*c^5*d^3 + 646*a^4*b^4*c^4*d^4 + 1640*a^5*b^3*c^3*d^5 - 900
*a^6*b^2*c^2*d^6 - 1000*a^7*b*c*d^7 + 625*a^8*d^8)/(a^7*b^9))^(3/4) + (3*a^
5*b^9*c^2 + 2*a^6*b^8*c*d - 5*a^7*b^7*d^2)*x*(-(81*b^8*c^8 + 216*a*b^7*c^7*
d - 324*a^2*b^6*c^6*d^2 - 984*a^3*b^5*c^5*d^3 + 646*a^4*b^4*c^4*d^4 + 1640*
a^5*b^3*c^3*d^5 - 900*a^6*b^2*c^2*d^6 - 1000*a^7*b*c*d^7 + 625*a^8*d^8)/(a^
7*b^9))^(3/4))/(81*b^8*c^8 + 216*a*b^7*c^7*d - 324*a^2*b^6*c^6*d^2 - 984*a^
3*b^5*c^5*d^3 + 646*a^4*b^4*c^4*d^4 + 1640*a^5*b^3*c^3*d^5 - 900*a^6*b^2*c^
2*d^6 - 1000*a^7*b*c*d^7 + 625*a^8*d^8)) - (a*b^3*x^4 + a^2*b^2)*(-(81*b^8*
c^8 + 216*a*b^7*c^7*d - 324*a^2*b^6*c^6*d^2 - 984*a^3*b^5*c^5*d^3 + 646*a^4
*b^4*c^4*d^4 + 1640*a^5*b^3*c^3*d^5 - 900*a^6*b^2*c^2*d^6 - 1000*a^7*b*c*d^
7 + 625*a^8*d^8)/(a^7*b^9))^(1/4)*log(a^2*b^2*(-(81*b^8*c^8 + 216*a*b^7*c^7
*d - 324*a^2*b^6*c^6*d^2 - 984*a^3*b^5*c^5*d^3 + 646*a^4*b^4*c^4*d^4 + 1640
*a^5*b^3*c^3*d^5 - 900*a^6*b^2*c^2*d^6 - 1000*a^7*b*c*d^7 + 625*a^8*d^8)/(a
^7*b^9))^(1/4) - (3*b^2*c^2 + 2*a*b*c*d - 5*a^2*d^2)*x) + (a*b^3*x^4 + a^2*
b^2)*(-(81*b^8*c^8 + 216*a*b^7*c^7*d - 324*a^2*b^6*c^6*d^2 - 984*a^3*b^5*c^
5*d^3 + 646*a^4*b^4*c^4*d^4 + 1640*a^5*b^3*c^3*d^5 - 900*a^6*b^2*c^2*d^6 -
1000*a^7*b*c*d^7 + 625*a^8*d^8)/(a^7*b^9))^(1/4)*log(-a^2*b^2*(-(81*b^8*c^8
 + 216*a*b^7*c^7*d - 324*a^2*b^6*c^6*d^2 - 984*a^3*b^5*c^5*d^3 + 646*a^4*b^
4*c^4*d^4 + 1640*a^5*b^3*c^3*d^5 - 900*a^6*b^2*c^2*d^6 - 1000*a^7*b*c*d^7 +
 625*a^8*d^8)/(a^7*b^9))^(1/4) - (3*b^2*c^2 + 2*a*b*c*d - 5*a^2*d^2)*x) + 4
*(b^2*c^2 - 2*a*b*c*d + 5*a^2*d^2)*x)/(a*b^3*x^4 + a^2*b^2)
```

Sympy [**A**]
time = 1.09, size = 219, normalized size = 0.75

$$\frac{x(a^2d^2-2abcd+b^2c^2)}{4a^2b^2+4ab^3x^4}+\operatorname{RootSum}\left(65536t^4a^7b^9+625a^8d^8-1000a^7bcd^7-900a^6b^2c^2d^6+1640a^5b^3c^3d^5+646a^4b^4c^4d^4-984a^3b^5c^5d^3-324a^2b^6c^6d^2+216ab^7c^7d+81b^8c^8,\left(t\mapsto t\log\left(-\frac{16ta^2b^2}{5a^2d^2-2abcd-3b^2c^2}+x\right)\right)\right)+\frac{d^2x}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**4+c)**2/(b*x**4+a)**2,x)
```

```
[Out] x*(a**2*d**2 - 2*a*b*c*d + b**2*c**2)/(4*a**2*b**2 + 4*a*b**3*x**4) + RootS
um(65536*_t**4*a**7*b**9 + 625*a**8*d**8 - 1000*a**7*b*c*d**7 - 900*a**6*b*
*2*c**2*d**6 + 1640*a**5*b**3*c**3*d**5 + 646*a**4*b**4*c**4*d**4 - 984*a**
3*b**5*c**5*d**3 - 324*a**2*b**6*c**6*d**2 + 216*a*b**7*c**7*d + 81*b**8*c*
*8, Lambda(_t, _t*log(-16*_t*a**2*b**2/(5*a**2*d**2 - 2*a*b*c*d - 3*b**2*c*
*2) + x))) + d**2*x/b**2
```

Giac [**A**]

time = 0.58, size = 376, normalized size = 1.29

$$\frac{d^{2} x}{b^{2}} + \frac{\sqrt{2} \left(3 \left(a b^{3}\right)^{\frac{1}{4}} b^{2} c^{2} + 2 \left(a b^{3}\right)^{\frac{1}{4}} a b c d - 5 \left(a b^{3}\right)^{\frac{1}{4}} a^{2} d^{2}\right) \arctan\left(\frac{\sqrt{2} \left(2 x + \sqrt{2} \left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2 \left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16 a^{2} b^{3}} + \frac{\sqrt{2} \left(3 \left(a b^{3}\right)^{\frac{1}{4}} b^{2} c^{2} + 2 \left(a b^{3}\right)^{\frac{1}{4}} a b c d - 5 \left(a b^{3}\right)^{\frac{1}{4}} a^{2} d^{2}\right) \arctan\left(\frac{\sqrt{2} \left(2 x - \sqrt{2} \left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2 \left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16 a^{2} b^{3}} + \frac{\sqrt{2} \left(3 \left(a b^{3}\right)^{\frac{1}{4}} b^{2} c^{2} + 2 \left(a b^{3}\right)^{\frac{1}{4}} a b c d - 5 \left(a b^{3}\right)^{\frac{1}{4}} a^{2} d^{2}\right) \log\left(x^{2} + \sqrt{2} x \left(\frac{a}{b}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{b}}\right)}{32 a^{2} b^{3}} - \frac{\sqrt{2} \left(3 \left(a b^{3}\right)^{\frac{1}{4}} b^{2} c^{2} + 2 \left(a b^{3}\right)^{\frac{1}{4}} a b c d - 5 \left(a b^{3}\right)^{\frac{1}{4}} a^{2} d^{2}\right) \log\left(x^{2} - \sqrt{2} x \left(\frac{a}{b}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{b}}\right)}{32 a^{2} b^{3}} + \frac{b^{2} c^{2} x - 2 a b c d x + a^{2} d^{2} x}{4 \left(b x^{4} + a\right) a b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^2/(b*x^4+a)^2,x, algorithm="giac")
```

```
[Out] d^2*x/b^2 + 1/16*sqrt(2)*(3*(a*b^3)^(1/4)*b^2*c^2 + 2*(a*b^3)^(1/4)*a*b*c*d
 - 5*(a*b^3)^(1/4)*a^2*d^2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/
(a/b)^(1/4))/(a^2*b^3) + 1/16*sqrt(2)*(3*(a*b^3)^(1/4)*b^2*c^2 + 2*(a*b^3)^
(1/4)*a*b*c*d - 5*(a*b^3)^(1/4)*a^2*d^2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*
(a/b)^(1/4))/(a/b)^(1/4))/(a^2*b^3) + 1/32*sqrt(2)*(3*(a*b^3)^(1/4)*b^2*c^2
 + 2*(a*b^3)^(1/4)*a*b*c*d - 5*(a*b^3)^(1/4)*a^2*d^2)*log(x^2 + sqrt(2)*x*(
a/b)^(1/4) + sqrt(a/b))/(a^2*b^3) - 1/32*sqrt(2)*(3*(a*b^3)^(1/4)*b^2*c^2 +
 2*(a*b^3)^(1/4)*a*b*c*d - 5*(a*b^3)^(1/4)*a^2*d^2)*log(x^2 - sqrt(2)*x*(a/
b)^(1/4) + sqrt(a/b))/(a^2*b^3) + 1/4*(b^2*c^2*x - 2*a*b*c*d*x + a^2*d^2*x)
/((b*x^4 + a)*a*b^2)
```

Mupad [B]
time = 0.30, size = 1254, normalized size = 4.31

$$\frac{d^2\,x}{b^2}+\frac{x\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}{4\,a\,\left(b^3\,x^4+a\,b^2\right)}+\frac{\mathrm{atan}\left(\frac{\frac{\left(a\,d-b\,c\right)\,\left(5\,a\,d+3\,b\,c\right)\,\left(\frac{x\,\left(25\,a^4\,d^4-20\,a^3\,b\,c\,d^3-26\,a^2\,b^2\,c^2\,d^2+12\,a\,b^3\,c^3\,d+9\,b^4\,c^4\right)}{4\,a^2\,b}-\frac{\left(a\,d-b\,c\right)\,\left(5\,a\,d+3\,b\,c\right)\,\left(-20\,a^2\,b\,d^2+8\,a\,b^2\,c\,d+12\,b^3\,c^2\right)}{16\,{\left(-a\right)}^{7/4}\,b^{9/4}}\right)\,1{}\mathrm{i}}{16\,{\left(-a\right)}^{7/4}\,b^{9/4}}+\frac{\left(a\,d-b\,c\right)\,\left(5\,a\,d+3\,b\,c\right)\,\left(\frac{x\,\left(25\,a^4\,d^4-20\,a^3\,b\,c\,d^3-26\,a^2\,b^2\,c^2\,d^2+12\,a\,b^3\,c^3\,d+9\,b^4\,c^4\right)}{4\,a^2\,b}+\frac{\left(a\,d-b\,c\right)\,\left(5\,a\,d+3\,b\,c\right)\,\left(-20\,a^2\,b\,d^2+8\,a\,b^2\,c\,d+12\,b^3\,c^2\right)}{16\,{\left(-a\right)}^{7/4}\,b^{9/4}}\right)\,1{}\mathrm{i}}{16\,{\left(-a\right)}^{7/4}\,b^{9/4}}}{\frac{\left(a\,d-b\,c\right)\,\left(5\,a\,d+3\,b\,c\right)\,\left(\frac{x\,\left(25\,a^4\,d^4-20\,a^3\,b\,c\,d^3-26\,a^2\,b^2\,c^2\,d^2+12\,a\,b^3\,c^3\,d+9\,b^4\,c^4\right)}{4\,a^2\,b}-\frac{\left(a\,d-b\,c\right)\,\left(5\,a\,d+3\,b\,c\right)\,\left(-20\,a^2\,b\,d^2+8\,a\,b^2\,c\,d+12\,b^3\,c^2\right)}{16\,{\left(-a\right)}^{7/4}\,b^{9/4}}\right)}{16\,{\left(-a\right)}^{7/4}\,b^{9/4}}-\frac{\left(a\,d-b\,c\right)\,\left(5\,a\,d+3\,b\,c\right)\,\left(\frac{x\,\left(25\,a^4\,d^4-20\,a^3\,b\,c\,d^3-26\,a^2\,b^2\,c^2\,d^2+12\,a\,b^3\,c^3\,d+9\,b^4\,c^4\right)}{4\,a^2\,b}+\frac{\left(a\,d-b\,c\right)\,\left(5\,a\,d+3\,b\,c\right)\,\left(-20\,a^2\,b\,d^2+8\,a\,b^2\,c\,d+12\,b^3\,c^2\right)}{16\,{\left(-a\right)}^{7/4}\,b^{9/4}}\right)}{16\,{\left(-a\right)}^{7/4}\,b^{9/4}}}\right)\,\left(a\,d-b\,c\right)\,\left(5\,a\,d+3\,b\,c\right)\,1{}\mathrm{i}}{8\,{\left(-a\right)}^{7/4}\,b^{9/4}}+\frac{\mathrm{atan}\left(\frac{\frac{\left(a\,d-b\,c\right)\,\left(5\,a\,d+3\,b\,c\right)\,\left(\frac{x\,\left(25\,a^4\,d^4-20\,a^3\,b\,c\,d^3-26\,a^2\,b^2\,c^2\,d^2+12\,a\,b^3\,c^3\,d+9\,b^4\,c^4\right)}{4\,a^2\,b}-\frac{\left(a\,d-b\,c\right)\,\left(5\,a\,d+3\,b\,c\right)\,\left(-20\,a^2\,b\,d^2+8\,a\,b^2\,c\,d+12\,b^3\,c^2\right)\,1{}\mathrm{i}}{16\,{\left(-a\right)}^{7/4}\,b^{9/4}}\right)}{16\,{\left(-a\right)}^{7/4}\,b^{9/4}}+\frac{\left(a\,d-b\,c\right)\,\left(5\,a\,d+3\,b\,c\right)\,\left(\frac{x\,\left(25\,a^4\,d^4-20\,a^3\,b\,c\,d^3-26\,a^2\,b^2\,c^2\,d^2+12\,a\,b^3\,c^3\,d+9\,b^4\,c^4\right)}{4\,a^2\,b}+\frac{\left(a\,d-b\,c\right)\,\left(5\,a\,d+3\,b\,c\right)\,\left(-20\,a^2\,b\,d^2+8\,a\,b^2\,c\,d+12\,b^3\,c^2\right)\,1{}\mathrm{i}}{16\,{\left(-a\right)}^{7/4}\,b^{9/4}}\right)}{16\,{\left(-a\right)}^{7/4}\,b^{9/4}}}{\frac{\left(a\,d-b\,c\right)\,\left(5\,a\,d+3\,b\,c\right)\,\left(\frac{x\,\left(25\,a^4\,d^4-20\,a^3\,b\,c\,d^3-26\,a^2\,b^2\,c^2\,d^2+12\,a\,b^3\,c^3\,d+9\,b^4\,c^4\right)}{4\,a^2\,b}-\frac{\left(a\,d-b\,c\right)\,\left(5\,a\,d+3\,b\,c\right)\,\left(-20\,a^2\,b\,d^2+8\,a\,b^2\,c\,d+12\,b^3\,c^2\right)\,1{}\mathrm{i}}{16\,{\left(-a\right)}^{7/4}\,b^{9/4}}\right)\,1{}\mathrm{i}}{16\,{\left(-a\right)}^{7/4}\,b^{9/4}}-\frac{\left(a\,d-b\,c\right)\,\left(5\,a\,d+3\,b\,c\right)\,\left(\frac{x\,\left(25\,a^4\,d^4-20\,a^3\,b\,c\,d^3-26\,a^2\,b^2\,c^2\,d^2+12\,a\,b^3\,c^3\,d+9\,b^4\,c^4\right)}{4\,a^2\,b}+\frac{\left(a\,d-b\,c\right)\,\left(5\,a\,d+3\,b\,c\right)\,\left(-20\,a^2\,b\,d^2+8\,a\,b^2\,c\,d+12\,b^3\,c^2\right)\,1{}\mathrm{i}}{16\,{\left(-a\right)}^{7/4}\,b^{9/4}}\right)\,1{}\mathrm{i}}{16\,{\left(-a\right)}^{7/4}\,b^{9/4}}}\right)\,\left(a\,d-b\,c\right)\,\left(5\,a\,d+3\,b\,c\right)}{8\,{\left(-a\right)}^{7/4}\,b^{9/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^4)^2/(a + b*x^4)^2,x)
```

```
[Out] (d^2*x)/b^2 + (x*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))/(4*a*(a*b^2 + b^3*x^4)) +
 (atan((((a*d - b*c)*(5*a*d + 3*b*c)*((x*(25*a^4*d^4 + 9*b^4*c^4 - 26*a^2*b
^2*c^2*d^2 + 12*a*b^3*c^3*d - 20*a^3*b*c*d^3))/(4*a^2*b) - ((a*d - b*c)*(5*
a*d + 3*b*c)*(12*b^3*c^2 - 20*a^2*b*d^2 + 8*a*b^2*c*d))/(16*(-a)^(7/4)*b^(9
/4)))*1i)/(16*(-a)^(7/4)*b^(9/4)) + ((a*d - b*c)*(5*a*d + 3*b*c)*((x*(25*a^
4*d^4 + 9*b^4*c^4 - 26*a^2*b^2*c^2*d^2 + 12*a*b^3*c^3*d - 20*a^3*b*c*d^3))/
(4*a^2*b) + ((a*d - b*c)*(5*a*d + 3*b*c)*(12*b^3*c^2 - 20*a^2*b*d^2 + 8*a*b
^2*c*d))/(16*(-a)^(7/4)*b^(9/4)))*1i)/(16*(-a)^(7/4)*b^(9/4)))/(((a*d - b*c
)*(5*a*d + 3*b*c)*((x*(25*a^4*d^4 + 9*b^4*c^4 - 26*a^2*b^2*c^2*d^2 + 12*a*b
^3*c^3*d - 20*a^3*b*c*d^3))/(4*a^2*b) - ((a*d - b*c)*(5*a*d + 3*b*c)*(12*b^
3*c^2 - 20*a^2*b*d^2 + 8*a*b^2*c*d))/(16*(-a)^(7/4)*b^(9/4))))/(16*(-a)^(7/
4)*b^(9/4)) - ((a*d - b*c)*(5*a*d + 3*b*c)*((x*(25*a^4*d^4 + 9*b^4*c^4 - 26
*a^2*b^2*c^2*d^2 + 12*a*b^3*c^3*d - 20*a^3*b*c*d^3))/(4*a^2*b) + ((a*d - b*
c)*(5*a*d + 3*b*c)*(12*b^3*c^2 - 20*a^2*b*d^2 + 8*a*b^2*c*d))/(16*(-a)^(7/4
)*b^(9/4))))/(16*(-a)^(7/4)*b^(9/4))))*(a*d - b*c)*(5*a*d + 3*b*c)*1i)/(8*(
-a)^(7/4)*b^(9/4)) + (atan((((a*d - b*c)*(5*a*d + 3*b*c)*((x*(25*a^4*d^4 +
9*b^4*c^4 - 26*a^2*b^2*c^2*d^2 + 12*a*b^3*c^3*d - 20*a^3*b*c*d^3))/(4*a^2*b
) - ((a*d - b*c)*(5*a*d + 3*b*c)*(12*b^3*c^2 - 20*a^2*b*d^2 + 8*a*b^2*c*d)*
1i)/(16*(-a)^(7/4)*b^(9/4))))/(16*(-a)^(7/4)*b^(9/4)) + ((a*d - b*c)*(5*a*d
```

```
 + 3*b*c)*((x*(25*a^4*d^4 + 9*b^4*c^4 - 26*a^2*b^2*c^2*d^2 + 12*a*b^3*c^3*d
 - 20*a^3*b*c*d^3))/(4*a^2*b) + ((a*d - b*c)*(5*a*d + 3*b*c)*(12*b^3*c^2 -
20*a^2*b*d^2 + 8*a*b^2*c*d)*1i)/(16*(-a)^(7/4)*b^(9/4))))/(16*(-a)^(7/4)*b^
(9/4)))/(((a*d - b*c)*(5*a*d + 3*b*c)*((x*(25*a^4*d^4 + 9*b^4*c^4 - 26*a^2*
b^2*c^2*d^2 + 12*a*b^3*c^3*d - 20*a^3*b*c*d^3))/(4*a^2*b) - ((a*d - b*c)*(5
*a*d + 3*b*c)*(12*b^3*c^2 - 20*a^2*b*d^2 + 8*a*b^2*c*d)*1i)/(16*(-a)^(7/4)*
b^(9/4)))*1i)/(16*(-a)^(7/4)*b^(9/4)) - ((a*d - b*c)*(5*a*d + 3*b*c)*((x*(2
5*a^4*d^4 + 9*b^4*c^4 - 26*a^2*b^2*c^2*d^2 + 12*a*b^3*c^3*d - 20*a^3*b*c*d^
3))/(4*a^2*b) + ((a*d - b*c)*(5*a*d + 3*b*c)*(12*b^3*c^2 - 20*a^2*b*d^2 + 8
*a*b^2*c*d)*1i)/(16*(-a)^(7/4)*b^(9/4)))*1i)/(16*(-a)^(7/4)*b^(9/4))))*(a*d
 - b*c)*(5*a*d + 3*b*c))/(8*(-a)^(7/4)*b^(9/4))
```

3.170 $\int \frac{c+dx^4}{(a+bx^4)^2}\,dx$

Optimal. Leaf size=245

$$\frac{(bc-ad)x}{4ab\,(a+bx^4)}-\frac{(3bc+ad)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{8\sqrt{2}\,a^{7/4}b^{5/4}}+\frac{(3bc+ad)\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{8\sqrt{2}\,a^{7/4}b^{5/4}}-\frac{(3bc+ad)\log\left(\sqrt{a}\right.}{16\sqrt{}}$$

```
[Out] 1/4*(-a*d+b*c)*x/a/b/(b*x^4+a)+1/16*(a*d+3*b*c)*arctan(-1+b^(1/4)*x*2^(1/2)
/a^(1/4))/a^(7/4)/b^(5/4)*2^(1/2)+1/16*(a*d+3*b*c)*arctan(1+b^(1/4)*x*2^(1/
2)/a^(1/4))/a^(7/4)/b^(5/4)*2^(1/2)-1/32*(a*d+3*b*c)*ln(-a^(1/4)*b^(1/4)*x*
2^(1/2)+a^(1/2)+x^2*b^(1/2))/a^(7/4)/b^(5/4)*2^(1/2)+1/32*(a*d+3*b*c)*ln(a^
(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))/a^(7/4)/b^(5/4)*2^(1/2)
```

Rubi [A]

time = 0.10, antiderivative size = 245, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {393, 217, 1179, 642, 1176, 631, 210}

$$-\frac{\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)(ad+3bc)}{8\sqrt{2}\,a^{7/4}b^{5/4}}+\frac{\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a}}+1\right)(ad+3bc)}{8\sqrt{2}\,a^{7/4}b^{5/4}}-\frac{(ad+3bc)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{16\sqrt{2}\,a^{7/4}b^{5/4}}+\frac{(ad+3bc)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{16\sqrt{2}\,a^{7/4}b^{5/4}}+\frac{x(bc-ad)}{4ab\,(a+bx^4)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^4)/(a + b*x^4)^2,x]
```

```
[Out] ((b*c - a*d)*x)/(4*a*b*(a + b*x^4)) - ((3*b*c + a*d)*ArcTan[1 - (Sqrt[2]*b^
(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*b^(5/4)) + ((3*b*c + a*d)*ArcTan[1 +
(Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*b^(5/4)) - ((3*b*c + a*d)*
Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)
*b^(5/4)) + ((3*b*c + a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b
]*x^2])/(16*Sqrt[2]*a^(7/4)*b^(5/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{c+dx^4}{(a+bx^4)^2}\,dx = \frac{(bc-ad)x}{4ab\,(a+bx^4)} + \frac{(3bc+ad)\int \frac{1}{a+bx^4}\,dx}{4ab}$$

$$= \frac{(bc-ad)x}{4ab\,(a+bx^4)} + \frac{(3bc+ad)\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx}{8a^{3/2}b} + \frac{(3bc+ad)\int \frac{\sqrt{a}+\sqrt{b}\,x^2}{a+bx^4}\,dx}{8a^{3/2}b}$$

$$= \frac{(bc-ad)x}{4ab\,(a+bx^4)} + \frac{(3bc+ad)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx}{16a^{3/2}b^{3/2}} + \frac{(3bc+ad)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx}{16a^{3/2}b^{3/2}}$$

$$= \frac{(bc-ad)x}{4ab\,(a+bx^4)} - \frac{(3bc+ad)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{b}\,x^2\right)}{16\sqrt{2}\,a^{7/4}b^{5/4}} + \frac{(3bc+ad)\log\left(\sqrt{a}+\right.}{16\sqrt{2}}$$

$$= \frac{(bc-ad)x}{4ab\,(a+bx^4)} - \frac{(3bc+ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{8\sqrt{2}\,a^{7/4}b^{5/4}} + \frac{(3bc+ad)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{8\sqrt{2}\,a^{7/4}b^{5/4}}$$

Mathematica [A]
time = 0.11, size = 212, normalized size = 0.87

$$\frac{-\frac{8a^{3/4}\sqrt[4]{b}(-bc+ad)x}{a+bx^4}-2\sqrt{2}(3bc+ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)+2\sqrt{2}(3bc+ad)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)-\sqrt{2}(3bc+ad)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{b}\,x^2\right)+\sqrt{2}(3bc+ad)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{b}\,x^2\right)}{32a^{7/4}b^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^4)/(a + b*x^4)^2,x]
```

```
[Out] ((-8*a^(3/4)*b^(1/4)*(-(b*c) + a*d)*x)/(a + b*x^4) - 2*Sqrt[2]*(3*b*c + a*d
)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 2*Sqrt[2]*(3*b*c + a*d)*ArcTan[
1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)] - Sqrt[2]*(3*b*c + a*d)*Log[Sqrt[a] - Sqrt
[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] + Sqrt[2]*(3*b*c + a*d)*Log[Sqrt[a] +
Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(32*a^(7/4)*b^(5/4))
```

Maple [A]
time = 0.24, size = 140, normalized size = 0.57

method	result	size
risch	$-\frac{(ad-bc)x}{4ab(b\,x^4+a)}+\frac{\sum_{\_R=\mathrm{RootOf}(\_Z^4 b+a)}\frac{(ad+3bc)\ln(x-\_R)}{\_R^3}}{16a\,b^2}$	65

default	$-\frac{(ad-bc)x}{4ab(b\,x^4+a)}+\frac{(ad+3bc)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{32a^2b}$	14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^4+c)/(b*x^4+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*(a*d-b*c)/a/b*x/(b*x^4+a)+1/32*(a*d+3*b*c)/a^2/b*(a/b)^(1/4)*2^(1/2)*(
ln((x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)
^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x-1
))
```

Maxima [A]
time = 0.55, size = 236, normalized size = 0.96

$$\frac{(bc-ad)x}{4\,(ab^2x^4+a^2b)}+\frac{\frac{2\sqrt{2}\,(3\,bc+ad)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}\,x+\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}}+\frac{2\sqrt{2}\,(3\,bc+ad)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}\,x-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}}+\frac{\sqrt{2}\,(3\,bc+ad)\log\left(\sqrt{b}\,x^2+\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}-\frac{\sqrt{2}\,(3\,bc+ad)\log\left(\sqrt{b}\,x^2-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}}{32\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)/(b*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(b*c - a*d)*x/(a*b^2*x^4 + a^2*b) + 1/32*(2*sqrt(2)*(3*b*c + a*d)*arcta
n(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b))
)/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqrt(2)*(3*b*c + a*d)*arctan(1/2*sqrt
(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)
*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*(3*b*c + a*d)*log(sqrt(b)*x^2 + sqrt(2)*a
^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(1/4)) - sqrt(2)*(3*b*c + a*d)*log(s
qrt(b)*x^2 - sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(1/4)))/(a*b)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 711 vs. $2(178) = 356$.
time = 2.68, size = 711, normalized size = 2.90

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)/(b*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] 1/16*(4*(a*b^2*x^4 + a^2*b)*(-(81*b^4*c^4 + 108*a*b^3*c^3*d + 54*a^2*b^2*c^
2*d^2 + 12*a^3*b*c*d^3 + a^4*d^4)/(a^7*b^5))^(1/4)*arctan((sqrt(a^4*b^2*sqr
t(-(81*b^4*c^4 + 108*a*b^3*c^3*d + 54*a^2*b^2*c^2*d^2 + 12*a^3*b*c*d^3 + a^
4*d^4)/(a^7*b^5)) + (9*b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^2)*a^5*b^4*(-(81*b^
4*c^4 + 108*a*b^3*c^3*d + 54*a^2*b^2*c^2*d^2 + 12*a^3*b*c*d^3 + a^4*d^4)/(a
^7*b^5))^(3/4) - (3*a^5*b^5*c + a^6*b^4*d)*x*(-(81*b^4*c^4 + 108*a*b^3*c^3*
```

```
d + 54*a^2*b^2*c^2*d^2 + 12*a^3*b*c*d^3 + a^4*d^4)/(a^7*b^5))^(3/4))/(81*b^
4*c^4 + 108*a*b^3*c^3*d + 54*a^2*b^2*c^2*d^2 + 12*a^3*b*c*d^3 + a^4*d^4)) +
 (a*b^2*x^4 + a^2*b)*(-(81*b^4*c^4 + 108*a*b^3*c^3*d + 54*a^2*b^2*c^2*d^2 +
 12*a^3*b*c*d^3 + a^4*d^4)/(a^7*b^5))^(1/4)*log(a^2*b*(-(81*b^4*c^4 + 108*a
*b^3*c^3*d + 54*a^2*b^2*c^2*d^2 + 12*a^3*b*c*d^3 + a^4*d^4)/(a^7*b^5))^(1/4
) + (3*b*c + a*d)*x) - (a*b^2*x^4 + a^2*b)*(-(81*b^4*c^4 + 108*a*b^3*c^3*d
+ 54*a^2*b^2*c^2*d^2 + 12*a^3*b*c*d^3 + a^4*d^4)/(a^7*b^5))^(1/4)*log(-a^2*
b*(-(81*b^4*c^4 + 108*a*b^3*c^3*d + 54*a^2*b^2*c^2*d^2 + 12*a^3*b*c*d^3 + a
^4*d^4)/(a^7*b^5))^(1/4) + (3*b*c + a*d)*x) + 4*(b*c - a*d)*x)/(a*b^2*x^4 +
 a^2*b)
```

Sympy [A]
time = 0.40, size = 112, normalized size = 0.46

$$\frac{x(-ad+bc)}{4a^2b+4ab^2x^4}+\operatorname{RootSum}\left(65536t^4a^7b^5+a^4d^4+12a^3bcd^3+54a^2b^2c^2d^2+108ab^3c^3d+81b^4c^4,\left(t\mapsto t\log\left(\frac{16ta^2b}{ad+3bc}+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**4+c)/(b*x**4+a)**2,x)
```

```
[Out] x*(-a*d + b*c)/(4*a**2*b + 4*a*b**2*x**4) + RootSum(65536*_t**4*a**7*b**5 +
 a**4*d**4 + 12*a**3*b*c*d**3 + 54*a**2*b**2*c**2*d**2 + 108*a*b**3*c**3*d
+ 81*b**4*c**4, Lambda(_t, _t*log(16*_t*a**2*b/(a*d + 3*b*c) + x)))
```

Giac [A]
time = 0.67, size = 266, normalized size = 1.09

$$\frac{\sqrt{2}\left(3\,(ab^3)^{\frac14}bc+(ab^3)^{\frac14}ad\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac14}\right)}{2\left(\frac{a}{b}\right)^{\frac14}}\right)}{16\,a^2b^2}+\frac{\sqrt{2}\left(3\,(ab^3)^{\frac14}bc+(ab^3)^{\frac14}ad\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac14}\right)}{2\left(\frac{a}{b}\right)^{\frac14}}\right)}{16\,a^2b^2}+\frac{\sqrt{2}\left(3\,(ab^3)^{\frac14}bc+(ab^3)^{\frac14}ad\right)\log\left(x^2+\sqrt{2}\,x\left(\frac{a}{b}\right)^{\frac14}+\sqrt{\frac{a}{b}}\right)}{32\,a^2b^2}-\frac{\sqrt{2}\left(3\,(ab^3)^{\frac14}bc+(ab^3)^{\frac14}ad\right)\log\left(x^2-\sqrt{2}\,x\left(\frac{a}{b}\right)^{\frac14}+\sqrt{\frac{a}{b}}\right)}{32\,a^2b^2}+\frac{bcx-adx}{4\,(bx^4+a)ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)/(b*x^4+a)^2,x, algorithm="giac")
```

```
[Out] 1/16*sqrt(2)*(3*(a*b^3)^(1/4)*b*c + (a*b^3)^(1/4)*a*d)*arctan(1/2*sqrt(2)*(
2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^2*b^2) + 1/16*sqrt(2)*(3*(a*b^3)
^(1/4)*b*c + (a*b^3)^(1/4)*a*d)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/
4))/(a/b)^(1/4))/(a^2*b^2) + 1/32*sqrt(2)*(3*(a*b^3)^(1/4)*b*c + (a*b^3)^(1
/4)*a*d)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^2*b^2) - 1/32*sqrt
(2)*(3*(a*b^3)^(1/4)*b*c + (a*b^3)^(1/4)*a*d)*log(x^2 - sqrt(2)*x*(a/b)^(1/
4) + sqrt(a/b))/(a^2*b^2) + 1/4*(b*c*x - a*d*x)/((b*x^4 + a)*a*b)
```

Mupad [B]
time = 1.53, size = 740, normalized size = 3.02

$$\frac{\mathrm{atan}\left(\frac{\frac{\left(\frac{x\,\left(a^2\,b\,d^2+6\,a\,b^2\,c\,d+9\,b^3\,c^2\right)}{4\,a^2}-\frac{\left(a\,d+3\,b\,c\right)\,\left(12\,c\,b^3+4\,a\,d\,b^2\right)\,1{}\mathrm{i}}{16\,{\left(-a\right)}^{7/4}\,b^{5/4}}\right)\,\left(a\,d+3\,b\,c\right)}{16\,{\left(-a\right)}^{7/4}\,b^{5/4}}+\frac{\left(\frac{x\,\left(a^2\,b\,d^2+6\,a\,b^2\,c\,d+9\,b^3\,c^2\right)}{4\,a^2}+\frac{\left(a\,d+3\,b\,c\right)\,\left(12\,c\,b^3+4\,a\,d\,b^2\right)\,1{}\mathrm{i}}{16\,{\left(-a\right)}^{7/4}\,b^{5/4}}\right)\,\left(a\,d+3\,b\,c\right)}{16\,{\left(-a\right)}^{7/4}\,b^{5/4}}}{\frac{\left(\frac{x\,\left(a^2\,b\,d^2+6\,a\,b^2\,c\,d+9\,b^3\,c^2\right)}{4\,a^2}-\frac{\left(a\,d+3\,b\,c\right)\,\left(12\,c\,b^3+4\,a\,d\,b^2\right)\,1{}\mathrm{i}}{16\,{\left(-a\right)}^{7/4}\,b^{5/4}}\right)\,\left(a\,d+3\,b\,c\right)\,1{}\mathrm{i}}{16\,{\left(-a\right)}^{7/4}\,b^{5/4}}-\frac{\left(\frac{x\,\left(a^2\,b\,d^2+6\,a\,b^2\,c\,d+9\,b^3\,c^2\right)}{4\,a^2}+\frac{\left(a\,d+3\,b\,c\right)\,\left(12\,c\,b^3+4\,a\,d\,b^2\right)\,1{}\mathrm{i}}{16\,{\left(-a\right)}^{7/4}\,b^{5/4}}\right)\,\left(a\,d+3\,b\,c\right)\,1{}\mathrm{i}}{16\,{\left(-a\right)}^{7/4}\,b^{5/4}}}\right)\,\left(a\,d+3\,b\,c\right)}{8\,{\left(-a\right)}^{7/4}\,b^{5/4}}-\frac{x\,\left(a\,d-b\,c\right)}{4\,a\,b\,\left(b\,x^4+a\right)}+\frac{\mathrm{atan}\left(\frac{\frac{\left(\frac{x\,\left(a^2\,b\,d^2+6\,a\,b^2\,c\,d+9\,b^3\,c^2\right)}{4\,a^2}-\frac{\left(a\,d+3\,b\,c\right)\,\left(12\,c\,b^3+4\,a\,d\,b^2\right)}{16\,{\left(-a\right)}^{7/4}\,b^{5/4}}\right)\,\left(a\,d+3\,b\,c\right)\,1{}\mathrm{i}}{16\,{\left(-a\right)}^{7/4}\,b^{5/4}}+\frac{\left(\frac{x\,\left(a^2\,b\,d^2+6\,a\,b^2\,c\,d+9\,b^3\,c^2\right)}{4\,a^2}+\frac{\left(a\,d+3\,b\,c\right)\,\left(12\,c\,b^3+4\,a\,d\,b^2\right)}{16\,{\left(-a\right)}^{7/4}\,b^{5/4}}\right)\,\left(a\,d+3\,b\,c\right)\,1{}\mathrm{i}}{16\,{\left(-a\right)}^{7/4}\,b^{5/4}}}{\frac{\left(\frac{x\,\left(a^2\,b\,d^2+6\,a\,b^2\,c\,d+9\,b^3\,c^2\right)}{4\,a^2}-\frac{\left(a\,d+3\,b\,c\right)\,\left(12\,c\,b^3+4\,a\,d\,b^2\right)}{16\,{\left(-a\right)}^{7/4}\,b^{5/4}}\right)\,\left(a\,d+3\,b\,c\right)}{16\,{\left(-a\right)}^{7/4}\,b^{5/4}}-\frac{\left(\frac{x\,\left(a^2\,b\,d^2+6\,a\,b^2\,c\,d+9\,b^3\,c^2\right)}{4\,a^2}+\frac{\left(a\,d+3\,b\,c\right)\,\left(12\,c\,b^3+4\,a\,d\,b^2\right)}{16\,{\left(-a\right)}^{7/4}\,b^{5/4}}\right)\,\left(a\,d+3\,b\,c\right)}{16\,{\left(-a\right)}^{7/4}\,b^{5/4}}}\right)\,\left(a\,d+3\,b\,c\right)\,1{}\mathrm{i}}{8\,{\left(-a\right)}^{7/4}\,b^{5/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^4)/(a + b*x^4)^2,x)
```

```
[Out] (atan(((((x*(9*b^3*c^2 + a^2*b*d^2 + 6*a*b^2*c*d))/(4*a^2) - ((a*d + 3*b*c)
*(12*b^3*c + 4*a*b^2*d))/(16*(-a)^(7/4)*b^(5/4)))*(a*d + 3*b*c)*1i)/(16*(-a
)^(7/4)*b^(5/4)) + (((x*(9*b^3*c^2 + a^2*b*d^2 + 6*a*b^2*c*d))/(4*a^2) + ((
a*d + 3*b*c)*(12*b^3*c + 4*a*b^2*d))/(16*(-a)^(7/4)*b^(5/4)))*(a*d + 3*b*c)
*1i)/(16*(-a)^(7/4)*b^(5/4)))/((((x*(9*b^3*c^2 + a^2*b*d^2 + 6*a*b^2*c*d))/
(4*a^2) - ((a*d + 3*b*c)*(12*b^3*c + 4*a*b^2*d))/(16*(-a)^(7/4)*b^(5/4)))*(
a*d + 3*b*c))/(16*(-a)^(7/4)*b^(5/4)) - (((x*(9*b^3*c^2 + a^2*b*d^2 + 6*a*b
^2*c*d))/(4*a^2) + ((a*d + 3*b*c)*(12*b^3*c + 4*a*b^2*d))/(16*(-a)^(7/4)*b^
(5/4)))*(a*d + 3*b*c))/(16*(-a)^(7/4)*b^(5/4))))*(a*d + 3*b*c)*1i)/(8*(-a)^
(7/4)*b^(5/4)) + (atan(((((x*(9*b^3*c^2 + a^2*b*d^2 + 6*a*b^2*c*d))/(4*a^2)
 - ((a*d + 3*b*c)*(12*b^3*c + 4*a*b^2*d)*1i)/(16*(-a)^(7/4)*b^(5/4)))*(a*d
+ 3*b*c))/(16*(-a)^(7/4)*b^(5/4)) + (((x*(9*b^3*c^2 + a^2*b*d^2 + 6*a*b^2*c
*d))/(4*a^2) + ((a*d + 3*b*c)*(12*b^3*c + 4*a*b^2*d)*1i)/(16*(-a)^(7/4)*b^(
5/4)))*(a*d + 3*b*c))/(16*(-a)^(7/4)*b^(5/4)))/((((x*(9*b^3*c^2 + a^2*b*d^2
 + 6*a*b^2*c*d))/(4*a^2) - ((a*d + 3*b*c)*(12*b^3*c + 4*a*b^2*d)*1i)/(16*(-
a)^(7/4)*b^(5/4)))*(a*d + 3*b*c)*1i)/(16*(-a)^(7/4)*b^(5/4)) - (((x*(9*b^3*
c^2 + a^2*b*d^2 + 6*a*b^2*c*d))/(4*a^2) + ((a*d + 3*b*c)*(12*b^3*c + 4*a*b^
2*d)*1i)/(16*(-a)^(7/4)*b^(5/4)))*(a*d + 3*b*c)*1i)/(16*(-a)^(7/4)*b^(5/4))
))*(a*d + 3*b*c))/(8*(-a)^(7/4)*b^(5/4)) - (x*(a*d - b*c))/(4*a*b*(a + b*x^
4))
```

3.171 $\int \frac{1}{(a+bx^4)^2(c+dx^4)}\,dx$

Optimal. Leaf size=513

$$\frac{bx}{4a(bc-ad)(a+bx^4)}-\frac{b^{3/4}(3bc-7ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{8\sqrt{2}\,a^{7/4}(bc-ad)^2}+\frac{b^{3/4}(3bc-7ad)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{8\sqrt{2}\,a^{7/4}(bc-ad)^2}-\frac{d^{7/}}{}$$

```
[Out] 1/4*b*x/a/(-a*d+b*c)/(b*x^4+a)+1/16*b^(3/4)*(-7*a*d+3*b*c)*arctan(-1+b^(1/4
)*x*2^(1/2)/a^(1/4))/a^(7/4)/(-a*d+b*c)^2*2^(1/2)+1/16*b^(3/4)*(-7*a*d+3*b*
c)*arctan(1+b^(1/4)*x*2^(1/2)/a^(1/4))/a^(7/4)/(-a*d+b*c)^2*2^(1/2)+1/4*d^(
7/4)*arctan(-1+d^(1/4)*x*2^(1/2)/c^(1/4))/c^(3/4)/(-a*d+b*c)^2*2^(1/2)+1/4*
d^(7/4)*arctan(1+d^(1/4)*x*2^(1/2)/c^(1/4))/c^(3/4)/(-a*d+b*c)^2*2^(1/2)-1/
32*b^(3/4)*(-7*a*d+3*b*c)*ln(-a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2)
)/a^(7/4)/(-a*d+b*c)^2*2^(1/2)+1/32*b^(3/4)*(-7*a*d+3*b*c)*ln(a^(1/4)*b^(1/
4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))/a^(7/4)/(-a*d+b*c)^2*2^(1/2)-1/8*d^(7/4)*
ln(-c^(1/4)*d^(1/4)*x*2^(1/2)+c^(1/2)+x^2*d^(1/2))/c^(3/4)/(-a*d+b*c)^2*2^(
1/2)+1/8*d^(7/4)*ln(c^(1/4)*d^(1/4)*x*2^(1/2)+c^(1/2)+x^2*d^(1/2))/c^(3/4)/
(-a*d+b*c)^2*2^(1/2)
```

Rubi [A]
time = 0.30, antiderivative size = 513, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {425, 536, 217, 1179, 642, 1176, 631, 210}

$$-\frac{b^{3/4}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)(3bc-7ad)}{8\sqrt{2}\,a^{7/4}(bc-ad)^2}+\frac{b^{3/4}\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}+1\right)(3bc-7ad)}{8\sqrt{2}\,a^{7/4}(bc-ad)^2}-\frac{b^{3/4}(3bc-7ad)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{16\sqrt{2}\,a^{7/4}(bc-ad)^2}+\frac{b^{3/4}(3bc-7ad)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{16\sqrt{2}\,a^{7/4}(bc-ad)^2}-\frac{d^{7/4}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right)}{2\sqrt{2}\,c^{3/4}(bc-ad)^2}+\frac{d^{7/4}\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{d}\,x}{\sqrt[4]{c}}+1\right)}{2\sqrt{2}\,c^{3/4}(bc-ad)^2}-\frac{d^{7/4}\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\,x+\sqrt{c}+\sqrt{d}\,x^2\right)}{4\sqrt{2}\,c^{3/4}(bc-ad)^2}+\frac{d^{7/4}\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\,x+\sqrt{c}+\sqrt{d}\,x^2\right)}{4\sqrt{2}\,c^{3/4}(bc-ad)^2}+\frac{bx}{4a(a+bx^4)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^4)^2*(c + d*x^4)),x]
```

```
[Out] (b*x)/(4*a*(b*c - a*d)*(a + b*x^4)) - (b^(3/4)*(3*b*c - 7*a*d)*ArcTan[1 - (
Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*(b*c - a*d)^2) + (b^(3/4)*(
3*b*c - 7*a*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*
(b*c - a*d)^2) - (d^(7/4)*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/(2*Sqrt[
2]*c^(3/4)*(b*c - a*d)^2) + (d^(7/4)*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)
])/(2*Sqrt[2]*c^(3/4)*(b*c - a*d)^2) - (b^(3/4)*(3*b*c - 7*a*d)*Log[Sqrt[a]
 - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)*(b*c - a*d
)^2) + (b^(3/4)*(3*b*c - 7*a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + S
qrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)*(b*c - a*d)^2) - (d^(7/4)*Log[Sqrt[c] - Sq
rt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(4*Sqrt[2]*c^(3/4)*(b*c - a*d)^2) +
 (d^(7/4)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(4*Sqrt[2
]*c^(3/4)*(b*c - a*d)^2)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
```

```
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{(a+bx^4)^2(c+dx^4)}\,dx = \frac{bx}{4a(bc-ad)(a+bx^4)} - \frac{\int \frac{-3bc+4ad-3bdx^4}{(a+bx^4)(c+dx^4)}\,dx}{4a(bc-ad)}$$

$$= \frac{bx}{4a(bc-ad)(a+bx^4)} + \frac{d^2\int \frac{1}{c+dx^4}\,dx}{(bc-ad)^2} + \frac{(b(3bc-7ad))\int \frac{1}{a+bx^4}\,dx}{4a(bc-ad)^2}$$

$$= \frac{bx}{4a(bc-ad)(a+bx^4)} + \frac{d^2\int \frac{\sqrt{c}-\sqrt{d}\,x^2}{c+dx^4}\,dx}{2\sqrt{c}(bc-ad)^2} + \frac{d^2\int \frac{\sqrt{c}+\sqrt{d}\,x^2}{c+dx^4}\,dx}{2\sqrt{c}(bc-ad)^2} + \frac{(b(3bc-\ldots}{8\ldots}$$

$$= \frac{bx}{4a(bc-ad)(a+bx^4)} + \frac{d^{3/2}\int \frac{1}{\frac{\sqrt{c}}{\sqrt{d}}-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{d}}+x^2}\,dx}{4\sqrt{c}(bc-ad)^2} + \frac{d^{3/2}\int \frac{1}{\frac{\sqrt{c}}{\sqrt{d}}+\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{d}}+\ldots}}{4\sqrt{c}(bc-ad)^2}$$

$$= \frac{bx}{4a(bc-ad)(a+bx^4)} - \frac{b^{3/4}(3bc-7ad)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{b}\,x^2\right)}{16\sqrt{2}\,a^{7/4}(bc-ad)^2} + \ldots$$

$$= \frac{bx}{4a(bc-ad)(a+bx^4)} - \frac{b^{3/4}(3bc-7ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{8\sqrt{2}\,a^{7/4}(bc-ad)^2} + \frac{b^{3/4}(3bc-7\ldots}{8\sqrt{\ldots}}$$

Mathematica [A]
time = 0.20, size = 499, normalized size = 0.97

$$\frac{8a^{3/4}bc^{3/4}(bc-ad)x - 2\sqrt{2}\,b^{3/4}c^{3/4}(3bc-7ad)(a+bx^4)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right) + 2\sqrt{2}\,b^{3/4}c^{3/4}(3bc-7ad)(a+bx^4)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right) - 8\sqrt{2}\,a^{7/4}d^{7/4}(a+bx^4)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right) + 8\sqrt{2}\,a^{7/4}d^{7/4}(a+bx^4)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right) - \sqrt{2}\,b^{3/4}c^{3/4}(3bc-7ad)(a+bx^4)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{b}\,x^2\right) + \sqrt{2}\,b^{3/4}c^{3/4}(3bc-7ad)(a+bx^4)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{b}\,x^2\right) - 4\sqrt{2}\,a^{7/4}d^{7/4}(a+bx^4)\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\,x+\sqrt{d}\,x^2\right) + 4\sqrt{2}\,a^{7/4}d^{7/4}(a+bx^4)\log\left(\sqrt{c}+\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\,x+\sqrt{d}\,x^2\right)}{32a^{7/4}c^{3/4}(bc-ad)^2(a+bx^4)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^4)^2*(c + d*x^4)),x]
```

```
[Out] (8*a^(3/4)*b*c^(3/4)*(b*c - a*d)*x - 2*Sqrt[2]*b^(3/4)*c^(3/4)*(3*b*c - 7*a
*d)*(a + b*x^4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 2*Sqrt[2]*b^(3/4)
```

```
*c^(3/4)*(3*b*c - 7*a*d)*(a + b*x^4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)
] - 8*Sqrt[2]*a^(7/4)*d^(7/4)*(a + b*x^4)*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^
(1/4)] + 8*Sqrt[2]*a^(7/4)*d^(7/4)*(a + b*x^4)*ArcTan[1 + (Sqrt[2]*d^(1/4)*
x)/c^(1/4)] - Sqrt[2]*b^(3/4)*c^(3/4)*(3*b*c - 7*a*d)*(a + b*x^4)*Log[Sqrt[
a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] + Sqrt[2]*b^(3/4)*c^(3/4)*(3*
b*c - 7*a*d)*(a + b*x^4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*
x^2] - 4*Sqrt[2]*a^(7/4)*d^(7/4)*(a + b*x^4)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*
d^(1/4)*x + Sqrt[d]*x^2] + 4*Sqrt[2]*a^(7/4)*d^(7/4)*(a + b*x^4)*Log[Sqrt[c
] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(32*a^(7/4)*c^(3/4)*(b*c - a*
d)^2*(a + b*x^4))
```

Maple [A]
time = 0.38, size = 263, normalized size = 0.51

method	result
default	$\frac{d^{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^{2}+\left(\frac{c}{d}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{c}{d}}}{x^{2}-\left(\frac{c}{d}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{c}{d}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)\right)}{8(ad-bc)^{2}c}-\frac{b\left(\frac{(ad-bc)x}{4a\left(b\,x^{4}+a\right)}+\frac{(7ad-3bc)\left(\frac{a}{b}\right.}{}\right.}{}$
risch	$-\frac{bx}{4a(ad-bc)(b\,x^{4}+a)}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(c^{3}d^{8}a^{8}-8a^{7}b\,c^{4}d^{7}+28a^{6}b^{2}c^{5}d^{6}-56a^{5}b^{3}c^{6}d^{5}+70a^{4}b^{4}c^{7}d^{4}-56a^{3}b^{5}c^{8}d^{3}+28a^{2}b^{6}c^{9}d^{2}-8a\,b^{7}c^{10}d\right.\right.}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^4+a)^2/(d*x^4+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*d^2/(a*d-b*c)^2*(c/d)^(1/4)/c*2^(1/2)*(ln((x^2+(c/d)^(1/4)*x*2^(1/2)+(c
/d)^(1/2))/(x^2-(c/d)^(1/4)*x*2^(1/2)+(c/d)^(1/2)))+2*arctan(2^(1/2)/(c/d)^
(1/4)*x+1)+2*arctan(2^(1/2)/(c/d)^(1/4)*x-1))-b/(a*d-b*c)^2*(1/4*(a*d-b*c)/
a*x/(b*x^4+a)+1/32*(7*a*d-3*b*c)/a^2*(a/b)^(1/4)*2^(1/2)*(ln((x^2+(a/b)^(1/
4)*x*2^(1/2)+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))+2*arctan
(2^(1/2)/(a/b)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x-1)))
```

Maxima [A]
time = 0.57, size = 470, normalized size = 0.92

$$\frac{\left(\frac{2\sqrt{2}(3bc-7ad)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{2\sqrt{2}(3bc-7ad)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{\sqrt{2}(3bc-7ad)\log\left(\sqrt{b}x^{2}+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}-\frac{\sqrt{2}(3bc-7ad)\log\left(\sqrt{b}x^{2}-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}\right)b}{32\left(ab^{2}c^{2}-2a^{2}bcd+a^{3}d^{2}\right)}+\frac{bx}{4\left((ab^{2}c-a^{2}bd)x^{4}+a^{2}bc-a^{3}d\right)}+\frac{\frac{2\sqrt{2}d^{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}x+\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}+\frac{2\sqrt{2}d^{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}x-\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}+\frac{\sqrt{2}d^{\frac{7}{4}}\log\left(\sqrt{d}x^{2}+\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}x+\sqrt{c}\right)}{c^{\frac{3}{4}}}-\frac{\sqrt{2}d^{\frac{7}{4}}\log\left(\sqrt{d}x^{2}-\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}x+\sqrt{c}\right)}{c^{\frac{3}{4}}}}{8\left(b^{2}c^{2}-2abcd+a^{2}d^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^2/(d*x^4+c),x, algorithm="maxima")
```

```
[Out] 1/32*(2*sqrt(2)*(3*b*c - 7*a*d)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a
^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*
```

```
sqrt(2)*(3*b*c - 7*a*d)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b
^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*(3
*b*c - 7*a*d)*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/
4)*b^(1/4)) - sqrt(2)*(3*b*c - 7*a*d)*log(sqrt(b)*x^2 - sqrt(2)*a^(1/4)*b^(
1/4)*x + sqrt(a))/(a^(3/4)*b^(1/4)))*b/(a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2)
+ 1/4*b*x/((a*b^2*c - a^2*b*d)*x^4 + a^2*b*c - a^3*d) + 1/8*(2*sqrt(2)*d^2*
arctan(1/2*sqrt(2)*(2*sqrt(d)*x + sqrt(2)*c^(1/4)*d^(1/4))/sqrt(sqrt(c)*sqr
t(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + 2*sqrt(2)*d^2*arctan(1/2*sqrt(2)*(
2*sqrt(d)*x - sqrt(2)*c^(1/4)*d^(1/4))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt
(sqrt(c)*sqrt(d))) + sqrt(2)*d^(7/4)*log(sqrt(d)*x^2 + sqrt(2)*c^(1/4)*d^(1
/4)*x + sqrt(c))/c^(3/4) - sqrt(2)*d^(7/4)*log(sqrt(d)*x^2 - sqrt(2)*c^(1/4
)*d^(1/4)*x + sqrt(c))/c^(3/4))/(b^2*c^2 - 2*a*b*c*d + a^2*d^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 3301 vs. $2(381) = 762$.
time = 41.39, size = 3301, normalized size = 6.43

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^2/(d*x^4+c),x, algorithm="fricas")
```

```
[Out] -1/16*(4*((a*b^2*c - a^2*b*d)*x^4 + a^2*b*c - a^3*d)*(-(81*b^7*c^4 - 756*a*
b^6*c^3*d + 2646*a^2*b^5*c^2*d^2 - 4116*a^3*b^4*c*d^3 + 2401*a^4*b^3*d^4)/(
a^7*b^8*c^8 - 8*a^8*b^7*c^7*d + 28*a^9*b^6*c^6*d^2 - 56*a^10*b^5*c^5*d^3 +
70*a^11*b^4*c^4*d^4 - 56*a^12*b^3*c^3*d^5 + 28*a^13*b^2*c^2*d^6 - 8*a^14*b*
c*d^7 + a^15*d^8))^(1/4)*arctan(((3*a^5*b^8*c^7 - 25*a^6*b^7*c^6*d + 87*a^7
*b^6*c^5*d^2 - 165*a^8*b^5*c^4*d^3 + 185*a^9*b^4*c^3*d^4 - 123*a^10*b^3*c^2
*d^5 + 45*a^11*b^2*c*d^6 - 7*a^12*b*d^7)*x*(-(81*b^7*c^4 - 756*a*b^6*c^3*d
+ 2646*a^2*b^5*c^2*d^2 - 4116*a^3*b^4*c*d^3 + 2401*a^4*b^3*d^4)/(a^7*b^8*c^
8 - 8*a^8*b^7*c^7*d + 28*a^9*b^6*c^6*d^2 - 56*a^10*b^5*c^5*d^3 + 70*a^11*b^
4*c^4*d^4 - 56*a^12*b^3*c^3*d^5 + 28*a^13*b^2*c^2*d^6 - 8*a^14*b*c*d^7 + a^
15*d^8))^(3/4) + (a^5*b^6*c^6 - 6*a^6*b^5*c^5*d + 15*a^7*b^4*c^4*d^2 - 20*a
^8*b^3*c^3*d^3 + 15*a^9*b^2*c^2*d^4 - 6*a^10*b*c*d^5 + a^11*d^6)*sqrt((9*b^
4*c^2 - 42*a*b^3*c*d + 49*a^2*b^2*d^2)*x^2 + (a^4*b^4*c^4 - 4*a^5*b^3*c^3*d
 + 6*a^6*b^2*c^2*d^2 - 4*a^7*b*c*d^3 + a^8*d^4)*sqrt(-(81*b^7*c^4 - 756*a*b
^6*c^3*d + 2646*a^2*b^5*c^2*d^2 - 4116*a^3*b^4*c*d^3 + 2401*a^4*b^3*d^4)/(a
^7*b^8*c^8 - 8*a^8*b^7*c^7*d + 28*a^9*b^6*c^6*d^2 - 56*a^10*b^5*c^5*d^3 + 7
0*a^11*b^4*c^4*d^4 - 56*a^12*b^3*c^3*d^5 + 28*a^13*b^2*c^2*d^6 - 8*a^14*b*c
*d^7 + a^15*d^8)))*(-(81*b^7*c^4 - 756*a*b^6*c^3*d + 2646*a^2*b^5*c^2*d^2 -
 4116*a^3*b^4*c*d^3 + 2401*a^4*b^3*d^4)/(a^7*b^8*c^8 - 8*a^8*b^7*c^7*d + 28
*a^9*b^6*c^6*d^2 - 56*a^10*b^5*c^5*d^3 + 70*a^11*b^4*c^4*d^4 - 56*a^12*b^3*
c^3*d^5 + 28*a^13*b^2*c^2*d^6 - 8*a^14*b*c*d^7 + a^15*d^8))^(3/4))/(81*b^7*
c^4 - 756*a*b^6*c^3*d + 2646*a^2*b^5*c^2*d^2 - 4116*a^3*b^4*c*d^3 + 2401*a^
4*b^3*d^4)) - 16*(-d^7/(b^8*c^11 - 8*a*b^7*c^10*d + 28*a^2*b^6*c^9*d^2 - 56
```

*a^3*b^5*c^8*d^3 + 70*a^4*b^4*c^7*d^4 - 56*a^5*b^3*c^6*d^5 + 28*a^6*b^2*c^5
*d^6 - 8*a^7*b*c^4*d^7 + a^8*c^3*d^8))^(1/4)*((a*b^2*c - a^2*b*d)*x^4 + a^2
*b*c - a^3*d)*arctan(-((b^6*c^8*d^2 - 6*a*b^5*c^7*d^3 + 15*a^2*b^4*c^6*d^4
- 20*a^3*b^3*c^5*d^5 + 15*a^4*b^2*c^4*d^6 - 6*a^5*b*c^3*d^7 + a^6*c^2*d^8)*
(-d^7/(b^8*c^11 - 8*a*b^7*c^10*d + 28*a^2*b^6*c^9*d^2 - 56*a^3*b^5*c^8*d^3
+ 70*a^4*b^4*c^7*d^4 - 56*a^5*b^3*c^6*d^5 + 28*a^6*b^2*c^5*d^6 - 8*a^7*b*c^
4*d^7 + a^8*c^3*d^8))^(3/4)*x - (b^6*c^8 - 6*a*b^5*c^7*d + 15*a^2*b^4*c^6*d
^2 - 20*a^3*b^3*c^5*d^3 + 15*a^4*b^2*c^4*d^4 - 6*a^5*b*c^3*d^5 + a^6*c^2*d^
6)*(-d^7/(b^8*c^11 - 8*a*b^7*c^10*d + 28*a^2*b^6*c^9*d^2 - 56*a^3*b^5*c^8*d
^3 + 70*a^4*b^4*c^7*d^4 - 56*a^5*b^3*c^6*d^5 + 28*a^6*b^2*c^5*d^6 - 8*a^7*b
*c^4*d^7 + a^8*c^3*d^8))^(3/4)*sqrt(d^4*x^2 + (b^4*c^6 - 4*a*b^3*c^5*d + 6*
a^2*b^2*c^4*d^2 - 4*a^3*b*c^3*d^3 + a^4*c^2*d^4)*sqrt(-d^7/(b^8*c^11 - 8*a*
b^7*c^10*d + 28*a^2*b^6*c^9*d^2 - 56*a^3*b^5*c^8*d^3 + 70*a^4*b^4*c^7*d^4 -
56*a^5*b^3*c^6*d^5 + 28*a^6*b^2*c^5*d^6 - 8*a^7*b*c^4*d^7 + a^8*c^3*d^8)))
)/d^7) - 4*(-d^7/(b^8*c^11 - 8*a*b^7*c^10*d + 28*a^2*b^6*c^9*d^2 - 56*a^3*b
^5*c^8*d^3 + 70*a^4*b^4*c^7*d^4 - 56*a^5*b^3*c^6*d^5 + 28*a^6*b^2*c^5*d^6 -
8*a^7*b*c^4*d^7 + a^8*c^3*d^8))^(1/4)*((a*b^2*c - a^2*b*d)*x^4 + a^2*b*c -
a^3*d)*log(d^2*x + (-d^7/(b^8*c^11 - 8*a*b^7*c^10*d + 28*a^2*b^6*c^9*d^2 -
56*a^3*b^5*c^8*d^3 + 70*a^4*b^4*c^7*d^4 - 56*a^5*b^3*c^6*d^5 + 28*a^6*b^2*
c^5*d^6 - 8*a^7*b*c^4*d^7 + a^8*c^3*d^8))^(1/4)*(b^2*c^3 - 2*a*b*c^2*d + a^
2*c*d^2)) + 4*(-d^7/(b^8*c^11 - 8*a*b^7*c^10*d + 28*a^2*b^6*c^9*d^2 - 56*a^
3*b^5*c^8*d^3 + 70*a^4*b^4*c^7*d^4 - 56*a^5*b^3*c^6*d^5 + 28*a^6*b^2*c^5*d^
6 - 8*a^7*b*c^4*d^7 + a^8*c^3*d^8))^(1/4)*((a*b^2*c - a^2*b*d)*x^4 + a^2*b*
c - a^3*d)*log(d^2*x - (-d^7/(b^8*c^11 - 8*a*b^7*c^10*d + 28*a^2*b^6*c^9*d^
2 - 56*a^3*b^5*c^8*d^3 + 70*a^4*b^4*c^7*d^4 - 56*a^5*b^3*c^6*d^5 + 28*a^6*b
^2*c^5*d^6 - 8*a^7*b*c^4*d^7 + a^8*c^3*d^8))^(1/4)*(b^2*c^3 - 2*a*b*c^2*d +
a^2*c*d^2)) + ((a*b^2*c - a^2*b*d)*x^4 + a^2*b*c - a^3*d)*(-(81*b^7*c^4 -
756*a*b^6*c^3*d + 2646*a^2*b^5*c^2*d^2 - 4116*a^3*b^4*c*d^3 + 2401*a^4*b^3*
d^4)/(a^7*b^8*c^8 - 8*a^8*b^7*c^7*d + 28*a^9*b^6*c^6*d^2 - 56*a^10*b^5*c^5*
d^3 + 70*a^11*b^4*c^4*d^4 - 56*a^12*b^3*c^3*d^5 + 28*a^13*b^2*c^2*d^6 - 8*a
^14*b*c*d^7 + a^15*d^8))^(1/4)*log(-(3*b^2*c - 7*a*b*d)*x + (a^2*b^2*c^2 -
2*a^3*b*c*d + a^4*d^2)*(-(81*b^7*c^4 - 756*a*b^6*c^3*d + 2646*a^2*b^5*c^2*d
^2 - 4116*a^3*b^4*c*d^3 + 2401*a^4*b^3*d^4)/(a^7*b^8*c^8 - 8*a^8*b^7*c^7*d
+ 28*a^9*b^6*c^6*d^2 - 56*a^10*b^5*c^5*d^3 + 70*a^11*b^4*c^4*d^4 - 56*a^12*
b^3*c^3*d^5 + 28*a^13*b^2*c^2*d^6 - 8*a^14*b*c*d^7 + a^15*d^8))^(1/4)) - ((
a*b^2*c - a^2*b*d)*x^4 + a^2*b*c - a^3*d)*(-(81*b^7*c^4 - 756*a*b^6*c^3*d +
2646*a^2*b^5*c^2*d^2 - 4116*a^3*b^4*c*d^3 + 2401*a^4*b^3*d^4)/(a^7*b^8*c^8
- 8*a^8*b^7*c^7*d + 28*a^9*b^6*c^6*d^2 - 56*a^10*b^5*c^5*d^3 + 70*a^11*b^4
*c^4*d^4 - 56*a^12*b^3*c^3*d^5 + 28*a^13*b^2*c^2*d^6 - 8*a^14*b*c*d^7 + a^1
5*d^8))^(1/4)*log(-(3*b^2*c - 7*a*b*d)*x - (a^2*b^2*c^2 - 2*a^3*b*c*d + a^4
d^2)(-(81*b^7*c^4 - 756*a*b^6*c^3*d + 2646*a^2*b^5*c^2*d^2 - 4116*a^3*b^4
*c*d^3 + 2401*a^4*b^3*d^4)/(a^7*b^8*c^8 - 8*a^8*b^7*c^7*d + 28*a^9*b^6*c^6*
d^2 - 56*a^10*b^5*c^5*d^3 + 70*a^11*b^4*c^4*d^4 - 56*a^12*b^3*c^3*d^5 + 28*
a^13*b^2*c^2*d^6 - 8*a^14*b*c*d^7 + a^15*d^8))^...

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**4+a)**2/(d*x**4+c),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.58, size = 667, normalized size = 1.30

$$\frac{(cd^3)^{\frac{1}{4}}\, d \arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}\, b^2c^3 - 2\sqrt{2}\, abc^2d + \sqrt{2}\, a^2cd^2\right)} + \frac{(cd^3)^{\frac{1}{4}}\, d \arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}\, b^2c^3 - 2\sqrt{2}\, abc^2d + \sqrt{2}\, a^2cd^2\right)} + \frac{(cd^3)^{\frac{1}{4}}\, d \log\left(x^2 + \sqrt{2}\, x\left(\frac{c}{d}\right)^{\frac{1}{4}} + \sqrt{\frac{c}{d}}\right)}{4\left(\sqrt{2}\, b^2c^3 - 2\sqrt{2}\, abc^2d + \sqrt{2}\, a^2cd^2\right)} - \frac{(cd^3)^{\frac{1}{4}}\, d \log\left(x^2 - \sqrt{2}\, x\left(\frac{c}{d}\right)^{\frac{1}{4}} + \sqrt{\frac{c}{d}}\right)}{4\left(\sqrt{2}\, b^2c^3 - 2\sqrt{2}\, abc^2d + \sqrt{2}\, a^2cd^2\right)} + \frac{\left(3\,(ab^3)^{\frac{1}{4}}\, bc - 7\,(ab^3)^{\frac{1}{4}}\, ad\right) \arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\left(\sqrt{2}\, a^2b^2c^2 - 2\sqrt{2}\, a^3bcd + \sqrt{2}\, a^4d^2\right)} + \frac{\left(3\,(ab^3)^{\frac{1}{4}}\, bc - 7\,(ab^3)^{\frac{1}{4}}\, ad\right) \arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\left(\sqrt{2}\, a^2b^2c^2 - 2\sqrt{2}\, a^3bcd + \sqrt{2}\, a^4d^2\right)} + \frac{\left(3\,(ab^3)^{\frac{1}{4}}\, bc - 7\,(ab^3)^{\frac{1}{4}}\, ad\right) \log\left(x^2 + \sqrt{2}\, x\left(\frac{a}{b}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{b}}\right)}{16\left(\sqrt{2}\, a^2b^2c^2 - 2\sqrt{2}\, a^3bcd + \sqrt{2}\, a^4d^2\right)} - \frac{\left(3\,(ab^3)^{\frac{1}{4}}\, bc - 7\,(ab^3)^{\frac{1}{4}}\, ad\right) \log\left(x^2 - \sqrt{2}\, x\left(\frac{a}{b}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{b}}\right)}{16\left(\sqrt{2}\, a^2b^2c^2 - 2\sqrt{2}\, a^3bcd + \sqrt{2}\, a^4d^2\right)} + \frac{bx}{4\,(bx^4+a)(abc-a^2d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^2/(d*x^4+c),x, algorithm="giac")
```

```
[Out] 1/2*(c*d^3)^(1/4)*d*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(c/d)^(1/4))/(c/d)^(1
/4))/(sqrt(2)*b^2*c^3 - 2*sqrt(2)*a*b*c^2*d + sqrt(2)*a^2*c*d^2) + 1/2*(c*d
^3)^(1/4)*d*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(c/d)^(1/4))/(c/d)^(1/4))/(sq
rt(2)*b^2*c^3 - 2*sqrt(2)*a*b*c^2*d + sqrt(2)*a^2*c*d^2) + 1/4*(c*d^3)^(1/4
)*d*log(x^2 + sqrt(2)*x*(c/d)^(1/4) + sqrt(c/d))/(sqrt(2)*b^2*c^3 - 2*sqrt(
2)*a*b*c^2*d + sqrt(2)*a^2*c*d^2) - 1/4*(c*d^3)^(1/4)*d*log(x^2 - sqrt(2)*x
*(c/d)^(1/4) + sqrt(c/d))/(sqrt(2)*b^2*c^3 - 2*sqrt(2)*a*b*c^2*d + sqrt(2)*
a^2*c*d^2) + 1/8*(3*(a*b^3)^(1/4)*b*c - 7*(a*b^3)^(1/4)*a*d)*arctan(1/2*sqr
t(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(sqrt(2)*a^2*b^2*c^2 - 2*sqrt
(2)*a^3*b*c*d + sqrt(2)*a^4*d^2) + 1/8*(3*(a*b^3)^(1/4)*b*c - 7*(a*b^3)^(1/
4)*a*d)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(sqrt(2
)*a^2*b^2*c^2 - 2*sqrt(2)*a^3*b*c*d + sqrt(2)*a^4*d^2) + 1/16*(3*(a*b^3)^(1
/4)*b*c - 7*(a*b^3)^(1/4)*a*d)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))
/(sqrt(2)*a^2*b^2*c^2 - 2*sqrt(2)*a^3*b*c*d + sqrt(2)*a^4*d^2) - 1/16*(3*(a
*b^3)^(1/4)*b*c - 7*(a*b^3)^(1/4)*a*d)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sq
rt(a/b))/(sqrt(2)*a^2*b^2*c^2 - 2*sqrt(2)*a^3*b*c*d + sqrt(2)*a^4*d^2) + 1/
4*b*x/((b*x^4 + a)*(a*b*c - a^2*d))
```

Mupad [B]
time = 3.82, size = 2500, normalized size = 4.87

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^4)^2*(c + d*x^4)),x)
```

```
[Out] 2*atan((((-(81*b^7*c^4 + 2401*a^4*b^3*d^4 - 4116*a^3*b^4*c*d^3 + 2646*a^2*b
^5*c^2*d^2 - 756*a*b^6*c^3*d)/(65536*a^15*d^8 + 65536*a^7*b^8*c^8 - 524288*
```

a^8*b^7*c^7*d + 1835008*a^9*b^6*c^6*d^2 - 3670016*a^10*b^5*c^5*d^3 + 458752
0*a^11*b^4*c^4*d^4 - 3670016*a^12*b^3*c^3*d^5 + 1835008*a^13*b^2*c^2*d^6 -
524288*a^14*b*c*d^7))^(1/4)*(((28*a^4*b^6*d^11 + (81*b^10*c^4*d^7)/16 - (67
5*a*b^9*c^3*d^8)/16 - (2145*a^3*b^7*c*d^10)/16 + (1971*a^2*b^8*c^2*d^9)/16)
*1i)/(a^7*d^3 - a^4*b^3*c^3 + 3*a^5*b^2*c^2*d - 3*a^6*b*c*d^2) + (-(81*b^7*
c^4 + 2401*a^4*b^3*d^4 - 4116*a^3*b^4*c*d^3 + 2646*a^2*b^5*c^2*d^2 - 756*a*
b^6*c^3*d)/(65536*a^15*d^8 + 65536*a^7*b^8*c^8 - 524288*a^8*b^7*c^7*d + 183
5008*a^9*b^6*c^6*d^2 - 3670016*a^10*b^5*c^5*d^3 + 4587520*a^11*b^4*c^4*d^4
- 3670016*a^12*b^3*c^3*d^5 + 1835008*a^13*b^2*c^2*d^6 - 524288*a^14*b*c*d^7
))^(3/4)*(((-(81*b^7*c^4 + 2401*a^4*b^3*d^4 - 4116*a^3*b^4*c*d^3 + 2646*a^2
*b^5*c^2*d^2 - 756*a*b^6*c^3*d)/(65536*a^15*d^8 + 65536*a^7*b^8*c^8 - 52428
8*a^8*b^7*c^7*d + 1835008*a^9*b^6*c^6*d^2 - 3670016*a^10*b^5*c^5*d^3 + 4587
520*a^11*b^4*c^4*d^4 - 3670016*a^12*b^3*c^3*d^5 + 1835008*a^13*b^2*c^2*d^6
- 524288*a^14*b*c*d^7))^(1/4)*(3072*a^4*b^14*c^11*d^4 - 4096*a^14*b^4*c*d^1
4 - 28672*a^5*b^13*c^10*d^5 + 114688*a^6*b^12*c^9*d^6 - 253952*a^7*b^11*c^8
*d^7 + 329728*a^8*b^10*c^7*d^8 - 229376*a^9*b^9*c^6*d^9 + 28672*a^10*b^8*c^
5*d^10 + 90112*a^11*b^7*c^4*d^11 - 78848*a^12*b^6*c^3*d^12 + 28672*a^13*b^5
*c^2*d^13))/(a^7*d^3 - a^4*b^3*c^3 + 3*a^5*b^2*c^2*d - 3*a^6*b*c*d^2) - (x*
(65536*a^15*b^4*d^17 - 524288*a^14*b^5*c*d^16 + 36864*a^2*b^17*c^13*d^4 - 4
66944*a^3*b^16*c^12*d^5 + 2609152*a^4*b^15*c^11*d^6 - 8486912*a^5*b^14*c^10
*d^7 + 17833984*a^6*b^13*c^9*d^8 - 25280512*a^7*b^12*c^8*d^9 + 24190976*a^8
*b^11*c^7*d^10 - 14516224*a^9*b^10*c^6*d^11 + 3362816*a^10*b^9*c^5*d^12 + 2
809856*a^11*b^8*c^4*d^13 - 3469312*a^12*b^7*c^3*d^14 + 1835008*a^13*b^6*c^2
*d^15)*1i)/(64*(a^10*d^6 + a^4*b^6*c^6 - 6*a^5*b^5*c^5*d + 15*a^6*b^4*c^4*d
^2 - 20*a^7*b^3*c^3*d^3 + 15*a^8*b^2*c^2*d^4 - 6*a^9*b*c*d^5)))*1i) + (x*(3
185*a^4*b^7*d^13 + 81*b^11*c^4*d^9 - 756*a*b^10*c^3*d^10 - 4788*a^3*b^8*c*d
^12 + 2790*a^2*b^9*c^2*d^11))/(64*(a^10*d^6 + a^4*b^6*c^6 - 6*a^5*b^5*c^5*d
 + 15*a^6*b^4*c^4*d^2 - 20*a^7*b^3*c^3*d^3 + 15*a^8*b^2*c^2*d^4 - 6*a^9*b*c
d^5)))(-(81*b^7*c^4 + 2401*a^4*b^3*d^4 - 4116*a^3*b^4*c*d^3 + 2646*a^2*b^
5*c^2*d^2 - 756*a*b^6*c^3*d)/(65536*a^15*d^8 + 65536*a^7*b^8*c^8 - 524288*a
^8*b^7*c^7*d + 1835008*a^9*b^6*c^6*d^2 - 3670016*a^10*b^5*c^5*d^3 + 4587520
*a^11*b^4*c^4*d^4 - 3670016*a^12*b^3*c^3*d^5 + 1835008*a^13*b^2*c^2*d^6 - 5
24288*a^14*b*c*d^7))^(1/4) - ((-(81*b^7*c^4 + 2401*a^4*b^3*d^4 - 4116*a^3*b
^4*c*d^3 + 2646*a^2*b^5*c^2*d^2 - 756*a*b^6*c^3*d)/(65536*a^15*d^8 + 65536*
a^7*b^8*c^8 - 524288*a^8*b^7*c^7*d + 1835008*a^9*b^6*c^6*d^2 - 3670016*a^10
*b^5*c^5*d^3 + 4587520*a^11*b^4*c^4*d^4 - 3670016*a^12*b^3*c^3*d^5 + 183500
8*a^13*b^2*c^2*d^6 - 524288*a^14*b*c*d^7))^(1/4)*(((28*a^4*b^6*d^11 + (81*b
^10*c^4*d^7)/16 - (675*a*b^9*c^3*d^8)/16 - (2145*a^3*b^7*c*d^10)/16 + (1971
*a^2*b^8*c^2*d^9)/16)*1i)/(a^7*d^3 - a^4*b^3*c^3 + 3*a^5*b^2*c^2*d - 3*a^6*
b*c*d^2) + (-(81*b^7*c^4 + 2401*a^4*b^3*d^4 - 4116*a^3*b^4*c*d^3 + 2646*a^2
*b^5*c^2*d^2 - 756*a*b^6*c^3*d)/(65536*a^15*d^8 + 65536*a^7*b^8*c^8 - 52428
8*a^8*b^7*c^7*d + 1835008*a^9*b^6*c^6*d^2 - 3670016*a^10*b^5*c^5*d^3 + 4587
520*a^11*b^4*c^4*d^4 - 3670016*a^12*b^3*c^3*d^5 + 1835008*a^13*b^2*c^2*d^6
- 524288*a^14*b*c*d^7))^(3/4)*(((-(81*b^7*c^4 + 2401*a^4*b^3*d^4 - 4116*a^3
*b^4*c*d^3 + 2646*a^2*b^5*c^2*d^2 - 756*a*b^6*c^3*d)/(65536*a^15*d^8 + 6553

```
6*a^7*b^8*c^8 - 524288*a^8*b^7*c^7*d + 1835008*a^9*b^6*c^6*d^2 - 3670016*a^
10*b^5*c^5*d^3 + 4587520*a^11*b^4*c^4*d^4 - 3670016*a^12*b^3*c^3*d^5 + 1835
008*a^13*b^2*c^2*d^6 - 524288*a^14*b*c*d^7))^(1/4)*(3072*a^4*b^14*c^11*d^4
- 4096*a^14*b^4*c*d^14 - 28672*a^5*b^13*c^10*d^5 + 114688*a^6*b^12*c^9*d^6
- 253952*a^7*b^11*c^8*d^7 + 329728*a^8*b^10*c^7*d^8 - 229376*a^9*b^9*c^6*d^
9 + 28672*a^10*b^8*c^5*d^10 + 90112*a^11*b^7*c^4*d^11 - 78848*a^12*b^6*c^3*
d^12 + 28672*a^13*b^5*c^2*d^13))/(a^7*d^3 - a^4*b^3*c^3 + 3*a^5*b^2*c^2*d -
 3*a^6*b*c*d^2) + (x*(65536*a^15*b^4*d^17 - 524288*a^14*b^5*c*d^16 + 36864*
a^2*b^17*c^13*d^4 - 466944*a^3*b^16*c^12*d^5 + 2609152*a^4*b^15*c^11*d^6 -
8486912*a^5*b^14*c^10*d^7 + 17833984*a^6*b^13*c^9*d^8 - 25280512*a^7*b^12*c
^8*d^9 + 24190976*a^8*b^11*c^7*d^10 - 14516224*a^9*b^10*c^6*d^11 + 3362816*
a^10*b^9*c^5*d^12 + 2809856*a^11*b^8*c^4*d^13 - 3469312*a^12*b^7*c^3*d^14 +
 1835008*a^13*b^6*c^2*d^15)*1i)/(64*(a^10*d^6 + a^4*b^6*c^6 - 6*a^5*b^5*c^5
*d + 15*a^6*b^4*c^4*d^2 - 20*a^7*b^3*c^3*d^3 + 15*a^8*b^2*c^2*d^4 - 6*a^9*b
*c*d^5)))*1i) - (x*(3185*a^4*b^7*d^13 + 81*b^11*c^4*d^9 - 756*a*b^10*c^3*d^
10 - 4788*a^3*b^8*c*d^12 + 2790*a^2*b^9*c^2*d^11))/(64*(a^10*d^6 + a^4*b^6*
c^6 - 6*a^5*b^5*c^5*d + 15*a^6*b^4*c^4*d^2 - 20*a^7*b^3*c^3*d^3 + 15*a^8*b^
2*c^2*d^4 - 6*a^9*b*c*d^5)))*(-(81*b^7*c^4 + 2401*a^4*b^3*d^4 - 4116*a^3*b^
4*c*d^3 + 2646*a^2*b^5*c^2*d^2 - 756*a*b^6*c^3*d)/(65536*a^15*d^8 + 65536*a
^7*b^8*c^8 - 524288*a^8*b^7*c^7*d + 1835008*a^9...
```

3.172 $\int \frac{1}{(a+bx^4)^2 (c+dx^4)^2} \, dx$

Optimal. Leaf size=596

$$\frac{d(bc+ad)x}{4ac(bc-ad)^2\left(c+dx^4\right)}+\frac{bx}{4a(bc-ad)\left(a+bx^4\right)\left(c+dx^4\right)}-\frac{b^{7/4}(3bc-11ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{8\sqrt{2}\,a^{7/4}(bc-ad)^3}+\frac{b^{7/4}(3}{}$$

[Out] 1/4*d*(a*d+b*c)*x/a/c/(-a*d+b*c)^2/(d*x^4+c)+1/4*b*x/a/(-a*d+b*c)/(b*x^4+a)/(d*x^4+c)+1/16*b^(7/4)*(-11*a*d+3*b*c)*arctan(-1+b^(1/4)*x*2^(1/2)/a^(1/4))/a^(7/4)/(-a*d+b*c)^3*2^(1/2)+1/16*b^(7/4)*(-11*a*d+3*b*c)*arctan(1+b^(1/4)*x*2^(1/2)/a^(1/4))/a^(7/4)/(-a*d+b*c)^3*2^(1/2)+1/16*d^(7/4)*(-3*a*d+11*b*c)*arctan(-1+d^(1/4)*x*2^(1/2)/c^(1/4))/c^(7/4)/(-a*d+b*c)^3*2^(1/2)+1/16*d^(7/4)*(-3*a*d+11*b*c)*arctan(1+d^(1/4)*x*2^(1/2)/c^(1/4))/c^(7/4)/(-a*d+b*c)^3*2^(1/2)-1/32*b^(7/4)*(-11*a*d+3*b*c)*ln(-a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))/a^(7/4)/(-a*d+b*c)^3*2^(1/2)+1/32*b^(7/4)*(-11*a*d+3*b*c)*ln(a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))/a^(7/4)/(-a*d+b*c)^3*2^(1/2)-1/32*d^(7/4)*(-3*a*d+11*b*c)*ln(-c^(1/4)*d^(1/4)*x*2^(1/2)+c^(1/2)+x^2*d^(1/2))/c^(7/4)/(-a*d+b*c)^3*2^(1/2)+1/32*d^(7/4)*(-3*a*d+11*b*c)*ln(c^(1/4)*d^(1/4)*x*2^(1/2)+c^(1/2)+x^2*d^(1/2))/c^(7/4)/(-a*d+b*c)^3*2^(1/2)

Rubi [A]
time = 0.50, antiderivative size = 596, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {425, 541, 536, 217, 1179, 642, 1176, 631, 210}

$$-\frac{b^{7/4}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)(3bc-11ad)}{8\sqrt{2}\,a^{7/4}(bc-ad)^3}+\frac{b^{7/4}\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}+1\right)(3bc-11ad)}{8\sqrt{2}\,a^{7/4}(bc-ad)^3}-\frac{b^{7/4}(3bc-11ad)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{16\sqrt{2}\,a^{7/4}(bc-ad)^3}+\frac{b^{7/4}(3bc-11ad)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{16\sqrt{2}\,a^{7/4}(bc-ad)^3}-\frac{d^{7/4}(11bc-3ad)\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right)}{8\sqrt{2}\,c^{7/4}(bc-ad)^3}+\frac{d^{7/4}(11bc-3ad)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{d}\,x}{\sqrt[4]{c}}+1\right)}{8\sqrt{2}\,c^{7/4}(bc-ad)^3}-\frac{d^{7/4}(11bc-3ad)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\,x+\sqrt{c}+\sqrt{d}\,x^2\right)}{16\sqrt{2}\,c^{7/4}(bc-ad)^3}+\frac{d^{7/4}(11bc-3ad)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\,x+\sqrt{c}+\sqrt{d}\,x^2\right)}{16\sqrt{2}\,c^{7/4}(bc-ad)^3}+\frac{bx}{4a\left(a+bx^4\right)\left(c+dx^4\right)(bc-ad)}+\frac{dx(ad+bc)}{4ac\left(c+dx^4\right)(bc-ad)^2}$$

Antiderivative was successfully verified.

[In] Int[1/((a + b*x^4)^2*(c + d*x^4)^2),x]

[Out] (d*(b*c + a*d)*x)/(4*a*c*(b*c - a*d)^2*(c + d*x^4)) + (b*x)/(4*a*(b*c - a*d)*(a + b*x^4)*(c + d*x^4)) - (b^(7/4)*(3*b*c - 11*a*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*(b*c - a*d)^3) + (b^(7/4)*(3*b*c - 11*a*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*(b*c - a*d)^3) - (d^(7/4)*(11*b*c - 3*a*d)*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/(8*Sqrt[2]*c^(7/4)*(b*c - a*d)^3) + (d^(7/4)*(11*b*c - 3*a*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/(8*Sqrt[2]*c^(7/4)*(b*c - a*d)^3) - (b^(7/4)*(3*b*c - 11*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)*(b*c - a*d)^3) + (b^(7/4)*(3*b*c - 11*a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)*(b*c - a*d)^3) - (d^(7/4)*(11*b*c - 3*a*d)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(16*Sqrt[2]*c^(7/4)*(b*c - a*d)^3) + (d^(7/4)*(11*b*c - 3*a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(16*Sqrt[2]*c^(7/4)*(b*c - a*d)^3)

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{(a+bx^4)^2 (c+dx^4)^2}\,dx = \frac{bx}{4a(bc-ad)(a+bx^4)(c+dx^4)} - \frac{\int \frac{-3bc+4ad-7bdx^4}{(a+bx^4)(c+dx^4)^2}\,dx}{4a(bc-ad)}$$

$$= \frac{d(bc+ad)x}{4ac(bc-ad)^2(c+dx^4)} + \frac{bx}{4a(bc-ad)(a+bx^4)(c+dx^4)} - \frac{\int \frac{-4(3b^2c^2-8abcd+3a}{(a+bx^4}}{16ac(b}$$

$$= \frac{d(bc+ad)x}{4ac(bc-ad)^2(c+dx^4)} + \frac{bx}{4a(bc-ad)(a+bx^4)(c+dx^4)} + \frac{(b^2(3bc-11ad))}{4a(bc-a}$$

$$= \frac{d(bc+ad)x}{4ac(bc-ad)^2(c+dx^4)} + \frac{bx}{4a(bc-ad)(a+bx^4)(c+dx^4)} + \frac{(b^2(3bc-11ad))}{8a^{3/2}(bc}$$

$$= \frac{d(bc+ad)x}{4ac(bc-ad)^2(c+dx^4)} + \frac{bx}{4a(bc-ad)(a+bx^4)(c+dx^4)} + \frac{(b^{3/2}(3bc-11ad)}{16a^}$$

$$= \frac{d(bc+ad)x}{4ac(bc-ad)^2(c+dx^4)} + \frac{bx}{4a(bc-ad)(a+bx^4)(c+dx^4)} - \frac{b^{7/4}(3bc-11ad)}{1}$$

$$= \frac{d(bc+ad)x}{4ac(bc-ad)^2(c+dx^4)} + \frac{bx}{4a(bc-ad)(a+bx^4)(c+dx^4)} - \frac{b^{7/4}(3bc-11ad)}{8\sqrt{2}\,a}$$

Mathematica [A]
time = 6.12, size = 629, normalized size = 1.06

$$\frac{b^2x}{4a(-bc+ad)^2(a+bx^4)} + \frac{d^2x}{4c(bc-ad)^2(c+dx^4)} - \frac{b^{7/4}(-3bc+11ad)\tan^{-1}\left(\frac{-\sqrt{2}\sqrt[4]{a}+2\sqrt[4]{b}x}{\sqrt{2}\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}(bc-ad)^3} - \frac{b^{7/4}(-3bc+11ad)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}+2\sqrt[4]{b}x}{\sqrt{2}\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}(bc-ad)^3} - \frac{d^{7/4}(11bc-3ad)\tan^{-1}\left(\frac{-\sqrt{2}\sqrt[4]{c}+2\sqrt[4]{d}x}{\sqrt{2}\sqrt[4]{c}}\right)}{8\sqrt{2}c^{7/4}(-bc+ad)^3} - \frac{d^{7/4}(11bc-3ad)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}+2\sqrt[4]{d}x}{\sqrt{2}\sqrt[4]{c}}\right)}{8\sqrt{2}c^{7/4}(-bc+ad)^3} + \frac{b^{7/4}(-3bc+11ad)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}(bc-ad)^3} - \frac{b^{7/4}(-3bc+11ad)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}(bc-ad)^3} + \frac{d^{7/4}(11bc-3ad)\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{d}x^2\right)}{16\sqrt{2}c^{7/4}(-bc+ad)^3} - \frac{d^{7/4}(11bc-3ad)\log\left(\sqrt{c}+\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{d}x^2\right)}{16\sqrt{2}c^{7/4}(-bc+ad)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^4)^2*(c + d*x^4)^2),x]
```

```
[Out] (b^2*x)/(4*a*(-(b*c) + a*d)^2*(a + b*x^4)) + (d^2*x)/(4*c*(b*c - a*d)^2*(c
+ d*x^4)) - (b^(7/4)*(-3*b*c + 11*a*d)*ArcTan[(-(Sqrt[2]*a^(1/4)) + 2*b^(1/
4)*x)/(Sqrt[2]*a^(1/4))])/(8*Sqrt[2]*a^(7/4)*(b*c - a*d)^3) - (b^(7/4)*(-3*
b*c + 11*a*d)*ArcTan[(Sqrt[2]*a^(1/4) + 2*b^(1/4)*x)/(Sqrt[2]*a^(1/4))])/(8
*Sqrt[2]*a^(7/4)*(b*c - a*d)^3) - (d^(7/4)*(11*b*c - 3*a*d)*ArcTan[(-(Sqrt[
2]*c^(1/4)) + 2*d^(1/4)*x)/(Sqrt[2]*c^(1/4))])/(8*Sqrt[2]*c^(7/4)*(-(b*c) +
 a*d)^3) - (d^(7/4)*(11*b*c - 3*a*d)*ArcTan[(Sqrt[2]*c^(1/4) + 2*d^(1/4)*x)
/(Sqrt[2]*c^(1/4))])/(8*Sqrt[2]*c^(7/4)*(-(b*c) + a*d)^3) + (b^(7/4)*(-3*b*
c + 11*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqr
t[2]*a^(7/4)*(b*c - a*d)^3) - (b^(7/4)*(-3*b*c + 11*a*d)*Log[Sqrt[a] + Sqrt
```

```
[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)*(b*c - a*d)^3) +
(d^(7/4)*(11*b*c - 3*a*d)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]
*x^2])/(16*Sqrt[2]*c^(7/4)*(-(b*c) + a*d)^3) - (d^(7/4)*(11*b*c - 3*a*d)*Lo
g[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(16*Sqrt[2]*c^(7/4)*(
-(b*c) + a*d)^3)
```

Maple [A]
time = 0.46, size = 298, normalized size = 0.50

method	result
default	$\frac{d^2\left(\frac{(ad-bc)x}{4c(d\,x^4+c)}+\frac{(3ad-11bc)\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{c}{d}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{c}{d}}}{x^2-\left(\frac{c}{d}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{c}{d}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)\right)}{32c^2}\right)}{(ad-bc)^3}+\frac{b^2\left(\frac{(\ldots}{4a\ldots}\right.}{\ldots}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^4+a)^2/(d*x^4+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] d^2/(a*d-b*c)^3*(1/4*(a*d-b*c)/c*x/(d*x^4+c)+1/32*(3*a*d-11*b*c)/c^2*(c/d)^
(1/4)*2^(1/2)*(ln((x^2+(c/d)^(1/4)*x*2^(1/2)+(c/d)^(1/2))/(x^2-(c/d)^(1/4)*
x*2^(1/2)+(c/d)^(1/2)))+2*arctan(2^(1/2)/(c/d)^(1/4)*x+1)+2*arctan(2^(1/2)/
(c/d)^(1/4)*x-1)))+b^2/(a*d-b*c)^3*(1/4*(a*d-b*c)/a*x/(b*x^4+a)+1/32*(11*a*
d-3*b*c)/a^2*(a/b)^(1/4)*2^(1/2)*(ln((x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)
)/(x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x+1
)+2*arctan(2^(1/2)/(a/b)^(1/4)*x-1)))
```

Maxima [A]
time = 0.54, size = 670, normalized size = 1.12

$$\frac{\left(\frac{2\sqrt{2}(3bc-11ad)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{2\sqrt{2}(3bc-11ad)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{\sqrt{2}(3bc-11ad)\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}-\frac{\sqrt{2}(3bc-11ad)\log\left(\sqrt{b}x^2-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}\right)b^2}{32\left(ab^3c^3-3a^2b^2c^2d+3a^3bcd^2-a^4d^3\right)}+\frac{(b^2cd+abd^2)x^5+(b^2c^2+a^2d^2)x}{4\left((ab^3c^3d-2a^2b^2c^2d^2+a^3bcd^3)x^8+a^2b^2c^4-2a^3bc^3d+a^4c^2d^2+(ab^3c^4-a^2b^2c^3d-a^3bc^2d^2+a^4cd^3)x^4\right)}+\frac{\frac{2\sqrt{2}(11bcd^2-3ad^3)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}x+\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}+\frac{2\sqrt{2}(11bcd^2-3ad^3)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}x-\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}+\frac{\sqrt{2}(11bcd^2-3ad^3)\log\left(\sqrt{d}x^2+\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}x+\sqrt{c}\right)}{c^{\frac{3}{4}}d^{\frac{1}{4}}}-\frac{\sqrt{2}(11bcd^2-3ad^3)\log\left(\sqrt{d}x^2-\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}x+\sqrt{c}\right)}{c^{\frac{3}{4}}d^{\frac{1}{4}}}}{32\left(b^3c^4-3ab^2c^3d+3a^2bc^2d^2-a^3cd^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^2/(d*x^4+c)^2,x, algorithm="maxima")
```

```
[Out] 1/32*(2*sqrt(2)*(3*b*c - 11*a*d)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*
a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2
*sqrt(2)*(3*b*c - 11*a*d)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)
*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*
(3*b*c - 11*a*d)*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^
(3/4)*b^(1/4)) - sqrt(2)*(3*b*c - 11*a*d)*log(sqrt(b)*x^2 - sqrt(2)*a^(1/4)
*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(1/4)))*b^2/(a*b^3*c^3 - 3*a^2*b^2*c^2*d +
 3*a^3*b*c*d^2 - a^4*d^3) + 1/4*((b^2*c*d + a*b*d^2)*x^5 + (b^2*c^2 + a^2*d
```

```
^2)*x)/((a*b^3*c^3*d - 2*a^2*b^2*c^2*d^2 + a^3*b*c*d^3)*x^8 + a^2*b^2*c^4 -
 2*a^3*b*c^3*d + a^4*c^2*d^2 + (a*b^3*c^4 - a^2*b^2*c^3*d - a^3*b*c^2*d^2 +
 a^4*c*d^3)*x^4) + 1/32*(2*sqrt(2)*(11*b*c*d^2 - 3*a*d^3)*arctan(1/2*sqrt(2
)*(2*sqrt(d)*x + sqrt(2)*c^(1/4)*d^(1/4))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*s
qrt(sqrt(c)*sqrt(d))) + 2*sqrt(2)*(11*b*c*d^2 - 3*a*d^3)*arctan(1/2*sqrt(2)
*(2*sqrt(d)*x - sqrt(2)*c^(1/4)*d^(1/4))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sq
rt(sqrt(c)*sqrt(d))) + sqrt(2)*(11*b*c*d^2 - 3*a*d^3)*log(sqrt(d)*x^2 + sqr
t(2)*c^(1/4)*d^(1/4)*x + sqrt(c))/(c^(3/4)*d^(1/4)) - sqrt(2)*(11*b*c*d^2 -
 3*a*d^3)*log(sqrt(d)*x^2 - sqrt(2)*c^(1/4)*d^(1/4)*x + sqrt(c))/(c^(3/4)*d
^(1/4)))/(b^3*c^4 - 3*a*b^2*c^3*d + 3*a^2*b*c^2*d^2 - a^3*c*d^3)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^2/(d*x^4+c)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**4+a)**2/(d*x**4+c)**2,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 967 vs. 2(462) = 924.
time = 0.69, size = 967, normalized size = 1.62

$$\frac{\left(3\,(ab^3)^{\frac{1}{4}}\,b^2c - 11\,(ab^3)^{\frac{1}{4}}\,abd\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\,(\frac{a}{b})^{\frac{1}{4}}\right)}{2\,(\frac{a}{b})^{\frac{1}{4}}}\right)}{8\left(\sqrt{2}\,a^2b^3c^3 - 3\sqrt{2}\,a^3b^2c^2d + 3\sqrt{2}\,a^4bcd^2 - \sqrt{2}\,a^5d^3\right)} + \frac{\left(3\,(ab^3)^{\frac{1}{4}}\,b^2c - 11\,(ab^3)^{\frac{1}{4}}\,abd\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\,(\frac{a}{b})^{\frac{1}{4}}\right)}{2\,(\frac{a}{b})^{\frac{1}{4}}}\right)}{8\left(\sqrt{2}\,a^2b^3c^3 - 3\sqrt{2}\,a^3b^2c^2d + 3\sqrt{2}\,a^4bcd^2 - \sqrt{2}\,a^5d^3\right)} + \frac{\left(11\,(cd^3)^{\frac{1}{4}}\,bcd - 3\,(cd^3)^{\frac{1}{4}}\,ad^2\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\,(\frac{c}{d})^{\frac{1}{4}}\right)}{2\,(\frac{c}{d})^{\frac{1}{4}}}\right)}{8\left(\sqrt{2}\,b^3c^5 - 3\sqrt{2}\,ab^2c^4d + 3\sqrt{2}\,a^2bc^3d^2 - \sqrt{2}\,a^3c^2d^3\right)} + \frac{\left(11\,(cd^3)^{\frac{1}{4}}\,bcd - 3\,(cd^3)^{\frac{1}{4}}\,ad^2\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\,(\frac{c}{d})^{\frac{1}{4}}\right)}{2\,(\frac{c}{d})^{\frac{1}{4}}}\right)}{8\left(\sqrt{2}\,b^3c^5 - 3\sqrt{2}\,ab^2c^4d + 3\sqrt{2}\,a^2bc^3d^2 - \sqrt{2}\,a^3c^2d^3\right)} + \frac{\left(3\,(ab^3)^{\frac{1}{4}}\,b^2c - 11\,(ab^3)^{\frac{1}{4}}\,abd\right)\log\left(x^2 + \sqrt{2}\,x(\frac{a}{b})^{\frac{1}{4}} + \sqrt{\frac{a}{b}}\right)}{16\left(\sqrt{2}\,a^2b^3c^3 - 3\sqrt{2}\,a^3b^2c^2d + 3\sqrt{2}\,a^4bcd^2 - \sqrt{2}\,a^5d^3\right)} - \frac{\left(3\,(ab^3)^{\frac{1}{4}}\,b^2c - 11\,(ab^3)^{\frac{1}{4}}\,abd\right)\log\left(x^2 - \sqrt{2}\,x(\frac{a}{b})^{\frac{1}{4}} + \sqrt{\frac{a}{b}}\right)}{16\left(\sqrt{2}\,a^2b^3c^3 - 3\sqrt{2}\,a^3b^2c^2d + 3\sqrt{2}\,a^4bcd^2 - \sqrt{2}\,a^5d^3\right)} + \frac{\left(11\,(cd^3)^{\frac{1}{4}}\,bcd - 3\,(cd^3)^{\frac{1}{4}}\,ad^2\right)\log\left(x^2 + \sqrt{2}\,x(\frac{c}{d})^{\frac{1}{4}} + \sqrt{\frac{c}{d}}\right)}{16\left(\sqrt{2}\,b^3c^5 - 3\sqrt{2}\,ab^2c^4d + 3\sqrt{2}\,a^2bc^3d^2 - \sqrt{2}\,a^3c^2d^3\right)} - \frac{\left(11\,(cd^3)^{\frac{1}{4}}\,bcd - 3\,(cd^3)^{\frac{1}{4}}\,ad^2\right)\log\left(x^2 - \sqrt{2}\,x(\frac{c}{d})^{\frac{1}{4}} + \sqrt{\frac{c}{d}}\right)}{16\left(\sqrt{2}\,b^3c^5 - 3\sqrt{2}\,ab^2c^4d + 3\sqrt{2}\,a^2bc^3d^2 - \sqrt{2}\,a^3c^2d^3\right)} + \frac{b^2cdx^5 + abd^2x^5 + b^2c^2x + a^2d^2x}{4\,(bdx^8 + bcx^4 + adx^4 + ac)(ab^2c^3 - 2\,a^2bc^2d + a^3cd^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^2/(d*x^4+c)^2,x, algorithm="giac")
```

```
[Out] 1/8*(3*(a*b^3)^(1/4)*b^2*c - 11*(a*b^3)^(1/4)*a*b*d)*arctan(1/2*sqrt(2)*(2*
x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(sqrt(2)*a^2*b^3*c^3 - 3*sqrt(2)*a^3*
b^2*c^2*d + 3*sqrt(2)*a^4*b*c*d^2 - sqrt(2)*a^5*d^3) + 1/8*(3*(a*b^3)^(1/4)
*b^2*c - 11*(a*b^3)^(1/4)*a*b*d)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1
/4))/(a/b)^(1/4))/(sqrt(2)*a^2*b^3*c^3 - 3*sqrt(2)*a^3*b^2*c^2*d + 3*sqrt(2
)*a^4*b*c*d^2 - sqrt(2)*a^5*d^3) + 1/8*(11*(c*d^3)^(1/4)*b*c*d - 3*(c*d^3)^
```

```
(1/4)*a*d^2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(c/d)^(1/4))/(c/d)^(1/4))/(s
qrt(2)*b^3*c^5 - 3*sqrt(2)*a*b^2*c^4*d + 3*sqrt(2)*a^2*b*c^3*d^2 - sqrt(2)*
a^3*c^2*d^3) + 1/8*(11*(c*d^3)^(1/4)*b*c*d - 3*(c*d^3)^(1/4)*a*d^2)*arctan(
1/2*sqrt(2)*(2*x - sqrt(2)*(c/d)^(1/4))/(c/d)^(1/4))/(sqrt(2)*b^3*c^5 - 3*s
qrt(2)*a*b^2*c^4*d + 3*sqrt(2)*a^2*b*c^3*d^2 - sqrt(2)*a^3*c^2*d^3) + 1/16*
(3*(a*b^3)^(1/4)*b^2*c - 11*(a*b^3)^(1/4)*a*b*d)*log(x^2 + sqrt(2)*x*(a/b)^
(1/4) + sqrt(a/b))/(sqrt(2)*a^2*b^3*c^3 - 3*sqrt(2)*a^3*b^2*c^2*d + 3*sqrt(
2)*a^4*b*c*d^2 - sqrt(2)*a^5*d^3) - 1/16*(3*(a*b^3)^(1/4)*b^2*c - 11*(a*b^3
)^(1/4)*a*b*d)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(sqrt(2)*a^2*b^
3*c^3 - 3*sqrt(2)*a^3*b^2*c^2*d + 3*sqrt(2)*a^4*b*c*d^2 - sqrt(2)*a^5*d^3)
+ 1/16*(11*(c*d^3)^(1/4)*b*c*d - 3*(c*d^3)^(1/4)*a*d^2)*log(x^2 + sqrt(2)*x
*(c/d)^(1/4) + sqrt(c/d))/(sqrt(2)*b^3*c^5 - 3*sqrt(2)*a*b^2*c^4*d + 3*sqrt
(2)*a^2*b*c^3*d^2 - sqrt(2)*a^3*c^2*d^3) - 1/16*(11*(c*d^3)^(1/4)*b*c*d - 3
*(c*d^3)^(1/4)*a*d^2)*log(x^2 - sqrt(2)*x*(c/d)^(1/4) + sqrt(c/d))/(sqrt(2)
*b^3*c^5 - 3*sqrt(2)*a*b^2*c^4*d + 3*sqrt(2)*a^2*b*c^3*d^2 - sqrt(2)*a^3*c^
2*d^3) + 1/4*(b^2*c*d*x^5 + a*b*d^2*x^5 + b^2*c^2*x + a^2*d^2*x)/((b*d*x^8
+ b*c*x^4 + a*d*x^4 + a*c)*(a*b^2*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2))
```

Mupad [B]
time = 5.62, size = 2500, normalized size = 4.19

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^4)^2*(c + d*x^4)^2),x)
```

```
[Out] ((x*(a^2*d^2 + b^2*c^2))/(4*a*c*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) + (b*d*x^5
*(a*d + b*c))/(4*a*c*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)))/(a*c + x^4*(a*d + b*
c) + b*d*x^8) - atan(((-(81*a^4*d^11 + 14641*b^4*c^4*d^7 - 15972*a*b^3*c^3*
d^8 + 6534*a^2*b^2*c^2*d^9 - 1188*a^3*b*c*d^10)/(65536*b^12*c^19 + 65536*a^
12*c^7*d^12 - 786432*a^11*b*c^8*d^11 + 4325376*a^2*b^10*c^17*d^2 - 14417920
*a^3*b^9*c^16*d^3 + 32440320*a^4*b^8*c^15*d^4 - 51904512*a^5*b^7*c^14*d^5 +
 60555264*a^6*b^6*c^13*d^6 - 51904512*a^7*b^5*c^12*d^7 + 32440320*a^8*b^4*c
^11*d^8 - 14417920*a^9*b^3*c^10*d^9 + 4325376*a^10*b^2*c^9*d^10 - 786432*a*
b^11*c^18*d))^(1/4)*((-(81*a^4*d^11 + 14641*b^4*c^4*d^7 - 15972*a*b^3*c^3*d
^8 + 6534*a^2*b^2*c^2*d^9 - 1188*a^3*b*c*d^10)/(65536*b^12*c^19 + 65536*a^1
2*c^7*d^12 - 786432*a^11*b*c^8*d^11 + 4325376*a^2*b^10*c^17*d^2 - 14417920*
a^3*b^9*c^16*d^3 + 32440320*a^4*b^8*c^15*d^4 - 51904512*a^5*b^7*c^14*d^5 +
60555264*a^6*b^6*c^13*d^6 - 51904512*a^7*b^5*c^12*d^7 + 32440320*a^8*b^4*c^
11*d^8 - 14417920*a^9*b^3*c^10*d^9 + 4325376*a^10*b^2*c^9*d^10 - 786432*a*b
^11*c^18*d))^(1/4)*(((891*a^8*b^7*d^15)/64 + (891*b^15*c^8*d^7)/64 - (3105*
a*b^14*c^7*d^8)/16 - (3105*a^7*b^8*c*d^14)/16 + (31509*a^2*b^13*c^6*d^9)/32
 - (33069*a^3*b^12*c^5*d^10)/16 + (60307*a^4*b^11*c^4*d^11)/32 - (33069*a^5
*b^10*c^3*d^12)/16 + (31509*a^6*b^9*c^2*d^13)/32)/(a^4*b^8*c^12 + a^12*c^4*
d^8 - 8*a^5*b^7*c^11*d - 8*a^11*b*c^5*d^7 + 28*a^6*b^6*c^10*d^2 - 56*a^7*b^
```

5*c^9*d^3 + 70*a^8*b^4*c^8*d^4 - 56*a^9*b^3*c^7*d^5 + 28*a^10*b^2*c^6*d^6)
+ (-(81*a^4*d^11 + 14641*b^4*c^4*d^7 - 15972*a*b^3*c^3*d^8 + 6534*a^2*b^2*c
^2*d^9 - 1188*a^3*b*c*d^10)/(65536*b^12*c^19 + 65536*a^12*c^7*d^12 - 786432
*a^11*b*c^8*d^11 + 4325376*a^2*b^10*c^17*d^2 - 14417920*a^3*b^9*c^16*d^3 +
32440320*a^4*b^8*c^15*d^4 - 51904512*a^5*b^7*c^14*d^5 + 60555264*a^6*b^6*c^
13*d^6 - 51904512*a^7*b^5*c^12*d^7 + 32440320*a^8*b^4*c^11*d^8 - 14417920*a
^9*b^3*c^10*d^9 + 4325376*a^10*b^2*c^9*d^10 - 786432*a*b^11*c^18*d))^(3/4)*
((x*(589824*a^2*b^23*c^21*d^4 - 11403264*a^3*b^22*c^20*d^5 + 98762752*a^4*b
^21*c^19*d^6 - 510394368*a^5*b^20*c^18*d^7 + 1766916096*a^6*b^19*c^17*d^8 -
4344840192*a^7*b^18*c^16*d^9 + 7796490240*a^8*b^17*c^15*d^10 - 10168369152
*a^9*b^16*c^14*d^11 + 9007726592*a^10*b^15*c^13*d^12 - 3635478528*a^11*b^14
*c^12*d^13 - 3635478528*a^12*b^13*c^11*d^14 + 9007726592*a^13*b^12*c^10*d^1
5 - 10168369152*a^14*b^11*c^9*d^16 + 7796490240*a^15*b^10*c^8*d^17 - 434484
0192*a^16*b^9*c^7*d^18 + 1766916096*a^17*b^8*c^6*d^19 - 510394368*a^18*b^7*
c^5*d^20 + 98762752*a^19*b^6*c^4*d^21 - 11403264*a^20*b^5*c^3*d^22 + 589824
*a^21*b^4*c^2*d^23))/(1024*(a^4*b^12*c^16 + a^16*c^4*d^12 - 12*a^5*b^11*c^1
5*d - 12*a^15*b*c^5*d^11 + 66*a^6*b^10*c^14*d^2 - 220*a^7*b^9*c^13*d^3 + 49
5*a^8*b^8*c^12*d^4 - 792*a^9*b^7*c^11*d^5 + 924*a^10*b^6*c^10*d^6 - 792*a^1
1*b^5*c^9*d^7 + 495*a^12*b^4*c^8*d^8 - 220*a^13*b^3*c^7*d^9 + 66*a^14*b^2*c
^6*d^10)) + ((-(81*a^4*d^11 + 14641*b^4*c^4*d^7 - 15972*a*b^3*c^3*d^8 + 653
4*a^2*b^2*c^2*d^9 - 1188*a^3*b*c*d^10)/(65536*b^12*c^19 + 65536*a^12*c^7*d^
12 - 786432*a^11*b*c^8*d^11 + 4325376*a^2*b^10*c^17*d^2 - 14417920*a^3*b^9*
c^16*d^3 + 32440320*a^4*b^8*c^15*d^4 - 51904512*a^5*b^7*c^14*d^5 + 60555264
*a^6*b^6*c^13*d^6 - 51904512*a^7*b^5*c^12*d^7 + 32440320*a^8*b^4*c^11*d^8 -
14417920*a^9*b^3*c^10*d^9 + 4325376*a^10*b^2*c^9*d^10 - 786432*a*b^11*c^18
d))^(1/4)(3072*a^4*b^19*c^19*d^4 - 45056*a^5*b^18*c^18*d^5 + 292864*a^6*b
^17*c^17*d^6 - 1115136*a^7*b^16*c^16*d^7 + 2745344*a^8*b^15*c^15*d^8 - 4483
072*a^9*b^14*c^14*d^9 + 4595712*a^10*b^13*c^13*d^10 - 1993728*a^11*b^12*c^1
2*d^11 - 1993728*a^12*b^11*c^11*d^12 + 4595712*a^13*b^10*c^10*d^13 - 448307
2*a^14*b^9*c^9*d^14 + 2745344*a^15*b^8*c^8*d^15 - 1115136*a^16*b^7*c^7*d^16
+ 292864*a^17*b^6*c^6*d^17 - 45056*a^18*b^5*c^5*d^18 + 3072*a^19*b^4*c^4*d
^19))/(a^4*b^8*c^12 + a^12*c^4*d^8 - 8*a^5*b^7*c^11*d - 8*a^11*b*c^5*d^7 +
28*a^6*b^6*c^10*d^2 - 56*a^7*b^5*c^9*d^3 + 70*a^8*b^4*c^8*d^4 - 56*a^9*b^3*
c^7*d^5 + 28*a^10*b^2*c^6*d^6))*1i + (x*(9801*a^8*b^9*d^17 + 9801*b^17*c^8
*d^9 - 149094*a*b^16*c^7*d^10 - 149094*a^7*b^10*c*d^16 + 1001520*a^2*b^15*c
^6*d^11 - 3484602*a^3*b^14*c^5*d^12 + 5769038*a^4*b^13*c^4*d^13 - 3484602*a
^5*b^12*c^3*d^14 + 1001520*a^6*b^11*c^2*d^15)*1i)/(1024*(a^4*b^12*c^16 + a^
16*c^4*d^12 - 12*a^5*b^11*c^15*d - 12*a^15*b*c^5*d^11 + 66*a^6*b^10*c^14*d^
2 - 220*a^7*b^9*c^13*d^3 + 495*a^8*b^8*c^12*d^4 - 792*a^9*b^7*c^11*d^5 + 92
4*a^10*b^6*c^10*d^6 - 792*a^11*b^5*c^9*d^7 + 495*a^12*b^4*c^8*d^8 - 220*a^1
3*b^3*c^7*d^9 + 66*a^14*b^2*c^6*d^10))) - (-(81*a^4*d^11 + 14641*b^4*c^4*d^
7 - 15972*a*b^3*c^3*d^8 + 6534*a^2*b^2*c^2*d^9 - 1188*a^3*b*c*d^10)/(65536*
b^12*c^19 + 65536*a^12*c^7*d^12 - 786432*a^11*b*c^8*d^11 + 4325376*a^2*b^10
*c^17*d^2 - 14417920*a^3*b^9*c^16*d^3 + 32440320*a^4*b^8*c^15*d^4 - 5190451
2*a^5*b^7*c^14*d^5 + 60555264*a^6*b^6*c^13*d^6 - 51904512*a^7*b^5*c^12*d^7

```
+ 32440320*a^8*b^4*c^11*d^8 - 14417920*a^9*b^3*c^10*d^9 + 4325376*a^10*b^2*
c^9*d^10 - 786432*a*b^11*c^18*d))^(1/4)*((-(81*a^4*d^11 + 14641*b^4*c^4*d^7
 - 15972*a*b^3*c^3*d^8 + 6534*a^2*b^2*c^2*d^9 -...
```

3.173 $\int \frac{(a-bx^4)^{5/2}}{c-dx^4}\,dx$

Optimal. Leaf size=321

$$-\frac{b(7bc-13ad)x\sqrt{a-bx^4}}{21d^2}+\frac{bx(a-bx^4)^{3/2}}{7d}+\frac{\sqrt[4]{a}\,b^{3/4}(21b^2c^2-56abcd+47a^2d^2)\sqrt{1-\frac{bx^4}{a}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}}{\sqrt[4]{a}}\right.\right.}{21d^3\sqrt{a-bx^4}}$$

```
[Out] 1/7*b*x*(-b*x^4+a)^(3/2)/d-1/21*b*(-13*a*d+7*b*c)*x*(-b*x^4+a)^(1/2)/d^2+1/
21*a^(1/4)*b^(3/4)*(47*a^2*d^2-56*a*b*c*d+21*b^2*c^2)*EllipticF(b^(1/4)*x/a
^(1/4),I)*(1-b*x^4/a)^(1/2)/d^3/(-b*x^4+a)^(1/2)-1/2*a^(1/4)*(-a*d+b*c)^3*E
llipticPi(b^(1/4)*x/a^(1/4),-a^(1/2)*d^(1/2)/b^(1/2)/c^(1/2),I)*(1-b*x^4/a)
^(1/2)/b^(1/4)/c/d^3/(-b*x^4+a)^(1/2)-1/2*a^(1/4)*(-a*d+b*c)^3*EllipticPi(b
^(1/4)*x/a^(1/4),a^(1/2)*d^(1/2)/b^(1/2)/c^(1/2),I)*(1-b*x^4/a)^(1/2)/b^(1/
4)/c/d^3/(-b*x^4+a)^(1/2)
```

Rubi [A]
time = 0.26, antiderivative size = 321, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {427, 542, 537, 230, 227, 418, 1233, 1232}

$$\frac{\sqrt[4]{a}\,b^{3/4}\sqrt{1-\frac{bx^4}{a}}\,(47a^2d^2-56abcd+21b^2c^2)\,F\left(\text{ArcSin}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{21d^3\sqrt{a-bx^4}}-\frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}\,(bc-ad)^3\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}};\text{ArcSin}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2\sqrt[4]{b}\,cd^3\sqrt{a-bx^4}}-\frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}\,(bc-ad)^3\Pi\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}};\text{ArcSin}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2\sqrt[4]{b}\,cd^3\sqrt{a-bx^4}}-\frac{bx\sqrt{a-bx^4}\,(7bc-13ad)}{21d^2}+\frac{bx(a-bx^4)^{3/2}}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^4)^(5/2)/(c - d*x^4),x]
```

```
[Out] -1/21*(b*(7*b*c - 13*a*d)*x*Sqrt[a - b*x^4])/d^2 + (b*x*(a - b*x^4)^(3/2))/
(7*d) + (a^(1/4)*b^(3/4)*(21*b^2*c^2 - 56*a*b*c*d + 47*a^2*d^2)*Sqrt[1 - (b
*x^4)/a]*EllipticF[ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(21*d^3*Sqrt[a - b*x^4
]) - (a^(1/4)*(b*c - a*d)^3*Sqrt[1 - (b*x^4)/a]*EllipticPi[-((Sqrt[a]*Sqrt[
d])/(Sqrt[b]*Sqrt[c])), ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(2*b^(1/4)*c*d^3*
Sqrt[a - b*x^4]) - (a^(1/4)*(b*c - a*d)^3*Sqrt[1 - (b*x^4)/a]*EllipticPi[(S
qrt[a]*Sqrt[d])/(Sqrt[b]*Sqrt[c]), ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(2*b^(
1/4)*c*d^3*Sqrt[a - b*x^4])
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
```

```
b/a] &&  !GtQ[a, 0]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Dist[1/(2*
c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 1233

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{(a-bx^4)^{5/2}}{c-dx^4}\,dx = \frac{bx(a-bx^4)^{3/2}}{7d} - \frac{\int \frac{\sqrt{a-bx^4}\,(a(bc-7ad)-b(7bc-13ad)x^4)}{c-dx^4}\,dx}{7d}$$

$$= -\frac{b(7bc-13ad)x\sqrt{a-bx^4}}{21d^2} + \frac{bx(a-bx^4)^{3/2}}{7d} + \frac{\int \frac{a(7b^2c^2-16abcd+21a^2d^2)-b(21b^2c^2-56abcd+47}{\sqrt{a-bx^4}\,(c-dx^4)}}{21d^2}$$

$$= -\frac{b(7bc-13ad)x\sqrt{a-bx^4}}{21d^2} + \frac{bx(a-bx^4)^{3/2}}{7d} - \frac{(bc-ad)^3\int \frac{1}{\sqrt{a-bx^4}\,(c-dx^4)}\,dx}{d^3} + \frac{(b(}{}$$

$$= -\frac{b(7bc-13ad)x\sqrt{a-bx^4}}{21d^2} + \frac{bx(a-bx^4)^{3/2}}{7d} - \frac{(bc-ad)^3\int \frac{1}{\left(1-\frac{\sqrt{d}\,x^2}{\sqrt{c}}\right)\sqrt{a-bx^4}}\,dx}{2cd^3}$$

$$= -\frac{b(7bc-13ad)x\sqrt{a-bx^4}}{21d^2} + \frac{bx(a-bx^4)^{3/2}}{7d} + \frac{\sqrt[4]{a}\,b^{3/4}(21b^2c^2-56abcd+47a^2d^2)\sqrt{}}{21d^3\sqrt{a-}}$$

$$= -\frac{b(7bc-13ad)x\sqrt{a-bx^4}}{21d^2} + \frac{bx(a-bx^4)^{3/2}}{7d} + \frac{\sqrt[4]{a}\,b^{3/4}(21b^2c^2-56abcd+47a^2d^2)\sqrt{}}{21d^3\sqrt{a-}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 10.49, size = 290, normalized size = 0.90

$$\frac{x\left(5b(-a+bx^4)(7bc-16ad+3bdx^4)-\frac{b(21b^2c^2-56abcd+47a^2d^2)x^4\sqrt{1-\frac{bx^4}{a}}\,F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)}{c}+\frac{25a^2c(7b^2c^2-16abcd+21a^2d^2)F_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)}{(c-dx^4)\left(5acF_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)+2x^4\left(2adF_1\left(\frac{5}{4};\frac{1}{2},2;\frac{9}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)+bcF_1\left(\frac{5}{4};\frac{3}{2},1;\frac{9}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)\right)\right)}\right)}{105d^2\sqrt{a-bx^4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a - b*x^4)^(5/2)/(c - d*x^4),x]
```

```
[Out] (x*(5*b*(-a + b*x^4)*(7*b*c - 16*a*d + 3*b*d*x^4) - (b*(21*b^2*c^2 - 56*a*b
*c*d + 47*a^2*d^2)*x^4*Sqrt[1 - (b*x^4)/a]*AppellF1[5/4, 1/2, 1, 9/4, (b*x^
4)/a, (d*x^4)/c])/c + (25*a^2*c*(7*b^2*c^2 - 16*a*b*c*d + 21*a^2*d^2)*Appel
lF1[1/4, 1/2, 1, 5/4, (b*x^4)/a, (d*x^4)/c])/((c - d*x^4)*(5*a*c*AppellF1[1
/4, 1/2, 1, 5/4, (b*x^4)/a, (d*x^4)/c] + 2*x^4*(2*a*d*AppellF1[5/4, 1/2, 2,
 9/4, (b*x^4)/a, (d*x^4)/c] + b*c*AppellF1[5/4, 3/2, 1, 9/4, (b*x^4)/a, (d*
x^4)/c])))))/(105*d^2*Sqrt[a - b*x^4])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.34, size = 408, normalized size = 1.27

method	result
risch	$\frac{bx\left(-3bd\,x^{4}+16ad-7bc\right)\sqrt{-b\,x^{4}+a}}{21d^{2}}+\frac{\frac{b\left(47a^{2}d^{2}-56abcd+21b^{2}c^{2}\right)\sqrt{1-\frac{x^{2}\sqrt{b}}{\sqrt{a}}}\,\sqrt{1+\frac{x^{2}\sqrt{b}}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\right.}{d\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^{4}+a}}}{}$
default	$-\frac{b^{2}x^{5}\sqrt{-b\,x^{4}+a}}{7d}-\frac{\left(-\frac{b^{2}(3ad-bc)}{d^{2}}+\frac{5b^{2}a}{7d}\right)x\sqrt{-b\,x^{4}+a}}{3b}+\frac{\left(\frac{b\left(3a^{2}d^{2}-3abcd+b^{2}c^{2}\right)}{d^{3}}+\frac{\left(-\frac{b^{2}(3ad-bc)}{d^{2}}+\frac{5b^{2}a}{7d}\right)a}{3b}\right)\sqrt{1}}{\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}}$
elliptic	$-\frac{b^{2}x^{5}\sqrt{-b\,x^{4}+a}}{7d}-\frac{\left(-\frac{b^{2}(3ad-bc)}{d^{2}}+\frac{5b^{2}a}{7d}\right)x\sqrt{-b\,x^{4}+a}}{3b}+\frac{\left(\frac{b\left(3a^{2}d^{2}-3abcd+b^{2}c^{2}\right)}{d^{3}}+\frac{\left(-\frac{b^{2}(3ad-bc)}{d^{2}}+\frac{5b^{2}a}{7d}\right)a}{3b}\right)\sqrt{1}}{\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^4+a)^(5/2)/(-d*x^4+c),x,method=_RETURNVERBOSE)
```

```
[Out] -1/7*b^2/d*x^5*(-b*x^4+a)^(1/2)-1/3*(-b^2/d^2*(3*a*d-b*c)+5/7*b^2/d*a)/b*x*
(-b*x^4+a)^(1/2)+(b*(3*a^2*d^2-3*a*b*c*d+b^2*c^2)/d^3+1/3*(-b^2/d^2*(3*a*d-
b*c)+5/7*b^2/d*a)/b*a)/(1/a^(1/2)*b^(1/2))^(1/2)*(1-x^2*b^(1/2)/a^(1/2))^(1
/2)*(1+x^2*b^(1/2)/a^(1/2))^(1/2)/(-b*x^4+a)^(1/2)*EllipticF(x*(1/a^(1/2)*b
^(1/2))^(1/2),I)-1/8/d^4*sum((a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/
_alpha^3*(-1/((a*d-b*c)/d)^(1/2)*arctanh(1/2*(-2*_alpha^2*b*x^2+2*a)/((a*d-
b*c)/d)^(1/2)/(-b*x^4+a)^(1/2))-2/(1/a^(1/2)*b^(1/2))^(1/2)*_alpha^3*d/c*(1
-x^2*b^(1/2)/a^(1/2))^(1/2)*(1+x^2*b^(1/2)/a^(1/2))^(1/2)/(-b*x^4+a)^(1/2)*
EllipticPi(x*(1/a^(1/2)*b^(1/2))^(1/2),a^(1/2)/b^(1/2)*_alpha^2/c*d,(-1/a^(
1/2)*b^(1/2))^(1/2)/(1/a^(1/2)*b^(1/2))^(1/2))),_alpha=RootOf(_Z^4*d-c))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^4+a)^(5/2)/(-d*x^4+c),x, algorithm="maxima")
```

```
[Out] -integrate((-b*x^4 + a)^(5/2)/(d*x^4 - c), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^4+a)^(5/2)/(-d*x^4+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{a^2\sqrt{a-bx^4}}{-c+dx^4}\,dx-\int \frac{b^2x^8\sqrt{a-bx^4}}{-c+dx^4}\,dx-\int \left(-\frac{2abx^4\sqrt{a-bx^4}}{-c+dx^4}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**4+a)**(5/2)/(-d*x**4+c),x)
```

```
[Out] -Integral(a**2*sqrt(a - b*x**4)/(-c + d*x**4), x) - Integral(b**2*x**8*sqrt
(a - b*x**4)/(-c + d*x**4), x) - Integral(-2*a*b*x**4*sqrt(a - b*x**4)/(-c
+ d*x**4), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^4+a)^(5/2)/(-d*x^4+c),x, algorithm="giac")
[Out] integrate(-(-b*x^4 + a)^(5/2)/(d*x^4 - c), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a-b\,x^4\right)}^{5/2}}{c-d\,x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x^4)^(5/2)/(c - d*x^4),x)
[Out] int((a - b*x^4)^(5/2)/(c - d*x^4), x)
```

3.174 $\int \frac{(a-bx^4)^{3/2}}{c-dx^4}\,dx$

Optimal. Leaf size=277

$$\frac{bx\sqrt{a-bx^4}}{3d}-\frac{\sqrt[4]{a}\,b^{3/4}(3bc-5ad)\sqrt{1-\frac{bx^4}{a}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{3d^2\sqrt{a-bx^4}}+\frac{\sqrt[4]{a}\,(bc-ad)^2\sqrt{1-\frac{bx^4}{a}}\,\Pi\left(-\frac{\sqrt{a}}{\sqrt{b}}\right.}{2\sqrt[4]{b}\,cd^2\sqrt{a-}}$$

```
[Out] 1/3*b*x*(-b*x^4+a)^(1/2)/d-1/3*a^(1/4)*b^(3/4)*(-5*a*d+3*b*c)*EllipticF(b^(
1/4)*x/a^(1/4),I)*(1-b*x^4/a)^(1/2)/d^2/(-b*x^4+a)^(1/2)+1/2*a^(1/4)*(-a*d+
b*c)^2*EllipticPi(b^(1/4)*x/a^(1/4),-a^(1/2)*d^(1/2)/b^(1/2)/c^(1/2),I)*(1-
b*x^4/a)^(1/2)/b^(1/4)/c/d^2/(-b*x^4+a)^(1/2)+1/2*a^(1/4)*(-a*d+b*c)^2*Elli
pticPi(b^(1/4)*x/a^(1/4),a^(1/2)*d^(1/2)/b^(1/2)/c^(1/2),I)*(1-b*x^4/a)^(1/
2)/b^(1/4)/c/d^2/(-b*x^4+a)^(1/2)
```

Rubi [A]

time = 0.18, antiderivative size = 277, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {427, 537, 230, 227, 418, 1233, 1232}

$$-\frac{\sqrt[4]{a}\,b^{3/4}\sqrt{1-\frac{bx^4}{a}}\,(3bc-5ad)F\left(\text{ArcSin}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{3d^2\sqrt{a-bx^4}}+\frac{\sqrt[4]{a}\,\sqrt{1-\frac{bx^4}{a}}\,(bc-ad)^2\Pi\left(-\frac{\sqrt{a}\,\sqrt{d}}{\sqrt{b}\,\sqrt{c}};\text{ArcSin}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2\sqrt[4]{b}\,cd^2\sqrt{a-bx^4}}+\frac{\sqrt[4]{a}\,\sqrt{1-\frac{bx^4}{a}}\,(bc-ad)^2\Pi\left(\frac{\sqrt{a}\,\sqrt{d}}{\sqrt{b}\,\sqrt{c}};\text{ArcSin}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2\sqrt[4]{b}\,cd^2\sqrt{a-bx^4}}+\frac{bx\sqrt{a-bx^4}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^4)^(3/2)/(c - d*x^4),x]
```

```
[Out] (b*x*Sqrt[a - b*x^4])/(3*d) - (a^(1/4)*b^(3/4)*(3*b*c - 5*a*d)*Sqrt[1 - (b*
x^4)/a]*EllipticF[ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(3*d^2*Sqrt[a - b*x^4])
 + (a^(1/4)*(b*c - a*d)^2*Sqrt[1 - (b*x^4)/a]*EllipticPi[-((Sqrt[a]*Sqrt[d]
)/(Sqrt[b]*Sqrt[c])), ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(2*b^(1/4)*c*d^2*Sq
rt[a - b*x^4]) + (a^(1/4)*(b*c - a*d)^2*Sqrt[1 - (b*x^4)/a]*EllipticPi[(Sqr
t[a]*Sqrt[d])/(Sqrt[b]*Sqrt[c]), ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(2*b^(1/
4)*c*d^2*Sqrt[a - b*x^4])
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Dist[1/(2*
c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 1233

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{(a-bx^4)^{3/2}}{c-dx^4}\,dx = \frac{bx\sqrt{a-bx^4}}{3d} - \frac{\int \frac{a(bc-3ad)-b(3bc-5ad)x^4}{\sqrt{a-bx^4}\,(c-dx^4)}\,dx}{3d}$$

$$= \frac{bx\sqrt{a-bx^4}}{3d} - \frac{(b(3bc-5ad))\int \frac{1}{\sqrt{a-bx^4}}\,dx}{3d^2} + \frac{(bc-ad)^2\int \frac{1}{\sqrt{a-bx^4}\,(c-dx^4)}\,dx}{d^2}$$

$$= \frac{bx\sqrt{a-bx^4}}{3d} + \frac{(bc-ad)^2\int \frac{1}{\left(1-\frac{\sqrt{d}\,x^2}{\sqrt{c}}\right)\sqrt{a-bx^4}}\,dx}{2cd^2} + \frac{(bc-ad)^2\int \frac{1}{\left(1+\frac{\sqrt{d}\,x^2}{\sqrt{c}}\right)\sqrt{a-}}}{2cd^2}$$

$$= \frac{bx\sqrt{a-bx^4}}{3d} - \frac{\sqrt[4]{a}\,b^{3/4}(3bc-5ad)\sqrt{1-\frac{bx^4}{a}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{3d^2\sqrt{a-bx^4}} + \frac{\left((bc-ad)^2\right.}{}$$

$$= \frac{bx\sqrt{a-bx^4}}{3d} - \frac{\sqrt[4]{a}\,b^{3/4}(3bc-5ad)\sqrt{1-\frac{bx^4}{a}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{3d^2\sqrt{a-bx^4}} + \frac{\sqrt[4]{a}\,(bc-a}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 10.26, size = 341, normalized size = 1.23

$$-\frac{x\left(\frac{b(-3bc+5ad)x^4\sqrt{1-\frac{bx^4}{a}}\,F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)}{c}+\frac{5\left(5ac(3a^2d-abdx^4+b^2x^4(-c+dx^4))F_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)+2bx^4(a-bx^4)(c-dx^4)\left(2adF_1\left(\frac{5}{4};\frac{1}{2},2;\frac{9}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)+bcF_1\left(\frac{5}{4};\frac{3}{2},1;\frac{9}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)\right)\right)}{(-c+dx^4)\left(5acF_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)+2x^4\left(2adF_1\left(\frac{5}{4};\frac{1}{2},2;\frac{9}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)+bcF_1\left(\frac{5}{4};\frac{3}{2},1;\frac{9}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)\right)\right)}\right)}{15d\sqrt{a-bx^4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a - b*x^4)^(3/2)/(c - d*x^4),x]
```

```
[Out] -1/15*(x*((b*(-3*b*c + 5*a*d)*x^4*Sqrt[1 - (b*x^4)/a]*AppellF1[5/4, 1/2, 1,
 9/4, (b*x^4)/a, (d*x^4)/c])/c + (5*(5*a*c*(3*a^2*d - a*b*d*x^4 + b^2*x^4*(
-c + d*x^4))*AppellF1[1/4, 1/2, 1, 5/4, (b*x^4)/a, (d*x^4)/c] + 2*b*x^4*(a
- b*x^4)*(c - d*x^4)*(2*a*d*AppellF1[5/4, 1/2, 2, 9/4, (b*x^4)/a, (d*x^4)/c
] + b*c*AppellF1[5/4, 3/2, 1, 9/4, (b*x^4)/a, (d*x^4)/c])))/((-c + d*x^4)*(
5*a*c*AppellF1[1/4, 1/2, 1, 5/4, (b*x^4)/a, (d*x^4)/c] + 2*x^4*(2*a*d*Appel
lF1[5/4, 1/2, 2, 9/4, (b*x^4)/a, (d*x^4)/c] + b*c*AppellF1[5/4, 3/2, 1, 9/4
, (b*x^4)/a, (d*x^4)/c])))))/(d*Sqrt[a - b*x^4])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.30, size = 307, normalized size = 1.11

method	result
risch	$\frac{bx\sqrt{-b\,x^4+a}}{3d}+\frac{\frac{b(5ad-3bc)\sqrt{1-\frac{x^2\sqrt{b}}{\sqrt{a}}}\sqrt{1+\frac{x^2\sqrt{b}}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)}{d\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\sqrt{-b\,x^4+a}}+\frac{3\left(-a^2d^2+2abcd-b^2c^2\right)\left(\_\alpha=\mathrm{Roo}\right.}{}}{}$
default	$\frac{bx\sqrt{-b\,x^4+a}}{3d}+\frac{\left(\frac{b(2ad-bc)}{d^2}-\frac{ba}{3d}\right)\sqrt{1-\frac{x^2\sqrt{b}}{\sqrt{a}}}\sqrt{1+\frac{x^2\sqrt{b}}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\sqrt{-b\,x^4+a}}-\frac{\sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z^4-\right.}}{}$
elliptic	$\frac{bx\sqrt{-b\,x^4+a}}{3d}+\frac{\left(\frac{b(2ad-bc)}{d^2}-\frac{ba}{3d}\right)\sqrt{1-\frac{x^2\sqrt{b}}{\sqrt{a}}}\sqrt{1+\frac{x^2\sqrt{b}}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\sqrt{-b\,x^4+a}}-\frac{\sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z^4-\right.}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^4+a)^(3/2)/(-d*x^4+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*b*x*(-b*x^4+a)^(1/2)/d+(b*(2*a*d-b*c)/d^2-1/3*b/d*a)/(1/a^(1/2)*b^(1/2)
)^(1/2)*(1-x^2*b^(1/2)/a^(1/2))^(1/2)*(1+x^2*b^(1/2)/a^(1/2))^(1/2)/(-b*x^4
+a)^(1/2)*EllipticF(x*(1/a^(1/2)*b^(1/2))^(1/2),I)-1/8/d^3*sum((a^2*d^2-2*a
*b*c*d+b^2*c^2)/_alpha^3*(-1/((a*d-b*c)/d)^(1/2)*arctanh(1/2*(-2*_alpha^2*b
*x^2+2*a)/((a*d-b*c)/d)^(1/2)/(-b*x^4+a)^(1/2))-2/(1/a^(1/2)*b^(1/2))^(1/2)
*_alpha^3*d/c*(1-x^2*b^(1/2)/a^(1/2))^(1/2)*(1+x^2*b^(1/2)/a^(1/2))^(1/2)/(
-b*x^4+a)^(1/2)*EllipticPi(x*(1/a^(1/2)*b^(1/2))^(1/2),a^(1/2)/b^(1/2)*_alp
ha^2/c*d,(-1/a^(1/2)*b^(1/2))^(1/2)/(1/a^(1/2)*b^(1/2))^(1/2))),_alpha=Root
Of(_Z^4*d-c))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^4+a)^(3/2)/(-d*x^4+c),x, algorithm="maxima")
```

```
[Out] -integrate((-b*x^4 + a)^(3/2)/(d*x^4 - c), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^4+a)^(3/2)/(-d*x^4+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{a\sqrt{a-bx^4}}{-c+dx^4}\,dx-\int \left(-\frac{bx^4\sqrt{a-bx^4}}{-c+dx^4}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**4+a)**(3/2)/(-d*x**4+c),x)
```

```
[Out] -Integral(a*sqrt(a - b*x**4)/(-c + d*x**4), x) - Integral(-b*x**4*sqrt(a -
b*x**4)/(-c + d*x**4), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^4+a)^(3/2)/(-d*x^4+c),x, algorithm="giac")
```

```
[Out] integrate(-(-b*x^4 + a)^(3/2)/(d*x^4 - c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a-b\,x^4\right)^{3/2}}{c-d\,x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x^4)^(3/2)/(c - d*x^4),x)
```

```
[Out] int((a - b*x^4)^(3/2)/(c - d*x^4), x)
```

3.175 $\int \frac{\sqrt{a-bx^4}}{c-dx^4}\,dx$

Optimal. Leaf size=240

$$\frac{\sqrt[4]{a}\,b^{3/4}\sqrt{1-\frac{bx^4}{a}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{d\sqrt{a-bx^4}}-\frac{\sqrt[4]{a}\,(bc-ad)\sqrt{1-\frac{bx^4}{a}}\,\Pi\left(-\frac{\sqrt{a}\,\sqrt{d}}{\sqrt{b}\,\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2\sqrt[4]{b}\,cd\sqrt{a-bx^4}}-$$

```
[Out] a^(1/4)*b^(3/4)*EllipticF(b^(1/4)*x/a^(1/4),I)*(1-b*x^4/a)^(1/2)/d/(-b*x^4+
a)^(1/2)-1/2*a^(1/4)*(-a*d+b*c)*EllipticPi(b^(1/4)*x/a^(1/4),-a^(1/2)*d^(1/
2)/b^(1/2)/c^(1/2),I)*(1-b*x^4/a)^(1/2)/b^(1/4)/c/d/(-b*x^4+a)^(1/2)-1/2*a^
(1/4)*(-a*d+b*c)*EllipticPi(b^(1/4)*x/a^(1/4),a^(1/2)*d^(1/2)/b^(1/2)/c^(1/
2),I)*(1-b*x^4/a)^(1/2)/b^(1/4)/c/d/(-b*x^4+a)^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 240, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {415, 230, 227, 418, 1233, 1232}

$$\frac{\sqrt[4]{a}\,b^{3/4}\sqrt{1-\frac{bx^4}{a}}\,F\left(\text{ArcSin}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{d\sqrt{a-bx^4}}-\frac{\sqrt[4]{a}\,\sqrt{1-\frac{bx^4}{a}}\,(bc-ad)\Pi\left(-\frac{\sqrt{a}\,\sqrt{d}}{\sqrt{b}\,\sqrt{c}};\text{ArcSin}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2\sqrt[4]{b}\,cd\sqrt{a-bx^4}}-\frac{\sqrt[4]{a}\,\sqrt{1-\frac{bx^4}{a}}\,(bc-ad)\Pi\left(\frac{\sqrt{a}\,\sqrt{d}}{\sqrt{b}\,\sqrt{c}};\text{ArcSin}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2\sqrt[4]{b}\,cd\sqrt{a-bx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a - b*x^4]/(c - d*x^4),x]
```

```
[Out] (a^(1/4)*b^(3/4)*Sqrt[1 - (b*x^4)/a]*EllipticF[ArcSin[(b^(1/4)*x)/a^(1/4)],
 -1])/(d*Sqrt[a - b*x^4]) - (a^(1/4)*(b*c - a*d)*Sqrt[1 - (b*x^4)/a]*Ellipt
icPi[-((Sqrt[a]*Sqrt[d])/(Sqrt[b]*Sqrt[c])), ArcSin[(b^(1/4)*x)/a^(1/4)], -
1])/(2*b^(1/4)*c*d*Sqrt[a - b*x^4]) - (a^(1/4)*(b*c - a*d)*Sqrt[1 - (b*x^4)
/a]*EllipticPi[(Sqrt[a]*Sqrt[d])/(Sqrt[b]*Sqrt[c]), ArcSin[(b^(1/4)*x)/a^(1
/4)], -1])/(2*b^(1/4)*c*d*Sqrt[a - b*x^4])
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 415

```
Int[Sqrt[(a_) + (b_.)*(x_)^4]/((c_) + (d_.)*(x_)^4), x_Symbol] :> Dist[b/d,
 Int[1/Sqrt[a + b*x^4], x], x] - Dist[(b*c - a*d)/d, Int[1/(Sqrt[a + b*x^4]
*(c + d*x^4)), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Dist[1/(2*
c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 1233

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sqrt{a-bx^4}}{c-dx^4}\,dx = \frac{b\int \frac{1}{\sqrt{a-bx^4}}\,dx}{d} + \frac{(-bc+ad)\int \frac{1}{\sqrt{a-bx^4}\,(c-dx^4)}\,dx}{d}$$

$$= \frac{(-bc+ad)\int \frac{1}{\left(1-\frac{\sqrt{d}\,x^2}{\sqrt{c}}\right)\sqrt{a-bx^4}}\,dx}{2cd} + \frac{(-bc+ad)\int \frac{1}{\left(1+\frac{\sqrt{d}\,x^2}{\sqrt{c}}\right)\sqrt{a-bx^4}}\,dx}{2cd} + \frac{\Big(b}{}$$

$$= \frac{\sqrt[4]{a}\,b^{3/4}\sqrt{1-\frac{bx^4}{a}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{d\sqrt{a-bx^4}} + \frac{\left((-bc+ad)\sqrt{1-\frac{bx^4}{a}}\right)\int \frac{}{\left(1-\frac{\sqrt{d}\,x^2}{\sqrt{c}}\right)}}{2cd\sqrt{a-bx^4}}$$

$$= \frac{\sqrt[4]{a}\,b^{3/4}\sqrt{1-\frac{bx^4}{a}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{d\sqrt{a-bx^4}} - \frac{\sqrt[4]{a}\,(bc-ad)\sqrt{1-\frac{bx^4}{a}}\,\Pi\Big(-\frac{\sqrt{a}\,\sqrt{d}}{\sqrt{b}\,\sqrt{c}};}{2\sqrt[4]{b}\,cd\sqrt{a-bx^4}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

time = 10.14, size = 155, normalized size = 0.65

$$-\frac{5acx\sqrt{a-bx^4}\,F_1\left(\frac{1}{4};-\frac{1}{2},1;\frac{5}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)}{(c-dx^4)\left(-5acF_1\left(\frac{1}{4};-\frac{1}{2},1;\frac{5}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)+2x^4\left(-2adF_1\left(\frac{5}{4};-\frac{1}{2},2;\frac{9}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)+bcF_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a - b*x^4]/(c - d*x^4),x]
```

```
[Out] (-5*a*c*x*Sqrt[a - b*x^4]*AppellF1[1/4, -1/2, 1, 5/4, (b*x^4)/a, (d*x^4)/c]
)/((c - d*x^4)*(-5*a*c*AppellF1[1/4, -1/2, 1, 5/4, (b*x^4)/a, (d*x^4)/c] +
2*x^4*(-2*a*d*AppellF1[5/4, -1/2, 2, 9/4, (b*x^4)/a, (d*x^4)/c] + b*c*Appel
lF1[5/4, 1/2, 1, 9/4, (b*x^4)/a, (d*x^4)/c])))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.27, size = 259, normalized size = 1.08

method	result
default	$\frac{b\sqrt{1-\frac{x^2\sqrt{b}}{\sqrt{a}}}\sqrt{1+\frac{x^2\sqrt{b}}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)}{d\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\sqrt{-b\,x^4+a}}-\sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z^4-c\right)}\frac{(ad-bc)\left(-\frac{\mathrm{arctanh}\left(\frac{-2b\,x^2\_\alpha^2+\cdots}{2\sqrt{\frac{ad-bc}{d}}\sqrt{-b\cdots}}\right)}{\sqrt{\frac{ad-bc}{d}}}\cdots\right.}{\cdots}$
elliptic	$\frac{b\sqrt{1-\frac{x^2\sqrt{b}}{\sqrt{a}}}\sqrt{1+\frac{x^2\sqrt{b}}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)}{d\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\sqrt{-b\,x^4+a}}-\sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z^4-c\right)}\frac{(ad-bc)\left(-\frac{\mathrm{arctanh}\left(\frac{-2b\,x^2\_\alpha^2+\cdots}{2\sqrt{\frac{ad-bc}{d}}\sqrt{-b\cdots}}\right)}{\sqrt{\frac{ad-bc}{d}}}\cdots\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^4+a)^(1/2)/(-d*x^4+c),x,method=_RETURNVERBOSE)
```

```
[Out] b/d/(1/a^(1/2)*b^(1/2))^(1/2)*(1-x^2*b^(1/2)/a^(1/2))^(1/2)*(1+x^2*b^(1/2)/
a^(1/2))^(1/2)/(-b*x^4+a)^(1/2)*EllipticF(x*(1/a^(1/2)*b^(1/2))^(1/2),I)-1/
8/d^2*sum((a*d-b*c)/_alpha^3*(-1/((a*d-b*c)/d)^(1/2)*arctanh(1/2*(-2*_alpha
^2*b*x^2+2*a)/((a*d-b*c)/d)^(1/2)/(-b*x^4+a)^(1/2))-2/(1/a^(1/2)*b^(1/2))^(
1/2)*_alpha^3*d/c*(1-x^2*b^(1/2)/a^(1/2))^(1/2)*(1+x^2*b^(1/2)/a^(1/2))^(1/
2)/(-b*x^4+a)^(1/2)*EllipticPi(x*(1/a^(1/2)*b^(1/2))^(1/2),a^(1/2)/b^(1/2)*
_alpha^2/c*d,(-1/a^(1/2)*b^(1/2))^(1/2)/(1/a^(1/2)*b^(1/2))^(1/2))),_alpha=
RootOf(_Z^4*d-c))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^4+a)^(1/2)/(-d*x^4+c),x, algorithm="maxima")
[Out] -integrate(sqrt(-b*x^4 + a)/(d*x^4 - c), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^4+a)^(1/2)/(-d*x^4+c),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt{a - bx^4}}{-c + dx^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**4+a)**(1/2)/(-d*x**4+c),x)
[Out] -Integral(sqrt(a - b*x**4)/(-c + d*x**4), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^4+a)^(1/2)/(-d*x^4+c),x, algorithm="giac")
```

```
[Out] integrate(-sqrt(-b*x^4 + a)/(d*x^4 - c), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{a-b\,x^4}}{c-d\,x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x^4)^(1/2)/(c - d*x^4),x)
```

```
[Out] int((a - b*x^4)^(1/2)/(c - d*x^4), x)
```

3.176 $\int \frac{1}{\sqrt{a-bx^4}\,(c-dx^4)}\,dx$

Optimal. Leaf size=162

$$\frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}\,\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2\sqrt[4]{b}\,c\sqrt{a-bx^4}}+\frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}\,\Pi\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2\sqrt[4]{b}\,c\sqrt{a-bx^4}}$$

```
[Out] 1/2*a^(1/4)*EllipticPi(b^(1/4)*x/a^(1/4),-a^(1/2)*d^(1/2)/b^(1/2)/c^(1/2),I
)*(1-b*x^4/a)^(1/2)/b^(1/4)/c/(-b*x^4+a)^(1/2)+1/2*a^(1/4)*EllipticPi(b^(1/
4)*x/a^(1/4),a^(1/2)*d^(1/2)/b^(1/2)/c^(1/2),I)*(1-b*x^4/a)^(1/2)/b^(1/4)/c
/(-b*x^4+a)^(1/2)
```

Rubi [A]

time = 0.07, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {418, 1233, 1232}

$$\frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}\,\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}};\text{ArcSin}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2\sqrt[4]{b}\,c\sqrt{a-bx^4}}+\frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}\,\Pi\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}};\text{ArcSin}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2\sqrt[4]{b}\,c\sqrt{a-bx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a - b*x^4]*(c - d*x^4)),x]
```

```
[Out] (a^(1/4)*Sqrt[1 - (b*x^4)/a]*EllipticPi[-((Sqrt[a]*Sqrt[d])/(Sqrt[b]*Sqrt[c
])), ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(2*b^(1/4)*c*Sqrt[a - b*x^4]) + (a^(
1/4)*Sqrt[1 - (b*x^4)/a]*EllipticPi[(Sqrt[a]*Sqrt[d])/(Sqrt[b]*Sqrt[c]), Ar
cSin[(b^(1/4)*x)/a^(1/4)], -1])/(2*b^(1/4)*c*Sqrt[a - b*x^4])
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Dist[1/(2*
c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 1233

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a-bx^4}\,(c-dx^4)}\,dx = \frac{\int \frac{1}{\left(1-\frac{\sqrt{d}\,x^2}{\sqrt{c}}\right)\sqrt{a-bx^4}}\,dx}{2c} + \frac{\int \frac{1}{\left(1+\frac{\sqrt{d}\,x^2}{\sqrt{c}}\right)\sqrt{a-bx^4}}\,dx}{2c}$$

$$= \frac{\sqrt{1-\frac{bx^4}{a}}\int \frac{1}{\left(1-\frac{\sqrt{d}\,x^2}{\sqrt{c}}\right)\sqrt{1-\frac{bx^4}{a}}}\,dx}{2c\sqrt{a-bx^4}} + \frac{\sqrt{1-\frac{bx^4}{a}}\int \frac{1}{\left(1+\frac{\sqrt{d}\,x^2}{\sqrt{c}}\right)\sqrt{1-\frac{bx^4}{a}}}}{2c\sqrt{a-bx^4}}$$

$$= \frac{\sqrt[4]{a}\,\sqrt{1-\frac{bx^4}{a}}\,\Pi\left(-\frac{\sqrt{a}\,\sqrt{d}}{\sqrt{b}\,\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2\sqrt[4]{b}\,c\sqrt{a-bx^4}} + \frac{\sqrt[4]{a}\,\sqrt{1-\frac{bx^4}{a}}\,\Pi\Big(\frac{\sqrt{a}}{\sqrt{b}}}{2\sqrt[4]{b}\,c}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 10.15, size = 156, normalized size = 0.96

$$-\frac{5acxF_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)}{\sqrt{a-bx^4}\,(-c+dx^4)\left(5acF_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)+2x^4\left(2adF_1\left(\frac{5}{4};\frac{1}{2},2;\frac{9}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)+bcF_1\left(\frac{5}{4};\frac{3}{2},1;\frac{9}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Sqrt[a - b*x^4]*(c - d*x^4)),x]
```

```
[Out] (-5*a*c*x*AppellF1[1/4, 1/2, 1, 5/4, (b*x^4)/a, (d*x^4)/c])/(Sqrt[a - b*x^4
]*(-c + d*x^4)*(5*a*c*AppellF1[1/4, 1/2, 1, 5/4, (b*x^4)/a, (d*x^4)/c] + 2*
x^4*(2*a*d*AppellF1[5/4, 1/2, 2, 9/4, (b*x^4)/a, (d*x^4)/c] + b*c*AppellF1[
5/4, 3/2, 1, 9/4, (b*x^4)/a, (d*x^4)/c])))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.27, size = 183, normalized size = 1.13

method	result

default	$-\frac{\sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z^4-c\right)}\frac{-\frac{\mathrm{arctanh}\left(\frac{-2b\,x^2\_\alpha^2+2a}{2\sqrt{\frac{ad-bc}{d}}\sqrt{-b\,x^4+a}}\right)}{\sqrt{\frac{ad-bc}{d}}}-\frac{2\_\alpha^3 d\sqrt{1-\frac{x^2\sqrt{b}}{\sqrt{a}}}\sqrt{1+\frac{x^2\sqrt{b}}{\sqrt{a}}}\,\mathrm{EllipticPi}\left(x\sqrt{\cdots}\right.}{\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,c\sqrt{-b\,x^4+a}}}{\_\alpha^3}}{8d}$
elliptic	$-\frac{\sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z^4-c\right)}\frac{-\frac{\mathrm{arctanh}\left(\frac{-2b\,x^2\_\alpha^2+2a}{2\sqrt{\frac{ad-bc}{d}}\sqrt{-b\,x^4+a}}\right)}{\sqrt{\frac{ad-bc}{d}}}-\frac{2\_\alpha^3 d\sqrt{1-\frac{x^2\sqrt{b}}{\sqrt{a}}}\sqrt{1+\frac{x^2\sqrt{b}}{\sqrt{a}}}\,\mathrm{EllipticPi}\left(x\sqrt{\cdots}\right.}{\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,c\sqrt{-b\,x^4+a}}}{\_\alpha^3}}{8d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x^4+a)^(1/2)/(-d*x^4+c),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8/d*sum(1/_alpha^3*(-1/((a*d-b*c)/d)^(1/2)*arctanh(1/2*(-2*_alpha^2*b*x^
2+2*a)/((a*d-b*c)/d)^(1/2)/(-b*x^4+a)^(1/2))-2/(1/a^(1/2)*b^(1/2))^(1/2)*_a
lpha^3*d/c*(1-x^2*b^(1/2)/a^(1/2))^(1/2)*(1+x^2*b^(1/2)/a^(1/2))^(1/2)/(-b*
x^4+a)^(1/2)*EllipticPi(x*(1/a^(1/2)*b^(1/2))^(1/2),a^(1/2)/b^(1/2)*_alpha^
2/c*d,(-1/a^(1/2)*b^(1/2))^(1/2)/(1/a^(1/2)*b^(1/2))^(1/2))),_alpha=RootOf(
_Z^4*d-c))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^4+a)^(1/2)/(-d*x^4+c),x, algorithm="maxima")
```

```
[Out] -integrate(1/(sqrt(-b*x^4 + a)*(d*x^4 - c)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^4+a)^(1/2)/(-d*x^4+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{-c\sqrt{a-bx^4}+dx^4\sqrt{a-bx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x**4+a)**(1/2)/(-d*x**4+c),x)
```

```
[Out] -Integral(1/(-c*sqrt(a - b*x**4) + d*x**4*sqrt(a - b*x**4)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^4+a)^(1/2)/(-d*x^4+c),x, algorithm="giac")
```

```
[Out] integrate(-1/(sqrt(-b*x^4 + a)*(d*x^4 - c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{a-b\,x^4}\,\left(c-d\,x^4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a - b*x^4)^(1/2)*(c - d*x^4)),x)
```

```
[Out] int(1/((a - b*x^4)^(1/2)*(c - d*x^4)), x)
```

3.177 $\int \frac{1}{(a-bx^4)^{3/2}(c-dx^4)}\,dx$

Optimal. Leaf size=281

$$\frac{bx}{2a(bc-ad)\sqrt{a-bx^4}}+\frac{b^{3/4}\sqrt{1-\frac{bx^4}{a}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2a^{3/4}(bc-ad)\sqrt{a-bx^4}}-\frac{\sqrt[4]{a}\,d\sqrt{1-\frac{bx^4}{a}}\,\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\right.\right.}{2\sqrt[4]{b}\,c(bc-ad)\sqrt{a-bx^4}}$$

```
[Out] 1/2*b*x/a/(-a*d+b*c)/(-b*x^4+a)^(1/2)+1/2*b^(3/4)*EllipticF(b^(1/4)*x/a^(1/
4),I)*(1-b*x^4/a)^(1/2)/a^(3/4)/(-a*d+b*c)/(-b*x^4+a)^(1/2)-1/2*a^(1/4)*d*E
llipticPi(b^(1/4)*x/a^(1/4),-a^(1/2)*d^(1/2)/b^(1/2)/c^(1/2),I)*(1-b*x^4/a)
^(1/2)/b^(1/4)/c/(-a*d+b*c)/(-b*x^4+a)^(1/2)-1/2*a^(1/4)*d*EllipticPi(b^(1/
4)*x/a^(1/4),a^(1/2)*d^(1/2)/b^(1/2)/c^(1/2),I)*(1-b*x^4/a)^(1/2)/b^(1/4)/c
/(-a*d+b*c)/(-b*x^4+a)^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 281, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {425, 537, 230, 227, 418, 1233, 1232}

$$\frac{b^{3/4}\sqrt{1-\frac{bx^4}{a}}\,F\left(\text{ArcSin}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2a^{3/4}\sqrt{a-bx^4}\,(bc-ad)}-\frac{\sqrt[4]{a}\,d\sqrt{1-\frac{bx^4}{a}}\,\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}};\text{ArcSin}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2\sqrt[4]{b}\,c\sqrt{a-bx^4}\,(bc-ad)}-\frac{\sqrt[4]{a}\,d\sqrt{1-\frac{bx^4}{a}}\,\Pi\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}};\text{ArcSin}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2\sqrt[4]{b}\,c\sqrt{a-bx^4}\,(bc-ad)}+\frac{bx}{2a\sqrt{a-bx^4}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - b*x^4)^(3/2)*(c - d*x^4)),x]
```

```
[Out] (b*x)/(2*a*(b*c - a*d)*Sqrt[a - b*x^4]) + (b^(3/4)*Sqrt[1 - (b*x^4)/a]*Elli
pticF[ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(2*a^(3/4)*(b*c - a*d)*Sqrt[a - b*x
^4]) - (a^(1/4)*d*Sqrt[1 - (b*x^4)/a]*EllipticPi[-((Sqrt[a]*Sqrt[d])/(Sqrt[
b]*Sqrt[c])), ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(2*b^(1/4)*c*(b*c - a*d)*Sq
rt[a - b*x^4]) - (a^(1/4)*d*Sqrt[1 - (b*x^4)/a]*EllipticPi[(Sqrt[a]*Sqrt[d]
)/(Sqrt[b]*Sqrt[c]), ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(2*b^(1/4)*c*(b*c -
a*d)*Sqrt[a - b*x^4])
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Dist[1/(2*
c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 1233

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{(a-bx^4)^{3/2}(c-dx^4)}dx = \frac{bx}{2a(bc-ad)\sqrt{a-bx^4}} + \frac{\int \frac{bc-2ad-bdx^4}{\sqrt{a-bx^4}(c-dx^4)}dx}{2a(bc-ad)}$$

$$= \frac{bx}{2a(bc-ad)\sqrt{a-bx^4}} + \frac{b\int \frac{1}{\sqrt{a-bx^4}}dx}{2a(bc-ad)} - \frac{d\int \frac{1}{\sqrt{a-bx^4}(c-dx^4)}dx}{bc-ad}$$

$$= \frac{bx}{2a(bc-ad)\sqrt{a-bx^4}} - \frac{d\int \frac{1}{\left(1-\frac{\sqrt{d}x^2}{\sqrt{c}}\right)\sqrt{a-bx^4}}dx}{2c(bc-ad)} - \frac{d\int \frac{1}{\left(1+\frac{\sqrt{d}x^2}{\sqrt{c}}\right)\sqrt{a}}}{2c(bc-a}$$

$$= \frac{bx}{2a(bc-ad)\sqrt{a-bx^4}} + \frac{b^{3/4}\sqrt{1-\frac{bx^4}{a}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2a^{3/4}(bc-ad)\sqrt{a-bx^4}} - \frac{\left(d\sqrt{1}\right.}{}$$

$$= \frac{bx}{2a(bc-ad)\sqrt{a-bx^4}} + \frac{b^{3/4}\sqrt{1-\frac{bx^4}{a}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2a^{3/4}(bc-ad)\sqrt{a-bx^4}} - \frac{\sqrt[4]{a}d\sqrt{}}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 10.19, size = 381, normalized size = 1.36

$$\frac{5acxF_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)\left(-5c(-2bc+2ad+bdx^4)+bdx^4\sqrt{1-\frac{bx^4}{a}}(-c+dx^4)F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)\right)+2bx^5(c-dx^4)\left(5c-dx^4\sqrt{1-\frac{bx^4}{a}}F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)\right)\left(2adF_1\left(\frac{5}{4};\frac{1}{2},2;\frac{9}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)+bcF_1\left(\frac{5}{4};\frac{3}{2},1;\frac{9}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)\right)}{10ac(-bc+ad)\sqrt{a-bx^4}(-c+dx^4)\left(5acF_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)+2x^4\left(2adF_1\left(\frac{5}{4};\frac{1}{2},2;\frac{9}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)+bcF_1\left(\frac{5}{4};\frac{3}{2},1;\frac{9}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a - b*x^4)^(3/2)*(c - d*x^4)),x]
```

```
[Out] (5*a*c*x*AppellF1[1/4, 1/2, 1, 5/4, (b*x^4)/a, (d*x^4)/c]*(-5*c*(-2*b*c + 2
*a*d + b*d*x^4) + b*d*x^4*Sqrt[1 - (b*x^4)/a]*(-c + d*x^4)*AppellF1[5/4, 1/
2, 1, 9/4, (b*x^4)/a, (d*x^4)/c]) + 2*b*x^5*(c - d*x^4)*(5*c - d*x^4*Sqrt[1
 - (b*x^4)/a]*AppellF1[5/4, 1/2, 1, 9/4, (b*x^4)/a, (d*x^4)/c])*(2*a*d*Appe
llF1[5/4, 1/2, 2, 9/4, (b*x^4)/a, (d*x^4)/c] + b*c*AppellF1[5/4, 3/2, 1, 9/
4, (b*x^4)/a, (d*x^4)/c]))/(10*a*c*(-(b*c) + a*d)*Sqrt[a - b*x^4]*(-c + d*x
^4)*(5*a*c*AppellF1[1/4, 1/2, 1, 5/4, (b*x^4)/a, (d*x^4)/c] + 2*x^4*(2*a*d*
AppellF1[5/4, 1/2, 2, 9/4, (b*x^4)/a, (d*x^4)/c] + b*c*AppellF1[5/4, 3/2, 1
, 9/4, (b*x^4)/a, (d*x^4)/c])))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.28, size = 301, normalized size = 1.07

method	result
default	$-\frac{bx}{2a(ad-bc)\sqrt{-\left(x^{4}-\frac{a}{b}\right)b}}-\frac{b\sqrt{1-\frac{x^{2}\sqrt{b}}{\sqrt{a}}}\sqrt{1+\frac{x^{2}\sqrt{b}}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)}{2a(ad-bc)\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\sqrt{-b\,x^{4}+a}}-\frac{\left(\sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z^{4}-c\right.}\right.}{}$
elliptic	$-\frac{bx}{2a(ad-bc)\sqrt{-\left(x^{4}-\frac{a}{b}\right)b}}-\frac{b\sqrt{1-\frac{x^{2}\sqrt{b}}{\sqrt{a}}}\sqrt{1+\frac{x^{2}\sqrt{b}}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)}{2a(ad-bc)\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\sqrt{-b\,x^{4}+a}}-\frac{\left(\sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z^{4}-c\right.}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x^4+a)^(3/2)/(-d*x^4+c),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*b/a*x/(a*d-b*c)/(-(x^4-a/b)*b)^(1/2)-1/2*b/a/(a*d-b*c)/(1/a^(1/2)*b^(1
/2))^(1/2)*(1-x^2*b^(1/2)/a^(1/2))^(1/2)*(1+x^2*b^(1/2)/a^(1/2))^(1/2)/(-b*
x^4+a)^(1/2)*EllipticF(x*(1/a^(1/2)*b^(1/2))^(1/2),I)-1/8*sum(1/_alpha^3/(a
*d-b*c)*(-1/((a*d-b*c)/d)^(1/2)*arctanh(1/2*(-2*_alpha^2*b*x^2+2*a)/((a*d-b
*c)/d)^(1/2)/(-b*x^4+a)^(1/2))-2/(1/a^(1/2)*b^(1/2))^(1/2)*_alpha^3*d/c*(1-
x^2*b^(1/2)/a^(1/2))^(1/2)*(1+x^2*b^(1/2)/a^(1/2))^(1/2)/(-b*x^4+a)^(1/2)*E
llipticPi(x*(1/a^(1/2)*b^(1/2))^(1/2),a^(1/2)/b^(1/2)*_alpha^2/c*d,(-1/a^(1
/2)*b^(1/2))^(1/2)/(1/a^(1/2)*b^(1/2))^(1/2))),_alpha=RootOf(_Z^4*d-c))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^4+a)^(3/2)/(-d*x^4+c),x, algorithm="maxima")
[Out] -integrate(1/((-b*x^4 + a)^(3/2)*(d*x^4 - c)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^4+a)^(3/2)/(-d*x^4+c),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{-ac\sqrt{a-bx^4}+adx^4\sqrt{a-bx^4}+bcx^4\sqrt{a-bx^4}-bdx^8\sqrt{a-bx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x**4+a)**(3/2)/(-d*x**4+c),x)
[Out] -Integral(1/(-a*c*sqrt(a - b*x**4) + a*d*x**4*sqrt(a - b*x**4) + b*c*x**4*s
qrt(a - b*x**4) - b*d*x**8*sqrt(a - b*x**4)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^4+a)^(3/2)/(-d*x^4+c),x, algorithm="giac")
[Out] integrate(-1/((-b*x^4 + a)^(3/2)*(d*x^4 - c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(a-b\,x^4\right)}^{3/2}\,\left(c-d\,x^4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a - b*x^4)^(3/2)*(c - d*x^4)),x)
[Out] int(1/((a - b*x^4)^(3/2)*(c - d*x^4)), x)
```

3.178 $\int \frac{1}{\left(a-bx^4\right)^{5/2}\left(c-dx^4\right)}\,dx$

Optimal. Leaf size=334

$$\frac{bx}{6a(bc-ad)\left(a-bx^4\right)^{3/2}}+\frac{b(5bc-11ad)x}{12a^2(bc-ad)^2\sqrt{a-bx^4}}+\frac{b^{3/4}(5bc-11ad)\sqrt{1-\frac{bx^4}{a}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{12a^{7/4}(bc-ad)^2\sqrt{a-bx^4}}+$$

```
[Out] 1/6*b*x/a/(-a*d+b*c)/(-b*x^4+a)^(3/2)+1/12*b*(-11*a*d+5*b*c)*x/a^2/(-a*d+b*
c)^2/(-b*x^4+a)^(1/2)+1/12*b^(3/4)*(-11*a*d+5*b*c)*EllipticF(b^(1/4)*x/a^(1
/4),I)*(1-b*x^4/a)^(1/2)/a^(7/4)/(-a*d+b*c)^2/(-b*x^4+a)^(1/2)+1/2*a^(1/4)*
d^2*EllipticPi(b^(1/4)*x/a^(1/4),-a^(1/2)*d^(1/2)/b^(1/2)/c^(1/2),I)*(1-b*x
^4/a)^(1/2)/b^(1/4)/c/(-a*d+b*c)^2/(-b*x^4+a)^(1/2)+1/2*a^(1/4)*d^2*Ellipti
cPi(b^(1/4)*x/a^(1/4),a^(1/2)*d^(1/2)/b^(1/2)/c^(1/2),I)*(1-b*x^4/a)^(1/2)/
b^(1/4)/c/(-a*d+b*c)^2/(-b*x^4+a)^(1/2)
```

Rubi [A]
time = 0.26, antiderivative size = 334, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {425, 541, 537, 230, 227, 418, 1233, 1232}

$$\frac{b^{3/4}\sqrt{1-\frac{bx^4}{a}}(5bc-11ad)F\left(\text{ArcSin}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{12a^{7/4}\sqrt{a-bx^4}\,(bc-ad)^2}+\frac{bx(5bc-11ad)}{12a^2\sqrt{a-bx^4}\,(bc-ad)^2}+\frac{\sqrt[4]{a}\,d^2\sqrt{1-\frac{bx^4}{a}}\,\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}};\text{ArcSin}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2\sqrt[4]{b}\,c\sqrt{a-bx^4}\,(bc-ad)^2}+\frac{\sqrt[4]{a}\,d^2\sqrt{1-\frac{bx^4}{a}}\,\Pi\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}};\text{ArcSin}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2\sqrt[4]{b}\,c\sqrt{a-bx^4}\,(bc-ad)^2}+\frac{bx}{6a\left(a-bx^4\right)^{3/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - b*x^4)^(5/2)*(c - d*x^4)),x]
```

```
[Out] (b*x)/(6*a*(b*c - a*d)*(a - b*x^4)^(3/2)) + (b*(5*b*c - 11*a*d)*x)/(12*a^2*
(b*c - a*d)^2*Sqrt[a - b*x^4]) + (b^(3/4)*(5*b*c - 11*a*d)*Sqrt[1 - (b*x^4)
/a]*EllipticF[ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(12*a^(7/4)*(b*c - a*d)^2*S
qrt[a - b*x^4]) + (a^(1/4)*d^2*Sqrt[1 - (b*x^4)/a]*EllipticPi[-((Sqrt[a]*Sq
rt[d])/(Sqrt[b]*Sqrt[c])), ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(2*b^(1/4)*c*(
b*c - a*d)^2*Sqrt[a - b*x^4]) + (a^(1/4)*d^2*Sqrt[1 - (b*x^4)/a]*EllipticPi
[(Sqrt[a]*Sqrt[d])/(Sqrt[b]*Sqrt[c]), ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(2*
b^(1/4)*c*(b*c - a*d)^2*Sqrt[a - b*x^4])
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
```

```
b/a] &&  !GtQ[a, 0]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Dist[1/(2*
c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 1233

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{(a-bx^4)^{5/2}(c-dx^4)}\,dx = \frac{bx}{6a(bc-ad)(a-bx^4)^{3/2}} + \frac{\int \frac{5bc-6ad-5bdx^4}{(a-bx^4)^{3/2}(c-dx^4)}\,dx}{6a(bc-ad)}$$

$$= \frac{bx}{6a(bc-ad)(a-bx^4)^{3/2}} + \frac{b(5bc-11ad)x}{12a^2(bc-ad)^2\sqrt{a-bx^4}} + \frac{\int \frac{5b^2c^2-11abcd+12a^2d^2-b\cdots}{\sqrt{a-bx^4}\,(c-\cdots}}{12a^2(bc-\cdots}$$

$$= \frac{bx}{6a(bc-ad)(a-bx^4)^{3/2}} + \frac{b(5bc-11ad)x}{12a^2(bc-ad)^2\sqrt{a-bx^4}} + \frac{d^2\int \frac{1}{\sqrt{a-bx^4}\,(c-dx^4)}\cdots}{(bc-ad)^2}$$

$$= \frac{bx}{6a(bc-ad)(a-bx^4)^{3/2}} + \frac{b(5bc-11ad)x}{12a^2(bc-ad)^2\sqrt{a-bx^4}} + \frac{d^2\int \frac{1}{\left(1-\frac{\sqrt{d}\,x^2}{\sqrt{c}}\right)\sqrt{a-\cdots}}}{2c(bc-ad)^2}$$

$$= \frac{bx}{6a(bc-ad)(a-bx^4)^{3/2}} + \frac{b(5bc-11ad)x}{12a^2(bc-ad)^2\sqrt{a-bx^4}} + \frac{b^{3/4}(5bc-11ad)\sqrt{1-\cdots}}{12a^{7/4}(b\cdots}$$

$$= \frac{bx}{6a(bc-ad)(a-bx^4)^{3/2}} + \frac{b(5bc-11ad)x}{12a^2(bc-ad)^2\sqrt{a-bx^4}} + \frac{b^{3/4}(5bc-11ad)\sqrt{1-\cdots}}{12a^{7/4}(b\cdots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 10.51, size = 422, normalized size = 1.26

$$\frac{x\left(\frac{bd(-5bc+11ad)x^4\sqrt{1-\frac{bx^4}{a}}\,F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)}{c}-\frac{5\left(5ac\left(12a^3d^2+a^2bd(-24c+dx^4)+5b^3cx^4(-2c+dx^4)+ab^2(12c^2+15cdx^4-11d^2x^8)\right)F_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)+2bx^4(-c+dx^4)(13a^2d+5b^2cx^4-ab(7c+11dx^4))\left(2adF_1\left(\frac{5}{4};\frac{1}{2},2;\frac{9}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)+bcF_1\left(\frac{5}{4};\frac{3}{2},1;\frac{9}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)\right)\right)}{(a-bx^4)(-c+dx^4)\left(5acF_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)+2x^4\left(2adF_1\left(\frac{5}{4};\frac{1}{2},2;\frac{9}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)+bcF_1\left(\frac{5}{4};\frac{3}{2},1;\frac{9}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)\right)\right)}\right)}{60a^2(bc-ad)^2\sqrt{a-bx^4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a - b*x^4)^(5/2)*(c - d*x^4)),x]
```

```
[Out] (x*((b*d*(-5*b*c + 11*a*d)*x^4*Sqrt[1 - (b*x^4)/a]*AppellF1[5/4, 1/2, 1, 9/
4, (b*x^4)/a, (d*x^4)/c])/c - (5*(5*a*c*(12*a^3*d^2 + a^2*b*d*(-24*c + d*x^
4) + 5*b^3*c*x^4*(-2*c + d*x^4) + a*b^2*(12*c^2 + 15*c*d*x^4 - 11*d^2*x^8))
*AppellF1[1/4, 1/2, 1, 5/4, (b*x^4)/a, (d*x^4)/c] + 2*b*x^4*(-c + d*x^4)*(1
3*a^2*d + 5*b^2*c*x^4 - a*b*(7*c + 11*d*x^4))*(2*a*d*AppellF1[5/4, 1/2, 2,
9/4, (b*x^4)/a, (d*x^4)/c] + b*c*AppellF1[5/4, 3/2, 1, 9/4, (b*x^4)/a, (d*x
^4)/c])))/((a - b*x^4)*(-c + d*x^4)*(5*a*c*AppellF1[1/4, 1/2, 1, 5/4, (b*x^
```

```
4)/a, (d*x^4)/c] + 2*x^4*(2*a*d*AppellF1[5/4, 1/2, 2, 9/4, (b*x^4)/a, (d*x^
4)/c] + b*c*AppellF1[5/4, 3/2, 1, 9/4, (b*x^4)/a, (d*x^4)/c])))))/(60*a^2*(
b*c - a*d)^2*Sqrt[a - b*x^4])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.28, size = 361, normalized size = 1.08

method	result
default	$-\frac{x\sqrt{-b\,x^{4}+a}}{6ab(ad-bc)\left(x^{4}-\frac{a}{b}\right)^{2}}-\frac{bx(11ad-5bc)}{12a^{2}(ad-bc)^{2}\sqrt{-\left(x^{4}-\frac{a}{b}\right)b}}-\frac{b(11ad-5bc)\sqrt{1-\frac{x^{2}\sqrt{b}}{\sqrt{a}}}\,\sqrt{1+\frac{x^{2}\sqrt{b}}{\sqrt{a}}}\ \mathrm{EllipticF}\Big(}{12a^{2}(ad-bc)^{2}\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^{4}+a}}$
elliptic	$-\frac{x\sqrt{-b\,x^{4}+a}}{6ab(ad-bc)\left(x^{4}-\frac{a}{b}\right)^{2}}-\frac{bx(11ad-5bc)}{12a^{2}(ad-bc)^{2}\sqrt{-\left(x^{4}-\frac{a}{b}\right)b}}-\frac{b(11ad-5bc)\sqrt{1-\frac{x^{2}\sqrt{b}}{\sqrt{a}}}\,\sqrt{1+\frac{x^{2}\sqrt{b}}{\sqrt{a}}}\ \mathrm{EllipticF}\Big(}{12a^{2}(ad-bc)^{2}\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^{4}+a}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x^4+a)^(5/2)/(-d*x^4+c),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6/a*x/b/(a*d-b*c)*(-b*x^4+a)^(1/2)/(x^4-a/b)^2-1/12*b/a^2*x*(11*a*d-5*b*
c)/(a*d-b*c)^2/(-(x^4-a/b)*b)^(1/2)-1/12*b/a^2*(11*a*d-5*b*c)/(a*d-b*c)^2/(
1/a^(1/2)*b^(1/2))^(1/2)*(1-x^2*b^(1/2)/a^(1/2))^(1/2)*(1+x^2*b^(1/2)/a^(1/
```

```
2))^(1/2)/(-b*x^4+a)^(1/2)*EllipticF(x*(1/a^(1/2)*b^(1/2))^(1/2),I)-1/8*d*s
um(1/(a*d-b*c)^2/_alpha^3*(-1/((a*d-b*c)/d)^(1/2)*arctanh(1/2*(-2*_alpha^2*
b*x^2+2*a)/((a*d-b*c)/d)^(1/2)/(-b*x^4+a)^(1/2))-2/(1/a^(1/2)*b^(1/2))^(1/2
)*_alpha^3*d/c*(1-x^2*b^(1/2)/a^(1/2))^(1/2)*(1+x^2*b^(1/2)/a^(1/2))^(1/2)/
(-b*x^4+a)^(1/2)*EllipticPi(x*(1/a^(1/2)*b^(1/2))^(1/2),a^(1/2)/b^(1/2)*_al
pha^2/c*d,(-1/a^(1/2)*b^(1/2))^(1/2)/(1/a^(1/2)*b^(1/2))^(1/2))),_alpha=Roo
tOf(_Z^4*d-c))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^4+a)^(5/2)/(-d*x^4+c),x, algorithm="maxima")
[Out] -integrate(1/((-b*x^4 + a)^(5/2)*(d*x^4 - c)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^4+a)^(5/2)/(-d*x^4+c),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{-a^2c\sqrt{a-bx^4}+a^2dx^4\sqrt{a-bx^4}+2abcx^4\sqrt{a-bx^4}-2abdx^8\sqrt{a-bx^4}-b^2cx^8\sqrt{a-bx^4}+b^2dx^{12}\sqrt{a-bx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x**4+a)**(5/2)/(-d*x**4+c),x)
[Out] -Integral(1/(-a**2*c*sqrt(a - b*x**4) + a**2*d*x**4*sqrt(a - b*x**4) + 2*a*
b*c*x**4*sqrt(a - b*x**4) - 2*a*b*d*x**8*sqrt(a - b*x**4) - b**2*c*x**8*sqr
t(a - b*x**4) + b**2*d*x**12*sqrt(a - b*x**4)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^4+a)^(5/2)/(-d*x^4+c),x, algorithm="giac")
```

```
[Out] integrate(-1/((-b*x^4 + a)^(5/2)*(d*x^4 - c)), x)
```

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(a-b\,x^4\right)^{5/2}\,\left(c-d\,x^4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a - b*x^4)^(5/2)*(c - d*x^4)),x)
```

```
[Out] int(1/((a - b*x^4)^(5/2)*(c - d*x^4)), x)
```

3.179 $\int \frac{\left(a+bx^4\right)^{3/2}}{c+dx^4}\,dx$

Optimal. Leaf size=926

$$\frac{bx\sqrt{a+bx^4}}{3d}-\frac{(bc-ad)^{3/2}\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt[4]{-c}\,\sqrt[4]{d}\,\sqrt{a+bx^4}}\right)}{4(-c)^{3/4}d^{7/4}}-\frac{(-bc+ad)^{3/2}\tan^{-1}\left(\frac{\sqrt{-bc+ad}\,x}{\sqrt[4]{-c}\,\sqrt[4]{d}\,\sqrt{a+bx^4}}\right)}{4(-c)^{3/4}d^{7/4}}-\ldots$$

```
[Out] -1/4*(-a*d+b*c)^(3/2)*arctan(x*(-a*d+b*c)^(1/2)/(-c)^(1/4)/d^(1/4)/(b*x^4+a
)^(1/2))/(-c)^(3/4)/d^(7/4)-1/4*(a*d-b*c)^(3/2)*arctan(x*(a*d-b*c)^(1/2)/(-
c)^(1/4)/d^(1/4)/(b*x^4+a)^(1/2))/(-c)^(3/4)/d^(7/4)+1/3*b*x*(b*x^4+a)^(1/2
)/d-1/6*b^(3/4)*(-5*a*d+3*b*c)*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/c
os(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x/a^(1/4))),
1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2
)/a^(1/4)/d^2/(b*x^4+a)^(1/2)+1/4*b^(1/4)*(-a*d+b*c)^2*(cos(2*arctan(b^(1/4
)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arc
tan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*(b^(1/2)*(-c)^(1
/2)-a^(1/2)*d^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/a^(1/4)/d^2/
(a*d+b*c)/(-c)^(1/2)/(b*x^4+a)^(1/2)+1/8*(-a*d+b*c)^2*(cos(2*arctan(b^(1/4)
*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticPi(sin(2*arc
tan(b^(1/4)*x/a^(1/4))),1/4*(b^(1/2)*(-c)^(1/2)+a^(1/2)*d^(1/2))^2/a^(1/2)/
b^(1/2)/(-c)^(1/2)/d^(1/2),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*(b^(1/2)*(-c)
^(1/2)-a^(1/2)*d^(1/2))^2*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/a^(1/4)
/b^(1/4)/c/d^2/(a*d+b*c)/(b*x^4+a)^(1/2)+1/4*b^(1/4)*(-a*d+b*c)^2*(cos(2*ar
ctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*Elliptic
F(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*(b^(1
/2)*(-c)^(1/2)+a^(1/2)*d^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/a
^(1/4)/d^2/(a*d+b*c)/(-c)^(1/2)/(b*x^4+a)^(1/2)+1/8*(-a*d+b*c)^2*(cos(2*arc
tan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticP
i(sin(2*arctan(b^(1/4)*x/a^(1/4))),-1/4*(b^(1/2)*(-c)^(1/2)-a^(1/2)*d^(1/2)
)^2/a^(1/2)/b^(1/2)/(-c)^(1/2)/d^(1/2),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*(
b^(1/2)*(-c)^(1/2)+a^(1/2)*d^(1/2))^2*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(
1/2)/a^(1/4)/b^(1/4)/c/d^2/(a*d+b*c)/(b*x^4+a)^(1/2)
```

Rubi [A]
time = 1.12, antiderivative size = 926, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {427, 537, 226, 418, 1231, 1721}

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)^(3/2)/(c + d*x^4),x]
```

```
[Out] (b*x*Sqrt[a + b*x^4])/(3*d) - ((b*c - a*d)^(3/2)*ArcTan[(Sqrt[b*c - a*d]*x)
/((-c)^(1/4)*d^(1/4)*Sqrt[a + b*x^4])])/(4*(-c)^(3/4)*d^(7/4)) - ((-(b*c) +
 a*d)^(3/2)*ArcTan[(Sqrt[-(b*c) + a*d]*x)/((-c)^(1/4)*d^(1/4)*Sqrt[a + b*x^
4])])/(4*(-c)^(3/4)*d^(7/4)) - (b^(3/4)*(3*b*c - 5*a*d)*(Sqrt[a] + Sqrt[b]*
x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4
)*x)/a^(1/4)], 1/2])/(6*a^(1/4)*d^2*Sqrt[a + b*x^4]) + (b^(1/4)*(Sqrt[b]*Sq
rt[-c] - Sqrt[a]*Sqrt[d])*(b*c - a*d)^2*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b
*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1
/2])/(4*a^(1/4)*Sqrt[-c]*d^2*(b*c + a*d)*Sqrt[a + b*x^4]) + (b^(1/4)*(Sqrt[
b]*Sqrt[-c] + Sqrt[a]*Sqrt[d])*(b*c - a*d)^2*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(
a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4
)], 1/2])/(4*a^(1/4)*Sqrt[-c]*d^2*(b*c + a*d)*Sqrt[a + b*x^4]) + ((Sqrt[b]*
Sqrt[-c] + Sqrt[a]*Sqrt[d])^2*(b*c - a*d)^2*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a
 + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticPi[-1/4*(Sqrt[b]*Sqrt[-c] - Sq
rt[a]*Sqrt[d])^2/(Sqrt[a]*Sqrt[b]*Sqrt[-c]*Sqrt[d]), 2*ArcTan[(b^(1/4)*x)/a
^(1/4)], 1/2])/(8*a^(1/4)*b^(1/4)*c*d^2*(b*c + a*d)*Sqrt[a + b*x^4]) + ((Sq
rt[b]*Sqrt[-c] - Sqrt[a]*Sqrt[d])^2*(b*c - a*d)^2*(Sqrt[a] + Sqrt[b]*x^2)*S
qrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticPi[(Sqrt[b]*Sqrt[-c] + S
qrt[a]*Sqrt[d])^2/(4*Sqrt[a]*Sqrt[b]*Sqrt[-c]*Sqrt[d]), 2*ArcTan[(b^(1/4)*x
)/a^(1/4)], 1/2])/(8*a^(1/4)*b^(1/4)*c*d^2*(b*c + a*d)*Sqrt[a + b*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Dist[1/(2*
c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 1231

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1721

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*((a + c*x^4)/(a*(A + B*x^2)^2))]/(
4*d*e*A*q*Sqrt[a + c*x^4]))*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)],
2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a+bx^4)^{3/2}}{c+dx^4}\,dx &= \frac{bx\sqrt{a+bx^4}}{3d} + \frac{\int \frac{-a(bc-3ad)-b(3bc-5ad)x^4}{\sqrt{a+bx^4}\,(c+dx^4)}\,dx}{3d} \\
&= \frac{bx\sqrt{a+bx^4}}{3d} - \frac{(b(3bc-5ad))\int \frac{1}{\sqrt{a+bx^4}}\,dx}{3d^2} + \frac{(bc-ad)^2\int \frac{1}{\sqrt{a+bx^4}\,(c+dx^4)}\,dx}{d^2} \\
&= \frac{bx\sqrt{a+bx^4}}{3d} - \frac{b^{3/4}(3bc-5ad)\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\cdots\right.\right.}{6\sqrt[4]{a}\,d^2\sqrt{a+bx^4}} \cdots \\
&= \frac{bx\sqrt{a+bx^4}}{3d} - \frac{b^{3/4}(3bc-5ad)\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\cdots\right.\right.}{6\sqrt[4]{a}\,d^2\sqrt{a+bx^4}} \cdots \\
&= \frac{bx\sqrt{a+bx^4}}{3d} - \frac{(bc-ad)^{3/2}\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt[4]{-c}\,\sqrt[4]{d}\,\sqrt{a+bx^4}}\right)}{4(-c)^{3/4}d^{7/4}} - \frac{(-bc+ad)^{3/2}\tan^{-1}\left(\cdots\right.}{4(-c)^{3/\cdots}} \cdots
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 10.32, size = 346, normalized size = 0.37

$$
\frac{x\left(\frac{b(-3bc+5ad)x^4\sqrt{1+\frac{bx^4}{a}}\,F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{c}+\frac{5\left(-5ac\left(3a^2d+abdx^4+b^2x^4(c+dx^4)\right)F_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)+2bx^4(a+bx^4)(c+dx^4)\left(2adF_1\left(\frac{5}{4};\frac{1}{2},2;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)+bcF_1\left(\frac{5}{4};\frac{3}{2},1;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)\right)\right)}{(c+dx^4)\left(-5acF_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)+2x^4\left(2adF_1\left(\frac{5}{4};\frac{1}{2},2;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)+bcF_1\left(\frac{5}{4};\frac{3}{2},1;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)\right)\right)}\right)}{15d\sqrt{a+bx^4}}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^4)^(3/2)/(c + d*x^4),x]
```

```
[Out] (x*((b*(-3*b*c + 5*a*d)*x^4*Sqrt[1 + (b*x^4)/a]*AppellF1[5/4, 1/2, 1, 9/4,
-((b*x^4)/a), -((d*x^4)/c)])/c + (5*(-5*a*c*(3*a^2*d + a*b*d*x^4 + b^2*x^4*
(c + d*x^4))*AppellF1[1/4, 1/2, 1, 5/4, -((b*x^4)/a), -((d*x^4)/c)] + 2*b*x
^4*(a + b*x^4)*(c + d*x^4)*(2*a*d*AppellF1[5/4, 1/2, 2, 9/4, -((b*x^4)/a),
-((d*x^4)/c)] + b*c*AppellF1[5/4, 3/2, 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)])
))/((c + d*x^4)*(-5*a*c*AppellF1[1/4, 1/2, 1, 5/4, -((b*x^4)/a), -((d*x^4)/
c)] + 2*x^4*(2*a*d*AppellF1[5/4, 1/2, 2, 9/4, -((b*x^4)/a), -((d*x^4)/c)] +
 b*c*AppellF1[5/4, 3/2, 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)])))))/(15*d*Sqrt
[a + b*x^4])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.32, size = 322, normalized size = 0.35

method	result
risch	$\frac{bx\sqrt{b\,x^4+a}}{3d}+\frac{\frac{b(5ad-3bc)\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{d\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}+\frac{3\left(a^2d^2-2abcd+b^2c^2\right)\left(\sum_{\_\alpha=\mathrm{RootOf}}\right.}{}}{}$
default	$\frac{bx\sqrt{b\,x^4+a}}{3d}+\frac{\left(\frac{b(2ad-bc)}{d^2}-\frac{ba}{3d}\right)\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}-\frac{\sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z^4+c\right)}}{}$
elliptic	$\frac{bx\sqrt{b\,x^4+a}}{3d}+\frac{\left(\frac{b(2ad-bc)}{d^2}-\frac{ba}{3d}\right)\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}-\frac{\sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z^4+c\right)}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)^(3/2)/(d*x^4+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*b*x*(b*x^4+a)^(1/2)/d+(b*(2*a*d-b*c)/d^2-1/3*b/d*a)/(I/a^(1/2)*b^(1/2))
^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x
^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-1/8/d^3*sum((-a^2*d^2+
2*a*b*c*d-b^2*c^2)/_alpha^3*(-1/((a*d-b*c)/d)^(1/2)*arctanh(1/2*(2*_alpha^2
*b*x^2+2*a)/((a*d-b*c)/d)^(1/2)/(b*x^4+a)^(1/2))+2/(I/a^(1/2)*b^(1/2))^(1/2
)*_alpha^3*d/c*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1
/2)/(b*x^4+a)^(1/2)*EllipticPi(x*(I/a^(1/2)*b^(1/2))^(1/2),I*a^(1/2)/b^(1/2
)*_alpha^2/c*d,(-I/a^(1/2)*b^(1/2))^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2))),_alph
a=RootOf(_Z^4*d+c))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(3/2)/(d*x^4+c),x, algorithm="maxima")
[Out] integrate((b*x^4 + a)^(3/2)/(d*x^4 + c), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(3/2)/(d*x^4+c),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + bx^4\right)^{\frac{3}{2}}}{c + dx^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)**(3/2)/(d*x**4+c),x)
[Out] Integral((a + b*x**4)**(3/2)/(c + d*x**4), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(3/2)/(d*x^4+c),x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)^(3/2)/(d*x^4 + c), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(b\,x^4+a)^{3/2}}{d\,x^4+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)^(3/2)/(c + d*x^4),x)
```

```
[Out] int((a + b*x^4)^(3/2)/(c + d*x^4), x)
```

3.180 $\int \frac{\sqrt{a+bx^4}}{c+dx^4}\,dx$

Optimal. Leaf size=881

$$\frac{\sqrt{bc-ad}\,\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt[4]{-c}\,\sqrt[4]{d}\,\sqrt{a+bx^4}}\right)}{4(-c)^{3/4}d^{3/4}}-\frac{\sqrt{-bc+ad}\,\tan^{-1}\left(\frac{\sqrt{-bc+ad}\,x}{\sqrt[4]{-c}\,\sqrt[4]{d}\,\sqrt{a+bx^4}}\right)}{4(-c)^{3/4}d^{3/4}}+\frac{b^{3/4}\left(\sqrt{a}+\sqrt{b}\,x^2\right.}{}$$

```
[Out] 1/4*arctan(x*(-a*d+b*c)^(1/2)/(-c)^(1/4)/d^(1/4)/(b*x^4+a)^(1/2))*(-a*d+b*c
)^(1/2)/(-c)^(3/4)/d^(3/4)-1/4*arctan(x*(a*d-b*c)^(1/2)/(-c)^(1/4)/d^(1/4)/
(b*x^4+a)^(1/2))*(a*d-b*c)^(1/2)/(-c)^(3/4)/d^(3/4)+1/2*b^(3/4)*(cos(2*arct
an(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticF(
sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4
+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/a^(1/4)/d/(b*x^4+a)^(1/2)-1/4*b^(1/4)*(-
a*d+b*c)*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/
a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+
x^2*b^(1/2))*(b^(1/2)*(-c)^(1/2)-a^(1/2)*d^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b
^(1/2))^2)^(1/2)/a^(1/4)/d/(a*d+b*c)/(-c)^(1/2)/(b*x^4+a)^(1/2)-1/8*(-a*d+b
*c)*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/
4)))*EllipticPi(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/4*(b^(1/2)*(-c)^(1/2)+a^
(1/2)*d^(1/2))^2/a^(1/2)/b^(1/2)/(-c)^(1/2)/d^(1/2),1/2*2^(1/2))*(a^(1/2)+x
^2*b^(1/2))*(b^(1/2)*(-c)^(1/2)-a^(1/2)*d^(1/2))^2*((b*x^4+a)/(a^(1/2)+x^2*
b^(1/2))^2)^(1/2)/a^(1/4)/b^(1/4)/c/d/(a*d+b*c)/(b*x^4+a)^(1/2)-1/8*(-a*d+b
*c)*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/
4)))*EllipticPi(sin(2*arctan(b^(1/4)*x/a^(1/4))),-1/4*(b^(1/2)*(-c)^(1/2)-a
^(1/2)*d^(1/2))^2/a^(1/2)/b^(1/2)/(-c)^(1/2)/d^(1/2),1/2*2^(1/2))*(a^(1/2)+
x^2*b^(1/2))*(b^(1/2)*(-c)^(1/2)+a^(1/2)*d^(1/2))^2*((b*x^4+a)/(a^(1/2)+x^2
*b^(1/2))^2)^(1/2)/a^(1/4)/b^(1/4)/c/d/(a*d+b*c)/(b*x^4+a)^(1/2)-1/4*b^(1/4
)*(-a*d+b*c)*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4
)*x/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1
/2)+x^2*b^(1/2))*(b^(1/2)+a^(1/2)*d^(1/2)/(-c)^(1/2))*((b*x^4+a)/(a^(1/2)+x
^2*b^(1/2))^2)^(1/2)/a^(1/4)/d/(a*d+b*c)/(b*x^4+a)^(1/2)
```

Rubi [A]

time = 0.57, antiderivative size = 881, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {415, 226, 418, 1231, 1721}

[illegible]

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^4]/(c + d*x^4),x]
```

```
[Out] (Sqrt[b*c - a*d]*ArcTan[(Sqrt[b*c - a*d]*x)/((-c)^(1/4)*d^(1/4)*Sqrt[a + b*
x^4])])/(4*(-c)^(3/4)*d^(3/4)) - (Sqrt[-(b*c) + a*d]*ArcTan[(Sqrt[-(b*c) +
a*d]*x)/((-c)^(1/4)*d^(1/4)*Sqrt[a + b*x^4])])/(4*(-c)^(3/4)*d^(3/4)) + (b^
(3/4)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*E
llipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*d*Sqrt[a + b*x^4])
 - (b^(1/4)*(Sqrt[b]*Sqrt[-c] - Sqrt[a]*Sqrt[d])*(b*c - a*d)*(Sqrt[a] + Sqr
t[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b
^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(1/4)*Sqrt[-c]*d*(b*c + a*d)*Sqrt[a + b*x^4]
) - (b^(1/4)*(Sqrt[b] + (Sqrt[a]*Sqrt[d])/Sqrt[-c])*(b*c - a*d)*(Sqrt[a] +
Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan
[(b^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(1/4)*d*(b*c + a*d)*Sqrt[a + b*x^4]) - ((
Sqrt[b]*Sqrt[-c] + Sqrt[a]*Sqrt[d])^2*(b*c - a*d)*(Sqrt[a] + Sqrt[b]*x^2)*S
qrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticPi[-1/4*(Sqrt[b]*Sqrt[-c
] - Sqrt[a]*Sqrt[d])^2/(Sqrt[a]*Sqrt[b]*Sqrt[-c]*Sqrt[d]), 2*ArcTan[(b^(1/4
)*x)/a^(1/4)], 1/2])/(8*a^(1/4)*b^(1/4)*c*d*(b*c + a*d)*Sqrt[a + b*x^4]) -
((Sqrt[b]*Sqrt[-c] - Sqrt[a]*Sqrt[d])^2*(b*c - a*d)*(Sqrt[a] + Sqrt[b]*x^2)
*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticPi[(Sqrt[b]*Sqrt[-c] +
 Sqrt[a]*Sqrt[d])^2/(4*Sqrt[a]*Sqrt[b]*Sqrt[-c]*Sqrt[d]), 2*ArcTan[(b^(1/4)
*x)/a^(1/4)], 1/2])/(8*a^(1/4)*b^(1/4)*c*d*(b*c + a*d)*Sqrt[a + b*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 415

```
Int[Sqrt[(a_) + (b_.)*(x_)^4]/((c_) + (d_.)*(x_)^4), x_Symbol] :> Dist[b/d,
 Int[1/Sqrt[a + b*x^4], x], x] - Dist[(b*c - a*d)/d, Int[1/(Sqrt[a + b*x^4]
*(c + d*x^4)), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Dist[1/(2*
c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 1231

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1721

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*((a + c*x^4)/(a*(A + B*x^2)^2))]/(
4*d*e*A*q*Sqrt[a + c*x^4]))*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)],
2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^4}}{c+dx^4}\,dx = \frac{b\int \frac{1}{\sqrt{a+bx^4}}\,dx}{d} - \frac{(bc-ad)\int \frac{1}{\sqrt{a+bx^4}\,(c+dx^4)}\,dx}{d}$$

$$= \frac{b^{3/4}\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\,d\sqrt{a+bx^4}} - \frac{(bc-ad)\int \frac{\sqrt{\ldots}}{\left(1-\frac{\sqrt{\ldots}}{\sqrt{\ldots}}\right.\ldots}}{2\ldots}$$

$$= \frac{b^{3/4}\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\,d\sqrt{a+bx^4}} - \frac{\left(\sqrt{b}\left(\sqrt{b}\sqrt{-c}\ldots\right.\right.}{\ldots}$$

$$= \frac{\sqrt{bc-ad}\,\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt[4]{-c}\,\sqrt[4]{d}\,\sqrt{a+bx^4}}\right)}{4(-c)^{3/4}d^{3/4}} - \frac{\sqrt{-bc+ad}\,\tan^{-1}\left(\frac{\sqrt{-bc+ad}\,x}{\sqrt[4]{-c}\,\sqrt[4]{d}\,\sqrt{a+bx^4}}\right)}{4(-c)^{3/4}d^{3/4}}\ldots$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 10.13, size = 161, normalized size = 0.18

$$\frac{5acx\sqrt{a+bx^4}\,F_1\left(\frac{1}{4};-\frac{1}{2},1;\frac{5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{(c+dx^4)\left(5acF_1\left(\frac{1}{4};-\frac{1}{2},1;\frac{5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)+2x^4\left(-2adF_1\left(\frac{5}{4};-\frac{1}{2},2;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)+bcF_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a + b*x^4]/(c + d*x^4),x]
```

```
[Out] (5*a*c*x*Sqrt[a + b*x^4]*AppellF1[1/4, -1/2, 1, 5/4, -((b*x^4)/a), -((d*x^4
)/c)])/((c + d*x^4)*(5*a*c*AppellF1[1/4, -1/2, 1, 5/4, -((b*x^4)/a), -((d*x
```

```
^4)/c)] + 2*x^4*(-2*a*d*AppellF1[5/4, -1/2, 2, 9/4, -((b*x^4)/a), -((d*x^4)
/c)] + b*c*AppellF1[5/4, 1/2, 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)])))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.29, size = 273, normalized size = 0.31

method	result
default	$\frac{b\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{d\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{b\,x^4+a}} - \sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z^4+c\right)} \frac{(-ad+bc)\left(-\frac{\operatorname{arctanh}\left(\frac{2b\,x^2\_\alpha^2}{2\sqrt{\frac{ad-bc}{d}}\sqrt{}}\right)}{\sqrt{\frac{ad-bc}{d}}}\right.}{}$
elliptic	$\frac{b\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{d\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{b\,x^4+a}} - \sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z^4+c\right)} \frac{(-ad+bc)\left(-\frac{\operatorname{arctanh}\left(\frac{2b\,x^2\_\alpha^2}{2\sqrt{\frac{ad-bc}{d}}\sqrt{}}\right)}{\sqrt{\frac{ad-bc}{d}}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)^(1/2)/(d*x^4+c),x,method=_RETURNVERBOSE)
```

```
[Out] b/d/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*
b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)
-1/8/d^2*sum((-a*d+b*c)/_alpha^3*(-1/((a*d-b*c)/d)^(1/2)*arctanh(1/2*(2*_al
pha^2*b*x^2+2*a)/((a*d-b*c)/d)^(1/2)/(b*x^4+a)^(1/2))+2/(I/a^(1/2)*b^(1/2))
^(1/2)*_alpha^3*d/c*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^
2)^(1/2)/(b*x^4+a)^(1/2)*EllipticPi(x*(I/a^(1/2)*b^(1/2))^(1/2),I*a^(1/2)/b
^(1/2)*_alpha^2/c*d,(-I/a^(1/2)*b^(1/2))^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2))),
_alpha=RootOf(_Z^4*d+c))
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(1/2)/(d*x^4+c),x, algorithm="maxima")
[Out] integrate(sqrt(b*x^4 + a)/(d*x^4 + c), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(1/2)/(d*x^4+c),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+bx^4}}{c+dx^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)**(1/2)/(d*x**4+c),x)
[Out] Integral(sqrt(a + b*x**4)/(c + d*x**4), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(1/2)/(d*x^4+c),x, algorithm="giac")
[Out] integrate(sqrt(b*x^4 + a)/(d*x^4 + c), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{b\,x^4+a}}{d\,x^4+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)^(1/2)/(c + d*x^4),x)
[Out] int((a + b*x^4)^(1/2)/(c + d*x^4), x)
```

3.181 $\int \frac{1}{\sqrt{a+bx^4}\,(c+dx^4)}\,dx$

Optimal. Leaf size=742

$$-\frac{\sqrt[4]{d}\,\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt[4]{-c}\,\sqrt[4]{d}\,\sqrt{a+bx^4}}\right)}{4(-c)^{3/4}\sqrt{bc-ad}}-\frac{\sqrt[4]{d}\,\tan^{-1}\left(\frac{\sqrt{-bc+ad}\,x}{\sqrt[4]{-c}\,\sqrt[4]{d}\,\sqrt{a+bx^4}}\right)}{4(-c)^{3/4}\sqrt{-bc+ad}}+\frac{\sqrt[4]{b}\left(\sqrt{b}+\frac{\sqrt{a}\sqrt{d}}{\sqrt{-c}}\right)\left(\sqrt{a}+\right.}{4}$$

```
[Out] -1/4*d^(1/4)*arctan(x*(-a*d+b*c)^(1/2)/(-c)^(1/4)/d^(1/4)/(b*x^4+a)^(1/2))/
(-c)^(3/4)/(-a*d+b*c)^(1/2)-1/4*d^(1/4)*arctan(x*(a*d-b*c)^(1/2)/(-c)^(1/4)
/d^(1/4)/(b*x^4+a)^(1/2))/(-c)^(3/4)/(a*d-b*c)^(1/2)+1/8*(cos(2*arctan(b^(1
/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticPi(sin(2*
arctan(b^(1/4)*x/a^(1/4))),1/4*(b^(1/2)*(-c)^(1/2)+a^(1/2)*d^(1/2))^2/a^(1/
2)/b^(1/2)/(-c)^(1/2)/d^(1/2),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*(b^(1/2)*(
-c)^(1/2)-a^(1/2)*d^(1/2))^2*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/a^(1
/4)/b^(1/4)/c/(a*d+b*c)/(b*x^4+a)^(1/2)+1/8*(cos(2*arctan(b^(1/4)*x/a^(1/4)
))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticPi(sin(2*arctan(b^(1/4
)*x/a^(1/4))),-1/4*(b^(1/2)*(-c)^(1/2)-a^(1/2)*d^(1/2))^2/a^(1/2)/b^(1/2)/(
-c)^(1/2)/d^(1/2),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*(b^(1/2)*(-c)^(1/2)+a^
(1/2)*d^(1/2))^2*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/a^(1/4)/b^(1/4)/
c/(a*d+b*c)/(b*x^4+a)^(1/2)+1/4*b^(1/4)*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2
)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x/a
^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*(b^(1/2)+a^(1/2)*d^(1/2)/(-c)^(
1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/a^(1/4)/(a*d+b*c)/(b*x^4+a)
^(1/2)+1/4*b^(1/4)*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(
b^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))
*(a^(1/2)+x^2*b^(1/2))*(c*b^(1/2)+a^(1/2)*(-c)^(1/2)*d^(1/2))*((b*x^4+a)/(a
^(1/2)+x^2*b^(1/2))^2)^(1/2)/a^(1/4)/c/(a*d+b*c)/(b*x^4+a)^(1/2)
```

Rubi [A]
time = 0.39, antiderivative size = 742, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {418, 1231, 226, 1721}

$$-\frac{\sqrt[4]{d}\,\text{ArcTan}\left(\frac{x\sqrt{bc-ad}}{\sqrt[4]{-c}\,\sqrt[4]{d}\,\sqrt{a+bx^4}}\right)}{4(-c)^{3/4}\sqrt{bc-ad}}-\frac{\sqrt[4]{d}\,\text{ArcTan}\left(\frac{x\sqrt{ad-bc}}{\sqrt[4]{-c}\,\sqrt[4]{d}\,\sqrt{a+bx^4}}\right)}{4(-c)^{3/4}\sqrt{ad-bc}}+\frac{\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{-c}}+\sqrt{b}\right)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4\sqrt[4]{a}\sqrt{a+bx^4}(ad+bc)}+\frac{\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{a}\sqrt{-c}\sqrt{d}+\sqrt{b}c\right)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4\sqrt[4]{a}c\sqrt{a+bx^4}(ad+bc)}+\frac{\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{b}\sqrt{-c}-\sqrt{a}\sqrt{d}\right)^2\Pi\left(\frac{\left(\sqrt{b}\sqrt{-c}+\sqrt{a}\sqrt{d}\right)^2}{4\sqrt{a}\sqrt{b}\sqrt{-c}\sqrt{d}};2\text{ArcTan}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{8\sqrt[4]{a}\sqrt[4]{b}c\sqrt{a+bx^4}(ad+bc)}+\frac{\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{-c}\right)^2\Pi\left(-\frac{\left(\sqrt{b}\sqrt{-c}-\sqrt{a}\sqrt{d}\right)^2}{4\sqrt{a}\sqrt{b}\sqrt{-c}\sqrt{d}};2\text{ArcTan}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{8\sqrt[4]{a}\sqrt[4]{b}c\sqrt{a+bx^4}(ad+bc)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b*x^4]*(c + d*x^4)),x]
```

```
[Out] -1/4*(d^(1/4)*ArcTan[(Sqrt[b*c - a*d]*x)/((-c)^(1/4)*d^(1/4)*Sqrt[a + b*x^4
])])/((-c)^(3/4)*Sqrt[b*c - a*d]) - (d^(1/4)*ArcTan[(Sqrt[-(b*c) + a*d]*x)/
((-c)^(1/4)*d^(1/4)*Sqrt[a + b*x^4])])/(4*(-c)^(3/4)*Sqrt[-(b*c) + a*d]) +
(b^(1/4)*(Sqrt[b] + (Sqrt[a]*Sqrt[d])/Sqrt[-c])*(Sqrt[a] + Sqrt[b]*x^2)*Sqr
```

```
t[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(
1/4)], 1/2])/(4*a^(1/4)*(b*c + a*d)*Sqrt[a + b*x^4]) + (b^(1/4)*(Sqrt[b]*c
+ Sqrt[a]*Sqrt[-c]*Sqrt[d])*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[
a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(1
/4)*c*(b*c + a*d)*Sqrt[a + b*x^4]) + ((Sqrt[b]*Sqrt[-c] + Sqrt[a]*Sqrt[d])^
2*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*Ellip
ticPi[-1/4*(Sqrt[b]*Sqrt[-c] - Sqrt[a]*Sqrt[d])^2/(Sqrt[a]*Sqrt[b]*Sqrt[-c]
*Sqrt[d]), 2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(8*a^(1/4)*b^(1/4)*c*(b*c +
 a*d)*Sqrt[a + b*x^4]) + ((Sqrt[b]*Sqrt[-c] - Sqrt[a]*Sqrt[d])^2*(Sqrt[a] +
 Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticPi[(Sqrt[
b]*Sqrt[-c] + Sqrt[a]*Sqrt[d])^2/(4*Sqrt[a]*Sqrt[b]*Sqrt[-c]*Sqrt[d]), 2*Ar
cTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(8*a^(1/4)*b^(1/4)*c*(b*c + a*d)*Sqrt[a +
b*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Dist[1/(2*
c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 1231

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1721

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*((a + c*x^4)/(a*(A + B*x^2)^2))]/(
4*d*e*A*q*Sqrt[a + c*x^4]))*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)],
2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+bx^4}\,(c+dx^4)}\,dx = \frac{\int \frac{1}{\left(1-\frac{\sqrt{d}\,x^2}{\sqrt{-c}}\right)\sqrt{a+bx^4}}\,dx}{2c} + \frac{\int \frac{1}{\left(1+\frac{\sqrt{d}\,x^2}{\sqrt{-c}}\right)\sqrt{a+bx^4}}\,dx}{2c}$$

$$= \frac{\left(\sqrt{b}\left(\sqrt{b}+\frac{\sqrt{a}\sqrt{d}}{\sqrt{-c}}\right)\right)\int \frac{1}{\sqrt{a+bx^4}}\,dx}{2(bc+ad)} + \frac{\left(\sqrt{b}\left(\sqrt{b}\,c+\sqrt{a}\sqrt{-c}\sqrt{d}\right)\right)}{2c(bc+ad)}$$

$$= -\frac{\sqrt[4]{d}\,\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt[4]{-c}\,\sqrt[4]{d}\,\sqrt{a+bx^4}}\right)}{4(-c)^{3/4}\sqrt{bc-ad}} - \frac{\sqrt[4]{d}\,\tan^{-1}\left(\frac{\sqrt{-bc+ad}\,x}{\sqrt[4]{-c}\,\sqrt[4]{d}\,\sqrt{a+bx^4}}\right)}{4(-c)^{3/4}\sqrt{-bc+ad}} + -$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 10.05, size = 161, normalized size = 0.22

$$-\frac{5acxF_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{\sqrt{a+bx^4}\,(c+dx^4)\left(-5acF_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)+2x^4\left(2adF_1\left(\frac{5}{4};\frac{1}{2},2;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)+bcF_1\left(\frac{5}{4};\frac{3}{2},1;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Sqrt[a + b*x^4]*(c + d*x^4)),x]
```

```
[Out] (-5*a*c*x*AppellF1[1/4, 1/2, 1, 5/4, -((b*x^4)/a), -((d*x^4)/c)])/(Sqrt[a +
 b*x^4]*(c + d*x^4)*(-5*a*c*AppellF1[1/4, 1/2, 1, 5/4, -((b*x^4)/a), -((d*x
^4)/c)] + 2*x^4*(2*a*d*AppellF1[5/4, 1/2, 2, 9/4, -((b*x^4)/a), -((d*x^4)/c
)] + b*c*AppellF1[5/4, 3/2, 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)])))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.27, size = 191, normalized size = 0.26

method	result
default	$\frac{\sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z^4+c\right)} \frac{-\frac{\mathrm{arctanh}\left(\frac{2b\,x^2\_\alpha^2+2a}{2\sqrt{\frac{ad-bc}{d}}\sqrt{b\,x^4+a}}\right)}{\sqrt{\frac{ad-bc}{d}}}+\frac{2\_\alpha^3 d\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticPi}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\right.}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,c\sqrt{b\,x^4+a}}}{\_\alpha^3}}{8d}$

elliptic	$\frac{\sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z^4+c\right)}\frac{-\frac{\operatorname{arctanh}\left(\frac{2b\,x^2\_\alpha^2+2a}{2\sqrt{\frac{ad-bc}{d}}\,\sqrt{b\,x^4+a}}\right)}{\sqrt{\frac{ad-bc}{d}}}+\frac{2\_\alpha^3 d\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticPi}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\right.}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,c\sqrt{b\,x^4+a}}}{\_\alpha^3}}{8d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^4+a)^(1/2)/(d*x^4+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/d*sum(1/_alpha^3*(-1/((a*d-b*c)/d)^(1/2)*arctanh(1/2*(2*_alpha^2*b*x^2+
2*a)/((a*d-b*c)/d)^(1/2)/(b*x^4+a)^(1/2))+2/(I/a^(1/2)*b^(1/2))^(1/2)*_alph
a^3*d/c*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*
x^4+a)^(1/2)*EllipticPi(x*(I/a^(1/2)*b^(1/2))^(1/2),I*a^(1/2)/b^(1/2)*_alph
a^2/c*d,(-I/a^(1/2)*b^(1/2))^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2))),_alpha=RootO
f(_Z^4*d+c))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(1/2)/(d*x^4+c),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x^4 + a)*(d*x^4 + c)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(1/2)/(d*x^4+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+bx^4}\,(c+dx^4)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**4+a)**(1/2)/(d*x**4+c),x)
```

```
[Out] Integral(1/(sqrt(a + b*x**4)*(c + d*x**4)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(1/2)/(d*x^4+c),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*x^4 + a)*(d*x^4 + c)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{b\,x^4+a}\,\left(d\,x^4+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^4)^(1/2)*(c + d*x^4)),x)
```

```
[Out] int(1/((a + b*x^4)^(1/2)*(c + d*x^4)), x)
```

3.182 $\int \frac{1}{\left(a+bx^4\right)^{3/2}\left(c+dx^4\right)}\,dx$

Optimal. Leaf size=913

$$\frac{bx}{2a(bc-ad)\sqrt{a+bx^4}}+\frac{d^{5/4}\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt[4]{-c}\,\sqrt[4]{d}\,\sqrt{a+bx^4}}\right)}{4(-c)^{3/4}(bc-ad)^{3/2}}-\frac{d^{5/4}\tan^{-1}\left(\frac{\sqrt{-bc+ad}\,x}{\sqrt[4]{-c}\,\sqrt[4]{d}\,\sqrt{a+bx^4}}\right)}{4(-c)^{3/4}(-bc+ad)^{3/2}}+\frac{b^{3/4}\left(\sqrt{a}\right.}{}$$

```
[Out] 1/4*d^(5/4)*arctan(x*(-a*d+b*c)^(1/2)/(-c)^(1/4)/d^(1/4)/(b*x^4+a)^(1/2))/(
-c)^(3/4)/(-a*d+b*c)^(3/2)-1/4*d^(5/4)*arctan(x*(a*d-b*c)^(1/2)/(-c)^(1/4)/
d^(1/4)/(b*x^4+a)^(1/2))/(-c)^(3/4)/(a*d-b*c)^(3/2)+1/2*b*x/a/(-a*d+b*c)/(b
*x^4+a)^(1/2)+1/4*b^(3/4)*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*
arctan(b^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2
^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/a^(
5/4)/(-a*d+b*c)/(b*x^4+a)^(1/2)-1/8*d*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^
(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticPi(sin(2*arctan(b^(1/4)*x/a^
(1/4))),1/4*(b^(1/2)*(-c)^(1/2)+a^(1/2)*d^(1/2))^2/a^(1/2)/b^(1/2)/(-c)^(1/
2)/d^(1/2),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*(b^(1/2)*(-c)^(1/2)-a^(1/2)*d
^(1/2))^2*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/a^(1/4)/b^(1/4)/c/(-a^2
*d^2+b^2*c^2)/(b*x^4+a)^(1/2)-1/8*d*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1
/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticPi(sin(2*arctan(b^(1/4)*x/a^(1
/4))),-1/4*(b^(1/2)*(-c)^(1/2)-a^(1/2)*d^(1/2))^2/a^(1/2)/b^(1/2)/(-c)^(1/2
)/d^(1/2),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*(b^(1/2)*(-c)^(1/2)+a^(1/2)*d^
(1/2))^2*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/a^(1/4)/b^(1/4)/c/(-a^2*
d^2+b^2*c^2)/(b*x^4+a)^(1/2)-1/4*b^(1/4)*d*(cos(2*arctan(b^(1/4)*x/a^(1/4))
)^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*
x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*(b^(1/2)+a^(1/2)*d^(1/2)/(-c
)^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/a^(1/4)/(-a*d+b*c)/(a*d+
b*c)/(b*x^4+a)^(1/2)-1/4*b^(1/4)*d*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/
2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x/a^(1/4
))),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*(c*b^(1/2)+a^(1/2)*(-c)^(1/2)*d^(1/2
))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/a^(1/4)/c/(-a^2*d^2+b^2*c^2)/(
b*x^4+a)^(1/2)
```

Rubi [A]
time = 0.86, antiderivative size = 913, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {425, 537, 226, 418, 1231, 1721}

$$-\frac{d\left(\sqrt{b}x^2+\sqrt{a}\right)\sqrt{\frac{bx^4+a}{\left(\sqrt{b}x^2+\sqrt{a}\right)^2}}\,\Pi\left(\frac{\left(\sqrt{b}\sqrt{-c}+\sqrt{a}\sqrt{d}\right)^2}{4\sqrt{a}\sqrt{b}\sqrt{-c}\sqrt{d}};2\text{ArcTan}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)\left(\sqrt{b}\sqrt{-c}-\sqrt{a}\sqrt{d}\right)^2}{8\sqrt[4]{a}\sqrt[4]{b}c(bc-ad)(bc+ad)\sqrt{bx^4+a}}+\frac{d^{5/4}\text{ArcTan}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt[4]{-c}\sqrt[4]{d}\sqrt{bx^4+a}}\right)}{4(-c)^{3/4}(bc-ad)^{3/2}}-\frac{d^{5/4}\text{ArcTan}\left(\frac{\sqrt{ad-bc}\,x}{\sqrt[4]{-c}\sqrt[4]{d}\sqrt{bx^4+a}}\right)}{4(-c)^{3/4}(ad-bc)^{3/2}}+\frac{b^{3/4}\left(\sqrt{b}x^2+\sqrt{a}\right)\sqrt{\frac{bx^4+a}{\left(\sqrt{b}x^2+\sqrt{a}\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4a^{5/4}(bc-ad)\sqrt{bx^4+a}}-\frac{\sqrt[4]{b}\left(\sqrt{b}+\frac{\sqrt{a}\sqrt{d}}{\sqrt{-c}}\right)d\left(\sqrt{b}x^2+\sqrt{a}\right)\sqrt{\frac{bx^4+a}{\left(\sqrt{b}x^2+\sqrt{a}\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4\sqrt[4]{a}(bc-ad)(bc+ad)\sqrt{bx^4+a}}-\frac{\sqrt[4]{b}\left(\sqrt{b}c+\sqrt{a}\sqrt{-c}\sqrt{d}\right)d\left(\sqrt{b}x^2+\sqrt{a}\right)\sqrt{\frac{bx^4+a}{\left(\sqrt{b}x^2+\sqrt{a}\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4\sqrt[4]{a}c\left(b^2c^2-a^2d^2\right)\sqrt{bx^4+a}}-\frac{\left(\sqrt{b}\sqrt{-c}+\sqrt{a}\sqrt{d}\right)^2d\left(\sqrt{b}x^2+\sqrt{a}\right)\sqrt{\frac{bx^4+a}{\left(\sqrt{b}x^2+\sqrt{a}\right)^2}}\,\Pi\left(-\frac{\left(\sqrt{b}\sqrt{-c}-\sqrt{a}\sqrt{d}\right)^2}{4\sqrt{a}\sqrt{b}\sqrt{-c}\sqrt{d}};2\text{ArcTan}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{8\sqrt[4]{a}\sqrt[4]{b}c(bc-ad)(bc+ad)\sqrt{bx^4+a}}+\frac{bx}{2a(bc-ad)\sqrt{bx^4+a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^4)^(3/2)*(c + d*x^4)),x]
```

```
[Out] (b*x)/(2*a*(b*c - a*d)*Sqrt[a + b*x^4]) + (d^(5/4)*ArcTan[(Sqrt[b*c - a*d]*
x)/((-c)^(1/4)*d^(1/4)*Sqrt[a + b*x^4])])/(4*(-c)^(3/4)*(b*c - a*d)^(3/2))
- (d^(5/4)*ArcTan[(Sqrt[-(b*c) + a*d]*x)/((-c)^(1/4)*d^(1/4)*Sqrt[a + b*x^4
])])/(4*(-c)^(3/4)*(-(b*c) + a*d)^(3/2)) + (b^(3/4)*(Sqrt[a] + Sqrt[b]*x^2)
*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)
/a^(1/4)], 1/2])/(4*a^(5/4)*(b*c - a*d)*Sqrt[a + b*x^4]) - (b^(1/4)*(Sqrt[b
] + (Sqrt[a]*Sqrt[d])/Sqrt[-c])*d*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/
(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(
4*a^(1/4)*(b*c - a*d)*(b*c + a*d)*Sqrt[a + b*x^4]) - (b^(1/4)*(Sqrt[b]*c +
Sqrt[a]*Sqrt[-c]*Sqrt[d])*d*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[
a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(1
/4)*c*(b^2*c^2 - a^2*d^2)*Sqrt[a + b*x^4]) - ((Sqrt[b]*Sqrt[-c] + Sqrt[a]*S
qrt[d])^2*d*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2
)^2]*EllipticPi[-1/4*(Sqrt[b]*Sqrt[-c] - Sqrt[a]*Sqrt[d])^2/(Sqrt[a]*Sqrt[b
]*Sqrt[-c]*Sqrt[d]), 2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(8*a^(1/4)*b^(1/4
)*c*(b*c - a*d)*(b*c + a*d)*Sqrt[a + b*x^4]) - ((Sqrt[b]*Sqrt[-c] - Sqrt[a]
*Sqrt[d])^2*d*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x
^2)^2]*EllipticPi[(Sqrt[b]*Sqrt[-c] + Sqrt[a]*Sqrt[d])^2/(4*Sqrt[a]*Sqrt[b]
*Sqrt[-c]*Sqrt[d]), 2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(8*a^(1/4)*b^(1/4)
*c*(b*c - a*d)*(b*c + a*d)*Sqrt[a + b*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Dist[1/(2*
c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 1231

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1721

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*((a + c*x^4)/(a*(A + B*x^2)^2))]/(
4*d*e*A*q*Sqrt[a + c*x^4]))*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)],
2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{1}{\left(a+bx^4\right)^{3/2}\left(c+dx^4\right)}\,dx = \frac{bx}{2a(bc-ad)\sqrt{a+bx^4}} - \frac{\int \frac{-bc+2ad-bdx^4}{\sqrt{a+bx^4}\,(c+dx^4)}\,dx}{2a(bc-ad)}$$

$$= \frac{bx}{2a(bc-ad)\sqrt{a+bx^4}} + \frac{b\int \frac{1}{\sqrt{a+bx^4}}\,dx}{2a(bc-ad)} - \frac{d\int \frac{1}{\sqrt{a+bx^4}\,(c+dx^4)}\,dx}{bc-ad}$$

$$= \frac{bx}{2a(bc-ad)\sqrt{a+bx^4}} + \frac{b^{3/4}\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,F\Big(2\tan^{-1}}{4a^{5/4}(bc-ad)\sqrt{a+bx^4}}$$

$$= \frac{bx}{2a(bc-ad)\sqrt{a+bx^4}} + \frac{b^{3/4}\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,F\Big(2\tan^{-1}}{4a^{5/4}(bc-ad)\sqrt{a+bx^4}}$$

$$= \frac{bx}{2a(bc-ad)\sqrt{a+bx^4}} + \frac{d^{5/4}\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt[4]{-c}\,\sqrt[4]{d}\,\sqrt{a+bx^4}}\right)}{4(-c)^{3/4}(bc-ad)^{3/2}} - \frac{d^{5/4}\tan^{-1}\Big(}{4(-c)^{3/4}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 10.19, size = 331, normalized size = 0.36

$$\frac{x\left(-\frac{bdx^4\sqrt{1+\frac{bx^4}{a}}\,F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{c}+\frac{5\left(5ac\left(2ad-b\left(2c+dx^4\right)\right)F_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)+2bx^4\left(c+dx^4\right)\left(2adF_1\left(\frac{5}{4};\frac{1}{2},2;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)+bcF_1\left(\frac{5}{4};\frac{3}{2},1;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)\right)\right)}{\left(c+dx^4\right)\left(5acF_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)-2x^4\left(2adF_1\left(\frac{5}{4};\frac{1}{2},2;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)+bcF_1\left(\frac{5}{4};\frac{3}{2},1;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)\right)\right)}\right)}{10a(-bc+ad)\sqrt{a+bx^4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^4)^(3/2)*(c + d*x^4)),x]
```

```
[Out] (x*(-((b*d*x^4*Sqrt[1 + (b*x^4)/a]*AppellF1[5/4, 1/2, 1, 9/4, -((b*x^4)/a),
 -((d*x^4)/c)])/c) + (5*(5*a*c*(2*a*d - b*(2*c + d*x^4))*AppellF1[1/4, 1/2,
 1, 5/4, -((b*x^4)/a), -((d*x^4)/c)] + 2*b*x^4*(c + d*x^4)*(2*a*d*AppellF1[
5/4, 1/2, 2, 9/4, -((b*x^4)/a), -((d*x^4)/c)] + b*c*AppellF1[5/4, 3/2, 1, 9
/4, -((b*x^4)/a), -((d*x^4)/c)])))/((c + d*x^4)*(5*a*c*AppellF1[1/4, 1/2, 1
, 5/4, -((b*x^4)/a), -((d*x^4)/c)] - 2*x^4*(2*a*d*AppellF1[5/4, 1/2, 2, 9/4
, -((b*x^4)/a), -((d*x^4)/c)] + b*c*AppellF1[5/4, 3/2, 1, 9/4, -((b*x^4)/a)
, -((d*x^4)/c)])))))/(10*a*(-(b*c) + a*d)*Sqrt[a + b*x^4])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.29, size = 313, normalized size = 0.34

method	result
default	$-\frac{bx}{2a(ad-bc)\sqrt{\left(x^4+\frac{a}{b}\right)b}}-\frac{b\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{2a(ad-bc)\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{b\,x^4+a}}+\frac{\left(\sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z^4+\right.} \right.}{}$
elliptic	$-\frac{bx}{2a(ad-bc)\sqrt{\left(x^4+\frac{a}{b}\right)b}}-\frac{b\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{2a(ad-bc)\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{b\,x^4+a}}+\frac{\left(\sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z^4+\right.} \right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^4+a)^(3/2)/(d*x^4+c),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*b/a*x/(a*d-b*c)/((x^4+a/b)*b)^(1/2)-1/2*b/a/(a*d-b*c)/(I/a^(1/2)*b^(1/
2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(
b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)+1/8*sum(1/_alpha^3/
(a*d-b*c)*(-1/((a*d-b*c)/d)^(1/2)*arctanh(1/2*(2*_alpha^2*b*x^2+2*a)/((a*d-
b*c)/d)^(1/2)/(b*x^4+a)^(1/2))+2/(I/a^(1/2)*b^(1/2))^(1/2)*_alpha^3*d/c*(1-
I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2
)*EllipticPi(x*(I/a^(1/2)*b^(1/2))^(1/2),I*a^(1/2)/b^(1/2)*_alpha^2/c*d,(-I
```

```
/a^(1/2)*b^(1/2))^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2))),_alpha=RootOf(_Z^4*d+c)
)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(3/2)/(d*x^4+c),x, algorithm="maxima")
[Out] integrate(1/((b*x^4 + a)^(3/2)*(d*x^4 + c)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(3/2)/(d*x^4+c),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx^4)^{\frac{3}{2}}\,(c+dx^4)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**4+a)**(3/2)/(d*x**4+c),x)
[Out] Integral(1/((a + b*x**4)**(3/2)*(c + d*x**4)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(3/2)/(d*x^4+c),x, algorithm="giac")
[Out] integrate(1/((b*x^4 + a)^(3/2)*(d*x^4 + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(b\,x^4+a\right)}^{3/2}\,\left(d\,x^4+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^4)^(3/2)*(c + d*x^4)),x)
[Out] int(1/((a + b*x^4)^(3/2)*(c + d*x^4)), x)
```

3.183 $\int \frac{1}{\left(a+bx^4\right)^{5/2}\left(c+dx^4\right)}\,dx$

Optimal. Leaf size=976

$$\frac{bx}{6a(bc-ad)\left(a+bx^4\right)^{3/2}}+\frac{b(5bc-11ad)x}{12a^2(bc-ad)^2\sqrt{a+bx^4}}-\frac{d^{9/4}\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt[4]{-c}\,\sqrt[4]{d}\,\sqrt{a+bx^4}}\right)}{4(-c)^{3/4}(bc-ad)^{5/2}}-\frac{d^{9/4}\tan^{-1}\left(\frac{\sqrt{\ }}{\sqrt[4]{-c}}\right.}{4(-c)^{3/4}(-}$$

```
[Out] 1/6*b*x/a/(-a*d+b*c)/(b*x^4+a)^(3/2)-1/4*d^(9/4)*arctan(x*(-a*d+b*c)^(1/2)/
(-c)^(1/4)/d^(1/4)/(b*x^4+a)^(1/2))/(-c)^(3/4)/(-a*d+b*c)^(5/2)-1/4*d^(9/4)
*arctan(x*(a*d-b*c)^(1/2)/(-c)^(1/4)/d^(1/4)/(b*x^4+a)^(1/2))/(-c)^(3/4)/(a
*d-b*c)^(5/2)+1/12*b*(-11*a*d+5*b*c)*x/a^2/(-a*d+b*c)^2/(b*x^4+a)^(1/2)+1/2
4*b^(3/4)*(-11*a*d+5*b*c)*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*
arctan(b^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2
^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/a^(
9/4)/(-a*d+b*c)^2/(b*x^4+a)^(1/2)+1/8*d^2*(cos(2*arctan(b^(1/4)*x/a^(1/4)))
^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticPi(sin(2*arctan(b^(1/4)*
x/a^(1/4))),1/4*(b^(1/2)*(-c)^(1/2)+a^(1/2)*d^(1/2))^2/a^(1/2)/b^(1/2)/(-c)
^(1/2)/d^(1/2),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*(b^(1/2)*(-c)^(1/2)-a^(1/
2)*d^(1/2))^2*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/a^(1/4)/b^(1/4)/c/(
-a*d+b*c)^2/(a*d+b*c)/(b*x^4+a)^(1/2)+1/8*d^2*(cos(2*arctan(b^(1/4)*x/a^(1/
4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticPi(sin(2*arctan(b^(1
/4)*x/a^(1/4))),-1/4*(b^(1/2)*(-c)^(1/2)-a^(1/2)*d^(1/2))^2/a^(1/2)/b^(1/2)
/(-c)^(1/2)/d^(1/2),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*(b^(1/2)*(-c)^(1/2)+
a^(1/2)*d^(1/2))^2*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/a^(1/4)/b^(1/4
)/c/(-a*d+b*c)^2/(a*d+b*c)/(b*x^4+a)^(1/2)+1/4*b^(1/4)*d^2*(cos(2*arctan(b^
(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticF(sin(2
*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*(c*b^(1/2)-a
^(1/2)*(-c)^(1/2)*d^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/a^(1/4
)/c/(-a*d+b*c)^2/(a*d+b*c)/(b*x^4+a)^(1/2)+1/4*b^(1/4)*d^2*(cos(2*arctan(b^
(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticF(sin(2
*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*(c*b^(1/2)+a
^(1/2)*(-c)^(1/2)*d^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/a^(1/4
)/c/(-a*d+b*c)^2/(a*d+b*c)/(b*x^4+a)^(1/2)
```

Rubi [A]
time = 1.10, antiderivative size = 976, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {425, 541, 537, 226, 418, 1231, 1721}

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^4)^(5/2)*(c + d*x^4)),x]
```

```
[Out] (b*x)/(6*a*(b*c - a*d)*(a + b*x^4)^(3/2)) + (b*(5*b*c - 11*a*d)*x)/(12*a^2*
(b*c - a*d)^2*Sqrt[a + b*x^4]) - (d^(9/4)*ArcTan[(Sqrt[b*c - a*d]*x)/((-c)^
(1/4)*d^(1/4)*Sqrt[a + b*x^4])])/(4*(-c)^(3/4)*(b*c - a*d)^(5/2)) - (d^(9/4
)*ArcTan[(Sqrt[-(b*c) + a*d]*x)/((-c)^(1/4)*d^(1/4)*Sqrt[a + b*x^4])])/(4*(
-c)^(3/4)*(-(b*c) + a*d)^(5/2)) + (b^(3/4)*(5*b*c - 11*a*d)*(Sqrt[a] + Sqrt
[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^
(1/4)*x)/a^(1/4)], 1/2])/(24*a^(9/4)*(b*c - a*d)^2*Sqrt[a + b*x^4]) + (b^(1
/4)*(Sqrt[b]*c - Sqrt[a]*Sqrt[-c]*Sqrt[d])*d^2*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt
[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1
/4)], 1/2])/(4*a^(1/4)*c*(b*c - a*d)^2*(b*c + a*d)*Sqrt[a + b*x^4]) + (b^(1
/4)*(Sqrt[b]*c + Sqrt[a]*Sqrt[-c]*Sqrt[d])*d^2*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt
[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1
/4)], 1/2])/(4*a^(1/4)*c*(b*c - a*d)^2*(b*c + a*d)*Sqrt[a + b*x^4]) + ((Sqr
t[b]*Sqrt[-c] + Sqrt[a]*Sqrt[d])^2*d^2*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*
x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticPi[-1/4*(Sqrt[b]*Sqrt[-c] - Sqrt[a]
*Sqrt[d])^2/(Sqrt[a]*Sqrt[b]*Sqrt[-c]*Sqrt[d]), 2*ArcTan[(b^(1/4)*x)/a^(1/4
)], 1/2])/(8*a^(1/4)*b^(1/4)*c*(b*c - a*d)^2*(b*c + a*d)*Sqrt[a + b*x^4]) +
 ((Sqrt[b]*Sqrt[-c] - Sqrt[a]*Sqrt[d])^2*d^2*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(
a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticPi[(Sqrt[b]*Sqrt[-c] + Sqrt[a
]*Sqrt[d])^2/(4*Sqrt[a]*Sqrt[b]*Sqrt[-c]*Sqrt[d]), 2*ArcTan[(b^(1/4)*x)/a^(
1/4)], 1/2])/(8*a^(1/4)*b^(1/4)*c*(b*c - a*d)^2*(b*c + a*d)*Sqrt[a + b*x^4]
)
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Dist[1/(2*
c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 1231

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1721

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*((a + c*x^4)/(a*(A + B*x^2)^2))]/(
4*d*e*A*q*Sqrt[a + c*x^4]))*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)],
2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{1}{(a+bx^4)^{5/2}(c+dx^4)}\,dx = \frac{bx}{6a(bc-ad)(a+bx^4)^{3/2}} - \frac{\int \frac{-5bc+6ad-5bdx^4}{(a+bx^4)^{3/2}(c+dx^4)}\,dx}{6a(bc-ad)}$$

$$= \frac{bx}{6a(bc-ad)(a+bx^4)^{3/2}} + \frac{b(5bc-11ad)x}{12a^2(bc-ad)^2\sqrt{a+bx^4}} + \frac{\int \frac{5b^2c^2-11abcd+12a^2d^2+}{\sqrt{a+bx^4}\,(}}{12a^2(bc-}$$

$$= \frac{bx}{6a(bc-ad)(a+bx^4)^{3/2}} + \frac{b(5bc-11ad)x}{12a^2(bc-ad)^2\sqrt{a+bx^4}} + \frac{d^2\int \frac{1}{\sqrt{a+bx^4}\,(c+dx^4}}{(bc-ad)^2}$$

$$= \frac{bx}{6a(bc-ad)(a+bx^4)^{3/2}} + \frac{b(5bc-11ad)x}{12a^2(bc-ad)^2\sqrt{a+bx^4}} + \frac{b^{3/4}(5bc-11ad)\left(\sqrt{\right.}{}$$

$$= \frac{bx}{6a(bc-ad)(a+bx^4)^{3/2}} + \frac{b(5bc-11ad)x}{12a^2(bc-ad)^2\sqrt{a+bx^4}} + \frac{b^{3/4}(5bc-11ad)\left(\sqrt{\right.}{}$$

$$= \frac{bx}{6a(bc-ad)(a+bx^4)^{3/2}} + \frac{b(5bc-11ad)x}{12a^2(bc-ad)^2\sqrt{a+bx^4}} - \frac{d^{9/4}\tan^{-1}\left(\frac{\sqrt{b}}{\sqrt[4]{-c}\,\sqrt[4]{}}\right.}{4(-c)^{3/4}(bc-}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 10.51, size = 429, normalized size = 0.44

$$-\frac{x\left(\frac{bd(-5bc+11ad)x^4\sqrt{1+\frac{bx^4}{a}}\,F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{c}+\frac{5\left(5ac\left(12a^3d^2+5b^3cx^4(2c+dx^4)-a^2bd(24c+dx^4)+ab^2(12c^2-15cdx^4-11d^2x^8)\right)F_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)+2bx^4(c+dx^4)(13a^2d-5b^2cx^4+ab(-7c+11dx^4))\left(2adF_1\left(\frac{5}{4};\frac{1}{2},2;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)+bcF_1\left(\frac{5}{4};\frac{3}{2},1;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)\right)\right)}{(a+bx^4)(c+dx^4)\left(-5acF_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)+2x^4\left(2adF_1\left(\frac{5}{4};\frac{1}{2},2;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)+bcF_1\left(\frac{5}{4};\frac{3}{2},1;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)\right)\right)}\right)}{60a^2(bc-ad)^2\sqrt{a+bx^4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^4)^(5/2)*(c + d*x^4)),x]
```

```
[Out] -1/60*(x*((b*d*(-5*b*c + 11*a*d)*x^4*Sqrt[1 + (b*x^4)/a]*AppellF1[5/4, 1/2,
 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)])/c + (5*(5*a*c*(12*a^3*d^2 + 5*b^3*c*x
^4*(2*c + d*x^4) - a^2*b*d*(24*c + d*x^4) + a*b^2*(12*c^2 - 15*c*d*x^4 - 11
*d^2*x^8))*AppellF1[1/4, 1/2, 1, 5/4, -((b*x^4)/a), -((d*x^4)/c)] + 2*b*x^4
*(c + d*x^4)*(13*a^2*d - 5*b^2*c*x^4 + a*b*(-7*c + 11*d*x^4))*(2*a*d*Appell
F1[5/4, 1/2, 2, 9/4, -((b*x^4)/a), -((d*x^4)/c)] + b*c*AppellF1[5/4, 3/2, 1
, 9/4, -((b*x^4)/a), -((d*x^4)/c)]))/((a + b*x^4)*(c + d*x^4)*(-5*a*c*Appe
```

```
llF1[1/4, 1/2, 1, 5/4, -((b*x^4)/a), -((d*x^4)/c)] + 2*x^4*(2*a*d*AppellF1[
5/4, 1/2, 2, 9/4, -((b*x^4)/a), -((d*x^4)/c)] + b*c*AppellF1[5/4, 3/2, 1, 9
/4, -((b*x^4)/a), -((d*x^4)/c)]))))))/(a^2*(b*c - a*d)^2*Sqrt[a + b*x^4])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.27, size = 371, normalized size = 0.38

method	result
default	$-\frac{x\sqrt{b\,x^4+a}}{6ab(ad-bc)\left(x^4+\frac{a}{b}\right)^2}-\frac{bx(11ad-5bc)}{12a^2(ad-bc)^2\sqrt{\left(x^4+\frac{a}{b}\right)b}}-\frac{b(11ad-5bc)\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\right.}}{12a^2(ad-bc)^2\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}$
elliptic	$-\frac{x\sqrt{b\,x^4+a}}{6ab(ad-bc)\left(x^4+\frac{a}{b}\right)^2}-\frac{bx(11ad-5bc)}{12a^2(ad-bc)^2\sqrt{\left(x^4+\frac{a}{b}\right)b}}-\frac{b(11ad-5bc)\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\right.}}{12a^2(ad-bc)^2\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^4+a)^(5/2)/(d*x^4+c),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6/a*x/b/(a*d-b*c)*(b*x^4+a)^(1/2)/(x^4+a/b)^2-1/12*b/a^2*x*(11*a*d-5*b*c
)/(a*d-b*c)^2/((x^4+a/b)*b)^(1/2)-1/12*b/a^2*(11*a*d-5*b*c)/(a*d-b*c)^2/(I/
a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)
```

```
*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)+1/8*d*
sum(1/(a*d-b*c)^2/_alpha^3*(-1/((a*d-b*c)/d)^(1/2)*arctanh(1/2*(2*_alpha^2*
b*x^2+2*a)/((a*d-b*c)/d)^(1/2)/(b*x^4+a)^(1/2))+2/(I/a^(1/2)*b^(1/2))^(1/2)
*_alpha^3*d/c*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/
2)/(b*x^4+a)^(1/2)*EllipticPi(x*(I/a^(1/2)*b^(1/2))^(1/2),I*a^(1/2)/b^(1/2)
*_alpha^2/c*d,(-I/a^(1/2)*b^(1/2))^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2))),_alpha
=RootOf(_Z^4*d+c))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(5/2)/(d*x^4+c),x, algorithm="maxima")
[Out] integrate(1/((b*x^4 + a)^(5/2)*(d*x^4 + c)), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(5/2)/(d*x^4+c),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx^4)^{\frac{5}{2}}\,(c+dx^4)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**4+a)**(5/2)/(d*x**4+c),x)
[Out] Integral(1/((a + b*x**4)**(5/2)*(c + d*x**4)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(5/2)/(d*x^4+c),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^4 + a)^(5/2)*(d*x^4 + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(b\,x^4+a\right)^{5/2}\,\left(d\,x^4+c\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^4)^(5/2)*(c + d*x^4)),x)
[Out] int(1/((a + b*x^4)^(5/2)*(c + d*x^4)), x)
```

3.184 $\int \frac{(a-bx^4)^{7/2}}{(c-dx^4)^2} \, dx$

Optimal. Leaf size=426

$$-\frac{b(77b^2c^2-122abcd+21a^2d^2)\,x\sqrt{a-bx^4}}{84cd^3}+\frac{b(11bc-7ad)x(a-bx^4)^{3/2}}{28cd^2}-\frac{(bc-ad)x(a-bx^4)^{5/2}}{4cd\,(c-dx^4)}+\frac{\sqrt[4]{a}\,b^{3/}}{}$$

```
[Out] 1/28*b*(-7*a*d+11*b*c)*x*(-b*x^4+a)^(3/2)/c/d^2-1/4*(-a*d+b*c)*x*(-b*x^4+a)
^(5/2)/c/d/(-d*x^4+c)-1/84*b*(21*a^2*d^2-122*a*b*c*d+77*b^2*c^2)*x*(-b*x^4+
a)^(1/2)/c/d^3+1/84*a^(1/4)*b^(3/4)*(21*a^3*d^3+349*a^2*b*c*d^2-553*a*b^2*c
^2*d+231*b^3*c^3)*EllipticF(b^(1/4)*x/a^(1/4),I)*(1-b*x^4/a)^(1/2)/c/d^4/(-
b*x^4+a)^(1/2)-1/8*a^(1/4)*(-a*d+b*c)^3*(3*a*d+11*b*c)*EllipticPi(b^(1/4)*x
/a^(1/4),-a^(1/2)*d^(1/2)/b^(1/2)/c^(1/2),I)*(1-b*x^4/a)^(1/2)/b^(1/4)/c^2/
d^4/(-b*x^4+a)^(1/2)-1/8*a^(1/4)*(-a*d+b*c)^3*(3*a*d+11*b*c)*EllipticPi(b^(
1/4)*x/a^(1/4),a^(1/2)*d^(1/2)/b^(1/2)/c^(1/2),I)*(1-b*x^4/a)^(1/2)/b^(1/4)
/c^2/d^4/(-b*x^4+a)^(1/2)
```

Rubi [A]

time = 0.34, antiderivative size = 426, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {424, 542, 537, 230, 227, 418, 1233, 1232}

$$-\frac{bx\sqrt{a-bx^4}\,(21a^2d^2-122abcd+77b^2c^2)}{84cd^3}+\frac{\sqrt[4]{a}\,b^{3/4}\sqrt{1-\frac{bx^4}{a}}\,(21a^3d^3+349a^2bcd^2-553ab^2c^2d+231b^3c^3)\,F\left(\text{ArcSin}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{84cd^4\sqrt{a-bx^4}}-\frac{\sqrt[4]{a}\,\sqrt{1-\frac{bx^4}{a}}\,(3ad+11bc)(bc-ad)^3\Pi\left(-\frac{\sqrt{a}\,\sqrt{d}}{\sqrt{b}\,\sqrt{c}};\text{ArcSin}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{8\sqrt[4]{b}\,c^2d^4\sqrt{a-bx^4}}-\frac{\sqrt[4]{a}\,\sqrt{1-\frac{bx^4}{a}}\,(3ad+11bc)(bc-ad)^3\Pi\left(\frac{\sqrt{a}\,\sqrt{d}}{\sqrt{b}\,\sqrt{c}};\text{ArcSin}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{8\sqrt[4]{b}\,c^2d^4\sqrt{a-bx^4}}+\frac{bx(a-bx^4)^{3/2}\,(11bc-7ad)}{28cd^2}-\frac{x(a-bx^4)^{5/2}\,(bc-ad)}{4cd\,(c-dx^4)}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^4)^(7/2)/(c - d*x^4)^2,x]
```

```
[Out] -1/84*(b*(77*b^2*c^2 - 122*a*b*c*d + 21*a^2*d^2)*x*Sqrt[a - b*x^4])/(c*d^3)
 + (b*(11*b*c - 7*a*d)*x*(a - b*x^4)^(3/2))/(28*c*d^2) - ((b*c - a*d)*x*(a
- b*x^4)^(5/2))/(4*c*d*(c - d*x^4)) + (a^(1/4)*b^(3/4)*(231*b^3*c^3 - 553*a
*b^2*c^2*d + 349*a^2*b*c*d^2 + 21*a^3*d^3)*Sqrt[1 - (b*x^4)/a]*EllipticF[Ar
cSin[(b^(1/4)*x)/a^(1/4)], -1])/(84*c*d^4*Sqrt[a - b*x^4]) - (a^(1/4)*(b*c
- a*d)^3*(11*b*c + 3*a*d)*Sqrt[1 - (b*x^4)/a]*EllipticPi[-((Sqrt[a]*Sqrt[d]
)/(Sqrt[b]*Sqrt[c])), ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(8*b^(1/4)*c^2*d^4*
Sqrt[a - b*x^4]) - (a^(1/4)*(b*c - a*d)^3*(11*b*c + 3*a*d)*Sqrt[1 - (b*x^4)
/a]*EllipticPi[(Sqrt[a]*Sqrt[d])/(Sqrt[b]*Sqrt[c]), ArcSin[(b^(1/4)*x)/a^(1
/4)], -1])/(8*b^(1/4)*c^2*d^4*Sqrt[a - b*x^4])
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Dist[1/(2*
c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 1233

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{(a-bx^4)^{7/2}}{(c-dx^4)^2}\,dx = -\frac{(bc-ad)x(a-bx^4)^{5/2}}{4cd\,(c-dx^4)} - \frac{\int \frac{(a-bx^4)^{3/2}(-a(bc+3ad)+b(11bc-7ad)x^4)}{c-dx^4}\,dx}{4cd}$$

$$= \frac{b(11bc-7ad)x(a-bx^4)^{3/2}}{28cd^2} - \frac{(bc-ad)x(a-bx^4)^{5/2}}{4cd\,(c-dx^4)} + \frac{\int \frac{\sqrt{a-bx^4}\,(-a(11b^2c^2-14abcd-\ldots}{c\ldots}}{2\ldots}$$

$$= -\frac{b(77b^2c^2-122abcd+21a^2d^2)\,x\sqrt{a-bx^4}}{84cd^3} + \frac{b(11bc-7ad)x(a-bx^4)^{3/2}}{28cd^2} - \frac{(bc-ad\ldots}{4cd\ldots}$$

$$= -\frac{b(77b^2c^2-122abcd+21a^2d^2)\,x\sqrt{a-bx^4}}{84cd^3} + \frac{b(11bc-7ad)x(a-bx^4)^{3/2}}{28cd^2} - \frac{(bc-ad\ldots}{4cd\ldots}$$

$$= -\frac{b(77b^2c^2-122abcd+21a^2d^2)\,x\sqrt{a-bx^4}}{84cd^3} + \frac{b(11bc-7ad)x(a-bx^4)^{3/2}}{28cd^2} - \frac{(bc-ad\ldots}{4cd\ldots}$$

$$= -\frac{b(77b^2c^2-122abcd+21a^2d^2)\,x\sqrt{a-bx^4}}{84cd^3} + \frac{b(11bc-7ad)x(a-bx^4)^{3/2}}{28cd^2} - \frac{(bc-ad\ldots}{4cd\ldots}$$

$$= -\frac{b(77b^2c^2-122abcd+21a^2d^2)\,x\sqrt{a-bx^4}}{84cd^3} + \frac{b(11bc-7ad)x(a-bx^4)^{3/2}}{28cd^2} - \frac{(bc-ad\ldots}{4cd\ldots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 10.59, size = 477, normalized size = 1.12

$$-\frac{b(231b^3c^3-553ab^2c^2d+349a^2bcd^2+21a^3d^3)\,x^5\sqrt{1-\frac{bx^4}{a}}\,F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)+\frac{5c\left(5acx(-84a^4d^3+29a^2b^2cd^2x^4+21a^3bd^3x^4+ab^3cdx^4(111c-104dx^4)+b^4cx^4(-77c^2+44cdx^4+12d^2x^8))F_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)+2x^5(-a+bx^4)(-63a^2bcd^2+21a^3d^3+ab^2cd(155c-92dx^4)+b^3c(-77c^2+44cdx^4+12d^2x^8))\left(2adF_1\left(\frac{5}{4};\frac{1}{2},2;\frac{9}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)+bcF_1\left(\frac{5}{4};\frac{3}{2},1;\frac{9}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)\right)\right)}{(c-dx^4)\left(5acF_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)+2x^4\left(2adF_1\left(\frac{5}{4};\frac{1}{2},2;\frac{9}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)+bcF_1\left(\frac{5}{4};\frac{3}{2},1;\frac{9}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)\right)\right)}}{420c^2d^3\sqrt{a-bx^4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a - b*x^4)^(7/2)/(c - d*x^4)^2,x]
```

```
[Out] -1/420*(b*(231*b^3*c^3 - 553*a*b^2*c^2*d + 349*a^2*b*c*d^2 + 21*a^3*d^3)*x^
5*Sqrt[1 - (b*x^4)/a]*AppellF1[5/4, 1/2, 1, 9/4, (b*x^4)/a, (d*x^4)/c] + (5
```

```
*c*(5*a*c*x*(-84*a^4*d^3 + 29*a^2*b^2*c*d^2*x^4 + 21*a^3*b*d^3*x^4 + a*b^3*
c*d*x^4*(111*c - 104*d*x^4) + b^4*c*x^4*(-77*c^2 + 44*c*d*x^4 + 12*d^2*x^8)
)*AppellF1[1/4, 1/2, 1, 5/4, (b*x^4)/a, (d*x^4)/c] + 2*x^5*(-a + b*x^4)*(-6
3*a^2*b*c*d^2 + 21*a^3*d^3 + a*b^2*c*d*(155*c - 92*d*x^4) + b^3*c*(-77*c^2
+ 44*c*d*x^4 + 12*d^2*x^8))*(2*a*d*AppellF1[5/4, 1/2, 2, 9/4, (b*x^4)/a, (d
*x^4)/c] + b*c*AppellF1[5/4, 3/2, 1, 9/4, (b*x^4)/a, (d*x^4)/c])))/((c - d*
x^4)*(5*a*c*AppellF1[1/4, 1/2, 1, 5/4, (b*x^4)/a, (d*x^4)/c] + 2*x^4*(2*a*d
*AppellF1[5/4, 1/2, 2, 9/4, (b*x^4)/a, (d*x^4)/c] + b*c*AppellF1[5/4, 3/2,
1, 9/4, (b*x^4)/a, (d*x^4)/c]))))/(c^2*d^3*Sqrt[a - b*x^4])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.35, size = 539, normalized size = 1.27 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^4+a)^(7/2)/(-d*x^4+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/c/d^3*x*(-b*x^4+a)^(1/2)/
(-d*x^4+c)-1/7*b^3/d^2*x^5*(-b*x^4+a)^(1/2)-1/3*(-2*b^3/d^3*(2*a*d-b*c)+5/7
*b^3/d^2*a)/b*x*(-b*x^4+a)^(1/2)+(b^2*(6*a^2*d^2-8*a*b*c*d+3*b^2*c^2)/d^4+1
/4*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/d^4*b/c+1/3*(-2*b^3/d^3*(2
*a*d-b*c)+5/7*b^3/d^2*a)/b*a)/(1/a^(1/2)*b^(1/2))^(1/2)*(1-x^2*b^(1/2)/a^(1
/2))^(1/2)*(1+x^2*b^(1/2)/a^(1/2))^(1/2)/(-b*x^4+a)^(1/2)*EllipticF(x*(1/a^
(1/2)*b^(1/2))^(1/2),I)-1/32/d^5/c*sum((3*a^4*d^4+2*a^3*b*c*d^3-24*a^2*b^2*
c^2*d^2+30*a*b^3*c^3*d-11*b^4*c^4)/_alpha^3*(-1/((a*d-b*c)/d)^(1/2)*arctanh
(1/2*(-2*_alpha^2*b*x^2+2*a)/((a*d-b*c)/d)^(1/2)/(-b*x^4+a)^(1/2))-2/(1/a^(
1/2)*b^(1/2))^(1/2)*_alpha^3*d/c*(1-x^2*b^(1/2)/a^(1/2))^(1/2)*(1+x^2*b^(1/
2)/a^(1/2))^(1/2)/(-b*x^4+a)^(1/2)*EllipticPi(x*(1/a^(1/2)*b^(1/2))^(1/2),a
^(1/2)/b^(1/2)*_alpha^2/c*d,(-1/a^(1/2)*b^(1/2))^(1/2)/(1/a^(1/2)*b^(1/2))^
(1/2))),_alpha=RootOf(_Z^4*d-c))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^4+a)^(7/2)/(-d*x^4+c)^2,x, algorithm="maxima")
```

```
[Out] integrate((-b*x^4 + a)^(7/2)/(d*x^4 - c)^2, x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^4+a)^(7/2)/(-d*x^4+c)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a-bx^4)^{\frac{7}{2}}}{(-c+dx^4)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**4+a)**(7/2)/(-d*x**4+c)**2,x)
```

```
[Out] Integral((a - b*x**4)**(7/2)/(-c + d*x**4)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^4+a)^(7/2)/(-d*x^4+c)^2,x, algorithm="giac")
```

```
[Out] integrate((-b*x^4 + a)^(7/2)/(d*x^4 - c)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a-b\,x^4)^{7/2}}{(c-d\,x^4)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x^4)^(7/2)/(c - d*x^4)^2,x)
```

```
[Out] int((a - b*x^4)^(7/2)/(c - d*x^4)^2, x)
```

3.185 $\int \frac{(a-bx^4)^{5/2}}{(c-dx^4)^2} \, dx$

Optimal. Leaf size=365

$$\frac{b(7bc-3ad)x\sqrt{a-bx^4}}{12cd^2}-\frac{(bc-ad)x(a-bx^4)^{3/2}}{4cd\,(c-dx^4)}-\frac{\sqrt[4]{a}\,b^{3/4}(21b^2c^2-26abcd-3a^2d^2)\sqrt{1-\frac{bx^4}{a}}\,F\Big(\sin^{-1}\Big(}{12cd^3\sqrt{a-bx^4}}$$

```
[Out] -1/4*(-a*d+b*c)*x*(-b*x^4+a)^(3/2)/c/d/(-d*x^4+c)+1/12*b*(-3*a*d+7*b*c)*x*(
-b*x^4+a)^(1/2)/c/d^2-1/12*a^(1/4)*b^(3/4)*(-3*a^2*d^2-26*a*b*c*d+21*b^2*c^
2)*EllipticF(b^(1/4)*x/a^(1/4),I)*(1-b*x^4/a)^(1/2)/c/d^3/(-b*x^4+a)^(1/2)+
1/8*a^(1/4)*(-a*d+b*c)^2*(3*a*d+7*b*c)*EllipticPi(b^(1/4)*x/a^(1/4),-a^(1/2
)*d^(1/2)/b^(1/2)/c^(1/2),I)*(1-b*x^4/a)^(1/2)/b^(1/4)/c^2/d^3/(-b*x^4+a)^(
1/2)+1/8*a^(1/4)*(-a*d+b*c)^2*(3*a*d+7*b*c)*EllipticPi(b^(1/4)*x/a^(1/4),a^
(1/2)*d^(1/2)/b^(1/2)/c^(1/2),I)*(1-b*x^4/a)^(1/2)/b^(1/4)/c^2/d^3/(-b*x^4+
a)^(1/2)
```

Rubi [A]
time = 0.25, antiderivative size = 365, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {424, 542, 537, 230, 227, 418, 1233, 1232}

$$-\frac{\sqrt[4]{a}\,b^{3/4}\sqrt{1-\frac{bx^4}{a}}\,(-3a^2d^2-26abcd+21b^2c^2)\,F\left(\text{ArcSin}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{12cd^3\sqrt{a-bx^4}}+\frac{\sqrt[4]{a}\,\sqrt{1-\frac{bx^4}{a}}\,(3ad+7bc)(bc-ad)^2\Pi\left(-\frac{\sqrt{a}\,\sqrt{d}}{\sqrt{b}\,\sqrt{c}};\text{ArcSin}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{8\sqrt[4]{b}\,c^2d^3\sqrt{a-bx^4}}+\frac{\sqrt[4]{a}\,\sqrt{1-\frac{bx^4}{a}}\,(3ad+7bc)(bc-ad)^2\Pi\left(\frac{\sqrt{a}\,\sqrt{d}}{\sqrt{b}\,\sqrt{c}};\text{ArcSin}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{8\sqrt[4]{b}\,c^2d^3\sqrt{a-bx^4}}+\frac{bx\sqrt{a-bx^4}\,(7bc-3ad)}{12cd^2}-\frac{x(a-bx^4)^{3/2}\,(bc-ad)}{4cd\,(c-dx^4)}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^4)^(5/2)/(c - d*x^4)^2,x]
```

```
[Out] (b*(7*b*c - 3*a*d)*x*Sqrt[a - b*x^4])/(12*c*d^2) - ((b*c - a*d)*x*(a - b*x^
4)^(3/2))/(4*c*d*(c - d*x^4)) - (a^(1/4)*b^(3/4)*(21*b^2*c^2 - 26*a*b*c*d -
 3*a^2*d^2)*Sqrt[1 - (b*x^4)/a]*EllipticF[ArcSin[(b^(1/4)*x)/a^(1/4)], -1])
/(12*c*d^3*Sqrt[a - b*x^4]) + (a^(1/4)*(b*c - a*d)^2*(7*b*c + 3*a*d)*Sqrt[1
 - (b*x^4)/a]*EllipticPi[-((Sqrt[a]*Sqrt[d])/(Sqrt[b]*Sqrt[c])), ArcSin[(b^
(1/4)*x)/a^(1/4)], -1])/(8*b^(1/4)*c^2*d^3*Sqrt[a - b*x^4]) + (a^(1/4)*(b*c
 - a*d)^2*(7*b*c + 3*a*d)*Sqrt[1 - (b*x^4)/a]*EllipticPi[(Sqrt[a]*Sqrt[d])/
(Sqrt[b]*Sqrt[c]), ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(8*b^(1/4)*c^2*d^3*Sqr
t[a - b*x^4])
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Dist[1/(2*
c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 1233

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)]
```

```
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a-bx^4)^{5/2}}{(c-dx^4)^2}\,dx &= -\frac{(bc-ad)x(a-bx^4)^{3/2}}{4cd\,(c-dx^4)} - \frac{\int \frac{\sqrt{a-bx^4}\,(-a(bc+3ad)+b(7bc-3ad)x^4)}{c-dx^4}\,dx}{4cd} \\
&= \frac{b(7bc-3ad)x\sqrt{a-bx^4}}{12cd^2} - \frac{(bc-ad)x(a-bx^4)^{3/2}}{4cd\,(c-dx^4)} + \frac{\int \frac{-a(7b^2c^2-6abcd-9a^2d^2)+b(21b^2c^2-26ab}{\sqrt{a-bx^4}\,(c-dx^4)}}{12cd^2} \\
&= \frac{b(7bc-3ad)x\sqrt{a-bx^4}}{12cd^2} - \frac{(bc-ad)x(a-bx^4)^{3/2}}{4cd\,(c-dx^4)} + \frac{((bc-ad)^2(7bc+3ad))\int \frac{}{\sqrt{a-b}}}{4cd^3} \\
&= \frac{b(7bc-3ad)x\sqrt{a-bx^4}}{12cd^2} - \frac{(bc-ad)x(a-bx^4)^{3/2}}{4cd\,(c-dx^4)} + \frac{((bc-ad)^2(7bc+3ad))\int \frac{}{\left(1-\frac{\sqrt{d}}{\sqrt{}}\right.}}{8c^2d^3} \\
&= \frac{b(7bc-3ad)x\sqrt{a-bx^4}}{12cd^2} - \frac{(bc-ad)x(a-bx^4)^{3/2}}{4cd\,(c-dx^4)} - \frac{\sqrt[4]{a}\,b^{3/4}(21b^2c^2-26abcd-3a^2d^2}{12cd^3} \\
&= \frac{b(7bc-3ad)x\sqrt{a-bx^4}}{12cd^2} - \frac{(bc-ad)x(a-bx^4)^{3/2}}{4cd\,(c-dx^4)} - \frac{\sqrt[4]{a}\,b^{3/4}(21b^2c^2-26abcd-3a^2d^2}{12cd^3}
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 10.42, size = 396, normalized size = 1.08

$$
-\frac{b(-21b^2c^2+26abcd+3a^2d^2)\,x^5\sqrt{1-\frac{bx^4}{a}}\,F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)+\frac{5c\left(5acx\left(12a^3d^2+2ab^2cdx^4-3a^2bd^2x^4+b^3cx^4(-7c+4dx^4)\right)F_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)+2x^5(a-bx^4)(-6abcd+3a^2d^2+b^2c(7c-4dx^4))\left(2adF_1\left(\frac{5}{4};\frac{1}{2},2;\frac{9}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)+bcF_1\left(\frac{5}{4};\frac{3}{2},1;\frac{9}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)\right)\right)}{(-c+dx^4)\left(5acF_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)+2x^4\left(2adF_1\left(\frac{5}{4};\frac{1}{2},2;\frac{9}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)+bcF_1\left(\frac{5}{4};\frac{3}{2},1;\frac{9}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)\right)\right)}}{60c^2d^2\sqrt{a-bx^4}}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a - b*x^4)^(5/2)/(c - d*x^4)^2,x]
```

```
[Out] -1/60*(b*(-21*b^2*c^2 + 26*a*b*c*d + 3*a^2*d^2)*x^5*Sqrt[1 - (b*x^4)/a]*App
ellF1[5/4, 1/2, 1, 9/4, (b*x^4)/a, (d*x^4)/c] + (5*c*(5*a*c*x*(12*a^3*d^2 +
 2*a*b^2*c*d*x^4 - 3*a^2*b*d^2*x^4 + b^3*c*x^4*(-7*c + 4*d*x^4))*AppellF1[1
/4, 1/2, 1, 5/4, (b*x^4)/a, (d*x^4)/c] + 2*x^5*(a - b*x^4)*(-6*a*b*c*d + 3*
a^2*d^2 + b^2*c*(7*c - 4*d*x^4))*(2*a*d*AppellF1[5/4, 1/2, 2, 9/4, (b*x^4)/
a, (d*x^4)/c] + b*c*AppellF1[5/4, 3/2, 1, 9/4, (b*x^4)/a, (d*x^4)/c])))/((-
```

```
c + d*x^4)*(5*a*c*AppellF1[1/4, 1/2, 1, 5/4, (b*x^4)/a, (d*x^4)/c] + 2*x^4*
(2*a*d*AppellF1[5/4, 1/2, 2, 9/4, (b*x^4)/a, (d*x^4)/c] + b*c*AppellF1[5/4,
 3/2, 1, 9/4, (b*x^4)/a, (d*x^4)/c]))))/(c^2*d^2*Sqrt[a - b*x^4])
```

Maple [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.32, size = 411, normalized size = 1.13 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^4+a)^(5/2)/(-d*x^4+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/d^2/c*(a^2*d^2-2*a*b*c*d+b^2*c^2)*x*(-b*x^4+a)^(1/2)/(-d*x^4+c)+1/3*b^2
/d^2*x*(-b*x^4+a)^(1/2)+(b^2*(3*a*d-2*b*c)/d^3+1/4*(a^2*d^2-2*a*b*c*d+b^2*c
^2)/d^3*b/c-1/3*b^2/d^2*a)/(1/a^(1/2)*b^(1/2))^(1/2)*(1-x^2*b^(1/2)/a^(1/2)
)^(1/2)*(1+x^2*b^(1/2)/a^(1/2))^(1/2)/(-b*x^4+a)^(1/2)*EllipticF(x*(1/a^(1/
2)*b^(1/2))^(1/2),I)-1/32/c/d^4*sum((3*a^3*d^3+a^2*b*c*d^2-11*a*b^2*c^2*d+7
*b^3*c^3)/_alpha^3*(-1/((a*d-b*c)/d)^(1/2)*arctanh(1/2*(-2*_alpha^2*b*x^2+2
*a)/((a*d-b*c)/d)^(1/2)/(-b*x^4+a)^(1/2))-2/(1/a^(1/2)*b^(1/2))^(1/2)*_alph
a^3*d/c*(1-x^2*b^(1/2)/a^(1/2))^(1/2)*(1+x^2*b^(1/2)/a^(1/2))^(1/2)/(-b*x^4
+a)^(1/2)*EllipticPi(x*(1/a^(1/2)*b^(1/2))^(1/2),a^(1/2)/b^(1/2)*_alpha^2/c
*d,(-1/a^(1/2)*b^(1/2))^(1/2)/(1/a^(1/2)*b^(1/2))^(1/2))),_alpha=RootOf(_Z^
4*d-c))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^4+a)^(5/2)/(-d*x^4+c)^2,x, algorithm="maxima")
```

```
[Out] integrate((-b*x^4 + a)^(5/2)/(d*x^4 - c)^2, x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^4+a)^(5/2)/(-d*x^4+c)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a - bx^{4}\right)^{\frac{5}{2}}}{\left(-c + dx^{4}\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**4+a)**(5/2)/(-d*x**4+c)**2,x)
```

```
[Out] Integral((a - b*x**4)**(5/2)/(-c + d*x**4)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^4+a)^(5/2)/(-d*x^4+c)^2,x, algorithm="giac")
```

```
[Out] integrate((-b*x^4 + a)^(5/2)/(d*x^4 - c)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a-b\,x^4\right)}^{5/2}}{{\left(c-d\,x^4\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x^4)^(5/2)/(c - d*x^4)^2,x)
```

```
[Out] int((a - b*x^4)^(5/2)/(c - d*x^4)^2, x)
```

3.186 $\int \frac{(a-bx^4)^{3/2}}{(c-dx^4)^2} dx$

Optimal. Leaf size=309

$$-\frac{(bc-ad)x\sqrt{a-bx^4}}{4cd\,(c-dx^4)}+\frac{\sqrt[4]{a}\,b^{3/4}(3bc+ad)\sqrt{1-\frac{bx^4}{a}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{4cd^2\sqrt{a-bx^4}}-\frac{3\sqrt[4]{a}\,(bc-ad)(bc+ad)\sqrt{}}{}$$

```
[Out] -1/4*(-a*d+b*c)*x*(-b*x^4+a)^(1/2)/c/d/(-d*x^4+c)+1/4*a^(1/4)*b^(3/4)*(a*d+
3*b*c)*EllipticF(b^(1/4)*x/a^(1/4),I)*(1-b*x^4/a)^(1/2)/c/d^2/(-b*x^4+a)^(1
/2)-3/8*a^(1/4)*(-a*d+b*c)*(a*d+b*c)*EllipticPi(b^(1/4)*x/a^(1/4),-a^(1/2)*
d^(1/2)/b^(1/2)/c^(1/2),I)*(1-b*x^4/a)^(1/2)/b^(1/4)/c^2/d^2/(-b*x^4+a)^(1/
2)-3/8*a^(1/4)*(-a*d+b*c)*(a*d+b*c)*EllipticPi(b^(1/4)*x/a^(1/4),a^(1/2)*d^
(1/2)/b^(1/2)/c^(1/2),I)*(1-b*x^4/a)^(1/2)/b^(1/4)/c^2/d^2/(-b*x^4+a)^(1/2)
```

Rubi [A]

time = 0.17, antiderivative size = 309, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {424, 537, 230, 227, 418, 1233, 1232}

$$\frac{\sqrt[4]{a}\,b^{3/4}\sqrt{1-\frac{bx^4}{a}}\,(ad+3bc)F\left(\text{ArcSin}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{4cd^2\sqrt{a-bx^4}}-\frac{3\sqrt[4]{a}\,\sqrt{1-\frac{bx^4}{a}}\,(bc-ad)(ad+bc)\Pi\left(-\frac{\sqrt{a}\,\sqrt{d}}{\sqrt{b}\,\sqrt{c}};\text{ArcSin}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{8\sqrt[4]{b}\,c^2d^2\sqrt{a-bx^4}}-\frac{3\sqrt[4]{a}\,\sqrt{1-\frac{bx^4}{a}}\,(bc-ad)(ad+bc)\Pi\left(\frac{\sqrt{a}\,\sqrt{d}}{\sqrt{b}\,\sqrt{c}};\text{ArcSin}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{8\sqrt[4]{b}\,c^2d^2\sqrt{a-bx^4}}-\frac{x\sqrt{a-bx^4}\,(bc-ad)}{4cd\,(c-dx^4)}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^4)^(3/2)/(c - d*x^4)^2,x]
```

```
[Out] -1/4*((b*c - a*d)*x*Sqrt[a - b*x^4])/(c*d*(c - d*x^4)) + (a^(1/4)*b^(3/4)*(
3*b*c + a*d)*Sqrt[1 - (b*x^4)/a]*EllipticF[ArcSin[(b^(1/4)*x)/a^(1/4)], -1]
)/(4*c*d^2*Sqrt[a - b*x^4]) - (3*a^(1/4)*(b*c - a*d)*(b*c + a*d)*Sqrt[1 - (
b*x^4)/a]*EllipticPi[-((Sqrt[a]*Sqrt[d])/(Sqrt[b]*Sqrt[c])), ArcSin[(b^(1/4
)*x)/a^(1/4)], -1])/(8*b^(1/4)*c^2*d^2*Sqrt[a - b*x^4]) - (3*a^(1/4)*(b*c -
 a*d)*(b*c + a*d)*Sqrt[1 - (b*x^4)/a]*EllipticPi[(Sqrt[a]*Sqrt[d])/(Sqrt[b]
*Sqrt[c]), ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(8*b^(1/4)*c^2*d^2*Sqrt[a - b*
x^4])
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
```

```
b/a] &&  !GtQ[a, 0]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Dist[1/(2*
c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 1233

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{(a-bx^4)^{3/2}}{(c-dx^4)^2}\,dx = -\frac{(bc-ad)x\sqrt{a-bx^4}}{4cd\,(c-dx^4)} - \frac{\int \frac{-a(bc+3ad)+b(3bc+ad)x^4}{\sqrt{a-bx^4}\,(c-dx^4)}\,dx}{4cd}$$

$$= -\frac{(bc-ad)x\sqrt{a-bx^4}}{4cd\,(c-dx^4)} + \frac{\left(3\left(a^2-\frac{b^2c^2}{d^2}\right)\right)\int \frac{1}{\sqrt{a-bx^4}\,(c-dx^4)}\,dx}{4c} + \frac{(b(3bc+ad))\int \ldots}{4cd^2}$$

$$= -\frac{(bc-ad)x\sqrt{a-bx^4}}{4cd\,(c-dx^4)} + \frac{\left(3\left(a^2-\frac{b^2c^2}{d^2}\right)\right)\int \frac{1}{\left(1-\frac{\sqrt{d}\,x^2}{\sqrt{c}}\right)\sqrt{a-bx^4}}\,dx}{8c^2} + \frac{\left(3\left(a^2-\frac{b^2c^2}{d^2}\right.\right.\ldots}{\ldots}$$

$$= -\frac{(bc-ad)x\sqrt{a-bx^4}}{4cd\,(c-dx^4)} + \frac{\sqrt[4]{a}\,b^{3/4}(3bc+ad)\sqrt{1-\frac{bx^4}{a}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{4cd^2\sqrt{a-bx^4}} + \ldots$$

$$= -\frac{(bc-ad)x\sqrt{a-bx^4}}{4cd\,(c-dx^4)} + \frac{\sqrt[4]{a}\,b^{3/4}(3bc+ad)\sqrt{1-\frac{bx^4}{a}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{4cd^2\sqrt{a-bx^4}} + \frac{3\ldots}{\ldots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 10.23, size = 342, normalized size = 1.11

$$\frac{x\left(-b(3bc+ad)x^4\sqrt{1-\frac{bx^4}{a}}\,(-c+dx^4)\,F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)+\frac{5c\left(-5ac(4a^2d+b^2cx^4-abdx^4)F_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)-2(-bc+ad)x^4(a-bx^4)\left(2adF_1\left(\frac{5}{4};\frac{1}{2},2;\frac{9}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)+bcF_1\left(\frac{5}{4};\frac{3}{2},1;\frac{9}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)\right)\right)}{5acF_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)+2x^4\left(2adF_1\left(\frac{5}{4};\frac{1}{2},2;\frac{9}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)+bcF_1\left(\frac{5}{4};\frac{3}{2},1;\frac{9}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)\right)}\right)}{20c^2d\sqrt{a-bx^4}\,(-c+dx^4)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a - b*x^4)^(3/2)/(c - d*x^4)^2,x]
```

```
[Out] (x*(-(b*(3*b*c + a*d)*x^4*Sqrt[1 - (b*x^4)/a]*(-c + d*x^4)*AppellF1[5/4, 1/
2, 1, 9/4, (b*x^4)/a, (d*x^4)/c]) + (5*c*(-5*a*c*(4*a^2*d + b^2*c*x^4 - a*b
*d*x^4)*AppellF1[1/4, 1/2, 1, 5/4, (b*x^4)/a, (d*x^4)/c] - 2*(-(b*c) + a*d)
*x^4*(a - b*x^4)*(2*a*d*AppellF1[5/4, 1/2, 2, 9/4, (b*x^4)/a, (d*x^4)/c] +
b*c*AppellF1[5/4, 3/2, 1, 9/4, (b*x^4)/a, (d*x^4)/c])))/(5*a*c*AppellF1[1/4
, 1/2, 1, 5/4, (b*x^4)/a, (d*x^4)/c] + 2*x^4*(2*a*d*AppellF1[5/4, 1/2, 2, 9
/4, (b*x^4)/a, (d*x^4)/c] + b*c*AppellF1[5/4, 3/2, 1, 9/4, (b*x^4)/a, (d*x^
4)/c]))))/(20*c^2*d*Sqrt[a - b*x^4]*(-c + d*x^4))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.27, size = 328, normalized size = 1.06

method	result
default	$\frac{(ad-bc)x\sqrt{-b\,x^4+a}}{4cd(-d\,x^4+c)}+\frac{\left(\frac{b^2}{d^2}+\frac{b(ad-bc)}{4d^2c}\right)\sqrt{1-\frac{x^2\sqrt{b}}{\sqrt{a}}}\,\sqrt{1+\frac{x^2\sqrt{b}}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^4+a}}-\frac{3\left(\sum_{\_\alpha=\mathrm{RootOf}(}\right.}{}$
elliptic	$\frac{(ad-bc)x\sqrt{-b\,x^4+a}}{4cd(-d\,x^4+c)}+\frac{\left(\frac{b^2}{d^2}+\frac{b(ad-bc)}{4d^2c}\right)\sqrt{1-\frac{x^2\sqrt{b}}{\sqrt{a}}}\,\sqrt{1+\frac{x^2\sqrt{b}}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^4+a}}-\frac{3\left(\sum_{\_\alpha=\mathrm{RootOf}(}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^4+a)^(3/2)/(-d*x^4+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(a*d-b*c)/c/d*x*(-b*x^4+a)^(1/2)/(-d*x^4+c)+(b^2/d^2+1/4*b*(a*d-b*c)/d^
```

```
2/c)/(1/a^(1/2)*b^(1/2))^(1/2)*(1-x^2*b^(1/2)/a^(1/2))^(1/2)*(1+x^2*b^(1/2)
/a^(1/2))^(1/2)/(-b*x^4+a)^(1/2)*EllipticF(x*(1/a^(1/2)*b^(1/2))^(1/2),I)-3
/32/c/d^3*sum((a^2*d^2-b^2*c^2)/_alpha^3*(-1/((a*d-b*c)/d)^(1/2)*arctanh(1/
2*(-2*_alpha^2*b*x^2+2*a)/((a*d-b*c)/d)^(1/2)/(-b*x^4+a)^(1/2))-2/(1/a^(1/2
)*b^(1/2))^(1/2)*_alpha^3*d/c*(1-x^2*b^(1/2)/a^(1/2))^(1/2)*(1+x^2*b^(1/2)/
a^(1/2))^(1/2)/(-b*x^4+a)^(1/2)*EllipticPi(x*(1/a^(1/2)*b^(1/2))^(1/2),a^(1
/2)/b^(1/2)*_alpha^2/c*d,(-1/a^(1/2)*b^(1/2))^(1/2)/(1/a^(1/2)*b^(1/2))^(1/
2))),_alpha=RootOf(_Z^4*d-c))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^4+a)^(3/2)/(-d*x^4+c)^2,x, algorithm="maxima")
[Out] integrate((-b*x^4 + a)^(3/2)/(d*x^4 - c)^2, x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^4+a)^(3/2)/(-d*x^4+c)^2,x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a-bx^4)^{\frac{3}{2}}}{(-c+dx^4)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**4+a)**(3/2)/(-d*x**4+c)**2,x)
[Out] Integral((a - b*x**4)**(3/2)/(-c + d*x**4)**2, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^4+a)^(3/2)/(-d*x^4+c)^2,x, algorithm="giac")
```

```
[Out] integrate((-b*x^4 + a)^(3/2)/(d*x^4 - c)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a-b\,x^4\right)^{3/2}}{\left(c-d\,x^4\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x^4)^(3/2)/(c - d*x^4)^2,x)
```

```
[Out] int((a - b*x^4)^(3/2)/(c - d*x^4)^2, x)
```

3.187 $\int \frac{\sqrt{a-bx^4}}{(c-dx^4)^2}\,dx$

Optimal. Leaf size=276

$$\frac{x\sqrt{a-bx^4}}{4c\left(c-dx^4\right)}+\frac{\sqrt[4]{a}\,b^{3/4}\sqrt{1-\frac{bx^4}{a}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{4cd\sqrt{a-bx^4}}-\frac{\sqrt[4]{a}\,(bc-3ad)\sqrt{1-\frac{bx^4}{a}}\,\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}};\sin^{-1}\right.}{8\sqrt[4]{b}\,c^2d\sqrt{a-bx^4}}$$

```
[Out] 1/4*x*(-b*x^4+a)^(1/2)/c/(-d*x^4+c)+1/4*a^(1/4)*b^(3/4)*EllipticF(b^(1/4)*x
/a^(1/4),I)*(1-b*x^4/a)^(1/2)/c/d/(-b*x^4+a)^(1/2)-1/8*a^(1/4)*(-3*a*d+b*c)
*EllipticPi(b^(1/4)*x/a^(1/4),-a^(1/2)*d^(1/2)/b^(1/2)/c^(1/2),I)*(1-b*x^4/
a)^(1/2)/b^(1/4)/c^2/d/(-b*x^4+a)^(1/2)-1/8*a^(1/4)*(-3*a*d+b*c)*EllipticPi
(b^(1/4)*x/a^(1/4),a^(1/2)*d^(1/2)/b^(1/2)/c^(1/2),I)*(1-b*x^4/a)^(1/2)/b^(
1/4)/c^2/d/(-b*x^4+a)^(1/2)
```

Rubi [A]

time = 0.14, antiderivative size = 276, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {423, 537, 230, 227, 418, 1233, 1232}

$$\frac{\sqrt[4]{a}\,b^{3/4}\sqrt{1-\frac{bx^4}{a}}\,F\left(\text{ArcSin}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{4cd\sqrt{a-bx^4}}-\frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}\,(bc-3ad)\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}};\text{ArcSin}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{8\sqrt[4]{b}\,c^2d\sqrt{a-bx^4}}-\frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}\,(bc-3ad)\Pi\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}};\text{ArcSin}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{8\sqrt[4]{b}\,c^2d\sqrt{a-bx^4}}+\frac{x\sqrt{a-bx^4}}{4c\left(c-dx^4\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a - b*x^4]/(c - d*x^4)^2,x]
```

```
[Out] (x*Sqrt[a - b*x^4])/(4*c*(c - d*x^4)) + (a^(1/4)*b^(3/4)*Sqrt[1 - (b*x^4)/a
]*EllipticF[ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(4*c*d*Sqrt[a - b*x^4]) - (a^
(1/4)*(b*c - 3*a*d)*Sqrt[1 - (b*x^4)/a]*EllipticPi[-((Sqrt[a]*Sqrt[d])/(Sqr
t[b]*Sqrt[c])), ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(8*b^(1/4)*c^2*d*Sqrt[a -
 b*x^4]) - (a^(1/4)*(b*c - 3*a*d)*Sqrt[1 - (b*x^4)/a]*EllipticPi[(Sqrt[a]*S
qrt[d])/(Sqrt[b]*Sqrt[c]), ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(8*b^(1/4)*c^2
*d*Sqrt[a - b*x^4])
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Dist[1/(2*
c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 423

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] + Dist[1
/(a*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(n*(p +
1) + 1) + d*(n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x
] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 1233

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sqrt{a-bx^4}}{(c-dx^4)^2}\,dx = \frac{x\sqrt{a-bx^4}}{4c(c-dx^4)} - \frac{\int \frac{-3a+bx^4}{\sqrt{a-bx^4}\,(c-dx^4)}\,dx}{4c}$$

$$= \frac{x\sqrt{a-bx^4}}{4c(c-dx^4)} + \frac{b\int \frac{1}{\sqrt{a-bx^4}}\,dx}{4cd} + \frac{(-bc+3ad)\int \frac{1}{\sqrt{a-bx^4}\,(c-dx^4)}\,dx}{4cd}$$

$$= \frac{x\sqrt{a-bx^4}}{4c(c-dx^4)} - \frac{(bc-3ad)\int \frac{1}{\left(1-\frac{\sqrt{d}\,x^2}{\sqrt{c}}\right)\sqrt{a-bx^4}}\,dx}{8c^2d} - \frac{(bc-3ad)\int \frac{1}{\left(1+\frac{\sqrt{d}\,x^2}{\sqrt{c}}\right)\sqrt{a-}}}{8c^2d}$$

$$= \frac{x\sqrt{a-bx^4}}{4c(c-dx^4)} + \frac{\sqrt[4]{a}\,b^{3/4}\sqrt{1-\frac{bx^4}{a}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{4cd\sqrt{a-bx^4}} - \frac{\left((bc-3ad)\sqrt{1-\frac{bx^4}{a}}\right.}{8c^2d}$$

$$= \frac{x\sqrt{a-bx^4}}{4c(c-dx^4)} + \frac{\sqrt[4]{a}\,b^{3/4}\sqrt{1-\frac{bx^4}{a}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{4cd\sqrt{a-bx^4}} - \frac{\sqrt[4]{a}\,(bc-3ad)\sqrt{1-\frac{bx^4}{a}}}{8\sqrt[4]{}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 10.12, size = 233, normalized size = 0.84

$$\frac{x\left(-\frac{5(a-bx^4)}{c}+\frac{bx^4\sqrt{1-\frac{bx^4}{a}}\,(c-dx^4)F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)}{c^2}-\frac{75a^2F_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)}{5acF_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)+2x^4\left(2adF_1\left(\frac{5}{4};\frac{1}{2},2;\frac{9}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)+bcF_1\left(\frac{5}{4};\frac{3}{2},1;\frac{9}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)\right)}\right)}{20\sqrt{a-bx^4}\,(-c+dx^4)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a - b*x^4]/(c - d*x^4)^2,x]
```

```
[Out] (x*((-5*(a - b*x^4))/c + (b*x^4*Sqrt[1 - (b*x^4)/a]*(c - d*x^4)*AppellF1[5/
4, 1/2, 1, 9/4, (b*x^4)/a, (d*x^4)/c])/c^2 - (75*a^2*AppellF1[1/4, 1/2, 1,
5/4, (b*x^4)/a, (d*x^4)/c])/(5*a*c*AppellF1[1/4, 1/2, 1, 5/4, (b*x^4)/a, (d
*x^4)/c] + 2*x^4*(2*a*d*AppellF1[5/4, 1/2, 2, 9/4, (b*x^4)/a, (d*x^4)/c] +
b*c*AppellF1[5/4, 3/2, 1, 9/4, (b*x^4)/a, (d*x^4)/c]))))/(20*Sqrt[a - b*x^4
]*(-c + d*x^4))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.27, size = 293, normalized size = 1.06

method	result
default	$\frac{x\sqrt{-b\,x^{4}+a}}{4c\left(-d\,x^{4}+c\right)}+\frac{b\sqrt{1-\frac{x^{2}\sqrt{b}}{\sqrt{a}}}\,\sqrt{1+\frac{x^{2}\sqrt{b}}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)}{4cd\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^{4}+a}}-\sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z^{4}-c\right)}\frac{(3ad-bc)\left(-\frac{\mathrm{arctan}\ldots}{\ldots}\right.}{\ldots}$
elliptic	$\frac{x\sqrt{-b\,x^{4}+a}}{4c\left(-d\,x^{4}+c\right)}+\frac{b\sqrt{1-\frac{x^{2}\sqrt{b}}{\sqrt{a}}}\,\sqrt{1+\frac{x^{2}\sqrt{b}}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)}{4cd\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^{4}+a}}-\sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z^{4}-c\right)}\frac{(3ad-bc)\left(-\frac{\mathrm{arctan}\ldots}{\ldots}\right.}{\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^4+a)^(1/2)/(-d*x^4+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*x*(-b*x^4+a)^(1/2)/c/(-d*x^4+c)+1/4/c/d*b/(1/a^(1/2)*b^(1/2))^(1/2)*(1-
x^2*b^(1/2)/a^(1/2))^(1/2)*(1+x^2*b^(1/2)/a^(1/2))^(1/2)/(-b*x^4+a)^(1/2)*E
llipticF(x*(1/a^(1/2)*b^(1/2))^(1/2),I)-1/32/c/d^2*sum((3*a*d-b*c)/_alpha^3
*(-1/((a*d-b*c)/d)^(1/2)*arctanh(1/2*(-2*_alpha^2*b*x^2+2*a)/((a*d-b*c)/d)^
(1/2)/(-b*x^4+a)^(1/2))-2/(1/a^(1/2)*b^(1/2))^(1/2)*_alpha^3*d/c*(1-x^2*b^(
1/2)/a^(1/2))^(1/2)*(1+x^2*b^(1/2)/a^(1/2))^(1/2)/(-b*x^4+a)^(1/2)*Elliptic
Pi(x*(1/a^(1/2)*b^(1/2))^(1/2),a^(1/2)/b^(1/2)*_alpha^2/c*d,(-1/a^(1/2)*b^(
1/2))^(1/2)/(1/a^(1/2)*b^(1/2))^(1/2))),_alpha=RootOf(_Z^4*d-c))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^4+a)^(1/2)/(-d*x^4+c)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-b*x^4 + a)/(d*x^4 - c)^2, x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^4+a)^(1/2)/(-d*x^4+c)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a-bx^4}}{\left(-c+dx^4\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**4+a)**(1/2)/(-d*x**4+c)**2,x)
```

```
[Out] Integral(sqrt(a - b*x**4)/(-c + d*x**4)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^4+a)^(1/2)/(-d*x^4+c)^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(-b*x^4 + a)/(d*x^4 - c)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{a-b\,x^4}}{{\left(c-d\,x^4\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x^4)^(1/2)/(c - d*x^4)^2,x)
```

```
[Out] int((a - b*x^4)^(1/2)/(c - d*x^4)^2, x)
```

3.188 $\int \frac{1}{\sqrt{a-bx^4}\,(c-dx^4)^2}\,dx$

Optimal. Leaf size=310

$$-\frac{dx\sqrt{a-bx^4}}{4c(bc-ad)\left(c-dx^4\right)}-\frac{\sqrt[4]{a}\,b^{3/4}\sqrt{1-\frac{bx^4}{a}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{4c(bc-ad)\sqrt{a-bx^4}}+\frac{\sqrt[4]{a}\,(5bc-3ad)\sqrt{1-\frac{bx^4}{a}}\,\Pi\left(-\frac{\sqrt{a}}{\sqrt{b}}\right.}{8\sqrt[4]{b}\,c^2(bc-ad)\sqrt{}}$$

```
[Out] -1/4*d*x*(-b*x^4+a)^(1/2)/c/(-a*d+b*c)/(-d*x^4+c)-1/4*a^(1/4)*b^(3/4)*Ellip
ticF(b^(1/4)*x/a^(1/4),I)*(1-b*x^4/a)^(1/2)/c/(-a*d+b*c)/(-b*x^4+a)^(1/2)+1
/8*a^(1/4)*(-3*a*d+5*b*c)*EllipticPi(b^(1/4)*x/a^(1/4),-a^(1/2)*d^(1/2)/b^(
1/2)/c^(1/2),I)*(1-b*x^4/a)^(1/2)/b^(1/4)/c^2/(-a*d+b*c)/(-b*x^4+a)^(1/2)+1
/8*a^(1/4)*(-3*a*d+5*b*c)*EllipticPi(b^(1/4)*x/a^(1/4),a^(1/2)*d^(1/2)/b^(1
/2)/c^(1/2),I)*(1-b*x^4/a)^(1/2)/b^(1/4)/c^2/(-a*d+b*c)/(-b*x^4+a)^(1/2)
```

Rubi [A]

time = 0.16, antiderivative size = 310, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {425, 537, 230, 227, 418, 1233, 1232}

$$-\frac{\sqrt[4]{a}\,b^{3/4}\sqrt{1-\frac{bx^4}{a}}\,F\left(\text{ArcSin}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{4c\sqrt{a-bx^4}\,(bc-ad)}+\frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}\,(5bc-3ad)\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}};\text{ArcSin}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{8\sqrt[4]{b}\,c^2\sqrt{a-bx^4}\,(bc-ad)}+\frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}\,(5bc-3ad)\Pi\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}};\text{ArcSin}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{8\sqrt[4]{b}\,c^2\sqrt{a-bx^4}\,(bc-ad)}-\frac{dx\sqrt{a-bx^4}}{4c\left(c-dx^4\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a - b*x^4]*(c - d*x^4)^2),x]
```

```
[Out] -1/4*(d*x*Sqrt[a - b*x^4])/(c*(b*c - a*d)*(c - d*x^4)) - (a^(1/4)*b^(3/4)*S
qrt[1 - (b*x^4)/a]*EllipticF[ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(4*c*(b*c -
a*d)*Sqrt[a - b*x^4]) + (a^(1/4)*(5*b*c - 3*a*d)*Sqrt[1 - (b*x^4)/a]*Ellipt
icPi[-((Sqrt[a]*Sqrt[d])/(Sqrt[b]*Sqrt[c])), ArcSin[(b^(1/4)*x)/a^(1/4)], -
1])/(8*b^(1/4)*c^2*(b*c - a*d)*Sqrt[a - b*x^4]) + (a^(1/4)*(5*b*c - 3*a*d)*
Sqrt[1 - (b*x^4)/a]*EllipticPi[(Sqrt[a]*Sqrt[d])/(Sqrt[b]*Sqrt[c]), ArcSin[
(b^(1/4)*x)/a^(1/4)], -1])/(8*b^(1/4)*c^2*(b*c - a*d)*Sqrt[a - b*x^4])
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Dist[1/(2*
c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 1233

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a-bx^4}\,(c-dx^4)^2}\,dx = -\frac{dx\sqrt{a-bx^4}}{4c(bc-ad)\,(c-dx^4)} - \frac{\int \frac{-4bc+3ad-bdx^4}{\sqrt{a-bx^4}\,(c-dx^4)}dx}{4c(bc-ad)}$$

$$= -\frac{dx\sqrt{a-bx^4}}{4c(bc-ad)\,(c-dx^4)} - \frac{b\int \frac{1}{\sqrt{a-bx^4}}dx}{4c(bc-ad)} + \frac{(5bc-3ad)\int \frac{1}{\sqrt{a-bx^4}\,(c-dx^4)}}{4c(bc-ad)}$$

$$= -\frac{dx\sqrt{a-bx^4}}{4c(bc-ad)\,(c-dx^4)} + \frac{(5bc-3ad)\int \frac{1}{\left(1-\frac{\sqrt{d}\,x^2}{\sqrt{c}}\right)\sqrt{a-bx^4}}dx}{8c^2(bc-ad)} + \frac{(5bc-3a}{}$$

$$= -\frac{dx\sqrt{a-bx^4}}{4c(bc-ad)\,(c-dx^4)} - \frac{\sqrt[4]{a}\,b^{3/4}\sqrt{1-\frac{bx^4}{a}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{4c(bc-ad)\sqrt{a-bx^4}} + \frac{\Big((5}{}$$

$$= -\frac{dx\sqrt{a-bx^4}}{4c(bc-ad)\,(c-dx^4)} - \frac{\sqrt[4]{a}\,b^{3/4}\sqrt{1-\frac{bx^4}{a}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{4c(bc-ad)\sqrt{a-bx^4}} + \frac{\sqrt[4]{a}}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 10.21, size = 386, normalized size = 1.25

$$\frac{5acxF_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)\left(-5c(4bc-4ad+bdx^4)+bdx^4\sqrt{1-\frac{bx^4}{a}}(-c+dx^4)F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)\right)+2dx^5\left(5c(a-bx^4)+bx^4\sqrt{1-\frac{bx^4}{a}}(-c+dx^4)F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)\right)\left(2adF_1\left(\frac{5}{4};\frac{1}{2},2;\frac{9}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)+bcF_1\left(\frac{5}{4};\frac{3}{2},1;\frac{9}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)\right)}{20c^2(bc-ad)\sqrt{a-bx^4}\,(-c+dx^4)\left(5acF_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)+2x^4\left(2adF_1\left(\frac{5}{4};\frac{1}{2},2;\frac{9}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)+bcF_1\left(\frac{5}{4};\frac{3}{2},1;\frac{9}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Sqrt[a - b*x^4]*(c - d*x^4)^2),x]
```

```
[Out] (5*a*c*x*AppellF1[1/4, 1/2, 1, 5/4, (b*x^4)/a, (d*x^4)/c]*(-5*c*(4*b*c - 4*
a*d + b*d*x^4) + b*d*x^4*Sqrt[1 - (b*x^4)/a]*(-c + d*x^4)*AppellF1[5/4, 1/2
, 1, 9/4, (b*x^4)/a, (d*x^4)/c]) + 2*d*x^5*(5*c*(a - b*x^4) + b*x^4*Sqrt[1
- (b*x^4)/a]*(-c + d*x^4)*AppellF1[5/4, 1/2, 1, 9/4, (b*x^4)/a, (d*x^4)/c])
*(2*a*d*AppellF1[5/4, 1/2, 2, 9/4, (b*x^4)/a, (d*x^4)/c] + b*c*AppellF1[5/4
, 3/2, 1, 9/4, (b*x^4)/a, (d*x^4)/c]))/(20*c^2*(b*c - a*d)*Sqrt[a - b*x^4]*
(-c + d*x^4)*(5*a*c*AppellF1[1/4, 1/2, 1, 5/4, (b*x^4)/a, (d*x^4)/c] + 2*x^
4*(2*a*d*AppellF1[5/4, 1/2, 2, 9/4, (b*x^4)/a, (d*x^4)/c] + b*c*AppellF1[5/
4, 3/2, 1, 9/4, (b*x^4)/a, (d*x^4)/c])))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.27, size = 321, normalized size = 1.04

method	result
default	$\frac{dx\sqrt{-b\,x^4+a}}{4c(ad-bc)(-d\,x^4+c)}+\frac{b\sqrt{1-\frac{x^2\sqrt{b}}{\sqrt{a}}}\,\sqrt{1+\frac{x^2\sqrt{b}}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)}{4c(ad-bc)\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^4+a}}-\frac{\sum_{\_\alpha=\mathrm{RootOf}(d\_Z^4-c)}\frac{(3ad-5bc)\left(-\right.}{}}{}$
elliptic	$\frac{dx\sqrt{-b\,x^4+a}}{4c(ad-bc)(-d\,x^4+c)}+\frac{b\sqrt{1-\frac{x^2\sqrt{b}}{\sqrt{a}}}\,\sqrt{1+\frac{x^2\sqrt{b}}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)}{4c(ad-bc)\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^4+a}}-\frac{\sum_{\_\alpha=\mathrm{RootOf}(d\_Z^4-c)}\frac{(3ad-5bc)\left(-\right.}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x^4+a)^(1/2)/(-d*x^4+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*d/c/(a*d-b*c)*x*(-b*x^4+a)^(1/2)/(-d*x^4+c)+1/4*b/c/(a*d-b*c)/(1/a^(1/2
)*b^(1/2))^(1/2)*(1-x^2*b^(1/2)/a^(1/2))^(1/2)*(1+x^2*b^(1/2)/a^(1/2))^(1/2
)/(-b*x^4+a)^(1/2)*EllipticF(x*(1/a^(1/2)*b^(1/2))^(1/2),I)-1/32/c/d*sum((3
*a*d-5*b*c)/(a*d-b*c)/_alpha^3*(-1/((a*d-b*c)/d)^(1/2)*arctanh(1/2*(-2*_alp
ha^2*b*x^2+2*a)/((a*d-b*c)/d)^(1/2)/(-b*x^4+a)^(1/2))-2/(1/a^(1/2)*b^(1/2))
^(1/2)*_alpha^3*d/c*(1-x^2*b^(1/2)/a^(1/2))^(1/2)*(1+x^2*b^(1/2)/a^(1/2))^(
1/2)/(-b*x^4+a)^(1/2)*EllipticPi(x*(1/a^(1/2)*b^(1/2))^(1/2),a^(1/2)/b^(1/2
)*_alpha^2/c*d,(-1/a^(1/2)*b^(1/2))^(1/2)/(1/a^(1/2)*b^(1/2))^(1/2))),_alph
a=RootOf(_Z^4*d-c))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^4+a)^(1/2)/(-d*x^4+c)^2,x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-b*x^4 + a)*(d*x^4 - c)^2), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^4+a)^(1/2)/(-d*x^4+c)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a - bx^4}\,(-c + dx^4)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x**4+a)**(1/2)/(-d*x**4+c)**2,x)
```

```
[Out] Integral(1/(sqrt(a - b*x**4)*(-c + d*x**4)**2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^4+a)^(1/2)/(-d*x^4+c)^2,x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-b*x^4 + a)*(d*x^4 - c)^2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{a - b\,x^4}\,(c - d\,x^4)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a - b*x^4)^(1/2)*(c - d*x^4)^2),x)
```

```
[Out] int(1/((a - b*x^4)^(1/2)*(c - d*x^4)^2), x)
```

3.189 $\int \frac{1}{(a-bx^4)^{3/2}(c-dx^4)^2}\,dx$

Optimal. Leaf size=362

$$\frac{b(2bc+ad)x}{4ac(bc-ad)^2\sqrt{a-bx^4}}-\frac{dx}{4c(bc-ad)\sqrt{a-bx^4}\,(c-dx^4)}+\frac{b^{3/4}(2bc+ad)\sqrt{1-\frac{bx^4}{a}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\middle|-\right.}{4a^{3/4}c(bc-ad)^2\sqrt{a-bx^4}}$$

```
[Out] 1/4*b*(a*d+2*b*c)*x/a/c/(-a*d+b*c)^2/(-b*x^4+a)^(1/2)-1/4*d*x/c/(-a*d+b*c)/
(-d*x^4+c)/(-b*x^4+a)^(1/2)+1/4*b^(3/4)*(a*d+2*b*c)*EllipticF(b^(1/4)*x/a^(
1/4),I)*(1-b*x^4/a)^(1/2)/a^(3/4)/c/(-a*d+b*c)^2/(-b*x^4+a)^(1/2)-3/8*a^(1/
4)*d*(-a*d+3*b*c)*EllipticPi(b^(1/4)*x/a^(1/4),-a^(1/2)*d^(1/2)/b^(1/2)/c^(
1/2),I)*(1-b*x^4/a)^(1/2)/b^(1/4)/c^2/(-a*d+b*c)^2/(-b*x^4+a)^(1/2)-3/8*a^(
1/4)*d*(-a*d+3*b*c)*EllipticPi(b^(1/4)*x/a^(1/4),a^(1/2)*d^(1/2)/b^(1/2)/c^
(1/2),I)*(1-b*x^4/a)^(1/2)/b^(1/4)/c^2/(-a*d+b*c)^2/(-b*x^4+a)^(1/2)
```

Rubi [A]
time = 0.44, antiderivative size = 362, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {425, 541, 537, 230, 227, 418, 1233, 1232}

$$\frac{b^{3/4}\sqrt{1-\frac{bx^4}{a}}\,(ad+2bc)F\left(\text{ArcSin}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{4a^{3/4}c\sqrt{a-bx^4}\,(bc-ad)^2}-\frac{3\sqrt[4]{a}\,d\sqrt{1-\frac{bx^4}{a}}\,(3bc-ad)\Pi\left(-\frac{\sqrt{a}\,\sqrt{d}}{\sqrt{b}\,\sqrt{c}};\text{ArcSin}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{8\sqrt[4]{b}\,c^2\sqrt{a-bx^4}\,(bc-ad)^2}-\frac{3\sqrt[4]{a}\,d\sqrt{1-\frac{bx^4}{a}}\,(3bc-ad)\Pi\left(\frac{\sqrt{a}\,\sqrt{d}}{\sqrt{b}\,\sqrt{c}};\text{ArcSin}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{8\sqrt[4]{b}\,c^2\sqrt{a-bx^4}\,(bc-ad)^2}+\frac{bx(ad+2bc)}{4ac\sqrt{a-bx^4}\,(bc-ad)^2}-\frac{dx}{4c\sqrt{a-bx^4}\,(c-dx^4)\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - b*x^4)^(3/2)*(c - d*x^4)^2),x]
```

```
[Out] (b*(2*b*c + a*d)*x)/(4*a*c*(b*c - a*d)^2*Sqrt[a - b*x^4]) - (d*x)/(4*c*(b*c
 - a*d)*Sqrt[a - b*x^4]*(c - d*x^4)) + (b^(3/4)*(2*b*c + a*d)*Sqrt[1 - (b*x
^4)/a]*EllipticF[ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(4*a^(3/4)*c*(b*c - a*d)
^2*Sqrt[a - b*x^4]) - (3*a^(1/4)*d*(3*b*c - a*d)*Sqrt[1 - (b*x^4)/a]*Ellipt
icPi[-((Sqrt[a]*Sqrt[d])/(Sqrt[b]*Sqrt[c])), ArcSin[(b^(1/4)*x)/a^(1/4)], -
1])/(8*b^(1/4)*c^2*(b*c - a*d)^2*Sqrt[a - b*x^4]) - (3*a^(1/4)*d*(3*b*c - a
*d)*Sqrt[1 - (b*x^4)/a]*EllipticPi[(Sqrt[a]*Sqrt[d])/(Sqrt[b]*Sqrt[c]), Arc
Sin[(b^(1/4)*x)/a^(1/4)], -1])/(8*b^(1/4)*c^2*(b*c - a*d)^2*Sqrt[a - b*x^4]
)
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
```

```
b/a] &&  !GtQ[a, 0]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Dist[1/(2*
c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 1233

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{(a-bx^4)^{3/2}(c-dx^4)^2}\,dx = -\frac{dx}{4c(bc-ad)\sqrt{a-bx^4}\,(c-dx^4)} - \frac{\int \frac{-4bc+3ad-5bdx^4}{(a-bx^4)^{3/2}(c-dx^4)}\,dx}{4c(bc-ad)}$$

$$= \frac{b(2bc+ad)x}{4ac(bc-ad)^2\sqrt{a-bx^4}} - \frac{dx}{4c(bc-ad)\sqrt{a-bx^4}\,(c-dx^4)} - \frac{\int \frac{-2(2b^2c^2-8a}{\sqrt{a}}}{8}$$

$$= \frac{b(2bc+ad)x}{4ac(bc-ad)^2\sqrt{a-bx^4}} - \frac{dx}{4c(bc-ad)\sqrt{a-bx^4}\,(c-dx^4)} - \frac{(3d(3bc-ad}{}$$

$$= \frac{b(2bc+ad)x}{4ac(bc-ad)^2\sqrt{a-bx^4}} - \frac{dx}{4c(bc-ad)\sqrt{a-bx^4}\,(c-dx^4)} - \frac{(3d(3bc-ad}{}$$

$$= \frac{b(2bc+ad)x}{4ac(bc-ad)^2\sqrt{a-bx^4}} - \frac{dx}{4c(bc-ad)\sqrt{a-bx^4}\,(c-dx^4)} + \frac{b^{3/4}(2bc+ad}{4}$$

$$= \frac{b(2bc+ad)x}{4ac(bc-ad)^2\sqrt{a-bx^4}} - \frac{dx}{4c(bc-ad)\sqrt{a-bx^4}\,(c-dx^4)} + \frac{b^{3/4}(2bc+ad}{4}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 10.38, size = 374, normalized size = 1.03

$$\frac{x\left(-bd(2bc+ad)x^4\sqrt{1-\frac{bx^4}{a}}\,F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)+\frac{c\left(25ac\left(4a^2d^2+2b^2c(2c-dx^4)-abd(8c+dx^4)\right)F_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)-10x^4\left(-a^2d^2+abd^2x^4-2b^2c(c-dx^4)\right)\left(2adF_1\left(\frac{5}{4};\frac{1}{2},2;\frac{9}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)+bcF_1\left(\frac{5}{4};\frac{3}{2},1;\frac{9}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)\right)\right)}{(c-dx^4)\left(5acF_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)+2x^4\left(2adF_1\left(\frac{5}{4};\frac{1}{2},2;\frac{9}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)+bcF_1\left(\frac{5}{4};\frac{3}{2},1;\frac{9}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)\right)\right)}\right)}{20ac^2(bc-ad)^2\sqrt{a-bx^4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a - b*x^4)^(3/2)*(c - d*x^4)^2),x]
```

```
[Out] (x*(-(b*d*(2*b*c + a*d)*x^4*Sqrt[1 - (b*x^4)/a]*AppellF1[5/4, 1/2, 1, 9/4,
(b*x^4)/a, (d*x^4)/c]) + (c*(25*a*c*(4*a^2*d^2 + 2*b^2*c*(2*c - d*x^4) - a*
b*d*(8*c + d*x^4))*AppellF1[1/4, 1/2, 1, 5/4, (b*x^4)/a, (d*x^4)/c] - 10*x^
4*(-(a^2*d^2) + a*b*d^2*x^4 - 2*b^2*c*(c - d*x^4))*(2*a*d*AppellF1[5/4, 1/2
, 2, 9/4, (b*x^4)/a, (d*x^4)/c] + b*c*AppellF1[5/4, 3/2, 1, 9/4, (b*x^4)/a,
 (d*x^4)/c])))/((c - d*x^4)*(5*a*c*AppellF1[1/4, 1/2, 1, 5/4, (b*x^4)/a, (d
*x^4)/c] + 2*x^4*(2*a*d*AppellF1[5/4, 1/2, 2, 9/4, (b*x^4)/a, (d*x^4)/c] +
```

b*c*AppellF1[5/4, 3/2, 1, 9/4, (b*x^4)/a, (d*x^4)/c])))))/(20*a*c^2*(b*c - a*d)^2*Sqrt[a - b*x^4])

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.27, size = 374, normalized size = 1.03

method	result
default	$\frac{d^{2} x \sqrt{-b\,x^{4}+a}}{4 c \left(a d -b c \right)^{2} \left(-d\,x^{4}+c \right)}+\frac{b^{2} x}{2 a \left(a d -b c \right)^{2} \sqrt{-\left(x^{4}-\frac{a}{b}\right) b}}+\frac{\left(\frac{b d}{4 c \left(a d -b c \right)^{2}}+\frac{b^{2}}{2 a \left(a d -b c \right)^{2}}\right) \sqrt{1-\frac{x^{2} \sqrt{b}}{\sqrt{a}}}\, \sqrt{1+\frac{x^{2} \sqrt{b}}{\sqrt{a}}}\ \mathrm{Elli}}{\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\, \sqrt{-b\,x^{4}+a}}$
elliptic	$\frac{d^{2} x \sqrt{-b\,x^{4}+a}}{4 c \left(a d -b c \right)^{2} \left(-d\,x^{4}+c \right)}+\frac{b^{2} x}{2 a \left(a d -b c \right)^{2} \sqrt{-\left(x^{4}-\frac{a}{b}\right) b}}+\frac{\left(\frac{b d}{4 c \left(a d -b c \right)^{2}}+\frac{b^{2}}{2 a \left(a d -b c \right)^{2}}\right) \sqrt{1-\frac{x^{2} \sqrt{b}}{\sqrt{a}}}\, \sqrt{1+\frac{x^{2} \sqrt{b}}{\sqrt{a}}}\ \mathrm{Elli}}{\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\, \sqrt{-b\,x^{4}+a}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x^4+a)^(3/2)/(-d*x^4+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*d^2/c/(a*d-b*c)^2*x*(-b*x^4+a)^(1/2)/(-d*x^4+c)+1/2*b^2/a*x/(a*d-b*c)^2
/(-(x^4-a/b)*b)^(1/2)+(1/4*b*d/c/(a*d-b*c)^2+1/2*b^2/a/(a*d-b*c)^2)/(1/a^(1
/2)*b^(1/2))^(1/2)*(1-x^2*b^(1/2)/a^(1/2))^(1/2)*(1+x^2*b^(1/2)/a^(1/2))^(1
/2)/(-b*x^4+a)^(1/2)*EllipticF(x*(1/a^(1/2)*b^(1/2))^(1/2),I)-3/32/c*sum((a
*d-3*b*c)/(a*d-b*c)^2/_alpha^3*(-1/((a*d-b*c)/d)^(1/2)*arctanh(1/2*(-2*_alp
ha^2*b*x^2+2*a)/((a*d-b*c)/d)^(1/2)/(-b*x^4+a)^(1/2))-2/(1/a^(1/2)*b^(1/2))
^(1/2)*_alpha^3*d/c*(1-x^2*b^(1/2)/a^(1/2))^(1/2)*(1+x^2*b^(1/2)/a^(1/2))^(
1/2)/(-b*x^4+a)^(1/2)*EllipticPi(x*(1/a^(1/2)*b^(1/2))^(1/2),a^(1/2)/b^(1/2
)*_alpha^2/c*d,(-1/a^(1/2)*b^(1/2))^(1/2)/(1/a^(1/2)*b^(1/2))^(1/2))),_alph
a=RootOf(_Z^4*d-c))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^4+a)^(3/2)/(-d*x^4+c)^2,x, algorithm="maxima")
```

```
[Out] integrate(1/((-b*x^4 + a)^(3/2)*(d*x^4 - c)^2), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^4+a)^(3/2)/(-d*x^4+c)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a - bx^{4}\right)^{\frac{3}{2}} \left(-c + dx^{4}\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x**4+a)**(3/2)/(-d*x**4+c)**2,x)
```

```
[Out] Integral(1/((a - b*x**4)**(3/2)*(-c + d*x**4)**2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^4+a)^(3/2)/(-d*x^4+c)^2,x, algorithm="giac")
```

```
[Out] integrate(1/((-b*x^4 + a)^(3/2)*(d*x^4 - c)^2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(a-b\,x^4\right)}^{3/2}\,{\left(c-d\,x^4\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a - b*x^4)^(3/2)*(c - d*x^4)^2),x)
```

```
[Out] int(1/((a - b*x^4)^(3/2)*(c - d*x^4)^2), x)
```

3.190 $\int \frac{1}{(a-bx^4)^{5/2}(c-dx^4)^2}\,dx$

Optimal. Leaf size=439

$$\frac{b(2bc+3ad)x}{12ac(bc-ad)^2\left(a-bx^4\right)^{3/2}}+\frac{b(5b^2c^2-17abcd-3a^2d^2)\,x}{12a^2c(bc-ad)^3\sqrt{a-bx^4}}-\frac{dx}{4c(bc-ad)\left(a-bx^4\right)^{3/2}(c-dx^4)}+\frac{b^{3/4}(5b^2c^2-}{}$$

```
[Out] 1/12*b*(3*a*d+2*b*c)*x/a/c/(-a*d+b*c)^2/(-b*x^4+a)^(3/2)-1/4*d*x/c/(-a*d+b*
c)/(-b*x^4+a)^(3/2)/(-d*x^4+c)+1/12*b*(-3*a^2*d^2-17*a*b*c*d+5*b^2*c^2)*x/a
^2/c/(-a*d+b*c)^3/(-b*x^4+a)^(1/2)+1/12*b^(3/4)*(-3*a^2*d^2-17*a*b*c*d+5*b^
2*c^2)*EllipticF(b^(1/4)*x/a^(1/4),I)*(1-b*x^4/a)^(1/2)/a^(7/4)/c/(-a*d+b*c
)^3/(-b*x^4+a)^(1/2)+1/8*a^(1/4)*d^2*(-3*a*d+13*b*c)*EllipticPi(b^(1/4)*x/a
^(1/4),-a^(1/2)*d^(1/2)/b^(1/2)/c^(1/2),I)*(1-b*x^4/a)^(1/2)/b^(1/4)/c^2/(-
a*d+b*c)^3/(-b*x^4+a)^(1/2)+1/8*a^(1/4)*d^2*(-3*a*d+13*b*c)*EllipticPi(b^(1
/4)*x/a^(1/4),a^(1/2)*d^(1/2)/b^(1/2)/c^(1/2),I)*(1-b*x^4/a)^(1/2)/b^(1/4)/
c^2/(-a*d+b*c)^3/(-b*x^4+a)^(1/2)
```

Rubi [A]
time = 0.35, antiderivative size = 439, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {425, 541, 537, 230, 227, 418, 1233, 1232}

$$\frac{bx(-3a^2d^2-17abcd+5b^2c^2)}{12a^2c\sqrt{a-bx^4}\,(bc-ad)^3}+\frac{b^{3/4}\sqrt{1-\frac{bx^4}{a}}\,(-3a^2d^2-17abcd+5b^2c^2)\,F\left(\text{ArcSin}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{12a^{7/4}c\sqrt{a-bx^4}\,(bc-ad)^3}+\frac{\sqrt[4]{a}\,d^2\sqrt{1-\frac{bx^4}{a}}\,(13bc-3ad)\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}};\text{ArcSin}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{8\sqrt[4]{b}\,c^2\sqrt{a-bx^4}\,(bc-ad)^3}+\frac{\sqrt[4]{a}\,d^2\sqrt{1-\frac{bx^4}{a}}\,(13bc-3ad)\Pi\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}};\text{ArcSin}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{8\sqrt[4]{b}\,c^2\sqrt{a-bx^4}\,(bc-ad)^3}-\frac{dx}{4c\left(a-bx^4\right)^{3/2}(c-dx^4)(bc-ad)}+\frac{bx(3ad+2bc)}{12ac\left(a-bx^4\right)^{3/2}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - b*x^4)^(5/2)*(c - d*x^4)^2),x]
```

```
[Out] (b*(2*b*c + 3*a*d)*x)/(12*a*c*(b*c - a*d)^2*(a - b*x^4)^(3/2)) + (b*(5*b^2*
c^2 - 17*a*b*c*d - 3*a^2*d^2)*x)/(12*a^2*c*(b*c - a*d)^3*Sqrt[a - b*x^4]) -
 (d*x)/(4*c*(b*c - a*d)*(a - b*x^4)^(3/2)*(c - d*x^4)) + (b^(3/4)*(5*b^2*c^
2 - 17*a*b*c*d - 3*a^2*d^2)*Sqrt[1 - (b*x^4)/a]*EllipticF[ArcSin[(b^(1/4)*x
)/a^(1/4)], -1])/(12*a^(7/4)*c*(b*c - a*d)^3*Sqrt[a - b*x^4]) + (a^(1/4)*d^
2*(13*b*c - 3*a*d)*Sqrt[1 - (b*x^4)/a]*EllipticPi[-((Sqrt[a]*Sqrt[d])/(Sqrt
[b]*Sqrt[c])), ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(8*b^(1/4)*c^2*(b*c - a*d)
^3*Sqrt[a - b*x^4]) + (a^(1/4)*d^2*(13*b*c - 3*a*d)*Sqrt[1 - (b*x^4)/a]*Ell
ipticPi[(Sqrt[a]*Sqrt[d])/(Sqrt[b]*Sqrt[c]), ArcSin[(b^(1/4)*x)/a^(1/4)], -
1])/(8*b^(1/4)*c^2*(b*c - a*d)^3*Sqrt[a - b*x^4])
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Dist[1/(2*
c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 1233

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{\left(a-bx^4\right)^{5/2}\left(c-dx^4\right)^2}\,dx = -\frac{dx}{4c(bc-ad)\left(a-bx^4\right)^{3/2}\left(c-dx^4\right)} - \frac{\int \frac{-4bc+3ad-9bdx^4}{\left(a-bx^4\right)^{5/2}\left(c-dx^4\right)}\,dx}{4c(bc-ad)}$$

$$= \frac{b(2bc+3ad)x}{12ac(bc-ad)^2\left(a-bx^4\right)^{3/2}} - \frac{dx}{4c(bc-ad)\left(a-bx^4\right)^{3/2}\left(c-dx^4\right)} - \frac{\int \frac{-2(10b}{}}{}$$

$$= \frac{b(2bc+3ad)x}{12ac(bc-ad)^2\left(a-bx^4\right)^{3/2}} + \frac{b\left(5b^2c^2-17abcd-3a^2d^2\right)x}{12a^2c(bc-ad)^3\sqrt{a-bx^4}} - \frac{}{4c(bc-ad)\,(}$$

$$= \frac{b(2bc+3ad)x}{12ac(bc-ad)^2\left(a-bx^4\right)^{3/2}} + \frac{b\left(5b^2c^2-17abcd-3a^2d^2\right)x}{12a^2c(bc-ad)^3\sqrt{a-bx^4}} - \frac{}{4c(bc-ad)\,(}$$

$$= \frac{b(2bc+3ad)x}{12ac(bc-ad)^2\left(a-bx^4\right)^{3/2}} + \frac{b\left(5b^2c^2-17abcd-3a^2d^2\right)x}{12a^2c(bc-ad)^3\sqrt{a-bx^4}} - \frac{}{4c(bc-ad)\,(}$$

$$= \frac{b(2bc+3ad)x}{12ac(bc-ad)^2\left(a-bx^4\right)^{3/2}} + \frac{b\left(5b^2c^2-17abcd-3a^2d^2\right)x}{12a^2c(bc-ad)^3\sqrt{a-bx^4}} - \frac{}{4c(bc-ad)\,(}$$

$$= \frac{b(2bc+3ad)x}{12ac(bc-ad)^2\left(a-bx^4\right)^{3/2}} + \frac{b\left(5b^2c^2-17abcd-3a^2d^2\right)x}{12a^2c(bc-ad)^3\sqrt{a-bx^4}} - \frac{}{4c(bc-ad)\,(}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 10.62, size = 382, normalized size = 0.87

$$\frac{x\left(\frac{bd\left(-5b^2c^2+17abcd+3a^2d^2\right)x^4\sqrt{1-\frac{bx^4}{a}}\,F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)}{a^2c^2}+5\left(\frac{5b^3c}{a^2}-\frac{17b^2d}{a}-\frac{2b^2d}{a-bx^4}+\frac{2b^3c}{a^2-abx^4}-\frac{3ad^3}{c^2-cdx^4}+\frac{3bd^3x^4}{c^2-cdx^4}+\frac{5\left(5b^3c^3-17ab^2c^2d+36a^2bcd^2-9a^3d^3\right)F_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)}{a\left(c-dx^4\right)\left(5acF_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)+2x^4\left(2adF_1\left(\frac{5}{4};\frac{1}{2},2;\frac{9}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)+bcF_1\left(\frac{5}{4};\frac{3}{2},1;\frac{9}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)\right)\right)}\right)\right)}{60(bc-ad)^3\sqrt{a-bx^4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a - b*x^4)^(5/2)*(c - d*x^4)^2),x]
```

```
[Out] (x*((b*d*(-5*b^2*c^2 + 17*a*b*c*d + 3*a^2*d^2)*x^4*Sqrt[1 - (b*x^4)/a]*Appe
llF1[5/4, 1/2, 1, 9/4, (b*x^4)/a, (d*x^4)/c])/(a^2*c^2) + 5*((5*b^3*c)/a^2
- (17*b^2*d)/a - (2*b^2*d)/(a - b*x^4) + (2*b^3*c)/(a^2 - a*b*x^4) - (3*a*d
^3)/(c^2 - c*d*x^4) + (3*b*d^3*x^4)/(c^2 - c*d*x^4) + (5*(5*b^3*c^3 - 17*a*
b^2*c^2*d + 36*a^2*b*c*d^2 - 9*a^3*d^3)*AppellF1[1/4, 1/2, 1, 5/4, (b*x^4)/
a, (d*x^4)/c])/(a*(c - d*x^4)*(5*a*c*AppellF1[1/4, 1/2, 1, 5/4, (b*x^4)/a,
(d*x^4)/c] + 2*x^4*(2*a*d*AppellF1[5/4, 1/2, 2, 9/4, (b*x^4)/a, (d*x^4)/c]
+ b*c*AppellF1[5/4, 3/2, 1, 9/4, (b*x^4)/a, (d*x^4)/c]))))))/(60*(b*c - a*d
)^3*Sqrt[a - b*x^4])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.27, size = 484, normalized size = 1.10

method	result
default	$-\frac{b\,d^{3} x\sqrt{-b\,x^{4}+a}}{4(ad-bc)c(a^{2}d^{2}-2abcd+b^{2}c^{2})(bd\,x^{4}-bc)}+\frac{x\sqrt{-b\,x^{4}+a}}{6(ad-bc)^{2}a\left(x^{4}-\frac{a}{b}\right)^{2}}+\frac{b^{2}x(17ad-5bc)}{12a^{2}(ad-bc)^{3}\sqrt{-\left(x^{4}-\frac{a}{b}\right)b}}+\frac{\left(\frac{b\,d^{2}}{4(ad-bc)c\left(a^{2}d^{2}-\right.}\right.}{}$

elliptic	$-\frac{b\,d^{3} x \sqrt{-b\,x^{4}+a}}{4(ad-bc)c(a^{2}d^{2}-2abcd+b^{2}c^{2})(bd\,x^{4}-bc)}+\frac{x\sqrt{-b\,x^{4}+a}}{6(ad-bc)^{2}a\left(x^{4}-\frac{a}{b}\right)^{2}}+\frac{b^{2}x(17ad-5bc)}{12a^{2}(ad-bc)^{3}\sqrt{-\left(x^{4}-\frac{a}{b}\right)b}}+\frac{\left(\frac{b\,d}{4(ad-bc)c\left(a^{2}d^{2}\right.}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x^4+a)^(5/2)/(-d*x^4+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*b*d^3/(a*d-b*c)/c/(a^2*d^2-2*a*b*c*d+b^2*c^2)*x*(-b*x^4+a)^(1/2)/(b*d*
x^4-b*c)+1/6/(a*d-b*c)^2/a*x*(-b*x^4+a)^(1/2)/(x^4-a/b)^2+1/12*b^2/a^2*x*(1
7*a*d-5*b*c)/(a*d-b*c)^3/(-(x^4-a/b)*b)^(1/2)+(1/4*b*d^2/(a*d-b*c)/c/(a^2*d
^2-2*a*b*c*d+b^2*c^2)+1/12*b^2/a^2*(17*a*d-5*b*c)/(a*d-b*c)^3)/(1/a^(1/2)*b
^(1/2))^(1/2)*(1-x^2*b^(1/2)/a^(1/2))^(1/2)*(1+x^2*b^(1/2)/a^(1/2))^(1/2)/(
-b*x^4+a)^(1/2)*EllipticF(x*(1/a^(1/2)*b^(1/2))^(1/2),I)-1/32*d/c*sum((3*a*
d-13*b*c)/(a*d-b*c)^3/_alpha^3*(-1/((a*d-b*c)/d)^(1/2)*arctanh(1/2*(-2*_alp
ha^2*b*x^2+2*a)/((a*d-b*c)/d)^(1/2)/(-b*x^4+a)^(1/2))-2/(1/a^(1/2)*b^(1/2))
^(1/2)*_alpha^3*d/c*(1-x^2*b^(1/2)/a^(1/2))^(1/2)*(1+x^2*b^(1/2)/a^(1/2))^(
1/2)/(-b*x^4+a)^(1/2)*EllipticPi(x*(1/a^(1/2)*b^(1/2))^(1/2),a^(1/2)/b^(1/2
)*_alpha^2/c*d,(-1/a^(1/2)*b^(1/2))^(1/2)/(1/a^(1/2)*b^(1/2))^(1/2))),_alph
a=RootOf(_Z^4*d-c))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^4+a)^(5/2)/(-d*x^4+c)^2,x, algorithm="maxima")
```

```
[Out] integrate(1/((-b*x^4 + a)^(5/2)*(d*x^4 - c)^2), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^4+a)^(5/2)/(-d*x^4+c)^2,x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a-bx^4)^{\frac{5}{2}}(-c+dx^4)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x**4+a)**(5/2)/(-d*x**4+c)**2,x)
[Out] Integral(1/((a - b*x**4)**(5/2)*(-c + d*x**4)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^4+a)^(5/2)/(-d*x^4+c)^2,x, algorithm="giac")
[Out] integrate(1/((-b*x^4 + a)^(5/2)*(d*x^4 - c)^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(a-b\,x^4)^{5/2}\,(c-d\,x^4)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a - b*x^4)^(5/2)*(c - d*x^4)^2),x)
[Out] int(1/((a - b*x^4)^(5/2)*(c - d*x^4)^2), x)
```

3.191 $\int \frac{\sqrt{a+bx^4}}{ac-bcx^4}\,dx$

Optimal. Leaf size=103

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x}{\sqrt{a+bx^4}}\right)}{2\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,c}+\frac{\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x}{\sqrt{a+bx^4}}\right)}{2\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,c}$$

```
[Out] 1/4*arctan(a^(1/4)*b^(1/4)*x*2^(1/2)/(b*x^4+a)^(1/2))/a^(1/4)/b^(1/4)/c*2^(
1/2)+1/4*arctanh(a^(1/4)*b^(1/4)*x*2^(1/2)/(b*x^4+a)^(1/2))/a^(1/4)/b^(1/4)
/c*2^(1/2)
```

Rubi [A]

time = 0.04, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {413, 218, 212, 209}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x}{\sqrt{a+bx^4}}\right)}{2\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,c}+\frac{\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x}{\sqrt{a+bx^4}}\right)}{2\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,c}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^4]/(a*c - b*c*x^4),x]
```

```
[Out] ArcTan[(Sqrt[2]*a^(1/4)*b^(1/4)*x)/Sqrt[a + b*x^4]]/(2*Sqrt[2]*a^(1/4)*b^(1
/4)*c) + ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*x)/Sqrt[a + b*x^4]]/(2*Sqrt[2]*a^
(1/4)*b^(1/4)*c)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
```

```
, 0]
```

Rule 413

```
Int[Sqrt[(a_) + (b_.)*(x_)^4]/((c_) + (d_.)*(x_)^4), x_Symbol] :> Dist[a/c,
 Subst[Int[1/(1 - 4*a*b*x^4), x], x, x/Sqrt[a + b*x^4]], x] /; FreeQ[{a, b,
 c, d}, x] && EqQ[b*c + a*d, 0] && PosQ[a*b]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a+bx^4}}{ac-bcx^4}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{1-4abx^4}\,dx, x, \frac{x}{\sqrt{a+bx^4}}\right)}{c}\\ &= \frac{\text{Subst}\left(\int \frac{1}{1-2\sqrt{a}\sqrt{b}\,x^2}\,dx, x, \frac{x}{\sqrt{a+bx^4}}\right)}{2c} + \frac{\text{Subst}\left(\int \frac{1}{1+2\sqrt{a}\sqrt{b}\,x^2}\,dx, x, \frac{x}{\sqrt{a+bx^4}}\right)}{2c}\\ &= \frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x}{\sqrt{a+bx^4}}\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,c} + \frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x}{\sqrt{a+bx^4}}\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,c}\end{aligned}$$

Mathematica [A]
time = 0.31, size = 81, normalized size = 0.79

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x}{\sqrt{a+bx^4}}\right)+\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x}{\sqrt{a+bx^4}}\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,c}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^4]/(a*c - b*c*x^4),x]
```

```
[Out] (ArcTan[(Sqrt[2]*a^(1/4)*b^(1/4)*x)/Sqrt[a + b*x^4]] + ArcTanh[(Sqrt[2]*a^(
1/4)*b^(1/4)*x)/Sqrt[a + b*x^4]])/(2*Sqrt[2]*a^(1/4)*b^(1/4)*c)
```

Maple [A]
time = 0.29, size = 99, normalized size = 0.96

method	result	size

default	$\frac{\left(-\frac{\arctan\left(\frac{\sqrt{b\,x^4+a}\,\sqrt{2}}{2x(ab)^{\frac{1}{4}}}\right)}{2(ab)^{\frac{1}{4}}}+\frac{\ln\left(\frac{\frac{\sqrt{b\,x^4+a}\,\sqrt{2}}{2x}+(ab)^{\frac{1}{4}}}{\frac{\sqrt{b\,x^4+a}\,\sqrt{2}}{2x}-(ab)^{\frac{1}{4}}}\right)}{4(ab)^{\frac{1}{4}}}\right)\sqrt{2}}{2c}$	99
elliptic	$\frac{\left(-\frac{\arctan\left(\frac{\sqrt{b\,x^4+a}\,\sqrt{2}}{2x(ab)^{\frac{1}{4}}}\right)}{2c(ab)^{\frac{1}{4}}}+\frac{\ln\left(\frac{\frac{\sqrt{b\,x^4+a}\,\sqrt{2}}{2x}+(ab)^{\frac{1}{4}}}{\frac{\sqrt{b\,x^4+a}\,\sqrt{2}}{2x}-(ab)^{\frac{1}{4}}}\right)}{4c(ab)^{\frac{1}{4}}}\right)\sqrt{2}}{2}$	102

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)^(1/2)/(-b*c*x^4+a*c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/c*(-1/2/(a*b)^(1/4)*arctan(1/2*(b*x^4+a)^(1/2)*2^(1/2)/x/(a*b)^(1/4))+1
/4/(a*b)^(1/4)*ln((1/2*(b*x^4+a)^(1/2)*2^(1/2)/x+(a*b)^(1/4))/(1/2*(b*x^4+a
)^(1/2)*2^(1/2)/x-(a*b)^(1/4))))*2^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(1/2)/(-b*c*x^4+a*c),x, algorithm="maxima")
```

```
[Out] -integrate(sqrt(b*x^4 + a)/(b*c*x^4 - a*c), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 315 vs. 2(71) = 142.
time = 4.38, size = 315, normalized size = 3.06

$$-\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(\frac{1}{abc^4}\right)^{\frac{1}{4}}\arctan\left(\frac{\left(\frac{1}{4}\right)^{\frac{1}{4}}\sqrt{bx^4+a}\,c\left(\frac{1}{abc^4}\right)^{\frac{1}{4}}-\frac{2\left(\frac{1}{4}\right)^{\frac{3}{4}}abc^3\left(\frac{1}{abc^4}\right)^{\frac{3}{4}}+\left(\frac{1}{4}\right)^{\frac{1}{4}}bcx^2\left(\frac{1}{abc^4}\right)^{\frac{1}{4}}}{\sqrt{b}}}{x}\right)+\frac{1}{4}\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(\frac{1}{abc^4}\right)^{\frac{1}{4}}\log\left(\frac{4\left(\frac{1}{4}\right)^{\frac{3}{4}}abc^3x^3\left(\frac{1}{abc^4}\right)^{\frac{3}{4}}+2\left(\frac{1}{4}\right)^{\frac{1}{4}}acx\left(\frac{1}{abc^4}\right)^{\frac{1}{4}}+\sqrt{bx^4+a}\left(ac^2\sqrt{\frac{1}{abc^4}}+x^2\right)}{bx^4-a}\right)-\frac{1}{4}\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(\frac{1}{abc^4}\right)^{\frac{1}{4}}\log\left(-\frac{4\left(\frac{1}{4}\right)^{\frac{3}{4}}abc^3x^3\left(\frac{1}{abc^4}\right)^{\frac{3}{4}}+2\left(\frac{1}{4}\right)^{\frac{1}{4}}acx\left(\frac{1}{abc^4}\right)^{\frac{1}{4}}-\sqrt{bx^4+a}\left(ac^2\sqrt{\frac{1}{abc^4}}+x^2\right)}{bx^4-a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(1/2)/(-b*c*x^4+a*c),x, algorithm="fricas")
```

```
[Out] -(1/4)^(1/4)*(1/(a*b*c^4))^(1/4)*arctan(((1/4)^(1/4)*sqrt(b*x^4 + a)*c*(1/(
a*b*c^4))^(1/4) - (2*(1/4)^(3/4)*a*b*c^3*(1/(a*b*c^4))^(3/4) + (1/4)^(1/4)*
b*c*x^2*(1/(a*b*c^4))^(1/4))/sqrt(b))/x) + 1/4*(1/4)^(1/4)*(1/(a*b*c^4))^(1
/4)*log((4*(1/4)^(3/4)*a*b*c^3*x^3*(1/(a*b*c^4))^(3/4) + 2*(1/4)^(1/4)*a*c*
x*(1/(a*b*c^4))^(1/4) + sqrt(b*x^4 + a)*(a*c^2*sqrt(1/(a*b*c^4)) + x^2))/(b
```

```
*x^4 - a)) - 1/4*(1/4)^(1/4)*(1/(a*b*c^4))^(1/4)*log(-(4*(1/4)^(3/4)*a*b*c^
3*x^3*(1/(a*b*c^4))^(3/4) + 2*(1/4)^(1/4)*a*c*x*(1/(a*b*c^4))^(1/4) - sqrt(
b*x^4 + a)*(a*c^2*sqrt(1/(a*b*c^4)) + x^2))/(b*x^4 - a))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{\sqrt{a+bx^4}}{-a+bx^4}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)**(1/2)/(-b*c*x**4+a*c),x)
```

```
[Out] -Integral(sqrt(a + b*x**4)/(-a + b*x**4), x)/c
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(1/2)/(-b*c*x^4+a*c),x, algorithm="giac")
```

```
[Out] integrate(-sqrt(b*x^4 + a)/(b*c*x^4 - a*c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{b\,x^4+a}}{a\,c-b\,c\,x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)^(1/2)/(a*c - b*c*x^4),x)
```

```
[Out] int((a + b*x^4)^(1/2)/(a*c - b*c*x^4), x)
```

3.192 $\int \frac{\sqrt{a-bx^4}}{ac+bcx^4}\,dx$

Optimal. Leaf size=116

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}\,x\left(\sqrt{a}+\sqrt{b}\,x^2\right)}{\sqrt[4]{a}\,\sqrt{a-bx^4}}\right)}{2\sqrt[4]{a}\,\sqrt[4]{b}\,c}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x\left(\sqrt{a}-\sqrt{b}\,x^2\right)}{\sqrt[4]{a}\,\sqrt{a-bx^4}}\right)}{2\sqrt[4]{a}\,\sqrt[4]{b}\,c}$$

```
[Out] 1/2*arctan(b^(1/4)*x*(a^(1/2)+x^2*b^(1/2))/a^(1/4)/(-b*x^4+a)^(1/2))/a^(1/4
)/b^(1/4)/c+1/2*arctanh(b^(1/4)*x*(a^(1/2)-x^2*b^(1/2))/a^(1/4)/(-b*x^4+a)^
(1/2))/a^(1/4)/b^(1/4)/c
```

Rubi [A]

time = 0.02, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {414}

$$\frac{\text{ArcTan}\left(\frac{\sqrt[4]{b}\,x\left(\sqrt{a}+\sqrt{b}\,x^2\right)}{\sqrt[4]{a}\,\sqrt{a-bx^4}}\right)}{2\sqrt[4]{a}\,\sqrt[4]{b}\,c}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x\left(\sqrt{a}-\sqrt{b}\,x^2\right)}{\sqrt[4]{a}\,\sqrt{a-bx^4}}\right)}{2\sqrt[4]{a}\,\sqrt[4]{b}\,c}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a - b*x^4]/(a*c + b*c*x^4),x]
```

```
[Out] ArcTan[(b^(1/4)*x*(Sqrt[a] + Sqrt[b]*x^2))/(a^(1/4)*Sqrt[a - b*x^4])]/(2*a^
(1/4)*b^(1/4)*c) + ArcTanh[(b^(1/4)*x*(Sqrt[a] - Sqrt[b]*x^2))/(a^(1/4)*Sqr
t[a - b*x^4])]/(2*a^(1/4)*b^(1/4)*c)
```

Rule 414

```
Int[Sqrt[(a_) + (b_.)*(x_)^4]/((c_) + (d_.)*(x_)^4), x_Symbol] :> With[{q =
 Rt[(-a)*b, 4]}, Simp[(a/(2*c*q))*ArcTan[q*x*((a + q^2*x^2)/(a*Sqrt[a + b*x
^4]))], x] + Simp[(a/(2*c*q))*ArcTanh[q*x*((a - q^2*x^2)/(a*Sqrt[a + b*x^4]
))], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c + a*d, 0] && NegQ[a*b]
```

Rubi steps

$$\int \frac{\sqrt{a-bx^4}}{ac+bcx^4}\,dx=\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}\,x\left(\sqrt{a}+\sqrt{b}\,x^2\right)}{\sqrt[4]{a}\,\sqrt{a-bx^4}}\right)}{2\sqrt[4]{a}\,\sqrt[4]{b}\,c}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x\left(\sqrt{a}-\sqrt{b}\,x^2\right)}{\sqrt[4]{a}\,\sqrt{a-bx^4}}\right)}{2\sqrt[4]{a}\,\sqrt[4]{b}\,c}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.31, size = 88, normalized size = 0.76

$$\frac{\left(\frac{1}{4}-\frac{i}{4}\right)\left(\tan^{-1}\left(\frac{(1+i)\sqrt[4]{a}\sqrt[4]{b}\,x}{\sqrt{a-bx^4}}\right)-i\tan^{-1}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{a-bx^4}}{\sqrt[4]{a}\sqrt[4]{b}\,x}\right)\right)}{\sqrt[4]{a}\sqrt[4]{b}\,c}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a - b*x^4]/(a*c + b*c*x^4),x]
```

```
[Out] ((1/4 - I/4)*(ArcTan[((1 + I)*a^(1/4)*b^(1/4)*x)/Sqrt[a - b*x^4]] - I*ArcTa
n[((1/2 + I/2)*Sqrt[a - b*x^4])/(a^(1/4)*b^(1/4)*x)]))/(a^(1/4)*b^(1/4)*c)
```

Maple [A]
time = 0.27, size = 165, normalized size = 1.42

method	result
default	$\frac{\left(-\frac{\sqrt{2}\,\ln\left(\frac{\frac{-b\,x^4+a}{2x^2}-\frac{(ab)^{\frac{1}{4}}\sqrt{-b\,x^4+a}}{x}+\sqrt{ab}}{\frac{-b\,x^4+a}{2x^2}+\frac{(ab)^{\frac{1}{4}}\sqrt{-b\,x^4+a}}{x}+\sqrt{ab}}\right)}{8(ab)^{\frac{1}{4}}}-\frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{-b\,x^4+a}}{(ab)^{\frac{1}{4}}x}+1\right)}{4(ab)^{\frac{1}{4}}}-\frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{-b\,x^4+a}}{(ab)^{\frac{1}{4}}x}-1\right.}{4(ab)^{\frac{1}{4}}}}{2c}$
elliptic	$\frac{\left(-\frac{\sqrt{2}\,\ln\left(\frac{\frac{-b\,x^4+a}{2x^2}-\frac{(ab)^{\frac{1}{4}}\sqrt{-b\,x^4+a}}{x}+\sqrt{ab}}{\frac{-b\,x^4+a}{2x^2}+\frac{(ab)^{\frac{1}{4}}\sqrt{-b\,x^4+a}}{x}+\sqrt{ab}}\right)}{8c(ab)^{\frac{1}{4}}}-\frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{-b\,x^4+a}}{(ab)^{\frac{1}{4}}x}+1\right)}{4c(ab)^{\frac{1}{4}}}-\frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{-b\,x^4+a}}{(ab)^{\frac{1}{4}}x}-1\right.}{4c(ab)^{\frac{1}{4}}}}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^4+a)^(1/2)/(b*c*x^4+a*c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/c*(-1/8/(a*b)^(1/4)*2^(1/2)*ln((1/2*(-b*x^4+a)/x^2-(a*b)^(1/4)*(-b*x^4+
a)^(1/2)/x+(a*b)^(1/2))/(1/2*(-b*x^4+a)/x^2+(a*b)^(1/4)*(-b*x^4+a)^(1/2)/x+
(a*b)^(1/2)))-1/4/(a*b)^(1/4)*2^(1/2)*arctan(1/(a*b)^(1/4)*(-b*x^4+a)^(1/2)
/x+1)-1/4/(a*b)^(1/4)*2^(1/2)*arctan(1/(a*b)^(1/4)*(-b*x^4+a)^(1/2)/x-1))*2
^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^4+a)^(1/2)/(b*c*x^4+a*c),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-b*x^4 + a)/(b*c*x^4 + a*c), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 339 vs. $2(86) = 172$.
time = 4.66, size = 339, normalized size = 2.92

$$-\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(-\frac{1}{abc^4}\right)^{\frac{1}{4}}\arctan\left(\frac{2\left(\frac{1}{4}\right)^{\frac{3}{4}}abc^3\sqrt{-\frac{1}{b}}\left(-\frac{1}{abc^4}\right)^{\frac{3}{4}}+\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(bcx^2\sqrt{-\frac{1}{b}}+\sqrt{-bx^4+a}\,c\right)\left(-\frac{1}{abc^4}\right)^{\frac{1}{4}}}{x}\right)-\frac{1}{4}\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(-\frac{1}{abc^4}\right)^{\frac{1}{4}}\log\left(-\frac{4\left(\frac{1}{4}\right)^{\frac{3}{4}}abc^3x^3\left(-\frac{1}{abc^4}\right)^{\frac{3}{4}}+\sqrt{-bx^4+a}\,ac^2\sqrt{-\frac{1}{abc^4}}-2\left(\frac{1}{4}\right)^{\frac{1}{4}}acx\left(-\frac{1}{abc^4}\right)^{\frac{1}{4}}+\sqrt{-bx^4+a}\,x^2}{bx^4+a}\right)+\frac{1}{4}\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(-\frac{1}{abc^4}\right)^{\frac{1}{4}}\log\left(\frac{4\left(\frac{1}{4}\right)^{\frac{3}{4}}abc^3x^3\left(-\frac{1}{abc^4}\right)^{\frac{3}{4}}-\sqrt{-bx^4+a}\,ac^2\sqrt{-\frac{1}{abc^4}}-2\left(\frac{1}{4}\right)^{\frac{1}{4}}acx\left(-\frac{1}{abc^4}\right)^{\frac{1}{4}}-\sqrt{-bx^4+a}\,x^2}{bx^4+a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^4+a)^(1/2)/(b*c*x^4+a*c),x, algorithm="fricas")
```

```
[Out] -(1/4)^(1/4)*(-1/(a*b*c^4))^(1/4)*arctan((2*(1/4)^(3/4)*a*b*c^3*sqrt(-1/b)*
(-1/(a*b*c^4))^(3/4) + (1/4)^(1/4)*(b*c*x^2*sqrt(-1/b) + sqrt(-b*x^4 + a)*c
)*(-1/(a*b*c^4))^(1/4))/x) - 1/4*(1/4)^(1/4)*(-1/(a*b*c^4))^(1/4)*log(-(4*(
1/4)^(3/4)*a*b*c^3*x^3*(-1/(a*b*c^4))^(3/4) + sqrt(-b*x^4 + a)*a*c^2*sqrt(-
1/(a*b*c^4)) - 2*(1/4)^(1/4)*a*c*x*(-1/(a*b*c^4))^(1/4) + sqrt(-b*x^4 + a)*
x^2)/(b*x^4 + a)) + 1/4*(1/4)^(1/4)*(-1/(a*b*c^4))^(1/4)*log((4*(1/4)^(3/4)
*a*b*c^3*x^3*(-1/(a*b*c^4))^(3/4) - sqrt(-b*x^4 + a)*a*c^2*sqrt(-1/(a*b*c^4
)) - 2*(1/4)^(1/4)*a*c*x*(-1/(a*b*c^4))^(1/4) - sqrt(-b*x^4 + a)*x^2)/(b*x^
4 + a))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sqrt{a-bx^4}}{a+bx^4}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**4+a)**(1/2)/(b*c*x**4+a*c),x)
```

```
[Out] Integral(sqrt(a - b*x**4)/(a + b*x**4), x)/c
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^4+a)^(1/2)/(b*c*x^4+a*c),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-b*x^4 + a)/(b*c*x^4 + a*c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a-b\,x^4}}{b\,c\,x^4+a\,c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x^4)^(1/2)/(a*c + b*c*x^4),x)
```

```
[Out] int((a - b*x^4)^(1/2)/(a*c + b*c*x^4), x)
```

3.193 $\int \frac{(a+bx^4)^{7/4}}{c+dx^4}\,dx$

Optimal. Leaf size=211

$$\frac{bx(a+bx^4)^{3/4}}{4d}-\frac{b^{3/4}(4bc-7ad)\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right)}{8d^2}+\frac{(bc-ad)^{7/4}\tan^{-1}\left(\frac{\sqrt[4]{bc-ad}\,x}{\sqrt[4]{c}\,\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}d^2}-\frac{b^{3/4}(4bc-7a}{}$$

```
[Out] 1/4*b*x*(b*x^4+a)^(3/4)/d-1/8*b^(3/4)*(-7*a*d+4*b*c)*arctan(b^(1/4)*x/(b*x^
4+a)^(1/4))/d^2+1/2*(-a*d+b*c)^(7/4)*arctan((-a*d+b*c)^(1/4)*x/c^(1/4)/(b*x
^4+a)^(1/4))/c^(3/4)/d^2-1/8*b^(3/4)*(-7*a*d+4*b*c)*arctanh(b^(1/4)*x/(b*x^
4+a)^(1/4))/d^2+1/2*(-a*d+b*c)^(7/4)*arctanh((-a*d+b*c)^(1/4)*x/c^(1/4)/(b*
x^4+a)^(1/4))/c^(3/4)/d^2
```

Rubi [A]
time = 0.15, antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {427, 544, 246, 218, 212, 209, 385, 214, 211}

$$-\frac{b^{3/4}\text{ArcTan}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right)(4bc-7ad)}{8d^2}+\frac{(bc-ad)^{7/4}\text{ArcTan}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\,\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}d^2}-\frac{b^{3/4}(4bc-7ad)\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right)}{8d^2}+\frac{(bc-ad)^{7/4}\tanh^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\,\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}d^2}+\frac{bx(a+bx^4)^{3/4}}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)^(7/4)/(c + d*x^4),x]
```

```
[Out] (b*x*(a + b*x^4)^(3/4))/(4*d) - (b^(3/4)*(4*b*c - 7*a*d)*ArcTan[(b^(1/4)*x)
/(a + b*x^4)^(1/4)])/(8*d^2) + ((b*c - a*d)^(7/4)*ArcTan[((b*c - a*d)^(1/4)
*x)/(c^(1/4)*(a + b*x^4)^(1/4))])/(2*c^(3/4)*d^2) - (b^(3/4)*(4*b*c - 7*a*d
)*ArcTanh[(b^(1/4)*x)/(a + b*x^4)^(1/4)])/(8*d^2) + ((b*c - a*d)^(7/4)*ArcT
anh[((b*c - a*d)^(1/4)*x)/(c^(1/4)*(a + b*x^4)^(1/4))])/(2*c^(3/4)*d^2)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 544

```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Dist[f/d, Int[(a + b*x^n)^p, x], x] + Dist[(d*e -
c*f)/d, Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, p
, n}, x]
```

Rubi steps

$$\int \frac{(a+bx^4)^{7/4}}{c+dx^4}\,dx = \frac{bx(a+bx^4)^{3/4}}{4d} + \frac{\int \frac{-a(bc-4ad)-b(4bc-7ad)x^4}{\sqrt[4]{a+bx^4}\,(c+dx^4)}\,dx}{4d}$$

$$= \frac{bx(a+bx^4)^{3/4}}{4d} - \frac{(b(4bc-7ad))\int \frac{1}{\sqrt[4]{a+bx^4}}\,dx}{4d^2} + \frac{(bc-ad)^2\int \frac{1}{\sqrt[4]{a+bx^4}\,(c+dx^4)}\,dx}{d^2}$$

$$= \frac{bx(a+bx^4)^{3/4}}{4d} - \frac{(b(4bc-7ad))\text{Subst}\left(\int \frac{1}{1-bx^4}\,dx, x, \frac{x}{\sqrt[4]{a+bx^4}}\right)}{4d^2} + \frac{(bc-ad)^2\text{Subst}}{}$$

$$= \frac{bx(a+bx^4)^{3/4}}{4d} - \frac{(b(4bc-7ad))\text{Subst}\left(\int \frac{1}{1-\sqrt{b}\,x^2}\,dx, x, \frac{x}{\sqrt[4]{a+bx^4}}\right)}{8d^2} - \frac{(b(4bc-7ad)}{}$$

$$= \frac{bx(a+bx^4)^{3/4}}{4d} - \frac{b^{3/4}(4bc-7ad)\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right)}{8d^2} + \frac{(bc-ad)^{7/4}\tan^{-1}\left(\frac{\sqrt[4]{bc-}}{\sqrt[4]{c}\,\sqrt[4]{a-}}\right.}{2c^{3/4}d^2}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.02, size = 288, normalized size = 1.36

$$\frac{2bdx(a+bx^4)^{3/4} - b^{3/4}(4bc-7ad)\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right) + \frac{(2+2i)(bc-ad)^{7/4}\tan^{-1}\left(\frac{\frac{(1-i)\sqrt[4]{bc-ad}\,x^2}{\sqrt[4]{c}\,\sqrt[4]{a+bx^4}} - \frac{(1+i)\sqrt[4]{c}\,\sqrt[4]{a+bx^4}}{\sqrt[4]{bc-ad}}}{2x}\right)}{c^{3/4}} - b^{3/4}(4bc-7ad)\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right) + \frac{(2+2i)(bc-ad)^{7/4}\tanh^{-1}\left(\frac{\frac{(1-i)\sqrt[4]{bc-ad}\,x^2}{\sqrt[4]{c}\,\sqrt[4]{a+bx^4}} + \frac{(1+i)\sqrt[4]{c}\,\sqrt[4]{a+bx^4}}{\sqrt[4]{bc-ad}}}{2x}\right)}{c^{3/4}}}{8d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^4)^(7/4)/(c + d*x^4),x]
```

```
[Out] (2*b*d*x*(a + b*x^4)^(3/4) - b^(3/4)*(4*b*c - 7*a*d)*ArcTan[(b^(1/4)*x)/(a
+ b*x^4)^(1/4)] + ((2 + 2*I)*(b*c - a*d)^(7/4)*ArcTan[(((1 - I)*(b*c - a*d)
^(1/4)*x^2)/(c^(1/4)*(a + b*x^4)^(1/4)) - ((1 + I)*c^(1/4)*(a + b*x^4)^(1/4
))/(b*c - a*d)^(1/4))/(2*x)])/c^(3/4) - b^(3/4)*(4*b*c - 7*a*d)*ArcTanh[(b^
(1/4)*x)/(a + b*x^4)^(1/4)] + ((2 + 2*I)*(b*c - a*d)^(7/4)*ArcTanh[(((1 - I
)*(b*c - a*d)^(1/4)*x^2)/(c^(1/4)*(a + b*x^4)^(1/4)) + ((1 + I)*c^(1/4)*(a
+ b*x^4)^(1/4))/(b*c - a*d)^(1/4))/(2*x)])/c^(3/4))/(8*d^2)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(b\,x^4+a)^{\frac{7}{4}}}{d\,x^4+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)^(7/4)/(d*x^4+c),x)
```

```
[Out] int((b*x^4+a)^(7/4)/(d*x^4+c),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(7/4)/(d*x^4+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^4 + a)^(7/4)/(d*x^4 + c), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2381 vs. $2(167) = 334$.
time = 6.76, size = 2381, normalized size = 11.28

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(7/4)/(d*x^4+c),x, algorithm="fricas")
```

```
[Out] 1/16*(4*(b*x^4 + a)^(3/4)*b*x + 16*d*((b^7*c^7 - 7*a*b^6*c^6*d + 21*a^2*b^5
*c^5*d^2 - 35*a^3*b^4*c^4*d^3 + 35*a^4*b^3*c^3*d^4 - 21*a^5*b^2*c^2*d^5 + 7
*a^6*b*c*d^6 - a^7*d^7)/(c^3*d^8))^(1/4)*arctan(-(c*d^2*x*sqrt(((b^7*c^8*d^
4 - 7*a*b^6*c^7*d^5 + 21*a^2*b^5*c^6*d^6 - 35*a^3*b^4*c^5*d^7 + 35*a^4*b^3*
c^4*d^8 - 21*a^5*b^2*c^3*d^9 + 7*a^6*b*c^2*d^10 - a^7*c*d^11)*x^2*sqrt((b^7
*c^7 - 7*a*b^6*c^6*d + 21*a^2*b^5*c^5*d^2 - 35*a^3*b^4*c^4*d^3 + 35*a^4*b^3
*c^3*d^4 - 21*a^5*b^2*c^2*d^5 + 7*a^6*b*c*d^6 - a^7*d^7)/(c^3*d^8)) + (b^10
*c^10 - 10*a*b^9*c^9*d + 45*a^2*b^8*c^8*d^2 - 120*a^3*b^7*c^7*d^3 + 210*a^4
*b^6*c^6*d^4 - 252*a^5*b^5*c^5*d^5 + 210*a^6*b^4*c^4*d^6 - 120*a^7*b^3*c^3*
d^7 + 45*a^8*b^2*c^2*d^8 - 10*a^9*b*c*d^9 + a^10*d^10)*sqrt(b*x^4 + a))/x^2
)*((b^7*c^7 - 7*a*b^6*c^6*d + 21*a^2*b^5*c^5*d^2 - 35*a^3*b^4*c^4*d^3 + 35*
a^4*b^3*c^3*d^4 - 21*a^5*b^2*c^2*d^5 + 7*a^6*b*c*d^6 - a^7*d^7)/(c^3*d^8))^
(1/4) + (b^5*c^6*d^2 - 5*a*b^4*c^5*d^3 + 10*a^2*b^3*c^4*d^4 - 10*a^3*b^2*c^
3*d^5 + 5*a^4*b*c^2*d^6 - a^5*c*d^7)*(b*x^4 + a)^(1/4)*((b^7*c^7 - 7*a*b^6*
c^6*d + 21*a^2*b^5*c^5*d^2 - 35*a^3*b^4*c^4*d^3 + 35*a^4*b^3*c^3*d^4 - 21*a
^5*b^2*c^2*d^5 + 7*a^6*b*c*d^6 - a^7*d^7)/(c^3*d^8))^(1/4))/((b^7*c^7 - 7*a
*b^6*c^6*d + 21*a^2*b^5*c^5*d^2 - 35*a^3*b^4*c^4*d^3 + 35*a^4*b^3*c^3*d^4 -
 21*a^5*b^2*c^2*d^5 + 7*a^6*b*c*d^6 - a^7*d^7)*x)) + 4*d*((256*b^7*c^4 - 17
92*a*b^6*c^3*d + 4704*a^2*b^5*c^2*d^2 - 5488*a^3*b^4*c*d^3 + 2401*a^4*b^3*d
^4)/d^8)^(1/4)*arctan((d^2*x*sqrt(((256*b^7*c^4*d^4 - 1792*a*b^6*c^3*d^5 +
4704*a^2*b^5*c^2*d^6 - 5488*a^3*b^4*c*d^7 + 2401*a^4*b^3*d^8)*x^2*sqrt((256
*b^7*c^4 - 1792*a*b^6*c^3*d + 4704*a^2*b^5*c^2*d^2 - 5488*a^3*b^4*c*d^3 + 2
401*a^4*b^3*d^4)/d^8) + (4096*b^10*c^6 - 43008*a*b^9*c^5*d + 188160*a^2*b^8
*c^4*d^2 - 439040*a^3*b^7*c^3*d^3 + 576240*a^4*b^6*c^2*d^4 - 403368*a^5*b^5
*c*d^5 + 117649*a^6*b^4*d^6)*sqrt(b*x^4 + a))/x^2)*((256*b^7*c^4 - 1792*a*b
```

```
^6*c^3*d + 4704*a^2*b^5*c^2*d^2 - 5488*a^3*b^4*c*d^3 + 2401*a^4*b^3*d^4)/d^
8)^(1/4) + (64*b^5*c^3*d^2 - 336*a*b^4*c^2*d^3 + 588*a^2*b^3*c*d^4 - 343*a^
3*b^2*d^5)*(b*x^4 + a)^(1/4)*((256*b^7*c^4 - 1792*a*b^6*c^3*d + 4704*a^2*b^
5*c^2*d^2 - 5488*a^3*b^4*c*d^3 + 2401*a^4*b^3*d^4)/d^8)^(1/4))/((256*b^7*c^
4 - 1792*a*b^6*c^3*d + 4704*a^2*b^5*c^2*d^2 - 5488*a^3*b^4*c*d^3 + 2401*a^4
*b^3*d^4)*x)) + 4*d*((b^7*c^7 - 7*a*b^6*c^6*d + 21*a^2*b^5*c^5*d^2 - 35*a^3
*b^4*c^4*d^3 + 35*a^4*b^3*c^3*d^4 - 21*a^5*b^2*c^2*d^5 + 7*a^6*b*c*d^6 - a^
7*d^7)/(c^3*d^8))^(1/4)*log(-(c^2*d^6*x*((b^7*c^7 - 7*a*b^6*c^6*d + 21*a^2*
b^5*c^5*d^2 - 35*a^3*b^4*c^4*d^3 + 35*a^4*b^3*c^3*d^4 - 21*a^5*b^2*c^2*d^5
+ 7*a^6*b*c*d^6 - a^7*d^7)/(c^3*d^8))^(3/4) + (b^5*c^5 - 5*a*b^4*c^4*d + 10
*a^2*b^3*c^3*d^2 - 10*a^3*b^2*c^2*d^3 + 5*a^4*b*c*d^4 - a^5*d^5)*(b*x^4 + a
)^(1/4))/x) - 4*d*((b^7*c^7 - 7*a*b^6*c^6*d + 21*a^2*b^5*c^5*d^2 - 35*a^3*b
^4*c^4*d^3 + 35*a^4*b^3*c^3*d^4 - 21*a^5*b^2*c^2*d^5 + 7*a^6*b*c*d^6 - a^7*
d^7)/(c^3*d^8))^(1/4)*log((c^2*d^6*x*((b^7*c^7 - 7*a*b^6*c^6*d + 21*a^2*b^5
*c^5*d^2 - 35*a^3*b^4*c^4*d^3 + 35*a^4*b^3*c^3*d^4 - 21*a^5*b^2*c^2*d^5 + 7
*a^6*b*c*d^6 - a^7*d^7)/(c^3*d^8))^(3/4) - (b^5*c^5 - 5*a*b^4*c^4*d + 10*a^
2*b^3*c^3*d^2 - 10*a^3*b^2*c^2*d^3 + 5*a^4*b*c*d^4 - a^5*d^5)*(b*x^4 + a)^(
1/4))/x) - d*((256*b^7*c^4 - 1792*a*b^6*c^3*d + 4704*a^2*b^5*c^2*d^2 - 5488
*a^3*b^4*c*d^3 + 2401*a^4*b^3*d^4)/d^8)^(1/4)*log(-(d^6*x*((256*b^7*c^4 - 1
792*a*b^6*c^3*d + 4704*a^2*b^5*c^2*d^2 - 5488*a^3*b^4*c*d^3 + 2401*a^4*b^3*
d^4)/d^8)^(3/4) + (64*b^5*c^3 - 336*a*b^4*c^2*d + 588*a^2*b^3*c*d^2 - 343*a
^3*b^2*d^3)*(b*x^4 + a)^(1/4))/x) + d*((256*b^7*c^4 - 1792*a*b^6*c^3*d + 47
04*a^2*b^5*c^2*d^2 - 5488*a^3*b^4*c*d^3 + 2401*a^4*b^3*d^4)/d^8)^(1/4)*log(
(d^6*x*((256*b^7*c^4 - 1792*a*b^6*c^3*d + 4704*a^2*b^5*c^2*d^2 - 5488*a^3*b
^4*c*d^3 + 2401*a^4*b^3*d^4)/d^8)^(3/4) - (64*b^5*c^3 - 336*a*b^4*c^2*d + 5
88*a^2*b^3*c*d^2 - 343*a^3*b^2*d^3)*(b*x^4 + a)^(1/4))/x))/d
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx^4)^{\frac{7}{4}}}{c+dx^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)**(7/4)/(d*x**4+c),x)
```

```
[Out] Integral((a + b*x**4)**(7/4)/(c + d*x**4), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(7/4)/(d*x^4+c),x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)^(7/4)/(d*x^4 + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(b\,x^4+a\right)^{7/4}}{d\,x^4+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)^(7/4)/(c + d*x^4),x)
[Out] int((a + b*x^4)^(7/4)/(c + d*x^4), x)
```

3.194 $\int \frac{(a+bx^4)^{3/4}}{c+dx^4} dx$

Optimal. Leaf size=173

$$\frac{b^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right)}{2d}-\frac{(bc-ad)^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{bc-ad}\,x}{\sqrt[4]{c}\,\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}d}+\frac{b^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right)}{2d}-\frac{(bc-ad)^{3/4}\,\mathrm{t}}{}$$

```
[Out] 1/2*b^(3/4)*arctan(b^(1/4)*x/(b*x^4+a)^(1/4))/d-1/2*(-a*d+b*c)^(3/4)*arctan
((-a*d+b*c)^(1/4)*x/c^(1/4)/(b*x^4+a)^(1/4))/c^(3/4)/d+1/2*b^(3/4)*arctanh(
b^(1/4)*x/(b*x^4+a)^(1/4))/d-1/2*(-a*d+b*c)^(3/4)*arctanh((-a*d+b*c)^(1/4)*
x/c^(1/4)/(b*x^4+a)^(1/4))/c^(3/4)/d
```

Rubi [A]

time = 0.07, antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {399, 246, 218, 212, 209, 385, 214, 211}

$$\frac{b^{3/4}\text{ArcTan}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right)}{2d}-\frac{(bc-ad)^{3/4}\text{ArcTan}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\,\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}d}+\frac{b^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right)}{2d}-\frac{(bc-ad)^{3/4}\tanh^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\,\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)^(3/4)/(c + d*x^4),x]
```

```
[Out] (b^(3/4)*ArcTan[(b^(1/4)*x)/(a + b*x^4)^(1/4)])/(2*d) - ((b*c - a*d)^(3/4)*
ArcTan[((b*c - a*d)^(1/4)*x)/(c^(1/4)*(a + b*x^4)^(1/4))])/(2*c^(3/4)*d) +
(b^(3/4)*ArcTanh[(b^(1/4)*x)/(a + b*x^4)^(1/4)])/(2*d) - ((b*c - a*d)^(3/4)
*ArcTanh[((b*c - a*d)^(1/4)*x)/(c^(1/4)*(a + b*x^4)^(1/4))])/(2*c^(3/4)*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 399

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Di
st[b/d, Int[(a + b*x^n)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^
n)^(p - 1)/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*
d, 0] && EqQ[n*(p - 1) + 1, 0] && IntegerQ[n]
```

Rubi steps

$$\int \frac{(a+bx^4)^{3/4}}{c+dx^4}\,dx = \frac{b\int \frac{1}{\sqrt[4]{a+bx^4}}\,dx}{d} - \frac{(bc-ad)\int \frac{1}{\sqrt[4]{a+bx^4}\,(c+dx^4)}\,dx}{d}$$

$$= \frac{b\text{Subst}\left(\int \frac{1}{1-bx^4}\,dx, x, \frac{x}{\sqrt[4]{a+bx^4}}\right)}{d} - \frac{(bc-ad)\text{Subst}\left(\int \frac{1}{c-(bc-ad)x^4}\,dx, x, \frac{x}{\sqrt[4]{a+bx^4}}\right)}{d}$$

$$= \frac{b\text{Subst}\left(\int \frac{1}{1-\sqrt{b}\,x^2}\,dx, x, \frac{x}{\sqrt[4]{a+bx^4}}\right)}{2d} + \frac{b\text{Subst}\left(\int \frac{1}{1+\sqrt{b}\,x^2}\,dx, x, \frac{x}{\sqrt[4]{a+bx^4}}\right)}{2d} - \frac{(bc\ \ldots}{}$$

$$= \frac{b^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right)}{2d} - \frac{(bc-ad)^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{bc-ad}\,x}{\sqrt[4]{c}\,\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}d} + \frac{b^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{\ldots}}{\sqrt[4]{a\ \ldots}}\right.}{2d}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.74, size = 255, normalized size = 1.47

$$-\frac{-2b^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right)+\frac{(1+i)\left((-1+i)b^{3/4}c^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right)+(bc-ad)^{3/4}\tan^{-1}\left(\frac{\frac{(1-i)\sqrt[4]{bc-ad}\,x^2}{\sqrt[4]{c}\,\sqrt[4]{a+bx^4}}-\frac{(1+i)\sqrt[4]{c}\,\sqrt[4]{a+bx^4}}{\sqrt[4]{bc-ad}}}{2x}\right)+(bc-ad)^{3/4}\tanh^{-1}\left(\frac{\frac{(1-i)\sqrt[4]{bc-ad}\,x^2}{\sqrt[4]{c}\,\sqrt[4]{a+bx^4}}+\frac{(1+i)\sqrt[4]{c}\,\sqrt[4]{a+bx^4}}{\sqrt[4]{bc-ad}}}{2x}\right)\right)}{c^{3/4}}}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^4)^(3/4)/(c + d*x^4),x]
```

```
[Out] -1/4*(-2*b^(3/4)*ArcTanh[(b^(1/4)*x)/(a + b*x^4)^(1/4)] + ((1 + I)*((-1 + I
)*b^(3/4)*c^(3/4)*ArcTan[(b^(1/4)*x)/(a + b*x^4)^(1/4)] + (b*c - a*d)^(3/4)
*ArcTan[(((1 - I)*(b*c - a*d)^(1/4)*x^2)/(c^(1/4)*(a + b*x^4)^(1/4)) - ((1
+ I)*c^(1/4)*(a + b*x^4)^(1/4))/(b*c - a*d)^(1/4))/(2*x)] + (b*c - a*d)^(3/
4)*ArcTanh[(((1 - I)*(b*c - a*d)^(1/4)*x^2)/(c^(1/4)*(a + b*x^4)^(1/4)) + (
(1 + I)*c^(1/4)*(a + b*x^4)^(1/4))/(b*c - a*d)^(1/4))/(2*x)]))/c^(3/4))/d
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(b\,x^4+a)^{\frac{3}{4}}}{d\,x^4+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)^(3/4)/(d*x^4+c),x)
```

```
[Out] int((b*x^4+a)^(3/4)/(d*x^4+c),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(3/4)/(d*x^4+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^4 + a)^(3/4)/(d*x^4 + c), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 844 vs. $2(133) = 266$.
time = 3.59, size = 844, normalized size = 4.88

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(3/4)/(d*x^4+c),x, algorithm="fricas")
```

```
[Out] ((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)/(c^3*d^4))^(1/4)*arcta
n(-(c*d*x*sqrt(((b^3*c^4*d^2 - 3*a*b^2*c^3*d^3 + 3*a^2*b*c^2*d^4 - a^3*c*d^
5)*x^2*sqrt((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)/(c^3*d^4))
+ (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*s
qrt(b*x^4 + a))/x^2)*((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)/(
c^3*d^4))^(1/4) - (b^2*c^3*d - 2*a*b*c^2*d^2 + a^2*c*d^3)*(b*x^4 + a)^(1/4)
*((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)/(c^3*d^4))^(1/4))/((b
^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*x)) + (b^3/d^4)^(1/4)*arc
tan(-((b*x^4 + a)^(1/4)*b^2*d*(b^3/d^4)^(1/4) - d*x*(b^3/d^4)^(1/4)*sqrt((b
^3*d^2*x^2*sqrt(b^3/d^4) + sqrt(b*x^4 + a)*b^4)/x^2))/(b^3*x)) - 1/4*((b^3*
c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)/(c^3*d^4))^(1/4)*log((c^2*d^
3*x*((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)/(c^3*d^4))^(3/4) +
 (b*x^4 + a)^(1/4)*(b^2*c^2 - 2*a*b*c*d + a^2*d^2))/x) + 1/4*((b^3*c^3 - 3*
a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)/(c^3*d^4))^(1/4)*log(-(c^2*d^3*x*((b
^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)/(c^3*d^4))^(3/4) - (b*x^4
 + a)^(1/4)*(b^2*c^2 - 2*a*b*c*d + a^2*d^2))/x) + 1/4*(b^3/d^4)^(1/4)*log((
d^3*x*(b^3/d^4)^(3/4) + (b*x^4 + a)^(1/4)*b^2)/x) - 1/4*(b^3/d^4)^(1/4)*log
(-(d^3*x*(b^3/d^4)^(3/4) - (b*x^4 + a)^(1/4)*b^2)/x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + bx^4)^{\frac{3}{4}}}{c + dx^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)**(3/4)/(d*x**4+c),x)
```

```
[Out] Integral((a + b*x**4)**(3/4)/(c + d*x**4), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(3/4)/(d*x^4+c),x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)^(3/4)/(d*x^4 + c), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(b\,x^4+a\right)}^{3/4}}{d\,x^4+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)^(3/4)/(c + d*x^4),x)
```

```
[Out] int((a + b*x^4)^(3/4)/(c + d*x^4), x)
```

3.195 $\int \frac{1}{\sqrt[4]{a+bx^4}\,(c+dx^4)}\,dx$

Optimal. Leaf size=105

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{bc-ad}\,x}{\sqrt[4]{c}\,\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}\sqrt[4]{bc-ad}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{bc-ad}\,x}{\sqrt[4]{c}\,\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}\sqrt[4]{bc-ad}}$$

```
[Out] 1/2*arctan((-a*d+b*c)^(1/4)*x/c^(1/4)/(b*x^4+a)^(1/4))/c^(3/4)/(-a*d+b*c)^(
1/4)+1/2*arctanh((-a*d+b*c)^(1/4)*x/c^(1/4)/(b*x^4+a)^(1/4))/c^(3/4)/(-a*d+
b*c)^(1/4)
```

Rubi [A]
time = 0.04, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {385, 218, 214, 211}

$$\frac{\text{ArcTan}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\,\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}\sqrt[4]{bc-ad}}+\frac{\tanh^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\,\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}\sqrt[4]{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^4)^(1/4)*(c + d*x^4)),x]
```

```
[Out] ArcTan[((b*c - a*d)^(1/4)*x)/(c^(1/4)*(a + b*x^4)^(1/4))]/(2*c^(3/4)*(b*c -
 a*d)^(1/4)) + ArcTanh[((b*c - a*d)^(1/4)*x)/(c^(1/4)*(a + b*x^4)^(1/4))]/(
2*c^(3/4)*(b*c - a*d)^(1/4))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt[4]{a+bx^4}\,(c+dx^4)}\,dx &= \text{Subst}\left(\int \frac{1}{c-(bc-ad)x^4}\,dx, x, \frac{x}{\sqrt[4]{a+bx^4}}\right)\\ &= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{c}-\sqrt{bc-ad}\,x^2}\,dx, x, \frac{x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt{c}} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{c}+\sqrt{bc-ad}\,x^2}\right.}{2\sqrt{c}}\\ &= \frac{\tan^{-1}\left(\frac{\sqrt[4]{bc-ad}\,x}{\sqrt[4]{c}\,\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}\sqrt[4]{bc-ad}} + \frac{\tanh^{-1}\left(\frac{\sqrt[4]{bc-ad}\,x}{\sqrt[4]{c}\,\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}\sqrt[4]{bc-ad}}\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.59, size = 178, normalized size = 1.70

$$\frac{\left(\frac{1}{4}+\frac{i}{4}\right)\left(\tan^{-1}\left(\frac{\frac{(1-i)\sqrt[4]{bc-ad}\,x^2}{\sqrt[4]{c}\,\sqrt[4]{a+bx^4}}-\frac{(1+i)\sqrt[4]{c}\,\sqrt[4]{a+bx^4}}{\sqrt[4]{bc-ad}}}{2x}\right)+\tanh^{-1}\left(\frac{\frac{(1-i)\sqrt[4]{bc-ad}\,x^2}{\sqrt[4]{c}\,\sqrt[4]{a+bx^4}}+\frac{(1+i)\sqrt[4]{c}\,\sqrt[4]{a+bx^4}}{\sqrt[4]{bc-ad}}}{2x}\right)\right)}{c^{3/4}\sqrt[4]{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^4)^(1/4)*(c + d*x^4)),x]
```

```
[Out] ((1/4 + I/4)*(ArcTan[(((1 - I)*(b*c - a*d)^(1/4)*x^2)/(c^(1/4)*(a + b*x^4)^
(1/4)) - ((1 + I)*c^(1/4)*(a + b*x^4)^(1/4))/(b*c - a*d)^(1/4))/(2*x)] + Ar
cTanh[(((1 - I)*(b*c - a*d)^(1/4)*x^2)/(c^(1/4)*(a + b*x^4)^(1/4)) + ((1 +
I)*c^(1/4)*(a + b*x^4)^(1/4))/(b*c - a*d)^(1/4))/(2*x)]))/(c^(3/4)*(b*c - a
*d)^(1/4))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\,x^4+a)^{\frac{1}{4}}\,(d\,x^4+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^4+a)^(1/4)/(d*x^4+c),x)
```

```
[Out] int(1/(b*x^4+a)^(1/4)/(d*x^4+c),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(1/4)/(d*x^4+c),x, algorithm="maxima")
[Out] integrate(1/((b*x^4 + a)^(1/4)*(d*x^4 + c)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(1/4)/(d*x^4+c),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[4]{a + bx^4}\,(c + dx^4)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**4+a)**(1/4)/(d*x**4+c),x)
[Out] Integral(1/((a + b*x**4)**(1/4)*(c + d*x**4)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(1/4)/(d*x^4+c),x, algorithm="giac")
[Out] integrate(1/((b*x^4 + a)^(1/4)*(d*x^4 + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(b\,x^4+a\right)}^{1/4}\,\left(d\,x^4+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^4)^(1/4)*(c + d*x^4)),x)
[Out] int(1/((a + b*x^4)^(1/4)*(c + d*x^4)), x)
```

3.196 $\int \frac{1}{(a+bx^4)^{5/4}(c+dx^4)}\,dx$

Optimal. Leaf size=134

$$\frac{bx}{a(bc-ad)\sqrt[4]{a+bx^4}}-\frac{d\tan^{-1}\left(\frac{\sqrt[4]{bc-ad}\,x}{\sqrt[4]{c}\,\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}(bc-ad)^{5/4}}-\frac{d\tanh^{-1}\left(\frac{\sqrt[4]{bc-ad}\,x}{\sqrt[4]{c}\,\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}(bc-ad)^{5/4}}$$

```
[Out] b*x/a/(-a*d+b*c)/(b*x^4+a)^(1/4)-1/2*d*arctan((-a*d+b*c)^(1/4)*x/c^(1/4)/(b
*x^4+a)^(1/4))/c^(3/4)/(-a*d+b*c)^(5/4)-1/2*d*arctanh((-a*d+b*c)^(1/4)*x/c^
(1/4)/(b*x^4+a)^(1/4))/c^(3/4)/(-a*d+b*c)^(5/4)
```

Rubi [A]
time = 0.06, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {390, 385, 218, 214, 211}

$$-\frac{d\text{ArcTan}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\,\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}(bc-ad)^{5/4}}-\frac{d\tanh^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\,\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}(bc-ad)^{5/4}}+\frac{bx}{a\sqrt[4]{a+bx^4}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^4)^(5/4)*(c + d*x^4)),x]
```

```
[Out] (b*x)/(a*(b*c - a*d)*(a + b*x^4)^(1/4)) - (d*ArcTan[((b*c - a*d)^(1/4)*x)/(
c^(1/4)*(a + b*x^4)^(1/4))])/(2*c^(3/4)*(b*c - a*d)^(5/4)) - (d*ArcTanh[((b
*c - a*d)^(1/4)*x)/(c^(1/4)*(a + b*x^4)^(1/4))])/(2*c^(3/4)*(b*c - a*d)^(5/
4))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)),
 Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q},
 x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !L
tQ[q, -1]) && NeQ[p, -1]
```

Rubi steps

$$\int \frac{1}{(a+bx^4)^{5/4}(c+dx^4)}dx = \frac{bx}{a(bc-ad)\sqrt[4]{a+bx^4}} - \frac{d\int \frac{1}{\sqrt[4]{a+bx^4}(c+dx^4)}dx}{bc-ad}$$

$$= \frac{bx}{a(bc-ad)\sqrt[4]{a+bx^4}} - \frac{d\text{Subst}\left(\int \frac{1}{c-(bc-ad)x^4}dx, x, \frac{x}{\sqrt[4]{a+bx^4}}\right)}{bc-ad}$$

$$= \frac{bx}{a(bc-ad)\sqrt[4]{a+bx^4}} - \frac{d\text{Subst}\left(\int \frac{1}{\sqrt{c}-\sqrt{bc-ad}\,x^2}dx, x, \frac{x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt{c}\,(bc-ad)} - \frac{dS}{}$$

$$= \frac{bx}{a(bc-ad)\sqrt[4]{a+bx^4}} - \frac{d\tan^{-1}\left(\frac{\sqrt[4]{bc-ad}\,x}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}(bc-ad)^{5/4}} - \frac{d\tanh^{-1}\left(\frac{\sqrt[4]{bc-ad}\,x}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}(bc-ad)^{5/4}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.17, size = 229, normalized size = 1.71

$$\frac{1}{4}\left(\frac{4bx}{(abc-a^2d)\sqrt[4]{a+bx^4}} - \frac{(1+i)d\tan^{-1}\left(\frac{\frac{(1-i)\sqrt[4]{bc-ad}\,x^2}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}-\frac{(1+i)\sqrt[4]{c}\sqrt[4]{a+bx^4}}{\sqrt[4]{bc-ad}}}{2x}\right)}{c^{3/4}(bc-ad)^{5/4}} - \frac{(1+i)d\tanh^{-1}\left(\frac{\frac{(1-i)\sqrt[4]{bc-ad}\,x^2}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}+\frac{(1+i)\sqrt[4]{c}\sqrt[4]{a+bx^4}}{\sqrt[4]{bc-ad}}}{2x}\right)}{c^{3/4}(bc-ad)^{5/4}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^4)^(5/4)*(c + d*x^4)),x]
```

```
[Out] ((4*b*x)/((a*b*c - a^2*d)*(a + b*x^4)^(1/4)) - ((1 + I)*d*ArcTan[(((1 - I)*
(b*c - a*d)^(1/4)*x^2)/(c^(1/4)*(a + b*x^4)^(1/4)) - ((1 + I)*c^(1/4)*(a +
b*x^4)^(1/4))/(b*c - a*d)^(1/4))/(2*x)])/(c^(3/4)*(b*c - a*d)^(5/4)) - ((1
+ I)*d*ArcTanh[(((1 - I)*(b*c - a*d)^(1/4)*x^2)/(c^(1/4)*(a + b*x^4)^(1/4))
 + ((1 + I)*c^(1/4)*(a + b*x^4)^(1/4))/(b*c - a*d)^(1/4))/(2*x)])/(c^(3/4)*
(b*c - a*d)^(5/4)))/4
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^{4}+a\right)^{\frac{5}{4}} \left(d\,x^{4}+c\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^4+a)^(5/4)/(d*x^4+c),x)
```

```
[Out] int(1/(b*x^4+a)^(5/4)/(d*x^4+c),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(5/4)/(d*x^4+c),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^4 + a)^(5/4)*(d*x^4 + c)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(5/4)/(d*x^4+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + bx^{4}\right)^{\frac{5}{4}} \left(c + dx^{4}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**4+a)**(5/4)/(d*x**4+c),x)
```

```
[Out] Integral(1/((a + b*x**4)**(5/4)*(c + d*x**4)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(5/4)/(d*x^4+c),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^4 + a)^(5/4)*(d*x^4 + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(b\,x^4+a\right)}^{5/4}\,\left(d\,x^4+c\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^4)^(5/4)*(c + d*x^4)),x)
```

```
[Out] int(1/((a + b*x^4)^(5/4)*(c + d*x^4)), x)
```

3.197 $\int \frac{1}{(a+bx^4)^{9/4}(c+dx^4)}\,dx$

Optimal. Leaf size=180

$$\frac{bx}{5a(bc-ad)(a+bx^4)^{5/4}}+\frac{b(4bc-9ad)x}{5a^2(bc-ad)^2\sqrt[4]{a+bx^4}}+\frac{d^2\tan^{-1}\left(\frac{\sqrt[4]{bc-ad}\,x}{\sqrt[4]{c}\,\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}(bc-ad)^{9/4}}+\frac{d^2\tanh^{-1}\left(\frac{\sqrt[4]{bc-ad}\,x}{\sqrt[4]{c}\,\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}(bc-ad)^{9/4}}$$

```
[Out] 1/5*b*x/a/(-a*d+b*c)/(b*x^4+a)^(5/4)+1/5*b*(-9*a*d+4*b*c)*x/a^2/(-a*d+b*c)^
2/(b*x^4+a)^(1/4)+1/2*d^2*arctan((-a*d+b*c)^(1/4)*x/c^(1/4)/(b*x^4+a)^(1/4)
)/c^(3/4)/(-a*d+b*c)^(9/4)+1/2*d^2*arctanh((-a*d+b*c)^(1/4)*x/c^(1/4)/(b*x^
4+a)^(1/4))/c^(3/4)/(-a*d+b*c)^(9/4)
```

Rubi [A]
time = 0.13, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {425, 541, 12, 385, 218, 214, 211}

$$\frac{bx(4bc-9ad)}{5a^2\sqrt[4]{a+bx^4}\,(bc-ad)^2}+\frac{d^2\,\text{ArcTan}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\,\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}(bc-ad)^{9/4}}+\frac{d^2\tanh^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\,\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}(bc-ad)^{9/4}}+\frac{bx}{5a\left(a+bx^4\right)^{5/4}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^4)^(9/4)*(c + d*x^4)),x]
```

```
[Out] (b*x)/(5*a*(b*c - a*d)*(a + b*x^4)^(5/4)) + (b*(4*b*c - 9*a*d)*x)/(5*a^2*(b
*c - a*d)^2*(a + b*x^4)^(1/4)) + (d^2*ArcTan[((b*c - a*d)^(1/4)*x)/(c^(1/4)
*(a + b*x^4)^(1/4))])/(2*c^(3/4)*(b*c - a*d)^(9/4)) + (d^2*ArcTanh[((b*c -
a*d)^(1/4)*x)/(c^(1/4)*(a + b*x^4)^(1/4))])/(2*c^(3/4)*(b*c - a*d)^(9/4))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{1}{(a+bx^4)^{9/4}(c+dx^4)}\,dx = \frac{bx}{5a(bc-ad)(a+bx^4)^{5/4}} - \frac{\int \frac{-4bc+5ad-4bdx^4}{(a+bx^4)^{5/4}(c+dx^4)}\,dx}{5a(bc-ad)}$$

$$= \frac{bx}{5a(bc-ad)(a+bx^4)^{5/4}} + \frac{b(4bc-9ad)x}{5a^2(bc-ad)^2\sqrt[4]{a+bx^4}} + \frac{\int \frac{5a^2d^2}{\sqrt[4]{a+bx^4}\,(c+dx^4)}\,dx \cdots}{5a^2(bc-ad)^2}$$

$$= \frac{bx}{5a(bc-ad)(a+bx^4)^{5/4}} + \frac{b(4bc-9ad)x}{5a^2(bc-ad)^2\sqrt[4]{a+bx^4}} + \frac{d^2\int \frac{1}{\sqrt[4]{a+bx^4}\,(c+dx^4)} \cdots}{(bc-ad)^2}$$

$$= \frac{bx}{5a(bc-ad)(a+bx^4)^{5/4}} + \frac{b(4bc-9ad)x}{5a^2(bc-ad)^2\sqrt[4]{a+bx^4}} + \frac{d^2\text{Subst}\left(\int \frac{1}{c-(bc-ad)x^{4}} \cdots\right.}{(bc- \cdots}$$

$$= \frac{bx}{5a(bc-ad)(a+bx^4)^{5/4}} + \frac{b(4bc-9ad)x}{5a^2(bc-ad)^2\sqrt[4]{a+bx^4}} + \frac{d^2\text{Subst}\left(\int \frac{1}{\sqrt{c}-\sqrt{bc \cdots}} \cdots\right.}{2\sqrt{c} \cdots}$$

$$= \frac{bx}{5a(bc-ad)(a+bx^4)^{5/4}} + \frac{b(4bc-9ad)x}{5a^2(bc-ad)^2\sqrt[4]{a+bx^4}} + \frac{d^2\tan^{-1}\left(\frac{\sqrt[4]{bc-ad \cdots}}{\sqrt[4]{c}\sqrt[4]{a+b \cdots}}\right.}{2c^{3/4}(bc-ad)^{9/4} \cdots}$$

Mathematica [C] Result contains complex when optimal does not.
time = 2.62, size = 264, normalized size = 1.47

$$\left(\frac{1}{20}+\frac{i}{20}\right)\left(-\frac{(2-2i)bx(-5abc+10a^2d-4b^2cx^4+9abdx^4)}{a^2(bc-ad)^2(a+bx^4)^{5/4}} + \frac{5d^2\tan^{-1}\left(\frac{\frac{(1-i)\sqrt[4]{bc-ad}\,x^2}{\sqrt[4]{c}\sqrt[4]{a+bx^4}} - \frac{(1+i)\sqrt[4]{c}\sqrt[4]{a+bx^4}}{\sqrt[4]{bc-ad}}}{2x}\right)}{c^{3/4}(bc-ad)^{9/4}} + \frac{5d^2\tanh^{-1}\left(\frac{\frac{(1-i)\sqrt[4]{bc-ad}\,x^2}{\sqrt[4]{c}\sqrt[4]{a+bx^4}} + \frac{(1+i)\sqrt[4]{c}\sqrt[4]{a+bx^4}}{\sqrt[4]{bc-ad}}}{2x}\right)}{c^{3/4}(bc-ad)^{9/4}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^4)^(9/4)*(c + d*x^4)),x]
```

```
[Out] (1/20 + I/20)*(((-2 + 2*I)*b*x*(-5*a*b*c + 10*a^2*d - 4*b^2*c*x^4 + 9*a*b*d
*x^4))/(a^2*(b*c - a*d)^2*(a + b*x^4)^(5/4)) + (5*d^2*ArcTan[(((1 - I)*(b*c
 - a*d)^(1/4)*x^2)/(c^(1/4)*(a + b*x^4)^(1/4)) - ((1 + I)*c^(1/4)*(a + b*x^
4)^(1/4))/(b*c - a*d)^(1/4))/(2*x)])/(c^(3/4)*(b*c - a*d)^(9/4)) + (5*d^2*A
rcTanh[(((1 - I)*(b*c - a*d)^(1/4)*x^2)/(c^(1/4)*(a + b*x^4)^(1/4)) + ((1 +
 I)*c^(1/4)*(a + b*x^4)^(1/4))/(b*c - a*d)^(1/4))/(2*x)])/(c^(3/4)*(b*c - a
*d)^(9/4)))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\,x^4+a)^{\frac{9}{4}}\,(d\,x^4+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^4+a)^(9/4)/(d*x^4+c),x)
```

```
[Out] int(1/(b*x^4+a)^(9/4)/(d*x^4+c),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(9/4)/(d*x^4+c),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^4 + a)^(9/4)*(d*x^4 + c)), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(9/4)/(d*x^4+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + bx^{4}\right)^{\frac{9}{4}} \left(c + dx^{4}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**4+a)**(9/4)/(d*x**4+c),x)
```

```
[Out] Integral(1/((a + b*x**4)**(9/4)*(c + d*x**4)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(9/4)/(d*x^4+c),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^4 + a)^(9/4)*(d*x^4 + c)), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(b\,x^4+a\right)^{9/4}\,\left(d\,x^4+c\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^4)^(9/4)*(c + d*x^4)),x)
[Out] int(1/((a + b*x^4)^(9/4)*(c + d*x^4)), x)
```

3.198 $\int \frac{1}{\left(a+bx^4\right)^{13/4}\left(c+dx^4\right)}\,dx$

Optimal. Leaf size=233

$$\frac{bx}{9a(bc-ad)\left(a+bx^4\right)^{9/4}}+\frac{b(8bc-17ad)x}{45a^2(bc-ad)^2\left(a+bx^4\right)^{5/4}}+\frac{b\left(32b^2c^2-100abcd+113a^2d^2\right)x}{45a^3(bc-ad)^3\sqrt[4]{a+bx^4}}-\frac{d^3\tan^{-1}\left(\frac{\sqrt[4]{bc-}}{\sqrt[4]{c}\sqrt[4]{a}}\right.}{2c^{3/4}(bc-ad}$$

```
[Out] 1/9*b*x/a/(-a*d+b*c)/(b*x^4+a)^(9/4)+1/45*b*(-17*a*d+8*b*c)*x/a^2/(-a*d+b*c
)^2/(b*x^4+a)^(5/4)+1/45*b*(113*a^2*d^2-100*a*b*c*d+32*b^2*c^2)*x/a^3/(-a*d
+b*c)^3/(b*x^4+a)^(1/4)-1/2*d^3*arctan((-a*d+b*c)^(1/4)*x/c^(1/4)/(b*x^4+a)
^(1/4))/c^(3/4)/(-a*d+b*c)^(13/4)-1/2*d^3*arctanh((-a*d+b*c)^(1/4)*x/c^(1/4
)/(b*x^4+a)^(1/4))/c^(3/4)/(-a*d+b*c)^(13/4)
```

Rubi [A]
time = 0.19, antiderivative size = 233, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {425, 541, 12, 385, 218, 214, 211}

$$\frac{bx(8bc-17ad)}{45a^2\left(a+bx^4\right)^{5/4}(bc-ad)^2}+\frac{bx\left(113a^2d^2-100abcd+32b^2c^2\right)}{45a^3\sqrt[4]{a+bx^4}(bc-ad)^3}-\frac{d^3\text{ArcTan}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}(bc-ad)^{13/4}}-\frac{d^3\tanh^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}(bc-ad)^{13/4}}+\frac{bx}{9a\left(a+bx^4\right)^{9/4}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^4)^(13/4)*(c + d*x^4)),x]
```

```
[Out] (b*x)/(9*a*(b*c - a*d)*(a + b*x^4)^(9/4)) + (b*(8*b*c - 17*a*d)*x)/(45*a^2*
(b*c - a*d)^2*(a + b*x^4)^(5/4)) + (b*(32*b^2*c^2 - 100*a*b*c*d + 113*a^2*d
^2)*x)/(45*a^3*(b*c - a*d)^3*(a + b*x^4)^(1/4)) - (d^3*ArcTan[((b*c - a*d)^
(1/4)*x)/(c^(1/4)*(a + b*x^4)^(1/4))])/(2*c^(3/4)*(b*c - a*d)^(13/4)) - (d^
3*ArcTanh[((b*c - a*d)^(1/4)*x)/(c^(1/4)*(a + b*x^4)^(1/4))])/(2*c^(3/4)*(b
*c - a*d)^(13/4))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{1}{(a+bx^4)^{13/4}(c+dx^4)}\,dx = \frac{bx}{9a(bc-ad)(a+bx^4)^{9/4}} - \frac{\int \frac{-8bc+9ad-8bdx^4}{(a+bx^4)^{9/4}(c+dx^4)}\,dx}{9a(bc-ad)}$$

$$= \frac{bx}{9a(bc-ad)(a+bx^4)^{9/4}} + \frac{b(8bc-17ad)x}{45a^2(bc-ad)^2(a+bx^4)^{5/4}} + \frac{\int \frac{32b^2c^2-68abcd+45a^2d \cdots}{(a+bx^4)^{5/4} \cdots}}{45a^2(bc \cdots}$$

$$= \frac{bx}{9a(bc-ad)(a+bx^4)^{9/4}} + \frac{b(8bc-17ad)x}{45a^2(bc-ad)^2(a+bx^4)^{5/4}} + \frac{b(32b^2c^2-100abcd \cdots}{45a^3(bc-ad)^3 \cdots}$$

$$= \frac{bx}{9a(bc-ad)(a+bx^4)^{9/4}} + \frac{b(8bc-17ad)x}{45a^2(bc-ad)^2(a+bx^4)^{5/4}} + \frac{b(32b^2c^2-100abcd \cdots}{45a^3(bc-ad)^3 \cdots}$$

$$= \frac{bx}{9a(bc-ad)(a+bx^4)^{9/4}} + \frac{b(8bc-17ad)x}{45a^2(bc-ad)^2(a+bx^4)^{5/4}} + \frac{b(32b^2c^2-100abcd \cdots}{45a^3(bc-ad)^3 \cdots}$$

$$= \frac{bx}{9a(bc-ad)(a+bx^4)^{9/4}} + \frac{b(8bc-17ad)x}{45a^2(bc-ad)^2(a+bx^4)^{5/4}} + \frac{b(32b^2c^2-100abcd \cdots}{45a^3(bc-ad)^3 \cdots}$$

$$= \frac{bx}{9a(bc-ad)(a+bx^4)^{9/4}} + \frac{b(8bc-17ad)x}{45a^2(bc-ad)^2(a+bx^4)^{5/4}} + \frac{b(32b^2c^2-100abcd \cdots}{45a^3(bc-ad)^3 \cdots}$$

Mathematica [**C**] Result contains complex when optimal does not.
time = 3.80, size = 320, normalized size = 1.37

$$\left(\frac{1}{180}+\frac{i}{180}\right)\left(\frac{(2-2i)bx(135a^4d^2+32b^4c^2x^8+4ab^3cx^4(18c-25dx^4)+27a^3bd(-5c+9dx^4)+a^2b^2(45c^2-225cdx^4+113d^2x^8))}{a^3(bc-ad)^3(a+bx^4)^{9/4}} - \frac{45d^3\tan^{-1}\left(\frac{\frac{(1-i)\sqrt[4]{bc-ad}\,x^2}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}-\frac{(1+i)\sqrt[4]{c}\sqrt[4]{a+bx^4}}{\sqrt[4]{bc-ad}}}{2x}\right)}{c^{3/4}(bc-ad)^{13/4}} - \frac{45d^3\tanh^{-1}\left(\frac{\frac{(1-i)\sqrt[4]{bc-ad}\,x^2}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}+\frac{(1+i)\sqrt[4]{c}\sqrt[4]{a+bx^4}}{\sqrt[4]{bc-ad}}}{2x}\right)}{c^{3/4}(bc-ad)^{13/4}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^4)^(13/4)*(c + d*x^4)),x]
```

```
[Out] (1/180 + I/180)*(((2 - 2*I)*b*x*(135*a^4*d^2 + 32*b^4*c^2*x^8 + 4*a*b^3*c*x
^4*(18*c - 25*d*x^4) + 27*a^3*b*d*(-5*c + 9*d*x^4) + a^2*b^2*(45*c^2 - 225*
c*d*x^4 + 113*d^2*x^8)))/(a^3*(b*c - a*d)^3*(a + b*x^4)^(9/4)) - (45*d^3*Ar
cTan[(((1 - I)*(b*c - a*d)^(1/4)*x^2)/(c^(1/4)*(a + b*x^4)^(1/4)) - ((1 + I
)*c^(1/4)*(a + b*x^4)^(1/4))/(b*c - a*d)^(1/4))/(2*x)])/(c^(3/4)*(b*c - a*d
)^(13/4)) - (45*d^3*ArcTanh[(((1 - I)*(b*c - a*d)^(1/4)*x^2)/(c^(1/4)*(a +
b*x^4)^(1/4)) + ((1 + I)*c^(1/4)*(a + b*x^4)^(1/4))/(b*c - a*d)^(1/4))/(2*x
)])/(c^(3/4)*(b*c - a*d)^(13/4)))
```

Maple [**F**]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^{4}+a\right)^{\frac{13}{4}} \left(d\,x^{4}+c\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^4+a)^(13/4)/(d*x^4+c),x)
[Out] int(1/(b*x^4+a)^(13/4)/(d*x^4+c),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(13/4)/(d*x^4+c),x, algorithm="maxima")
[Out] integrate(1/((b*x^4 + a)^(13/4)*(d*x^4 + c)), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(13/4)/(d*x^4+c),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + bx^{4}\right)^{\frac{13}{4}} \left(c + dx^{4}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**4+a)**(13/4)/(d*x**4+c),x)
[Out] Integral(1/((a + b*x**4)**(13/4)*(c + d*x**4)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(13/4)/(d*x^4+c),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^4 + a)^(13/4)*(d*x^4 + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(b\,x^4+a\right)}^{13/4}\,\left(d\,x^4+c\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^4)^(13/4)*(c + d*x^4)),x)
```

```
[Out] int(1/((a + b*x^4)^(13/4)*(c + d*x^4)), x)
```

3.199 $\int \frac{(a+bx^4)^{9/4}}{c+dx^4} dx$

Optimal. Leaf size=316

$$-\frac{b(6bc-11ad)x\sqrt[4]{a+bx^4}}{12d^2}+\frac{bx(a+bx^4)^{5/4}}{6d}+\frac{\sqrt{a}\,b^{3/2}(6bc-11ad)\left(1+\frac{a}{bx^4}\right)^{3/4}x^3F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{12d^2\left(a+bx^4\right)^{3/4}}$$

```
[Out] -1/12*b*(-11*a*d+6*b*c)*x*(b*x^4+a)^(1/4)/d^2+1/6*b*x*(b*x^4+a)^(5/4)/d+1/1
2*b^(3/2)*(-11*a*d+6*b*c)*(1+a/b/x^4)^(3/4)*x^3*(cos(1/2*arccot(x^2*b^(1/2)
/a^(1/2)))^2)^(1/2)/cos(1/2*arccot(x^2*b^(1/2)/a^(1/2)))*EllipticF(sin(1/2*
arccot(x^2*b^(1/2)/a^(1/2))),2^(1/2))*a^(1/2)/d^2/(b*x^4+a)^(3/4)+1/2*(-a*d
+b*c)^2*EllipticPi(b^(1/4)*x/(b*x^4+a)^(1/4),-(-a*d+b*c)^(1/2)/b^(1/2)/c^(1
/2),I)*(a/(b*x^4+a))^(1/2)*(b*x^4+a)^(1/2)/b^(1/4)/c/d^2+1/2*(-a*d+b*c)^2*E
llipticPi(b^(1/4)*x/(b*x^4+a)^(1/4),(-a*d+b*c)^(1/2)/b^(1/2)/c^(1/2),I)*(a/
(b*x^4+a))^(1/2)*(b*x^4+a)^(1/2)/b^(1/4)/c/d^2
```

Rubi [A]
time = 0.21, antiderivative size = 316, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {427, 542, 543, 243, 342, 281, 237, 416, 418, 1232}

$$\frac{\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,(bc-ad)^2\Pi\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}};\text{ArcSin}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{bx^4+a}}\right)\middle|-1\right)}{2\sqrt[4]{b}\,cd^2}+\frac{\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,(bc-ad)^2\Pi\left(\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}};\text{ArcSin}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{bx^4+a}}\right)\middle|-1\right)}{2\sqrt[4]{b}\,cd^2}+\frac{\sqrt{a}\,b^{3/2}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}(6bc-11ad)F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{12d^2\left(a+bx^4\right)^{3/4}}-\frac{bx\sqrt[4]{a+bx^4}\,(6bc-11ad)}{12d^2}+\frac{bx(a+bx^4)^{5/4}}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)^(9/4)/(c + d*x^4),x]
```

```
[Out] -1/12*(b*(6*b*c - 11*a*d)*x*(a + b*x^4)^(1/4))/d^2 + (b*x*(a + b*x^4)^(5/4)
)/(6*d) + (Sqrt[a]*b^(3/2)*(6*b*c - 11*a*d)*(1 + a/(b*x^4))^(3/4)*x^3*Ellip
ticF[ArcCot[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/(12*d^2*(a + b*x^4)^(3/4)) + ((b*
c - a*d)^2*Sqrt[a/(a + b*x^4)]*Sqrt[a + b*x^4]*EllipticPi[-(Sqrt[b*c - a*d]
/(Sqrt[b]*Sqrt[c])), ArcSin[(b^(1/4)*x)/(a + b*x^4)^(1/4)], -1])/(2*b^(1/4)
*c*d^2) + ((b*c - a*d)^2*Sqrt[a/(a + b*x^4)]*Sqrt[a + b*x^4]*EllipticPi[Sqr
t[b*c - a*d]/(Sqrt[b]*Sqrt[c]), ArcSin[(b^(1/4)*x)/(a + b*x^4)^(1/4)], -1])
/(2*b^(1/4)*c*d^2)
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 243

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Dist[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4)), Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ[
```

```
{a, b}, x]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^4)^(1/4)/((c_) + (d_.)*(x_)^4), x_Symbol] :> Dist[Sq
rt[a + b*x^4]*Sqrt[a/(a + b*x^4)], Subst[Int[1/(Sqrt[1 - b*x^4]*(c - (b*c -
 a*d)*x^4)), x], x, x/(a + b*x^4)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] && N
eQ[b*c - a*d, 0]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Dist[1/(2*
c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 543

```
Int[((e_) + (f_.)*(x_)^4)/(((a_) + (b_.)*(x_)^4)^(3/4)*((c_) + (d_.)*(x_)^4
)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^4)^(3/4), x],
 x] - Dist[(d*e - c*f)/(b*c - a*d), Int[(a + b*x^4)^(1/4)/(c + d*x^4), x],
x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{(a+bx^4)^{9/4}}{c+dx^4}\,dx = \frac{bx(a+bx^4)^{5/4}}{6d} + \frac{\int \frac{\sqrt[4]{a+bx^4}\,(-a(bc-6ad)-b(6bc-11ad)x^4)}{c+dx^4}\,dx}{6d}$$

$$= -\frac{b(6bc-11ad)x\sqrt[4]{a+bx^4}}{12d^2} + \frac{bx(a+bx^4)^{5/4}}{6d} + \frac{\int \frac{a(6b^2c^2-13abcd+12a^2d^2)+b(12b^2c^2-30abcd+2}{(a+bx^4)^{3/4}(c+dx^4)}}{12d^2}$$

$$= -\frac{b(6bc-11ad)x\sqrt[4]{a+bx^4}}{12d^2} + \frac{bx(a+bx^4)^{5/4}}{6d} - \frac{(ab(6bc-11ad))\int \frac{1}{(a+bx^4)^{3/4}}\,dx}{12d^2} + (bc$$

$$= -\frac{b(6bc-11ad)x\sqrt[4]{a+bx^4}}{12d^2} + \frac{bx(a+bx^4)^{5/4}}{6d} - \frac{\left(ab(6bc-11ad)\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\right)\int \frac{}{(1}}{12d^2(a+bx^4)^{3/4}}$$

$$= -\frac{b(6bc-11ad)x\sqrt[4]{a+bx^4}}{12d^2} + \frac{bx(a+bx^4)^{5/4}}{6d} + \frac{\left(ab(6bc-11ad)\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\right)\text{Sub}}{12d^2(a+bx^4)}$$

$$= -\frac{b(6bc-11ad)x\sqrt[4]{a+bx^4}}{12d^2} + \frac{bx(a+bx^4)^{5/4}}{6d} + \frac{(bc-ad)^2\sqrt{\frac{a}{a+bx^4}}\,\sqrt{a+bx^4}\,\Pi\Big(-}{2\sqrt[4]{}}$$

$$= -\frac{b(6bc-11ad)x\sqrt[4]{a+bx^4}}{12d^2} + \frac{bx(a+bx^4)^{5/4}}{6d} + \frac{\sqrt{a}\,b^{3/2}(6bc-11ad)\left(1+\frac{a}{bx^4}\right)^{3/4}x^3F}{12d^2(a+bx^4)^{3}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

time = 10.47, size = 294, normalized size = 0.93

$$\frac{x\left(5b(a+bx^4)\left(-6bc+13ad+2bdx^4\right)+\frac{b(12b^2c^2-30abcd+23a^2d^2)x^4\left(1+\frac{bx^4}{a}\right)^{3/4}F_1\left(\frac{5}{4};\frac{3}{4},1;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{c}-\frac{25a^2c(6b^2c^2-13abcd+12a^2d^2)F_1\left(\frac{1}{4};\frac{3}{4},1;\frac{5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{(c+dx^4)\left(-5acF_1\left(\frac{1}{4};\frac{3}{4},1;\frac{5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)+x^4\left(4adF_1\left(\frac{5}{4};\frac{3}{4},2;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)+3bcF_1\left(\frac{5}{4};\frac{7}{4},1;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)\right)\right)}\right)}{60d^2\left(a+bx^4\right)^{3/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^4)^(9/4)/(c + d*x^4),x]
```

```
[Out] (x*(5*b*(a + b*x^4)*(-6*b*c + 13*a*d + 2*b*d*x^4) + (b*(12*b^2*c^2 - 30*a*b
*c*d + 23*a^2*d^2)*x^4*(1 + (b*x^4)/a)^(3/4)*AppellF1[5/4, 3/4, 1, 9/4, -((
b*x^4)/a), -((d*x^4)/c)])/c - (25*a^2*c*(6*b^2*c^2 - 13*a*b*c*d + 12*a^2*d^
2)*AppellF1[1/4, 3/4, 1, 5/4, -((b*x^4)/a), -((d*x^4)/c)])/((c + d*x^4)*(-5
*a*c*AppellF1[1/4, 3/4, 1, 5/4, -((b*x^4)/a), -((d*x^4)/c)] + x^4*(4*a*d*Ap
pellF1[5/4, 3/4, 2, 9/4, -((b*x^4)/a), -((d*x^4)/c)] + 3*b*c*AppellF1[5/4,
7/4, 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)])))))/(60*d^2*(a + b*x^4)^(3/4))
```

Maple [**F**]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^{4}+a\right)^{\frac{9}{4}}}{d\,x^{4}+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)^(9/4)/(d*x^4+c),x)
```

```
[Out] int((b*x^4+a)^(9/4)/(d*x^4+c),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(9/4)/(d*x^4+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^4 + a)^(9/4)/(d*x^4 + c), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(9/4)/(d*x^4+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx^4)^{\frac{9}{4}}}{c+dx^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)**(9/4)/(d*x**4+c),x)
[Out] Integral((a + b*x**4)**(9/4)/(c + d*x**4), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(9/4)/(d*x^4+c),x, algorithm="giac")
[Out] integrate((b*x^4 + a)^(9/4)/(d*x^4 + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(b\,x^4+a\right)}^{9/4}}{d\,x^4+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)^(9/4)/(c + d*x^4),x)
[Out] int((a + b*x^4)^(9/4)/(c + d*x^4), x)
```

3.200 $\int \frac{(a+bx^4)^{5/4}}{c+dx^4}\,dx$

Optimal. Leaf size=274

$$\frac{bx\sqrt[4]{a+bx^4}}{2d}-\frac{\sqrt{a}\,b^{3/2}\left(1+\frac{a}{bx^4}\right)^{3/4}x^3F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{2d\,(a+bx^4)^{3/4}}-\frac{(bc-ad)\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,\Pi\left(-\frac{\sqrt{bc-}}{\sqrt{b}\,\sqrt{}}\right.}{2\sqrt[4]{b}\,cd}$$

```
[Out] 1/2*b*x*(b*x^4+a)^(1/4)/d-1/2*b^(3/2)*(1+a/b/x^4)^(3/4)*x^3*(cos(1/2*arccot
(x^2*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arccot(x^2*b^(1/2)/a^(1/2)))*Ellipt
icF(sin(1/2*arccot(x^2*b^(1/2)/a^(1/2))),2^(1/2))*a^(1/2)/d/(b*x^4+a)^(3/4)
-1/2*(-a*d+b*c)*EllipticPi(b^(1/4)*x/(b*x^4+a)^(1/4),-(-a*d+b*c)^(1/2)/b^(1
/2)/c^(1/2),I)*(a/(b*x^4+a))^(1/2)*(b*x^4+a)^(1/2)/b^(1/4)/c/d-1/2*(-a*d+b*
c)*EllipticPi(b^(1/4)*x/(b*x^4+a)^(1/4),(-a*d+b*c)^(1/2)/b^(1/2)/c^(1/2),I)
*(a/(b*x^4+a))^(1/2)*(b*x^4+a)^(1/2)/b^(1/4)/c/d
```

Rubi [A]
time = 0.10, antiderivative size = 274, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {417, 201, 243, 342, 281, 237, 416, 418, 1232}

$$-\frac{\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,(bc-ad)\Pi\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}};\text{ArcSin}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{bx^4+a}}\right)\middle|-1\right)}{2\sqrt[4]{b}\,cd}-\frac{\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,(bc-ad)\Pi\left(\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}};\text{ArcSin}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{bx^4+a}}\right)\middle|-1\right)}{2\sqrt[4]{b}\,cd}-\frac{\sqrt{a}\,b^{3/2}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{2d\,(a+bx^4)^{3/4}}+\frac{bx\sqrt[4]{a+bx^4}}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)^(5/4)/(c + d*x^4),x]
```

```
[Out] (b*x*(a + b*x^4)^(1/4))/(2*d) - (Sqrt[a]*b^(3/2)*(1 + a/(b*x^4))^(3/4)*x^3*
EllipticF[ArcCot[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/(2*d*(a + b*x^4)^(3/4)) - ((
b*c - a*d)*Sqrt[a/(a + b*x^4)]*Sqrt[a + b*x^4]*EllipticPi[-(Sqrt[b*c - a*d]
/(Sqrt[b]*Sqrt[c])), ArcSin[(b^(1/4)*x)/(a + b*x^4)^(1/4)], -1])/(2*b^(1/4)
*c*d) - ((b*c - a*d)*Sqrt[a/(a + b*x^4)]*Sqrt[a + b*x^4]*EllipticPi[Sqrt[b*
c - a*d]/(Sqrt[b]*Sqrt[c]), ArcSin[(b^(1/4)*x)/(a + b*x^4)^(1/4)], -1])/(2*
b^(1/4)*c*d)
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
```

```
, 0] && PosQ[b/a]
```

Rule 243

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Dist[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4)), Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ[
{a, b}, x]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^4)^(1/4)/((c_) + (d_.)*(x_)^4), x_Symbol] :> Dist[Sq
rt[a + b*x^4]*Sqrt[a/(a + b*x^4)], Subst[Int[1/(Sqrt[1 - b*x^4]*(c - (b*c -
 a*d)*x^4)), x], x, x/(a + b*x^4)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] && N
eQ[b*c - a*d, 0]
```

Rule 417

```
Int[((a_) + (b_.)*(x_)^4)^(5/4)/((c_) + (d_.)*(x_)^4), x_Symbol] :> Dist[b/
d, Int[(a + b*x^4)^(1/4), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^4)^(1/4
)/(c + d*x^4), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Dist[1/(2*
c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{\left(a+bx^4\right)^{5/4}}{c+dx^4}\,dx = \frac{b\int \sqrt[4]{a+bx^4}\,dx}{d} - \frac{(bc-ad)\int \frac{\sqrt[4]{a+bx^4}}{c+dx^4}\,dx}{d}$$

$$= \frac{bx\sqrt[4]{a+bx^4}}{2d} + \frac{(ab)\int \frac{1}{(a+bx^4)^{3/4}}\,dx}{2d} - \frac{\left((bc-ad)\sqrt{\frac{a}{a+bx^4}}\,\sqrt{a+bx^4}\right)\operatorname{Subst}\left(\int \frac{}{\sqrt{1-}}\right.}{d}$$

$$= \frac{bx\sqrt[4]{a+bx^4}}{2d} + \frac{\left(ab\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\right)\int \frac{1}{\left(1+\frac{a}{bx^4}\right)^{3/4}x^3}\,dx}{2d\left(a+bx^4\right)^{3/4}} - \frac{\left((bc-ad)\sqrt{\frac{a}{a+bx^4}}\,\sqrt{a+bx\right.}{}$$

$$= \frac{bx\sqrt[4]{a+bx^4}}{2d} - \frac{(bc-ad)\sqrt{\frac{a}{a+bx^4}}\,\sqrt{a+bx^4}\,\Pi\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\,\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right)\middle|\, -\right.}{2\sqrt[4]{b}\,cd}$$

$$= \frac{bx\sqrt[4]{a+bx^4}}{2d} - \frac{(bc-ad)\sqrt{\frac{a}{a+bx^4}}\,\sqrt{a+bx^4}\,\Pi\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\,\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right)\middle|\, -\right.}{2\sqrt[4]{b}\,cd}$$

$$= \frac{bx\sqrt[4]{a+bx^4}}{2d} - \frac{\sqrt{a}\,b^{3/2}\left(1+\frac{a}{bx^4}\right)^{3/4}x^3F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|\,2\right)}{2d\left(a+bx^4\right)^{3/4}} - \frac{(bc-ad)\sqrt{\frac{a}{a+bx^4}}}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 10.24, size = 346, normalized size = 1.26

$$\frac{x\left(\frac{b(-2bc+3ad)x^4\left(1+\frac{bx^4}{a}\right)^{3/4}F_1\left(\frac{5}{4};\frac{3}{4},1;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{c}+\frac{5\left(-5ac\left(2a^2d+abdx^4+b^2x^4\left(c+dx^4\right)\right)F_1\left(\frac{1}{4};\frac{3}{4},1;\frac{5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)+bx^4\left(a+bx^4\right)\left(c+dx^4\right)\left(4adF_1\left(\frac{5}{4};\frac{3}{4},2;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)+3bcF_1\left(\frac{5}{4};\frac{7}{4},1;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)\right)\right)}{\left(c+dx^4\right)\left(-5acF_1\left(\frac{1}{4};\frac{3}{4},1;\frac{5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)+x^4\left(4adF_1\left(\frac{5}{4};\frac{3}{4},2;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)+3bcF_1\left(\frac{5}{4};\frac{7}{4},1;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)\right)\right)}\right)}{10d\left(a+bx^4\right)^{3/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^4)^(5/4)/(c + d*x^4),x]
```

```
[Out] (x*((b*(-2*b*c + 3*a*d)*x^4*(1 + (b*x^4)/a)^(3/4)*AppellF1[5/4, 3/4, 1, 9/4
, -((b*x^4)/a), -((d*x^4)/c)])/c + (5*(-5*a*c*(2*a^2*d + a*b*d*x^4 + b^2*x^
4*(c + d*x^4))*AppellF1[1/4, 3/4, 1, 5/4, -((b*x^4)/a), -((d*x^4)/c)] + b*x
^4*(a + b*x^4)*(c + d*x^4)*(4*a*d*AppellF1[5/4, 3/4, 2, 9/4, -((b*x^4)/a),
-((d*x^4)/c)] + 3*b*c*AppellF1[5/4, 7/4, 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)
])))/((c + d*x^4)*(-5*a*c*AppellF1[1/4, 3/4, 1, 5/4, -((b*x^4)/a), -((d*x^4
)/c)] + x^4*(4*a*d*AppellF1[5/4, 3/4, 2, 9/4, -((b*x^4)/a), -((d*x^4)/c)] +
```

```
3*b*c*AppellF1[5/4, 7/4, 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)])))))/(10*d*(a
+ b*x^4)^(3/4))
```

Maple [**F**]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^{4}+a\right)^{\frac{5}{4}}}{d\,x^{4}+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)^(5/4)/(d*x^4+c),x)
[Out] int((b*x^4+a)^(5/4)/(d*x^4+c),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(5/4)/(d*x^4+c),x, algorithm="maxima")
[Out] integrate((b*x^4 + a)^(5/4)/(d*x^4 + c), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(5/4)/(d*x^4+c),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + bx^{4}\right)^{\frac{5}{4}}}{c + dx^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)**(5/4)/(d*x**4+c),x)
[Out] Integral((a + b*x**4)**(5/4)/(c + d*x**4), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(5/4)/(d*x^4+c),x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)^(5/4)/(d*x^4 + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(b\,x^4+a)^{5/4}}{d\,x^4+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)^(5/4)/(c + d*x^4),x)
```

```
[Out] int((a + b*x^4)^(5/4)/(c + d*x^4), x)
```

3.201 $\int \frac{\sqrt[4]{a+bx^4}}{c+dx^4}\,dx$

Optimal. Leaf size=166

$$\frac{\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,\Pi\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right)\middle|-1\right)}{2\sqrt[4]{b}\,c}+\frac{\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,\Pi\left(\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}};\text{si}\right.}{2\sqrt[4]{b}\,c}$$

```
[Out] 1/2*EllipticPi(b^(1/4)*x/(b*x^4+a)^(1/4),-(-a*d+b*c)^(1/2)/b^(1/2)/c^(1/2),
I)*(a/(b*x^4+a))^(1/2)*(b*x^4+a)^(1/2)/b^(1/4)/c+1/2*EllipticPi(b^(1/4)*x/(
b*x^4+a)^(1/4),(-a*d+b*c)^(1/2)/b^(1/2)/c^(1/2),I)*(a/(b*x^4+a))^(1/2)*(b*x
^4+a)^(1/2)/b^(1/4)/c
```

Rubi [A]

time = 0.06, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {416, 418, 1232}

$$\frac{\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,\Pi\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}};\text{ArcSin}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{bx^4+a}}\right)\middle|-1\right)}{2\sqrt[4]{b}\,c}+\frac{\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,\Pi\left(\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}};\text{ArcSin}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{bx^4+a}}\right)\middle|-1\right)}{2\sqrt[4]{b}\,c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)^(1/4)/(c + d*x^4),x]
```

```
[Out] (Sqrt[a/(a + b*x^4)]*Sqrt[a + b*x^4]*EllipticPi[-(Sqrt[b*c - a*d]/(Sqrt[b]*
Sqrt[c])), ArcSin[(b^(1/4)*x)/(a + b*x^4)^(1/4)], -1])/(2*b^(1/4)*c) + (Sqr
t[a/(a + b*x^4)]*Sqrt[a + b*x^4]*EllipticPi[Sqrt[b*c - a*d]/(Sqrt[b]*Sqrt[c
]), ArcSin[(b^(1/4)*x)/(a + b*x^4)^(1/4)], -1])/(2*b^(1/4)*c)
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^4)^(1/4)/((c_) + (d_.)*(x_)^4), x_Symbol] :> Dist[Sq
rt[a + b*x^4]*Sqrt[a/(a + b*x^4)], Subst[Int[1/(Sqrt[1 - b*x^4]*(c - (b*c -
 a*d)*x^4)), x], x, x/(a + b*x^4)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] && N
eQ[b*c - a*d, 0]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Dist[1/(2*
c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sqrt[4]{a+bx^4}}{c+dx^4}\,dx = \left(\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{1-bx^4}\,(c-(bc-ad)x^4)}\,dx, x, \frac{x}{\sqrt[4]{a+bx^4}}\right)$$

$$= \frac{\left(\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\right)\operatorname{Subst}\left(\int \frac{1}{\left(1-\frac{\sqrt{bc-ad}\,x^2}{\sqrt{c}}\right)\sqrt{1-bx^4}}\,dx, x, \frac{x}{\sqrt[4]{a+bx^4}}\right)}{2c} + \ldots$$

$$= \frac{\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,\Pi\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right)\middle|-1\right)}{2\sqrt[4]{b}\,c} + \frac{\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\ldots}{\ldots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 10.13, size = 160, normalized size = 0.96

$$\frac{5acx\sqrt[4]{a+bx^4}\,F_1\left(\frac{1}{4};-\frac{1}{4},1;\frac{5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{(c+dx^4)\left(5acF_1\left(\frac{1}{4};-\frac{1}{4},1;\frac{5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)+x^4\left(-4adF_1\left(\frac{5}{4};-\frac{1}{4},2;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)+bcF_1\left(\frac{5}{4};\frac{3}{4},1;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^4)^(1/4)/(c + d*x^4),x]
[Out] (5*a*c*x*(a + b*x^4)^(1/4)*AppellF1[1/4, -1/4, 1, 5/4, -((b*x^4)/a), -((d*x
^4)/c)])/((c + d*x^4)*(5*a*c*AppellF1[1/4, -1/4, 1, 5/4, -((b*x^4)/a), -((d
*x^4)/c)] + x^4*(-4*a*d*AppellF1[5/4, -1/4, 2, 9/4, -((b*x^4)/a), -((d*x^4)
/c)] + b*c*AppellF1[5/4, 3/4, 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)])))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(b\,x^4+a)^{\frac{1}{4}}}{d\,x^4+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)^(1/4)/(d*x^4+c),x)
[Out] int((b*x^4+a)^(1/4)/(d*x^4+c),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(1/4)/(d*x^4+c),x, algorithm="maxima")
[Out] integrate((b*x^4 + a)^(1/4)/(d*x^4 + c), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(1/4)/(d*x^4+c),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[4]{a + bx^4}}{c + dx^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)**(1/4)/(d*x**4+c),x)
[Out] Integral((a + b*x**4)**(1/4)/(c + d*x**4), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(1/4)/(d*x^4+c),x, algorithm="giac")
[Out] integrate((b*x^4 + a)^(1/4)/(d*x^4 + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(b\,x^4+a\right)}^{1/4}}{d\,x^4+c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)^(1/4)/(c + d*x^4),x)
[Out] int((a + b*x^4)^(1/4)/(c + d*x^4), x)
```

3.202 $\int \frac{1}{\left(a+bx^4\right)^{3/4}\left(c+dx^4\right)}\,dx$

Optimal. Leaf size=259

$$-\frac{b^{3/2}\left(1+\frac{a}{bx^4}\right)^{3/4}x^3F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}\,(bc-ad)\left(a+bx^4\right)^{3/4}}-\frac{d\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,\Pi\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right)\right.}{2\sqrt[4]{b}\,c(bc-ad)}$$

```
[Out] -b^(3/2)*(1+a/b/x^4)^(3/4)*x^3*(cos(1/2*arccot(x^2*b^(1/2)/a^(1/2)))^2)^(1/
2)/cos(1/2*arccot(x^2*b^(1/2)/a^(1/2)))*EllipticF(sin(1/2*arccot(x^2*b^(1/2
)/a^(1/2))),2^(1/2))/(-a*d+b*c)/(b*x^4+a)^(3/4)/a^(1/2)-1/2*d*EllipticPi(b^
(1/4)*x/(b*x^4+a)^(1/4),-(-a*d+b*c)^(1/2)/b^(1/2)/c^(1/2),I)*(a/(b*x^4+a))^
(1/2)*(b*x^4+a)^(1/2)/b^(1/4)/c/(-a*d+b*c)-1/2*d*EllipticPi(b^(1/4)*x/(b*x^
4+a)^(1/4),(-a*d+b*c)^(1/2)/b^(1/2)/c^(1/2),I)*(a/(b*x^4+a))^(1/2)*(b*x^4+a
)^(1/2)/b^(1/4)/c/(-a*d+b*c)
```

Rubi [A]

time = 0.10, antiderivative size = 259, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {419, 243, 342, 281, 237, 416, 418, 1232}

$$-\frac{d\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,\Pi\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}};\text{ArcSin}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{bx^4+a}}\right)\middle|-1\right)}{2\sqrt[4]{b}\,c(bc-ad)}-\frac{d\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,\Pi\left(\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}};\text{ArcSin}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{bx^4+a}}\right)\middle|-1\right)}{2\sqrt[4]{b}\,c(bc-ad)}-\frac{b^{3/2}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}\,\left(a+bx^4\right)^{3/4}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^4)^(3/4)*(c + d*x^4)),x]
```

```
[Out] -((b^(3/2)*(1 + a/(b*x^4))^(3/4)*x^3*EllipticF[ArcCot[(Sqrt[b]*x^2)/Sqrt[a]
]/2, 2])/(Sqrt[a]*(b*c - a*d)*(a + b*x^4)^(3/4))) - (d*Sqrt[a/(a + b*x^4)]*
Sqrt[a + b*x^4]*EllipticPi[-(Sqrt[b*c - a*d]/(Sqrt[b]*Sqrt[c])), ArcSin[(b^
(1/4)*x)/(a + b*x^4)^(1/4)], -1])/(2*b^(1/4)*c*(b*c - a*d)) - (d*Sqrt[a/(a
+ b*x^4)]*Sqrt[a + b*x^4]*EllipticPi[Sqrt[b*c - a*d]/(Sqrt[b]*Sqrt[c]), Arc
Sin[(b^(1/4)*x)/(a + b*x^4)^(1/4)], -1])/(2*b^(1/4)*c*(b*c - a*d))
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 243

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Dist[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4)), Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ[
{a, b}, x]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^4)^(1/4)/((c_) + (d_.)*(x_)^4), x_Symbol] :> Dist[Sq
rt[a + b*x^4]*Sqrt[a/(a + b*x^4)], Subst[Int[1/(Sqrt[1 - b*x^4]*(c - (b*c -
 a*d)*x^4)), x], x, x/(a + b*x^4)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] && N
eQ[b*c - a*d, 0]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Dist[1/(2*
c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 419

```
Int[1/(((a_) + (b_.)*(x_)^4)^(3/4)*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^4)^(3/4), x], x] - Dist[d/(b*c - a*d), Int[
(a + b*x^4)^(1/4)/(c + d*x^4), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{(a+bx^4)^{3/4}(c+dx^4)}\,dx = \frac{b\int \frac{1}{(a+bx^4)^{3/4}}\,dx}{bc-ad} - \frac{d\int \frac{\sqrt[4]{a+bx^4}}{c+dx^4}\,dx}{bc-ad}$$

$$= \frac{\left(b\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\right)\int \frac{1}{\left(1+\frac{a}{bx^4}\right)^{3/4}x^3}\,dx}{(bc-ad)(a+bx^4)^{3/4}} - \frac{\left(d\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\right)\text{Subst}\left(\int \frac{}{\sqrt{}}\right.}{bc-}$$

$$= -\frac{\left(b\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\right)\text{Subst}\left(\int \frac{x}{\left(1+\frac{ax^4}{b}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)}{(bc-ad)(a+bx^4)^{3/4}} - \frac{\left(d\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\right.}{}$$

$$= -\frac{d\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,\Pi\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right)\middle|-1\right)}{2\sqrt[4]{b}\,c(bc-ad)} - \frac{d\sqrt{\frac{}{a}}}{}$$

$$= -\frac{b^{3/2}\left(1+\frac{a}{bx^4}\right)^{3/4}x^3 F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}\,(bc-ad)(a+bx^4)^{3/4}} - \frac{d\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,\Pi\left(-\right.}{2\sqrt[4]{}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 10.04, size = 161, normalized size = 0.62

$$-\frac{5acxF_1\left(\frac{1}{4};\frac{3}{4},1;\frac{5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{(a+bx^4)^{3/4}(c+dx^4)\left(-5acF_1\left(\frac{1}{4};\frac{3}{4},1;\frac{5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)+x^4\left(4adF_1\left(\frac{5}{4};\frac{3}{4},2;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)+3bcF_1\left(\frac{5}{4};\frac{7}{4},1;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^4)^(3/4)*(c + d*x^4)),x]
```

```
[Out] (-5*a*c*x*AppellF1[1/4, 3/4, 1, 5/4, -((b*x^4)/a), -((d*x^4)/c)])/((a + b*x
^4)^(3/4)*(c + d*x^4)*(-5*a*c*AppellF1[1/4, 3/4, 1, 5/4, -((b*x^4)/a), -((d
*x^4)/c)] + x^4*(4*a*d*AppellF1[5/4, 3/4, 2, 9/4, -((b*x^4)/a), -((d*x^4)/c
)] + 3*b*c*AppellF1[5/4, 7/4, 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)])))
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\,x^4+a)^{\frac{3}{4}}(d\,x^4+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^4+a)^(3/4)/(d*x^4+c),x)
[Out] int(1/(b*x^4+a)^(3/4)/(d*x^4+c),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(3/4)/(d*x^4+c),x, algorithm="maxima")
[Out] integrate(1/((b*x^4 + a)^(3/4)*(d*x^4 + c)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(3/4)/(d*x^4+c),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + bx^{4}\right)^{\frac{3}{4}} \left(c + dx^{4}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**4+a)**(3/4)/(d*x**4+c),x)
[Out] Integral(1/((a + b*x**4)**(3/4)*(c + d*x**4)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(3/4)/(d*x^4+c),x, algorithm="giac")
[Out] integrate(1/((b*x^4 + a)^(3/4)*(d*x^4 + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(b\,x^4+a\right)}^{3/4}\,\left(d\,x^4+c\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^4)^(3/4)*(c + d*x^4)),x)
[Out] int(1/((a + b*x^4)^(3/4)*(c + d*x^4)), x)
```

3.203 $\int \frac{1}{(a+bx^4)^{7/4}(c+dx^4)}\,dx$

Optimal. Leaf size=304

$$\frac{bx}{3a(bc-ad)\left(a+bx^4\right)^{3/4}}-\frac{b^{3/2}(2bc-5ad)\left(1+\frac{a}{bx^4}\right)^{3/4}x^3F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{3a^{3/2}(bc-ad)^2\left(a+bx^4\right)^{3/4}}+\frac{d^2\sqrt{\frac{a}{a+bx^4}}\,\sqrt{a+bx^4}}{}$$

[Out] 1/3*b*x/a/(-a*d+b*c)/(b*x^4+a)^(3/4)-1/3*b^(3/2)*(-5*a*d+2*b*c)*(1+a/b/x^4)^(3/4)*x^3*(cos(1/2*arccot(x^2*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arccot(x^2*b^(1/2)/a^(1/2)))*EllipticF(sin(1/2*arccot(x^2*b^(1/2)/a^(1/2))),2^(1/2))/a^(3/2)/(-a*d+b*c)^2/(b*x^4+a)^(3/4)+1/2*d^2*EllipticPi(b^(1/4)*x/(b*x^4+a)^(1/4),-(-a*d+b*c)^(1/2)/b^(1/2)/c^(1/2),I)*(a/(b*x^4+a))^(1/2)*(b*x^4+a)^(1/2)/b^(1/4)/c/(-a*d+b*c)^2+1/2*d^2*EllipticPi(b^(1/4)*x/(b*x^4+a)^(1/4),(-a*d+b*c)^(1/2)/b^(1/2)/c^(1/2),I)*(a/(b*x^4+a))^(1/2)*(b*x^4+a)^(1/2)/b^(1/4)/c/(-a*d+b*c)^2

Rubi [A]

time = 0.15, antiderivative size = 304, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {425, 543, 243, 342, 281, 237, 416, 418, 1232}

$$-\frac{b^{3/2}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}(2bc-5ad)F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{3a^{3/2}\left(a+bx^4\right)^{3/4}(bc-ad)^2}+\frac{d^2\sqrt{\frac{a}{a+bx^4}}\,\sqrt{a+bx^4}\,\Pi\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\,\sqrt{c}};\text{ArcSin}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{bx^4+a}}\right)\middle|-1\right)}{2\sqrt[4]{b}\,c(bc-ad)^2}+\frac{d^2\sqrt{\frac{a}{a+bx^4}}\,\sqrt{a+bx^4}\,\Pi\left(\frac{\sqrt{bc-ad}}{\sqrt{b}\,\sqrt{c}};\text{ArcSin}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{bx^4+a}}\right)\middle|-1\right)}{2\sqrt[4]{b}\,c(bc-ad)^2}+\frac{bx}{3a\left(a+bx^4\right)^{3/4}(bc-ad)}$$

Antiderivative was successfully verified.

[In] Int[1/((a + b*x^4)^(7/4)*(c + d*x^4)),x]

[Out] (b*x)/(3*a*(b*c - a*d)*(a + b*x^4)^(3/4)) - (b^(3/2)*(2*b*c - 5*a*d)*(1 + a/(b*x^4))^(3/4)*x^3*EllipticF[ArcCot[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/(3*a^(3/2)*(b*c - a*d)^2*(a + b*x^4)^(3/4)) + (d^2*Sqrt[a/(a + b*x^4)]*Sqrt[a + b*x^4]*EllipticPi[-(Sqrt[b*c - a*d]/(Sqrt[b]*Sqrt[c])), ArcSin[(b^(1/4)*x)/(a + b*x^4)^(1/4)], -1])/(2*b^(1/4)*c*(b*c - a*d)^2) + (d^2*Sqrt[a/(a + b*x^4)]*Sqrt[a + b*x^4]*EllipticPi[Sqrt[b*c - a*d]/(Sqrt[b]*Sqrt[c]), ArcSin[(b^(1/4)*x)/(a + b*x^4)^(1/4)], -1])/(2*b^(1/4)*c*(b*c - a*d)^2)

Rule 237

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2]))*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b/a]
```

Rule 243

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Dist[x^3*((1 + a/(b*x^4))^(3/4)/(a + b*x^4)^(3/4)), Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ[
```

```
{a, b}, x]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^4)^(1/4)/((c_) + (d_.)*(x_)^4), x_Symbol] :> Dist[Sq
rt[a + b*x^4]*Sqrt[a/(a + b*x^4)], Subst[Int[1/(Sqrt[1 - b*x^4]*(c - (b*c -
 a*d)*x^4)), x], x, x/(a + b*x^4)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] && N
eQ[b*c - a*d, 0]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Dist[1/(2*
c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 543

```
Int[((e_) + (f_.)*(x_)^4)/(((a_) + (b_.)*(x_)^4)^(3/4)*((c_) + (d_.)*(x_)^4
)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^4)^(3/4), x],
 x] - Dist[(d*e - c*f)/(b*c - a*d), Int[(a + b*x^4)^(1/4)/(c + d*x^4), x],
x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{(a+bx^4)^{7/4}(c+dx^4)}\,dx &= \frac{bx}{3a(bc-ad)(a+bx^4)^{3/4}} - \frac{\int \frac{-2bc+3ad-2bdx^4}{(a+bx^4)^{3/4}(c+dx^4)}\,dx}{3a(bc-ad)} \\
&= \frac{bx}{3a(bc-ad)(a+bx^4)^{3/4}} + \frac{d^2\int \frac{\sqrt[4]{a+bx^4}}{c+dx^4}\,dx}{(bc-ad)^2} + \frac{(b(2bc-5ad))\int \frac{1}{(a+bx^4)^{3/4}}\,d}{3a(bc-ad)^2} \\
&= \frac{bx}{3a(bc-ad)(a+bx^4)^{3/4}} + \frac{\left(b(2bc-5ad)\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\right)\int \frac{1}{\left(1+\frac{a}{bx^4}\right)^{3/4}x^3}\,dx}{3a(bc-ad)^2(a+bx^4)^{3/4}} \\
&= \frac{bx}{3a(bc-ad)(a+bx^4)^{3/4}} - \frac{\left(b(2bc-5ad)\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\right)\text{Subst}\left(\int \frac{x}{\left(1+\frac{ax^4}{b}\right)^{3}}\right.}{3a(bc-ad)^2(a+bx^4)^{3/4}} \\
&= \frac{bx}{3a(bc-ad)(a+bx^4)^{3/4}} + \frac{d^2\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,\Pi\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\right.\right.}{2\sqrt[4]{b}\,c(bc-ad)^2} \\
&= \frac{bx}{3a(bc-ad)(a+bx^4)^{3/4}} - \frac{b^{3/2}(2bc-5ad)\left(1+\frac{a}{bx^4}\right)^{3/4}x^3F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right.\right.}{3a^{3/2}(bc-ad)^2(a+bx^4)^{3/4}}
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 10.19, size = 332, normalized size = 1.09

$$
\frac{x\left(-\frac{2bdx^4\left(1+\frac{bx^4}{a}\right)^{3/4}F_1\left(\frac{5}{4};\frac{3}{4},1;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{c}+\frac{5\left(5ac(3ad-b(3c+dx^4))F_1\left(\frac{1}{4};\frac{3}{4},1;\frac{5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)+bx^4(c+dx^4)\left(4adF_1\left(\frac{5}{4};\frac{3}{4},2;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)+3bcF_1\left(\frac{5}{4};\frac{7}{4},1;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)\right)\right)}{(c+dx^4)\left(5acF_1\left(\frac{1}{4};\frac{3}{4},1;\frac{5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)-x^4\left(4adF_1\left(\frac{5}{4};\frac{3}{4},2;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)+3bcF_1\left(\frac{5}{4};\frac{7}{4},1;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)\right)\right)}\right)}{15a(-bc+ad)(a+bx^4)^{3/4}}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^4)^(7/4)*(c + d*x^4)),x]
```

```
[Out] (x*((-2*b*d*x^4*(1 + (b*x^4)/a)^(3/4)*AppellF1[5/4, 3/4, 1, 9/4, -((b*x^4)/
a), -((d*x^4)/c)])/c + (5*(5*a*c*(3*a*d - b*(3*c + d*x^4))*AppellF1[1/4, 3/
4, 1, 5/4, -((b*x^4)/a), -((d*x^4)/c)] + b*x^4*(c + d*x^4)*(4*a*d*AppellF1[
```

```
5/4, 3/4, 2, 9/4, -((b*x^4)/a), -((d*x^4)/c)] + 3*b*c*AppellF1[5/4, 7/4, 1,
 9/4, -((b*x^4)/a), -((d*x^4)/c)])))/((c + d*x^4)*(5*a*c*AppellF1[1/4, 3/4,
 1, 5/4, -((b*x^4)/a), -((d*x^4)/c)] - x^4*(4*a*d*AppellF1[5/4, 3/4, 2, 9/4
, -((b*x^4)/a), -((d*x^4)/c)] + 3*b*c*AppellF1[5/4, 7/4, 1, 9/4, -((b*x^4)/
a), -((d*x^4)/c)])))))/(15*a*(-(b*c) + a*d)*(a + b*x^4)^(3/4))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^{4}+a\right)^{\frac{7}{4}} \left(d\,x^{4}+c\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^4+a)^(7/4)/(d*x^4+c),x)
[Out] int(1/(b*x^4+a)^(7/4)/(d*x^4+c),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(7/4)/(d*x^4+c),x, algorithm="maxima")
[Out] integrate(1/((b*x^4 + a)^(7/4)*(d*x^4 + c)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(7/4)/(d*x^4+c),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+bx^{4}\right)^{\frac{7}{4}} \left(c+dx^{4}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**4+a)**(7/4)/(d*x**4+c),x)
```

```
[Out] Integral(1/((a + b*x**4)**(7/4)*(c + d*x**4)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(7/4)/(d*x^4+c),x, algorithm="giac")
[Out] integrate(1/((b*x^4 + a)^(7/4)*(d*x^4 + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(b\,x^4+a\right)}^{7/4}\,\left(d\,x^4+c\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^4)^(7/4)*(c + d*x^4)),x)
[Out] int(1/((a + b*x^4)^(7/4)*(c + d*x^4)), x)
```

3.204 $\int \frac{1}{(a+bx^4)^{11/4}(c+dx^4)}\,dx$

Optimal. Leaf size=357

$$\frac{bx}{7a(bc-ad)\left(a+bx^4\right)^{7/4}}+\frac{b(6bc-13ad)x}{21a^2(bc-ad)^2\left(a+bx^4\right)^{3/4}}-\frac{b^{3/2}(12b^2c^2-38abcd+47a^2d^2)\left(1+\frac{a}{bx^4}\right)^{3/4}x^3F\left(\frac{1}{2}\mathrm{c}\right.}{21a^{5/2}(bc-ad)^3\left(a+bx^4\right)^{3/4}}$$

```
[Out] 1/7*b*x/a/(-a*d+b*c)/(b*x^4+a)^(7/4)+1/21*b*(-13*a*d+6*b*c)*x/a^2/(-a*d+b*c
)^2/(b*x^4+a)^(3/4)-1/21*b^(3/2)*(47*a^2*d^2-38*a*b*c*d+12*b^2*c^2)*(1+a/b/
x^4)^(3/4)*x^3*(cos(1/2*arccot(x^2*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arcco
t(x^2*b^(1/2)/a^(1/2)))*EllipticF(sin(1/2*arccot(x^2*b^(1/2)/a^(1/2))),2^(1
/2))/a^(5/2)/(-a*d+b*c)^3/(b*x^4+a)^(3/4)-1/2*d^3*EllipticPi(b^(1/4)*x/(b*x
^4+a)^(1/4),-(-a*d+b*c)^(1/2)/b^(1/2)/c^(1/2),I)*(a/(b*x^4+a))^(1/2)*(b*x^4
+a)^(1/2)/b^(1/4)/c/(-a*d+b*c)^3-1/2*d^3*EllipticPi(b^(1/4)*x/(b*x^4+a)^(1/
4),(-a*d+b*c)^(1/2)/b^(1/2)/c^(1/2),I)*(a/(b*x^4+a))^(1/2)*(b*x^4+a)^(1/2)/
b^(1/4)/c/(-a*d+b*c)^3
```

Rubi [A]

time = 0.26, antiderivative size = 357, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {425, 541, 543, 243, 342, 281, 237, 416, 418, 1232}

$$\frac{bx(6bc-13ad)}{21a^2\left(a+bx^4\right)^{3/4}(bc-ad)^2}-\frac{b^{3/2}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}\left(47a^2d^2-38abcd+12b^2c^2\right)F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{21a^{5/2}\left(a+bx^4\right)^{3/4}(bc-ad)^3}-\frac{d^3\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,\Pi\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}};\mathrm{ArcSin}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{bx^4+a}}\right)\middle|-1\right)}{2\sqrt[4]{b}c(bc-ad)^3}-\frac{d^3\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,\Pi\left(\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}};\mathrm{ArcSin}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{bx^4+a}}\right)\middle|-1\right)}{2\sqrt[4]{b}c(bc-ad)^3}+\frac{bx}{7a\left(a+bx^4\right)^{7/4}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^4)^(11/4)*(c + d*x^4)),x]
```

```
[Out] (b*x)/(7*a*(b*c - a*d)*(a + b*x^4)^(7/4)) + (b*(6*b*c - 13*a*d)*x)/(21*a^2*
(b*c - a*d)^2*(a + b*x^4)^(3/4)) - (b^(3/2)*(12*b^2*c^2 - 38*a*b*c*d + 47*a
^2*d^2)*(1 + a/(b*x^4))^(3/4)*x^3*EllipticF[ArcCot[(Sqrt[b]*x^2)/Sqrt[a]]/2
, 2])/(21*a^(5/2)*(b*c - a*d)^3*(a + b*x^4)^(3/4)) - (d^3*Sqrt[a/(a + b*x^4
)]*Sqrt[a + b*x^4]*EllipticPi[-(Sqrt[b*c - a*d]/(Sqrt[b]*Sqrt[c])), ArcSin[
(b^(1/4)*x)/(a + b*x^4)^(1/4)], -1])/(2*b^(1/4)*c*(b*c - a*d)^3) - (d^3*Sqr
t[a/(a + b*x^4)]*Sqrt[a + b*x^4]*EllipticPi[Sqrt[b*c - a*d]/(Sqrt[b]*Sqrt[c
]), ArcSin[(b^(1/4)*x)/(a + b*x^4)^(1/4)], -1])/(2*b^(1/4)*c*(b*c - a*d)^3)
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 243

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Dist[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4)), Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ[
{a, b}, x]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^4)^(1/4)/((c_) + (d_.)*(x_)^4), x_Symbol] :> Dist[Sq
rt[a + b*x^4]*Sqrt[a/(a + b*x^4)], Subst[Int[1/(Sqrt[1 - b*x^4]*(c - (b*c -
 a*d)*x^4)), x], x, x/(a + b*x^4)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] && N
eQ[b*c - a*d, 0]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Dist[1/(2*
c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
```

```
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 543

```
Int[((e_) + (f_.)*(x_)^4)/(((a_) + (b_.)*(x_)^4)^(3/4)*((c_) + (d_.)*(x_)^4
)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^4)^(3/4), x],
 x] - Dist[(d*e - c*f)/(b*c - a*d), Int[(a + b*x^4)^(1/4)/(c + d*x^4), x],
x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{(a+bx^4)^{11/4}(c+dx^4)}\,dx = \frac{bx}{7a(bc-ad)(a+bx^4)^{7/4}} - \frac{\int \frac{-6bc+7ad-6bdx^4}{(a+bx^4)^{7/4}(c+dx^4)}\,dx}{7a(bc-ad)}$$

$$= \frac{bx}{7a(bc-ad)(a+bx^4)^{7/4}} + \frac{b(6bc-13ad)x}{21a^2(bc-ad)^2(a+bx^4)^{3/4}} + \frac{\int \frac{12b^2c^2-26abcd+21a^2d\ldots}{(a+bx^4)^{3/4}\ldots}}{21a^2(bc\ldots}$$

$$= \frac{bx}{7a(bc-ad)(a+bx^4)^{7/4}} + \frac{b(6bc-13ad)x}{21a^2(bc-ad)^2(a+bx^4)^{3/4}} - \frac{d^3\int \frac{\sqrt[4]{a+bx^4}}{c+dx^4}\,dx}{(bc-ad)^3}\ldots$$

$$= \frac{bx}{7a(bc-ad)(a+bx^4)^{7/4}} + \frac{b(6bc-13ad)x}{21a^2(bc-ad)^2(a+bx^4)^{3/4}} + \frac{\left(b(12b^2c^2-38abc\ldots\right.}{2\ldots}$$

$$= \frac{bx}{7a(bc-ad)(a+bx^4)^{7/4}} + \frac{b(6bc-13ad)x}{21a^2(bc-ad)^2(a+bx^4)^{3/4}} - \frac{\left(b(12b^2c^2-38abc\ldots\right.}{\ldots}$$

$$= \frac{bx}{7a(bc-ad)(a+bx^4)^{7/4}} + \frac{b(6bc-13ad)x}{21a^2(bc-ad)^2(a+bx^4)^{3/4}} - \frac{d^3\sqrt{\frac{a}{a+bx^4}}\,\sqrt{a+\ldots}}{\ldots}$$

$$= \frac{bx}{7a(bc-ad)(a+bx^4)^{7/4}} + \frac{b(6bc-13ad)x}{21a^2(bc-ad)^2(a+bx^4)^{3/4}} - \frac{b^{3/2}(12b^2c^2-38ab\ldots}{\ldots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 10.47, size = 430, normalized size = 1.20

$$\frac{x\left(-\frac{2bd(-6bc+13ad)x^4\left(1+\frac{bx^4}{a}\right)^{3/4}F_1\left(\frac{5}{4};\frac{3}{4},1;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{c}-\frac{5\left(5ac\left(21a^3d^2+6b^3cx^4\left(3c+dx^4\right)+a^2bd\left(-42c+5dx^4\right)+ab^2\left(21c^2-30cdx^4-13d^2x^8\right)\right)F_1\left(\frac{1}{4};\frac{3}{4},1;\frac{5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)+bx^4\left(c+dx^4\right)\left(16a^2d-6b^2cx^4+ab\left(-9c+13dx^4\right)\right)\left(4adF_1\left(\frac{5}{4};\frac{3}{4},2;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)+3bcF_1\left(\frac{5}{4};\frac{7}{4},1;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)\right)\right)}{\left(a+bx^4\right)\left(c+dx^4\right)\left(-5acF_1\left(\frac{1}{4};\frac{3}{4},1;\frac{5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)+x^4\left(4adF_1\left(\frac{5}{4};\frac{3}{4},2;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)+3bcF_1\left(\frac{5}{4};\frac{7}{4},1;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)\right)\right)}\right)}{105a^2(bc-ad)^2\left(a+bx^4\right)^{3/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^4)^(11/4)*(c + d*x^4)),x]
```

```
[Out] (x*((-2*b*d*(-6*b*c + 13*a*d)*x^4*(1 + (b*x^4)/a)^(3/4)*AppellF1[5/4, 3/4,
1, 9/4, -((b*x^4)/a), -((d*x^4)/c)])/c - (5*(5*a*c*(21*a^3*d^2 + 6*b^3*c*x^
4*(3*c + d*x^4) + a^2*b*d*(-42*c + 5*d*x^4) + a*b^2*(21*c^2 - 30*c*d*x^4 -
13*d^2*x^8))*AppellF1[1/4, 3/4, 1, 5/4, -((b*x^4)/a), -((d*x^4)/c)] + b*x^4
*(c + d*x^4)*(16*a^2*d - 6*b^2*c*x^4 + a*b*(-9*c + 13*d*x^4))*(4*a*d*Appell
F1[5/4, 3/4, 2, 9/4, -((b*x^4)/a), -((d*x^4)/c)] + 3*b*c*AppellF1[5/4, 7/4,
 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)])))/((a + b*x^4)*(c + d*x^4)*(-5*a*c*Ap
pellF1[1/4, 3/4, 1, 5/4, -((b*x^4)/a), -((d*x^4)/c)] + x^4*(4*a*d*AppellF1[
5/4, 3/4, 2, 9/4, -((b*x^4)/a), -((d*x^4)/c)] + 3*b*c*AppellF1[5/4, 7/4, 1,
 9/4, -((b*x^4)/a), -((d*x^4)/c)])))))/(105*a^2*(b*c - a*d)^2*(a + b*x^4)^(
3/4))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^{4}+a\right)^{\frac{11}{4}}\left(d\,x^{4}+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^4+a)^(11/4)/(d*x^4+c),x)
```

```
[Out] int(1/(b*x^4+a)^(11/4)/(d*x^4+c),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(11/4)/(d*x^4+c),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^4 + a)^(11/4)*(d*x^4 + c)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(11/4)/(d*x^4+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx^4)^{\frac{11}{4}}\,(c+dx^4)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**4+a)**(11/4)/(d*x**4+c),x)
```

```
[Out] Integral(1/((a + b*x**4)**(11/4)*(c + d*x**4)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(11/4)/(d*x^4+c),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^4 + a)^(11/4)*(d*x^4 + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(b\,x^4+a\right)}^{11/4}\,\left(d\,x^4+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^4)^(11/4)*(c + d*x^4)),x)
```

```
[Out] int(1/((a + b*x^4)^(11/4)*(c + d*x^4)), x)
```

3.205 $\int \frac{(a+bx^4)^{11/4}}{(c+dx^4)^2}\,dx$

Optimal. Leaf size=280

$$\frac{b(2bc-ad)x(a+bx^4)^{3/4}}{4cd^2}-\frac{(bc-ad)x(a+bx^4)^{7/4}}{4cd\,(c+dx^4)}-\frac{b^{7/4}(8bc-11ad)\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right)}{8d^3}+\frac{(bc-ad)^{7/4}(8bc}{}$$

```
[Out] 1/4*b*(-a*d+2*b*c)*x*(b*x^4+a)^(3/4)/c/d^2-1/4*(-a*d+b*c)*x*(b*x^4+a)^(7/4)
/c/d/(d*x^4+c)-1/8*b^(7/4)*(-11*a*d+8*b*c)*arctan(b^(1/4)*x/(b*x^4+a)^(1/4)
)/d^3+1/8*(-a*d+b*c)^(7/4)*(3*a*d+8*b*c)*arctan((-a*d+b*c)^(1/4)*x/c^(1/4)/
(b*x^4+a)^(1/4))/c^(7/4)/d^3-1/8*b^(7/4)*(-11*a*d+8*b*c)*arctanh(b^(1/4)*x/
(b*x^4+a)^(1/4))/d^3+1/8*(-a*d+b*c)^(7/4)*(3*a*d+8*b*c)*arctanh((-a*d+b*c)^
(1/4)*x/c^(1/4)/(b*x^4+a)^(1/4))/c^(7/4)/d^3
```

Rubi [A]

time = 0.23, antiderivative size = 280, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {424, 542, 544, 246, 218, 212, 209, 385, 214, 211}

$$-\frac{b^{7/4}\text{ArcTan}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right)(8bc-11ad)}{8d^3}+\frac{(bc-ad)^{7/4}(3ad+8bc)\text{ArcTan}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\,\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}d^3}-\frac{b^{7/4}(8bc-11ad)\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right)}{8d^3}+\frac{(bc-ad)^{7/4}(3ad+8bc)\tanh^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\,\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}d^3}+\frac{bx(a+bx^4)^{3/4}(2bc-ad)}{4cd^2}-\frac{x(a+bx^4)^{7/4}(bc-ad)}{4cd\,(c+dx^4)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)^(11/4)/(c + d*x^4)^2,x]
```

```
[Out] (b*(2*b*c - a*d)*x*(a + b*x^4)^(3/4))/(4*c*d^2) - ((b*c - a*d)*x*(a + b*x^4
)^(7/4))/(4*c*d*(c + d*x^4)) - (b^(7/4)*(8*b*c - 11*a*d)*ArcTan[(b^(1/4)*x)
/(a + b*x^4)^(1/4)])/(8*d^3) + ((b*c - a*d)^(7/4)*(8*b*c + 3*a*d)*ArcTan[((
b*c - a*d)^(1/4)*x)/(c^(1/4)*(a + b*x^4)^(1/4))])/(8*c^(7/4)*d^3) - (b^(7/4
)*(8*b*c - 11*a*d)*ArcTanh[(b^(1/4)*x)/(a + b*x^4)^(1/4)])/(8*d^3) + ((b*c
- a*d)^(7/4)*(8*b*c + 3*a*d)*ArcTanh[((b*c - a*d)^(1/4)*x)/(c^(1/4)*(a + b*
x^4)^(1/4))])/(8*c^(7/4)*d^3)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 544

```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Dist[f/d, Int[(a + b*x^n)^p, x], x] + Dist[(d*e -
c*f)/d, Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, p
, n}, x]
```

Rubi steps

$$\int \frac{(a+bx^4)^{11/4}}{(c+dx^4)^2}\,dx = -\frac{(bc-ad)x(a+bx^4)^{7/4}}{4cd\,(c+dx^4)} + \frac{\int \frac{(a+bx^4)^{3/4}(a(bc+3ad)+4b(2bc-ad)x^4)}{c+dx^4}\,dx}{4cd}$$

$$= \frac{b(2bc-ad)x(a+bx^4)^{3/4}}{4cd^2} - \frac{(bc-ad)x(a+bx^4)^{7/4}}{4cd\,(c+dx^4)} + \frac{\int \frac{-4a(2b^2c^2-2abcd-3a^2d^2)-4b^2c(8bc-}{\sqrt[4]{a+bx^4}\,(c+dx^4)}}{16cd^2}$$

$$= \frac{b(2bc-ad)x(a+bx^4)^{3/4}}{4cd^2} - \frac{(bc-ad)x(a+bx^4)^{7/4}}{4cd\,(c+dx^4)} - \frac{(b^2(8bc-11ad))\int \frac{1}{\sqrt[4]{a+bx^4}}\,d}{4d^3}$$

$$= \frac{b(2bc-ad)x(a+bx^4)^{3/4}}{4cd^2} - \frac{(bc-ad)x(a+bx^4)^{7/4}}{4cd\,(c+dx^4)} - \frac{(b^2(8bc-11ad))\,\mathrm{Subst}\left(\int \frac{1}{1-bx}\right.}{4d^3}$$

$$= \frac{b(2bc-ad)x(a+bx^4)^{3/4}}{4cd^2} - \frac{(bc-ad)x(a+bx^4)^{7/4}}{4cd\,(c+dx^4)} - \frac{(b^2(8bc-11ad))\,\mathrm{Subst}\left(\int \frac{1}{1-\sqrt{}}\right.}{8d^3}$$

$$= \frac{b(2bc-ad)x(a+bx^4)^{3/4}}{4cd^2} - \frac{(bc-ad)x(a+bx^4)^{7/4}}{4cd\,(c+dx^4)} - \frac{b^{7/4}(8bc-11ad)\tan^{-1}\left(\frac{\sqrt[4]{b}}{\sqrt[4]{a+}}\right.}{8d^3}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.99, size = 349, normalized size = 1.25

$$\frac{\left(\frac{1}{16}+\frac{i}{16}\right)\left(\frac{(2-2i)dx(a+bx^4)^{3/4}(-2abcd+a^2d^2+b^2c(2c+dx^4))}{c(c+dx^4)} - (1-i)b^{7/4}(8bc-11ad)\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right) + \frac{(bc-ad)^{7/4}(8bc+3ad)\tan^{-1}\left(\frac{\frac{(1-i)\sqrt[4]{bc-ad}\,x^2}{\sqrt[4]{c}\sqrt[4]{a+bx^4}} - \frac{(1+i)\sqrt[4]{c}\sqrt[4]{a+bx^4}}{\sqrt[4]{bc-ad}}}{2x}\right)}{c^{7/4}} - (1-i)b^{7/4}(8bc-11ad)\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right) + \frac{(bc-ad)^{7/4}(8bc+3ad)\tanh^{-1}\left(\frac{\frac{(1-i)\sqrt[4]{bc-ad}\,x^2}{\sqrt[4]{c}\sqrt[4]{a+bx^4}} + \frac{(1+i)\sqrt[4]{c}\sqrt[4]{a+bx^4}}{\sqrt[4]{bc-ad}}}{2x}\right)}{c^{7/4}}\right)}{d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^4)^(11/4)/(c + d*x^4)^2,x]
```

```
[Out] ((1/16 + I/16)*(((2 - 2*I)*d*x*(a + b*x^4)^(3/4)*(-2*a*b*c*d + a^2*d^2 + b^
2*c*(2*c + d*x^4)))/(c*(c + d*x^4)) - (1 - I)*b^(7/4)*(8*b*c - 11*a*d)*ArcT
an[(b^(1/4)*x)/(a + b*x^4)^(1/4)] + ((b*c - a*d)^(7/4)*(8*b*c + 3*a*d)*ArcT
an[(((1 - I)*(b*c - a*d)^(1/4)*x^2)/(c^(1/4)*(a + b*x^4)^(1/4)) - ((1 + I)*
```

```
c^(1/4)*(a + b*x^4)^(1/4))/(b*c - a*d)^(1/4))/(2*x)])/c^(7/4) - (1 - I)*b^(
7/4)*(8*b*c - 11*a*d)*ArcTanh[(b^(1/4)*x)/(a + b*x^4)^(1/4)] + ((b*c - a*d)
^(7/4)*(8*b*c + 3*a*d)*ArcTanh[(((1 - I)*(b*c - a*d)^(1/4)*x^2)/(c^(1/4)*(a
 + b*x^4)^(1/4)) + ((1 + I)*c^(1/4)*(a + b*x^4)^(1/4))/(b*c - a*d)^(1/4))/(
2*x)])/c^(7/4)))/d^3
```

Maple [**F**]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^{4}+a\right)^{\frac{11}{4}}}{\left(d\,x^{4}+c\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)^(11/4)/(d*x^4+c)^2,x)
```

```
[Out] int((b*x^4+a)^(11/4)/(d*x^4+c)^2,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(11/4)/(d*x^4+c)^2,x, algorithm="maxima")
```

```
[Out] integrate((b*x^4 + a)^(11/4)/(d*x^4 + c)^2, x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 3308 vs. 2(232) = 464.
time = 41.73, size = 3308, normalized size = 11.81

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(11/4)/(d*x^4+c)^2,x, algorithm="fricas")
```

```
[Out] 1/16*(4*(c*d^3*x^4 + c^2*d^2)*((4096*b^11*c^11 - 22528*a*b^10*c^10*d + 4646
4*a^2*b^9*c^9*d^2 - 37664*a^3*b^8*c^8*d^3 - 5071*a^4*b^7*c^7*d^4 + 25641*a^
5*b^6*c^6*d^5 - 7931*a^6*b^5*c^5*d^6 - 6259*a^7*b^4*c^4*d^7 + 2739*a^8*b^3*
c^3*d^8 + 891*a^9*b^2*c^2*d^9 - 297*a^10*b*c*d^10 - 81*a^11*d^11)/(c^7*d^12
))^(1/4)*arctan(-(c^2*d^3*x*sqrt(((4096*b^11*c^14*d^6 - 22528*a*b^10*c^13*d
^7 + 46464*a^2*b^9*c^12*d^8 - 37664*a^3*b^8*c^11*d^9 - 5071*a^4*b^7*c^10*d^
10 + 25641*a^5*b^6*c^9*d^11 - 7931*a^6*b^5*c^8*d^12 - 6259*a^7*b^4*c^7*d^13
 + 2739*a^8*b^3*c^6*d^14 + 891*a^9*b^2*c^5*d^15 - 297*a^10*b*c^4*d^16 - 81*
a^11*c^3*d^17)*x^2*sqrt((4096*b^11*c^11 - 22528*a*b^10*c^10*d + 46464*a^2*b
^9*c^9*d^2 - 37664*a^3*b^8*c^8*d^3 - 5071*a^4*b^7*c^7*d^4 + 25641*a^5*b^6*c
```

^6*d^5 - 7931*a^6*b^5*c^5*d^6 - 6259*a^7*b^4*c^4*d^7 + 2739*a^8*b^3*c^3*d^8 + 891*a^9*b^2*c^2*d^9 - 297*a^10*b*c*d^10 - 81*a^11*d^11)/(c^7*d^12)) + (2 62144*b^16*c^16 - 2031616*a*b^15*c^15*d + 6451200*a^2*b^14*c^14*d^2 - 10168 320*a^3*b^13*c^13*d^3 + 6467520*a^4*b^12*c^12*d^4 + 3123216*a^5*b^11*c^11*d ^5 - 7258119*a^6*b^10*c^10*d^6 + 2307030*a^7*b^9*c^9*d^7 + 2428965*a^8*b^8* c^8*d^8 - 1607320*a^9*b^7*c^7*d^9 - 387134*a^10*b^6*c^6*d^10 + 436356*a^11* b^5*c^5*d^11 + 40770*a^12*b^4*c^4*d^12 - 63720*a^13*b^3*c^3*d^13 - 6075*a^1 4*b^2*c^2*d^14 + 4374*a^15*b*c*d^15 + 729*a^16*d^16)*sqrt(b*x^4 + a))/x^2)* ((4096*b^11*c^11 - 22528*a*b^10*c^10*d + 46464*a^2*b^9*c^9*d^2 - 37664*a^3* b^8*c^8*d^3 - 5071*a^4*b^7*c^7*d^4 + 25641*a^5*b^6*c^6*d^5 - 7931*a^6*b^5*c ^5*d^6 - 6259*a^7*b^4*c^4*d^7 + 2739*a^8*b^3*c^3*d^8 + 891*a^9*b^2*c^2*d^9 - 297*a^10*b*c*d^10 - 81*a^11*d^11)/(c^7*d^12))^(1/4) + (512*b^8*c^10*d^3 - 1984*a*b^7*c^9*d^4 + 2456*a^2*b^6*c^8*d^5 - 413*a^3*b^5*c^7*d^6 - 1175*a^4 *b^4*c^6*d^7 + 478*a^5*b^3*c^5*d^8 + 234*a^6*b^2*c^4*d^9 - 81*a^7*b*c^3*d^1 0 - 27*a^8*c^2*d^11)*(b*x^4 + a)^(1/4)*((4096*b^11*c^11 - 22528*a*b^10*c^10 *d + 46464*a^2*b^9*c^9*d^2 - 37664*a^3*b^8*c^8*d^3 - 5071*a^4*b^7*c^7*d^4 + 25641*a^5*b^6*c^6*d^5 - 7931*a^6*b^5*c^5*d^6 - 6259*a^7*b^4*c^4*d^7 + 2739 *a^8*b^3*c^3*d^8 + 891*a^9*b^2*c^2*d^9 - 297*a^10*b*c*d^10 - 81*a^11*d^11)/ (c^7*d^12))^(1/4))/((4096*b^11*c^11 - 22528*a*b^10*c^10*d + 46464*a^2*b^9*c ^9*d^2 - 37664*a^3*b^8*c^8*d^3 - 5071*a^4*b^7*c^7*d^4 + 25641*a^5*b^6*c^6*d ^5 - 7931*a^6*b^5*c^5*d^6 - 6259*a^7*b^4*c^4*d^7 + 2739*a^8*b^3*c^3*d^8 + 8 91*a^9*b^2*c^2*d^9 - 297*a^10*b*c*d^10 - 81*a^11*d^11)*x)) + 4*(c*d^3*x^4 + c^2*d^2)*((4096*b^11*c^4 - 22528*a*b^10*c^3*d + 46464*a^2*b^9*c^2*d^2 - 42 592*a^3*b^8*c*d^3 + 14641*a^4*b^7*d^4)/d^12)^(1/4)*arctan((d^3*x*sqrt(((409 6*b^11*c^4*d^6 - 22528*a*b^10*c^3*d^7 + 46464*a^2*b^9*c^2*d^8 - 42592*a^3*b ^8*c*d^9 + 14641*a^4*b^7*d^10)*x^2*sqrt((4096*b^11*c^4 - 22528*a*b^10*c^3*d + 46464*a^2*b^9*c^2*d^2 - 42592*a^3*b^8*c*d^3 + 14641*a^4*b^7*d^4)/d^12) + (262144*b^16*c^6 - 2162688*a*b^15*c^5*d + 7434240*a^2*b^14*c^4*d^2 - 13629 440*a^3*b^13*c^3*d^3 + 14055360*a^4*b^12*c^2*d^4 - 7730448*a^5*b^11*c*d^5 + 1771561*a^6*b^10*d^6)*sqrt(b*x^4 + a))/x^2)*((4096*b^11*c^4 - 22528*a*b^10 *c^3*d + 46464*a^2*b^9*c^2*d^2 - 42592*a^3*b^8*c*d^3 + 14641*a^4*b^7*d^4)/d ^12)^(1/4) + (512*b^8*c^3*d^3 - 2112*a*b^7*c^2*d^4 + 2904*a^2*b^6*c*d^5 - 1 331*a^3*b^5*d^6)*(b*x^4 + a)^(1/4)*((4096*b^11*c^4 - 22528*a*b^10*c^3*d + 4 6464*a^2*b^9*c^2*d^2 - 42592*a^3*b^8*c*d^3 + 14641*a^4*b^7*d^4)/d^12)^(1/4))/((4096*b^11*c^4 - 22528*a*b^10*c^3*d + 46464*a^2*b^9*c^2*d^2 - 42592*a^3* b^8*c*d^3 + 14641*a^4*b^7*d^4)*x)) + (c*d^3*x^4 + c^2*d^2)*((4096*b^11*c^11 - 22528*a*b^10*c^10*d + 46464*a^2*b^9*c^9*d^2 - 37664*a^3*b^8*c^8*d^3 - 50 71*a^4*b^7*c^7*d^4 + 25641*a^5*b^6*c^6*d^5 - 7931*a^6*b^5*c^5*d^6 - 6259*a^ 7*b^4*c^4*d^7 + 2739*a^8*b^3*c^3*d^8 + 891*a^9*b^2*c^2*d^9 - 297*a^10*b*c*d ^10 - 81*a^11*d^11)/(c^7*d^12))^(1/4)*log(-(c^5*d^9*x*((4096*b^11*c^11 - 22 528*a*b^10*c^10*d + 46464*a^2*b^9*c^9*d^2 - 37664*a^3*b^8*c^8*d^3 - 5071*a^ 4*b^7*c^7*d^4 + 25641*a^5*b^6*c^6*d^5 - 7931*a^6*b^5*c^5*d^6 - 6259*a^7*b^4 *c^4*d^7 + 2739*a^8*b^3*c^3*d^8 + 891*a^9*b^2*c^2*d^9 - 297*a^10*b*c*d^10 - 81*a^11*d^11)/(c^7*d^12))^(3/4) + (512*b^8*c^8 - 1984*a*b^7*c^7*d + 2456*a ^2*b^6*c^6*d^2 - 413*a^3*b^5*c^5*d^3 - 1175*a^4*b^4*c^4*d^4 + 478*a^5*b^3*c

```
^3*d^5 + 234*a^6*b^2*c^2*d^6 - 81*a^7*b*c*d^7 - 27*a^8*d^8)*(b*x^4 + a)^(1/
4))/x) - (c*d^3*x^4 + c^2*d^2)*((4096*b^11*c^11 - 22528*a*b^10*c^10*d + 464
64*a^2*b^9*c^9*d^2 - 37664*a^3*b^8*c^8*d^3 - 5071*a^4*b^7*c^7*d^4 + 25641*a
^5*b^6*c^6*d^5 - 7931*a^6*b^5*c^5*d^6 - 6259*a^7*b^4*c^4*d^7 + 2739*a^8*b^3
*c^3*d^8 + 891*a^9*b^2*c^2*d^9 - 297*a^10*b*c*d^10 - 81*a^11*d^11)/(c^7*d^1
2))^(1/4)*log((c^5*d^9*x*((4096*b^11*c^11 - 22528*a*b^10*c^10*d + 46464*a^2
*b^9*c^9*d^2 - 37664*a^3*b^8*c^8*d^3 - 5071*a^4*b^7*c^7*d^4 + 25641*a^5*b^6
*c^6*d^5 - 7931*a^6*b^5*c^5*d^6 - 6259*a^7*b^4*c^4*d^7 + 2739*a^8*b^3*c^3*d
^8 + 891*a^9*b^2*c^2*d^9 - 297*a^10*b*c*d^10 - 81*a^11*d^11)/(c^7*d^12))^(3
/4) - (512*b^8*c^8 - 1984*a*b^7*c^7*d + 2456*a^2*b^6*c^6*d^2 - 413*a^3*b^5*
c^5*d^3 - 1175*a^4*b^4*c^4*d^4 + 478*a^5*b^3*c^3*d^5 + 234*a^6*b^2*c^2*d^6
- 81*a^7*b*c*d^7 - 27*a^8*d^8)*(b*x^4 + a)^(1/4...
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)**(11/4)/(d*x**4+c)**2,x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(11/4)/(d*x^4+c)^2,x, algorithm="giac")
[Out] integrate((b*x^4 + a)^(11/4)/(d*x^4 + c)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(b\,x^4+a\right)}^{11/4}}{{\left(d\,x^4+c\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)^(11/4)/(c + d*x^4)^2,x)
[Out] int((a + b*x^4)^(11/4)/(c + d*x^4)^2, x)
```

3.206 $\int \frac{(a+bx^4)^{7/4}}{(c+dx^4)^2}\,dx$

Optimal. Leaf size=230

$$-\frac{(bc-ad)x(a+bx^4)^{3/4}}{4cd\,(c+dx^4)}+\frac{b^{7/4}\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right)}{2d^2}-\frac{(bc-ad)^{3/4}(4bc+3ad)\tan^{-1}\left(\frac{\sqrt[4]{bc-ad}\,x}{\sqrt[4]{c}\,\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}d^2}+\frac{b^{7/4}\cdots}{}$$

```
[Out] -1/4*(-a*d+b*c)*x*(b*x^4+a)^(3/4)/c/d/(d*x^4+c)+1/2*b^(7/4)*arctan(b^(1/4)*
x/(b*x^4+a)^(1/4))/d^2-1/8*(-a*d+b*c)^(3/4)*(3*a*d+4*b*c)*arctan((-a*d+b*c)
^(1/4)*x/c^(1/4)/(b*x^4+a)^(1/4))/c^(7/4)/d^2+1/2*b^(7/4)*arctanh(b^(1/4)*x
/(b*x^4+a)^(1/4))/d^2-1/8*(-a*d+b*c)^(3/4)*(3*a*d+4*b*c)*arctanh((-a*d+b*c)
^(1/4)*x/c^(1/4)/(b*x^4+a)^(1/4))/c^(7/4)/d^2
```

Rubi [A]

time = 0.11, antiderivative size = 230, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {424, 544, 246, 218, 212, 209, 385, 214, 211}

$$\frac{b^{7/4}\text{ArcTan}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right)}{2d^2}-\frac{(bc-ad)^{3/4}(3ad+4bc)\text{ArcTan}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\,\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}d^2}+\frac{b^{7/4}\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right)}{2d^2}-\frac{(bc-ad)^{3/4}(3ad+4bc)\tanh^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\,\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}d^2}-\frac{x(a+bx^4)^{3/4}(bc-ad)}{4cd\,(c+dx^4)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)^(7/4)/(c + d*x^4)^2,x]
```

```
[Out] -1/4*((b*c - a*d)*x*(a + b*x^4)^(3/4))/(c*d*(c + d*x^4)) + (b^(7/4)*ArcTan[
(b^(1/4)*x)/(a + b*x^4)^(1/4)])/(2*d^2) - ((b*c - a*d)^(3/4)*(4*b*c + 3*a*d
)*ArcTan[((b*c - a*d)^(1/4)*x)/(c^(1/4)*(a + b*x^4)^(1/4))])/(8*c^(7/4)*d^2
) + (b^(7/4)*ArcTanh[(b^(1/4)*x)/(a + b*x^4)^(1/4)])/(2*d^2) - ((b*c - a*d)
^(3/4)*(4*b*c + 3*a*d)*ArcTanh[((b*c - a*d)^(1/4)*x)/(c^(1/4)*(a + b*x^4)^(
1/4))])/(8*c^(7/4)*d^2)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 544

```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Dist[f/d, Int[(a + b*x^n)^p, x], x] + Dist[(d*e -
c*f)/d, Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, p
, n}, x]
```

Rubi steps

$$\int \frac{(a+bx^4)^{7/4}}{(c+dx^4)^2}\,dx = -\frac{(bc-ad)x(a+bx^4)^{3/4}}{4cd\,(c+dx^4)} + \frac{\int \frac{a(bc+3ad)+4b^2cx^4}{\sqrt[4]{a+bx^4}\,(c+dx^4)}\,dx}{4cd}$$

$$= -\frac{(bc-ad)x(a+bx^4)^{3/4}}{4cd\,(c+dx^4)} + \frac{b^2\int \frac{1}{\sqrt[4]{a+bx^4}}\,dx}{d^2} - \frac{((bc-ad)(4bc+3ad))\int \frac{1}{\sqrt[4]{a+bx^4}\,(\ldots}}{4cd^2}$$

$$= -\frac{(bc-ad)x(a+bx^4)^{3/4}}{4cd\,(c+dx^4)} + \frac{b^2\text{Subst}\left(\int \frac{1}{1-bx^4}\,dx, x, \frac{x}{\sqrt[4]{a+bx^4}}\right)}{d^2} - \frac{((bc-ad)(4bc+3a\ldots}{\ldots}$$

$$= -\frac{(bc-ad)x(a+bx^4)^{3/4}}{4cd\,(c+dx^4)} + \frac{b^2\text{Subst}\left(\int \frac{1}{1-\sqrt{b}\,x^2}\,dx, x, \frac{x}{\sqrt[4]{a+bx^4}}\right)}{2d^2} + \frac{b^2\text{Subst}\left(\int \frac{1}{1+\sqrt{\ldots}}\ldots\right.}{\ldots}$$

$$= -\frac{(bc-ad)x(a+bx^4)^{3/4}}{4cd\,(c+dx^4)} + \frac{b^{7/4}\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right)}{2d^2} - \frac{(bc-ad)^{3/4}(4bc+3ad)\tan^{-1}\ldots}{8c^{7/4}d^2}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.26, size = 341, normalized size = 1.48

$$\frac{\left(\frac{1}{16}+\frac{i}{16}\right)\left(-\frac{(2-2i)d(bc-ad)x(a+bx^4)^{3/4}}{c(c+dx^4)} + (4-4i)b^{7/4}\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right) - \frac{(4b^2c^2-abcd-3a^2d^2)\tan^{-1}\left(\frac{\frac{(1-i)\sqrt[4]{bc-ad}\,x^2}{\sqrt[4]{c}\sqrt[4]{a+bx^4}} - \frac{(1+i)\sqrt[4]{c}\sqrt[4]{a+bx^4}}{\sqrt[4]{bc-ad}}}{2x}\right)}{c^{7/4}\sqrt[4]{bc-ad}} + (4-4i)b^{7/4}\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right) - \frac{(4b^2c^2-abcd-3a^2d^2)\tanh^{-1}\left(\frac{\frac{(1-i)\sqrt[4]{bc-ad}\,x^2}{\sqrt[4]{c}\sqrt[4]{a+bx^4}} + \frac{(1+i)\sqrt[4]{c}\sqrt[4]{a+bx^4}}{\sqrt[4]{bc-ad}}}{2x}\right)}{c^{7/4}\sqrt[4]{bc-ad}}\right)}{d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^4)^(7/4)/(c + d*x^4)^2,x]
```

```
[Out] ((1/16 + I/16)*(((-2 + 2*I)*d*(b*c - a*d)*x*(a + b*x^4)^(3/4))/(c*(c + d*x^
4)) + (4 - 4*I)*b^(7/4)*ArcTan[(b^(1/4)*x)/(a + b*x^4)^(1/4)] - ((4*b^2*c^2
 - a*b*c*d - 3*a^2*d^2)*ArcTan[(((1 - I)*(b*c - a*d)^(1/4)*x^2)/(c^(1/4)*(a
 + b*x^4)^(1/4)) - ((1 + I)*c^(1/4)*(a + b*x^4)^(1/4))/(b*c - a*d)^(1/4))/(
2*x)])/(c^(7/4)*(b*c - a*d)^(1/4)) + (4 - 4*I)*b^(7/4)*ArcTanh[(b^(1/4)*x)/
(a + b*x^4)^(1/4)] - ((4*b^2*c^2 - a*b*c*d - 3*a^2*d^2)*ArcTanh[(((1 - I)*(
b*c - a*d)^(1/4)*x^2)/(c^(1/4)*(a + b*x^4)^(1/4)) + ((1 + I)*c^(1/4)*(a + b
*x^4)^(1/4))/(b*c - a*d)^(1/4))/(2*x)])/(c^(7/4)*(b*c - a*d)^(1/4))))/d^2
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(b\,x^4+a)^{\frac{7}{4}}}{(d\,x^4+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)^(7/4)/(d*x^4+c)^2,x)
[Out] int((b*x^4+a)^(7/4)/(d*x^4+c)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(7/4)/(d*x^4+c)^2,x, algorithm="maxima")
[Out] integrate((b*x^4 + a)^(7/4)/(d*x^4 + c)^2, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1667 vs. $2(186) = 372$.
time = 4.18, size = 1667, normalized size = 7.25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(7/4)/(d*x^4+c)^2,x, algorithm="fricas")
[Out] -1/16*(4*(b*x^4 + a)^(3/4)*(b*c - a*d)*x - 4*(c*d^2*x^4 + c^2*d)*((256*b^7*
c^7 - 672*a^2*b^5*c^5*d^2 - 112*a^3*b^4*c^4*d^3 + 609*a^4*b^3*c^3*d^4 + 189
*a^5*b^2*c^2*d^5 - 189*a^6*b*c*d^6 - 81*a^7*d^7)/(c^7*d^8))^(1/4)*arctan(-(
c^2*d^2*x*sqrt(((256*b^7*c^10*d^4 - 672*a^2*b^5*c^8*d^6 - 112*a^3*b^4*c^7*d
^7 + 609*a^4*b^3*c^6*d^8 + 189*a^5*b^2*c^5*d^9 - 189*a^6*b*c^4*d^10 - 81*a^
7*c^3*d^11)*x^2*sqrt((256*b^7*c^7 - 672*a^2*b^5*c^5*d^2 - 112*a^3*b^4*c^4*d
^3 + 609*a^4*b^3*c^3*d^4 + 189*a^5*b^2*c^2*d^5 - 189*a^6*b*c*d^6 - 81*a^7*d
^7)/(c^7*d^8)) + (4096*b^10*c^10 + 2048*a*b^9*c^9*d - 14592*a^2*b^8*c^8*d^2
 - 9472*a^3*b^7*c^7*d^3 + 18928*a^4*b^6*c^6*d^4 + 15624*a^5*b^5*c^5*d^5 - 9
639*a^6*b^4*c^4*d^6 - 11124*a^7*b^3*c^3*d^7 + 486*a^8*b^2*c^2*d^8 + 2916*a^
9*b*c*d^9 + 729*a^10*d^10)*sqrt(b*x^4 + a))/x^2)*((256*b^7*c^7 - 672*a^2*b^
5*c^5*d^2 - 112*a^3*b^4*c^4*d^3 + 609*a^4*b^3*c^3*d^4 + 189*a^5*b^2*c^2*d^5
 - 189*a^6*b*c*d^6 - 81*a^7*d^7)/(c^7*d^8))^(1/4) - (64*b^5*c^7*d^2 + 16*a*
b^4*c^6*d^3 - 116*a^2*b^3*c^5*d^4 - 45*a^3*b^2*c^4*d^5 + 54*a^4*b*c^3*d^6 +
 27*a^5*c^2*d^7)*(b*x^4 + a)^(1/4)*((256*b^7*c^7 - 672*a^2*b^5*c^5*d^2 - 11
2*a^3*b^4*c^4*d^3 + 609*a^4*b^3*c^3*d^4 + 189*a^5*b^2*c^2*d^5 - 189*a^6*b*c
*d^6 - 81*a^7*d^7)/(c^7*d^8))^(1/4))/((256*b^7*c^7 - 672*a^2*b^5*c^5*d^2 -
112*a^3*b^4*c^4*d^3 + 609*a^4*b^3*c^3*d^4 + 189*a^5*b^2*c^2*d^5 - 189*a^6*b
*c*d^6 - 81*a^7*d^7)*x)) - 16*(c*d^2*x^4 + c^2*d)*(b^7/d^8)^(1/4)*arctan(-(
(b*x^4 + a)^(1/4)*b^5*d^2*(b^7/d^8)^(1/4) - d^2*x*(b^7/d^8)^(1/4)*sqrt((b^7
*d^4*x^2*sqrt(b^7/d^8) + sqrt(b*x^4 + a)*b^10)/x^2))/(b^7*x)) + (c*d^2*x^4
+ c^2*d)*((256*b^7*c^7 - 672*a^2*b^5*c^5*d^2 - 112*a^3*b^4*c^4*d^3 + 609*a^
4*b^3*c^3*d^4 + 189*a^5*b^2*c^2*d^5 - 189*a^6*b*c*d^6 - 81*a^7*d^7)/(c^7*d^
```

```
8))^(1/4)*log((c^5*d^6*x*((256*b^7*c^7 - 672*a^2*b^5*c^5*d^2 - 112*a^3*b^4*
c^4*d^3 + 609*a^4*b^3*c^3*d^4 + 189*a^5*b^2*c^2*d^5 - 189*a^6*b*c*d^6 - 81*
a^7*d^7)/(c^7*d^8))^(3/4) + (64*b^5*c^5 + 16*a*b^4*c^4*d - 116*a^2*b^3*c^3*
d^2 - 45*a^3*b^2*c^2*d^3 + 54*a^4*b*c*d^4 + 27*a^5*d^5)*(b*x^4 + a)^(1/4))/
x) - (c*d^2*x^4 + c^2*d)*((256*b^7*c^7 - 672*a^2*b^5*c^5*d^2 - 112*a^3*b^4*
c^4*d^3 + 609*a^4*b^3*c^3*d^4 + 189*a^5*b^2*c^2*d^5 - 189*a^6*b*c*d^6 - 81*
a^7*d^7)/(c^7*d^8))^(1/4)*log(-(c^5*d^6*x*((256*b^7*c^7 - 672*a^2*b^5*c^5*d
^2 - 112*a^3*b^4*c^4*d^3 + 609*a^4*b^3*c^3*d^4 + 189*a^5*b^2*c^2*d^5 - 189*
a^6*b*c*d^6 - 81*a^7*d^7)/(c^7*d^8))^(3/4) - (64*b^5*c^5 + 16*a*b^4*c^4*d -
 116*a^2*b^3*c^3*d^2 - 45*a^3*b^2*c^2*d^3 + 54*a^4*b*c*d^4 + 27*a^5*d^5)*(b
*x^4 + a)^(1/4))/x) - 4*(c*d^2*x^4 + c^2*d)*(b^7/d^8)^(1/4)*log((d^6*x*(b^7
/d^8)^(3/4) + (b*x^4 + a)^(1/4)*b^5)/x) + 4*(c*d^2*x^4 + c^2*d)*(b^7/d^8)^(
1/4)*log(-(d^6*x*(b^7/d^8)^(3/4) - (b*x^4 + a)^(1/4)*b^5)/x))/(c*d^2*x^4 +
c^2*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx^4)^{\frac{7}{4}}}{(c+dx^4)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)**(7/4)/(d*x**4+c)**2,x)
[Out] Integral((a + b*x**4)**(7/4)/(c + d*x**4)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(7/4)/(d*x^4+c)^2,x, algorithm="giac")
[Out] integrate((b*x^4 + a)^(7/4)/(d*x^4 + c)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(b\,x^4+a\right)}^{7/4}}{{\left(d\,x^4+c\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)^(7/4)/(c + d*x^4)^2,x)
[Out] int((a + b*x^4)^(7/4)/(c + d*x^4)^2, x)
```

3.207 $\int \frac{(a+bx^4)^{3/4}}{(c+dx^4)^2} dx$

Optimal. Leaf size=135

$$\frac{x(a+bx^4)^{3/4}}{4c\,(c+dx^4)}+\frac{3a\tan^{-1}\left(\frac{\sqrt[4]{bc-ad}\,x}{\sqrt[4]{c}\,\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}\sqrt[4]{bc-ad}}+\frac{3a\tanh^{-1}\left(\frac{\sqrt[4]{bc-ad}\,x}{\sqrt[4]{c}\,\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}\sqrt[4]{bc-ad}}$$

```
[Out] 1/4*x*(b*x^4+a)^(3/4)/c/(d*x^4+c)+3/8*a*arctan((-a*d+b*c)^(1/4)*x/c^(1/4)/(
b*x^4+a)^(1/4))/c^(7/4)/(-a*d+b*c)^(1/4)+3/8*a*arctanh((-a*d+b*c)^(1/4)*x/c
^(1/4)/(b*x^4+a)^(1/4))/c^(7/4)/(-a*d+b*c)^(1/4)
```

Rubi [A]
time = 0.05, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {386, 385, 218, 214, 211}

$$\frac{3a\text{ArcTan}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\,\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}\sqrt[4]{bc-ad}}+\frac{3a\tanh^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\,\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}\sqrt[4]{bc-ad}}+\frac{x(a+bx^4)^{3/4}}{4c\,(c+dx^4)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)^(3/4)/(c + d*x^4)^2,x]
```

```
[Out] (x*(a + b*x^4)^(3/4))/(4*c*(c + d*x^4)) + (3*a*ArcTan[((b*c - a*d)^(1/4)*x)
/(c^(1/4)*(a + b*x^4)^(1/4))])/(8*c^(7/4)*(b*c - a*d)^(1/4)) + (3*a*ArcTanh
[((b*c - a*d)^(1/4)*x)/(c^(1/4)*(a + b*x^4)^(1/4))])/(8*c^(7/4)*(b*c - a*d)
^(1/4))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 386

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] - Dist[
c*(q/(a*(p + 1))), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1,
0] && GtQ[q, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\left(a+bx^4\right)^{3/4}}{\left(c+dx^4\right)^2}\,dx = \frac{x\left(a+bx^4\right)^{3/4}}{4c\left(c+dx^4\right)}+\frac{(3a)\int \frac{1}{\sqrt[4]{a+bx^4}\,(c+dx^4)}\,dx}{4c}$$

$$= \frac{x\left(a+bx^4\right)^{3/4}}{4c\left(c+dx^4\right)}+\frac{(3a)\text{Subst}\left(\int \frac{1}{c-(bc-ad)x^4}\,dx,x,\frac{x}{\sqrt[4]{a+bx^4}}\right)}{4c}$$

$$= \frac{x\left(a+bx^4\right)^{3/4}}{4c\left(c+dx^4\right)}+\frac{(3a)\text{Subst}\left(\int \frac{1}{\sqrt{c}-\sqrt{bc-ad}\,x^2}\,dx,x,\frac{x}{\sqrt[4]{a+bx^4}}\right)}{8c^{3/2}}+\frac{(3a)\text{Subst}\left(\int -\right.}{}$$

$$= \frac{x\left(a+bx^4\right)^{3/4}}{4c\left(c+dx^4\right)}+\frac{3a\tan^{-1}\left(\frac{\sqrt[4]{bc-ad}\,x}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}\sqrt[4]{bc-ad}}+\frac{3a\tanh^{-1}\left(\frac{\sqrt[4]{bc-ad}\,x}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}\sqrt[4]{bc-ad}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.88, size = 238, normalized size = 1.76

$$\frac{4c^{3/4}\sqrt[4]{bc-ad}\,x\left(a+bx^4\right)^{3/4}+(3+3i)a\left(c+dx^4\right)\tan^{-1}\left(\frac{\frac{(1-i)\sqrt[4]{bc-ad}\,x^2}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}-\frac{(1+i)\sqrt[4]{c}\sqrt[4]{a+bx^4}}{\sqrt[4]{bc-ad}}}{2x}\right)+(3+3i)a\left(c+dx^4\right)\tanh^{-1}\left(\frac{\frac{(1-i)\sqrt[4]{bc-ad}\,x^2}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}+\frac{(1+i)\sqrt[4]{c}\sqrt[4]{a+bx^4}}{\sqrt[4]{bc-ad}}}{2x}\right)}{16c^{7/4}\sqrt[4]{bc-ad}\,\left(c+dx^4\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^4)^(3/4)/(c + d*x^4)^2,x]
```

```
[Out] (4*c^(3/4)*(b*c - a*d)^(1/4)*x*(a + b*x^4)^(3/4) + (3 + 3*I)*a*(c + d*x^4)*
ArcTan[(((1 - I)*(b*c - a*d)^(1/4)*x^2)/(c^(1/4)*(a + b*x^4)^(1/4)) - ((1 +
 I)*c^(1/4)*(a + b*x^4)^(1/4))/(b*c - a*d)^(1/4))/(2*x)] + (3 + 3*I)*a*(c +
```

```
d*x^4)*ArcTanh[(((1 - I)*(b*c - a*d)^(1/4)*x^2)/(c^(1/4)*(a + b*x^4)^(1/4)
) + ((1 + I)*c^(1/4)*(a + b*x^4)^(1/4))/(b*c - a*d)^(1/4))/(2*x)])/(16*c^(7
/4)*(b*c - a*d)^(1/4)*(c + d*x^4))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^{4}+a\right)^{\frac{3}{4}}}{\left(d\,x^{4}+c\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)^(3/4)/(d*x^4+c)^2,x)
[Out] int((b*x^4+a)^(3/4)/(d*x^4+c)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(3/4)/(d*x^4+c)^2,x, algorithm="maxima")
[Out] integrate((b*x^4 + a)^(3/4)/(d*x^4 + c)^2, x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(3/4)/(d*x^4+c)^2,x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + bx^{4}\right)^{\frac{3}{4}}}{\left(c + dx^{4}\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)**(3/4)/(d*x**4+c)**2,x)
[Out] Integral((a + b*x**4)**(3/4)/(c + d*x**4)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(3/4)/(d*x^4+c)^2,x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)^(3/4)/(d*x^4 + c)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(b\,x^4+a)^{3/4}}{(d\,x^4+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)^(3/4)/(c + d*x^4)^2,x)
```

```
[Out] int((a + b*x^4)^(3/4)/(c + d*x^4)^2, x)
```

3.208 $\int \frac{1}{\sqrt[4]{a+bx^4}\,(c+dx^4)^2}\,dx$

Optimal. Leaf size=162

$$-\frac{dx(a+bx^4)^{3/4}}{4c(bc-ad)\,(c+dx^4)}+\frac{(4bc-3ad)\tan^{-1}\left(\frac{\sqrt[4]{bc-ad}\,x}{\sqrt[4]{c}\,\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}(bc-ad)^{5/4}}+\frac{(4bc-3ad)\tanh^{-1}\left(\frac{\sqrt[4]{bc-ad}\,x}{\sqrt[4]{c}\,\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}(bc-ad)^{5/4}}$$

```
[Out] -1/4*d*x*(b*x^4+a)^(3/4)/c/(-a*d+b*c)/(d*x^4+c)+1/8*(-3*a*d+4*b*c)*arctan((
-a*d+b*c)^(1/4)*x/c^(1/4)/(b*x^4+a)^(1/4))/c^(7/4)/(-a*d+b*c)^(5/4)+1/8*(-3
*a*d+4*b*c)*arctanh((-a*d+b*c)^(1/4)*x/c^(1/4)/(b*x^4+a)^(1/4))/c^(7/4)/(-a
*d+b*c)^(5/4)
```

Rubi [A]
time = 0.07, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {390, 385, 218, 214, 211}

$$\frac{(4bc-3ad)\text{ArcTan}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\,\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}(bc-ad)^{5/4}}+\frac{(4bc-3ad)\tanh^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\,\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}(bc-ad)^{5/4}}-\frac{dx(a+bx^4)^{3/4}}{4c\,(c+dx^4)\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^4)^(1/4)*(c + d*x^4)^2),x]
```

```
[Out] -1/4*(d*x*(a + b*x^4)^(3/4))/(c*(b*c - a*d)*(c + d*x^4)) + ((4*b*c - 3*a*d)
*ArcTan[((b*c - a*d)^(1/4)*x)/(c^(1/4)*(a + b*x^4)^(1/4))])/(8*c^(7/4)*(b*c
 - a*d)^(5/4)) + ((4*b*c - 3*a*d)*ArcTanh[((b*c - a*d)^(1/4)*x)/(c^(1/4)*(a
 + b*x^4)^(1/4))])/(8*c^(7/4)*(b*c - a*d)^(5/4))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
```

```
, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)),
 Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q},
 x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !L
tQ[q, -1]) && NeQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\sqrt[4]{a+bx^4}\,(c+dx^4)^2}\,dx &= -\frac{dx(a+bx^4)^{3/4}}{4c(bc-ad)(c+dx^4)}+\frac{(4bc-3ad)\int \frac{1}{\sqrt[4]{a+bx^4}\,(c+dx^4)}\,dx}{4c(bc-ad)}\\
&= -\frac{dx(a+bx^4)^{3/4}}{4c(bc-ad)(c+dx^4)}+\frac{(4bc-3ad)\mathrm{Subst}\left(\int \frac{1}{c-(bc-ad)x^4}\,dx,x,\frac{x}{\sqrt[4]{a+bx^4}}\right)}{4c(bc-ad)}\\
&= -\frac{dx(a+bx^4)^{3/4}}{4c(bc-ad)(c+dx^4)}+\frac{(4bc-3ad)\mathrm{Subst}\left(\int \frac{1}{\sqrt{c}-\sqrt{bc-ad}\,x^2}\,dx,x,\frac{}{\sqrt[4]{a-}}\right.}{8c^{3/2}(bc-ad)}\\
&= -\frac{dx(a+bx^4)^{3/4}}{4c(bc-ad)(c+dx^4)}+\frac{(4bc-3ad)\tan^{-1}\left(\frac{\sqrt[4]{bc-ad}\,x}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}(bc-ad)^{5/4}}+\frac{(4bc-3ad}{}
\end{aligned}
$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.62, size = 251, normalized size = 1.55

$$
\frac{\left(\frac{1}{16}+\frac{i}{16}\right)\left(-\frac{(2-2i)c^{3/4}dx(a+bx^4)^{3/4}}{(bc-ad)(c+dx^4)}+\frac{(4bc-3ad)\tan^{-1}\left(\frac{\frac{(1-i)\sqrt[4]{bc-ad}\,x^2}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}-\frac{(1+i)\sqrt[4]{c}\sqrt[4]{a+bx^4}}{\sqrt[4]{bc-ad}}}{2x}\right)}{(bc-ad)^{5/4}}+\frac{(4bc-3ad)\tanh^{-1}\left(\frac{\frac{(1-i)\sqrt[4]{bc-ad}\,x^2}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}+\frac{(1+i)\sqrt[4]{c}\sqrt[4]{a+bx^4}}{\sqrt[4]{bc-ad}}}{2x}\right)}{(bc-ad)^{5/4}}\right)}{c^{7/4}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^4)^(1/4)*(c + d*x^4)^2),x]
```

```
[Out] ((1/16 + I/16)*(((-2 + 2*I)*c^(3/4)*d*x*(a + b*x^4)^(3/4))/((b*c - a*d)*(c
+ d*x^4)) + ((4*b*c - 3*a*d)*ArcTan[(((1 - I)*(b*c - a*d)^(1/4)*x^2)/(c^(1/
4)*(a + b*x^4)^(1/4)) - ((1 + I)*c^(1/4)*(a + b*x^4)^(1/4))/(b*c - a*d)^(1/
4))/(2*x)])/(b*c - a*d)^(5/4) + ((4*b*c - 3*a*d)*ArcTanh[(((1 - I)*(b*c - a
*d)^(1/4)*x^2)/(c^(1/4)*(a + b*x^4)^(1/4)) + ((1 + I)*c^(1/4)*(a + b*x^4)^(
1/4))/(b*c - a*d)^(1/4))/(2*x)])/(b*c - a*d)^(5/4)))/c^(7/4)
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^{4}+a\right)^{\frac{1}{4}} \left(d\,x^{4}+c\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^4+a)^(1/4)/(d*x^4+c)^2,x)
```

```
[Out] int(1/(b*x^4+a)^(1/4)/(d*x^4+c)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(1/4)/(d*x^4+c)^2,x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^4 + a)^(1/4)*(d*x^4 + c)^2), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(1/4)/(d*x^4+c)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[4]{a + bx^{4}}\, \left(c + dx^{4}\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**4+a)**(1/4)/(d*x**4+c)**2,x)
```

```
[Out] Integral(1/((a + b*x**4)**(1/4)*(c + d*x**4)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(1/4)/(d*x^4+c)^2,x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^4 + a)^(1/4)*(d*x^4 + c)^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(b\,x^4+a\right)}^{1/4}\,{\left(d\,x^4+c\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^4)^(1/4)*(c + d*x^4)^2),x)
```

```
[Out] int(1/((a + b*x^4)^(1/4)*(c + d*x^4)^2), x)
```

3.209 $\int \frac{1}{(a+bx^4)^{5/4}(c+dx^4)^2}\,dx$

Optimal. Leaf size=205

$$\frac{b(4bc+ad)x}{4ac(bc-ad)^2\sqrt[4]{a+bx^4}}-\frac{dx}{4c(bc-ad)\sqrt[4]{a+bx^4}\,(c+dx^4)}-\frac{d(8bc-3ad)\tan^{-1}\left(\frac{\sqrt[4]{bc-ad}\,x}{\sqrt[4]{c}\,\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}(bc-ad)^{9/4}}-\frac{d(8bc-\cdots}{}$$

```
[Out] 1/4*b*(a*d+4*b*c)*x/a/c/(-a*d+b*c)^2/(b*x^4+a)^(1/4)-1/4*d*x/c/(-a*d+b*c)/(
b*x^4+a)^(1/4)/(d*x^4+c)-1/8*d*(-3*a*d+8*b*c)*arctan((-a*d+b*c)^(1/4)*x/c^(
1/4)/(b*x^4+a)^(1/4))/c^(7/4)/(-a*d+b*c)^(9/4)-1/8*d*(-3*a*d+8*b*c)*arctanh
((-a*d+b*c)^(1/4)*x/c^(1/4)/(b*x^4+a)^(1/4))/c^(7/4)/(-a*d+b*c)^(9/4)
```

Rubi [A]
time = 0.13, antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {425, 541, 12, 385, 218, 214, 211}

$$-\frac{d(8bc-3ad)\text{ArcTan}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\,\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}(bc-ad)^{9/4}}-\frac{d(8bc-3ad)\tanh^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\,\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}(bc-ad)^{9/4}}+\frac{bx(ad+4bc)}{4ac\sqrt[4]{a+bx^4}\,(bc-ad)^2}-\frac{dx}{4c\sqrt[4]{a+bx^4}\,(c+dx^4)\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^4)^(5/4)*(c + d*x^4)^2),x]
```

```
[Out] (b*(4*b*c + a*d)*x)/(4*a*c*(b*c - a*d)^2*(a + b*x^4)^(1/4)) - (d*x)/(4*c*(b
*c - a*d)*(a + b*x^4)^(1/4)*(c + d*x^4)) - (d*(8*b*c - 3*a*d)*ArcTan[((b*c
- a*d)^(1/4)*x)/(c^(1/4)*(a + b*x^4)^(1/4))])/(8*c^(7/4)*(b*c - a*d)^(9/4))
 - (d*(8*b*c - 3*a*d)*ArcTanh[((b*c - a*d)^(1/4)*x)/(c^(1/4)*(a + b*x^4)^(1
/4))])/(8*c^(7/4)*(b*c - a*d)^(9/4))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{1}{(a+bx^4)^{5/4}(c+dx^4)^2}\,dx = -\frac{dx}{4c(bc-ad)\sqrt[4]{a+bx^4}\,(c+dx^4)} + \frac{\int \frac{4bc-3ad-4bdx^4}{(a+bx^4)^{5/4}(c+dx^4)}\,dx}{4c(bc-ad)}$$

$$= \frac{b(4bc+ad)x}{4ac(bc-ad)^2\sqrt[4]{a+bx^4}} - \frac{dx}{4c(bc-ad)\sqrt[4]{a+bx^4}\,(c+dx^4)} - \frac{\int \frac{ad(8bc-3ac}{\sqrt[4]{a+bx^4}\,(c}}{4ac(bc-a}$$

$$= \frac{b(4bc+ad)x}{4ac(bc-ad)^2\sqrt[4]{a+bx^4}} - \frac{dx}{4c(bc-ad)\sqrt[4]{a+bx^4}\,(c+dx^4)} - \frac{(d(8bc-3ad)}{4c}$$

$$= \frac{b(4bc+ad)x}{4ac(bc-ad)^2\sqrt[4]{a+bx^4}} - \frac{dx}{4c(bc-ad)\sqrt[4]{a+bx^4}\,(c+dx^4)} - \frac{(d(8bc-3ad)}{}$$

$$= \frac{b(4bc+ad)x}{4ac(bc-ad)^2\sqrt[4]{a+bx^4}} - \frac{dx}{4c(bc-ad)\sqrt[4]{a+bx^4}\,(c+dx^4)} - \frac{(d(8bc-3ad)}{}$$

$$= \frac{b(4bc+ad)x}{4ac(bc-ad)^2\sqrt[4]{a+bx^4}} - \frac{dx}{4c(bc-ad)\sqrt[4]{a+bx^4}\,(c+dx^4)} - \frac{d(8bc-3ad)\,\mathrm{t}}{8c^{7/}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 2.92, size = 285, normalized size = 1.39

$$\frac{\left(\frac{1}{16}+\frac{i}{16}\right)\left(\frac{(2-2i)c^{3/4}x(a^2d^2+abd^2x^4+4b^2c(c+dx^4))}{a(bc-ad)^2\sqrt[4]{a+bx^4}\,(c+dx^4)} + \frac{d(-8bc+3ad)\tan^{-1}\left(\frac{\frac{(1-i)\sqrt[4]{bc-ad}\,x^2}{\sqrt[4]{c}\sqrt[4]{a+bx^4}} - \frac{(1+i)\sqrt[4]{c}\sqrt[4]{a+bx^4}}{\sqrt[4]{bc-ad}}}{2x}\right)}{(bc-ad)^{9/4}} + \frac{d(-8bc+3ad)\tanh^{-1}\left(\frac{\frac{(1-i)\sqrt[4]{bc-ad}\,x^2}{\sqrt[4]{c}\sqrt[4]{a+bx^4}} + \frac{(1+i)\sqrt[4]{c}\sqrt[4]{a+bx^4}}{\sqrt[4]{bc-ad}}}{2x}\right)}{(bc-ad)^{9/4}}\right)}{c^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^4)^(5/4)*(c + d*x^4)^2),x]
```

```
[Out] ((1/16 + I/16)*(((2 - 2*I)*c^(3/4)*x*(a^2*d^2 + a*b*d^2*x^4 + 4*b^2*c*(c +
d*x^4)))/(a*(b*c - a*d)^2*(a + b*x^4)^(1/4)*(c + d*x^4)) + (d*(-8*b*c + 3*a
*d)*ArcTan[(((1 - I)*(b*c - a*d)^(1/4)*x^2)/(c^(1/4)*(a + b*x^4)^(1/4)) - (
(1 + I)*c^(1/4)*(a + b*x^4)^(1/4))/(b*c - a*d)^(1/4))/(2*x)])/(b*c - a*d)^(
9/4) + (d*(-8*b*c + 3*a*d)*ArcTanh[(((1 - I)*(b*c - a*d)^(1/4)*x^2)/(c^(1/4
)*(a + b*x^4)^(1/4)) + ((1 + I)*c^(1/4)*(a + b*x^4)^(1/4))/(b*c - a*d)^(1/4
))/(2*x)])/(b*c - a*d)^(9/4)))/c^(7/4)
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\,x^4+a)^{\frac{5}{4}}\,(d\,x^4+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^4+a)^(5/4)/(d*x^4+c)^2,x)
```

```
[Out] int(1/(b*x^4+a)^(5/4)/(d*x^4+c)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(5/4)/(d*x^4+c)^2,x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^4 + a)^(5/4)*(d*x^4 + c)^2), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(5/4)/(d*x^4+c)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx^4)^{\frac{5}{4}}(c+dx^4)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**4+a)**(5/4)/(d*x**4+c)**2,x)
```

```
[Out] Integral(1/((a + b*x**4)**(5/4)*(c + d*x**4)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(5/4)/(d*x^4+c)^2,x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^4 + a)^(5/4)*(d*x^4 + c)^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(b\,x^4+a\right)^{5/4}\,\left(d\,x^4+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^4)^(5/4)*(c + d*x^4)^2),x)
[Out] int(1/((a + b*x^4)^(5/4)*(c + d*x^4)^2), x)
```

3.210 $\int \frac{1}{(a+bx^4)^{9/4}(c+dx^4)^2}\,dx$

Optimal. Leaf size=266

$$\frac{b(4bc+5ad)x}{20ac(bc-ad)^2\,(a+bx^4)^{5/4}}+\frac{b(16b^2c^2-56abcd-5a^2d^2)\,x}{20a^2c(bc-ad)^3\sqrt[4]{a+bx^4}}-\frac{dx}{4c(bc-ad)\,(a+bx^4)^{5/4}\,(c+dx^4)}+\frac{3d^2(4bc-$$

```
[Out] 1/20*b*(5*a*d+4*b*c)*x/a/c/(-a*d+b*c)^2/(b*x^4+a)^(5/4)+1/20*b*(-5*a^2*d^2-
56*a*b*c*d+16*b^2*c^2)*x/a^2/c/(-a*d+b*c)^3/(b*x^4+a)^(1/4)-1/4*d*x/c/(-a*d
+b*c)/(b*x^4+a)^(5/4)/(d*x^4+c)+3/8*d^2*(-a*d+4*b*c)*arctan((-a*d+b*c)^(1/4
)*x/c^(1/4)/(b*x^4+a)^(1/4))/c^(7/4)/(-a*d+b*c)^(13/4)+3/8*d^2*(-a*d+4*b*c)
*arctanh((-a*d+b*c)^(1/4)*x/c^(1/4)/(b*x^4+a)^(1/4))/c^(7/4)/(-a*d+b*c)^(13
/4)
```

Rubi [A]

time = 0.22, antiderivative size = 266, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {425, 541, 12, 385, 218, 214, 211}

$$\frac{bx(-5a^2d^2-56abcd+16b^2c^2)}{20a^2c\sqrt[4]{a+bx^4}\,(bc-ad)^3}+\frac{3d^2(4bc-ad)\text{ArcTan}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\,\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}(bc-ad)^{13/4}}+\frac{3d^2(4bc-ad)\tanh^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\,\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}(bc-ad)^{13/4}}-\frac{dx}{4c\,(a+bx^4)^{5/4}\,(c+dx^4)\,(bc-ad)}+\frac{bx(5ad+4bc)}{20ac\,(a+bx^4)^{5/4}\,(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^4)^(9/4)*(c + d*x^4)^2),x]
```

```
[Out] (b*(4*b*c + 5*a*d)*x)/(20*a*c*(b*c - a*d)^2*(a + b*x^4)^(5/4)) + (b*(16*b^2
*c^2 - 56*a*b*c*d - 5*a^2*d^2)*x)/(20*a^2*c*(b*c - a*d)^3*(a + b*x^4)^(1/4)
) - (d*x)/(4*c*(b*c - a*d)*(a + b*x^4)^(5/4)*(c + d*x^4)) + (3*d^2*(4*b*c -
 a*d)*ArcTan[((b*c - a*d)^(1/4)*x)/(c^(1/4)*(a + b*x^4)^(1/4))])/(8*c^(7/4)
*(b*c - a*d)^(13/4)) + (3*d^2*(4*b*c - a*d)*ArcTanh[((b*c - a*d)^(1/4)*x)/(
c^(1/4)*(a + b*x^4)^(1/4))])/(8*c^(7/4)*(b*c - a*d)^(13/4))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{1}{(a+bx^4)^{9/4}(c+dx^4)^2}\,dx = -\frac{dx}{4c(bc-ad)(a+bx^4)^{5/4}(c+dx^4)} + \frac{\int \frac{4bc-3ad-8bdx^4}{(a+bx^4)^{9/4}(c+dx^4)}\,dx}{4c(bc-ad)}$$

$$= \frac{b(4bc+5ad)x}{20ac(bc-ad)^2(a+bx^4)^{5/4}} - \frac{dx}{4c(bc-ad)(a+bx^4)^{5/4}(c+dx^4)} - \frac{\int \frac{-16b^2c}{}}{}$$

$$= \frac{b(4bc+5ad)x}{20ac(bc-ad)^2(a+bx^4)^{5/4}} + \frac{b(16b^2c^2-56abcd-5a^2d^2)x}{20a^2c(bc-ad)^3\sqrt[4]{a+bx^4}} - \frac{}{4c(bc-ad)}$$

$$= \frac{b(4bc+5ad)x}{20ac(bc-ad)^2(a+bx^4)^{5/4}} + \frac{b(16b^2c^2-56abcd-5a^2d^2)x}{20a^2c(bc-ad)^3\sqrt[4]{a+bx^4}} - \frac{}{4c(bc-ad)}$$

$$= \frac{b(4bc+5ad)x}{20ac(bc-ad)^2(a+bx^4)^{5/4}} + \frac{b(16b^2c^2-56abcd-5a^2d^2)x}{20a^2c(bc-ad)^3\sqrt[4]{a+bx^4}} - \frac{}{4c(bc-ad)}$$

$$= \frac{b(4bc+5ad)x}{20ac(bc-ad)^2(a+bx^4)^{5/4}} + \frac{b(16b^2c^2-56abcd-5a^2d^2)x}{20a^2c(bc-ad)^3\sqrt[4]{a+bx^4}} - \frac{}{4c(bc-ad)}$$

$$= \frac{b(4bc+5ad)x}{20ac(bc-ad)^2(a+bx^4)^{5/4}} + \frac{b(16b^2c^2-56abcd-5a^2d^2)x}{20a^2c(bc-ad)^3\sqrt[4]{a+bx^4}} - \frac{}{4c(bc-ad)}$$

Mathematica [C] Result contains complex when optimal does not.
time = 4.68, size = 357, normalized size = 1.34

$$\frac{\left(\frac{1}{80}+\frac{i}{80}\right)\left(\frac{(2-2i)c^{3/4}x\left(5a^4d^3+10a^3bd^3x^4-16b^4c^2x^4(c+dx^4)+5a^2b^2d(12c^2+12cdx^4+d^2x^8)+4ab^3c(-5c^2+9cdx^4+14d^2x^8)\right)}{a^2(-bc+ad)^3(a+bx^4)^{5/4}(c+dx^4)} + \frac{15d^2(4bc-ad)\tan^{-1}\left(\frac{\frac{(1-i)\sqrt[4]{bc-ad}\,x^2}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}-\frac{(1+i)\sqrt[4]{c}\sqrt[4]{a+bx^4}}{\sqrt[4]{bc-ad}}}{2x}\right)}{(bc-ad)^{13/4}} + \frac{15d^2(4bc-ad)\tanh^{-1}\left(\frac{\frac{(1-i)\sqrt[4]{bc-ad}\,x^2}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}+\frac{(1+i)\sqrt[4]{c}\sqrt[4]{a+bx^4}}{\sqrt[4]{bc-ad}}}{2x}\right)}{(bc-ad)^{13/4}}\right)}{c^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^4)^(9/4)*(c + d*x^4)^2),x]
```

```
[Out] ((1/80 + I/80)*(((2 - 2*I)*c^(3/4)*x*(5*a^4*d^3 + 10*a^3*b*d^3*x^4 - 16*b^4
*c^2*x^4*(c + d*x^4) + 5*a^2*b^2*d*(12*c^2 + 12*c*d*x^4 + d^2*x^8) + 4*a*b^
3*c*(-5*c^2 + 9*c*d*x^4 + 14*d^2*x^8)))/(a^2*(-(b*c) + a*d)^3*(a + b*x^4)^(
5/4)*(c + d*x^4)) + (15*d^2*(4*b*c - a*d)*ArcTan[(((1 - I)*(b*c - a*d)^(1/4
)*x^2)/(c^(1/4)*(a + b*x^4)^(1/4)) - ((1 + I)*c^(1/4)*(a + b*x^4)^(1/4))/(b
*c - a*d)^(1/4))/(2*x)])/(b*c - a*d)^(13/4) + (15*d^2*(4*b*c - a*d)*ArcTanh
[(((1 - I)*(b*c - a*d)^(1/4)*x^2)/(c^(1/4)*(a + b*x^4)^(1/4)) + ((1 + I)*c^
```

```
(1/4)*(a + b*x^4)^(1/4))/(b*c - a*d)^(1/4))/(2*x)])/(b*c - a*d)^(13/4)))/c^
(7/4)
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^{4}+a\right)^{\frac{9}{4}} \left(d\,x^{4}+c\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^4+a)^(9/4)/(d*x^4+c)^2,x)
[Out] int(1/(b*x^4+a)^(9/4)/(d*x^4+c)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(9/4)/(d*x^4+c)^2,x, algorithm="maxima")
[Out] integrate(1/((b*x^4 + a)^(9/4)*(d*x^4 + c)^2), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(9/4)/(d*x^4+c)^2,x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+bx^{4}\right)^{\frac{9}{4}} \left(c+dx^{4}\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**4+a)**(9/4)/(d*x**4+c)**2,x)
[Out] Integral(1/((a + b*x**4)**(9/4)*(c + d*x**4)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(9/4)/(d*x^4+c)^2,x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^4 + a)^(9/4)*(d*x^4 + c)^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(b\,x^4+a\right)^{9/4}\,\left(d\,x^4+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^4)^(9/4)*(c + d*x^4)^2),x)
```

```
[Out] int(1/((a + b*x^4)^(9/4)*(c + d*x^4)^2), x)
```

3.211 $\int \frac{(a+bx^4)^{9/4}}{(c+dx^4)^2}\,dx$

Optimal. Leaf size=353

$$\frac{b(3bc-ad)x\sqrt[4]{a+bx^4}}{4cd^2}-\frac{(bc-ad)x(a+bx^4)^{5/4}}{4cd\,(c+dx^4)}-\frac{\sqrt{a}\,b^{3/2}(3bc-ad)\left(1+\frac{a}{bx^4}\right)^{3/4}x^3F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{4cd^2\,(a+bx^4)^{3/4}}$$

```
[Out] 1/4*b*(-a*d+3*b*c)*x*(b*x^4+a)^(1/4)/c/d^2-1/4*(-a*d+b*c)*x*(b*x^4+a)^(5/4)
/c/d/(d*x^4+c)-1/4*b^(3/2)*(-a*d+3*b*c)*(1+a/b/x^4)^(3/4)*x^3*(cos(1/2*arcc
ot(x^2*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arccot(x^2*b^(1/2)/a^(1/2)))*Elli
pticF(sin(1/2*arccot(x^2*b^(1/2)/a^(1/2))),2^(1/2))*a^(1/2)/c/d^2/(b*x^4+a)
^(3/4)-3/8*(-a*d+b*c)*(a*d+2*b*c)*EllipticPi(b^(1/4)*x/(b*x^4+a)^(1/4),-(-a
*d+b*c)^(1/2)/b^(1/2)/c^(1/2),I)*(a/(b*x^4+a))^(1/2)*(b*x^4+a)^(1/2)/b^(1/4
)/c^2/d^2-3/8*(-a*d+b*c)*(a*d+2*b*c)*EllipticPi(b^(1/4)*x/(b*x^4+a)^(1/4),(
-a*d+b*c)^(1/2)/b^(1/2)/c^(1/2),I)*(a/(b*x^4+a))^(1/2)*(b*x^4+a)^(1/2)/b^(1
/4)/c^2/d^2
```

Rubi [A]
time = 0.23, antiderivative size = 353, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {424, 542, 543, 243, 342, 281, 237, 416, 418, 1232}

$$-\frac{3\sqrt{\frac{a}{a+bx^4}}\,\sqrt{a+bx^4}\,(bc-ad)(ad+2bc)\Pi\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\,\sqrt{c}};\text{ArcSin}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{bx^4+a}}\right)\middle|-1\right)}{8\sqrt[4]{b}\,c^2d^2}-\frac{3\sqrt{\frac{a}{a+bx^4}}\,\sqrt{a+bx^4}\,(bc-ad)(ad+2bc)\Pi\left(\frac{\sqrt{bc-ad}}{\sqrt{b}\,\sqrt{c}};\text{ArcSin}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{bx^4+a}}\right)\middle|-1\right)}{8\sqrt[4]{b}\,c^2d^2}-\frac{\sqrt{a}\,b^{3/2}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}(3bc-ad)F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{4cd^2\,(a+bx^4)^{3/4}}+\frac{bx\sqrt[4]{a+bx^4}\,(3bc-ad)}{4cd^2}-\frac{x(a+bx^4)^{5/4}\,(bc-ad)}{4cd\,(c+dx^4)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)^(9/4)/(c + d*x^4)^2,x]
```

```
[Out] (b*(3*b*c - a*d)*x*(a + b*x^4)^(1/4))/(4*c*d^2) - ((b*c - a*d)*x*(a + b*x^4
)^(5/4))/(4*c*d*(c + d*x^4)) - (Sqrt[a]*b^(3/2)*(3*b*c - a*d)*(1 + a/(b*x^4
))^(3/4)*x^3*EllipticF[ArcCot[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/(4*c*d^2*(a + b
*x^4)^(3/4)) - (3*(b*c - a*d)*(2*b*c + a*d)*Sqrt[a/(a + b*x^4)]*Sqrt[a + b*
x^4]*EllipticPi[-(Sqrt[b*c - a*d]/(Sqrt[b]*Sqrt[c])), ArcSin[(b^(1/4)*x)/(a
 + b*x^4)^(1/4)], -1])/(8*b^(1/4)*c^2*d^2) - (3*(b*c - a*d)*(2*b*c + a*d)*S
qrt[a/(a + b*x^4)]*Sqrt[a + b*x^4]*EllipticPi[Sqrt[b*c - a*d]/(Sqrt[b]*Sqrt
[c]), ArcSin[(b^(1/4)*x)/(a + b*x^4)^(1/4)], -1])/(8*b^(1/4)*c^2*d^2)
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 243

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Dist[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4)), Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ[
{a, b}, x]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^4)^(1/4)/((c_) + (d_.)*(x_)^4), x_Symbol] :> Dist[Sq
rt[a + b*x^4]*Sqrt[a/(a + b*x^4)], Subst[Int[1/(Sqrt[1 - b*x^4]*(c - (b*c -
 a*d)*x^4)), x], x, x/(a + b*x^4)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] && N
eQ[b*c - a*d, 0]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Dist[1/(2*
c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
```

```
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 543

```
Int[((e_) + (f_.)*(x_)^4)/(((a_) + (b_.)*(x_)^4)^(3/4)*((c_) + (d_.)*(x_)^4
)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^4)^(3/4), x],
 x] - Dist[(d*e - c*f)/(b*c - a*d), Int[(a + b*x^4)^(1/4)/(c + d*x^4), x],
x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{(a+bx^4)^{9/4}}{(c+dx^4)^2}\,dx = -\frac{(bc-ad)x(a+bx^4)^{5/4}}{4cd\,(c+dx^4)} + \frac{\int \frac{\sqrt[4]{a+bx^4}\,(a(bc+3ad)+2b(3bc-ad)x^4)}{c+dx^4}\,dx}{4cd}$$

$$= \frac{b(3bc-ad)x\sqrt[4]{a+bx^4}}{4cd^2} - \frac{(bc-ad)x(a+bx^4)^{5/4}}{4cd\,(c+dx^4)} + \frac{\int \frac{-2a(3b^2c^2-2abcd-3a^2d^2)-4b(3b^2c^2-3abc}{(a+bx^4)^{3/4}(c+dx^4)}}{8cd^2}$$

$$= \frac{b(3bc-ad)x\sqrt[4]{a+bx^4}}{4cd^2} - \frac{(bc-ad)x(a+bx^4)^{5/4}}{4cd\,(c+dx^4)} + \frac{(ab(3bc-ad))\int \frac{1}{(a+bx^4)^{3/4}}\,dx}{4cd^2} - \frac{(3}{}$$

$$= \frac{b(3bc-ad)x\sqrt[4]{a+bx^4}}{4cd^2} - \frac{(bc-ad)x(a+bx^4)^{5/4}}{4cd\,(c+dx^4)} + \frac{\left(ab(3bc-ad)\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\right)\int \frac{}{(1}}{4cd^2\,(a+bx^4)^{3/4}}$$

$$= \frac{b(3bc-ad)x\sqrt[4]{a+bx^4}}{4cd^2} - \frac{(bc-ad)x(a+bx^4)^{5/4}}{4cd\,(c+dx^4)} - \frac{\left(ab(3bc-ad)\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\right)\text{Sub}}{4cd^2\,(a+bx^4}$$

$$= \frac{b(3bc-ad)x\sqrt[4]{a+bx^4}}{4cd^2} - \frac{(bc-ad)x(a+bx^4)^{5/4}}{4cd\,(c+dx^4)} - \frac{3(bc-ad)(2bc+ad)\sqrt{\frac{a}{a+bx^4}}\,\sqrt{}}{}$$

$$= \frac{b(3bc-ad)x\sqrt[4]{a+bx^4}}{4cd^2} - \frac{(bc-ad)x(a+bx^4)^{5/4}}{4cd\,(c+dx^4)} - \frac{\sqrt{a}\,b^{3/2}(3bc-ad)\left(1+\frac{a}{bx^4}\right)^{3/4}x^3F}{4cd^2\,(a+bx^4)}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 10.35, size = 392, normalized size = 1.11

$$\frac{2b(-3b^2c^2+3abcd+a^2d^2)\,x^5\left(1+\frac{bx^4}{a}\right)^{3/4}F_1\left(\frac{5}{4};\frac{3}{4},1;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)+\frac{5c\left(-5acx\left(4a^3d^2+a^2bd^2x^4+b^3cx^4(3c+2dx^4)\right)F_1\left(\frac{1}{4};\frac{3}{4},1;\frac{5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)+x^5(a+bx^4)\left(-2abcd+a^2d^2+b^2c(3c+2dx^4)\right)\left(4adF_1\left(\frac{5}{4};\frac{3}{4},2;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)+3bcF_1\left(\frac{5}{4};\frac{7}{4},1;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)\right)\right)}{(c+dx^4)\left(-5acF_1\left(\frac{1}{4};\frac{3}{4},1;\frac{5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)+x^4\left(4adF_1\left(\frac{5}{4};\frac{3}{4},2;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)+3bcF_1\left(\frac{5}{4};\frac{7}{4},1;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)\right)\right)}}{20c^2d^2\left(a+bx^4\right)^{3/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^4)^(9/4)/(c + d*x^4)^2,x]
```

```
[Out] (2*b*(-3*b^2*c^2 + 3*a*b*c*d + a^2*d^2)*x^5*(1 + (b*x^4)/a)^(3/4)*AppellF1[
5/4, 3/4, 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)] + (5*c*(-5*a*c*x*(4*a^3*d^2 +
 a^2*b*d^2*x^4 + b^3*c*x^4*(3*c + 2*d*x^4))*AppellF1[1/4, 3/4, 1, 5/4, -((b
*x^4)/a), -((d*x^4)/c)] + x^5*(a + b*x^4)*(-2*a*b*c*d + a^2*d^2 + b^2*c*(3*
c + 2*d*x^4))*(4*a*d*AppellF1[5/4, 3/4, 2, 9/4, -((b*x^4)/a), -((d*x^4)/c)]
 + 3*b*c*AppellF1[5/4, 7/4, 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)])))/((c + d*
x^4)*(-5*a*c*AppellF1[1/4, 3/4, 1, 5/4, -((b*x^4)/a), -((d*x^4)/c)] + x^4*(
4*a*d*AppellF1[5/4, 3/4, 2, 9/4, -((b*x^4)/a), -((d*x^4)/c)] + 3*b*c*Appell
F1[5/4, 7/4, 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)]))))/(20*c^2*d^2*(a + b*x^4
)^(3/4))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^{4}+a\right)^{\frac{9}{4}}}{\left(d\,x^{4}+c\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)^(9/4)/(d*x^4+c)^2,x)
```

```
[Out] int((b*x^4+a)^(9/4)/(d*x^4+c)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(9/4)/(d*x^4+c)^2,x, algorithm="maxima")
```

```
[Out] integrate((b*x^4 + a)^(9/4)/(d*x^4 + c)^2, x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^4+a)^(9/4)/(d*x^4+c)^2,x, algorithm="fricas")

[Out] Timed out

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + bx^4\right)^{\frac{9}{4}}}{\left(c + dx^4\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**4+a)**(9/4)/(d*x**4+c)**2,x)

[Out] Integral((a + b*x**4)**(9/4)/(c + d*x**4)**2, x)

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^4+a)^(9/4)/(d*x^4+c)^2,x, algorithm="giac")

[Out] integrate((b*x^4 + a)^(9/4)/(d*x^4 + c)^2, x)

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(b\,x^4+a\right)}^{9/4}}{{\left(d\,x^4+c\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*x^4)^(9/4)/(c + d*x^4)^2,x)

[Out] int((a + b*x^4)^(9/4)/(c + d*x^4)^2, x)

3.212 $\int \frac{(a+bx^4)^{5/4}}{(c+dx^4)^2}\,dx$

Optimal. Leaf size=298

$$-\frac{(bc-ad)x\sqrt[4]{a+bx^4}}{4cd\,(c+dx^4)}+\frac{\sqrt{a}\,b^{3/2}\left(1+\frac{a}{bx^4}\right)^{3/4}x^3F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{4cd\,(a+bx^4)^{3/4}}+\frac{(2bc+3ad)\sqrt{\frac{a}{a+bx^4}}\,\sqrt{a+bx^4}}{}$$

```
[Out] -1/4*(-a*d+b*c)*x*(b*x^4+a)^(1/4)/c/d/(d*x^4+c)+1/4*b^(3/2)*(1+a/b/x^4)^(3/
4)*x^3*(cos(1/2*arccot(x^2*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arccot(x^2*b^
(1/2)/a^(1/2)))*EllipticF(sin(1/2*arccot(x^2*b^(1/2)/a^(1/2))),2^(1/2))*a^(
1/2)/c/d/(b*x^4+a)^(3/4)+1/8*(3*a*d+2*b*c)*EllipticPi(b^(1/4)*x/(b*x^4+a)^(
1/4),-(-a*d+b*c)^(1/2)/b^(1/2)/c^(1/2),I)*(a/(b*x^4+a))^(1/2)*(b*x^4+a)^(1/
2)/b^(1/4)/c^2/d+1/8*(3*a*d+2*b*c)*EllipticPi(b^(1/4)*x/(b*x^4+a)^(1/4),(-a
*d+b*c)^(1/2)/b^(1/2)/c^(1/2),I)*(a/(b*x^4+a))^(1/2)*(b*x^4+a)^(1/2)/b^(1/4
)/c^2/d
```

Rubi [A]
time = 0.15, antiderivative size = 298, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {424, 543, 243, 342, 281, 237, 416, 418, 1232}

$$\frac{\sqrt{\frac{a}{a+bx^4}}\,\sqrt{a+bx^4}\,(3ad+2bc)\Pi\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\,\sqrt{c}};\text{ArcSin}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{bx^4+a}}\right)\middle|-1\right)}{8\sqrt[4]{b}\,c^2d}+\frac{\sqrt{\frac{a}{a+bx^4}}\,\sqrt{a+bx^4}\,(3ad+2bc)\Pi\left(\frac{\sqrt{bc-ad}}{\sqrt{b}\,\sqrt{c}};\text{ArcSin}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{bx^4+a}}\right)\middle|-1\right)}{8\sqrt[4]{b}\,c^2d}+\frac{\sqrt{a}\,b^{3/2}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{4cd\,(a+bx^4)^{3/4}}-\frac{x\sqrt[4]{a+bx^4}\,(bc-ad)}{4cd\,(c+dx^4)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)^(5/4)/(c + d*x^4)^2,x]
```

```
[Out] -1/4*((b*c - a*d)*x*(a + b*x^4)^(1/4))/(c*d*(c + d*x^4)) + (Sqrt[a]*b^(3/2)
*(1 + a/(b*x^4))^(3/4)*x^3*EllipticF[ArcCot[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/(
4*c*d*(a + b*x^4)^(3/4)) + ((2*b*c + 3*a*d)*Sqrt[a/(a + b*x^4)]*Sqrt[a + b*
x^4]*EllipticPi[-(Sqrt[b*c - a*d]/(Sqrt[b]*Sqrt[c])), ArcSin[(b^(1/4)*x)/(a
 + b*x^4)^(1/4)], -1])/(8*b^(1/4)*c^2*d) + ((2*b*c + 3*a*d)*Sqrt[a/(a + b*x
^4)]*Sqrt[a + b*x^4]*EllipticPi[Sqrt[b*c - a*d]/(Sqrt[b]*Sqrt[c]), ArcSin[(
b^(1/4)*x)/(a + b*x^4)^(1/4)], -1])/(8*b^(1/4)*c^2*d)
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 243

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Dist[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4)), Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ[
```

{a, b}, x]

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^4)^(1/4)/((c_) + (d_.)*(x_)^4), x_Symbol] :> Dist[Sq
rt[a + b*x^4]*Sqrt[a/(a + b*x^4)], Subst[Int[1/(Sqrt[1 - b*x^4]*(c - (b*c -
 a*d)*x^4)), x], x, x/(a + b*x^4)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] && N
eQ[b*c - a*d, 0]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Dist[1/(2*
c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 543

```
Int[((e_) + (f_.)*(x_)^4)/(((a_) + (b_.)*(x_)^4)^(3/4)*((c_) + (d_.)*(x_)^4
)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^4)^(3/4), x],
 x] - Dist[(d*e - c*f)/(b*c - a*d), Int[(a + b*x^4)^(1/4)/(c + d*x^4), x],
x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{(a+bx^4)^{5/4}}{(c+dx^4)^2}\,dx = -\frac{(bc-ad)x\sqrt[4]{a+bx^4}}{4cd\,(c+dx^4)} + \frac{\int \frac{a(bc+3ad)+2b(bc+ad)x^4}{(a+bx^4)^{3/4}(c+dx^4)}\,dx}{4cd}$$

$$= -\frac{(bc-ad)x\sqrt[4]{a+bx^4}}{4cd\,(c+dx^4)} - \frac{(ab)\int \frac{1}{(a+bx^4)^{3/4}}\,dx}{4cd} - \frac{(-2bc-3ad)\int \frac{\sqrt[4]{a+bx^4}}{c+dx^4}\,dx}{4cd}$$

$$= -\frac{(bc-ad)x\sqrt[4]{a+bx^4}}{4cd\,(c+dx^4)} - \frac{\left(ab\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\right)\int \frac{1}{\left(1+\frac{a}{bx^4}\right)^{3/4}x^3}\,dx}{4cd\,(a+bx^4)^{3/4}} - \frac{\left((-2bc-3ad)\sqrt{\frac{\ }{a}\ \cdots}\right.}{\cdots}$$

$$= -\frac{(bc-ad)x\sqrt[4]{a+bx^4}}{4cd\,(c+dx^4)} + \frac{\left(ab\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\right)\text{Subst}\left(\int \frac{x}{\left(1+\frac{ax^4}{b}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)}{4cd\,(a+bx^4)^{3/4}} - \frac{\left((-2b\cdots\right.}{\cdots}$$

$$= -\frac{(bc-ad)x\sqrt[4]{a+bx^4}}{4cd\,(c+dx^4)} + \frac{(2bc+3ad)\sqrt{\frac{a}{a+bx^4}}\,\sqrt{a+bx^4}\,\Pi\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\,\sqrt{c}};\sin^{-1}\left(\cdots\right.\right.}{8\sqrt[4]{b}\,c^2d}$$

$$= -\frac{(bc-ad)x\sqrt[4]{a+bx^4}}{4cd\,(c+dx^4)} + \frac{\sqrt{a}\,b^{3/2}\left(1+\frac{a}{bx^4}\right)^{3/4}x^3F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{4cd\,(a+bx^4)^{3/4}} + \frac{(2bc+3\cdots}{\cdots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 10.25, size = 341, normalized size = 1.14

$$\frac{x\left(2b(bc+ad)x^4\left(1+\frac{bx^4}{a}\right)^{3/4}F_1\left(\frac{5}{4};\frac{3}{4},1;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)+\frac{5c\left(-5ac(4a^2d-b^2cx^4+abdx^4)F_1\left(\frac{1}{4};\frac{3}{4},1;\frac{5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)+(-bc+ad)x^4(a+bx^4)\left(4adF_1\left(\frac{5}{4};\frac{3}{4},2;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)+3bcF_1\left(\frac{5}{4};\frac{7}{4},1;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)\right)\right)}{(c+dx^4)\left(-5acF_1\left(\frac{1}{4};\frac{3}{4},1;\frac{5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)+x^4\left(4adF_1\left(\frac{5}{4};\frac{3}{4},2;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)+3bcF_1\left(\frac{5}{4};\frac{7}{4},1;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)\right)\right)}\right)}{20c^2d\,(a+bx^4)^{3/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^4)^(5/4)/(c + d*x^4)^2,x]
```

```
[Out] (x*(2*b*(b*c + a*d)*x^4*(1 + (b*x^4)/a)^(3/4)*AppellF1[5/4, 3/4, 1, 9/4, -(
(b*x^4)/a), -((d*x^4)/c)] + (5*c*(-5*a*c*(4*a^2*d - b^2*c*x^4 + a*b*d*x^4)*
AppellF1[1/4, 3/4, 1, 5/4, -((b*x^4)/a), -((d*x^4)/c)] + (-(b*c) + a*d)*x^4
```

```
*(a + b*x^4)*(4*a*d*AppellF1[5/4, 3/4, 2, 9/4, -((b*x^4)/a), -((d*x^4)/c)]
+ 3*b*c*AppellF1[5/4, 7/4, 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)])))/((c + d*x
^4)*(-5*a*c*AppellF1[1/4, 3/4, 1, 5/4, -((b*x^4)/a), -((d*x^4)/c)] + x^4*(4
*a*d*AppellF1[5/4, 3/4, 2, 9/4, -((b*x^4)/a), -((d*x^4)/c)] + 3*b*c*AppellF
1[5/4, 7/4, 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)]))))/(20*c^2*d*(a + b*x^4)^
(3/4))
```

Maple [**F**]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^{4}+a\right)^{\frac{5}{4}}}{\left(d\,x^{4}+c\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)^(5/4)/(d*x^4+c)^2,x)
```

```
[Out] int((b*x^4+a)^(5/4)/(d*x^4+c)^2,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(5/4)/(d*x^4+c)^2,x, algorithm="maxima")
```

```
[Out] integrate((b*x^4 + a)^(5/4)/(d*x^4 + c)^2, x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(5/4)/(d*x^4+c)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^{4}\right)^{\frac{5}{4}}}{\left(c+dx^{4}\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)**(5/4)/(d*x**4+c)**2,x)
```

```
[Out] Integral((a + b*x**4)**(5/4)/(c + d*x**4)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(5/4)/(d*x^4+c)^2,x, algorithm="giac")
[Out] integrate((b*x^4 + a)^(5/4)/(d*x^4 + c)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(b\,x^4+a\right)}^{5/4}}{{\left(d\,x^4+c\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)^(5/4)/(c + d*x^4)^2,x)
[Out] int((a + b*x^4)^(5/4)/(c + d*x^4)^2, x)
```

3.213 $\int \frac{\sqrt[4]{a+bx^4}}{(c+dx^4)^2} \, dx$

Optimal. Leaf size=308

$$\frac{x\sqrt[4]{a+bx^4}}{4c\,(c+dx^4)}-\frac{\sqrt{a}\,b^{3/2}\left(1+\frac{a}{bx^4}\right)^{3/4}x^3F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{4c(bc-ad)\,(a+bx^4)^{3/4}}+\frac{(2bc-3ad)\sqrt{\frac{a}{a+bx^4}}\,\sqrt{a+bx^4}\,\Pi\left(-\frac{\sqrt{bc-}}{\sqrt{b}}\right.}{8\sqrt[4]{b}\,c^2(bc-a}$$

[Out] 1/4*x*(b*x^4+a)^(1/4)/c/(d*x^4+c)-1/4*b^(3/2)*(1+a/b/x^4)^(3/4)*x^3*(cos(1/2*arccot(x^2*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arccot(x^2*b^(1/2)/a^(1/2)))*EllipticF(sin(1/2*arccot(x^2*b^(1/2)/a^(1/2))),2^(1/2))*a^(1/2)/c/(-a*d+b*c)/(b*x^4+a)^(3/4)+1/8*(-3*a*d+2*b*c)*EllipticPi(b^(1/4)*x/(b*x^4+a)^(1/4),-(-a*d+b*c)^(1/2)/b^(1/2)/c^(1/2),I)*(a/(b*x^4+a))^(1/2)*(b*x^4+a)^(1/2)/b^(1/4)/c^2/(-a*d+b*c)+1/8*(-3*a*d+2*b*c)*EllipticPi(b^(1/4)*x/(b*x^4+a)^(1/4),(-a*d+b*c)^(1/2)/b^(1/2)/c^(1/2),I)*(a/(b*x^4+a))^(1/2)*(b*x^4+a)^(1/2)/b^(1/4)/c^2/(-a*d+b*c)

Rubi [A]
time = 0.13, antiderivative size = 308, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {423, 543, 243, 342, 281, 237, 416, 418, 1232}

$$\frac{\sqrt{\frac{a}{a+bx^4}}\,\sqrt{a+bx^4}\,(2bc-3ad)\Pi\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\,\sqrt{c}};\text{ArcSin}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{bx^4+a}}\right)\middle|-1\right)}{8\sqrt[4]{b}\,c^2(bc-ad)}+\frac{\sqrt{\frac{a}{a+bx^4}}\,\sqrt{a+bx^4}\,(2bc-3ad)\Pi\left(\frac{\sqrt{bc-ad}}{\sqrt{b}\,\sqrt{c}};\text{ArcSin}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{bx^4+a}}\right)\middle|-1\right)}{8\sqrt[4]{b}\,c^2(bc-ad)}-\frac{\sqrt{a}\,b^{3/2}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{4c\,(a+bx^4)^{3/4}\,(bc-ad)}+\frac{x\sqrt[4]{a+bx^4}}{4c\,(c+dx^4)}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x^4)^(1/4)/(c + d*x^4)^2,x]

[Out] (x*(a + b*x^4)^(1/4))/(4*c*(c + d*x^4)) - (Sqrt[a]*b^(3/2)*(1 + a/(b*x^4))^(3/4)*x^3*EllipticF[ArcCot[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/(4*c*(b*c - a*d)*(a + b*x^4)^(3/4)) + ((2*b*c - 3*a*d)*Sqrt[a/(a + b*x^4)]*Sqrt[a + b*x^4]*EllipticPi[-(Sqrt[b*c - a*d]/(Sqrt[b]*Sqrt[c])), ArcSin[(b^(1/4)*x)/(a + b*x^4)^(1/4)], -1])/(8*b^(1/4)*c^2*(b*c - a*d)) + ((2*b*c - 3*a*d)*Sqrt[a/(a + b*x^4)]*Sqrt[a + b*x^4]*EllipticPi[Sqrt[b*c - a*d]/(Sqrt[b]*Sqrt[c]), ArcSin[(b^(1/4)*x)/(a + b*x^4)^(1/4)], -1])/(8*b^(1/4)*c^2*(b*c - a*d))

Rule 237

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2]))*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b/a]
```

Rule 243

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Dist[x^3*((1 + a/(b*x^4))^(3/4)/(a + b*x^4)^(3/4)), Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ[
```

```
{a, b}, x]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^4)^(1/4)/((c_) + (d_.)*(x_)^4), x_Symbol] :> Dist[Sq
rt[a + b*x^4]*Sqrt[a/(a + b*x^4)], Subst[Int[1/(Sqrt[1 - b*x^4]*(c - (b*c -
 a*d)*x^4)), x], x, x/(a + b*x^4)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] && N
eQ[b*c - a*d, 0]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Dist[1/(2*
c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 423

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] + Dist[1
/(a*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(n*(p +
1) + 1) + d*(n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x
] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 543

```
Int[((e_) + (f_.)*(x_)^4)/(((a_) + (b_.)*(x_)^4)^(3/4)*((c_) + (d_.)*(x_)^4
)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^4)^(3/4), x],
 x] - Dist[(d*e - c*f)/(b*c - a*d), Int[(a + b*x^4)^(1/4)/(c + d*x^4), x],
x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt[4]{a+bx^4}}{(c+dx^4)^2}\,dx &= \frac{x\sqrt[4]{a+bx^4}}{4c\,(c+dx^4)} - \frac{\int \frac{-3a-2bx^4}{(a+bx^4)^{3/4}(c+dx^4)}\,dx}{4c} \\
&= \frac{x\sqrt[4]{a+bx^4}}{4c\,(c+dx^4)} + \frac{(ab)\int \frac{1}{(a+bx^4)^{3/4}}\,dx}{4c(bc-ad)} + \frac{(2bc-3ad)\int \frac{\sqrt[4]{a+bx^4}}{c+dx^4}\,dx}{4c(bc-ad)} \\
&= \frac{x\sqrt[4]{a+bx^4}}{4c\,(c+dx^4)} + \frac{\left(ab\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\right)\int \frac{1}{\left(1+\frac{a}{bx^4}\right)^{3/4}x^3}\,dx}{4c(bc-ad)\,(a+bx^4)^{3/4}} + \frac{\left((2bc-3ad)\sqrt{\frac{a}{a+bx^4}}\,\sqrt{a+bx\right.}{} \\
&= \frac{x\sqrt[4]{a+bx^4}}{4c\,(c+dx^4)} - \frac{\left(ab\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\right)\mathrm{Subst}\left(\int \frac{x}{\left(1+\frac{ax^4}{b}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)}{4c(bc-ad)\,(a+bx^4)^{3/4}} + \frac{\left((2bc-3ad)\sqrt{\frac{a}{a+\right.}}{} \\
&= \frac{x\sqrt[4]{a+bx^4}}{4c\,(c+dx^4)} + \frac{(2bc-3ad)\sqrt{\frac{a}{a+bx^4}}\,\sqrt{a+bx^4}\,\Pi\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\,\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right)\right|-}{8\sqrt[4]{b}\,c^2(bc-ad)} \\
&= \frac{x\sqrt[4]{a+bx^4}}{4c\,(c+dx^4)} - \frac{\sqrt{a}\,b^{3/2}\left(1+\frac{a}{bx^4}\right)^{3/4}x^3F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{4c(bc-ad)\,(a+bx^4)^{3/4}} + \frac{(2bc-3ad)\sqrt{\frac{a}{a+bx^4}}}{}
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 10.16, size = 233, normalized size = 0.76

$$
\frac{x\left(\frac{2bx^4\left(1+\frac{bx^4}{a}\right)^{3/4}F_1\left(\frac{5}{4};\frac{3}{4},1;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{c^2}+\frac{5\left(\frac{a+bx^4}{c}-\frac{15a^2F_1\left(\frac{1}{4};\frac{3}{4},1;\frac{5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{-5acF_1\left(\frac{1}{4};\frac{3}{4},1;\frac{5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)+x^4\left(4adF_1\left(\frac{5}{4};\frac{3}{4},2;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)+3bcF_1\left(\frac{5}{4};\frac{7}{4},1;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)\right)}\right)}{c+dx^4}\right)}{20\,(a+bx^4)^{3/4}}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^4)^(1/4)/(c + d*x^4)^2,x]
```

```
[Out] (x*((2*b*x^4*(1 + (b*x^4)/a)^(3/4)*AppellF1[5/4, 3/4, 1, 9/4, -((b*x^4)/a),
 -((d*x^4)/c)])/c^2 + (5*((a + b*x^4)/c - (15*a^2*AppellF1[1/4, 3/4, 1, 5/4
```

```
, -((b*x^4)/a), -((d*x^4)/c)])/(-5*a*c*AppellF1[1/4, 3/4, 1, 5/4, -((b*x^4)
/a), -((d*x^4)/c)] + x^4*(4*a*d*AppellF1[5/4, 3/4, 2, 9/4, -((b*x^4)/a), -(
(d*x^4)/c)] + 3*b*c*AppellF1[5/4, 7/4, 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)])
)))/(c + d*x^4)))/(20*(a + b*x^4)^(3/4))
```

Maple [**F**]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^{4}+a\right)^{\frac{1}{4}}}{\left(d\,x^{4}+c\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)^(1/4)/(d*x^4+c)^2,x)
[Out] int((b*x^4+a)^(1/4)/(d*x^4+c)^2,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(1/4)/(d*x^4+c)^2,x, algorithm="maxima")
[Out] integrate((b*x^4 + a)^(1/4)/(d*x^4 + c)^2, x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(1/4)/(d*x^4+c)^2,x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[4]{a+bx^{4}}}{\left(c+dx^{4}\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)**(1/4)/(d*x**4+c)**2,x)
[Out] Integral((a + b*x**4)**(1/4)/(c + d*x**4)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(1/4)/(d*x^4+c)^2,x, algorithm="giac")
[Out] integrate((b*x^4 + a)^(1/4)/(d*x^4 + c)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(b\,x^4+a\right)}^{1/4}}{{\left(d\,x^4+c\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)^(1/4)/(c + d*x^4)^2,x)
[Out] int((a + b*x^4)^(1/4)/(c + d*x^4)^2, x)
```

3.214 $\int \frac{1}{(a+bx^4)^{3/4}(c+dx^4)^2}\,dx$

Optimal. Leaf size=330

$$-\frac{dx\sqrt[4]{a+bx^4}}{4c(bc-ad)\left(c+dx^4\right)}-\frac{b^{3/2}(4bc-ad)\left(1+\frac{a}{bx^4}\right)^{3/4}x^3F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{4\sqrt{a}\,c(bc-ad)^2\left(a+bx^4\right)^{3/4}}-\frac{3d(2bc-ad)\sqrt{\frac{a}{a+bx^4}}}{}$$

```
[Out] -1/4*d*x*(b*x^4+a)^(1/4)/c/(-a*d+b*c)/(d*x^4+c)-1/4*b^(3/2)*(-a*d+4*b*c)*(1
+a/b/x^4)^(3/4)*x^3*(cos(1/2*arccot(x^2*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*
arccot(x^2*b^(1/2)/a^(1/2)))*EllipticF(sin(1/2*arccot(x^2*b^(1/2)/a^(1/2)))
,2^(1/2))/c/(-a*d+b*c)^2/(b*x^4+a)^(3/4)/a^(1/2)-3/8*d*(-a*d+2*b*c)*Ellipti
cPi(b^(1/4)*x/(b*x^4+a)^(1/4),-(-a*d+b*c)^(1/2)/b^(1/2)/c^(1/2),I)*(a/(b*x^
4+a))^(1/2)*(b*x^4+a)^(1/2)/b^(1/4)/c^2/(-a*d+b*c)^2-3/8*d*(-a*d+2*b*c)*Ell
ipticPi(b^(1/4)*x/(b*x^4+a)^(1/4),(-a*d+b*c)^(1/2)/b^(1/2)/c^(1/2),I)*(a/(b
*x^4+a))^(1/2)*(b*x^4+a)^(1/2)/b^(1/4)/c^2/(-a*d+b*c)^2
```

Rubi [A]
time = 0.16, antiderivative size = 330, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {425, 543, 243, 342, 281, 237, 416, 418, 1232}

$$-\frac{3d\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}(2bc-ad)\Pi\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}};\text{ArcSin}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{bx^4+a}}\right)\middle|-1\right)}{8\sqrt[4]{b}\,c^2(bc-ad)^2}-\frac{3d\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}(2bc-ad)\Pi\left(\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}};\text{ArcSin}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{bx^4+a}}\right)\middle|-1\right)}{8\sqrt[4]{b}\,c^2(bc-ad)^2}-\frac{b^{3/2}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}(4bc-ad)F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{4\sqrt{a}\,c\left(a+bx^4\right)^{3/4}(bc-ad)^2}-\frac{dx\sqrt[4]{a+bx^4}}{4c\left(c+dx^4\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^4)^(3/4)*(c + d*x^4)^2),x]
```

```
[Out] -1/4*(d*x*(a + b*x^4)^(1/4))/(c*(b*c - a*d)*(c + d*x^4)) - (b^(3/2)*(4*b*c
- a*d)*(1 + a/(b*x^4))^(3/4)*x^3*EllipticF[ArcCot[(Sqrt[b]*x^2)/Sqrt[a]]/2,
 2])/(4*Sqrt[a]*c*(b*c - a*d)^2*(a + b*x^4)^(3/4)) - (3*d*(2*b*c - a*d)*Sqr
t[a/(a + b*x^4)]*Sqrt[a + b*x^4]*EllipticPi[-(Sqrt[b*c - a*d]/(Sqrt[b]*Sqrt
[c])), ArcSin[(b^(1/4)*x)/(a + b*x^4)^(1/4)], -1])/(8*b^(1/4)*c^2*(b*c - a*
d)^2) - (3*d*(2*b*c - a*d)*Sqrt[a/(a + b*x^4)]*Sqrt[a + b*x^4]*EllipticPi[S
qrt[b*c - a*d]/(Sqrt[b]*Sqrt[c]), ArcSin[(b^(1/4)*x)/(a + b*x^4)^(1/4)], -1
])/(8*b^(1/4)*c^2*(b*c - a*d)^2)
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 243

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Dist[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4)), Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ[
```

```
{a, b}, x]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^4)^(1/4)/((c_) + (d_.)*(x_)^4), x_Symbol] :> Dist[Sq
rt[a + b*x^4]*Sqrt[a/(a + b*x^4)], Subst[Int[1/(Sqrt[1 - b*x^4]*(c - (b*c -
 a*d)*x^4)), x], x, x/(a + b*x^4)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] && N
eQ[b*c - a*d, 0]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Dist[1/(2*
c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 543

```
Int[((e_) + (f_.)*(x_)^4)/(((a_) + (b_.)*(x_)^4)^(3/4)*((c_) + (d_.)*(x_)^4
)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^4)^(3/4), x],
 x] - Dist[(d*e - c*f)/(b*c - a*d), Int[(a + b*x^4)^(1/4)/(c + d*x^4), x],
x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{(a+bx^4)^{3/4}(c+dx^4)^2}\,dx = -\frac{dx\sqrt[4]{a+bx^4}}{4c(bc-ad)(c+dx^4)} + \frac{\int \frac{4bc-3ad-2bdx^4}{(a+bx^4)^{3/4}(c+dx^4)}\,dx}{4c(bc-ad)}$$

$$= -\frac{dx\sqrt[4]{a+bx^4}}{4c(bc-ad)(c+dx^4)} - \frac{(3d(2bc-ad))\int \frac{\sqrt[4]{a+bx^4}}{c+dx^4}\,dx}{4c(bc-ad)^2} + \frac{(b(4bc-ad))\int}{4c(bc-}$$

$$= -\frac{dx\sqrt[4]{a+bx^4}}{4c(bc-ad)(c+dx^4)} + \frac{\left(b(4bc-ad)\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\right)\int \frac{1}{\left(1+\frac{a}{bx^4}\right)^{3/4}x^3}\,dx}{4c(bc-ad)^2(a+bx^4)^{3/4}} -$$

$$= -\frac{dx\sqrt[4]{a+bx^4}}{4c(bc-ad)(c+dx^4)} - \frac{\left(b(4bc-ad)\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\right)\text{Subst}\left(\int \frac{x}{\left(1+\frac{ax^4}{b}\right)^{3/4}}\right.}{4c(bc-ad)^2(a+bx^4)^{3/4}}$$

$$= -\frac{dx\sqrt[4]{a+bx^4}}{4c(bc-ad)(c+dx^4)} - \frac{3d(2bc-ad)\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,\Pi\left(-\frac{\sqrt{bc-a}}{\sqrt{b}\sqrt{c}}\right.}{8\sqrt[4]{b}\,c^2(bc-ad)^2}$$

$$= -\frac{dx\sqrt[4]{a+bx^4}}{4c(bc-ad)(c+dx^4)} - \frac{b^{3/2}(4bc-ad)\left(1+\frac{a}{bx^4}\right)^{3/4}x^3F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right.\right.}{4\sqrt{a}\,c(bc-ad)^2(a+bx^4)^{3/4}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 10.25, size = 337, normalized size = 1.02

$$\frac{x\left(\frac{2bdx^4\left(1+\frac{bx^4}{a}\right)^{3/4}F_1\left(\frac{5}{4};\frac{3}{4},1;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{-bc+ad}+\frac{c\left(25ac(-4bc+4ad+bdx^4)F_1\left(\frac{1}{4};\frac{3}{4},1;\frac{5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)-5dx^4(a+bx^4)\left(4adF_1\left(\frac{5}{4};\frac{3}{4},2;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)+3bcF_1\left(\frac{5}{4};\frac{7}{4},1;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)\right)\right)}{(bc-ad)(c+dx^4)\left(-5acF_1\left(\frac{1}{4};\frac{3}{4},1;\frac{5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)+x^4\left(4adF_1\left(\frac{5}{4};\frac{3}{4},2;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)+3bcF_1\left(\frac{5}{4};\frac{7}{4},1;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)\right)\right)}\right)}{20c^2(a+bx^4)^{3/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^4)^(3/4)*(c + d*x^4)^2),x]
```

```
[Out] (x*((2*b*d*x^4*(1 + (b*x^4)/a)^(3/4)*AppellF1[5/4, 3/4, 1, 9/4, -((b*x^4)/a
), -((d*x^4)/c)])/(-(b*c) + a*d) + (c*(25*a*c*(-4*b*c + 4*a*d + b*d*x^4)*Ap
pellF1[1/4, 3/4, 1, 5/4, -((b*x^4)/a), -((d*x^4)/c)] - 5*d*x^4*(a + b*x^4)*
```

```
(4*a*d*AppellF1[5/4, 3/4, 2, 9/4, -((b*x^4)/a), -((d*x^4)/c)] + 3*b*c*Appel
lF1[5/4, 7/4, 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)])))/((b*c - a*d)*(c + d*x^
4)*(-5*a*c*AppellF1[1/4, 3/4, 1, 5/4, -((b*x^4)/a), -((d*x^4)/c)] + x^4*(4*
a*d*AppellF1[5/4, 3/4, 2, 9/4, -((b*x^4)/a), -((d*x^4)/c)] + 3*b*c*AppellF1
[5/4, 7/4, 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)]))))/(20*c^2*(a + b*x^4)^(3/
4))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^{4}+a\right)^{\frac{3}{4}} \left(d\,x^{4}+c\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^4+a)^(3/4)/(d*x^4+c)^2,x)
[Out] int(1/(b*x^4+a)^(3/4)/(d*x^4+c)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(3/4)/(d*x^4+c)^2,x, algorithm="maxima")
[Out] integrate(1/((b*x^4 + a)^(3/4)*(d*x^4 + c)^2), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(3/4)/(d*x^4+c)^2,x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + bx^{4}\right)^{\frac{3}{4}} \left(c + dx^{4}\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**4+a)**(3/4)/(d*x**4+c)**2,x)
```

```
[Out] Integral(1/((a + b*x**4)**(3/4)*(c + d*x**4)**2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(3/4)/(d*x^4+c)^2,x, algorithm="giac")
[Out] integrate(1/((b*x^4 + a)^(3/4)*(d*x^4 + c)^2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(b\,x^4+a\right)}^{3/4}\,{\left(d\,x^4+c\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^4)^(3/4)*(c + d*x^4)^2),x)
[Out] int(1/((a + b*x^4)^(3/4)*(c + d*x^4)^2), x)
```

3.215 $\int \frac{1}{\left(a+bx^4\right)^{7/4}\left(c+dx^4\right)^2}\,dx$

Optimal. Leaf size=390

$$\frac{b(4bc+3ad)x}{12ac(bc-ad)^2\left(a+bx^4\right)^{3/4}}-\frac{dx}{4c(bc-ad)\left(a+bx^4\right)^{3/4}\left(c+dx^4\right)}-\frac{b^{3/2}\left(8b^2c^2-32abcd+3a^2d^2\right)\left(1+\frac{a}{bx^4}\right)^{3/4}x^3}{12a^{3/2}c(bc-ad)^3\left(a+b\right.}$$

```
[Out] 1/12*b*(3*a*d+4*b*c)*x/a/c/(-a*d+b*c)^2/(b*x^4+a)^(3/4)-1/4*d*x/c/(-a*d+b*c
)/(b*x^4+a)^(3/4)/(d*x^4+c)-1/12*b^(3/2)*(3*a^2*d^2-32*a*b*c*d+8*b^2*c^2)*(
1+a/b/x^4)^(3/4)*x^3*(cos(1/2*arccot(x^2*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2
*arccot(x^2*b^(1/2)/a^(1/2)))*EllipticF(sin(1/2*arccot(x^2*b^(1/2)/a^(1/2))
),2^(1/2))/a^(3/2)/c/(-a*d+b*c)^3/(b*x^4+a)^(3/4)+1/8*d^2*(-3*a*d+10*b*c)*E
llipticPi(b^(1/4)*x/(b*x^4+a)^(1/4),-(-a*d+b*c)^(1/2)/b^(1/2)/c^(1/2),I)*(a
/(b*x^4+a))^(1/2)*(b*x^4+a)^(1/2)/b^(1/4)/c^2/(-a*d+b*c)^3+1/8*d^2*(-3*a*d+
10*b*c)*EllipticPi(b^(1/4)*x/(b*x^4+a)^(1/4),(-a*d+b*c)^(1/2)/b^(1/2)/c^(1/
2),I)*(a/(b*x^4+a))^(1/2)*(b*x^4+a)^(1/2)/b^(1/4)/c^2/(-a*d+b*c)^3
```

Rubi [A]

time = 0.35, antiderivative size = 390, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {425, 541, 543, 243, 342, 281, 237, 416, 418, 1232}

$$-\frac{b^{3/2}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}\left(3a^2d^2-32abcd+8b^2c^2\right)F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{12a^{3/2}c\left(a+bx^4\right)^{3/4}(bc-ad)^3}+\frac{d^2\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,(10bc-3ad)\Pi\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}};\text{ArcSin}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{bx^4+a}}\right)\middle|-1\right)}{8\sqrt[4]{b}\,c^2(bc-ad)^3}+\frac{d^2\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,(10bc-3ad)\Pi\left(\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}};\text{ArcSin}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{bx^4+a}}\right)\middle|-1\right)}{8\sqrt[4]{b}\,c^2(bc-ad)^3}+\frac{bx(3ad+4bc)}{12ac\left(a+bx^4\right)^{3/4}(bc-ad)^2}-\frac{dx}{4c\left(a+bx^4\right)^{3/4}\left(c+dx^4\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^4)^(7/4)*(c + d*x^4)^2),x]
```

```
[Out] (b*(4*b*c + 3*a*d)*x)/(12*a*c*(b*c - a*d)^2*(a + b*x^4)^(3/4)) - (d*x)/(4*c
*(b*c - a*d)*(a + b*x^4)^(3/4)*(c + d*x^4)) - (b^(3/2)*(8*b^2*c^2 - 32*a*b*
c*d + 3*a^2*d^2)*(1 + a/(b*x^4))^(3/4)*x^3*EllipticF[ArcCot[(Sqrt[b]*x^2)/S
qrt[a]]/2, 2])/(12*a^(3/2)*c*(b*c - a*d)^3*(a + b*x^4)^(3/4)) + (d^2*(10*b*
c - 3*a*d)*Sqrt[a/(a + b*x^4)]*Sqrt[a + b*x^4]*EllipticPi[-(Sqrt[b*c - a*d]
/(Sqrt[b]*Sqrt[c])), ArcSin[(b^(1/4)*x)/(a + b*x^4)^(1/4)], -1])/(8*b^(1/4)
*c^2*(b*c - a*d)^3) + (d^2*(10*b*c - 3*a*d)*Sqrt[a/(a + b*x^4)]*Sqrt[a + b*
x^4]*EllipticPi[Sqrt[b*c - a*d]/(Sqrt[b]*Sqrt[c]), ArcSin[(b^(1/4)*x)/(a +
b*x^4)^(1/4)], -1])/(8*b^(1/4)*c^2*(b*c - a*d)^3)
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 243

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Dist[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4)), Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ[
{a, b}, x]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^4)^(1/4)/((c_) + (d_.)*(x_)^4), x_Symbol] :> Dist[Sq
rt[a + b*x^4]*Sqrt[a/(a + b*x^4)], Subst[Int[1/(Sqrt[1 - b*x^4]*(c - (b*c -
 a*d)*x^4)), x], x, x/(a + b*x^4)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] && N
eQ[b*c - a*d, 0]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Dist[1/(2*
c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
```

```
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 543

```
Int[((e_) + (f_.)*(x_)^4)/(((a_) + (b_.)*(x_)^4)^(3/4)*((c_) + (d_.)*(x_)^4
)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^4)^(3/4), x],
 x] - Dist[(d*e - c*f)/(b*c - a*d), Int[(a + b*x^4)^(1/4)/(c + d*x^4), x],
x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{\left(a+bx^4\right)^{7/4}\left(c+dx^4\right)^2}\,dx = -\frac{dx}{4c(bc-ad)\left(a+bx^4\right)^{3/4}\left(c+dx^4\right)} + \frac{\int \frac{4bc-3ad-6bdx^4}{\left(a+bx^4\right)^{7/4}\left(c+dx^4\right)}\,dx}{4c(bc-ad)}$$

$$= \frac{b(4bc+3ad)x}{12ac(bc-ad)^2\left(a+bx^4\right)^{3/4}} - \frac{dx}{4c(bc-ad)\left(a+bx^4\right)^{3/4}\left(c+dx^4\right)} - \frac{\int \frac{-8b^2c^2+\cdots}{\cdots}}{\cdots}$$

$$= \frac{b(4bc+3ad)x}{12ac(bc-ad)^2\left(a+bx^4\right)^{3/4}} - \frac{dx}{4c(bc-ad)\left(a+bx^4\right)^{3/4}\left(c+dx^4\right)} + \frac{\left(d^2(10bc\cdots\right.}{\cdots}$$

$$= \frac{b(4bc+3ad)x}{12ac(bc-ad)^2\left(a+bx^4\right)^{3/4}} - \frac{dx}{4c(bc-ad)\left(a+bx^4\right)^{3/4}\left(c+dx^4\right)} + \frac{\left(b(8b^2c^2\cdots\right.}{\cdots}$$

$$= \frac{b(4bc+3ad)x}{12ac(bc-ad)^2\left(a+bx^4\right)^{3/4}} - \frac{dx}{4c(bc-ad)\left(a+bx^4\right)^{3/4}\left(c+dx^4\right)} - \frac{\left(b(8b^2c^2\cdots\right.}{\cdots}$$

$$= \frac{b(4bc+3ad)x}{12ac(bc-ad)^2\left(a+bx^4\right)^{3/4}} - \frac{dx}{4c(bc-ad)\left(a+bx^4\right)^{3/4}\left(c+dx^4\right)} + \frac{d^2(10bc-\cdots}{\cdots}$$

$$= \frac{b(4bc+3ad)x}{12ac(bc-ad)^2\left(a+bx^4\right)^{3/4}} - \frac{dx}{4c(bc-ad)\left(a+bx^4\right)^{3/4}\left(c+dx^4\right)} - \frac{b^{3/2}(8b^2c^2\cdots}{\cdots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 10.37, size = 387, normalized size = 0.99

$$\frac{x\left(2bd(4bc+3ad)x^4\left(1+\frac{bx^4}{a}\right)^{3/4}F_1\left(\frac{5}{4};\frac{3}{4},1;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)+\frac{c\left(25ac(12a^2d^2+3abd(-8c+dx^4)+4b^2c(3c+dx^4))F_1\left(\frac{1}{4};\frac{3}{4},1;\frac{5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)-5x^4(3a^2d^2+3abd^2x^4+4b^2c(c+dx^4))\left(4adF_1\left(\frac{5}{4};\frac{3}{4},2;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)+3bcF_1\left(\frac{5}{4};\frac{7}{4},1;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)\right)\right)}{(c+dx^4)\left(5acF_1\left(\frac{1}{4};\frac{3}{4},1;\frac{5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)-x^4\left(4adF_1\left(\frac{5}{4};\frac{3}{4},2;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)+3bcF_1\left(\frac{5}{4};\frac{7}{4},1;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)\right)\right)}\right)}{60ac^2(bc-ad)^2\left(a+bx^4\right)^{3/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^4)^(7/4)*(c + d*x^4)^2),x]
```

```
[Out] (x*(2*b*d*(4*b*c + 3*a*d)*x^4*(1 + (b*x^4)/a)^(3/4)*AppellF1[5/4, 3/4, 1, 9
/4, -((b*x^4)/a), -((d*x^4)/c)] + (c*(25*a*c*(12*a^2*d^2 + 3*a*b*d*(-8*c +
d*x^4) + 4*b^2*c*(3*c + d*x^4))*AppellF1[1/4, 3/4, 1, 5/4, -((b*x^4)/a), -(
(d*x^4)/c)] - 5*x^4*(3*a^2*d^2 + 3*a*b*d^2*x^4 + 4*b^2*c*(c + d*x^4))*(4*a*
d*AppellF1[5/4, 3/4, 2, 9/4, -((b*x^4)/a), -((d*x^4)/c)] + 3*b*c*AppellF1[5
/4, 7/4, 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)])))/((c + d*x^4)*(5*a*c*AppellF
1[1/4, 3/4, 1, 5/4, -((b*x^4)/a), -((d*x^4)/c)] - x^4*(4*a*d*AppellF1[5/4,
3/4, 2, 9/4, -((b*x^4)/a), -((d*x^4)/c)] + 3*b*c*AppellF1[5/4, 7/4, 1, 9/4,
 -((b*x^4)/a), -((d*x^4)/c)])))))/(60*a*c^2*(b*c - a*d)^2*(a + b*x^4)^(3/4)
)
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^{4}+a\right)^{\frac{7}{4}}\left(d\,x^{4}+c\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^4+a)^(7/4)/(d*x^4+c)^2,x)
```

```
[Out] int(1/(b*x^4+a)^(7/4)/(d*x^4+c)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(7/4)/(d*x^4+c)^2,x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^4 + a)^(7/4)*(d*x^4 + c)^2), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(7/4)/(d*x^4+c)^2,x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + bx^{4}\right)^{\frac{7}{4}} \left(c + dx^{4}\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**4+a)**(7/4)/(d*x**4+c)**2,x)
[Out] Integral(1/((a + b*x**4)**(7/4)*(c + d*x**4)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(7/4)/(d*x^4+c)^2,x, algorithm="giac")
[Out] integrate(1/((b*x^4 + a)^(7/4)*(d*x^4 + c)^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(b\,x^4+a\right)}^{7/4}\,{\left(d\,x^4+c\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^4)^(7/4)*(c + d*x^4)^2),x)
[Out] int(1/((a + b*x^4)^(7/4)*(c + d*x^4)^2), x)
```

3.216 $\int \frac{1}{\sqrt[4]{1+x^4}\,(2+x^4)}\,dx$

Optimal. Leaf size=53

$$\frac{\tan^{-1}\left(\frac{x}{\sqrt[4]{2}\,\sqrt[4]{1+x^4}}\right)}{2\,2^{3/4}}+\frac{\tanh^{-1}\left(\frac{x}{\sqrt[4]{2}\,\sqrt[4]{1+x^4}}\right)}{2\,2^{3/4}}$$

```
[Out] 1/4*arctan(1/2*x*2^(3/4)/(x^4+1)^(1/4))*2^(1/4)+1/4*arctanh(1/2*x*2^(3/4)/(
x^4+1)^(1/4))*2^(1/4)
```

Rubi [A]

time = 0.01, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {385, 218, 212, 209}

$$\frac{\text{ArcTan}\left(\frac{x}{\sqrt[4]{2}\,\sqrt[4]{x^4+1}}\right)}{2\,2^{3/4}}+\frac{\tanh^{-1}\left(\frac{x}{\sqrt[4]{2}\,\sqrt[4]{x^4+1}}\right)}{2\,2^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 + x^4)^(1/4)*(2 + x^4)),x]
```

```
[Out] ArcTan[x/(2^(1/4)*(1 + x^4)^(1/4))]/(2*2^(3/4)) + ArcTanh[x/(2^(1/4)*(1 + x
^4)^(1/4))]/(2*2^(3/4))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\sqrt[4]{1+x^4}\,(2+x^4)}\,dx &= \text{Subst}\left(\int \frac{1}{2-x^4}\,dx, x, \frac{x}{\sqrt[4]{1+x^4}}\right)\\
&= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{2}-x^2}\,dx, x, \frac{x}{\sqrt[4]{1+x^4}}\right)}{2\sqrt{2}} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{2}+x^2}\,dx, x, \frac{x}{\sqrt[4]{1+x^4}}\right)}{2\sqrt{2}}\\
&= \frac{\tan^{-1}\left(\frac{x}{\sqrt[4]{2}\,\sqrt[4]{1+x^4}}\right)}{2\,2^{3/4}} + \frac{\tanh^{-1}\left(\frac{x}{\sqrt[4]{2}\,\sqrt[4]{1+x^4}}\right)}{2\,2^{3/4}}
\end{aligned}
$$

Mathematica [A]
time = 0.18, size = 44, normalized size = 0.83

$$
\frac{\tan^{-1}\left(\frac{x}{\sqrt[4]{2}\,\sqrt[4]{1+x^4}}\right)+\tanh^{-1}\left(\frac{x}{\sqrt[4]{2}\,\sqrt[4]{1+x^4}}\right)}{2\,2^{3/4}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 + x^4)^(1/4)*(2 + x^4)),x]
```

```
[Out] (ArcTan[x/(2^(1/4)*(1 + x^4)^(1/4))] + ArcTanh[x/(2^(1/4)*(1 + x^4)^(1/4))]
)/(2*2^(3/4))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 1.58, size = 211, normalized size = 3.98

method	result
trager	$-\frac{\text{RootOf}\left(\_Z^2+\text{RootOf}\left(\_Z^4-2\right)^2\right)\ln\left(\frac{2\sqrt{x^4+1}\,\text{RootOf}\left(\_Z^4-2\right)^2\text{RootOf}\left(\_Z^2+\text{RootOf}\left(\_Z^4-2\right)^2\right)x^2-2\left(x^4+1\right)^{\frac{1}{4}}\text{Roo}}{}\right.}{8}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4+1)^(1/4)/(x^4+2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*RootOf(_Z^2+RootOf(_Z^4-2)^2)*ln((2*(x^4+1)^(1/2)*RootOf(_Z^4-2)^2*Roo
tOf(_Z^2+RootOf(_Z^4-2)^2)*x^2-2*(x^4+1)^(1/4)*RootOf(_Z^4-2)^2*x^3-3*RootO
```

```
f(_Z^2+RootOf(_Z^4-2)^2)*x^4+4*(x^4+1)^(3/4)*x-2*RootOf(_Z^2+RootOf(_Z^4-2)
^2))/(x^4+2))-1/8*RootOf(_Z^4-2)*ln(-(-2*(x^4+1)^(1/2)*RootOf(_Z^4-2)^3*x^2
+2*(x^4+1)^(1/4)*RootOf(_Z^4-2)^2*x^3-3*RootOf(_Z^4-2)*x^4+4*(x^4+1)^(3/4)*
x-2*RootOf(_Z^4-2))/(x^4+2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+1)^(1/4)/(x^4+2),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^4 + 2)*(x^4 + 1)^(1/4)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 208 vs. $2(39) = 78$.
time = 9.17, size = 208, normalized size = 3.92

$$-\frac{1}{16}\cdot 8^{\frac{3}{4}}\arctan\left(-\frac{8^{\frac{3}{4}}(x^4+1)^{\frac{1}{4}}x^3+4\cdot 8^{\frac{1}{4}}(x^4+1)^{\frac{3}{4}}x-2^{\frac{1}{4}}\left(8^{\frac{3}{4}}\sqrt{x^4+1}\,x^2+8^{\frac{1}{4}}(3x^4+2)\right)}{2(x^4+2)}\right)+\frac{1}{64}\cdot 8^{\frac{3}{4}}\log\left(\frac{8\sqrt{2}(x^4+1)^{\frac{1}{4}}x^3+8\cdot 8^{\frac{1}{4}}\sqrt{x^4+1}\,x^2+8^{\frac{3}{4}}(3x^4+2)+16(x^4+1)^{\frac{3}{4}}x}{x^4+2}\right)-\frac{1}{64}\cdot 8^{\frac{3}{4}}\log\left(\frac{8\sqrt{2}(x^4+1)^{\frac{1}{4}}x^3-8\cdot 8^{\frac{1}{4}}\sqrt{x^4+1}\,x^2-8^{\frac{3}{4}}(3x^4+2)+16(x^4+1)^{\frac{3}{4}}x}{x^4+2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+1)^(1/4)/(x^4+2),x, algorithm="fricas")
```

```
[Out] -1/16*8^(3/4)*arctan(-1/2*(8^(3/4)*(x^4 + 1)^(1/4)*x^3 + 4*8^(1/4)*(x^4 + 1
)^(3/4)*x - 2^(1/4)*(8^(3/4)*sqrt(x^4 + 1)*x^2 + 8^(1/4)*(3*x^4 + 2)))/(x^4
 + 2)) + 1/64*8^(3/4)*log((8*sqrt(2)*(x^4 + 1)^(1/4)*x^3 + 8*8^(1/4)*sqrt(x
^4 + 1)*x^2 + 8^(3/4)*(3*x^4 + 2) + 16*(x^4 + 1)^(3/4)*x)/(x^4 + 2)) - 1/64
*8^(3/4)*log((8*sqrt(2)*(x^4 + 1)^(1/4)*x^3 - 8*8^(1/4)*sqrt(x^4 + 1)*x^2 -
 8^(3/4)*(3*x^4 + 2) + 16*(x^4 + 1)^(3/4)*x)/(x^4 + 2))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[4]{x^4+1}\,(x^4+2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**4+1)**(1/4)/(x**4+2),x)
```

```
[Out] Integral(1/((x**4 + 1)**(1/4)*(x**4 + 2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+1)^(1/4)/(x^4+2),x, algorithm="giac")
[Out] integrate(1/((x^4 + 2)*(x^4 + 1)^(1/4)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{(x^4+1)^{1/4}\,(x^4+2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x^4 + 1)^(1/4)*(x^4 + 2)),x)
[Out] int(1/((x^4 + 1)^(1/4)*(x^4 + 2)), x)
```

3.217 $\int \frac{1}{(a-(a-b)x^4)\sqrt[4]{a+bx^4}}\,dx$

Optimal. Leaf size=57

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{a+bx^4}}\right)}{2a^{5/4}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{a+bx^4}}\right)}{2a^{5/4}}$$

[Out] 1/2*arctan(a^(1/4)*x/(b*x^4+a)^(1/4))/a^(5/4)+1/2*arctanh(a^(1/4)*x/(b*x^4+a)^(1/4))/a^(5/4)

Rubi [A]
time = 0.01, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {385, 218, 212, 209}

$$\frac{\text{ArcTan}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{a+bx^4}}\right)}{2a^{5/4}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{a+bx^4}}\right)}{2a^{5/4}}$$

Antiderivative was successfully verified.

[In] Int[1/((a - (a - b)*x^4)*(a + b*x^4)^(1/4)),x]

[Out] ArcTan[(a^(1/4)*x)/(a + b*x^4)^(1/4)]/(2*a^(5/4)) + ArcTanh[(a^(1/4)*x)/(a + b*x^4)^(1/4)]/(2*a^(5/4))

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a-(a-b)x^4)\sqrt[4]{a+bx^4}}\,dx &= \text{Subst}\left(\int \frac{1}{a-(ab-a(-a+b))x^4}\,dx, x, \frac{x}{\sqrt[4]{a+bx^4}}\right)\\ &= \frac{\text{Subst}\left(\int \frac{1}{1-\sqrt{a}\,x^2}\,dx, x, \frac{x}{\sqrt[4]{a+bx^4}}\right)}{2a} + \frac{\text{Subst}\left(\int \frac{1}{1+\sqrt{a}\,x^2}\,dx, x, \frac{x}{\sqrt[4]{a+bx^4}}\right)}{2a}\\ &= \frac{\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{a+bx^4}}\right)}{2a^{5/4}} + \frac{\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{a+bx^4}}\right)}{2a^{5/4}}\end{aligned}$$

Mathematica [**A**]
time = 0.31, size = 48, normalized size = 0.84

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{a+bx^4}}\right)+\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{a+bx^4}}\right)}{2a^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a - (a - b)*x^4)*(a + b*x^4)^(1/4)),x]
[Out] (ArcTan[(a^(1/4)*x)/(a + b*x^4)^(1/4)] + ArcTanh[(a^(1/4)*x)/(a + b*x^4)^(1
/4)])/(2*a^(5/4))
```

Maple [**F**]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{(a-(a-b)\,x^4)\,(b\,x^4+a)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-(a-b)*x^4)/(b*x^4+a)^(1/4),x)
[Out] int(1/(a-(a-b)*x^4)/(b*x^4+a)^(1/4),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-(a-b)*x^4)/(b*x^4+a)^(1/4),x, algorithm="maxima")
```

```
[Out] -integrate(1/(((a - b)*x^4 - a)*(b*x^4 + a)^(1/4)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-(a-b)*x^4)/(b*x^4+a)^(1/4),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{ax^4\sqrt[4]{a+bx^4}-a\sqrt[4]{a+bx^4}-bx^4\sqrt[4]{a+bx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-(a-b)*x**4)/(b*x**4+a)**(1/4),x)
```

```
[Out] -Integral(1/(a*x**4*(a + b*x**4)**(1/4) - a*(a + b*x**4)**(1/4) - b*x**4*(a
 + b*x**4)**(1/4)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-(a-b)*x^4)/(b*x^4+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(-1/(((a - b)*x^4 - a)*(b*x^4 + a)^(1/4)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{{\left(b\,x^4+a\right)}^{1/4}\,\left(a-x^4\,\left(a-b\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^4)^(1/4)*(a - x^4*(a - b))),x)
```

```
[Out] int(1/((a + b*x^4)^(1/4)*(a - x^4*(a - b))), x)
```

3.218 $\int (a+bx^4)^p (c+dx^4)^q \, dx$

Optimal. Leaf size=79

$$x(a+bx^4)^p \left(1+\frac{bx^4}{a}\right)^{-p} (c+dx^4)^q \left(1+\frac{dx^4}{c}\right)^{-q} F_1\left(\frac{1}{4};-p,-q;\frac{5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)$$

```
[Out] x*(b*x^4+a)^p*(d*x^4+c)^q*AppellF1(1/4,-p,-q,5/4,-b*x^4/a,-d*x^4/c)/((1+b*x
^4/a)^p)/((1+d*x^4/c)^q)
```

Rubi [A]
time = 0.03, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {441, 440}

$$x(a+bx^4)^p \left(\frac{bx^4}{a}+1\right)^{-p} (c+dx^4)^q \left(\frac{dx^4}{c}+1\right)^{-q} F_1\left(\frac{1}{4};-p,-q;\frac{5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)^p*(c + d*x^4)^q,x]
```

```
[Out] (x*(a + b*x^4)^p*(c + d*x^4)^q*AppellF1[1/4, -p, -q, 5/4, -((b*x^4)/a), -((
d*x^4)/c)])/((1 + (b*x^4)/a)^p*(1 + (d*x^4)/c)^q)
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \left(a+bx^4\right)^p \left(c+dx^4\right)^q \,dx = \left(\left(a+bx^4\right)^p \left(1+\frac{bx^4}{a}\right)^{-p}\right) \int \left(1+\frac{bx^4}{a}\right)^p \left(c+dx^4\right)^q \,dx$$

$$= \left(\left(a+bx^4\right)^p \left(1+\frac{bx^4}{a}\right)^{-p} \left(c+dx^4\right)^q \left(1+\frac{dx^4}{c}\right)^{-q}\right) \int \left(1+\frac{bx^4}{a}\right)^p \left(1+ \right.$$

$$= x\left(a+bx^4\right)^p \left(1+\frac{bx^4}{a}\right)^{-p} \left(c+dx^4\right)^q \left(1+\frac{dx^4}{c}\right)^{-q} F_1\left(\frac{1}{4};-p,-q;\frac{5}{4};-\frac{bx^4}{a}, \right.$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 172 vs. 2(79) = 158.
time = 0.20, size = 172, normalized size = 2.18

$$\frac{5acx(a+bx^4)^p \left(c+dx^4\right)^q F_1\left(\frac{1}{4};-p,-q;\frac{5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{5acF_1\left(\frac{1}{4};-p,-q;\frac{5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)+4x^4\left(bcpF_1\left(\frac{5}{4};1-p,-q;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)+adqF_1\left(\frac{5}{4};-p,1-q;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^4)^p*(c + d*x^4)^q,x]
```

```
[Out] (5*a*c*x*(a + b*x^4)^p*(c + d*x^4)^q*AppellF1[1/4, -p, -q, 5/4, -((b*x^4)/a
), -((d*x^4)/c)])/(5*a*c*AppellF1[1/4, -p, -q, 5/4, -((b*x^4)/a), -((d*x^4)
/c)] + 4*x^4*(b*c*p*AppellF1[5/4, 1 - p, -q, 9/4, -((b*x^4)/a), -((d*x^4)/c
)] + a*d*q*AppellF1[5/4, -p, 1 - q, 9/4, -((b*x^4)/a), -((d*x^4)/c)]))
```

Maple [F]
time = 0.10, size = 0, normalized size = 0.00

$$\int \left(b\,x^4+a\right)^p \left(d\,x^4+c\right)^q \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)^p*(d*x^4+c)^q,x)
```

```
[Out] int((b*x^4+a)^p*(d*x^4+c)^q,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^p*(d*x^4+c)^q,x, algorithm="maxima")
```

```
[Out] integrate((b*x^4 + a)^p*(d*x^4 + c)^q, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^p*(d*x^4+c)^q,x, algorithm="fricas")
[Out] integral((b*x^4 + a)^p*(d*x^4 + c)^q, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)**p*(d*x**4+c)**q,x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^p*(d*x^4+c)^q,x, algorithm="giac")
[Out] integrate((b*x^4 + a)^p*(d*x^4 + c)^q, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(b\,x^4+a\right)}^p\,{\left(d\,x^4+c\right)}^q \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)^p*(c + d*x^4)^q,x)
[Out] int((a + b*x^4)^p*(c + d*x^4)^q, x)
```

3.219 $\int \left(a+bx^4\right)^2 \left(c+dx^4\right)^q \, dx$

Optimal. Leaf size=176

$$-\frac{b(5bc-ad(13+4q))x\left(c+dx^4\right)^{1+q}}{d^2(5+4q)(9+4q)}+\frac{bx\left(a+bx^4\right)\left(c+dx^4\right)^{1+q}}{d(9+4q)}+\frac{\left(5b^2c^2-2abcd(9+4q)+a^2d^2(45+56q+\right.}{d^2(5}$$

```
[Out] -b*(5*b*c-a*d*(13+4*q))*x*(d*x^4+c)^(1+q)/d^2/(16*q^2+56*q+45)+b*x*(b*x^4+a
)*(d*x^4+c)^(1+q)/d/(9+4*q)+(5*b^2*c^2-2*a*b*c*d*(9+4*q)+a^2*d^2*(16*q^2+56
*q+45))*x*(d*x^4+c)^q*hypergeom([1/4, -q],[5/4],-d*x^4/c)/d^2/(16*q^2+56*q+
45)/((1+d*x^4/c)^q)
```

Rubi [A]
time = 0.09, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {427, 396, 252, 251}

$$\frac{x\left(c+dx^4\right)^q \left(\frac{dx^4}{c}+1\right)^{-q} \left(a^2d^2\left(16q^2+56q+45\right)-2abcd(4q+9)+5b^2c^2\right) {}_2F_1\left(\frac{1}{4},-q;\frac{5}{4};-\frac{dx^4}{c}\right)}{d^2(4q+5)(4q+9)}-\frac{bx\left(c+dx^4\right)^{q+1}(5bc-ad(4q+13))}{d^2(4q+5)(4q+9)}+\frac{bx\left(a+bx^4\right)\left(c+dx^4\right)^{q+1}}{d(4q+9)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)^2*(c + d*x^4)^q,x]
```

```
[Out] -((b*(5*b*c - a*d*(13 + 4*q))*x*(c + d*x^4)^(1 + q))/(d^2*(5 + 4*q)*(9 + 4*
q))) + (b*x*(a + b*x^4)*(c + d*x^4)^(1 + q))/(d*(9 + 4*q)) + ((5*b^2*c^2 -
2*a*b*c*d*(9 + 4*q) + a^2*d^2*(45 + 56*q + 16*q^2))*x*(c + d*x^4)^q*Hyperge
ometric2F1[1/4, -q, 5/4, -((d*x^4)/c)])/(d^2*(5 + 4*q)*(9 + 4*q)*(1 + (d*x^
4)/c)^q)
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
```

```
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rubi steps

$$\int (a+bx^4)^2 (c+dx^4)^q \, dx = \frac{bx(a+bx^4)(c+dx^4)^{1+q}}{d(9+4q)} + \frac{\int (c+dx^4)^q (-a(bc-ad(9+4q)) - b(5bc-ad(13$$

$$= -\frac{b(5bc-ad(13+4q))x(c+dx^4)^{1+q}}{d^2(5+4q)(9+4q)} + \frac{bx(a+bx^4)(c+dx^4)^{1+q}}{d(9+4q)} + \frac{(5b^2c^2-2$$

$$= -\frac{b(5bc-ad(13+4q))x(c+dx^4)^{1+q}}{d^2(5+4q)(9+4q)} + \frac{bx(a+bx^4)(c+dx^4)^{1+q}}{d(9+4q)} + \frac{\Big((5b^2c^2-$$

$$= -\frac{b(5bc-ad(13+4q))x(c+dx^4)^{1+q}}{d^2(5+4q)(9+4q)} + \frac{bx(a+bx^4)(c+dx^4)^{1+q}}{d(9+4q)} + \frac{(5b^2c^2-2$$

Mathematica [A]
time = 0.19, size = 106, normalized size = 0.60

$$\frac{1}{45}x(c+dx^4)^q \left(1+\frac{dx^4}{c}\right)^{-q} \left(45a^2\,{}_2F_1\left(\frac{1}{4},-q;\frac{5}{4};-\frac{dx^4}{c}\right)+bx^4\left(18a\,{}_2F_1\left(\frac{5}{4},-q;\frac{9}{4};-\frac{dx^4}{c}\right)+5bx^4\,{}_2F_1\left(\frac{9}{4},-q;\frac{13}{4};-\frac{dx^4}{c}\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^4)^2*(c + d*x^4)^q,x]
```

```
[Out] (x*(c + d*x^4)^q*(45*a^2*Hypergeometric2F1[1/4, -q, 5/4, -((d*x^4)/c)] + b*
x^4*(18*a*Hypergeometric2F1[5/4, -q, 9/4, -((d*x^4)/c)] + 5*b*x^4*Hypergeom
etric2F1[9/4, -q, 13/4, -((d*x^4)/c)])))/(45*(1 + (d*x^4)/c)^q)
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \left(b\,x^4+a\right)^2 \left(d\,x^4+c\right)^q\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)^2*(d*x^4+c)^q,x)
[Out] int((b*x^4+a)^2*(d*x^4+c)^q,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^2*(d*x^4+c)^q,x, algorithm="maxima")
[Out] integrate((b*x^4 + a)^2*(d*x^4 + c)^q, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^2*(d*x^4+c)^q,x, algorithm="fricas")
[Out] integral((b^2*x^8 + 2*a*b*x^4 + a^2)*(d*x^4 + c)^q, x)
```

Sympy [C] Result contains complex when optimal does not.
time = 83.88, size = 119, normalized size = 0.68

$$\frac{a^2c^qx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},-q\\ \frac{5}{4}\end{matrix}\middle|\frac{dx^4e^{i\pi}}{c}\right)}{4\Gamma\left(\frac{5}{4}\right)}+\frac{abc^qx^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{5}{4},-q\\ \frac{9}{4}\end{matrix}\middle|\frac{dx^4e^{i\pi}}{c}\right)}{2\Gamma\left(\frac{9}{4}\right)}+\frac{b^2c^qx^9\Gamma\left(\frac{9}{4}\right){}_2F_1\left(\begin{matrix}\frac{9}{4},-q\\ \frac{13}{4}\end{matrix}\middle|\frac{dx^4e^{i\pi}}{c}\right)}{4\Gamma\left(\frac{13}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)**2*(d*x**4+c)**q,x)
[Out] a**2*c**q*x*gamma(1/4)*hyper((1/4, -q), (5/4,), d*x**4*exp_polar(I*pi)/c)/(
4*gamma(5/4)) + a*b*c**q*x**5*gamma(5/4)*hyper((5/4, -q), (9/4,), d*x**4*ex
p_polar(I*pi)/c)/(2*gamma(9/4)) + b**2*c**q*x**9*gamma(9/4)*hyper((9/4, -q)
, (13/4,), d*x**4*exp_polar(I*pi)/c)/(4*gamma(13/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^2*(d*x^4+c)^q,x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)^2*(d*x^4 + c)^q, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(b\,x^4+a\right)^2\,\left(d\,x^4+c\right)^q\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)^2*(c + d*x^4)^q,x)
```

```
[Out] int((a + b*x^4)^2*(c + d*x^4)^q, x)
```

3.220 $\int (a+bx^4)(c+dx^4)^q \, dx$

Optimal. Leaf size=93

$$\frac{bx(c+dx^4)^{1+q}}{d(5+4q)} - \frac{(bc-ad(5+4q))x(c+dx^4)^q\left(1+\frac{dx^4}{c}\right)^{-q} {}_2F_1\left(\frac{1}{4},-q;\frac{5}{4};-\frac{dx^4}{c}\right)}{d(5+4q)}$$

```
[Out] b*x*(d*x^4+c)^(1+q)/d/(5+4*q)-(b*c-a*d*(5+4*q))*x*(d*x^4+c)^q*hypergeom([1/
4, -q],[5/4],-d*x^4/c)/d/(5+4*q)/((1+d*x^4/c)^q)
```

Rubi [A]

time = 0.03, antiderivative size = 85, normalized size of antiderivative = 0.91, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {396, 252, 251}

$$x(c+dx^4)^q\left(\frac{dx^4}{c}+1\right)^{-q}\left(a-\frac{bc}{4dq+5d}\right) {}_2F_1\left(\frac{1}{4},-q;\frac{5}{4};-\frac{dx^4}{c}\right)+\frac{bx(c+dx^4)^{q+1}}{d(4q+5)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)*(c + d*x^4)^q,x]
```

```
[Out] (b*x*(c + d*x^4)^(1 + q))/(d*(5 + 4*q)) + ((a - (b*c)/(5*d + 4*d*q))*x*(c +
 d*x^4)^q*Hypergeometric2F1[1/4, -q, 5/4, -((d*x^4)/c)])/(1 + (d*x^4)/c)^q
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rubi steps

$$\int (a+bx^4)(c+dx^4)^q\,dx = \frac{bx(c+dx^4)^{1+q}}{d(5+4q)} - \left(-a+\frac{bc}{5d+4dq}\right)\int (c+dx^4)^q\,dx$$
$$= \frac{bx(c+dx^4)^{1+q}}{d(5+4q)} - \left(\left(-a+\frac{bc}{5d+4dq}\right)(c+dx^4)^q\left(1+\frac{dx^4}{c}\right)^{-q}\right)\int \left(1+\frac{dx^4}{c}\right.$$
$$= \frac{bx(c+dx^4)^{1+q}}{d(5+4q)} + \left(a-\frac{bc}{5d+4dq}\right)x(c+dx^4)^q\left(1+\frac{dx^4}{c}\right)^{-q} {}_2F_1\left(\frac{1}{4},-q;\frac{5}{4};-\right.$$

Mathematica [A]
time = 0.13, size = 90, normalized size = 0.97

$$\frac{x(c+dx^4)^q\left(1+\frac{dx^4}{c}\right)^{-q}\left(b(c+dx^4)\left(1+\frac{dx^4}{c}\right)^q+(-bc+ad(5+4q))\,{}_2F_1\left(\frac{1}{4},-q;\frac{5}{4};-\frac{dx^4}{c}\right)\right)}{d(5+4q)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^4)*(c + d*x^4)^q,x]
[Out] (x*(c + d*x^4)^q*(b*(c + d*x^4)*(1 + (d*x^4)/c)^q + (-(b*c) + a*d*(5 + 4*q)
)*Hypergeometric2F1[1/4, -q, 5/4, -((d*x^4)/c)]))/(d*(5 + 4*q)*(1 + (d*x^4)
/c)^q)
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \left(b\,x^4+a\right)\left(d\,x^4+c\right)^q\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)*(d*x^4+c)^q,x)
[Out] int((b*x^4+a)*(d*x^4+c)^q,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)*(d*x^4+c)^q,x, algorithm="maxima")
[Out] integrate((b*x^4 + a)*(d*x^4 + c)^q, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)*(d*x^4+c)^q,x, algorithm="fricas")
```

```
[Out] integral((b*x^4 + a)*(d*x^4 + c)^q, x)
```

Sympy [C] Result contains complex when optimal does not.
time = 30.54, size = 75, normalized size = 0.81

$$\frac{ac^{q}x\Gamma\left(\frac{1}{4}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{4},-q\\\frac{5}{4}\end{matrix}\middle|\frac{dx^{4}e^{i\pi}}{c}\right)}{4\Gamma\left(\frac{5}{4}\right)}+\frac{bc^{q}x^{5}\Gamma\left(\frac{5}{4}\right){}_{2}F_{1}\left(\begin{matrix}\frac{5}{4},-q\\\frac{9}{4}\end{matrix}\middle|\frac{dx^{4}e^{i\pi}}{c}\right)}{4\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)*(d*x**4+c)**q,x)
```

```
[Out] a*c**q*x*gamma(1/4)*hyper((1/4, -q), (5/4,), d*x**4*exp_polar(I*pi)/c)/(4*g
amma(5/4)) + b*c**q*x**5*gamma(5/4)*hyper((5/4, -q), (9/4,), d*x**4*exp_pol
ar(I*pi)/c)/(4*gamma(9/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)*(d*x^4+c)^q,x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)*(d*x^4 + c)^q, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(b\,x^4+a\right)\,{\left(d\,x^4+c\right)}^q\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)*(c + d*x^4)^q,x)
```

```
[Out] int((a + b*x^4)*(c + d*x^4)^q, x)
```

3.221 $\int \frac{(c+dx^4)^q}{a+bx^4}\,dx$

Optimal. Leaf size=57

$$\frac{x(c+dx^4)^q\left(1+\frac{dx^4}{c}\right)^{-q}F_1\left(\frac{1}{4};1,-q;\frac{5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{a}$$

```
[Out] x*(d*x^4+c)^q*AppellF1(1/4,1,-q,5/4,-b*x^4/a,-d*x^4/c)/a/((1+d*x^4/c)^q)
```

Rubi [A]
time = 0.02, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {441, 440}

$$\frac{x(c+dx^4)^q\left(\frac{dx^4}{c}+1\right)^{-q}F_1\left(\frac{1}{4};1,-q;\frac{5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^4)^q/(a + b*x^4),x]
```

```
[Out] (x*(c + d*x^4)^q*AppellF1[1/4, 1, -q, 5/4, -((b*x^4)/a), -((d*x^4)/c)])/(a*
(1 + (d*x^4)/c)^q)
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx^4)^q}{a+bx^4}\,dx &= \left((c+dx^4)^q\left(1+\frac{dx^4}{c}\right)^{-q}\right)\int \frac{\left(1+\frac{dx^4}{c}\right)^q}{a+bx^4}\,dx\\ &= \frac{x(c+dx^4)^q\left(1+\frac{dx^4}{c}\right)^{-q}F_1\left(\frac{1}{4};1,-q;\frac{5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{a}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 162 vs. 2(57) = 114.
time = 0.32, size = 162, normalized size = 2.84

$$\frac{5acx(c+dx^4)^q F_1\left(\frac{1}{4};-q,1;\frac{5}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)}{(a+bx^4)\left(5acF_1\left(\frac{1}{4};-q,1;\frac{5}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)+4x^4\left(adqF_1\left(\frac{5}{4};1-q,1;\frac{9}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)-bcF_1\left(\frac{5}{4};-q,2;\frac{9}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x^4)^q/(a + b*x^4),x]
```

```
[Out] (5*a*c*x*(c + d*x^4)^q*AppellF1[1/4, -q, 1, 5/4, -((d*x^4)/c), -((b*x^4)/a)
])/((a + b*x^4)*(5*a*c*AppellF1[1/4, -q, 1, 5/4, -((d*x^4)/c), -((b*x^4)/a)
] + 4*x^4*(a*d*q*AppellF1[5/4, 1 - q, 1, 9/4, -((d*x^4)/c), -((b*x^4)/a)] -
 b*c*AppellF1[5/4, -q, 2, 9/4, -((d*x^4)/c), -((b*x^4)/a)])))
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{(d\,x^4+c)^q}{b\,x^4+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^4+c)^q/(b*x^4+a),x)
```

```
[Out] int((d*x^4+c)^q/(b*x^4+a),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^q/(b*x^4+a),x, algorithm="maxima")
```

```
[Out] integrate((d*x^4 + c)^q/(b*x^4 + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^q/(b*x^4+a),x, algorithm="fricas")
```

```
[Out] integral((d*x^4 + c)^q/(b*x^4 + a), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**4+c)**q/(b*x**4+a),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^q/(b*x^4+a),x, algorithm="giac")
```

```
[Out] integrate((d*x^4 + c)^q/(b*x^4 + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{{\left(d\,x^4+c\right)}^q}{b\,x^4+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^4)^q/(a + b*x^4),x)
```

```
[Out] int((c + d*x^4)^q/(a + b*x^4), x)
```

3.222 $\int \frac{(c+dx^4)^q}{(a+bx^4)^2}\,dx$

Optimal. Leaf size=57

$$\frac{x(c+dx^4)^q\left(1+\frac{dx^4}{c}\right)^{-q}F_1\left(\frac{1}{4};2,-q;\frac{5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{a^2}$$

```
[Out] x*(d*x^4+c)^q*AppellF1(1/4,2,-q,5/4,-b*x^4/a,-d*x^4/c)/a^2/((1+d*x^4/c)^q)
```

Rubi [A]
time = 0.02, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {441, 440}

$$\frac{x(c+dx^4)^q\left(\frac{dx^4}{c}+1\right)^{-q}F_1\left(\frac{1}{4};2,-q;\frac{5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^4)^q/(a + b*x^4)^2,x]
```

```
[Out] (x*(c + d*x^4)^q*AppellF1[1/4, 2, -q, 5/4, -((b*x^4)/a), -((d*x^4)/c)])/(a^
2*(1 + (d*x^4)/c)^q)
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx^4)^q}{(a+bx^4)^2}\,dx &= \left((c+dx^4)^q\left(1+\frac{dx^4}{c}\right)^{-q}\right)\int \frac{\left(1+\frac{dx^4}{c}\right)^q}{(a+bx^4)^2}\,dx\\ &= \frac{x(c+dx^4)^q\left(1+\frac{dx^4}{c}\right)^{-q}F_1\left(\frac{1}{4};2,-q;\frac{5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{a^2}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 162 vs. 2(57) = 114.
time = 0.43, size = 162, normalized size = 2.84

$$\frac{5acx(c+dx^4)^q F_1\left(\frac{1}{4};2,-q;\frac{5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{(a+bx^4)^2\left(5acF_1\left(\frac{1}{4};2,-q;\frac{5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)+4x^4\left(adqF_1\left(\frac{5}{4};2,1-q;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)-2bcF_1\left(\frac{5}{4};3,-q;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x^4)^q/(a + b*x^4)^2,x]
```

```
[Out] (5*a*c*x*(c + d*x^4)^q*AppellF1[1/4, 2, -q, 5/4, -((b*x^4)/a), -((d*x^4)/c)
])/((a + b*x^4)^2*(5*a*c*AppellF1[1/4, 2, -q, 5/4, -((b*x^4)/a), -((d*x^4)/
c)] + 4*x^4*(a*d*q*AppellF1[5/4, 2, 1 - q, 9/4, -((b*x^4)/a), -((d*x^4)/c)]
 - 2*b*c*AppellF1[5/4, 3, -q, 9/4, -((b*x^4)/a), -((d*x^4)/c)])))
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{(d\,x^4+c)^q}{(b\,x^4+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^4+c)^q/(b*x^4+a)^2,x)
```

```
[Out] int((d*x^4+c)^q/(b*x^4+a)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^q/(b*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*x^4 + c)^q/(b*x^4 + a)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^q/(b*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] integral((d*x^4 + c)^q/(b^2*x^8 + 2*a*b*x^4 + a^2), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**4+c)**q/(b*x**4+a)**2,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^q/(b*x^4+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x^4 + c)^q/(b*x^4 + a)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(d\,x^4+c\right)^q}{\left(b\,x^4+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^4)^q/(a + b*x^4)^2,x)
```

```
[Out] int((c + d*x^4)^q/(a + b*x^4)^2, x)
```

3.223 $\int \frac{1}{\sqrt[5]{a+bx^5}\,(c+dx^5)}\,dx$

Optimal. Leaf size=545

$$-\frac{\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\,\tan^{-1}\left(\sqrt{\frac{1}{5}\left(5-2\sqrt{5}\right)}-\frac{2\sqrt{\frac{2}{5+\sqrt{5}}}\,\sqrt[5]{bc-ad}\,x}{\sqrt[5]{c}\,\sqrt[5]{a+bx^5}}\right)}{5c^{4/5}\sqrt[5]{bc-ad}}+\frac{\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\,\tan^{-1}\left(\sqrt{\frac{1}{5}\left(5\right.\right.}}{5}$$

```
[Out] -1/5*ln(c^(1/5)-(-a*d+b*c)^(1/5)*x/(b*x^5+a)^(1/5))/c^(4/5)/(-a*d+b*c)^(1/5
)+1/20*ln((2*(-a*d+b*c)^(2/5)*x^2+c^(1/5)*(-a*d+b*c)^(1/5)*x*(b*x^5+a)^(1/5
)+2*c^(2/5)*(b*x^5+a)^(2/5)-c^(1/5)*(-a*d+b*c)^(1/5)*x*(b*x^5+a)^(1/5)*5^(1
/2))/(b*x^5+a)^(2/5))*(-5^(1/2)+1)/c^(4/5)/(-a*d+b*c)^(1/5)+1/20*ln((2*(-a*
d+b*c)^(2/5)*x^2+c^(1/5)*(-a*d+b*c)^(1/5)*x*(b*x^5+a)^(1/5)+2*c^(2/5)*(b*x^
5+a)^(2/5)+c^(1/5)*(-a*d+b*c)^(1/5)*x*(b*x^5+a)^(1/5)*5^(1/2))/(b*x^5+a)^(2
/5))*(5^(1/2)+1)/c^(4/5)/(-a*d+b*c)^(1/5)+1/10*arctan(1/5*(-a*d+b*c)^(1/5)*
x*(50+10*5^(1/2))^(1/2)/c^(1/5)/(b*x^5+a)^(1/5)+1/5*(25+10*5^(1/2))^(1/2))*
(10-2*5^(1/2))^(1/2)/c^(4/5)/(-a*d+b*c)^(1/5)+1/10*arctan(-1/5*(25-10*5^(1/
2))^(1/2)+2*(-a*d+b*c)^(1/5)*x*2^(1/2)/(5+5^(1/2))^(1/2)/c^(1/5)/(b*x^5+a)^
(1/5))*(10+2*5^(1/2))^(1/2)/c^(4/5)/(-a*d+b*c)^(1/5)
```

Rubi [A]
time = 0.76, antiderivative size = 545, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {385, 208, 648, 632, 210, 642, 31}

$$-\frac{\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\,\text{ArcTan}\left(\sqrt{\frac{1}{5}\left(5-2\sqrt{5}\right)}-\frac{2\sqrt{\frac{2}{5+\sqrt{5}}}\,x\sqrt[5]{bc-ad}}{\sqrt[5]{c}\,\sqrt[5]{a+bx^5}}\right)}{5c^{4/5}\sqrt[5]{bc-ad}}+\frac{\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\,\text{ArcTan}\left(\frac{\sqrt{\frac{2}{5}\left(5+\sqrt{5}\right)}\,x\sqrt[5]{bc-ad}}{\sqrt[5]{c}\,\sqrt[5]{a+bx^5}}+\sqrt{\frac{1}{5}\left(5+2\sqrt{5}\right)}\right)}{5c^{4/5}\sqrt[5]{bc-ad}}-\frac{\log\left(\sqrt[5]{c}-\frac{x\sqrt[5]{bc-ad}}{\sqrt[5]{a+bx^5}}\right)}{5c^{4/5}\sqrt[5]{bc-ad}}+\frac{\left(1-\sqrt{5}\right)\log\left(\frac{2c^{2/5}(a+bx^5)^{2/5}-\sqrt{5}\,\sqrt[5]{c}\,x\sqrt[5]{a+bx^5}\,\sqrt[5]{bc-ad}+\sqrt[5]{c}\,x\sqrt[5]{a+bx^5}\,\sqrt[5]{bc-ad}+2x^2(bc-ad)^{2/5}}{(a+bx^5)^{2/5}}\right)}{20c^{4/5}\sqrt[5]{bc-ad}}+\frac{\left(1+\sqrt{5}\right)\log\left(\frac{2c^{2/5}(a+bx^5)^{2/5}+\sqrt{5}\,\sqrt[5]{c}\,x\sqrt[5]{a+bx^5}\,\sqrt[5]{bc-ad}+\sqrt[5]{c}\,x\sqrt[5]{a+bx^5}\,\sqrt[5]{bc-ad}+2x^2(bc-ad)^{2/5}}{(a+bx^5)^{2/5}}\right)}{20c^{4/5}\sqrt[5]{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^5)^(1/5)*(c + d*x^5)),x]
```

```
[Out] -1/5*(Sqrt[(5 + Sqrt[5])/2]*ArcTan[Sqrt[(5 - 2*Sqrt[5])/5] - (2*Sqrt[2/(5 +
 Sqrt[5])]*(b*c - a*d)^(1/5)*x)/(c^(1/5)*(a + b*x^5)^(1/5))])/(c^(4/5)*(b*c
 - a*d)^(1/5)) + (Sqrt[(5 - Sqrt[5])/2]*ArcTan[Sqrt[(5 + 2*Sqrt[5])/5] + (S
qrt[(2*(5 + Sqrt[5]))/5]*(b*c - a*d)^(1/5)*x)/(c^(1/5)*(a + b*x^5)^(1/5))])
/(5*c^(4/5)*(b*c - a*d)^(1/5)) - Log[c^(1/5) - ((b*c - a*d)^(1/5)*x)/(a + b
*x^5)^(1/5)]/(5*c^(4/5)*(b*c - a*d)^(1/5)) + ((1 - Sqrt[5])*Log[(2*(b*c - a
*d)^(2/5)*x^2 + c^(1/5)*(b*c - a*d)^(1/5)*x*(a + b*x^5)^(1/5) - Sqrt[5]*c^(
1/5)*(b*c - a*d)^(1/5)*x*(a + b*x^5)^(1/5) + 2*c^(2/5)*(a + b*x^5)^(2/5))/(
a + b*x^5)^(2/5)])/(20*c^(4/5)*(b*c - a*d)^(1/5)) + ((1 + Sqrt[5])*Log[(2*(
b*c - a*d)^(2/5)*x^2 + c^(1/5)*(b*c - a*d)^(1/5)*x*(a + b*x^5)^(1/5) + Sqrt
```

[5]*c^(1/5)*(b*c - a*d)^(1/5)*x*(a + b*x^5)^(1/5) + 2*c^(2/5)*(a + b*x^5)^(2/5))/(a + b*x^5)^(2/5)])/(20*c^(4/5)*(b*c - a*d)^(1/5))

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-a
/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r + s*Cos[(2*k
- 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x]; (r/(a*
n))*Int[1/(r - s*x), x] + Dist[2*(r/(a*n)), Sum[u, {k, 1, (n - 1)/2}], x],
x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 3)/2, 0] && NegQ[a/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{\sqrt[5]{a+bx^5}\,(c+dx^5)}\,dx = \text{Subst}\left(\int \frac{1}{c-(bc-ad)x^5}\,dx, x, \frac{x}{\sqrt[5]{a+bx^5}}\right)$$

$$= \frac{2\text{Subst}\left(\int \frac{\sqrt[5]{c}+\frac{1}{4}\left(1-\sqrt{5}\right)\sqrt[5]{bc-ad}\,x}{c^{2/5}+\frac{1}{2}\left(1-\sqrt{5}\right)\sqrt[5]{c}\sqrt[5]{bc-ad}\,x+(bc-ad)^{2/5}x^2}\,dx, x, \frac{x}{\sqrt[5]{a+bx^5}}\right)}{5c^{4/5}} + \text{2Sub}$$

$$= -\frac{\log\left(\sqrt[5]{c}-\frac{\sqrt[5]{bc-ad}\,x}{\sqrt[5]{a+bx^5}}\right)}{5c^{4/5}\sqrt[5]{bc-ad}} + \frac{\left(5-\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{c^{2/5}+\frac{1}{2}\left(1+\sqrt{5}\right)\sqrt[5]{c}\sqrt[5]{bc-a}}\right.}{20c^{3/5}}$$

$$= -\frac{\log\left(\sqrt[5]{c}-\frac{\sqrt[5]{bc-ad}\,x}{\sqrt[5]{a+bx^5}}\right)}{5c^{4/5}\sqrt[5]{bc-ad}} + \frac{\left(1-\sqrt{5}\right)\log\left(2c^{2/5}+\frac{2(bc-ad)^{2/5}x^2}{(a+bx^5)^{2/5}}+\frac{\sqrt[5]{c}\sqrt[5]{bc-}}{\sqrt[5]{a+b}}\right.}{20c^{4/5}\sqrt[5]{bc-ad}}$$

$$= \frac{\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\tan^{-1}\left(\frac{\left(1-\sqrt{5}\right)\sqrt[5]{c}+\frac{4\sqrt[5]{bc-ad}\,x}{\sqrt[5]{a+bx^5}}}{\sqrt{2\left(5+\sqrt{5}\right)}\sqrt[5]{c}}\right)}{5c^{4/5}\sqrt[5]{bc-ad}} + \frac{\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\tan^{-}}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 10.03, size = 48, normalized size = 0.09

$$\frac{x\,{}_2F_1\left(\frac{1}{5},1;\frac{6}{5};\frac{(bc-ad)x^5}{c(a+bx^5)}\right)}{c\sqrt[5]{a+bx^5}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^5)^(1/5)*(c + d*x^5)),x]
[Out] (x*Hypergeometric2F1[1/5, 1, 6/5, ((b*c - a*d)*x^5)/(c*(a + b*x^5))])/(c*(a
 + b*x^5)^(1/5))
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\,x^5+a)^{\frac{1}{5}}\,(d\,x^5+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^5+a)^(1/5)/(d*x^5+c),x)
```

```
[Out] int(1/(b*x^5+a)^(1/5)/(d*x^5+c),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^5+a)^(1/5)/(d*x^5+c),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^5 + a)^(1/5)*(d*x^5 + c)), x)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^5+a)^(1/5)/(d*x^5+c),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (trace 0)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[5]{a+bx^5}\,(c+dx^5)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**5+a)**(1/5)/(d*x**5+c),x)
```

```
[Out] Integral(1/((a + b*x**5)**(1/5)*(c + d*x**5)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^5+a)^(1/5)/(d*x^5+c),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^5 + a)^(1/5)*(d*x^5 + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(b\,x^5+a\right)^{1/5}\,\left(d\,x^5+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^5)^(1/5)*(c + d*x^5)),x)
[Out] int(1/((a + b*x^5)^(1/5)*(c + d*x^5)), x)
```

3.224 $\int \sqrt{a+\frac{b}{x}} \left(c+\frac{d}{x}\right)^3 dx$

Optimal. Leaf size=143

$$-\frac{7}{5}d\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2-\frac{d\sqrt{a+\frac{b}{x}}\left(2(57b^2c^2+15abcd-2a^2d^2)+\frac{bd(33bc+2ad)}{x}\right)}{15b^2}+\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^3x+\frac{c}{}$$

[Out] c^2*(6*a*d+b*c)*arctanh((a+b/x)^(1/2)/a^(1/2))/a^(1/2)-7/5*d*(c+d/x)^2*(a+b/x)^(1/2)-1/15*d*(-4*a^2*d^2+30*a*b*c*d+114*b^2*c^2+b*d*(2*a*d+33*b*c)/x)*(a+b/x)^(1/2)/b^2+(c+d/x)^3*x*(a+b/x)^(1/2)

Rubi [A]
time = 0.08, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {382, 99, 158, 152, 65, 214}

$$-\frac{d\sqrt{a+\frac{b}{x}}\left(2(-2a^2d^2+15abcd+57b^2c^2)+\frac{bd(2ad+33bc)}{x}\right)}{15b^2}+\frac{c^2(6ad+bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}}+x\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^3-\frac{7}{5}d\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b/x]*(c + d/x)^3,x]
```

```
[Out] (-7*d*Sqrt[a + b/x]*(c + d/x)^2)/5 - (d*Sqrt[a + b/x]*(2*(57*b^2*c^2 + 15*a
*b*c*d - 2*a^2*d^2) + (b*d*(33*b*c + 2*a*d))/x))/(15*b^2) + Sqrt[a + b/x]*(
c + d/x)^3*x + (c^2*(b*c + 6*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/Sqrt[a]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^p/(b*(
m + 1))), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
```

```
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m +
 1)*((c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Dist[(a^2*d
^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m
+ n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n +
 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)),
Int[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}
, x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 158

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegerQ[m]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^3 dx = -\text{Subst}\left(\int \frac{\sqrt{a+bx}\,(c+dx)^3}{x^2}\,dx, x, \frac{1}{x}\right)$$

$$= \sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^3 x - \text{Subst}\left(\int \frac{(c+dx)^2\left(\frac{1}{2}(bc+6ad)+\frac{7bdx}{2}\right)}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{7}{5}d\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2 + \sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^3 x - \frac{2\text{Subst}\left(\int \frac{(c+dx)\left(\frac{5}{4}bc(bc+6ad)\ldots}{x\sqrt{a+\ldots}}\right.}{5b\ldots}$$

$$= -\frac{7}{5}d\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2 - \frac{d\sqrt{a+\frac{b}{x}}\left(2(57b^2c^2+15abcd-2a^2d^2)+\frac{bd(33bc+2a\ldots}{x}\right.}{15b^2}$$

$$= -\frac{7}{5}d\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2 - \frac{d\sqrt{a+\frac{b}{x}}\left(2(57b^2c^2+15abcd-2a^2d^2)+\frac{bd(33bc+2a\ldots}{x}\right.}{15b^2}$$

$$= -\frac{7}{5}d\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2 - \frac{d\sqrt{a+\frac{b}{x}}\left(2(57b^2c^2+15abcd-2a^2d^2)+\frac{bd(33bc+2a\ldots}{x}\right.}{15b^2}$$

Mathematica [A]
time = 0.21, size = 118, normalized size = 0.83

$$\frac{\sqrt{a+\frac{b}{x}}\,(4a^2d^3x^2-2abd^2x(d+15cx)-3b^2(2d^3+10cd^2x+30c^2dx^2-5c^3x^3))}{15b^2x^2}+\frac{c^2(bc+6ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b/x]*(c + d/x)^3,x]
```

```
[Out] (Sqrt[a + b/x]*(4*a^2*d^3*x^2 - 2*a*b*d^2*x*(d + 15*c*x) - 3*b^2*(2*d^3 + 1
0*c*d^2*x + 30*c^2*d*x^2 - 5*c^3*x^3)))/(15*b^2*x^2) + (c^2*(b*c + 6*a*d)*A
rcTanh[Sqrt[a + b/x]/Sqrt[a]])/Sqrt[a]
```

Maple [A]
time = 0.07, size = 248, normalized size = 1.73

method	result

risch	$\frac{(15b^2c^3x^3+4a^2d^3x^2-30ac\,d^2x^2b-90d\,c^2b^2x^2-2ax\,d^3b-30c\,d^2x\,b^2-6b^2d^3)\sqrt{\frac{ax+b}{x}}}{15x^2b^2}+\frac{\left(3c^2\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^2+bx}\right)\sqrt{a}\ c\right.}{}$
default	$\frac{\sqrt{\frac{ax+b}{x}}\left(180\sqrt{a\,x^2+bx}\ a^{\frac{3}{2}}b\,c^2d\,x^4+30\sqrt{a\,x^2+bx}\ \sqrt{a}\ b^2c^3x^4+90\ln\left(\frac{2\sqrt{a\,x^2+bx}\ \sqrt{a}+2ax+b}{2\sqrt{a}}\right)a\,b^2c^2d\,x^4+1\right.}{30x^3}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c+d/x)^3*(a+1/x*b)^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/30*((a*x+b)/x)^(1/2)/x^3*(180*(a*x^2+b*x)^(1/2)*a^(3/2)*b*c^2*d*x^4+30*(a*x^2+b*x)^(1/2)*a^(1/2)*b^2*c^3*x^4+90*ln(1/2*(2*(a*x^2+b*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a*b^2*c^2*d*x^4+15*ln(1/2*(2*(a*x^2+b*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*b^3*c^3*x^4-180*(a*x^2+b*x)^(3/2)*a^(1/2)*b*c^2*d*x^2+8*(a*x^2+b*x)^(3/2)*a^(3/2)*d^3*x-60*d^2*c*(a*x^2+b*x)^(3/2)*b*x*a^(1/2)-12*(a*x^2+b*x)^(3/2)*a^(1/2)*b*d^3)/(x*(a*x+b))^(1/2)/b^2/a^(1/2)

Maxima [A]
time = 0.49, size = 164, normalized size = 1.15

$$\frac{1}{2}\left(2\sqrt{a+\frac{b}{x}}\,x-\frac{b\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{\sqrt{a}}\right)c^3-3\left(\sqrt{a}\,\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)+2\sqrt{a+\frac{b}{x}}\right)c^2d-\frac{2}{15}d^3\left(\frac{3\left(a+\frac{b}{x}\right)^{\frac{5}{2}}}{b^2}-\frac{5\left(a+\frac{b}{x}\right)^{\frac{3}{2}}a}{b^2}\right)-\frac{2\left(a+\frac{b}{x}\right)^{\frac{3}{2}}cd^2}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d/x)^3*(a+b/x)^(1/2),x, algorithm="maxima")

[Out] 1/2*(2*sqrt(a + b/x)*x - b*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/x) + sqrt(a)))/sqrt(a))*c^3 - 3*(sqrt(a)*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/x) + sqrt(a))) + 2*sqrt(a + b/x))*c^2*d - 2/15*d^3*(3*(a + b/x)^(5/2)/b^2 - 5*(a + b/x)^(3/2)*a/b^2) - 2*(a + b/x)^(3/2)*c*d^2/b

Fricas [A]
time = 3.24, size = 306, normalized size = 2.14

$$\left[\frac{15\,(b^3c^3+6\,ab^2c^2d)\sqrt{a}\,x^2\log\left(2\,ax+2\,\sqrt{a}\,x\sqrt{\frac{ax+b}{x}}+b\right)+2\,(15\,ab^2c^3x^3-6\,ab^2d^3-2\,(45\,ab^2c^2d+15\,a^2bcd^2-2\,a^3d^3)x^2-2\,(15\,ab^2cd^2+a^2bd^3)x)\sqrt{\frac{ax+b}{x}}}{30\,ab^2x^2},\ -\frac{15\,(b^3c^3+6\,ab^2c^2d)\sqrt{-a}\,x^2\arctan\left(\frac{\sqrt{-a}\,\sqrt{\frac{ax+b}{x}}}{a}\right)-(15\,ab^2c^3x^3-6\,ab^2d^3-2\,(45\,ab^2c^2d+15\,a^2bcd^2-2\,a^3d^3)x^2-2\,(15\,ab^2cd^2+a^2bd^3)x)\sqrt{\frac{ax+b}{x}}}{15\,ab^2x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d/x)^3*(a+b/x)^(1/2),x, algorithm="fricas")

```
[Out] [1/30*(15*(b^3*c^3 + 6*a*b^2*c^2*d)*sqrt(a)*x^2*log(2*a*x + 2*sqrt(a)*x*sqr
t((a*x + b)/x) + b) + 2*(15*a*b^2*c^3*x^3 - 6*a*b^2*d^3 - 2*(45*a*b^2*c^2*d
 + 15*a^2*b*c*d^2 - 2*a^3*d^3)*x^2 - 2*(15*a*b^2*c*d^2 + a^2*b*d^3)*x)*sqrt
((a*x + b)/x))/(a*b^2*x^2), -1/15*(15*(b^3*c^3 + 6*a*b^2*c^2*d)*sqrt(-a)*x^
2*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) - (15*a*b^2*c^3*x^3 - 6*a*b^2*d^3 -
2*(45*a*b^2*c^2*d + 15*a^2*b*c*d^2 - 2*a^3*d^3)*x^2 - 2*(15*a*b^2*c*d^2 + a
^2*b*d^3)*x)*sqrt((a*x + b)/x))/(a*b^2*x^2)]
```

Sympy [A]
time = 30.67, size = 454, normalized size = 3.17

$$\frac{4a^{\frac{11}{2}}b^{\frac{3}{2}}d^3x^3\sqrt{\frac{ax}{b}+1}}{15a^{\frac{7}{2}}b^3x^{\frac{7}{2}}+15a^{\frac{5}{2}}b^4x^{\frac{5}{2}}}+\frac{2a^{\frac{9}{2}}b^{\frac{5}{2}}d^3x^2\sqrt{\frac{ax}{b}+1}}{15a^{\frac{7}{2}}b^3x^{\frac{7}{2}}+15a^{\frac{5}{2}}b^4x^{\frac{5}{2}}}-\frac{8a^{\frac{7}{2}}b^{\frac{7}{2}}d^3x\sqrt{\frac{ax}{b}+1}}{15a^{\frac{7}{2}}b^3x^{\frac{7}{2}}+15a^{\frac{5}{2}}b^4x^{\frac{5}{2}}}-\frac{6a^{\frac{5}{2}}b^{\frac{9}{2}}d^3\sqrt{\frac{ax}{b}+1}}{15a^{\frac{7}{2}}b^3x^{\frac{7}{2}}+15a^{\frac{5}{2}}b^4x^{\frac{5}{2}}}-\frac{4a^6bd^3x^{\frac{7}{2}}}{15a^{\frac{7}{2}}b^3x^{\frac{7}{2}}+15a^{\frac{5}{2}}b^4x^{\frac{5}{2}}}-\frac{4a^5b^2d^3x^{\frac{5}{2}}}{15a^{\frac{7}{2}}b^3x^{\frac{7}{2}}+15a^{\frac{5}{2}}b^4x^{\frac{5}{2}}}-\frac{6ac^2d\,\mathrm{atan}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{-a}}\right)}{\sqrt{-a}}+\sqrt{b}\,c^3\sqrt{x}\,\sqrt{\frac{ax}{b}+1}-6c^2d\sqrt{a+\frac{b}{x}}+3cd^2\left(\begin{cases}-\frac{\sqrt{a}}{x} & \text{for}\ b=0\\ -\frac{2\left(a+\frac{b}{x}\right)^{\frac{3}{2}}}{3b} & \text{otherwise}\end{cases}\right)+\frac{bc^3\,\mathrm{asinh}\left(\frac{\sqrt{a}\,\sqrt{x}}{\sqrt{b}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)**3*(a+b/x)**(1/2),x)
```

```
[Out] 4*a**(11/2)*b**(3/2)*d**3*x**3*sqrt(a*x/b + 1)/(15*a**(7/2)*b**3*x**(7/2) +
 15*a**(5/2)*b**4*x**(5/2)) + 2*a**(9/2)*b**(5/2)*d**3*x**2*sqrt(a*x/b + 1)
/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) - 8*a**(7/2)*b**(7
/2)*d**3*x*sqrt(a*x/b + 1)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x*
*(5/2)) - 6*a**(5/2)*b**(9/2)*d**3*sqrt(a*x/b + 1)/(15*a**(7/2)*b**3*x**(7/
2) + 15*a**(5/2)*b**4*x**(5/2)) - 4*a**6*b*d**3*x**(7/2)/(15*a**(7/2)*b**3*
x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) - 4*a**5*b**2*d**3*x**(5/2)/(15*a**(7
/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) - 6*a*c**2*d*atan(sqrt(a + b
/x)/sqrt(-a))/sqrt(-a) + sqrt(b)*c**3*sqrt(x)*sqrt(a*x/b + 1) - 6*c**2*d*sq
rt(a + b/x) + 3*c*d**2*Piecewise((-sqrt(a)/x, Eq(b, 0)), (-2*(a + b/x)**(3/
2)/(3*b), True)) + b*c**3*asinh(sqrt(a)*sqrt(x)/sqrt(b))/sqrt(a)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)^3*(a+b/x)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [B]
time = 2.59, size = 173, normalized size = 1.21

$${\left(a+\frac{b}{x}\right)}^{3/2}\,\left(\frac{6\,a\,d^3-6\,b\,c\,d^2}{3\,b^2}-\frac{4\,a\,d^3}{3\,b^2}\right)+\sqrt{a+\frac{b}{x}}\,\left(2\,a\,\left(\frac{6\,a\,d^3-6\,b\,c\,d^2}{b^2}-\frac{4\,a\,d^3}{b^2}\right)-\frac{6\,d\,{\left(a\,d-b\,c\right)}^2}{b^2}+\frac{2\,a^2\,d^3}{b^2}\right)+c^3\,x\,\sqrt{a+\frac{b}{x}}-\frac{2\,d^3\,{\left(a+\frac{b}{x}\right)}^{5/2}}{5\,b^2}-\frac{c^2\,\mathrm{atan}\left(\frac{\sqrt{a+\frac{b}{x}}\,1{}\mathrm{i}}{\sqrt{a}}\right)\,\left(6\,a\,d+b\,c\right)\,1{}\mathrm{i}}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x)^(1/2)*(c + d/x)^3,x)
```

```
[Out] (a + b/x)^(3/2)*((6*a*d^3 - 6*b*c*d^2)/(3*b^2) - (4*a*d^3)/(3*b^2)) + (a +
b/x)^(1/2)*(2*a*((6*a*d^3 - 6*b*c*d^2)/b^2 - (4*a*d^3)/b^2) - (6*d*(a*d - b
*c)^2)/b^2 + (2*a^2*d^3)/b^2) + c^3*x*(a + b/x)^(1/2) - (2*d^3*(a + b/x)^(5
/2))/(5*b^2) - (c^2*atan(((a + b/x)^(1/2)*1i)/a^(1/2))*(6*a*d + b*c)*1i)/a^
(1/2)
```

3.225 $\int \sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2 dx$

Optimal. Leaf size=99

$$-\frac{c(bc+4ad)\sqrt{a+\frac{b}{x}}}{a}-\frac{2d^2\left(a+\frac{b}{x}\right)^{3/2}}{3b}+\frac{c^2\left(a+\frac{b}{x}\right)^{3/2}x}{a}+\frac{c(bc+4ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}}$$

```
[Out] -2/3*d^2*(a+b/x)^(3/2)/b+c^2*(a+b/x)^(3/2)*x/a+c*(4*a*d+b*c)*arctanh((a+b/x
)^(1/2)/a^(1/2))/a^(1/2)-c*(4*a*d+b*c)*(a+b/x)^(1/2)/a
```

Rubi [A]
time = 0.04, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {382, 91, 81, 52, 65, 214}

$$\frac{c^2x\left(a+\frac{b}{x}\right)^{3/2}}{a}-\frac{c\sqrt{a+\frac{b}{x}}\,(4ad+bc)}{a}+\frac{c(4ad+bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{2d^2\left(a+\frac{b}{x}\right)^{3/2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b/x]*(c + d/x)^2,x]
```

```
[Out] -((c*(b*c + 4*a*d)*Sqrt[a + b/x])/a) - (2*d^2*(a + b/x)^(3/2))/(3*b) + (c^2
*(a + b/x)^(3/2)*x)/a + (c*(b*c + 4*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/Sq
rt[a]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2 dx = -\text{Subst}\left(\int \frac{\sqrt{a+bx}\,(c+dx)^2}{x^2}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{c^2\left(a+\frac{b}{x}\right)^{3/2}x}{a} - \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx}\,\left(\frac{1}{2}c(bc+4ad)+ad^2x\right)}{x}\,dx, x, \frac{1}{x}\right)}{a}$$

$$= -\frac{2d^2\left(a+\frac{b}{x}\right)^{3/2}}{3b} + \frac{c^2\left(a+\frac{b}{x}\right)^{3/2}x}{a} - \frac{(c(bc+4ad))\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx, x, \frac{1}{x}\right)}{2a}$$

$$= -\frac{c(bc+4ad)\sqrt{a+\frac{b}{x}}}{a} - \frac{2d^2\left(a+\frac{b}{x}\right)^{3/2}}{3b} + \frac{c^2\left(a+\frac{b}{x}\right)^{3/2}x}{a} - \frac{1}{2}(c(bc+4ad))\text{Su}$$

$$= -\frac{c(bc+4ad)\sqrt{a+\frac{b}{x}}}{a} - \frac{2d^2\left(a+\frac{b}{x}\right)^{3/2}}{3b} + \frac{c^2\left(a+\frac{b}{x}\right)^{3/2}x}{a} - \frac{(c(bc+4ad))\text{Sub}}{}$$

$$= -\frac{c(bc+4ad)\sqrt{a+\frac{b}{x}}}{a} - \frac{2d^2\left(a+\frac{b}{x}\right)^{3/2}}{3b} + \frac{c^2\left(a+\frac{b}{x}\right)^{3/2}x}{a} + \frac{c(bc+4ad)\tanh^{-}}{\sqrt{a}}$$

Mathematica [A]
time = 0.18, size = 84, normalized size = 0.85

$$\frac{\sqrt{a+\frac{b}{x}}\,(-2ad^2x+b(-2d^2-12cdx+3c^2x^2))}{3bx} + \frac{c(bc+4ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b/x]*(c + d/x)^2,x]
```

```
[Out] (Sqrt[a + b/x]*(-2*a*d^2*x + b*(-2*d^2 - 12*c*d*x + 3*c^2*x^2)))/(3*b*x) +
(c*(b*c + 4*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/Sqrt[a]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 190 vs. 2(85) = 170.
time = 0.05, size = 191, normalized size = 1.93

method	result
risch	$-\frac{\left(-3b\,c^2x^2+2ax\,d^2+12xbcd+2b\,d^2\right)\sqrt{\frac{ax+b}{x}}}{3xb}+\frac{\left(2c\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^2+bx}\right)\sqrt{a}\,d+\frac{c^2\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^2+bx}\right)b}{2\sqrt{a}}\right)}{ax+b}$
default	$-\frac{\sqrt{\frac{ax+b}{x}}\left(-24\sqrt{a\,x^2+bx}\,a^{\frac{3}{2}}cd\,x^3-6\sqrt{a\,x^2+bx}\,\sqrt{a}\,b\,c^2x^3-12\ln\left(\frac{2\sqrt{a\,x^2+bx}\,\sqrt{a}+2ax+b}{2\sqrt{a}}\right)abcd\,x^3-3\ln\left(\right.\right.}{6x^2\sqrt{x\,(ax+b)}\,b\sqrt{a}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d/x)^2*(a+1/x*b)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*((a*x+b)/x)^(1/2)/x^2*(-24*(a*x^2+b*x)^(1/2)*a^(3/2)*c*d*x^3-6*(a*x^2+
b*x)^(1/2)*a^(1/2)*b*c^2*x^3-12*ln(1/2*(2*(a*x^2+b*x)^(1/2)*a^(1/2)+2*a*x+b
)/a^(1/2))*a*b*c*d*x^3-3*ln(1/2*(2*(a*x^2+b*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/
2))*b^2*c^2*x^3+24*(a*x^2+b*x)^(3/2)*a^(1/2)*c*d*x+4*d^2*(a*x^2+b*x)^(3/2)*
a^(1/2))/(x*(a*x+b))^(1/2)/b/a^(1/2)
```

Maxima [A]
time = 0.50, size = 126, normalized size = 1.27

$$\frac{1}{2}\left(2\sqrt{a+\frac{b}{x}}\,x-\frac{b\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{\sqrt{a}}\right)c^2-2\left(\sqrt{a}\,\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)+2\sqrt{a+\frac{b}{x}}\right)cd-\frac{2\left(a+\frac{b}{x}\right)^{\frac{3}{2}}d^2}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)^2*(a+b/x)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*(2*sqrt(a + b/x)*x - b*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/x) + s
qrt(a)))/sqrt(a))*c^2 - 2*(sqrt(a)*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a +
b/x) + sqrt(a))) + 2*sqrt(a + b/x))*c*d - 2/3*(a + b/x)^(3/2)*d^2/b
```

Fricas [A]
time = 2.97, size = 208, normalized size = 2.10

$$\left[\frac{3\,(b^2c^2+4\,abcd)\sqrt{a}\,x\log\left(2\,ax+2\,\sqrt{a}\,x\sqrt{\frac{ax+b}{x}}+b\right)+2\,(3\,abc^2x^2-2\,abd^2-2\,(6\,abcd+a^2d^2)x)\sqrt{\frac{ax+b}{x}}}{6\,abx},\ -\frac{3\,(b^2c^2+4\,abcd)\sqrt{-a}\,x\arctan\left(\frac{\sqrt{-a}\,\sqrt{\frac{ax+b}{x}}}{a}\right)-(3\,abc^2x^2-2\,abd^2-2\,(6\,abcd+a^2d^2)x)\sqrt{\frac{ax+b}{x}}}{3\,abx}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)^2*(a+b/x)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*(b^2*c^2 + 4*a*b*c*d)*sqrt(a)*x*log(2*a*x + 2*sqrt(a)*x*sqrt((a*x +
 b)/x) + b) + 2*(3*a*b*c^2*x^2 - 2*a*b*d^2 - 2*(6*a*b*c*d + a^2*d^2)*x)*sqr
t((a*x + b)/x))/(a*b*x), -1/3*(3*(b^2*c^2 + 4*a*b*c*d)*sqrt(-a)*x*arctan(sq
rt(-a)*sqrt((a*x + b)/x)/a) - (3*a*b*c^2*x^2 - 2*a*b*d^2 - 2*(6*a*b*c*d + a
^2*d^2)*x)*sqrt((a*x + b)/x))/(a*b*x)]
```

Sympy [A]
time = 20.78, size = 121, normalized size = 1.22

$$-\frac{4acd\operatorname{atan}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{-a}}\right)}{\sqrt{-a}}+\sqrt{b}\,c^2\sqrt{x}\,\sqrt{\frac{ax}{b}+1}-4cd\sqrt{a+\frac{b}{x}}+d^2\left(\begin{cases}-\frac{\sqrt{a}}{x} & \text{for } b=0\\ -\frac{2\left(a+\frac{b}{x}\right)^{\frac{3}{2}}}{3b} & \text{otherwise}\end{cases}\right)+\frac{bc^2\operatorname{asinh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)**2*(a+b/x)**(1/2),x)
```

```
[Out] -4*a*c*d*atan(sqrt(a + b/x)/sqrt(-a))/sqrt(-a) + sqrt(b)*c**2*sqrt(x)*sqrt(
a*x/b + 1) - 4*c*d*sqrt(a + b/x) + d**2*Piecewise((-sqrt(a)/x, Eq(b, 0)), (
-2*(a + b/x)**(3/2)/(3*b), True)) + b*c**2*asinh(sqrt(a)*sqrt(x)/sqrt(b))/s
qrt(a)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)^2*(a+b/x)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [B]
time = 1.90, size = 99, normalized size = 1.00

$$\left(\frac{4\,a\,d^2-4\,b\,c\,d}{b}-\frac{4\,a\,d^2}{b}\right)\,\sqrt{a+\frac{b}{x}}+c^2\,x\,\sqrt{a+\frac{b}{x}}-\frac{2\,d^2\,{\left(a+\frac{b}{x}\right)}^{3/2}}{3\,b}-\frac{c\operatorname{atan}\left(\frac{\sqrt{a+\frac{b}{x}}\,1\mathrm{i}}{\sqrt{a}}\right)\,\left(4\,a\,d+b\,c\right)\,1\mathrm{i}}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x)^(1/2)*(c + d/x)^2,x)
```

```
[Out] ((4*a*d^2 - 4*b*c*d)/b - (4*a*d^2)/b)*(a + b/x)^(1/2) + c^2*x*(a + b/x)^(1/
2) - (2*d^2*(a + b/x)^(3/2))/(3*b) - (c*atan(((a + b/x)^(1/2)*1i)/a^(1/2))*
(4*a*d + b*c)*1i)/a^(1/2)
```

3.226 $\int \sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right) dx$

Optimal. Leaf size=74

$$-\frac{(bc+2ad)\sqrt{a+\frac{b}{x}}}{a}+\frac{c\left(a+\frac{b}{x}\right)^{3/2}x}{a}+\frac{(bc+2ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}}$$

```
[Out] c*(a+b/x)^(3/2)*x/a+(2*a*d+b*c)*arctanh((a+b/x)^(1/2)/a^(1/2))/a^(1/2)-(2*a
*d+b*c)*(a+b/x)^(1/2)/a
```

Rubi [A]
time = 0.03, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {382, 79, 52, 65, 214}

$$-\frac{\sqrt{a+\frac{b}{x}}\,(2ad+bc)}{a}+\frac{(2ad+bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}}+\frac{cx\left(a+\frac{b}{x}\right)^{3/2}}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b/x]*(c + d/x),x]
```

```
[Out] -(((b*c + 2*a*d)*Sqrt[a + b/x])/a) + (c*(a + b/x)^(3/2)*x)/a + ((b*c + 2*a*
d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/Sqrt[a]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int \sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)\,dx &= -\text{Subst}\left(\int \frac{\sqrt{a+bx}\,(c+dx)}{x^2}\,dx, x, \frac{1}{x}\right)\\
&= \frac{c\left(a+\frac{b}{x}\right)^{3/2}x}{a} - \frac{\left(\frac{bc}{2}+ad\right)\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx, x, \frac{1}{x}\right)}{a}\\
&= -\frac{(bc+2ad)\sqrt{a+\frac{b}{x}}}{a} + \frac{c\left(a+\frac{b}{x}\right)^{3/2}x}{a} - \frac{1}{2}(bc+2ad)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x\right.\\
&= -\frac{(bc+2ad)\sqrt{a+\frac{b}{x}}}{a} + \frac{c\left(a+\frac{b}{x}\right)^{3/2}x}{a} - \frac{(bc+2ad)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\right.}{b}\\
&= -\frac{(bc+2ad)\sqrt{a+\frac{b}{x}}}{a} + \frac{c\left(a+\frac{b}{x}\right)^{3/2}x}{a} + \frac{(bc+2ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}}
\end{aligned}
$$

Mathematica [A]
time = 0.11, size = 52, normalized size = 0.70

$$\sqrt{a+\frac{b}{x}}\,(-2d+cx)+\frac{(bc+2ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b/x]*(c + d/x),x]
```

```
[Out] Sqrt[a + b/x]*(-2*d + c*x) + ((b*c + 2*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])
/Sqrt[a]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 163 vs. $2(64) = 128$.
time = 0.04, size = 164, normalized size = 2.22

method	result
risch	$(cx-2d)\sqrt{\frac{ax+b}{x}}+\frac{\left(\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^2+bx}\right)\sqrt{a}\,d+\frac{\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^2+bx}\right)bc}{2\sqrt{a}}\right)\sqrt{\frac{ax+b}{x}}\,\sqrt{x\,(ax+b)}}{ax+b}$
default	$-\frac{\sqrt{\frac{ax+b}{x}}\left(-4\sqrt{a\,x^2+bx}\,a^{\frac{3}{2}}d\,x^2-2\sqrt{a\,x^2+bx}\,\sqrt{a}\,bc\,x^2-2\ln\left(\frac{2\sqrt{a\,x^2+bx}\,\sqrt{a}+2ax+b}{2\sqrt{a}}\right)abd\,x^2-\ln\left(\frac{2\sqrt{}}{}\right.\right.}{2x\sqrt{x\,(ax+b)}\,b\sqrt{a}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d/x)*(a+1/x*b)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*((a*x+b)/x)^(1/2)/x*(-4*(a*x^2+b*x)^(1/2)*a^(3/2)*d*x^2-2*(a*x^2+b*x)^
(1/2)*a^(1/2)*b*c*x^2-2*ln(1/2*(2*(a*x^2+b*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2
))*a*b*d*x^2-ln(1/2*(2*(a*x^2+b*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*b^2*c*x^
2+4*(a*x^2+b*x)^(3/2)*a^(1/2)*d)/(x*(a*x+b))^(1/2)/b/a^(1/2)
```

Maxima [A]
time = 0.49, size = 106, normalized size = 1.43

$$\frac{1}{2}\left(2\sqrt{a+\frac{b}{x}}\,x-\frac{b\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{\sqrt{a}}\right)c-\left(\sqrt{a}\,\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)+2\sqrt{a+\frac{b}{x}}\right)d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)*(a+b/x)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*(2*sqrt(a + b/x)*x - b*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/x) + s
qrt(a)))/sqrt(a))*c - (sqrt(a)*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/x)
 + sqrt(a))) + 2*sqrt(a + b/x))*d
```

Fricas [A]
time = 2.44, size = 128, normalized size = 1.73

$$\left[\frac{(bc+2\,ad)\sqrt{a}\,\log\left(2\,ax+2\,\sqrt{a}\,x\sqrt{\frac{ax+b}{x}}+b\right)+2\,(acx-2\,ad)\sqrt{\frac{ax+b}{x}}}{2\,a},-\frac{(bc+2\,ad)\sqrt{-a}\,\arctan\left(\frac{\sqrt{-a}\,\sqrt{\frac{ax+b}{x}}}{a}\right)-(acx-2\,ad)\sqrt{\frac{ax+b}{x}}}{a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)*(a+b/x)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*((b*c + 2*a*d)*sqrt(a)*log(2*a*x + 2*sqrt(a)*x*sqrt((a*x + b)/x) + b)
+ 2*(a*c*x - 2*a*d)*sqrt((a*x + b)/x))/a, -((b*c + 2*a*d)*sqrt(-a)*arctan(s
qrt(-a)*sqrt((a*x + b)/x)/a) - (a*c*x - 2*a*d)*sqrt((a*x + b)/x))/a]
```

Sympy [A]
time = 24.71, size = 87, normalized size = 1.18

$$-\frac{2ad\,\mathrm{atan}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{-a}}\right)}{\sqrt{-a}}+\sqrt{b}\,c\sqrt{x}\,\sqrt{\frac{ax}{b}+1}-2d\sqrt{a+\frac{b}{x}}+\frac{bc\,\mathrm{asinh}\left(\frac{\sqrt{a}\,\sqrt{x}}{\sqrt{b}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)*(a+b/x)**(1/2),x)
```

```
[Out] -2*a*d*atan(sqrt(a + b/x)/sqrt(-a))/sqrt(-a) + sqrt(b)*c*sqrt(x)*sqrt(a*x/b
 + 1) - 2*d*sqrt(a + b/x) + b*c*asinh(sqrt(a)*sqrt(x)/sqrt(b))/sqrt(a)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)*(a+b/x)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [B]
time = 1.96, size = 92, normalized size = 1.24

$$2\sqrt{a}\,d\,\mathrm{atanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)-2\,d\,\sqrt{a+\frac{b}{x}}+c\,x\,\sqrt{a\,x^2+b\,x}\,\sqrt{\frac{1}{x^2}}+\frac{b\,c\,x\,\ln\left(\frac{\frac{b}{2}+a\,x+\sqrt{a}\,\sqrt{a\,x^2+b\,x}}{\sqrt{a}}\right)\sqrt{\frac{1}{x^2}}}{2\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x)^(1/2)*(c + d/x),x)
```

```
[Out] 2*a^(1/2)*d*atanh((a + b/x)^(1/2)/a^(1/2)) - 2*d*(a + b/x)^(1/2) + c*x*(b*x
 + a*x^2)^(1/2)*(1/x^2)^(1/2) + (b*c*x*log((b/2 + a*x + a^(1/2)*(b*x + a*x^
2)^(1/2))/a^(1/2))*(1/x^2)^(1/2))/(2*a^(1/2))
```

3.227 $\int \sqrt{a+\frac{b}{x}}\,dx$

Optimal. Leaf size=39

$$\sqrt{a+\frac{b}{x}}\,x+\frac{b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}}$$

```
[Out] b*arctanh((a+b/x)^(1/2)/a^(1/2))/a^(1/2)+x*(a+b/x)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {248, 43, 65, 214}

$$x\sqrt{a+\frac{b}{x}}+\frac{b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b/x],x]
```

```
[Out] Sqrt[a + b/x]*x + (b*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/Sqrt[a]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 248

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{a+\frac{b}{x}}\,dx &= -\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x^2}\,dx, x, \frac{1}{x}\right)\\ &= \sqrt{a+\frac{b}{x}}\,x-\frac{1}{2}b\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)\\ &= \sqrt{a+\frac{b}{x}}\,x-\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x}}\right)\\ &= \sqrt{a+\frac{b}{x}}\,x+\frac{b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 39, normalized size = 1.00

$$\sqrt{a+\frac{b}{x}}\,x+\frac{b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b/x],x]
```

```
[Out] Sqrt[a + b/x]*x + (b*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/Sqrt[a]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 73 vs. $2(31) = 62$.
time = 0.02, size = 74, normalized size = 1.90

method	result	size
risch	$x\sqrt{\frac{ax+b}{x}}+\frac{b\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^{2}+bx}\right)\sqrt{\frac{ax+b}{x}}\,\sqrt{x\,(ax+b)}}{2\sqrt{a}\,(ax+b)}$	72
default	$\frac{\sqrt{\frac{ax+b}{x}}\,x\left(2\sqrt{a\,x^{2}+bx}\,\sqrt{a}+b\ln\left(\frac{2\sqrt{a\,x^{2}+bx}\,\sqrt{a}+2ax+b}{2\sqrt{a}}\right)\right)}{2\sqrt{x\,(ax+b)}\,\sqrt{a}}$	74

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+1/x*b)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*((a*x+b)/x)^(1/2)*x*(2*(a*x^2+b*x)^(1/2)*a^(1/2)+b*ln(1/2*(2*(a*x^2+b*x
)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2)))/(x*(a*x+b))^(1/2)/a^(1/2)
```

Maxima [A]
time = 0.48, size = 50, normalized size = 1.28

$$\sqrt{a+\frac{b}{x}}\,x-\frac{b\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{2\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(a + b/x)*x - 1/2*b*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/x) + sqrt
(a)))/sqrt(a)
```

Fricas [A]
time = 3.50, size = 99, normalized size = 2.54

$$\left[\frac{2\,ax\sqrt{\frac{ax+b}{x}}+\sqrt{a}\,b\log\left(2\,ax+2\,\sqrt{a}\,x\sqrt{\frac{ax+b}{x}}+b\right)}{2\,a},\frac{ax\sqrt{\frac{ax+b}{x}}-\sqrt{-a}\,b\arctan\left(\frac{\sqrt{-a}\,\sqrt{\frac{ax+b}{x}}}{a}\right)}{a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(2*a*x*sqrt((a*x + b)/x) + sqrt(a)*b*log(2*a*x + 2*sqrt(a)*x*sqrt((a*x
 + b)/x) + b))/a, (a*x*sqrt((a*x + b)/x) - sqrt(-a)*b*arctan(sqrt(-a)*sqrt(
(a*x + b)/x)/a))/a]
```

Sympy [A]
time = 0.92, size = 42, normalized size = 1.08

$$\sqrt{b}\,\sqrt{x}\,\sqrt{\frac{ax}{b}+1}+\frac{b\operatorname{asinh}\left(\frac{\sqrt{a}\,\sqrt{x}}{\sqrt{b}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**(1/2),x)
```

```
[Out] sqrt(b)*sqrt(x)*sqrt(a*x/b + 1) + b*asinh(sqrt(a)*sqrt(x)/sqrt(b))/sqrt(a)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 64 vs. 2(31) = 62.
time = 1.50, size = 64, normalized size = 1.64

$$-\frac{b\log\left(\left|-2\left(\sqrt{a}\,x-\sqrt{ax^2+bx}\right)\sqrt{a}\,-b\right|\right)\operatorname{sgn}(x)}{2\,\sqrt{a}}+\frac{b\log\left(|b|\right)\operatorname{sgn}(x)}{2\,\sqrt{a}}+\sqrt{ax^2+bx}\,\operatorname{sgn}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*b*log(abs(-2*(sqrt(a)*x - sqrt(a*x^2 + b*x))*sqrt(a) - b))*sgn(x)/sqrt
(a) + 1/2*b*log(abs(b))*sgn(x)/sqrt(a) + sqrt(a*x^2 + b*x)*sgn(x)
```

Mupad [B]
time = 0.08, size = 58, normalized size = 1.49

$$x\,\sqrt{a\,x^2+b\,x}\,\sqrt{\frac{1}{x^2}}+\frac{b\,x\,\ln\left(\frac{\frac{b}{2}+a\,x+\sqrt{a}\,\sqrt{a\,x^2+b\,x}}{\sqrt{a}}\right)\,\sqrt{\frac{1}{x^2}}}{2\,\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x)^(1/2),x)
```

```
[Out] x*(b*x + a*x^2)^(1/2)*(1/x^2)^(1/2) + (b*x*log((b/2 + a*x + a^(1/2)*(b*x +
a*x^2)^(1/2))/a^(1/2))*(1/x^2)^(1/2))/(2*a^(1/2))
```

3.228 $\int \frac{\sqrt{a+\frac{b}{x}}}{c+\frac{d}{x}} dx$

Optimal. Leaf size=104

$$\frac{\sqrt{a+\frac{b}{x}}\,x}{c}+\frac{2\sqrt{d}\,\sqrt{bc-ad}\,\tan^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^2}+\frac{(bc-2ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}\,c^2}$$

```
[Out] (-2*a*d+b*c)*arctanh((a+b/x)^(1/2)/a^(1/2))/c^2/a^(1/2)+2*arctan(d^(1/2)*(a
+b/x)^(1/2)/(-a*d+b*c)^(1/2))*d^(1/2)*(-a*d+b*c)^(1/2)/c^2+x*(a+b/x)^(1/2)/
c
```

Rubi [A]
time = 0.07, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {382, 101, 162, 65, 214, 211}

$$\frac{2\sqrt{d}\,\sqrt{bc-ad}\,\text{ArcTan}\left(\frac{\sqrt{d}\,\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^2}+\frac{(bc-2ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}\,c^2}+\frac{x\sqrt{a+\frac{b}{x}}}{c}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b/x]/(c + d/x),x]
```

```
[Out] (Sqrt[a + b/x]*x)/c + (2*Sqrt[d]*Sqrt[b*c - a*d]*ArcTan[(Sqrt[d]*Sqrt[a + b
/x])/Sqrt[b*c - a*d]])/c^2 + ((b*c - 2*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])
/(Sqrt[a]*c^2)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 101

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
```

```
)/((m + 1)*(b*e - a*f))), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+\frac{b}{x}}}{c+\frac{d}{x}}\,dx = -\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x^2(c+dx)}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{\sqrt{a+\frac{b}{x}}\,x}{c} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(bc-2ad)-\frac{bdx}{2}}{x\sqrt{a+bx}\,(c+dx)}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= \frac{\sqrt{a+\frac{b}{x}}\,x}{c} - \frac{(bc-2ad)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)}{2c^2} + \frac{(d(bc-ad))\text{Subst}\left(\int \frac{1}{\sqrt{a+bx}\,(\cdots}\right.}{c^2}$$

$$= \frac{\sqrt{a+\frac{b}{x}}\,x}{c} - \frac{(bc-2ad)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x}}\right)}{bc^2} + \frac{(2d(bc-ad))\text{Subst}\left(\int \frac{1}{c-\frac{ad}{b}\cdots}\right.}{bc^2}$$

$$= \frac{\sqrt{a+\frac{b}{x}}\,x}{c} + \frac{2\sqrt{d}\,\sqrt{bc-ad}\,\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^2} + \frac{(bc-2ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}\,c^2}$$

Mathematica [**A**]
time = 0.22, size = 100, normalized size = 0.96

$$\frac{c\sqrt{a+\frac{b}{x}}\,x + 2\sqrt{d}\,\sqrt{bc-ad}\,\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right) + \frac{(bc-2ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}}}{c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b/x]/(c + d/x),x]
```

```
[Out] (c*Sqrt[a + b/x]*x + 2*Sqrt[d]*Sqrt[b*c - a*d]*ArcTan[(Sqrt[d]*Sqrt[a + b/x
])/Sqrt[b*c - a*d]] + ((b*c - 2*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/Sqrt[a
])/c^2
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 285 vs. $2(86) = 172$.
time = 0.10, size = 286, normalized size = 2.75

method	result
default	$\frac{\sqrt{\frac{ax+b}{x}}\, x \left(2\sqrt{x(ax+b)}\, c^2\sqrt{a}\,\sqrt{\frac{d(ad-bc)}{c^2}} - 2\ln\left(\frac{2\sqrt{x(ax+b)}\,\sqrt{a}+2ax+b}{2\sqrt{a}}\right)\sqrt{\frac{d(ad-bc)}{c^2}}\, acd + \ln\left(\frac{2\sqrt{x \cdots}}{\cdots}\right) \cdots\right)}{\cdots}$
risch	$\frac{x\sqrt{\frac{ax+b}{x}}}{c} + \frac{\left(-\frac{\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^2+bx}\right)\sqrt{a}\, d}{c^2} + \frac{\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^2+bx}\right) b}{2c\sqrt{a}} - \frac{d^2\ln\left(\frac{\frac{2d(ad-bc)}{c^2} - \frac{(2ad-bc)\left(x+\frac{d}{c}\right)}{c} + 2\sqrt{\cdots}}{\cdots}\right)}{\cdots} \cdots\right)}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+1/x*b)^(1/2)/(c+d/x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*((a*x+b)/x)^(1/2)*x*(2*(x*(a*x+b))^(1/2)*c^2*a^(1/2)*(d*(a*d-b*c)/c^2)^
(1/2)-2*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*(d*(a*d-b*c)/
c^2)^(1/2)*a*c*d+ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*(d*(
a*d-b*c)/c^2)^(1/2)*b*c^2-2*ln((2*(x*(a*x+b))^(1/2)*(d*(a*d-b*c)/c^2)^(1/2)
*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*a^(3/2)*d^2+2*ln((2*(x*(a*x+b))^(1/2)*(d*(a*
d-b*c)/c^2)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*a^(1/2)*b*c*d)/(x*(a*x+b))^
(1/2)/c^3/a^(1/2)/(d*(a*d-b*c)/c^2)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(1/2)/(c+d/x),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a + b/x)/(c + d/x), x)
```

Fricas [A]
time = 3.45, size = 482, normalized size = 4.63

$$\left[\frac{2\,acx\sqrt{\frac{ax+b}{x}} - (bc-2\,ad)\sqrt{a}\log\left(2\,ax-2\,\sqrt{a}\,x\sqrt{\frac{ax+b}{x}}+b\right) + 2\,\sqrt{-bcd+ad^2}\,a\log\left(\frac{bd-(bc-2\,ad)x+2\,\sqrt{-bcd+ad^2}\,x\sqrt{\frac{ax+b}{x}}}{cx+d}\right)}{2\,ac^2}, \frac{2\,acx\sqrt{\frac{ax+b}{x}} - 4\,\sqrt{bcd-ad^2}\,a\arctan\left(\frac{\sqrt{bcd-ad^2}\,x\sqrt{\frac{ax+b}{x}}}{adx+bd}\right) - (bc-2\,ad)\sqrt{a}\log\left(2\,ax-2\,\sqrt{a}\,x\sqrt{\frac{ax+b}{x}}+b\right)}{2\,ac^2}, \frac{acx\sqrt{\frac{ax+b}{x}} - (bc-2\,ad)\sqrt{-a}\arctan\left(\frac{\sqrt{-a}\,\sqrt{\frac{ax+b}{x}}}{a}\right) + \sqrt{-bcd+ad^2}\,a\log\left(\frac{bd-(bc-2\,ad)x+2\,\sqrt{-bcd+ad^2}\,x\sqrt{\frac{ax+b}{x}}}{cx+d}\right)}{ac^2}, \frac{acx\sqrt{\frac{ax+b}{x}} - 2\,\sqrt{bcd-ad^2}\,a\arctan\left(\frac{\sqrt{bcd-ad^2}\,x\sqrt{\frac{ax+b}{x}}}{adx+bd}\right) - (bc-2\,ad)\sqrt{-a}\arctan\left(\frac{\sqrt{-a}\,\sqrt{\frac{ax+b}{x}}}{a}\right)}{ac^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(1/2)/(c+d/x),x, algorithm="fricas")
```

```
[Out] [1/2*(2*a*c*x*sqrt((a*x + b)/x) - (b*c - 2*a*d)*sqrt(a)*log(2*a*x - 2*sqrt(
a)*x*sqrt((a*x + b)/x) + b) + 2*sqrt(-b*c*d + a*d^2)*a*log((b*d - (b*c - 2*
a*d)*x + 2*sqrt(-b*c*d + a*d^2)*x*sqrt((a*x + b)/x))/(c*x + d)))/(a*c^2), 1
/2*(2*a*c*x*sqrt((a*x + b)/x) - 4*sqrt(b*c*d - a*d^2)*a*arctan(sqrt(b*c*d -
 a*d^2)*x*sqrt((a*x + b)/x)/(a*d*x + b*d)) - (b*c - 2*a*d)*sqrt(a)*log(2*a*
x - 2*sqrt(a)*x*sqrt((a*x + b)/x) + b))/(a*c^2), (a*c*x*sqrt((a*x + b)/x) -
 (b*c - 2*a*d)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) + sqrt(-b*c*d
+ a*d^2)*a*log((b*d - (b*c - 2*a*d)*x + 2*sqrt(-b*c*d + a*d^2)*x*sqrt((a*x
+ b)/x))/(c*x + d)))/(a*c^2), (a*c*x*sqrt((a*x + b)/x) - 2*sqrt(b*c*d - a*d
^2)*a*arctan(sqrt(b*c*d - a*d^2)*x*sqrt((a*x + b)/x)/(a*d*x + b*d)) - (b*c
- 2*a*d)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a))/(a*c^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt{a+\frac{b}{x}}}{cx+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**(1/2)/(c+d/x),x)
```

```
[Out] Integral(x*sqrt(a + b/x)/(c*x + d), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(1/2)/(c+d/x),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [B]
time = 1.63, size = 149, normalized size = 1.43

$$\frac{x\sqrt{a+\frac{b}{x}}}{c}+\frac{\ln\left(\sqrt{a+\frac{b}{x}}-\sqrt{a}\right)\left(a\,d-\frac{b\,c}{2}\right)}{\sqrt{a}\,c^2}-\frac{\ln\left(\sqrt{a+\frac{b}{x}}+\sqrt{a}\right)\left(2\,a\,d-b\,c\right)}{2\,\sqrt{a}\,c^2}-\frac{\mathrm{atan}\left(\frac{b^4\,d^3\,\sqrt{a+\frac{b}{x}}\,\sqrt{a\,d^2-b\,c\,d}\,4{}\mathrm{i}}{4\,a\,b^4\,d^4-4\,b^5\,c\,d^3}\right)\,\sqrt{a\,d^2-b\,c\,d}\,2{}\mathrm{i}}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x)^(1/2)/(c + d/x),x)
```

```
[Out] (x*(a + b/x)^(1/2))/c - (atan((b^4*d^3*(a + b/x)^(1/2)*(a*d^2 - b*c*d)^(1/2
)*4i)/(4*a*b^4*d^4 - 4*b^5*c*d^3))*(a*d^2 - b*c*d)^(1/2)*2i)/c^2 + (log((a
+ b/x)^(1/2) - a^(1/2))*(a*d - (b*c)/2))/(a^(1/2)*c^2) - (log((a + b/x)^(1/
2) + a^(1/2))*(2*a*d - b*c))/(2*a^(1/2)*c^2)
```

3.229 $\int \frac{\sqrt{a+\frac{b}{x}}}{\left(c+\frac{d}{x}\right)^2} dx$

Optimal. Leaf size=147

$$\frac{2d\sqrt{a+\frac{b}{x}}}{c^2\left(c+\frac{d}{x}\right)}+\frac{\sqrt{a+\frac{b}{x}}\,x}{c\left(c+\frac{d}{x}\right)}+\frac{\sqrt{d}\,(3bc-4ad)\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^3\sqrt{bc-ad}}+\frac{(bc-4ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}\,c^3}$$

```
[Out] (-4*a*d+b*c)*arctanh((a+b/x)^(1/2)/a^(1/2))/c^3/a^(1/2)+(-4*a*d+3*b*c)*arct
an(d^(1/2)*(a+b/x)^(1/2)/(-a*d+b*c)^(1/2))*d^(1/2)/c^3/(-a*d+b*c)^(1/2)+2*d
*(a+b/x)^(1/2)/c^2/(c+d/x)+x*(a+b/x)^(1/2)/c/(c+d/x)
```

Rubi [A]
time = 0.13, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {382, 101, 156, 162, 65, 214, 211}

$$\frac{\sqrt{d}\,(3bc-4ad)\text{ArcTan}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^3\sqrt{bc-ad}}+\frac{(bc-4ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}\,c^3}+\frac{2d\sqrt{a+\frac{b}{x}}}{c^2\left(c+\frac{d}{x}\right)}+\frac{x\sqrt{a+\frac{b}{x}}}{c\left(c+\frac{d}{x}\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b/x]/(c + d/x)^2,x]
```

```
[Out] (2*d*Sqrt[a + b/x])/(c^2*(c + d/x)) + (Sqrt[a + b/x]*x)/(c*(c + d/x)) + (Sq
rt[d]*(3*b*c - 4*a*d)*ArcTan[(Sqrt[d]*Sqrt[a + b/x])/Sqrt[b*c - a*d]])/(c^3
*Sqrt[b*c - a*d]) + ((b*c - 4*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/(Sqrt[a]
*c^3)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 101

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+\frac{b}{x}}}{\left(c+\frac{d}{x}\right)^2}\,dx = -\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x^2(c+dx)^2}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{\sqrt{a+\frac{b}{x}}\,x}{c\left(c+\frac{d}{x}\right)} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(bc-4ad)-\frac{3bdx}{2}}{x\sqrt{a+bx}\,(c+dx)^2}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= \frac{2d\sqrt{a+\frac{b}{x}}}{c^2\left(c+\frac{d}{x}\right)} + \frac{\sqrt{a+\frac{b}{x}}\,x}{c\left(c+\frac{d}{x}\right)} + \frac{\text{Subst}\left(\int \frac{-\frac{1}{2}(bc-4ad)(bc-ad)+bd(bc-ad)x}{x\sqrt{a+bx}\,(c+dx)}\,dx, x, \frac{1}{x}\right)}{c^2(bc-ad)}$$

$$= \frac{2d\sqrt{a+\frac{b}{x}}}{c^2\left(c+\frac{d}{x}\right)} + \frac{\sqrt{a+\frac{b}{x}}\,x}{c\left(c+\frac{d}{x}\right)} - \frac{(bc-4ad)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)}{2c^3} + \frac{(d(3bc-4ad))\text{Su}}{}$$

$$= \frac{2d\sqrt{a+\frac{b}{x}}}{c^2\left(c+\frac{d}{x}\right)} + \frac{\sqrt{a+\frac{b}{x}}\,x}{c\left(c+\frac{d}{x}\right)} - \frac{(bc-4ad)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x}}\right)}{bc^3} + \frac{(d(3bc-4ad}{}$$

$$= \frac{2d\sqrt{a+\frac{b}{x}}}{c^2\left(c+\frac{d}{x}\right)} + \frac{\sqrt{a+\frac{b}{x}}\,x}{c\left(c+\frac{d}{x}\right)} + \frac{\sqrt{d}\,(3bc-4ad)\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^3\sqrt{bc-ad}} + \frac{(bc-4ad)\tanh^{-}}{\sqrt{a}}$$

Mathematica [A]
time = 0.37, size = 122, normalized size = 0.83

$$\frac{\frac{c\sqrt{a+\frac{b}{x}}\,x(2d+cx)}{d+cx} + \frac{\sqrt{d}\,(3bc-4ad)\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{\sqrt{bc-ad}} + \frac{(bc-4ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}}}{c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b/x]/(c + d/x)^2,x]
```

```
[Out] ((c*Sqrt[a + b/x]*x*(2*d + c*x))/(d + c*x) + (Sqrt[d]*(3*b*c - 4*a*d)*ArcTa
n[(Sqrt[d]*Sqrt[a + b/x])/Sqrt[b*c - a*d]])/Sqrt[b*c - a*d] + ((b*c - 4*a*d
)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/Sqrt[a])/c^3
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 944 vs. $2(127) = 254$.
time = 0.08, size = 945, normalized size = 6.43 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+1/x*b)^(1/2)/(c+d/x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*((a*x+b)/x)^(1/2)*x*(-2*(x*(a*x+b))^(1/2)*a^(5/2)*(d*(a*d-b*c)/c^2)^(1/
2)*c^4*x^2-4*a^(7/2)*ln((2*(x*(a*x+b))^(1/2)*(d*(a*d-b*c)/c^2)^(1/2)*c-2*a*
d*x+b*c*x-b*d)/(c*x+d))*c*d^3*x+2*(x*(a*x+b))^(1/2)*a^(5/2)*(d*(a*d-b*c)/c^
2)^(1/2)*c^3*d*x-4*a^(7/2)*ln((2*(x*(a*x+b))^(1/2)*(d*(a*d-b*c)/c^2)^(1/2)*
c-2*a*d*x+b*c*x-b*d)/(c*x+d))*d^4+7*a^(5/2)*ln((2*(x*(a*x+b))^(1/2)*(d*(a*d
-b*c)/c^2)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*b*c^2*d^2*x+2*c^4*(x*(a*x+b)
)^(3/2)*a^(3/2)*(d*(a*d-b*c)/c^2)^(1/2)+4*(x*(a*x+b))^(1/2)*a^(5/2)*(d*(a*d
-b*c)/c^2)^(1/2)*c^2*d^2-4*(x*(a*x+b))^(1/2)*a^(3/2)*(d*(a*d-b*c)/c^2)^(1/2
)*b*c^4*x+7*a^(5/2)*ln((2*(x*(a*x+b))^(1/2)*(d*(a*d-b*c)/c^2)^(1/2)*c-2*a*d
*x+b*c*x-b*d)/(c*x+d))*b*c*d^3-3*a^(3/2)*ln((2*(x*(a*x+b))^(1/2)*(d*(a*d-b*
c)/c^2)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*b^2*c^3*d*x-4*(x*(a*x+b))^(1/2)
*a^(3/2)*(d*(a*d-b*c)/c^2)^(1/2)*b*c^3*d-4*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1
/2)+2*a*x+b)/a^(1/2))*a^3*(d*(a*d-b*c)/c^2)^(1/2)*c^2*d^2*x+5*ln(1/2*(2*(x*
(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^2*(d*(a*d-b*c)/c^2)^(1/2)*b*c^3*
d*x-ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a*(d*(a*d-b*c)/c^
2)^(1/2)*b^2*c^4*x-3*a^(3/2)*ln((2*(x*(a*x+b))^(1/2)*(d*(a*d-b*c)/c^2)^(1/2
)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*b^2*c^2*d^2-4*ln(1/2*(2*(x*(a*x+b))^(1/2)*a
^(1/2)+2*a*x+b)/a^(1/2))*a^3*(d*(a*d-b*c)/c^2)^(1/2)*c*d^3+5*ln(1/2*(2*(x*(
a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^2*(d*(a*d-b*c)/c^2)^(1/2)*b*c^2*d
^2-ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a*(d*(a*d-b*c)/c^2
)^(1/2)*b^2*c^3*d)/c^4/(x*(a*x+b))^(1/2)/(a*d-b*c)/(c*x+d)/a^(3/2)/(d*(a*d-
b*c)/c^2)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(1/2)/(c+d/x)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a + b/x)/(c + d/x)^2, x)
```

Fricas [A]
time = 4.22, size = 801, normalized size = 5.45

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(1/2)/(c+d/x)^2,x, algorithm="fricas")
```

```
[Out] [-1/2*((b*c*d - 4*a*d^2 + (b*c^2 - 4*a*c*d)*x)*sqrt(a)*log(2*a*x - 2*sqrt(a
)*x*sqrt((a*x + b)/x) + b) + (3*a*b*c*d - 4*a^2*d^2 + (3*a*b*c^2 - 4*a^2*c*
```

```
d)*x)*sqrt(-d/(b*c - a*d))*log(-(2*(b*c - a*d)*x*sqrt(-d/(b*c - a*d))*sqrt(
(a*x + b)/x) - b*d + (b*c - 2*a*d)*x)/(c*x + d)) - 2*(a*c^2*x^2 + 2*a*c*d*x
)*sqrt((a*x + b)/x))/(a*c^4*x + a*c^3*d), -1/2*(2*(b*c*d - 4*a*d^2 + (b*c^2
 - 4*a*c*d)*x)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) + (3*a*b*c*d -
 4*a^2*d^2 + (3*a*b*c^2 - 4*a^2*c*d)*x)*sqrt(-d/(b*c - a*d))*log(-(2*(b*c -
 a*d)*x*sqrt(-d/(b*c - a*d))*sqrt((a*x + b)/x) - b*d + (b*c - 2*a*d)*x)/(c*
x + d)) - 2*(a*c^2*x^2 + 2*a*c*d*x)*sqrt((a*x + b)/x))/(a*c^4*x + a*c^3*d),
 1/2*(2*(3*a*b*c*d - 4*a^2*d^2 + (3*a*b*c^2 - 4*a^2*c*d)*x)*sqrt(d/(b*c - a
*d))*arctan(-(b*c - a*d)*x*sqrt(d/(b*c - a*d))*sqrt((a*x + b)/x)/(a*d*x + b
*d)) - (b*c*d - 4*a*d^2 + (b*c^2 - 4*a*c*d)*x)*sqrt(a)*log(2*a*x - 2*sqrt(a
)*x*sqrt((a*x + b)/x) + b) + 2*(a*c^2*x^2 + 2*a*c*d*x)*sqrt((a*x + b)/x))/(
a*c^4*x + a*c^3*d), ((3*a*b*c*d - 4*a^2*d^2 + (3*a*b*c^2 - 4*a^2*c*d)*x)*sq
rt(d/(b*c - a*d))*arctan(-(b*c - a*d)*x*sqrt(d/(b*c - a*d))*sqrt((a*x + b)/
x)/(a*d*x + b*d)) - (b*c*d - 4*a*d^2 + (b*c^2 - 4*a*c*d)*x)*sqrt(-a)*arctan
(sqrt(-a)*sqrt((a*x + b)/x)/a) + (a*c^2*x^2 + 2*a*c*d*x)*sqrt((a*x + b)/x))
/(a*c^4*x + a*c^3*d)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\sqrt{a+\frac{b}{x}}}{\left(cx+d\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**(1/2)/(c+d/x)**2,x)
```

```
[Out] Integral(x**2*sqrt(a + b/x)/(c*x + d)**2, x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(1/2)/(c+d/x)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [**B**]
time = 2.26, size = 1195, normalized size = 8.13

$$-\frac{\frac{2\,b\,d\,{\left(a+\frac{b}{x}\right)}^{3/2}}{c^2}-\frac{b\,\sqrt{a+\frac{b}{x}}\,\left(2\,a\,d-b\,c\right)}{c^2}}{\left(a+\frac{b}{x}\right)\,\left(2\,a\,d-b\,c\right)-d\,{\left(a+\frac{b}{x}\right)}^2-a^2\,d+a\,b\,c}-\frac{\mathrm{atanh}\left(\frac{8\,b^5\,d^3\,\sqrt{a+\frac{b}{x}}}{\sqrt{a}\,\left(8\,b^5\,d^3-\frac{2\,b^6\,c\,d^2}{a}\right)}+\frac{2\,b^6\,d^2\,\sqrt{a+\frac{b}{x}}}{a^{3/2}\,\left(\frac{2\,b^6\,d^2}{a}-\frac{8\,b^5\,d^3}{c}\right)}\right)\,\left(4\,a\,d-b\,c\right)}{\sqrt{a}\,c^3}-\frac{\mathrm{atan}\left(\frac{\frac{\sqrt{d\,\left(a\,d-b\,c\right)}\,\left(\frac{4\,\sqrt{a+\frac{b}{x}}\,\left(16\,a^2\,b^2\,d^5-16\,a\,b^3\,c\,d^4+5\,b^4\,c^2\,d^3\right)}{c^4}-\frac{\left(\frac{2\,\left(2\,b^4\,c^7\,d^2-4\,a\,b^3\,c^6\,d^3\right)}{c^6}-\frac{2\,\left(2\,b^3\,c^7\,d^2-4\,a\,b^2\,c^6\,d^3\right)\,\sqrt{a+\frac{b}{x}}\,\sqrt{d\,\left(a\,d-b\,c\right)}\,\left(4\,a\,d-3\,b\,c\right)}{c^4\,\left(b\,c^4-a\,c^3\,d\right)}\right)\,\sqrt{d\,\left(a\,d-b\,c\right)}\,\left(4\,a\,d-3\,b\,c\right)}{2\,\left(b\,c^4-a\,c^3\,d\right)}\right)\,\left(4\,a\,d-3\,b\,c\right)\,1{}\mathrm{i}}{2\,\left(b\,c^4-a\,c^3\,d\right)}+\frac{\sqrt{d\,\left(a\,d-b\,c\right)}\,\left(\frac{4\,\sqrt{a+\frac{b}{x}}\,\left(16\,a^2\,b^2\,d^5-16\,a\,b^3\,c\,d^4+5\,b^4\,c^2\,d^3\right)}{c^4}+\frac{\left(\frac{2\,\left(2\,b^4\,c^7\,d^2-4\,a\,b^3\,c^6\,d^3\right)}{c^6}+\frac{2\,\left(2\,b^3\,c^7\,d^2-4\,a\,b^2\,c^6\,d^3\right)\,\sqrt{a+\frac{b}{x}}\,\sqrt{d\,\left(a\,d-b\,c\right)}\,\left(4\,a\,d-3\,b\,c\right)}{c^4\,\left(b\,c^4-a\,c^3\,d\right)}\right)\,\sqrt{d\,\left(a\,d-b\,c\right)}\,\left(4\,a\,d-3\,b\,c\right)}{2\,\left(b\,c^4-a\,c^3\,d\right)}\right)\,\left(4\,a\,d-3\,b\,c\right)\,1{}\mathrm{i}}{2\,\left(b\,c^4-a\,c^3\,d\right)}}{\frac{4\,\left(16\,a^2\,b^3\,d^5-16\,a\,b^4\,c\,d^4+3\,b^5\,c^2\,d^3\right)}{c^6}-\frac{\sqrt{d\,\left(a\,d-b\,c\right)}\,\left(\frac{4\,\sqrt{a+\frac{b}{x}}\,\left(16\,a^2\,b^2\,d^5-16\,a\,b^3\,c\,d^4+5\,b^4\,c^2\,d^3\right)}{c^4}-\frac{\left(\frac{2\,\left(2\,b^4\,c^7\,d^2-4\,a\,b^3\,c^6\,d^3\right)}{c^6}-\frac{2\,\left(2\,b^3\,c^7\,d^2-4\,a\,b^2\,c^6\,d^3\right)\,\sqrt{a+\frac{b}{x}}\,\sqrt{d\,\left(a\,d-b\,c\right)}\,\left(4\,a\,d-3\,b\,c\right)}{c^4\,\left(b\,c^4-a\,c^3\,d\right)}\right)\,\sqrt{d\,\left(a\,d-b\,c\right)}\,\left(4\,a\,d-3\,b\,c\right)}{2\,\left(b\,c^4-a\,c^3\,d\right)}\right)\,\left(4\,a\,d-3\,b\,c\right)}{2\,\left(b\,c^4-a\,c^3\,d\right)}+\frac{\sqrt{d\,\left(a\,d-b\,c\right)}\,\left(\frac{4\,\sqrt{a+\frac{b}{x}}\,\left(16\,a^2\,b^2\,d^5-16\,a\,b^3\,c\,d^4+5\,b^4\,c^2\,d^3\right)}{c^4}+\frac{\left(\frac{2\,\left(2\,b^4\,c^7\,d^2-4\,a\,b^3\,c^6\,d^3\right)}{c^6}+\frac{2\,\left(2\,b^3\,c^7\,d^2-4\,a\,b^2\,c^6\,d^3\right)\,\sqrt{a+\frac{b}{x}}\,\sqrt{d\,\left(a\,d-b\,c\right)}\,\left(4\,a\,d-3\,b\,c\right)}{c^4\,\left(b\,c^4-a\,c^3\,d\right)}\right)\,\sqrt{d\,\left(a\,d-b\,c\right)}\,\left(4\,a\,d-3\,b\,c\right)}{2\,\left(b\,c^4-a\,c^3\,d\right)}\right)\,\left(4\,a\,d-3\,b\,c\right)}{2\,\left(b\,c^4-a\,c^3\,d\right)}}\right)\,\sqrt{d\,\left(a\,d-b\,c\right)}\,\left(4\,a\,d-3\,b\,c\right)\,1{}\mathrm{i}}{b\,c^4-a\,c^3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b/x)^(1/2)/(c + d/x)^2,x)

[Out] - ((2*b*d*(a + b/x)^(3/2))/c^2 - (b*(a + b/x)^(1/2)*(2*a*d - b*c))/c^2)/((a + b/x)*(2*a*d - b*c) - d*(a + b/x)^2 - a^2*d + a*b*c) - (atanh((8*b^5*d^3*(a + b/x)^(1/2))/(a^(1/2)*(8*b^5*d^3 - (2*b^6*c*d^2)/a)) + (2*b^6*d^2*(a + b/x)^(1/2))/(a^(3/2)*((2*b^6*d^2)/a - (8*b^5*d^3)/c)))*(4*a*d - b*c))/(a^(1/2)*c^3) - (atan((((d*(a*d - b*c))^(1/2)*((4*(a + b/x)^(1/2)*(16*a^2*b^2*d^5 + 5*b^4*c^2*d^3 - 16*a*b^3*c*d^4))/c^4 - (((2*(2*b^4*c^7*d^2 - 4*a*b^3*c^6*d^3))/c^6 - (2*(2*b^3*c^7*d^2 - 4*a*b^2*c^6*d^3)*(a + b/x)^(1/2)*(d*(a*d - b*c))^(1/2)*(4*a*d - 3*b*c))/(c^4*(b*c^4 - a*c^3*d)))*(d*(a*d - b*c))^(1/2)*(4*a*d - 3*b*c))/(2*(b*c^4 - a*c^3*d)))*(4*a*d - 3*b*c)*1i)/(2*(b*c^4 - a*c^3*d)) + ((d*(a*d - b*c))^(1/2)*((4*(a + b/x)^(1/2)*(16*a^2*b^2*d^5 + 5*b^4*c^2*d^3 - 16*a*b^3*c*d^4))/c^4 + (((2*(2*b^4*c^7*d^2 - 4*a*b^3*c^6*d^3))/c^6 + (2*(2*b^3*c^7*d^2 - 4*a*b^2*c^6*d^3)*(a + b/x)^(1/2)*(d*(a*d - b*c))^(1/2)*(4*a*d - 3*b*c))/(c^4*(b*c^4 - a*c^3*d)))*(d*(a*d - b*c))^(1/2)*(4*a*d - 3*b*c))/(2*(b*c^4 - a*c^3*d)))*(4*a*d - 3*b*c)*1i)/(2*(b*c^4 - a*c^3*d)))/((4*(16*a^2*b^3*d^5 + 3*b^5*c^2*d^3 - 16*a*b^4*c*d^4))/c^6 - ((d*(a*d - b*c))^(1/2)*((4*(a + b/x)^(1/2)*(16*a^2*b^2*d^5 + 5*b^4*c^2*d^3 - 16*a*b^3*c*d^4))/c^4 - (((2*(2*b^4*c^7*d^2 - 4*a*b^3*c^6*d^3))/c^6 - (2*(2*b^3*c^7*d^2 - 4*a*b^2*c^6*d^3)*(a + b/x)^(1/2)*(d*(a*d - b*c))^(1/2)*(4*a*d - 3*b*c))/(c^4*(b*c^4 - a*c^3*d)))*(d*(a*d - b*c))^(1/2)*(4*a*d - 3*b*c))/(2*(b*c^4 - a*c^3*d)))*(4*a*d - 3*b*c))/(2*(b*c^4 - a*c^3*d)) + ((d*(a*d - b*c))^(1/2)*((4*(a + b/x)^(1/2)*(16*a^2*b^2*d^5 + 5*b^4*c^2*d^3 - 16*a*b^3*c*d^4))/c^4 + (((2*(2*b^4*c^7*d^2 - 4*a*b^3*c^6*d^3))/c^6 + (2*(2*b^3*c^7*d^2 - 4*a*b^2*c^6*d^3)*(a + b/x)^(1/2)*(d*(a*d - b*c))^(1/2)*(4*a*d - 3*b*c))/(c^4*(b*c^4 - a*c^3*d)))*(d*(a*d - b*c))^(1/2)*(4*a*d - 3*b*c))/(2*(b*c^4 - a*c^3*d)))*(4*a*d - 3*b*c))/(2*(b*c^4 - a*c^3*d))))*(d*(a*d - b*c))^(1/2)*(4*a*d - 3*b*c)*1i)/(b*c^4 - a*c^3*d)

3.230 $\int \frac{\sqrt{a+\frac{b}{x}}}{\left(c+\frac{d}{x}\right)^3} \, dx$

Optimal. Leaf size=213

$$\frac{3d\sqrt{a+\frac{b}{x}}}{2c^2\left(c+\frac{d}{x}\right)^2}+\frac{d(11bc-12ad)\sqrt{a+\frac{b}{x}}}{4c^3(bc-ad)\left(c+\frac{d}{x}\right)}+\frac{\sqrt{a+\frac{b}{x}}\,x}{c\left(c+\frac{d}{x}\right)^2}+\frac{\sqrt{d}\,(15b^2c^2-40abcd+24a^2d^2)\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{4c^4(bc-ad)^{3/2}}$$

```
[Out] (-6*a*d+b*c)*arctanh((a+b/x)^(1/2)/a^(1/2))/c^4/a^(1/2)+1/4*(24*a^2*d^2-40*
a*b*c*d+15*b^2*c^2)*arctan(d^(1/2)*(a+b/x)^(1/2)/(-a*d+b*c)^(1/2))*d^(1/2)/
c^4/(-a*d+b*c)^(3/2)+3/2*d*(a+b/x)^(1/2)/c^2/(c+d/x)^2+1/4*d*(-12*a*d+11*b*
c)*(a+b/x)^(1/2)/c^3/(-a*d+b*c)/(c+d/x)+x*(a+b/x)^(1/2)/c/(c+d/x)^2
```

Rubi [A]

time = 0.22, antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {382, 101, 156, 162, 65, 214, 211}

$$\frac{\sqrt{d}\,(24a^2d^2-40abcd+15b^2c^2)\,\text{ArcTan}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{4c^4(bc-ad)^{3/2}}+\frac{(bc-6ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}\,c^4}+\frac{d\sqrt{a+\frac{b}{x}}\,(11bc-12ad)}{4c^3\left(c+\frac{d}{x}\right)(bc-ad)}+\frac{3d\sqrt{a+\frac{b}{x}}}{2c^2\left(c+\frac{d}{x}\right)^2}+\frac{x\sqrt{a+\frac{b}{x}}}{c\left(c+\frac{d}{x}\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b/x]/(c + d/x)^3,x]
```

```
[Out] (3*d*Sqrt[a + b/x])/(2*c^2*(c + d/x)^2) + (d*(11*b*c - 12*a*d)*Sqrt[a + b/x
])/(4*c^3*(b*c - a*d)*(c + d/x)) + (Sqrt[a + b/x]*x)/(c*(c + d/x)^2) + (Sqr
t[d]*(15*b^2*c^2 - 40*a*b*c*d + 24*a^2*d^2)*ArcTan[(Sqrt[d]*Sqrt[a + b/x])/
Sqrt[b*c - a*d]])/(4*c^4*(b*c - a*d)^(3/2)) + ((b*c - 6*a*d)*ArcTanh[Sqrt[a
 + b/x]/Sqrt[a]])/(Sqrt[a]*c^4)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 101

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+\frac{b}{x}}}{\left(c+\frac{d}{x}\right)^3}\,dx = -\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x^2(c+dx)^3}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{\sqrt{a+\frac{b}{x}}\,x}{c\left(c+\frac{d}{x}\right)^2} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(bc-6ad)-\frac{5bdx}{2}}{x\sqrt{a+bx}\,(c+dx)^3}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= \frac{3d\sqrt{a+\frac{b}{x}}}{2c^2\left(c+\frac{d}{x}\right)^2} + \frac{\sqrt{a+\frac{b}{x}}\,x}{c\left(c+\frac{d}{x}\right)^2} + \frac{\text{Subst}\left(\int \frac{-(bc-6ad)(bc-ad)+\frac{9}{2}bd(bc-ad)x}{x\sqrt{a+bx}\,(c+dx)^2}\,dx, x, \frac{1}{x}\right)}{2c^2(bc-ad)}$$

$$= \frac{3d\sqrt{a+\frac{b}{x}}}{2c^2\left(c+\frac{d}{x}\right)^2} + \frac{d(11bc-12ad)\sqrt{a+\frac{b}{x}}}{4c^3(bc-ad)\left(c+\frac{d}{x}\right)} + \frac{\sqrt{a+\frac{b}{x}}\,x}{c\left(c+\frac{d}{x}\right)^2} - \frac{\text{Subst}\left(\int \frac{(bc-6ad)(bc-ad)^2-\frac{1}{4}bd(11bc-}{x\sqrt{a+bx}\,(c+}\right.}{2c^3(bc-ad)}$$

$$= \frac{3d\sqrt{a+\frac{b}{x}}}{2c^2\left(c+\frac{d}{x}\right)^2} + \frac{d(11bc-12ad)\sqrt{a+\frac{b}{x}}}{4c^3(bc-ad)\left(c+\frac{d}{x}\right)} + \frac{\sqrt{a+\frac{b}{x}}\,x}{c\left(c+\frac{d}{x}\right)^2} - \frac{(bc-6ad)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,d\right.}{2c^4}$$

$$= \frac{3d\sqrt{a+\frac{b}{x}}}{2c^2\left(c+\frac{d}{x}\right)^2} + \frac{d(11bc-12ad)\sqrt{a+\frac{b}{x}}}{4c^3(bc-ad)\left(c+\frac{d}{x}\right)} + \frac{\sqrt{a+\frac{b}{x}}\,x}{c\left(c+\frac{d}{x}\right)^2} - \frac{(bc-6ad)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x\right.}{bc^4}$$

$$= \frac{3d\sqrt{a+\frac{b}{x}}}{2c^2\left(c+\frac{d}{x}\right)^2} + \frac{d(11bc-12ad)\sqrt{a+\frac{b}{x}}}{4c^3(bc-ad)\left(c+\frac{d}{x}\right)} + \frac{\sqrt{a+\frac{b}{x}}\,x}{c\left(c+\frac{d}{x}\right)^2} + \frac{\sqrt{d}\,(15b^2c^2-40abcd+24a^2d^2)}{4c^4(bc-ad}$$

Mathematica [A]
time = 1.03, size = 189, normalized size = 0.89

$$\frac{\frac{c\sqrt{a+\frac{b}{x}}\,x(-2ad(6d^2+9cdx+2c^2x^2)+bc(11d^2+17cdx+4c^2x^2))}{(bc-ad)(d+cx)^2} + \frac{\sqrt{d}\,(15b^2c^2-40abcd+24a^2d^2)\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{3/2}} + \frac{4(bc-6ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}}}{4c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b/x]/(c + d/x)^3,x]
```

```
[Out] ((c*Sqrt[a + b/x]*x*(-2*a*d*(6*d^2 + 9*c*d*x + 2*c^2*x^2) + b*c*(11*d^2 + 1
7*c*d*x + 4*c^2*x^2)))/((b*c - a*d)*(d + c*x)^2) + (Sqrt[d]*(15*b^2*c^2 - 4
```

```
0*a*b*c*d + 24*a^2*d^2)*ArcTan[(Sqrt[d]*Sqrt[a + b/x])/Sqrt[b*c - a*d]])/(b
*c - a*d)^(3/2) + (4*(b*c - 6*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/Sqrt[a])
/(4*c^4)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1971 vs. $2(185) = 370$.
time = 0.09, size = 1972, normalized size = 9.26

method	result	size
risch	Expression too large to display	1949
default	Expression too large to display	1972

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+1/x*b)^(1/2)/(c+d/x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*((a*x+b)/x)^(1/2)*x*(-64*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a
^(1/2))*a^2*(d*(a*d-b*c)/c^2)^(1/2)*b^2*c^4*d^2*x-48*ln(1/2*(2*(x*(a*x+b))^
(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^4*(d*(a*d-b*c)/c^2)^(1/2)*c^2*d^4*x+8*ln(
1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a*(d*(a*d-b*c)/c^2)^(1/2
)*b^3*c^5*d*x+128*a^(7/2)*ln((2*(x*(a*x+b))^(1/2)*(d*(a*d-b*c)/c^2)^(1/2)*c
-2*a*d*x+b*c*x-b*d)/(c*x+d))*b*c^2*d^4*x-110*a^(5/2)*ln((2*(x*(a*x+b))^(1/2
)*(d*(a*d-b*c)/c^2)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*b^2*c^3*d^3*x-46*(x
*(a*x+b))^(1/2)*a^(5/2)*(d*(a*d-b*c)/c^2)^(1/2)*b*c^3*d^3+52*ln(1/2*(2*(x*(
a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^3*(d*(a*d-b*c)/c^2)^(1/2)*b*c^2*d
^4-32*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^2*(d*(a*d-b*c
)/c^2)^(1/2)*b^2*c^3*d^3+52*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^
(1/2))*a^3*(d*(a*d-b*c)/c^2)^(1/2)*b*c^4*d^2*x^2-32*ln(1/2*(2*(x*(a*x+b))^(
1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^2*(d*(a*d-b*c)/c^2)^(1/2)*b^2*c^5*d*x^2-78
*(x*(a*x+b))^(1/2)*a^(5/2)*(d*(a*d-b*c)/c^2)^(1/2)*b*c^4*d^2*x+104*ln(1/2*(
2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^3*(d*(a*d-b*c)/c^2)^(1/2)*b
*c^3*d^3*x-18*(x*(a*x+b))^(1/2)*a^(5/2)*(d*(a*d-b*c)/c^2)^(1/2)*b*c^5*d*x^2
+15*a^(3/2)*ln((2*(x*(a*x+b))^(1/2)*(d*(a*d-b*c)/c^2)^(1/2)*c-2*a*d*x+b*c*x
-b*d)/(c*x+d))*b^3*c^3*d^3-12*(x*(a*x+b))^(1/2)*a^(7/2)*(d*(a*d-b*c)/c^2)^(
1/2)*c^5*d*x^3+14*(x*(a*x+b))^(1/2)*a^(5/2)*(d*(a*d-b*c)/c^2)^(1/2)*b*c^6*x
^3-48*a^(9/2)*ln((2*(x*(a*x+b))^(1/2)*(d*(a*d-b*c)/c^2)^(1/2)*c-2*a*d*x+b*c
*x-b*d)/(c*x+d))*c*d^5*x+30*a^(3/2)*ln((2*(x*(a*x+b))^(1/2)*(d*(a*d-b*c)/c^
2)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*b^3*c^4*d^2*x+24*(x*(a*x+b))^(1/2)*a
^(7/2)*(d*(a*d-b*c)/c^2)^(1/2)*c^2*d^4+22*(x*(a*x+b))^(1/2)*a^(3/2)*(d*(a*d
-b*c)/c^2)^(1/2)*b^2*c^4*d^2-24*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b
)/a^(1/2))*a^4*(d*(a*d-b*c)/c^2)^(1/2)*c*d^5+4*ln(1/2*(2*(x*(a*x+b))^(1/2)*
a^(1/2)+2*a*x+b)/a^(1/2))*a*(d*(a*d-b*c)/c^2)^(1/2)*b^3*c^4*d^2+64*a^(7/2)*
ln((2*(x*(a*x+b))^(1/2)*(d*(a*d-b*c)/c^2)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d
))*b*c*d^5-55*a^(5/2)*ln((2*(x*(a*x+b))^(1/2)*(d*(a*d-b*c)/c^2)^(1/2)*c-2*a
*d*x+b*c*x-b*d)/(c*x+d))*b^2*c^2*d^4-14*(x*(a*x+b))^(3/2)*a^(3/2)*(d*(a*d-b
*c)/c^2)^(1/2)*b*c^6*x+22*(x*(a*x+b))^(1/2)*a^(3/2)*(d*(a*d-b*c)/c^2)^(1/2)
```

```
*b^2*c^6*x^2+4*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a*(d*(
a*d-b*c)/c^2)^(1/2)*b^3*c^6*x^2-24*a^(9/2)*ln((2*(x*(a*x+b))^(1/2)*(d*(a*d-
b*c)/c^2)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*c^2*d^4*x^2+15*a^(3/2)*ln((2*
(x*(a*x+b))^(1/2)*(d*(a*d-b*c)/c^2)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*b^3
*c^5*d*x^2+8*(x*(a*x+b))^(3/2)*a^(5/2)*(d*(a*d-b*c)/c^2)^(1/2)*c^4*d^2-10*(
x*(a*x+b))^(3/2)*a^(3/2)*(d*(a*d-b*c)/c^2)^(1/2)*b*c^5*d+12*(x*(a*x+b))^(3/
2)*a^(5/2)*(d*(a*d-b*c)/c^2)^(1/2)*c^5*d*x-24*ln(1/2*(2*(x*(a*x+b))^(1/2)*a
^(1/2)+2*a*x+b)/a^(1/2))*a^4*(d*(a*d-b*c)/c^2)^(1/2)*c^3*d^3*x^2+64*a^(7/2)
*ln((2*(x*(a*x+b))^(1/2)*(d*(a*d-b*c)/c^2)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+
d))*b*c^3*d^3*x^2-55*a^(5/2)*ln((2*(x*(a*x+b))^(1/2)*(d*(a*d-b*c)/c^2)^(1/2
)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*b^2*c^4*d^2*x^2+36*(x*(a*x+b))^(1/2)*a^(7/2
)*(d*(a*d-b*c)/c^2)^(1/2)*c^3*d^3*x+44*(x*(a*x+b))^(1/2)*a^(3/2)*(d*(a*d-b*
c)/c^2)^(1/2)*b^2*c^5*d*x-24*a^(9/2)*ln((2*(x*(a*x+b))^(1/2)*(d*(a*d-b*c)/c
^2)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*d^6)/c^5/(x*(a*x+b))^(1/2)/(a*d-b*c
)^2/(c*x+d)^2/a^(3/2)/(d*(a*d-b*c)/c^2)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(1/2)/(c+d/x)^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a + b/x)/(c + d/x)^3, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 428 vs. $2(185) = 370$.
time = 3.46, size = 1749, normalized size = 8.21

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(1/2)/(c+d/x)^3,x, algorithm="fricas")
```

```
[Out] [-1/8*(4*(b^2*c^2*d^2 - 7*a*b*c*d^3 + 6*a^2*d^4 + (b^2*c^4 - 7*a*b*c^3*d +
6*a^2*c^2*d^2)*x^2 + 2*(b^2*c^3*d - 7*a*b*c^2*d^2 + 6*a^2*c*d^3)*x)*sqrt(a)
*log(2*a*x - 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) + (15*a*b^2*c^2*d^2 - 40*a^
2*b*c*d^3 + 24*a^3*d^4 + (15*a*b^2*c^4 - 40*a^2*b*c^3*d + 24*a^3*c^2*d^2)*x
^2 + 2*(15*a*b^2*c^3*d - 40*a^2*b*c^2*d^2 + 24*a^3*c*d^3)*x)*sqrt(-d/(b*c -
 a*d))*log(-(2*(b*c - a*d)*x*sqrt(-d/(b*c - a*d))*sqrt((a*x + b)/x) - b*d +
 (b*c - 2*a*d)*x)/(c*x + d)) - 2*(4*(a*b*c^4 - a^2*c^3*d)*x^3 + (17*a*b*c^3
*d - 18*a^2*c^2*d^2)*x^2 + (11*a*b*c^2*d^2 - 12*a^2*c*d^3)*x)*sqrt((a*x + b
)/x))/(a*b*c^5*d^2 - a^2*c^4*d^3 + (a*b*c^7 - a^2*c^6*d)*x^2 + 2*(a*b*c^6*d
 - a^2*c^5*d^2)*x), 1/4*((15*a*b^2*c^2*d^2 - 40*a^2*b*c*d^3 + 24*a^3*d^4 +
(15*a*b^2*c^4 - 40*a^2*b*c^3*d + 24*a^3*c^2*d^2)*x^2 + 2*(15*a*b^2*c^3*d -
```

```
40*a^2*b*c^2*d^2 + 24*a^3*c*d^3)*x)*sqrt(d/(b*c - a*d))*arctan(-(b*c - a*d)
*x*sqrt(d/(b*c - a*d))*sqrt((a*x + b)/x)/(a*d*x + b*d)) - 2*(b^2*c^2*d^2 -
7*a*b*c*d^3 + 6*a^2*d^4 + (b^2*c^4 - 7*a*b*c^3*d + 6*a^2*c^2*d^2)*x^2 + 2*(
b^2*c^3*d - 7*a*b*c^2*d^2 + 6*a^2*c*d^3)*x)*sqrt(a)*log(2*a*x - 2*sqrt(a)*x
*sqrt((a*x + b)/x) + b) + (4*(a*b*c^4 - a^2*c^3*d)*x^3 + (17*a*b*c^3*d - 18
*a^2*c^2*d^2)*x^2 + (11*a*b*c^2*d^2 - 12*a^2*c*d^3)*x)*sqrt((a*x + b)/x))/(
a*b*c^5*d^2 - a^2*c^4*d^3 + (a*b*c^7 - a^2*c^6*d)*x^2 + 2*(a*b*c^6*d - a^2*
c^5*d^2)*x), -1/8*(8*(b^2*c^2*d^2 - 7*a*b*c*d^3 + 6*a^2*d^4 + (b^2*c^4 - 7*
a*b*c^3*d + 6*a^2*c^2*d^2)*x^2 + 2*(b^2*c^3*d - 7*a*b*c^2*d^2 + 6*a^2*c*d^3
)*x)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) + (15*a*b^2*c^2*d^2 - 40
*a^2*b*c*d^3 + 24*a^3*d^4 + (15*a*b^2*c^4 - 40*a^2*b*c^3*d + 24*a^3*c^2*d^2
)*x^2 + 2*(15*a*b^2*c^3*d - 40*a^2*b*c^2*d^2 + 24*a^3*c*d^3)*x)*sqrt(-d/(b*
c - a*d))*log(-(2*(b*c - a*d)*x*sqrt(-d/(b*c - a*d))*sqrt((a*x + b)/x) - b*
d + (b*c - 2*a*d)*x)/(c*x + d)) - 2*(4*(a*b*c^4 - a^2*c^3*d)*x^3 + (17*a*b*
c^3*d - 18*a^2*c^2*d^2)*x^2 + (11*a*b*c^2*d^2 - 12*a^2*c*d^3)*x)*sqrt((a*x
+ b)/x))/(a*b*c^5*d^2 - a^2*c^4*d^3 + (a*b*c^7 - a^2*c^6*d)*x^2 + 2*(a*b*c^
6*d - a^2*c^5*d^2)*x), 1/4*((15*a*b^2*c^2*d^2 - 40*a^2*b*c*d^3 + 24*a^3*d^4
 + (15*a*b^2*c^4 - 40*a^2*b*c^3*d + 24*a^3*c^2*d^2)*x^2 + 2*(15*a*b^2*c^3*d
 - 40*a^2*b*c^2*d^2 + 24*a^3*c*d^3)*x)*sqrt(d/(b*c - a*d))*arctan(-(b*c - a
*d)*x*sqrt(d/(b*c - a*d))*sqrt((a*x + b)/x)/(a*d*x + b*d)) - 4*(b^2*c^2*d^2
 - 7*a*b*c*d^3 + 6*a^2*d^4 + (b^2*c^4 - 7*a*b*c^3*d + 6*a^2*c^2*d^2)*x^2 +
2*(b^2*c^3*d - 7*a*b*c^2*d^2 + 6*a^2*c*d^3)*x)*sqrt(-a)*arctan(sqrt(-a)*sqr
t((a*x + b)/x)/a) + (4*(a*b*c^4 - a^2*c^3*d)*x^3 + (17*a*b*c^3*d - 18*a^2*c
^2*d^2)*x^2 + (11*a*b*c^2*d^2 - 12*a^2*c*d^3)*x)*sqrt((a*x + b)/x))/(a*b*c^
5*d^2 - a^2*c^4*d^3 + (a*b*c^7 - a^2*c^6*d)*x^2 + 2*(a*b*c^6*d - a^2*c^5*d^
2)*x)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3\sqrt{a+\frac{b}{x}}}{(cx+d)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**(1/2)/(c+d/x)**3,x)
```

```
[Out] Integral(x**3*sqrt(a + b/x)/(c*x + d)**3, x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 820 vs. 2(185) = 370.
time = 2.00, size = 820, normalized size = 3.85

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(1/2)/(c+d/x)^3,x, algorithm="giac")
```

```
[Out] -1/4*(15*sqrt(a)*b^2*c^2*d*arctan(sqrt(a)*d/sqrt(b*c*d - a*d^2)) - 40*a^(3/
2)*b*c*d^2*arctan(sqrt(a)*d/sqrt(b*c*d - a*d^2)) + 24*a^(5/2)*d^3*arctan(sq
rt(a)*d/sqrt(b*c*d - a*d^2)) - 2*sqrt(b*c*d - a*d^2)*b^2*c^2*log(abs(b)) +
14*sqrt(b*c*d - a*d^2)*a*b*c*d*log(abs(b)) - 12*sqrt(b*c*d - a*d^2)*a^2*d^2
*log(abs(b)) + 9*sqrt(b*c*d - a*d^2)*a*b*c*d - 10*sqrt(b*c*d - a*d^2)*a^2*d
^2)*sgn(x)/(sqrt(b*c*d - a*d^2)*sqrt(a)*b*c^5 - sqrt(b*c*d - a*d^2)*a^(3/2)
*c^4*d) - 1/4*(15*b^2*c^2*d*sgn(x) - 40*a*b*c*d^2*sgn(x) + 24*a^2*d^3*sgn(x
))*arctan(-((sqrt(a)*x - sqrt(a*x^2 + b*x))*c + sqrt(a)*d)/sqrt(b*c*d - a*d
^2))/((b*c^5 - a*c^4*d)*sqrt(b*c*d - a*d^2)) + sqrt(a*x^2 + b*x)*sgn(x)/c^3
 - 1/4*(9*(sqrt(a)*x - sqrt(a*x^2 + b*x))^3*sqrt(a)*b^2*c^3*d*sgn(x) - 32*(
sqrt(a)*x - sqrt(a*x^2 + b*x))^3*a^(3/2)*b*c^2*d^2*sgn(x) + 24*(sqrt(a)*x -
 sqrt(a*x^2 + b*x))^3*a^(5/2)*c*d^3*sgn(x) + 3*(sqrt(a)*x - sqrt(a*x^2 + b*
x))^2*a*b^2*c^2*d^2*sgn(x) - 40*(sqrt(a)*x - sqrt(a*x^2 + b*x))^2*a^2*b*c*d
^3*sgn(x) + 40*(sqrt(a)*x - sqrt(a*x^2 + b*x))^2*a^3*d^4*sgn(x) + 7*(sqrt(a
)*x - sqrt(a*x^2 + b*x))*sqrt(a)*b^3*c^2*d^2*sgn(x) - 44*(sqrt(a)*x - sqrt(
a*x^2 + b*x))*a^(3/2)*b^2*c*d^3*sgn(x) + 40*(sqrt(a)*x - sqrt(a*x^2 + b*x))
*a^(5/2)*b*d^4*sgn(x) - 9*a*b^3*c*d^3*sgn(x) + 10*a^2*b^2*d^4*sgn(x))/((sqr
t(a)*b*c^5 - a^(3/2)*c^4*d)*((sqrt(a)*x - sqrt(a*x^2 + b*x))^2*c + 2*(sqrt(
a)*x - sqrt(a*x^2 + b*x))*sqrt(a)*d + b*d)^2) - 1/2*(b*c*sgn(x) - 6*a*d*sgn
(x))*log(abs(2*(sqrt(a)*x - sqrt(a*x^2 + b*x))*sqrt(a) + b))/(sqrt(a)*c^4)
```

Mupad [**B**]
time = 3.73, size = 1895, normalized size = 8.90

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x)^(1/2)/(c + d/x)^3,x)
```

```
[Out] (log((a + b/x)^(1/2)*(d*(a*d - b*c)^3)^(1/2) - a^2*d^2 - b^2*c^2 + 2*a*b*c*
d)*(d*(a*d - b*c)^3)^(1/2)*(3*a^2*d^2 + (15*b^2*c^2)/8 - 5*a*b*c*d))/(b^3*c
^7 - a^3*c^4*d^3 + 3*a^2*b*c^5*d^2 - 3*a*b^2*c^6*d) - ((b*(a + b/x)^(1/2)*(
12*a^2*d^2 + 4*b^2*c^2 - 17*a*b*c*d))/(4*c^3) + (b*(a + b/x)^(5/2)*(12*a*d^
3 - 11*b*c*d^2))/(4*c^3*(a*d - b*c)) - (d*(a + b/x)^(3/2)*(17*b^3*c^2 + 24*
a^2*b*d^2 - 40*a*b^2*c*d))/(4*c^3*(a*d - b*c)))/((a + b/x)^2*(3*a*d^2 - 2*b
*c*d) - (a + b/x)*(3*a^2*d^2 + b^2*c^2 - 4*a*b*c*d) - d^2*(a + b/x)^3 + a^3
*d^2 + a*b^2*c^2 - 2*a^2*b*c*d) - (log((a + b/x)^(1/2)*(d*(a*d - b*c)^3)^(1
/2) + a^2*d^2 + b^2*c^2 - 2*a*b*c*d)*(d*(a*d - b*c)^3)^(1/2)*(24*a^2*d^2 +
15*b^2*c^2 - 40*a*b*c*d))/(8*(b^3*c^7 - a^3*c^4*d^3 + 3*a^2*b*c^5*d^2 - 3*a
*b^2*c^6*d)) - (atan((((((a + b/x)^(1/2)*(1152*a^4*b^2*d^7 + 241*b^6*c^4*d^
3 - 1424*a*b^5*c^3*d^4 - 3264*a^3*b^3*c*d^6 + 3296*a^2*b^4*c^2*d^5))/(8*(b^
2*c^8 + a^2*c^6*d^2 - 2*a*b*c^7*d)) - ((6*a*d - b*c)*((4*b^6*c^11*d^2 - 21*
```

a*b^5*c^10*d^3 + 29*a^2*b^4*c^9*d^4 - 12*a^3*b^3*c^8*d^5)/(b^2*c^11 + a^2*c^9*d^2 - 2*a*b*c^10*d) - ((a + b/x)^(1/2)*(6*a*d - b*c)*(64*b^5*c^11*d^2 - 256*a*b^4*c^10*d^3 + 320*a^2*b^3*c^9*d^4 - 128*a^3*b^2*c^8*d^5))/(16*a^(1/2)*c^4*(b^2*c^8 + a^2*c^6*d^2 - 2*a*b*c^7*d))))/(2*a^(1/2)*c^4))*(6*a*d - b*c)*1i)/(2*a^(1/2)*c^4) + ((((a + b/x)^(1/2)*(1152*a^4*b^2*d^7 + 241*b^6*c^4*d^3 - 1424*a*b^5*c^3*d^4 - 3264*a^3*b^3*c*d^6 + 3296*a^2*b^4*c^2*d^5))/(8*(b^2*c^8 + a^2*c^6*d^2 - 2*a*b*c^7*d)) + ((6*a*d - b*c)*((4*b^6*c^11*d^2 - 21*a*b^5*c^10*d^3 + 29*a^2*b^4*c^9*d^4 - 12*a^3*b^3*c^8*d^5)/(b^2*c^11 + a^2*c^9*d^2 - 2*a*b*c^10*d) + ((a + b/x)^(1/2)*(6*a*d - b*c)*(64*b^5*c^11*d^2 - 256*a*b^4*c^10*d^3 + 320*a^2*b^3*c^9*d^4 - 128*a^3*b^2*c^8*d^5))/(16*a^(1/2)*c^4*(b^2*c^8 + a^2*c^6*d^2 - 2*a*b*c^7*d))))/(2*a^(1/2)*c^4))*(6*a*d - b*c)*1i)/(2*a^(1/2)*c^4))/((216*a^4*b^3*d^7 + (165*b^7*c^4*d^3)/8 - (805*a*b^6*c^3*d^4)/4 - 594*a^3*b^4*c*d^6 + 558*a^2*b^5*c^2*d^5)/(b^2*c^11 + a^2*c^9*d^2 - 2*a*b*c^10*d) - ((((a + b/x)^(1/2)*(1152*a^4*b^2*d^7 + 241*b^6*c^4*d^3 - 1424*a*b^5*c^3*d^4 - 3264*a^3*b^3*c*d^6 + 3296*a^2*b^4*c^2*d^5))/(8*(b^2*c^8 + a^2*c^6*d^2 - 2*a*b*c^7*d)) - ((6*a*d - b*c)*((4*b^6*c^11*d^2 - 21*a*b^5*c^10*d^3 + 29*a^2*b^4*c^9*d^4 - 12*a^3*b^3*c^8*d^5)/(b^2*c^11 + a^2*c^9*d^2 - 2*a*b*c^10*d) - ((a + b/x)^(1/2)*(6*a*d - b*c)*(64*b^5*c^11*d^2 - 256*a*b^4*c^10*d^3 + 320*a^2*b^3*c^9*d^4 - 128*a^3*b^2*c^8*d^5))/(16*a^(1/2)*c^4*(b^2*c^8 + a^2*c^6*d^2 - 2*a*b*c^7*d))))/(2*a^(1/2)*c^4))*(6*a*d - b*c))/(2*a^(1/2)*c^4) + ((((a + b/x)^(1/2)*(1152*a^4*b^2*d^7 + 241*b^6*c^4*d^3 - 1424*a*b^5*c^3*d^4 - 3264*a^3*b^3*c*d^6 + 3296*a^2*b^4*c^2*d^5))/(8*(b^2*c^8 + a^2*c^6*d^2 - 2*a*b*c^7*d)) + ((6*a*d - b*c)*((4*b^6*c^11*d^2 - 21*a*b^5*c^10*d^3 + 29*a^2*b^4*c^9*d^4 - 12*a^3*b^3*c^8*d^5)/(b^2*c^11 + a^2*c^9*d^2 - 2*a*b*c^10*d) + ((a + b/x)^(1/2)*(6*a*d - b*c)*(64*b^5*c^11*d^2 - 256*a*b^4*c^10*d^3 + 320*a^2*b^3*c^9*d^4 - 128*a^3*b^2*c^8*d^5))/(16*a^(1/2)*c^4*(b^2*c^8 + a^2*c^6*d^2 - 2*a*b*c^7*d))))/(2*a^(1/2)*c^4))*(6*a*d - b*c))/(2*a^(1/2)*c^4)))*(6*a*d - b*c)*1i)/(a^(1/2)*c^4)

3.231 $\int \left(a+\frac{b}{x}\right)^{3/2} \left(c+\frac{d}{x}\right)^3 \, dx$

Optimal. Leaf size=164

$$-3c^2(bc+2ad)\sqrt{a+\frac{b}{x}}-\frac{9}{7}d\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2-\frac{d\left(a+\frac{b}{x}\right)^{3/2}\left(2(13bc-ad)(5bc+2ad)+\frac{3bd(19bc+2ad)}{x}\right)}{35b^2}+$$

```
[Out] -9/7*d*(a+b/x)^(3/2)*(c+d/x)^2-1/35*d*(a+b/x)^(3/2)*(2*(-a*d+13*b*c)*(2*a*d
+5*b*c)+3*b*d*(2*a*d+19*b*c)/x)/b^2+(a+b/x)^(3/2)*(c+d/x)^3*x+3*c^2*(2*a*d+
b*c)*arctanh((a+b/x)^(1/2)/a^(1/2))*a^(1/2)-3*c^2*(2*a*d+b*c)*(a+b/x)^(1/2)
```

Rubi [A]

time = 0.10, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {382, 99, 158, 152, 52, 65, 214}

$$-\frac{d\left(a+\frac{b}{x}\right)^{3/2}\left(\frac{3bd(2ad+19bc)}{x}+2(13bc-ad)(2ad+5bc)\right)}{35b^2}-3c^2\sqrt{a+\frac{b}{x}}(2ad+bc)+3\sqrt{a}c^2(2ad+bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)+x\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^3-\frac{9}{7}d\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^(3/2)*(c + d/x)^3,x]
```

```
[Out] -3*c^2*(b*c + 2*a*d)*Sqrt[a + b/x] - (9*d*(a + b/x)^(3/2)*(c + d/x)^2)/7 -
(d*(a + b/x)^(3/2)*(2*(13*b*c - a*d)*(5*b*c + 2*a*d) + (3*b*d*(19*b*c + 2*a
*d))/x))/(35*b^2) + (a + b/x)^(3/2)*(c + d/x)^3*x + 3*Sqrt[a]*c^2*(b*c + 2*
a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^p/(b*(
m + 1))), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m +
 1)*((c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Dist[(a^2*d
^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m
+ n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n +
 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)),
Int[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}
, x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 158

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegerQ[m]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^3 dx = -\text{Subst}\left(\int \frac{(a+bx)^{3/2}(c+dx)^3}{x^2}dx, x, \frac{1}{x}\right)$$

$$= \left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^3 x - \text{Subst}\left(\int \frac{\sqrt{a+bx}\,(c+dx)^2\left(\frac{3}{2}(bc+2ad)+\frac{9bdx}{2}\right.}{x}\right.$$

$$= -\frac{9}{7}d\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2 + \left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^3 x - \frac{2\text{Subst}\left(\int \frac{\sqrt{a+b}}{}\right.}{}$$

$$= -\frac{9}{7}d\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2 - \frac{d\left(a+\frac{b}{x}\right)^{3/2}\left(2(13bc-ad)(5bc+2ad)+\frac{3bd(19}{}\right.}{35b^2}$$

$$= -3c^2(bc+2ad)\sqrt{a+\frac{b}{x}} - \frac{9}{7}d\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2 - \frac{d\left(a+\frac{b}{x}\right)^{3/2}\left(2(13bc\right.}{}$$

$$= -3c^2(bc+2ad)\sqrt{a+\frac{b}{x}} - \frac{9}{7}d\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2 - \frac{d\left(a+\frac{b}{x}\right)^{3/2}\left(2(13bc\right.}{}$$

$$= -3c^2(bc+2ad)\sqrt{a+\frac{b}{x}} - \frac{9}{7}d\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2 - \frac{d\left(a+\frac{b}{x}\right)^{3/2}\left(2(13bc\right.}{}$$

Mathematica [**A**]
time = 0.26, size = 159, normalized size = 0.97

$$\frac{\sqrt{a+\frac{b}{x}}\left(4a^3d^3x^3-2a^2bd^2x^2(d+21cx)+ab^2x\left(-16d^3-84cd^2x-280c^2dx^2+35c^3x^3\right)-2b^3\left(5d^3+21cd^2x+35c^2dx^2+35c^3x^3\right)\right)}{35b^2x^3}+3\sqrt{a}\,c^2(bc+2ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^(3/2)*(c + d/x)^3,x]
```

```
[Out] (Sqrt[a + b/x]*(4*a^3*d^3*x^3 - 2*a^2*b*d^2*x^2*(d + 21*c*x) + a*b^2*x*(-16
*d^3 - 84*c*d^2*x - 280*c^2*d*x^2 + 35*c^3*x^3) - 2*b^3*(5*d^3 + 21*c*d^2*x
 + 35*c^2*d*x^2 + 35*c^3*x^3)))/(35*b^2*x^3) + 3*Sqrt[a]*c^2*(b*c + 2*a*d)*
ArcTanh[Sqrt[a + b/x]/Sqrt[a]]
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 352 vs. $2(146) = 292$.
time = 0.07, size = 353, normalized size = 2.15

method	result
risch	$\frac{(35a\,b^2c^3x^4+4a^3d^3x^3-42a^2c\,d^2x^3b-280a\,b^2c^2d\,x^3-70b^3c^3x^3-2a^2d^3x^2b-84ac\,d^2x^2b^2-70b^3c^2d\,x^2-16ax\,d^3b^2-42x\,b^3c\,d^2-10b^3d^3}{35x^3b^2}$
default	$\frac{\sqrt{\frac{ax+b}{x}}\left(420\sqrt{a\,x^2+bx}\,a^{\frac{5}{2}}b\,c^2d\,x^5+210\sqrt{a\,x^2+bx}\,a^{\frac{3}{2}}b^2c^3x^5+210\ln\left(\frac{2\sqrt{a\,x^2+bx}\,\sqrt{a}+2ax+b}{2\sqrt{a}}\right)a^2b^2c^2d\,x^5+\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+1/x*b)^(3/2)*(c+d/x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/70*((a*x+b)/x)^(1/2)/x^4/b^2*(420*(a*x^2+b*x)^(1/2)*a^(5/2)*b*c^2*d*x^5+2
10*(a*x^2+b*x)^(1/2)*a^(3/2)*b^2*c^3*x^5+210*ln(1/2*(2*(a*x^2+b*x)^(1/2)*a^
(1/2)+2*a*x+b)/a^(1/2))*a^2*b^2*c^2*d*x^5+105*ln(1/2*(2*(a*x^2+b*x)^(1/2)*a
^(1/2)+2*a*x+b)/a^(1/2))*a*b^3*c^3*x^5-420*(a*x^2+b*x)^(3/2)*a^(3/2)*b*c^2*
d*x^3-140*(a*x^2+b*x)^(3/2)*a^(1/2)*b^2*c^3*x^3+8*(a*x^2+b*x)^(3/2)*a^(5/2)
*d^3*x^2-84*(a*x^2+b*x)^(3/2)*a^(3/2)*b*c*d^2*x^2-140*(a*x^2+b*x)^(3/2)*a^(
1/2)*b^2*c^2*d*x^2-12*(a*x^2+b*x)^(3/2)*a^(3/2)*b*d^3*x-84*(a*x^2+b*x)^(3/2
)*a^(1/2)*b^2*c*d^2*x-20*(a*x^2+b*x)^(3/2)*a^(1/2)*b^2*d^3)/(x*(a*x+b))^(1/
2)/a^(1/2)
```

Maxima [A]
time = 0.49, size = 190, normalized size = 1.16

$$-\frac{6\left(a+\frac{b}{x}\right)^{\frac{5}{2}}cd^2}{5\,b}+\frac{1}{2}\left(2\sqrt{a+\frac{b}{x}}\,ax-3\sqrt{a}\,b\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)-4\sqrt{a+\frac{b}{x}}\,b\right)c^3-\left(3\,a^{\frac{3}{2}}\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)+2\left(a+\frac{b}{x}\right)^{\frac{3}{2}}+6\sqrt{a+\frac{b}{x}}\,a\right)c^2d-\frac{2}{35}\left(\frac{5\left(a+\frac{b}{x}\right)^{\frac{7}{2}}}{b^2}-\frac{7\left(a+\frac{b}{x}\right)^{\frac{5}{2}}a}{b^2}\right)d^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(3/2)*(c+d/x)^3,x, algorithm="maxima")
```

```
[Out] -6/5*(a + b/x)^(5/2)*c*d^2/b + 1/2*(2*sqrt(a + b/x)*a*x - 3*sqrt(a)*b*log((
sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/x) + sqrt(a))) - 4*sqrt(a + b/x)*b)*c^
3 - (3*a^(3/2)*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/x) + sqrt(a))) + 2
*(a + b/x)^(3/2) + 6*sqrt(a + b/x)*a)*c^2*d - 2/35*(5*(a + b/x)^(7/2)/b^2 -
 7*(a + b/x)^(5/2)*a/b^2)*d^3
```

Fricas [A]
time = 2.46, size = 380, normalized size = 2.32

$$\left[\frac{105\,(b^3c^3+2\,ab^2c^2d)\sqrt{a}\,x^3\log\left(2\,ax+2\,\sqrt{a}\,x\sqrt{\frac{ax+b}{x}}+b\right)+2\,(35\,ab^2c^3x^4-10\,b^3d^3-2\,(35\,b^3c^3+140\,ab^2c^2d+21\,a^2bcd^2-2\,a^3d^3)x^3-2\,(35\,b^3c^2d+42\,ab^2cd^2+a^2bd^3)x^2-2\,(21\,b^3cd^2+8\,ab^2d^3)x)\sqrt{\frac{ax+b}{x}}}{70\,b^2x^3},-\frac{105\,(b^3c^3+2\,ab^2c^2d)\sqrt{-a}\,x^3\arctan\left(\frac{\sqrt{-a}\sqrt{\frac{ax+b}{x}}}{a}\right)-(35\,ab^2c^3x^4-10\,b^3d^3-2\,(35\,b^3c^3+140\,ab^2c^2d+21\,a^2bcd^2-2\,a^3d^3)x^3-2\,(35\,b^3c^2d+42\,ab^2cd^2+a^2bd^3)x^2-2\,(21\,b^3cd^2+8\,ab^2d^3)x)\sqrt{\frac{ax+b}{x}}}{35\,b^2x^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(3/2)*(c+d/x)^3,x, algorithm="fricas")
```

```
[Out] [1/70*(105*(b^3*c^3 + 2*a*b^2*c^2*d)*sqrt(a)*x^3*log(2*a*x + 2*sqrt(a)*x*sq
rt((a*x + b)/x) + b) + 2*(35*a*b^2*c^3*x^4 - 10*b^3*d^3 - 2*(35*b^3*c^3 + 1
40*a*b^2*c^2*d + 21*a^2*b*c*d^2 - 2*a^3*d^3)*x^3 - 2*(35*b^3*c^2*d + 42*a*b
^2*c*d^2 + a^2*b*d^3)*x^2 - 2*(21*b^3*c*d^2 + 8*a*b^2*d^3)*x)*sqrt((a*x + b
)/x))/(b^2*x^3), -1/35*(105*(b^3*c^3 + 2*a*b^2*c^2*d)*sqrt(-a)*x^3*arctan(s
qrt(-a)*sqrt((a*x + b)/x)/a) - (35*a*b^2*c^3*x^4 - 10*b^3*d^3 - 2*(35*b^3*c
^3 + 140*a*b^2*c^2*d + 21*a^2*b*c*d^2 - 2*a^3*d^3)*x^3 - 2*(35*b^3*c^2*d +
42*a*b^2*c*d^2 + a^2*b*d^3)*x^2 - 2*(21*b^3*c*d^2 + 8*a*b^2*d^3)*x)*sqrt((a
*x + b)/x))/(b^2*x^3)]
```

Sympy [A]
time = 74.17, size = 1817, normalized size = 11.08

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**(3/2)*(c+d/x)**3,x)
```

```
[Out] -16*a**(19/2)*b**(11/2)*d**3*x**6*sqrt(a*x/b + 1)/(105*a**(13/2)*b**7*x**(1
3/2) + 315*a**(11/2)*b**8*x**(11/2) + 315*a**(9/2)*b**9*x**(9/2) + 105*a**(
7/2)*b**10*x**(7/2)) - 40*a**(17/2)*b**(13/2)*d**3*x**5*sqrt(a*x/b + 1)/(10
5*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(11/2) + 315*a**(9/2)*b*
*9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) - 30*a**(15/2)*b**(15/2)*d**3*x*
*4*sqrt(a*x/b + 1)/(105*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(1
1/2) + 315*a**(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) - 40*a**(1
3/2)*b**(17/2)*d**3*x**3*sqrt(a*x/b + 1)/(105*a**(13/2)*b**7*x**(13/2) + 31
5*a**(11/2)*b**8*x**(11/2) + 315*a**(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b**1
0*x**(7/2)) + 4*a**(13/2)*b**(3/2)*d**3*x**3*sqrt(a*x/b + 1)/(15*a**(7/2)*b
**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) - 100*a**(11/2)*b**(19/2)*d**3*x*
*2*sqrt(a*x/b + 1)/(105*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(1
1/2) + 315*a**(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) + 12*a**(1
1/2)*b**(5/2)*c*d**2*x**3*sqrt(a*x/b + 1)/(15*a**(7/2)*b**3*x**(7/2) + 15*a
**(5/2)*b**4*x**(5/2)) + 2*a**(11/2)*b**(5/2)*d**3*x**2*sqrt(a*x/b + 1)/(15
*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) - 96*a**(9/2)*b**(21/2
)*d**3*x*sqrt(a*x/b + 1)/(105*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8
*x**(11/2) + 315*a**(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) + 6*
a**(9/2)*b**(7/2)*c*d**2*x**2*sqrt(a*x/b + 1)/(15*a**(7/2)*b**3*x**(7/2) +
15*a**(5/2)*b**4*x**(5/2)) - 8*a**(9/2)*b**(7/2)*d**3*x*sqrt(a*x/b + 1)/(15
*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) - 30*a**(7/2)*b**(23/2
)*d**3*sqrt(a*x/b + 1)/(105*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x
**(11/2) + 315*a**(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) - 24*a
**(7/2)*b**(9/2)*c*d**2*x*sqrt(a*x/b + 1)/(15*a**(7/2)*b**3*x**(7/2) + 15*a
**(5/2)*b**4*x**(5/2)) - 6*a**(7/2)*b**(9/2)*d**3*sqrt(a*x/b + 1)/(15*a**(7
/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) - 18*a**(5/2)*b**(11/2)*c*d*
```

```
*2*sqrt(a*x/b + 1)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2))
+ sqrt(a)*b*c**3*asinh(sqrt(a)*sqrt(x)/sqrt(b)) + 16*a**10*b**5*d**3*x**(13
/2)/(105*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(11/2) + 315*a**(
9/2)*b**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) + 48*a**9*b**6*d**3*x**(1
1/2)/(105*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(11/2) + 315*a**
(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) + 48*a**8*b**7*d**3*x**(
9/2)/(105*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(11/2) + 315*a**
(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) + 16*a**7*b**8*d**3*x**(
7/2)/(105*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(11/2) + 315*a**
(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) - 4*a**7*b*d**3*x**(7/2)
/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) - 12*a**6*b**2*c*d
**2*x**(7/2)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) - 4*a*
*6*b**2*d**3*x**(5/2)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2
)) - 12*a**5*b**3*c*d**2*x**(5/2)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*
b**4*x**(5/2)) - 6*a**2*c**2*d*atan(sqrt(a + b/x)/sqrt(-a))/sqrt(-a) + a*sq
rt(b)*c**3*sqrt(x)*sqrt(a*x/b + 1) - 2*a*b*c**3*atan(sqrt(a + b/x)/sqrt(-a)
)/sqrt(-a) - 6*a*c**2*d*sqrt(a + b/x) + 3*a*c*d**2*Piecewise((-sqrt(a)/x, E
q(b, 0)), (-2*(a + b/x)**(3/2)/(3*b), True)) - 2*b*c**3*sqrt(a + b/x) + 3*b
*c**2*d*Piecewise((-sqrt(a)/x, Eq(b, 0)), (-2*(a + b/x)**(3/2)/(3*b), True)
)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(3/2)*(c+d/x)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [**B**]
time = 3.88, size = 327, normalized size = 1.99

$$\left(a+\frac{b}{x}\right)^{5/2}\left(\frac{6\,a\,d^3-6\,b\,c\,d^2}{5\,b^2}-\frac{4\,a\,d^3}{5\,b^2}\right)+\sqrt{a+\frac{b}{x}}\,\left(\frac{2\,{\left(a\,d-b\,c\right)}^3}{b^2}+2\,a\,\left(2\,a\,\left(\frac{6\,a\,d^3-6\,b\,c\,d^2}{b^2}-\frac{4\,a\,d^3}{b^2}\right)-\frac{6\,d\,{\left(a\,d-b\,c\right)}^2}{b^2}+\frac{2\,a^2\,d^3}{b^2}\right)-a^2\,\left(\frac{6\,a\,d^3-6\,b\,c\,d^2}{b^2}-\frac{4\,a\,d^3}{b^2}\right)\right)+\left(a+\frac{b}{x}\right)^{3/2}\,\left(\frac{2\,a\,\left(\frac{6\,a\,d^3-6\,b\,c\,d^2}{b^2}-\frac{4\,a\,d^3}{b^2}\right)}{3}-\frac{2\,d\,{\left(a\,d-b\,c\right)}^2}{b^2}+\frac{2\,a^2\,d^3}{3\,b^2}\right)-\frac{2\,d^3\,{\left(a+\frac{b}{x}\right)}^{7/2}}{7\,b^2}+a\,c^3\,x\,\sqrt{a+\frac{b}{x}}-2\,c^2\,\mathrm{atan}\left(\frac{2\,c^2\,\sqrt{a+\frac{b}{x}}\,\left(2\,a\,d+b\,c\right)\,\sqrt{-\frac{9\,a}{4}}}{6\,d\,a^2\,c^2+3\,b\,a\,c^3}\right)\,\left(2\,a\,d+b\,c\right)\,\sqrt{-\frac{9\,a}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x)^(3/2)*(c + d/x)^3,x)
```

```
[Out] (a + b/x)^(5/2)*((6*a*d^3 - 6*b*c*d^2)/(5*b^2) - (4*a*d^3)/(5*b^2)) + (a +
b/x)^(1/2)*((2*(a*d - b*c)^3)/b^2 + 2*a*(2*a*((6*a*d^3 - 6*b*c*d^2)/b^2 - (
4*a*d^3)/b^2) - (6*d*(a*d - b*c)^2)/b^2 + (2*a^2*d^3)/b^2) - a^2*((6*a*d^3
```

```
- 6*b*c*d^2)/b^2 - (4*a*d^3)/b^2)) + (a + b/x)^(3/2)*((2*a*((6*a*d^3 - 6*b*
c*d^2)/b^2 - (4*a*d^3)/b^2))/3 - (2*d*(a*d - b*c)^2)/b^2 + (2*a^2*d^3)/(3*b
^2)) - (2*d^3*(a + b/x)^(7/2))/(7*b^2) + a*c^3*x*(a + b/x)^(1/2) - 2*c^2*at
an((2*c^2*(a + b/x)^(1/2)*(2*a*d + b*c)*(-(9*a)/4)^(1/2))/(6*a^2*c^2*d + 3*
a*b*c^3))*(2*a*d + b*c)*(-(9*a)/4)^(1/2)
```

3.232 $\int \left(a+\frac{b}{x}\right)^{3/2} \left(c+\frac{d}{x}\right)^2 \, dx$

Optimal. Leaf size=126

$$-c(3bc+4ad)\sqrt{a+\frac{b}{x}}-\frac{c(3bc+4ad)\left(a+\frac{b}{x}\right)^{3/2}}{3a}-\frac{2d^2\left(a+\frac{b}{x}\right)^{5/2}}{5b}+\frac{c^2\left(a+\frac{b}{x}\right)^{5/2}x}{a}+\sqrt{a}\,c(3bc+4ad)\tanh^{-1}$$

```
[Out] -1/3*c*(4*a*d+3*b*c)*(a+b/x)^(3/2)/a-2/5*d^2*(a+b/x)^(5/2)/b+c^2*(a+b/x)^(5
/2)*x/a+c*(4*a*d+3*b*c)*arctanh((a+b/x)^(1/2)/a^(1/2))*a^(1/2)-c*(4*a*d+3*b
*c)*(a+b/x)^(1/2)
```

Rubi [A]

time = 0.06, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {382, 91, 81, 52, 65, 214}

$$\frac{c^2x\left(a+\frac{b}{x}\right)^{5/2}}{a}-\frac{c\left(a+\frac{b}{x}\right)^{3/2}(4ad+3bc)}{3a}-c\sqrt{a+\frac{b}{x}}\,(4ad+3bc)+\sqrt{a}\,c(4ad+3bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)-\frac{2d^2\left(a+\frac{b}{x}\right)^{5/2}}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^(3/2)*(c + d/x)^2,x]
```

```
[Out] -(c*(3*b*c + 4*a*d)*Sqrt[a + b/x]) - (c*(3*b*c + 4*a*d)*(a + b/x)^(3/2))/(3
*a) - (2*d^2*(a + b/x)^(5/2))/(5*b) + (c^2*(a + b/x)^(5/2)*x)/a + Sqrt[a]*c
*(3*b*c + 4*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2 dx = -\text{Subst}\left(\int \frac{(a+bx)^{3/2}(c+dx)^2}{x^2}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{c^2\left(a+\frac{b}{x}\right)^{5/2}x}{a} - \frac{\text{Subst}\left(\int \frac{(a+bx)^{3/2}\left(\frac{1}{2}c(3bc+4ad)+ad^2x\right)}{x}\,dx, x, \frac{1}{x}\right)}{a}$$

$$= -\frac{2d^2\left(a+\frac{b}{x}\right)^{5/2}}{5b} + \frac{c^2\left(a+\frac{b}{x}\right)^{5/2}x}{a} - \frac{(c(3bc+4ad))\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x}\,dx, x,\right.}{2a}$$

$$= -\frac{c(3bc+4ad)\left(a+\frac{b}{x}\right)^{3/2}}{3a} - \frac{2d^2\left(a+\frac{b}{x}\right)^{5/2}}{5b} + \frac{c^2\left(a+\frac{b}{x}\right)^{5/2}x}{a} - \frac{1}{2}(c(3bc+4$$

$$= -c(3bc+4ad)\sqrt{a+\frac{b}{x}} - \frac{c(3bc+4ad)\left(a+\frac{b}{x}\right)^{3/2}}{3a} - \frac{2d^2\left(a+\frac{b}{x}\right)^{5/2}}{5b} + \frac{c^2(a}{}$$

$$= -c(3bc+4ad)\sqrt{a+\frac{b}{x}} - \frac{c(3bc+4ad)\left(a+\frac{b}{x}\right)^{3/2}}{3a} - \frac{2d^2\left(a+\frac{b}{x}\right)^{5/2}}{5b} + \frac{c^2(a}{}$$

$$= -c(3bc+4ad)\sqrt{a+\frac{b}{x}} - \frac{c(3bc+4ad)\left(a+\frac{b}{x}\right)^{3/2}}{3a} - \frac{2d^2\left(a+\frac{b}{x}\right)^{5/2}}{5b} + \frac{c^2(a}{}$$

Mathematica [A]
time = 0.23, size = 115, normalized size = 0.91

$$\frac{\sqrt{a+\frac{b}{x}}\left(-6a^2d^2x^2+abx(-12d^2-80cdx+15c^2x^2)-2b^2(3d^2+10cdx+15c^2x^2)\right)}{15bx^2} + \sqrt{a}\,c(3bc+4ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^(3/2)*(c + d/x)^2,x]
```

```
[Out] (Sqrt[a + b/x]*(-6*a^2*d^2*x^2 + a*b*x*(-12*d^2 - 80*c*d*x + 15*c^2*x^2) -
2*b^2*(3*d^2 + 10*c*d*x + 15*c^2*x^2)))/(15*b*x^2) + Sqrt[a]*c*(3*b*c + 4*a
*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 259 vs. $2(108) = 216$.
time = 0.06, size = 260, normalized size = 2.06

method	result

risch	$-\frac{(-15ab c^2x^3+6a^2d^2x^2+80abcd x^2+30b^2c^2x^2+12ab d^2x+20b^2cdx+6b^2d^2)\sqrt{\frac{ax+b}{x}}}{15x^2b}+\frac{\left(2a^{\frac{3}{2}}\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a x^2+bx}\right)cd+\right.}{}$
default	$-\frac{\sqrt{\frac{ax+b}{x}}\left(-120\sqrt{a x^2+bx}\,a^{\frac{5}{2}}cd x^4-90\sqrt{a x^2+bx}\,a^{\frac{3}{2}}bc^2x^4-60\ln\left(\frac{2\sqrt{a x^2+bx}\sqrt{a}+2ax+b}{2\sqrt{a}}\right)a^2bcd x^4-45\ln\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+1/x*b)^(3/2)*(c+d/x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/30*((a*x+b)/x)^(1/2)/x^3/b*(-120*(a*x^2+b*x)^(1/2)*a^(5/2)*c*d*x^4-90*(a
*x^2+b*x)^(1/2)*a^(3/2)*b*c^2*x^4-60*ln(1/2*(2*(a*x^2+b*x)^(1/2)*a^(1/2)+2*
a*x+b)/a^(1/2))*a^2*b*c*d*x^4-45*ln(1/2*(2*(a*x^2+b*x)^(1/2)*a^(1/2)+2*a*x+
b)/a^(1/2))*a*b^2*c^2*x^4+120*(a*x^2+b*x)^(3/2)*a^(3/2)*c*d*x^2+60*(a*x^2+b
*x)^(3/2)*a^(1/2)*b*c^2*x^2+12*(a*x^2+b*x)^(3/2)*a^(3/2)*d^2*x+40*(a*x^2+b*
x)^(3/2)*a^(1/2)*b*c*d*x+12*(a*x^2+b*x)^(3/2)*a^(1/2)*b*d^2)/(x*(a*x+b))^(1
/2)/a^(1/2)
```

Maxima [A]
time = 0.49, size = 152, normalized size = 1.21

$$-\frac{2\left(a+\frac{b}{x}\right)^{\frac{5}{2}}d^2}{5\,b}+\frac{1}{2}\left(2\sqrt{a+\frac{b}{x}}\,ax-3\sqrt{a}\,b\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)-4\sqrt{a+\frac{b}{x}}\,b\right)c^2-\frac{2}{3}\left(3\,a^{\frac{3}{2}}\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)+2\left(a+\frac{b}{x}\right)^{\frac{3}{2}}+6\sqrt{a+\frac{b}{x}}\,a\right)cd$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(3/2)*(c+d/x)^2,x, algorithm="maxima")
```

```
[Out] -2/5*(a + b/x)^(5/2)*d^2/b + 1/2*(2*sqrt(a + b/x)*a*x - 3*sqrt(a)*b*log((sq
rt(a + b/x) - sqrt(a))/(sqrt(a + b/x) + sqrt(a))) - 4*sqrt(a + b/x)*b)*c^2
- 2/3*(3*a^(3/2)*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/x) + sqrt(a))) +
 2*(a + b/x)^(3/2) + 6*sqrt(a + b/x)*a)*c*d
```

Fricas [A]
time = 3.04, size = 268, normalized size = 2.13

$$\left[\frac{15\,(3\,b^2c^2+4\,abcd)\sqrt{a}\,x^2\log\left(2\,ax+2\,\sqrt{a}\,x\sqrt{\frac{ax+b}{x}}+b\right)+2\,(15\,abc^2x^3-6\,b^2d^2-2\,(15\,b^2c^2+40\,abcd+3\,a^2d^2)x^2-4\,(5\,b^2cd+3\,abd^2)x)\sqrt{\frac{ax+b}{x}}}{30\,bx^2},\ -\frac{15\,(3\,b^2c^2+4\,abcd)\sqrt{-a}\,x^2\arctan\left(\frac{\sqrt{-a}\sqrt{\frac{ax+b}{x}}}{a}\right)-(15\,abc^2x^3-6\,b^2d^2-2\,(15\,b^2c^2+40\,abcd+3\,a^2d^2)x^2-4\,(5\,b^2cd+3\,abd^2)x)\sqrt{\frac{ax+b}{x}}}{15\,bx^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(3/2)*(c+d/x)^2,x, algorithm="fricas")
```

```
[Out] [1/30*(15*(3*b^2*c^2 + 4*a*b*c*d)*sqrt(a)*x^2*log(2*a*x + 2*sqrt(a)*x*sqrt(
(a*x + b)/x) + b) + 2*(15*a*b*c^2*x^3 - 6*b^2*d^2 - 2*(15*b^2*c^2 + 40*a*b*
c*d + 3*a^2*d^2)*x^2 - 4*(5*b^2*c*d + 3*a*b*d^2)*x)*sqrt((a*x + b)/x))/(b*x
^2), -1/15*(15*(3*b^2*c^2 + 4*a*b*c*d)*sqrt(-a)*x^2*arctan(sqrt(-a)*sqrt((a
*x + b)/x)/a) - (15*a*b*c^2*x^3 - 6*b^2*d^2 - 2*(15*b^2*c^2 + 40*a*b*c*d +
3*a^2*d^2)*x^2 - 4*(5*b^2*c*d + 3*a*b*d^2)*x)*sqrt((a*x + b)/x))/(b*x^2)]
```

Sympy [**A**]
time = 50.32, size = 534, normalized size = 4.24

$$\frac{4a^{\frac{11}{2}}b^{\frac{5}{2}}d^2x^3\sqrt{\frac{ax}{b}+1}}{15a^{\frac{7}{2}}b^3x^{\frac{7}{2}}+15a^{\frac{5}{2}}b^4x^{\frac{5}{2}}}+\frac{2a^{\frac{9}{2}}b^{\frac{7}{2}}d^2x^2\sqrt{\frac{ax}{b}+1}}{15a^{\frac{7}{2}}b^3x^{\frac{7}{2}}+15a^{\frac{5}{2}}b^4x^{\frac{5}{2}}}-\frac{8a^{\frac{7}{2}}b^{\frac{9}{2}}d^2x\sqrt{\frac{ax}{b}+1}}{15a^{\frac{7}{2}}b^3x^{\frac{7}{2}}+15a^{\frac{5}{2}}b^4x^{\frac{5}{2}}}-\frac{6a^{\frac{5}{2}}b^{\frac{11}{2}}d^2\sqrt{\frac{ax}{b}+1}}{15a^{\frac{7}{2}}b^3x^{\frac{7}{2}}+15a^{\frac{5}{2}}b^4x^{\frac{5}{2}}}+\sqrt{a}\,bc^2\operatorname{asinh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)-\frac{4a^6b^2d^2x^{\frac{7}{2}}}{15a^{\frac{7}{2}}b^3x^{\frac{7}{2}}+15a^{\frac{5}{2}}b^4x^{\frac{5}{2}}}-\frac{4a^5b^3d^2x^{\frac{5}{2}}}{15a^{\frac{7}{2}}b^3x^{\frac{7}{2}}+15a^{\frac{5}{2}}b^4x^{\frac{5}{2}}}-\frac{4a^2cd\operatorname{atan}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{-a}}\right)}{\sqrt{-a}}+a\sqrt{b}\,c^2\sqrt{x}\sqrt{\frac{ax}{b}+1}-\frac{2abc^2\operatorname{atan}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{-a}}\right)}{\sqrt{-a}}-4acd\sqrt{a+\frac{b}{x}}+ad^2\left(\begin{cases}-\frac{\sqrt{a}}{x} & \text{for}\ b=0\\ -\frac{2\left(a+\frac{b}{x}\right)^{\frac{3}{2}}}{3b} & \text{otherwise}\end{cases}\right)-2bc^2\sqrt{a+\frac{b}{x}}+2bcd\left(\begin{cases}-\frac{\sqrt{a}}{x} & \text{for}\ b=0\\ -\frac{2\left(a+\frac{b}{x}\right)^{\frac{3}{2}}}{3b} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**(3/2)*(c+d/x)**2,x)
```

```
[Out] 4*a**(11/2)*b**(5/2)*d**2*x**3*sqrt(a*x/b + 1)/(15*a**(7/2)*b**3*x**(7/2) +
 15*a**(5/2)*b**4*x**(5/2)) + 2*a**(9/2)*b**(7/2)*d**2*x**2*sqrt(a*x/b + 1)
/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) - 8*a**(7/2)*b**(9
/2)*d**2*x*sqrt(a*x/b + 1)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x*
*(5/2)) - 6*a**(5/2)*b**(11/2)*d**2*sqrt(a*x/b + 1)/(15*a**(7/2)*b**3*x**(7
/2) + 15*a**(5/2)*b**4*x**(5/2)) + sqrt(a)*b*c**2*asinh(sqrt(a)*sqrt(x)/sqr
t(b)) - 4*a**6*b**2*d**2*x**(7/2)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*
b**4*x**(5/2)) - 4*a**5*b**3*d**2*x**(5/2)/(15*a**(7/2)*b**3*x**(7/2) + 15*
a**(5/2)*b**4*x**(5/2)) - 4*a**2*c*d*atan(sqrt(a + b/x)/sqrt(-a))/sqrt(-a)
+ a*sqrt(b)*c**2*sqrt(x)*sqrt(a*x/b + 1) - 2*a*b*c**2*atan(sqrt(a + b/x)/sq
rt(-a))/sqrt(-a) - 4*a*c*d*sqrt(a + b/x) + a*d**2*Piecewise((-sqrt(a)/x, Eq
(b, 0)), (-2*(a + b/x)**(3/2)/(3*b), True)) - 2*b*c**2*sqrt(a + b/x) + 2*b*
c*d*Piecewise((-sqrt(a)/x, Eq(b, 0)), (-2*(a + b/x)**(3/2)/(3*b), True))
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(3/2)*(c+d/x)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [**B**]
time = 2.58, size = 197, normalized size = 1.56

$$\sqrt{a+\frac{b}{x}}\,\left(2\,a\,\left(\frac{4\,a\,d^2-4\,b\,c\,d}{b}-\frac{4\,a\,d^2}{b}\right)-\frac{2\,{\left(a\,d-b\,c\right)}^2}{b}+\frac{2\,a^2\,d^2}{b}\right)+\left(\frac{4\,a\,d^2-4\,b\,c\,d}{3\,b}-\frac{4\,a\,d^2}{3\,b}\right)\,{\left(a+\frac{b}{x}\right)}^{3/2}-\frac{2\,d^2\,{\left(a+\frac{b}{x}\right)}^{5/2}}{5\,b}+a\,c^2\,x\,\sqrt{a+\frac{b}{x}}-2\,c\,\mathrm{atan}\left(\frac{2\,c\,\sqrt{a+\frac{b}{x}}\,\left(4\,a\,d+3\,b\,c\right)\,\sqrt{-\frac{a}{4}}}{4\,d\,a^2\,c+3\,b\,a\,c^2}\right)\,\left(4\,a\,d+3\,b\,c\right)\,\sqrt{-\frac{a}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x)^(3/2)*(c + d/x)^2,x)
```

```
[Out] (a + b/x)^(1/2)*(2*a*((4*a*d^2 - 4*b*c*d)/b - (4*a*d^2)/b) - (2*(a*d - b*c)
^2)/b + (2*a^2*d^2)/b) + ((4*a*d^2 - 4*b*c*d)/(3*b) - (4*a*d^2)/(3*b))*(a +
 b/x)^(3/2) - (2*d^2*(a + b/x)^(5/2))/(5*b) + a*c^2*x*(a + b/x)^(1/2) - 2*c
*atan((2*c*(a + b/x)^(1/2)*(4*a*d + 3*b*c)*(-a/4)^(1/2))/(3*a*b*c^2 + 4*a^2
*c*d))*(4*a*d + 3*b*c)*(-a/4)^(1/2)
```

3.233 $\int \left(a+\frac{b}{x}\right)^{3/2} \left(c+\frac{d}{x}\right) \, dx$

Optimal. Leaf size=100

$$-\left((3bc+2ad)\sqrt{a+\frac{b}{x}}\right)-\frac{(3bc+2ad)\left(a+\frac{b}{x}\right)^{3/2}}{3a}+\frac{c\left(a+\frac{b}{x}\right)^{5/2}x}{a}+\sqrt{a}\,(3bc+2ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)$$

```
[Out] -1/3*(2*a*d+3*b*c)*(a+b/x)^(3/2)/a+c*(a+b/x)^(5/2)*x/a+(2*a*d+3*b*c)*arctan
h((a+b/x)^(1/2)/a^(1/2))*a^(1/2)-(2*a*d+3*b*c)*(a+b/x)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {382, 79, 52, 65, 214}

$$-\frac{\left(a+\frac{b}{x}\right)^{3/2}(2ad+3bc)}{3a}-\sqrt{a+\frac{b}{x}}\,(2ad+3bc)+\sqrt{a}\,(2ad+3bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)+\frac{cx\left(a+\frac{b}{x}\right)^{5/2}}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^(3/2)*(c + d/x),x]
```

```
[Out] -((3*b*c + 2*a*d)*Sqrt[a + b/x]) - ((3*b*c + 2*a*d)*(a + b/x)^(3/2))/(3*a)
+ (c*(a + b/x)^(5/2)*x)/a + Sqrt[a]*(3*b*c + 2*a*d)*ArcTanh[Sqrt[a + b/x]/S
qrt[a]]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)dx = -\text{Subst}\left(\int \frac{(a+bx)^{3/2}(c+dx)}{x^2}\,dx,x,\frac{1}{x}\right)$$
$$= \frac{c\left(a+\frac{b}{x}\right)^{5/2}x}{a} - \frac{\left(\frac{3bc}{2}+ad\right)\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x}\,dx,x,\frac{1}{x}\right)}{a}$$
$$= -\frac{(3bc+2ad)\left(a+\frac{b}{x}\right)^{3/2}}{3a} + \frac{c\left(a+\frac{b}{x}\right)^{5/2}x}{a} - \frac{1}{2}(3bc+2ad)\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\right.$$
$$= -(3bc+2ad)\sqrt{a+\frac{b}{x}} - \frac{(3bc+2ad)\left(a+\frac{b}{x}\right)^{3/2}}{3a} + \frac{c\left(a+\frac{b}{x}\right)^{5/2}x}{a} - \frac{1}{2}(a(3bc +$$
$$= -(3bc+2ad)\sqrt{a+\frac{b}{x}} - \frac{(3bc+2ad)\left(a+\frac{b}{x}\right)^{3/2}}{3a} + \frac{c\left(a+\frac{b}{x}\right)^{5/2}x}{a} - \frac{(a(3bc +}{}$$
$$= -(3bc+2ad)\sqrt{a+\frac{b}{x}} - \frac{(3bc+2ad)\left(a+\frac{b}{x}\right)^{3/2}}{3a} + \frac{c\left(a+\frac{b}{x}\right)^{5/2}x}{a} + \sqrt{a}\,(3bc$$

Mathematica [A]

time = 0.17, size = 73, normalized size = 0.73

$$\frac{\sqrt{a+\frac{b}{x}}\,(ax(-8d+3cx)-2b(d+3cx))}{3x}+\sqrt{a}\,(3bc+2ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^(3/2)*(c + d/x),x]
```

```
[Out] (Sqrt[a + b/x]*(a*x*(-8*d + 3*c*x) - 2*b*(d + 3*c*x)))/(3*x) + Sqrt[a]*(3*b
*c + 2*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 204 vs. 2(86) = 172.
time = 0.05, size = 205, normalized size = 2.05

method	result
risch	$\frac{(3c\,x^2a-8adx-6bcx-2bd)\sqrt{\frac{ax+b}{x}}}{3x}+\frac{\left(a^{\frac{3}{2}}\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^2+bx}\right)d+\frac{3\sqrt{a}\,\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^2+bx}\right)bc}{2}\right)\sqrt{\frac{ax+b}{x}}}{ax+b}$
default	$-\frac{\sqrt{\frac{ax+b}{x}}\left(-12\sqrt{a\,x^2+bx}\,a^{\frac{5}{2}}d\,x^3-18\sqrt{a\,x^2+bx}\,a^{\frac{3}{2}}bc\,x^3-6\ln\left(\frac{2\sqrt{a\,x^2+bx}\,\sqrt{a}+2ax+b}{2\sqrt{a}}\right)a^2bd\,x^3-9\ln\left(\frac{2\sqrt{}}{}\right.\right.}{6x^2\sqrt{x\,(ax+b)}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+1/x*b)^(3/2)*(c+d/x),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*((a*x+b)/x)^(1/2)/x^2*(-12*(a*x^2+b*x)^(1/2)*a^(5/2)*d*x^3-18*(a*x^2+b
*x)^(1/2)*a^(3/2)*b*c*x^3-6*ln(1/2*(2*(a*x^2+b*x)^(1/2)*a^(1/2)+2*a*x+b)/a^
(1/2))*a^2*b*d*x^3-9*ln(1/2*(2*(a*x^2+b*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*
a*b^2*c*x^3+12*(a*x^2+b*x)^(3/2)*a^(3/2)*d*x+12*(a*x^2+b*x)^(3/2)*a^(1/2)*b
*c*x+4*d*(a*x^2+b*x)^(3/2)*a^(1/2)*b)/(x*(a*x+b))^(1/2)/a^(1/2)/b
```

Maxima [A]
time = 0.49, size = 132, normalized size = 1.32

$$\frac{1}{2}\left(2\sqrt{a+\frac{b}{x}}\,ax-3\sqrt{a}\,b\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)-4\sqrt{a+\frac{b}{x}}\,b\right)c-\frac{1}{3}\left(3a^{\frac{3}{2}}\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)+2\left(a+\frac{b}{x}\right)^{\frac{3}{2}}+6\sqrt{a+\frac{b}{x}}\,a\right)d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(3/2)*(c+d/x),x, algorithm="maxima")
```

```
[Out] 1/2*(2*sqrt(a + b/x)*a*x - 3*sqrt(a)*b*log((sqrt(a + b/x) - sqrt(a))/(sqrt(
a + b/x) + sqrt(a))) - 4*sqrt(a + b/x)*b)*c - 1/3*(3*a^(3/2)*log((sqrt(a +
b/x) - sqrt(a))/(sqrt(a + b/x) + sqrt(a))) + 2*(a + b/x)^(3/2) + 6*sqrt(a +
 b/x)*a)*d
```

Fricas [A]
time = 2.80, size = 164, normalized size = 1.64

$$\left[\frac{3\,(3\,bc+2\,ad)\sqrt{a}\,x\log\left(2\,ax+2\,\sqrt{a}\,x\sqrt{\frac{ax+b}{x}}+b\right)+2\,(3\,acx^2-2\,bd-2\,(3\,bc+4\,ad)x)\sqrt{\frac{ax+b}{x}}}{6\,x},-\frac{3\,(3\,bc+2\,ad)\sqrt{-a}\,x\arctan\left(\frac{\sqrt{-a}\,\sqrt{\frac{ax+b}{x}}}{a}\right)-(3\,acx^2-2\,bd-2\,(3\,bc+4\,ad)x)\sqrt{\frac{ax+b}{x}}}{3\,x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(3/2)*(c+d/x),x, algorithm="fricas")
```

```
[Out] [1/6*(3*(3*b*c + 2*a*d)*sqrt(a)*x*log(2*a*x + 2*sqrt(a)*x*sqrt((a*x + b)/x)
 + b) + 2*(3*a*c*x^2 - 2*b*d - 2*(3*b*c + 4*a*d)*x)*sqrt((a*x + b)/x))/x, -
1/3*(3*(3*b*c + 2*a*d)*sqrt(-a)*x*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) - (3
*a*c*x^2 - 2*b*d - 2*(3*b*c + 4*a*d)*x)*sqrt((a*x + b)/x))/x]
```

Sympy [A]
time = 32.84, size = 163, normalized size = 1.63

$$\sqrt{a}\,bc\operatorname{asinh}\left(\frac{\sqrt{a}\,\sqrt{x}}{\sqrt{b}}\right)-\frac{2a^2d\operatorname{atan}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{-a}}\right)}{\sqrt{-a}}+a\sqrt{b}\,c\sqrt{x}\,\sqrt{\frac{ax}{b}+1}-\frac{2abc\operatorname{atan}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{-a}}\right)}{\sqrt{-a}}-2ad\sqrt{a+\frac{b}{x}}-2bc\sqrt{a+\frac{b}{x}}+bd\left(\begin{cases}-\frac{\sqrt{a}}{x} & \text{for}\ b=0\\ -\frac{2\left(a+\frac{b}{x}\right)^{\frac{3}{2}}}{3b} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**(3/2)*(c+d/x),x)
```

```
[Out] sqrt(a)*b*c*asinh(sqrt(a)*sqrt(x)/sqrt(b)) - 2*a**2*d*atan(sqrt(a + b/x)/sq
rt(-a))/sqrt(-a) + a*sqrt(b)*c*sqrt(x)*sqrt(a*x/b + 1) - 2*a*b*c*atan(sqrt(
a + b/x)/sqrt(-a))/sqrt(-a) - 2*a*d*sqrt(a + b/x) - 2*b*c*sqrt(a + b/x) + b
*d*Piecewise((-sqrt(a)/x, Eq(b, 0)), (-2*(a + b/x)**(3/2)/(3*b), True))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(3/2)*(c+d/x),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
```

```
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [B]
time = 2.51, size = 81, normalized size = 0.81

$$2\,a^{3/2}\,d\,\mathrm{atanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)-\frac{2\,d\,{\left(a+\frac{b}{x}\right)}^{3/2}}{3}-2\,a\,d\,\sqrt{a+\frac{b}{x}}-\frac{2\,c\,x\,{\left(a+\frac{b}{x}\right)}^{3/2}\,{}_2\mathrm{F}_1\left(-\frac{3}{2},-\frac{1}{2};\ \frac{1}{2};\ -\frac{a\,x}{b}\right)}{{\left(\frac{a\,x}{b}+1\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x)^(3/2)*(c + d/x),x)
```

```
[Out] 2*a^(3/2)*d*atanh((a + b/x)^(1/2)/a^(1/2)) - (2*d*(a + b/x)^(3/2))/3 - 2*a*
d*(a + b/x)^(1/2) - (2*c*x*(a + b/x)^(3/2)*hypergeom([-3/2, -1/2], 1/2, -(a
*x)/b))/((a*x)/b + 1)^(3/2)
```

3.234 $\int \left(a+\frac{b}{x}\right)^{3/2} dx$

Optimal. Leaf size=54

$$-3b\sqrt{a+\frac{b}{x}}+\left(a+\frac{b}{x}\right)^{3/2}x+3\sqrt{a}\,b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)$$

```
[Out] (a+b/x)^(3/2)*x+3*b*arctanh((a+b/x)^(1/2)/a^(1/2))*a^(1/2)-3*b*(a+b/x)^(1/2
)
```

Rubi [A]
time = 0.02, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.454$, Rules used = {248, 43, 52, 65, 214}

$$x\left(a+\frac{b}{x}\right)^{3/2}-3b\sqrt{a+\frac{b}{x}}+3\sqrt{a}\,b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^(3/2),x]
```

```
[Out] -3*b*Sqrt[a + b/x] + (a + b/x)^(3/2)*x + 3*Sqrt[a]*b*ArcTanh[Sqrt[a + b/x]/
Sqrt[a]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 248

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \left(a+\frac{b}{x}\right)^{3/2} dx &= -\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x^2}\,dx, x, \frac{1}{x}\right)\\
&= \left(a+\frac{b}{x}\right)^{3/2} x - \frac{1}{2}(3b)\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx, x, \frac{1}{x}\right)\\
&= -3b\sqrt{a+\frac{b}{x}} + \left(a+\frac{b}{x}\right)^{3/2} x - \frac{1}{2}(3ab)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)\\
&= -3b\sqrt{a+\frac{b}{x}} + \left(a+\frac{b}{x}\right)^{3/2} x - (3a)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x}}\right)\\
&= -3b\sqrt{a+\frac{b}{x}} + \left(a+\frac{b}{x}\right)^{3/2} x + 3\sqrt{a}\,b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 46, normalized size = 0.85

$$\sqrt{a+\frac{b}{x}}\,(-2b+ax)+3\sqrt{a}\,b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^(3/2),x]
```

```
[Out] Sqrt[a + b/x]*(-2*b + a*x) + 3*Sqrt[a]*b*ArcTanh[Sqrt[a + b/x]/Sqrt[a]]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 99 vs. $2(44) = 88$.

time = 0.02, size = 100, normalized size = 1.85

method	result	size
risch	$(ax-2b)\sqrt{\frac{ax+b}{x}}+\frac{3\sqrt{a}\,b\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^{2}+bx}\right)\sqrt{\frac{ax+b}{x}}\,\sqrt{x\,(ax+b)}}{2(ax+b)}$	78
default	$-\frac{\sqrt{\frac{ax+b}{x}}\left(-6\sqrt{a\,x^{2}+bx}\,a^{\frac{3}{2}}x^{2}-3\ln\left(\frac{2\sqrt{a\,x^{2}+bx}\,\sqrt{a}+2ax+b}{2\sqrt{a}}\right)ab\,x^{2}+4\left(a\,x^{2}+bx\right)^{\frac{3}{2}}\sqrt{a}\right)}{2x\sqrt{x\,(ax+b)}\,\sqrt{a}}$	100

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+1/x*b)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*((a*x+b)/x)^(1/2)/x*(-6*(a*x^2+b*x)^(1/2)*a^(3/2)*x^2-3*ln(1/2*(2*(a*x
^2+b*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a*b*x^2+4*(a*x^2+b*x)^(3/2)*a^(1/2)
)/(x*(a*x+b))^(1/2)/a^(1/2)
```

Maxima [A]

time = 0.48, size = 63, normalized size = 1.17

$$\sqrt{a+\frac{b}{x}}\,ax-\frac{3}{2}\sqrt{a}\,b\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)-2\sqrt{a+\frac{b}{x}}\,b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(3/2),x, algorithm="maxima")
```

```
[Out] sqrt(a + b/x)*a*x - 3/2*sqrt(a)*b*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a + b
/x) + sqrt(a))) - 2*sqrt(a + b/x)*b
```

Fricas [A]

time = 2.87, size = 100, normalized size = 1.85

$$\left[\frac{3}{2}\sqrt{a}\,b\log\left(2\,ax+2\,\sqrt{a}\,x\sqrt{\frac{ax+b}{x}}+b\right)+(ax-2\,b)\sqrt{\frac{ax+b}{x}},\,-3\,\sqrt{-a}\,b\arctan\left(\frac{\sqrt{-a}\,\sqrt{\frac{ax+b}{x}}}{a}\right)+(ax-2\,b)\sqrt{\frac{ax+b}{x}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(3/2),x, algorithm="fricas")
```

```
[Out] [3/2*sqrt(a)*b*log(2*a*x + 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) + (a*x - 2*b)
*sqrt((a*x + b)/x), -3*sqrt(-a)*b*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) + (a
*x - 2*b)*sqrt((a*x + b)/x)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 92 vs. $2(44) = 88$.
time = 2.25, size = 92, normalized size = 1.70

$$3\sqrt{a}\,b\,\mathrm{asinh}\left(\frac{\sqrt{a}\,\sqrt{x}}{\sqrt{b}}\right)+\frac{a^2x^{\frac{3}{2}}}{\sqrt{b}\,\sqrt{\frac{ax}{b}+1}}-\frac{a\sqrt{b}\,\sqrt{x}}{\sqrt{\frac{ax}{b}+1}}-\frac{2b^{\frac{3}{2}}}{\sqrt{x}\,\sqrt{\frac{ax}{b}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**(3/2),x)
```

```
[Out] 3*sqrt(a)*b*asinh(sqrt(a)*sqrt(x)/sqrt(b)) + a**2*x**(3/2)/(sqrt(b)*sqrt(a*
x/b + 1)) - a*sqrt(b)*sqrt(x)/sqrt(a*x/b + 1) - 2*b**(3/2)/(sqrt(x)*sqrt(a*
x/b + 1))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [B]
time = 1.50, size = 34, normalized size = 0.63

$$-\frac{2\,x\,{\left(a+\frac{b}{x}\right)}^{3/2}\,{}_2\mathrm{F}_1\left(-\frac{3}{2},-\frac{1}{2};\ \frac{1}{2};\ -\frac{a\,x}{b}\right)}{{\left(\frac{a\,x}{b}+1\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x)^(3/2),x)
```

```
[Out] -(2*x*(a + b/x)^(3/2)*hypergeom([-3/2, -1/2], 1/2, -(a*x)/b))/((a*x)/b + 1)
^(3/2)
```

3.235 $\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{c+\frac{d}{x}}\,dx$

Optimal. Leaf size=106

$$\frac{a\sqrt{a+\frac{b}{x}}\,x}{c}-\frac{2(bc-ad)^{3/2}\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^2\sqrt{d}}+\frac{\sqrt{a}\,(3bc-2ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{c^2}$$

```
[Out] (-2*a*d+3*b*c)*arctanh((a+b/x)^(1/2)/a^(1/2))*a^(1/2)/c^2-2*(-a*d+b*c)^(3/2
)*arctan(d^(1/2)*(a+b/x)^(1/2)/(-a*d+b*c)^(1/2))/c^2/d^(1/2)+a*x*(a+b/x)^(1
/2)/c
```

Rubi [A]

time = 0.08, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {382, 100, 162, 65, 214, 211}

$$-\frac{2(bc-ad)^{3/2}\text{ArcTan}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^2\sqrt{d}}+\frac{\sqrt{a}\,(3bc-2ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{c^2}+\frac{ax\sqrt{a+\frac{b}{x}}}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^(3/2)/(c + d/x),x]
```

```
[Out] (a*Sqrt[a + b/x]*x)/c - (2*(b*c - a*d)^(3/2)*ArcTan[(Sqrt[d]*Sqrt[a + b/x])
/Sqrt[b*c - a*d]])/(c^2*Sqrt[d]) + (Sqrt[a]*(3*b*c - 2*a*d)*ArcTanh[Sqrt[a
+ b/x]/Sqrt[a]])/c^2
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
```

```
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{c+\frac{d}{x}}\,dx = -\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x^2(c+dx)}\,dx, x, \frac{1}{x}\right)$$
$$= \frac{a\sqrt{a+\frac{b}{x}}\,x}{c} + \frac{\text{Subst}\left(\int \frac{-\frac{1}{2}a(3bc-2ad)-\frac{1}{2}b(2bc-ad)x}{x\sqrt{a+bx}\,(c+dx)}\,dx, x, \frac{1}{x}\right)}{c}$$
$$= \frac{a\sqrt{a+\frac{b}{x}}\,x}{c} - \frac{(a(3bc-2ad))\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)}{2c^2} - \frac{(bc-ad)^2\text{Subst}\left(\int \frac{}{\sqrt{a+}}\right.}{c^2}$$
$$= \frac{a\sqrt{a+\frac{b}{x}}\,x}{c} - \frac{(a(3bc-2ad))\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x}}\right)}{bc^2} - \frac{(2(bc-ad)^2)\,\text{Subst}\left(\right.}{}$$
$$= \frac{a\sqrt{a+\frac{b}{x}}\,x}{c} - \frac{2(bc-ad)^{3/2}\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^2\sqrt{d}} + \frac{\sqrt{a}\,(3bc-2ad)\tanh^{-1}\left(\frac{\sqrt{a+}}{\sqrt{a}}\right.}{c^2}$$

Mathematica [A]
time = 0.27, size = 103, normalized size = 0.97

$$\frac{ac\sqrt{a+\frac{b}{x}}\,x - \frac{2(bc-ad)^{3/2}\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{\sqrt{d}} - \sqrt{a}\,(-3bc+2ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^(3/2)/(c + d/x),x]
```

```
[Out] (a*c*Sqrt[a + b/x]*x - (2*(b*c - a*d)^(3/2)*ArcTan[(Sqrt[d]*Sqrt[a + b/x])/
Sqrt[b*c - a*d]])/Sqrt[d] - Sqrt[a]*(-3*b*c + 2*a*d)*ArcTanh[Sqrt[a + b/x]/
Sqrt[a]])/c^2
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 527 vs. $2(88) = 176$.
time = 0.06, size = 528, normalized size = 4.98

method	result

risch	$\frac{xa\sqrt{\frac{ax+b}{x}}}{c}+\frac{\left(-\frac{a^{\frac{3}{2}}\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^{2}+bx}\right)d}{c^{2}}+\frac{3\sqrt{a}\,\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^{2}+bx}\right)b}{2c}-\frac{\ln\left(\frac{\frac{2d(ad-bc)}{c^{2}}-\frac{(2ad-bc)\left(x+\frac{d}{c}\right)}{c}+2 \cdots\right.}{\cdots}\right.}{\cdots}$
default	$\frac{\sqrt{\frac{ax+b}{x}}\,x\left(2\sqrt{x\,(ax+b)}\,a^{\frac{3}{2}}\sqrt{\frac{d(ad-bc)}{c^{2}}}\,c^{2}d-2\sqrt{x\,(ax+b)}\,\sqrt{a}\,\sqrt{\frac{d(ad-bc)}{c^{2}}}\,b\,c^{3}-2\ln\left(\frac{2\sqrt{x\,(ax+b)}\,\sqrt{\cdots}}{2\sqrt{a}}\cdots\right.\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+1/x*b)^(3/2)/(c+d/x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*((a*x+b)/x)^(1/2)*x*(2*(x*(a*x+b))^(1/2)*a^(3/2)*(d*(a*d-b*c)/c^2)^(1/2
)*c^2*d-2*(x*(a*x+b))^(1/2)*a^(1/2)*(d*(a*d-b*c)/c^2)^(1/2)*b*c^3-2*ln(1/2*
(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*(d*(a*d-b*c)/c^2)^(1/2)*a^2*
c*d^2+3*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*(d*(a*d-b*c)/
c^2)^(1/2)*a*b*c^2*d-ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*
(d*(a*d-b*c)/c^2)^(1/2)*b^2*c^3+2*a^(1/2)*(d*(a*d-b*c)/c^2)^(1/2)*(a*x^2+b*
x)^(1/2)*b*c^3-2*a^(5/2)*ln((2*(x*(a*x+b))^(1/2)*(d*(a*d-b*c)/c^2)^(1/2)*c-
2*a*d*x+b*c*x-b*d)/(c*x+d))*d^3+4*a^(3/2)*ln((2*(x*(a*x+b))^(1/2)*(d*(a*d-b
*c)/c^2)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*b*c*d^2-2*a^(1/2)*ln((2*(x*(a*
x+b))^(1/2)*(d*(a*d-b*c)/c^2)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*b^2*c^2*d
+(d*(a*d-b*c)/c^2)^(1/2)*ln(1/2*(2*(a*x^2+b*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/
2))*b^2*c^3)/(x*(a*x+b))^(1/2)/d/c^3/a^(1/2)/(d*(a*d-b*c)/c^2)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(3/2)/(c+d/x),x, algorithm="maxima")
```

```
[Out] integrate((a + b/x)^(3/2)/(c + d/x), x)
```

Fricas [A]
time = 2.55, size = 519, normalized size = 4.90

$$\left[\frac{2acx\sqrt{\frac{ax+b}{x}}-(3bc-2ad)\sqrt{a}\log\left(2ax-2\sqrt{a}\,x\sqrt{\frac{ax+b}{x}}+b\right)-2(bc-ad)\sqrt{-\frac{bc-ad}{d}}\log\left(\frac{2dx\sqrt{-\frac{bc-ad}{d}}\sqrt{\frac{ax+b}{x}}+bd-(bc-2ad)x}{cx+d}\right)}{2c^{2}},\ \frac{acx\sqrt{\frac{ax+b}{x}}-(3bc-2ad)\sqrt{-a}\arctan\left(\frac{\sqrt{-a}\sqrt{\frac{ax+b}{x}}}{a}\right)-(bc-ad)\sqrt{-\frac{bc-ad}{d}}\log\left(\frac{2dx\sqrt{-\frac{bc-ad}{d}}\sqrt{\frac{ax+b}{x}}+bd-(bc-2ad)x}{cx+d}\right)}{c^{2}},\ \frac{2acx\sqrt{\frac{ax+b}{x}}+4(bc-ad)\sqrt{\frac{bc-ad}{d}}\arctan\left(-\frac{d\sqrt{\frac{bc-ad}{d}}\sqrt{\frac{ax+b}{x}}}{bc-ad}\right)-(3bc-2ad)\sqrt{a}\log\left(2ax-2\sqrt{a}\,x\sqrt{\frac{ax+b}{x}}+b\right)}{2c^{2}},\ \frac{acx\sqrt{\frac{ax+b}{x}}+2(bc-ad)\sqrt{\frac{bc-ad}{d}}\arctan\left(-\frac{d\sqrt{\frac{bc-ad}{d}}\sqrt{\frac{ax+b}{x}}}{bc-ad}\right)-(3bc-2ad)\sqrt{-a}\arctan\left(\frac{\sqrt{-a}\sqrt{\frac{ax+b}{x}}}{a}\right)}{c^{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(3/2)/(c+d/x),x, algorithm="fricas")
```

```
[Out] [1/2*(2*a*c*x*sqrt((a*x + b)/x) - (3*b*c - 2*a*d)*sqrt(a)*log(2*a*x - 2*sqr
t(a)*x*sqrt((a*x + b)/x) + b) - 2*(b*c - a*d)*sqrt(-(b*c - a*d)/d)*log((2*d
*x*sqrt(-(b*c - a*d)/d)*sqrt((a*x + b)/x) + b*d - (b*c - 2*a*d)*x)/(c*x + d
)))/c^2, (a*c*x*sqrt((a*x + b)/x) - (3*b*c - 2*a*d)*sqrt(-a)*arctan(sqrt(-a
)*sqrt((a*x + b)/x)/a) - (b*c - a*d)*sqrt(-(b*c - a*d)/d)*log((2*d*x*sqrt(-
(b*c - a*d)/d)*sqrt((a*x + b)/x) + b*d - (b*c - 2*a*d)*x)/(c*x + d)))/c^2,
1/2*(2*a*c*x*sqrt((a*x + b)/x) + 4*(b*c - a*d)*sqrt((b*c - a*d)/d)*arctan(-
d*sqrt((b*c - a*d)/d)*sqrt((a*x + b)/x)/(b*c - a*d)) - (3*b*c - 2*a*d)*sqrt
(a)*log(2*a*x - 2*sqrt(a)*x*sqrt((a*x + b)/x) + b))/c^2, (a*c*x*sqrt((a*x +
 b)/x) + 2*(b*c - a*d)*sqrt((b*c - a*d)/d)*arctan(-d*sqrt((b*c - a*d)/d)*sq
rt((a*x + b)/x)/(b*c - a*d)) - (3*b*c - 2*a*d)*sqrt(-a)*arctan(sqrt(-a)*sqr
t((a*x + b)/x)/a))/c^2]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\left(a+\frac{b}{x}\right)^{\frac{3}{2}}}{cx+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**(3/2)/(c+d/x),x)
```

```
[Out] Integral(x*(a + b/x)**(3/2)/(c*x + d), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(3/2)/(c+d/x),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [B]
time = 1.68, size = 556, normalized size = 5.25

$$\frac{a\,x\,\sqrt{a+\frac{b}{x}}}{c}-\frac{\sqrt{a}\,\mathrm{atanh}\left(\frac{58\,a^{3/2}\,b^6\,d^2\,\sqrt{a+\frac{b}{x}}}{58\,a^2\,b^6\,d^2-24\,a\,b^7\,c\,d-\frac{46\,a^3\,b^5\,d^3}{c}+\frac{12\,a^4\,b^4\,d^4}{c^2}}+\frac{46\,a^{5/2}\,b^5\,d^3\,\sqrt{a+\frac{b}{x}}}{46\,a^3\,b^5\,d^3-58\,a^2\,b^6\,c\,d^2-\frac{12\,a^4\,b^4\,d^4}{c}+24\,a\,b^7\,c^2\,d}+\frac{12\,a^{7/2}\,b^4\,d^4\,\sqrt{a+\frac{b}{x}}}{12\,a^4\,b^4\,d^4-46\,a^3\,b^5\,c\,d^3+58\,a^2\,b^6\,c^2\,d^2-24\,a\,b^7\,c^3\,d}-\frac{24\,\sqrt{a}\,b^7\,c\,d\,\sqrt{a+\frac{b}{x}}}{58\,a^2\,b^6\,d^2-24\,a\,b^7\,c\,d-\frac{46\,a^3\,b^5\,d^3}{c}+\frac{12\,a^4\,b^4\,d^4}{c^2}}\right)\,\left(2\,a\,d-3\,b\,c\right)}{c^2}+\frac{2\,\mathrm{atanh}\left(\frac{12\,a^2\,b^4\,d^2\,\sqrt{a+\frac{b}{x}}\,\sqrt{a^3\,d^4-3\,a^2\,b\,c\,d^3+3\,a\,b^2\,c^2\,d^2-b^3\,c^3\,d}}{12\,a^4\,b^4\,d^4-40\,a^3\,b^5\,c\,d^3+44\,a^2\,b^6\,c^2\,d^2-16\,a\,b^7\,c^3\,d}+\frac{16\,a\,b^5\,d\,\sqrt{a+\frac{b}{x}}\,\sqrt{a^3\,d^4-3\,a^2\,b\,c\,d^3+3\,a\,b^2\,c^2\,d^2-b^3\,c^3\,d}}{40\,a^3\,b^5\,d^3-44\,a^2\,b^6\,c\,d^2-\frac{12\,a^4\,b^4\,d^4}{c}+16\,a\,b^7\,c^2\,d}\right)\,\sqrt{d\,{\left(a\,d-b\,c\right)}^3}}{c^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x)^(3/2)/(c + d/x),x)
```

```
[Out] (a*x*(a + b/x)^(1/2))/c - (a^(1/2)*atanh((58*a^(3/2)*b^6*d^2*(a + b/x)^(1/2
))/(58*a^2*b^6*d^2 - 24*a*b^7*c*d - (46*a^3*b^5*d^3)/c + (12*a^4*b^4*d^4)/c
^2) + (46*a^(5/2)*b^5*d^3*(a + b/x)^(1/2))/(46*a^3*b^5*d^3 - 58*a^2*b^6*c*d
^2 - (12*a^4*b^4*d^4)/c + 24*a*b^7*c^2*d) + (12*a^(7/2)*b^4*d^4*(a + b/x)^(
1/2))/(12*a^4*b^4*d^4 - 46*a^3*b^5*c*d^3 + 58*a^2*b^6*c^2*d^2 - 24*a*b^7*c^
3*d) - (24*a^(1/2)*b^7*c*d*(a + b/x)^(1/2))/(58*a^2*b^6*d^2 - 24*a*b^7*c*d
- (46*a^3*b^5*d^3)/c + (12*a^4*b^4*d^4)/c^2))*(2*a*d - 3*b*c))/c^2 + (2*ata
nh((12*a^2*b^4*d^2*(a + b/x)^(1/2)*(a^3*d^4 - b^3*c^3*d + 3*a*b^2*c^2*d^2 -
 3*a^2*b*c*d^3)^(1/2))/(12*a^4*b^4*d^4 - 40*a^3*b^5*c*d^3 + 44*a^2*b^6*c^2*
d^2 - 16*a*b^7*c^3*d) + (16*a*b^5*d*(a + b/x)^(1/2)*(a^3*d^4 - b^3*c^3*d +
3*a*b^2*c^2*d^2 - 3*a^2*b*c*d^3)^(1/2))/(40*a^3*b^5*d^3 - 44*a^2*b^6*c*d^2
- (12*a^4*b^4*d^4)/c + 16*a*b^7*c^2*d))*(d*(a*d - b*c)^3)^(1/2))/(c^2*d)
```

3.236 $\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{\left(c+\frac{d}{x}\right)^2}\,dx$

Optimal. Leaf size=156

$$-\frac{(bc-2ad)\sqrt{a+\frac{b}{x}}}{c^2\left(c+\frac{d}{x}\right)}+\frac{a\sqrt{a+\frac{b}{x}}\,x}{c\left(c+\frac{d}{x}\right)}-\frac{(bc-4ad)\sqrt{bc-ad}\,\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^3\sqrt{d}}+\frac{\sqrt{a}\,(3bc-4ad)\tanh^{-}}{c^3}$$

```
[Out] (-4*a*d+3*b*c)*arctanh((a+b/x)^(1/2)/a^(1/2))*a^(1/2)/c^3-(-4*a*d+b*c)*arct
an(d^(1/2)*(a+b/x)^(1/2)/(-a*d+b*c)^(1/2))*(-a*d+b*c)^(1/2)/c^3/d^(1/2)-(-2
*a*d+b*c)*(a+b/x)^(1/2)/c^2/(c+d/x)+a*x*(a+b/x)^(1/2)/c/(c+d/x)
```

Rubi [A]
time = 0.15, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {382, 100, 156, 162, 65, 214, 211}

$$-\frac{(bc-4ad)\sqrt{bc-ad}\,\text{ArcTan}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^3\sqrt{d}}+\frac{\sqrt{a}\,(3bc-4ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{c^3}-\frac{\sqrt{a+\frac{b}{x}}\,(bc-2ad)}{c^2\left(c+\frac{d}{x}\right)}+\frac{ax\sqrt{a+\frac{b}{x}}}{c\left(c+\frac{d}{x}\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^(3/2)/(c + d/x)^2,x]
```

```
[Out] -(((b*c - 2*a*d)*Sqrt[a + b/x])/(c^2*(c + d/x))) + (a*Sqrt[a + b/x]*x)/(c*(
c + d/x)) - ((b*c - 4*a*d)*Sqrt[b*c - a*d]*ArcTan[(Sqrt[d]*Sqrt[a + b/x])/S
qrt[b*c - a*d]])/(c^3*Sqrt[d]) + (Sqrt[a]*(3*b*c - 4*a*d)*ArcTanh[Sqrt[a +
b/x]/Sqrt[a]])/c^3
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
```

```
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{\left(c+\frac{d}{x}\right)^2}\,dx = -\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x^2(c+dx)^2}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{a\sqrt{a+\frac{b}{x}}\,x}{c\left(c+\frac{d}{x}\right)} + \frac{\text{Subst}\left(\int \frac{-\frac{1}{2}a(3bc-4ad)-\frac{1}{2}b(2bc-3ad)x}{x\sqrt{a+bx}\,(c+dx)^2}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= -\frac{(bc-2ad)\sqrt{a+\frac{b}{x}}}{c^2\left(c+\frac{d}{x}\right)} + \frac{a\sqrt{a+\frac{b}{x}}\,x}{c\left(c+\frac{d}{x}\right)} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}a(3bc-4ad)(bc-ad)+\frac{1}{2}b(bc-2ad)(bc-ad)x}{x\sqrt{a+bx}\,(c+dx)}\,dx, x, \right.}{c^2(bc-ad)}$$

$$= -\frac{(bc-2ad)\sqrt{a+\frac{b}{x}}}{c^2\left(c+\frac{d}{x}\right)} + \frac{a\sqrt{a+\frac{b}{x}}\,x}{c\left(c+\frac{d}{x}\right)} - \frac{(a(3bc-4ad))\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)}{2c^3} - $$

$$= -\frac{(bc-2ad)\sqrt{a+\frac{b}{x}}}{c^2\left(c+\frac{d}{x}\right)} + \frac{a\sqrt{a+\frac{b}{x}}\,x}{c\left(c+\frac{d}{x}\right)} - \frac{(a(3bc-4ad))\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x}}\right)}{bc^3}$$

$$= -\frac{(bc-2ad)\sqrt{a+\frac{b}{x}}}{c^2\left(c+\frac{d}{x}\right)} + \frac{a\sqrt{a+\frac{b}{x}}\,x}{c\left(c+\frac{d}{x}\right)} - \frac{(bc-4ad)\sqrt{bc-ad}\,\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^3\sqrt{d}} + $$

__Mathematica__ [A]
time = 0.36, size = 144, normalized size = 0.92

$$\frac{\frac{c\sqrt{a+\frac{b}{x}}\,x(-bc+2ad+acx)}{d+cx} - \frac{(b^2c^2-5abcd+4a^2d^2)\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{\sqrt{d}\sqrt{bc-ad}} - \sqrt{a}\,(-3bc+4ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^(3/2)/(c + d/x)^2,x]
```

```
[Out] ((c*Sqrt[a + b/x]*x*(-(b*c) + 2*a*d + a*c*x))/(d + c*x) - ((b^2*c^2 - 5*a*b
*c*d + 4*a^2*d^2)*ArcTan[(Sqrt[d]*Sqrt[a + b/x])/Sqrt[b*c - a*d]])/(Sqrt[d]
*Sqrt[b*c - a*d]) - Sqrt[a]*(-3*b*c + 4*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]]
)/c^3
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 835 vs. $2(136) = 272$.

time = 0.08, size = 836, normalized size = 5.36

method	result
default	$\frac{\left(-2\sqrt{x\,(ax+b)}\,a^{\frac{5}{2}}\sqrt{\frac{d(ad-bc)}{c^2}}\,c^4x^2-4a^{\frac{7}{2}}\ln\left(\frac{2\sqrt{x\,(ax+b)}\,\sqrt{\frac{d(ad-bc)}{c^2}}\,c-2adx+bcx-bd}{cx+d}\right)c\,d^3x+2\sqrt{x\,(ax+ \right.}{}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+1/x*b)^(3/2)/(c+d/x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(-2*(x*(a*x+b))^(1/2)*a^(5/2)*(d*(a*d-b*c)/c^2)^(1/2)*c^4*x^2-4*a^(7/2)
*ln((2*(x*(a*x+b))^(1/2)*(d*(a*d-b*c)/c^2)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+
d))*c*d^3*x+2*(x*(a*x+b))^(1/2)*a^(5/2)*(d*(a*d-b*c)/c^2)^(1/2)*c^3*d*x-4*a
^(7/2)*ln((2*(x*(a*x+b))^(1/2)*(d*(a*d-b*c)/c^2)^(1/2)*c-2*a*d*x+b*c*x-b*d)
/(c*x+d))*d^4+5*a^(5/2)*ln((2*(x*(a*x+b))^(1/2)*(d*(a*d-b*c)/c^2)^(1/2)*c-2
*a*d*x+b*c*x-b*d)/(c*x+d))*b*c^2*d^2*x+2*c^4*(x*(a*x+b))^(3/2)*a^(3/2)*(d*(
a*d-b*c)/c^2)^(1/2)+4*(x*(a*x+b))^(1/2)*a^(5/2)*(d*(a*d-b*c)/c^2)^(1/2)*c^2
*d^2-2*(x*(a*x+b))^(1/2)*a^(3/2)*(d*(a*d-b*c)/c^2)^(1/2)*b*c^4*x+5*a^(5/2)*
ln((2*(x*(a*x+b))^(1/2)*(d*(a*d-b*c)/c^2)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d
))*b*c*d^3-a^(3/2)*ln((2*(x*(a*x+b))^(1/2)*(d*(a*d-b*c)/c^2)^(1/2)*c-2*a*d*
x+b*c*x-b*d)/(c*x+d))*b^2*c^3*d*x-2*(x*(a*x+b))^(1/2)*a^(3/2)*(d*(a*d-b*c)/
c^2)^(1/2)*b*c^3*d-4*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*
a^3*(d*(a*d-b*c)/c^2)^(1/2)*c^2*d^2*x+3*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)
+2*a*x+b)/a^(1/2))*a^2*(d*(a*d-b*c)/c^2)^(1/2)*b*c^3*d*x-a^(3/2)*ln((2*(x*(
a*x+b))^(1/2)*(d*(a*d-b*c)/c^2)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*b^2*c^2
*d^2-4*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^3*(d*(a*d-b*
c)/c^2)^(1/2)*c*d^3+3*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))
*a^2*(d*(a*d-b*c)/c^2)^(1/2)*b*c^2*d^2)*x*((a*x+b)/x)^(1/2)/c^4/(d*(a*d-b*c
)/c^2)^(1/2)/a^(3/2)/(c*x+d)/d/(x*(a*x+b))^(1/2)
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(3/2)/(c+d/x)^2,x, algorithm="maxima")
```

```
[Out] integrate((a + b/x)^(3/2)/(c + d/x)^2, x)
```

Fricas [A]
time = 3.27, size = 769, normalized size = 4.93

$$\left[-\frac{\left(3\,bcd-4\,ad^{2}+\left(3\,bc^{2}-4\,acd\right)x\right)\sqrt{a}\log\left(2\,ax-2\,\sqrt{a}x\sqrt{\frac{ax+b}{x}}+b\right)+\left(bcd-4\,ad^{2}+\left(bc^{2}-4\,acd\right)x\right)\sqrt{-\frac{bc-ad}{d}}\log\left(\frac{2\,dx\sqrt{-\frac{bc-ad}{d}}\sqrt{\frac{ax+b}{x}}+bd-\left(bc-2\,ad\right)x}{cx+d}\right)-2\,\left(ac^{2}x^{2}-\left(bc^{2}-2\,acd\right)x\right)\sqrt{\frac{ax+b}{x}}}{2\,\left(c^{4}x+c^{3}d\right)}, \frac{2\,\left(bcd-4\,ad^{2}+\left(bc^{2}-4\,acd\right)x\right)\sqrt{\frac{bc-ad}{d}}\arctan\left(-\frac{d\sqrt{\frac{bc-ad}{d}}\sqrt{\frac{ax+b}{x}}}{bc-ad}\right)-\left(3\,bcd-4\,ad^{2}+\left(3\,bc^{2}-4\,acd\right)x\right)\sqrt{a}\log\left(2\,ax-2\,\sqrt{a}x\sqrt{\frac{ax+b}{x}}+b\right)+2\,\left(ac^{2}x^{2}-\left(bc^{2}-2\,acd\right)x\right)\sqrt{\frac{ax+b}{x}}}{2\,\left(c^{4}x+c^{3}d\right)}, -\frac{2\,\left(3\,bcd-4\,ad^{2}+\left(3\,bc^{2}-4\,acd\right)x\right)\sqrt{-a}\arctan\left(\frac{\sqrt{-a}\sqrt{\frac{ax+b}{x}}}{a}\right)+\left(bcd-4\,ad^{2}+\left(bc^{2}-4\,acd\right)x\right)\sqrt{-\frac{bc-ad}{d}}\log\left(\frac{2\,dx\sqrt{-\frac{bc-ad}{d}}\sqrt{\frac{ax+b}{x}}+bd-\left(bc-2\,ad\right)x}{cx+d}\right)-2\,\left(ac^{2}x^{2}-\left(bc^{2}-2\,acd\right)x\right)\sqrt{\frac{ax+b}{x}}}{2\,\left(c^{4}x+c^{3}d\right)}, \frac{\left(bcd-4\,ad^{2}+\left(bc^{2}-4\,acd\right)x\right)\sqrt{\frac{bc-ad}{d}}\arctan\left(-\frac{d\sqrt{\frac{bc-ad}{d}}\sqrt{\frac{ax+b}{x}}}{bc-ad}\right)-\left(3\,bcd-4\,ad^{2}+\left(3\,bc^{2}-4\,acd\right)x\right)\sqrt{-a}\arctan\left(\frac{\sqrt{-a}\sqrt{\frac{ax+b}{x}}}{a}\right)+\left(ac^{2}x^{2}-\left(bc^{2}-2\,acd\right)x\right)\sqrt{\frac{ax+b}{x}}}{c^{4}x+c^{3}d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(3/2)/(c+d/x)^2,x, algorithm="fricas")
```

```
[Out] [-1/2*((3*b*c*d - 4*a*d^2 + (3*b*c^2 - 4*a*c*d)*x)*sqrt(a)*log(2*a*x - 2*sq
rt(a)*x*sqrt((a*x + b)/x) + b) + (b*c*d - 4*a*d^2 + (b*c^2 - 4*a*c*d)*x)*sq
rt(-(b*c - a*d)/d)*log((2*d*x*sqrt(-(b*c - a*d)/d)*sqrt((a*x + b)/x) + b*d
- (b*c - 2*a*d)*x)/(c*x + d)) - 2*(a*c^2*x^2 - (b*c^2 - 2*a*c*d)*x)*sqrt((a
*x + b)/x))/(c^4*x + c^3*d), 1/2*(2*(b*c*d - 4*a*d^2 + (b*c^2 - 4*a*c*d)*x)
*sqrt((b*c - a*d)/d)*arctan(-d*sqrt((b*c - a*d)/d)*sqrt((a*x + b)/x)/(b*c -
 a*d)) - (3*b*c*d - 4*a*d^2 + (3*b*c^2 - 4*a*c*d)*x)*sqrt(a)*log(2*a*x - 2*
sqrt(a)*x*sqrt((a*x + b)/x) + b) + 2*(a*c^2*x^2 - (b*c^2 - 2*a*c*d)*x)*sqrt
((a*x + b)/x))/(c^4*x + c^3*d), -1/2*(2*(3*b*c*d - 4*a*d^2 + (3*b*c^2 - 4*a
*c*d)*x)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) + (b*c*d - 4*a*d^2 +
 (b*c^2 - 4*a*c*d)*x)*sqrt(-(b*c - a*d)/d)*log((2*d*x*sqrt(-(b*c - a*d)/d)*
sqrt((a*x + b)/x) + b*d - (b*c - 2*a*d)*x)/(c*x + d)) - 2*(a*c^2*x^2 - (b*c
^2 - 2*a*c*d)*x)*sqrt((a*x + b)/x))/(c^4*x + c^3*d), ((b*c*d - 4*a*d^2 + (b
*c^2 - 4*a*c*d)*x)*sqrt((b*c - a*d)/d)*arctan(-d*sqrt((b*c - a*d)/d)*sqrt((
a*x + b)/x)/(b*c - a*d)) - (3*b*c*d - 4*a*d^2 + (3*b*c^2 - 4*a*c*d)*x)*sqrt
(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) + (a*c^2*x^2 - (b*c^2 - 2*a*c*d)*
x)*sqrt((a*x + b)/x))/(c^4*x + c^3*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2}\left(a+\frac{b}{x}\right)^{\frac{3}{2}}}{\left(cx+d\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**(3/2)/(c+d/x)**2,x)
```

```
[Out] Integral(x**2*(a + b/x)**(3/2)/(c*x + d)**2, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(3/2)/(c+d/x)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [B]

time = 2.16, size = 448, normalized size = 2.87

$$\frac{\operatorname{atanh}\left(\frac{8\,a^2\,b^5\,d^2\,\sqrt{a+\frac{b}{x}}\,\sqrt{a\,d^2-b\,c\,d}}{8\,a^3\,b^5\,d^3-10\,a^2\,b^6\,c\,d^2+2\,a\,b^7\,c^2\,d}-\frac{2\,a\,b^6\,d\,\sqrt{a+\frac{b}{x}}\,\sqrt{a\,d^2-b\,c\,d}}{2\,a\,b^7\,c\,d-10\,a^2\,b^6\,d^2+\frac{8\,a^3\,b^5\,d^3}{c}}\right)\,\sqrt{d\,\left(a\,d-b\,c\right)}\,\left(4\,a\,d-b\,c\right)}{c^3\,d}-\frac{\sqrt{a}\,\operatorname{atanh}\left(\frac{6\,\sqrt{a}\,b^7\,d\,\sqrt{a+\frac{b}{x}}}{6\,a\,b^7\,d-\frac{14\,a^2\,b^6\,d^2}{c}+\frac{8\,a^3\,b^5\,d^3}{c^2}}-\frac{14\,a^{3/2}\,b^6\,d^2\,\sqrt{a+\frac{b}{x}}}{6\,a\,b^7\,c\,d-14\,a^2\,b^6\,d^2+\frac{8\,a^3\,b^5\,d^3}{c}}+\frac{8\,a^{5/2}\,b^5\,d^3\,\sqrt{a+\frac{b}{x}}}{8\,a^3\,b^5\,d^3-14\,a^2\,b^6\,c\,d^2+6\,a\,b^7\,c^2\,d}\right)\,\left(4\,a\,d-3\,b\,c\right)}{c^3}-\frac{\frac{2\,\left(a\,b^2\,c-a^2\,b\,d\right)\,\sqrt{a+\frac{b}{x}}}{c^2}+\frac{b\,{\left(a+\frac{b}{x}\right)}^{3/2}\,\left(2\,a\,d-b\,c\right)}{c^2}}{\left(a+\frac{b}{x}\right)\,\left(2\,a\,d-b\,c\right)-d\,{\left(a+\frac{b}{x}\right)}^2-a^2\,d+a\,b\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x)^(3/2)/(c + d/x)^2,x)
```

```
[Out] (atanh((8*a^2*b^5*d^2*(a + b/x)^(1/2)*(a*d^2 - b*c*d)^(1/2))/(8*a^3*b^5*d^3
 - 10*a^2*b^6*c*d^2 + 2*a*b^7*c^2*d) - (2*a*b^6*d*(a + b/x)^(1/2)*(a*d^2 -
b*c*d)^(1/2))/(2*a*b^7*c*d - 10*a^2*b^6*d^2 + (8*a^3*b^5*d^3)/c))*(d*(a*d -
 b*c))^(1/2)*(4*a*d - b*c))/(c^3*d) - (a^(1/2)*atanh((6*a^(1/2)*b^7*d*(a +
b/x)^(1/2))/(6*a*b^7*d - (14*a^2*b^6*d^2)/c + (8*a^3*b^5*d^3)/c^2) - (14*a^
(3/2)*b^6*d^2*(a + b/x)^(1/2))/(6*a*b^7*c*d - 14*a^2*b^6*d^2 + (8*a^3*b^5*d
^3)/c) + (8*a^(5/2)*b^5*d^3*(a + b/x)^(1/2))/(8*a^3*b^5*d^3 - 14*a^2*b^6*c*
d^2 + 6*a*b^7*c^2*d))*(4*a*d - 3*b*c))/c^3 - ((2*(a*b^2*c - a^2*b*d)*(a + b
/x)^(1/2))/c^2 + (b*(a + b/x)^(3/2)*(2*a*d - b*c))/c^2)/((a + b/x)*(2*a*d -
 b*c) - d*(a + b/x)^2 - a^2*d + a*b*c)
```

3.237 $\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{\left(c+\frac{d}{x}\right)^3} dx$

Optimal. Leaf size=209

$$-\frac{(bc-3ad)\sqrt{a+\frac{b}{x}}}{2c^2\left(c+\frac{d}{x}\right)^2}-\frac{3(bc-4ad)\sqrt{a+\frac{b}{x}}}{4c^3\left(c+\frac{d}{x}\right)}+\frac{a\sqrt{a+\frac{b}{x}}\,x}{c\left(c+\frac{d}{x}\right)^2}-\frac{3(b^2c^2-8abcd+8a^2d^2)\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{4c^4\sqrt{d}\sqrt{bc-ad}}$$

```
[Out] 3*(-2*a*d+b*c)*arctanh((a+b/x)^(1/2)/a^(1/2))*a^(1/2)/c^4-3/4*(8*a^2*d^2-8*
a*b*c*d+b^2*c^2)*arctan(d^(1/2)*(a+b/x)^(1/2)/(-a*d+b*c)^(1/2))/c^4/d^(1/2)
/(-a*d+b*c)^(1/2)-1/2*(-3*a*d+b*c)*(a+b/x)^(1/2)/c^2/(c+d/x)^2-3/4*(-4*a*d+
b*c)*(a+b/x)^(1/2)/c^3/(c+d/x)+a*x*(a+b/x)^(1/2)/c/(c+d/x)^2
```

Rubi [A]
time = 0.23, antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {382, 100, 156, 162, 65, 214, 211}

$$-\frac{3(8a^2d^2-8abcd+b^2c^2)\,\text{ArcTan}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{4c^4\sqrt{d}\sqrt{bc-ad}}+\frac{3\sqrt{a}\,(bc-2ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{c^4}-\frac{3\sqrt{a+\frac{b}{x}}\,(bc-4ad)}{4c^3\left(c+\frac{d}{x}\right)}-\frac{\sqrt{a+\frac{b}{x}}\,(bc-3ad)}{2c^2\left(c+\frac{d}{x}\right)^2}+\frac{ax\sqrt{a+\frac{b}{x}}}{c\left(c+\frac{d}{x}\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^(3/2)/(c + d/x)^3,x]
```

```
[Out] -1/2*((b*c - 3*a*d)*Sqrt[a + b/x])/(c^2*(c + d/x)^2) - (3*(b*c - 4*a*d)*Sqr
t[a + b/x])/(4*c^3*(c + d/x)) + (a*Sqrt[a + b/x]*x)/(c*(c + d/x)^2) - (3*(b
^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*ArcTan[(Sqrt[d]*Sqrt[a + b/x])/Sqrt[b*c - a
*d]])/(4*c^4*Sqrt[d]*Sqrt[b*c - a*d]) + (3*Sqrt[a]*(b*c - 2*a*d)*ArcTanh[Sq
rt[a + b/x]/Sqrt[a]])/c^4
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{\left(c+\frac{d}{x}\right)^3}\,dx = -\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x^2(c+dx)^3}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{a\sqrt{a+\frac{b}{x}}\,x}{c\left(c+\frac{d}{x}\right)^2} + \frac{\text{Subst}\left(\int \frac{-\frac{3}{2}a(bc-2ad)-\frac{1}{2}b(2bc-5ad)x}{x\sqrt{a+bx}\,(c+dx)^3}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= -\frac{(bc-3ad)\sqrt{a+\frac{b}{x}}}{2c^2\left(c+\frac{d}{x}\right)^2} + \frac{a\sqrt{a+\frac{b}{x}}\,x}{c\left(c+\frac{d}{x}\right)^2} - \frac{\text{Subst}\left(\int \frac{3a(bc-2ad)(bc-ad)+\frac{3}{2}b(bc-3ad)(bc-ad)x}{x\sqrt{a+bx}\,(c+dx)^2}\,dx, x, \ldots}{2c^2(bc-ad)}$$

$$= -\frac{(bc-3ad)\sqrt{a+\frac{b}{x}}}{2c^2\left(c+\frac{d}{x}\right)^2} - \frac{3(bc-4ad)\sqrt{a+\frac{b}{x}}}{4c^3\left(c+\frac{d}{x}\right)} + \frac{a\sqrt{a+\frac{b}{x}}\,x}{c\left(c+\frac{d}{x}\right)^2} + \frac{\text{Subst}\left(\int \frac{-3a(bc-2ad)(bc-a\ldots}{x\sqrt{a\ldots}}\right.}{2c^3(\ldots}$$

$$= -\frac{(bc-3ad)\sqrt{a+\frac{b}{x}}}{2c^2\left(c+\frac{d}{x}\right)^2} - \frac{3(bc-4ad)\sqrt{a+\frac{b}{x}}}{4c^3\left(c+\frac{d}{x}\right)} + \frac{a\sqrt{a+\frac{b}{x}}\,x}{c\left(c+\frac{d}{x}\right)^2} - \frac{(3a(bc-2ad))\text{Subst}\left(\int \ldots\right.}{2c^4\ldots}$$

$$= -\frac{(bc-3ad)\sqrt{a+\frac{b}{x}}}{2c^2\left(c+\frac{d}{x}\right)^2} - \frac{3(bc-4ad)\sqrt{a+\frac{b}{x}}}{4c^3\left(c+\frac{d}{x}\right)} + \frac{a\sqrt{a+\frac{b}{x}}\,x}{c\left(c+\frac{d}{x}\right)^2} - \frac{(3a(bc-2ad))\text{Subst}\left(\int \ldots\right.}{b\ldots}$$

$$= -\frac{(bc-3ad)\sqrt{a+\frac{b}{x}}}{2c^2\left(c+\frac{d}{x}\right)^2} - \frac{3(bc-4ad)\sqrt{a+\frac{b}{x}}}{4c^3\left(c+\frac{d}{x}\right)} + \frac{a\sqrt{a+\frac{b}{x}}\,x}{c\left(c+\frac{d}{x}\right)^2} - \frac{3(b^2c^2-8abcd+8a^2d^2)\ldots}{4c^4\sqrt{d}\,\ldots}$$

Mathematica [A]
time = 0.46, size = 169, normalized size = 0.81

$$\frac{\frac{c\sqrt{a+\frac{b}{x}}\,x\left(-bc(3d+5cx)+2a\left(6d^2+9cdx+2c^2x^2\right)\right)}{(d+cx)^2} - \frac{3\left(b^2c^2-8abcd+8a^2d^2\right)\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{\sqrt{d}\sqrt{bc-ad}} - 12\sqrt{a}\,(-bc+2ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{4c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^(3/2)/(c + d/x)^3,x]
```

```
[Out] ((c*Sqrt[a + b/x]*x*(-(b*c*(3*d + 5*c*x)) + 2*a*(6*d^2 + 9*c*d*x + 2*c^2*x^
2)))/(d + c*x)^2 - (3*(b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*ArcTan[(Sqrt[d]*Sqr
```

```
t[a + b/x])/Sqrt[b*c - a*d]])/(Sqrt[d]*Sqrt[b*c - a*d]) - 12*Sqrt[a]*(-(b*c
) + 2*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/(4*c^4)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1816 vs. $2(181) = 362$.
time = 0.10, size = 1817, normalized size = 8.69

method	result	size
default	Expression too large to display	1817
risch	Expression too large to display	2618

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+1/x*b)^(3/2)/(c+d/x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*(-24*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^2*(d*(a*d-
b*c)/c^2)^(1/2)*b^2*c^4*d^2*x-48*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+
b)/a^(1/2))*a^4*(d*(a*d-b*c)/c^2)^(1/2)*c^2*d^4*x+96*a^(7/2)*ln((2*(x*(a*x+
b))^(1/2)*(d*(a*d-b*c)/c^2)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*b*c^2*d^4*x
-54*a^(5/2)*ln((2*(x*(a*x+b))^(1/2)*(d*(a*d-b*c)/c^2)^(1/2)*c-2*a*d*x+b*c*x
-b*d)/(c*x+d))*b^2*c^3*d^3*x-30*(x*(a*x+b))^(1/2)*a^(5/2)*(d*(a*d-b*c)/c^2)
^(1/2)*b*c^3*d^3+36*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a
^3*(d*(a*d-b*c)/c^2)^(1/2)*b*c^2*d^4-12*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)
+2*a*x+b)/a^(1/2))*a^2*(d*(a*d-b*c)/c^2)^(1/2)*b^2*c^3*d^3+36*ln(1/2*(2*(x*
(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^3*(d*(a*d-b*c)/c^2)^(1/2)*b*c^4*
d^2*x^2-12*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^2*(d*(a*
d-b*c)/c^2)^(1/2)*b^2*c^5*d*x^2-54*(x*(a*x+b))^(1/2)*a^(5/2)*(d*(a*d-b*c)/c
^2)^(1/2)*b*c^4*d^2*x+72*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/
2))*a^3*(d*(a*d-b*c)/c^2)^(1/2)*b*c^3*d^3*x-18*(x*(a*x+b))^(1/2)*a^(5/2)*(d
*(a*d-b*c)/c^2)^(1/2)*b*c^5*d*x^2+3*a^(3/2)*ln((2*(x*(a*x+b))^(1/2)*(d*(a*d
-b*c)/c^2)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*b^3*c^3*d^3-12*(x*(a*x+b))^(
1/2)*a^(7/2)*(d*(a*d-b*c)/c^2)^(1/2)*c^5*d*x^3+6*(x*(a*x+b))^(1/2)*a^(5/2)*
(d*(a*d-b*c)/c^2)^(1/2)*b*c^6*x^3-48*a^(9/2)*ln((2*(x*(a*x+b))^(1/2)*(d*(a*
d-b*c)/c^2)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*c*d^5*x+6*a^(3/2)*ln((2*(x*
(a*x+b))^(1/2)*(d*(a*d-b*c)/c^2)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*b^3*c^
4*d^2*x+24*(x*(a*x+b))^(1/2)*a^(7/2)*(d*(a*d-b*c)/c^2)^(1/2)*c^2*d^4+6*(x*(
a*x+b))^(1/2)*a^(3/2)*(d*(a*d-b*c)/c^2)^(1/2)*b^2*c^4*d^2-24*ln(1/2*(2*(x*(
a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^4*(d*(a*d-b*c)/c^2)^(1/2)*c*d^5+4
8*a^(7/2)*ln((2*(x*(a*x+b))^(1/2)*(d*(a*d-b*c)/c^2)^(1/2)*c-2*a*d*x+b*c*x-b
*d)/(c*x+d))*b*c*d^5-27*a^(5/2)*ln((2*(x*(a*x+b))^(1/2)*(d*(a*d-b*c)/c^2)^(
1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*b^2*c^2*d^4-6*(x*(a*x+b))^(3/2)*a^(3/2)*
(d*(a*d-b*c)/c^2)^(1/2)*b*c^6*x+6*(x*(a*x+b))^(1/2)*a^(3/2)*(d*(a*d-b*c)/c^
2)^(1/2)*b^2*c^6*x^2-24*a^(9/2)*ln((2*(x*(a*x+b))^(1/2)*(d*(a*d-b*c)/c^2)^(
1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*c^2*d^4*x^2+3*a^(3/2)*ln((2*(x*(a*x+b))^
(1/2)*(d*(a*d-b*c)/c^2)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*b^3*c^5*d*x^2+8
*(x*(a*x+b))^(3/2)*a^(5/2)*(d*(a*d-b*c)/c^2)^(1/2)*c^4*d^2-2*(x*(a*x+b))^(3
```

```
/2)*a^(3/2)*(d*(a*d-b*c)/c^2)^(1/2)*b*c^5*d+12*(x*(a*x+b))^(3/2)*a^(5/2)*(d
*(a*d-b*c)/c^2)^(1/2)*c^5*d*x-24*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+
b)/a^(1/2))*a^4*(d*(a*d-b*c)/c^2)^(1/2)*c^3*d^3*x^2+48*a^(7/2)*ln((2*(x*(a*
x+b))^(1/2)*(d*(a*d-b*c)/c^2)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*b*c^3*d^3
*x^2-27*a^(5/2)*ln((2*(x*(a*x+b))^(1/2)*(d*(a*d-b*c)/c^2)^(1/2)*c-2*a*d*x+b
*c*x-b*d)/(c*x+d))*b^2*c^4*d^2*x^2+36*(x*(a*x+b))^(1/2)*a^(7/2)*(d*(a*d-b*c
)/c^2)^(1/2)*c^3*d^3*x+12*(x*(a*x+b))^(1/2)*a^(3/2)*(d*(a*d-b*c)/c^2)^(1/2)
*b^2*c^5*d*x-24*a^(9/2)*ln((2*(x*(a*x+b))^(1/2)*(d*(a*d-b*c)/c^2)^(1/2)*c-2
*a*d*x+b*c*x-b*d)/(c*x+d))*d^6)*x*((a*x+b)/x)^(1/2)/c^5/d/(d*(a*d-b*c)/c^2)
^(1/2)/a^(3/2)/(c*x+d)^2/(a*d-b*c)/(x*(a*x+b))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(3/2)/(c+d/x)^3,x, algorithm="maxima")
[Out] integrate((a + b/x)^(3/2)/(c + d/x)^3, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 433 vs. 2(181) = 362.
time = 3.28, size = 1765, normalized size = 8.44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(3/2)/(c+d/x)^3,x, algorithm="fricas")
[Out] [-1/8*(12*(b^2*c^2*d^3 - 3*a*b*c*d^4 + 2*a^2*d^5 + (b^2*c^4*d - 3*a*b*c^3*d
^2 + 2*a^2*c^2*d^3)*x^2 + 2*(b^2*c^3*d^2 - 3*a*b*c^2*d^3 + 2*a^2*c*d^4)*x)*
sqrt(a)*log(2*a*x - 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) + 3*(b^2*c^2*d^2 - 8
*a*b*c*d^3 + 8*a^2*d^4 + (b^2*c^4 - 8*a*b*c^3*d + 8*a^2*c^2*d^2)*x^2 + 2*(b
^2*c^3*d - 8*a*b*c^2*d^2 + 8*a^2*c*d^3)*x)*sqrt(-b*c*d + a*d^2)*log((b*d -
(b*c - 2*a*d)*x + 2*sqrt(-b*c*d + a*d^2)*x*sqrt((a*x + b)/x))/(c*x + d)) -
2*(4*(a*b*c^4*d - a^2*c^3*d^2)*x^3 - (5*b^2*c^4*d - 23*a*b*c^3*d^2 + 18*a^2
*c^2*d^3)*x^2 - 3*(b^2*c^3*d^2 - 5*a*b*c^2*d^3 + 4*a^2*c*d^4)*x)*sqrt((a*x
+ b)/x))/(b*c^5*d^3 - a*c^4*d^4 + (b*c^7*d - a*c^6*d^2)*x^2 + 2*(b*c^6*d^2
- a*c^5*d^3)*x), -1/8*(24*(b^2*c^2*d^3 - 3*a*b*c*d^4 + 2*a^2*d^5 + (b^2*c^4
*d - 3*a*b*c^3*d^2 + 2*a^2*c^2*d^3)*x^2 + 2*(b^2*c^3*d^2 - 3*a*b*c^2*d^3 +
2*a^2*c*d^4)*x)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) + 3*(b^2*c^2*
d^2 - 8*a*b*c*d^3 + 8*a^2*d^4 + (b^2*c^4 - 8*a*b*c^3*d + 8*a^2*c^2*d^2)*x^2
 + 2*(b^2*c^3*d - 8*a*b*c^2*d^2 + 8*a^2*c*d^3)*x)*sqrt(-b*c*d + a*d^2)*log(
(b*d - (b*c - 2*a*d)*x + 2*sqrt(-b*c*d + a*d^2)*x*sqrt((a*x + b)/x))/(c*x +
 d)) - 2*(4*(a*b*c^4*d - a^2*c^3*d^2)*x^3 - (5*b^2*c^4*d - 23*a*b*c^3*d^2 +
```

```
18*a^2*c^2*d^3)*x^2 - 3*(b^2*c^3*d^2 - 5*a*b*c^2*d^3 + 4*a^2*c*d^4)*x)*sqr
t((a*x + b)/x))/(b*c^5*d^3 - a*c^4*d^4 + (b*c^7*d - a*c^6*d^2)*x^2 + 2*(b*c
^6*d^2 - a*c^5*d^3)*x), 1/4*(3*(b^2*c^2*d^2 - 8*a*b*c*d^3 + 8*a^2*d^4 + (b^
2*c^4 - 8*a*b*c^3*d + 8*a^2*c^2*d^2)*x^2 + 2*(b^2*c^3*d - 8*a*b*c^2*d^2 + 8
*a^2*c*d^3)*x)*sqrt(b*c*d - a*d^2)*arctan(sqrt(b*c*d - a*d^2)*x*sqrt((a*x +
 b)/x)/(a*d*x + b*d)) - 6*(b^2*c^2*d^3 - 3*a*b*c*d^4 + 2*a^2*d^5 + (b^2*c^4
*d - 3*a*b*c^3*d^2 + 2*a^2*c^2*d^3)*x^2 + 2*(b^2*c^3*d^2 - 3*a*b*c^2*d^3 +
2*a^2*c*d^4)*x)*sqrt(a)*log(2*a*x - 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) + (4
*(a*b*c^4*d - a^2*c^3*d^2)*x^3 - (5*b^2*c^4*d - 23*a*b*c^3*d^2 + 18*a^2*c^2
*d^3)*x^2 - 3*(b^2*c^3*d^2 - 5*a*b*c^2*d^3 + 4*a^2*c*d^4)*x)*sqrt((a*x + b)
/x))/(b*c^5*d^3 - a*c^4*d^4 + (b*c^7*d - a*c^6*d^2)*x^2 + 2*(b*c^6*d^2 - a*
c^5*d^3)*x), 1/4*(3*(b^2*c^2*d^2 - 8*a*b*c*d^3 + 8*a^2*d^4 + (b^2*c^4 - 8*a
*b*c^3*d + 8*a^2*c^2*d^2)*x^2 + 2*(b^2*c^3*d - 8*a*b*c^2*d^2 + 8*a^2*c*d^3)
*x)*sqrt(b*c*d - a*d^2)*arctan(sqrt(b*c*d - a*d^2)*x*sqrt((a*x + b)/x)/(a*d
*x + b*d)) - 12*(b^2*c^2*d^3 - 3*a*b*c*d^4 + 2*a^2*d^5 + (b^2*c^4*d - 3*a*b
*c^3*d^2 + 2*a^2*c^2*d^3)*x^2 + 2*(b^2*c^3*d^2 - 3*a*b*c^2*d^3 + 2*a^2*c*d^
4)*x)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) + (4*(a*b*c^4*d - a^2*c
^3*d^2)*x^3 - (5*b^2*c^4*d - 23*a*b*c^3*d^2 + 18*a^2*c^2*d^3)*x^2 - 3*(b^2*
c^3*d^2 - 5*a*b*c^2*d^3 + 4*a^2*c*d^4)*x)*sqrt((a*x + b)/x))/(b*c^5*d^3 - a
*c^4*d^4 + (b*c^7*d - a*c^6*d^2)*x^2 + 2*(b*c^6*d^2 - a*c^5*d^3)*x)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**(3/2)/(c+d/x)**3,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 727 vs. 2(181) = 362.
time = 1.13, size = 727, normalized size = 3.48

[illegible]

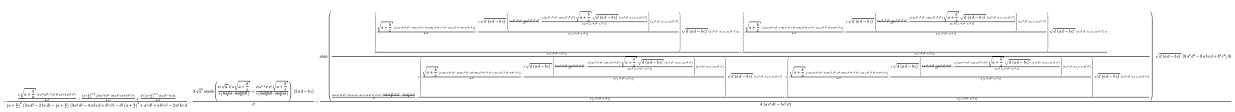
Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(3/2)/(c+d/x)^3,x, algorithm="giac")
```

```
[Out] sqrt(a*x^2 + b*x)*a*sgn(x)/c^3 + 3/4*(b^2*c^2*sgn(x) - 8*a*b*c*d*sgn(x) + 8
*a^2*d^2*sgn(x))*arctan(-((sqrt(a)*x - sqrt(a*x^2 + b*x))*c + sqrt(a)*d)/sq
rt(b*c*d - a*d^2))/(sqrt(b*c*d - a*d^2)*c^4) - 3/2*(a*b*c*sgn(x) - 2*a^2*d*
sgn(x))*log(abs(2*(sqrt(a)*x - sqrt(a*x^2 + b*x))*sqrt(a) + b))/(sqrt(a)*c^
4) + 1/4*(3*sqrt(a)*b^2*c^2*arctan(sqrt(a)*d/sqrt(b*c*d - a*d^2)) - 24*a^(3
/2)*b*c*d*arctan(sqrt(a)*d/sqrt(b*c*d - a*d^2)) + 24*a^(5/2)*d^2*arctan(sqr
```

```
t(a)*d/sqrt(b*c*d - a*d^2)) + 6*sqrt(b*c*d - a*d^2)*a*b*c*log(abs(b)) - 12*
sqrt(b*c*d - a*d^2)*a^2*d*log(abs(b)) + 5*sqrt(b*c*d - a*d^2)*a*b*c - 10*sq
rt(b*c*d - a*d^2)*a^2*d)*sgn(x)/(sqrt(b*c*d - a*d^2)*sqrt(a)*c^4) + 1/4*(5*
(sqrt(a)*x - sqrt(a*x^2 + b*x))^3*sqrt(a)*b^2*c^3*sgn(x) - 24*(sqrt(a)*x -
sqrt(a*x^2 + b*x))^3*a^(3/2)*b*c^2*d*sgn(x) + 24*(sqrt(a)*x - sqrt(a*x^2 +
b*x))^3*a^(5/2)*c*d^2*sgn(x) - (sqrt(a)*x - sqrt(a*x^2 + b*x))^2*a*b^2*c^2*
d*sgn(x) - 24*(sqrt(a)*x - sqrt(a*x^2 + b*x))^2*a^2*b*c*d^2*sgn(x) + 40*(sq
rt(a)*x - sqrt(a*x^2 + b*x))^2*a^3*d^3*sgn(x) + 3*(sqrt(a)*x - sqrt(a*x^2 +
 b*x))*sqrt(a)*b^3*c^2*d*sgn(x) - 28*(sqrt(a)*x - sqrt(a*x^2 + b*x))*a^(3/2
)*b^2*c*d^2*sgn(x) + 40*(sqrt(a)*x - sqrt(a*x^2 + b*x))*a^(5/2)*b*d^3*sgn(x
) - 5*a*b^3*c*d^2*sgn(x) + 10*a^2*b^2*d^3*sgn(x))/(((sqrt(a)*x - sqrt(a*x^2
 + b*x))^2*c + 2*(sqrt(a)*x - sqrt(a*x^2 + b*x))*sqrt(a)*d + b*d)^2*sqrt(a)
*c^4)
```

Mupad [B]
time = 3.47, size = 1664, normalized size = 7.96



Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x)^(3/2)/(c + d/x)^3,x)
```

```
[Out] - ((3*(a + b/x)^(1/2)*(3*a*b^3*c^2 + 4*a^3*b*d^2 - 7*a^2*b^2*c*d))/(4*c^3)
- ((a + b/x)^(3/2)*(5*b^3*c^2 + 24*a^2*b*d^2 - 24*a*b^2*c*d))/(4*c^3) + (3*
b*(a + b/x)^(5/2)*(4*a*d^2 - b*c*d))/(4*c^3))/((a + b/x)^2*(3*a*d^2 - 2*b*c
*d) - (a + b/x)*(3*a^2*d^2 + b^2*c^2 - 4*a*b*c*d) - d^2*(a + b/x)^3 + a^3*d
^2 + a*b^2*c^2 - 2*a^2*b*c*d) - (3*a^(1/2)*atanh((27*a^(1/2)*b^7*d*(a + b/x
)^(1/2))/(8*((27*a*b^7*d)/8 - (27*a^2*b^6*d^2)/(4*c))) + (27*a^(3/2)*b^6*d^
2*(a + b/x)^(1/2))/(4*((27*a^2*b^6*d^2)/4 - (27*a*b^7*c*d)/8)))*(2*a*d - b*
c))/c^4 - (atan((((((a + b/x)^(1/2)*(9*b^6*c^4*d + 1152*a^4*b^2*d^5 - 144*a
*b^5*c^3*d^2 - 1728*a^3*b^3*c*d^4 + 864*a^2*b^4*c^2*d^3))/(8*c^6) - (3*(d*(
a*d - b*c))^(1/2)*((9*a*b^4*c^9*d^2 - 12*a^2*b^3*c^8*d^3)/c^9 - (3*(64*b^3*
c^9*d^2 - 128*a*b^2*c^8*d^3)*(a + b/x)^(1/2)*(d*(a*d - b*c))^(1/2)*(8*a^2*d
^2 + b^2*c^2 - 8*a*b*c*d))/(64*c^6*(a*c^4*d^2 - b*c^5*d)))*(8*a^2*d^2 + b^2
*c^2 - 8*a*b*c*d))/(8*(a*c^4*d^2 - b*c^5*d)))*(d*(a*d - b*c))^(1/2)*(8*a^2*
d^2 + b^2*c^2 - 8*a*b*c*d)*3i)/(8*(a*c^4*d^2 - b*c^5*d)) + ((((a + b/x)^(1/
2)*(9*b^6*c^4*d + 1152*a^4*b^2*d^5 - 144*a*b^5*c^3*d^2 - 1728*a^3*b^3*c*d^4
 + 864*a^2*b^4*c^2*d^3))/(8*c^6) + (3*(d*(a*d - b*c))^(1/2)*((9*a*b^4*c^9*d
^2 - 12*a^2*b^3*c^8*d^3)/c^9 + (3*(64*b^3*c^9*d^2 - 128*a*b^2*c^8*d^3)*(a +
 b/x)^(1/2)*(d*(a*d - b*c))^(1/2)*(8*a^2*d^2 + b^2*c^2 - 8*a*b*c*d))/(64*c^
6*(a*c^4*d^2 - b*c^5*d)))*(8*a^2*d^2 + b^2*c^2 - 8*a*b*c*d))/(8*(a*c^4*d^2
- b*c^5*d)))*(d*(a*d - b*c))^(1/2)*(8*a^2*d^2 + b^2*c^2 - 8*a*b*c*d)*3i)/(8
*(a*c^4*d^2 - b*c^5*d)))/((216*a^5*b^3*d^5 - 378*a^4*b^4*c*d^4 - (189*a^2*b
^6*c^3*d^2)/4 + 216*a^3*b^5*c^2*d^3 + (27*a*b^7*c^4*d)/8)/c^9 - (3*(((a + b
```

```
/x)^(1/2)*(9*b^6*c^4*d + 1152*a^4*b^2*d^5 - 144*a*b^5*c^3*d^2 - 1728*a^3*b^
3*c*d^4 + 864*a^2*b^4*c^2*d^3))/(8*c^6) - (3*(d*(a*d - b*c))^(1/2)*((9*a*b^
4*c^9*d^2 - 12*a^2*b^3*c^8*d^3)/c^9 - (3*(64*b^3*c^9*d^2 - 128*a*b^2*c^8*d^
3)*(a + b/x)^(1/2)*(d*(a*d - b*c))^(1/2)*(8*a^2*d^2 + b^2*c^2 - 8*a*b*c*d))
/(64*c^6*(a*c^4*d^2 - b*c^5*d)))*(8*a^2*d^2 + b^2*c^2 - 8*a*b*c*d))/(8*(a*c
^4*d^2 - b*c^5*d)))*(d*(a*d - b*c))^(1/2)*(8*a^2*d^2 + b^2*c^2 - 8*a*b*c*d)
)/(8*(a*c^4*d^2 - b*c^5*d)) + (3*(((a + b/x)^(1/2)*(9*b^6*c^4*d + 1152*a^4*
b^2*d^5 - 144*a*b^5*c^3*d^2 - 1728*a^3*b^3*c*d^4 + 864*a^2*b^4*c^2*d^3))/(8
*c^6) + (3*(d*(a*d - b*c))^(1/2)*((9*a*b^4*c^9*d^2 - 12*a^2*b^3*c^8*d^3)/c^
9 + (3*(64*b^3*c^9*d^2 - 128*a*b^2*c^8*d^3)*(a + b/x)^(1/2)*(d*(a*d - b*c))
^(1/2)*(8*a^2*d^2 + b^2*c^2 - 8*a*b*c*d))/(64*c^6*(a*c^4*d^2 - b*c^5*d)))*(
8*a^2*d^2 + b^2*c^2 - 8*a*b*c*d))/(8*(a*c^4*d^2 - b*c^5*d)))*(d*(a*d - b*c)
)^(1/2)*(8*a^2*d^2 + b^2*c^2 - 8*a*b*c*d))/(8*(a*c^4*d^2 - b*c^5*d))))*(d*(
a*d - b*c))^(1/2)*(8*a^2*d^2 + b^2*c^2 - 8*a*b*c*d)*3i)/(4*(a*c^4*d^2 - b*c
^5*d))
```

3.238 $\int \left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^3\,dx$

Optimal. Leaf size=198

$$-ac^2(5bc+6ad)\sqrt{a+\frac{b}{x}}-\frac{1}{3}c^2(5bc+6ad)\left(a+\frac{b}{x}\right)^{3/2}-\frac{11}{9}d\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^2-\frac{d\left(a+\frac{b}{x}\right)^{5/2}\left(2(469b^2c^2\right.}{}$$

```
[Out] -1/3*c^2*(6*a*d+5*b*c)*(a+b/x)^(3/2)-11/9*d*(a+b/x)^(5/2)*(c+d/x)^2-1/315*d
*(a+b/x)^(5/2)*(-20*a^2*d^2+270*a*b*c*d+938*b^2*c^2+5*b*d*(10*a*d+89*b*c)/x
)/b^2+(a+b/x)^(5/2)*(c+d/x)^3*x+a^(3/2)*c^2*(6*a*d+5*b*c)*arctanh((a+b/x)^(
1/2)/a^(1/2))-a*c^2*(6*a*d+5*b*c)*(a+b/x)^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {382, 99, 158, 152, 52, 65, 214}

$$a^{3/2}c^2(6ad+5bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)-\frac{d\left(a+\frac{b}{x}\right)^{5/2}\left(2(-10a^2d^2+135abcd+469b^2c^2)+\frac{5bd(10ad+89bc)}{x}\right)}{315b^2}-\frac{1}{3}c^2\left(a+\frac{b}{x}\right)^{3/2}(6ad+5bc)-ac^2\sqrt{a+\frac{b}{x}}(6ad+5bc)+x\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^3-\frac{11}{9}d\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^2$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^(5/2)*(c + d/x)^3,x]
```

```
[Out] -(a*c^2*(5*b*c + 6*a*d)*Sqrt[a + b/x]) - (c^2*(5*b*c + 6*a*d)*(a + b/x)^(3/
2))/3 - (11*d*(a + b/x)^(5/2)*(c + d/x)^2)/9 - (d*(a + b/x)^(5/2)*(2*(469*b
^2*c^2 + 135*a*b*c*d - 10*a^2*d^2) + (5*b*d*(89*b*c + 10*a*d))/x))/(315*b^2
) + (a + b/x)^(5/2)*(c + d/x)^3*x + a^(3/2)*c^2*(5*b*c + 6*a*d)*ArcTanh[Sqr
t[a + b/x]/Sqrt[a]]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^p/(b*(
m + 1))), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m +
 1)*((c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Dist[(a^2*d
^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m
+ n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n +
 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)),
Int[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}
, x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 158

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegerQ[m]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \left(a+\frac{b}{x}\right)^{5/2} \left(c+\frac{d}{x}\right)^3 \, dx = -\text{Subst}\left(\int \frac{(a+bx)^{5/2}(c+dx)^3}{x^2} \, dx, x, \frac{1}{x}\right)$$

$$= \left(a+\frac{b}{x}\right)^{5/2} \left(c+\frac{d}{x}\right)^3 x - \text{Subst}\left(\int \frac{(a+bx)^{3/2}(c+dx)^2 \left(\frac{1}{2}(5bc+6ad)+ \frac{1}{\vphantom{x}}\right.}{x}\right.$$

$$= -\frac{11}{9}d\left(a+\frac{b}{x}\right)^{5/2} \left(c+\frac{d}{x}\right)^2 + \left(a+\frac{b}{x}\right)^{5/2} \left(c+\frac{d}{x}\right)^3 x - \frac{2\text{Subst}\left(\int \frac{(a+bx)^{3/}}{}\right.}{}$$

$$= -\frac{11}{9}d\left(a+\frac{b}{x}\right)^{5/2} \left(c+\frac{d}{x}\right)^2 - \frac{d\left(a+\frac{b}{x}\right)^{5/2} \left(2(469b^2c^2+135abcd-10a^2d^2)\right.}{315b^2}$$

$$= -\frac{1}{3}c^2(5bc+6ad)\left(a+\frac{b}{x}\right)^{3/2} - \frac{11}{9}d\left(a+\frac{b}{x}\right)^{5/2} \left(c+\frac{d}{x}\right)^2 - \frac{d\left(a+\frac{b}{x}\right)^{5/2} \left(2\right.}{}$$

$$= -ac^2(5bc+6ad)\sqrt{a+\frac{b}{x}} - \frac{1}{3}c^2(5bc+6ad)\left(a+\frac{b}{x}\right)^{3/2} - \frac{11}{9}d\left(a+\frac{b}{x}\right)^{5/2}$$

$$= -ac^2(5bc+6ad)\sqrt{a+\frac{b}{x}} - \frac{1}{3}c^2(5bc+6ad)\left(a+\frac{b}{x}\right)^{3/2} - \frac{11}{9}d\left(a+\frac{b}{x}\right)^{5/2}$$

$$= -ac^2(5bc+6ad)\sqrt{a+\frac{b}{x}} - \frac{1}{3}c^2(5bc+6ad)\left(a+\frac{b}{x}\right)^{3/2} - \frac{11}{9}d\left(a+\frac{b}{x}\right)^{5/2}$$

Mathematica [A]
time = 0.32, size = 201, normalized size = 1.02

$$\frac{\sqrt{a+\frac{b}{x}}\,(20a^4d^3x^4-10a^3bd^2x^3(d+27cx)-3a^2b^2x^2(50d^3+270cd^2x+966c^2dx^2-105c^3x^3)-2b^4(35d^3+135cd^2x+189c^2dx^2+105c^3x^3)-2ab^3x(95d^3+405cd^2x+693c^2dx^2+735c^3x^3))}{315b^2x^4}+a^{3/2}c^2(5bc+6ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^(5/2)*(c + d/x)^3,x]
```

```
[Out] (Sqrt[a + b/x]*(20*a^4*d^3*x^4 - 10*a^3*b*d^2*x^3*(d + 27*c*x) - 3*a^2*b^2*
x^2*(50*d^3 + 270*c*d^2*x + 966*c^2*d*x^2 - 105*c^3*x^3) - 2*b^4*(35*d^3 +
135*c*d^2*x + 189*c^2*d*x^2 + 105*c^3*x^3) - 2*a*b^3*x*(95*d^3 + 405*c*d^2*
x + 693*c^2*d*x^2 + 735*c^3*x^3)))/(315*b^2*x^4) + a^(3/2)*c^2*(5*b*c + 6*a
*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 456 vs. $2(173) = 346$.
time = 0.09, size = 457, normalized size = 2.31

method	result
risch	$\frac{(315a^2b^2c^3x^5+20a^4d^3x^4-270a^3bc\,d^2x^4-2898a^2b^2c^2d\,x^4-1470a\,b^3c^3x^4-10a^3b\,d^3x^3-810a^2b^2c\,d^2x^3-1386a\,b^3c^2d\,x^3-210b^4c^3x^3-}{315x^4b^2}$
default	$\frac{\sqrt{\frac{ax+b}{x}}\,\left(3780\sqrt{a\,x^2+bx}\;a^{\frac{7}{2}}b\,c^2d\,x^6+3150\sqrt{a\,x^2+bx}\;a^{\frac{5}{2}}b^2c^3x^6+1890\ln\left(\frac{2\sqrt{a\,x^2+bx}\,\sqrt{a}+2ax+b}{2\sqrt{a}}\right)a^3b^2c^2d}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+1/x*b)^(5/2)*(c+d/x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/630*((a*x+b)/x)^(1/2)/x^5/b^2*(3780*(a*x^2+b*x)^(1/2)*a^(7/2)*b*c^2*d*x^6
+3150*(a*x^2+b*x)^(1/2)*a^(5/2)*b^2*c^3*x^6+1890*ln(1/2*(2*(a*x^2+b*x)^(1/2
)*a^(1/2)+2*a*x+b)/a^(1/2))*a^3*b^2*c^2*d*x^6+1575*ln(1/2*(2*(a*x^2+b*x)^(1
/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^2*b^3*c^3*x^6-3780*(a*x^2+b*x)^(3/2)*a^(5/2
)*b*c^2*d*x^4-2520*(a*x^2+b*x)^(3/2)*a^(3/2)*b^2*c^3*x^4+40*(a*x^2+b*x)^(3/
2)*a^(7/2)*d^3*x^3-540*(a*x^2+b*x)^(3/2)*a^(5/2)*b*c*d^2*x^3-2016*(a*x^2+b*
x)^(3/2)*a^(3/2)*b^2*c^2*d*x^3-420*(a*x^2+b*x)^(3/2)*a^(1/2)*b^3*c^3*x^3-60
*(a*x^2+b*x)^(3/2)*a^(5/2)*b*d^3*x^2-1080*(a*x^2+b*x)^(3/2)*a^(3/2)*b^2*c*d
^2*x^2-756*(a*x^2+b*x)^(3/2)*a^(1/2)*b^3*c^2*d*x^2-240*(a*x^2+b*x)^(3/2)*a^
(3/2)*b^2*d^3*x-540*(a*x^2+b*x)^(3/2)*a^(1/2)*b^3*c*d^2*x-140*(a*x^2+b*x)^(
3/2)*a^(1/2)*b^3*d^3)/(x*(a*x+b))^(1/2)/a^(1/2)
```

Maxima [A]
time = 0.49, size = 219, normalized size = 1.11

$$-\frac{6\left(a+\frac{b}{x}\right)^{\frac{7}{2}}cd^2}{7b}+\frac{1}{6}\left(6\sqrt{a+\frac{b}{x}}\,a^2x-15a^{\frac{3}{2}}b\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)-4\left(a+\frac{b}{x}\right)^{\frac{3}{2}}b-24\sqrt{a+\frac{b}{x}}\,ab\right)c^3-\frac{1}{5}\left(15a^{\frac{5}{2}}\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)+6\left(a+\frac{b}{x}\right)^{\frac{5}{2}}+10\left(a+\frac{b}{x}\right)^{\frac{3}{2}}a+30\sqrt{a+\frac{b}{x}}\,a^2\right)c^2d-\frac{2}{63}\left(\frac{7\left(a+\frac{b}{x}\right)^{\frac{9}{2}}}{b^2}-\frac{9\left(a+\frac{b}{x}\right)^{\frac{7}{2}}a}{b^2}\right)d^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(5/2)*(c+d/x)^3,x, algorithm="maxima")
```

```
[Out] -6/7*(a + b/x)^(7/2)*c*d^2/b + 1/6*(6*sqrt(a + b/x)*a^2*x - 15*a^(3/2)*b*lo
g((sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/x) + sqrt(a))) - 4*(a + b/x)^(3/2)*
b - 24*sqrt(a + b/x)*a*b)*c^3 - 1/5*(15*a^(5/2)*log((sqrt(a + b/x) - sqrt(a
))/(sqrt(a + b/x) + sqrt(a))) + 6*(a + b/x)^(5/2) + 10*(a + b/x)^(3/2)*a +
30*sqrt(a + b/x)*a^2)*c^2*d - 2/63*(7*(a + b/x)^(9/2)/b^2 - 9*(a + b/x)^(7/
2)*a/b^2)*d^3
```

Fricas [A]
time = 3.05, size = 494, normalized size = 2.49

$$\left[\frac{315\left(5ab^3c^3+6a^2b^2c^2d\right)\sqrt{a}\,x^4\log\left(2ax+2\sqrt{a}\,x\sqrt{\frac{ax+b}{x}}+b\right)+2\left(315a^2b^2c^3x^5-70b^4d^3-2\left(735ab^3c^3+1449a^2b^2c^2d+135a^3bcd^2-10a^4d^3\right)x^4-2\left(105b^4c^3+693ab^3c^2d+405a^2b^2cd^2+5a^3bd^3\right)x^3-6\left(63b^4c^2d+135ab^3cd^2+25a^2b^2d^3\right)x^2-10\left(27b^4cd^2+19ab^3d^3\right)x\right)\sqrt{\frac{ax+b}{x}}}{630b^2x^4},\ -\frac{315\left(5ab^3c^3+6a^2b^2c^2d\right)\sqrt{-a}\,x^4\arctan\left(\frac{\sqrt{-a}\sqrt{\frac{ax+b}{x}}}{a}\right)-\left(315a^2b^2c^3x^5-70b^4d^3-2\left(735ab^3c^3+1449a^2b^2c^2d+135a^3bcd^2-10a^4d^3\right)x^4-2\left(105b^4c^3+693ab^3c^2d+405a^2b^2cd^2+5a^3bd^3\right)x^3-6\left(63b^4c^2d+135ab^3cd^2+25a^2b^2d^3\right)x^2-10\left(27b^4cd^2+19ab^3d^3\right)x\right)\sqrt{\frac{ax+b}{x}}}{315b^2x^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x)^(5/2)*(c+d/x)^3,x, algorithm="fricas")

[Out] [1/630*(315*(5*a*b^3*c^3 + 6*a^2*b^2*c^2*d)*sqrt(a)*x^4*log(2*a*x + 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) + 2*(315*a^2*b^2*c^3*x^5 - 70*b^4*d^3 - 2*(735*a*b^3*c^3 + 1449*a^2*b^2*c^2*d + 135*a^3*b*c*d^2 - 10*a^4*d^3)*x^4 - 2*(105*b^4*c^3 + 693*a*b^3*c^2*d + 405*a^2*b^2*c*d^2 + 5*a^3*b*d^3)*x^3 - 6*(63*b^4*c^2*d + 135*a*b^3*c*d^2 + 25*a^2*b^2*d^3)*x^2 - 10*(27*b^4*c*d^2 + 19*a*b^3*d^3)*x)*sqrt((a*x + b)/x))/(b^2*x^4), -1/315*(315*(5*a*b^3*c^3 + 6*a^2*b^2*c^2*d)*sqrt(-a)*x^4*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) - (315*a^2*b^2*c^3*x^5 - 70*b^4*d^3 - 2*(735*a*b^3*c^3 + 1449*a^2*b^2*c^2*d + 135*a^3*b*c*d^2 - 10*a^4*d^3)*x^4 - 2*(105*b^4*c^3 + 693*a*b^3*c^2*d + 405*a^2*b^2*c*d^2 + 5*a^3*b*d^3)*x^3 - 6*(63*b^4*c^2*d + 135*a*b^3*c*d^2 + 25*a^2*b^2*d^3)*x^2 - 10*(27*b^4*c*d^2 + 19*a*b^3*d^3)*x)*sqrt((a*x + b)/x))/(b^2*x^4)]

Sympy [**A**]
time = 85.57, size = 5513, normalized size = 27.84

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x)**(5/2)*(c+d/x)**3,x)

[Out] 32*a**(29/2)*b**(27/2)*d**3*x**10*sqrt(a*x/b + 1)/(315*a**(21/2)*b**15*x**(21/2) + 1890*a**(19/2)*b**16*x**(19/2) + 4725*a**(17/2)*b**17*x**(17/2) + 6300*a**(15/2)*b**18*x**(15/2) + 4725*a**(13/2)*b**19*x**(13/2) + 1890*a**(11/2)*b**20*x**(11/2) + 315*a**(9/2)*b**21*x**(9/2)) + 176*a**(27/2)*b**(29/2)*d**3*x**9*sqrt(a*x/b + 1)/(315*a**(21/2)*b**15*x**(21/2) + 1890*a**(19/2)*b**16*x**(19/2) + 4725*a**(17/2)*b**17*x**(17/2) + 6300*a**(15/2)*b**18*x**(15/2) + 4725*a**(13/2)*b**19*x**(13/2) + 1890*a**(11/2)*b**20*x**(11/2) + 315*a**(9/2)*b**21*x**(9/2)) + 396*a**(25/2)*b**(31/2)*d**3*x**8*sqrt(a*x/b + 1)/(315*a**(21/2)*b**15*x**(21/2) + 1890*a**(19/2)*b**16*x**(19/2) + 4725*a**(17/2)*b**17*x**(17/2) + 6300*a**(15/2)*b**18*x**(15/2) + 4725*a**(13/2)*b**19*x**(13/2) + 1890*a**(11/2)*b**20*x**(11/2) + 315*a**(9/2)*b**21*x**(9/2)) + 462*a**(23/2)*b**(33/2)*d**3*x**7*sqrt(a*x/b + 1)/(315*a**(21/2)*b**15*x**(21/2) + 1890*a**(19/2)*b**16*x**(19/2) + 4725*a**(17/2)*b**17*x**(17/2) + 6300*a**(15/2)*b**18*x**(15/2) + 4725*a**(13/2)*b**19*x**(13/2) + 1890*a**(11/2)*b**20*x**(11/2) + 315*a**(9/2)*b**21*x**(9/2)) + 210*a**(21/2)*b**(35/2)*d**3*x**6*sqrt(a*x/b + 1)/(315*a**(21/2)*b**15*x**(21/2) + 1890*a**(19/2)*b**16*x**(19/2) + 4725*a**(17/2)*b**17*x**(17/2) + 6300*a**(15/2)*b**18*x**(15/2) + 4725*a**(13/2)*b**19*x**(13/2) + 1890*a**(11/2)*b**20*x**(11/2) + 315*a**(9/2)*b**21*x**(9/2)) - 32*a**(21/2)*b**(11/2)*d**3*x**6*sqrt(a*x/b + 1)/(105*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(11/2) + 315*a**(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) - 378*a**(19/2)*b**(37/2)*d**3*x**5*sqrt(a*x/b + 1)/(315*a**(21/2)*b**15*x**(21/2) +

```
1890*a**(19/2)*b**16*x**(19/2) + 4725*a**(17/2)*b**17*x**(17/2) + 6300*a**(
15/2)*b**18*x**(15/2) + 4725*a**(13/2)*b**19*x**(13/2) + 1890*a**(11/2)*b**
20*x**(11/2) + 315*a**(9/2)*b**21*x**(9/2)) - 48*a**(19/2)*b**(13/2)*c*d**2
*x**6*sqrt(a*x/b + 1)/(105*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x*
*(11/2) + 315*a**(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) - 80*a*
*(19/2)*b**(13/2)*d**3*x**5*sqrt(a*x/b + 1)/(105*a**(13/2)*b**7*x**(13/2) +
 315*a**(11/2)*b**8*x**(11/2) + 315*a**(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b
**10*x**(7/2)) - 1134*a**(17/2)*b**(39/2)*d**3*x**4*sqrt(a*x/b + 1)/(315*a*
*(21/2)*b**15*x**(21/2) + 1890*a**(19/2)*b**16*x**(19/2) + 4725*a**(17/2)*b
**17*x**(17/2) + 6300*a**(15/2)*b**18*x**(15/2) + 4725*a**(13/2)*b**19*x**(
13/2) + 1890*a**(11/2)*b**20*x**(11/2) + 315*a**(9/2)*b**21*x**(9/2)) - 120
*a**(17/2)*b**(15/2)*c*d**2*x**5*sqrt(a*x/b + 1)/(105*a**(13/2)*b**7*x**(13
/2) + 315*a**(11/2)*b**8*x**(11/2) + 315*a**(9/2)*b**9*x**(9/2) + 105*a**(7
/2)*b**10*x**(7/2)) - 60*a**(17/2)*b**(15/2)*d**3*x**4*sqrt(a*x/b + 1)/(105
*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(11/2) + 315*a**(9/2)*b**
9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) - 1494*a**(15/2)*b**(41/2)*d**3*x
**3*sqrt(a*x/b + 1)/(315*a**(21/2)*b**15*x**(21/2) + 1890*a**(19/2)*b**16*x
**(19/2) + 4725*a**(17/2)*b**17*x**(17/2) + 6300*a**(15/2)*b**18*x**(15/2)
+ 4725*a**(13/2)*b**19*x**(13/2) + 1890*a**(11/2)*b**20*x**(11/2) + 315*a**
(9/2)*b**21*x**(9/2)) - 90*a**(15/2)*b**(17/2)*c*d**2*x**4*sqrt(a*x/b + 1)/
(105*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(11/2) + 315*a**(9/2)
*b**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) - 80*a**(15/2)*b**(17/2)*d**3
*x**3*sqrt(a*x/b + 1)/(105*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x*
*(11/2) + 315*a**(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) + 4*a**
(15/2)*b**(3/2)*d**3*x**3*sqrt(a*x/b + 1)/(15*a**(7/2)*b**3*x**(7/2) + 15*a
**(5/2)*b**4*x**(5/2)) - 1098*a**(13/2)*b**(43/2)*d**3*x**2*sqrt(a*x/b + 1)
/(315*a**(21/2)*b**15*x**(21/2) + 1890*a**(19/2)*b**16*x**(19/2) + 4725*a**
(17/2)*b**17*x**(17/2) + 6300*a**(15/2)*b**18*x**(15/2) + 4725*a**(13/2)*b*
*19*x**(13/2) + 1890*a**(11/2)*b**20*x**(11/2) + 315*a**(9/2)*b**21*x**(9/2
)) - 120*a**(13/2)*b**(19/2)*c*d**2*x**3*sqrt(a*x/b + 1)/(105*a**(13/2)*b**
7*x**(13/2) + 315*a**(11/2)*b**8*x**(11/2) + 315*a**(9/2)*b**9*x**(9/2) + 1
05*a**(7/2)*b**10*x**(7/2)) - 200*a**(13/2)*b**(19/2)*d**3*x**2*sqrt(a*x/b
+ 1)/(105*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(11/2) + 315*a**
(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) + 24*a**(13/2)*b**(5/2)*
c*d**2*x**3*sqrt(a*x/b + 1)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x
**(5/2)) + 2*a**(13/2)*b**(5/2)*d**3*x**2*sqrt(a*x/b + 1)/(15*a**(7/2)*b**3
*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) - 430*a**(11/2)*b**(45/2)*d**3*x*sqr
t(a*x/b + 1)/(315*a**(21/2)*b**15*x**(21/2) + 1890*a**(19/2)*b**16*x**(19/2
) + 4725*a**(17/2)*b**17*x**(17/2) + 6300*a**(15/2)*b**18*x**(15/2) + 4725*
a**(13/2)*b**19*x**(13/2) + 1890*a**(11/2)*b**20*x**(11/2) + 315*a**(9/2)*b
**21*x**(9/2)) - 300*a**(11/2)*b**(21/2)*c*d**2*x**2*sqrt(a*x/b + 1)/(105*a
**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(11/2) + 315*a**(9/2)*b**9*
x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) - 192*a**(11/2)*b**(21/2)*d**3*x*sq
rt(a*x/b + 1)/(105*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(11/2)
+ 315*a**(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b**...
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(5/2)*(c+d/x)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [**B**]
time = 6.05, size = 487, normalized size = 2.46

$$\left(a+\frac{b}{x}\right)^{7/2}\left(\frac{6\,a\,d^3-6\,b\,c\,d^2}{7\,b^2}-\frac{4\,a\,d^3}{7\,b^2}\right)-\sqrt{a+\frac{b}{x}}\,\left(a^2\,\left(2\,a\,\left(\frac{6\,a\,d^3-6\,b\,c\,d^2}{b^2}-\frac{4\,a\,d^3}{b^2}\right)-\frac{6\,d\,{\left(a\,d-b\,c\right)}^2}{b^2}+\frac{2\,a^2\,d^3}{b^2}\right)-2\,a\,\left(\frac{2\,{\left(a\,d-b\,c\right)}^3}{b^2}+2\,a\,\left(2\,a\,\left(\frac{6\,a\,d^3-6\,b\,c\,d^2}{b^2}-\frac{4\,a\,d^3}{b^2}\right)-\frac{6\,d\,{\left(a\,d-b\,c\right)}^2}{b^2}+\frac{2\,a^2\,d^3}{b^2}\right)-a^2\,\left(\frac{6\,a\,d^3-6\,b\,c\,d^2}{b^2}-\frac{4\,a\,d^3}{b^2}\right)\right)\right)+\left(a+\frac{b}{x}\right)^{3/2}\left(\frac{2\,{\left(a\,d-b\,c\right)}^3}{3\,b^2}+\frac{2\,a\,\left(2\,a\,\left(\frac{6\,a\,d^3-6\,b\,c\,d^2}{b^2}-\frac{4\,a\,d^3}{b^2}\right)-\frac{6\,d\,{\left(a\,d-b\,c\right)}^2}{b^2}+\frac{2\,a^2\,d^3}{b^2}\right)}{3}-\frac{a^2\,\left(\frac{6\,a\,d^3-6\,b\,c\,d^2}{b^2}-\frac{4\,a\,d^3}{b^2}\right)}{3}\right)+\left(a+\frac{b}{x}\right)^{5/2}\left(\frac{2\,a\,\left(\frac{6\,a\,d^3-6\,b\,c\,d^2}{b^2}-\frac{4\,a\,d^3}{b^2}\right)}{5}-\frac{6\,d\,{\left(a\,d-b\,c\right)}^2}{5\,b^2}+\frac{2\,a^2\,d^3}{5\,b^2}\right)-\frac{2\,d^3\,{\left(a+\frac{b}{x}\right)}^{9/2}}{9\,b^2}+a^2\,c^3\,x\,\sqrt{a+\frac{b}{x}}-a^{3/2}\,c^2\,\mathrm{atan}\left(\frac{\sqrt{a+\frac{b}{x}}\,1{}\mathrm{i}}{\sqrt{a}}\right)\,\left(6\,a\,d+5\,b\,c\right)\,1{}\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x)^(5/2)*(c + d/x)^3,x)
```

```
[Out] (a + b/x)^(7/2)*((6*a*d^3 - 6*b*c*d^2)/(7*b^2) - (4*a*d^3)/(7*b^2)) - (a +
b/x)^(1/2)*(a^2*(2*a*((6*a*d^3 - 6*b*c*d^2)/b^2 - (4*a*d^3)/b^2) - (6*d*(a*
d - b*c)^2)/b^2 + (2*a^2*d^3)/b^2) - 2*a*((2*(a*d - b*c)^3)/b^2 + 2*a*(2*a*
((6*a*d^3 - 6*b*c*d^2)/b^2 - (4*a*d^3)/b^2) - (6*d*(a*d - b*c)^2)/b^2 + (2*
a^2*d^3)/b^2) - a^2*((6*a*d^3 - 6*b*c*d^2)/b^2 - (4*a*d^3)/b^2))) + (a + b/
x)^(3/2)*((2*(a*d - b*c)^3)/(3*b^2) + (2*a*(2*a*((6*a*d^3 - 6*b*c*d^2)/b^2
- (4*a*d^3)/b^2) - (6*d*(a*d - b*c)^2)/b^2 + (2*a^2*d^3)/b^2))/3 - (a^2*((6
*a*d^3 - 6*b*c*d^2)/b^2 - (4*a*d^3)/b^2))/3) + (a + b/x)^(5/2)*((2*a*((6*a*
d^3 - 6*b*c*d^2)/b^2 - (4*a*d^3)/b^2))/5 - (6*d*(a*d - b*c)^2)/(5*b^2) + (2
*a^2*d^3)/(5*b^2)) - (2*d^3*(a + b/x)^(9/2))/(9*b^2) + a^2*c^3*x*(a + b/x)^
(1/2) - a^(3/2)*c^2*atan(((a + b/x)^(1/2)*1i)/a^(1/2))*(6*a*d + 5*b*c)*1i
```

3.239 $\int \left(a+\frac{b}{x}\right)^{5/2} \left(c+\frac{d}{x}\right)^2 \, dx$

Optimal. Leaf size=152

$$-ac(5bc+4ad)\sqrt{a+\frac{b}{x}}-\frac{1}{3}c(5bc+4ad)\left(a+\frac{b}{x}\right)^{3/2}-\frac{c(5bc+4ad)\left(a+\frac{b}{x}\right)^{5/2}}{5a}-\frac{2d^2\left(a+\frac{b}{x}\right)^{7/2}}{7b}+\frac{c^2\left(a+\frac{b}{x}\right)^{7/2}}{a}$$

```
[Out] -1/3*c*(4*a*d+5*b*c)*(a+b/x)^(3/2)-1/5*c*(4*a*d+5*b*c)*(a+b/x)^(5/2)/a-2/7*
d^2*(a+b/x)^(7/2)/b+c^2*(a+b/x)^(7/2)*x/a+a^(3/2)*c*(4*a*d+5*b*c)*arctanh((
a+b/x)^(1/2)/a^(1/2))-a*c*(4*a*d+5*b*c)*(a+b/x)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {382, 91, 81, 52, 65, 214}

$$a^{3/2}c(4ad+5bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)+\frac{c^2x\left(a+\frac{b}{x}\right)^{7/2}}{a}-\frac{c\left(a+\frac{b}{x}\right)^{5/2}(4ad+5bc)}{5a}-\frac{1}{3}c\left(a+\frac{b}{x}\right)^{3/2}(4ad+5bc)-ac\sqrt{a+\frac{b}{x}}(4ad+5bc)-\frac{2d^2\left(a+\frac{b}{x}\right)^{7/2}}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^(5/2)*(c + d/x)^2,x]
```

```
[Out] -(a*c*(5*b*c + 4*a*d)*Sqrt[a + b/x]) - (c*(5*b*c + 4*a*d)*(a + b/x)^(3/2))/
3 - (c*(5*b*c + 4*a*d)*(a + b/x)^(5/2))/(5*a) - (2*d^2*(a + b/x)^(7/2))/(7*
b) + (c^2*(a + b/x)^(7/2)*x)/a + a^(3/2)*c*(5*b*c + 4*a*d)*ArcTanh[Sqrt[a +
 b/x]/Sqrt[a]]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^2 dx = -\text{Subst}\left(\int \frac{(a+bx)^{5/2}(c+dx)^2}{x^2}dx, x, \frac{1}{x}\right)$$

$$= \frac{c^2\left(a+\frac{b}{x}\right)^{7/2}x}{a} - \frac{\text{Subst}\left(\int \frac{(a+bx)^{5/2}\left(\frac{1}{2}c(5bc+4ad)+ad^2x\right)}{x}dx, x, \frac{1}{x}\right)}{a}$$

$$= -\frac{2d^2\left(a+\frac{b}{x}\right)^{7/2}}{7b} + \frac{c^2\left(a+\frac{b}{x}\right)^{7/2}x}{a} - \frac{(c(5bc+4ad))\text{Subst}\left(\int \frac{(a+bx)^{5/2}}{x}dx, x,\right.}{2a}$$

$$= -\frac{c(5bc+4ad)\left(a+\frac{b}{x}\right)^{5/2}}{5a} - \frac{2d^2\left(a+\frac{b}{x}\right)^{7/2}}{7b} + \frac{c^2\left(a+\frac{b}{x}\right)^{7/2}x}{a} - \frac{1}{2}(c(5bc+4$$

$$= -\frac{1}{3}c(5bc+4ad)\left(a+\frac{b}{x}\right)^{3/2} - \frac{c(5bc+4ad)\left(a+\frac{b}{x}\right)^{5/2}}{5a} - \frac{2d^2\left(a+\frac{b}{x}\right)^{7/2}}{7b} +$$

$$= -ac(5bc+4ad)\sqrt{a+\frac{b}{x}} - \frac{1}{3}c(5bc+4ad)\left(a+\frac{b}{x}\right)^{3/2} - \frac{c(5bc+4ad)\left(a+\right.}{5a}$$

$$= -ac(5bc+4ad)\sqrt{a+\frac{b}{x}} - \frac{1}{3}c(5bc+4ad)\left(a+\frac{b}{x}\right)^{3/2} - \frac{c(5bc+4ad)\left(a+\right.}{5a}$$

$$= -ac(5bc+4ad)\sqrt{a+\frac{b}{x}} - \frac{1}{3}c(5bc+4ad)\left(a+\frac{b}{x}\right)^{3/2} - \frac{c(5bc+4ad)\left(a+\right.}{5a}$$

Mathematica [A]
time = 0.27, size = 145, normalized size = 0.95

$$\frac{\sqrt{a+\frac{b}{x}}\left(-30a^3d^2x^3-2b^3(15d^2+42cdx+35c^2x^2)+a^2bx^2(-90d^2-644cdx+105c^2x^2)-2ab^2x(45d^2+154cdx+245c^2x^2)\right)}{105bx^3}+a^{3/2}c(5bc+4ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^(5/2)*(c + d/x)^2,x]
```

```
[Out] (Sqrt[a + b/x]*(-30*a^3*d^2*x^3 - 2*b^3*(15*d^2 + 42*c*d*x + 35*c^2*x^2) +
a^2*b*x^2*(-90*d^2 - 644*c*d*x + 105*c^2*x^2) - 2*a*b^2*x*(45*d^2 + 154*c*d
*x + 245*c^2*x^2)))/(105*b*x^3) + a^(3/2)*c*(5*b*c + 4*a*d)*ArcTanh[Sqrt[a
+ b/x]/Sqrt[a]]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 335 vs. $2(130) = 260$.
time = 0.06, size = 336, normalized size = 2.21

method	result
risch	$-\frac{\left(-105a^{2}b\,c^{2}x^{4}+30a^{3}d^{2}x^{3}+644a^{2}bcd\,x^{3}+490a\,b^{2}c^{2}x^{3}+90x^{2}a^{2}b\,d^{2}+308x^{2}a\,b^{2}cd+70x^{2}b^{3}c^{2}+90a\,b^{2}d^{2}x+84b^{3}cdx+30b^{3}d^{2}\right)\sqrt{\frac{ax+}{x}}}{105x^{3}b}$
default	$-\frac{\sqrt{\frac{ax+b}{x}}\,\left(-840\sqrt{a\,x^{2}+bx}\,a^{\frac{7}{2}}cd\,x^{5}-1050\sqrt{a\,x^{2}+bx}\,a^{\frac{5}{2}}b\,c^{2}x^{5}-420\ln\left(\frac{2\sqrt{a\,x^{2}+bx}\,\sqrt{a}+2ax+b}{2\sqrt{a}}\right)a^{3}bcd\,x^{5}-5\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+1/x*b)^(5/2)*(c+d/x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/210*((a*x+b)/x)^(1/2)/x^4/b*(-840*(a*x^2+b*x)^(1/2)*a^(7/2)*c*d*x^5-1050
*(a*x^2+b*x)^(1/2)*a^(5/2)*b*c^2*x^5-420*ln(1/2*(2*(a*x^2+b*x)^(1/2)*a^(1/2
)+2*a*x+b)/a^(1/2))*a^3*b*c*d*x^5-525*ln(1/2*(2*(a*x^2+b*x)^(1/2)*a^(1/2)+2
*a*x+b)/a^(1/2))*a^2*b^2*c^2*x^5+840*(a*x^2+b*x)^(3/2)*a^(5/2)*c*d*x^3+840*
(a*x^2+b*x)^(3/2)*a^(3/2)*b*c^2*x^3+60*(a*x^2+b*x)^(3/2)*a^(5/2)*d^2*x^2+44
8*(a*x^2+b*x)^(3/2)*a^(3/2)*b*c*d*x^2+140*(a*x^2+b*x)^(3/2)*a^(1/2)*b^2*c^2
*x^2+120*(a*x^2+b*x)^(3/2)*a^(3/2)*b*d^2*x+168*(a*x^2+b*x)^(3/2)*a^(1/2)*b^
2*c*d*x+60*(a*x^2+b*x)^(3/2)*a^(1/2)*b^2*d^2)/(x*(a*x+b))^(1/2)/a^(1/2)
```

Maxima [A]

time = 0.49, size = 181, normalized size = 1.19

$$-\frac{2\left(a+\frac{b}{x}\right)^{\frac{7}{2}}d^{2}}{7\,b}+\frac{1}{6}\left(6\sqrt{a+\frac{b}{x}}\,a^{2}x-15\,a^{\frac{3}{2}}b\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)-4\left(a+\frac{b}{x}\right)^{\frac{3}{2}}b-24\sqrt{a+\frac{b}{x}}\,ab\right)c^{2}-\frac{2}{15}\left(15\,a^{\frac{5}{2}}\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)+6\left(a+\frac{b}{x}\right)^{\frac{5}{2}}+10\left(a+\frac{b}{x}\right)^{\frac{3}{2}}a+30\sqrt{a+\frac{b}{x}}\,a^{2}\right)cd$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(5/2)*(c+d/x)^2,x, algorithm="maxima")
```

```
[Out] -2/7*(a + b/x)^(7/2)*d^2/b + 1/6*(6*sqrt(a + b/x)*a^2*x - 15*a^(3/2)*b*log(
(sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/x) + sqrt(a))) - 4*(a + b/x)^(3/2)*b
- 24*sqrt(a + b/x)*a*b)*c^2 - 2/15*(15*a^(5/2)*log((sqrt(a + b/x) - sqrt(a)
)/(sqrt(a + b/x) + sqrt(a))) + 6*(a + b/x)^(5/2) + 10*(a + b/x)^(3/2)*a + 3
0*sqrt(a + b/x)*a^2)*c*d
```

Fricas [A]

time = 3.26, size = 350, normalized size = 2.30

$$\left[\frac{105\,(5\,ab^{2}c^{2}+4\,a^{2}bcd)\sqrt{a}\,x^{3}\log\left(2\,ax+2\,\sqrt{a}\,x\sqrt{\frac{ax+b}{x}}+b\right)+2\,(105\,a^{2}bc^{2}x^{4}-30\,b^{3}d^{2}-2\,(245\,ab^{2}c^{2}+322\,a^{2}bcd+15\,a^{3}d^{2})x^{3}-2\,(35\,b^{3}c^{2}+154\,ab^{2}cd+45\,a^{2}bd^{2})x^{2}-6\,(14\,b^{3}cd+15\,ab^{2}d^{2})x)\sqrt{\frac{ax+b}{x}}}{210\,bx^{3}},\ -\frac{105\,(5\,ab^{2}c^{2}+4\,a^{2}bcd)\sqrt{-a}\,x^{3}\arctan\left(\frac{\sqrt{-a}\sqrt{\frac{ax+b}{x}}}{a}\right)-(105\,a^{2}bc^{2}x^{4}-30\,b^{3}d^{2}-2\,(245\,ab^{2}c^{2}+322\,a^{2}bcd+15\,a^{3}d^{2})x^{3}-2\,(35\,b^{3}c^{2}+154\,ab^{2}cd+45\,a^{2}bd^{2})x^{2}-6\,(14\,b^{3}cd+15\,ab^{2}d^{2})x)\sqrt{\frac{ax+b}{x}}}{105\,bx^{3}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(5/2)*(c+d/x)^2,x, algorithm="fricas")
```

```
[Out] [1/210*(105*(5*a*b^2*c^2 + 4*a^2*b*c*d)*sqrt(a)*x^3*log(2*a*x + 2*sqrt(a)*x
*sqrt((a*x + b)/x) + b) + 2*(105*a^2*b*c^2*x^4 - 30*b^3*d^2 - 2*(245*a*b^2*
c^2 + 322*a^2*b*c*d + 15*a^3*d^2)*x^3 - 2*(35*b^3*c^2 + 154*a*b^2*c*d + 45*
a^2*b*d^2)*x^2 - 6*(14*b^3*c*d + 15*a*b^2*d^2)*x)*sqrt((a*x + b)/x))/(b*x^3
), -1/105*(105*(5*a*b^2*c^2 + 4*a^2*b*c*d)*sqrt(-a)*x^3*arctan(sqrt(-a)*sqr
t((a*x + b)/x)/a) - (105*a^2*b*c^2*x^4 - 30*b^3*d^2 - 2*(245*a*b^2*c^2 + 32
2*a^2*b*c*d + 15*a^3*d^2)*x^3 - 2*(35*b^3*c^2 + 154*a*b^2*c*d + 45*a^2*b*d^
2)*x^2 - 6*(14*b^3*c*d + 15*a*b^2*d^2)*x)*sqrt((a*x + b)/x))/(b*x^3)]
```

Sympy [**A**]
time = 46.54, size = 1841, normalized size = 12.11

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**(5/2)*(c+d/x)**2,x)
```

```
[Out] -16*a**(19/2)*b**(13/2)*d**2*x**6*sqrt(a*x/b + 1)/(105*a**(13/2)*b**7*x**(1
3/2) + 315*a**(11/2)*b**8*x**(11/2) + 315*a**(9/2)*b**9*x**(9/2) + 105*a**(
7/2)*b**10*x**(7/2)) - 40*a**(17/2)*b**(15/2)*d**2*x**5*sqrt(a*x/b + 1)/(10
5*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(11/2) + 315*a**(9/2)*b*
*9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) - 30*a**(15/2)*b**(17/2)*d**2*x*
*4*sqrt(a*x/b + 1)/(105*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(1
1/2) + 315*a**(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) - 40*a**(1
3/2)*b**(19/2)*d**2*x**3*sqrt(a*x/b + 1)/(105*a**(13/2)*b**7*x**(13/2) + 31
5*a**(11/2)*b**8*x**(11/2) + 315*a**(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b**1
0*x**(7/2)) + 8*a**(13/2)*b**(5/2)*d**2*x**3*sqrt(a*x/b + 1)/(15*a**(7/2)*b
**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) - 100*a**(11/2)*b**(21/2)*d**2*x*
*2*sqrt(a*x/b + 1)/(105*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(1
1/2) + 315*a**(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) + 8*a**(11
/2)*b**(7/2)*c*d*x**3*sqrt(a*x/b + 1)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5
/2)*b**4*x**(5/2)) + 4*a**(11/2)*b**(7/2)*d**2*x**2*sqrt(a*x/b + 1)/(15*a**
(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) - 96*a**(9/2)*b**(23/2)*d*
*2*x*sqrt(a*x/b + 1)/(105*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**
(11/2) + 315*a**(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) + 4*a**(
9/2)*b**(9/2)*c*d*x**2*sqrt(a*x/b + 1)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(
5/2)*b**4*x**(5/2)) - 16*a**(9/2)*b**(9/2)*d**2*x*sqrt(a*x/b + 1)/(15*a**(7
/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) - 30*a**(7/2)*b**(25/2)*d**2
*sqrt(a*x/b + 1)/(105*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(11/
2) + 315*a**(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) - 16*a**(7/2
)*b**(11/2)*c*d*x*sqrt(a*x/b + 1)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*
b**4*x**(5/2)) - 12*a**(7/2)*b**(11/2)*d**2*sqrt(a*x/b + 1)/(15*a**(7/2)*b*
*3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) - 12*a**(5/2)*b**(13/2)*c*d*sqrt(a
*x/b + 1)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) + a**(3/2
```

```
)*b*c**2*asinh(sqrt(a)*sqrt(x)/sqrt(b)) + 16*a**10*b**6*d**2*x**(13/2)/(105
*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(11/2) + 315*a**(9/2)*b**
9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) + 48*a**9*b**7*d**2*x**(11/2)/(10
5*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(11/2) + 315*a**(9/2)*b*
*9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) + 48*a**8*b**8*d**2*x**(9/2)/(10
5*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(11/2) + 315*a**(9/2)*b*
*9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) + 16*a**7*b**9*d**2*x**(7/2)/(10
5*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(11/2) + 315*a**(9/2)*b*
*9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) - 8*a**7*b**2*d**2*x**(7/2)/(15*
a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) - 8*a**6*b**3*c*d*x**(7
/2)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) - 8*a**6*b**3*d
**2*x**(5/2)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) - 8*a*
*5*b**4*c*d*x**(5/2)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)
) - 4*a**3*c*d*atan(sqrt(a + b/x)/sqrt(-a))/sqrt(-a) + a**2*sqrt(b)*c**2*sq
rt(x)*sqrt(a*x/b + 1) - 4*a**2*b*c**2*atan(sqrt(a + b/x)/sqrt(-a))/sqrt(-a)
 - 4*a**2*c*d*sqrt(a + b/x) + a**2*d**2*Piecewise((-sqrt(a)/x, Eq(b, 0)), (
-2*(a + b/x)**(3/2)/(3*b), True)) - 4*a*b*c**2*sqrt(a + b/x) + 4*a*b*c*d*Pi
ecewise((-sqrt(a)/x, Eq(b, 0)), (-2*(a + b/x)**(3/2)/(3*b), True)) + b**2*c
**2*Piecewise((-sqrt(a)/x, Eq(b, 0)), (-2*(a + b/x)**(3/2)/(3*b), True))
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(5/2)*(c+d/x)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [**B**]
time = 3.79, size = 271, normalized size = 1.78

$$\left(a+\frac{b}{x}\right)^{3/2}\left(\frac{2\,a\,\left(\frac{4\,a\,d^2-4\,b\,c\,d}{b}-\frac{4\,a\,d^2}{b}\right)}{3}-\frac{2\,{\left(a\,d-b\,c\right)}^2}{3\,b}+\frac{2\,a^2\,d^2}{3\,b}\right)+\left(\frac{4\,a\,d^2-4\,b\,c\,d}{5\,b}-\frac{4\,a\,d^2}{5\,b}\right)\,\left(a+\frac{b}{x}\right)^{5/2}-\sqrt{a+\frac{b}{x}}\,\left(a^2\,\left(\frac{4\,a\,d^2-4\,b\,c\,d}{b}-\frac{4\,a\,d^2}{b}\right)-2\,a\,\left(2\,a\,\left(\frac{4\,a\,d^2-4\,b\,c\,d}{b}-\frac{4\,a\,d^2}{b}\right)-\frac{2\,{\left(a\,d-b\,c\right)}^2}{b}+\frac{2\,a^2\,d^2}{b}\right)\right)-\frac{2\,d^2\,{\left(a+\frac{b}{x}\right)}^{7/2}}{7\,b}+a^2\,c^2\,x\,\sqrt{a+\frac{b}{x}}-a^{3/2}\,c\,\mathrm{atan}\left(\frac{\sqrt{a+\frac{b}{x}}\,1{}\mathrm{i}}{\sqrt{a}}\right)\,\left(4\,a\,d+5\,b\,c\right)\,1{}\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x)^(5/2)*(c + d/x)^2,x)
```

```
[Out] (a + b/x)^(3/2)*((2*a*((4*a*d^2 - 4*b*c*d)/b - (4*a*d^2)/b))/3 - (2*(a*d -
b*c)^2)/(3*b) + (2*a^2*d^2)/(3*b)) + ((4*a*d^2 - 4*b*c*d)/(5*b) - (4*a*d^2)
/(5*b))*(a + b/x)^(5/2) - (a + b/x)^(1/2)*(a^2*((4*a*d^2 - 4*b*c*d)/b - (4*
a*d^2)/b) - 2*a*(2*a*((4*a*d^2 - 4*b*c*d)/b - (4*a*d^2)/b) - (2*(a*d - b*c)
^2)/b + (2*a^2*d^2)/b)) - (2*d^2*(a + b/x)^(7/2))/(7*b) + a^2*c^2*x*(a + b/
x)^(1/2) - a^(3/2)*c*atan(((a + b/x)^(1/2)*1i)/a^(1/2))*(4*a*d + 5*b*c)*1i
```

3.240 $\int \left(a+\frac{b}{x}\right)^{5/2} \left(c+\frac{d}{x}\right) dx$

Optimal. Leaf size=125

$$-a(5bc+2ad)\sqrt{a+\frac{b}{x}}-\frac{1}{3}(5bc+2ad)\left(a+\frac{b}{x}\right)^{3/2}-\frac{(5bc+2ad)\left(a+\frac{b}{x}\right)^{5/2}}{5a}+\frac{c\left(a+\frac{b}{x}\right)^{7/2}x}{a}+a^{3/2}(5bc+2ad)\,\mathrm{t}$$

```
[Out] -1/3*(2*a*d+5*b*c)*(a+b/x)^(3/2)-1/5*(2*a*d+5*b*c)*(a+b/x)^(5/2)/a+c*(a+b/x
)^(7/2)*x/a+a^(3/2)*(2*a*d+5*b*c)*arctanh((a+b/x)^(1/2)/a^(1/2))-a*(2*a*d+5
*b*c)*(a+b/x)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {382, 79, 52, 65, 214}

$$a^{3/2}(2ad+5bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)-\frac{\left(a+\frac{b}{x}\right)^{5/2}(2ad+5bc)}{5a}-\frac{1}{3}\left(a+\frac{b}{x}\right)^{3/2}(2ad+5bc)-a\sqrt{a+\frac{b}{x}}\,(2ad+5bc)+\frac{cx\left(a+\frac{b}{x}\right)^{7/2}}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^(5/2)*(c + d/x),x]
```

```
[Out] -(a*(5*b*c + 2*a*d)*Sqrt[a + b/x]) - ((5*b*c + 2*a*d)*(a + b/x)^(3/2))/3 -
((5*b*c + 2*a*d)*(a + b/x)^(5/2))/(5*a) + (c*(a + b/x)^(7/2)*x)/a + a^(3/2)
*(5*b*c + 2*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int \left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)dx &= -\text{Subst}\left(\int \frac{(a+bx)^{5/2}(c+dx)}{x^2}\,dx, x, \frac{1}{x}\right)\\
&= \frac{c\left(a+\frac{b}{x}\right)^{7/2}x}{a}-\frac{\left(\frac{5bc}{2}+ad\right)\text{Subst}\left(\int \frac{(a+bx)^{5/2}}{x}\,dx, x, \frac{1}{x}\right)}{a}\\
&= -\frac{(5bc+2ad)\left(a+\frac{b}{x}\right)^{5/2}}{5a}+\frac{c\left(a+\frac{b}{x}\right)^{7/2}x}{a}-\frac{1}{2}(5bc+2ad)\text{Subst}\left(\int \frac{(a+bx)^{3}}{x}\right.\\
&= -\frac{1}{3}(5bc+2ad)\left(a+\frac{b}{x}\right)^{3/2}-\frac{(5bc+2ad)\left(a+\frac{b}{x}\right)^{5/2}}{5a}+\frac{c\left(a+\frac{b}{x}\right)^{7/2}x}{a}-\frac{1}{2}(a(\\
&= -a(5bc+2ad)\sqrt{a+\frac{b}{x}}-\frac{1}{3}(5bc+2ad)\left(a+\frac{b}{x}\right)^{3/2}-\frac{(5bc+2ad)\left(a+\frac{b}{x}\right)^{5/}}{5a}\\
&= -a(5bc+2ad)\sqrt{a+\frac{b}{x}}-\frac{1}{3}(5bc+2ad)\left(a+\frac{b}{x}\right)^{3/2}-\frac{(5bc+2ad)\left(a+\frac{b}{x}\right)^{5/}}{5a}\\
&= -a(5bc+2ad)\sqrt{a+\frac{b}{x}}-\frac{1}{3}(5bc+2ad)\left(a+\frac{b}{x}\right)^{3/2}-\frac{(5bc+2ad)\left(a+\frac{b}{x}\right)^{5/}}{5a}
\end{aligned}
$$

Mathematica [A]
time = 0.20, size = 94, normalized size = 0.75

$$\frac{\sqrt{a+\frac{b}{x}}\left(-2b^2(3d+5cx)+a^2x^2(-46d+15cx)-2abx(11d+35cx)\right)}{15x^2}+a^{3/2}(5bc+2ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^(5/2)*(c + d/x),x]
```

```
[Out] (Sqrt[a + b/x]*(-2*b^2*(3*d + 5*c*x) + a^2*x^2*(-46*d + 15*c*x) - 2*a*b*x*(
11*d + 35*c*x)))/(15*x^2) + a^(3/2)*(5*b*c + 2*a*d)*ArcTanh[Sqrt[a + b/x]/S
qrt[a]]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 252 vs. $2(107) = 214$.
time = 0.05, size = 253, normalized size = 2.02

method	result
risch	$\frac{\left(15a^2cx^3-46a^2dx^2-70abcx^2-22abdx-10b^2cx-6b^2d\right)\sqrt{\frac{ax+b}{x}}}{15x^2}+\frac{\left(a^{\frac{5}{2}}\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{ax^2+bx}\right)d+\frac{5a^{\frac{3}{2}}\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x\right.}\right.}{2}}{ax+b}$
default	$-\frac{\sqrt{\frac{ax+b}{x}}\left(-60\sqrt{ax^2+bx}\,a^{\frac{7}{2}}dx^4-150\sqrt{ax^2+bx}\,a^{\frac{5}{2}}bcx^4-30\ln\left(\frac{2\sqrt{ax^2+bx}\sqrt{a}+2ax+b}{2\sqrt{a}}\right)a^3bdx^4-75\ln\left(\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+1/x*b)^(5/2)*(c+d/x),x,method=_RETURNVERBOSE)
```

```
[Out] -1/30*((a*x+b)/x)^(1/2)/x^3/b*(-60*(a*x^2+b*x)^(1/2)*a^(7/2)*d*x^4-150*(a*x
^2+b*x)^(1/2)*a^(5/2)*b*c*x^4-30*ln(1/2*(2*(a*x^2+b*x)^(1/2)*a^(1/2)+2*a*x+
b)/a^(1/2))*a^3*b*d*x^4-75*ln(1/2*(2*(a*x^2+b*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(
1/2))*a^2*b^2*c*x^4+60*(a*x^2+b*x)^(3/2)*a^(5/2)*d*x^2+120*(a*x^2+b*x)^(3/2
)*a^(3/2)*b*c*x^2+32*(a*x^2+b*x)^(3/2)*a^(3/2)*b*d*x+20*(a*x^2+b*x)^(3/2)*a
^(1/2)*b^2*c*x+12*(a*x^2+b*x)^(3/2)*a^(1/2)*b^2*d)/(x*(a*x+b))^(1/2)/a^(1/2
)
```

Maxima [A]
time = 0.49, size = 161, normalized size = 1.29

$$\frac{1}{6}\left(6\sqrt{a+\frac{b}{x}}\,a^2x-15a^{\frac{3}{2}}b\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)-4\left(a+\frac{b}{x}\right)^{\frac{3}{2}}b-24\sqrt{a+\frac{b}{x}}\,ab\right)c-\frac{1}{15}\left(15a^{\frac{5}{2}}\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)+6\left(a+\frac{b}{x}\right)^{\frac{5}{2}}+10\left(a+\frac{b}{x}\right)^{\frac{3}{2}}a+30\sqrt{a+\frac{b}{x}}\,a^2\right)d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(5/2)*(c+d/x),x, algorithm="maxima")
```

```
[Out] 1/6*(6*sqrt(a + b/x)*a^2*x - 15*a^(3/2)*b*log((sqrt(a + b/x) - sqrt(a))/(sq
rt(a + b/x) + sqrt(a))) - 4*(a + b/x)^(3/2)*b - 24*sqrt(a + b/x)*a*b)*c - 1
/15*(15*a^(5/2)*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/x) + sqrt(a))) +
6*(a + b/x)^(5/2) + 10*(a + b/x)^(3/2)*a + 30*sqrt(a + b/x)*a^2)*d
```

Fricas [A]
time = 3.54, size = 222, normalized size = 1.78

$$\left[\frac{15\,(5\,abc + 2\,a^2d)\sqrt{a}\,x^2\log\left(2\,ax + 2\,\sqrt{a}\,x\sqrt{\frac{ax+b}{x}} + b\right) + 2\,(15\,a^2cx^3 - 6\,b^2d - 2\,(35\,abc + 23\,a^2d)x^2 - 2\,(5\,b^2c + 11\,abd)x)\sqrt{\frac{ax+b}{x}}}{30\,x^2}, -\frac{15\,(5\,abc + 2\,a^2d)\sqrt{-a}\,x^2\arctan\left(\frac{\sqrt{-a}\,\sqrt{\frac{ax+b}{x}}}{a}\right) - (15\,a^2cx^3 - 6\,b^2d - 2\,(35\,abc + 23\,a^2d)x^2 - 2\,(5\,b^2c + 11\,abd)x)\sqrt{\frac{ax+b}{x}}}{15\,x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(5/2)*(c+d/x),x, algorithm="fricas")
```

```
[Out] [1/30*(15*(5*a*b*c + 2*a^2*d)*sqrt(a)*x^2*log(2*a*x + 2*sqrt(a)*x*sqrt((a*x
 + b)/x) + b) + 2*(15*a^2*c*x^3 - 6*b^2*d - 2*(35*a*b*c + 23*a^2*d)*x^2 - 2
*(5*b^2*c + 11*a*b*d)*x)*sqrt((a*x + b)/x))/x^2, -1/15*(15*(5*a*b*c + 2*a^2
*d)*sqrt(-a)*x^2*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) - (15*a^2*c*x^3 - 6*b
^2*d - 2*(35*a*b*c + 23*a^2*d)*x^2 - 2*(5*b^2*c + 11*a*b*d)*x)*sqrt((a*x +
b)/x))/x^2]
```

Sympy [A]
time = 29.41, size = 520, normalized size = 4.16

$$\frac{4a^{\frac{11}{2}}b^{\frac{7}{2}}dx^3\sqrt{\frac{ax}{b}+1}}{15a^{\frac{7}{2}}b^3x^{\frac{7}{2}} + 15a^{\frac{5}{2}}b^4x^{\frac{5}{2}}} + \frac{2a^{\frac{9}{2}}b^{\frac{9}{2}}dx^2\sqrt{\frac{ax}{b}+1}}{15a^{\frac{7}{2}}b^3x^{\frac{7}{2}} + 15a^{\frac{5}{2}}b^4x^{\frac{5}{2}}} - \frac{8a^{\frac{7}{2}}b^{\frac{11}{2}}dx\sqrt{\frac{ax}{b}+1}}{15a^{\frac{7}{2}}b^3x^{\frac{7}{2}} + 15a^{\frac{5}{2}}b^4x^{\frac{5}{2}}} - \frac{6a^{\frac{5}{2}}b^{\frac{13}{2}}d\sqrt{\frac{ax}{b}+1}}{15a^{\frac{7}{2}}b^3x^{\frac{7}{2}} + 15a^{\frac{5}{2}}b^4x^{\frac{5}{2}}} + a^{\frac{3}{2}}bc\operatorname{asinh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right) - \frac{4a^6b^3dx^{\frac{7}{2}}}{15a^{\frac{7}{2}}b^3x^{\frac{7}{2}} + 15a^{\frac{5}{2}}b^4x^{\frac{5}{2}}} - \frac{4a^5b^4dx^{\frac{5}{2}}}{15a^{\frac{7}{2}}b^3x^{\frac{7}{2}} + 15a^{\frac{5}{2}}b^4x^{\frac{5}{2}}} - \frac{2a^3d\operatorname{atan}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{-a}}\right)}{\sqrt{-a}} + a^2\sqrt{b}\,c\sqrt{x}\,\sqrt{\frac{ax}{b}+1} - \frac{4a^2bc\operatorname{atan}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{-a}}\right)}{\sqrt{-a}} - 2a^2d\sqrt{a+\frac{b}{x}} - 4abc\sqrt{a+\frac{b}{x}} + 2abd\left(\begin{cases} -\frac{\sqrt{a}}{x} & \text{for}\: b = 0 \\ -\frac{2\left(a+\frac{b}{x}\right)^{\frac{3}{2}}}{3b} & \text{otherwise} \end{cases}\right) + b^2c\left(\begin{cases} -\frac{\sqrt{a}}{x} & \text{for}\: b = 0 \\ -\frac{2\left(a+\frac{b}{x}\right)^{\frac{3}{2}}}{3b} & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**(5/2)*(c+d/x),x)
```

```
[Out] 4*a**(11/2)*b**(7/2)*d*x**3*sqrt(a*x/b + 1)/(15*a**(7/2)*b**3*x**(7/2) + 15
*a**(5/2)*b**4*x**(5/2)) + 2*a**(9/2)*b**(9/2)*d*x**2*sqrt(a*x/b + 1)/(15*a
**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) - 8*a**(7/2)*b**(11/2)*d
*x*sqrt(a*x/b + 1)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2))
- 6*a**(5/2)*b**(13/2)*d*sqrt(a*x/b + 1)/(15*a**(7/2)*b**3*x**(7/2) + 15*a*
*(5/2)*b**4*x**(5/2)) + a**(3/2)*b*c*asinh(sqrt(a)*sqrt(x)/sqrt(b)) - 4*a**
6*b**3*d*x**(7/2)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) -
 4*a**5*b**4*d*x**(5/2)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5
/2)) - 2*a**3*d*atan(sqrt(a + b/x)/sqrt(-a))/sqrt(-a) + a**2*sqrt(b)*c*sqrt
(x)*sqrt(a*x/b + 1) - 4*a**2*b*c*atan(sqrt(a + b/x)/sqrt(-a))/sqrt(-a) - 2*
a**2*d*sqrt(a + b/x) - 4*a*b*c*sqrt(a + b/x) + 2*a*b*d*Piecewise((-sqrt(a)/
x, Eq(b, 0)), (-2*(a + b/x)**(3/2)/(3*b), True)) + b**2*c*Piecewise((-sqrt(
a)/x, Eq(b, 0)), (-2*(a + b/x)**(3/2)/(3*b), True))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(5/2)*(c+d/x),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [B]
time = 3.48, size = 99, normalized size = 0.79

$$-\frac{2\,d\,{\left(a+\frac{b}{x}\right)}^{5/2}}{5}-2\,a^2\,d\,\sqrt{a+\frac{b}{x}}-\frac{2\,a\,d\,{\left(a+\frac{b}{x}\right)}^{3/2}}{3}-\frac{2\,c\,x\,{\left(a+\frac{b}{x}\right)}^{5/2}\,{}_2\mathrm{F}_1\left(-\frac{5}{2},-\frac{3}{2};\ -\frac{1}{2};\ -\frac{a\,x}{b}\right)}{3\,{\left(\frac{a\,x}{b}+1\right)}^{5/2}}-a^{5/2}\,d\,\mathrm{atan}\left(\frac{\sqrt{a+\frac{b}{x}}\,1{}\mathrm{i}}{\sqrt{a}}\right)\,2{}\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x)^(5/2)*(c + d/x),x)
```

```
[Out] - (2*d*(a + b/x)^(5/2))/5 - 2*a^2*d*(a + b/x)^(1/2) - a^(5/2)*d*atan(((a +
b/x)^(1/2)*1i)/a^(1/2))*2i - (2*a*d*(a + b/x)^(3/2))/3 - (2*c*x*(a + b/x)^(
5/2)*hypergeom([-5/2, -3/2], -1/2, -(a*x)/b))/(3*((a*x)/b + 1)^(5/2))
```

3.241 $\int \left(a+\frac{b}{x}\right)^{5/2} dx$

Optimal. Leaf size=71

$$-5ab\sqrt{a+\frac{b}{x}}-\frac{5}{3}b\left(a+\frac{b}{x}\right)^{3/2}+\left(a+\frac{b}{x}\right)^{5/2}x+5a^{3/2}b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)$$

```
[Out] -5/3*b*(a+b/x)^(3/2)+(a+b/x)^(5/2)*x+5*a^(3/2)*b*arctanh((a+b/x)^(1/2)/a^(1
/2))-5*a*b*(a+b/x)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.454$, Rules used = {248, 43, 52, 65, 214}

$$5a^{3/2}b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)+x\left(a+\frac{b}{x}\right)^{5/2}-\frac{5}{3}b\left(a+\frac{b}{x}\right)^{3/2}-5ab\sqrt{a+\frac{b}{x}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^(5/2),x]
```

```
[Out] -5*a*b*Sqrt[a + b/x] - (5*b*(a + b/x)^(3/2))/3 + (a + b/x)^(5/2)*x + 5*a^(3
/2)*b*ArcTanh[Sqrt[a + b/x]/Sqrt[a]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 248

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int \left(a+\frac{b}{x}\right)^{5/2} dx &= -\text{Subst}\left(\int \frac{(a+bx)^{5/2}}{x^2}\,dx, x, \frac{1}{x}\right)\\
&= \left(a+\frac{b}{x}\right)^{5/2} x - \frac{1}{2}(5b)\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x}\,dx, x, \frac{1}{x}\right)\\
&= -\frac{5}{3}b\left(a+\frac{b}{x}\right)^{3/2} + \left(a+\frac{b}{x}\right)^{5/2} x - \frac{1}{2}(5ab)\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx, x, \frac{1}{x}\right)\\
&= -5ab\sqrt{a+\frac{b}{x}} - \frac{5}{3}b\left(a+\frac{b}{x}\right)^{3/2} + \left(a+\frac{b}{x}\right)^{5/2} x - \frac{1}{2}\left(5a^2b\right)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx,\right.\\
&= -5ab\sqrt{a+\frac{b}{x}} - \frac{5}{3}b\left(a+\frac{b}{x}\right)^{3/2} + \left(a+\frac{b}{x}\right)^{5/2} x - \left(5a^2\right)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{\right.\\
&= -5ab\sqrt{a+\frac{b}{x}} - \frac{5}{3}b\left(a+\frac{b}{x}\right)^{3/2} + \left(a+\frac{b}{x}\right)^{5/2} x + 5a^{3/2}b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)
\end{aligned}
$$

Mathematica [A]
time = 0.04, size = 64, normalized size = 0.90

$$
\frac{\sqrt{a+\frac{b}{x}}\,\left(-2b^2-14abx+3a^2x^2\right)}{3x} + 5a^{3/2}b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^(5/2),x]
```

```
[Out] (Sqrt[a + b/x]*(-2*b^2 - 14*a*b*x + 3*a^2*x^2))/(3*x) + 5*a^(3/2)*b*ArcTanh
[Sqrt[a + b/x]/Sqrt[a]]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 119 vs. $2(57) = 114$.
time = 0.00, size = 120, normalized size = 1.69

method	result
risch	$\frac{(3a^2x^2-14abx-2b^2)\sqrt{\frac{ax+b}{x}}}{3x}+\frac{5a^{\frac{3}{2}}b\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^2+bx}\right)\sqrt{\frac{ax+b}{x}}\,\sqrt{x\,(ax+b)}}{2(ax+b)}$
default	$-\frac{\sqrt{\frac{ax+b}{x}}\,\left(-30\sqrt{a\,x^2+bx}\,a^{\frac{5}{2}}x^3-15\ln\left(\frac{2\sqrt{a\,x^2+bx}\,\sqrt{a}+2ax+b}{2\sqrt{a}}\right)a^2b\,x^3+24(a\,x^2+bx)^{\frac{3}{2}}a^{\frac{3}{2}}x+4(a\,x^2+bx)^{\frac{3}{2}}b\sqrt{a}\right.}{6x^2\sqrt{x\,(ax+b)}\,\sqrt{a}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+1/x*b)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*((a*x+b)/x)^(1/2)/x^2*(-30*(a*x^2+b*x)^(1/2)*a^(5/2)*x^3-15*ln(1/2*(2*
(a*x^2+b*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^2*b*x^3+24*(a*x^2+b*x)^(3/2)*
a^(3/2)*x+4*(a*x^2+b*x)^(3/2)*b*a^(1/2))/(x*(a*x+b))^(1/2)/a^(1/2)
```

Maxima [A]
time = 0.49, size = 78, normalized size = 1.10

$$\sqrt{a+\frac{b}{x}}\,a^2x-\frac{5}{2}\,a^{\frac{3}{2}}b\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)-\frac{2}{3}\left(a+\frac{b}{x}\right)^{\frac{3}{2}}b-4\sqrt{a+\frac{b}{x}}\,ab$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(5/2),x, algorithm="maxima")
```

```
[Out] sqrt(a + b/x)*a^2*x - 5/2*a^(3/2)*b*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a +
 b/x) + sqrt(a))) - 2/3*(a + b/x)^(3/2)*b - 4*sqrt(a + b/x)*a*b
```

Fricas [A]
time = 3.22, size = 139, normalized size = 1.96

$$\left[\frac{15\,a^{\frac{3}{2}}bx\log\left(2\,ax+2\,\sqrt{a}\,x\sqrt{\frac{ax+b}{x}}+b\right)+2\,(3\,a^2x^2-14\,abx-2\,b^2)\sqrt{\frac{ax+b}{x}}}{6\,x},-\frac{15\,\sqrt{-a}\,abx\arctan\left(\frac{\sqrt{-a}\,\sqrt{\frac{ax+b}{x}}}{a}\right)-(3\,a^2x^2-14\,abx-2\,b^2)\sqrt{\frac{ax+b}{x}}}{3\,x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/6*(15*a^(3/2)*b*x*log(2*a*x + 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) + 2*(3*
a^2*x^2 - 14*a*b*x - 2*b^2)*sqrt((a*x + b)/x))/x, -1/3*(15*sqrt(-a)*a*b*x*a
rctan(sqrt(-a)*sqrt((a*x + b)/x)/a) - (3*a^2*x^2 - 14*a*b*x - 2*b^2)*sqrt((
a*x + b)/x))/x]
```

Sympy [A]
time = 2.14, size = 99, normalized size = 1.39

$$a^{\frac{5}{2}}x\sqrt{1+\frac{b}{ax}}-\frac{14a^{\frac{3}{2}}b\sqrt{1+\frac{b}{ax}}}{3}-\frac{5a^{\frac{3}{2}}b\log\left(\frac{b}{ax}\right)}{2}+5a^{\frac{3}{2}}b\log\left(\sqrt{1+\frac{b}{ax}}+1\right)-\frac{2\sqrt{a}\,b^2\sqrt{1+\frac{b}{ax}}}{3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**(5/2),x)
```

```
[Out] a**(5/2)*x*sqrt(1 + b/(a*x)) - 14*a**(3/2)*b*sqrt(1 + b/(a*x))/3 - 5*a**(3/
2)*b*log(b/(a*x))/2 + 5*a**(3/2)*b*log(sqrt(1 + b/(a*x)) + 1) - 2*sqrt(a)*b
**2*sqrt(1 + b/(a*x))/(3*x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [B]
time = 1.63, size = 34, normalized size = 0.48

$$-\frac{2\,x\,{\left(a+\frac{b}{x}\right)}^{5/2}\,{}_2\mathrm{F}_1\left(-\frac{5}{2},-\frac{3}{2};\ -\frac{1}{2};\ -\frac{a\,x}{b}\right)}{3\,{\left(\frac{a\,x}{b}+1\right)}^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x)^(5/2),x)
```

```
[Out] -(2*x*(a + b/x)^(5/2)*hypergeom([-5/2, -3/2], -1/2, -(a*x)/b))/(3*((a*x)/b
+ 1)^(5/2))
```

3.242 $\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{c+\frac{d}{x}} \, dx$

Optimal. Leaf size=134

$$-\frac{b(2bc+ad)\sqrt{a+\frac{b}{x}}}{cd}+\frac{a\left(a+\frac{b}{x}\right)^{3/2}x}{c}+\frac{2(bc-ad)^{5/2}\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^2d^{3/2}}+\frac{a^{3/2}(5bc-2ad)\tanh^{-1}\left(\sqrt{\ldots}\right.}{c^2}$$

```
[Out] a*(a+b/x)^(3/2)*x/c+2*(-a*d+b*c)^(5/2)*arctan(d^(1/2)*(a+b/x)^(1/2)/(-a*d+b
*c)^(1/2))/c^2/d^(3/2)+a^(3/2)*(-2*a*d+5*b*c)*arctanh((a+b/x)^(1/2)/a^(1/2)
)/c^2-b*(a*d+2*b*c)*(a+b/x)^(1/2)/c/d
```

Rubi [A]

time = 0.14, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {382, 100, 159, 162, 65, 214, 211}

$$\frac{a^{3/2}(5bc-2ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{c^2}+\frac{2(bc-ad)^{5/2}\text{ArcTan}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^2d^{3/2}}-\frac{b\sqrt{a+\frac{b}{x}}(ad+2bc)}{cd}+\frac{ax\left(a+\frac{b}{x}\right)^{3/2}}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^(5/2)/(c + d/x),x]
```

```
[Out] -((b*(2*b*c + a*d)*Sqrt[a + b/x])/(c*d)) + (a*(a + b/x)^(3/2)*x)/c + (2*(b*
c - a*d)^(5/2)*ArcTan[(Sqrt[d]*Sqrt[a + b/x])/Sqrt[b*c - a*d]])/(c^2*d^(3/2
)) + (a^(3/2)*(5*b*c - 2*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/c^2
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
```

```
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 159

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{c+\frac{d}{x}}\,dx = -\text{Subst}\left(\int \frac{(a+bx)^{5/2}}{x^2(c+dx)}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{a\left(a+\frac{b}{x}\right)^{3/2}x}{c} + \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx}\left(-\frac{1}{2}a(5bc-2ad)-\frac{1}{2}b(2bc+ad)x\right)}{x(c+dx)}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= -\frac{b(2bc+ad)\sqrt{a+\frac{b}{x}}}{cd} + \frac{a\left(a+\frac{b}{x}\right)^{3/2}x}{c} + \frac{2\text{Subst}\left(\int \frac{-\frac{1}{4}a^2d(5bc-2ad)+\frac{1}{4}b(2b^2c^2-6abcd+a^2d^2)x}{x\sqrt{a+bx}\,(c+dx)}\,d\right.}{cd}$$

$$= -\frac{b(2bc+ad)\sqrt{a+\frac{b}{x}}}{cd} + \frac{a\left(a+\frac{b}{x}\right)^{3/2}x}{c} - \frac{(a^2(5bc-2ad))\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)}{2c^2}$$

$$= -\frac{b(2bc+ad)\sqrt{a+\frac{b}{x}}}{cd} + \frac{a\left(a+\frac{b}{x}\right)^{3/2}x}{c} - \frac{(a^2(5bc-2ad))\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\right.}{bc^2}$$

$$= -\frac{b(2bc+ad)\sqrt{a+\frac{b}{x}}}{cd} + \frac{a\left(a+\frac{b}{x}\right)^{3/2}x}{c} + \frac{2(bc-ad)^{5/2}\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^2d^{3/2}} + \frac{a^{3/2}(}{}$$

Mathematica [A]
time = 0.25, size = 117, normalized size = 0.87

$$\frac{\frac{c\sqrt{a+\frac{b}{x}}\,(-2b^2c+a^2dx)}{d} + \frac{2(bc-ad)^{5/2}\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{d^{3/2}} - a^{3/2}(-5bc+2ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^(5/2)/(c + d/x),x]
```

```
[Out] ((c*Sqrt[a + b/x]*(-2*b^2*c + a^2*d*x))/d + (2*(b*c - a*d)^(5/2)*ArcTan[(Sq
rt[d]*Sqrt[a + b/x])/Sqrt[b*c - a*d]])/d^(3/2) - a^(3/2)*(-5*b*c + 2*a*d)*A
rcTanh[Sqrt[a + b/x]/Sqrt[a]])/c^2
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 858 vs. $2(114) = 228$.
time = 0.08, size = 859, normalized size = 6.41

method	result
risch	$\frac{(a^2dx-2b^2c)\sqrt{\frac{ax+b}{x}}}{dc}+\frac{\left(-\frac{d\,a^{\frac{5}{2}}\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^2+bx}\right)}{c^2}+\frac{5a^{\frac{3}{2}}\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^2+bx}\right)b}{2c}-\frac{d^2\ln\left(\frac{\frac{2d(ad-bc)}{c^2}-\frac{(2ad-bc}{}}{}\right.}{}\right.}{}$
default	$\frac{\sqrt{\frac{ax+b}{x}}\left(2\sqrt{x\,(ax+b)}\,\sqrt{\frac{d(ad-bc)}{c^2}}\,a^{\frac{5}{2}}c^2d^2x^2-4\sqrt{x\,(ax+b)}\,\sqrt{\frac{d(ad-bc)}{c^2}}\,a^{\frac{3}{2}}bc^3dx^2+2\sqrt{x\,(ax+b)}\,\sqrt{\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+1/x*b)^(5/2)/(c+d/x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*((a*x+b)/x)^(1/2)/x*(2*(x*(a*x+b))^(1/2)*(d*(a*d-b*c)/c^2)^(1/2)*a^(5/2
)*c^2*d^2*x^2-4*(x*(a*x+b))^(1/2)*(d*(a*d-b*c)/c^2)^(1/2)*a^(3/2)*b*c^3*d*x
^2+2*(x*(a*x+b))^(1/2)*(d*(a*d-b*c)/c^2)^(1/2)*a^(1/2)*b^2*c^4*x^2+8*(d*(a*
d-b*c)/c^2)^(1/2)*(a*x^2+b*x)^(1/2)*a^(3/2)*b*c^3*d*x^2-2*(d*(a*d-b*c)/c^2)
^(1/2)*(a*x^2+b*x)^(1/2)*a^(1/2)*b^2*c^4*x^2+4*(d*(a*d-b*c)/c^2)^(1/2)*ln(1
/2*(2*(a*x^2+b*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a*b^2*c^3*d*x^2-(d*(a*d-b
*c)/c^2)^(1/2)*ln(1/2*(2*(a*x^2+b*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*b^3*c^
4*x^2-2*(d*(a*d-b*c)/c^2)^(1/2)*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b
)/a^(1/2))*a^3*c*d^3*x^2+5*(d*(a*d-b*c)/c^2)^(1/2)*ln(1/2*(2*(x*(a*x+b))^(1
/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^2*b*c^2*d^2*x^2-4*(d*(a*d-b*c)/c^2)^(1/2)*l
n(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a*b^2*c^3*d*x^2+(d*(a*
d-b*c)/c^2)^(1/2)*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*b^3
*c^4*x^2-2*ln((2*(x*(a*x+b))^(1/2)*(d*(a*d-b*c)/c^2)^(1/2)*c-2*a*d*x+b*c*x-
b*d)/(c*x+d))*a^(7/2)*d^4*x^2+6*ln((2*(x*(a*x+b))^(1/2)*(d*(a*d-b*c)/c^2)^(
1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*a^(5/2)*b*c*d^3*x^2-6*ln((2*(x*(a*x+b))^
(1/2)*(d*(a*d-b*c)/c^2)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*a^(3/2)*b^2*c^2
*d^2*x^2+2*ln((2*(x*(a*x+b))^(1/2)*(d*(a*d-b*c)/c^2)^(1/2)*c-2*a*d*x+b*c*x-
b*d)/(c*x+d))*a^(1/2)*b^3*c^3*d*x^2-4*(d*(a*d-b*c)/c^2)^(1/2)*(a*x^2+b*x)^(
3/2)*a^(1/2)*b*c^3*d)/(x*(a*x+b))^(1/2)/d^2/c^3/a^(1/2)/(d*(a*d-b*c)/c^2)^(
1/2)
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(5/2)/(c+d/x),x, algorithm="maxima")
```

```
[Out] integrate((a + b/x)^(5/2)/(c + d/x), x)
```

Fricas [A]
time = 3.96, size = 659, normalized size = 4.92

$$\left[-\frac{(5\,abcd-2\,a^2d^2)\sqrt{a}\log\left(2\,ax-2\,\sqrt{a}x\sqrt{\frac{ax+b}{x}}+b\right)-2\,(b^2c^2-2\,abcd+a^2d^2)\sqrt{-\frac{bc-ad}{d}}\log\left(\frac{2\,dx\sqrt{-\frac{bc-ad}{d}}\sqrt{\frac{ax+b}{x}}+bd-(bc-2\,ad)x}{cx+d}\right)-2\,(a^2cdx-2\,b^2c^2)\sqrt{\frac{ax+b}{x}}}{2\,c^2d},\ -\frac{(5\,abcd-2\,a^2d^2)\sqrt{-a}\arctan\left(\frac{\sqrt{-a}\sqrt{\frac{ax+b}{x}}}{a}\right)-(b^2c^2-2\,abcd+a^2d^2)\sqrt{-\frac{bc-ad}{d}}\log\left(\frac{2\,dx\sqrt{-\frac{bc-ad}{d}}\sqrt{\frac{ax+b}{x}}+bd-(bc-2\,ad)x}{cx+d}\right)-(a^2cdx-2\,b^2c^2)\sqrt{\frac{ax+b}{x}}}{c^2d},\ -\frac{4\,(b^2c^2-2\,abcd+a^2d^2)\sqrt{\frac{bc-ad}{d}}\arctan\left(-\frac{d\sqrt{\frac{bc-ad}{d}}\sqrt{\frac{ax+b}{x}}}{bc-ad}\right)+(5\,abcd-2\,a^2d^2)\sqrt{a}\log\left(2\,ax-2\,\sqrt{a}x\sqrt{\frac{ax+b}{x}}+b\right)-2\,(a^2cdx-2\,b^2c^2)\sqrt{\frac{ax+b}{x}}}{2\,c^2d},\ -\frac{2\,(b^2c^2-2\,abcd+a^2d^2)\sqrt{\frac{bc-ad}{d}}\arctan\left(-\frac{d\sqrt{\frac{bc-ad}{d}}\sqrt{\frac{ax+b}{x}}}{bc-ad}\right)+(5\,abcd-2\,a^2d^2)\sqrt{-a}\arctan\left(\frac{\sqrt{-a}\sqrt{\frac{ax+b}{x}}}{a}\right)-(a^2cdx-2\,b^2c^2)\sqrt{\frac{ax+b}{x}}}{c^2d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(5/2)/(c+d/x),x, algorithm="fricas")
```

```
[Out] [-1/2*((5*a*b*c*d - 2*a^2*d^2)*sqrt(a)*log(2*a*x - 2*sqrt(a)*x*sqrt((a*x +
b)/x) + b) - 2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(-(b*c - a*d)/d)*log((2*
d*x*sqrt(-(b*c - a*d)/d)*sqrt((a*x + b)/x) + b*d - (b*c - 2*a*d)*x)/(c*x +
d)) - 2*(a^2*c*d*x - 2*b^2*c^2)*sqrt((a*x + b)/x))/(c^2*d), -((5*a*b*c*d -
2*a^2*d^2)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) - (b^2*c^2 - 2*a*b
*c*d + a^2*d^2)*sqrt(-(b*c - a*d)/d)*log((2*d*x*sqrt(-(b*c - a*d)/d)*sqrt((
a*x + b)/x) + b*d - (b*c - 2*a*d)*x)/(c*x + d)) - (a^2*c*d*x - 2*b^2*c^2)*s
qrt((a*x + b)/x))/(c^2*d), -1/2*(4*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt((b*
c - a*d)/d)*arctan(-d*sqrt((b*c - a*d)/d)*sqrt((a*x + b)/x)/(b*c - a*d)) +
(5*a*b*c*d - 2*a^2*d^2)*sqrt(a)*log(2*a*x - 2*sqrt(a)*x*sqrt((a*x + b)/x) +
 b) - 2*(a^2*c*d*x - 2*b^2*c^2)*sqrt((a*x + b)/x))/(c^2*d), -(2*(b^2*c^2 -
2*a*b*c*d + a^2*d^2)*sqrt((b*c - a*d)/d)*arctan(-d*sqrt((b*c - a*d)/d)*sqrt
((a*x + b)/x)/(b*c - a*d)) + (5*a*b*c*d - 2*a^2*d^2)*sqrt(-a)*arctan(sqrt(-
a)*sqrt((a*x + b)/x)/a) - (a^2*c*d*x - 2*b^2*c^2)*sqrt((a*x + b)/x))/(c^2*d
)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\left(a+\frac{b}{x}\right)^{\frac{5}{2}}}{cx+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**(5/2)/(c+d/x),x)
```

```
[Out] Integral(x*(a + b/x)**(5/2)/(c*x + d), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(5/2)/(c+d/x),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [B]
time = 2.16, size = 1427, normalized size = 10.65

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x)^(5/2)/(c + d/x),x)
```

```
[Out] (atan((a^3*b^5*(a + b/x)^(1/2)*(a^5*d^8 - b^5*c^5*d^3 + 5*a*b^4*c^4*d^4 - 1
0*a^2*b^3*c^3*d^5 + 10*a^3*b^2*c^2*d^6 - 5*a^4*b*c*d^7)^(1/2)*160i)/(448*a^
3*b^8*c^3*d - 340*a^6*b^5*d^4 - 128*a^2*b^9*c^4 + 740*a^5*b^6*c*d^3 + (16*a
*b^10*c^5)/d - 796*a^4*b^7*c^2*d^2 + (60*a^7*b^4*d^5)/c) - (a^2*b^6*(a + b/
x)^(1/2)*(a^5*d^8 - b^5*c^5*d^3 + 5*a*b^4*c^4*d^4 - 10*a^2*b^3*c^3*d^5 + 10
*a^3*b^2*c^2*d^6 - 5*a^4*b*c*d^7)^(1/2)*80i)/(16*a*b^10*c^4 + 740*a^5*b^6*d
^4 - 128*a^2*b^9*c^3*d - 796*a^4*b^7*c*d^3 + 448*a^3*b^8*c^2*d^2 - (340*a^6
*b^5*d^5)/c + (60*a^7*b^4*d^6)/c^2) - (a^4*b^4*(a + b/x)^(1/2)*(a^5*d^8 - b
^5*c^5*d^3 + 5*a*b^4*c^4*d^4 - 10*a^2*b^3*c^3*d^5 + 10*a^3*b^2*c^2*d^6 - 5*
a^4*b*c*d^7)^(1/2)*60i)/(448*a^3*b^8*c^4 + 60*a^7*b^4*d^4 - 796*a^4*b^7*c^3
*d - 340*a^6*b^5*c*d^3 + (16*a*b^10*c^6)/d^2 + 740*a^5*b^6*c^2*d^2 - (128*a
^2*b^9*c^5)/d) + (a*b^7*c*(a + b/x)^(1/2)*(a^5*d^8 - b^5*c^5*d^3 + 5*a*b^4*
c^4*d^4 - 10*a^2*b^3*c^3*d^5 + 10*a^3*b^2*c^2*d^6 - 5*a^4*b*c*d^7)^(1/2)*16
i)/(740*a^5*b^6*d^5 - 796*a^4*b^7*c*d^4 - 128*a^2*b^9*c^3*d^2 + 448*a^3*b^8
*c^2*d^3 - (340*a^6*b^5*d^6)/c + (60*a^7*b^4*d^7)/c^2 + 16*a*b^10*c^4*d))*(
d^3*(a*d - b*c)^5)^(1/2)*2i)/(c^2*d^3) - (2*b^2*(a + b/x)^(1/2))/d + (atan(
(b^9*c^3*(a + b/x)^(1/2)*(a^3)^(1/2)*40i)/(40*a^2*b^9*c^3 - 790*a^5*b^6*d^3
 - 256*a^3*b^8*c^2*d + 696*a^4*b^7*c*d^2 + (370*a^6*b^5*d^4)/c - (60*a^7*b^
4*d^5)/c^2) + (a*b^8*c^2*(a + b/x)^(1/2)*(a^3)^(1/2)*256i)/(256*a^3*b^8*c^2
 + 790*a^5*b^6*d^2 - (40*a^2*b^9*c^3)/d - (370*a^6*b^5*d^3)/c + (60*a^7*b^4
*d^4)/c^2 - 696*a^4*b^7*c*d) + (a^3*b^6*d^2*(a + b/x)^(1/2)*(a^3)^(1/2)*790
i)/(256*a^3*b^8*c^2 + 790*a^5*b^6*d^2 - (40*a^2*b^9*c^3)/d - (370*a^6*b^5*d
^3)/c + (60*a^7*b^4*d^4)/c^2 - 696*a^4*b^7*c*d) - (a^4*b^5*d^3*(a + b/x)^(1
/2)*(a^3)^(1/2)*370i)/(256*a^3*b^8*c^3 - 370*a^6*b^5*d^3 - 696*a^4*b^7*c^2*
d + 790*a^5*b^6*c*d^2 - (40*a^2*b^9*c^4)/d + (60*a^7*b^4*d^4)/c) + (a^5*b^4
*d^4*(a + b/x)^(1/2)*(a^3)^(1/2)*60i)/(256*a^3*b^8*c^4 + 60*a^7*b^4*d^4 - 6
96*a^4*b^7*c^3*d - 370*a^6*b^5*c*d^3 + 790*a^5*b^6*c^2*d^2 - (40*a^2*b^9*c^
5)/d) - (a^2*b^7*c*d*(a + b/x)^(1/2)*(a^3)^(1/2)*696i)/(256*a^3*b^8*c^2 + 7
90*a^5*b^6*d^2 - (40*a^2*b^9*c^3)/d - (370*a^6*b^5*d^3)/c + (60*a^7*b^4*d^4
)/c^2 - 696*a^4*b^7*c*d))*(2*a*d - 5*b*c)*(a^3)^(1/2)*1i)/c^2 + (a^2*b*d*(a
 + b/x)^(1/2))/(c*(d*(a + b/x) - a*d))
```

3.243 $\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{\left(c+\frac{d}{x}\right)^2}\,dx$

Optimal. Leaf size=166

$$\frac{(bc-2ad)(bc-ad)\sqrt{a+\frac{b}{x}}}{c^2d\left(c+\frac{d}{x}\right)}+\frac{a\left(a+\frac{b}{x}\right)^{3/2}x}{c\left(c+\frac{d}{x}\right)}-\frac{(bc-ad)^{3/2}(bc+4ad)\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^3d^{3/2}}+\frac{a^{3/2}(5bc-4}{}$$

```
[Out] a*(a+b/x)^(3/2)*x/c/(c+d/x)-(-a*d+b*c)^(3/2)*(4*a*d+b*c)*arctan(d^(1/2)*(a+
b/x)^(1/2)/(-a*d+b*c)^(1/2))/c^3/d^(3/2)+a^(3/2)*(-4*a*d+5*b*c)*arctanh((a+
b/x)^(1/2)/a^(1/2))/c^3+(-2*a*d+b*c)*(-a*d+b*c)*(a+b/x)^(1/2)/c^2/d/(c+d/x)
```

Rubi [A]
time = 0.15, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {382, 100, 154, 162, 65, 214, 211}

$$\frac{a^{3/2}(5bc-4ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{c^3}-\frac{(bc-ad)^{3/2}(4ad+bc)\text{ArcTan}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^3d^{3/2}}+\frac{\sqrt{a+\frac{b}{x}}(bc-2ad)(bc-ad)}{c^2d\left(c+\frac{d}{x}\right)}+\frac{ax\left(a+\frac{b}{x}\right)^{3/2}}{c\left(c+\frac{d}{x}\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^(5/2)/(c + d/x)^2,x]
```

```
[Out] ((b*c - 2*a*d)*(b*c - a*d)*Sqrt[a + b/x])/(c^2*d*(c + d/x)) + (a*(a + b/x)^
(3/2)*x)/(c*(c + d/x)) - ((b*c - a*d)^(3/2)*(b*c + 4*a*d)*ArcTan[(Sqrt[d]*S
qrt[a + b/x])/Sqrt[b*c - a*d]])/(c^3*d^(3/2)) + (a^(3/2)*(5*b*c - 4*a*d)*Ar
cTanh[Sqrt[a + b/x]/Sqrt[a]])/c^3
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
```

```
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && ILtQ[m, -1] && GtQ[n, 0]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{\left(c+\frac{d}{x}\right)^2}\,dx = -\text{Subst}\left(\int \frac{(a+bx)^{5/2}}{x^2(c+dx)^2}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{a\left(a+\frac{b}{x}\right)^{3/2}x}{c\left(c+\frac{d}{x}\right)} + \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx}\,\left(-\frac{1}{2}a(5bc-4ad)-\frac{1}{2}b(2bc-ad)x\right)}{x(c+dx)^2}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= \frac{(bc-2ad)(bc-ad)\sqrt{a+\frac{b}{x}}}{c^2d\left(c+\frac{d}{x}\right)} + \frac{a\left(a+\frac{b}{x}\right)^{3/2}x}{c\left(c+\frac{d}{x}\right)} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}a^2d(5bc-4ad)+\frac{1}{2}b(b^2c^2+2abcd-2a^2d^2}{x\sqrt{a+bx}\,(c+dx)}\right.}{c^2d}$$

$$= \frac{(bc-2ad)(bc-ad)\sqrt{a+\frac{b}{x}}}{c^2d\left(c+\frac{d}{x}\right)} + \frac{a\left(a+\frac{b}{x}\right)^{3/2}x}{c\left(c+\frac{d}{x}\right)} - \frac{(a^2(5bc-4ad))\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx\right.}{2c^3}$$

$$= \frac{(bc-2ad)(bc-ad)\sqrt{a+\frac{b}{x}}}{c^2d\left(c+\frac{d}{x}\right)} + \frac{a\left(a+\frac{b}{x}\right)^{3/2}x}{c\left(c+\frac{d}{x}\right)} - \frac{(a^2(5bc-4ad))\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x,\right.}{bc^3}$$

$$= \frac{(bc-2ad)(bc-ad)\sqrt{a+\frac{b}{x}}}{c^2d\left(c+\frac{d}{x}\right)} + \frac{a\left(a+\frac{b}{x}\right)^{3/2}x}{c\left(c+\frac{d}{x}\right)} - \frac{(bc-ad)^{3/2}(bc+4ad)\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a}}{\sqrt{bc-}}\right.}{c^3d^{3/2}}$$

Mathematica [A]
time = 0.33, size = 146, normalized size = 0.88

$$\frac{\frac{c\sqrt{a+\frac{b}{x}}\,x(b^2c^2-2abcd+a^2d(2d+cx))}{d(d+cx)} - \frac{(bc-ad)^{3/2}(bc+4ad)\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{d^{3/2}} - a^{3/2}(-5bc+4ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^(5/2)/(c + d/x)^2,x]
```

```
[Out] ((c*Sqrt[a + b/x]*x*(b^2*c^2 - 2*a*b*c*d + a^2*d*(2*d + c*x)))/(d*(d + c*x)
) - ((b*c - a*d)^(3/2)*(b*c + 4*a*d)*ArcTan[(Sqrt[d]*Sqrt[a + b/x])/Sqrt[b*
c - a*d]])/d^(3/2) - a^(3/2)*(-5*b*c + 4*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]
])/c^3
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1324 vs. 2(146) = 292.
time = 0.08, size = 1325, normalized size = 7.98

method	result	size
default	Expression too large to display	1325
risch	Expression too large to display	1568

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+1/x*b)^(5/2)/(c+d/x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(-2*(x*(a*x+b))^(1/2)*a^(5/2)*(d*(a*d-b*c)/c^2)^(1/2)*b*c^4*d*x+5*ln(1/
2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^3*(d*(a*d-b*c)/c^2)^(1/2
)*b*c^3*d^2*x-2*(x*(a*x+b))^(3/2)*a^(3/2)*(d*(a*d-b*c)/c^2)^(1/2)*b*c^5-a^(
3/2)*ln((2*(x*(a*x+b))^(1/2)*(d*(a*d-b*c)/c^2)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(
c*x+d))*b^3*c^3*d^2-2*(x*(a*x+b))^(1/2)*a^(7/2)*(d*(a*d-b*c)/c^2)^(1/2)*c^4
*d*x^2+2*(x*(a*x+b))^(1/2)*a^(5/2)*(d*(a*d-b*c)/c^2)^(1/2)*b*c^5*x^2+2*(x*(
a*x+b))^(1/2)*a^(7/2)*(d*(a*d-b*c)/c^2)^(1/2)*c^3*d^2*x+a*(d*(a*d-b*c)/c^2)
^(1/2)*ln(1/2*(2*(a*x^2+b*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*b^3*c^4*d-4*a^
(9/2)*ln((2*(x*(a*x+b))^(1/2)*(d*(a*d-b*c)/c^2)^(1/2)*c-2*a*d*x+b*c*x-b*d)/
(c*x+d))*d^5-ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a*(d*(a*
d-b*c)/c^2)^(1/2)*b^3*c^5*x+2*a^(3/2)*(d*(a*d-b*c)/c^2)^(1/2)*(a*x^2+b*x)^(
1/2)*b^2*c^5*x-4*a^(9/2)*ln((2*(x*(a*x+b))^(1/2)*(d*(a*d-b*c)/c^2)^(1/2)*c-
2*a*d*x+b*c*x-b*d)/(c*x+d))*c*d^4*x-4*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2
*a*x+b)/a^(1/2))*a^4*(d*(a*d-b*c)/c^2)^(1/2)*c^2*d^3*x+7*a^(7/2)*ln((2*(x*(
a*x+b))^(1/2)*(d*(a*d-b*c)/c^2)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*b*c^2*d
^3*x-2*a^(5/2)*ln((2*(x*(a*x+b))^(1/2)*(d*(a*d-b*c)/c^2)^(1/2)*c-2*a*d*x+b*
c*x-b*d)/(c*x+d))*b^2*c^3*d^2*x-4*(x*(a*x+b))^(1/2)*a^(5/2)*(d*(a*d-b*c)/c^
2)^(1/2)*b*c^3*d^2+5*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*
a^3*(d*(a*d-b*c)/c^2)^(1/2)*b*c^2*d^3+a*(d*(a*d-b*c)/c^2)^(1/2)*ln(1/2*(2*(
a*x^2+b*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*b^3*c^5*x+4*(x*(a*x+b))^(1/2)*a^
(7/2)*(d*(a*d-b*c)/c^2)^(1/2)*c^2*d^3-4*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)
+2*a*x+b)/a^(1/2))*a^4*(d*(a*d-b*c)/c^2)^(1/2)*c*d^4-a^(3/2)*ln((2*(x*(a*x+
b))^(1/2)*(d*(a*d-b*c)/c^2)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*b^3*c^4*d*x
+2*(x*(a*x+b))^(3/2)*a^(5/2)*(d*(a*d-b*c)/c^2)^(1/2)*c^4*d-ln(1/2*(2*(x*(a*
x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a*(d*(a*d-b*c)/c^2)^(1/2)*b^3*c^4*d+2
*a^(3/2)*(d*(a*d-b*c)/c^2)^(1/2)*(a*x^2+b*x)^(1/2)*b^2*c^4*d+7*a^(7/2)*ln((
2*(x*(a*x+b))^(1/2)*(d*(a*d-b*c)/c^2)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*b
*c*d^4-2*a^(5/2)*ln((2*(x*(a*x+b))^(1/2)*(d*(a*d-b*c)/c^2)^(1/2)*c-2*a*d*x+
b*c*x-b*d)/(c*x+d))*b^2*c^2*d^3)*x*((a*x+b)/x)^(1/2)/c^4/(d*(a*d-b*c)/c^2)^
(1/2)/a^(3/2)/(c*x+d)/d^2/(x*(a*x+b))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(5/2)/(c+d/x)^2,x, algorithm="maxima")
```

```
[Out] integrate((a + b/x)^(5/2)/(c + d/x)^2, x)
```

Fricas [A]
time = 3.41, size = 1001, normalized size = 6.03

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(5/2)/(c+d/x)^2,x, algorithm="fricas")
```

```
[Out] [-1/2*((5*a*b*c*d^2 - 4*a^2*d^3 + (5*a*b*c^2*d - 4*a^2*c*d^2)*x)*sqrt(a)*lo
g(2*a*x - 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) + (b^2*c^2*d + 3*a*b*c*d^2 - 4
*a^2*d^3 + (b^2*c^3 + 3*a*b*c^2*d - 4*a^2*c*d^2)*x)*sqrt(-(b*c - a*d)/d)*lo
g((2*d*x*sqrt(-(b*c - a*d)/d)*sqrt((a*x + b)/x) + b*d - (b*c - 2*a*d)*x)/(c
*x + d)) - 2*(a^2*c^2*d*x^2 + (b^2*c^3 - 2*a*b*c^2*d + 2*a^2*c*d^2)*x)*sqrt
((a*x + b)/x))/(c^4*d*x + c^3*d^2), -1/2*(2*(5*a*b*c*d^2 - 4*a^2*d^3 + (5*a
*b*c^2*d - 4*a^2*c*d^2)*x)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) +
(b^2*c^2*d + 3*a*b*c*d^2 - 4*a^2*d^3 + (b^2*c^3 + 3*a*b*c^2*d - 4*a^2*c*d^2
)*x)*sqrt(-(b*c - a*d)/d)*log((2*d*x*sqrt(-(b*c - a*d)/d)*sqrt((a*x + b)/x)
 + b*d - (b*c - 2*a*d)*x)/(c*x + d)) - 2*(a^2*c^2*d*x^2 + (b^2*c^3 - 2*a*b*
c^2*d + 2*a^2*c*d^2)*x)*sqrt((a*x + b)/x))/(c^4*d*x + c^3*d^2), 1/2*(2*(b^2
*c^2*d + 3*a*b*c*d^2 - 4*a^2*d^3 + (b^2*c^3 + 3*a*b*c^2*d - 4*a^2*c*d^2)*x)
*sqrt((b*c - a*d)/d)*arctan(-d*sqrt((b*c - a*d)/d)*sqrt((a*x + b)/x)/(b*c -
 a*d)) - (5*a*b*c*d^2 - 4*a^2*d^3 + (5*a*b*c^2*d - 4*a^2*c*d^2)*x)*sqrt(a)*
log(2*a*x - 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) + 2*(a^2*c^2*d*x^2 + (b^2*c^
3 - 2*a*b*c^2*d + 2*a^2*c*d^2)*x)*sqrt((a*x + b)/x))/(c^4*d*x + c^3*d^2), (
(b^2*c^2*d + 3*a*b*c*d^2 - 4*a^2*d^3 + (b^2*c^3 + 3*a*b*c^2*d - 4*a^2*c*d^2
)*x)*sqrt((b*c - a*d)/d)*arctan(-d*sqrt((b*c - a*d)/d)*sqrt((a*x + b)/x)/(b
*c - a*d)) - (5*a*b*c*d^2 - 4*a^2*d^3 + (5*a*b*c^2*d - 4*a^2*c*d^2)*x)*sqrt
(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) + (a^2*c^2*d*x^2 + (b^2*c^3 - 2*a
*b*c^2*d + 2*a^2*c*d^2)*x)*sqrt((a*x + b)/x))/(c^4*d*x + c^3*d^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \left(a + \frac{b}{x}\right)^{\frac{5}{2}}}{\left(cx + d\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**(5/2)/(c+d/x)**2,x)
```

```
[Out] Integral(x**2*(a + b/x)**(5/2)/(c*x + d)**2, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(5/2)/(c+d/x)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [B]
time = 2.31, size = 1153, normalized size = 6.95

$$\frac{\frac{\sqrt{a+\frac{b}{x}}\,(2a^3bd^2-3a^2b^2cd+ab^3c^2)}{c^2d}-\frac{b\left(a+\frac{b}{x}\right)^{3/2}(2a^2d^2-2abcd+b^2c^2)}{c^2d}}{\left(a+\frac{b}{x}\right)(2ad-bc)-d\left(a+\frac{b}{x}\right)^2-a^2d+abc}-\frac{\mathrm{atanh}\left(\frac{10b^9\sqrt{a+\frac{b}{x}}\sqrt{a^3}}{10a^2b^9+\frac{32a^3b^8d}{c}-\frac{132a^4b^7d^2}{c^2}+\frac{130a^5b^6d^3}{c^3}-\frac{40a^6b^5d^4}{c^4}}+\frac{32ab^8\sqrt{a+\frac{b}{x}}\sqrt{a^3}}{32a^3b^8+\frac{10a^2b^9c}{d}-\frac{132a^4b^7d}{c}+\frac{130a^5b^6d^2}{c^2}-\frac{40a^6b^5d^3}{c^3}}-\frac{132a^2b^7d\sqrt{a+\frac{b}{x}}\sqrt{a^3}}{32a^3b^8c-132a^4b^7d+\frac{10a^2b^9c^2}{d}+\frac{130a^5b^6d^2}{c}-\frac{40a^6b^5d^3}{c^2}}+\frac{130a^3b^6d^2\sqrt{a+\frac{b}{x}}\sqrt{a^3}}{32a^3b^8c^2+130a^5b^6d^2+\frac{10a^2b^9c^3}{d}-\frac{40a^6b^5d^3}{c}-132a^4b^7cd}-\frac{40a^4b^5d^3\sqrt{a+\frac{b}{x}}\sqrt{a^3}}{32a^3b^8c^3-40a^6b^5d^3-132a^4b^7c^2d+130a^5b^6cd^2+\frac{10a^2b^9c^4}{d}}\right)(4ad-5bc)\sqrt{a^3}}{c^3}+\frac{\mathrm{atanh}\left(\frac{30a^3b^6\sqrt{a+\frac{b}{x}}\sqrt{a^3d^6-3a^2bcd^5+3ab^2c^2d^4-b^3c^3d^3}}{14a^2b^9c^3+110a^5b^6d^3-4a^3b^8c^2d-82a^4b^7cd^2+\frac{2ab^{10}c^4}{d}-\frac{40a^6b^5d^4}{c}}+\frac{18a^2b^7\sqrt{a+\frac{b}{x}}\sqrt{a^3d^6-3a^2bcd^5+3ab^2c^2d^4-b^3c^3d^3}}{2ab^{10}c^3-82a^4b^7d^3+14a^2b^9c^2d-4a^3b^8cd^2+\frac{110a^5b^6d^4}{c}-\frac{40a^6b^5d^5}{c^2}}+\frac{40a^4b^5\sqrt{a+\frac{b}{x}}\sqrt{a^3d^6-3a^2bcd^5+3ab^2c^2d^4-b^3c^3d^3}}{4a^3b^8c^3+40a^6b^5d^3+82a^4b^7c^2d-110a^5b^6cd^2-\frac{2ab^{10}c^5}{d^2}-\frac{14a^2b^9c^4}{d}}-\frac{2ab^8\sqrt{a+\frac{b}{x}}\sqrt{a^3d^6-3a^2bcd^5+3ab^2c^2d^4-b^3c^3d^3}}{4a^3b^8d^3-14a^2b^9cd^2+\frac{82a^4b^7d^4}{c}-\frac{110a^5b^6d^5}{c^2}+\frac{40a^6b^5d^6}{c^3}-2ab^{10}c^2d}\right)\sqrt{d^3(ad-bc)^3}\,(4ad+bc)}{c^3d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x)^(5/2)/(c + d/x)^2,x)
```

```
[Out] (((a + b/x)^(1/2)*(a*b^3*c^2 + 2*a^3*b*d^2 - 3*a^2*b^2*c*d))/(c^2*d) - (b*(
a + b/x)^(3/2)*(2*a^2*d^2 + b^2*c^2 - 2*a*b*c*d))/(c^2*d))/((a + b/x)*(2*a*
d - b*c) - d*(a + b/x)^2 - a^2*d + a*b*c) - (atanh((10*b^9*(a + b/x)^(1/2)*
(a^3)^(1/2))/(10*a^2*b^9 + (32*a^3*b^8*d)/c - (132*a^4*b^7*d^2)/c^2 + (130*
a^5*b^6*d^3)/c^3 - (40*a^6*b^5*d^4)/c^4) + (32*a*b^8*(a + b/x)^(1/2)*(a^3)^
(1/2))/(32*a^3*b^8 + (10*a^2*b^9*c)/d - (132*a^4*b^7*d)/c + (130*a^5*b^6*d^
2)/c^2 - (40*a^6*b^5*d^3)/c^3) - (132*a^2*b^7*d*(a + b/x)^(1/2)*(a^3)^(1/2)
)/(32*a^3*b^8*c - 132*a^4*b^7*d + (10*a^2*b^9*c^2)/d + (130*a^5*b^6*d^2)/c
- (40*a^6*b^5*d^3)/c^2) + (130*a^3*b^6*d^2*(a + b/x)^(1/2)*(a^3)^(1/2))/(32
*a^3*b^8*c^2 + 130*a^5*b^6*d^2 + (10*a^2*b^9*c^3)/d - (40*a^6*b^5*d^3)/c -
132*a^4*b^7*c*d) - (40*a^4*b^5*d^3*(a + b/x)^(1/2)*(a^3)^(1/2))/(32*a^3*b^8
*c^3 - 40*a^6*b^5*d^3 - 132*a^4*b^7*c^2*d + 130*a^5*b^6*c*d^2 + (10*a^2*b^9
*c^4)/d))*(4*a*d - 5*b*c)*(a^3)^(1/2))/c^3 + (atanh((30*a^3*b^6*(a + b/x)^(
1/2)*(a^3*d^6 - b^3*c^3*d^3 + 3*a*b^2*c^2*d^4 - 3*a^2*b*c*d^5)^(1/2))/(14*a
^2*b^9*c^3 + 110*a^5*b^6*d^3 - 4*a^3*b^8*c^2*d - 82*a^4*b^7*c*d^2 + (2*a*b^
10*c^4)/d - (40*a^6*b^5*d^4)/c) + (18*a^2*b^7*(a + b/x)^(1/2)*(a^3*d^6 - b^
3*c^3*d^3 + 3*a*b^2*c^2*d^4 - 3*a^2*b*c*d^5)^(1/2))/(2*a*b^10*c^3 - 82*a^4*
b^7*d^3 + 14*a^2*b^9*c^2*d - 4*a^3*b^8*c*d^2 + (110*a^5*b^6*d^4)/c - (40*a^
6*b^5*d^5)/c^2) + (40*a^4*b^5*(a + b/x)^(1/2)*(a^3*d^6 - b^3*c^3*d^3 + 3*a*
b^2*c^2*d^4 - 3*a^2*b*c*d^5)^(1/2))/(4*a^3*b^8*c^3 + 40*a^6*b^5*d^3 + 82*a^
4*b^7*c^2*d - 110*a^5*b^6*c*d^2 - (2*a*b^10*c^5)/d^2 - (14*a^2*b^9*c^4)/d)
- (2*a*b^8*(a + b/x)^(1/2)*(a^3*d^6 - b^3*c^3*d^3 + 3*a*b^2*c^2*d^4 - 3*a^2
*b*c*d^5)^(1/2))/(4*a^3*b^8*d^3 - 14*a^2*b^9*c*d^2 + (82*a^4*b^7*d^4)/c - (
110*a^5*b^6*d^5)/c^2 + (40*a^6*b^5*d^6)/c^3 - 2*a*b^10*c^2*d))*(d^3*(a*d -
b*c)^3)^(1/2)*(4*a*d + b*c))/(c^3*d^3)
```

3.244 $\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{\left(c+\frac{d}{x}\right)^3} \, dx$

Optimal. Leaf size=237

$$\frac{(bc-3ad)(bc-ad)\sqrt{a+\frac{b}{x}}}{2c^2d\left(c+\frac{d}{x}\right)^2}-\frac{\left(b^2c^2+7abcd-12a^2d^2\right)\sqrt{a+\frac{b}{x}}}{4c^3d\left(c+\frac{d}{x}\right)}+\frac{a\left(a+\frac{b}{x}\right)^{3/2}x}{c\left(c+\frac{d}{x}\right)^2}-\frac{\sqrt{bc-ad}\left(b^2c^2+8abcd-\right.}{}$$

```
[Out] a*(a+b/x)^(3/2)*x/c/(c+d/x)^2+a^(3/2)*(-6*a*d+5*b*c)*arctanh((a+b/x)^(1/2)/
a^(1/2))/c^4-1/4*(-24*a^2*d^2+8*a*b*c*d+b^2*c^2)*arctan(d^(1/2)*(a+b/x)^(1/
2)/(-a*d+b*c)^(1/2))*(-a*d+b*c)^(1/2)/c^4/d^(3/2)+1/2*(-3*a*d+b*c)*(-a*d+b*
c)*(a+b/x)^(1/2)/c^2/d/(c+d/x)^2-1/4*(-12*a^2*d^2+7*a*b*c*d+b^2*c^2)*(a+b/x
)^(1/2)/c^3/d/(c+d/x)
```

Rubi [A]
time = 0.24, antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {382, 100, 154, 156, 162, 65, 214, 211}

$$\frac{a^{3/2}(5bc-6ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{c^4}-\frac{\sqrt{bc-ad}\left(-24a^2d^2+8abcd+b^2c^2\right)\text{ArcTan}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{4c^4d^{3/2}}-\frac{\sqrt{a+\frac{b}{x}}\left(-12a^2d^2+7abcd+b^2c^2\right)}{4c^3d\left(c+\frac{d}{x}\right)}+\frac{\sqrt{a+\frac{b}{x}}(bc-3ad)(bc-ad)}{2c^2d\left(c+\frac{d}{x}\right)^2}+\frac{ax\left(a+\frac{b}{x}\right)^{3/2}}{c\left(c+\frac{d}{x}\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^(5/2)/(c + d/x)^3,x]
```

```
[Out] ((b*c - 3*a*d)*(b*c - a*d)*Sqrt[a + b/x])/(2*c^2*d*(c + d/x)^2) - ((b^2*c^2
 + 7*a*b*c*d - 12*a^2*d^2)*Sqrt[a + b/x])/(4*c^3*d*(c + d/x)) + (a*(a + b/x
)^(3/2)*x)/(c*(c + d/x)^2) - (Sqrt[b*c - a*d]*(b^2*c^2 + 8*a*b*c*d - 24*a^2
*d^2)*ArcTan[(Sqrt[d]*Sqrt[a + b/x])/Sqrt[b*c - a*d]])/(4*c^4*d^(3/2)) + (a
^(3/2)*(5*b*c - 6*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/c^4
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && ILtQ[m, -1] && GtQ[n, 0]
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{\left(c+\frac{d}{x}\right)^3}\,dx = -\text{Subst}\left(\int \frac{(a+bx)^{5/2}}{x^2(c+dx)^3}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{a\left(a+\frac{b}{x}\right)^{3/2}x}{c\left(c+\frac{d}{x}\right)^2} + \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx}\left(-\frac{1}{2}a(5bc-6ad)-\frac{1}{2}b(2bc-3ad)x\right)}{x(c+dx)^3}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= \frac{(bc-3ad)(bc-ad)\sqrt{a+\frac{b}{x}}}{2c^2d\left(c+\frac{d}{x}\right)^2} + \frac{a\left(a+\frac{b}{x}\right)^{3/2}x}{c\left(c+\frac{d}{x}\right)^2} - \frac{\text{Subst}\left(\int \frac{a^2d(5bc-6ad)+\frac{1}{2}b\left(b^2c^2+6abcd-9a^2d^2\right)x}{x\sqrt{a+bx}\,(c+dx)^2}\right.}{2c^2d}$$

$$= \frac{(bc-3ad)(bc-ad)\sqrt{a+\frac{b}{x}}}{2c^2d\left(c+\frac{d}{x}\right)^2} - \frac{\left(b^2c^2+7abcd-12a^2d^2\right)\sqrt{a+\frac{b}{x}}}{4c^3d\left(c+\frac{d}{x}\right)} + \frac{a\left(a+\frac{b}{x}\right)^{3/2}x}{c\left(c+\frac{d}{x}\right)^2} + \frac{\text{Su}}{}$$

$$= \frac{(bc-3ad)(bc-ad)\sqrt{a+\frac{b}{x}}}{2c^2d\left(c+\frac{d}{x}\right)^2} - \frac{\left(b^2c^2+7abcd-12a^2d^2\right)\sqrt{a+\frac{b}{x}}}{4c^3d\left(c+\frac{d}{x}\right)} + \frac{a\left(a+\frac{b}{x}\right)^{3/2}x}{c\left(c+\frac{d}{x}\right)^2} - \frac{(a}{}$$

$$= \frac{(bc-3ad)(bc-ad)\sqrt{a+\frac{b}{x}}}{2c^2d\left(c+\frac{d}{x}\right)^2} - \frac{\left(b^2c^2+7abcd-12a^2d^2\right)\sqrt{a+\frac{b}{x}}}{4c^3d\left(c+\frac{d}{x}\right)} + \frac{a\left(a+\frac{b}{x}\right)^{3/2}x}{c\left(c+\frac{d}{x}\right)^2} - \frac{(a}{}$$

$$= \frac{(bc-3ad)(bc-ad)\sqrt{a+\frac{b}{x}}}{2c^2d\left(c+\frac{d}{x}\right)^2} - \frac{\left(b^2c^2+7abcd-12a^2d^2\right)\sqrt{a+\frac{b}{x}}}{4c^3d\left(c+\frac{d}{x}\right)} + \frac{a\left(a+\frac{b}{x}\right)^{3/2}x}{c\left(c+\frac{d}{x}\right)^2} - \frac{\sqrt{}}{}$$

Mathematica [A]
time = 0.44, size = 205, normalized size = 0.86

$$\frac{\frac{c\sqrt{a+\frac{b}{x}}\,x\left(b^2c^2(-d+cx)-abcd(7d+11cx)+2a^2d\left(6d^2+9cdx+2c^2x^2\right)\right)}{d(d+cx)^2} - \frac{\left(b^3c^3+7ab^2c^2d-32a^2bcd^2+24a^3d^3\right)\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{d^{3/2}\sqrt{bc-ad}} - 4a^{3/2}(-5bc+6ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{4c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^(5/2)/(c + d/x)^3,x]
```

```
[Out] ((c*Sqrt[a + b/x]*x*(b^2*c^2*(-d + c*x) - a*b*c*d*(7*d + 11*c*x) + 2*a^2*d*
(6*d^2 + 9*c*d*x + 2*c^2*x^2)))/(d*(d + c*x)^2) - ((b^3*c^3 + 7*a*b^2*c^2*d
 - 32*a^2*b*c*d^2 + 24*a^3*d^3)*ArcTan[(Sqrt[d]*Sqrt[a + b/x])/Sqrt[b*c - a
*d]])/(d^(3/2)*Sqrt[b*c - a*d]) - 4*a^(3/2)*(-5*b*c + 6*a*d)*ArcTanh[Sqrt[a
 + b/x]/Sqrt[a]])/(4*c^4)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1637 vs. $2(209) = 418$.

time = 0.09, size = 1638, normalized size = 6.91

method	result	size
default	Expression too large to display	1638
risch	Expression too large to display	3160

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+1/x*b)^(5/2)/(c+d/x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*(48*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^4*(d*(a*d-
b*c)/c^2)^(1/2)*c^2*d^4*x-64*a^(7/2)*ln((2*(x*(a*x+b))^(1/2)*(d*(a*d-b*c)/c
^2)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*b*c^2*d^4*x+14*a^(5/2)*ln((2*(x*(a*
x+b))^(1/2)*(d*(a*d-b*c)/c^2)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*b^2*c^3*d
^3*x+14*(x*(a*x+b))^(1/2)*a^(5/2)*(d*(a*d-b*c)/c^2)^(1/2)*b*c^3*d^3-20*ln(1
/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^3*(d*(a*d-b*c)/c^2)^(1/
2)*b*c^2*d^4-20*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^3*(
d*(a*d-b*c)/c^2)^(1/2)*b*c^4*d^2*x^2+30*(x*(a*x+b))^(1/2)*a^(5/2)*(d*(a*d-b
*c)/c^2)^(1/2)*b*c^4*d^2*x-40*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/
a^(1/2))*a^3*(d*(a*d-b*c)/c^2)^(1/2)*b*c^3*d^3*x+18*(x*(a*x+b))^(1/2)*a^(5/
2)*(d*(a*d-b*c)/c^2)^(1/2)*b*c^5*d*x^2+a^(3/2)*ln((2*(x*(a*x+b))^(1/2)*(d*(
a*d-b*c)/c^2)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*b^3*c^3*d^3+12*(x*(a*x+b)
)^(1/2)*a^(7/2)*(d*(a*d-b*c)/c^2)^(1/2)*c^5*d*x^3+2*(x*(a*x+b))^(1/2)*a^(5/
2)*(d*(a*d-b*c)/c^2)^(1/2)*b*c^6*x^3+48*a^(9/2)*ln((2*(x*(a*x+b))^(1/2)*(d*
(a*d-b*c)/c^2)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*c*d^5*x+2*a^(3/2)*ln((2*
(x*(a*x+b))^(1/2)*(d*(a*d-b*c)/c^2)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*b^3
*c^4*d^2*x-24*(x*(a*x+b))^(1/2)*a^(7/2)*(d*(a*d-b*c)/c^2)^(1/2)*c^2*d^4+2*(
x*(a*x+b))^(1/2)*a^(3/2)*(d*(a*d-b*c)/c^2)^(1/2)*b^2*c^4*d^2+24*ln(1/2*(2*(
x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^4*(d*(a*d-b*c)/c^2)^(1/2)*c*d^
5-32*a^(7/2)*ln((2*(x*(a*x+b))^(1/2)*(d*(a*d-b*c)/c^2)^(1/2)*c-2*a*d*x+b*c*
x-b*d)/(c*x+d))*b*c*d^5+7*a^(5/2)*ln((2*(x*(a*x+b))^(1/2)*(d*(a*d-b*c)/c^2)
^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*b^2*c^2*d^4-2*(x*(a*x+b))^(3/2)*a^(3/2
)*(d*(a*d-b*c)/c^2)^(1/2)*b*c^6*x+2*(x*(a*x+b))^(1/2)*a^(3/2)*(d*(a*d-b*c)/
c^2)^(1/2)*b^2*c^6*x^2+24*a^(9/2)*ln((2*(x*(a*x+b))^(1/2)*(d*(a*d-b*c)/c^2)
```

```
^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*c^2*d^4*x^2+a^(3/2)*ln((2*(x*(a*x+b))^
(1/2)*(d*(a*d-b*c)/c^2)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*b^3*c^5*d*x^2-8
*(x*(a*x+b))^(3/2)*a^(5/2)*(d*(a*d-b*c)/c^2)^(1/2)*c^4*d^2-6*(x*(a*x+b))^(3
/2)*a^(3/2)*(d*(a*d-b*c)/c^2)^(1/2)*b*c^5*d-12*(x*(a*x+b))^(3/2)*a^(5/2)*(d
*(a*d-b*c)/c^2)^(1/2)*c^5*d*x+24*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+
b)/a^(1/2))*a^4*(d*(a*d-b*c)/c^2)^(1/2)*c^3*d^3*x^2-32*a^(7/2)*ln((2*(x*(a*
x+b))^(1/2)*(d*(a*d-b*c)/c^2)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*b*c^3*d^3
*x^2+7*a^(5/2)*ln((2*(x*(a*x+b))^(1/2)*(d*(a*d-b*c)/c^2)^(1/2)*c-2*a*d*x+b*
c*x-b*d)/(c*x+d))*b^2*c^4*d^2*x^2-36*(x*(a*x+b))^(1/2)*a^(7/2)*(d*(a*d-b*c)
/c^2)^(1/2)*c^3*d^3*x+4*(x*(a*x+b))^(1/2)*a^(3/2)*(d*(a*d-b*c)/c^2)^(1/2)*b
^2*c^5*d*x+24*a^(9/2)*ln((2*(x*(a*x+b))^(1/2)*(d*(a*d-b*c)/c^2)^(1/2)*c-2*a
*d*x+b*c*x-b*d)/(c*x+d))*d^6)*x*((a*x+b)/x)^(1/2)/c^5/(d*(a*d-b*c)/c^2)^(1/
2)/a^(3/2)/(c*x+d)^2/d^2/(x*(a*x+b))^(1/2)
```

<u>Maxima</u> [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(5/2)/(c+d/x)^3,x, algorithm="maxima")
```

```
[Out] integrate((a + b/x)^(5/2)/(c + d/x)^3, x)
```

<u>Fricas</u> [A]
time = 3.13, size = 1445, normalized size = 6.10

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(5/2)/(c+d/x)^3,x, algorithm="fricas")
```

```
[Out] [-1/8*(4*(5*a*b*c*d^3 - 6*a^2*d^4 + (5*a*b*c^3*d - 6*a^2*c^2*d^2)*x^2 + 2*(
5*a*b*c^2*d^2 - 6*a^2*c*d^3)*x)*sqrt(a)*log(2*a*x - 2*sqrt(a)*x*sqrt((a*x +
 b)/x) + b) + (b^2*c^2*d^2 + 8*a*b*c*d^3 - 24*a^2*d^4 + (b^2*c^4 + 8*a*b*c^
3*d - 24*a^2*c^2*d^2)*x^2 + 2*(b^2*c^3*d + 8*a*b*c^2*d^2 - 24*a^2*c*d^3)*x)
*sqrt(-(b*c - a*d)/d)*log((2*d*x*sqrt(-(b*c - a*d)/d)*sqrt((a*x + b)/x) + b
*d - (b*c - 2*a*d)*x)/(c*x + d)) - 2*(4*a^2*c^3*d*x^3 + (b^2*c^4 - 11*a*b*c
^3*d + 18*a^2*c^2*d^2)*x^2 - (b^2*c^3*d + 7*a*b*c^2*d^2 - 12*a^2*c*d^3)*x)*
sqrt((a*x + b)/x))/(c^6*d*x^2 + 2*c^5*d^2*x + c^4*d^3), 1/4*((b^2*c^2*d^2 +
 8*a*b*c*d^3 - 24*a^2*d^4 + (b^2*c^4 + 8*a*b*c^3*d - 24*a^2*c^2*d^2)*x^2 +
2*(b^2*c^3*d + 8*a*b*c^2*d^2 - 24*a^2*c*d^3)*x)*sqrt((b*c - a*d)/d)*arctan(
-d*sqrt((b*c - a*d)/d)*sqrt((a*x + b)/x)/(b*c - a*d)) - 2*(5*a*b*c*d^3 - 6*
a^2*d^4 + (5*a*b*c^3*d - 6*a^2*c^2*d^2)*x^2 + 2*(5*a*b*c^2*d^2 - 6*a^2*c*d^
3)*x)*sqrt(a)*log(2*a*x - 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) + (4*a^2*c^3*d
*x^3 + (b^2*c^4 - 11*a*b*c^3*d + 18*a^2*c^2*d^2)*x^2 - (b^2*c^3*d + 7*a*b*c
```

```
^2*d^2 - 12*a^2*c*d^3)*x)*sqrt((a*x + b)/x))/(c^6*d*x^2 + 2*c^5*d^2*x + c^4
*d^3), -1/8*(8*(5*a*b*c*d^3 - 6*a^2*d^4 + (5*a*b*c^3*d - 6*a^2*c^2*d^2)*x^2
 + 2*(5*a*b*c^2*d^2 - 6*a^2*c*d^3)*x)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x +
b)/x)/a) + (b^2*c^2*d^2 + 8*a*b*c*d^3 - 24*a^2*d^4 + (b^2*c^4 + 8*a*b*c^3*d
 - 24*a^2*c^2*d^2)*x^2 + 2*(b^2*c^3*d + 8*a*b*c^2*d^2 - 24*a^2*c*d^3)*x)*sq
rt(-(b*c - a*d)/d)*log((2*d*x*sqrt(-(b*c - a*d)/d)*sqrt((a*x + b)/x) + b*d
- (b*c - 2*a*d)*x)/(c*x + d)) - 2*(4*a^2*c^3*d*x^3 + (b^2*c^4 - 11*a*b*c^3*
d + 18*a^2*c^2*d^2)*x^2 - (b^2*c^3*d + 7*a*b*c^2*d^2 - 12*a^2*c*d^3)*x)*sqr
t((a*x + b)/x))/(c^6*d*x^2 + 2*c^5*d^2*x + c^4*d^3), 1/4*((b^2*c^2*d^2 + 8*
a*b*c*d^3 - 24*a^2*d^4 + (b^2*c^4 + 8*a*b*c^3*d - 24*a^2*c^2*d^2)*x^2 + 2*(
b^2*c^3*d + 8*a*b*c^2*d^2 - 24*a^2*c*d^3)*x)*sqrt((b*c - a*d)/d)*arctan(-d*
sqrt((b*c - a*d)/d)*sqrt((a*x + b)/x)/(b*c - a*d)) - 4*(5*a*b*c*d^3 - 6*a^2
*d^4 + (5*a*b*c^3*d - 6*a^2*c^2*d^2)*x^2 + 2*(5*a*b*c^2*d^2 - 6*a^2*c*d^3)*
x)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) + (4*a^2*c^3*d*x^3 + (b^2*
c^4 - 11*a*b*c^3*d + 18*a^2*c^2*d^2)*x^2 - (b^2*c^3*d + 7*a*b*c^2*d^2 - 12*
a^2*c*d^3)*x)*sqrt((a*x + b)/x))/(c^6*d*x^2 + 2*c^5*d^2*x + c^4*d^3)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**(5/2)/(c+d/x)**3,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 945 vs. 2(209) = 418.
time = 1.82, size = 945, normalized size = 3.99

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

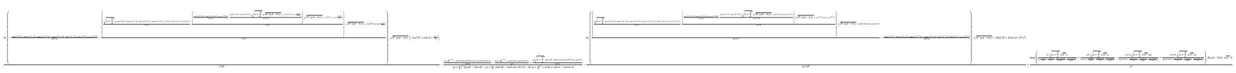
```
[In] integrate((a+b/x)^(5/2)/(c+d/x)^3,x, algorithm="giac")
```

```
[Out] sqrt(a*x^2 + b*x)*a^2*sgn(x)/c^3 - 1/2*(5*a^2*b*c*sgn(x) - 6*a^3*d*sgn(x))*
log(abs(2*(sqrt(a)*x - sqrt(a*x^2 + b*x))*sqrt(a) + b))/(sqrt(a)*c^4) + 1/4
*(b^3*c^3*sgn(x) + 7*a*b^2*c^2*d*sgn(x) - 32*a^2*b*c*d^2*sgn(x) + 24*a^3*d^
3*sgn(x))*arctan(-((sqrt(a)*x - sqrt(a*x^2 + b*x))*c + sqrt(a)*d)/sqrt(b*c*
d - a*d^2))/(sqrt(b*c*d - a*d^2)*c^4*d) + 1/4*(sqrt(a)*b^3*c^3*arctan(sqrt(
a)*d/sqrt(b*c*d - a*d^2)) + 7*a^(3/2)*b^2*c^2*d*arctan(sqrt(a)*d/sqrt(b*c*d
 - a*d^2)) - 32*a^(5/2)*b*c*d^2*arctan(sqrt(a)*d/sqrt(b*c*d - a*d^2)) + 24*
a^(7/2)*d^3*arctan(sqrt(a)*d/sqrt(b*c*d - a*d^2)) + 10*sqrt(b*c*d - a*d^2)*
a^2*b*c*d*log(abs(b)) - 12*sqrt(b*c*d - a*d^2)*a^3*d^2*log(abs(b)) - sqrt(b
*c*d - a*d^2)*a*b^2*c^2 + 11*sqrt(b*c*d - a*d^2)*a^2*b*c*d - 10*sqrt(b*c*d
```

```
- a*d^2)*a^3*d^2)*sgn(x)/(sqrt(b*c*d - a*d^2)*sqrt(a)*c^4*d) - 1/4*((sqrt(a
)*x - sqrt(a*x^2 + b*x))^3*sqrt(a)*b^3*c^4*sgn(x) - 17*(sqrt(a)*x - sqrt(a*
x^2 + b*x))^3*a^(3/2)*b^2*c^3*d*sgn(x) + 40*(sqrt(a)*x - sqrt(a*x^2 + b*x))
^3*a^(5/2)*b*c^2*d^2*sgn(x) - 24*(sqrt(a)*x - sqrt(a*x^2 + b*x))^3*a^(7/2)*
c*d^3*sgn(x) - 5*(sqrt(a)*x - sqrt(a*x^2 + b*x))^2*a*b^3*c^3*d*sgn(x) - 3*(
sqrt(a)*x - sqrt(a*x^2 + b*x))^2*a^2*b^2*c^2*d^2*sgn(x) + 48*(sqrt(a)*x - s
qrt(a*x^2 + b*x))^2*a^3*b*c*d^3*sgn(x) - 40*(sqrt(a)*x - sqrt(a*x^2 + b*x))
^2*a^4*d^4*sgn(x) - (sqrt(a)*x - sqrt(a*x^2 + b*x))*sqrt(a)*b^4*c^3*d*sgn(x
) - 11*(sqrt(a)*x - sqrt(a*x^2 + b*x))*a^(3/2)*b^3*c^2*d^2*sgn(x) + 52*(sqr
t(a)*x - sqrt(a*x^2 + b*x))*a^(5/2)*b^2*c*d^3*sgn(x) - 40*(sqrt(a)*x - sqrt
(a*x^2 + b*x))*a^(7/2)*b*d^4*sgn(x) - a*b^4*c^2*d^2*sgn(x) + 11*a^2*b^3*c*d
^3*sgn(x) - 10*a^3*b^2*d^4*sgn(x))/(((sqrt(a)*x - sqrt(a*x^2 + b*x))^2*c +
2*(sqrt(a)*x - sqrt(a*x^2 + b*x))*sqrt(a)*d + b*d)^2*sqrt(a)*c^4*d)
```

Mupad [B]

time = 3.44, size = 1476, normalized size = 6.23



Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x)^(5/2)/(c + d/x)^3,x)
```

```
[Out] (atan((b^9*(a + b/x)^(1/2)*(a^3)^(1/2)*5i)/(8*((5*a^2*b^9)/8 + (8*a^3*b^8*d
)/c - (159*a^4*b^7*d^2)/(8*c^2) + (45*a^5*b^6*d^3)/(4*c^3))) + (a*b^8*(a +
b/x)^(1/2)*(a^3)^(1/2)*8i)/(8*a^3*b^8 + (5*a^2*b^9*c)/(8*d) - (159*a^4*b^7*
d)/(8*c) + (45*a^5*b^6*d^2)/(4*c^2)) - (a^2*b^7*d*(a + b/x)^(1/2)*(a^3)^(1/
2)*159i)/(8*(8*a^3*b^8*c - (159*a^4*b^7*d)/8 + (5*a^2*b^9*c^2)/(8*d) + (45*
a^5*b^6*d^2)/(4*c))) + (a^3*b^6*d^2*(a + b/x)^(1/2)*(a^3)^(1/2)*45i)/(4*(8*
a^3*b^8*c^2 + (45*a^5*b^6*d^2)/4 + (5*a^2*b^9*c^3)/(8*d) - (159*a^4*b^7*c*d
)/8)))*(6*a*d - 5*b*c)*(a^3)^(1/2)*1i)/c^4 - (((a + b/x)^(3/2)*(b^4*c^3 - 2
4*a^3*b*d^3 + 32*a^2*b^2*c*d^2 - 9*a*b^3*c^2*d))/(4*c^3*d) - (b*(a + b/x)^(
5/2)*(b^2*c^2 - 12*a^2*d^2 + 7*a*b*c*d))/(4*c^3) + (b*(a + b/x)^(1/2)*(12*a
^4*d^3 - a*b^3*c^3 + 14*a^2*b^2*c^2*d - 25*a^3*b*c*d^2))/(4*c^3*d))/((a + b
/x)^2*(3*a*d^2 - 2*b*c*d) - (a + b/x)*(3*a^2*d^2 + b^2*c^2 - 4*a*b*c*d) - d
^2*(a + b/x)^3 + a^3*d^2 + a*b^2*c^2 - 2*a^2*b*c*d) + (log(- (5*a^2*b^9*c^6
 + 1728*a^8*b^3*d^6 + 64*a^3*b^8*c^5*d - 4752*a^7*b^4*c*d^5 - 59*a^4*b^7*c^
4*d^2 - 1450*a^5*b^6*c^3*d^3 + 4464*a^6*b^5*c^2*d^4)/(16*c^9*d) - ((((a + b
/x)^(1/2)*(b^8*c^6 + 1152*a^6*b^2*d^6 - 2496*a^5*b^3*c*d^5 - 15*a^2*b^6*c^4
*d^2 - 400*a^3*b^5*c^3*d^3 + 1760*a^4*b^4*c^2*d^4 + 14*a*b^7*c^5*d))/(8*c^6
*d) - (((16*a*b^5*c^10*d^2 - 208*a^2*b^4*c^9*d^3 + 192*a^3*b^3*c^8*d^4)/(16
*c^9*d) - ((64*b^3*c^9*d^3 - 128*a*b^2*c^8*d^4)*(a + b/x)^(1/2)*(d^3*(a*d -
 b*c))^(1/2)*((b^2*c^2)/8 - 3*a^2*d^2 + a*b*c*d))/(8*c^10*d^4))*(d^3*(a*d -
 b*c))^(1/2)*((b^2*c^2)/8 - 3*a^2*d^2 + a*b*c*d))/(c^4*d^3))*(d^3*(a*d - b*
c))^(1/2)*((b^2*c^2)/8 - 3*a^2*d^2 + a*b*c*d))/(c^4*d^3))*(d^3*(a*d - b*c))
^(1/2)*((b^2*c^2)/8 - 3*a^2*d^2 + a*b*c*d))/(c^4*d^3) - (log((((a + b/x)^(
```

```
1/2)*(b^8*c^6 + 1152*a^6*b^2*d^6 - 2496*a^5*b^3*c*d^5 - 15*a^2*b^6*c^4*d^2
- 400*a^3*b^5*c^3*d^3 + 1760*a^4*b^4*c^2*d^4 + 14*a*b^7*c^5*d))/(8*c^6*d) +
 (((16*a*b^5*c^10*d^2 - 208*a^2*b^4*c^9*d^3 + 192*a^3*b^3*c^8*d^4)/(16*c^9*
d) + ((64*b^3*c^9*d^3 - 128*a*b^2*c^8*d^4)*(a + b/x)^(1/2)*(d^3*(a*d - b*c)
)^(1/2)*(b^2*c^2 - 24*a^2*d^2 + 8*a*b*c*d))/(64*c^10*d^4))*(d^3*(a*d - b*c)
)^(1/2)*(b^2*c^2 - 24*a^2*d^2 + 8*a*b*c*d))/(8*c^4*d^3))*(d^3*(a*d - b*c))^
(1/2)*(b^2*c^2 - 24*a^2*d^2 + 8*a*b*c*d))/(8*c^4*d^3) - (5*a^2*b^9*c^6 + 17
28*a^8*b^3*d^6 + 64*a^3*b^8*c^5*d - 4752*a^7*b^4*c*d^5 - 59*a^4*b^7*c^4*d^2
 - 1450*a^5*b^6*c^3*d^3 + 4464*a^6*b^5*c^2*d^4)/(16*c^9*d))*(d^3*(a*d - b*c
))^(1/2)*(b^2*c^2 - 24*a^2*d^2 + 8*a*b*c*d))/(8*c^4*d^3)
```

3.245 $\int \frac{\left(c+\frac{d}{x}\right)^3}{\sqrt{a+\frac{b}{x}}}\,dx$

Optimal. Leaf size=126

$$-\frac{d\sqrt{a+\frac{b}{x}}\left(2(3b^2c^2+9abcd-2a^2d^2)+\frac{bd(3bc+2ad)}{x}\right)}{3ab^2}+\frac{c\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2x}{a}-\frac{c^2(bc-6ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}}$$

```
[Out] -c^2*(-6*a*d+b*c)*arctanh((a+b/x)^(1/2)/a^(1/2))/a^(3/2)-1/3*d*(-4*a^2*d^2+
18*a*b*c*d+6*b^2*c^2+b*d*(2*a*d+3*b*c)/x)*(a+b/x)^(1/2)/a/b^2+c*(c+d/x)^2*x
*(a+b/x)^(1/2)/a
```

Rubi [A]
time = 0.06, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {382, 100, 152, 65, 214}

$$-\frac{c^2(bc-6ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{d\sqrt{a+\frac{b}{x}}\left(2(-2a^2d^2+9abcd+3b^2c^2)+\frac{bd(2ad+3bc)}{x}\right)}{3ab^2}+\frac{cx\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d/x)^3/Sqrt[a + b/x],x]
```

```
[Out] -1/3*(d*Sqrt[a + b/x]*(2*(3*b^2*c^2 + 9*a*b*c*d - 2*a^2*d^2) + (b*d*(3*b*c
+ 2*a*d))/x))/(a*b^2) + (c*Sqrt[a + b/x]*(c + d/x)^2*x)/a - (c^2*(b*c - 6*a
*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/a^(3/2)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
```

```
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m +
 1)*((c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Dist[(a^2*d
^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m
+ n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n +
 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)),
Int[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}
, x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{\left(c+\frac{d}{x}\right)^3}{\sqrt{a+\frac{b}{x}}}\,dx = -\text{Subst}\left(\int \frac{(c+dx)^3}{x^2\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{c\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2 x}{a} + \frac{\text{Subst}\left(\int \frac{(c+dx)\left(\frac{1}{2}c(bc-6ad)-\frac{1}{2}d(3bc+2ad)x\right)}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)}{a}$$

$$= -\frac{d\sqrt{a+\frac{b}{x}}\left(2(3b^2c^2+9abcd-2a^2d^2)+\frac{bd(3bc+2ad)}{x}\right)}{3ab^2} + \frac{c\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2 x}{a} + \frac{(c^2(bc-6\ldots}{\ldots}$$

$$= -\frac{d\sqrt{a+\frac{b}{x}}\left(2(3b^2c^2+9abcd-2a^2d^2)+\frac{bd(3bc+2ad)}{x}\right)}{3ab^2} + \frac{c\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2 x}{a} + \frac{(c^2(bc-6\ldots}{\ldots}$$

$$= -\frac{d\sqrt{a+\frac{b}{x}}\left(2(3b^2c^2+9abcd-2a^2d^2)+\frac{bd(3bc+2ad)}{x}\right)}{3ab^2} + \frac{c\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2 x}{a} - \frac{c^2(bc-6\ldots}{\ldots}$$

<u>**Mathematica**</u> [A]
time = 0.22, size = 95, normalized size = 0.75

$$\frac{\sqrt{a+\frac{b}{x}}\,(4a^2d^3x+3b^2c^3x^2-2abd^2(d+9cx))}{3ab^2x} + \frac{c^2(-bc+6ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d/x)^3/Sqrt[a + b/x],x]
```

```
[Out] (Sqrt[a + b/x]*(4*a^2*d^3*x + 3*b^2*c^3*x^2 - 2*a*b*d^2*(d + 9*c*x)))/(3*a*
b^2*x) + (c^2*(-(b*c) + 6*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/a^(3/2)
```

<u>**Maple**</u> [B] Leaf count of result is larger than twice the leaf count of optimal. 534 vs. $2(111) = 222$.
time = 0.06, size = 535, normalized size = 4.25

method	result

risch	$\frac{(ax+b)(3b^2c^3x^2+4a^2d^3x-18axbc\,d^2-2ab\,d^3)}{3b^2x^2a\sqrt{\frac{ax+b}{x}}}+\frac{\left(\frac{3c^2\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^2+bx}\right)d}{\sqrt{a}}-\frac{c^3\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^2+bx}\right)b}{2a^{\frac{3}{2}}}\right)\sqrt{x(}}{x\sqrt{\frac{ax+b}{x}}}$
default	$-\frac{\sqrt{\frac{ax+b}{x}}\left(6\sqrt{x\,(ax+b)}\,a^{\frac{7}{2}}d^3x^3-18\sqrt{x\,(ax+b)}\,a^{\frac{5}{2}}bc\,d^2x^3+18\sqrt{x\,(ax+b)}\,a^{\frac{3}{2}}b^2c^2d\,x^3-6\sqrt{x\,(ax+b)}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d/x)^3/(a+1/x*b)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*((a*x+b)/x)^(1/2)/x^2*(6*(x*(a*x+b))^(1/2)*a^(7/2)*d^3*x^3-18*(x*(a*x+
b))^(1/2)*a^(5/2)*b*c*d^2*x^3+18*(x*(a*x+b))^(1/2)*a^(3/2)*b^2*c^2*d*x^3-6*
(x*(a*x+b))^(1/2)*a^(1/2)*b^3*c^3*x^3+6*(a*x^2+b*x)^(1/2)*a^(7/2)*d^3*x^3-1
8*(a*x^2+b*x)^(1/2)*a^(5/2)*b*c*d^2*x^3-18*(a*x^2+b*x)^(1/2)*a^(3/2)*b^2*c^
2*d*x^3+3*ln(1/2*(2*(a*x^2+b*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^3*b*d^3*x
^3-9*ln(1/2*(2*(a*x^2+b*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^2*b^2*c*d^2*x^
3-9*ln(1/2*(2*(a*x^2+b*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a*b^3*c^2*d*x^3-3
*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^3*b*d^3*x^3+9*ln(1
/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^2*b^2*c*d^2*x^3-9*ln(1/
2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a*b^3*c^2*d*x^3+3*ln(1/2*(
2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*b^4*c^3*x^3-12*(a*x^2+b*x)^(3
/2)*a^(5/2)*d^3*x+36*(a*x^2+b*x)^(3/2)*a^(3/2)*b*c*d^2*x+4*d^3*(a*x^2+b*x)^
(3/2)*b*a^(3/2))/(x*(a*x+b))^(1/2)/b^3/a^(3/2)
```

Maxima [A]
time = 0.47, size = 166, normalized size = 1.32

$$\frac{1}{2}c^3\left(\frac{2\sqrt{a+\frac{b}{x}}\,b}{\left(a+\frac{b}{x}\right)a-a^2}+\frac{b\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{a^{\frac{3}{2}}}\right)-\frac{2}{3}d^3\left(\frac{\left(a+\frac{b}{x}\right)^{\frac{3}{2}}}{b^2}-\frac{3\sqrt{a+\frac{b}{x}}\,a}{b^2}\right)-\frac{3c^2d\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{\sqrt{a}}-\frac{6\sqrt{a+\frac{b}{x}}\,cd^2}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)^3/(a+b/x)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*c^3*(2*sqrt(a + b/x)*b/((a + b/x)*a - a^2) + b*log((sqrt(a + b/x) - sqr
t(a))/(sqrt(a + b/x) + sqrt(a)))/a^(3/2)) - 2/3*d^3*((a + b/x)^(3/2)/b^2 -
3*sqrt(a + b/x)*a/b^2) - 3*c^2*d*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/
x) + sqrt(a)))/sqrt(a) - 6*sqrt(a + b/x)*c*d^2/b
```

Fricas [A]
time = 2.87, size = 233, normalized size = 1.85

$$\left[-\frac{3\left(b^{3}c^{3}-6ab^{2}c^{2}d\right)\sqrt{a}\,x\log\left(2ax+2\sqrt{a}\,x\sqrt{\frac{ax+b}{x}}+b\right)-2\left(3ab^{2}c^{3}x^{2}-2a^{2}bd^{3}-2\left(9a^{2}bcd^{2}-2a^{3}d^{3}\right)x\right)\sqrt{\frac{ax+b}{x}}}{6a^{2}b^{2}x},\ \frac{3\left(b^{3}c^{3}-6ab^{2}c^{2}d\right)\sqrt{-a}\,x\arctan\left(\frac{\sqrt{-a}\sqrt{\frac{ax+b}{x}}}{a}\right)+\left(3ab^{2}c^{3}x^{2}-2a^{2}bd^{3}-2\left(9a^{2}bcd^{2}-2a^{3}d^{3}\right)x\right)\sqrt{\frac{ax+b}{x}}}{3a^{2}b^{2}x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)^3/(a+b/x)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/6*(3*(b^3*c^3 - 6*a*b^2*c^2*d)*sqrt(a)*x*log(2*a*x + 2*sqrt(a)*x*sqrt((
a*x + b)/x) + b) - 2*(3*a*b^2*c^3*x^2 - 2*a^2*b*d^3 - 2*(9*a^2*b*c*d^2 - 2*
a^3*d^3)*x)*sqrt((a*x + b)/x))/(a^2*b^2*x), 1/3*(3*(b^3*c^3 - 6*a*b^2*c^2*d
)*sqrt(-a)*x*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) + (3*a*b^2*c^3*x^2 - 2*a^
2*b*d^3 - 2*(9*a^2*b*c*d^2 - 2*a^3*d^3)*x)*sqrt((a*x + b)/x))/(a^2*b^2*x)]
```

Sympy [A]
time = 37.56, size = 386, normalized size = 3.06

$$\frac{4a^{\frac{7}{2}}b^{\frac{3}{2}}d^{3}x^{2}\sqrt{\frac{ax}{b}+1}}{3a^{\frac{5}{2}}b^{3}x^{\frac{5}{2}}+3a^{\frac{3}{2}}b^{4}x^{\frac{3}{2}}}+\frac{2a^{\frac{5}{2}}b^{\frac{5}{2}}d^{3}x\sqrt{\frac{ax}{b}+1}}{3a^{\frac{5}{2}}b^{3}x^{\frac{5}{2}}+3a^{\frac{3}{2}}b^{4}x^{\frac{3}{2}}}-\frac{2a^{\frac{3}{2}}b^{\frac{7}{2}}d^{3}\sqrt{\frac{ax}{b}+1}}{3a^{\frac{5}{2}}b^{3}x^{\frac{5}{2}}+3a^{\frac{3}{2}}b^{4}x^{\frac{3}{2}}}-\frac{4a^{4}bd^{3}x^{\frac{5}{2}}}{3a^{\frac{5}{2}}b^{3}x^{\frac{5}{2}}+3a^{\frac{3}{2}}b^{4}x^{\frac{3}{2}}}-\frac{4a^{3}b^{2}d^{3}x^{\frac{3}{2}}}{3a^{\frac{5}{2}}b^{3}x^{\frac{5}{2}}+3a^{\frac{3}{2}}b^{4}x^{\frac{3}{2}}}+3cd^{2}\left(\begin{cases}-\frac{1}{\sqrt{a}\,x} & \text{for}\ b=0\\ -\frac{2\sqrt{a+\frac{b}{x}}}{b} & \text{otherwise}\end{cases}\right)+\frac{\sqrt{b}\,c^{3}\sqrt{x}\sqrt{\frac{ax}{b}+1}}{a}-\frac{6c^{2}d\operatorname{atan}\left(\frac{1}{\sqrt{-\frac{1}{a}}\sqrt{a+\frac{b}{x}}}\right)}{a\sqrt{-\frac{1}{a}}}-\frac{bc^{3}\operatorname{asinh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)**3/(a+b/x)**(1/2),x)
```

```
[Out] 4*a**(7/2)*b**(3/2)*d**3*x**2*sqrt(a*x/b + 1)/(3*a**(5/2)*b**3*x**(5/2) + 3
*a**(3/2)*b**4*x**(3/2)) + 2*a**(5/2)*b**(5/2)*d**3*x*sqrt(a*x/b + 1)/(3*a*
*(5/2)*b**3*x**(5/2) + 3*a**(3/2)*b**4*x**(3/2)) - 2*a**(3/2)*b**(7/2)*d**3
*sqrt(a*x/b + 1)/(3*a**(5/2)*b**3*x**(5/2) + 3*a**(3/2)*b**4*x**(3/2)) - 4*
a**4*b*d**3*x**(5/2)/(3*a**(5/2)*b**3*x**(5/2) + 3*a**(3/2)*b**4*x**(3/2))
- 4*a**3*b**2*d**3*x**(3/2)/(3*a**(5/2)*b**3*x**(5/2) + 3*a**(3/2)*b**4*x**
(3/2)) + 3*c*d**2*Piecewise((-1/(sqrt(a)*x), Eq(b, 0)), (-2*sqrt(a + b/x)/b
, True)) + sqrt(b)*c**3*sqrt(x)*sqrt(a*x/b + 1)/a - 6*c**2*d*atan(1/(sqrt(-
1/a)*sqrt(a + b/x)))/(a*sqrt(-1/a)) - b*c**3*asinh(sqrt(a)*sqrt(x)/sqrt(b))
/a**(3/2)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)^3/(a+b/x)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
```

```
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [B]
time = 1.73, size = 107, normalized size = 0.85

$$\sqrt{a+\frac{b}{x}}\,\left(\frac{6\,a\,d^3-6\,b\,c\,d^2}{b^2}-\frac{4\,a\,d^3}{b^2}\right)-\frac{2\,d^3\,{\left(a+\frac{b}{x}\right)}^{3/2}}{3\,b^2}+\frac{c^3\,x\,\sqrt{a+\frac{b}{x}}}{a}-\frac{c^2\,\mathrm{atan}\left(\frac{\sqrt{a+\frac{b}{x}}\,1{}\mathrm{i}}{\sqrt{a}}\right)\,\left(6\,a\,d-b\,c\right)\,1{}\mathrm{i}}{a^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d/x)^3/(a + b/x)^(1/2),x)
```

```
[Out] (a + b/x)^(1/2)*((6*a*d^3 - 6*b*c*d^2)/b^2 - (4*a*d^3)/b^2) - (2*d^3*(a + b
/x)^(3/2))/(3*b^2) + (c^3*x*(a + b/x)^(1/2))/a - (c^2*atan(((a + b/x)^(1/2)
*1i)/a^(1/2))*(6*a*d - b*c)*1i)/a^(3/2)
```

3.246 $\int \frac{\left(c+\frac{d}{x}\right)^2}{\sqrt{a+\frac{b}{x}}}\,dx$

Optimal. Leaf size=73

$$-\frac{2d^2\sqrt{a+\frac{b}{x}}}{b}+\frac{c^2\sqrt{a+\frac{b}{x}}\,x}{a}-\frac{c(bc-4ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}}$$

```
[Out] -c*(-4*a*d+b*c)*arctanh((a+b/x)^(1/2)/a^(1/2))/a^(3/2)-2*d^2*(a+b/x)^(1/2)/
b+c^2*x*(a+b/x)^(1/2)/a
```

Rubi [A]
time = 0.04, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {382, 91, 81, 65, 214}

$$-\frac{c(bc-4ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}}+\frac{c^2x\sqrt{a+\frac{b}{x}}}{a}-\frac{2d^2\sqrt{a+\frac{b}{x}}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d/x)^2/Sqrt[a + b/x],x]
```

```
[Out] (-2*d^2*Sqrt[a + b/x])/b + (c^2*Sqrt[a + b/x]*x)/a - (c*(b*c - 4*a*d)*ArcTa
nh[Sqrt[a + b/x]/Sqrt[a]])/a^(3/2)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
```

```
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(c+\frac{d}{x}\right)^2}{\sqrt{a+\frac{b}{x}}}\,dx &= -\text{Subst}\left(\int \frac{(c+dx)^2}{x^2\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)\\
&= \frac{c^2\sqrt{a+\frac{b}{x}}\,x}{a} - \frac{\text{Subst}\left(\int \frac{-\frac{1}{2}c(bc-4ad)+ad^2x}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)}{a}\\
&= -\frac{2d^2\sqrt{a+\frac{b}{x}}}{b} + \frac{c^2\sqrt{a+\frac{b}{x}}\,x}{a} + \frac{(c(bc-4ad))\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)}{2a}\\
&= -\frac{2d^2\sqrt{a+\frac{b}{x}}}{b} + \frac{c^2\sqrt{a+\frac{b}{x}}\,x}{a} + \frac{(c(bc-4ad))\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x}}\right)}{ab}\\
&= -\frac{2d^2\sqrt{a+\frac{b}{x}}}{b} + \frac{c^2\sqrt{a+\frac{b}{x}}\,x}{a} - \frac{c(bc-4ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}}
\end{aligned}
$$

Mathematica [A]
time = 0.15, size = 66, normalized size = 0.90

$$
\frac{\sqrt{a+\frac{b}{x}}\,(-2ad^2+bc^2x)}{ab} + \frac{c(-bc+4ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d/x)^2/Sqrt[a + b/x],x]
```

```
[Out] (Sqrt[a + b/x]*(-2*a*d^2 + b*c^2*x))/(a*b) + (c*(-(b*c) + 4*a*d)*ArcTanh[Sq
rt[a + b/x]/Sqrt[a]])/a^(3/2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 347 vs. 2(63) = 126.
time = 0.04, size = 348, normalized size = 4.77

method	result

risch	$-\frac{(ax+b)(-b c^2 x+2a d^2)}{bax\sqrt{\frac{ax+b}{x}}}+\frac{\left(\frac{2c\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a x^2+bx}\right)d}{\sqrt{a}}-\frac{c^2\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a x^2+bx}\right)b}{2a^{\frac{3}{2}}}\right)\sqrt{x(ax+b)}}{x\sqrt{\frac{ax+b}{x}}}$
default	$-\frac{\sqrt{\frac{ax+b}{x}}\left(-2\sqrt{a x^2+bx}\,a^{\frac{5}{2}}d^2x^2-4\sqrt{a x^2+bx}\,a^{\frac{3}{2}}bcd\,x^2-\ln\left(\frac{2\sqrt{a x^2+bx}\,\sqrt{a}+2ax+b}{2\sqrt{a}}\right)a^2bd^2x^2-2\ln\left(\frac{2\sqrt{}}{}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d/x)^2/(a+1/x*b)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*((a*x+b)/x)^(1/2)/x*(-2*(a*x^2+b*x)^(1/2)*a^(5/2)*d^2*x^2-4*(a*x^2+b*x
)^(1/2)*a^(3/2)*b*c*d*x^2-ln(1/2*(2*(a*x^2+b*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1
/2))*a^2*b*d^2*x^2-2*ln(1/2*(2*(a*x^2+b*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*
a*b^2*c*d*x^2-2*a^(5/2)*(x*(a*x+b))^(1/2)*d^2*x^2+4*a^(3/2)*(x*(a*x+b))^(1/
2)*b*c*d*x^2-2*a^(1/2)*(x*(a*x+b))^(1/2)*b^2*c^2*x^2+ln(1/2*(2*(x*(a*x+b))^
(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^2*b*d^2*x^2-2*ln(1/2*(2*(x*(a*x+b))^(1/2)
*a^(1/2)+2*a*x+b)/a^(1/2))*a*b^2*c*d*x^2+ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2
)+2*a*x+b)/a^(1/2))*b^3*c^2*x^2+4*(a*x^2+b*x)^(3/2)*a^(3/2)*d^2)/(x*(a*x+b)
)^(1/2)/b^2/a^(3/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 129 vs. $2(63) = 126$.
time = 0.50, size = 129, normalized size = 1.77

$$\frac{1}{2}c^2\left(\frac{2\sqrt{a+\frac{b}{x}}\,b}{(a+\frac{b}{x})a-a^2}+\frac{b\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{a^{\frac{3}{2}}}\right)-\frac{2\,cd\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{\sqrt{a}}-\frac{2\sqrt{a+\frac{b}{x}}\,d^2}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)^2/(a+b/x)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*c^2*(2*sqrt(a + b/x)*b/((a + b/x)*a - a^2) + b*log((sqrt(a + b/x) - sqr
t(a))/(sqrt(a + b/x) + sqrt(a)))/a^(3/2)) - 2*c*d*log((sqrt(a + b/x) - sqrt
(a))/(sqrt(a + b/x) + sqrt(a)))/sqrt(a) - 2*sqrt(a + b/x)*d^2/b
```

Fricas [A]
time = 2.50, size = 158, normalized size = 2.16

$$\left[-\frac{\left(b^{2} c^{2} - 4 \, a b c d\right) \sqrt{a} \log\left(2 \, a x + 2 \, \sqrt{a} x \sqrt{\frac{a x + b}{x}} + b\right) - 2 \, \left(a b c^{2} x - 2 \, a^{2} d^{2}\right) \sqrt{\frac{a x + b}{x}}}{2 \, a^{2} b}, \frac{\left(b^{2} c^{2} - 4 \, a b c d\right) \sqrt{-a} \arctan\left(\frac{\sqrt{-a} \sqrt{\frac{a x + b}{x}}}{a}\right) + \left(a b c^{2} x - 2 \, a^{2} d^{2}\right) \sqrt{\frac{a x + b}{x}}}{a^{2} b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)^2/(a+b/x)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*((b^2*c^2 - 4*a*b*c*d)*sqrt(a)*log(2*a*x + 2*sqrt(a)*x*sqrt((a*x + b)
/x) + b) - 2*(a*b*c^2*x - 2*a^2*d^2)*sqrt((a*x + b)/x))/(a^2*b), ((b^2*c^2
- 4*a*b*c*d)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) + (a*b*c^2*x - 2
*a^2*d^2)*sqrt((a*x + b)/x))/(a^2*b)]
```

Sympy [A]
time = 24.78, size = 114, normalized size = 1.56

$$d^{2} \left(\begin{cases} - \frac{1}{\sqrt{a} x} & \text{for}\: b = 0 \\ - \frac{2 \sqrt{a + \frac{b}{x}}}{b} & \text{otherwise} \end{cases}\right) + \frac{\sqrt{b} c^{2} \sqrt{x} \sqrt{\frac{a x}{b} + 1}}{a} - \frac{4 c d \operatorname{atan}{\left(\frac{1}{\sqrt{- \frac{1}{a}} \sqrt{a + \frac{b}{x}}} \right)}}{a \sqrt{- \frac{1}{a}}} - \frac{b c^{2} \operatorname{asinh}{\left(\frac{\sqrt{a} \sqrt{x}}{\sqrt{b}} \right)}}{a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)**2/(a+b/x)**(1/2),x)
```

```
[Out] d**2*Piecewise((-1/(sqrt(a)*x), Eq(b, 0)), (-2*sqrt(a + b/x)/b, True)) + sq
rt(b)*c**2*sqrt(x)*sqrt(a*x/b + 1)/a - 4*c*d*atan(1/(sqrt(-1/a)*sqrt(a + b/
x)))/(a*sqrt(-1/a)) - b*c**2*asinh(sqrt(a)*sqrt(x)/sqrt(b))/a**(3/2)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)^2/(a+b/x)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [B]
time = 1.62, size = 63, normalized size = 0.86

$$\frac{c^2\,x\,\sqrt{a+\frac{b}{x}}}{a}-\frac{2\,d^2\,\sqrt{a+\frac{b}{x}}}{b}+\frac{c\,\mathrm{atanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)\,\left(4\,a\,d-b\,c\right)}{a^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d/x)^2/(a + b/x)^(1/2),x)
```

```
[Out] (c^2*x*(a + b/x)^(1/2))/a - (2*d^2*(a + b/x)^(1/2))/b + (c*atanh((a + b/x)^
(1/2)/a^(1/2))*(4*a*d - b*c))/a^(3/2)
```

3.247 $\int \frac{c+\frac{d}{x}}{\sqrt{a+\frac{b}{x}}}\,dx$

Optimal. Leaf size=51

$$\frac{c\sqrt{a+\frac{b}{x}}\,x}{a}-\frac{(bc-2ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}}$$

```
[Out] -(-2*a*d+b*c)*arctanh((a+b/x)^(1/2)/a^(1/2))/a^(3/2)+c*x*(a+b/x)^(1/2)/a
```

Rubi [A]
time = 0.02, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {382, 79, 65, 214}

$$\frac{cx\sqrt{a+\frac{b}{x}}}{a}-\frac{(bc-2ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d/x)/Sqrt[a + b/x],x]
```

```
[Out] (c*Sqrt[a + b/x]*x)/a - ((b*c - 2*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/a^(3
/2)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
```

```
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{c+\frac{d}{x}}{\sqrt{a+\frac{b}{x}}}\,dx &= -\text{Subst}\left(\int \frac{c+dx}{x^2\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)\\
&= \frac{c\sqrt{a+\frac{b}{x}}\,x}{a} - \frac{\left(-\frac{bc}{2}+ad\right)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)}{a}\\
&= \frac{c\sqrt{a+\frac{b}{x}}\,x}{a} - \frac{\left(2\left(-\frac{bc}{2}+ad\right)\right)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x}}\right)}{ab}\\
&= \frac{c\sqrt{a+\frac{b}{x}}\,x}{a} - \frac{(bc-2ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}}
\end{aligned}$$

Mathematica [A]
time = 0.10, size = 51, normalized size = 1.00

$$\frac{c\sqrt{a+\frac{b}{x}}\,x}{a} + \frac{(-bc+2ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d/x)/Sqrt[a + b/x],x]
```

```
[Out] (c*Sqrt[a + b/x]*x)/a + ((-(b*c) + 2*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/a
^(3/2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 173 vs. $2(43) = 86$.
time = 0.04, size = 174, normalized size = 3.41

method	result
risch	$\frac{c(ax+b)}{a\sqrt{\frac{ax+b}{x}}}+\frac{\left(\frac{\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^{2}+bx}\right)d}{\sqrt{a}}-\frac{\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^{2}+bx}\right)bc}{2a^{\frac{3}{2}}}\right)\sqrt{x\,(ax+b)}}{x\sqrt{\frac{ax+b}{x}}}$
default	$-\frac{\sqrt{\frac{ax+b}{x}}\,x\left(2\sqrt{x\,(ax+b)}\,a^{\frac{3}{2}}d-2\sqrt{x\,(ax+b)}\,\sqrt{a}\,bc-2\sqrt{a\,x^{2}+bx}\,a^{\frac{3}{2}}d-\ln\left(\frac{2\sqrt{a\,x^{2}+bx}\,\sqrt{a}+2ax+b}{2\sqrt{a}}\right)\right.}{2\sqrt{x\,(ax+b)}\,b\,a^{\frac{3}{2}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d/x)/(a+1/x*b)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*((a*x+b)/x)^(1/2)*x*(2*(x*(a*x+b))^(1/2)*a^(3/2)*d-2*(x*(a*x+b))^(1/2)
*a^(1/2)*b*c-2*(a*x^2+b*x)^(1/2)*a^(3/2)*d-ln(1/2*(2*(a*x^2+b*x)^(1/2)*a^(1
/2)+2*a*x+b)/a^(1/2))*a*b*d-ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^
(1/2))*a*b*d+ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*b^2*c)/(
x*(a*x+b))^(1/2)/b/a^(3/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 109 vs. $2(43) = 86$.
time = 0.50, size = 109, normalized size = 2.14

$$\frac{1}{2}\,c\left(\frac{2\,\sqrt{a+\frac{b}{x}}\,b}{\left(a+\frac{b}{x}\right)a-a^{2}}+\frac{b\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{a^{\frac{3}{2}}}\right)-\frac{d\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)/(a+b/x)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*c*(2*sqrt(a + b/x)*b/((a + b/x)*a - a^2) + b*log((sqrt(a + b/x) - sqrt(
a))/(sqrt(a + b/x) + sqrt(a)))/a^(3/2)) - d*log((sqrt(a + b/x) - sqrt(a))/(
sqrt(a + b/x) + sqrt(a)))/sqrt(a)
```

Fricas [A]
time = 2.64, size = 115, normalized size = 2.25

$$\left[\frac{2\,acx\sqrt{\frac{ax+b}{x}}-(bc-2\,ad)\sqrt{a}\,\log\left(2\,ax+2\,\sqrt{a}\,x\sqrt{\frac{ax+b}{x}}+b\right)}{2\,a^2},\frac{acx\sqrt{\frac{ax+b}{x}}+(bc-2\,ad)\sqrt{-a}\,\arctan\left(\frac{\sqrt{-a}\,\sqrt{\frac{ax+b}{x}}}{a}\right)}{a^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)/(a+b/x)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(2*a*c*x*sqrt((a*x + b)/x) - (b*c - 2*a*d)*sqrt(a)*log(2*a*x + 2*sqrt(
a)*x*sqrt((a*x + b)/x) + b))/a^2, (a*c*x*sqrt((a*x + b)/x) + (b*c - 2*a*d)*
sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a))/a^2]
```

Sympy [A]
time = 23.03, size = 82, normalized size = 1.61

$$\frac{\sqrt{b}\,c\sqrt{x}\,\sqrt{\frac{ax}{b}+1}}{a}-\frac{2d\,\mathrm{atan}\left(\frac{1}{\sqrt{-\frac{1}{a}}\,\sqrt{a+\frac{b}{x}}}\right)}{a\sqrt{-\frac{1}{a}}}-\frac{bc\,\mathrm{asinh}\left(\frac{\sqrt{a}\,\sqrt{x}}{\sqrt{b}}\right)}{a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)/(a+b/x)**(1/2),x)
```

```
[Out] sqrt(b)*c*sqrt(x)*sqrt(a*x/b + 1)/a - 2*d*atan(1/(sqrt(-1/a)*sqrt(a + b/x))
)/(a*sqrt(-1/a)) - b*c*asinh(sqrt(a)*sqrt(x)/sqrt(b))/a**(3/2)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 89 vs. $2(43) = 86$.
time = 0.60, size = 89, normalized size = 1.75

$$-\frac{(bc\log(|b|)-2\,ad\log(|b|))\mathrm{sgn}(x)}{2\,a^{\frac{3}{2}}}+\frac{\sqrt{ax^2+bx}\,c}{a\mathrm{sgn}(x)}+\frac{(bc-2\,ad)\log\left(\left|-2\left(\sqrt{a}\,x-\sqrt{ax^2+bx}\right)\sqrt{a}\,-b\right|\right)}{2\,a^{\frac{3}{2}}\mathrm{sgn}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)/(a+b/x)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*(b*c*log(abs(b)) - 2*a*d*log(abs(b)))*sgn(x)/a^(3/2) + sqrt(a*x^2 + b*
x)*c/(a*sgn(x)) + 1/2*(b*c - 2*a*d)*log(abs(-2*(sqrt(a)*x - sqrt(a*x^2 + b*
x))*sqrt(a) - b))/(a^(3/2)*sgn(x))
```

Mupad [B]
time = 1.98, size = 88, normalized size = 1.73

$$\frac{2\,d\,\mathrm{atanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}}+\frac{2\,c\,x\,\left(\frac{3\,\sqrt{b}\,\sqrt{b+a\,x}}{2\,a\,x}+\frac{b^{3/2}\,\mathrm{asin}\left(\frac{\sqrt{a}\,\sqrt{x}\,1\mathrm{i}}{\sqrt{b}}\right)\,3\mathrm{i}}{2\,a^{3/2}\,x^{3/2}}\right)\,\sqrt{\frac{a\,x}{b}+1}}{3\,\sqrt{a+\frac{b}{x}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d/x)/(a + b/x)^(1/2),x)
```

```
[Out] (2*d*atanh((a + b/x)^(1/2)/a^(1/2)))/a^(1/2) + (2*c*x*((3*b^(1/2)*(b + a*x)
^(1/2))/(2*a*x) + (b^(3/2)*asin((a^(1/2)*x^(1/2)*1i)/b^(1/2))*3i)/(2*a^(3/2
)*x^(3/2)))*((a*x)/b + 1)^(1/2))/(3*(a + b/x)^(1/2))
```

3.248 $\int \frac{1}{\sqrt{a+\frac{b}{x}}}\,dx$

Optimal. Leaf size=43

$$\frac{\sqrt{a+\frac{b}{x}}\,x}{a}-\frac{b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}}$$

```
[Out] -b*arctanh((a+b/x)^(1/2)/a^(1/2))/a^(3/2)+x*(a+b/x)^(1/2)/a
```

Rubi [A]
time = 0.01, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {248, 44, 65, 214}

$$\frac{x\sqrt{a+\frac{b}{x}}}{a}-\frac{b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b/x],x]
```

```
[Out] (Sqrt[a + b/x]*x)/a - (b*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/a^(3/2)
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 248

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{a+\frac{b}{x}}}\,dx &= -\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)\\
&= \frac{\sqrt{a+\frac{b}{x}}\,x}{a} + \frac{b\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)}{2a}\\
&= \frac{\sqrt{a+\frac{b}{x}}\,x}{a} + \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x}}\right)}{a}\\
&= \frac{\sqrt{a+\frac{b}{x}}\,x}{a} - \frac{b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}}
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 43, normalized size = 1.00

$$\frac{\sqrt{a+\frac{b}{x}}\,x}{a} - \frac{b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + b/x],x]
[Out] (Sqrt[a + b/x]*x)/a - (b*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/a^(3/2)
```

Maple [A]
time = 0.00, size = 71, normalized size = 1.65

method	result	size
default	$\frac{\sqrt{\frac{ax+b}{x}}\, x \left(2\sqrt{x(ax+b)}\,\sqrt{a}-b\ln\left(\frac{2\sqrt{x(ax+b)}\,\sqrt{a}+2ax+b}{2\sqrt{a}}\right)\right)}{2\sqrt{x(ax+b)}\, a^{\frac{3}{2}}}$	71
risch	$\frac{ax+b}{a\sqrt{\frac{ax+b}{x}}}-\frac{b\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^2+bx}\right)\sqrt{x(ax+b)}}{2a^{\frac{3}{2}}x\sqrt{\frac{ax+b}{x}}}$	75

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+1/x*b)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*((a*x+b)/x)^(1/2)*x*(2*(x*(a*x+b))^(1/2)*a^(1/2)-b*ln(1/2*(2*(x*(a*x+b)
)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2)))/(x*(a*x+b))^(1/2)/a^(3/2)
```

Maxima [A]
time = 0.56, size = 67, normalized size = 1.56

$$\frac{\sqrt{a+\frac{b}{x}}\,b}{(a+\frac{b}{x})a-a^2}+\frac{b\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{2\,a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(a + b/x)*b/((a + b/x)*a - a^2) + 1/2*b*log((sqrt(a + b/x) - sqrt(a))/(
sqrt(a + b/x) + sqrt(a)))/a^(3/2)
```

Fricas [A]
time = 3.22, size = 98, normalized size = 2.28

$$\left[\frac{2\,ax\sqrt{\frac{ax+b}{x}}+\sqrt{a}\,b\log\left(2\,ax-2\sqrt{a}\,x\sqrt{\frac{ax+b}{x}}+b\right)}{2\,a^2}, \frac{ax\sqrt{\frac{ax+b}{x}}+\sqrt{-a}\,b\arctan\left(\frac{\sqrt{-a}\sqrt{\frac{ax+b}{x}}}{a}\right)}{a^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(2*a*x*sqrt((a*x + b)/x) + sqrt(a)*b*log(2*a*x - 2*sqrt(a)*x*sqrt((a*x
 + b)/x) + b))/a^2, (a*x*sqrt((a*x + b)/x) + sqrt(-a)*b*arctan(sqrt(-a)*sqr
t((a*x + b)/x)/a))/a^2]
```

Sympy [A]
time = 1.17, size = 44, normalized size = 1.02

$$\frac{\sqrt{b}\,\sqrt{x}\,\sqrt{\frac{ax}{b}+1}}{a}-\frac{b\,\operatorname{asinh}\left(\frac{\sqrt{a}\,\sqrt{x}}{\sqrt{b}}\right)}{a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)**(1/2),x)
```

```
[Out] sqrt(b)*sqrt(x)*sqrt(a*x/b + 1)/a - b*asinh(sqrt(a)*sqrt(x)/sqrt(b))/a**(3/
2)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 71 vs. 2(35) = 70.
time = 1.13, size = 71, normalized size = 1.65

$$-\frac{b\log\left(|b|\right)\operatorname{sgn}(x)}{2\,a^{\frac{3}{2}}}+\frac{b\log\left(\left|-2\left(\sqrt{a}\,x-\sqrt{ax^2+bx}\right)\sqrt{a}\,-b\right|\right)}{2\,a^{\frac{3}{2}}\operatorname{sgn}\left(x\right)}+\frac{\sqrt{ax^2+bx}}{a\operatorname{sgn}\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*b*log(abs(b))*sgn(x)/a^(3/2) + 1/2*b*log(abs(-2*(sqrt(a)*x - sqrt(a*x^
2 + b*x))*sqrt(a) - b))/(a^(3/2)*sgn(x)) + sqrt(a*x^2 + b*x)/(a*sgn(x))
```

Mupad [B]
time = 1.44, size = 66, normalized size = 1.53

$$\frac{2\,x\left(\frac{3\,\sqrt{b}\,\sqrt{b+a\,x}}{2\,a\,x}+\frac{b^{3/2}\,\mathrm{asin}\left(\frac{\sqrt{a}\,\sqrt{x}\,1\mathrm{i}}{\sqrt{b}}\right)\,3\mathrm{i}}{2\,a^{3/2}\,x^{3/2}}\right)\sqrt{\frac{a\,x}{b}+1}}{3\,\sqrt{a+\frac{b}{x}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b/x)^(1/2),x)
```

```
[Out] (2*x*((3*b^(1/2)*(b + a*x)^(1/2))/(2*a*x) + (b^(3/2)*asin((a^(1/2)*x^(1/2)*
1i)/b^(1/2))*3i)/(2*a^(3/2)*x^(3/2)))*((a*x)/b + 1)^(1/2))/(3*(a + b/x)^(1/
2))
```

3.249 $\int \frac{1}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}\,dx$

Optimal. Leaf size=108

$$\frac{\sqrt{a+\frac{b}{x}}\,x}{ac}-\frac{2d^{3/2}\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^2\sqrt{bc-ad}}-\frac{(bc+2ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}c^2}$$

```
[Out] -(2*a*d+b*c)*arctanh((a+b/x)^(1/2)/a^(1/2))/a^(3/2)/c^2-2*d^(3/2)*arctan(d^
(1/2)*(a+b/x)^(1/2)/(-a*d+b*c)^(1/2))/c^2/(-a*d+b*c)^(1/2)+x*(a+b/x)^(1/2)/
a/c
```

Rubi [A]
time = 0.06, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {382, 105, 162, 65, 214, 211}

$$-\frac{(2ad+bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}c^2}-\frac{2d^{3/2}\text{ArcTan}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^2\sqrt{bc-ad}}+\frac{x\sqrt{a+\frac{b}{x}}}{ac}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b/x]*(c + d/x)),x]
```

```
[Out] (Sqrt[a + b/x]*x)/(a*c) - (2*d^(3/2)*ArcTan[(Sqrt[d]*Sqrt[a + b/x])/Sqrt[b*
c - a*d]])/(c^2*Sqrt[b*c - a*d]) - ((b*c + 2*a*d)*ArcTanh[Sqrt[a + b/x]/Sqr
t[a]])/(a^(3/2)*c^2)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
```

```
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}\,dx = -\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx}\,(c+dx)}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{\sqrt{a+\frac{b}{x}}\,x}{ac} + \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(bc+2ad)+\frac{bdx}{2}}{x\sqrt{a+bx}\,(c+dx)}\,dx, x, \frac{1}{x}\right)}{ac}$$

$$= \frac{\sqrt{a+\frac{b}{x}}\,x}{ac} - \frac{d^2\text{Subst}\left(\int \frac{1}{\sqrt{a+bx}\,(c+dx)}\,dx, x, \frac{1}{x}\right)}{c^2} + \frac{(bc+2ad)\text{Subst}\left(\int \frac{1}{x\sqrt{a+}}\right.}{2ac^2}$$

$$= \frac{\sqrt{a+\frac{b}{x}}\,x}{ac} - \frac{(2d^2)\,\text{Subst}\left(\int \frac{1}{c-\frac{ad}{b}+\frac{dx^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x}}\right)}{bc^2} + \frac{(bc+2ad)\text{Subst}\left(\int\right.}{a}$$

$$= \frac{\sqrt{a+\frac{b}{x}}\,x}{ac} - \frac{2d^{3/2}\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^2\sqrt{bc-ad}} - \frac{(bc+2ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}c^2}$$

Mathematica [A]
time = 0.24, size = 104, normalized size = 0.96

$$\frac{\frac{c\sqrt{a+\frac{b}{x}}\,x}{a} - \frac{2d^{3/2}\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{\sqrt{bc-ad}} - \frac{(bc+2ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}}}{c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b/x]*(c + d/x)),x]
[Out] ((c*Sqrt[a + b/x]*x)/a - (2*d^(3/2)*ArcTan[(Sqrt[d]*Sqrt[a + b/x])/Sqrt[b*c
 - a*d]])/Sqrt[b*c - a*d] - ((b*c + 2*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/
a^(3/2))/c^2
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 228 vs. 2(90) = 180.
time = 0.05, size = 229, normalized size = 2.12

method	result

default	$\frac{\left(2\sqrt{x\left(ax+b\right)}\, c^{2}\sqrt{a}\, \sqrt{\frac{d(ad-bc)}{c^{2}}}-2\ln\left(\frac{2\sqrt{x\left(ax+b\right)}\,\sqrt{a}+2ax+b}{2\sqrt{a}}\right)\sqrt{\frac{d(ad-bc)}{c^{2}}}\, acd-\ln\left(\frac{2\sqrt{x\left(ax+b\right)}\,\sqrt{}}{2\sqrt{a}}\right.\right.}{2a^{\frac{3}{2}}\sqrt{\frac{d(ad-bc)}{c^{2}}}\, c^{3}\sqrt{x\left(ax\right.}}$
risch	$\frac{ax+b}{ca\sqrt{\frac{ax+b}{x}}}+\frac{\left(-\frac{\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^{2}+bx}\right)d}{\sqrt{a}\, c^{2}}-\frac{\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^{2}+bx}\right)b}{2a^{\frac{3}{2}}c}-\frac{d^{2}\ln\left(\frac{\frac{2d(ad-bc)}{c^{2}}-\frac{(2ad-bc)\left(x+\frac{d}{c}\right)}{c}+2\sqrt{\frac{d(ad-}{c^{2}}}}{}\right.}{c}\right.}{x\sqrt{\frac{ax+b}{x}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c+d/x)/(a+1/x*b)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(2*(x*(a*x+b))^(1/2)*c^2*a^(1/2)*(d*(a*d-b*c)/c^2)^(1/2)-2*ln(1/2*(2*(x
*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*(d*(a*d-b*c)/c^2)^(1/2)*a*c*d-ln(
1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*(d*(a*d-b*c)/c^2)^(1/2)*
b*c^2-2*ln((2*(x*(a*x+b))^(1/2)*(d*(a*d-b*c)/c^2)^(1/2)*c-2*a*d*x+b*c*x-b*d
)/(c*x+d))*a^(3/2)*d^2)*x*((a*x+b)/x)^(1/2)/a^(3/2)/(d*(a*d-b*c)/c^2)^(1/2)
/c^3/(x*(a*x+b))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d/x)/(a+b/x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(a + b/x)*(c + d/x)), x)
```

Fricas [A]
time = 3.53, size = 542, normalized size = 5.02

$$\left[\frac{2a^{2}d\sqrt{-\frac{d}{bc-ad}}\log\left(-\frac{2(bc-ad)x\sqrt{-\frac{d}{bc-ad}}\sqrt{\frac{ax+b}{x}}-bd+(bc-2ad)x}{cx+d}\right)+2acx\sqrt{\frac{ax+b}{x}}+(bc+2ad)\sqrt{a}\log\left(2ax-2\sqrt{a}\,x\sqrt{\frac{ax+b}{x}}+b\right)}{2a^{2}c^{2}}, \frac{a^{2}d\sqrt{-\frac{d}{bc-ad}}\log\left(-\frac{2(bc-ad)x\sqrt{-\frac{d}{bc-ad}}\sqrt{\frac{ax+b}{x}}-bd+(bc-2ad)x}{cx+d}\right)+acx\sqrt{\frac{ax+b}{x}}+(bc+2ad)\sqrt{-a}\arctan\left(\frac{\sqrt{-a}\sqrt{\frac{ax+b}{x}}}{a}\right)}{a^{2}c^{2}}, -\frac{4a^{2}d\sqrt{\frac{d}{bc-ad}}\arctan\left(-\frac{(bc-ad)x\sqrt{\frac{d}{bc-ad}}\sqrt{\frac{ax+b}{x}}}{adx+bd}\right)-2acx\sqrt{\frac{ax+b}{x}}-(bc+2ad)\sqrt{a}\log\left(2ax-2\sqrt{a}\,x\sqrt{\frac{ax+b}{x}}+b\right)}{2a^{2}c^{2}}, -\frac{2a^{2}d\sqrt{\frac{d}{bc-ad}}\arctan\left(-\frac{(bc-ad)x\sqrt{\frac{d}{bc-ad}}\sqrt{\frac{ax+b}{x}}}{adx+bd}\right)-acx\sqrt{\frac{ax+b}{x}}-(bc+2ad)\sqrt{-a}\arctan\left(\frac{\sqrt{-a}\sqrt{\frac{ax+b}{x}}}{a}\right)}{a^{2}c^{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d/x)/(a+b/x)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(2*a^2*d*sqrt(-d/(b*c - a*d))*log(-(2*(b*c - a*d)*x*sqrt(-d/(b*c - a*d
))*sqrt((a*x + b)/x) - b*d + (b*c - 2*a*d)*x)/(c*x + d)) + 2*a*c*x*sqrt((a*
x + b)/x) + (b*c + 2*a*d)*sqrt(a)*log(2*a*x - 2*sqrt(a)*x*sqrt((a*x + b)/x)
 + b))/(a^2*c^2), (a^2*d*sqrt(-d/(b*c - a*d))*log(-(2*(b*c - a*d)*x*sqrt(-d
/(b*c - a*d))*sqrt((a*x + b)/x) - b*d + (b*c - 2*a*d)*x)/(c*x + d)) + a*c*x
*sqrt((a*x + b)/x) + (b*c + 2*a*d)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/
x)/a))/(a^2*c^2), -1/2*(4*a^2*d*sqrt(d/(b*c - a*d))*arctan(-(b*c - a*d)*x*s
qrt(d/(b*c - a*d))*sqrt((a*x + b)/x)/(a*d*x + b*d)) - 2*a*c*x*sqrt((a*x + b
)/x) - (b*c + 2*a*d)*sqrt(a)*log(2*a*x - 2*sqrt(a)*x*sqrt((a*x + b)/x) + b)
)/(a^2*c^2), -(2*a^2*d*sqrt(d/(b*c - a*d))*arctan(-(b*c - a*d)*x*sqrt(d/(b*
c - a*d))*sqrt((a*x + b)/x)/(a*d*x + b*d)) - a*c*x*sqrt((a*x + b)/x) - (b*c
 + 2*a*d)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a))/(a^2*c^2)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{a+\frac{b}{x}}\,(cx+d)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d/x)/(a+b/x)**(1/2),x)
```

```
[Out] Integral(x/(sqrt(a + b/x)*(c*x + d)), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d/x)/(a+b/x)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [**B**]
time = 1.98, size = 1183, normalized size = 10.95

$$\frac{x\sqrt{a+\frac{b}{x}}}{a\,c}-\frac{\mathrm{atanh}\left(\frac{12\,b^4\,d^4\,\sqrt{a+\frac{b}{x}}}{\sqrt{a^3}\,\left(\frac{12\,b^4\,d^4}{a}+\frac{10\,b^5\,c\,d^3}{a^2}+\frac{2\,b^6\,c^2\,d^2}{a^3}\right)}+\frac{10\,b^5\,d^3\,\sqrt{a+\frac{b}{x}}}{\sqrt{a^3}\,\left(\frac{10\,b^5\,d^3}{a}+\frac{12\,b^4\,d^4}{c}+\frac{2\,b^6\,c\,d^2}{a^2}\right)}+\frac{2\,b^6\,d^2\,\sqrt{a+\frac{b}{x}}}{\sqrt{a^3}\,\left(\frac{2\,b^6\,d^2}{a}+\frac{10\,b^5\,d^3}{c}+\frac{12\,a\,b^4\,d^4}{c^2}\right)}\right)\,\left(2\,a\,d+b\,c\right)}{c^2\,\sqrt{a^3}}-\frac{\mathrm{atan}\left(\frac{\frac{\left(\frac{\left(\frac{2\,\left(2\,a^2\,b^3\,c^4\,d^3+2\,a\,b^4\,c^5\,d^2\right)}{a^2\,c^3}-\frac{2\,\left(4\,a^2\,b^3\,c^5\,d^2-8\,a^3\,b^2\,c^4\,d^3\right)\,\sqrt{a+\frac{b}{x}}\,\sqrt{a\,d^4-b\,c\,d^3}}{a^2\,c^2\,\left(b\,c^3-a\,c^2\,d\right)}\right)\,\sqrt{a\,d^4-b\,c\,d^3}}{b\,c^3-a\,c^2\,d}-\frac{2\,\sqrt{a+\frac{b}{x}}\,\left(8\,a^2\,b^2\,d^5+4\,a\,b^3\,c\,d^4+b^4\,c^2\,d^3\right)}{a^2\,c^2}\right)\,\sqrt{a\,d^4-b\,c\,d^3}\,1{}\mathrm{i}}{b\,c^3-a\,c^2\,d}-\frac{\left(\frac{\left(\frac{2\,\left(2\,a^2\,b^3\,c^4\,d^3+2\,a\,b^4\,c^5\,d^2\right)}{a^2\,c^3}+\frac{2\,\left(4\,a^2\,b^3\,c^5\,d^2-8\,a^3\,b^2\,c^4\,d^3\right)\,\sqrt{a+\frac{b}{x}}\,\sqrt{a\,d^4-b\,c\,d^3}}{a^2\,c^2\,\left(b\,c^3-a\,c^2\,d\right)}\right)\,\sqrt{a\,d^4-b\,c\,d^3}}{b\,c^3-a\,c^2\,d}+\frac{2\,\sqrt{a+\frac{b}{x}}\,\left(8\,a^2\,b^2\,d^5+4\,a\,b^3\,c\,d^4+b^4\,c^2\,d^3\right)}{a^2\,c^2}\right)\,\sqrt{a\,d^4-b\,c\,d^3}\,1{}\mathrm{i}}{b\,c^3-a\,c^2\,d}}{\frac{\left(\frac{\left(\frac{2\,\left(2\,a^2\,b^3\,c^4\,d^3+2\,a\,b^4\,c^5\,d^2\right)}{a^2\,c^3}-\frac{2\,\left(4\,a^2\,b^3\,c^5\,d^2-8\,a^3\,b^2\,c^4\,d^3\right)\,\sqrt{a+\frac{b}{x}}\,\sqrt{a\,d^4-b\,c\,d^3}}{a^2\,c^2\,\left(b\,c^3-a\,c^2\,d\right)}\right)\,\sqrt{a\,d^4-b\,c\,d^3}}{b\,c^3-a\,c^2\,d}-\frac{2\,\sqrt{a+\frac{b}{x}}\,\left(8\,a^2\,b^2\,d^5+4\,a\,b^3\,c\,d^4+b^4\,c^2\,d^3\right)}{a^2\,c^2}\right)\,\sqrt{a\,d^4-b\,c\,d^3}}{b\,c^3-a\,c^2\,d}-\frac{4\,\left(c\,b^4\,d^4+2\,a\,b^3\,d^5\right)}{a^2\,c^3}+\frac{\left(\frac{\left(\frac{2\,\left(2\,a^2\,b^3\,c^4\,d^3+2\,a\,b^4\,c^5\,d^2\right)}{a^2\,c^3}+\frac{2\,\left(4\,a^2\,b^3\,c^5\,d^2-8\,a^3\,b^2\,c^4\,d^3\right)\,\sqrt{a+\frac{b}{x}}\,\sqrt{a\,d^4-b\,c\,d^3}}{a^2\,c^2\,\left(b\,c^3-a\,c^2\,d\right)}\right)\,\sqrt{a\,d^4-b\,c\,d^3}}{b\,c^3-a\,c^2\,d}+\frac{2\,\sqrt{a+\frac{b}{x}}\,\left(8\,a^2\,b^2\,d^5+4\,a\,b^3\,c\,d^4+b^4\,c^2\,d^3\right)}{a^2\,c^2}\right)\,\sqrt{a\,d^4-b\,c\,d^3}}{b\,c^3-a\,c^2\,d}}\right)\,\sqrt{a\,d^4-b\,c\,d^3}\,2{}\mathrm{i}}{b\,c^3-a\,c^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b/x)^(1/2)*(c + d/x)),x)

[Out] (x*(a + b/x)^(1/2))/(a*c) - (atan(((((((2*(2*a*b^4*c^5*d^2 + 2*a^2*b^3*c^4*
d^3))/(a^2*c^3) - (2*(4*a^2*b^3*c^5*d^2 - 8*a^3*b^2*c^4*d^3)*(a + b/x)^(1/2
)*(a*d^4 - b*c*d^3)^(1/2))/(a^2*c^2*(b*c^3 - a*c^2*d)))*(a*d^4 - b*c*d^3)^(
1/2))/(b*c^3 - a*c^2*d) - (2*(a + b/x)^(1/2)*(8*a^2*b^2*d^5 + b^4*c^2*d^3 +
 4*a*b^3*c*d^4))/(a^2*c^2))*(a*d^4 - b*c*d^3)^(1/2)*1i)/(b*c^3 - a*c^2*d) -
 (((((2*(2*a*b^4*c^5*d^2 + 2*a^2*b^3*c^4*d^3))/(a^2*c^3) + (2*(4*a^2*b^3*c^
5*d^2 - 8*a^3*b^2*c^4*d^3)*(a + b/x)^(1/2)*(a*d^4 - b*c*d^3)^(1/2))/(a^2*c^
2*(b*c^3 - a*c^2*d)))*(a*d^4 - b*c*d^3)^(1/2))/(b*c^3 - a*c^2*d) + (2*(a +
b/x)^(1/2)*(8*a^2*b^2*d^5 + b^4*c^2*d^3 + 4*a*b^3*c*d^4))/(a^2*c^2))*(a*d^4
 - b*c*d^3)^(1/2)*1i)/(b*c^3 - a*c^2*d))/((((((2*(2*a*b^4*c^5*d^2 + 2*a^2*b
^3*c^4*d^3))/(a^2*c^3) - (2*(4*a^2*b^3*c^5*d^2 - 8*a^3*b^2*c^4*d^3)*(a + b/
x)^(1/2)*(a*d^4 - b*c*d^3)^(1/2))/(a^2*c^2*(b*c^3 - a*c^2*d)))*(a*d^4 - b*c
*d^3)^(1/2))/(b*c^3 - a*c^2*d) - (2*(a + b/x)^(1/2)*(8*a^2*b^2*d^5 + b^4*c^
2*d^3 + 4*a*b^3*c*d^4))/(a^2*c^2))*(a*d^4 - b*c*d^3)^(1/2))/(b*c^3 - a*c^2*
d) - (4*(2*a*b^3*d^5 + b^4*c*d^4))/(a^2*c^3) + (((((2*(2*a*b^4*c^5*d^2 + 2*
a^2*b^3*c^4*d^3))/(a^2*c^3) + (2*(4*a^2*b^3*c^5*d^2 - 8*a^3*b^2*c^4*d^3)*(a
 + b/x)^(1/2)*(a*d^4 - b*c*d^3)^(1/2))/(a^2*c^2*(b*c^3 - a*c^2*d)))*(a*d^4
- b*c*d^3)^(1/2))/(b*c^3 - a*c^2*d) + (2*(a + b/x)^(1/2)*(8*a^2*b^2*d^5 + b
^4*c^2*d^3 + 4*a*b^3*c*d^4))/(a^2*c^2))*(a*d^4 - b*c*d^3)^(1/2))/(b*c^3 - a
*c^2*d)))*(a*d^4 - b*c*d^3)^(1/2)*2i)/(b*c^3 - a*c^2*d) - (atanh((12*b^4*d^
4*(a + b/x)^(1/2))/((a^3)^(1/2)*((12*b^4*d^4)/a + (10*b^5*c*d^3)/a^2 + (2*b
^6*c^2*d^2)/a^3)) + (10*b^5*d^3*(a + b/x)^(1/2))/((a^3)^(1/2)*((10*b^5*d^3)
/a + (12*b^4*d^4)/c + (2*b^6*c*d^2)/a^2)) + (2*b^6*d^2*(a + b/x)^(1/2))/((a
^3)^(1/2)*((2*b^6*d^2)/a + (10*b^5*d^3)/c + (12*a*b^4*d^4)/c^2)))*(2*a*d +
b*c))/(c^2*(a^3)^(1/2))
```

3.250 $\int \frac{1}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2}\,dx$

Optimal. Leaf size=172

$$\frac{d(bc-2ad)\sqrt{a+\frac{b}{x}}}{ac^2(bc-ad)\left(c+\frac{d}{x}\right)}+\frac{\sqrt{a+\frac{b}{x}}\,x}{ac\left(c+\frac{d}{x}\right)}-\frac{d^{3/2}(5bc-4ad)\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^3(bc-ad)^{3/2}}-\frac{(bc+4ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}c^3}$$

```
[Out] -d^(3/2)*(-4*a*d+5*b*c)*arctan(d^(1/2)*(a+b/x)^(1/2)/(-a*d+b*c)^(1/2))/c^3/
(-a*d+b*c)^(3/2)-(4*a*d+b*c)*arctanh((a+b/x)^(1/2)/a^(1/2))/a^(3/2)/c^3+d*(
-2*a*d+b*c)*(a+b/x)^(1/2)/a/c^2/(-a*d+b*c)/(c+d/x)+x*(a+b/x)^(1/2)/a/c/(c+d
/x)
```

Rubi [A]
time = 0.14, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {382, 105, 156, 162, 65, 214, 211}

$$-\frac{(4ad+bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}c^3}-\frac{d^{3/2}(5bc-4ad)\mathrm{ArcTan}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^3(bc-ad)^{3/2}}+\frac{d\sqrt{a+\frac{b}{x}}\,(bc-2ad)}{ac^2\left(c+\frac{d}{x}\right)(bc-ad)}+\frac{x\sqrt{a+\frac{b}{x}}}{ac\left(c+\frac{d}{x}\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b/x]*(c + d/x)^2),x]
```

```
[Out] (d*(b*c - 2*a*d)*Sqrt[a + b/x])/(a*c^2*(b*c - a*d)*(c + d/x)) + (Sqrt[a + b
/x]*x)/(a*c*(c + d/x)) - (d^(3/2)*(5*b*c - 4*a*d)*ArcTan[(Sqrt[d]*Sqrt[a +
b/x])/Sqrt[b*c - a*d]])/(c^3*(b*c - a*d)^(3/2)) - ((b*c + 4*a*d)*ArcTanh[Sq
rt[a + b/x]/Sqrt[a]])/(a^(3/2)*c^3)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
```

```
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2}\,dx = -\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx}\,(c+dx)^2}\,dx, x, \frac{1}{x}\right)$$
$$= \frac{\sqrt{a+\frac{b}{x}}\,x}{ac\left(c+\frac{d}{x}\right)} + \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(bc+4ad)+\frac{3bdx}{2}}{x\sqrt{a+bx}\,(c+dx)^2}\,dx, x, \frac{1}{x}\right)}{ac}$$
$$= \frac{d(bc-2ad)\sqrt{a+\frac{b}{x}}}{ac^2(bc-ad)\left(c+\frac{d}{x}\right)} + \frac{\sqrt{a+\frac{b}{x}}\,x}{ac\left(c+\frac{d}{x}\right)} - \frac{\text{Subst}\left(\int \frac{-\frac{1}{2}(bc-ad)(bc+4ad)-\frac{1}{2}bd(bc-2ad)x}{x\sqrt{a+bx}\,(c+dx)}\,dx,\right.}{ac^2(bc-ad)}$$
$$= \frac{d(bc-2ad)\sqrt{a+\frac{b}{x}}}{ac^2(bc-ad)\left(c+\frac{d}{x}\right)} + \frac{\sqrt{a+\frac{b}{x}}\,x}{ac\left(c+\frac{d}{x}\right)} - \frac{\left(d^2(5bc-4ad)\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx}\,(c+dx)}\right.}{2c^3(bc-ad)}$$
$$= \frac{d(bc-2ad)\sqrt{a+\frac{b}{x}}}{ac^2(bc-ad)\left(c+\frac{d}{x}\right)} + \frac{\sqrt{a+\frac{b}{x}}\,x}{ac\left(c+\frac{d}{x}\right)} - \frac{\left(d^2(5bc-4ad)\right)\text{Subst}\left(\int \frac{1}{c-\frac{ad}{b}+\frac{dx^2}{b}}\,dx, x,\right.}{bc^3(bc-ad)}$$
$$= \frac{d(bc-2ad)\sqrt{a+\frac{b}{x}}}{ac^2(bc-ad)\left(c+\frac{d}{x}\right)} + \frac{\sqrt{a+\frac{b}{x}}\,x}{ac\left(c+\frac{d}{x}\right)} - \frac{d^{3/2}(5bc-4ad)\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^3(bc-ad)^{3/2}}$$

Mathematica [A]
time = 0.55, size = 149, normalized size = 0.87

$$\frac{\frac{c\sqrt{a+\frac{b}{x}}\,x(-bc(d+cx)+ad(2d+cx))}{a(-bc+ad)(d+cx)} + \frac{d^{3/2}(-5bc+4ad)\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{3/2}} - \frac{(bc+4ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}}}{c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b/x]*(c + d/x)^2),x]
```

```
[Out] ((c*Sqrt[a + b/x]*x*(-(b*c*(d + c*x)) + a*d*(2*d + c*x)))/(a*(-(b*c) + a*d)
*(d + c*x)) + (d^(3/2)*(-5*b*c + 4*a*d)*ArcTan[(Sqrt[d]*Sqrt[a + b/x])/Sqrt
[b*c - a*d]])/(b*c - a*d)^(3/2) - ((b*c + 4*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt
[a]])/a^(3/2))/c^3
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1136 vs. $2(152) = 304$.

time = 0.08, size = 1137, normalized size = 6.61 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c+d/x)^2/(a+1/x*b)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*((a*x+b)/x)^(1/2)*x*(-2*(x*(a*x+b))^(1/2)*a^(7/2)*(d*(a*d-b*c)/c^2)^(1/
2)*c^4*d*x^2-4*a^(9/2)*ln((2*(x*(a*x+b))^(1/2)*(d*(a*d-b*c)/c^2)^(1/2)*c-2*
a*d*x+b*c*x-b*d)/(c*x+d))*c*d^4*x+2*(x*(a*x+b))^(1/2)*a^(7/2)*(d*(a*d-b*c)/
c^2)^(1/2)*c^3*d^2*x-4*a^(9/2)*ln((2*(x*(a*x+b))^(1/2)*(d*(a*d-b*c)/c^2)^(1
/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*d^5+9*a^(7/2)*ln((2*(x*(a*x+b))^(1/2)*(d*
(a*d-b*c)/c^2)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*b*c^2*d^3*x+2*(x*(a*x+b)
)^(3/2)*a^(5/2)*(d*(a*d-b*c)/c^2)^(1/2)*c^4*d+4*(x*(a*x+b))^(1/2)*a^(7/2)*(
d*(a*d-b*c)/c^2)^(1/2)*c^2*d^3-6*(x*(a*x+b))^(1/2)*a^(5/2)*(d*(a*d-b*c)/c^2
)^(1/2)*b*c^4*d*x+9*a^(7/2)*ln((2*(x*(a*x+b))^(1/2)*(d*(a*d-b*c)/c^2)^(1/2)
*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*b*c*d^4-5*a^(5/2)*ln((2*(x*(a*x+b))^(1/2)*(d
*(a*d-b*c)/c^2)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*b^2*c^3*d^2*x-6*(x*(a*x
+b))^(1/2)*a^(5/2)*(d*(a*d-b*c)/c^2)^(1/2)*b*c^3*d^2+2*(x*(a*x+b))^(1/2)*a^
(3/2)*(d*(a*d-b*c)/c^2)^(1/2)*b^2*c^5*x-5*a^(5/2)*ln((2*(x*(a*x+b))^(1/2)*(
d*(a*d-b*c)/c^2)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*b^2*c^2*d^3+2*(x*(a*x+
b))^(1/2)*a^(3/2)*(d*(a*d-b*c)/c^2)^(1/2)*b^2*c^4*d-4*ln(1/2*(2*(x*(a*x+b))
^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^4*(d*(a*d-b*c)/c^2)^(1/2)*c^2*d^3*x+7*ln
(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^3*(d*(a*d-b*c)/c^2)^(
1/2)*b*c^3*d^2*x-2*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^
2*(d*(a*d-b*c)/c^2)^(1/2)*b^2*c^4*d*x-ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2
*a*x+b)/a^(1/2))*a*(d*(a*d-b*c)/c^2)^(1/2)*b^3*c^5*x-4*ln(1/2*(2*(x*(a*x+b)
)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^4*(d*(a*d-b*c)/c^2)^(1/2)*c*d^4+7*ln(1/
2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^3*(d*(a*d-b*c)/c^2)^(1/2
)*b*c^2*d^3-2*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^2*(d*
(a*d-b*c)/c^2)^(1/2)*b^2*c^3*d^2-ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+
b)/a^(1/2))*a*(d*(a*d-b*c)/c^2)^(1/2)*b^3*c^4*d)/c^4/(x*(a*x+b))^(1/2)/(a*d
-b*c)^2/(c*x+d)/a^(5/2)/(d*(a*d-b*c)/c^2)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d/x)^2/(a+b/x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(a + b/x)*(c + d/x)^2), x)
```

Fricas [A]
time = 3.39, size = 1163, normalized size = 6.76

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d/x)^2/(a+b/x)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*((b^2*c^2*d + 3*a*b*c*d^2 - 4*a^2*d^3 + (b^2*c^3 + 3*a*b*c^2*d - 4*a^2
*c*d^2)*x)*sqrt(a)*log(2*a*x - 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) + (5*a^2*
b*c*d^2 - 4*a^3*d^3 + (5*a^2*b*c^2*d - 4*a^3*c*d^2)*x)*sqrt(-d/(b*c - a*d))
*log(-(2*(b*c - a*d)*x*sqrt(-d/(b*c - a*d))*sqrt((a*x + b)/x) - b*d + (b*c
- 2*a*d)*x)/(c*x + d)) + 2*((a*b*c^3 - a^2*c^2*d)*x^2 + (a*b*c^2*d - 2*a^2*
c*d^2)*x)*sqrt((a*x + b)/x))/(a^2*b*c^4*d - a^3*c^3*d^2 + (a^2*b*c^5 - a^3*
c^4*d)*x), -1/2*(2*(5*a^2*b*c*d^2 - 4*a^3*d^3 + (5*a^2*b*c^2*d - 4*a^3*c*d^
2)*x)*sqrt(d/(b*c - a*d))*arctan(-(b*c - a*d)*x*sqrt(d/(b*c - a*d))*sqrt((a
*x + b)/x)/(a*d*x + b*d)) - (b^2*c^2*d + 3*a*b*c*d^2 - 4*a^2*d^3 + (b^2*c^3
 + 3*a*b*c^2*d - 4*a^2*c*d^2)*x)*sqrt(a)*log(2*a*x - 2*sqrt(a)*x*sqrt((a*x
+ b)/x) + b) - 2*((a*b*c^3 - a^2*c^2*d)*x^2 + (a*b*c^2*d - 2*a^2*c*d^2)*x)*
sqrt((a*x + b)/x))/(a^2*b*c^4*d - a^3*c^3*d^2 + (a^2*b*c^5 - a^3*c^4*d)*x),
 1/2*(2*(b^2*c^2*d + 3*a*b*c*d^2 - 4*a^2*d^3 + (b^2*c^3 + 3*a*b*c^2*d - 4*a
^2*c*d^2)*x)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) + (5*a^2*b*c*d^2
 - 4*a^3*d^3 + (5*a^2*b*c^2*d - 4*a^3*c*d^2)*x)*sqrt(-d/(b*c - a*d))*log(-(
2*(b*c - a*d)*x*sqrt(-d/(b*c - a*d))*sqrt((a*x + b)/x) - b*d + (b*c - 2*a*d
)*x)/(c*x + d)) + 2*((a*b*c^3 - a^2*c^2*d)*x^2 + (a*b*c^2*d - 2*a^2*c*d^2)*
x)*sqrt((a*x + b)/x))/(a^2*b*c^4*d - a^3*c^3*d^2 + (a^2*b*c^5 - a^3*c^4*d)*
x), -((5*a^2*b*c*d^2 - 4*a^3*d^3 + (5*a^2*b*c^2*d - 4*a^3*c*d^2)*x)*sqrt(d/
(b*c - a*d))*arctan(-(b*c - a*d)*x*sqrt(d/(b*c - a*d))*sqrt((a*x + b)/x)/(a
*d*x + b*d)) - (b^2*c^2*d + 3*a*b*c*d^2 - 4*a^2*d^3 + (b^2*c^3 + 3*a*b*c^2*
d - 4*a^2*c*d^2)*x)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) - ((a*b*c
^3 - a^2*c^2*d)*x^2 + (a*b*c^2*d - 2*a^2*c*d^2)*x)*sqrt((a*x + b)/x))/(a^2*
b*c^4*d - a^3*c^3*d^2 + (a^2*b*c^5 - a^3*c^4*d)*x)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{a + \frac{b}{x}}\,(cx+d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d/x)**2/(a+b/x)**(1/2),x)
```

```
[Out] Integral(x**2/(sqrt(a + b/x)*(c*x + d)**2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 501 vs. 2(152) = 304.
time = 1.88, size = 501, normalized size = 2.91

$$\frac{\left(10\,a^{\frac{3}{2}}bcd^2\arctan\left(\frac{\sqrt{a}\,d}{\sqrt{bcd-ad^2}}\right)-8\,a^{\frac{5}{2}}d^3\arctan\left(\frac{\sqrt{a}\,d}{\sqrt{bcd-ad^2}}\right)-\sqrt{bcd-ad^2}\,b^2c^2\log\left(|b|\right)-3\,\sqrt{bcd-ad^2}\,abcd\log\left(|b|\right)+4\,\sqrt{bcd-ad^2}\,a^2d^2\log\left(|b|\right)+2\,\sqrt{bcd-ad^2}\,a^2d^2\right)\mathrm{sgn}(x)}{2\left(\sqrt{bcd-ad^2}\,a^{\frac{3}{2}}bc^4-\sqrt{bcd-ad^2}\,a^{\frac{5}{2}}c^3d\right)}+\frac{(5\,bcd^2-4\,ad^3)\arctan\left(-\frac{\left(\sqrt{a}\,x-\sqrt{ax^2+bx}\right)c+\sqrt{a}\,d}{\sqrt{bcd-ad^2}}\right)}{\left(bc^4\mathrm{sgn}\left(x\right)-ac^3d\mathrm{sgn}\left(x\right)\right)\sqrt{bcd-ad^2}}+\frac{\left(\sqrt{a}\,x-\sqrt{ax^2+bx}\right)\sqrt{a}\,bcd^2-2\left(\sqrt{a}\,x-\sqrt{ax^2+bx}\right)a^{\frac{3}{2}}d^3-abd^3}{\left(\sqrt{a}\,bc^4\mathrm{sgn}\left(x\right)-a^{\frac{3}{2}}c^3d\mathrm{sgn}\left(x\right)\right)\left(\left(\sqrt{a}\,x-\sqrt{ax^2+bx}\right)^2c+2\left(\sqrt{a}\,x-\sqrt{ax^2+bx}\right)\sqrt{a}\,d+bd\right)}+\frac{\sqrt{ax^2+bx}}{ac^2\mathrm{sgn}\left(x\right)}+\frac{(bc+4\,ad)\log\left(\left|2\left(\sqrt{a}\,x-\sqrt{ax^2+bx}\right)\sqrt{a}+b\right|\right)}{2\,a^{\frac{3}{2}}c^3\mathrm{sgn}\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c+d/x)^2/(a+b/x)^(1/2),x, algorithm="giac")

[Out] 1/2*(10*a^(3/2)*b*c*d^2*arctan(sqrt(a)*d/sqrt(b*c*d - a*d^2)) - 8*a^(5/2)*d^3*arctan(sqrt(a)*d/sqrt(b*c*d - a*d^2)) - sqrt(b*c*d - a*d^2)*b^2*c^2*log(abs(b)) - 3*sqrt(b*c*d - a*d^2)*a*b*c*d*log(abs(b)) + 4*sqrt(b*c*d - a*d^2)*a^2*d^2*log(abs(b)) + 2*sqrt(b*c*d - a*d^2)*a^2*d^2)*sgn(x)/(sqrt(b*c*d - a*d^2)*a^(3/2)*b*c^4 - sqrt(b*c*d - a*d^2)*a^(5/2)*c^3*d) + (5*b*c*d^2 - 4*a*d^3)*arctan(-((sqrt(a)*x - sqrt(a*x^2 + b*x))*c + sqrt(a)*d)/sqrt(b*c*d - a*d^2))/((b*c^4*sgn(x) - a*c^3*d*sgn(x))*sqrt(b*c*d - a*d^2)) + ((sqrt(a)*x - sqrt(a*x^2 + b*x))*sqrt(a)*b*c*d^2 - 2*(sqrt(a)*x - sqrt(a*x^2 + b*x))*a^(3/2)*d^3 - a*b*d^3)/((sqrt(a)*b*c^4*sgn(x) - a^(3/2)*c^3*d*sgn(x))*((sqrt(a)*x - sqrt(a*x^2 + b*x))^2*c + 2*(sqrt(a)*x - sqrt(a*x^2 + b*x))*sqrt(a)*d + b*d)) + sqrt(a*x^2 + b*x)/(a*c^2*sgn(x)) + 1/2*(b*c + 4*a*d)*log(abs(2*(sqrt(a)*x - sqrt(a*x^2 + b*x))*sqrt(a) + b))/(a^(3/2)*c^3*sgn(x))

Mupad [B]
time = 3.54, size = 2500, normalized size = 14.53

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a + b/x)^(1/2)*(c + d/x)^2),x)

[Out] (((a + b/x)^(1/2)*(b^3*c^2 + 2*a^2*b*d^2 - 2*a*b^2*c*d))/(c^2*(a^2*d - a*b*c)) + (d*(a + b/x)^(3/2)*(b^2*c - 2*a*b*d))/(c^2*(a^2*d - a*b*c)))/((a + b/x)*(2*a*d - b*c) - d*(a + b/x)^2 - a^2*d + a*b*c) - (atan(((((2*(a + b/x)^(1/2)*(32*a^4*b^2*d^7 + b^6*c^4*d^3 + 6*a*b^5*c^3*d^4 - 64*a^3*b^3*c*d^6 + 26*a^2*b^4*c^2*d^5))/(a^2*b^2*c^6 + a^4*c^4*d^2 - 2*a^3*b*c^5*d) + (((4*a*b^6*c^9*d^2 + 4*a^2*b^5*c^8*d^3 - 16*a^3*b^4*c^7*d^4 + 8*a^4*b^3*c^6*d^5)/(a^2*b^2*c^8 + a^4*c^6*d^2 - 2*a^3*b*c^7*d) + ((a + b/x)^(1/2)*(4*a*d + b*c)*(4*a^2*b^5*c^9*d^2 - 16*a^3*b^4*c^8*d^3 + 20*a^4*b^3*c^7*d^4 - 8*a^5*b^2*c^6*d^5))/(c^3*(a^3)^(1/2)*(a^2*b^2*c^6 + a^4*c^4*d^2 - 2*a^3*b*c^5*d)))*(4*a*d + b*c))/(2*c^3*(a^3)^(1/2)))*(4*a*d + b*c)*1i)/(2*c^3*(a^3)^(1/2)) + (((2*(a + b/x)^(1/2)*(32*a^4*b^2*d^7 + b^6*c^4*d^3 + 6*a*b^5*c^3*d^4 - 64*a^3*b^3*c*d^6 + 26*a^2*b^4*c^2*d^5))/(a^2*b^2*c^6 + a^4*c^4*d^2 - 2*a^3*b*c^5*d) - (((4*a*b^6*c^9*d^2 + 4*a^2*b^5*c^8*d^3 - 16*a^3*b^4*c^7*d^4 + 8*a^4*b^3*c^6*d^5)/(a^2*b^2*c^8 + a^4*c^6*d^2 - 2*a^3*b*c^7*d) - ((a + b/x)^(1/2)*(4*a*d + b*c)*(4*a^2*b^5*c^9*d^2 - 16*a^3*b^4*c^8*d^3 + 20*a^4*b^3*c^7*d^4 - 8*a^5*b^2*c^6*d^5))/(c^3*(a^3)^(1/2)*(a^2*b^2*c^6 + a^4*c^4*d^2 - 2*a^3*b*c^5*d)))*(4*a*d + b*c))/(2*c^3*(a^3)^(1/2)))*(4*a*d + b*c)*1i)/(2*c^3*(a^3)^(1/2)))/((2*(32*a^3*b^3*d^7 + 5*b^6*c^3*d^4 + 6*a*b^5*c^2*d^5 - 48*a^2*b^4*c*d^6))/(a^2*b^2*c^8 + a^4*c^6*d^2 - 2*a^3*b*c^7*d) - (((2*(a + b/x)^(1/2)*(32*a^4*b^2*d^7 + b^6*c^4*d^3 + 6*a*b^5*c^3*d^4 - 64*a^3*b^3*c*d^6 + 26*a^2*b^4*c^2*d^5))/(a^2*b^2*c^6 + a^4*c^4*d^2 - 2*a^3*b*c^5*d) + (((4*a*b^6*c^9*

d^2 + 4*a^2*b^5*c^8*d^3 - 16*a^3*b^4*c^7*d^4 + 8*a^4*b^3*c^6*d^5)/(a^2*b^2*
c^8 + a^4*c^6*d^2 - 2*a^3*b*c^7*d) + ((a + b/x)^(1/2)*(4*a*d + b*c)*(4*a^2*
b^5*c^9*d^2 - 16*a^3*b^4*c^8*d^3 + 20*a^4*b^3*c^7*d^4 - 8*a^5*b^2*c^6*d^5))
/(c^3*(a^3)^(1/2)*(a^2*b^2*c^6 + a^4*c^4*d^2 - 2*a^3*b*c^5*d)))*(4*a*d + b*
c))/(2*c^3*(a^3)^(1/2)))*(4*a*d + b*c))/(2*c^3*(a^3)^(1/2)) + (((2*(a + b/x
)^(1/2)*(32*a^4*b^2*d^7 + b^6*c^4*d^3 + 6*a*b^5*c^3*d^4 - 64*a^3*b^3*c*d^6
+ 26*a^2*b^4*c^2*d^5))/(a^2*b^2*c^6 + a^4*c^4*d^2 - 2*a^3*b*c^5*d) - (((4*a
*b^6*c^9*d^2 + 4*a^2*b^5*c^8*d^3 - 16*a^3*b^4*c^7*d^4 + 8*a^4*b^3*c^6*d^5)/
(a^2*b^2*c^8 + a^4*c^6*d^2 - 2*a^3*b*c^7*d) - ((a + b/x)^(1/2)*(4*a*d + b*c
)*(4*a^2*b^5*c^9*d^2 - 16*a^3*b^4*c^8*d^3 + 20*a^4*b^3*c^7*d^4 - 8*a^5*b^2*
c^6*d^5))/(c^3*(a^3)^(1/2)*(a^2*b^2*c^6 + a^4*c^4*d^2 - 2*a^3*b*c^5*d)))*(4
*a*d + b*c))/(2*c^3*(a^3)^(1/2)))*(4*a*d + b*c))/(2*c^3*(a^3)^(1/2))))*(4*a
*d + b*c)*1i)/(c^3*(a^3)^(1/2)) - (atan((((d^3*(a*d - b*c)^3)^(1/2)*(4*a*d
- 5*b*c)*((2*(a + b/x)^(1/2)*(32*a^4*b^2*d^7 + b^6*c^4*d^3 + 6*a*b^5*c^3*d^
4 - 64*a^3*b^3*c*d^6 + 26*a^2*b^4*c^2*d^5))/(a^2*b^2*c^6 + a^4*c^4*d^2 - 2*
a^3*b*c^5*d) + ((d^3*(a*d - b*c)^3)^(1/2)*(4*a*d - 5*b*c)*((4*a*b^6*c^9*d^2
+ 4*a^2*b^5*c^8*d^3 - 16*a^3*b^4*c^7*d^4 + 8*a^4*b^3*c^6*d^5)/(a^2*b^2*c^8
+ a^4*c^6*d^2 - 2*a^3*b*c^7*d) + ((d^3*(a*d - b*c)^3)^(1/2)*(a + b/x)^(1/2
)*(4*a*d - 5*b*c)*(4*a^2*b^5*c^9*d^2 - 16*a^3*b^4*c^8*d^3 + 20*a^4*b^3*c^7*
d^4 - 8*a^5*b^2*c^6*d^5))/((a^2*b^2*c^6 + a^4*c^4*d^2 - 2*a^3*b*c^5*d)*(b^3
*c^6 - a^3*c^3*d^3 + 3*a^2*b*c^4*d^2 - 3*a*b^2*c^5*d))))/(2*(b^3*c^6 - a^3*
c^3*d^3 + 3*a^2*b*c^4*d^2 - 3*a*b^2*c^5*d)))*1i)/(2*(b^3*c^6 - a^3*c^3*d^3
+ 3*a^2*b*c^4*d^2 - 3*a*b^2*c^5*d)) + ((d^3*(a*d - b*c)^3)^(1/2)*(4*a*d - 5
*b*c)*((2*(a + b/x)^(1/2)*(32*a^4*b^2*d^7 + b^6*c^4*d^3 + 6*a*b^5*c^3*d^4 -
64*a^3*b^3*c*d^6 + 26*a^2*b^4*c^2*d^5))/(a^2*b^2*c^6 + a^4*c^4*d^2 - 2*a^3
*b*c^5*d) - ((d^3*(a*d - b*c)^3)^(1/2)*(4*a*d - 5*b*c)*((4*a*b^6*c^9*d^2 +
4*a^2*b^5*c^8*d^3 - 16*a^3*b^4*c^7*d^4 + 8*a^4*b^3*c^6*d^5)/(a^2*b^2*c^8 +
a^4*c^6*d^2 - 2*a^3*b*c^7*d) - ((d^3*(a*d - b*c)^3)^(1/2)*(a + b/x)^(1/2)*(
4*a*d - 5*b*c)*(4*a^2*b^5*c^9*d^2 - 16*a^3*b^4*c^8*d^3 + 20*a^4*b^3*c^7*d^4
- 8*a^5*b^2*c^6*d^5))/((a^2*b^2*c^6 + a^4*c^4*d^2 - 2*a^3*b*c^5*d)*(b^3*c^
6 - a^3*c^3*d^3 + 3*a^2*b*c^4*d^2 - 3*a*b^2*c^5*d))))/(2*(b^3*c^6 - a^3*c^3
*d^3 + 3*a^2*b*c^4*d^2 - 3*a*b^2*c^5*d)))*1i)/(2*(b^3*c^6 - a^3*c^3*d^3 + 3
*a^2*b*c^4*d^2 - 3*a*b^2*c^5*d)))/((2*(32*a^3*b^3*d^7 + 5*b^6*c^3*d^4 + 6*a
*b^5*c^2*d^5 - 48*a^2*b^4*c*d^6))/(a^2*b^2*c^8 + a^4*c^6*d^2 - 2*a^3*b*c^7*
d) - ((d^3*(a*d - b*c)^3)^(1/2)*(4*a*d - 5*b*c)*((2*(a + b/x)^(1/2)*(32*a^4
*b^2*d^7 + b^6*c^4*d^3 + 6*a*b^5*c^3*d^4 - 64*a^3*b^3*c*d^6 + 26*a^2*b^4*c^
2*d^5))/(a^2*b^2*c^6 + a^4*c^4*d^2 - 2*a^3*b*c^5*d) + ((d^3*(a*d - b*c)^3)^
(1/2)*(4*a*d - 5*b*c)*((4*a*b^6*c^9*d^2 + 4*a^2*b^5*c^8*d^3 - 16*a^3*b^4*c^
7*d^4 + 8*a^4*b^3*c^6*d^5)/(a^2*b^2*c^8 + a^4*c^6*d^2 - 2*a^3*b*c^7*d) + ((
d^3*(a*d - b*c)^3)^(1/2)*(a + b/x)^(1/2)*(4*a*d - 5*b*c)*(4*a^2*b^5*c^9*d^2
- 16*a^3*b^4*c^8*d^3 + 20*a^4*b^3*c^7*d^4 - 8*a^5*b^2*c^6*d^5))/((a^2*b^2*
c^6 + a^4*c^4*d^2 - 2*a^3*b*c^5*d)*(b^3*c^6 - a^3*c^3*d^3 + 3*a^2*b*c^4*d^2
- 3*a*b^2*c^5*d))))/(2*(b^3*c^6 - a^3*c^3*d^3 + 3*a^2*b*c^4*d^2 - 3*a*b^2*
c^5*d))))/(2*(b^3*c^6 - a^3*c^3*d^3 + 3*a^2*b*c^4*d^2 - 3*a*b^2*c^5*d)) + (
(d^3*(a*d - b*c)^3)^(1/2)*(4*a*d - 5*b*c)*((2*(a + b/x)^(1/2)*(32*a^4*b^2*d

```
^7 + b^6*c^4*d^3 + 6*a*b^5*c^3*d^4 - 64*a^3*b^3...
```

3.251 $\int \frac{1}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^3}\,dx$

Optimal. Leaf size=250

$$\frac{d(2bc-3ad)\sqrt{a+\frac{b}{x}}}{2ac^2(bc-ad)\left(c+\frac{d}{x}\right)^2}+\frac{d(bc-4ad)(4bc-3ad)\sqrt{a+\frac{b}{x}}}{4ac^3(bc-ad)^2\left(c+\frac{d}{x}\right)}+\frac{\sqrt{a+\frac{b}{x}}\,x}{ac\left(c+\frac{d}{x}\right)^2}-\frac{d^{3/2}(35b^2c^2-56abcd+24a^2d^2)\,\text{ta}}{4c^4(bc-ad)^5}$$

```
[Out] -1/4*d^(3/2)*(24*a^2*d^2-56*a*b*c*d+35*b^2*c^2)*arctan(d^(1/2)*(a+b/x)^(1/2
)/(-a*d+b*c)^(1/2))/c^4/(-a*d+b*c)^(5/2)-(6*a*d+b*c)*arctanh((a+b/x)^(1/2)/
a^(1/2))/a^(3/2)/c^4+1/2*d*(-3*a*d+2*b*c)*(a+b/x)^(1/2)/a/c^2/(-a*d+b*c)/(c
+d/x)^2+1/4*d*(-4*a*d+b*c)*(-3*a*d+4*b*c)*(a+b/x)^(1/2)/a/c^3/(-a*d+b*c)^2/
(c+d/x)+x*(a+b/x)^(1/2)/a/c/(c+d/x)^2
```

Rubi [A]

time = 0.25, antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {382, 105, 156, 162, 65, 214, 211}

$$-\frac{(6ad+bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}c^4}-\frac{d^{3/2}(24a^2d^2-56abcd+35b^2c^2)\,\text{ArcTan}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{4c^4(bc-ad)^{5/2}}+\frac{d\sqrt{a+\frac{b}{x}}\,(bc-4ad)(4bc-3ad)}{4ac^3\left(c+\frac{d}{x}\right)(bc-ad)^2}+\frac{d\sqrt{a+\frac{b}{x}}\,(2bc-3ad)}{2ac^2\left(c+\frac{d}{x}\right)^2(bc-ad)}+\frac{x\sqrt{a+\frac{b}{x}}}{ac\left(c+\frac{d}{x}\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b/x]*(c + d/x)^3),x]
```

```
[Out] (d*(2*b*c - 3*a*d)*Sqrt[a + b/x])/(2*a*c^2*(b*c - a*d)*(c + d/x)^2) + (d*(b
*c - 4*a*d)*(4*b*c - 3*a*d)*Sqrt[a + b/x])/(4*a*c^3*(b*c - a*d)^2*(c + d/x)
) + (Sqrt[a + b/x]*x)/(a*c*(c + d/x)^2) - (d^(3/2)*(35*b^2*c^2 - 56*a*b*c*d
 + 24*a^2*d^2)*ArcTan[(Sqrt[d]*Sqrt[a + b/x])/Sqrt[b*c - a*d]])/(4*c^4*(b*c
 - a*d)^(5/2)) - ((b*c + 6*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/(a^(3/2)*c^
4)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^3}\,dx = -\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx}\,(c+dx)^3}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{\sqrt{a+\frac{b}{x}}\,x}{ac\left(c+\frac{d}{x}\right)^2} + \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(bc+6ad)+\frac{5bdx}{2}}{x\sqrt{a+bx}\,(c+dx)^3}\,dx, x, \frac{1}{x}\right)}{ac}$$

$$= \frac{d(2bc-3ad)\sqrt{a+\frac{b}{x}}}{2ac^2(bc-ad)\left(c+\frac{d}{x}\right)^2} + \frac{\sqrt{a+\frac{b}{x}}\,x}{ac\left(c+\frac{d}{x}\right)^2} - \frac{\text{Subst}\left(\int \frac{-(bc-ad)(bc+6ad)-\frac{3}{2}bd(2bc-3ad)x}{x\sqrt{a+bx}\,(c+dx)^2}\,dx \right.}{2ac^2(bc-ad)}$$

$$= \frac{d(2bc-3ad)\sqrt{a+\frac{b}{x}}}{2ac^2(bc-ad)\left(c+\frac{d}{x}\right)^2} + \frac{d(bc-4ad)(4bc-3ad)\sqrt{a+\frac{b}{x}}}{4ac^3(bc-ad)^2\left(c+\frac{d}{x}\right)} + \frac{\sqrt{a+\frac{b}{x}}\,x}{ac\left(c+\frac{d}{x}\right)^2} + \frac{\text{Su}\ldots}{\ldots}$$

$$= \frac{d(2bc-3ad)\sqrt{a+\frac{b}{x}}}{2ac^2(bc-ad)\left(c+\frac{d}{x}\right)^2} + \frac{d(bc-4ad)(4bc-3ad)\sqrt{a+\frac{b}{x}}}{4ac^3(bc-ad)^2\left(c+\frac{d}{x}\right)} + \frac{\sqrt{a+\frac{b}{x}}\,x}{ac\left(c+\frac{d}{x}\right)^2} + \frac{(bc\ldots}{\ldots}$$

$$= \frac{d(2bc-3ad)\sqrt{a+\frac{b}{x}}}{2ac^2(bc-ad)\left(c+\frac{d}{x}\right)^2} + \frac{d(bc-4ad)(4bc-3ad)\sqrt{a+\frac{b}{x}}}{4ac^3(bc-ad)^2\left(c+\frac{d}{x}\right)} + \frac{\sqrt{a+\frac{b}{x}}\,x}{ac\left(c+\frac{d}{x}\right)^2} + \frac{(bc\ldots}{\ldots}$$

$$= \frac{d(2bc-3ad)\sqrt{a+\frac{b}{x}}}{2ac^2(bc-ad)\left(c+\frac{d}{x}\right)^2} + \frac{d(bc-4ad)(4bc-3ad)\sqrt{a+\frac{b}{x}}}{4ac^3(bc-ad)^2\left(c+\frac{d}{x}\right)} + \frac{\sqrt{a+\frac{b}{x}}\,x}{ac\left(c+\frac{d}{x}\right)^2} - \frac{d^{3/}\ldots}{\ldots}$$

<u>**Mathematica**</u> [**A**]
time = 1.25, size = 215, normalized size = 0.86

$$\frac{\frac{c\sqrt{a+\frac{b}{x}}\,x\left(4b^2c^2(d+cx)^2+2a^2d^2\left(6d^2+9cdx+2c^2x^2\right)-abcd\left(19d^2+29cdx+8c^2x^2\right)\right)}{a(bc-ad)^2(d+cx)^2}-\frac{d^{3/2}\left(35b^2c^2-56abcd+24a^2d^2\right)\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{5/2}}-\frac{4(bc+6ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}}}{4c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b/x]*(c + d/x)^3),x]
```

```
[Out] ((c*Sqrt[a + b/x]*x*(4*b^2*c^2*(d + c*x)^2 + 2*a^2*d^2*(6*d^2 + 9*c*d*x + 2
*c^2*x^2) - a*b*c*d*(19*d^2 + 29*c*d*x + 8*c^2*x^2)))/(a*(b*c - a*d)^2*(d +
```

```
c*x)^2) - (d^(3/2)*(35*b^2*c^2 - 56*a*b*c*d + 24*a^2*d^2)*ArcTan[(Sqrt[d]*
Sqrt[a + b/x])/Sqrt[b*c - a*d]])/(b*c - a*d)^(5/2) - (4*(b*c + 6*a*d)*ArcTa
nh[Sqrt[a + b/x]/Sqrt[a]])/a^(3/2))/(4*c^4)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2268 vs. 2(222) = 444.
time = 0.10, size = 2269, normalized size = 9.08

method	result	size
risch	Expression too large to display	1288
default	Expression too large to display	2269

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c+d/x)^3/(a+1/x*b)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*((a*x+b)/x)^(1/2)*x*(-91*a^(7/2)*ln((2*(x*(a*x+b))^(1/2)*(d*(a*d-b*c)/c
^2)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*b^2*c^2*d^5+35*a^(5/2)*ln((2*(x*(a*
x+b))^(1/2)*(d*(a*d-b*c)/c^2)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*b^3*c^3*d
^4+24*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^2*(d*(a*d-b*c
)/c^2)^(1/2)*b^3*c^5*d^2*x+46*(x*(a*x+b))^(1/2)*a^(5/2)*(d*(a*d-b*c)/c^2)^(
1/2)*b^2*c^6*d*x^2+68*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))
*a^4*(d*(a*d-b*c)/c^2)^(1/2)*b*c^4*d^3*x^2-60*ln(1/2*(2*(x*(a*x+b))^(1/2)*a
^(1/2)+2*a*x+b)/a^(1/2))*a^3*(d*(a*d-b*c)/c^2)^(1/2)*b^2*c^5*d^2*x^2+12*ln(
1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^2*(d*(a*d-b*c)/c^2)^(1
/2)*b^3*c^6*d*x^2-102*(x*(a*x+b))^(1/2)*a^(7/2)*(d*(a*d-b*c)/c^2)^(1/2)*b*c
^4*d^3*x+92*(x*(a*x+b))^(1/2)*a^(5/2)*(d*(a*d-b*c)/c^2)^(1/2)*b^2*c^5*d^2*x
+136*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^4*(d*(a*d-b*c)
/c^2)^(1/2)*b*c^3*d^4*x-120*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^
(1/2))*a^3*(d*(a*d-b*c)/c^2)^(1/2)*b^2*c^4*d^3*x+22*(x*(a*x+b))^(1/2)*a^(7/
2)*(d*(a*d-b*c)/c^2)^(1/2)*b*c^6*d*x^3-22*(x*(a*x+b))^(3/2)*a^(5/2)*(d*(a*d
-b*c)/c^2)^(1/2)*b*c^6*d*x-18*(x*(a*x+b))^(1/2)*a^(7/2)*(d*(a*d-b*c)/c^2)^(
1/2)*b*c^5*d^2*x^2-12*(x*(a*x+b))^(1/2)*a^(9/2)*(d*(a*d-b*c)/c^2)^(1/2)*c^5
*d^2*x^3+12*(x*(a*x+b))^(3/2)*a^(7/2)*(d*(a*d-b*c)/c^2)^(1/2)*c^5*d^2*x-24*
ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^5*(d*(a*d-b*c)/c^2)
^(1/2)*c^3*d^4*x^2+80*a^(9/2)*ln((2*(x*(a*x+b))^(1/2)*(d*(a*d-b*c)/c^2)^(1/
2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*b*c^3*d^4*x^2-91*a^(7/2)*ln((2*(x*(a*x+b))
^(1/2)*(d*(a*d-b*c)/c^2)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*b^2*c^4*d^3*x^
2+35*a^(5/2)*ln((2*(x*(a*x+b))^(1/2)*(d*(a*d-b*c)/c^2)^(1/2)*c-2*a*d*x+b*c*
x-b*d)/(c*x+d))*b^3*c^5*d^2*x^2-18*(x*(a*x+b))^(3/2)*a^(5/2)*(d*(a*d-b*c)/c
^2)^(1/2)*b*c^5*d^2+36*(x*(a*x+b))^(1/2)*a^(9/2)*(d*(a*d-b*c)/c^2)^(1/2)*c^
3*d^4*x-16*(x*(a*x+b))^(1/2)*a^(3/2)*(d*(a*d-b*c)/c^2)^(1/2)*b^3*c^6*d*x-48
*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^5*(d*(a*d-b*c)/c^2
)^(1/2)*c^2*d^5*x+8*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a
*(d*(a*d-b*c)/c^2)^(1/2)*b^4*c^6*d*x+160*a^(9/2)*ln((2*(x*(a*x+b))^(1/2)*(d
*(a*d-b*c)/c^2)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*b*c^2*d^5*x-8*(x*(a*x+b
```

```
))^(1/2)*a^(3/2)*(d*(a*d-b*c)/c^2)^(1/2)*b^3*c^7*x^2-182*a^(7/2)*ln((2*(x*(
a*x+b))^(1/2)*(d*(a*d-b*c)/c^2)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*b^2*c^3
*d^4*x+70*a^(5/2)*ln((2*(x*(a*x+b))^(1/2)*(d*(a*d-b*c)/c^2)^(1/2)*c-2*a*d*x
+b*c*x-b*d)/(c*x+d))*b^3*c^4*d^3*x-62*(x*(a*x+b))^(1/2)*a^(7/2)*(d*(a*d-b*c
)/c^2)^(1/2)*b*c^3*d^4+46*(x*(a*x+b))^(1/2)*a^(5/2)*(d*(a*d-b*c)/c^2)^(1/2)
*b^2*c^4*d^3+68*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^4*(
d*(a*d-b*c)/c^2)^(1/2)*b*c^2*d^5-60*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a
*x+b)/a^(1/2))*a^3*(d*(a*d-b*c)/c^2)^(1/2)*b^2*c^3*d^4+12*ln(1/2*(2*(x*(a*x
+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^2*(d*(a*d-b*c)/c^2)^(1/2)*b^3*c^4*d^
3+4*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a*(d*(a*d-b*c)/c^
2)^(1/2)*b^4*c^7*x^2-24*a^(11/2)*ln((2*(x*(a*x+b))^(1/2)*(d*(a*d-b*c)/c^2)^
(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*c^2*d^5*x^2+8*(x*(a*x+b))^(3/2)*a^(7/2)
*(d*(a*d-b*c)/c^2)^(1/2)*c^4*d^3-48*a^(11/2)*ln((2*(x*(a*x+b))^(1/2)*(d*(a*
d-b*c)/c^2)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*c*d^6*x+24*(x*(a*x+b))^(1/2
)*a^(9/2)*(d*(a*d-b*c)/c^2)^(1/2)*c^2*d^5-8*(x*(a*x+b))^(1/2)*a^(3/2)*(d*(a
*d-b*c)/c^2)^(1/2)*b^3*c^5*d^2-24*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x
+b)/a^(1/2))*a^5*(d*(a*d-b*c)/c^2)^(1/2)*c*d^6+4*ln(1/2*(2*(x*(a*x+b))^(1/2
)*a^(1/2)+2*a*x+b)/a^(1/2))*a*(d*(a*d-b*c)/c^2)^(1/2)*b^4*c^5*d^2+80*a^(9/2
)*ln((2*(x*(a*x+b))^(1/2)*(d*(a*d-b*c)/c^2)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x
+d))*b*c*d^6-24*a^(11/2)*ln((2*(x*(a*x+b))^(1/2)*(d*(a*d-b*c)/c^2)^(1/2)*c-
2*a*d*x+b*c*x-b*d)/(c*x+d))*d^7)/c^5/(x*(a*x+b))^(1/2)/(a*d-b*c)^3/(c*x+d)^
2/a^(5/2)/(d*(a*d-b*c)/c^2)^(1/2)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d/x)^3/(a+b/x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(a + b/x)*(c + d/x)^3), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 568 vs. 2(222) = 444.
time = 4.36, size = 2307, normalized size = 9.23

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d/x)^3/(a+b/x)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(4*(b^3*c^3*d^2 + 4*a*b^2*c^2*d^3 - 11*a^2*b*c*d^4 + 6*a^3*d^5 + (b^3*
c^5 + 4*a*b^2*c^4*d - 11*a^2*b*c^3*d^2 + 6*a^3*c^2*d^3)*x^2 + 2*(b^3*c^4*d
+ 4*a*b^2*c^3*d^2 - 11*a^2*b*c^2*d^3 + 6*a^3*c*d^4)*x)*sqrt(a)*log(2*a*x -
2*sqrt(a)*x*sqrt((a*x + b)/x) + b) + (35*a^2*b^2*c^2*d^3 - 56*a^3*b*c*d^4 +
```

```
24*a^4*d^5 + (35*a^2*b^2*c^4*d - 56*a^3*b*c^3*d^2 + 24*a^4*c^2*d^3)*x^2 +
2*(35*a^2*b^2*c^3*d^2 - 56*a^3*b*c^2*d^3 + 24*a^4*c*d^4)*x)*sqrt(-d/(b*c -
a*d))*log(-(2*(b*c - a*d)*x*sqrt(-d/(b*c - a*d))*sqrt((a*x + b)/x) - b*d +
(b*c - 2*a*d)*x)/(c*x + d)) + 2*(4*(a*b^2*c^5 - 2*a^2*b*c^4*d + a^3*c^3*d^2
)*x^3 + (8*a*b^2*c^4*d - 29*a^2*b*c^3*d^2 + 18*a^3*c^2*d^3)*x^2 + (4*a*b^2*
c^3*d^2 - 19*a^2*b*c^2*d^3 + 12*a^3*c*d^4)*x)*sqrt((a*x + b)/x))/(a^2*b^2*c
^6*d^2 - 2*a^3*b*c^5*d^3 + a^4*c^4*d^4 + (a^2*b^2*c^8 - 2*a^3*b*c^7*d + a^4
*c^6*d^2)*x^2 + 2*(a^2*b^2*c^7*d - 2*a^3*b*c^6*d^2 + a^4*c^5*d^3)*x), 1/8*(
8*(b^3*c^3*d^2 + 4*a*b^2*c^2*d^3 - 11*a^2*b*c*d^4 + 6*a^3*d^5 + (b^3*c^5 +
4*a*b^2*c^4*d - 11*a^2*b*c^3*d^2 + 6*a^3*c^2*d^3)*x^2 + 2*(b^3*c^4*d + 4*a*
b^2*c^3*d^2 - 11*a^2*b*c^2*d^3 + 6*a^3*c*d^4)*x)*sqrt(-a)*arctan(sqrt(-a)*s
qrt((a*x + b)/x)/a) + (35*a^2*b^2*c^2*d^3 - 56*a^3*b*c*d^4 + 24*a^4*d^5 + (
35*a^2*b^2*c^4*d - 56*a^3*b*c^3*d^2 + 24*a^4*c^2*d^3)*x^2 + 2*(35*a^2*b^2*c
^3*d^2 - 56*a^3*b*c^2*d^3 + 24*a^4*c*d^4)*x)*sqrt(-d/(b*c - a*d))*log(-(2*(
b*c - a*d)*x*sqrt(-d/(b*c - a*d))*sqrt((a*x + b)/x) - b*d + (b*c - 2*a*d)*x
)/(c*x + d)) + 2*(4*(a*b^2*c^5 - 2*a^2*b*c^4*d + a^3*c^3*d^2)*x^3 + (8*a*b^
2*c^4*d - 29*a^2*b*c^3*d^2 + 18*a^3*c^2*d^3)*x^2 + (4*a*b^2*c^3*d^2 - 19*a^
2*b*c^2*d^3 + 12*a^3*c*d^4)*x)*sqrt((a*x + b)/x))/(a^2*b^2*c^6*d^2 - 2*a^3*
b*c^5*d^3 + a^4*c^4*d^4 + (a^2*b^2*c^8 - 2*a^3*b*c^7*d + a^4*c^6*d^2)*x^2 +
 2*(a^2*b^2*c^7*d - 2*a^3*b*c^6*d^2 + a^4*c^5*d^3)*x), -1/4*((35*a^2*b^2*c^
2*d^3 - 56*a^3*b*c*d^4 + 24*a^4*d^5 + (35*a^2*b^2*c^4*d - 56*a^3*b*c^3*d^2
+ 24*a^4*c^2*d^3)*x^2 + 2*(35*a^2*b^2*c^3*d^2 - 56*a^3*b*c^2*d^3 + 24*a^4*c
*d^4)*x)*sqrt(d/(b*c - a*d))*arctan(-(b*c - a*d)*x*sqrt(d/(b*c - a*d))*sqrt
((a*x + b)/x)/(a*d*x + b*d)) - 2*(b^3*c^3*d^2 + 4*a*b^2*c^2*d^3 - 11*a^2*b*
c*d^4 + 6*a^3*d^5 + (b^3*c^5 + 4*a*b^2*c^4*d - 11*a^2*b*c^3*d^2 + 6*a^3*c^2
*d^3)*x^2 + 2*(b^3*c^4*d + 4*a*b^2*c^3*d^2 - 11*a^2*b*c^2*d^3 + 6*a^3*c*d^4
)*x)*sqrt(a)*log(2*a*x - 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) - (4*(a*b^2*c^5
 - 2*a^2*b*c^4*d + a^3*c^3*d^2)*x^3 + (8*a*b^2*c^4*d - 29*a^2*b*c^3*d^2 + 1
8*a^3*c^2*d^3)*x^2 + (4*a*b^2*c^3*d^2 - 19*a^2*b*c^2*d^3 + 12*a^3*c*d^4)*x)
*sqrt((a*x + b)/x))/(a^2*b^2*c^6*d^2 - 2*a^3*b*c^5*d^3 + a^4*c^4*d^4 + (a^2
*b^2*c^8 - 2*a^3*b*c^7*d + a^4*c^6*d^2)*x^2 + 2*(a^2*b^2*c^7*d - 2*a^3*b*c^
6*d^2 + a^4*c^5*d^3)*x), -1/4*((35*a^2*b^2*c^2*d^3 - 56*a^3*b*c*d^4 + 24*a^
4*d^5 + (35*a^2*b^2*c^4*d - 56*a^3*b*c^3*d^2 + 24*a^4*c^2*d^3)*x^2 + 2*(35*
a^2*b^2*c^3*d^2 - 56*a^3*b*c^2*d^3 + 24*a^4*c*d^4)*x)*sqrt(d/(b*c - a*d))*a
rctan(-(b*c - a*d)*x*sqrt(d/(b*c - a*d))*sqrt((a*x + b)/x)/(a*d*x + b*d)) -
 4*(b^3*c^3*d^2 + 4*a*b^2*c^2*d^3 - 11*a^2*b*c*d^4 + 6*a^3*d^5 + (b^3*c^5 +
 4*a*b^2*c^4*d - 11*a^2*b*c^3*d^2 + 6*a^3*c^2*d^3)*x^2 + 2*(b^3*c^4*d + 4*a
*b^2*c^3*d^2 - 11*a^2*b*c^2*d^3 + 6*a^3*c*d^4)*x)*sqrt(-a)*arctan(sqrt(-a)*
sqrt((a*x + b)/x)/a) - (4*(a*b^2*c^5 - 2*a^2*b*c^4*d + a^3*c^3*d^2)*x^3 + (
8*a*b^2*c^4*d - 29*a^2*b*c^3*d^2 + 18*a^3*c^2*d^3)*x^2 + (4*a*b^2*c^3*d^2 -
 19*a^2*b*c^2*d^3 + 12*a^3*c*d^4)*x)*sqrt((a*x + b)/x))/(a^2*b^2*c^6*d^2 -
2*a^3*b*c^5*d^3 + a^4*c^4*d^4 + (a^2*b^2*c^8 - 2*a^3*b*c^7*d + a^4*c^6*d^2)
*x^2 + 2*(a^2*b^2*c^7*d - 2*a^3*b*c^6*d^2 + a^4*c^5*d^3)*x)]
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{a+\frac{b}{x}}\,(cx+d)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d/x)**3/(a+b/x)**(1/2),x)
```

```
[Out] Integral(x**3/(sqrt(a + b/x)*(c*x + d)**3), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 901 vs. 2(222) = 444.
time = 2.23, size = 901, normalized size = 3.60

$$\frac{\left(35 a^{\frac{3}{2}} b^2 c^2 d^2 \arctan\left(\frac{\sqrt{a}d}{\sqrt{bcd-ad^2}}\right) - 56 a^{\frac{5}{2}} b c d^3 \arctan\left(\frac{\sqrt{a}d}{\sqrt{bcd-ad^2}}\right) + 24 a^{\frac{7}{2}} d^4 \arctan\left(\frac{\sqrt{a}d}{\sqrt{bcd-ad^2}}\right) - 2\sqrt{bcd-ad^2}\,b^3c^3\log\left(|b|\right) - 8\sqrt{bcd-ad^2}\,ab^2c^2d\log\left(|b|\right) + 22\sqrt{bcd-ad^2}\,a^2bcd^2\log\left(|b|\right) - 12\sqrt{bcd-ad^2}\,a^3d^3\log\left(|b|\right) + 13\sqrt{bcd-ad^2}\,a^2bcd^2 - 10\sqrt{bcd-ad^2}\,a^3d^3\right)\operatorname{sgn}(x)}{4\left(\sqrt{bcd-ad^2}\,a^{\frac{3}{2}}b^2c^6 - 2\sqrt{bcd-ad^2}\,a^{\frac{5}{2}}bc^5d + \sqrt{bcd-ad^2}\,a^{\frac{7}{2}}c^4d^2\right)} + \frac{\left(35b^2c^2d^2 - 56abcd^3 + 24a^2d^4\right)\arctan\left(-\frac{\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)c+\sqrt{a}d}{\sqrt{bcd-ad^2}}\right)}{4\left(b^2c^6\operatorname{sgn}(x) - 2abc^5d\operatorname{sgn}(x) + a^2c^4d^2\operatorname{sgn}(x)\right)\sqrt{bcd-ad^2}} + \frac{13\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)^3\sqrt{a}b^2c^3d^2 - 40\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)^3a^{\frac{3}{2}}bc^2d^3 + 24\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)^3a^{\frac{5}{2}}cd^4 + 7\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)^2ab^2c^2d^3 - 56\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)^2a^2bcd^4 + 40\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)^2a^3d^5 + 11\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)\sqrt{a}b^3c^2d^3 - 60\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)a^{\frac{3}{2}}b^2cd^4 + 40\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)a^{\frac{5}{2}}bd^5 - 13ab^3cd^4 + 10a^2b^2d^5}{4\left(\sqrt{a}b^2c^6\operatorname{sgn}(x) - 2a^{\frac{3}{2}}bc^5d\operatorname{sgn}(x) + a^{\frac{5}{2}}c^4d^2\operatorname{sgn}(x)\right)\left(\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)^2c + 2\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)\sqrt{a}d + bd\right)^2} + \frac{\sqrt{ax^2+bx}}{ac^3\operatorname{sgn}(x)} + \frac{(bc+6ad)\log\left(\left|2\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)\sqrt{a}+b\right|\right)}{2a^{\frac{3}{2}}c^4\operatorname{sgn}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d/x)^3/(a+b/x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*(35*a^(3/2)*b^2*c^2*d^2*arctan(sqrt(a)*d/sqrt(b*c*d - a*d^2)) - 56*a^(5
/2)*b*c*d^3*arctan(sqrt(a)*d/sqrt(b*c*d - a*d^2)) + 24*a^(7/2)*d^4*arctan(s
qrt(a)*d/sqrt(b*c*d - a*d^2)) - 2*sqrt(b*c*d - a*d^2)*b^3*c^3*log(abs(b)) -
 8*sqrt(b*c*d - a*d^2)*a*b^2*c^2*d*log(abs(b)) + 22*sqrt(b*c*d - a*d^2)*a^2
*b*c*d^2*log(abs(b)) - 12*sqrt(b*c*d - a*d^2)*a^3*d^3*log(abs(b)) + 13*sqrt
(b*c*d - a*d^2)*a^2*b*c*d^2 - 10*sqrt(b*c*d - a*d^2)*a^3*d^3)*sgn(x)/(sqrt(
b*c*d - a*d^2)*a^(3/2)*b^2*c^6 - 2*sqrt(b*c*d - a*d^2)*a^(5/2)*b*c^5*d + sq
rt(b*c*d - a*d^2)*a^(7/2)*c^4*d^2) + 1/4*(35*b^2*c^2*d^2 - 56*a*b*c*d^3 + 2
4*a^2*d^4)*arctan(-((sqrt(a)*x - sqrt(a*x^2 + b*x))*c + sqrt(a)*d)/sqrt(b*c
*d - a*d^2))/((b^2*c^6*sgn(x) - 2*a*b*c^5*d*sgn(x) + a^2*c^4*d^2*sgn(x))*sq
rt(b*c*d - a*d^2)) + 1/4*(13*(sqrt(a)*x - sqrt(a*x^2 + b*x))^3*sqrt(a)*b^2*
c^3*d^2 - 40*(sqrt(a)*x - sqrt(a*x^2 + b*x))^3*a^(3/2)*b*c^2*d^3 + 24*(sqrt
(a)*x - sqrt(a*x^2 + b*x))^3*a^(5/2)*c*d^4 + 7*(sqrt(a)*x - sqrt(a*x^2 + b*
x))^2*a*b^2*c^2*d^3 - 56*(sqrt(a)*x - sqrt(a*x^2 + b*x))^2*a^2*b*c*d^4 + 40
*(sqrt(a)*x - sqrt(a*x^2 + b*x))^2*a^3*d^5 + 11*(sqrt(a)*x - sqrt(a*x^2 + b
*x))*sqrt(a)*b^3*c^2*d^3 - 60*(sqrt(a)*x - sqrt(a*x^2 + b*x))*a^(3/2)*b^2*c
*d^4 + 40*(sqrt(a)*x - sqrt(a*x^2 + b*x))*a^(5/2)*b*d^5 - 13*a*b^3*c*d^4 +
10*a^2*b^2*d^5)/((sqrt(a)*b^2*c^6*sgn(x) - 2*a^(3/2)*b*c^5*d*sgn(x) + a^(5/
2)*c^4*d^2*sgn(x))*((sqrt(a)*x - sqrt(a*x^2 + b*x))^2*c + 2*(sqrt(a)*x - sq
rt(a*x^2 + b*x))*sqrt(a)*d + b*d)^2) + sqrt(a*x^2 + b*x)/(a*c^3*sgn(x)) + 1
/2*(b*c + 6*a*d)*log(abs(2*(sqrt(a)*x - sqrt(a*x^2 + b*x))*sqrt(a) + b))/(a
^(3/2)*c^4*sgn(x))
```

Mupad [B]
time = 5.48, size = 2890, normalized size = 11.56

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b/x)^(1/2)*(c + d/x)^3),x)
```

```
[Out] (log((d^3*(a*d - b*c)^5)^(1/2)*(a + b/x)^(1/2) - a^3*d^4 + b^3*c^3*d - 3*a*
b^2*c^2*d^2 + 3*a^2*b*c*d^3)*(d^3*(a*d - b*c)^5)^(1/2)*(3*a^2*d^2 + (35*b^2
*c^2)/8 - 7*a*b*c*d))/(b^5*c^9 - a^5*c^4*d^5 + 5*a^4*b*c^5*d^4 + 10*a^2*b^3
*c^7*d^2 - 10*a^3*b^2*c^6*d^3 - 5*a*b^4*c^8*d) - ((b*(a + b/x)^(5/2)*(12*a^
2*d^4 + 4*b^2*c^2*d^2 - 19*a*b*c*d^3))/(4*a*c^3*(a*d - b*c)^2) - ((a + b/x)
^(1/2)*(4*b^4*c^3 - 12*a^3*b*d^3 + 25*a^2*b^2*c*d^2 - 12*a*b^3*c^2*d))/(4*a
*c^3*(a*d - b*c)) + (d*(a + b/x)^(3/2)*(8*b^4*c^3 - 24*a^3*b*d^3 + 56*a^2*b
^2*c*d^2 - 37*a*b^3*c^2*d))/(4*c^3*(a^2*d - a*b*c)*(a*d - b*c)))/((a + b/x)
^2*(3*a*d^2 - 2*b*c*d) - (a + b/x)*(3*a^2*d^2 + b^2*c^2 - 4*a*b*c*d) - d^2*
(a + b/x)^3 + a^3*d^2 + a*b^2*c^2 - 2*a^2*b*c*d) - (log((d^3*(a*d - b*c)^5)
^(1/2)*(a + b/x)^(1/2) + a^3*d^4 - b^3*c^3*d + 3*a*b^2*c^2*d^2 - 3*a^2*b*c*
d^3)*(d^3*(a*d - b*c)^5)^(1/2)*(24*a^2*d^2 + 35*b^2*c^2 - 56*a*b*c*d))/(8*(
b^5*c^9 - a^5*c^4*d^5 + 5*a^4*b*c^5*d^4 + 10*a^2*b^3*c^7*d^2 - 10*a^3*b^2*c
^6*d^3 - 5*a*b^4*c^8*d)) - (atan((((((a + b/x)^(1/2)*(1152*a^6*b^2*d^9 + 16
*b^8*c^6*d^3 + 128*a*b^7*c^5*d^4 - 4800*a^5*b^3*c*d^8 + 1129*a^2*b^6*c^4*d^
5 - 5136*a^3*b^5*c^3*d^6 + 7520*a^4*b^4*c^2*d^7))/(8*(a^2*b^4*c^10 + a^6*c^
6*d^4 - 4*a^3*b^3*c^9*d - 4*a^5*b*c^7*d^3 + 6*a^4*b^2*c^8*d^2)) - (((4*a*b^
8*c^13*d^2 + 4*a^2*b^7*c^12*d^3 - 45*a^3*b^6*c^11*d^4 + 74*a^4*b^5*c^10*d^5
 - 49*a^5*b^4*c^9*d^6 + 12*a^6*b^3*c^8*d^7)/(a^2*b^4*c^13 + a^6*c^9*d^4 - 4
*a^3*b^3*c^12*d - 4*a^5*b*c^10*d^3 + 6*a^4*b^2*c^11*d^2) - ((a + b/x)^(1/2)
*(6*a*d + b*c)*(64*a^2*b^7*c^13*d^2 - 384*a^3*b^6*c^12*d^3 + 896*a^4*b^5*c^
11*d^4 - 1024*a^5*b^4*c^10*d^5 + 576*a^6*b^3*c^9*d^6 - 128*a^7*b^2*c^8*d^7)
)/(16*c^4*(a^3)^(1/2)*(a^2*b^4*c^10 + a^6*c^6*d^4 - 4*a^3*b^3*c^9*d - 4*a^5
*b*c^7*d^3 + 6*a^4*b^2*c^8*d^2)))*(6*a*d + b*c))/(2*c^4*(a^3)^(1/2)))*(6*a*
d + b*c)*1i)/(2*c^4*(a^3)^(1/2)) + ((((a + b/x)^(1/2)*(1152*a^6*b^2*d^9 + 1
6*b^8*c^6*d^3 + 128*a*b^7*c^5*d^4 - 4800*a^5*b^3*c*d^8 + 1129*a^2*b^6*c^4*d
^5 - 5136*a^3*b^5*c^3*d^6 + 7520*a^4*b^4*c^2*d^7))/(8*(a^2*b^4*c^10 + a^6*c
^6*d^4 - 4*a^3*b^3*c^9*d - 4*a^5*b*c^7*d^3 + 6*a^4*b^2*c^8*d^2)) + (((4*a*b
^8*c^13*d^2 + 4*a^2*b^7*c^12*d^3 - 45*a^3*b^6*c^11*d^4 + 74*a^4*b^5*c^10*d^
5 - 49*a^5*b^4*c^9*d^6 + 12*a^6*b^3*c^8*d^7)/(a^2*b^4*c^13 + a^6*c^9*d^4 -
4*a^3*b^3*c^12*d - 4*a^5*b*c^10*d^3 + 6*a^4*b^2*c^11*d^2) + ((a + b/x)^(1/2
)*(6*a*d + b*c)*(64*a^2*b^7*c^13*d^2 - 384*a^3*b^6*c^12*d^3 + 896*a^4*b^5*c
^11*d^4 - 1024*a^5*b^4*c^10*d^5 + 576*a^6*b^3*c^9*d^6 - 128*a^7*b^2*c^8*d^7
))/(16*c^4*(a^3)^(1/2)*(a^2*b^4*c^10 + a^6*c^6*d^4 - 4*a^3*b^3*c^9*d - 4*a^
5*b*c^7*d^3 + 6*a^4*b^2*c^8*d^2)))*(6*a*d + b*c))/(2*c^4*(a^3)^(1/2)))*(6*a
*d + b*c)*1i)/(2*c^4*(a^3)^(1/2)))/((216*a^5*b^3*d^9 + (35*b^8*c^5*d^4)/2 -
 (49*a*b^7*c^4*d^5)/8 - 810*a^4*b^4*c*d^8 - (1877*a^2*b^6*c^3*d^6)/4 + 1044
*a^3*b^5*c^2*d^7)/(a^2*b^4*c^13 + a^6*c^9*d^4 - 4*a^3*b^3*c^12*d - 4*a^5*b*
c^10*d^3 + 6*a^4*b^2*c^11*d^2) + ((((a + b/x)^(1/2)*(1152*a^6*b^2*d^9 + 16*
b^8*c^6*d^3 + 128*a*b^7*c^5*d^4 - 4800*a^5*b^3*c*d^8 + 1129*a^2*b^6*c^4*d^5
 - 5136*a^3*b^5*c^3*d^6 + 7520*a^4*b^4*c^2*d^7))/(8*(a^2*b^4*c^10 + a^6*c^6
```

*d^4 - 4*a^3*b^3*c^9*d - 4*a^5*b*c^7*d^3 + 6*a^4*b^2*c^8*d^2)) - (((4*a*b^8
*c^13*d^2 + 4*a^2*b^7*c^12*d^3 - 45*a^3*b^6*c^11*d^4 + 74*a^4*b^5*c^10*d^5
- 49*a^5*b^4*c^9*d^6 + 12*a^6*b^3*c^8*d^7)/(a^2*b^4*c^13 + a^6*c^9*d^4 - 4*
a^3*b^3*c^12*d - 4*a^5*b*c^10*d^3 + 6*a^4*b^2*c^11*d^2) - ((a + b/x)^(1/2)*
(6*a*d + b*c)*(64*a^2*b^7*c^13*d^2 - 384*a^3*b^6*c^12*d^3 + 896*a^4*b^5*c^1
1*d^4 - 1024*a^5*b^4*c^10*d^5 + 576*a^6*b^3*c^9*d^6 - 128*a^7*b^2*c^8*d^7))
/(16*c^4*(a^3)^(1/2)*(a^2*b^4*c^10 + a^6*c^6*d^4 - 4*a^3*b^3*c^9*d - 4*a^5*
b*c^7*d^3 + 6*a^4*b^2*c^8*d^2)))*(6*a*d + b*c))/(2*c^4*(a^3)^(1/2)))*(6*a*d
 + b*c))/(2*c^4*(a^3)^(1/2)) - ((((a + b/x)^(1/2)*(1152*a^6*b^2*d^9 + 16*b^
8*c^6*d^3 + 128*a*b^7*c^5*d^4 - 4800*a^5*b^3*c*d^8 + 1129*a^2*b^6*c^4*d^5 -
 5136*a^3*b^5*c^3*d^6 + 7520*a^4*b^4*c^2*d^7))/(8*(a^2*b^4*c^10 + a^6*c^6*d
^4 - 4*a^3*b^3*c^9*d - 4*a^5*b*c^7*d^3 + 6*a^4*b^2*c^8*d^2)) + (((4*a*b^8*c
^13*d^2 + 4*a^2*b^7*c^12*d^3 - 45*a^3*b^6*c^11*d^4 + 74*a^4*b^5*c^10*d^5 -
49*a^5*b^4*c^9*d^6 + 12*a^6*b^3*c^8*d^7)/(a^2*b^4*c^13 + a^6*c^9*d^4 - 4*a^
3*b^3*c^12*d - 4*a^5*b*c^10*d^3 + 6*a^4*b^2*c^11*d^2) + ((a + b/x)^(1/2)*(6
*a*d + b*c)*(64*a^2*b^7*c^13*d^2 - 384*a^3*b^6*c^12*d^3 + 896*a^4*b^5*c^11*
d^4 - 1024*a^5*b^4*c^10*d^5 + 576*a^6*b^3*c^9*d^6 - 128*a^7*b^2*c^8*d^7))/(
16*c^4*(a^3)^(1/2)*(a^2*b^4*c^10 + a^6*c^6*d^4 - 4*a^3*b^3*c^9*d - 4*a^5*b*
c^7*d^3 + 6*a^4*b^2*c^8*d^2)))*(6*a*d + b*c))/(2*c^4*(a^3)^(1/2)))*(6*a*d +
 b*c))/(2*c^4*(a^3)^(1/2))))*(6*a*d + b*c)*1i)/(c^4*(a^3)^(1/2))

3.252 $\int \frac{\left(c+\frac{d}{x}\right)^3}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx$

Optimal. Leaf size=132

$$\frac{(bc-2ad)\left(3b^2c^2-2abcd+2a^2d^2\right)-\frac{abd^2(bc+2ad)}{x}}{a^2b^2\sqrt{a+\frac{b}{x}}}+\frac{c\left(c+\frac{d}{x}\right)^2x}{a\sqrt{a+\frac{b}{x}}}-\frac{3c^2(bc-2ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}}$$

```
[Out] -3*c^2*(-2*a*d+b*c)*arctanh((a+b/x)^(1/2)/a^(1/2))/a^(5/2)+((-2*a*d+b*c)*(2
*a^2*d^2-2*a*b*c*d+3*b^2*c^2)-a*b*d^2*(2*a*d+b*c)/x)/a^2/b^2/(a+b/x)^(1/2)+
c*(c+d/x)^2*x/a/(a+b/x)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {382, 100, 151, 65, 214}

$$-\frac{3c^2(bc-2ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}}+\frac{(bc-2ad)\left(2a^2d^2-2abcd+3b^2c^2\right)-\frac{abd^2(2ad+bc)}{x}}{a^2b^2\sqrt{a+\frac{b}{x}}}+\frac{cx\left(c+\frac{d}{x}\right)^2}{a\sqrt{a+\frac{b}{x}}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d/x)^3/(a + b/x)^(3/2),x]
```

```
[Out] ((b*c - 2*a*d)*(3*b^2*c^2 - 2*a*b*c*d + 2*a^2*d^2) - (a*b*d^2*(b*c + 2*a*d)
)/x)/(a^2*b^2*Sqrt[a + b/x]) + (c*(c + d/x)^2*x)/(a*Sqrt[a + b/x]) - (3*c^2
*(b*c - 2*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/a^(5/2)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((a^2*d*f*h*(n + 2) + b^2*d*e*g*(
m + n + 3) + a*b*(c*f*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b*f*h*(b*c -
 a*d)*(m + 1)*x)/(b^2*d*(b*c - a*d)*(m + 1)*(m + n + 3)))*(a + b*x)^(m + 1)
*(c + d*x)^(n + 1), x] - Dist[(a^2*d^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*
(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2
) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))
/(b^2*d*(b*c - a*d)*(m + 1)*(m + n + 3)), Int[(a + b*x)^(m + 1)*(c + d*x)^n
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && ((GeQ[m, -2] && LtQ
[m, -1]) || SumSimplerQ[m, 1]) && NeQ[m, -1] && NeQ[m + n + 3, 0]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{\left(c+\frac{d}{x}\right)^3}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx = -\text{Subst}\left(\int \frac{(c+dx)^3}{x^2(a+bx)^{3/2}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{c\left(c+\frac{d}{x}\right)^2 x}{a\sqrt{a+\frac{b}{x}}} + \frac{\text{Subst}\left(\int \frac{(c+dx)\left(\frac{3}{2}c(bc-2ad)-\frac{1}{2}d(bc+2ad)x\right)}{x(a+bx)^{3/2}}\,dx, x, \frac{1}{x}\right)}{a}$$

$$= \frac{(bc-2ad)\left(3b^2c^2-2abcd+2a^2d^2\right)-\frac{abd^2(bc+2ad)}{x}}{a^2b^2\sqrt{a+\frac{b}{x}}} + \frac{c\left(c+\frac{d}{x}\right)^2 x}{a\sqrt{a+\frac{b}{x}}} + \frac{\left(3c^2(bc-2ad)\right)\text{Subst}(}{2}$$

$$= \frac{(bc-2ad)\left(3b^2c^2-2abcd+2a^2d^2\right)-\frac{abd^2(bc+2ad)}{x}}{a^2b^2\sqrt{a+\frac{b}{x}}} + \frac{c\left(c+\frac{d}{x}\right)^2 x}{a\sqrt{a+\frac{b}{x}}} + \frac{\left(3c^2(bc-2ad)\right)\text{Subst}(}{}$$

$$= \frac{(bc-2ad)\left(3b^2c^2-2abcd+2a^2d^2\right)-\frac{abd^2(bc+2ad)}{x}}{a^2b^2\sqrt{a+\frac{b}{x}}} + \frac{c\left(c+\frac{d}{x}\right)^2 x}{a\sqrt{a+\frac{b}{x}}} - \frac{3c^2(bc-2ad)\tanh^{-1}}{a^{5/2}}$$

Mathematica [A]
time = 0.21, size = 113, normalized size = 0.86

$$\frac{\sqrt{a+\frac{b}{x}}\left(3b^3c^3x-4a^3d^3x-2a^2bd^2(d-3cx)+ab^2c^2x(-6d+cx)\right)}{a^2b^2(b+ax)} + \frac{3c^2(-bc+2ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d/x)^3/(a + b/x)^(3/2),x]
```

```
[Out] (Sqrt[a + b/x]*(3*b^3*c^3*x - 4*a^3*d^3*x - 2*a^2*b*d^2*(d - 3*c*x) + a*b^2
*c^2*x*(-6*d + c*x)))/(a^2*b^2*(b + a*x)) + (3*c^2*(-(b*c) + 2*a*d)*ArcTanh
[Sqrt[a + b/x]/Sqrt[a]])/a^(5/2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 968 vs. $2(122) = 244$.
time = 0.09, size = 969, normalized size = 7.34

method	result
risch	$-\frac{(ax+b)(-b^2c^3x+2a^2d^3)}{b^2a^2x\sqrt{\frac{ax+b}{x}}}+\frac{\left(\frac{3c^2d\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^2+bx}\right)}{a^{\frac{3}{2}}}-\frac{3b\,c^3\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^2+bx}\right)}{2a^{\frac{5}{2}}}-\frac{2\sqrt{a\left(x+\frac{b}{a}\right)^2-b}}{b^2\left(x+\frac{b}{a}\right)}\right.}{}$
default	$\frac{\sqrt{\frac{ax+b}{x}}\left(6\sqrt{a\,x^2+bx}\,a^{\frac{5}{2}}b^3c\,d^2x^2+3\ln\left(\frac{2\sqrt{a\,x^2+bx}\,\sqrt{a}+2ax+b}{2\sqrt{a}}\right)a^4b^2c\,d^2x^4-4(x(ax+b))^{\frac{3}{2}}a^{\frac{3}{2}}b^3c^3x^2+12\sqrt{x(a}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d/x)^3/(a+1/x*b)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*((a*x+b)/x)^(1/2)/x/a^(5/2)*(6*(a*x^2+b*x)^(1/2)*a^(5/2)*b^3*c*d^2*x^2+
3*ln(1/2*(2*(a*x^2+b*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^4*b^2*c*d^2*x^4-4
*(x*(a*x+b))^(3/2)*a^(3/2)*b^3*c^3*x^2+12*(x*(a*x+b))^(1/2)*a^(3/2)*b^4*c^3
*x^3-3*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*b^6*c^3*x^2-8*
(a*x^2+b*x)^(3/2)*a^(7/2)*b*d^3*x-3*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a
*x+b)/a^(1/2))*a^4*b^2*c*d^2*x^4+6*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*
x+b)/a^(1/2))*a^3*b^3*c^2*d*x^4+6*(x*(a*x+b))^(1/2)*a^(5/2)*b^3*c^3*x^4+6*(
x*(a*x+b))^(1/2)*a^(9/2)*b*c*d^2*x^4-12*(x*(a*x+b))^(1/2)*a^(7/2)*b^2*c^2*d
*x^4-3*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^2*b^4*c^3*x^
4+6*(a*x^2+b*x)^(1/2)*a^(9/2)*b*c*d^2*x^4-12*(x*(a*x+b))^(3/2)*a^(7/2)*b*c*
d^2*x^2+12*(x*(a*x+b))^(3/2)*a^(5/2)*b^2*c^2*d*x^2+12*(x*(a*x+b))^(1/2)*a^(
7/2)*b^2*c*d^2*x^3-24*(x*(a*x+b))^(1/2)*a^(5/2)*b^3*c^2*d*x^3+12*(a*x^2+b*x
)^(1/2)*a^(7/2)*b^2*c*d^2*x^3+6*(x*(a*x+b))^(1/2)*a^(5/2)*b^3*c*d^2*x^2-12*
(x*(a*x+b))^(1/2)*a^(3/2)*b^4*c^2*d*x^2+4*(x*(a*x+b))^(3/2)*a^(9/2)*d^3*x^2
-6*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a*b^5*c^3*x^3-4*(a
*x^2+b*x)^(3/2)*a^(9/2)*d^3*x^2+6*(x*(a*x+b))^(1/2)*a^(1/2)*b^5*c^3*x^2-4*(
a*x^2+b*x)^(3/2)*a^(5/2)*b^2*d^3-6*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*
x+b)/a^(1/2))*a^3*b^3*c*d^2*x^3+12*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*
x+b)/a^(1/2))*a^2*b^4*c^2*d*x^3+6*ln(1/2*(2*(a*x^2+b*x)^(1/2)*a^(1/2)+2*a*x
+b)/a^(1/2))*a^3*b^3*c*d^2*x^3-3*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+
b)/a^(1/2))*a^2*b^4*c*d^2*x^2+6*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b
)/a^(1/2))*a*b^5*c^2*d*x^2+3*ln(1/2*(2*(a*x^2+b*x)^(1/2)*a^(1/2)+2*a*x+b)/a
^(1/2))*a^2*b^4*c*d^2*x^2)/(x*(a*x+b))^(1/2)/b^3/(a*x+b)^2
```

Maxima [A]

time = 0.58, size = 200, normalized size = 1.52

$$\frac{1}{2}\,c^{3}\left(\frac{2\,\left(3\,\left(a+\frac{b}{x}\right)b-2\,ab\right)}{\left(a+\frac{b}{x}\right)^{\frac{3}{2}}a^{2}-\sqrt{a+\frac{b}{x}}\,a^{3}}+\frac{3\,b\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{a^{\frac{5}{2}}}\right)-3\,c^{2}d\left(\frac{\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{a^{\frac{3}{2}}}+\frac{2}{\sqrt{a+\frac{b}{x}}\,a}\right)-2\,d^{3}\left(\frac{\sqrt{a+\frac{b}{x}}}{b^{2}}+\frac{a}{\sqrt{a+\frac{b}{x}}\,b^{2}}\right)+\frac{6\,cd^{2}}{\sqrt{a+\frac{b}{x}}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)^3/(a+b/x)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*c^3*(2*(3*(a + b/x)*b - 2*a*b)/((a + b/x)^(3/2)*a^2 - sqrt(a + b/x)*a^3
) + 3*b*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/x) + sqrt(a)))/a^(5/2)) -
 3*c^2*d*(log((sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/x) + sqrt(a)))/a^(3/2)
+ 2/(sqrt(a + b/x)*a)) - 2*d^3*(sqrt(a + b/x)/b^2 + a/(sqrt(a + b/x)*b^2))
+ 6*c*d^2/(sqrt(a + b/x)*b)
```

Fricas [A]
time = 2.03, size = 336, normalized size = 2.55

$$\left[-\frac{3\,\left(b^{4}c^{3}-2\,ab^{3}c^{2}d+\left(ab^{3}c^{3}-2\,a^{2}b^{2}c^{2}d\right)x\right)\sqrt{a}\,\log\left(2\,ax+2\,\sqrt{a}\,x\sqrt{\frac{ax+b}{x}}+b\right)-2\,\left(a^{2}b^{2}c^{3}x^{2}-2\,a^{3}bd^{3}+\left(3\,ab^{3}c^{3}-6\,a^{2}b^{2}c^{2}d+6\,a^{3}bcd^{2}-4\,a^{4}d^{3}\right)x\right)\sqrt{\frac{ax+b}{x}}}{2\,\left(a^{4}b^{2}x+a^{3}b^{3}\right)},\ \frac{3\,\left(b^{4}c^{3}-2\,ab^{3}c^{2}d+\left(ab^{3}c^{3}-2\,a^{2}b^{2}c^{2}d\right)x\right)\sqrt{-a}\,\arctan\left(\frac{\sqrt{-a}\sqrt{\frac{ax+b}{x}}}{a}\right)+\left(a^{2}b^{2}c^{3}x^{2}-2\,a^{3}bd^{3}+\left(3\,ab^{3}c^{3}-6\,a^{2}b^{2}c^{2}d+6\,a^{3}bcd^{2}-4\,a^{4}d^{3}\right)x\right)\sqrt{\frac{ax+b}{x}}}{a^{4}b^{2}x+a^{3}b^{3}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)^3/(a+b/x)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/2*(3*(b^4*c^3 - 2*a*b^3*c^2*d + (a*b^3*c^3 - 2*a^2*b^2*c^2*d)*x)*sqrt(a
)*log(2*a*x + 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) - 2*(a^2*b^2*c^3*x^2 - 2*a
^3*b*d^3 + (3*a*b^3*c^3 - 6*a^2*b^2*c^2*d + 6*a^3*b*c*d^2 - 4*a^4*d^3)*x)*s
qrt((a*x + b)/x))/(a^4*b^2*x + a^3*b^3), (3*(b^4*c^3 - 2*a*b^3*c^2*d + (a*b
^3*c^3 - 2*a^2*b^2*c^2*d)*x)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a)
+ (a^2*b^2*c^3*x^2 - 2*a^3*b*d^3 + (3*a*b^3*c^3 - 6*a^2*b^2*c^2*d + 6*a^3*b
*c*d^2 - 4*a^4*d^3)*x)*sqrt((a*x + b)/x))/(a^4*b^2*x + a^3*b^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx+d\right)^{3}}{x^{3}\left(a+\frac{b}{x}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)**3/(a+b/x)**(3/2),x)
```

```
[Out] Integral((c*x + d)**3/(x**3*(a + b/x)**(3/2)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)^3/(a+b/x)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [B]
time = 1.91, size = 172, normalized size = 1.30

$$\frac{\frac{2\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3\right)}{a}-\frac{\left(a+\frac{b}{x}\right)\,\left(2\,a^3\,d^3-6\,a^2\,b\,c\,d^2+6\,a\,b^2\,c^2\,d-3\,b^3\,c^3\right)}{a^2}}{b^2\,{\left(a+\frac{b}{x}\right)}^{3/2}-a\,b^2\,\sqrt{a+\frac{b}{x}}}-\frac{2\,d^3\,\sqrt{a+\frac{b}{x}}}{b^2}+\frac{3\,c^2\,\mathrm{atanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)\,\left(2\,a\,d-b\,c\right)}{a^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d/x)^3/(a + b/x)^(3/2),x)
```

```
[Out] ((2*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))/a - ((a + b/x)*(2*
a^3*d^3 - 3*b^3*c^3 + 6*a*b^2*c^2*d - 6*a^2*b*c*d^2))/a^2)/(b^2*(a + b/x)^(
3/2) - a*b^2*(a + b/x)^(1/2)) - (2*d^3*(a + b/x)^(1/2))/b^2 + (3*c^2*atanh(
(a + b/x)^(1/2)/a^(1/2))*(2*a*d - b*c))/a^(5/2)
```

3.253 $\int \frac{\left(c+\frac{d}{x}\right)^2}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx$

Optimal. Leaf size=94

$$\frac{2a^2d^2+bc(3bc-4ad)}{a^2b\sqrt{a+\frac{b}{x}}}+\frac{c^2x}{a\sqrt{a+\frac{b}{x}}}-\frac{c(3bc-4ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}}$$

```
[Out] -c*(-4*a*d+3*b*c)*arctanh((a+b/x)^(1/2)/a^(1/2))/a^(5/2)+(2*a^2*d^2+b*c*(-4
*a*d+3*b*c))/a^2/b/(a+b/x)^(1/2)+c^2*x/a/(a+b/x)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 93, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {382, 91, 79, 65, 214}

$$-\frac{c(3bc-4ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}}+\frac{\frac{3bc^2}{a}+\frac{2ad^2}{b}-4cd}{a\sqrt{a+\frac{b}{x}}}+\frac{c^2x}{a\sqrt{a+\frac{b}{x}}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d/x)^2/(a + b/x)^(3/2),x]
```

```
[Out] ((3*b*c^2)/a - 4*c*d + (2*a*d^2)/b)/(a*Sqrt[a + b/x]) + (c^2*x)/(a*Sqrt[a +
 b/x]) - (c*(3*b*c - 4*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/a^(5/2)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
```

```
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{\left(c+\frac{d}{x}\right)^2}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx = -\text{Subst}\left(\int \frac{(c+dx)^2}{x^2(a+bx)^{3/2}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{c^2x}{a\sqrt{a+\frac{b}{x}}} - \frac{\text{Subst}\left(\int \frac{-\frac{1}{2}c(3bc-4ad)+ad^2x}{x(a+bx)^{3/2}}\,dx, x, \frac{1}{x}\right)}{a}$$

$$= \frac{\frac{2d^2}{b}+\frac{c(3bc-4ad)}{a^2}}{\sqrt{a+\frac{b}{x}}} + \frac{c^2x}{a\sqrt{a+\frac{b}{x}}} + \frac{(c(3bc-4ad))\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)}{2a^2}$$

$$= \frac{\frac{2d^2}{b}+\frac{c(3bc-4ad)}{a^2}}{\sqrt{a+\frac{b}{x}}} + \frac{c^2x}{a\sqrt{a+\frac{b}{x}}} + \frac{(c(3bc-4ad))\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x}}\right)}{a^2b}$$

$$= \frac{\frac{2d^2}{b}+\frac{c(3bc-4ad)}{a^2}}{\sqrt{a+\frac{b}{x}}} + \frac{c^2x}{a\sqrt{a+\frac{b}{x}}} - \frac{c(3bc-4ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}}$$

Mathematica [A]
time = 0.21, size = 89, normalized size = 0.95

$$\frac{\sqrt{a+\frac{b}{x}}\,x(3b^2c^2+2a^2d^2+abc(-4d+cx))}{a^2b(b+ax)} + \frac{c(-3bc+4ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d/x)^2/(a + b/x)^(3/2),x]
```

```
[Out] (Sqrt[a + b/x]*x*(3*b^2*c^2 + 2*a^2*d^2 + a*b*c*(-4*d + c*x)))/(a^2*b*(b +
a*x)) + (c*(-3*b*c + 4*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/a^(5/2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 788 vs. 2(84) = 168.
time = 0.07, size = 789, normalized size = 8.39

method	result
risch	$\frac{c^2(ax+b)}{a^2\sqrt{\frac{ax+b}{x}}}+\frac{\left(\frac{2c\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^2+bx}\right)d}{a^{\frac{3}{2}}}-\frac{3c^2\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^2+bx}\right)b}{2a^{\frac{5}{2}}}+\frac{2\sqrt{a\left(x+\frac{b}{a}\right)^2-b\left(x+\frac{b}{a}\right)}\,d^2}{ab\left(x+\frac{b}{a}\right)}\right.}{x\sqrt{\frac{ax+b}{x}}}$
default	$\frac{\sqrt{\frac{ax+b}{x}}\,x\left(-4a^{\frac{3}{2}}(x(ax+b))^{\frac{3}{2}}b^2c^2+2\sqrt{a\,x^2+bx}\,a^{\frac{9}{2}}d^2x^2+8\ln\left(\frac{2\sqrt{x(ax+b)}\sqrt{a}+2ax+b}{2\sqrt{a}}\right)a^2b^3cdx-8a^{\frac{3}{2}}\sqrt{x(ax\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c+d/x)^2/(a+1/x*b)^(3/2),x,method=_RETURNVERBOSE)

[Out] 1/2*((a*x+b)/x)^(1/2)*x/a^(5/2)*(-4*a^(3/2)*(x*(a*x+b))^(3/2)*b^2*c^2+2*(a*x^2+b*x)^(1/2)*a^(9/2)*d^2*x^2+8*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^2*b^3*c*d*x-8*a^(3/2)*(x*(a*x+b))^(1/2)*b^3*c*d+8*a^(5/2)*(x*(a*x+b))^(3/2)*b*c*d+4*a^(7/2)*(x*(a*x+b))^(1/2)*b*d^2*x+12*a^(3/2)*(x*(a*x+b))^(1/2)*b^3*c^2*x+4*(a*x^2+b*x)^(1/2)*a^(7/2)*b*d^2*x+6*a^(5/2)*(x*(a*x+b))^(1/2)*b^2*c^2*x^2+4*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^3*b^2*c*d*x^2-3*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^2*b^3*c^2*x^2+2*ln(1/2*(2*(a*x^2+b*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^3*b^2*d^2*x-8*a^(7/2)*(x*(a*x+b))^(1/2)*b*c*d*x^2-4*a^(7/2)*(x*(a*x+b))^(3/2)*d^2+ln(1/2*(2*(a*x^2+b*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^2*b^3*d^2+6*a^(1/2)*(x*(a*x+b))^(1/2)*b^4*c^2-ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^2*b^3*d^2-3*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*b^5*c^2-16*a^(5/2)*(x*(a*x+b))^(1/2)*b^2*c*d*x+4*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a*b^4*c*d-2*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^3*b^2*d^2*x+ln(1/2*(2*(a*x^2+b*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^4*b*d^2*x^2+2*a^(9/2)*(x*(a*x+b))^(1/2)*d^2*x^2-ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^4*b*d^2*x^2-6*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a*b^4*c^2*x+2*(a*x^2+b*x)^(1/2)*a^(5/2)*b^2*d^2+2*a^(5/2)*(x*(a*x+b))^(1/2)*b^2*d^2)/(x*(a*x+b))^(1/2)/b^2/(a*x+b)^2

Maxima [A]

time = 0.49, size = 164, normalized size = 1.74

$$\frac{1}{2}\,c^2\left(\frac{2\left(3\left(a+\frac{b}{x}\right)b-2\,ab\right)}{\left(a+\frac{b}{x}\right)^{\frac{3}{2}}a^2-\sqrt{a+\frac{b}{x}}\,a^3}+\frac{3\,b\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{a^{\frac{5}{2}}}\right)-2\,cd\left(\frac{\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{a^{\frac{3}{2}}}+\frac{2}{\sqrt{a+\frac{b}{x}}\,a}\right)+\frac{2\,d^2}{\sqrt{a+\frac{b}{x}}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)^2/(a+b/x)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*c^2*(2*(3*(a + b/x)*b - 2*a*b)/((a + b/x)^(3/2)*a^2 - sqrt(a + b/x)*a^3
) + 3*b*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/x) + sqrt(a)))/a^(5/2)) -
 2*c*d*(log((sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/x) + sqrt(a)))/a^(3/2) +
2/(sqrt(a + b/x)*a)) + 2*d^2/(sqrt(a + b/x)*b)
```

Fricas [A]
time = 2.58, size = 272, normalized size = 2.89

$$\left[-\frac{(3\,b^3c^2 - 4\,ab^2cd + (3\,ab^2c^2 - 4\,a^2bcd)x)\sqrt{a}\,\log\left(2\,ax + 2\,\sqrt{a}\,x\sqrt{\frac{ax+b}{x}} + b\right) - 2\,(a^2bc^2x^2 + (3\,ab^2c^2 - 4\,a^2bcd + 2\,a^3d^2)x)\sqrt{\frac{ax+b}{x}}}{2\,(a^4bx + a^3b^2)}, \frac{(3\,b^3c^2 - 4\,ab^2cd + (3\,ab^2c^2 - 4\,a^2bcd)x)\sqrt{-a}\,\arctan\left(\frac{\sqrt{-a}\,\sqrt{\frac{ax+b}{x}}}{a}\right) + (a^2bc^2x^2 + (3\,ab^2c^2 - 4\,a^2bcd + 2\,a^3d^2)x)\sqrt{\frac{ax+b}{x}}}{a^4bx + a^3b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)^2/(a+b/x)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/2*((3*b^3*c^2 - 4*a*b^2*c*d + (3*a*b^2*c^2 - 4*a^2*b*c*d)*x)*sqrt(a)*lo
g(2*a*x + 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) - 2*(a^2*b*c^2*x^2 + (3*a*b^2*
c^2 - 4*a^2*b*c*d + 2*a^3*d^2)*x)*sqrt((a*x + b)/x))/(a^4*b*x + a^3*b^2), (
(3*b^3*c^2 - 4*a*b^2*c*d + (3*a*b^2*c^2 - 4*a^2*b*c*d)*x)*sqrt(-a)*arctan(s
qrt(-a)*sqrt((a*x + b)/x)/a) + (a^2*b*c^2*x^2 + (3*a*b^2*c^2 - 4*a^2*b*c*d
+ 2*a^3*d^2)*x)*sqrt((a*x + b)/x))/(a^4*b*x + a^3*b^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx+d)^2}{x^2\left(a+\frac{b}{x}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)**2/(a+b/x)**(3/2),x)
```

```
[Out] Integral((c*x + d)**2/(x**2*(a + b/x)**(3/2)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 186 vs. 2(84) = 168.
time = 1.48, size = 186, normalized size = 1.98

$$\frac{\sqrt{ax^2+bx}\,c^2}{a^2\mathrm{sgn}\,(x)} + \frac{(3\,bc^2 - 4\,acd)\log\left(\left|-2\left(\sqrt{a}\,x - \sqrt{ax^2+bx}\right)\sqrt{a} - b\right|\right)}{2\,a^{\frac{5}{2}}\mathrm{sgn}\,(x)} - \frac{(3\,b^2c^2\log\left(|b|\right) - 4\,abcd\log\left(|b|\right) + 4\,b^2c^2 - 8\,abcd + 4\,a^2d^2)\mathrm{sgn}(x)}{2\,a^{\frac{5}{2}}b} + \frac{2\,(b^2c^2 - 2\,abcd + a^2d^2)}{\left(\left(\sqrt{a}\,x - \sqrt{ax^2+bx}\right)a + \sqrt{a}\,b\right)a^2\mathrm{sgn}\,(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)^2/(a+b/x)^(3/2),x, algorithm="giac")
```

```
[Out] sqrt(a*x^2 + b*x)*c^2/(a^2*sgn(x)) + 1/2*(3*b*c^2 - 4*a*c*d)*log(abs(-2*(sq
rt(a)*x - sqrt(a*x^2 + b*x))*sqrt(a) - b))/(a^(5/2)*sgn(x)) - 1/2*(3*b^2*c^
2*log(abs(b)) - 4*a*b*c*d*log(abs(b)) + 4*b^2*c^2 - 8*a*b*c*d + 4*a^2*d^2)*
sgn(x)/(a^(5/2)*b) + 2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)/(((sqrt(a)*x - sqrt(
a*x^2 + b*x))*a + sqrt(a)*b)*a^2*sgn(x))
```

Mupad [B]
time = 1.83, size = 120, normalized size = 1.28

$$\frac{c\,\mathrm{atanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)\,(4\,a\,d-3\,b\,c)}{a^{5/2}}-\frac{\frac{2\,(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2)}{a}-\frac{\left(a+\frac{b}{x}\right)\,(2\,a^2\,d^2-4\,a\,b\,c\,d+3\,b^2\,c^2)}{a^2}}{b\,\left(a+\frac{b}{x}\right)^{3/2}-a\,b\,\sqrt{a+\frac{b}{x}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d/x)^2/(a + b/x)^(3/2),x)
```

```
[Out] (c*atanh((a + b/x)^(1/2)/a^(1/2))*(4*a*d - 3*b*c))/a^(5/2) - ((2*(a^2*d^2 +
 b^2*c^2 - 2*a*b*c*d))/a - ((a + b/x)*(2*a^2*d^2 + 3*b^2*c^2 - 4*a*b*c*d))/
a^2)/(b*(a + b/x)^(3/2) - a*b*(a + b/x)^(1/2))
```

3.254 $\int \frac{c+\frac{d}{x}}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx$

Optimal. Leaf size=76

$$\frac{3bc-2ad}{a^2\sqrt{a+\frac{b}{x}}}+\frac{cx}{a\sqrt{a+\frac{b}{x}}}-\frac{(3bc-2ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}}$$

```
[Out] -(-2*a*d+3*b*c)*arctanh((a+b/x)^(1/2)/a^(1/2))/a^(5/2)+(-2*a*d+3*b*c)/a^2/(
a+b/x)^(1/2)+c*x/a/(a+b/x)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {382, 79, 53, 65, 214}

$$-\frac{(3bc-2ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}}+\frac{3bc-2ad}{a^2\sqrt{a+\frac{b}{x}}}+\frac{cx}{a\sqrt{a+\frac{b}{x}}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d/x)/(a + b/x)^(3/2),x]
```

```
[Out] (3*b*c - 2*a*d)/(a^2*Sqrt[a + b/x]) + (c*x)/(a*Sqrt[a + b/x]) - ((3*b*c - 2
*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/a^(5/2)
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{c+\frac{d}{x}}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx = -\text{Subst}\left(\int \frac{c+dx}{x^2(a+bx)^{3/2}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{cx}{a\sqrt{a+\frac{b}{x}}} - \frac{\left(-\frac{3bc}{2}+ad\right)\text{Subst}\left(\int \frac{1}{x(a+bx)^{3/2}}\,dx, x, \frac{1}{x}\right)}{a}$$

$$= \frac{3bc-2ad}{a^2\sqrt{a+\frac{b}{x}}} + \frac{cx}{a\sqrt{a+\frac{b}{x}}} + \frac{(3bc-2ad)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)}{2a^2}$$

$$= \frac{3bc-2ad}{a^2\sqrt{a+\frac{b}{x}}} + \frac{cx}{a\sqrt{a+\frac{b}{x}}} + \frac{(3bc-2ad)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x}}\right)}{a^2b}$$

$$= \frac{3bc-2ad}{a^2\sqrt{a+\frac{b}{x}}} + \frac{cx}{a\sqrt{a+\frac{b}{x}}} - \frac{(3bc-2ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}}$$

Mathematica [**A**]
time = 0.14, size = 70, normalized size = 0.92

$$\frac{\sqrt{a+\frac{b}{x}}\,x(3bc-2ad+acx)}{a^2(b+ax)} + \frac{(-3bc+2ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d/x)/(a + b/x)^(3/2),x]
```

```
[Out] (Sqrt[a + b/x]*x*(3*b*c - 2*a*d + a*c*x))/(a^2*(b + a*x)) + ((-3*b*c + 2*a*
d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/a^(5/2)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 386 vs. $2(66) = 132$.
time = 0.06, size = 387, normalized size = 5.09

method	result
risch	$\frac{c(ax+b)}{a^{2}\sqrt{\frac{ax+b}{x}}}+\frac{\left(\frac{\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^{2}+bx}\right)d}{a^{\frac{3}{2}}}-\frac{3\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^{2}+bx}\right)bc}{2a^{\frac{5}{2}}}-\frac{2\sqrt{a\left(x+\frac{b}{a}\right)^{2}-b\left(x+\frac{b}{a}\right)}\,d}{a^{2}\left(x+\frac{b}{a}\right)}+\frac{2\sqrt{}}{}\cdots\right.}{x\sqrt{\frac{ax+b}{x}}}$
default	$\frac{\sqrt{\frac{ax+b}{x}}\,x\left(-4\sqrt{x\,(ax+b)}\,a^{\frac{7}{2}}d\,x^{2}+6\sqrt{x\,(ax+b)}\,a^{\frac{5}{2}}bc\,x^{2}+2\ln\left(\frac{2\sqrt{x\,(ax+b)}\,\sqrt{a}+2ax+b}{2\sqrt{a}}\right)a^{3}bd\,x^{2}-3\ln\left(\cdots\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d/x)/(a+1/x*b)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*((a*x+b)/x)^(1/2)*x/a^(5/2)*(-4*(x*(a*x+b))^(1/2)*a^(7/2)*d*x^2+6*(x*(a
*x+b))^(1/2)*a^(5/2)*b*c*x^2+2*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)
/a^(1/2))*a^3*b*d*x^2-3*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2
))*a^2*b^2*c*x^2+4*(x*(a*x+b))^(3/2)*a^(5/2)*d-4*(x*(a*x+b))^(3/2)*a^(3/2)*
b*c-8*(x*(a*x+b))^(1/2)*a^(5/2)*b*d*x+12*(x*(a*x+b))^(1/2)*a^(3/2)*b^2*c*x+
4*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^2*b^2*d*x-6*ln(1/
2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a*b^3*c*x-4*(x*(a*x+b))^(1
/2)*a^(3/2)*b^2*d+6*(x*(a*x+b))^(1/2)*a^(1/2)*b^3*c+2*ln(1/2*(2*(x*(a*x+b))
^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a*b^3*d-3*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1
/2)+2*a*x+b)/a^(1/2))*b^4*c)/(x*(a*x+b))^(1/2)/b/(a*x+b)^2
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 144 vs. $2(66) = 132$.
time = 0.49, size = 144, normalized size = 1.89

$$\frac{1}{2}\,c\left(\frac{2\,\left(3\,\left(a+\frac{b}{x}\right)b-2\,ab\right)}{\left(a+\frac{b}{x}\right)^{\frac{3}{2}}a^{2}-\sqrt{a+\frac{b}{x}}\,a^{3}}+\frac{3\,b\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{a^{\frac{5}{2}}}\right)-d\left(\frac{\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{a^{\frac{3}{2}}}+\frac{2}{\sqrt{a+\frac{b}{x}}\,a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)/(a+b/x)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*c*(2*(3*(a + b/x)*b - 2*a*b)/((a + b/x)^(3/2)*a^2 - sqrt(a + b/x)*a^3)
+ 3*b*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/x) + sqrt(a)))/a^(5/2)) - d
*(log((sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/x) + sqrt(a)))/a^(3/2) + 2/(sqr
t(a + b/x)*a))
```

Fricas [A]
time = 2.16, size = 210, normalized size = 2.76

$$\left[-\frac{\left(3\,b^{2}c-2\,abd+\left(3\,abc-2\,a^{2}d\right)x\right)\sqrt{a}\,\log\left(2\,ax+2\,\sqrt{a}\,x\sqrt{\frac{ax+b}{x}}+b\right)-2\,\left(a^{2}cx^{2}+\left(3\,abc-2\,a^{2}d\right)x\right)\sqrt{\frac{ax+b}{x}}}{2\,\left(a^{4}x+a^{3}b\right)},\ \frac{\left(3\,b^{2}c-2\,abd+\left(3\,abc-2\,a^{2}d\right)x\right)\sqrt{-a}\,\arctan\left(\frac{\sqrt{-a}\,\sqrt{\frac{ax+b}{x}}}{a}\right)+\left(a^{2}cx^{2}+\left(3\,abc-2\,a^{2}d\right)x\right)\sqrt{\frac{ax+b}{x}}}{a^{4}x+a^{3}b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)/(a+b/x)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/2*((3*b^2*c - 2*a*b*d + (3*a*b*c - 2*a^2*d)*x)*sqrt(a)*log(2*a*x + 2*sq
rt(a)*x*sqrt((a*x + b)/x) + b) - 2*(a^2*c*x^2 + (3*a*b*c - 2*a^2*d)*x)*sqrt
((a*x + b)/x))/(a^4*x + a^3*b), ((3*b^2*c - 2*a*b*d + (3*a*b*c - 2*a^2*d)*x
)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) + (a^2*c*x^2 + (3*a*b*c - 2
*a^2*d)*x)*sqrt((a*x + b)/x))/(a^4*x + a^3*b)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 224 vs. $2(65) = 130$.
time = 20.95, size = 224, normalized size = 2.95

$$c\left(\frac{x^{\frac{3}{2}}}{a\sqrt{b}\,\sqrt{\frac{ax}{b}+1}}+\frac{3\sqrt{b}\,\sqrt{x}}{a^{2}\sqrt{\frac{ax}{b}+1}}-\frac{3b\operatorname{asinh}\left(\frac{\sqrt{a}\,\sqrt{x}}{\sqrt{b}}\right)}{a^{\frac{5}{2}}}\right)+d\left(-\frac{2a^{3}x\sqrt{1+\frac{b}{ax}}}{a^{\frac{9}{2}}x+a^{\frac{7}{2}}b}-\frac{a^{3}x\log\left(\frac{b}{ax}\right)}{a^{\frac{9}{2}}x+a^{\frac{7}{2}}b}+\frac{2a^{3}x\log\left(\sqrt{1+\frac{b}{ax}}+1\right)}{a^{\frac{9}{2}}x+a^{\frac{7}{2}}b}-\frac{a^{2}b\log\left(\frac{b}{ax}\right)}{a^{\frac{9}{2}}x+a^{\frac{7}{2}}b}+\frac{2a^{2}b\log\left(\sqrt{1+\frac{b}{ax}}+1\right)}{a^{\frac{9}{2}}x+a^{\frac{7}{2}}b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)/(a+b/x)**(3/2),x)
```

```
[Out] c*(x**(3/2)/(a*sqrt(b)*sqrt(a*x/b + 1)) + 3*sqrt(b)*sqrt(x)/(a**2*sqrt(a*x/
b + 1)) - 3*b*asinh(sqrt(a)*sqrt(x)/sqrt(b))/a**(5/2)) + d*(-2*a**3*x*sqrt(
1 + b/(a*x))/(a**(9/2)*x + a**(7/2)*b) - a**3*x*log(b/(a*x))/(a**(9/2)*x +
a**(7/2)*b) + 2*a**3*x*log(sqrt(1 + b/(a*x)) + 1)/(a**(9/2)*x + a**(7/2)*b)
 - a**2*b*log(b/(a*x))/(a**(9/2)*x + a**(7/2)*b) + 2*a**2*b*log(sqrt(1 + b/
(a*x)) + 1)/(a**(9/2)*x + a**(7/2)*b))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 148 vs. $2(66) = 132$.
time = 1.60, size = 148, normalized size = 1.95

$$-\frac{\left(3\,bc\log\left(|b|\right)-2\,ad\log\left(|b|\right)+4\,bc-4\,ad\right)\operatorname{sgn}\left(x\right)}{2\,a^{\frac{5}{2}}}+\frac{\sqrt{ax^{2}+bx}\,c}{a^{2}\operatorname{sgn}\left(x\right)}+\frac{\left(3\,bc-2\,ad\right)\log\left(\left|-2\,\left(\sqrt{a}\,x-\sqrt{ax^{2}+bx}\right)\sqrt{a}-b\right|\right)}{2\,a^{\frac{5}{2}}\operatorname{sgn}\left(x\right)}+\frac{2\,\left(b^{2}c-abd\right)}{\left(\left(\sqrt{a}\,x-\sqrt{ax^{2}+bx}\right)a+\sqrt{a}\,b\right)a^{2}\operatorname{sgn}\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)/(a+b/x)^(3/2),x, algorithm="giac")
```

```
[Out] -1/2*(3*b*c*log(abs(b)) - 2*a*d*log(abs(b)) + 4*b*c - 4*a*d)*sgn(x)/a^(5/2)
 + sqrt(a*x^2 + b*x)*c/(a^2*sgn(x)) + 1/2*(3*b*c - 2*a*d)*log(abs(-2*(sqrt(
```

```
a)*x - sqrt(a*x^2 + b*x))*sqrt(a) - b))/(a^(5/2)*sgn(x)) + 2*(b^2*c - a*b*d
)/(((sqrt(a)*x - sqrt(a*x^2 + b*x))*a + sqrt(a)*b)*a^2*sgn(x))
```

Mupad [B]
time = 2.44, size = 71, normalized size = 0.93

$$\frac{2\,d\,\mathrm{atanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{2\,d}{a\,\sqrt{a+\frac{b}{x}}}+\frac{2\,c\,x\,\left(\frac{a\,x}{b}+1\right)^{3/2}\,{}_2\mathrm{F}_1\left(\frac{3}{2},\frac{5}{2};\ \frac{7}{2};\ -\frac{a\,x}{b}\right)}{5\,\left(a+\frac{b}{x}\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d/x)/(a + b/x)^(3/2),x)
```

```
[Out] (2*d*atanh((a + b/x)^(1/2)/a^(1/2)))/a^(3/2) - (2*d)/(a*(a + b/x)^(1/2)) +
(2*c*x*((a*x)/b + 1)^(3/2)*hypergeom([3/2, 5/2], 7/2, -(a*x)/b))/(5*(a + b/
x)^(3/2))
```

3.255 $\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx$

Optimal. Leaf size=60

$$\frac{3b}{a^2\sqrt{a+\frac{b}{x}}}+\frac{x}{a\sqrt{a+\frac{b}{x}}}-\frac{3b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}}$$

```
[Out] -3*b*arctanh((a+b/x)^(1/2)/a^(1/2))/a^(5/2)+3*b/a^2/(a+b/x)^(1/2)+x/a/(a+b/
x)^(1/2)
```

Rubi [A]

time = 0.02, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {248, 44, 53, 65, 214}

$$-\frac{3b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}}+\frac{3b}{a^2\sqrt{a+\frac{b}{x}}}+\frac{x}{a\sqrt{a+\frac{b}{x}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^(-3/2),x]
```

```
[Out] (3*b)/(a^2*Sqrt[a + b/x]) + x/(a*Sqrt[a + b/x]) - (3*b*ArcTanh[Sqrt[a + b/x
]/Sqrt[a]])/a^(5/2)
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
```

```
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 248

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx = -\text{Subst}\left(\int \frac{1}{x^2(a+bx)^{3/2}}\,dx, x, \frac{1}{x}\right)$$
$$= -\frac{2x}{a\sqrt{a+\frac{b}{x}}} - \frac{3\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)}{a}$$
$$= -\frac{2x}{a\sqrt{a+\frac{b}{x}}} + \frac{3\sqrt{a+\frac{b}{x}}\,x}{a^2} + \frac{(3b)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)}{2a^2}$$
$$= -\frac{2x}{a\sqrt{a+\frac{b}{x}}} + \frac{3\sqrt{a+\frac{b}{x}}\,x}{a^2} + \frac{3\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x}}\right)}{a^2}$$
$$= -\frac{2x}{a\sqrt{a+\frac{b}{x}}} + \frac{3\sqrt{a+\frac{b}{x}}\,x}{a^2} - \frac{3b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}}$$

Mathematica [A]
time = 0.04, size = 57, normalized size = 0.95

$$\frac{\sqrt{a+\frac{b}{x}}\,x(3b+ax)}{a^2(b+ax)} - \frac{3b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^(-3/2),x]
```

```
[Out] (Sqrt[a + b/x]*x*(3*b + a*x))/(a^2*(b + a*x)) - (3*b*ArcTanh[Sqrt[a + b/x]/
Sqrt[a]])/a^(5/2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 197 vs. $2(50) = 100$.
time = 0.03, size = 198, normalized size = 3.30

method	result
risch	$\frac{ax+b}{a^{2}\sqrt{\frac{ax+b}{x}}}+\frac{\left(-\frac{3b\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^{2}+bx}\right)}{2a^{\frac{5}{2}}}+\frac{2b\sqrt{a\left(x+\frac{b}{a}\right)^{2}-b\left(x+\frac{b}{a}\right)}}{a^{3}\left(x+\frac{b}{a}\right)}\right)\sqrt{x\left(ax+b\right)}}{x\sqrt{\frac{ax+b}{x}}}$
default	$-\frac{\sqrt{\frac{ax+b}{x}}\,x\left(-6\sqrt{x\left(ax+b\right)}\,a^{\frac{5}{2}}x^{2}+3\ln\left(\frac{2\sqrt{x\left(ax+b\right)}\,\sqrt{a}+2ax+b}{2\sqrt{a}}\right)a^{2}bx^{2}+4a^{\frac{3}{2}}\left(x\left(ax+b\right)\right)^{\frac{3}{2}}-12\sqrt{x\left(ax+b\right.}}{2a^{\frac{5}{2}}\sqrt{x\left(a\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+1/x*b)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*((a*x+b)/x)^(1/2)*x/a^(5/2)*(-6*(x*(a*x+b))^(1/2)*a^(5/2)*x^2+3*ln(1/2
*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^2*b*x^2+4*a^(3/2)*(x*(a*x
+b))^(3/2)-12*(x*(a*x+b))^(1/2)*a^(3/2)*b*x+6*ln(1/2*(2*(x*(a*x+b))^(1/2)*a
^(1/2)+2*a*x+b)/a^(1/2))*a*b^2*x-6*(x*(a*x+b))^(1/2)*a^(1/2)*b^2+3*ln(1/2*(
2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*b^3)/(x*(a*x+b))^(1/2)/(a*x+b
)^2
```

Maxima [A]
time = 0.54, size = 85, normalized size = 1.42

$$\frac{3\left(a+\frac{b}{x}\right)b-2\,ab}{\left(a+\frac{b}{x}\right)^{\frac{3}{2}}a^{2}-\sqrt{a+\frac{b}{x}}\,a^{3}}+\frac{3\,b\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{2\,a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(3/2),x, algorithm="maxima")
```

```
[Out] (3*(a + b/x)*b - 2*a*b)/((a + b/x)^(3/2)*a^2 - sqrt(a + b/x)*a^3) + 3/2*b*l
og((sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/x) + sqrt(a)))/a^(5/2)
```

Fricas [A]
time = 2.10, size = 156, normalized size = 2.60

$$\left[\frac{3\,(abx+b^{2})\sqrt{a}\,\log\left(2\,ax-2\,\sqrt{a}\,x\sqrt{\frac{ax+b}{x}}+b\right)+2\,(a^{2}x^{2}+3\,abx)\sqrt{\frac{ax+b}{x}}}{2\,(a^{4}x+a^{3}b)},\ \frac{3\,(abx+b^{2})\sqrt{-a}\,\arctan\left(\frac{\sqrt{-a}\,\sqrt{\frac{ax+b}{x}}}{a}\right)+(a^{2}x^{2}+3\,abx)\sqrt{\frac{ax+b}{x}}}{a^{4}x+a^{3}b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*(3*(a*b*x + b^2)*sqrt(a)*log(2*a*x - 2*sqrt(a)*x*sqrt((a*x + b)/x) + b
) + 2*(a^2*x^2 + 3*a*b*x)*sqrt((a*x + b)/x))/(a^4*x + a^3*b), (3*(a*b*x + b
^2)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) + (a^2*x^2 + 3*a*b*x)*sqr
t((a*x + b)/x))/(a^4*x + a^3*b)]
```

Sympy [A]
time = 1.73, size = 71, normalized size = 1.18

$$\frac{x^{\frac{3}{2}}}{a\sqrt{b}\,\sqrt{\frac{ax}{b}+1}}+\frac{3\sqrt{b}\,\sqrt{x}}{a^{2}\sqrt{\frac{ax}{b}+1}}-\frac{3b\operatorname{asinh}\left(\frac{\sqrt{a}\,\sqrt{x}}{\sqrt{b}}\right)}{a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)**(3/2),x)
```

```
[Out] x**(3/2)/(a*sqrt(b)*sqrt(a*x/b + 1)) + 3*sqrt(b)*sqrt(x)/(a**2*sqrt(a*x/b +
 1)) - 3*b*asinh(sqrt(a)*sqrt(x)/sqrt(b))/a**(5/2)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 118 vs. 2(50) = 100.
time = 1.72, size = 118, normalized size = 1.97

$$-\frac{(3\,b\log\left(|b|\right)+4\,b)\mathrm{sgn}(x)}{2\,a^{\frac{5}{2}}}+\frac{3\,b\log\left(\left|-2\left(\sqrt{a}\,x-\sqrt{ax^{2}+bx}\right)\sqrt{a}-b\right|\right)}{2\,a^{\frac{5}{2}}\mathrm{sgn}\left(x\right)}+\frac{2\,b^{2}}{\left(\left(\sqrt{a}\,x-\sqrt{ax^{2}+bx}\right)a+\sqrt{a}\,b\right)a^{2}\mathrm{sgn}\left(x\right)}+\frac{\sqrt{ax^{2}+bx}}{a^{2}\mathrm{sgn}\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(3/2),x, algorithm="giac")
```

```
[Out] -1/2*(3*b*log(abs(b)) + 4*b)*sgn(x)/a^(5/2) + 3/2*b*log(abs(-2*(sqrt(a)*x -
 sqrt(a*x^2 + b*x))*sqrt(a) - b))/(a^(5/2)*sgn(x)) + 2*b^2/(((sqrt(a)*x - s
qrt(a*x^2 + b*x))*a + sqrt(a)*b)*a^2*sgn(x)) + sqrt(a*x^2 + b*x)/(a^2*sgn(x
))
```

Mupad [B]
time = 1.87, size = 34, normalized size = 0.57

$$\frac{2\,x\,{\left(\frac{a\,x}{b}+1\right)}^{3/2}\,{}_2\mathrm{F}_1\left(\frac{3}{2},\frac{5}{2};\ \frac{7}{2};\ -\frac{a\,x}{b}\right)}{5\,{\left(a+\frac{b}{x}\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b/x)^(3/2),x)
```

```
[Out] (2*x*((a*x)/b + 1)^(3/2)*hypergeom([3/2, 5/2], 7/2, -(a*x)/b))/(5*(a + b/x)
^(3/2))
```

3.256 $\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)}\,dx$

Optimal. Leaf size=147

$$\frac{b(3bc-ad)}{a^2c(bc-ad)\sqrt{a+\frac{b}{x}}}+\frac{x}{ac\sqrt{a+\frac{b}{x}}}+\frac{2d^{5/2}\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^2(bc-ad)^{3/2}}-\frac{(3bc+2ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}c^2}$$

```
[Out] 2*d^(5/2)*arctan(d^(1/2)*(a+b/x)^(1/2)/(-a*d+b*c)^(1/2))/c^2/(-a*d+b*c)^(3/
2)-(2*a*d+3*b*c)*arctanh((a+b/x)^(1/2)/a^(1/2))/a^(5/2)/c^2+b*(-a*d+3*b*c)/
a^2/c/(-a*d+b*c)/(a+b/x)^(1/2)+x/a/c/(a+b/x)^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {382, 105, 157, 162, 65, 214, 211}

$$-\frac{(2ad+3bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}c^2}+\frac{b(3bc-ad)}{a^2c\sqrt{a+\frac{b}{x}}\,(bc-ad)}+\frac{2d^{5/2}\text{ArcTan}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^2(bc-ad)^{3/2}}+\frac{x}{ac\sqrt{a+\frac{b}{x}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x)^(3/2)*(c + d/x)),x]
```

```
[Out] (b*(3*b*c - a*d))/(a^2*c*(b*c - a*d)*Sqrt[a + b/x]) + x/(a*c*Sqrt[a + b/x])
 + (2*d^(5/2)*ArcTan[(Sqrt[d]*Sqrt[a + b/x])/Sqrt[b*c - a*d]])/(c^2*(b*c -
a*d)^(3/2)) - ((3*b*c + 2*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/(a^(5/2)*c^2
)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 157

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)}\,dx = -\text{Subst}\left(\int \frac{1}{x^2(a+bx)^{3/2}(c+dx)}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{x}{ac\sqrt{a+\frac{b}{x}}} + \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(3bc+2ad)+\frac{3bdx}{2}}{x(a+bx)^{3/2}(c+dx)}\,dx, x, \frac{1}{x}\right)}{ac}$$

$$= \frac{b(3bc-ad)}{a^2c(bc-ad)\sqrt{a+\frac{b}{x}}} + \frac{x}{ac\sqrt{a+\frac{b}{x}}} + \frac{2\text{Subst}\left(\int \frac{\frac{1}{4}(bc-ad)(3bc+2ad)+\frac{1}{4}bd(3bc-ad)x}{x\sqrt{a+bx}\,(c+dx)}\,d\right.}{a^2c(bc-ad)}$$

$$= \frac{b(3bc-ad)}{a^2c(bc-ad)\sqrt{a+\frac{b}{x}}} + \frac{x}{ac\sqrt{a+\frac{b}{x}}} + \frac{d^3\text{Subst}\left(\int \frac{1}{\sqrt{a+bx}\,(c+dx)}\,dx, x, \frac{1}{x}\right)}{c^2(bc-ad)} +$$

$$= \frac{b(3bc-ad)}{a^2c(bc-ad)\sqrt{a+\frac{b}{x}}} + \frac{x}{ac\sqrt{a+\frac{b}{x}}} + \frac{(2d^3)\,\text{Subst}\left(\int \frac{1}{c-\frac{ad}{b}+\frac{dx^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x}}\right.}{bc^2(bc-ad)}$$

$$= \frac{b(3bc-ad)}{a^2c(bc-ad)\sqrt{a+\frac{b}{x}}} + \frac{x}{ac\sqrt{a+\frac{b}{x}}} + \frac{2d^{5/2}\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^2(bc-ad)^{3/2}} - \frac{(3bc+}{}$$

Mathematica [A]
time = 0.46, size = 144, normalized size = 0.98

$$\frac{\frac{c\sqrt{a+\frac{b}{x}}\,x\left(-3b^2c+a^2dx+ab(d-cx)\right)}{a^2(-bc+ad)(b+ax)} + \frac{2d^{5/2}\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{3/2}} - \frac{(3bc+2ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}}}{c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x)^(3/2)*(c + d/x)),x]
[Out] ((c*Sqrt[a + b/x]*x*(-3*b^2*c + a^2*d*x + a*b*(d - c*x)))/(a^2*(-(b*c) + a*
d)*(b + a*x)) + (2*d^(5/2)*ArcTan[(Sqrt[d]*Sqrt[a + b/x])/Sqrt[b*c - a*d]])
```

```
/(b*c - a*d)^(3/2) - ((3*b*c + 2*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/a^(5/
2))/c^2
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 963 vs. $2(127) = 254$.

time = 0.07, size = 964, normalized size = 6.56

method	result
risch	$\frac{ax+b}{a^2 c\sqrt{\frac{ax+b}{x}}} + \frac{\left(-\frac{\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^2+bx}\right)d}{a^{\frac{3}{2}}c^2} - \frac{3\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^2+bx}\right)b}{2a^{\frac{5}{2}}c} - \frac{d^3\ln\left(\frac{\frac{2d(ad-bc)}{c^2}-\frac{(2ad-bc)\left(x+\frac{d}{c}\right)}{c}+2\sqrt{\frac{d(a}{}}}{}\right)}{c^3}\right.}{}$
default	$\frac{\left(2\sqrt{x\,(ax+b)}\,a^{\frac{7}{2}}\sqrt{\frac{d(ad-bc)}{c^2}}\,c^2 d\,x^2 - 6\sqrt{x\,(ax+b)}\,a^{\frac{5}{2}}\sqrt{\frac{d(ad-bc)}{c^2}}\,b\,c^3 x^2 - 2\ln\left(\frac{2\sqrt{x\,(ax+b)}\,\sqrt{a}+2ax+b}{2\sqrt{a}}\right)\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+1/x*b)^(3/2)/(c+d/x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(2*(x*(a*x+b))^(1/2)*a^(7/2)*(d*(a*d-b*c)/c^2)^(1/2)*c^2*d*x^2-6*(x*(a*
x+b))^(1/2)*a^(5/2)*(d*(a*d-b*c)/c^2)^(1/2)*b*c^3*x^2-2*ln(1/2*(2*(x*(a*x+b
))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*(d*(a*d-b*c)/c^2)^(1/2)*a^4*c*d^2*x^2-ln
(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*(d*(a*d-b*c)/c^2)^(1/2)
*a^3*b*c^2*d*x^2+3*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*(d
*(a*d-b*c)/c^2)^(1/2)*a^2*b^2*c^3*x^2-2*a^(9/2)*ln((2*(x*(a*x+b))^(1/2)*(d*
(a*d-b*c)/c^2)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*d^3*x^2+4*(x*(a*x+b))^(3
/2)*a^(3/2)*(d*(a*d-b*c)/c^2)^(1/2)*b*c^3+4*(x*(a*x+b))^(1/2)*a^(5/2)*(d*(a
*d-b*c)/c^2)^(1/2)*b*c^2*d*x-12*(x*(a*x+b))^(1/2)*a^(3/2)*(d*(a*d-b*c)/c^2)
^(1/2)*b^2*c^3*x-4*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*(d
*(a*d-b*c)/c^2)^(1/2)*a^3*b*c*d^2*x-2*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2
*a*x+b)/a^(1/2))*(d*(a*d-b*c)/c^2)^(1/2)*a^2*b^2*c^2*d*x+6*ln(1/2*(2*(x*(a*
x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*(d*(a*d-b*c)/c^2)^(1/2)*a*b^3*c^3*x-4
*a^(7/2)*ln((2*(x*(a*x+b))^(1/2)*(d*(a*d-b*c)/c^2)^(1/2)*c-2*a*d*x+b*c*x-b*
d)/(c*x+d))*b*d^3*x+2*(x*(a*x+b))^(1/2)*a^(3/2)*(d*(a*d-b*c)/c^2)^(1/2)*b^2
*c^2*d-6*(x*(a*x+b))^(1/2)*a^(1/2)*(d*(a*d-b*c)/c^2)^(1/2)*b^3*c^3-2*ln(1/2
*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*(d*(a*d-b*c)/c^2)^(1/2)*a^2
*b^2*c*d^2-ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*(d*(a*d-b*
```

```
c)/c^2)^(1/2)*a*b^3*c^2*d+3*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^
(1/2))*(d*(a*d-b*c)/c^2)^(1/2)*b^4*c^3-2*a^(5/2)*ln((2*(x*(a*x+b))^(1/2)*(d
*(a*d-b*c)/c^2)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*b^2*d^3)/a^(5/2)*x*((a*
x+b)/x)^(1/2)/(a*x+b)^2/(d*(a*d-b*c)/c^2)^(1/2)/c^3/(a*d-b*c)/(x*(a*x+b))^(
1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(3/2)/(c+d/x),x, algorithm="maxima")
```

```
[Out] integrate(1/((a + b/x)^(3/2)*(c + d/x)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 259 vs. 2(127) = 254.
time = 2.54, size = 1075, normalized size = 7.31

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(3/2)/(c+d/x),x, algorithm="fricas")
```

```
[Out] [1/2*((3*b^3*c^2 - a*b^2*c*d - 2*a^2*b*d^2 + (3*a*b^2*c^2 - a^2*b*c*d - 2*a
^3*d^2)*x)*sqrt(a)*log(2*a*x - 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) - 2*(a^4*
d^2*x + a^3*b*d^2)*sqrt(-d/(b*c - a*d))*log(-(2*(b*c - a*d)*x*sqrt(-d/(b*c
- a*d))*sqrt((a*x + b)/x) - b*d + (b*c - 2*a*d)*x)/(c*x + d)) + 2*((a^2*b*c
^2 - a^3*c*d)*x^2 + (3*a*b^2*c^2 - a^2*b*c*d)*x)*sqrt((a*x + b)/x))/(a^3*b^
2*c^3 - a^4*b*c^2*d + (a^4*b*c^3 - a^5*c^2*d)*x), ((3*b^3*c^2 - a*b^2*c*d -
 2*a^2*b*d^2 + (3*a*b^2*c^2 - a^2*b*c*d - 2*a^3*d^2)*x)*sqrt(-a)*arctan(sqr
t(-a)*sqrt((a*x + b)/x)/a) - (a^4*d^2*x + a^3*b*d^2)*sqrt(-d/(b*c - a*d))*l
og(-(2*(b*c - a*d)*x*sqrt(-d/(b*c - a*d))*sqrt((a*x + b)/x) - b*d + (b*c -
2*a*d)*x)/(c*x + d)) + ((a^2*b*c^2 - a^3*c*d)*x^2 + (3*a*b^2*c^2 - a^2*b*c*
d)*x)*sqrt((a*x + b)/x))/(a^3*b^2*c^3 - a^4*b*c^2*d + (a^4*b*c^3 - a^5*c^2*
d)*x), 1/2*(4*(a^4*d^2*x + a^3*b*d^2)*sqrt(d/(b*c - a*d))*arctan(-(b*c - a*
d)*x*sqrt(d/(b*c - a*d))*sqrt((a*x + b)/x)/(a*d*x + b*d)) + (3*b^3*c^2 - a*
b^2*c*d - 2*a^2*b*d^2 + (3*a*b^2*c^2 - a^2*b*c*d - 2*a^3*d^2)*x)*sqrt(a)*lo
g(2*a*x - 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) + 2*((a^2*b*c^2 - a^3*c*d)*x^2
 + (3*a*b^2*c^2 - a^2*b*c*d)*x)*sqrt((a*x + b)/x))/(a^3*b^2*c^3 - a^4*b*c^2
*d + (a^4*b*c^3 - a^5*c^2*d)*x), (2*(a^4*d^2*x + a^3*b*d^2)*sqrt(d/(b*c - a
*d))*arctan(-(b*c - a*d)*x*sqrt(d/(b*c - a*d))*sqrt((a*x + b)/x)/(a*d*x + b
*d)) + (3*b^3*c^2 - a*b^2*c*d - 2*a^2*b*d^2 + (3*a*b^2*c^2 - a^2*b*c*d - 2*
a^3*d^2)*x)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) + ((a^2*b*c^2 - a
```

```
^3*c*d)*x^2 + (3*a*b^2*c^2 - a^2*b*c*d)*x)*sqrt((a*x + b)/x))/(a^3*b^2*c^3
- a^4*b*c^2*d + (a^4*b*c^3 - a^5*c^2*d)*x)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(a + \frac{b}{x}\right)^{\frac{3}{2}} (cx + d)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)**(3/2)/(c+d/x),x)
```

```
[Out] Integral(x/((a + b/x)**(3/2)*(c*x + d)), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(3/2)/(c+d/x),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [**B**]
time = 2.68, size = 3000, normalized size = 20.41

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b/x)^(3/2)*(c + d/x)),x)
```

```
[Out] (atan((((d^5*(a*d - b*c)^3)^(1/2)*((a + b/x)^(1/2)*(18*a^6*b^9*c^10*d^3 - 6
6*a^7*b^8*c^9*d^4 + 68*a^8*b^7*c^8*d^5 + 20*a^9*b^6*c^7*d^6 - 62*a^10*b^5*c
^6*d^7 - 2*a^11*b^4*c^5*d^8 + 40*a^12*b^3*c^4*d^9 - 16*a^13*b^2*c^3*d^10) +
 ((d^5*(a*d - b*c)^3)^(1/2)*(64*a^9*b^8*c^11*d^3 - 12*a^8*b^9*c^12*d^2 - 13
2*a^10*b^7*c^10*d^4 + 128*a^11*b^6*c^9*d^5 - 52*a^12*b^5*c^8*d^6 + 4*a^14*b
^3*c^6*d^8 + ((d^5*(a*d - b*c)^3)^(1/2)*(a + b/x)^(1/2)*(8*a^10*b^8*c^13*d^
2 - 56*a^11*b^7*c^12*d^3 + 160*a^12*b^6*c^11*d^4 - 240*a^13*b^5*c^10*d^5 +
200*a^14*b^4*c^9*d^6 - 88*a^15*b^3*c^8*d^7 + 16*a^16*b^2*c^7*d^8))/(c^2*(a*
d - b*c)^3)))/(c^2*(a*d - b*c)^3))*1i)/(c^2*(a*d - b*c)^3) + ((d^5*(a*d - b
*c)^3)^(1/2)*((a + b/x)^(1/2)*(18*a^6*b^9*c^10*d^3 - 66*a^7*b^8*c^9*d^4 + 6
8*a^8*b^7*c^8*d^5 + 20*a^9*b^6*c^7*d^6 - 62*a^10*b^5*c^6*d^7 - 2*a^11*b^4*c
```

^5*d^8 + 40*a^12*b^3*c^4*d^9 - 16*a^13*b^2*c^3*d^10) + ((d^5*(a*d - b*c)^3)^(1/2)*(12*a^8*b^9*c^12*d^2 - 64*a^9*b^8*c^11*d^3 + 132*a^10*b^7*c^10*d^4 - 128*a^11*b^6*c^9*d^5 + 52*a^12*b^5*c^8*d^6 - 4*a^14*b^3*c^6*d^8 + ((d^5*(a*d - b*c)^3)^(1/2)*(a + b/x)^(1/2)*(8*a^10*b^8*c^13*d^2 - 56*a^11*b^7*c^12*d^3 + 160*a^12*b^6*c^11*d^4 - 240*a^13*b^5*c^10*d^5 + 200*a^14*b^4*c^9*d^6 - 88*a^15*b^3*c^8*d^7 + 16*a^16*b^2*c^7*d^8))/(c^2*(a*d - b*c)^3)))/(c^2*(a*d - b*c)^3))*1i)/(c^2*(a*d - b*c)^3))/(36*a^6*b^8*c^7*d^5 - 96*a^7*b^7*c^6*d^6 + 64*a^8*b^6*c^5*d^7 + 24*a^9*b^5*c^4*d^8 - 36*a^10*b^4*c^3*d^9 + 8*a^11*b^3*c^2*d^10 - ((d^5*(a*d - b*c)^3)^(1/2)*((a + b/x)^(1/2)*(18*a^6*b^9*c^10*d^3 - 66*a^7*b^8*c^9*d^4 + 68*a^8*b^7*c^8*d^5 + 20*a^9*b^6*c^7*d^6 - 62*a^10*b^5*c^6*d^7 - 2*a^11*b^4*c^5*d^8 + 40*a^12*b^3*c^4*d^9 - 16*a^13*b^2*c^3*d^10) + ((d^5*(a*d - b*c)^3)^(1/2)*(64*a^9*b^8*c^11*d^3 - 12*a^8*b^9*c^12*d^2 - 132*a^10*b^7*c^10*d^4 + 128*a^11*b^6*c^9*d^5 - 52*a^12*b^5*c^8*d^6 + 4*a^14*b^3*c^6*d^8 + ((d^5*(a*d - b*c)^3)^(1/2)*(a + b/x)^(1/2)*(8*a^10*b^8*c^13*d^2 - 56*a^11*b^7*c^12*d^3 + 160*a^12*b^6*c^11*d^4 - 240*a^13*b^5*c^10*d^5 + 200*a^14*b^4*c^9*d^6 - 88*a^15*b^3*c^8*d^7 + 16*a^16*b^2*c^7*d^8))/(c^2*(a*d - b*c)^3)))/(c^2*(a*d - b*c)^3)))/(c^2*(a*d - b*c)^3) + ((d^5*(a*d - b*c)^3)^(1/2)*((a + b/x)^(1/2)*(18*a^6*b^9*c^10*d^3 - 66*a^7*b^8*c^9*d^4 + 68*a^8*b^7*c^8*d^5 + 20*a^9*b^6*c^7*d^6 - 62*a^10*b^5*c^6*d^7 - 2*a^11*b^4*c^5*d^8 + 40*a^12*b^3*c^4*d^9 - 16*a^13*b^2*c^3*d^10) + ((d^5*(a*d - b*c)^3)^(1/2)*(12*a^8*b^9*c^12*d^2 - 64*a^9*b^8*c^11*d^3 + 132*a^10*b^7*c^10*d^4 - 128*a^11*b^6*c^9*d^5 + 52*a^12*b^5*c^8*d^6 - 4*a^14*b^3*c^6*d^8 + ((d^5*(a*d - b*c)^3)^(1/2)*(a + b/x)^(1/2)*(8*a^10*b^8*c^13*d^2 - 56*a^11*b^7*c^12*d^3 + 160*a^12*b^6*c^11*d^4 - 240*a^13*b^5*c^10*d^5 + 200*a^14*b^4*c^9*d^6 - 88*a^15*b^3*c^8*d^7 + 16*a^16*b^2*c^7*d^8))/(c^2*(a*d - b*c)^3)))/(c^2*(a*d - b*c)^3)))/(c^2*(a*d - b*c)^3)))*(d^5*(a*d - b*c)^3)^(1/2)*2i)/(c^2*(a*d - b*c)^3) - (atanh((54*a^5*b^11*c^10*d^2*(a + b/x)^(1/2))/((a^5)^(1/2)*(54*a^3*b^11*c^10*d^2 - 216*a^4*b^10*c^9*d^3 + 234*a^5*b^9*c^8*d^4 + 124*a^6*b^8*c^7*d^5 - 366*a^7*b^7*c^6*d^6 + 120*a^8*b^6*c^5*d^7 + 110*a^9*b^5*c^4*d^8 - 60*a^10*b^4*c^3*d^9)) - (216*a^6*b^10*c^9*d^3*(a + b/x)^(1/2))/((a^5)^(1/2)*(54*a^3*b^11*c^10*d^2 - 216*a^4*b^10*c^9*d^3 + 234*a^5*b^9*c^8*d^4 + 124*a^6*b^8*c^7*d^5 - 366*a^7*b^7*c^6*d^6 + 120*a^8*b^6*c^5*d^7 + 110*a^9*b^5*c^4*d^8 - 60*a^10*b^4*c^3*d^9)) + (234*a^7*b^9*c^8*d^4*(a + b/x)^(1/2))/((a^5)^(1/2)*(54*a^3*b^11*c^10*d^2 - 216*a^4*b^10*c^9*d^3 + 234*a^5*b^9*c^8*d^4 + 124*a^6*b^8*c^7*d^5 - 366*a^7*b^7*c^6*d^6 + 120*a^8*b^6*c^5*d^7 + 110*a^9*b^5*c^4*d^8 - 60*a^10*b^4*c^3*d^9)) + (124*a^8*b^8*c^7*d^5*(a + b/x)^(1/2))/((a^5)^(1/2)*(54*a^3*b^11*c^10*d^2 - 216*a^4*b^10*c^9*d^3 + 234*a^5*b^9*c^8*d^4 + 124*a^6*b^8*c^7*d^5 - 366*a^7*b^7*c^6*d^6 + 120*a^8*b^6*c^5*d^7 + 110*a^9*b^5*c^4*d^8 - 60*a^10*b^4*c^3*d^9)) - (366*a^9*b^7*c^6*d^6*(a + b/x)^(1/2))/((a^5)^(1/2)*(54*a^3*b^11*c^10*d^2 - 216*a^4*b^10*c^9*d^3 + 234*a^5*b^9*c^8*d^4 + 124*a^6*b^8*c^7*d^5 - 366*a^7*b^7*c^6*d^6 + 120*a^8*b^6*c^5*d^7 + 110*a^9*b^5*c^4*d^8 - 60*a^10*b^4*c^3*d^9)) + (120*a^10*b^6*c^5*d^7*(a + b/x)^(1/2))/((a^5)^(1/2)*(54*a^3*b^11*c^10*d^2 - 216*a^4*b^10*c^9*d^3 + 234*a^5*b^9*c^8*d^4 + 124*a^6*b^8*c^7*d^5 - 366*a^7*b^7*c^6*d^6 + 120*a^8*b^6*c^5*d^7 + 110*a^9*b^5*c^4*d^8 - 60*a^10*b^4*c^3*d^9)) + (

110*a^11*b^5*c^4*d^8*(a + b/x)^(1/2))/((a^5)^(1/2)*(54*a^3*b^11*c^10*d^2 - 216*a^4*b^10*c^9*d^3 + 234*a^5*b^9*c^8*d^4 + 124*a^6*b^8*c^7*d^5 - 366*a^7*b^7*c^6*d^6 + 120*a^8*b^6*c^5*d^7 + 110*a^9*b^5*c^4*d^8 - 60*a^10*b^4*c^3*d^9)) - (60*a^12*b^4*c^3*d^9*(a + b/x)^(1/2))/((a^5)^(1/2)*(54*a^3*b^11*c^10*d^2 - 216*a^4*b^10*c^9*d^3 + 234*a^5*b^9*c^8*d^4 + 124*a^6*b^8*c^7*d^5 - 366*a^7*b^7*c^6*d^6 + 120*a^8*b^6*c^5*d^7 + 110*a^9*b^5*c^4*d^8 - 60*a^10*b^4*c^3*d^9)))*(2*a*d + 3*b*c))/(c^2*(a^5)^(1/2)) - ((2*b^2)/(a^2*d - a*b*c) + (b*(a + b/x)*(a*d - 3*b*c))/(a^2*c*(a*d - b*c)))/(a*(a + b/x)^(1/2) - (a + b/x)^(3/2))

3.257 $\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2} \, dx$

Optimal. Leaf size=224

$$\frac{b(3b^2c^2-2abcd+2a^2d^2)}{a^2c^2(bc-ad)^2\sqrt{a+\frac{b}{x}}}+\frac{d(bc-2ad)}{ac^2(bc-ad)\sqrt{a+\frac{b}{x}}\,\left(c+\frac{d}{x}\right)}+\frac{x}{ac\sqrt{a+\frac{b}{x}}\,\left(c+\frac{d}{x}\right)}+\frac{d^{5/2}(7bc-4ad)\tan^{-1}\left(\sqrt{\cdots}\right)}{c^3(bc-ad)^{5/\cdots}}$$

```
[Out] d^(5/2)*(-4*a*d+7*b*c)*arctan(d^(1/2)*(a+b/x)^(1/2)/(-a*d+b*c)^(1/2))/c^3/(
-a*d+b*c)^(5/2)-(4*a*d+3*b*c)*arctanh((a+b/x)^(1/2)/a^(1/2))/a^(5/2)/c^3+b*
(2*a^2*d^2-2*a*b*c*d+3*b^2*c^2)/a^2/c^2/(-a*d+b*c)^2/(a+b/x)^(1/2)+d*(-2*a*
d+b*c)/a/c^2/(-a*d+b*c)/(c+d/x)/(a+b/x)^(1/2)+x/a/c/(c+d/x)/(a+b/x)^(1/2)
```

Rubi [A]
time = 0.21, antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {382, 105, 156, 157, 162, 65, 214, 211}

$$-\frac{(4ad+3bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}c^3}+\frac{b(2a^2d^2-2abcd+3b^2c^2)}{a^2c^2\sqrt{a+\frac{b}{x}}\,(bc-ad)^2}+\frac{d^{5/2}(7bc-4ad)\text{ArcTan}\left(\frac{\sqrt{d}\,\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^3(bc-ad)^{5/2}}+\frac{d(bc-2ad)}{ac^2\sqrt{a+\frac{b}{x}}\,\left(c+\frac{d}{x}\right)(bc-ad)}+\frac{x}{ac\sqrt{a+\frac{b}{x}}\,\left(c+\frac{d}{x}\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x)^(3/2)*(c + d/x)^2),x]
```

```
[Out] (b*(3*b^2*c^2 - 2*a*b*c*d + 2*a^2*d^2))/(a^2*c^2*(b*c - a*d)^2*Sqrt[a + b/x
]) + (d*(b*c - 2*a*d))/(a*c^2*(b*c - a*d)*Sqrt[a + b/x]*(c + d/x)) + x/(a*c
*Sqrt[a + b/x]*(c + d/x)) + (d^(5/2)*(7*b*c - 4*a*d)*ArcTan[(Sqrt[d]*Sqrt[a
 + b/x])/Sqrt[b*c - a*d]])/(c^3*(b*c - a*d)^(5/2)) - ((3*b*c + 4*a*d)*ArcTa
nh[Sqrt[a + b/x]/Sqrt[a]])/(a^(5/2)*c^3)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 157

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2}\,dx = -\mathrm{Subst}\left(\int \frac{1}{x^2(a+bx)^{3/2}(c+dx)^2}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{x}{ac\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)} + \frac{\mathrm{Subst}\left(\int \frac{\frac{1}{2}(3bc+4ad)+\frac{5bdx}{2}}{x(a+bx)^{3/2}(c+dx)^2}\,dx, x, \frac{1}{x}\right)}{ac}$$

$$= \frac{d(bc-2ad)}{ac^2(bc-ad)\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)} + \frac{x}{ac\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)} - \frac{\mathrm{Subst}\left(\int \frac{-\frac{1}{2}(bc-ad)(3bc+}{x(a+bx}\right.}{ac^2(b}$$

$$= \frac{b(3b^2c^2-2abcd+2a^2d^2)}{a^2c^2(bc-ad)^2\sqrt{a+\frac{b}{x}}} + \frac{d(bc-2ad)}{ac^2(bc-ad)\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)} + \frac{x}{ac\sqrt{a+\frac{b}{x}}\left(c+\right.}$$

$$= \frac{b(3b^2c^2-2abcd+2a^2d^2)}{a^2c^2(bc-ad)^2\sqrt{a+\frac{b}{x}}} + \frac{d(bc-2ad)}{ac^2(bc-ad)\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)} + \frac{x}{ac\sqrt{a+\frac{b}{x}}\left(c+\right.}$$

$$= \frac{b(3b^2c^2-2abcd+2a^2d^2)}{a^2c^2(bc-ad)^2\sqrt{a+\frac{b}{x}}} + \frac{d(bc-2ad)}{ac^2(bc-ad)\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)} + \frac{x}{ac\sqrt{a+\frac{b}{x}}\left(c+\right.}$$

$$= \frac{b(3b^2c^2-2abcd+2a^2d^2)}{a^2c^2(bc-ad)^2\sqrt{a+\frac{b}{x}}} + \frac{d(bc-2ad)}{ac^2(bc-ad)\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)} + \frac{x}{ac\sqrt{a+\frac{b}{x}}\left(c+\right.}$$

Mathematica [A]

time = 0.78, size = 215, normalized size = 0.96

$$\frac{\frac{c\sqrt{a+\frac{b}{x}}\,x(3b^3c^2(d+cx)+a^3d^2x(2d+cx)+a^2bd(2d^2-cdx-2c^2x^2)+ab^2c(-2d^2-cdx+c^2x^2))}{a^2(bc-ad)^2(b+ax)(d+cx)}+\frac{d^{5/2}(7bc-4ad)\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{5/2}}-\frac{(3bc+4ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}}}{c^3}$$

Antiderivative was successfully verified.

[In] Integrate[1/((a + b/x)^(3/2)*(c + d/x)^2),x]

[Out] ((c*Sqrt[a + b/x]*x*(3*b^3*c^2*(d + c*x) + a^3*d^2*x*(2*d + c*x) + a^2*b*d*(2*d^2 - c*d*x - 2*c^2*x^2) + a*b^2*c*(-2*d^2 - c*d*x + c^2*x^2)))/(a^2*(b*c - a*d)^2*(b + a*x)*(d + c*x)) + (d^(5/2)*(7*b*c - 4*a*d)*ArcTan[(Sqrt[d]*Sqrt[a + b/x])/Sqrt[b*c - a*d]])/(b*c - a*d)^(5/2) - ((3*b*c + 4*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/a^(5/2))/c^3

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 3118 vs. $2(202) = 404$.
time = 0.11, size = 3119, normalized size = 13.92

method	result
risch	$\frac{ax+b}{c^2a^2\sqrt{\frac{ax+b}{x}}}+\frac{\left(-\frac{2\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^2+bx}\right)d}{a^{\frac{3}{2}}c^3}-\frac{3\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^2+bx}\right)b}{2a^{\frac{5}{2}}c^2}+\frac{d^3\sqrt{a\left(x+\frac{d}{c}\right)^2-\frac{(2ad-bc)\left(x+\frac{d}{c}\right)}{c}}}{c^3(ad-bc)^2\left(x+\frac{d}{c}\right)}\right.}{}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+1/x*b)^(3/2)/(c+d/x)^2,x,method=_RETURNVERBOSE)

[Out] -1/2*((a*x+b)/x)^(1/2)*x*(4*a^(15/2)*ln((2*(x*(a*x+b))^(1/2)*(d*(a*d-b*c)/c^2)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*d^6*x^2+2*a^(13/2)*(d*(a*d-b*c)/c^2)^(1/2)*(x*(a*x+b))^(1/2)*c^4*d^2*x^4+4*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^7*(d*(a*d-b*c)/c^2)^(1/2)*c^2*d^4*x^3-3*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^3*(d*(a*d-b*c)/c^2)^(1/2)*b^4*c^6*x^3-2*a^(11/2)*(d*(a*d-b*c)/c^2)^(1/2)*(x*(a*x+b))^(3/2)*c^4*d^2*x^2-2*a^(13/2)*(d*(a*d-b*c)/c^2)^(1/2)*(x*(a*x+b))^(1/2)*c^3*d^3*x^3+6*a^(7/2)*(d*(a*d-b*c)/c^2)^(1/2)*(x*(a*x+b))^(1/2)*b^3*c^6*x^3-11*a^(13/2)*ln((2*(x*(a*x+b))^(1/2)*(d*(a*d-b*c)/c^2)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*b*c^2*d^4*x

^3+7*a^(11/2)*ln((2*(x*(a*x+b))^(1/2)*(d*(a*d-b*c)/c^2)^(1/2)*c-2*a*d*x+b*c
*x-b*d)/(c*x+d))*b^2*c^3*d^3*x^3+4*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*
x+b)/a^(1/2))*a^7*(d*(a*d-b*c)/c^2)^(1/2)*c*d^5*x^2-8*a^(11/2)*(d*(a*d-b*c)
/c^2)^(1/2)*(x*(a*x+b))^(1/2)*b*c^2*d^4*x+14*a^(9/2)*(d*(a*d-b*c)/c^2)^(1/2
)*(x*(a*x+b))^(1/2)*b^2*c^3*d^3*x-12*a^(7/2)*(d*(a*d-b*c)/c^2)^(1/2)*(x*(a*
x+b))^(1/2)*b^3*c^4*d^2*x+2*a^(5/2)*(d*(a*d-b*c)/c^2)^(1/2)*(x*(a*x+b))^(1/
2)*b^4*c^5*d*x-14*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^5
(d(a*d-b*c)/c^2)^(1/2)*b^2*c^2*d^4*x-3*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2
)+2*a*x+b)/a^(1/2))*a^4*(d*(a*d-b*c)/c^2)^(1/2)*b^3*c^3*d^3*x+13*ln(1/2*(2*
(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^3*(d*(a*d-b*c)/c^2)^(1/2)*b^4
*c^4*d^2*x-ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^2*(d*(a*
d-b*c)/c^2)^(1/2)*b^5*c^5*d*x+7*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b
)/a^(1/2))*a^3*(d*(a*d-b*c)/c^2)^(1/2)*b^4*c^5*d*x^2-4*a^(9/2)*(d*(a*d-b*c)
/c^2)^(1/2)*(x*(a*x+b))^(3/2)*b*c^4*d^2*x+4*a^(7/2)*(d*(a*d-b*c)/c^2)^(1/2)
(x(a*x+b))^(3/2)*b^2*c^5*d*x+4*a^(11/2)*(d*(a*d-b*c)/c^2)^(1/2)*(x*(a*x+b
))^(1/2)*b*c^3*d^3*x^2+8*a^(9/2)*(d*(a*d-b*c)/c^2)^(1/2)*(x*(a*x+b))^(1/2)*
b^2*c^4*d^2*x^2-14*a^(7/2)*(d*(a*d-b*c)/c^2)^(1/2)*(x*(a*x+b))^(1/2)*b^3*c^
5*d*x^2+8*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^6*(d*(a*d
-b*c)/c^2)^(1/2)*b*c*d^5*x+12*a^(11/2)*(d*(a*d-b*c)/c^2)^(1/2)*(x*(a*x+b))^
(1/2)*b*c^4*d^2*x^3-10*a^(9/2)*(d*(a*d-b*c)/c^2)^(1/2)*(x*(a*x+b))^(1/2)*b^
2*c^5*d*x^3-ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^6*(d*(a
*d-b*c)/c^2)^(1/2)*b*c^2*d^4*x^2-15*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a
*x+b)/a^(1/2))*a^5*(d*(a*d-b*c)/c^2)^(1/2)*b^2*c^3*d^3*x^2+11*ln(1/2*(2*(x*
(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^4*(d*(a*d-b*c)/c^2)^(1/2)*b^3*c^
4*d^2*x^2-9*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^6*(d*(a
*d-b*c)/c^2)^(1/2)*b*c^3*d^3*x^3+3*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*
x+b)/a^(1/2))*a^5*(d*(a*d-b*c)/c^2)^(1/2)*b^2*c^4*d^2*x^3+5*ln(1/2*(2*(x*(a
*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^4*(d*(a*d-b*c)/c^2)^(1/2)*b^3*c^5*
d*x^3-6*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^2*(d*(a*d-b
*c)/c^2)^(1/2)*b^5*c^6*x^2-4*a^(5/2)*(d*(a*d-b*c)/c^2)^(1/2)*(x*(a*x+b))^(3
/2)*b^3*c^6*x-4*a^(13/2)*(d*(a*d-b*c)/c^2)^(1/2)*(x*(a*x+b))^(1/2)*c^2*d^4*
x^2+12*a^(5/2)*(d*(a*d-b*c)/c^2)^(1/2)*(x*(a*x+b))^(1/2)*b^4*c^6*x^2-3*a^(1
3/2)*ln((2*(x*(a*x+b))^(1/2)*(d*(a*d-b*c)/c^2)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(
c*x+d))*b*c*d^5*x^2-15*a^(11/2)*ln((2*(x*(a*x+b))^(1/2)*(d*(a*d-b*c)/c^2)^(
1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*b^2*c^2*d^4*x^2+14*a^(9/2)*ln((2*(x*(a*x
+b))^(1/2)*(d*(a*d-b*c)/c^2)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*b^3*c^3*d^
3*x^2+4*a^(15/2)*ln((2*(x*(a*x+b))^(1/2)*(d*(a*d-b*c)/c^2)^(1/2)*c-2*a*d*x+
b*c*x-b*d)/(c*x+d))*c*d^5*x^3-3*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b
)/a^(1/2))*a*(d*(a*d-b*c)/c^2)^(1/2)*b^6*c^6*x+6*a^(3/2)*(d*(a*d-b*c)/c^2)^
(1/2)*(x*(a*x+b))^(1/2)*b^5*c^6*x+8*a^(13/2)*ln((2*(x*(a*x+b))^(1/2)*(d*(a*
d-b*c)/c^2)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*b*d^6*x-3*ln(1/2*(2*(x*(a*x
+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a*(d*(a*d-b*c)/c^2)^(1/2)*b^6*c^5*d+6*
a^(3/2)*(d*(a*d-b*c)/c^2)^(1/2)*(x*(a*x+b))^(1/2)*b^5*c^5*d-11*a^(9/2)*ln((
2*(x*(a*x+b))^(1/2)*(d*(a*d-b*c)/c^2)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*b
^3*c*d^5+7*a^(7/2)*ln((2*(x*(a*x+b))^(1/2)*(d*(a*d-b*c)/c^2)^(1/2)*c-2*a*d*

```
x+b*c*x-b*d)/(c*x+d))*b^4*c^2*d^4+5*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a
*x+b)/a^(1/2))*a^2*(d*(a*d-b*c)/c^2)^(1/2)*b^5*c^4*d^2-4*a^(9/2)*(d*(a*d-b*
c)/c^2)^(1/2)*(x*(a*x+b))^(1/2)*b^2*c^2*d^4+8*a^(7/2)*(d*(a*d-b*c)/c^2)^(1/
2)*(x*(a*x+b))^(1/2)*b^3*c^3*d^3-10*a^(5/2)*(d*(a*d-b*c)/c^2)^(1/2)*(x*(a*x
+b))^(1/2)*b^4*c^4*d^2+4*a^(11/2)*ln((2*(x*(a*x+b))^(1/2)*(d*(a*d-b*c)/c^2)
^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*b^2*d^6+2*a^(7/2)*(d*(a*d-b*c)/c^2)^(1
/2)*(x*(a*x+b))^(3/2)*b^2*c^4*d^2-4*a^(5/2)*(d*(a*d-b*c)/c^2)^(1/2)*(x*(a*x
+b))^(3/2)*b^3*c^5*d-18*a^(11/2)*ln((2*(x*(a*x+b))^(1/2)*(d*(a*d-b*c)/c^2)^
(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*b^2*c*d^5*x+3*a^(9/2)*ln((2*(x*(a*x+b))
^(1/2)*(d*(a*d-b*c)/c^2)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*b^3*c^2*d^4*x+
7*a^(7/2)*ln((2*(x*(a*x+b))^(1/2)*(d*(a*d-b*c)/c^2)^(1/2)*c-2*a*d*x+b*c*x-b
*d)/(c*x+d))*b^4*c^3*d^3*x+4*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a
^(1/2))*a^5*(d*(a*d-b*c)/c^2)^(1/2)*b^2*c*d^5-9...
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(3/2)/(c+d/x)^2,x, algorithm="maxima")
```

```
[Out] integrate(1/((a + b/x)^(3/2)*(c + d/x)^2), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 569 vs. $2(202) = 404$.
time = 4.16, size = 2321, normalized size = 10.36

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(3/2)/(c+d/x)^2,x, algorithm="fricas")
```

```
[Out] [1/2*((3*b^4*c^3*d - 2*a*b^3*c^2*d^2 - 5*a^2*b^2*c*d^3 + 4*a^3*b*d^4 + (3*a
*b^3*c^4 - 2*a^2*b^2*c^3*d - 5*a^3*b*c^2*d^2 + 4*a^4*c*d^3)*x^2 + (3*b^4*c^
4 + a*b^3*c^3*d - 7*a^2*b^2*c^2*d^2 - a^3*b*c*d^3 + 4*a^4*d^4)*x)*sqrt(a)*l
og(2*a*x - 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) - (7*a^3*b^2*c*d^3 - 4*a^4*b*
d^4 + (7*a^4*b*c^2*d^2 - 4*a^5*c*d^3)*x^2 + (7*a^3*b^2*c^2*d^2 + 3*a^4*b*c*
d^3 - 4*a^5*d^4)*x)*sqrt(-d/(b*c - a*d))*log(-(2*(b*c - a*d)*x*sqrt(-d/(b*c
 - a*d))*sqrt((a*x + b)/x) - b*d + (b*c - 2*a*d)*x)/(c*x + d)) + 2*((a^2*b^
2*c^4 - 2*a^3*b*c^3*d + a^4*c^2*d^2)*x^3 + (3*a*b^3*c^4 - a^2*b^2*c^3*d - a
^3*b*c^2*d^2 + 2*a^4*c*d^3)*x^2 + (3*a*b^3*c^3*d - 2*a^2*b^2*c^2*d^2 + 2*a^
3*b*c*d^3)*x)*sqrt((a*x + b)/x))/(a^3*b^3*c^5*d - 2*a^4*b^2*c^4*d^2 + a^5*b
*c^3*d^3 + (a^4*b^2*c^6 - 2*a^5*b*c^5*d + a^6*c^4*d^2)*x^2 + (a^3*b^3*c^6 -
 a^4*b^2*c^5*d - a^5*b*c^4*d^2 + a^6*c^3*d^3)*x), 1/2*(2*(7*a^3*b^2*c*d^3 -
 4*a^4*b*d^4 + (7*a^4*b*c^2*d^2 - 4*a^5*c*d^3)*x^2 + (7*a^3*b^2*c^2*d^2 + 3
```

```
*a^4*b*c*d^3 - 4*a^5*d^4)*x)*sqrt(d/(b*c - a*d))*arctan(-(b*c - a*d)*x*sqrt
(d/(b*c - a*d))*sqrt((a*x + b)/x)/(a*d*x + b*d)) + (3*b^4*c^3*d - 2*a*b^3*c
^2*d^2 - 5*a^2*b^2*c*d^3 + 4*a^3*b*d^4 + (3*a*b^3*c^4 - 2*a^2*b^2*c^3*d - 5
*a^3*b*c^2*d^2 + 4*a^4*c*d^3)*x^2 + (3*b^4*c^4 + a*b^3*c^3*d - 7*a^2*b^2*c^
2*d^2 - a^3*b*c*d^3 + 4*a^4*d^4)*x)*sqrt(a)*log(2*a*x - 2*sqrt(a)*x*sqrt((a
*x + b)/x) + b) + 2*((a^2*b^2*c^4 - 2*a^3*b*c^3*d + a^4*c^2*d^2)*x^3 + (3*a
*b^3*c^4 - a^2*b^2*c^3*d - a^3*b*c^2*d^2 + 2*a^4*c*d^3)*x^2 + (3*a*b^3*c^3*
d - 2*a^2*b^2*c^2*d^2 + 2*a^3*b*c*d^3)*x)*sqrt((a*x + b)/x))/(a^3*b^3*c^5*d
 - 2*a^4*b^2*c^4*d^2 + a^5*b*c^3*d^3 + (a^4*b^2*c^6 - 2*a^5*b*c^5*d + a^6*c
^4*d^2)*x^2 + (a^3*b^3*c^6 - a^4*b^2*c^5*d - a^5*b*c^4*d^2 + a^6*c^3*d^3)*x
), 1/2*(2*(3*b^4*c^3*d - 2*a*b^3*c^2*d^2 - 5*a^2*b^2*c*d^3 + 4*a^3*b*d^4 +
(3*a*b^3*c^4 - 2*a^2*b^2*c^3*d - 5*a^3*b*c^2*d^2 + 4*a^4*c*d^3)*x^2 + (3*b^
4*c^4 + a*b^3*c^3*d - 7*a^2*b^2*c^2*d^2 - a^3*b*c*d^3 + 4*a^4*d^4)*x)*sqrt(
-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) - (7*a^3*b^2*c*d^3 - 4*a^4*b*d^4 +
 (7*a^4*b*c^2*d^2 - 4*a^5*c*d^3)*x^2 + (7*a^3*b^2*c^2*d^2 + 3*a^4*b*c*d^3 -
 4*a^5*d^4)*x)*sqrt(-d/(b*c - a*d))*log(-(2*(b*c - a*d)*x*sqrt(-d/(b*c - a*
d))*sqrt((a*x + b)/x) - b*d + (b*c - 2*a*d)*x)/(c*x + d)) + 2*((a^2*b^2*c^4
 - 2*a^3*b*c^3*d + a^4*c^2*d^2)*x^3 + (3*a*b^3*c^4 - a^2*b^2*c^3*d - a^3*b*
c^2*d^2 + 2*a^4*c*d^3)*x^2 + (3*a*b^3*c^3*d - 2*a^2*b^2*c^2*d^2 + 2*a^3*b*c
*d^3)*x)*sqrt((a*x + b)/x))/(a^3*b^3*c^5*d - 2*a^4*b^2*c^4*d^2 + a^5*b*c^3*
d^3 + (a^4*b^2*c^6 - 2*a^5*b*c^5*d + a^6*c^4*d^2)*x^2 + (a^3*b^3*c^6 - a^4*
b^2*c^5*d - a^5*b*c^4*d^2 + a^6*c^3*d^3)*x), ((7*a^3*b^2*c*d^3 - 4*a^4*b*d^
4 + (7*a^4*b*c^2*d^2 - 4*a^5*c*d^3)*x^2 + (7*a^3*b^2*c^2*d^2 + 3*a^4*b*c*d^
3 - 4*a^5*d^4)*x)*sqrt(d/(b*c - a*d))*arctan(-(b*c - a*d)*x*sqrt(d/(b*c - a
*d))*sqrt((a*x + b)/x)/(a*d*x + b*d)) + (3*b^4*c^3*d - 2*a*b^3*c^2*d^2 - 5*
a^2*b^2*c*d^3 + 4*a^3*b*d^4 + (3*a*b^3*c^4 - 2*a^2*b^2*c^3*d - 5*a^3*b*c^2*
d^2 + 4*a^4*c*d^3)*x^2 + (3*b^4*c^4 + a*b^3*c^3*d - 7*a^2*b^2*c^2*d^2 - a^3
*b*c*d^3 + 4*a^4*d^4)*x)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) + ((
a^2*b^2*c^4 - 2*a^3*b*c^3*d + a^4*c^2*d^2)*x^3 + (3*a*b^3*c^4 - a^2*b^2*c^3
*d - a^3*b*c^2*d^2 + 2*a^4*c*d^3)*x^2 + (3*a*b^3*c^3*d - 2*a^2*b^2*c^2*d^2
+ 2*a^3*b*c*d^3)*x)*sqrt((a*x + b)/x))/(a^3*b^3*c^5*d - 2*a^4*b^2*c^4*d^2 +
 a^5*b*c^3*d^3 + (a^4*b^2*c^6 - 2*a^5*b*c^5*d + a^6*c^4*d^2)*x^2 + (a^3*b^3
*c^6 - a^4*b^2*c^5*d - a^5*b*c^4*d^2 + a^6*c^3*d^3)*x)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(a+\frac{b}{x}\right)^{\frac{3}{2}}(cx+d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)**(3/2)/(c+d/x)**2,x)
```

```
[Out] Integral(x**2/((a + b/x)**(3/2)*(c*x + d)**2), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(3/2)/(c+d/x)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [**B**]
time = 6.20, size = 2500, normalized size = 11.16

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b/x)^(3/2)*(c + d/x)^2),x)
```

```
[Out] ((2*b^3)/(a^2*d - a*b*c) + (b*(a + b/x)^2*(2*a^2*d^3 + 3*b^2*c^2*d - 2*a*b*
c*d^2))/(c^2*(a^2*d - a*b*c)^2) - (b*(a + b/x)*(2*a*d - b*c)*(a^2*d^2 + 3*b
^2*c^2 - a*b*c*d))/(c^2*(a^2*d - a*b*c)^2))/(d*(a + b/x)^(5/2) + (a + b/x)^
(1/2)*(a^2*d - a*b*c) - (a + b/x)^(3/2)*(2*a*d - b*c)) + (atan((a^13*b^11*c
^11*d^3*(a + b/x)^(1/2)*35i - a^12*b^12*c^12*d^2*(a + b/x)^(1/2)*441i - a^1
0*b^14*c^14*(a + b/x)^(1/2)*27i + a^14*b^10*c^10*d^4*(a + b/x)^(1/2)*1694i
- a^15*b^9*c^9*d^5*(a + b/x)^(1/2)*3073i + a^16*b^8*c^8*d^6*(a + b/x)^(1/2)
*1316i + a^17*b^7*c^7*d^7*(a + b/x)^(1/2)*2561i - a^18*b^6*c^6*d^8*(a + b/x
)^(1/2)*4375i + a^19*b^5*c^5*d^9*(a + b/x)^(1/2)*2996i - a^20*b^4*c^4*d^10*
(a + b/x)^(1/2)*1015i + a^21*b^3*c^3*d^11*(a + b/x)^(1/2)*140i + a^11*b^13*
c^13*d*(a + b/x)^(1/2)*189i)/(a^5*(a^5)^(1/2)*(a^5*(a^5*(2561*b^7*c^7*d^7 -
 4375*a*b^6*c^6*d^8 + 2996*a^2*b^5*c^5*d^9 - 1015*a^3*b^4*c^4*d^10 + 140*a^
4*b^3*c^3*d^11) - 441*b^12*c^12*d^2 + 35*a*b^11*c^11*d^3 + 1694*a^2*b^10*c^
10*d^4 - 3073*a^3*b^9*c^9*d^5 + 1316*a^4*b^8*c^8*d^6) - 27*a^3*b^14*c^14 +
189*a^4*b^13*c^13*d)))*(4*a*d + 3*b*c)*1i)/(c^3*(a^5)^(1/2)) - (atan((((d^5
*(a*d - b*c)^5)^(1/2)*(4*a*d - 7*b*c)*((a + b/x)^(1/2)*(18*a^6*b^14*c^18*d^
3 - 132*a^7*b^13*c^17*d^4 + 362*a^8*b^12*c^16*d^5 - 320*a^9*b^11*c^15*d^6 -
 442*a^10*b^10*c^14*d^7 + 1004*a^11*b^9*c^13*d^8 + 578*a^12*b^8*c^12*d^9 -
3976*a^13*b^7*c^11*d^10 + 5960*a^14*b^6*c^10*d^11 - 4768*a^15*b^5*c^9*d^12
+ 2228*a^16*b^4*c^8*d^13 - 576*a^17*b^3*c^7*d^14 + 64*a^18*b^2*c^6*d^15) -
((d^5*(a*d - b*c)^5)^(1/2)*(4*a*d - 7*b*c)*(12*a^8*b^14*c^21*d^2 - 116*a^9*
b^13*c^20*d^3 + 484*a^10*b^12*c^19*d^4 - 1128*a^11*b^11*c^18*d^5 + 1560*a^1
2*b^10*c^17*d^6 - 1176*a^13*b^9*c^16*d^7 + 168*a^14*b^8*c^15*d^8 + 576*a^15
*b^7*c^14*d^9 - 612*a^16*b^6*c^13*d^10 + 300*a^17*b^5*c^12*d^11 - 76*a^18*b
```

^4*c^11*d^12 + 8*a^19*b^3*c^10*d^13 - ((d^5*(a*d - b*c)^5)^(1/2)*(a + b/x)^(1/2)*(4*a*d - 7*b*c)*(8*a^10*b^13*c^23*d^2 - 96*a^11*b^12*c^22*d^3 + 520*a^12*b^11*c^21*d^4 - 1680*a^13*b^10*c^20*d^5 + 3600*a^14*b^9*c^19*d^6 - 5376*a^15*b^8*c^18*d^7 + 5712*a^16*b^7*c^17*d^8 - 4320*a^17*b^6*c^16*d^9 + 2280*a^18*b^5*c^15*d^10 - 800*a^19*b^4*c^14*d^11 + 168*a^20*b^3*c^13*d^12 - 16*a^21*b^2*c^12*d^13))/(2*(b^5*c^8 - a^5*c^3*d^5 + 5*a^4*b*c^4*d^4 + 10*a^2*b^3*c^6*d^2 - 10*a^3*b^2*c^5*d^3 - 5*a*b^4*c^7*d))))/(2*(b^5*c^8 - a^5*c^3*d^5 + 5*a^4*b*c^4*d^4 + 10*a^2*b^3*c^6*d^2 - 10*a^3*b^2*c^5*d^3 - 5*a*b^4*c^7*d)))*1i)/(2*(b^5*c^8 - a^5*c^3*d^5 + 5*a^4*b*c^4*d^4 + 10*a^2*b^3*c^6*d^2 - 10*a^3*b^2*c^5*d^3 - 5*a*b^4*c^7*d)) + ((d^5*(a*d - b*c)^5)^(1/2)*(4*a*d - 7*b*c)*((a + b/x)^(1/2)*(18*a^6*b^14*c^18*d^3 - 132*a^7*b^13*c^17*d^4 + 362*a^8*b^12*c^16*d^5 - 320*a^9*b^11*c^15*d^6 - 442*a^10*b^10*c^14*d^7 + 1004*a^11*b^9*c^13*d^8 + 578*a^12*b^8*c^12*d^9 - 3976*a^13*b^7*c^11*d^10 + 5960*a^14*b^6*c^10*d^11 - 4768*a^15*b^5*c^9*d^12 + 2228*a^16*b^4*c^8*d^13 - 576*a^17*b^3*c^7*d^14 + 64*a^18*b^2*c^6*d^15) + ((d^5*(a*d - b*c)^5)^(1/2)*(4*a*d - 7*b*c)*(12*a^8*b^14*c^21*d^2 - 116*a^9*b^13*c^20*d^3 + 484*a^10*b^12*c^19*d^4 - 1128*a^11*b^11*c^18*d^5 + 1560*a^12*b^10*c^17*d^6 - 1176*a^13*b^9*c^16*d^7 + 168*a^14*b^8*c^15*d^8 + 576*a^15*b^7*c^14*d^9 - 612*a^16*b^6*c^13*d^10 + 300*a^17*b^5*c^12*d^11 - 76*a^18*b^4*c^11*d^12 + 8*a^19*b^3*c^10*d^13 + ((d^5*(a*d - b*c)^5)^(1/2)*(a + b/x)^(1/2)*(4*a*d - 7*b*c)*(8*a^10*b^13*c^23*d^2 - 96*a^11*b^12*c^22*d^3 + 520*a^12*b^11*c^21*d^4 - 1680*a^13*b^10*c^20*d^5 + 3600*a^14*b^9*c^19*d^6 - 5376*a^15*b^8*c^18*d^7 + 5712*a^16*b^7*c^17*d^8 - 4320*a^17*b^6*c^16*d^9 + 2280*a^18*b^5*c^15*d^10 - 800*a^19*b^4*c^14*d^11 + 168*a^20*b^3*c^13*d^12 - 16*a^21*b^2*c^12*d^13))/(2*(b^5*c^8 - a^5*c^3*d^5 + 5*a^4*b*c^4*d^4 + 10*a^2*b^3*c^6*d^2 - 10*a^3*b^2*c^5*d^3 - 5*a*b^4*c^7*d))))/(2*(b^5*c^8 - a^5*c^3*d^5 + 5*a^4*b*c^4*d^4 + 10*a^2*b^3*c^6*d^2 - 10*a^3*b^2*c^5*d^3 - 5*a*b^4*c^7*d)))*1i)/(2*(b^5*c^8 - a^5*c^3*d^5 + 5*a^4*b*c^4*d^4 + 10*a^2*b^3*c^6*d^2 - 10*a^3*b^2*c^5*d^3 - 5*a*b^4*c^7*d)))/(((d^5*(a*d - b*c)^5)^(1/2)*(4*a*d - 7*b*c)*((a + b/x)^(1/2)*(18*a^6*b^14*c^18*d^3 - 132*a^7*b^13*c^17*d^4 + 362*a^8*b^12*c^16*d^5 - 320*a^9*b^11*c^15*d^6 - 442*a^10*b^10*c^14*d^7 + 1004*a^11*b^9*c^13*d^8 + 578*a^12*b^8*c^12*d^9 - 3976*a^13*b^7*c^11*d^10 + 5960*a^14*b^6*c^10*d^11 - 4768*a^15*b^5*c^9*d^12 + 2228*a^16*b^4*c^8*d^13 - 576*a^17*b^3*c^7*d^14 + 64*a^18*b^2*c^6*d^15) - ((d^5*(a*d - b*c)^5)^(1/2)*(4*a*d - 7*b*c)*(12*a^8*b^14*c^21*d^2 - 116*a^9*b^13*c^20*d^3 + 484*a^10*b^12*c^19*d^4 - 1128*a^11*b^11*c^18*d^5 + 1560*a^12*b^10*c^17*d^6 - 1176*a^13*b^9*c^16*d^7 + 168*a^14*b^8*c^15*d^8 + 576*a^15*b^7*c^14*d^9 - 612*a^16*b^6*c^13*d^10 + 300*a^17*b^5*c^12*d^11 - 76*a^18*b^4*c^11*d^12 + 8*a^19*b^3*c^10*d^13 - ((d^5*(a*d - b*c)^5)^(1/2)*(a + b/x)^(1/2)*(4*a*d - 7*b*c)*(8*a^10*b^13*c^23*d^2 - 96*a^11*b^12*c^22*d^3 + 520*a^12*b^11*c^21*d^4 - 1680*a^13*b^10*c^20*d^5 + 3600*a^14*b^9*c^19*d^6 - 5376*a^15*b^8*c^18*d^7 + 5712*a^16*b^7*c^17*d^8 - 4320*a^17*b^6*c^16*d^9 + 2280*a^18*b^5*c^15*d^10 - 800*a^19*b^4*c^14*d^11 + 168*a^20*b^3*c^13*d^12 - 16*a^21*b^2*c^12*d^13))/(2*(b^5...

3.258 $\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^3}\,dx$

Optimal. Leaf size=320

$$\frac{3b(2bc-ad)\left(2b^2c^2-abcd+4a^2d^2\right)}{4a^2c^3(bc-ad)^3\sqrt{a+\frac{b}{x}}}+\frac{d(2bc-3ad)}{2ac^2(bc-ad)\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2}+\frac{d\left(4b^2c^2-21abcd+12a^2d^2\right)}{4ac^3(bc-ad)^2\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}+\frac{\cdots}{ac\cdots}$$

```
[Out] 3/4*d^(5/2)*(8*a^2*d^2-24*a*b*c*d+21*b^2*c^2)*arctan(d^(1/2)*(a+b/x)^(1/2)/
(-a*d+b*c)^(1/2))/c^4/(-a*d+b*c)^(7/2)-3*(2*a*d+b*c)*arctanh((a+b/x)^(1/2)/
a^(1/2))/a^(5/2)/c^4+3/4*b*(-a*d+2*b*c)*(4*a^2*d^2-a*b*c*d+2*b^2*c^2)/a^2/c
^3/(-a*d+b*c)^3/(a+b/x)^(1/2)+1/2*d*(-3*a*d+2*b*c)/a/c^2/(-a*d+b*c)/(c+d/x)
^2/(a+b/x)^(1/2)+1/4*d*(12*a^2*d^2-21*a*b*c*d+4*b^2*c^2)/a/c^3/(-a*d+b*c)^2
/(c+d/x)/(a+b/x)^(1/2)+x/a/c/(c+d/x)^2/(a+b/x)^(1/2)
```

Rubi [A]

time = 0.34, antiderivative size = 320, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {382, 105, 156, 157, 162, 65, 214, 211}

$$-\frac{3(2ad+bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}c^4}+\frac{3d^{5/2}\left(8a^2d^2-24abcd+21b^2c^2\right)\text{ArcTan}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{4c^4(bc-ad)^{7/2}}+\frac{d\left(12a^2d^2-21abcd+4b^2c^2\right)}{4ac^3\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)(bc-ad)^2}+\frac{3b(2bc-ad)\left(4a^2d^2-abcd+2b^2c^2\right)}{4a^2c^3\sqrt{a+\frac{b}{x}}(bc-ad)^3}+\frac{d(2bc-3ad)}{2ac^2\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2(bc-ad)}+\frac{x}{ac\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x)^(3/2)*(c + d/x)^3),x]
```

```
[Out] (3*b*(2*b*c - a*d)*(2*b^2*c^2 - a*b*c*d + 4*a^2*d^2))/(4*a^2*c^3*(b*c - a*d
)^3*Sqrt[a + b/x]) + (d*(2*b*c - 3*a*d))/(2*a*c^2*(b*c - a*d)*Sqrt[a + b/x]
*(c + d/x)^2) + (d*(4*b^2*c^2 - 21*a*b*c*d + 12*a^2*d^2))/(4*a*c^3*(b*c - a
*d)^2*Sqrt[a + b/x]*(c + d/x)) + x/(a*c*Sqrt[a + b/x]*(c + d/x)^2) + (3*d^(
5/2)*(21*b^2*c^2 - 24*a*b*c*d + 8*a^2*d^2)*ArcTan[(Sqrt[d]*Sqrt[a + b/x])/S
qrt[b*c - a*d]])/(4*c^4*(b*c - a*d)^(7/2)) - (3*(b*c + 2*a*d)*ArcTanh[Sqrt[
a + b/x]/Sqrt[a]])/(a^(5/2)*c^4)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 157

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
```

```
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^3}\,dx = -\text{Subst}\left(\int \frac{1}{x^2(a+bx)^{3/2}(c+dx)^3}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{x}{ac\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2} + \frac{\text{Subst}\left(\int \frac{\frac{3}{2}(bc+2ad)+\frac{7bdx}{2}}{x(a+bx)^{3/2}(c+dx)^3}\,dx, x, \frac{1}{x}\right)}{ac}$$

$$= \frac{d(2bc-3ad)}{2ac^2(bc-ad)\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2} + \frac{x}{ac\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2} - \frac{\text{Subst}\left(\int \frac{-3(bc-ad)(b}{x(a-}\right.}{2a}$$

$$= \frac{d(2bc-3ad)}{2ac^2(bc-ad)\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2} + \frac{d(4b^2c^2-21abcd+12a^2d^2)}{4ac^3(bc-ad)^2\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)} + \frac{}{ac\sqrt{a+}}$$

$$= \frac{3b(2bc-ad)(2b^2c^2-abcd+4a^2d^2)}{4a^2c^3(bc-ad)^3\sqrt{a+\frac{b}{x}}} + \frac{d(2bc-3ad)}{2ac^2(bc-ad)\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2} + \frac{d(}{4ac}$$

$$= \frac{3b(2bc-ad)(2b^2c^2-abcd+4a^2d^2)}{4a^2c^3(bc-ad)^3\sqrt{a+\frac{b}{x}}} + \frac{d(2bc-3ad)}{2ac^2(bc-ad)\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2} + \frac{d(}{4ac}$$

$$= \frac{3b(2bc-ad)(2b^2c^2-abcd+4a^2d^2)}{4a^2c^3(bc-ad)^3\sqrt{a+\frac{b}{x}}} + \frac{d(2bc-3ad)}{2ac^2(bc-ad)\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2} + \frac{d(}{4ac}$$

$$= \frac{3b(2bc-ad)(2b^2c^2-abcd+4a^2d^2)}{4a^2c^3(bc-ad)^3\sqrt{a+\frac{b}{x}}} + \frac{d(2bc-3ad)}{2ac^2(bc-ad)\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2} + \frac{d(}{4ac}$$

<u>Mathematica</u> [A]

time = 1.26, size = 298, normalized size = 0.93

$$\frac{\frac{c\sqrt{a+\frac{b}{x}}\,x\left(-12b^4c^3(d+cx)^2-4ab^3c^2(-3d+cx)(d+cx)^2+2a^4d^3x(6d^2+9cdx+2c^2x^2)+a^3bd^2(12d^3-9cd^2x-37c^2dx^2-12c^3x^3)+a^2b^2cd(-27d^3-29cd^2x+12c^2dx^2+12c^3x^3)\right)}{a^2(-bc+ad)^3(b+ax)(d+cx)^2}+\frac{3d^{5/2}(21b^2c^2-24abcd+8a^2d^2)\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{7/2}}-\frac{12(bc+2ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}}}{4c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x)^(3/2)*(c + d/x)^3),x]
```

```
[Out] ((c*Sqrt[a + b/x]*x*(-12*b^4*c^3*(d + c*x)^2 - 4*a*b^3*c^2*(-3*d + c*x)*(d
+ c*x)^2 + 2*a^4*d^3*x*(6*d^2 + 9*c*d*x + 2*c^2*x^2) + a^3*b*d^2*(12*d^3 -
9*c*d^2*x - 37*c^2*d*x^2 - 12*c^3*x^3) + a^2*b^2*c*d*(-27*d^3 - 29*c*d^2*x
+ 12*c^2*d*x^2 + 12*c^3*x^3)))/(a^2*(-(b*c) + a*d)^3*(b + a*x)*(d + c*x)^2)
 + (3*d^(5/2)*(21*b^2*c^2 - 24*a*b*c*d + 8*a^2*d^2)*ArcTan[(Sqrt[d]*Sqrt[a
+ b/x])/Sqrt[b*c - a*d]])/(b*c - a*d)^(7/2) - (12*(b*c + 2*a*d)*ArcTanh[Sqr
t[a + b/x]/Sqrt[a]])/a^(5/2))/(4*c^4)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 5157 vs. $2(288) = 576$.
time = 0.14, size = 5158, normalized size = 16.12

method	result
risch	$\frac{ax+b}{c^3a^2\sqrt{\frac{ax+b}{x}}} + \frac{\left(-\frac{3\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^2+bx}\right)d}{a^{\frac{3}{2}}c^4} - \frac{3\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^2+bx}\right)b}{2a^{\frac{5}{2}}c^3} - \frac{d^4\sqrt{a\left(x+\frac{d}{c}\right)^2 - \frac{(2ad-bc)\left(x+\frac{d}{c}\right)}{c}}}{2c^5(ad-bc)^2\left(x+\frac{d}{c}\right)^2}\right.}{}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+1/x*b)^(3/2)/(c+d/x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(3/2)/(c+d/x)^3,x, algorithm="maxima")
```

```
[Out] integrate(1/((a + b/x)^(3/2)*(c + d/x)^3), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1014 vs. $2(288) = 576$.
time = 4.81, size = 4093, normalized size = 12.79

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b/x)^(3/2)/(c+d/x)^3,x, algorithm="fricas")

[Out] [1/8*(12*(b^5*c^4*d^2 - a*b^4*c^3*d^3 - 3*a^2*b^3*c^2*d^4 + 5*a^3*b^2*c*d^5 - 2*a^4*b*d^6 + (a*b^4*c^6 - a^2*b^3*c^5*d - 3*a^3*b^2*c^4*d^2 + 5*a^4*b*c^3*d^3 - 2*a^5*c^2*d^4)*x^3 + (b^5*c^6 + a*b^4*c^5*d - 5*a^2*b^3*c^4*d^2 - a^3*b^2*c^3*d^3 + 8*a^4*b*c^2*d^4 - 4*a^5*c*d^5)*x^2 + (2*b^5*c^5*d - a*b^4*c^4*d^2 - 7*a^2*b^3*c^3*d^3 + 7*a^3*b^2*c^2*d^4 + a^4*b*c*d^5 - 2*a^5*d^6)*x)*sqrt(a)*log(2*a*x - 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) - 3*(21*a^3*b^3*c^2*d^4 - 24*a^4*b^2*c*d^5 + 8*a^5*b*d^6 + (21*a^4*b^2*c^4*d^2 - 24*a^5*b*c^3*d^3 + 8*a^6*c^2*d^4)*x^3 + (21*a^3*b^3*c^4*d^2 + 18*a^4*b^2*c^3*d^3 - 40*a^5*b*c^2*d^4 + 16*a^6*c*d^5)*x^2 + (42*a^3*b^3*c^3*d^3 - 27*a^4*b^2*c^2*d^4 - 8*a^5*b*c*d^5 + 8*a^6*d^6)*x)*sqrt(-d/(b*c - a*d))*log(-(2*(b*c - a*d)*x*sqrt(-d/(b*c - a*d))*sqrt((a*x + b)/x) - b*d + (b*c - 2*a*d)*x)/(c*x + d)) + 2*(4*(a^2*b^3*c^6 - 3*a^3*b^2*c^5*d + 3*a^4*b*c^4*d^2 - a^5*c^3*d^3)*x^4 + (12*a*b^4*c^6 - 4*a^2*b^3*c^5*d - 12*a^3*b^2*c^4*d^2 + 37*a^4*b*c^3*d^3 - 18*a^5*c^2*d^4)*x^3 + (24*a*b^4*c^5*d - 20*a^2*b^3*c^4*d^2 + 29*a^3*b^2*c^3*d^3 + 9*a^4*b*c^2*d^4 - 12*a^5*c*d^5)*x^2 + 3*(4*a*b^4*c^4*d^2 - 4*a^2*b^3*c^3*d^3 + 9*a^3*b^2*c^2*d^4 - 4*a^4*b*c*d^5)*x)*sqrt((a*x + b)/x))/(a^3*b^4*c^7*d^2 - 3*a^4*b^3*c^6*d^3 + 3*a^5*b^2*c^5*d^4 - a^6*b*c^4*d^5 + (a^4*b^3*c^9 - 3*a^5*b^2*c^8*d + 3*a^6*b*c^7*d^2 - a^7*c^6*d^3)*x^3 + (a^3*b^4*c^9 - a^4*b^3*c^8*d - 3*a^5*b^2*c^7*d^2 + 5*a^6*b*c^6*d^3 - 2*a^7*c^5*d^4)*x^2 + (2*a^3*b^4*c^8*d - 5*a^4*b^3*c^7*d^2 + 3*a^5*b^2*c^6*d^3 + a^6*b*c^5*d^4 - a^7*c^4*d^5)*x), 1/8*(24*(b^5*c^4*d^2 - a*b^4*c^3*d^3 - 3*a^2*b^3*c^2*d^4 + 5*a^3*b^2*c*d^5 - 2*a^4*b*d^6 + (a*b^4*c^6 - a^2*b^3*c^5*d - 3*a^3*b^2*c^4*d^2 + 5*a^4*b*c^3*d^3 - 2*a^5*c^2*d^4)*x^3 + (b^5*c^6 + a*b^4*c^5*d - 5*a^2*b^3*c^4*d^2 - a^3*b^2*c^3*d^3 + 8*a^4*b*c^2*d^4 - 4*a^5*c*d^5)*x^2 + (2*b^5*c^5*d - a*b^4*c^4*d^2 - 7*a^2*b^3*c^3*d^3 + 7*a^3*b^2*c^2*d^4 + a^4*b*c*d^5 - 2*a^5*d^6)*x)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) - 3*(21*a^3*b^3*c^2*d^4 - 24*a^4*b^2*c*d^5 + 8*a^5*b*d^6 + (21*a^4*b^2*c^4*d^2 - 24*a^5*b*c^3*d^3 + 8*a^6*c^2*d^4)*x^3 + (21*a^3*b^3*c^4*d^2 + 18*a^4*b^2*c^3*d^3 - 40*a^5*b*c^2*d^4 + 16*a^6*c*d^5)*x^2 + (42*a^3*b^3*c^3*d^3 - 27*a^4*b^2*c^2*d^4 - 8*a^5*b*c*d^5 + 8*a^6*d^6)*x)*sqrt(-d/(b*c - a*d))*log(-(2*(b*c - a*d)*x*sqrt(-d/(b*c - a*d))*sqrt((a*x + b)/x) - b*d + (b*c - 2*a*d)*x)/(c*x + d)) + 2*(4*(a^2*b^3*c^6 - 3*a^3*b^2*c^5*d + 3*a^4*b*c^4*d^2 - a^5*c^3*d^3)*x^4 + (12*a*b^4*c^6 - 4*a^2*b^3*c^5*d - 12*a^3*b^2*c^4*d^2 + 37*a^4*b*c^3*d^3 - 18*a^5*c^2*d^4)*x^3 + (24*a*b^4*c^5*d - 20*a^2*b^3*c^4*d^2 + 29*a^3*b^2*c^3*d^3 + 9*a^4*b*c^2*d^4 - 12*a^5*c*d^5)*x^2 + 3*(4*a*b^4*c^4*d^2 - 4*a^2*b^3*c^3*d^3 + 9*a^3*b^2*c^2*d^4 - 4*a^4*b*c*d^5)*x)*sqrt((a*x + b)/x))/(a^3*b^4*c^7*d^2 - 3*a^4*b^3*c^6*d^3 + 3*a^5*b^2*c^5*d^4 - a^6*b*c^4*d^5 + (a^4*b^3*c^9 - 3*a^5*b^2*c^8*d + 3*a^6*b*c^7*d^2 - a^7*c^6*d^3)*x^3 + (a^3*b^4*c^9 - a^4*b^3*c^8*d - 3*a^5*b^2*c^7*d^2 + 5*a^6*b*c^6*d^3 - 2*a^7*c^5*d^4)*x^2 + (2*a^3*b^4*c^8*d - 5*a^4*b^3*c^7*d^2 + 3*a^5*b^2*c^6*d^3 + a^6*b*c^5*d^4 - a^7*c^4*d^5)*x), 1/4*(3*(21*a^3*b^3*c^2*d^4 - 24*a^4*b

```
^2*c*d^5 + 8*a^5*b*d^6 + (21*a^4*b^2*c^4*d^2 - 24*a^5*b*c^3*d^3 + 8*a^6*c^2
*d^4)*x^3 + (21*a^3*b^3*c^4*d^2 + 18*a^4*b^2*c^3*d^3 - 40*a^5*b*c^2*d^4 + 1
6*a^6*c*d^5)*x^2 + (42*a^3*b^3*c^3*d^3 - 27*a^4*b^2*c^2*d^4 - 8*a^5*b*c*d^5
 + 8*a^6*d^6)*x)*sqrt(d/(b*c - a*d))*arctan(-(b*c - a*d)*x*sqrt(d/(b*c - a*
d))*sqrt((a*x + b)/x)/(a*d*x + b*d)) + 6*(b^5*c^4*d^2 - a*b^4*c^3*d^3 - 3*a
^2*b^3*c^2*d^4 + 5*a^3*b^2*c*d^5 - 2*a^4*b*d^6 + (a*b^4*c^6 - a^2*b^3*c^5*d
 - 3*a^3*b^2*c^4*d^2 + 5*a^4*b*c^3*d^3 - 2*a^5*c^2*d^4)*x^3 + (b^5*c^6 + a*
b^4*c^5*d - 5*a^2*b^3*c^4*d^2 - a^3*b^2*c^3*d^3 + 8*a^4*b*c^2*d^4 - 4*a^5*c
*d^5)*x^2 + (2*b^5*c^5*d - a*b^4*c^4*d^2 - 7*a^2*b^3*c^3*d^3 + 7*a^3*b^2*c^
2*d^4 + a^4*b*c*d^5 - 2*a^5*d^6)*x)*sqrt(a)*log(2*a*x - 2*sqrt(a)*x*sqrt((a
*x + b)/x) + b) + (4*(a^2*b^3*c^6 - 3*a^3*b^2*c^5*d + 3*a^4*b*c^4*d^2 - a^5
*c^3*d^3)*x^4 + (12*a*b^4*c^6 - 4*a^2*b^3*c^5*d - 12*a^3*b^2*c^4*d^2 + 37*a
^4*b*c^3*d^3 - 18*a^5*c^2*d^4)*x^3 + (24*a*b^4*c^5*d - 20*a^2*b^3*c^4*d^2 +
 29*a^3*b^2*c^3*d^3 + 9*a^4*b*c^2*d^4 - 12*a^5*c*d^5)*x^2 + 3*(4*a*b^4*c^4*
d^2 - 4*a^2*b^3*c^3*d^3 + 9*a^3*b^2*c^2*d^4 - 4*a^4*b*c*d^5)*x)*sqrt((a*x +
 b)/x))/(a^3*b^4*c^7*d^2 - 3*a^4*b^3*c^6*d^3 + 3*a^5*b^2*c^5*d^4 - a^6*b*c^
4*d^5 + (a^4*b^3*c^9 - 3*a^5*b^2*c^8*d + 3*a^6*b*c^7*d^2 - a^7*c^6*d^3)*x^3
 + (a^3*b^4*c^9 - a^4*b^3*c^8*d - 3*a^5*b^2*c^7*d^2 + 5*a^6*b*c^6*d^3 - 2*a
^7*c^5*d^4)*x^2 + (2*a^3*b^4*c^8*d - 5*a^4*b^3*c^7*d^2 + 3*a^5*b^2*c^6*d^3
+ a^6*b*c^5*d^4 - a^7*c^4*d^5)*x), 1/4*(3*(21*a^3*b^3*c^2*d^4 - 24*a^4*b^2*
c*d^5 + 8*a^5*b*d^6 + (21*a^4*b^2*c^4*d^2 - 24*a^5*b*c^3*d^3 + 8*a^6*c^2*d^
4)*x^3 + (21*a^3*b^3*c^4*d^2 + 18*a^4*b^2*c^3*d^3 - 40*a^5*b*c^2*d^4 + 16*a
^6*c*d^5)*x^2 + (42*a^3*b^3*c^3*d^3 - 27*a^4*b^2*c^2*d^4 - 8*a^5*b*c*d^5 +
8*a^6*d^6)*x)*sqrt(d/(b*c - a*d))*arctan(-(b*c - a*d)*x*sqrt(d/(b*c - a*d))
*sqrt((a*x + b)/x)/(a*d*x + b*d)) + 12*(b^5*c^4*d^2 - a*b^4*c^3*d^3 - 3*a^2
*b^3*c^2*d^4 + 5*a^3*b^2*c*d^5 - 2*a^4*b*d^6 + ...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)**(3/2)/(c+d/x)**3,x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(3/2)/(c+d/x)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [B]
time = 9.49, size = 2500, normalized size = 7.81

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b/x)^(3/2)*(c + d/x)^3),x)
```

```
[Out] ((2*b^4)/(a^2*d - a*b*c) + (b*(a + b/x)*(12*a^4*d^4 + 12*b^4*c^4 + 24*a^2*b
^2*c^2*d^2 - 40*a*b^3*c^3*d - 33*a^3*b*c*d^3))/(4*a*c^3*(a^2*d - a*b*c)*(a*
d - b*c)) + (3*b*(a + b/x)^3*(4*a^3*d^5 - 4*b^3*c^3*d^2 + 4*a*b^2*c^2*d^3 -
 9*a^2*b*c*d^4))/(4*a*c^3*(a^2*d - a*b*c)*(a*d - b*c)^2) - (b*(a + b/x)^2*(
24*a^4*d^5 + 24*b^4*c^4*d - 56*a*b^3*c^3*d^2 + 65*a^2*b^2*c^2*d^3 - 72*a^3*
b*c*d^4))/(4*a*c^3*(a^2*d - a*b*c)*(a*d - b*c)^2))/((a + b/x)^(3/2)*(3*a^2*
d^2 + b^2*c^2 - 4*a*b*c*d) - (a + b/x)^(5/2)*(3*a*d^2 - 2*b*c*d) + d^2*(a +
 b/x)^(7/2) - (a + b/x)^(1/2)*(a^3*d^2 + a*b^2*c^2 - 2*a^2*b*c*d)) + (atan(
((((a + b/x)^(1/2)*(18432*a^6*b^19*c^26*d^3 - 202752*a^7*b^18*c^25*d^4 + 90
3168*a^8*b^17*c^24*d^5 - 1751040*a^9*b^16*c^23*d^6 - 137088*a^10*b^15*c^22*
d^7 + 6007680*a^11*b^14*c^21*d^8 + 1276416*a^12*b^13*c^20*d^9 - 65382912*a^
13*b^12*c^19*d^10 + 216610560*a^14*b^11*c^18*d^11 - 407418624*a^15*b^10*c^1
7*d^12 + 521961984*a^16*b^9*c^16*d^13 - 482904576*a^17*b^8*c^15*d^14 + 3288
09600*a^18*b^7*c^14*d^15 - 164257920*a^19*b^6*c^13*d^16 + 58816512*a^20*b^5
*c^12*d^17 - 14340096*a^21*b^4*c^11*d^18 + 2138112*a^22*b^3*c^10*d^19 - 147
456*a^23*b^2*c^9*d^20) - (3*(d^5*(a*d - b*c)^7)^(1/2)*(8*a^2*d^2 + 21*b^2*c
^2 - 24*a*b*c*d)*(12288*a^8*b^19*c^30*d^2 - 172032*a^9*b^18*c^29*d^3 + 1081
344*a^10*b^17*c^28*d^4 - 3996672*a^11*b^16*c^27*d^5 + 9449472*a^12*b^15*c^2
6*d^6 - 14112768*a^13*b^14*c^25*d^7 + 10407936*a^14*b^13*c^24*d^8 + 6454272
*a^15*b^12*c^23*d^9 - 30007296*a^16*b^11*c^22*d^10 + 45551616*a^17*b^10*c^2
1*d^11 - 44064768*a^18*b^9*c^20*d^12 + 30096384*a^19*b^8*c^19*d^13 - 148316
16*a^20*b^7*c^18*d^14 + 5203968*a^21*b^6*c^17*d^15 - 1241088*a^22*b^5*c^16*
d^16 + 181248*a^23*b^4*c^15*d^17 - 12288*a^24*b^3*c^14*d^18 - (3*(d^5*(a*d
- b*c)^7)^(1/2)*(a + b/x)^(1/2)*(8*a^2*d^2 + 21*b^2*c^2 - 24*a*b*c*d)*(8192
*a^10*b^18*c^33*d^2 - 139264*a^11*b^17*c^32*d^3 + 1105920*a^12*b^16*c^31*d^
4 - 5447680*a^13*b^15*c^30*d^5 + 18636800*a^14*b^14*c^29*d^6 - 46964736*a^1
5*b^13*c^28*d^7 + 90202112*a^16*b^12*c^27*d^8 - 134717440*a^17*b^11*c^26*d^
9 + 158146560*a^18*b^10*c^25*d^10 - 146432000*a^19*b^9*c^24*d^11 + 10660249
6*a^20*b^8*c^23*d^12 - 60383232*a^21*b^7*c^22*d^13 + 26091520*a^22*b^6*c^21
*d^14 - 8314880*a^23*b^5*c^20*d^15 + 1843200*a^24*b^4*c^19*d^16 - 253952*a^
25*b^3*c^18*d^17 + 16384*a^26*b^2*c^17*d^18))/(8*(b^7*c^11 - a^7*c^4*d^7 +
7*a^6*b*c^5*d^6 + 21*a^2*b^5*c^9*d^2 - 35*a^3*b^4*c^8*d^3 + 35*a^4*b^3*c^7*
```

d^4 - 21*a^5*b^2*c^6*d^5 - 7*a*b^6*c^10*d))))/(8*(b^7*c^11 - a^7*c^4*d^7 + 7*a^6*b*c^5*d^6 + 21*a^2*b^5*c^9*d^2 - 35*a^3*b^4*c^8*d^3 + 35*a^4*b^3*c^7*d^4 - 21*a^5*b^2*c^6*d^5 - 7*a*b^6*c^10*d)))*(d^5*(a*d - b*c)^7)^(1/2)*(8*a^2*d^2 + 21*b^2*c^2 - 24*a*b*c*d)*3i)/(8*(b^7*c^11 - a^7*c^4*d^7 + 7*a^6*b*c^5*d^6 + 21*a^2*b^5*c^9*d^2 - 35*a^3*b^4*c^8*d^3 + 35*a^4*b^3*c^7*d^4 - 21*a^5*b^2*c^6*d^5 - 7*a*b^6*c^10*d)) + (((a + b/x)^(1/2)*(18432*a^6*b^19*c^26*d^3 - 202752*a^7*b^18*c^25*d^4 + 903168*a^8*b^17*c^24*d^5 - 1751040*a^9*b^16*c^23*d^6 - 137088*a^10*b^15*c^22*d^7 + 6007680*a^11*b^14*c^21*d^8 + 1276416*a^12*b^13*c^20*d^9 - 65382912*a^13*b^12*c^19*d^10 + 216610560*a^14*b^11*c^18*d^11 - 407418624*a^15*b^10*c^17*d^12 + 521961984*a^16*b^9*c^16*d^13 - 482904576*a^17*b^8*c^15*d^14 + 328809600*a^18*b^7*c^14*d^15 - 164257920*a^19*b^6*c^13*d^16 + 58816512*a^20*b^5*c^12*d^17 - 14340096*a^21*b^4*c^11*d^18 + 2138112*a^22*b^3*c^10*d^19 - 147456*a^23*b^2*c^9*d^20) + (3*(d^5*(a*d - b*c)^7)^(1/2)*(8*a^2*d^2 + 21*b^2*c^2 - 24*a*b*c*d)*(12288*a^8*b^19*c^30*d^2 - 172032*a^9*b^18*c^29*d^3 + 1081344*a^10*b^17*c^28*d^4 - 3996672*a^11*b^16*c^27*d^5 + 9449472*a^12*b^15*c^26*d^6 - 14112768*a^13*b^14*c^25*d^7 + 10407936*a^14*b^13*c^24*d^8 + 6454272*a^15*b^12*c^23*d^9 - 30007296*a^16*b^11*c^22*d^10 + 45551616*a^17*b^10*c^21*d^11 - 44064768*a^18*b^9*c^20*d^12 + 30096384*a^19*b^8*c^19*d^13 - 14831616*a^20*b^7*c^18*d^14 + 5203968*a^21*b^6*c^17*d^15 - 1241088*a^22*b^5*c^16*d^16 + 181248*a^23*b^4*c^15*d^17 - 12288*a^24*b^3*c^14*d^18 + (3*(d^5*(a*d - b*c)^7)^(1/2)*(a + b/x)^(1/2)*(8*a^2*d^2 + 21*b^2*c^2 - 24*a*b*c*d)*(8192*a^10*b^18*c^33*d^2 - 139264*a^11*b^17*c^32*d^3 + 1105920*a^12*b^16*c^31*d^4 - 5447680*a^13*b^15*c^30*d^5 + 18636800*a^14*b^14*c^29*d^6 - 46964736*a^15*b^13*c^28*d^7 + 90202112*a^16*b^12*c^27*d^8 - 134717440*a^17*b^11*c^26*d^9 + 158146560*a^18*b^10*c^25*d^10 - 146432000*a^19*b^9*c^24*d^11 + 106602496*a^20*b^8*c^23*d^12 - 60383232*a^21*b^7*c^22*d^13 + 26091520*a^22*b^6*c^21*d^14 - 8314880*a^23*b^5*c^20*d^15 + 1843200*a^24*b^4*c^19*d^16 - 253952*a^25*b^3*c^18*d^17 + 16384*a^26*b^2*c^17*d^18))/(8*(b^7*c^11 - a^7*c^4*d^7 + 7*a^6*b*c^5*d^6 + 21*a^2*b^5*c^9*d^2 - 35*a^3*b^4*c^8*d^3 + 35*a^4*b^3*c^7*d^4 - 21*a^5*b^2*c^6*d^5 - 7*a*b^6*c^10*d))))/(8*(b^7*c^11 - a^7*c^4*d^7 + 7*a^6*b*c^5*d^6 + 21*a^2*b^5*c^9*d^2 - 35*a^3*b^4*c^8*d^3 + 35*a^4*b^3*c^7*d^4 - 21*a^5*b^2*c^6*d^5 - 7*a*b^6*c^10*d)))*(d^5*(a*d - b*c)^7)^(1/2)*(8*a^2*d^2 + 21*b^2*c^2 - 24*a*b*c*d)*3i)/(8*(b^7*c^11 - a^7*c^4*d^7 + 7*a^6*b*c^5*d^6 + 21*a^2*b^5*c^9*d^2 - 35*a^3*b^4*c^8*d^3 + 35*a^4*b^3*c^7*d^4 - 21*a^5*b^2*c...

3.259 $\int \frac{\left(c+\frac{d}{x}\right)^3}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx$

Optimal. Leaf size=143

$$\frac{c\left(c+\frac{d}{x}\right)^2 x}{a\left(a+\frac{b}{x}\right)^{3/2}}+\frac{(bc-ad)\left(15b^3c^2-4a^3d^2x-2a^2bd(3d+5cx)+ab^2c(-3d+20cx)\right)}{3a^3b^2\left(a+\frac{b}{x}\right)^{3/2}x}-\frac{c^2(5bc-6ad)\tanh^{-1}\left(\dots\right.}{a^{7/2}}$$

```
[Out] c*(c+d/x)^2*x/a/(a+b/x)^(3/2)+1/3*(-a*d+b*c)*(15*b^3*c^2-4*a^3*d^2*x-2*a^2*
b*d*(5*c*x+3*d)+a*b^2*c*(20*c*x-3*d))/a^3/b^2/(a+b/x)^(3/2)/x-c^2*(-6*a*d+5
*b*c)*arctanh((a+b/x)^(1/2)/a^(1/2))/a^(7/2)
```

Rubi [A]
time = 0.10, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {382, 100, 150, 65, 214}

$$-\frac{c^2(5bc-6ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{7/2}}+\frac{(bc-ad)\left(-4a^3d^2x-2a^2bd(5cx+3d)+ab^2c(20cx-3d)+15b^3c^2\right)}{3a^3b^2x\left(a+\frac{b}{x}\right)^{3/2}}+\frac{cx\left(c+\frac{d}{x}\right)^2}{a\left(a+\frac{b}{x}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d/x)^3/(a + b/x)^(5/2),x]
```

```
[Out] (c*(c + d/x)^2*x)/(a*(a + b/x)^(3/2)) + ((b*c - a*d)*(15*b^3*c^2 - 4*a^3*d^
2*x - 2*a^2*b*d*(3*d + 5*c*x) + a*b^2*c*(-3*d + 20*c*x)))/(3*a^3*b^2*(a + b
/x)^(3/2)*x) - (c^2*(5*b*c - 6*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/a^(7/2)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
```

```
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 150

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b^3*c*e*g*(m + 2) - a^3*d*f*h*(
n + 2) - a^2*b*(c*f*h*m - d*(f*g + e*h)*(m + n + 3)) - a*b^2*(c*(f*g + e*h)
 + d*e*g*(2*m + n + 4)) + b*(a^2*d*f*h*(m - n) - a*b*(2*c*f*h*(m + 1) - d*(
f*g + e*h)*(n + 1)) + b^2*(c*(f*g + e*h)*(m + 1) - d*e*g*(m + n + 2)))*x)/(
b^2*(b*c - a*d)^2*(m + 1)*(m + 2)))*(a + b*x)^(m + 1)*(c + d*x)^(n + 1), x]
 + Dist[f*(h/b^2) - (d*(m + n + 3)*(a^2*d*f*h*(m - n) - a*b*(2*c*f*h*(m + 1
) - d*(f*g + e*h)*(n + 1)) + b^2*(c*(f*g + e*h)*(m + 1) - d*e*g*(m + n + 2)
)))/(b^2*(b*c - a*d)^2*(m + 1)*(m + 2)), Int[(a + b*x)^(m + 2)*(c + d*x)^n,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && (LtQ[m, -2] || (EqQ[
m + n + 3, 0] &&  !LtQ[n, -2]))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{\left(c+\frac{d}{x}\right)^3}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx = -\text{Subst}\left(\int \frac{(c+dx)^3}{x^2(a+bx)^{5/2}}\,dx,x,\frac{1}{x}\right)$$

$$= \frac{c\left(c+\frac{d}{x}\right)^2 x}{a\left(a+\frac{b}{x}\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{(c+dx)\left(\frac{1}{2}c(5bc-6ad)+\frac{1}{2}d(bc-2ad)x\right)}{x(a+bx)^{5/2}}\,dx,x,\frac{1}{x}\right)}{a}$$

$$= \frac{c\left(c+\frac{d}{x}\right)^2 x}{a\left(a+\frac{b}{x}\right)^{3/2}} + \frac{(bc-ad)\left(15b^3c^2-4a^3d^2x-ab^2c(3d-20cx)-2a^2bd(3d+5cx)\right)}{3a^3b^2\left(a+\frac{b}{x}\right)^{3/2}x} + \frac{(c^2}{}$$

$$= \frac{c\left(c+\frac{d}{x}\right)^2 x}{a\left(a+\frac{b}{x}\right)^{3/2}} + \frac{(bc-ad)\left(15b^3c^2-4a^3d^2x-ab^2c(3d-20cx)-2a^2bd(3d+5cx)\right)}{3a^3b^2\left(a+\frac{b}{x}\right)^{3/2}x} + \frac{(c^2}{}$$

$$= \frac{c\left(c+\frac{d}{x}\right)^2 x}{a\left(a+\frac{b}{x}\right)^{3/2}} + \frac{(bc-ad)\left(15b^3c^2-4a^3d^2x-ab^2c(3d-20cx)-2a^2bd(3d+5cx)\right)}{3a^3b^2\left(a+\frac{b}{x}\right)^{3/2}x} - \frac{c^2(}{}$$

Mathematica [A]
time = 0.23, size = 134, normalized size = 0.94

$$\frac{\sqrt{a+\frac{b}{x}}\,x(15b^4c^3+4a^4d^3x+3a^2b^2c^2x(-8d+cx)+6a^3bd^2(d+cx)+2ab^3c^2(-9d+10cx))}{3a^3b^2(b+ax)^2} + \frac{c^2(-5bc+6ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d/x)^3/(a + b/x)^(5/2),x]
```

```
[Out] (Sqrt[a + b/x]*x*(15*b^4*c^3 + 4*a^4*d^3*x + 3*a^2*b^2*c^2*x*(-8*d + c*x) +
 6*a^3*b*d^2*(d + c*x) + 2*a*b^3*c^2*(-9*d + 10*c*x)))/(3*a^3*b^2*(b + a*x)
^2) + (c^2*(-5*b*c + 6*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/a^(7/2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1149 vs. $2(131) = 262$.
time = 0.10, size = 1150, normalized size = 8.04 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d/x)^3/(a+1/x*b)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*((a*x+b)/x)^(1/2)*x/a^(7/2)*(12*(x*(a*x+b))^(3/2)*a^(11/2)*d^3*x-9*ln(
1/2*(2*(a*x^2+b*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^5*b^2*d^3*x^2+9*ln(1/2
```

```
*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^5*b^2*d^3*x^2+45*ln(1/2*(
2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^2*b^5*c^3*x^2+16*(x*(a*x+b)
)^(3/2)*a^(9/2)*b*d^3-6*(x*(a*x+b))^(1/2)*a^(13/2)*d^3*x^3-6*(a*x^2+b*x)^(1
/2)*a^(13/2)*d^3*x^3-3*ln(1/2*(2*(a*x^2+b*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2)
)*a^6*b*d^3*x^3+3*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^6
*b*d^3*x^3+15*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^3*b^4
*c^3*x^3+20*(x*(a*x+b))^(3/2)*a^(3/2)*b^4*c^3-9*ln(1/2*(2*(a*x^2+b*x)^(1/2)
*a^(1/2)+2*a*x+b)/a^(1/2))*a^4*b^3*d^3*x+9*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1
/2)+2*a*x+b)/a^(1/2))*a^4*b^3*d^3*x+45*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+
2*a*x+b)/a^(1/2))*a*b^6*c^3*x-6*(x*(a*x+b))^(1/2)*a^(7/2)*b^3*d^3-6*(a*x^2+
b*x)^(1/2)*a^(7/2)*b^3*d^3-18*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/
a^(1/2))*a*b^6*c^2*d-30*(x*(a*x+b))^(1/2)*a^(7/2)*b^3*c^3*x^3-18*ln(1/2*(2*
(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^4*b^3*c^2*d*x^3+24*(x*(a*x+b)
)^(3/2)*a^(5/2)*b^3*c^3*x-18*(x*(a*x+b))^(1/2)*a^(11/2)*b*d^3*x^2-90*(x*(a*
x+b))^(1/2)*a^(5/2)*b^4*c^3*x^2+15*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*
x+b)/a^(1/2))*b^7*c^3-18*(a*x^2+b*x)^(1/2)*a^(11/2)*b*d^3*x^2-54*ln(1/2*(2*
(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^3*b^4*c^2*d*x^2-12*(x*(a*x+b)
)^(3/2)*a^(7/2)*b^2*c*d^2-24*(x*(a*x+b))^(3/2)*a^(5/2)*b^3*c^2*d-30*(x*(a*x
+b))^(1/2)*a^(1/2)*b^6*c^3+36*(x*(a*x+b))^(1/2)*a^(9/2)*b^2*c^2*d*x^3-36*(x
*(a*x+b))^(3/2)*a^(7/2)*b^2*c^2*d*x+108*(x*(a*x+b))^(1/2)*a^(7/2)*b^3*c^2*d
*x^2+108*(x*(a*x+b))^(1/2)*a^(5/2)*b^4*c^2*d*x-3*ln(1/2*(2*(a*x^2+b*x)^(1/2
)*a^(1/2)+2*a*x+b)/a^(1/2))*a^3*b^4*d^3+3*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/
2)+2*a*x+b)/a^(1/2))*a^3*b^4*d^3-90*(x*(a*x+b))^(1/2)*a^(3/2)*b^5*c^3*x-18*
(a*x^2+b*x)^(1/2)*a^(9/2)*b^2*d^3*x-54*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+
2*a*x+b)/a^(1/2))*a^2*b^5*c^2*d*x+36*(x*(a*x+b))^(1/2)*a^(3/2)*b^5*c^2*d-18
*(x*(a*x+b))^(1/2)*a^(9/2)*b^2*d^3*x)/(x*(a*x+b))^(1/2)/b^3/(a*x+b)^3
```

<u>Maxima</u> [A]
time = 0.51, size = 228, normalized size = 1.59

$$\frac{1}{6}c^3\left(\frac{2\left(15\left(a+\frac{b}{x}\right)^2 b-10\left(a+\frac{b}{x}\right)ab-2a^2b\right)}{\left(a+\frac{b}{x}\right)^{\frac{5}{2}}a^3-\left(a+\frac{b}{x}\right)^{\frac{3}{2}}a^4}+\frac{15b\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{a^{\frac{7}{2}}}\right)-c^2d\left(\frac{3\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{a^{\frac{5}{2}}}+\frac{2\left(4a+\frac{3b}{x}\right)}{\left(a+\frac{b}{x}\right)^{\frac{3}{2}}a^2}\right)+\frac{2}{3}d^3\left(\frac{3}{\sqrt{a+\frac{b}{x}}\,b^2}-\frac{a}{\left(a+\frac{b}{x}\right)^{\frac{3}{2}}b^2}\right)+\frac{2cd^2}{\left(a+\frac{b}{x}\right)^{\frac{3}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)^3/(a+b/x)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/6*c^3*(2*(15*(a + b/x)^2*b - 10*(a + b/x)*a*b - 2*a^2*b)/((a + b/x)^(5/2)
*a^3 - (a + b/x)^(3/2)*a^4) + 15*b*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a +
b/x) + sqrt(a)))/a^(7/2)) - c^2*d*(3*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a
+ b/x) + sqrt(a)))/a^(5/2) + 2*(4*a + 3*b/x)/((a + b/x)^(3/2)*a^2)) + 2/3*d
^3*(3/(sqrt(a + b/x)*b^2) - a/((a + b/x)^(3/2)*b^2)) + 2*c*d^2/((a + b/x)^(
3/2)*b)
```

Fricas [A]
time = 2.48, size = 483, normalized size = 3.38

$$\left[-\frac{3\,{\left(5\,b^{5}c^{3} - 6\,ab^{4}c^{2}d + {\left(5\,a^{2}b^{3}c^{3} - 6\,a^{3}b^{2}c^{2}d\right)}x^{2} + 2\,{\left(5\,ab^{4}c^{3} - 6\,a^{2}b^{3}c^{2}d\right)}x\right)}\sqrt{a}\log\left(2\,ax + 2\,\sqrt{a}x\sqrt{\frac{ax + b}{x}} + b\right) - 2\,{\left(3\,a^{3}b^{2}c^{3}x^{3} + 2\,{\left(10\,a^{2}b^{3}c^{3} - 12\,a^{3}b^{2}c^{2}d + 3\,a^{4}bcd^{2} + 2\,a^{5}d^{3}\right)}x^{2} + 3\,{\left(5\,ab^{4}c^{3} - 6\,a^{2}b^{3}c^{2}d + 2\,a^{4}bd^{3}\right)}x\right)}\sqrt{\frac{ax + b}{x}}}{6\,{\left(a^{6}b^{2}x^{2} + 2\,a^{5}b^{3}x + a^{4}b^{4}\right)}}, \frac{3\,{\left(5\,b^{5}c^{3} - 6\,ab^{4}c^{2}d + {\left(5\,a^{2}b^{3}c^{3} - 6\,a^{3}b^{2}c^{2}d\right)}x^{2} + 2\,{\left(5\,ab^{4}c^{3} - 6\,a^{2}b^{3}c^{2}d\right)}x\right)}\sqrt{-a}\arctan\left(\frac{\sqrt{-a}\sqrt{\frac{ax + b}{x}}}{a}\right) + {\left(3\,a^{3}b^{2}c^{3}x^{3} + 2\,{\left(10\,a^{2}b^{3}c^{3} - 12\,a^{3}b^{2}c^{2}d + 3\,a^{4}bcd^{2} + 2\,a^{5}d^{3}\right)}x^{2} + 3\,{\left(5\,ab^{4}c^{3} - 6\,a^{2}b^{3}c^{2}d + 2\,a^{4}bd^{3}\right)}x\right)}\sqrt{\frac{ax + b}{x}}}{3\,{\left(a^{6}b^{2}x^{2} + 2\,a^{5}b^{3}x + a^{4}b^{4}\right)}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)^3/(a+b/x)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/6*(3*(5*b^5*c^3 - 6*a*b^4*c^2*d + (5*a^2*b^3*c^3 - 6*a^3*b^2*c^2*d)*x^2
 + 2*(5*a*b^4*c^3 - 6*a^2*b^3*c^2*d)*x)*sqrt(a)*log(2*a*x + 2*sqrt(a)*x*sqr
t((a*x + b)/x) + b) - 2*(3*a^3*b^2*c^3*x^3 + 2*(10*a^2*b^3*c^3 - 12*a^3*b^2
*c^2*d + 3*a^4*b*c*d^2 + 2*a^5*d^3)*x^2 + 3*(5*a*b^4*c^3 - 6*a^2*b^3*c^2*d
+ 2*a^4*b*d^3)*x)*sqrt((a*x + b)/x))/(a^6*b^2*x^2 + 2*a^5*b^3*x + a^4*b^4),
 1/3*(3*(5*b^5*c^3 - 6*a*b^4*c^2*d + (5*a^2*b^3*c^3 - 6*a^3*b^2*c^2*d)*x^2
+ 2*(5*a*b^4*c^3 - 6*a^2*b^3*c^2*d)*x)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x +
 b)/x)/a) + (3*a^3*b^2*c^3*x^3 + 2*(10*a^2*b^3*c^3 - 12*a^3*b^2*c^2*d + 3*a
^4*b*c*d^2 + 2*a^5*d^3)*x^2 + 3*(5*a*b^4*c^3 - 6*a^2*b^3*c^2*d + 2*a^4*b*d^
3)*x)*sqrt((a*x + b)/x))/(a^6*b^2*x^2 + 2*a^5*b^3*x + a^4*b^4)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx + d\right)^{3}}{x^{3}\left(a + \frac{b}{x}\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)**3/(a+b/x)**(5/2),x)
```

```
[Out] Integral((c*x + d)**3/(x**3*(a + b/x)**(5/2)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 436 vs. 2(132) = 264.
time = 2.02, size = 436, normalized size = 3.05

$$\frac{\sqrt{ax^{2} + bx}\,c^{3}}{a^{3}\mathrm{sgn}\left(x\right)} + \frac{{\left(5\,bc^{3} - 6\,ac^{2}d\right)}\log\left({\left| -2\,{\left(\sqrt{a}x - \sqrt{ax^{2} + bx}\right)}\sqrt{a} - b \right|}\right)}{2\,a^{\frac{7}{2}}\mathrm{sgn}\left(x\right)} - \frac{{\left(15\,b^{3}c^{3}\log\left({\left| b \right|}\right) - 18\,ab^{2}c^{2}d\log\left({\left| b \right|}\right) + 28\,b^{3}c^{3} - 48\,ab^{2}c^{2}d + 12\,a^{2}bcd^{2} + 8\,a^{3}d^{3}\right)}\mathrm{sgn}(x)}{6\,a^{\frac{7}{2}}b^{2}} + \frac{2\,{\left(9\,{\left(\sqrt{a}x - \sqrt{ax^{2} + bx}\right)}^{2}a^{\frac{3}{2}}b^{2}c^{3} - 18\,{\left(\sqrt{a}x - \sqrt{ax^{2} + bx}\right)}^{2}a^{\frac{5}{2}}bc^{2}d + 9\,{\left(\sqrt{a}x - \sqrt{ax^{2} + bx}\right)}^{2}a^{\frac{7}{2}}cd^{2} + 15\,{\left(\sqrt{a}x - \sqrt{ax^{2} + bx}\right)}ab^{3}c^{3} - 27\,{\left(\sqrt{a}x - \sqrt{ax^{2} + bx}\right)}a^{2}b^{2}c^{2}d + 9\,{\left(\sqrt{a}x - \sqrt{ax^{2} + bx}\right)}a^{3}bcd^{2} + 3\,{\left(\sqrt{a}x - \sqrt{ax^{2} + bx}\right)}a^{4}d^{3} + 7\,\sqrt{a}b^{4}c^{3} - 12\,a^{\frac{3}{2}}b^{3}c^{2}d + 3\,a^{\frac{5}{2}}b^{2}cd^{2} + 2\,a^{\frac{7}{2}}bd^{3}\right)}}{3\,{\left({\left(\sqrt{a}x - \sqrt{ax^{2} + bx}\right)}\sqrt{a} + b\right)}^{3}a^{4}\mathrm{sgn}\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)^3/(a+b/x)^(5/2),x, algorithm="giac")
```

```
[Out] sqrt(a*x^2 + b*x)*c^3/(a^3*sgn(x)) + 1/2*(5*b*c^3 - 6*a*c^2*d)*log(abs(-2*(
sqrt(a)*x - sqrt(a*x^2 + b*x))*sqrt(a) - b))/(a^(7/2)*sgn(x)) - 1/6*(15*b^3
*c^3*log(abs(b)) - 18*a*b^2*c^2*d*log(abs(b)) + 28*b^3*c^3 - 48*a*b^2*c^2*d
 + 12*a^2*b*c*d^2 + 8*a^3*d^3)*sgn(x)/(a^(7/2)*b^2) + 2/3*(9*(sqrt(a)*x - s
qrt(a*x^2 + b*x))^2*a^(3/2)*b^2*c^3 - 18*(sqrt(a)*x - sqrt(a*x^2 + b*x))^2*
```

```
a^(5/2)*b*c^2*d + 9*(sqrt(a)*x - sqrt(a*x^2 + b*x))^2*a^(7/2)*c*d^2 + 15*(s
qrt(a)*x - sqrt(a*x^2 + b*x))*a*b^3*c^3 - 27*(sqrt(a)*x - sqrt(a*x^2 + b*x)
)*a^2*b^2*c^2*d + 9*(sqrt(a)*x - sqrt(a*x^2 + b*x))*a^3*b*c*d^2 + 3*(sqrt(a
)*x - sqrt(a*x^2 + b*x))*a^4*d^3 + 7*sqrt(a)*b^4*c^3 - 12*a^(3/2)*b^3*c^2*d
 + 3*a^(5/2)*b^2*c*d^2 + 2*a^(7/2)*b*d^3)/(((sqrt(a)*x - sqrt(a*x^2 + b*x))
*sqrt(a) + b)^3*a^4*sgn(x))
```

Mupad [B]
time = 2.05, size = 194, normalized size = 1.36

$$\frac{\frac{2\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3\right)}{3\,a}+\frac{{\left(a+\frac{b}{x}\right)}^2\,\left(2\,a^3\,d^3-6\,a\,b^2\,c^2\,d+5\,b^3\,c^3\right)}{a^3}-\frac{2\,\left(a+\frac{b}{x}\right)\,\left(4\,a^3\,d^3-3\,a^2\,b\,c\,d^2-6\,a\,b^2\,c^2\,d+5\,b^3\,c^3\right)}{3\,a^2}}{b^2\,{\left(a+\frac{b}{x}\right)}^{5/2}-a\,b^2\,{\left(a+\frac{b}{x}\right)}^{3/2}}+\frac{c^2\,\mathrm{atanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)\,\left(6\,a\,d-5\,b\,c\right)}{a^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d/x)^3/(a + b/x)^(5/2),x)
```

```
[Out] ((2*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))/(3*a) + ((a + b/x)
^2*(2*a^3*d^3 + 5*b^3*c^3 - 6*a*b^2*c^2*d))/a^3 - (2*(a + b/x)*(4*a^3*d^3 +
 5*b^3*c^3 - 6*a*b^2*c^2*d - 3*a^2*b*c*d^2))/(3*a^2))/(b^2*(a + b/x)^(5/2)
- a*b^2*(a + b/x)^(3/2)) + (c^2*atanh((a + b/x)^(1/2)/a^(1/2))*(6*a*d - 5*b
*c))/a^(7/2)
```

3.260 $\int \frac{\left(c+\frac{d}{x}\right)^2}{\left(a+\frac{b}{x}\right)^{5/2}} dx$

Optimal. Leaf size=122

$$\frac{2a^2d^2+bc(5bc-4ad)}{3a^2b\left(a+\frac{b}{x}\right)^{3/2}}+\frac{c(5bc-4ad)}{a^3\sqrt{a+\frac{b}{x}}}+\frac{c^2x}{a\left(a+\frac{b}{x}\right)^{3/2}}-\frac{c(5bc-4ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{7/2}}$$

```
[Out] 1/3*(2*a^2*d^2+b*c*(-4*a*d+5*b*c))/a^2/b/(a+b/x)^(3/2)+c^2*x/a/(a+b/x)^(3/2
)-c*(-4*a*d+5*b*c)*arctanh((a+b/x)^(1/2)/a^(1/2))/a^(7/2)+c*(-4*a*d+5*b*c)/
a^3/(a+b/x)^(1/2)
```

Rubi [A]

time = 0.06, antiderivative size = 121, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {382, 91, 79, 53, 65, 214}

$$-\frac{c(5bc-4ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{7/2}}+\frac{c(5bc-4ad)}{a^3\sqrt{a+\frac{b}{x}}}+\frac{\frac{5bc^2}{a}+\frac{2ad^2}{b}-4cd}{3a\left(a+\frac{b}{x}\right)^{3/2}}+\frac{c^2x}{a\left(a+\frac{b}{x}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d/x)^2/(a + b/x)^(5/2),x]
```

```
[Out] ((5*b*c^2)/a - 4*c*d + (2*a*d^2)/b)/(3*a*(a + b/x)^(3/2)) + (c*(5*b*c - 4*a
*d))/(a^3*Sqrt[a + b/x]) + (c^2*x)/(a*(a + b/x)^(3/2)) - (c*(5*b*c - 4*a*d)
*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/a^(7/2)
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{\left(c+\frac{d}{x}\right)^2}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx = -\text{Subst}\left(\int \frac{(c+dx)^2}{x^2(a+bx)^{5/2}}\,dx,x,\frac{1}{x}\right)$$

$$= \frac{c^2x}{a\left(a+\frac{b}{x}\right)^{3/2}} - \frac{\text{Subst}\left(\int \frac{-\frac{1}{2}c(5bc-4ad)+ad^2x}{x(a+bx)^{5/2}}\,dx,x,\frac{1}{x}\right)}{a}$$

$$= \frac{\frac{2d^2}{b}+\frac{c(5bc-4ad)}{a^2}}{3\left(a+\frac{b}{x}\right)^{3/2}} + \frac{c^2x}{a\left(a+\frac{b}{x}\right)^{3/2}} + \frac{(c(5bc-4ad))\text{Subst}\left(\int \frac{1}{x(a+bx)^{3/2}}\,dx,x,\frac{1}{x}\right)}{2a^2}$$

$$= \frac{\frac{2d^2}{b}+\frac{c(5bc-4ad)}{a^2}}{3\left(a+\frac{b}{x}\right)^{3/2}} + \frac{c(5bc-4ad)}{a^3\sqrt{a+\frac{b}{x}}} + \frac{c^2x}{a\left(a+\frac{b}{x}\right)^{3/2}} + \frac{(c(5bc-4ad))\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx\right.}{2a^3}$$

$$= \frac{\frac{2d^2}{b}+\frac{c(5bc-4ad)}{a^2}}{3\left(a+\frac{b}{x}\right)^{3/2}} + \frac{c(5bc-4ad)}{a^3\sqrt{a+\frac{b}{x}}} + \frac{c^2x}{a\left(a+\frac{b}{x}\right)^{3/2}} + \frac{(c(5bc-4ad))\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\right.}{a^3b}$$

$$= \frac{\frac{2d^2}{b}+\frac{c(5bc-4ad)}{a^2}}{3\left(a+\frac{b}{x}\right)^{3/2}} + \frac{c(5bc-4ad)}{a^3\sqrt{a+\frac{b}{x}}} + \frac{c^2x}{a\left(a+\frac{b}{x}\right)^{3/2}} - \frac{c(5bc-4ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{7/2}}$$

Mathematica [A]
time = 0.20, size = 112, normalized size = 0.92

$$\frac{\sqrt{a+\frac{b}{x}}\,x(15b^3c^2+2a^3d^2x+a^2bcx(-16d+3cx)+4ab^2c(-3d+5cx))}{3a^3b(b+ax)^2} + \frac{c(-5bc+4ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d/x)^2/(a + b/x)^(5/2),x]
```

```
[Out] (Sqrt[a + b/x]*x*(15*b^3*c^2 + 2*a^3*d^2*x + a^2*b*c*x*(-16*d + 3*c*x) + 4*
a*b^2*c*(-3*d + 5*c*x)))/(3*a^3*b*(b + a*x)^2) + (c*(-5*b*c + 4*a*d)*ArcTan
h[Sqrt[a + b/x]/Sqrt[a]])/a^(7/2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 587 vs. $2(108) = 216$.
time = 0.08, size = 588, normalized size = 4.82

method	result
risch	$\frac{c^2(ax+b)}{a^3\sqrt{\frac{ax+b}{x}}}+\frac{\left(\frac{2c\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^2+bx}\right)d}{a^{\frac{5}{2}}}-\frac{5c^2\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^2+bx}\right)b}{2a^{\frac{7}{2}}}-\frac{2\sqrt{a\left(x+\frac{b}{a}\right)^2-b\left(x+\frac{b}{a}\right)}\,d^2}{3a^3\left(x+\frac{b}{a}\right)^2}+\right.}{}$
default	$\frac{\sqrt{\frac{ax+b}{x}}\,x\left(12\ln\left(\frac{2\sqrt{x(ax+b)}\,\sqrt{a}+2ax+b}{2\sqrt{a}}\right)a^4bcd\,x^3-15\ln\left(\frac{2\sqrt{x(ax+b)}\,\sqrt{a}+2ax+b}{2\sqrt{a}}\right)a^3b^2c^2x^3-24a^{\frac{9}{2}}\sqrt{x(a\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d/x)^2/(a+1/x*b)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*((a*x+b)/x)^(1/2)*x/a^(7/2)*(12*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a
*x+b)/a^(1/2))*a^4*b*c*d*x^3-15*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b
)/a^(1/2))*a^3*b^2*c^2*x^3-24*a^(9/2)*(x*(a*x+b))^(1/2)*c*d*x^3+30*a^(7/2)*
(x*(a*x+b))^(1/2)*b*c^2*x^3+36*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)
/a^(1/2))*a^3*b^2*c*d*x^2-45*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a
^(1/2))*a^2*b^3*c^2*x^2+24*a^(7/2)*(x*(a*x+b))^(3/2)*c*d*x-24*a^(5/2)*(x*(a
*x+b))^(3/2)*b*c^2*x-72*a^(7/2)*(x*(a*x+b))^(1/2)*b*c*d*x^2+90*a^(5/2)*(x*(
a*x+b))^(1/2)*b^2*c^2*x^2+36*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a
^(1/2))*a^2*b^3*c*d*x-45*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/
2))*a*b^4*c^2*x+4*a^(7/2)*(x*(a*x+b))^(3/2)*d^2+16*a^(5/2)*(x*(a*x+b))^(3/2
)*b*c*d-20*a^(3/2)*(x*(a*x+b))^(3/2)*b^2*c^2-72*a^(5/2)*(x*(a*x+b))^(1/2)*b
^2*c*d*x+90*a^(3/2)*(x*(a*x+b))^(1/2)*b^3*c^2*x+12*ln(1/2*(2*(x*(a*x+b))^(1
/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a*b^4*c*d-15*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1
/2)+2*a*x+b)/a^(1/2))*b^5*c^2-24*a^(3/2)*(x*(a*x+b))^(1/2)*b^3*c*d+30*a^(1/
2)*(x*(a*x+b))^(1/2)*b^4*c^2)/(x*(a*x+b))^(1/2)/b/(a*x+b)^3
```

Maxima [A]
time = 0.48, size = 190, normalized size = 1.56

$$\frac{1}{6}c^2\left(\frac{2\left(15\left(a+\frac{b}{x}\right)^2b-10\left(a+\frac{b}{x}\right)ab-2a^2b\right)}{\left(a+\frac{b}{x}\right)^{\frac{5}{2}}a^3-\left(a+\frac{b}{x}\right)^{\frac{3}{2}}a^4}+\frac{15\,b\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{a^{\frac{7}{2}}}\right)-\frac{2}{3}cd\left(\frac{3\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{a^{\frac{5}{2}}}+\frac{2\left(4a+\frac{3b}{x}\right)}{\left(a+\frac{b}{x}\right)^{\frac{3}{2}}a^2}\right)+\frac{2\,d^2}{3\left(a+\frac{b}{x}\right)^{\frac{3}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)^2/(a+b/x)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/6*c^2*(2*(15*(a + b/x)^2*b - 10*(a + b/x)*a*b - 2*a^2*b)/((a + b/x)^(5/2)
*a^3 - (a + b/x)^(3/2)*a^4) + 15*b*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a +
```

```
b/x) + sqrt(a)))/a^(7/2)) - 2/3*c*d*(3*log((sqrt(a + b/x) - sqrt(a))/(sqrt(
a + b/x) + sqrt(a)))/a^(5/2) + 2*(4*a + 3*b/x)/((a + b/x)^(3/2)*a^2)) + 2/3
*d^2/((a + b/x)^(3/2)*b)
```

Fricas [A]
time = 2.38, size = 407, normalized size = 3.34

$$\left[-\frac{3\,(5\,b^4c^2-4\,ab^3cd+(5\,a^2b^2c^2-4\,a^3bcd)x^2+2\,(5\,ab^3c^2-4\,a^2b^2cd)x)\sqrt{a}\log\left(2\,ax+2\,\sqrt{a}x\sqrt{\frac{ax+b}{x}}+b\right)-2\,(3\,a^3bc^2x^3+2\,(10\,a^2b^2c^2-8\,a^3bcd+a^4d^2)x^2+3\,(5\,ab^3c^2-4\,a^2b^2cd)x)\sqrt{\frac{ax+b}{x}}}{6\,(a^6bx^2+2\,a^5b^2x+a^4b^3)},\ \frac{3\,(5\,b^4c^2-4\,ab^3cd+(5\,a^2b^2c^2-4\,a^3bcd)x^2+2\,(5\,ab^3c^2-4\,a^2b^2cd)x)\sqrt{-a}\arctan\left(\frac{\sqrt{-a}\sqrt{\frac{ax+b}{x}}}{a}\right)+(3\,a^3bc^2x^3+2\,(10\,a^2b^2c^2-8\,a^3bcd+a^4d^2)x^2+3\,(5\,ab^3c^2-4\,a^2b^2cd)x)\sqrt{\frac{ax+b}{x}}}{3\,(a^6bx^2+2\,a^5b^2x+a^4b^3)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)^2/(a+b/x)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/6*(3*(5*b^4*c^2 - 4*a*b^3*c*d + (5*a^2*b^2*c^2 - 4*a^3*b*c*d)*x^2 + 2*(
5*a*b^3*c^2 - 4*a^2*b^2*c*d)*x)*sqrt(a)*log(2*a*x + 2*sqrt(a)*x*sqrt((a*x +
 b)/x) + b) - 2*(3*a^3*b*c^2*x^3 + 2*(10*a^2*b^2*c^2 - 8*a^3*b*c*d + a^4*d^
2)*x^2 + 3*(5*a*b^3*c^2 - 4*a^2*b^2*c*d)*x)*sqrt((a*x + b)/x))/(a^6*b*x^2 +
 2*a^5*b^2*x + a^4*b^3), 1/3*(3*(5*b^4*c^2 - 4*a*b^3*c*d + (5*a^2*b^2*c^2 -
 4*a^3*b*c*d)*x^2 + 2*(5*a*b^3*c^2 - 4*a^2*b^2*c*d)*x)*sqrt(-a)*arctan(sqrt
(-a)*sqrt((a*x + b)/x)/a) + (3*a^3*b*c^2*x^3 + 2*(10*a^2*b^2*c^2 - 8*a^3*b*
c*d + a^4*d^2)*x^2 + 3*(5*a*b^3*c^2 - 4*a^2*b^2*c*d)*x)*sqrt((a*x + b)/x))/
(a^6*b*x^2 + 2*a^5*b^2*x + a^4*b^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx+d)^2}{x^2\left(a+\frac{b}{x}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)**2/(a+b/x)**(5/2),x)
```

```
[Out] Integral((c*x + d)**2/(x**2*(a + b/x)**(5/2)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 370 vs. 2(108) = 216.
time = 3.41, size = 370, normalized size = 3.03

$$\frac{\sqrt{ax^2+bx}\,c^2}{a^3\mathrm{sgn}(x)}+\frac{(5\,bc^2-4\,acd)\log\left(\left|-2\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)\sqrt{a}-b\right|\right)}{2\,a^{\frac{7}{2}}\mathrm{sgn}(x)}-\frac{(15\,b^2c^2\log(|b|)-12\,abcd\log(|b|)+28\,b^2c^2-32\,abcd+4\,a^2d^2)\mathrm{sgn}(x)}{6\,a^{\frac{7}{2}}b}+\frac{2\left(9\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)^2a^{\frac{3}{2}}b^2c^2-12\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)^2a^{\frac{5}{2}}bcd+3\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)^2a^{\frac{7}{2}}d^2+15\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)ab^3c^2-18\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)a^2b^2cd+3\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)a^3bd^2+7\,\sqrt{a}b^4c^2-8\,a^{\frac{3}{2}}b^3cd+a^{\frac{5}{2}}b^2d^2\right)}{3\left(\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)\sqrt{a}+b\right)^3a^4\mathrm{sgn}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)^2/(a+b/x)^(5/2),x, algorithm="giac")
```

```
[Out] sqrt(a*x^2 + b*x)*c^2/(a^3*sgn(x)) + 1/2*(5*b*c^2 - 4*a*c*d)*log(abs(-2*(sq
rt(a)*x - sqrt(a*x^2 + b*x))*sqrt(a) - b))/(a^(7/2)*sgn(x)) - 1/6*(15*b^2*c
^2*log(abs(b)) - 12*a*b*c*d*log(abs(b)) + 28*b^2*c^2 - 32*a*b*c*d + 4*a^2*d
```

```
^2)*sgn(x)/(a^(7/2)*b) + 2/3*(9*(sqrt(a)*x - sqrt(a*x^2 + b*x))^2*a^(3/2)*b
^2*c^2 - 12*(sqrt(a)*x - sqrt(a*x^2 + b*x))^2*a^(5/2)*b*c*d + 3*(sqrt(a)*x
- sqrt(a*x^2 + b*x))^2*a^(7/2)*d^2 + 15*(sqrt(a)*x - sqrt(a*x^2 + b*x))*a*b
^3*c^2 - 18*(sqrt(a)*x - sqrt(a*x^2 + b*x))*a^2*b^2*c*d + 3*(sqrt(a)*x - sq
rt(a*x^2 + b*x))*a^3*b*d^2 + 7*sqrt(a)*b^4*c^2 - 8*a^(3/2)*b^3*c*d + a^(5/2
)*b^2*d^2)/(((sqrt(a)*x - sqrt(a*x^2 + b*x))*sqrt(a) + b)^3*a^4*sgn(x))
```

Mupad [B]
time = 2.22, size = 144, normalized size = 1.18

$$\frac{\frac{2\left(a+\frac{b}{x}\right)\left(a^2\,d^2+4\,a\,b\,c\,d-5\,b^2\,c^2\right)}{3\,a^2}-\frac{2\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}{3\,a}+\frac{b\left(a+\frac{b}{x}\right)^2\left(5\,b\,c^2-4\,a\,c\,d\right)}{a^3}}{b\left(a+\frac{b}{x}\right)^{5/2}-a\,b\left(a+\frac{b}{x}\right)^{3/2}}+\frac{c\,\mathrm{atanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)\left(4\,a\,d-5\,b\,c\right)}{a^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d/x)^2/(a + b/x)^(5/2),x)
```

```
[Out] ((2*(a + b/x)*(a^2*d^2 - 5*b^2*c^2 + 4*a*b*c*d))/(3*a^2) - (2*(a^2*d^2 + b^
2*c^2 - 2*a*b*c*d))/(3*a) + (b*(a + b/x)^2*(5*b*c^2 - 4*a*c*d))/a^3)/(b*(a
+ b/x)^(5/2) - a*b*(a + b/x)^(3/2)) + (c*atanh((a + b/x)^(1/2)/a^(1/2))*(4*
a*d - 5*b*c))/a^(7/2)
```

3.261 $\int \frac{c+\frac{d}{x}}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx$

Optimal. Leaf size=103

$$\frac{5bc-2ad}{3a^2\left(a+\frac{b}{x}\right)^{3/2}}+\frac{5bc-2ad}{a^3\sqrt{a+\frac{b}{x}}}+\frac{cx}{a\left(a+\frac{b}{x}\right)^{3/2}}-\frac{(5bc-2ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{7/2}}$$

```
[Out] 1/3*(-2*a*d+5*b*c)/a^2/(a+b/x)^(3/2)+c*x/a/(a+b/x)^(3/2)-(-2*a*d+5*b*c)*arc
tanh((a+b/x)^(1/2)/a^(1/2))/a^(7/2)+(-2*a*d+5*b*c)/a^3/(a+b/x)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {382, 79, 53, 65, 214}

$$-\frac{(5bc-2ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{7/2}}+\frac{5bc-2ad}{a^3\sqrt{a+\frac{b}{x}}}+\frac{5bc-2ad}{3a^2\left(a+\frac{b}{x}\right)^{3/2}}+\frac{cx}{a\left(a+\frac{b}{x}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d/x)/(a + b/x)^(5/2),x]
```

```
[Out] (5*b*c - 2*a*d)/(3*a^2*(a + b/x)^(3/2)) + (5*b*c - 2*a*d)/(a^3*Sqrt[a + b/x
]) + (c*x)/(a*(a + b/x)^(3/2)) - ((5*b*c - 2*a*d)*ArcTanh[Sqrt[a + b/x]/Sqr
t[a]])/a^(7/2)
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{c+\frac{d}{x}}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx = -\text{Subst}\left(\int \frac{c+dx}{x^2(a+bx)^{5/2}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{cx}{a\left(a+\frac{b}{x}\right)^{3/2}} - \frac{\left(-\frac{5bc}{2}+ad\right)\text{Subst}\left(\int \frac{1}{x(a+bx)^{5/2}}\,dx, x, \frac{1}{x}\right)}{a}$$

$$= \frac{5bc-2ad}{3a^2\left(a+\frac{b}{x}\right)^{3/2}} + \frac{cx}{a\left(a+\frac{b}{x}\right)^{3/2}} + \frac{(5bc-2ad)\text{Subst}\left(\int \frac{1}{x(a+bx)^{3/2}}\,dx, x, \frac{1}{x}\right)}{2a^2}$$

$$= \frac{5bc-2ad}{3a^2\left(a+\frac{b}{x}\right)^{3/2}} + \frac{5bc-2ad}{a^3\sqrt{a+\frac{b}{x}}} + \frac{cx}{a\left(a+\frac{b}{x}\right)^{3/2}} + \frac{(5bc-2ad)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right.}{2a^3}$$

$$= \frac{5bc-2ad}{3a^2\left(a+\frac{b}{x}\right)^{3/2}} + \frac{5bc-2ad}{a^3\sqrt{a+\frac{b}{x}}} + \frac{cx}{a\left(a+\frac{b}{x}\right)^{3/2}} + \frac{(5bc-2ad)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\cdots}\right.}{a^3b}$$

$$= \frac{5bc-2ad}{3a^2\left(a+\frac{b}{x}\right)^{3/2}} + \frac{5bc-2ad}{a^3\sqrt{a+\frac{b}{x}}} + \frac{cx}{a\left(a+\frac{b}{x}\right)^{3/2}} - \frac{(5bc-2ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{7/2}}$$

Mathematica [A]
time = 0.15, size = 91, normalized size = 0.88

$$\frac{\sqrt{a+\frac{b}{x}}\,x(15b^2c+a^2x(-8d+3cx)+ab(-6d+20cx))}{3a^3(b+ax)^2} + \frac{(-5bc+2ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d/x)/(a + b/x)^(5/2),x]
```

```
[Out] (Sqrt[a + b/x]*x*(15*b^2*c + a^2*x*(-8*d + 3*c*x) + a*b*(-6*d + 20*c*x)))/(
3*a^3*(b + a*x)^2) + ((-5*b*c + 2*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/a^(7
/2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 540 vs. $2(89) = 178$.
time = 0.07, size = 541, normalized size = 5.25

method	result
risch	$\frac{c(ax+b)}{a^3\sqrt{\frac{ax+b}{x}}}+\frac{\left(\frac{\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^2+bx}\right)d}{a^{\frac{5}{2}}}-\frac{5\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^2+bx}\right)bc}{2a^{\frac{7}{2}}}+\frac{2b\sqrt{a\left(x+\frac{b}{a}\right)^2-b\left(x+\frac{b}{a}\right)}\,d}{3a^4\left(x+\frac{b}{a}\right)^2}-\frac{2b^2\cdots}{}\right.}{}$
default	$\frac{\sqrt{\frac{ax+b}{x}}\,x\left(-12\sqrt{x\,(ax+b)}\,a^{\frac{9}{2}}d\,x^3+30\sqrt{x\,(ax+b)}\,a^{\frac{7}{2}}bc\,x^3+6\ln\left(\frac{2\sqrt{x\,(ax+b)}\,\sqrt{a}+2ax+b}{2\sqrt{a}}\right)a^4bd\,x^3-15\ln\left(\cdots\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d/x)/(a+1/x*b)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*((a*x+b)/x)^(1/2)*x/a^(7/2)*(-12*(x*(a*x+b))^(1/2)*a^(9/2)*d*x^3+30*(x*
(a*x+b))^(1/2)*a^(7/2)*b*c*x^3+6*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+
b)/a^(1/2))*a^4*b*d*x^3-15*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(
1/2))*a^3*b^2*c*x^3+12*(x*(a*x+b))^(3/2)*a^(7/2)*d*x-24*(x*(a*x+b))^(3/2)*a
^(5/2)*b*c*x-36*(x*(a*x+b))^(1/2)*a^(7/2)*b*d*x^2+90*(x*(a*x+b))^(1/2)*a^(5
/2)*b^2*c*x^2+18*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^3*
b^2*d*x^2-45*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^2*b^3*
c*x^2+8*(x*(a*x+b))^(3/2)*a^(5/2)*b*d-20*(x*(a*x+b))^(3/2)*a^(3/2)*b^2*c-36
*(x*(a*x+b))^(1/2)*a^(5/2)*b^2*d*x+90*(x*(a*x+b))^(1/2)*a^(3/2)*b^3*c*x+18*
ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^2*b^3*d*x-45*ln(1/2
*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a*b^4*c*x-12*(x*(a*x+b))^(1
/2)*a^(3/2)*b^3*d+30*(x*(a*x+b))^(1/2)*a^(1/2)*b^4*c+6*ln(1/2*(2*(x*(a*x+b)
)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a*b^4*d-15*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^
(1/2)+2*a*x+b)/a^(1/2))*b^5*c)/(x*(a*x+b))^(1/2)/(a*x+b)^3/b
```

Maxima [A]
time = 0.50, size = 170, normalized size = 1.65

$$\frac{1}{6}\,c\left(\frac{2\left(15\left(a+\frac{b}{x}\right)^2b-10\left(a+\frac{b}{x}\right)ab-2\,a^2b\right)}{\left(a+\frac{b}{x}\right)^{\frac{5}{2}}a^3-\left(a+\frac{b}{x}\right)^{\frac{3}{2}}a^4}+\frac{15\,b\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{a^{\frac{7}{2}}}\right)-\frac{1}{3}\,d\left(\frac{3\,\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{a^{\frac{5}{2}}}+\frac{2\left(4\,a+\frac{3\,b}{x}\right)}{\left(a+\frac{b}{x}\right)^{\frac{3}{2}}a^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)/(a+b/x)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/6*c*(2*(15*(a + b/x)^2*b - 10*(a + b/x)*a*b - 2*a^2*b)/((a + b/x)^(5/2)*a
^3 - (a + b/x)^(3/2)*a^4) + 15*b*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/
x) + sqrt(a)))/a^(7/2)) - 1/3*d*(3*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a +
b/x) + sqrt(a)))/a^(5/2) + 2*(4*a + 3*b/x)/((a + b/x)^(3/2)*a^2))
```

Fricas [A]
time = 2.33, size = 331, normalized size = 3.21

$$\left[-\frac{3\,{\left(5\,b^{3} c - 2\,a b^{2} d + {\left(5\,a^{2} b c - 2\,a^{3} d\right)} x^{2} + 2\,{\left(5\,a b^{2} c - 2\,a^{2} b d\right)} x\right)} \sqrt{a} \log\left(2\,a x + 2\,\sqrt{a} x \sqrt{\frac{a x + b}{x}} + b\right) - 2\,{\left(3\,a^{3} c x^{3} + 4\,{\left(5\,a^{2} b c - 2\,a^{3} d\right)} x^{2} + 3\,{\left(5\,a b^{2} c - 2\,a^{2} b d\right)} x\right)} \sqrt{\frac{a x + b}{x}}}{6\,{\left(a^{6} x^{2} + 2\,a^{5} b x + a^{4} b^{2}\right)}}, \frac{3\,{\left(5\,b^{3} c - 2\,a b^{2} d + {\left(5\,a^{2} b c - 2\,a^{3} d\right)} x^{2} + 2\,{\left(5\,a b^{2} c - 2\,a^{2} b d\right)} x\right)} \sqrt{-a} \arctan\left(\frac{\sqrt{-a} \sqrt{\frac{a x + b}{x}}}{a}\right) + {\left(3\,a^{3} c x^{3} + 4\,{\left(5\,a^{2} b c - 2\,a^{3} d\right)} x^{2} + 3\,{\left(5\,a b^{2} c - 2\,a^{2} b d\right)} x\right)} \sqrt{\frac{a x + b}{x}}}{3\,{\left(a^{6} x^{2} + 2\,a^{5} b x + a^{4} b^{2}\right)}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)/(a+b/x)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/6*(3*(5*b^3*c - 2*a*b^2*d + (5*a^2*b*c - 2*a^3*d)*x^2 + 2*(5*a*b^2*c -
2*a^2*b*d)*x)*sqrt(a)*log(2*a*x + 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) - 2*(3
*a^3*c*x^3 + 4*(5*a^2*b*c - 2*a^3*d)*x^2 + 3*(5*a*b^2*c - 2*a^2*b*d)*x)*sqr
t((a*x + b)/x))/(a^6*x^2 + 2*a^5*b*x + a^4*b^2), 1/3*(3*(5*b^3*c - 2*a*b^2*
d + (5*a^2*b*c - 2*a^3*d)*x^2 + 2*(5*a*b^2*c - 2*a^2*b*d)*x)*sqrt(-a)*arcta
n(sqrt(-a)*sqrt((a*x + b)/x)/a) + (3*a^3*c*x^3 + 4*(5*a^2*b*c - 2*a^3*d)*x^
2 + 3*(5*a*b^2*c - 2*a^2*b*d)*x)*sqrt((a*x + b)/x))/(a^6*x^2 + 2*a^5*b*x +
a^4*b^2)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1479 vs. $2(90) = 180$.
time = 49.75, size = 1479, normalized size = 14.36

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)/(a+b/x)**(5/2),x)
```

```
[Out] c*(6*a**17*x**4*sqrt(1 + b/(a*x))/(6*a**(39/2)*x**3 + 18*a**(37/2)*b*x**2 +
 18*a**(35/2)*b**2*x + 6*a**(33/2)*b**3) + 46*a**16*b*x**3*sqrt(1 + b/(a*x)
)/(6*a**(39/2)*x**3 + 18*a**(37/2)*b*x**2 + 18*a**(35/2)*b**2*x + 6*a**(33/
2)*b**3) + 15*a**16*b*x**3*log(b/(a*x))/(6*a**(39/2)*x**3 + 18*a**(37/2)*b*
x**2 + 18*a**(35/2)*b**2*x + 6*a**(33/2)*b**3) - 30*a**16*b*x**3*log(sqrt(1
 + b/(a*x)) + 1)/(6*a**(39/2)*x**3 + 18*a**(37/2)*b*x**2 + 18*a**(35/2)*b**
2*x + 6*a**(33/2)*b**3) + 70*a**15*b**2*x**2*sqrt(1 + b/(a*x))/(6*a**(39/2)
*x**3 + 18*a**(37/2)*b*x**2 + 18*a**(35/2)*b**2*x + 6*a**(33/2)*b**3) + 45*
a**15*b**2*x**2*log(b/(a*x))/(6*a**(39/2)*x**3 + 18*a**(37/2)*b*x**2 + 18*a
**(35/2)*b**2*x + 6*a**(33/2)*b**3) - 90*a**15*b**2*x**2*log(sqrt(1 + b/(a*
x)) + 1)/(6*a**(39/2)*x**3 + 18*a**(37/2)*b*x**2 + 18*a**(35/2)*b**2*x + 6*
a**(33/2)*b**3) + 30*a**14*b**3*x*sqrt(1 + b/(a*x))/(6*a**(39/2)*x**3 + 18*
a**(37/2)*b*x**2 + 18*a**(35/2)*b**2*x + 6*a**(33/2)*b**3) + 45*a**14*b**3*
```

```
x*log(b/(a*x))/(6*a**(39/2)*x**3 + 18*a**(37/2)*b*x**2 + 18*a**(35/2)*b**2*
x + 6*a**(33/2)*b**3) - 90*a**14*b**3*x*log(sqrt(1 + b/(a*x)) + 1)/(6*a**(3
9/2)*x**3 + 18*a**(37/2)*b*x**2 + 18*a**(35/2)*b**2*x + 6*a**(33/2)*b**3) +
 15*a**13*b**4*log(b/(a*x))/(6*a**(39/2)*x**3 + 18*a**(37/2)*b*x**2 + 18*a*
*(35/2)*b**2*x + 6*a**(33/2)*b**3) - 30*a**13*b**4*log(sqrt(1 + b/(a*x)) +
1)/(6*a**(39/2)*x**3 + 18*a**(37/2)*b*x**2 + 18*a**(35/2)*b**2*x + 6*a**(33
/2)*b**3)) + d*(-8*a**7*x**3*sqrt(1 + b/(a*x))/(3*a**(19/2)*x**3 + 9*a**(17
/2)*b*x**2 + 9*a**(15/2)*b**2*x + 3*a**(13/2)*b**3) - 3*a**7*x**3*log(b/(a*
x))/(3*a**(19/2)*x**3 + 9*a**(17/2)*b*x**2 + 9*a**(15/2)*b**2*x + 3*a**(13/
2)*b**3) + 6*a**7*x**3*log(sqrt(1 + b/(a*x)) + 1)/(3*a**(19/2)*x**3 + 9*a**
(17/2)*b*x**2 + 9*a**(15/2)*b**2*x + 3*a**(13/2)*b**3) - 14*a**6*b*x**2*sqr
t(1 + b/(a*x))/(3*a**(19/2)*x**3 + 9*a**(17/2)*b*x**2 + 9*a**(15/2)*b**2*x
+ 3*a**(13/2)*b**3) - 9*a**6*b*x**2*log(b/(a*x))/(3*a**(19/2)*x**3 + 9*a**(
17/2)*b*x**2 + 9*a**(15/2)*b**2*x + 3*a**(13/2)*b**3) + 18*a**6*b*x**2*log(
sqrt(1 + b/(a*x)) + 1)/(3*a**(19/2)*x**3 + 9*a**(17/2)*b*x**2 + 9*a**(15/2)
*b**2*x + 3*a**(13/2)*b**3) - 6*a**5*b**2*x*sqrt(1 + b/(a*x))/(3*a**(19/2)*
x**3 + 9*a**(17/2)*b*x**2 + 9*a**(15/2)*b**2*x + 3*a**(13/2)*b**3) - 9*a**5
*b**2*x*log(b/(a*x))/(3*a**(19/2)*x**3 + 9*a**(17/2)*b*x**2 + 9*a**(15/2)*b
**2*x + 3*a**(13/2)*b**3) + 18*a**5*b**2*x*log(sqrt(1 + b/(a*x)) + 1)/(3*a*
*(19/2)*x**3 + 9*a**(17/2)*b*x**2 + 9*a**(15/2)*b**2*x + 3*a**(13/2)*b**3)
- 3*a**4*b**3*log(b/(a*x))/(3*a**(19/2)*x**3 + 9*a**(17/2)*b*x**2 + 9*a**(1
5/2)*b**2*x + 3*a**(13/2)*b**3) + 6*a**4*b**3*log(sqrt(1 + b/(a*x)) + 1)/(3
*a**(19/2)*x**3 + 9*a**(17/2)*b*x**2 + 9*a**(15/2)*b**2*x + 3*a**(13/2)*b**
3))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 266 vs. 2(89) = 178.

time = 2.01, size = 266, normalized size = 2.58

$$-\frac{(15\,bc\log(|b|) - 6\,ad\log(|b|) + 28\,bc - 16\,ad)\mathrm{sgn}(x)}{6\,a^{\frac{7}{2}}} + \frac{\sqrt{ax^2+bx}\,c}{a^3\mathrm{sgn}(x)} + \frac{(5\,bc - 2\,ad)\log\left(\left|-2\left(\sqrt{a}x - \sqrt{ax^2+bx}\right)\sqrt{a} - b\right|\right)}{2\,a^{\frac{7}{2}}\mathrm{sgn}(x)} + \frac{2\left(9\left(\sqrt{a}x - \sqrt{ax^2+bx}\right)^2 a^{\frac{3}{2}}b^2c - 6\left(\sqrt{a}x - \sqrt{ax^2+bx}\right)^2 a^{\frac{5}{2}}bd + 15\left(\sqrt{a}x - \sqrt{ax^2+bx}\right)ab^3c - 9\left(\sqrt{a}x - \sqrt{ax^2+bx}\right)a^2b^2d + 7\sqrt{a}\,b^4c - 4\,a^{\frac{3}{2}}b^3d\right)}{3\left(\left(\sqrt{a}x - \sqrt{ax^2+bx}\right)\sqrt{a} + b\right)^3 a^4\mathrm{sgn}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)/(a+b/x)^(5/2),x, algorithm="giac")
```

```
[Out] -1/6*(15*b*c*log(abs(b)) - 6*a*d*log(abs(b)) + 28*b*c - 16*a*d)*sgn(x)/a^(7
/2) + sqrt(a*x^2 + b*x)*c/(a^3*sgn(x)) + 1/2*(5*b*c - 2*a*d)*log(abs(-2*(sq
rt(a)*x - sqrt(a*x^2 + b*x))*sqrt(a) - b))/(a^(7/2)*sgn(x)) + 2/3*(9*(sqrt(
a)*x - sqrt(a*x^2 + b*x))^2*a^(3/2)*b^2*c - 6*(sqrt(a)*x - sqrt(a*x^2 + b*x
))^2*a^(5/2)*b*d + 15*(sqrt(a)*x - sqrt(a*x^2 + b*x))*a*b^3*c - 9*(sqrt(a)*
x - sqrt(a*x^2 + b*x))*a^2*b^2*d + 7*sqrt(a)*b^4*c - 4*a^(3/2)*b^3*d)/(((sq
rt(a)*x - sqrt(a*x^2 + b*x))*sqrt(a) + b)^3*a^4*sgn(x))
```

Mupad [B]

time = 2.91, size = 87, normalized size = 0.84

$$\frac{2\,d\,\mathrm{atanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}}-\frac{\frac{2\,d}{3\,a}+\frac{2\,d\,\left(a+\frac{b}{x}\right)}{a^2}}{\left(a+\frac{b}{x}\right)^{3/2}}+\frac{2\,c\,x\,\left(\frac{a\,x}{b}+1\right)^{5/2}\,{}_2\mathrm{F}_1\left(\frac{5}{2},\frac{7}{2};\ \frac{9}{2};\ -\frac{a\,x}{b}\right)}{7\,\left(a+\frac{b}{x}\right)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d/x)/(a + b/x)^(5/2),x)
```

```
[Out] (2*d*atanh((a + b/x)^(1/2)/a^(1/2)))/a^(5/2) - ((2*d)/(3*a) + (2*d*(a + b/x
))/a^2)/(a + b/x)^(3/2) + (2*c*x*((a*x)/b + 1)^(5/2)*hypergeom([5/2, 7/2],
9/2, -(a*x)/b))/(7*(a + b/x)^(5/2))
```

3.262 $\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx$

Optimal. Leaf size=79

$$\frac{5b}{3a^2\left(a+\frac{b}{x}\right)^{3/2}}+\frac{5b}{a^3\sqrt{a+\frac{b}{x}}}+\frac{x}{a\left(a+\frac{b}{x}\right)^{3/2}}-\frac{5b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{7/2}}$$

```
[Out] 5/3*b/a^2/(a+b/x)^(3/2)+x/a/(a+b/x)^(3/2)-5*b*arctanh((a+b/x)^(1/2)/a^(1/2)
)/a^(7/2)+5*b/a^3/(a+b/x)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {248, 44, 53, 65, 214}

$$-\frac{5b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{7/2}}+\frac{5b}{a^3\sqrt{a+\frac{b}{x}}}+\frac{5b}{3a^2\left(a+\frac{b}{x}\right)^{3/2}}+\frac{x}{a\left(a+\frac{b}{x}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^(-5/2),x]
```

```
[Out] (5*b)/(3*a^2*(a + b/x)^(3/2)) + (5*b)/(a^3*Sqrt[a + b/x]) + x/(a*(a + b/x)^
(3/2)) - (5*b*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/a^(7/2)
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
```

```
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 248

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx = -\text{Subst}\left(\int \frac{1}{x^2(a+bx)^{5/2}}\,dx, x, \frac{1}{x}\right)$$
$$= -\frac{2x}{3a\left(a+\frac{b}{x}\right)^{3/2}} - \frac{5\text{Subst}\left(\int \frac{1}{x^2(a+bx)^{3/2}}\,dx, x, \frac{1}{x}\right)}{3a}$$
$$= -\frac{2x}{3a\left(a+\frac{b}{x}\right)^{3/2}} - \frac{10x}{3a^2\sqrt{a+\frac{b}{x}}} - \frac{5\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)}{a^2}$$
$$= -\frac{2x}{3a\left(a+\frac{b}{x}\right)^{3/2}} - \frac{10x}{3a^2\sqrt{a+\frac{b}{x}}} + \frac{5\sqrt{a+\frac{b}{x}}\,x}{a^3} + \frac{(5b)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)}{2a^3}$$
$$= -\frac{2x}{3a\left(a+\frac{b}{x}\right)^{3/2}} - \frac{10x}{3a^2\sqrt{a+\frac{b}{x}}} + \frac{5\sqrt{a+\frac{b}{x}}\,x}{a^3} + \frac{5\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x}}\right)}{a^3}$$
$$= -\frac{2x}{3a\left(a+\frac{b}{x}\right)^{3/2}} - \frac{10x}{3a^2\sqrt{a+\frac{b}{x}}} + \frac{5\sqrt{a+\frac{b}{x}}\,x}{a^3} - \frac{5b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{7/2}}$$

Mathematica [A]
time = 0.03, size = 72, normalized size = 0.91

$$\frac{\sqrt{a+\frac{b}{x}}\,x(15b^2+20abx+3a^2x^2)}{3a^3(b+ax)^2} - \frac{5b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^(-5/2),x]
```

```
[Out] (Sqrt[a + b/x]*x*(15*b^2 + 20*a*b*x + 3*a^2*x^2))/(3*a^3*(b + a*x)^2) - (5*
b*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/a^(7/2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 270 vs. $2(65) = 130$.
time = 0.06, size = 271, normalized size = 3.43

method	result
risch	$\frac{ax+b}{a^3\sqrt{\frac{ax+b}{x}}}+\frac{\left(-\frac{5b\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^2+bx}\right)}{2a^{\frac{7}{2}}}-\frac{2b^2\sqrt{a\left(x+\frac{b}{a}\right)^2-b\left(x+\frac{b}{a}\right)}}{3a^5\left(x+\frac{b}{a}\right)^2}+\frac{14b\sqrt{a\left(x+\frac{b}{a}\right)^2-b\left(x+\right.}}{3a^4\left(x+\frac{b}{a}\right)}\right.}{x\sqrt{\frac{ax+b}{x}}}$
default	$-\frac{\sqrt{\frac{ax+b}{x}}\,x\left(-30\sqrt{x\left(ax+b\right)}\,a^{\frac{7}{2}}x^3+15\ln\left(\frac{2\sqrt{x\left(ax+b\right)}\,\sqrt{a}+2ax+b}{2\sqrt{a}}\right)a^3bx^3+24(x(ax+b))^{\frac{3}{2}}a^{\frac{5}{2}}x-90\sqrt{x\left(ax\right.}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+1/x*b)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*((a*x+b)/x)^(1/2)*x/a^(7/2)*(-30*(x*(a*x+b))^(1/2)*a^(7/2)*x^3+15*ln(1
/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^3*b*x^3+24*(x*(a*x+b))^
(3/2)*a^(5/2)*x-90*(x*(a*x+b))^(1/2)*a^(5/2)*b*x^2+45*ln(1/2*(2*(x*(a*x+b))
^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^2*b^2*x^2+20*b*a^(3/2)*(x*(a*x+b))^(3/2)
-90*(x*(a*x+b))^(1/2)*a^(3/2)*b^2*x+45*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+
2*a*x+b)/a^(1/2))*a*b^3*x-30*(x*(a*x+b))^(1/2)*a^(1/2)*b^3+15*ln(1/2*(2*(x*
(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*b^4)/(x*(a*x+b))^(1/2)/(a*x+b)^3
```

Maxima [A]
time = 0.50, size = 101, normalized size = 1.28

$$\frac{15\left(a+\frac{b}{x}\right)^2b-10\left(a+\frac{b}{x}\right)ab-2\,a^2b}{3\left(\left(a+\frac{b}{x}\right)^{\frac{5}{2}}a^3-\left(a+\frac{b}{x}\right)^{\frac{3}{2}}a^4\right)}+\frac{5\,b\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{2\,a^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/3*(15*(a + b/x)^2*b - 10*(a + b/x)*a*b - 2*a^2*b)/((a + b/x)^(5/2)*a^3 -
(a + b/x)^(3/2)*a^4) + 5/2*b*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/x) +
 sqrt(a)))/a^(7/2)
```

Fricas [A]

time = 2.48, size = 225, normalized size = 2.85

$$\left[\frac{15\,(a^{2}bx^{2}+2\,ab^{2}x+b^{3})\sqrt{a}\,\log\left(2\,ax-2\,\sqrt{a}\,x\sqrt{\frac{ax+b}{x}}+b\right)+2\,(3\,a^{3}x^{3}+20\,a^{2}bx^{2}+15\,ab^{2}x)\sqrt{\frac{ax+b}{x}}}{6\,(a^{6}x^{2}+2\,a^{5}bx+a^{4}b^{2})},\ \frac{15\,(a^{2}bx^{2}+2\,ab^{2}x+b^{3})\sqrt{-a}\,\arctan\left(\frac{\sqrt{-a}\,\sqrt{\frac{ax+b}{x}}}{a}\right)+(3\,a^{3}x^{3}+20\,a^{2}bx^{2}+15\,ab^{2}x)\sqrt{\frac{ax+b}{x}}}{3\,(a^{6}x^{2}+2\,a^{5}bx+a^{4}b^{2})}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/6*(15*(a^2*b*x^2 + 2*a*b^2*x + b^3)*sqrt(a)*log(2*a*x - 2*sqrt(a)*x*sqrt
((a*x + b)/x) + b) + 2*(3*a^3*x^3 + 20*a^2*b*x^2 + 15*a*b^2*x)*sqrt((a*x +
b)/x))/(a^6*x^2 + 2*a^5*b*x + a^4*b^2), 1/3*(15*(a^2*b*x^2 + 2*a*b^2*x + b^
3)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) + (3*a^3*x^3 + 20*a^2*b*x^
2 + 15*a*b^2*x)*sqrt((a*x + b)/x))/(a^6*x^2 + 2*a^5*b*x + a^4*b^2)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 774 vs. $2(66) = 132$.
time = 3.75, size = 774, normalized size = 9.80

$$\frac{6a^{17}x^{4}\sqrt{1+\frac{b}{ax}}}{6a^{\frac{39}{2}}x^{3}+18a^{\frac{37}{2}}bx^{2}+18a^{\frac{35}{2}}b^{2}x+6a^{\frac{33}{2}}b^{3}}+\frac{46a^{16}bx^{3}\sqrt{1+\frac{b}{ax}}}{6a^{\frac{39}{2}}x^{3}+18a^{\frac{37}{2}}bx^{2}+18a^{\frac{35}{2}}b^{2}x+6a^{\frac{33}{2}}b^{3}}+\frac{15a^{16}bx^{3}\log\left(\frac{b}{ax}\right)}{6a^{\frac{39}{2}}x^{3}+18a^{\frac{37}{2}}bx^{2}+18a^{\frac{35}{2}}b^{2}x+6a^{\frac{33}{2}}b^{3}}-\frac{30a^{16}bx^{3}\log\left(\sqrt{1+\frac{b}{ax}}+1\right)}{6a^{\frac{39}{2}}x^{3}+18a^{\frac{37}{2}}bx^{2}+18a^{\frac{35}{2}}b^{2}x+6a^{\frac{33}{2}}b^{3}}+\frac{70a^{15}b^{2}x^{2}\sqrt{1+\frac{b}{ax}}}{6a^{\frac{39}{2}}x^{3}+18a^{\frac{37}{2}}bx^{2}+18a^{\frac{35}{2}}b^{2}x+6a^{\frac{33}{2}}b^{3}}+\frac{45a^{15}b^{2}x^{2}\log\left(\frac{b}{ax}\right)}{6a^{\frac{39}{2}}x^{3}+18a^{\frac{37}{2}}bx^{2}+18a^{\frac{35}{2}}b^{2}x+6a^{\frac{33}{2}}b^{3}}-\frac{90a^{15}b^{2}x^{2}\log\left(\sqrt{1+\frac{b}{ax}}+1\right)}{6a^{\frac{39}{2}}x^{3}+18a^{\frac{37}{2}}bx^{2}+18a^{\frac{35}{2}}b^{2}x+6a^{\frac{33}{2}}b^{3}}+\frac{30a^{14}b^{3}x\sqrt{1+\frac{b}{ax}}}{6a^{\frac{39}{2}}x^{3}+18a^{\frac{37}{2}}bx^{2}+18a^{\frac{35}{2}}b^{2}x+6a^{\frac{33}{2}}b^{3}}+\frac{45a^{14}b^{3}x\log\left(\frac{b}{ax}\right)}{6a^{\frac{39}{2}}x^{3}+18a^{\frac{37}{2}}bx^{2}+18a^{\frac{35}{2}}b^{2}x+6a^{\frac{33}{2}}b^{3}}-\frac{90a^{14}b^{3}x\log\left(\sqrt{1+\frac{b}{ax}}+1\right)}{6a^{\frac{39}{2}}x^{3}+18a^{\frac{37}{2}}bx^{2}+18a^{\frac{35}{2}}b^{2}x+6a^{\frac{33}{2}}b^{3}}+\frac{15a^{13}b^{4}\log\left(\frac{b}{ax}\right)}{6a^{\frac{39}{2}}x^{3}+18a^{\frac{37}{2}}bx^{2}+18a^{\frac{35}{2}}b^{2}x+6a^{\frac{33}{2}}b^{3}}-\frac{30a^{13}b^{4}\log\left(\sqrt{1+\frac{b}{ax}}+1\right)}{6a^{\frac{39}{2}}x^{3}+18a^{\frac{37}{2}}bx^{2}+18a^{\frac{35}{2}}b^{2}x+6a^{\frac{33}{2}}b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)**(5/2),x)
```

```
[Out] 6*a**17*x**4*sqrt(1 + b/(a*x))/(6*a**(39/2)*x**3 + 18*a**(37/2)*b*x**2 + 18
*a**(35/2)*b**2*x + 6*a**(33/2)*b**3) + 46*a**16*b*x**3*sqrt(1 + b/(a*x))/(
6*a**(39/2)*x**3 + 18*a**(37/2)*b*x**2 + 18*a**(35/2)*b**2*x + 6*a**(33/2)*
b**3) + 15*a**16*b*x**3*log(b/(a*x))/(6*a**(39/2)*x**3 + 18*a**(37/2)*b*x**
2 + 18*a**(35/2)*b**2*x + 6*a**(33/2)*b**3) - 30*a**16*b*x**3*log(sqrt(1 +
b/(a*x)) + 1)/(6*a**(39/2)*x**3 + 18*a**(37/2)*b*x**2 + 18*a**(35/2)*b**2*x
 + 6*a**(33/2)*b**3) + 70*a**15*b**2*x**2*sqrt(1 + b/(a*x))/(6*a**(39/2)*x*
*3 + 18*a**(37/2)*b*x**2 + 18*a**(35/2)*b**2*x + 6*a**(33/2)*b**3) + 45*a**
15*b**2*x**2*log(b/(a*x))/(6*a**(39/2)*x**3 + 18*a**(37/2)*b*x**2 + 18*a**(
35/2)*b**2*x + 6*a**(33/2)*b**3) - 90*a**15*b**2*x**2*log(sqrt(1 + b/(a*x))
 + 1)/(6*a**(39/2)*x**3 + 18*a**(37/2)*b*x**2 + 18*a**(35/2)*b**2*x + 6*a**
(33/2)*b**3) + 30*a**14*b**3*x*sqrt(1 + b/(a*x))/(6*a**(39/2)*x**3 + 18*a**
(37/2)*b*x**2 + 18*a**(35/2)*b**2*x + 6*a**(33/2)*b**3) + 45*a**14*b**3*x*l
og(b/(a*x))/(6*a**(39/2)*x**3 + 18*a**(37/2)*b*x**2 + 18*a**(35/2)*b**2*x +
 6*a**(33/2)*b**3) - 90*a**14*b**3*x*log(sqrt(1 + b/(a*x)) + 1)/(6*a**(39/2
)*x**3 + 18*a**(37/2)*b*x**2 + 18*a**(35/2)*b**2*x + 6*a**(33/2)*b**3) + 15
*a**13*b**4*log(b/(a*x))/(6*a**(39/2)*x**3 + 18*a**(37/2)*b*x**2 + 18*a**(3
5/2)*b**2*x + 6*a**(33/2)*b**3) - 30*a**13*b**4*log(sqrt(1 + b/(a*x)) + 1)/
(6*a**(39/2)*x**3 + 18*a**(37/2)*b*x**2 + 18*a**(35/2)*b**2*x + 6*a**(33/2)
*b**3)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 173 vs. 2(65) = 130.
time = 1.98, size = 173, normalized size = 2.19

$$-\frac{(15\,b\log\left(|b|\right)+28\,b)\mathrm{sgn}(x)}{6\,a^{\frac{7}{2}}}+\frac{5\,b\log\left(\left|-2\left(\sqrt{a}\,x-\sqrt{ax^2+bx}\right)\sqrt{a}-b\right|\right)}{2\,a^{\frac{7}{2}}\mathrm{sgn}\left(x\right)}+\frac{\sqrt{ax^2+bx}}{a^3\mathrm{sgn}\left(x\right)}+\frac{2\left(9\left(\sqrt{a}\,x-\sqrt{ax^2+bx}\right)^2ab^2+15\left(\sqrt{a}\,x-\sqrt{ax^2+bx}\right)\sqrt{a}\,b^3+7\,b^4\right)}{3\left(\left(\sqrt{a}\,x-\sqrt{ax^2+bx}\right)\sqrt{a}+b\right)^3a^{\frac{7}{2}}\mathrm{sgn}\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(5/2),x, algorithm="giac")
```

```
[Out] -1/6*(15*b*log(abs(b)) + 28*b)*sgn(x)/a^(7/2) + 5/2*b*log(abs(-2*(sqrt(a)*x
 - sqrt(a*x^2 + b*x))*sqrt(a) - b))/(a^(7/2)*sgn(x)) + sqrt(a*x^2 + b*x)/(a
^3*sgn(x)) + 2/3*(9*(sqrt(a)*x - sqrt(a*x^2 + b*x))^2*a*b^2 + 15*(sqrt(a)*x
 - sqrt(a*x^2 + b*x))*sqrt(a)*b^3 + 7*b^4)/(((sqrt(a)*x - sqrt(a*x^2 + b*x)
)*sqrt(a) + b)^3*a^(7/2)*sgn(x))
```

Mupad [B]
time = 1.72, size = 34, normalized size = 0.43

$$\frac{2\,x\,{\left(\frac{a\,x}{b}+1\right)}^{5/2}\,{}_2\mathrm{F}_1\left(\frac{5}{2},\frac{7}{2};\ \frac{9}{2};\ -\frac{a\,x}{b}\right)}{7\,{\left(a+\frac{b}{x}\right)}^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b/x)^(5/2),x)
```

```
[Out] (2*x*((a*x)/b + 1)^(5/2)*hypergeom([5/2, 7/2], 9/2, -(a*x)/b))/(7*(a + b/x)
^(5/2))
```

3.263 $\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)}\,dx$

Optimal. Leaf size=201

$$\frac{b(5bc-3ad)}{3a^2c(bc-ad)\left(a+\frac{b}{x}\right)^{3/2}}+\frac{b(5b^2c^2-8abcd+a^2d^2)}{a^3c(bc-ad)^2\sqrt{a+\frac{b}{x}}}+\frac{x}{ac\left(a+\frac{b}{x}\right)^{3/2}}-\frac{2d^{7/2}\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^2(bc-ad)^{5/2}}-\frac{(5bc+2$$

[Out] 1/3*b*(-3*a*d+5*b*c)/a^2/c/(-a*d+b*c)/(a+b/x)^(3/2)+x/a/c/(a+b/x)^(3/2)-2*d^(7/2)*arctan(d^(1/2)*(a+b/x)^(1/2)/(-a*d+b*c)^(1/2))/c^2/(-a*d+b*c)^(5/2)-(2*a*d+5*b*c)*arctanh((a+b/x)^(1/2)/a^(1/2))/a^(7/2)/c^2+b*(a^2*d^2-8*a*b*c*d+5*b^2*c^2)/a^3/c/(-a*d+b*c)^2/(a+b/x)^(1/2)

Rubi [A]
time = 0.20, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {382, 105, 157, 162, 65, 214, 211}

$$-\frac{(2ad+5bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{7/2}c^2}+\frac{b(5bc-3ad)}{3a^2c\left(a+\frac{b}{x}\right)^{3/2}(bc-ad)}+\frac{b(a^2d^2-8abcd+5b^2c^2)}{a^3c\sqrt{a+\frac{b}{x}}(bc-ad)^2}-\frac{2d^{7/2}\text{ArcTan}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^2(bc-ad)^{5/2}}+\frac{x}{ac\left(a+\frac{b}{x}\right)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[1/((a + b/x)^(5/2)*(c + d/x)),x]

[Out] (b*(5*b*c - 3*a*d))/(3*a^2*c*(b*c - a*d)*(a + b/x)^(3/2)) + (b*(5*b^2*c^2 - 8*a*b*c*d + a^2*d^2))/(a^3*c*(b*c - a*d)^2*Sqrt[a + b/x]) + x/(a*c*(a + b/x)^(3/2)) - (2*d^(7/2)*ArcTan[(Sqrt[d]*Sqrt[a + b/x])/Sqrt[b*c - a*d]])/(c^2*(b*c - a*d)^(5/2)) - ((5*b*c + 2*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/(a^(7/2)*c^2)

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 157

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)}\,dx = -\text{Subst}\left(\int \frac{1}{x^2(a+bx)^{5/2}(c+dx)}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{x}{ac\left(a+\frac{b}{x}\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(5bc+2ad)+\frac{5bdx}{2}}{x(a+bx)^{5/2}(c+dx)}\,dx, x, \frac{1}{x}\right)}{ac}$$

$$= \frac{b(5bc-3ad)}{3a^2c(bc-ad)\left(a+\frac{b}{x}\right)^{3/2}} + \frac{x}{ac\left(a+\frac{b}{x}\right)^{3/2}} + \frac{2\text{Subst}\left(\int \frac{\frac{3}{4}(bc-ad)(5bc+2ad)+\frac{3}{4}bd(5bc-3ad}{x(a+bx)^{3/2}(c+dx)}\right.}{3a^2c(bc-ad)}$$

$$= \frac{b(5bc-3ad)}{3a^2c(bc-ad)\left(a+\frac{b}{x}\right)^{3/2}} + \frac{b(5b^2c^2-8abcd+a^2d^2)}{a^3c(bc-ad)^2\sqrt{a+\frac{b}{x}}} + \frac{x}{ac\left(a+\frac{b}{x}\right)^{3/2}} + \frac{4\text{Subst}\left(\right.}{}$$

$$= \frac{b(5bc-3ad)}{3a^2c(bc-ad)\left(a+\frac{b}{x}\right)^{3/2}} + \frac{b(5b^2c^2-8abcd+a^2d^2)}{a^3c(bc-ad)^2\sqrt{a+\frac{b}{x}}} + \frac{x}{ac\left(a+\frac{b}{x}\right)^{3/2}} - \frac{d^4\text{Subst}\left(\right.}{}$$

$$= \frac{b(5bc-3ad)}{3a^2c(bc-ad)\left(a+\frac{b}{x}\right)^{3/2}} + \frac{b(5b^2c^2-8abcd+a^2d^2)}{a^3c(bc-ad)^2\sqrt{a+\frac{b}{x}}} + \frac{x}{ac\left(a+\frac{b}{x}\right)^{3/2}} - \frac{(2d^4)\,\text{Sub}}{}$$

$$= \frac{b(5bc-3ad)}{3a^2c(bc-ad)\left(a+\frac{b}{x}\right)^{3/2}} + \frac{b(5b^2c^2-8abcd+a^2d^2)}{a^3c(bc-ad)^2\sqrt{a+\frac{b}{x}}} + \frac{x}{ac\left(a+\frac{b}{x}\right)^{3/2}} - \frac{2d^{7/2}\tan}{c}$$

Mathematica [A]
time = 0.51, size = 200, normalized size = 1.00

$$\frac{\frac{c\sqrt{a+\frac{b}{x}}\,x\left(15b^4c^2+3a^4d^2x^2+6a^3bdx(d-cx)+4ab^3c(-6d+5cx)+a^2b^2\left(3d^2-32cdx+3c^2x^2\right)\right)}{a^3(bc-ad)^2(b+ax)^2} - \frac{6d^{7/2}\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{5/2}} - \frac{3(5bc+2ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{7/2}}}{3c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x)^(5/2)*(c + d/x)),x]
```

```
[Out] ((c*Sqrt[a + b/x]*x*(15*b^4*c^2 + 3*a^4*d^2*x^2 + 6*a^3*b*d*x*(d - c*x) + 4
*a*b^3*c*(-6*d + 5*c*x) + a^2*b^2*(3*d^2 - 32*c*d*x + 3*c^2*x^2)))/(a^3*(b*
c - a*d)^2*(b + a*x)^2) - (6*d^(7/2)*ArcTan[(Sqrt[d]*Sqrt[a + b/x])/Sqrt[b*
c - a*d]])/(b*c - a*d)^(5/2) - (3*(5*b*c + 2*a*d)*ArcTanh[Sqrt[a + b/x]/Sqr
t[a]])/a^(7/2))/(3*c^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1766 vs. $2(177) = 354$.

time = 0.10, size = 1767, normalized size = 8.79

method	result
risch	$\frac{ax+b}{a^{3} c \sqrt{\frac{ax+b}{x}}}+\frac{\left(-\frac{\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^{2}+bx}\right) d}{a^{\frac{5}{2}} c^{2}}-\frac{5\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^{2}+bx}\right) b}{2a^{\frac{7}{2}} c}+\frac{2b^{3}\sqrt{a\left(x+\frac{b}{a}\right)^{2}-b\left(x+\frac{b}{a}\right)}}{3a^{5}(ad-bc)\left(x+\frac{b}{a}\right)^{2}}+\cdots\right.}{}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+1/x*b)^(5/2)/(c+d/x),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*(15*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*(d*(a*d-b*c)
/c^2)^(1/2)*b^6*c^4+18*a^(11/2)*ln((2*(x*(a*x+b))^(1/2)*(d*(a*d-b*c)/c^2)^(
1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*b*d^4*x^2+45*ln(1/2*(2*(x*(a*x+b))^(1/2)
*a^(1/2)+2*a*x+b)/a^(1/2))*(d*(a*d-b*c)/c^2)^(1/2)*a*b^5*c^4*x+20*a^(3/2)*(
d*(a*d-b*c)/c^2)^(1/2)*(x*(a*x+b))^(3/2)*b^3*c^4+18*a^(9/2)*ln((2*(x*(a*x+b
))^(1/2)*(d*(a*d-b*c)/c^2)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*b^2*d^4*x+6*
ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*(d*(a*d-b*c)/c^2)^(1/
2)*a^3*b^3*c*d^3+3*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*(d
*(a*d-b*c)/c^2)^(1/2)*a^2*b^4*c^2*d^2+9*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)
+2*a*x+b)/a^(1/2))*(d*(a*d-b*c)/c^2)^(1/2)*a^3*b^3*c^2*d^2*x-18*a^(9/2)*(d*
(a*d-b*c)/c^2)^(1/2)*(x*(a*x+b))^(1/2)*b*c^2*d^2*x^2+144*a^(7/2)*(d*(a*d-b*
c)/c^2)^(1/2)*(x*(a*x+b))^(1/2)*b^2*c^3*d*x^2-18*a^(7/2)*(d*(a*d-b*c)/c^2)^
(1/2)*(x*(a*x+b))^(1/2)*b^2*c^2*d^2*x+144*a^(5/2)*(d*(a*d-b*c)/c^2)^(1/2)*(
x*(a*x+b))^(1/2)*b^3*c^3*d*x+48*a^(9/2)*(d*(a*d-b*c)/c^2)^(1/2)*(x*(a*x+b))
^(1/2)*b*c^3*d*x^3-36*a^(7/2)*(d*(a*d-b*c)/c^2)^(1/2)*(x*(a*x+b))^(3/2)*b*c
^3*d*x+6*a^(13/2)*ln((2*(x*(a*x+b))^(1/2)*(d*(a*d-b*c)/c^2)^(1/2)*c-2*a*d*x
+b*c*x-b*d)/(c*x+d))*d^4*x^3-72*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b
)/a^(1/2))*(d*(a*d-b*c)/c^2)^(1/2)*a^2*b^4*c^3*d*x-24*ln(1/2*(2*(x*(a*x+b))
^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*(d*(a*d-b*c)/c^2)^(1/2)*a*b^5*c^3*d-6*a^(1
```

```
1/2)*(d*(a*d-b*c)/c^2)^(1/2)*(x*(a*x+b))^(1/2)*c^2*d^2*x^3-30*a^(7/2)*(d*(a
*d-b*c)/c^2)^(1/2)*(x*(a*x+b))^(1/2)*b^2*c^4*x^3+18*ln(1/2*(2*(x*(a*x+b))^(
1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*(d*(a*d-b*c)/c^2)^(1/2)*a^5*b*c*d^3*x^2+9*ln
(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*(d*(a*d-b*c)/c^2)^(1/2)
*a^4*b^2*c^2*d^2*x^2-72*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2
))*(d*(a*d-b*c)/c^2)^(1/2)*a^3*b^3*c^3*d*x^2+24*a^(5/2)*(d*(a*d-b*c)/c^2)^(
1/2)*(x*(a*x+b))^(3/2)*b^2*c^4*x-90*a^(5/2)*(d*(a*d-b*c)/c^2)^(1/2)*(x*(a*x
+b))^(1/2)*b^3*c^4*x^2+18*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1
/2))*(d*(a*d-b*c)/c^2)^(1/2)*a^4*b^2*c*d^3*x+3*ln(1/2*(2*(x*(a*x+b))^(1/2)*
a^(1/2)+2*a*x+b)/a^(1/2))*(d*(a*d-b*c)/c^2)^(1/2)*a^5*b*c^2*d^2*x^3-24*ln(1
/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*(d*(a*d-b*c)/c^2)^(1/2)*a
^4*b^2*c^3*d*x^3-30*a^(1/2)*(d*(a*d-b*c)/c^2)^(1/2)*(x*(a*x+b))^(1/2)*b^5*c
^4-6*a^(5/2)*(d*(a*d-b*c)/c^2)^(1/2)*(x*(a*x+b))^(1/2)*b^3*c^2*d^2+48*a^(3/
2)*(d*(a*d-b*c)/c^2)^(1/2)*(x*(a*x+b))^(1/2)*b^4*c^3*d+6*a^(7/2)*ln((2*(x*(
a*x+b))^(1/2)*(d*(a*d-b*c)/c^2)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*b^3*d^4
-32*a^(5/2)*(d*(a*d-b*c)/c^2)^(1/2)*(x*(a*x+b))^(3/2)*b^2*c^3*d-90*a^(3/2)*
(d*(a*d-b*c)/c^2)^(1/2)*(x*(a*x+b))^(1/2)*b^4*c^4*x+6*ln(1/2*(2*(x*(a*x+b))
^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*(d*(a*d-b*c)/c^2)^(1/2)*a^6*c*d^3*x^3+15*l
n(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*(d*(a*d-b*c)/c^2)^(1/2
)*a^3*b^3*c^4*x^3+45*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*
(d*(a*d-b*c)/c^2)^(1/2)*a^2*b^4*c^4*x^2)/a^(7/2)*x*((a*x+b)/x)^(1/2)/(a*x+b
)^3/(d*(a*d-b*c)/c^2)^(1/2)/c^3/(a*d-b*c)^2/(x*(a*x+b))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(5/2)/(c+d/x),x, algorithm="maxima")
```

```
[Out] integrate(1/((a + b/x)^(5/2)*(c + d/x)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 489 vs. $2(177) = 354$.
time = 7.51, size = 1990, normalized size = 9.90

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(5/2)/(c+d/x),x, algorithm="fricas")
```

```
[Out] [1/6*(3*(5*b^5*c^3 - 8*a*b^4*c^2*d + a^2*b^3*c*d^2 + 2*a^3*b^2*d^3 + (5*a^2
*b^3*c^3 - 8*a^3*b^2*c^2*d + a^4*b*c*d^2 + 2*a^5*d^3)*x^2 + 2*(5*a*b^4*c^3
- 8*a^2*b^3*c^2*d + a^3*b^2*c*d^2 + 2*a^4*b*d^3)*x)*sqrt(a)*log(2*a*x - 2*s
qrt(a)*x*sqrt((a*x + b)/x) + b) + 6*(a^6*d^3*x^2 + 2*a^5*b*d^3*x + a^4*b^2*
```

```
d^3)*sqrt(-d/(b*c - a*d))*log(-(2*(b*c - a*d)*x*sqrt(-d/(b*c - a*d))*sqrt((
a*x + b)/x) - b*d + (b*c - 2*a*d)*x)/(c*x + d)) + 2*(3*(a^3*b^2*c^3 - 2*a^4
*b*c^2*d + a^5*c*d^2)*x^3 + 2*(10*a^2*b^3*c^3 - 16*a^3*b^2*c^2*d + 3*a^4*b*
c*d^2)*x^2 + 3*(5*a*b^4*c^3 - 8*a^2*b^3*c^2*d + a^3*b^2*c*d^2)*x)*sqrt((a*x
 + b)/x))/(a^4*b^4*c^4 - 2*a^5*b^3*c^3*d + a^6*b^2*c^2*d^2 + (a^6*b^2*c^4 -
 2*a^7*b*c^3*d + a^8*c^2*d^2)*x^2 + 2*(a^5*b^3*c^4 - 2*a^6*b^2*c^3*d + a^7*
b*c^2*d^2)*x), 1/3*(3*(5*b^5*c^3 - 8*a*b^4*c^2*d + a^2*b^3*c*d^2 + 2*a^3*b^
2*d^3 + (5*a^2*b^3*c^3 - 8*a^3*b^2*c^2*d + a^4*b*c*d^2 + 2*a^5*d^3)*x^2 + 2
*(5*a*b^4*c^3 - 8*a^2*b^3*c^2*d + a^3*b^2*c*d^2 + 2*a^4*b*d^3)*x)*sqrt(-a)*
arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) + 3*(a^6*d^3*x^2 + 2*a^5*b*d^3*x + a^4
*b^2*d^3)*sqrt(-d/(b*c - a*d))*log(-(2*(b*c - a*d)*x*sqrt(-d/(b*c - a*d))*s
qrt((a*x + b)/x) - b*d + (b*c - 2*a*d)*x)/(c*x + d)) + (3*(a^3*b^2*c^3 - 2*
a^4*b*c^2*d + a^5*c*d^2)*x^3 + 2*(10*a^2*b^3*c^3 - 16*a^3*b^2*c^2*d + 3*a^4
*b*c*d^2)*x^2 + 3*(5*a*b^4*c^3 - 8*a^2*b^3*c^2*d + a^3*b^2*c*d^2)*x)*sqrt((
a*x + b)/x))/(a^4*b^4*c^4 - 2*a^5*b^3*c^3*d + a^6*b^2*c^2*d^2 + (a^6*b^2*c^
4 - 2*a^7*b*c^3*d + a^8*c^2*d^2)*x^2 + 2*(a^5*b^3*c^4 - 2*a^6*b^2*c^3*d + a
^7*b*c^2*d^2)*x), -1/6*(12*(a^6*d^3*x^2 + 2*a^5*b*d^3*x + a^4*b^2*d^3)*sqrt
(d/(b*c - a*d))*arctan(-(b*c - a*d)*x*sqrt(d/(b*c - a*d))*sqrt((a*x + b)/x)
/(a*d*x + b*d)) - 3*(5*b^5*c^3 - 8*a*b^4*c^2*d + a^2*b^3*c*d^2 + 2*a^3*b^2*
d^3 + (5*a^2*b^3*c^3 - 8*a^3*b^2*c^2*d + a^4*b*c*d^2 + 2*a^5*d^3)*x^2 + 2*(
5*a*b^4*c^3 - 8*a^2*b^3*c^2*d + a^3*b^2*c*d^2 + 2*a^4*b*d^3)*x)*sqrt(a)*log
(2*a*x - 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) - 2*(3*(a^3*b^2*c^3 - 2*a^4*b*c
^2*d + a^5*c*d^2)*x^3 + 2*(10*a^2*b^3*c^3 - 16*a^3*b^2*c^2*d + 3*a^4*b*c*d^
2)*x^2 + 3*(5*a*b^4*c^3 - 8*a^2*b^3*c^2*d + a^3*b^2*c*d^2)*x)*sqrt((a*x + b
)/x))/(a^4*b^4*c^4 - 2*a^5*b^3*c^3*d + a^6*b^2*c^2*d^2 + (a^6*b^2*c^4 - 2*a
^7*b*c^3*d + a^8*c^2*d^2)*x^2 + 2*(a^5*b^3*c^4 - 2*a^6*b^2*c^3*d + a^7*b*c^
2*d^2)*x), -1/3*(6*(a^6*d^3*x^2 + 2*a^5*b*d^3*x + a^4*b^2*d^3)*sqrt(d/(b*c
- a*d))*arctan(-(b*c - a*d)*x*sqrt(d/(b*c - a*d))*sqrt((a*x + b)/x)/(a*d*x
+ b*d)) - 3*(5*b^5*c^3 - 8*a*b^4*c^2*d + a^2*b^3*c*d^2 + 2*a^3*b^2*d^3 + (5
*a^2*b^3*c^3 - 8*a^3*b^2*c^2*d + a^4*b*c*d^2 + 2*a^5*d^3)*x^2 + 2*(5*a*b^4*
c^3 - 8*a^2*b^3*c^2*d + a^3*b^2*c*d^2 + 2*a^4*b*d^3)*x)*sqrt(-a)*arctan(sqr
t(-a)*sqrt((a*x + b)/x)/a) - (3*(a^3*b^2*c^3 - 2*a^4*b*c^2*d + a^5*c*d^2)*x
^3 + 2*(10*a^2*b^3*c^3 - 16*a^3*b^2*c^2*d + 3*a^4*b*c*d^2)*x^2 + 3*(5*a*b^4
*c^3 - 8*a^2*b^3*c^2*d + a^3*b^2*c*d^2)*x)*sqrt((a*x + b)/x))/(a^4*b^4*c^4
- 2*a^5*b^3*c^3*d + a^6*b^2*c^2*d^2 + (a^6*b^2*c^4 - 2*a^7*b*c^3*d + a^8*c^
2*d^2)*x^2 + 2*(a^5*b^3*c^4 - 2*a^6*b^2*c^3*d + a^7*b*c^2*d^2)*x)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(a + \frac{b}{x}\right)^{\frac{5}{2}} (cx + d)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)**(5/2)/(c+d/x),x)
```

```
[Out] Integral(x/((a + b/x)**(5/2)*(c*x + d)), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(5/2)/(c+d/x),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [**B**]
time = 4.62, size = 2500, normalized size = 12.44

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b/x)^(5/2)*(c + d/x)),x)
```

```
[Out] - ((2*b^2)/(3*(a^2*d - a*b*c)) + (2*b^2*(a + b/x)*(8*a*d - 5*b*c))/(3*(a^2*
d - a*b*c)^2) + (b*(a + b/x)^2*(a^2*d^2 + 5*b^2*c^2 - 8*a*b*c*d))/(a^2*c*(a
^2*d - a*b*c)*(a*d - b*c)))/(a*(a + b/x)^(3/2) - (a + b/x)^(5/2)) - (atan((
(((a + b/x)^(1/2)*(50*a^9*b^14*c^15*d^3 - 460*a^10*b^13*c^14*d^4 + 1858*a^1
1*b^12*c^13*d^5 - 4280*a^12*b^11*c^12*d^6 + 6060*a^13*b^10*c^11*d^7 - 5160*
a^14*b^9*c^10*d^8 + 2108*a^15*b^8*c^9*d^9 + 336*a^16*b^7*c^8*d^10 - 750*a^1
7*b^6*c^7*d^11 + 180*a^18*b^5*c^6*d^12 + 130*a^19*b^4*c^5*d^13 - 88*a^20*b^
3*c^4*d^14 + 16*a^21*b^2*c^3*d^15) - ((2*a*d + 5*b*c)*(20*a^12*b^14*c^17*d^
2 - 212*a^13*b^13*c^16*d^3 + 1012*a^14*b^12*c^15*d^4 - 2860*a^15*b^11*c^14*
d^5 + 5288*a^16*b^10*c^13*d^6 - 6664*a^17*b^9*c^12*d^7 + 5768*a^18*b^8*c^11
*d^8 - 3352*a^19*b^7*c^10*d^9 + 1220*a^20*b^6*c^9*d^10 - 228*a^21*b^5*c^8*d
^11 + 4*a^22*b^4*c^7*d^12 + 4*a^23*b^3*c^6*d^13 - ((a + b/x)^(1/2)*(2*a*d +
 5*b*c)*(8*a^15*b^13*c^18*d^2 - 96*a^16*b^12*c^17*d^3 + 520*a^17*b^11*c^16*
d^4 - 1680*a^18*b^10*c^15*d^5 + 3600*a^19*b^9*c^14*d^6 - 5376*a^20*b^8*c^13
*d^7 + 5712*a^21*b^7*c^12*d^8 - 4320*a^22*b^6*c^11*d^9 + 2280*a^23*b^5*c^10
*d^10 - 800*a^24*b^4*c^9*d^11 + 168*a^25*b^3*c^8*d^12 - 16*a^26*b^2*c^7*d^1
3))/(2*c^2*(a^7)^(1/2))))/(2*c^2*(a^7)^(1/2)))*(2*a*d + 5*b*c)*1i)/(2*c^2*(
a^7)^(1/2)) + (((a + b/x)^(1/2)*(50*a^9*b^14*c^15*d^3 - 460*a^10*b^13*c^14*
d^4 + 1858*a^11*b^12*c^13*d^5 - 4280*a^12*b^11*c^12*d^6 + 6060*a^13*b^10*c^
11*d^7 - 5160*a^14*b^9*c^10*d^8 + 2108*a^15*b^8*c^9*d^9 + 336*a^16*b^7*c^8*
d^10 - 750*a^17*b^6*c^7*d^11 + 180*a^18*b^5*c^6*d^12 + 130*a^19*b^4*c^5*d^1
3 - 88*a^20*b^3*c^4*d^14 + 16*a^21*b^2*c^3*d^15) + ((2*a*d + 5*b*c)*(20*a^1
2*b^14*c^17*d^2 - 212*a^13*b^13*c^16*d^3 + 1012*a^14*b^12*c^15*d^4 - 2860*a
```

^15*b^11*c^14*d^5 + 5288*a^16*b^10*c^13*d^6 - 6664*a^17*b^9*c^12*d^7 + 5768
*a^18*b^8*c^11*d^8 - 3352*a^19*b^7*c^10*d^9 + 1220*a^20*b^6*c^9*d^10 - 228*
a^21*b^5*c^8*d^11 + 4*a^22*b^4*c^7*d^12 + 4*a^23*b^3*c^6*d^13 + ((a + b/x)^
(1/2)*(2*a*d + 5*b*c)*(8*a^15*b^13*c^18*d^2 - 96*a^16*b^12*c^17*d^3 + 520*a
^17*b^11*c^16*d^4 - 1680*a^18*b^10*c^15*d^5 + 3600*a^19*b^9*c^14*d^6 - 5376
*a^20*b^8*c^13*d^7 + 5712*a^21*b^7*c^12*d^8 - 4320*a^22*b^6*c^11*d^9 + 2280
*a^23*b^5*c^10*d^10 - 800*a^24*b^4*c^9*d^11 + 168*a^25*b^3*c^8*d^12 - 16*a^
26*b^2*c^7*d^13))/(2*c^2*(a^7)^(1/2))))/(2*c^2*(a^7)^(1/2)))*(2*a*d + 5*b*c
)*1i)/(2*c^2*(a^7)^(1/2)))/(100*a^9*b^12*c^11*d^6 - 720*a^10*b^11*c^10*d^7
+ 2176*a^11*b^10*c^9*d^8 - 3528*a^12*b^9*c^8*d^9 + 3192*a^13*b^8*c^7*d^10 -
1400*a^14*b^7*c^6*d^11 + 264*a^16*b^5*c^4*d^13 - 92*a^17*b^4*c^3*d^14 + 8*
a^18*b^3*c^2*d^15 + (((a + b/x)^(1/2)*(50*a^9*b^14*c^15*d^3 - 460*a^10*b^13
*c^14*d^4 + 1858*a^11*b^12*c^13*d^5 - 4280*a^12*b^11*c^12*d^6 + 6060*a^13*b
^10*c^11*d^7 - 5160*a^14*b^9*c^10*d^8 + 2108*a^15*b^8*c^9*d^9 + 336*a^16*b^
7*c^8*d^10 - 750*a^17*b^6*c^7*d^11 + 180*a^18*b^5*c^6*d^12 + 130*a^19*b^4*c
^5*d^13 - 88*a^20*b^3*c^4*d^14 + 16*a^21*b^2*c^3*d^15) - ((2*a*d + 5*b*c)*(
20*a^12*b^14*c^17*d^2 - 212*a^13*b^13*c^16*d^3 + 1012*a^14*b^12*c^15*d^4 -
2860*a^15*b^11*c^14*d^5 + 5288*a^16*b^10*c^13*d^6 - 6664*a^17*b^9*c^12*d^7
+ 5768*a^18*b^8*c^11*d^8 - 3352*a^19*b^7*c^10*d^9 + 1220*a^20*b^6*c^9*d^10
- 228*a^21*b^5*c^8*d^11 + 4*a^22*b^4*c^7*d^12 + 4*a^23*b^3*c^6*d^13 - ((a +
b/x)^(1/2)*(2*a*d + 5*b*c)*(8*a^15*b^13*c^18*d^2 - 96*a^16*b^12*c^17*d^3 +
520*a^17*b^11*c^16*d^4 - 1680*a^18*b^10*c^15*d^5 + 3600*a^19*b^9*c^14*d^6
- 5376*a^20*b^8*c^13*d^7 + 5712*a^21*b^7*c^12*d^8 - 4320*a^22*b^6*c^11*d^9
+ 2280*a^23*b^5*c^10*d^10 - 800*a^24*b^4*c^9*d^11 + 168*a^25*b^3*c^8*d^12 -
16*a^26*b^2*c^7*d^13))/(2*c^2*(a^7)^(1/2))))/(2*c^2*(a^7)^(1/2)))*(2*a*d +
5*b*c))/(2*c^2*(a^7)^(1/2)) - (((a + b/x)^(1/2)*(50*a^9*b^14*c^15*d^3 - 46
0*a^10*b^13*c^14*d^4 + 1858*a^11*b^12*c^13*d^5 - 4280*a^12*b^11*c^12*d^6 +
6060*a^13*b^10*c^11*d^7 - 5160*a^14*b^9*c^10*d^8 + 2108*a^15*b^8*c^9*d^9 +
336*a^16*b^7*c^8*d^10 - 750*a^17*b^6*c^7*d^11 + 180*a^18*b^5*c^6*d^12 + 130
*a^19*b^4*c^5*d^13 - 88*a^20*b^3*c^4*d^14 + 16*a^21*b^2*c^3*d^15) + ((2*a*d
+ 5*b*c)*(20*a^12*b^14*c^17*d^2 - 212*a^13*b^13*c^16*d^3 + 1012*a^14*b^12*
c^15*d^4 - 2860*a^15*b^11*c^14*d^5 + 5288*a^16*b^10*c^13*d^6 - 6664*a^17*b^
9*c^12*d^7 + 5768*a^18*b^8*c^11*d^8 - 3352*a^19*b^7*c^10*d^9 + 1220*a^20*b^
6*c^9*d^10 - 228*a^21*b^5*c^8*d^11 + 4*a^22*b^4*c^7*d^12 + 4*a^23*b^3*c^6*d
^13 + ((a + b/x)^(1/2)*(2*a*d + 5*b*c)*(8*a^15*b^13*c^18*d^2 - 96*a^16*b^12
*c^17*d^3 + 520*a^17*b^11*c^16*d^4 - 1680*a^18*b^10*c^15*d^5 + 3600*a^19*b^
9*c^14*d^6 - 5376*a^20*b^8*c^13*d^7 + 5712*a^21*b^7*c^12*d^8 - 4320*a^22*b^
6*c^11*d^9 + 2280*a^23*b^5*c^10*d^10 - 800*a^24*b^4*c^9*d^11 + 168*a^25*b^3
*c^8*d^12 - 16*a^26*b^2*c^7*d^13))/(2*c^2*(a^7)^(1/2))))/(2*c^2*(a^7)^(1/2)
))*(2*a*d + 5*b*c))/(2*c^2*(a^7)^(1/2))))*(2*a*d + 5*b*c)*1i)/(c^2*(a^7)^(1
/2)) - (atan((((d^7*(a*d - b*c)^5)^(1/2)*((a + b/x)^(1/2)*(50*a^9*b^14*c^15
*d^3 - 460*a^10*b^13*c^14*d^4 + 1858*a^11*b^12*c^13*d^5 - 4280*a^12*b^11*c^
12*d^6 + 6060*a^13*b^10*c^11*d^7 - 5160*a^14*b^9*c^10*d^8 + 2108*a^15*b^8*c
^9*d^9 + 336*a^16*b^7*c^8*d^10 - 750*a^17*b^6*c...

3.264 $\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^2}\,dx$

Optimal. Leaf size=287

$$\frac{b(5b^2c^2-6abcd+6a^2d^2)}{3a^2c^2(bc-ad)^2\left(a+\frac{b}{x}\right)^{3/2}}+\frac{b(bc-2ad)\left(5b^2c^2-abcd+a^2d^2\right)}{a^3c^2(bc-ad)^3\sqrt{a+\frac{b}{x}}}+\frac{d(bc-2ad)}{ac^2(bc-ad)\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)}+\frac{x}{ac\left(a+\frac{b}{x}\right)\cdots}$$

```
[Out] 1/3*b*(6*a^2*d^2-6*a*b*c*d+5*b^2*c^2)/a^2/c^2/(-a*d+b*c)^2/(a+b/x)^(3/2)+d*
(-2*a*d+b*c)/a/c^2/(-a*d+b*c)/(a+b/x)^(3/2)/(c+d/x)+x/a/c/(a+b/x)^(3/2)/(c+
d/x)-d^(7/2)*(-4*a*d+9*b*c)*arctan(d^(1/2)*(a+b/x)^(1/2)/(-a*d+b*c)^(1/2))/
c^3/(-a*d+b*c)^(7/2)-(4*a*d+5*b*c)*arctanh((a+b/x)^(1/2)/a^(1/2))/a^(7/2)/c
^3+b*(-2*a*d+b*c)*(a^2*d^2-a*b*c*d+5*b^2*c^2)/a^3/c^2/(-a*d+b*c)^3/(a+b/x)^
(1/2)
```

Rubi [A]
time = 0.36, antiderivative size = 287, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {382, 105, 156, 157, 162, 65, 214, 211}

$$-\frac{(4ad+5bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{7/2}c^3}+\frac{b(6a^2d^2-6abcd+5b^2c^2)}{3a^2c^2\left(a+\frac{b}{x}\right)^{3/2}(bc-ad)^2}+\frac{b(bc-2ad)\left(a^2d^2-abcd+5b^2c^2\right)}{a^3c^2\sqrt{a+\frac{b}{x}}\,(bc-ad)^3}-\frac{d^{7/2}(9bc-4ad)\text{ArcTan}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^3(bc-ad)^{7/2}}+\frac{d(bc-2ad)}{ac^2\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)(bc-ad)}+\frac{x}{ac\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x)^(5/2)*(c + d/x)^2),x]
```

```
[Out] (b*(5*b^2*c^2 - 6*a*b*c*d + 6*a^2*d^2))/(3*a^2*c^2*(b*c - a*d)^2*(a + b/x)^
(3/2)) + (b*(b*c - 2*a*d)*(5*b^2*c^2 - a*b*c*d + a^2*d^2))/(a^3*c^2*(b*c -
a*d)^3*Sqrt[a + b/x]) + (d*(b*c - 2*a*d))/(a*c^2*(b*c - a*d)*(a + b/x)^(3/2
)*(c + d/x)) + x/(a*c*(a + b/x)^(3/2)*(c + d/x)) - (d^(7/2)*(9*b*c - 4*a*d)
*ArcTan[(Sqrt[d]*Sqrt[a + b/x])/Sqrt[b*c - a*d]])/(c^3*(b*c - a*d)^(7/2)) -
 ((5*b*c + 4*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/(a^(7/2)*c^3)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 157

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
```

```
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^2}\,dx = -\text{Subst}\left(\int \frac{1}{x^2(a+bx)^{5/2}(c+dx)^2}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{x}{ac\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)} + \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(5bc+4ad)+\frac{7bdx}{2}}{x(a+bx)^{5/2}(c+dx)^2}\,dx, x, \frac{1}{x}\right)}{ac}$$

$$= \frac{d(bc-2ad)}{ac^2(bc-ad)\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)} + \frac{x}{ac\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)} - \frac{\text{Subst}\left(\int \frac{-\frac{1}{2}(bc-ad)(5bc}{x(a+}\right.}{ac^2}$$

$$= \frac{b(5b^2c^2-6abcd+6a^2d^2)}{3a^2c^2(bc-ad)^2\left(a+\frac{b}{x}\right)^{3/2}} + \frac{d(bc-2ad)}{ac^2(bc-ad)\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)} + \frac{x}{ac\left(a+\frac{b}{x}\right)^{3/2}(c}$$

$$= \frac{b(5b^2c^2-6abcd+6a^2d^2)}{3a^2c^2(bc-ad)^2\left(a+\frac{b}{x}\right)^{3/2}} + \frac{b(bc-2ad)(5b^2c^2-abcd+a^2d^2)}{a^3c^2(bc-ad)^3\sqrt{a+\frac{b}{x}}} + \frac{d(b}{ac^2(bc-ad)}$$

$$= \frac{b(5b^2c^2-6abcd+6a^2d^2)}{3a^2c^2(bc-ad)^2\left(a+\frac{b}{x}\right)^{3/2}} + \frac{b(bc-2ad)(5b^2c^2-abcd+a^2d^2)}{a^3c^2(bc-ad)^3\sqrt{a+\frac{b}{x}}} + \frac{d(b}{ac^2(bc-ad)}$$

$$= \frac{b(5b^2c^2-6abcd+6a^2d^2)}{3a^2c^2(bc-ad)^2\left(a+\frac{b}{x}\right)^{3/2}} + \frac{b(bc-2ad)(5b^2c^2-abcd+a^2d^2)}{a^3c^2(bc-ad)^3\sqrt{a+\frac{b}{x}}} + \frac{d(b}{ac^2(bc-ad)}$$

$$= \frac{b(5b^2c^2-6abcd+6a^2d^2)}{3a^2c^2(bc-ad)^2\left(a+\frac{b}{x}\right)^{3/2}} + \frac{b(bc-2ad)(5b^2c^2-abcd+a^2d^2)}{a^3c^2(bc-ad)^3\sqrt{a+\frac{b}{x}}} + \frac{d(b}{ac^2(bc-ad)}$$

Mathematica [A]
time = 1.11, size = 305, normalized size = 1.06

$$\frac{\frac{c\sqrt{a+\frac{b}{x}}\,x(-15b^5c^3(d+cx)+3a^5d^3x^2(2d+cx)+ab^4c^2(33d^2+13cdx-20c^2x^2)-3a^4bd^2x(-4d^2+cdx+3c^2x^2)+a^2b^3c(-9d^3+35cd^2x+41c^2dx^2-3c^3x^3)+3a^3b^2d(2d^3-5cd^2x-3c^2dx^2+3c^3x^3))}{a^3(-bc+ad)^3(b+ax)^2(d+cx)}+\frac{3d^{7/2}(-9bc+4ad)\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{7/2}}-\frac{3(5bc+4ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{7/2}}}{3c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x)^(5/2)*(c + d/x)^2),x]
```

```
[Out] ((c*Sqrt[a + b/x]*x*(-15*b^5*c^3*(d + c*x) + 3*a^5*d^3*x^2*(2*d + c*x) + a*
b^4*c^2*(33*d^2 + 13*c*d*x - 20*c^2*x^2) - 3*a^4*b*d^2*x*(-4*d^2 + c*d*x +
3*c^2*x^2) + a^2*b^3*c*(-9*d^3 + 35*c*d^2*x + 41*c^2*d*x^2 - 3*c^3*x^3) + 3
*a^3*b^2*d*(2*d^3 - 5*c*d^2*x - 3*c^2*d*x^2 + 3*c^3*x^3)))/(a^3*(-(b*c) + a
*d)^3*(b + a*x)^2*(d + c*x)) + (3*d^(7/2)*(-9*b*c + 4*a*d)*ArcTan[(Sqrt[d]*
Sqrt[a + b/x])/Sqrt[b*c - a*d]])/(b*c - a*d)^(7/2) - (3*(5*b*c + 4*a*d)*Arc
Tanh[Sqrt[a + b/x]/Sqrt[a]])/a^(7/2))/(3*c^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 4643 vs. $2(261) = 522$.
time = 0.13, size = 4644, normalized size = 16.18

method	result
risch	$\frac{ax+b}{a^3c^2\sqrt{\frac{ax+b}{x}}}+\frac{\left(-\frac{2\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^2+bx}\right)d}{a^{\frac{5}{2}}c^3}-\frac{5\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^2+bx}\right)b}{2a^{\frac{7}{2}}c^2}-\frac{2b^4\sqrt{a\left(x+\frac{b}{a}\right)^2-b\left(x+\frac{b}{a}\right)}}{3a^5(ad-bc)^2\left(x+\frac{b}{a}\right)^2}\right.}{}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+1/x*b)^(5/2)/(c+d/x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*((a*x+b)/x)^(1/2)*x*(-12*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a
^(1/2))*a^6*(d*(a*d-b*c)/c^2)^(1/2)*b^3*c*d^6+33*ln(1/2*(2*(x*(a*x+b))^(1/2
)*a^(1/2)+2*a*x+b)/a^(1/2))*a^5*(d*(a*d-b*c)/c^2)^(1/2)*b^4*c^2*d^5-12*ln(1
/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^4*(d*(a*d-b*c)/c^2)^(1/
2)*b^5*c^3*d^4-42*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^3
*(d*(a*d-b*c)/c^2)^(1/2)*b^6*c^4*d^3+48*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)
+2*a*x+b)/a^(1/2))*a^2*(d*(a*d-b*c)/c^2)^(1/2)*b^7*c^5*d^2-12*a^(19/2)*ln((
```

```
2*(x*(a*x+b))^(1/2)*(d*(a*d-b*c)/c^2)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*c
*d^6*x^4-36*a^(17/2)*ln((2*(x*(a*x+b))^(1/2)*(d*(a*d-b*c)/c^2)^(1/2)*c-2*a*
d*x+b*c*x-b*d)/(c*x+d))*b*d^7*x^2+30*(x*(a*x+b))^(1/2)*a^(3/2)*(d*(a*d-b*c)
/c^2)^(1/2)*b^7*c^7*x-15*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/
2))*a*(d*(a*d-b*c)/c^2)^(1/2)*b^8*c^7*x-36*a^(15/2)*ln((2*(x*(a*x+b))^(1/2)
*(d*(a*d-b*c)/c^2)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*b^2*d^7*x-84*(x*(a*x
+b))^(1/2)*a^(11/2)*(d*(a*d-b*c)/c^2)^(1/2)*b^3*c^3*d^4*x+222*(x*(a*x+b))^(
1/2)*a^(9/2)*(d*(a*d-b*c)/c^2)^(1/2)*b^4*c^4*d^3*x-204*(x*(a*x+b))^(1/2)*a^
(7/2)*(d*(a*d-b*c)/c^2)^(1/2)*b^5*c^5*d^2*x-6*(x*(a*x+b))^(1/2)*a^(5/2)*(d*
(a*d-b*c)/c^2)^(1/2)*b^6*c^6*d*x-36*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a
*x+b)/a^(1/2))*a^7*(d*(a*d-b*c)/c^2)^(1/2)*b^2*c*d^6*x+30*(x*(a*x+b))^(1/2)
*a^(3/2)*(d*(a*d-b*c)/c^2)^(1/2)*b^7*c^6*d-15*ln(1/2*(2*(x*(a*x+b))^(1/2)*a
^(1/2)+2*a*x+b)/a^(1/2))*a*(d*(a*d-b*c)/c^2)^(1/2)*b^8*c^6*d+39*a^(11/2)*ln
((2*(x*(a*x+b))^(1/2)*(d*(a*d-b*c)/c^2)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))
*b^4*c*d^6-27*a^(9/2)*ln((2*(x*(a*x+b))^(1/2)*(d*(a*d-b*c)/c^2)^(1/2)*c-2*a
*d*x+b*c*x-b*d)/(c*x+d))*b^5*c^2*d^5+33*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)
+2*a*x+b)/a^(1/2))*a^8*(d*(a*d-b*c)/c^2)^(1/2)*b*c^3*d^4*x^4-12*ln(1/2*(2*(
x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^7*(d*(a*d-b*c)/c^2)^(1/2)*b^2*
c^4*d^3*x^4-42*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^6*(d
*(a*d-b*c)/c^2)^(1/2)*b^3*c^5*d^2*x^4+48*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2
)+2*a*x+b)/a^(1/2))*a^5*(d*(a*d-b*c)/c^2)^(1/2)*b^4*c^6*d*x^4+18*(x*(a*x+b)
)^(3/2)*a^(13/2)*(d*(a*d-b*c)/c^2)^(1/2)*b*c^4*d^3*x^2-48*(x*(a*x+b))^(3/2)
*a^(11/2)*(d*(a*d-b*c)/c^2)^(1/2)*b^2*c^5*d^2*x^2+72*(x*(a*x+b))^(3/2)*a^(9
/2)*(d*(a*d-b*c)/c^2)^(1/2)*b^3*c^6*d*x^2-12*(x*(a*x+b))^(1/2)*a^(15/2)*(d*
(a*d-b*c)/c^2)^(1/2)*b*c^3*d^4*x^3-24*(x*(a*x+b))^(1/2)*a^(13/2)*(d*(a*d-b*
c)/c^2)^(1/2)*b^2*c^4*d^3*x^3+156*(x*(a*x+b))^(1/2)*a^(11/2)*(d*(a*d-b*c)/c
^2)^(1/2)*b^3*c^5*d^2*x^3-258*(x*(a*x+b))^(1/2)*a^(9/2)*(d*(a*d-b*c)/c^2)^(
1/2)*b^4*c^6*d*x^3-3*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*
a^8*(d*(a*d-b*c)/c^2)^(1/2)*b*c^2*d^5*x^3+36*a^(13/2)*ln((2*(x*(a*x+b))^(1/
2)*(d*(a*d-b*c)/c^2)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*b^3*c^2*d^5*x^2-81
*a^(11/2)*ln((2*(x*(a*x+b))^(1/2)*(d*(a*d-b*c)/c^2)^(1/2)*c-2*a*d*x+b*c*x-b
*d)/(c*x+d))*b^4*c^3*d^4*x^2-38*(x*(a*x+b))^(3/2)*a^(9/2)*(d*(a*d-b*c)/c^2)
^(1/2)*b^3*c^4*d^3+64*(x*(a*x+b))^(3/2)*a^(7/2)*(d*(a*d-b*c)/c^2)^(1/2)*b^4
*c^5*d^2-20*(x*(a*x+b))^(3/2)*a^(5/2)*(d*(a*d-b*c)/c^2)^(1/2)*b^5*c^6*d+105
*a^(13/2)*ln((2*(x*(a*x+b))^(1/2)*(d*(a*d-b*c)/c^2)^(1/2)*c-2*a*d*x+b*c*x-b
*d)/(c*x+d))*b^3*c*d^6*x-42*a^(11/2)*ln((2*(x*(a*x+b))^(1/2)*(d*(a*d-b*c)/c
^2)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*b^4*c^2*d^5*x-27*a^(9/2)*ln((2*(x*(
a*x+b))^(1/2)*(d*(a*d-b*c)/c^2)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*b^5*c^3
*d^4*x+12*(x*(a*x+b))^(1/2)*a^(11/2)*(d*(a*d-b*c)/c^2)^(1/2)*b^3*c^2*d^5-30
*(x*(a*x+b))^(1/2)*a^(9/2)*(d*(a*d-b*c)/c^2)^(1/2)*b^4*c^3*d^4+84*(x*(a*x+b
))^(1/2)*a^(7/2)*(d*(a*d-b*c)/c^2)^(1/2)*b^5*c^4*d^3-96*(x*(a*x+b))^(1/2)*a
^(5/2)*(d*(a*d-b*c)/c^2)^(1/2)*b^6*c^5*d^2+28*(x*(a*x+b))^(3/2)*a^(9/2)*(d*
(a*d-b*c)/c^2)^(1/2)*b^3*c^5*d^2*x+40*(x*(a*x+b))^(3/2)*a^(7/2)*(d*(a*d-b*c
)/c^2)^(1/2)*b^4*c^6*d*x+36*(x*(a*x+b))^(1/2)*a^(15/2)*(d*(a*d-b*c)/c^2)^(1
/2)*b*c^2*d^5*x^2-72*(x*(a*x+b))^(1/2)*a^(13/2)*(d*(a*d-b*c)/c^2)^(1/2)*b^2
```

```
*c^3*d^4*x^2+156*(x*(a*x+b))^(1/2)*a^(11/2)*(d*(a*d-b*c)/c^2)^(1/2)*b^3*c^4
*d^3*x^2-36*(x*(a*x+b))^(1/2)*a^(9/2)*(d*(a*d-b*c)/c^2)^(1/2)*b^4*c^5*d^2*x
^2-198*(x*(a*x+b))^(1/2)*a^(7/2)*(d*(a*d-b*c)/c^2)^(1/2)*b^5*c^6*d*x^2-36*l
n(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^8*(d*(a*d-b*c)/c^2)^
(1/2)*b*c*d^6*x^2+63*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*
a^7*(d*(a*d-b*c)/c^2)^(1/2)*b^2*c^2*d^5*x^2+63*ln(1/2*(2*(x*(a*x+b))^(1/2)*
a^(1/2)+2*a*x+b)/a^(1/2))*a^6*(d*(a*d-b*c)/c^2)^(1/2)*b^3*c^3*d^4*x^2-162*l
n(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^5*(d*(a*d-b*c)/c^2)^
(1/2)*b^4*c^4*d^3*x^2+18*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/
2))*a^4*(d*(a*d-b*c)/c^2)^(1/2)*b^5*c^5*d^2*x^2+99*ln(1/2*(2*(x*(a*x+b))^(1
/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^3*(d*(a*d-b*c)/c^2)^(1/2)*b^6*c^6*d*x^2+36*
(x*(a*x+b))^(1/2)*a^(13/2)*(d*(a*d-b*c)/c^2)^(1/2)*b^2*c^2*d^5*x+87*ln(1/2*
(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^7*(d*(a*d-b*c)/c^2)^(1/2)*
b^2*c^3*d^4*x^3-78*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^
6*(d*(a*d-b*c)/c^2)^(1/2)*b^3*c^4*d^3*x^3-78*ln...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(5/2)/(c+d/x)^2,x, algorithm="maxima")
```

```
[Out] integrate(1/((a + b/x)^(5/2)*(c + d/x)^2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 963 vs. $2(261) = 522$.
time = 7.73, size = 3887, normalized size = 13.54

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(5/2)/(c+d/x)^2,x, algorithm="fricas")
```

```
[Out] [1/6*(3*(5*b^6*c^4*d - 11*a*b^5*c^3*d^2 + 3*a^2*b^4*c^2*d^3 + 7*a^3*b^3*c*d
^4 - 4*a^4*b^2*d^5 + (5*a^2*b^4*c^5 - 11*a^3*b^3*c^4*d + 3*a^4*b^2*c^3*d^2
+ 7*a^5*b*c^2*d^3 - 4*a^6*c*d^4)*x^3 + (10*a*b^5*c^5 - 17*a^2*b^4*c^4*d - 5
*a^3*b^3*c^3*d^2 + 17*a^4*b^2*c^2*d^3 - a^5*b*c*d^4 - 4*a^6*d^5)*x^2 + (5*b
^6*c^5 - a*b^5*c^4*d - 19*a^2*b^4*c^3*d^2 + 13*a^3*b^3*c^2*d^3 + 10*a^4*b^2
*c*d^4 - 8*a^5*b*d^5)*x)*sqrt(a)*log(2*a*x - 2*sqrt(a)*x*sqrt((a*x + b)/x)
+ b) + 3*(9*a^4*b^3*c*d^4 - 4*a^5*b^2*d^5 + (9*a^6*b*c^2*d^3 - 4*a^7*c*d^4)
*x^3 + (18*a^5*b^2*c^2*d^3 + a^6*b*c*d^4 - 4*a^7*d^5)*x^2 + (9*a^4*b^3*c^2*
d^3 + 14*a^5*b^2*c*d^4 - 8*a^6*b*d^5)*x)*sqrt(-d/(b*c - a*d))*log(-(2*(b*c
- a*d)*x*sqrt(-d/(b*c - a*d))*sqrt((a*x + b)/x) - b*d + (b*c - 2*a*d)*x)/(c
*x + d)) + 2*(3*(a^3*b^3*c^5 - 3*a^4*b^2*c^4*d + 3*a^5*b*c^3*d^2 - a^6*c^2*
```

d^3)*x^4 + (20*a^2*b^4*c^5 - 41*a^3*b^3*c^4*d + 9*a^4*b^2*c^3*d^2 + 3*a^5*b *c^2*d^3 - 6*a^6*c*d^4)*x^3 + (15*a*b^5*c^5 - 13*a^2*b^4*c^4*d - 35*a^3*b^3 *c^3*d^2 + 15*a^4*b^2*c^2*d^3 - 12*a^5*b*c*d^4)*x^2 + 3*(5*a*b^5*c^4*d - 11 *a^2*b^4*c^3*d^2 + 3*a^3*b^3*c^2*d^3 - 2*a^4*b^2*c*d^4)*x)*sqrt((a*x + b)/x))/(a^4*b^5*c^6*d - 3*a^5*b^4*c^5*d^2 + 3*a^6*b^3*c^4*d^3 - a^7*b^2*c^3*d^4 + (a^6*b^3*c^7 - 3*a^7*b^2*c^6*d + 3*a^8*b*c^5*d^2 - a^9*c^4*d^3)*x^3 + (2 *a^5*b^4*c^7 - 5*a^6*b^3*c^6*d + 3*a^7*b^2*c^5*d^2 + a^8*b*c^4*d^3 - a^9*c^ 3*d^4)*x^2 + (a^4*b^5*c^7 - a^5*b^4*c^6*d - 3*a^6*b^3*c^5*d^2 + 5*a^7*b^2*c ^4*d^3 - 2*a^8*b*c^3*d^4)*x), -1/6*(6*(9*a^4*b^3*c*d^4 - 4*a^5*b^2*d^5 + (9 *a^6*b*c^2*d^3 - 4*a^7*c*d^4)*x^3 + (18*a^5*b^2*c^2*d^3 + a^6*b*c*d^4 - 4*a ^7*d^5)*x^2 + (9*a^4*b^3*c^2*d^3 + 14*a^5*b^2*c*d^4 - 8*a^6*b*d^5)*x)*sqrt(d/(b*c - a*d))*arctan(-(b*c - a*d)*x*sqrt(d/(b*c - a*d))*sqrt((a*x + b)/x)/ (a*d*x + b*d)) - 3*(5*b^6*c^4*d - 11*a*b^5*c^3*d^2 + 3*a^2*b^4*c^2*d^3 + 7* a^3*b^3*c*d^4 - 4*a^4*b^2*d^5 + (5*a^2*b^4*c^5 - 11*a^3*b^3*c^4*d + 3*a^4*b ^2*c^3*d^2 + 7*a^5*b*c^2*d^3 - 4*a^6*c*d^4)*x^3 + (10*a*b^5*c^5 - 17*a^2*b^ 4*c^4*d - 5*a^3*b^3*c^3*d^2 + 17*a^4*b^2*c^2*d^3 - a^5*b*c*d^4 - 4*a^6*d^5) *x^2 + (5*b^6*c^5 - a*b^5*c^4*d - 19*a^2*b^4*c^3*d^2 + 13*a^3*b^3*c^2*d^3 + 10*a^4*b^2*c*d^4 - 8*a^5*b*d^5)*x)*sqrt(a)*log(2*a*x - 2*sqrt(a)*x*sqrt((a *x + b)/x) + b) - 2*(3*(a^3*b^3*c^5 - 3*a^4*b^2*c^4*d + 3*a^5*b*c^3*d^2 - a ^6*c^2*d^3)*x^4 + (20*a^2*b^4*c^5 - 41*a^3*b^3*c^4*d + 9*a^4*b^2*c^3*d^2 + 3*a^5*b*c^2*d^3 - 6*a^6*c*d^4)*x^3 + (15*a*b^5*c^5 - 13*a^2*b^4*c^4*d - 35* a^3*b^3*c^3*d^2 + 15*a^4*b^2*c^2*d^3 - 12*a^5*b*c*d^4)*x^2 + 3*(5*a*b^5*c^4 *d - 11*a^2*b^4*c^3*d^2 + 3*a^3*b^3*c^2*d^3 - 2*a^4*b^2*c*d^4)*x)*sqrt((a*x + b)/x))/(a^4*b^5*c^6*d - 3*a^5*b^4*c^5*d^2 + 3*a^6*b^3*c^4*d^3 - a^7*b^2* c^3*d^4 + (a^6*b^3*c^7 - 3*a^7*b^2*c^6*d + 3*a^8*b*c^5*d^2 - a^9*c^4*d^3)*x ^3 + (2*a^5*b^4*c^7 - 5*a^6*b^3*c^6*d + 3*a^7*b^2*c^5*d^2 + a^8*b*c^4*d^3 - a^9*c^3*d^4)*x^2 + (a^4*b^5*c^7 - a^5*b^4*c^6*d - 3*a^6*b^3*c^5*d^2 + 5*a^ 7*b^2*c^4*d^3 - 2*a^8*b*c^3*d^4)*x), 1/6*(6*(5*b^6*c^4*d - 11*a*b^5*c^3*d^2 + 3*a^2*b^4*c^2*d^3 + 7*a^3*b^3*c*d^4 - 4*a^4*b^2*d^5 + (5*a^2*b^4*c^5 - 1 1*a^3*b^3*c^4*d + 3*a^4*b^2*c^3*d^2 + 7*a^5*b*c^2*d^3 - 4*a^6*c*d^4)*x^3 + (10*a*b^5*c^5 - 17*a^2*b^4*c^4*d - 5*a^3*b^3*c^3*d^2 + 17*a^4*b^2*c^2*d^3 - a^5*b*c*d^4 - 4*a^6*d^5)*x^2 + (5*b^6*c^5 - a*b^5*c^4*d - 19*a^2*b^4*c^3*d ^2 + 13*a^3*b^3*c^2*d^3 + 10*a^4*b^2*c*d^4 - 8*a^5*b*d^5)*x)*sqrt(-a)*arcta n(sqrt(-a)*sqrt((a*x + b)/x)/a) + 3*(9*a^4*b^3*c*d^4 - 4*a^5*b^2*d^5 + (9*a ^6*b*c^2*d^3 - 4*a^7*c*d^4)*x^3 + (18*a^5*b^2*c^2*d^3 + a^6*b*c*d^4 - 4*a^7 *d^5)*x^2 + (9*a^4*b^3*c^2*d^3 + 14*a^5*b^2*c*d^4 - 8*a^6*b*d^5)*x)*sqrt(-d /(b*c - a*d))*log(-(2*(b*c - a*d)*x*sqrt(-d/(b*c - a*d))*sqrt((a*x + b)/x) - b*d + (b*c - 2*a*d)*x)/(c*x + d)) + 2*(3*(a^3*b^3*c^5 - 3*a^4*b^2*c^4*d + 3*a^5*b*c^3*d^2 - a^6*c^2*d^3)*x^4 + (20*a^2*b^4*c^5 - 41*a^3*b^3*c^4*d + 9*a^4*b^2*c^3*d^2 + 3*a^5*b*c^2*d^3 - 6*a^6*c*d^4)*x^3 + (15*a*b^5*c^5 - 13 *a^2*b^4*c^4*d - 35*a^3*b^3*c^3*d^2 + 15*a^4*b^2*c^2*d^3 - 12*a^5*b*c*d^4)* x^2 + 3*(5*a*b^5*c^4*d - 11*a^2*b^4*c^3*d^2 + 3*a^3*b^3*c^2*d^3 - 2*a^4*b^2 *c*d^4)*x)*sqrt((a*x + b)/x))/(a^4*b^5*c^6*d - 3*a^5*b^4*c^5*d^2 + 3*a^6*b^ 3*c^4*d^3 - a^7*b^2*c^3*d^4 + (a^6*b^3*c^7 - 3*a^7*b^2*c^6*d + 3*a^8*b*c^5* d^2 - a^9*c^4*d^3)*x^3 + (2*a^5*b^4*c^7 - 5*a^6*b^3*c^6*d + 3*a^7*b^2*c^5*d

```
^2 + a^8*b*c^4*d^3 - a^9*c^3*d^4)*x^2 + (a^4*b^5*c^7 - a^5*b^4*c^6*d - 3*a^
6*b^3*c^5*d^2 + 5*a^7*b^2*c^4*d^3 - 2*a^8*b*c^3*d^4)*x), -1/3*(3*(9*a^4*b^3
*c*d^4 - 4*a^5*b^2*d^5 + (9*a^6*b*c^2*d^3 - 4*a^7*c*d^4)*x^3 + (18*a^5*b^2*
c^2*d^3 + a^6*b*c*d^4 - 4*a^7*d^5)*x^2 + (9*a^4*b^3*c^2*d^3 + 14*a^5*b^2*c*
d^4 - 8*a^6*b*d^5)*x)*sqrt(d/(b*c - a*d))*arctan(-(b*c - a*d)*x*sqrt(d/(b*c
 - a*d))*sqrt((a*x + b)/x)/(a*d*x + b*d)) - 3*(5*b^6*c^4*d - 11*a*b^5*c^3*d
^2 + 3*a^2*b^4*c^2*d^3 + 7*a^3*b^3*c*d^4 - 4*a^4*b^2*d^5 + (5*a^2*b^4*c^5 -
 11*a^3*b^3*c^4*d + 3*a^4*b^2*c^3*d^2 + 7*a^5*b*c^2*d^3 - 4*a^6*c*d^4)*x^3
+ (10*a*b^5*c^5 - 17*a^2*b^4*c^4*d - 5*a^3*b^3*c^3*d^2 + 17*a^4*b^2*c^2*d^3
 - a^5*b*c*d^4 - 4*a^6*d^5)*x^2 + (5*b^6*c^5 - a*b^5*c^4*d - 19*a^2*b^4*c^3
*d^2 + 13*a^3*b^3*c^2*d^3 + 10*a^4*b^2*c*d^4 - ...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(a+\frac{b}{x}\right)^{\frac{5}{2}}(cx+d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)**(5/2)/(c+d/x)**2,x)
```

```
[Out] Integral(x**2/((a + b/x)**(5/2)*(c*x + d)**2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(5/2)/(c+d/x)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [B]
time = 8.73, size = 2500, normalized size = 8.71

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b/x)^(5/2)*(c + d/x)^2),x)
```

```
[Out] ((2*b^3)/(3*(a^2*d - a*b*c)) + (10*b^3*(a + b/x)*(2*a*d - b*c))/(3*(a^2*d -
 a*b*c)^2) - (b*(a + b/x)^2*(6*a^4*d^4 + 15*b^4*c^4 + 64*a^2*b^2*c^2*d^2 -
```

58*a*b^3*c^3*d - 12*a^3*b*c*d^3))/(3*c^2*(a^2*d - a*b*c)^3) + (b*(a + b/x)^
3*(2*a*d - b*c)*(a^2*d^3 + 5*b^2*c^2*d - a*b*c*d^2))/(c^2*(a^2*d - a*b*c)^3
))/(d*(a + b/x)^(7/2) + (a + b/x)^(3/2)*(a^2*d - a*b*c) - (a + b/x)^(5/2)*(
2*a*d - b*c)) + (atan((a^15*b^19*c^19*(a + b/x)^(1/2)*125i + a^17*b^17*c^17
d^2(a + b/x)^(1/2)*10440i - a^18*b^16*c^16*d^3*(a + b/x)^(1/2)*37776i + a
^19*b^15*c^15*d^4*(a + b/x)^(1/2)*87276i - a^20*b^14*c^14*d^5*(a + b/x)^(1/
2)*126720i + a^21*b^13*c^13*d^6*(a + b/x)^(1/2)*91560i + a^22*b^12*c^12*d^7
*(a + b/x)^(1/2)*40965i - a^23*b^11*c^11*d^8*(a + b/x)^(1/2)*184563i + a^24
*b^10*c^10*d^9*(a + b/x)^(1/2)*212608i - a^25*b^9*c^9*d^10*(a + b/x)^(1/2)*
107740i - a^26*b^8*c^8*d^11*(a + b/x)^(1/2)*19530i + a^27*b^7*c^7*d^12*(a +
b/x)^(1/2)*71070i - a^28*b^6*c^6*d^13*(a + b/x)^(1/2)*52836i + a^29*b^5*c^
5*d^14*(a + b/x)^(1/2)*20916i - a^30*b^4*c^4*d^15*(a + b/x)^(1/2)*4515i + a
^31*b^3*c^3*d^16*(a + b/x)^(1/2)*420i - a^16*b^18*c^18*d*(a + b/x)^(1/2)*17
00i)/(a^7*(a^7)^(1/2)*(a^7*(a^7*(212608*b^10*c^10*d^9 - 107740*a*b^9*c^9*d^
10 - 19530*a^2*b^8*c^8*d^11 + 71070*a^3*b^7*c^7*d^12 - 52836*a^4*b^6*c^6*d^
13 + 20916*a^5*b^5*c^5*d^14 - 4515*a^6*b^4*c^4*d^15 + 420*a^7*b^3*c^3*d^16)
+ 10440*b^17*c^17*d^2 - 37776*a*b^16*c^16*d^3 + 87276*a^2*b^15*c^15*d^4 -
126720*a^3*b^14*c^14*d^5 + 91560*a^4*b^13*c^13*d^6 + 40965*a^5*b^12*c^12*d^
7 - 184563*a^6*b^11*c^11*d^8) + 125*a^5*b^19*c^19 - 1700*a^6*b^18*c^18*d)))
*(4*a*d + 5*b*c)*1i)/(c^3*(a^7)^(1/2)) - (atan((((d^7*(a*d - b*c)^7)^(1/2)*
((a + b/x)^(1/2)*(670*a^10*b^18*c^22*d^4 - 50*a^9*b^19*c^23*d^3 - 4082*a^11
*b^17*c^21*d^5 + 14830*a^12*b^16*c^20*d^6 - 35210*a^13*b^15*c^19*d^7 + 5551
0*a^14*b^14*c^18*d^8 - 53852*a^15*b^13*c^17*d^9 + 19048*a^16*b^12*c^16*d^10
+ 25730*a^17*b^11*c^15*d^11 - 39550*a^18*b^10*c^14*d^12 + 10670*a^19*b^9*c
^13*d^13 + 29414*a^20*b^8*c^12*d^14 - 45430*a^21*b^7*c^11*d^15 + 34490*a^22
*b^6*c^10*d^16 - 16240*a^23*b^5*c^9*d^17 + 4820*a^24*b^4*c^8*d^18 - 832*a^2
5*b^3*c^7*d^19 + 64*a^26*b^2*c^6*d^20) - ((d^7*(a*d - b*c)^7)^(1/2)*(4*a*d
- 9*b*c)*(304*a^13*b^18*c^25*d^3 - 20*a^12*b^19*c^26*d^2 - 2144*a^14*b^17*c
^24*d^4 + 9280*a^15*b^16*c^23*d^5 - 27476*a^16*b^15*c^22*d^6 + 58688*a^17*b
^14*c^21*d^7 - 92840*a^18*b^13*c^20*d^8 + 109648*a^19*b^12*c^19*d^9 - 95700
*a^20*b^11*c^18*d^10 + 59312*a^21*b^10*c^17*d^11 - 23056*a^22*b^9*c^16*d^12
+ 2528*a^23*b^8*c^15*d^13 + 2996*a^24*b^7*c^14*d^14 - 2080*a^25*b^6*c^13*d
^15 + 664*a^26*b^5*c^12*d^16 - 112*a^27*b^4*c^11*d^17 + 8*a^28*b^3*c^10*d^1
8 + ((d^7*(a*d - b*c)^7)^(1/2)*(a + b/x)^(1/2)*(4*a*d - 9*b*c)*(8*a^15*b^18
*c^28*d^2 - 136*a^16*b^17*c^27*d^3 + 1080*a^17*b^16*c^26*d^4 - 5320*a^18*b^
15*c^25*d^5 + 18200*a^19*b^14*c^24*d^6 - 45864*a^20*b^13*c^23*d^7 + 88088*a
^21*b^12*c^22*d^8 - 131560*a^22*b^11*c^21*d^9 + 154440*a^23*b^10*c^20*d^10
- 143000*a^24*b^9*c^19*d^11 + 104104*a^25*b^8*c^18*d^12 - 58968*a^26*b^7*c^
17*d^13 + 25480*a^27*b^6*c^16*d^14 - 8120*a^28*b^5*c^15*d^15 + 1800*a^29*b^
4*c^14*d^16 - 248*a^30*b^3*c^13*d^17 + 16*a^31*b^2*c^12*d^18))/(2*(b^7*c^10
- a^7*c^3*d^7 + 7*a^6*b*c^4*d^6 + 21*a^2*b^5*c^8*d^2 - 35*a^3*b^4*c^7*d^3
+ 35*a^4*b^3*c^6*d^4 - 21*a^5*b^2*c^5*d^5 - 7*a*b^6*c^9*d))))/(2*(b^7*c^10
- a^7*c^3*d^7 + 7*a^6*b*c^4*d^6 + 21*a^2*b^5*c^8*d^2 - 35*a^3*b^4*c^7*d^3 +
35*a^4*b^3*c^6*d^4 - 21*a^5*b^2*c^5*d^5 - 7*a*b^6*c^9*d)))*(4*a*d - 9*b*c)
1i)/(2(b^7*c^10 - a^7*c^3*d^7 + 7*a^6*b*c^4*d^6 + 21*a^2*b^5*c^8*d^2 - 35

```
*a^3*b^4*c^7*d^3 + 35*a^4*b^3*c^6*d^4 - 21*a^5*b^2*c^5*d^5 - 7*a*b^6*c^9*d)
) + ((d^7*(a*d - b*c)^7)^(1/2)*((a + b/x)^(1/2)*(670*a^10*b^18*c^22*d^4 - 5
0*a^9*b^19*c^23*d^3 - 4082*a^11*b^17*c^21*d^5 + 14830*a^12*b^16*c^20*d^6 -
35210*a^13*b^15*c^19*d^7 + 55510*a^14*b^14*c^18*d^8 - 53852*a^15*b^13*c^17*
d^9 + 19048*a^16*b^12*c^16*d^10 + 25730*a^17*b^11*c^15*d^11 - 39550*a^18*b^
10*c^14*d^12 + 10670*a^19*b^9*c^13*d^13 + 29414*a^20*b^8*c^12*d^14 - 45430*
a^21*b^7*c^11*d^15 + 34490*a^22*b^6*c^10*d^16 - 16240*a^23*b^5*c^9*d^17 + 4
820*a^24*b^4*c^8*d^18 - 832*a^25*b^3*c^7*d^19 + 64*a^26*b^2*c^6*d^20) - ((d
^7*(a*d - b*c)^7)^(1/2)*(4*a*d - 9*b*c)*(20*a^12*b^19*c^26*d^2 - 304*a^13*b
^18*c^25*d^3 + 2144*a^14*b^17*c^24*d^4 - 9280*a^15*b^16*c^23*d^5 + 27476*a^
16*b^15*c^22*d^6 - 58688*a^17*b^14*c^21*d^7 + 92840*a^18*b^13*c^20*d^8 - 10
9648*a^19*b^12*c^19*d^9 + 95700*a^20*b^11*c^18*d^10 - 59312*a^21*b^10*c^17*
d^11 + 23056*a^22*b^9*c^16*d^12 - 2528*a^23*b^8*c^15*d^13 - 2996*a^24*b^7*c
^14*d^14 + 2080*a^25*b^6*c^13*d^15 - 664*a^26*b^5*c^12*d^16 + 112*a^27*b^4*
c^11*d^17 - 8*a^28*b^3*c^10*d^18 + ((d^7*(a*d - b*c)^7)^(1/2)*(a + b/x)^(1/
2)*(4*a*d - 9*b*c)*(8*a^15*b^18*c^28*d^2 - 136*a^16*b^17*c^27*d^3 + 1080*a^
17*b^16*c^26*d^4 - 5320*a^18*b^15*c^25*d^5 + 18200*a^19*b^14*c^24*d^6 - 458
64*a^20*b^13*c^23*d^7 + 88088*a^21*b^12*c^22*d^8 - 131560*a^22*b^11*c^21*d^
9 + 154440*a^23*b^10*c^20*d^10 - 143000*a^24*b^9*c^19*d^11 + 104104*a^25*b^
8*c^18*d^12 - 58968*a^26*b^7*c^17*d^13 + 25480*...
```

3.265 $\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^3}\,dx$

Optimal. Leaf size=409

$$\frac{b(20b^3c^3-36ab^2c^2d+87a^2bcd^2-36a^3d^3)}{12a^2c^3(bc-ad)^3\left(a+\frac{b}{x}\right)^{3/2}}+\frac{b(20b^4c^4-56ab^3c^3d+24a^2b^2c^2d^2-35a^3bcd^3+12a^4d^4)}{4a^3c^3(bc-ad)^4\sqrt{a+\frac{b}{x}}}+\frac{}{2ac^2(b}$$

```
[Out] 1/12*b*(-36*a^3*d^3+87*a^2*b*c*d^2-36*a*b^2*c^2*d+20*b^3*c^3)/a^2/c^3/(-a*d
+b*c)^3/(a+b/x)^(3/2)+1/2*d*(-3*a*d+2*b*c)/a/c^2/(-a*d+b*c)/(a+b/x)^(3/2)/(
c+d/x)^2+1/4*d*(12*a^2*d^2-23*a*b*c*d+4*b^2*c^2)/a/c^3/(-a*d+b*c)^2/(a+b/x)
^(3/2)/(c+d/x)+x/a/c/(a+b/x)^(3/2)/(c+d/x)^2-1/4*d^(7/2)*(24*a^2*d^2-88*a*b
*c*d+99*b^2*c^2)*arctan(d^(1/2)*(a+b/x)^(1/2)/(-a*d+b*c)^(1/2))/c^4/(-a*d+b
*c)^(9/2)-(6*a*d+5*b*c)*arctanh((a+b/x)^(1/2)/a^(1/2))/a^(7/2)/c^4+1/4*b*(1
2*a^4*d^4-35*a^3*b*c*d^3+24*a^2*b^2*c^2*d^2-56*a*b^3*c^3*d+20*b^4*c^4)/a^3/
c^3/(-a*d+b*c)^4/(a+b/x)^(1/2)
```

Rubi [A]
time = 0.48, antiderivative size = 409, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {382, 105, 156, 157, 162, 65, 214, 211}

$$-\frac{(6ad+5bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{7/2}c^4}-\frac{d^{7/2}(24a^2d^2-88abcd+99b^2c^2)\,\text{ArcTan}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{4c^4(bc-ad)^{9/2}}+\frac{d(12a^2d^2-23abcd+4b^2c^2)}{4ac^3\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)(bc-ad)^2}+\frac{b(-36a^3d^3+87a^2bcd^2-36ab^2c^2d+20b^3c^3)}{12a^2c^3\left(a+\frac{b}{x}\right)^{3/2}(bc-ad)^3}+\frac{b(12a^4d^4-35a^3bcd^3+24a^2b^2c^2d^2-56ab^3c^3d+20b^4c^4)}{4a^3c^3\sqrt{a+\frac{b}{x}}\,(bc-ad)^4}+\frac{d(2bc-3ad)}{2ac^2\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2(bc-ad)}+\frac{x}{ac\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x)^(5/2)*(c + d/x)^3),x]
```

```
[Out] (b*(20*b^3*c^3 - 36*a*b^2*c^2*d + 87*a^2*b*c*d^2 - 36*a^3*d^3))/(12*a^2*c^3
*(b*c - a*d)^3*(a + b/x)^(3/2)) + (b*(20*b^4*c^4 - 56*a*b^3*c^3*d + 24*a^2*
b^2*c^2*d^2 - 35*a^3*b*c*d^3 + 12*a^4*d^4))/(4*a^3*c^3*(b*c - a*d)^4*Sqrt[a
 + b/x]) + (d*(2*b*c - 3*a*d))/(2*a*c^2*(b*c - a*d)*(a + b/x)^(3/2)*(c + d/
x)^2) + (d*(4*b^2*c^2 - 23*a*b*c*d + 12*a^2*d^2))/(4*a*c^3*(b*c - a*d)^2*(a
 + b/x)^(3/2)*(c + d/x)) + x/(a*c*(a + b/x)^(3/2)*(c + d/x)^2) - (d^(7/2)*(
99*b^2*c^2 - 88*a*b*c*d + 24*a^2*d^2)*ArcTan[(Sqrt[d]*Sqrt[a + b/x])/Sqrt[b
*c - a*d]])/(4*c^4*(b*c - a*d)^(9/2)) - ((5*b*c + 6*a*d)*ArcTanh[Sqrt[a + b
/x]/Sqrt[a]])/(a^(7/2)*c^4)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 157

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^3}\,dx = -\text{Subst}\left(\int \frac{1}{x^2(a+bx)^{5/2}(c+dx)^3}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{x}{ac\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2} + \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(5bc+6ad)+\frac{9bdx}{2}}{x(a+bx)^{5/2}(c+dx)^3}\,dx, x, \frac{1}{x}\right)}{ac}$$

$$= \frac{d(2bc-3ad)}{2ac^2(bc-ad)\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2} + \frac{x}{ac\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2} - \frac{\text{Subst}\left(\int \frac{-(bc-ad \cdots}{\cdots}\right.}{\cdots}$$

$$= \frac{d(2bc-3ad)}{2ac^2(bc-ad)\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2} + \frac{d(4b^2c^2-23abcd+12a^2d^2)}{4ac^3(bc-ad)^2\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)} + \frac{\cdots}{ac\,(a \cdots}$$

$$= \frac{b(20b^3c^3-36ab^2c^2d+87a^2bcd^2-36a^3d^3)}{12a^2c^3(bc-ad)^3\left(a+\frac{b}{x}\right)^{3/2}} + \frac{d(2bc-3ad)}{2ac^2(bc-ad)\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^{\cdots}}$$

$$= \frac{b(20b^3c^3-36ab^2c^2d+87a^2bcd^2-36a^3d^3)}{12a^2c^3(bc-ad)^3\left(a+\frac{b}{x}\right)^{3/2}} + \frac{b(20b^4c^4-56ab^3c^3d+24a^2b^2c^2d^{\cdots}}{4a^3c^3(bc-ad)^4\sqrt{\cdots}}$$

$$= \frac{b(20b^3c^3-36ab^2c^2d+87a^2bcd^2-36a^3d^3)}{12a^2c^3(bc-ad)^3\left(a+\frac{b}{x}\right)^{3/2}} + \frac{b(20b^4c^4-56ab^3c^3d+24a^2b^2c^2d^{\cdots}}{4a^3c^3(bc-ad)^4\sqrt{\cdots}}$$

$$= \frac{b(20b^3c^3-36ab^2c^2d+87a^2bcd^2-36a^3d^3)}{12a^2c^3(bc-ad)^3\left(a+\frac{b}{x}\right)^{3/2}} + \frac{b(20b^4c^4-56ab^3c^3d+24a^2b^2c^2d^{\cdots}}{4a^3c^3(bc-ad)^4\sqrt{\cdots}}$$

$$= \frac{b(20b^3c^3-36ab^2c^2d+87a^2bcd^2-36a^3d^3)}{12a^2c^3(bc-ad)^3\left(a+\frac{b}{x}\right)^{3/2}} + \frac{b(20b^4c^4-56ab^3c^3d+24a^2b^2c^2d^{\cdots}}{4a^3c^3(bc-ad)^4\sqrt{\cdots}}$$

Mathematica [A]
time = 1.66, size = 404, normalized size = 0.99

$$\frac{\frac{c\sqrt{a+\frac{b}{x}}\,x\left(60b^6c^4(d+cx)^2+8ab^5c^3(d+cx)^2(-21d+10cx)+6a^6d^4x^2(6d^2+9cdx+2c^2x^2)+4a^2b^4c^2(d+cx)^2(18d^2-56cdx+3c^2x^2)+3a^5bd^3x(24d^3+cd^2x-45c^2dx^2-16c^3x^3)+6a^4b^2d^2(6d^4-26cd^3x-39c^2d^2x^2+8c^3dx^3+12c^4x^4)-3a^3b^3cd(35d^4+5cd^3x-64c^2d^2x^2-16c^3dx^3+16c^4x^4)\right)}{a^3(bc-ad)^4(b+ax)^2(d+cx)^2}-\frac{3d^{7/2}(99b^2c^2-88abcd+24a^2d^2)\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{9/2}}-\frac{12(5bc+6ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{7/2}}}{12c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x)^(5/2)*(c + d/x)^3),x]
```

```
[Out] ((c*Sqrt[a + b/x]*x*(60*b^6*c^4*(d + c*x)^2 + 8*a*b^5*c^3*(d + c*x)^2*(-21*
d + 10*c*x) + 6*a^6*d^4*x^2*(6*d^2 + 9*c*d*x + 2*c^2*x^2) + 4*a^2*b^4*c^2*(
d + c*x)^2*(18*d^2 - 56*c*d*x + 3*c^2*x^2) + 3*a^5*b*d^3*x*(24*d^3 + c*d^2*
x - 45*c^2*d*x^2 - 16*c^3*x^3) + 6*a^4*b^2*d^2*(6*d^4 - 26*c*d^3*x - 39*c^2
*d^2*x^2 + 8*c^3*d*x^3 + 12*c^4*x^4) - 3*a^3*b^3*c*d*(35*d^4 + 5*c*d^3*x -
64*c^2*d^2*x^2 - 16*c^3*d*x^3 + 16*c^4*x^4)))/(a^3*(b*c - a*d)^4*(b + a*x)^
2*(d + c*x)^2) - (3*d^(7/2)*(99*b^2*c^2 - 88*a*b*c*d + 24*a^2*d^2)*ArcTan[(
Sqrt[d]*Sqrt[a + b/x])/Sqrt[b*c - a*d]])/(b*c - a*d)^(9/2) - (12*(5*b*c + 6
*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/a^(7/2))/(12*c^4)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 7299 vs. $2(373) = 746$.
time = 0.23, size = 7300, normalized size = 17.85

method	result
risch	$\frac{ax+b}{a^3c^3\sqrt{\frac{ax+b}{x}}}+\frac{\left(-\frac{3\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^2+bx}\right)d}{a^{\frac{5}{2}}c^4}-\frac{5\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^2+bx}\right)b}{2a^{\frac{7}{2}}c^3}+\frac{2b^5\sqrt{a\left(x+\frac{b}{a}\right)^2-b\left(x+\frac{b}{a}\right)}}{3a^5(ad-bc)^3\left(x+\frac{b}{a}\right)^2}+\right.}{}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+1/x*b)^(5/2)/(c+d/x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(5/2)/(c+d/x)^3,x, algorithm="maxima")
```

```
[Out] integrate(1/((a + b/x)^(5/2)*(c + d/x)^3), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1534 vs. $2(373) = 746$.

time = 14.15, size = 6171, normalized size = 15.09

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(5/2)/(c+d/x)^3,x, algorithm="fricas")
```

```
[Out] [1/24*(12*(5*b^7*c^5*d^2 - 14*a*b^6*c^4*d^3 + 6*a^2*b^5*c^3*d^4 + 16*a^3*b^
4*c^2*d^5 - 19*a^4*b^3*c*d^6 + 6*a^5*b^2*d^7 + (5*a^2*b^5*c^7 - 14*a^3*b^4*
c^6*d + 6*a^4*b^3*c^5*d^2 + 16*a^5*b^2*c^4*d^3 - 19*a^6*b*c^3*d^4 + 6*a^7*c
^2*d^5)*x^4 + 2*(5*a*b^6*c^7 - 9*a^2*b^5*c^6*d - 8*a^3*b^4*c^5*d^2 + 22*a^4
*b^3*c^4*d^3 - 3*a^5*b^2*c^3*d^4 - 13*a^6*b*c^2*d^5 + 6*a^7*c*d^6)*x^3 + (5
*b^7*c^7 + 6*a*b^6*c^6*d - 45*a^2*b^5*c^5*d^2 + 26*a^3*b^4*c^4*d^3 + 51*a^4
*b^3*c^3*d^4 - 54*a^5*b^2*c^2*d^5 + 5*a^6*b*c*d^6 + 6*a^7*d^7)*x^2 + 2*(5*b
^7*c^6*d - 9*a*b^6*c^5*d^2 - 8*a^2*b^5*c^4*d^3 + 22*a^3*b^4*c^3*d^4 - 3*a^4
*b^3*c^2*d^5 - 13*a^5*b^2*c*d^6 + 6*a^6*b*d^7)*x)*sqrt(a)*log(2*a*x - 2*sqr
t(a)*x*sqrt((a*x + b)/x) + b) + 3*(99*a^4*b^4*c^2*d^5 - 88*a^5*b^3*c*d^6 +
24*a^6*b^2*d^7 + (99*a^6*b^2*c^4*d^3 - 88*a^7*b*c^3*d^4 + 24*a^8*c^2*d^5)*x
^4 + 2*(99*a^5*b^3*c^4*d^3 + 11*a^6*b^2*c^3*d^4 - 64*a^7*b*c^2*d^5 + 24*a^8
*c*d^6)*x^3 + (99*a^4*b^4*c^4*d^3 + 308*a^5*b^3*c^3*d^4 - 229*a^6*b^2*c^2*d
^5 + 8*a^7*b*c*d^6 + 24*a^8*d^7)*x^2 + 2*(99*a^4*b^4*c^3*d^4 + 11*a^5*b^3*c
^2*d^5 - 64*a^6*b^2*c*d^6 + 24*a^7*b*d^7)*x)*sqrt(-d/(b*c - a*d))*log(-(2*(
b*c - a*d)*x*sqrt(-d/(b*c - a*d))*sqrt((a*x + b)/x) - b*d + (b*c - 2*a*d)*x
)/(c*x + d)) + 2*(12*(a^3*b^4*c^7 - 4*a^4*b^3*c^6*d + 6*a^5*b^2*c^5*d^2 - 4
*a^6*b*c^4*d^3 + a^7*c^3*d^4)*x^5 + (80*a^2*b^5*c^7 - 200*a^3*b^4*c^6*d + 4
8*a^4*b^3*c^5*d^2 + 48*a^5*b^2*c^4*d^3 - 135*a^6*b*c^3*d^4 + 54*a^7*c^2*d^5
)*x^4 + (60*a*b^6*c^7 - 8*a^2*b^5*c^6*d - 364*a^3*b^4*c^5*d^2 + 192*a^4*b^3
*c^4*d^3 - 234*a^5*b^2*c^3*d^4 + 3*a^6*b*c^2*d^5 + 36*a^7*c*d^6)*x^3 + (120
*a*b^6*c^6*d - 256*a^2*b^5*c^5*d^2 - 80*a^3*b^4*c^4*d^3 - 15*a^4*b^3*c^3*d^
4 - 156*a^5*b^2*c^2*d^5 + 72*a^6*b*c*d^6)*x^2 + 3*(20*a*b^6*c^5*d^2 - 56*a^
2*b^5*c^4*d^3 + 24*a^3*b^4*c^3*d^4 - 35*a^4*b^3*c^2*d^5 + 12*a^5*b^2*c*d^6)
*x)*sqrt((a*x + b)/x))/(a^4*b^6*c^8*d^2 - 4*a^5*b^5*c^7*d^3 + 6*a^6*b^4*c^6
*d^4 - 4*a^7*b^3*c^5*d^5 + a^8*b^2*c^4*d^6 + (a^6*b^4*c^10 - 4*a^7*b^3*c^9*
d + 6*a^8*b^2*c^8*d^2 - 4*a^9*b*c^7*d^3 + a^10*c^6*d^4)*x^4 + 2*(a^5*b^5*c^
10 - 3*a^6*b^4*c^9*d + 2*a^7*b^3*c^8*d^2 + 2*a^8*b^2*c^7*d^3 - 3*a^9*b*c^6*
d^4 + a^10*c^5*d^5)*x^3 + (a^4*b^6*c^10 - 9*a^6*b^4*c^8*d^2 + 16*a^7*b^3*c^
7*d^3 - 9*a^8*b^2*c^6*d^4 + a^10*c^4*d^6)*x^2 + 2*(a^4*b^6*c^9*d - 3*a^5*b^
5*c^8*d^2 + 2*a^6*b^4*c^7*d^3 + 2*a^7*b^3*c^6*d^4 - 3*a^8*b^2*c^5*d^5 + a^9
*b*c^4*d^6)*x), 1/24*(24*(5*b^7*c^5*d^2 - 14*a*b^6*c^4*d^3 + 6*a^2*b^5*c^3*
d^4 + 16*a^3*b^4*c^2*d^5 - 19*a^4*b^3*c*d^6 + 6*a^5*b^2*d^7 + (5*a^2*b^5*c^
7 - 14*a^3*b^4*c^6*d + 6*a^4*b^3*c^5*d^2 + 16*a^5*b^2*c^4*d^3 - 19*a^6*b*c^
3*d^4 + 6*a^7*c^2*d^5)*x^4 + 2*(5*a*b^6*c^7 - 9*a^2*b^5*c^6*d - 8*a^3*b^4*c
^5*d^2 + 22*a^4*b^3*c^4*d^3 - 3*a^5*b^2*c^3*d^4 - 13*a^6*b*c^2*d^5 + 6*a^7*
```

```
c*d^6)*x^3 + (5*b^7*c^7 + 6*a*b^6*c^6*d - 45*a^2*b^5*c^5*d^2 + 26*a^3*b^4*c
^4*d^3 + 51*a^4*b^3*c^3*d^4 - 54*a^5*b^2*c^2*d^5 + 5*a^6*b*c*d^6 + 6*a^7*d^
7)*x^2 + 2*(5*b^7*c^6*d - 9*a*b^6*c^5*d^2 - 8*a^2*b^5*c^4*d^3 + 22*a^3*b^4*
c^3*d^4 - 3*a^4*b^3*c^2*d^5 - 13*a^5*b^2*c*d^6 + 6*a^6*b*d^7)*x)*sqrt(-a)*a
rctan(sqrt(-a)*sqrt((a*x + b)/x)/a) + 3*(99*a^4*b^4*c^2*d^5 - 88*a^5*b^3*c*
d^6 + 24*a^6*b^2*d^7 + (99*a^6*b^2*c^4*d^3 - 88*a^7*b*c^3*d^4 + 24*a^8*c^2*
d^5)*x^4 + 2*(99*a^5*b^3*c^4*d^3 + 11*a^6*b^2*c^3*d^4 - 64*a^7*b*c^2*d^5 +
24*a^8*c*d^6)*x^3 + (99*a^4*b^4*c^4*d^3 + 308*a^5*b^3*c^3*d^4 - 229*a^6*b^2
*c^2*d^5 + 8*a^7*b*c*d^6 + 24*a^8*d^7)*x^2 + 2*(99*a^4*b^4*c^3*d^4 + 11*a^5
*b^3*c^2*d^5 - 64*a^6*b^2*c*d^6 + 24*a^7*b*d^7)*x)*sqrt(-d/(b*c - a*d))*log
(-(2*(b*c - a*d)*x*sqrt(-d/(b*c - a*d))*sqrt((a*x + b)/x) - b*d + (b*c - 2*
a*d)*x)/(c*x + d)) + 2*(12*(a^3*b^4*c^7 - 4*a^4*b^3*c^6*d + 6*a^5*b^2*c^5*d
^2 - 4*a^6*b*c^4*d^3 + a^7*c^3*d^4)*x^5 + (80*a^2*b^5*c^7 - 200*a^3*b^4*c^6
*d + 48*a^4*b^3*c^5*d^2 + 48*a^5*b^2*c^4*d^3 - 135*a^6*b*c^3*d^4 + 54*a^7*c
^2*d^5)*x^4 + (60*a*b^6*c^7 - 8*a^2*b^5*c^6*d - 364*a^3*b^4*c^5*d^2 + 192*a
^4*b^3*c^4*d^3 - 234*a^5*b^2*c^3*d^4 + 3*a^6*b*c^2*d^5 + 36*a^7*c*d^6)*x^3
+ (120*a*b^6*c^6*d - 256*a^2*b^5*c^5*d^2 - 80*a^3*b^4*c^4*d^3 - 15*a^4*b^3*
c^3*d^4 - 156*a^5*b^2*c^2*d^5 + 72*a^6*b*c*d^6)*x^2 + 3*(20*a*b^6*c^5*d^2 -
 56*a^2*b^5*c^4*d^3 + 24*a^3*b^4*c^3*d^4 - 35*a^4*b^3*c^2*d^5 + 12*a^5*b^2*
c*d^6)*x)*sqrt((a*x + b)/x))/(a^4*b^6*c^8*d^2 - 4*a^5*b^5*c^7*d^3 + 6*a^6*b
^4*c^6*d^4 - 4*a^7*b^3*c^5*d^5 + a^8*b^2*c^4*d^6 + (a^6*b^4*c^10 - 4*a^7*b^
3*c^9*d + 6*a^8*b^2*c^8*d^2 - 4*a^9*b*c^7*d^3 + a^10*c^6*d^4)*x^4 + 2*(a^5*
b^5*c^10 - 3*a^6*b^4*c^9*d + 2*a^7*b^3*c^8*d^2 + 2*a^8*b^2*c^7*d^3 - 3*a^9*
b*c^6*d^4 + a^10*c^5*d^5)*x^3 + (a^4*b^6*c^10 - 9*a^6*b^4*c^8*d^2 + 16*a^7*
b^3*c^7*d^3 - 9*a^8*b^2*c^6*d^4 + a^10*c^4*d^6)*x^2 + 2*(a^4*b^6*c^9*d - 3*
a^5*b^5*c^8*d^2 + 2*a^6*b^4*c^7*d^3 + 2*a^7*b^3*c^6*d^4 - 3*a^8*b^2*c^5*d^5
 + a^9*b*c^4*d^6)*x), -1/12*(3*(99*a^4*b^4*c^2*d^5 - 88*a^5*b^3*c*d^6 + 24*
a^6*b^2*d^7 + (99*a^6*b^2*c^4*d^3 - 88*a^7*b*c^3*d^4 + 24*a^8*c^2*d^5)*x^4
+ 2*(99*a^5*b^3*c^4*d^3 + 11*a^6*b^2*c^3*d^4 - 64*a^7*b*c^2*d^5 + 24*a^8*c*
d^6)*x^3 + (99*a^4*b^4*c^4*d^3 + 308*a^5*b^3*c^3*d^4 - 229*a^6*b^2*c^2*d^5
+ 8*a^7*b*c*d^6 + 24*a^8*d^7)*x^2 + 2*(99*a^4*b...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)**(5/2)/(c+d/x)**3,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1354 vs. 2(373) = 746.
time = 1.03, size = 1354, normalized size = 3.31

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b/x)^(5/2)/(c+d/x)^3,x, algorithm="giac")

[Out] 1/12*(297*a^(7/2)*b^2*c^2*d^4*arctan(sqrt(a)*d/sqrt(b*c*d - a*d^2)) - 264*a^(9/2)*b*c*d^5*arctan(sqrt(a)*d/sqrt(b*c*d - a*d^2)) + 72*a^(11/2)*d^6*arctan(sqrt(a)*d/sqrt(b*c*d - a*d^2)) - 30*sqrt(b*c*d - a*d^2)*b^5*c^5*log(abs(b)) + 84*sqrt(b*c*d - a*d^2)*a*b^4*c^4*d*log(abs(b)) - 36*sqrt(b*c*d - a*d^2)*a^2*b^3*c^3*d^2*log(abs(b)) - 96*sqrt(b*c*d - a*d^2)*a^3*b^2*c^2*d^3*log(abs(b)) + 114*sqrt(b*c*d - a*d^2)*a^4*b*c*d^4*log(abs(b)) - 36*sqrt(b*c*d - a*d^2)*a^5*d^5*log(abs(b)) - 56*sqrt(b*c*d - a*d^2)*b^5*c^5 + 128*sqrt(b*c*d - a*d^2)*a*b^4*c^4*d + 63*sqrt(b*c*d - a*d^2)*a^4*b*c*d^4 - 30*sqrt(b*c*d - a*d^2)*a^5*d^5)*sgn(x)/(sqrt(b*c*d - a*d^2)*a^(7/2)*b^4*c^8 - 4*sqrt(b*c*d - a*d^2)*a^(9/2)*b^3*c^7*d + 6*sqrt(b*c*d - a*d^2)*a^(11/2)*b^2*c^6*d^2 - 4*sqrt(b*c*d - a*d^2)*a^(13/2)*b*c^5*d^3 + sqrt(b*c*d - a*d^2)*a^(15/2)*c^4*d^4) + 1/4*(99*b^2*c^2*d^4 - 88*a*b*c*d^5 + 24*a^2*d^6)*arctan(-((sqrt(a)*x - sqrt(a*x^2 + b*x))*c + sqrt(a)*d)/sqrt(b*c*d - a*d^2))/((b^4*c^8*sgn(x) - 4*a*b^3*c^7*d*sgn(x) + 6*a^2*b^2*c^6*d^2*sgn(x) - 4*a^3*b*c^5*d^3*sgn(x) + a^4*c^4*d^4*sgn(x))*sqrt(b*c*d - a*d^2)) + 1/4*(21*(sqrt(a)*x - sqrt(a*x^2 + b*x))^3*sqrt(a)*b^2*c^3*d^4 - 56*(sqrt(a)*x - sqrt(a*x^2 + b*x))^3*a^(3/2)*b*c^2*d^5 + 24*(sqrt(a)*x - sqrt(a*x^2 + b*x))^3*a^(5/2)*c*d^6 + 15*(sqrt(a)*x - sqrt(a*x^2 + b*x))^2*a*b^2*c^2*d^5 - 88*(sqrt(a)*x - sqrt(a*x^2 + b*x))^2*a^2*b*c*d^6 + 40*(sqrt(a)*x - sqrt(a*x^2 + b*x))^2*a^3*d^7 + 19*(sqrt(a)*x - sqrt(a*x^2 + b*x))*sqrt(a)*b^3*c^2*d^5 - 92*(sqrt(a)*x - sqrt(a*x^2 + b*x))*a^(3/2)*b^2*c*d^6 + 40*(sqrt(a)*x - sqrt(a*x^2 + b*x))*a^(5/2)*b*d^7 - 21*a*b^3*c*d^6 + 10*a^2*b^2*d^7)/((sqrt(a)*b^4*c^8*sgn(x) - 4*a^(3/2)*b^3*c^7*d*sgn(x) + 6*a^(5/2)*b^2*c^6*d^2*sgn(x) - 4*a^(7/2)*b*c^5*d^3*sgn(x) + a^(9/2)*c^4*d^4*sgn(x))*((sqrt(a)*x - sqrt(a*x^2 + b*x))^2*c + 2*(sqrt(a)*x - sqrt(a*x^2 + b*x))*sqrt(a)*d + b*d)^2) + 2/3*(9*(sqrt(a)*x - sqrt(a*x^2 + b*x))^2*a^(3/2)*b^6*c - 18*(sqrt(a)*x - sqrt(a*x^2 + b*x))^2*a^(5/2)*b^5*d + 15*(sqrt(a)*x - sqrt(a*x^2 + b*x))*a*b^7*c - 33*(sqrt(a)*x - sqrt(a*x^2 + b*x))*a^2*b^6*d + 7*sqrt(a)*b^8*c - 16*a^(3/2)*b^7*d)/((a^(5/2)*b^4*c^4*sgn(x) - 4*a^(7/2)*b^3*c^3*d*sgn(x) + 6*a^(9/2)*b^2*c^2*d^2*sgn(x) - 4*a^(11/2)*b*c*d^3*sgn(x) + a^(13/2)*d^4*sgn(x))*((sqrt(a)*x - sqrt(a*x^2 + b*x))*a + sqrt(a)*b)^3) + sqrt(a*x^2 + b*x)/(a^3*c^3*sgn(x)) + 1/2*(5*b*c + 6*a*d)*log(abs(2*(sqrt(a)*x - sqrt(a*x^2 + b*x))*sqrt(a) + b))/(a^(7/2)*c^4*sgn(x))

Mupad [B]
time = 8.23, size = 2500, normalized size = 6.11

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a + b/x)^(5/2)*(c + d/x)^3),x)

```
[Out] ((2*b^4)/(3*(a^2*d - a*b*c)) + (2*b^4*(a + b/x)*(12*a*d - 5*b*c))/(3*(a^2*d
 - a*b*c)^2) + (b*(a + b/x)^2*(36*a^5*d^5 - 60*b^5*c^5 - 456*a^2*b^3*c^3*d^
2 + 120*a^3*b^2*c^2*d^3 + 308*a*b^4*c^4*d - 123*a^4*b*c*d^4))/(12*a^2*c^3*(
a^2*d - a*b*c)*(a*d - b*c)^2) + (b*(a + b/x)^4*(12*a^4*d^6 + 20*b^4*c^4*d^2
 - 56*a*b^3*c^3*d^3 + 24*a^2*b^2*c^2*d^4 - 35*a^3*b*c*d^5))/(4*a^2*c^3*(a^2
*d - a*b*c)*(a*d - b*c)^3) - (b*(a + b/x)^3*(72*a^5*d^6 - 120*b^5*c^5*d + 4
96*a*b^4*c^4*d^2 - 592*a^2*b^3*c^3*d^3 + 303*a^3*b^2*c^2*d^4 - 264*a^4*b*c*
d^5))/(12*a^2*c^3*(a^2*d - a*b*c)*(a*d - b*c)^3))/((a + b/x)^(5/2)*(3*a^2*d
^2 + b^2*c^2 - 4*a*b*c*d) - (a + b/x)^(7/2)*(3*a*d^2 - 2*b*c*d) + d^2*(a +
b/x)^(9/2) - (a + b/x)^(3/2)*(a^3*d^2 + a*b^2*c^2 - 2*a^2*b*c*d)) + (atan((
a^15*b^24*c^24*(a + b/x)^(1/2)*2000i + a^17*b^22*c^22*d^2*(a + b/x)^(1/2)*2
77440i - a^18*b^21*c^21*d^3*(a + b/x)^(1/2)*1325984i + a^19*b^20*c^20*d^4*(
a + b/x)^(1/2)*4135824i - a^20*b^19*c^19*d^5*(a + b/x)^(1/2)*8371440i + a^2
1*b^18*c^18*d^6*(a + b/x)^(1/2)*9129120i + a^22*b^17*c^17*d^7*(a + b/x)^(1/
2)*3058605i - a^23*b^16*c^16*d^8*(a + b/x)^(1/2)*32337558i + a^24*b^15*c^15
*d^9*(a + b/x)^(1/2)*63677218i - a^25*b^14*c^14*d^10*(a + b/x)^(1/2)*666652
80i + a^26*b^13*c^13*d^11*(a + b/x)^(1/2)*24871035i + a^27*b^12*c^12*d^12*(
a + b/x)^(1/2)*40203170i - a^28*b^11*c^11*d^13*(a + b/x)^(1/2)*85652532i +
a^29*b^10*c^10*d^14*(a + b/x)^(1/2)*88170192i - a^30*b^9*c^9*d^15*(a + b/x)
^(1/2)*60362445i + a^31*b^8*c^8*d^16*(a + b/x)^(1/2)*29178270i - a^32*b^7*c
^7*d^17*(a + b/x)^(1/2)*9940590i + a^33*b^6*c^6*d^18*(a + b/x)^(1/2)*228782
4i - a^34*b^5*c^5*d^19*(a + b/x)^(1/2)*320859i + a^35*b^4*c^4*d^20*(a + b/x
)^(1/2)*20790i - a^16*b^23*c^23*d*(a + b/x)^(1/2)*34800i)/(a^7*(a^7)^(1/2)*
(a^7*(a^7*(a^7*(29178270*b^8*c^8*d^16 - 9940590*a*b^7*c^7*d^17 + 2287824*a^
2*b^6*c^6*d^18 - 320859*a^3*b^5*c^5*d^19 + 20790*a^4*b^4*c^4*d^20) + 636772
18*b^15*c^15*d^9 - 66665280*a*b^14*c^14*d^10 + 24871035*a^2*b^13*c^13*d^11
+ 40203170*a^3*b^12*c^12*d^12 - 85652532*a^4*b^11*c^11*d^13 + 88170192*a^5*
b^10*c^10*d^14 - 60362445*a^6*b^9*c^9*d^15) + 277440*b^22*c^22*d^2 - 132598
4*a*b^21*c^21*d^3 + 4135824*a^2*b^20*c^20*d^4 - 8371440*a^3*b^19*c^19*d^5 +
 9129120*a^4*b^18*c^18*d^6 + 3058605*a^5*b^17*c^17*d^7 - 32337558*a^6*b^16*
c^16*d^8) + 2000*a^5*b^24*c^24 - 34800*a^6*b^23*c^23*d)))*(6*a*d + 5*b*c)*1
i)/(c^4*(a^7)^(1/2)) + (log(400*b^25*c^25*d^4 - 8240*a*b^24*c^24*d^5 - 1152
*a^11*d^5*(d^7*(a*d - b*c)^9)^(3/2)*(a + b/x)^(1/2) + 1152*a^20*d^21*(d^7*(
a*d - b*c)^9)^(1/2)*(a + b/x)^(1/2) + 79696*a^2*b^23*c^23*d^6 - 478768*a^3*
b^22*c^22*d^7 + 1987568*a^4*b^21*c^21*d^8 - 5978896*a^5*b^20*c^20*d^9 + 131
76240*a^6*b^19*c^19*d^10 - 20525703*a^7*b^18*c^18*d^11 + 18765714*a^8*b^17*
c^17*d^12 + 3763331*a^9*b^16*c^16*d^13 - 49787452*a^10*b^15*c^15*d^14 + 104
120705*a^11*b^14*c^14*d^15 - 140185682*a^12*b^13*c^13*d^16 + 139985251*a^13
*b^12*c^12*d^17 - 108046616*a^14*b^11*c^11*d^18 + 65184867*a^15*b^10*c^10*d
^19 - 30607170*a^16*b^9*c^9*d^20 + 10996689*a^17*b^8*c^8*d^21 - 2926572*a^1
8*b^7*c^7*d^22 + 544467*a^19*b^6*c^6*d^23 - 63294*a^20*b^5*c^5*d^24 + 3465*
a^21*b^4*c^4*d^25 + 400*b^20*c^20*d*(d^7*(a*d - b*c)^9)^(1/2)*(a + b/x)^(1/
2) + 9801*a^6*b^5*c^5*(d^7*(a*d - b*c)^9)^(3/2)*(a + b/x)^(1/2) - 37026*a^7
*b^4*c^4*d*(d^7*(a*d - b*c)^9)^(3/2)*(a + b/x)^(1/2) - 6240*a*b^19*c^19*d^2
*(d^7*(a*d - b*c)^9)^(1/2)*(a + b/x)^(1/2) + 47344*a^8*b^3*c^3*d^2*(d^7*(a*
```

```
d - b*c)^9)^(3/2)*(a + b/x)^(1/2) - 29216*a^9*b^2*c^2*d^3*(d^7*(a*d - b*c)^
9)^(3/2)*(a + b/x)^(1/2) + 44496*a^2*b^18*c^18*d^3*(d^7*(a*d - b*c)^9)^(1/2
)*(a + b/x)^(1/2) - 189888*a^3*b^17*c^17*d^4*(d^7*(a*d - b*c)^9)^(1/2)*(a +
 b/x)^(1/2) + 528768*a^4*b^16*c^16*d^5*(d^7*(a*d - b*c)^9)^(1/2)*(a + b/x)^
(1/2) - 959616*a^5*b^15*c^15*d^6*(d^7*(a*d - b*c)^9)^(1/2)*(a + b/x)^(1/2)
+ 972681*a^6*b^14*c^14*d^7*(d^7*(a*d - b*c)^9)^(1/2)*(a + b/x)^(1/2) + 4123
8*a^7*b^13*c^13*d^8*(d^7*(a*d - b*c)^9)^(1/2)*(a + b/x)^(1/2) - 1727195*a^8
*b^12*c^12*d^9*(d^7*(a*d - b*c)^9)^(1/2)*(a + b/x)^(1/2) + 2139672*a^9*b^11
*c^11*d^10*(d^7*(a*d - b*c)^9)^(1/2)*(a + b/x)^(1/2) + 786834*a^10*b^10*c^1
0*d^11*(d^7*(a*d - b*c)^9)^(1/2)*(a + b/x)^(1/2) - 6551292*a^11*b^9*c^9*d^1
2*(d^7*(a*d - b*c)^9)^(1/2)*(a + b/x)^(1/2) + 11685186*a^12*b^8*c^8*d^13*(d
^7*(a*d - b*c)^9)^(1/2)*(a + b/x)^(1/2) - 12876696*a^13*b^7*c^7*d^14*(d^7*(
a*d - b*c)^9)^(1/2)*(a + b/x)^(1/2) + 10033077*a^14*b^6*c^6*d^15*(d^7*(a*d
- b*c)^9)^(1/2)*(a + b/x)^(1/2) - 5737770*a^15*b^5*c^5*d^16*(d^7*(a*d - b*c
)^9)^(1/2)*(a + b/x)^(1/2) + 2414601*a^16*b^4*c^4*d^17*(d^7*(a*d - b*c)^9)^
(1/2)*(a + b/x)^(1/2) - 731920*a^17*b^3*c^3*d^18*(d^7*(a*d - b*c)^9)^(1/2)*
(a + b/x)^(1/2) + 151904*a^18*b^2*c^2*d^19*(d^7*(a*d - b*c)^9)^(1/2)*(a + b
/x)^(1/2) + 9024*a^10*b*c*d^4*(d^7*(a*d - b*c)^9)^(3/2)*(a + b/x)^(1/2) - 1
9392*a^19*b*c*d^20*(d^7*(a*d - b*c)^9)^(1/2)*(a + b/x)^(1/2))*(d^7*(a*d - b
*c)^9)^(1/2)*(3*a^2*d^2 + (99*b^2*c^2)/8 - 11*a*b*c*d))/(b^9*c^13 - a^9*c^4
*d^9 + 9*a^8*b*c^5*d^8 + 36*a^2*b^7*c^11*d^2 - 84*a^3*b^6*c^10*d^3 + 126*a^
4*b^5*c^9*d^4 - 126*a^5*b^4*c^8*d^5 + 84*a^6*b^...
```

3.266 $\int \sqrt{a+\frac{b}{x}} \sqrt{c+\frac{d}{x}} \, dx$

Optimal. Leaf size=123

$$\sqrt{a+\frac{b}{x}} \sqrt{c+\frac{d}{x}}\, x+\frac{(bc+ad)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+\frac{b}{x}}}{\sqrt{a}\sqrt{c+\frac{d}{x}}}\right)}{\sqrt{a}\sqrt{c}}-2\sqrt{b}\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{b}\sqrt{c+\frac{d}{x}}}\right)$$

```
[Out] (a*d+b*c)*arctanh(c^(1/2)*(a+b/x)^(1/2)/a^(1/2)/(c+d/x)^(1/2))/a^(1/2)/c^(1
/2)-2*arctanh(d^(1/2)*(a+b/x)^(1/2)/b^(1/2)/(c+d/x)^(1/2))*b^(1/2)*d^(1/2)+
x*(a+b/x)^(1/2)*(c+d/x)^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {382, 99, 163, 65, 223, 212, 95, 214}

$$x\sqrt{a+\frac{b}{x}} \sqrt{c+\frac{d}{x}}+\frac{(ad+bc)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+\frac{b}{x}}}{\sqrt{a}\sqrt{c+\frac{d}{x}}}\right)}{\sqrt{a}\sqrt{c}}-2\sqrt{b}\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{b}\sqrt{c+\frac{d}{x}}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b/x]*Sqrt[c + d/x],x]
```

```
[Out] Sqrt[a + b/x]*Sqrt[c + d/x]*x + ((b*c + a*d)*ArcTanh[(Sqrt[c]*Sqrt[a + b/x]
)/(Sqrt[a]*Sqrt[c + d/x])])/(Sqrt[a]*Sqrt[c]) - 2*Sqrt[b]*Sqrt[d]*ArcTanh[(
Sqrt[d]*Sqrt[a + b/x])/(Sqrt[b]*Sqrt[c + d/x])]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
```

```
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^p/(b*(
m + 1))), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 163

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int \sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}\,dx &= -\text{Subst}\left(\int \frac{\sqrt{a+bx}\sqrt{c+dx}}{x^2}\,dx, x, \frac{1}{x}\right)\\
&= \sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}\,x - \text{Subst}\left(\int \frac{\frac{1}{2}(bc+ad)+bdx}{x\sqrt{a+bx}\sqrt{c+dx}}\,dx, x, \frac{1}{x}\right)\\
&= \sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}\,x - (bd)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}}\,dx, x, \frac{1}{x}\right) - \frac{1}{2}(bc+ad)\text{S}\\
&= \sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}\,x - (2d)\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{ad}{b}+\frac{dx^2}{b}}}\,dx, x, \sqrt{a+\frac{b}{x}}\right) - (bc+\\
&= \sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}\,x + \frac{(bc+ad)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+\frac{b}{x}}}{\sqrt{a}\sqrt{c+\frac{d}{x}}}\right)}{\sqrt{a}\sqrt{c}} - (2d)\text{Subst}\left(\int \frac{1}{1-\cdots}\right.\\
&= \sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}\,x + \frac{(bc+ad)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+\frac{b}{x}}}{\sqrt{a}\sqrt{c+\frac{d}{x}}}\right)}{\sqrt{a}\sqrt{c}} - 2\sqrt{b}\sqrt{d}\tanh^{-1}\left(\cdots\right.
\end{aligned}
$$

Mathematica [A]
time = 0.35, size = 176, normalized size = 1.43

$$
\frac{\sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}\,x\left((bc+ad)\sqrt{b+ax}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d+cx}}{\sqrt{c}\sqrt{b+ax}}\right)+\sqrt{a}\sqrt{c}\left((b+ax)\sqrt{d+cx}-2\sqrt{b}\sqrt{d}\sqrt{b+ax}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+cx}}{\sqrt{d}\sqrt{b+ax}}\right)\right)\right)}{\sqrt{a}\sqrt{c}(b+ax)\sqrt{d+cx}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b/x]*Sqrt[c + d/x],x]
```

```
[Out] (Sqrt[a + b/x]*Sqrt[c + d/x]*x*((b*c + a*d)*Sqrt[b + a*x]*ArcTanh[(Sqrt[a]*
Sqrt[d + c*x])/(Sqrt[c]*Sqrt[b + a*x])] + Sqrt[a]*Sqrt[c]*((b + a*x)*Sqrt[d
 + c*x] - 2*Sqrt[b]*Sqrt[d]*Sqrt[b + a*x]*ArcTanh[(Sqrt[b]*Sqrt[d + c*x])/(
Sqrt[d]*Sqrt[b + a*x])])))/(Sqrt[a]*Sqrt[c]*(b + a*x)*Sqrt[d + c*x])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 217 vs. $2(95) = 190$.
time = 0.05, size = 218, normalized size = 1.77

method	result
default	$\frac{\sqrt{\frac{ax+b}{x}}\, x\sqrt{\frac{cx+d}{x}}\left(\ln\left(\frac{2acx+2\sqrt{(cx+d)(ax+b)}\,\sqrt{ac}+ad+bc}{2\sqrt{ac}}\right)\sqrt{bd}\, ad+\ln\left(\frac{2acx+2\sqrt{(cx+d)(ax+b)}}{2\sqrt{ac}}\right.\right.}{2\sqrt{(cx+d)}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d/x)^(1/2)*(a+1/x*b)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*((a*x+b)/x)^(1/2)*x*((c*x+d)/x)^(1/2)*(ln(1/2*(2*a*c*x+2*((c*x+d)*(a*x+
b))^(1/2)*(a*c)^(1/2)+a*d+b*c)/(a*c)^(1/2))*(b*d)^(1/2)*a*d+ln(1/2*(2*a*c*x
+2*((c*x+d)*(a*x+b))^(1/2)*(a*c)^(1/2)+a*d+b*c)/(a*c)^(1/2))*(b*d)^(1/2)*b*
c-2*b*d*ln((a*d*x+b*c*x+2*(b*d)^(1/2)*((c*x+d)*(a*x+b))^(1/2)+2*b*d)/x)*(a*
c)^(1/2)+2*((c*x+d)*(a*x+b))^(1/2)*(a*c)^(1/2)*(b*d)^(1/2))/((c*x+d)*(a*x+b
))^(1/2)/(a*c)^(1/2)/(b*d)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)^(1/2)*(a+b/x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a + b/x)*sqrt(c + d/x), x)
```

Fricas [A]
time = 4.41, size = 890, normalized size = 7.24

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)^(1/2)*(a+b/x)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(4*a*c*x*sqrt((a*x + b)/x)*sqrt((c*x + d)/x) + 2*sqrt(b*d)*a*c*log(-(8
*b^2*d^2 + (b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^2 - 4*(2*b*d*x + (b*c + a*d)*x
^2)*sqrt(b*d)*sqrt((a*x + b)/x)*sqrt((c*x + d)/x) + 8*(b^2*c*d + a*b*d^2)*x
)/x^2) + sqrt(a*c)*(b*c + a*d)*log(-8*a^2*c^2*x^2 - b^2*c^2 - 6*a*b*c*d - a
^2*d^2 - 4*(2*a*c*x^2 + (b*c + a*d)*x)*sqrt(a*c)*sqrt((a*x + b)/x)*sqrt((c*
x + d)/x) - 8*(a*b*c^2 + a^2*c*d)*x))/(a*c), 1/4*(4*a*c*x*sqrt((a*x + b)/x)
*sqrt((c*x + d)/x) + 4*sqrt(-b*d)*a*c*arctan(1/2*(2*b*d*x + (b*c + a*d)*x^2
)*sqrt(-b*d)*sqrt((a*x + b)/x)*sqrt((c*x + d)/x)/(a*b*c*d*x^2 + b^2*d^2 + (
b^2*c*d + a*b*d^2)*x)) + sqrt(a*c)*(b*c + a*d)*log(-8*a^2*c^2*x^2 - b^2*c^2
 - 6*a*b*c*d - a^2*d^2 - 4*(2*a*c*x^2 + (b*c + a*d)*x)*sqrt(a*c)*sqrt((a*x
```

```
+ b)/x)*sqrt((c*x + d)/x) - 8*(a*b*c^2 + a^2*c*d)*x))/(a*c), 1/2*(2*a*c*x*s
qrt((a*x + b)/x)*sqrt((c*x + d)/x) + sqrt(b*d)*a*c*log(-(8*b^2*d^2 + (b^2*c
^2 + 6*a*b*c*d + a^2*d^2)*x^2 - 4*(2*b*d*x + (b*c + a*d)*x^2)*sqrt(b*d)*sqr
t((a*x + b)/x)*sqrt((c*x + d)/x) + 8*(b^2*c*d + a*b*d^2)*x)/x^2) - sqrt(-a*
c)*(b*c + a*d)*arctan(2*sqrt(-a*c)*x*sqrt((a*x + b)/x)*sqrt((c*x + d)/x)/(2
*a*c*x + b*c + a*d)))/(a*c), 1/2*(2*a*c*x*sqrt((a*x + b)/x)*sqrt((c*x + d)/
x) + 2*sqrt(-b*d)*a*c*arctan(1/2*(2*b*d*x + (b*c + a*d)*x^2)*sqrt(-b*d)*sqr
t((a*x + b)/x)*sqrt((c*x + d)/x)/(a*b*c*d*x^2 + b^2*d^2 + (b^2*c*d + a*b*d^
2)*x)) - sqrt(-a*c)*(b*c + a*d)*arctan(2*sqrt(-a*c)*x*sqrt((a*x + b)/x)*sqr
t((c*x + d)/x)/(2*a*c*x + b*c + a*d)))/(a*c)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+\frac{b}{x}}\,\sqrt{c+\frac{d}{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)**(1/2)*(a+b/x)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b/x)*sqrt(c + d/x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)^(1/2)*(a+b/x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a + b/x)*sqrt(c + d/x), x)
```

Mupad [B]
time = 22.22, size = 2500, normalized size = 20.33

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x)^(1/2)*(c + d/x)^(1/2),x)
```

```
[Out] atan(((b*d)^(1/2)*(2*(b*d)^(1/2)*(2*(b*d)^(1/2)*(2*(b*d)^(1/2)*((2*(4*a^(9/
2)*b^9*c^(19/2) - 4*a^(13/2)*b^7*c^(15/2)*d^2 - 4*a^(15/2)*b^6*c^(13/2)*d^3
 + 4*a^(19/2)*b^4*c^(9/2)*d^5))/(a^7*c^7*d^9) - (((a + b/x)^(1/2) - a^(1/2)
)*(32*a^4*b^9*c^10 - 120*a^5*b^8*c^9*d + 288*a^6*b^7*c^8*d^2 - 400*a^7*b^6*
c^7*d^3 + 288*a^8*b^5*c^6*d^4 - 120*a^9*b^4*c^5*d^5 + 32*a^10*b^3*c^4*d^6))
/(2*a^7*c^7*d^9*((c + d/x)^(1/2) - c^(1/2)))) - (2*(8*a^5*b^9*c^9*d + 16*a^
```

6*b^8*c^8*d^2 - 48*a^7*b^7*c^7*d^3 + 16*a^8*b^6*c^6*d^4 + 8*a^9*b^5*c^5*d^5
))/(a^7*c^7*d^9) + (((a + b/x)^(1/2) - a^(1/2))*(16*a^(7/2)*b^10*c^(21/2) -
 76*a^(9/2)*b^9*c^(19/2)*d + 228*a^(11/2)*b^8*c^(17/2)*d^2 - 168*a^(13/2)*b
^7*c^(15/2)*d^3 - 168*a^(15/2)*b^6*c^(13/2)*d^4 + 228*a^(17/2)*b^5*c^(11/2)
*d^5 - 76*a^(19/2)*b^4*c^(9/2)*d^6 + 16*a^(21/2)*b^3*c^(7/2)*d^7))/(2*a^7*c
^7*d^9*((c + d/x)^(1/2) - c^(1/2)))) - (2*(a^(7/2)*b^11*c^(21/2) + 16*a^(9/
2)*b^10*c^(19/2)*d - 42*a^(11/2)*b^9*c^(17/2)*d^2 + 25*a^(13/2)*b^8*c^(15/2
)*d^3 + 25*a^(15/2)*b^7*c^(13/2)*d^4 - 42*a^(17/2)*b^6*c^(11/2)*d^5 + 16*a^
(19/2)*b^5*c^(9/2)*d^6 + a^(21/2)*b^4*c^(7/2)*d^7))/(a^7*c^7*d^9) + (((a +
b/x)^(1/2) - a^(1/2))*(146*a^4*b^10*c^10*d - 556*a^5*b^9*c^9*d^2 + 1006*a^6
*b^8*c^8*d^3 - 1192*a^7*b^7*c^7*d^4 + 1006*a^8*b^6*c^6*d^5 - 556*a^9*b^5*c^
5*d^6 + 146*a^10*b^4*c^4*d^7))/(2*a^7*c^7*d^9*((c + d/x)^(1/2) - c^(1/2))))
 + (2*(2*a^4*b^11*c^10*d + 8*a^5*b^10*c^9*d^2 - 2*a^6*b^9*c^8*d^3 - 16*a^7*
b^8*c^7*d^4 - 2*a^8*b^7*c^6*d^5 + 8*a^9*b^6*c^5*d^6 + 2*a^10*b^5*c^4*d^7))/
(a^7*c^7*d^9) - (((a + b/x)^(1/2) - a^(1/2))*(65*a^(7/2)*b^11*c^(21/2)*d -
297*a^(9/2)*b^10*c^(19/2)*d^2 + 597*a^(11/2)*b^9*c^(17/2)*d^3 - 365*a^(13/2
)*b^8*c^(15/2)*d^4 - 365*a^(15/2)*b^7*c^(13/2)*d^5 + 597*a^(17/2)*b^6*c^(11
/2)*d^6 - 297*a^(19/2)*b^5*c^(9/2)*d^7 + 65*a^(21/2)*b^4*c^(7/2)*d^8))/(2*a
^7*c^7*d^9*((c + d/x)^(1/2) - c^(1/2))))*1i - (b*d)^(1/2)*(2*(b*d)^(1/2)*(2
*(b*d)^(1/2)*(2*(b*d)^(1/2)*((2*(4*a^(9/2)*b^9*c^(19/2) - 4*a^(13/2)*b^7*c^
(15/2)*d^2 - 4*a^(15/2)*b^6*c^(13/2)*d^3 + 4*a^(19/2)*b^4*c^(9/2)*d^5))/(a^
7*c^7*d^9) - (((a + b/x)^(1/2) - a^(1/2))*(32*a^4*b^9*c^10 - 120*a^5*b^8*c^
9*d + 288*a^6*b^7*c^8*d^2 - 400*a^7*b^6*c^7*d^3 + 288*a^8*b^5*c^6*d^4 - 120
*a^9*b^4*c^5*d^5 + 32*a^10*b^3*c^4*d^6))/(2*a^7*c^7*d^9*((c + d/x)^(1/2) -
c^(1/2)))) + (2*(8*a^5*b^9*c^9*d + 16*a^6*b^8*c^8*d^2 - 48*a^7*b^7*c^7*d^3
+ 16*a^8*b^6*c^6*d^4 + 8*a^9*b^5*c^5*d^5))/(a^7*c^7*d^9) - (((a + b/x)^(1/2
) - a^(1/2))*(16*a^(7/2)*b^10*c^(21/2) - 76*a^(9/2)*b^9*c^(19/2)*d + 228*a^
(11/2)*b^8*c^(17/2)*d^2 - 168*a^(13/2)*b^7*c^(15/2)*d^3 - 168*a^(15/2)*b^6*
c^(13/2)*d^4 + 228*a^(17/2)*b^5*c^(11/2)*d^5 - 76*a^(19/2)*b^4*c^(9/2)*d^6
+ 16*a^(21/2)*b^3*c^(7/2)*d^7))/(2*a^7*c^7*d^9*((c + d/x)^(1/2) - c^(1/2)))
) - (2*(a^(7/2)*b^11*c^(21/2) + 16*a^(9/2)*b^10*c^(19/2)*d - 42*a^(11/2)*b^
9*c^(17/2)*d^2 + 25*a^(13/2)*b^8*c^(15/2)*d^3 + 25*a^(15/2)*b^7*c^(13/2)*d^
4 - 42*a^(17/2)*b^6*c^(11/2)*d^5 + 16*a^(19/2)*b^5*c^(9/2)*d^6 + a^(21/2)*b
^4*c^(7/2)*d^7))/(a^7*c^7*d^9) + (((a + b/x)^(1/2) - a^(1/2))*(146*a^4*b^10
*c^10*d - 556*a^5*b^9*c^9*d^2 + 1006*a^6*b^8*c^8*d^3 - 1192*a^7*b^7*c^7*d^4
 + 1006*a^8*b^6*c^6*d^5 - 556*a^9*b^5*c^5*d^6 + 146*a^10*b^4*c^4*d^7))/(2*a
^7*c^7*d^9*((c + d/x)^(1/2) - c^(1/2)))) - (2*(2*a^4*b^11*c^10*d + 8*a^5*b^
10*c^9*d^2 - 2*a^6*b^9*c^8*d^3 - 16*a^7*b^8*c^7*d^4 - 2*a^8*b^7*c^6*d^5 + 8
*a^9*b^6*c^5*d^6 + 2*a^10*b^5*c^4*d^7))/(a^7*c^7*d^9) + (((a + b/x)^(1/2) -
 a^(1/2))*(65*a^(7/2)*b^11*c^(21/2)*d - 297*a^(9/2)*b^10*c^(19/2)*d^2 + 597
*a^(11/2)*b^9*c^(17/2)*d^3 - 365*a^(13/2)*b^8*c^(15/2)*d^4 - 365*a^(15/2)*b
^7*c^(13/2)*d^5 + 597*a^(17/2)*b^6*c^(11/2)*d^6 - 297*a^(19/2)*b^5*c^(9/2)*
d^7 + 65*a^(21/2)*b^4*c^(7/2)*d^8))/(2*a^7*c^7*d^9*((c + d/x)^(1/2) - c^(1/
2))))*1i)/((b*d)^(1/2)*(2*(b*d)^(1/2)*(2*(b*d)^(1/2)*(2*(b*d)^(1/2)*((2*(4*
a^(9/2)*b^9*c^(19/2) - 4*a^(13/2)*b^7*c^(15/2)*d^2 - 4*a^(15/2)*b^6*c^(13/2

)*d^3 + 4*a^(19/2)*b^4*c^(9/2)*d^5))/(a^7*c^7*d^9) - (((a + b/x)^(1/2) - a^(1/2))*(32*a^4*b^9*c^10 - 120*a^5*b^8*c^9*d + 288*a^6*b^7*c^8*d^2 - 400*a^7*b^6*c^7*d^3 + 288*a^8*b^5*c^6*d^4 - 120*a^9*b^4*c^5*d^5 + 32*a^10*b^3*c^4*d^6))/(2*a^7*c^7*d^9*((c + d/x)^(1/2) - c^(1/2)))) - (2*(8*a^5*b^9*c^9*d + 16*a^6*b^8*c^8*d^2 - 48*a^7*b^7*c^7*d^3 + 16*a^8*b^6*c^6*d^4 + 8*a^9*b^5*c^5*d^5))/(a^7*c^7*d^9) + (((a + b/x)^(1/2) - a^(1/2))*(16*a^(7/2)*b^10*c^(21/2) - 76*a^(9/2)*b^9*c^(19/2)*d + 228*a^(11/2)*b^8*c^(17/2)*d^2 - 168*a^(13/2)*b^7*c^(15/2)*d^3 - 168*a^(15/2)*b^6*c^(13/2)*d^4 + 228*a^(17/2)*b^5*c^(11/2)*d^5 - 76*a^(19/2)*b^4*c^(9/2)*d^6 + 16*a^(21/2)*b^3*c^(7/2)*d^7))/(2*a^7*c^7*d^9*((c + d/x)^(1/2) - c^(1/2)))) - (2*(a^(7/2)*b^11*c^(21/2) + 16*a^(9/2)*b^10*c^(19/2)*d - 42*a^(11/2)*b^9*c^(17/2)*d^2 + 25*a^(13/2)*b^8*c^(15/2)*d^3 + 25*a^(15/2)*b^7*c^(13/2)*d^4 - 42*a^(17/2)*b^6*c^(11/2)*d^5 + 16*a^(19/2)*b^5*c^(9/2)*d^6 + a^(21/2)*b^4*c^(7/2)*d^7))/(a^7*c^7*d^9) + (((a + b/x)^(1/2) - a^(1/2))*(146*a^4*b^10*c^10*d - 556*a^5*b^9*c^9*d^2 + 1006*a^6*b^8*c^8*d^3 - 1192*a^7*b^7*c^7*d^4 + 1006*a^8*b^6*c^6*d^5 - 556*a^9*b^5*c^5*d^6 + 146*a^10*b^4*c^4*d^7))/(2*a^7*c^7*...

3.267 $\int \frac{\sqrt{a+\frac{b}{x}}}{\sqrt{c+\frac{d}{x}}}\,dx$

Optimal. Leaf size=81

$$\frac{\sqrt{a+\frac{b}{x}}\,\sqrt{c+\frac{d}{x}}\,x}{c}+\frac{(bc-ad)\tanh^{-1}\left(\frac{\sqrt{c}\,\sqrt{a+\frac{b}{x}}}{\sqrt{a}\,\sqrt{c+\frac{d}{x}}}\right)}{\sqrt{a}\,c^{3/2}}$$

```
[Out] (-a*d+b*c)*arctanh(c^(1/2)*(a+b/x)^(1/2)/a^(1/2)/(c+d/x)^(1/2))/c^(3/2)/a^(
1/2)+x*(a+b/x)^(1/2)*(c+d/x)^(1/2)/c
```

Rubi [A]
time = 0.07, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {382, 96, 95, 214}

$$\frac{(bc-ad)\tanh^{-1}\left(\frac{\sqrt{c}\,\sqrt{a+\frac{b}{x}}}{\sqrt{a}\,\sqrt{c+\frac{d}{x}}}\right)}{\sqrt{a}\,c^{3/2}}+\frac{x\sqrt{a+\frac{b}{x}}\,\sqrt{c+\frac{d}{x}}}{c}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b/x]/Sqrt[c + d/x],x]
```

```
[Out] (Sqrt[a + b/x]*Sqrt[c + d/x]*x)/c + ((b*c - a*d)*ArcTanh[(Sqrt[c]*Sqrt[a +
b/x])/(Sqrt[a]*Sqrt[c + d/x])])/(Sqrt[a]*c^(3/2))
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
```

```
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+\frac{b}{x}}}{\sqrt{c+\frac{d}{x}}}\,dx = -\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x^2\sqrt{c+dx}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{\sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}\,x}{c} - \frac{(bc-ad)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}\sqrt{c+dx}}\,dx, x, \frac{1}{x}\right)}{2c}$$

$$= \frac{\sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}\,x}{c} - \frac{(bc-ad)\text{Subst}\left(\int \frac{1}{-a+cx^2}\,dx, x, \frac{\sqrt{a+\frac{b}{x}}}{\sqrt{c+\frac{d}{x}}}\right)}{c}$$

$$= \frac{\sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}\,x}{c} + \frac{(bc-ad)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+\frac{b}{x}}}{\sqrt{a}\sqrt{c+\frac{d}{x}}}\right)}{\sqrt{a}\,c^{3/2}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 179 vs. 2(81) = 162.

time = 0.40, size = 179, normalized size = 2.21

$$\frac{\sqrt{a+\frac{b}{x}}\sqrt{d+cx}\left(\sqrt{b+ax}-\sqrt{\frac{a}{c}}\sqrt{d+cx}\right)\left(\sqrt{\frac{a}{c}}\,c\sqrt{b+ax}\sqrt{d+cx}+(-bc+ad)\log\left(\sqrt{b+ax}-\sqrt{\frac{a}{c}}\sqrt{d+cx}\right)\right)}{c\sqrt{c+\frac{d}{x}}\sqrt{b+ax}\left(\sqrt{\frac{a}{c}}\,c\sqrt{b+ax}-a\sqrt{d+cx}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b/x]/Sqrt[c + d/x],x]
```

```
[Out] (Sqrt[a + b/x]*Sqrt[d + c*x]*(Sqrt[b + a*x] - Sqrt[a/c]*Sqrt[d + c*x])*(Sqr
t[a/c]*c*Sqrt[b + a*x]*Sqrt[d + c*x] + (-(b*c) + a*d)*Log[Sqrt[b + a*x] - S
qrt[a/c]*Sqrt[d + c*x]]))/(c*Sqrt[c + d/x]*Sqrt[b + a*x]*(Sqrt[a/c]*c*Sqrt[
b + a*x] - a*Sqrt[d + c*x]))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 154 vs. $2(65) = 130$.
time = 0.03, size = 155, normalized size = 1.91

method	result
default	$\frac{\sqrt{\frac{ax+b}{x}}\, x \sqrt{\frac{cx+d}{x}} \left(-\ln\left(\frac{2acx+2\sqrt{(cx+d)(ax+b)}\,\sqrt{ac}+ad+bc}{2\sqrt{ac}}\right)ad+\ln\left(\frac{2acx+2\sqrt{(cx+d)(ax+b)}\,\sqrt{a}}{2\sqrt{ac}}\right.\right.}{2\sqrt{(cx+d)(ax+b)}\, c\sqrt{ac}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+1/x*b)^(1/2)/(c+d/x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*((a*x+b)/x)^(1/2)*x*((c*x+d)/x)^(1/2)*(-ln(1/2*(2*a*c*x+2*((c*x+d)*(a*x
+b))^(1/2)*(a*c)^(1/2)+a*d+b*c)/(a*c)^(1/2))*a*d+ln(1/2*(2*a*c*x+2*((c*x+d)
*(a*x+b))^(1/2)*(a*c)^(1/2)+a*d+b*c)/(a*c)^(1/2))*b*c+2*((c*x+d)*(a*x+b))^(
1/2)*(a*c)^(1/2))/((c*x+d)*(a*x+b))^(1/2)/c/(a*c)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(1/2)/(c+d/x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a + b/x)/sqrt(c + d/x), x)
```

Fricas [A]
time = 3.15, size = 247, normalized size = 3.05

$$\left[\frac{4\,acx\sqrt{\frac{ax+b}{x}}\sqrt{\frac{cx+d}{x}} - \sqrt{ac}\,(bc-ad)\log\left(-8\,a^2c^2x^2 - b^2c^2 - 6\,abcd - a^2d^2 + 4\,(2\,acx^2 + (bc+ad)x)\sqrt{ac}\sqrt{\frac{ax+b}{x}}\sqrt{\frac{cx+d}{x}} - 8\,(abc^2 + a^2cd)x\right)}{4\,ac^2}, \frac{2\,acx\sqrt{\frac{ax+b}{x}}\sqrt{\frac{cx+d}{x}} - \sqrt{-ac}\,(bc-ad)\arctan\left(\frac{2\sqrt{-ac}\,x\sqrt{\frac{ax+b}{x}}\sqrt{\frac{cx+d}{x}}}{2\,acx+bc+ad}\right)}{2\,ac^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(1/2)/(c+d/x)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(4*a*c*x*sqrt((a*x + b)/x)*sqrt((c*x + d)/x) - sqrt(a*c)*(b*c - a*d)*l
og(-8*a^2*c^2*x^2 - b^2*c^2 - 6*a*b*c*d - a^2*d^2 + 4*(2*a*c*x^2 + (b*c + a
*d)*x)*sqrt(a*c)*sqrt((a*x + b)/x)*sqrt((c*x + d)/x) - 8*(a*b*c^2 + a^2*c*d
)*x))/(a*c^2), 1/2*(2*a*c*x*sqrt((a*x + b)/x)*sqrt((c*x + d)/x) - sqrt(-a*c
)*(b*c - a*d)*arctan(2*sqrt(-a*c)*x*sqrt((a*x + b)/x)*sqrt((c*x + d)/x)/(2*
a*c*x + b*c + a*d)))/(a*c^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+\frac{b}{x}}}{\sqrt{c+\frac{d}{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**(1/2)/(c+d/x)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b/x)/sqrt(c + d/x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(1/2)/(c+d/x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a + b/x)/sqrt(c + d/x), x)
```

Mupad [B]
time = 6.58, size = 478, normalized size = 5.90

$$\frac{d\left(\sqrt{a+\frac{b}{x}}-\sqrt{a}\right)}{4\,c\left(\sqrt{c+\frac{d}{x}}-\sqrt{c}\right)}-\frac{\frac{\left(\sqrt{a+\frac{b}{x}}-\sqrt{a}\right)\left(\frac{c\,b^2}{4}+\frac{a\,d\,b}{4}\right)}{\sqrt{a}\,c^{3/2}\,d\left(\sqrt{c+\frac{d}{x}}-\sqrt{c}\right)}-\frac{b^2}{4\,c\,d}+\frac{{\left(\sqrt{a+\frac{b}{x}}-\sqrt{a}\right)}^2\left(\frac{a^2\,d^2}{4}-\frac{3\,a\,b\,c\,d}{4}+\frac{b^2\,c^2}{4}\right)}{a\,c^2\,d\,{\left(\sqrt{c+\frac{d}{x}}-\sqrt{c}\right)}^2}}{\frac{{\left(\sqrt{a+\frac{b}{x}}-\sqrt{a}\right)}^3}{{\left(\sqrt{c+\frac{d}{x}}-\sqrt{c}\right)}^3}+\frac{b\left(\sqrt{a+\frac{b}{x}}-\sqrt{a}\right)}{d\left(\sqrt{c+\frac{d}{x}}-\sqrt{c}\right)}-\frac{{\left(\sqrt{a+\frac{b}{x}}-\sqrt{a}\right)}^2\,\left(a\,d+b\,c\right)}{\sqrt{a}\,\sqrt{c}\,d\,{\left(\sqrt{c+\frac{d}{x}}-\sqrt{c}\right)}^2}}+\frac{\ln\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{c+\frac{d}{x}}-\sqrt{c}}\right)\left(\sqrt{a}\,b\,c^{3/2}-a^{3/2}\,\sqrt{c}\,d\right)}{2\,a\,c^2}-\frac{\ln\left(\frac{\left(\sqrt{c}\,\sqrt{a+\frac{b}{x}}-\sqrt{a}\,\sqrt{c+\frac{d}{x}}\right)\left(b\,\sqrt{c}-\frac{\sqrt{a}\,d\left(\sqrt{a+\frac{b}{x}}-\sqrt{a}\right)}{\sqrt{c+\frac{d}{x}}-\sqrt{c}}\right)}{\sqrt{c+\frac{d}{x}}-\sqrt{c}}\right)\left(\sqrt{a}\,b\,c^{3/2}-a^{3/2}\,\sqrt{c}\,d\right)}{2\,a\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x)^(1/2)/(c + d/x)^(1/2),x)
```

```
[Out] (d*((a + b/x)^(1/2) - a^(1/2)))/(4*c*((c + d/x)^(1/2) - c^(1/2))) - ((((a +
 b/x)^(1/2) - a^(1/2))*((b^2*c)/4 + (a*b*d)/4))/(a^(1/2)*c^(3/2)*d*((c + d/
x)^(1/2) - c^(1/2))) - b^2/(4*c*d) + (((a + b/x)^(1/2) - a^(1/2))^2*((a^2*d
```

```
^2)/4 + (b^2*c^2)/4 - (3*a*b*c*d)/4))/(a*c^2*d*((c + d/x)^(1/2) - c^(1/2))^
2))/(((a + b/x)^(1/2) - a^(1/2))^3/((c + d/x)^(1/2) - c^(1/2))^3 + (b*((a +
 b/x)^(1/2) - a^(1/2)))/(d*((c + d/x)^(1/2) - c^(1/2))) - (((a + b/x)^(1/2)
 - a^(1/2))^2*(a*d + b*c))/(a^(1/2)*c^(1/2)*d*((c + d/x)^(1/2) - c^(1/2))^2
)) + (log(((a + b/x)^(1/2) - a^(1/2))/((c + d/x)^(1/2) - c^(1/2)))*(a^(1/2)
*b*c^(3/2) - a^(3/2)*c^(1/2)*d))/(2*a*c^2) - (log(((c^(1/2)*(a + b/x)^(1/2)
 - a^(1/2)*(c + d/x)^(1/2))*(b*c^(1/2) - (a^(1/2)*d*((a + b/x)^(1/2) - a^(1
/2)))/((c + d/x)^(1/2) - c^(1/2))))/((c + d/x)^(1/2) - c^(1/2)))*(a^(1/2)*b
*c^(3/2) - a^(3/2)*c^(1/2)*d))/(2*a*c^2)
```

3.268 $\int \frac{\sqrt{a+\frac{b}{x}}}{\left(c+\frac{d}{x}\right)^{3/2}}\,dx$

Optimal. Leaf size=122

$$-\frac{(bc-3ad)\sqrt{a+\frac{b}{x}}}{ac^2\sqrt{c+\frac{d}{x}}}+\frac{\left(a+\frac{b}{x}\right)^{3/2}x}{ac\sqrt{c+\frac{d}{x}}}+\frac{(bc-3ad)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+\frac{b}{x}}}{\sqrt{a}\sqrt{c+\frac{d}{x}}}\right)}{\sqrt{a}\,c^{5/2}}$$

```
[Out] (-3*a*d+b*c)*arctanh(c^(1/2)*(a+b/x)^(1/2)/a^(1/2)/(c+d/x)^(1/2))/c^(5/2)/a
^(1/2)+(a+b/x)^(3/2)*x/a/c/(c+d/x)^(1/2)-(-3*a*d+b*c)*(a+b/x)^(1/2)/a/c^2/(
c+d/x)^(1/2)
```

Rubi [A]
time = 0.17, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {382, 98, 96, 95, 214}

$$\frac{(bc-3ad)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+\frac{b}{x}}}{\sqrt{a}\sqrt{c+\frac{d}{x}}}\right)}{\sqrt{a}\,c^{5/2}}-\frac{\sqrt{a+\frac{b}{x}}\,(bc-3ad)}{ac^2\sqrt{c+\frac{d}{x}}}+\frac{x\left(a+\frac{b}{x}\right)^{3/2}}{ac\sqrt{c+\frac{d}{x}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b/x]/(c + d/x)^(3/2),x]
```

```
[Out] -(((b*c - 3*a*d)*Sqrt[a + b/x])/(a*c^2*Sqrt[c + d/x])) + ((a + b/x)^(3/2)*x
)/(a*c*Sqrt[c + d/x]) + ((b*c - 3*a*d)*ArcTanh[(Sqrt[c]*Sqrt[a + b/x])/(Sqr
t[a]*Sqrt[c + d/x])])/(Sqrt[a]*c^(5/2))
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+\frac{b}{x}}}{\left(c+\frac{d}{x}\right)^{3/2}}\,dx = -\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x^2(c+dx)^{3/2}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{\left(a+\frac{b}{x}\right)^{3/2}x}{ac\sqrt{c+\frac{d}{x}}} + \frac{\left(-\frac{bc}{2}+\frac{3ad}{2}\right)\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x(c+dx)^{3/2}}\,dx, x, \frac{1}{x}\right)}{ac}$$

$$= -\frac{(bc-3ad)\sqrt{a+\frac{b}{x}}}{ac^2\sqrt{c+\frac{d}{x}}} + \frac{\left(a+\frac{b}{x}\right)^{3/2}x}{ac\sqrt{c+\frac{d}{x}}} - \frac{(bc-3ad)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}\sqrt{c+dx}}\,dx, x, \frac{1}{x}\right)}{2c^2}$$

$$= -\frac{(bc-3ad)\sqrt{a+\frac{b}{x}}}{ac^2\sqrt{c+\frac{d}{x}}} + \frac{\left(a+\frac{b}{x}\right)^{3/2}x}{ac\sqrt{c+\frac{d}{x}}} - \frac{(bc-3ad)\text{Subst}\left(\int \frac{1}{-a+cx^2}\,dx, x, \frac{\sqrt{a+\frac{b}{x}}}{\sqrt{c+\frac{d}{x}}}\right)}{c^2}$$

$$= -\frac{(bc-3ad)\sqrt{a+\frac{b}{x}}}{ac^2\sqrt{c+\frac{d}{x}}} + \frac{\left(a+\frac{b}{x}\right)^{3/2}x}{ac\sqrt{c+\frac{d}{x}}} + \frac{(bc-3ad)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+\frac{b}{x}}}{\sqrt{a}\sqrt{c+\frac{d}{x}}}\right)}{\sqrt{a}\,c^{5/2}}$$

Mathematica [A]
time = 0.31, size = 126, normalized size = 1.03

$$\frac{\sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}\,x\left(\sqrt{a}\sqrt{c}\sqrt{b+ax}\,(3d+cx)+(bc-3ad)\sqrt{d+cx}\,\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{b+ax}}{\sqrt{a}\sqrt{d+cx}}\right)\right)}{\sqrt{a}\,c^{5/2}\sqrt{b+ax}\,(d+cx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b/x]/(c + d/x)^(3/2),x]
[Out] (Sqrt[a + b/x]*Sqrt[c + d/x]*x*(Sqrt[a]*Sqrt[c]*Sqrt[b + a*x]*(3*d + c*x) +
 (b*c - 3*a*d)*Sqrt[d + c*x]*ArcTanh[(Sqrt[c]*Sqrt[b + a*x])/(Sqrt[a]*Sqrt[
d + c*x])]))/(Sqrt[a]*c^(5/2)*Sqrt[b + a*x]*(d + c*x))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 279 vs. 2(102) = 204.
time = 0.04, size = 280, normalized size = 2.30

method	result
default	$\frac{\sqrt{\frac{ax+b}{x}}\, x\sqrt{\frac{cx+d}{x}}\left(-3\ln\left(\frac{2acx+2\sqrt{(cx+d)(ax+b)}\,\sqrt{ac}+ad+bc}{2\sqrt{ac}}\right)acdx+\ln\left(\frac{2acx+2\sqrt{(cx+d)(ax+b)}}{2\sqrt{ac}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+1/x*b)^(1/2)/(c+d/x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*((a*x+b)/x)^(1/2)*x*((c*x+d)/x)^(1/2)*(-3*ln(1/2*(2*a*c*x+2*((c*x+d)*(a
*x+b))^(1/2)*(a*c)^(1/2)+a*d+b*c)/(a*c)^(1/2))*a*c*d*x+ln(1/2*(2*a*c*x+2*((
c*x+d)*(a*x+b))^(1/2)*(a*c)^(1/2)+a*d+b*c)/(a*c)^(1/2))*b*c^2*x+2*c*x*((c*x
+d)*(a*x+b))^(1/2)*(a*c)^(1/2)-3*ln(1/2*(2*a*c*x+2*((c*x+d)*(a*x+b))^(1/2)*
(a*c)^(1/2)+a*d+b*c)/(a*c)^(1/2))*a*d^2+ln(1/2*(2*a*c*x+2*((c*x+d)*(a*x+b))
^(1/2)*(a*c)^(1/2)+a*d+b*c)/(a*c)^(1/2))*b*c*d+6*d*((c*x+d)*(a*x+b))^(1/2)*
(a*c)^(1/2))/(a*c)^(1/2)/(c*x+d)/((c*x+d)*(a*x+b))^(1/2)/c^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(1/2)/(c+d/x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a + b/x)/(c + d/x)^(3/2), x)
```

Fricas [A]
time = 2.76, size = 319, normalized size = 2.61

$$\left[-\frac{(bcd-3\,ad^2+(bc^2-3\,acd)x)\sqrt{ac}\,\log\left(-8\,a^2c^2x^2-b^2c^2-6\,abcd-a^2d^2+4\,(2\,acx^2+(bc+ad)x)\sqrt{ac}\,\sqrt{\frac{ax+b}{x}}\,\sqrt{\frac{cx+d}{x}}-8\,(abc^2+a^2cd)x\right)-4\,(ac^2x^2+3\,acdx)\sqrt{\frac{ax+b}{x}}\,\sqrt{\frac{cx+d}{x}}}{4\,(ac^4x+ac^3d)},\ -\frac{(bcd-3\,ad^2+(bc^2-3\,acd)x)\sqrt{-ac}\,\arctan\left(\frac{2\sqrt{-ac}\,x\sqrt{\frac{ax+b}{x}}\,\sqrt{\frac{cx+d}{x}}}{2\,acx+bc+ad}\right)-2\,(ac^2x^2+3\,acdx)\sqrt{\frac{ax+b}{x}}\,\sqrt{\frac{cx+d}{x}}}{2\,(ac^4x+ac^3d)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(1/2)/(c+d/x)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*((b*c*d - 3*a*d^2 + (b*c^2 - 3*a*c*d)*x)*sqrt(a*c)*log(-8*a^2*c^2*x^2
 - b^2*c^2 - 6*a*b*c*d - a^2*d^2 + 4*(2*a*c*x^2 + (b*c + a*d)*x)*sqrt(a*c)*
sqrt((a*x + b)/x)*sqrt((c*x + d)/x) - 8*(a*b*c^2 + a^2*c*d)*x) - 4*(a*c^2*x
^2 + 3*a*c*d*x)*sqrt((a*x + b)/x)*sqrt((c*x + d)/x))/(a*c^4*x + a*c^3*d), -
1/2*((b*c*d - 3*a*d^2 + (b*c^2 - 3*a*c*d)*x)*sqrt(-a*c)*arctan(2*sqrt(-a*c)
*x*sqrt((a*x + b)/x)*sqrt((c*x + d)/x)/(2*a*c*x + b*c + a*d)) - 2*(a*c^2*x^
2 + 3*a*c*d*x)*sqrt((a*x + b)/x)*sqrt((c*x + d)/x))/(a*c^4*x + a*c^3*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+\frac{b}{x}}}{\left(c+\frac{d}{x}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**(1/2)/(c+d/x)**(3/2),x)
[Out] Integral(sqrt(a + b/x)/(c + d/x)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(1/2)/(c+d/x)^(3/2),x, algorithm="giac")
[Out] integrate(sqrt(a + b/x)/(c + d/x)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a+\frac{b}{x}}}{{\left(c+\frac{d}{x}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x)^(1/2)/(c + d/x)^(3/2),x)
[Out] int((a + b/x)^(1/2)/(c + d/x)^(3/2), x)
```

3.269 $\int \left(a+\frac{b}{x}\right)^p \left(c+\frac{d}{x}\right)^q dx$

Optimal. Leaf size=96

$$-\frac{b\left(a+\frac{b}{x}\right)^{1+p}\left(c+\frac{d}{x}\right)^q\left(\frac{b\left(c+\frac{d}{x}\right)}{bc-ad}\right)^{-q}F_1\left(1+p;-q,2;2+p;-\frac{d\left(a+\frac{b}{x}\right)}{bc-ad},\frac{a+\frac{b}{x}}{a}\right)}{a^2(1+p)}$$

```
[Out] -b*(a+b/x)^(1+p)*(c+d/x)^q*AppellF1(1+p,2,-q,2+p,(a+b/x)/a,-d*(a+b/x)/(-a*d
+b*c))/a^2/(1+p)/((b*(c+d/x)/(-a*d+b*c))^q)
```

Rubi [A]
time = 0.05, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {382, 142, 141}

$$-\frac{b\left(a+\frac{b}{x}\right)^{p+1}\left(c+\frac{d}{x}\right)^q\left(\frac{b\left(c+\frac{d}{x}\right)}{bc-ad}\right)^{-q}F_1\left(p+1;-q,2;p+2;-\frac{d\left(a+\frac{b}{x}\right)}{bc-ad},\frac{a+\frac{b}{x}}{a}\right)}{a^2(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^p*(c + d/x)^q,x]
```

```
[Out] -((b*(a + b/x)^(1 + p)*(c + d/x)^q*AppellF1[1 + p, -q, 2, 2 + p, -((d*(a +
b/x))/(b*c - a*d)), (a + b/x)/a])/(a^2*(1 + p)*((b*(c + d/x))/(b*c - a*d))^
q))
```

Rule 141

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1
)*(b/(b*c - a*d))^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c -
 a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 142

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
(b*((c + d*x)/(b*c - a*d)))^FracPart[n]), Int[(a + b*x)^m*(b*(c/(b*c - a*d)
) + b*d*(x/(b*c - a*d)))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] &&  !GtQ[b/(b*c
- a*d), 0] &&  !SimplerQ[c + d*x, a + b*x]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int \left(a+\frac{b}{x}\right)^p \left(c+\frac{d}{x}\right)^q dx &= -\text{Subst}\left(\int \frac{(a+bx)^p(c+dx)^q}{x^2}\,dx, x, \frac{1}{x}\right) \\
&= -\left(\left(\left(c+\frac{d}{x}\right)^q \left(\frac{b\left(c+\frac{d}{x}\right)}{bc-ad}\right)^{-q}\right)\text{Subst}\left(\int \frac{(a+bx)^p\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^q}{x^2}\,dx, x, \frac{1}{x}\right.\right. \\
&= -\frac{b\left(a+\frac{b}{x}\right)^{1+p}\left(c+\frac{d}{x}\right)^q\left(\frac{b\left(c+\frac{d}{x}\right)}{bc-ad}\right)^{-q} F_1\left(1+p;-q,2;2+p;-\frac{d\left(a+\frac{b}{x}\right)}{bc-ad},\frac{a+\frac{b}{x}}{a}\right)}{a^2(1+p)}
\end{aligned}
$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 206 vs. 2(96) = 192.
time = 0.30, size = 206, normalized size = 2.15

$$
\frac{bd(-2+p+q)\left(a+\frac{b}{x}\right)^p\left(c+\frac{d}{x}\right)^q xF_1\left(1-p-q;-p,-q;2-p-q;-\frac{ax}{b},-\frac{cx}{d}\right)}{(-1+p+q)\left(-bd(-2+p+q)F_1\left(1-p-q;-p,-q;2-p-q;-\frac{ax}{b},-\frac{cx}{d}\right)+x\left(adpF_1\left(2-p-q;1-p,-q;3-p-q;-\frac{ax}{b},-\frac{cx}{d}\right)+bcqF_1\left(2-p-q;-p,1-q;3-p-q;-\frac{ax}{b},-\frac{cx}{d}\right)\right)\right)}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b/x)^p*(c + d/x)^q,x]
```

```
[Out] (b*d*(-2 + p + q)*(a + b/x)^p*(c + d/x)^q*x*AppellF1[1 - p - q, -p, -q, 2 -
 p - q, -((a*x)/b), -((c*x)/d)])/((-1 + p + q)*(-(b*d*(-2 + p + q)*AppellF1
[1 - p - q, -p, -q, 2 - p - q, -((a*x)/b), -((c*x)/d)]) + x*(a*d*p*AppellF1
[2 - p - q, 1 - p, -q, 3 - p - q, -((a*x)/b), -((c*x)/d)] + b*c*q*AppellF1[
2 - p - q, -p, 1 - q, 3 - p - q, -((a*x)/b), -((c*x)/d)])))
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$
\int \left(a+\frac{b}{x}\right)^p \left(c+\frac{d}{x}\right)^q dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+1/x*b)^p*(c+d/x)^q,x)
```

```
[Out] int((a+1/x*b)^p*(c+d/x)^q,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^p*(c+d/x)^q,x, algorithm="maxima")
[Out] integrate((a + b/x)^p*(c + d/x)^q, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^p*(c+d/x)^q,x, algorithm="fricas")
[Out] integral(((a*x + b)/x)^p*((c*x + d)/x)^q, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + \frac{b}{x}\right)^p \left(c + \frac{d}{x}\right)^q dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**p*(c+d/x)**q,x)
[Out] Integral((a + b/x)**p*(c + d/x)**q, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^p*(c+d/x)^q,x, algorithm="giac")
[Out] integrate((a + b/x)^p*(c + d/x)^q, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a + \frac{b}{x}\right)^p \left(c + \frac{d}{x}\right)^q dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x)^p*(c + d/x)^q,x)
[Out] int((a + b/x)^p*(c + d/x)^q, x)
```

3.270 $\int \frac{a+\frac{b}{x^2}}{c+\frac{d}{x^2}}\,dx$

Optimal. Leaf size=39

$$\frac{ax}{c}+\frac{(bc-ad)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{d}}\right)}{c^{3/2}\sqrt{d}}$$

```
[Out] a*x/c+(-a*d+b*c)*arctan(x*c^(1/2)/d^(1/2))/c^(3/2)/d^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}=0.176$, Rules used = {381, 396, 211}

$$\frac{(bc-ad)\text{ArcTan}\left(\frac{\sqrt{c}\,x}{\sqrt{d}}\right)}{c^{3/2}\sqrt{d}}+\frac{ax}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)/(c + d/x^2),x]
```

```
[Out] (a*x)/c + ((b*c - a*d)*ArcTan[(Sqrt[c]*x)/Sqrt[d]])/(c^(3/2)*Sqrt[d])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 381

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[x^(n*(p + q))*(b + a/x^n)^p*(d + c/x^n)^q, x] /; FreeQ[{a, b, c, d
, n}, x] && NeQ[b*c - a*d, 0] && IntegersQ[p, q] && NegQ[n]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rubi steps

$$\int \frac{a+\frac{b}{x^2}}{c+\frac{d}{x^2}}\,dx = \int \frac{b+ax^2}{d+cx^2}\,dx$$
$$= \frac{ax}{c} - \frac{(-bc+ad)\int \frac{1}{d+cx^2}\,dx}{c}$$
$$= \frac{ax}{c} + \frac{(bc-ad)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{d}}\right)}{c^{3/2}\sqrt{d}}$$

Mathematica [A]
time = 0.03, size = 40, normalized size = 1.03

$$\frac{ax}{c} - \frac{(-bc+ad)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{d}}\right)}{c^{3/2}\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)/(c + d/x^2),x]
[Out] (a*x)/c - ((-(b*c) + a*d)*ArcTan[(Sqrt[c]*x)/Sqrt[d]])/(c^(3/2)*Sqrt[d])
```

Maple [A]
time = 0.04, size = 34, normalized size = 0.87

method	result	size
default	$\frac{ax}{c}+\frac{(-ad+bc)\arctan\left(\frac{cx}{\sqrt{cd}}\right)}{c\sqrt{cd}}$	34
risch	$\frac{ax}{c}-\frac{\ln\left(cx-\sqrt{-cd}\right)ad}{2c\sqrt{-cd}}+\frac{\ln\left(cx-\sqrt{-cd}\right)b}{2\sqrt{-cd}}+\frac{\ln\left(-cx-\sqrt{-cd}\right)ad}{2c\sqrt{-cd}}-\frac{\ln\left(-cx-\sqrt{-cd}\right)b}{2\sqrt{-cd}}$	106

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/x^2+a)/(c+d/x^2),x,method=_RETURNVERBOSE)
[Out] a*x/c+(-a*d+b*c)/c/(c*d)^(1/2)*arctan(c*x/(c*d)^(1/2))
```

Maxima [A]
time = 0.48, size = 33, normalized size = 0.85

$$\frac{ax}{c} + \frac{(bc-ad)\arctan\left(\frac{cx}{\sqrt{cd}}\right)}{\sqrt{cd}\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/(c+d/x^2),x, algorithm="maxima")
```

```
[Out] a*x/c + (b*c - a*d)*arctan(c*x/sqrt(c*d))/(sqrt(c*d)*c)
```

Fricas [A]
time = 2.14, size = 98, normalized size = 2.51

$$\left[\frac{2\,acdx + (bc - ad)\sqrt{-cd}\,\log\left(\frac{cx^2+2\,\sqrt{-cd}\,x-d}{cx^2+d}\right)}{2\,c^2d}, \frac{acdx + (bc - ad)\sqrt{cd}\,\arctan\left(\frac{\sqrt{cd}\,x}{d}\right)}{c^2d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/(c+d/x^2),x, algorithm="fricas")
```

```
[Out] [1/2*(2*a*c*d*x + (b*c - a*d)*sqrt(-c*d)*log((c*x^2 + 2*sqrt(-c*d)*x - d)/(
c*x^2 + d)))/(c^2*d), (a*c*d*x + (b*c - a*d)*sqrt(c*d)*arctan(sqrt(c*d)*x/d
))/(c^2*d)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 82 vs. $2(34) = 68$.
time = 0.30, size = 82, normalized size = 2.10

$$\frac{ax}{c} + \frac{\sqrt{-\frac{1}{c^3d}}\,(ad - bc)\log\left(-cd\sqrt{-\frac{1}{c^3d}} + x\right)}{2} - \frac{\sqrt{-\frac{1}{c^3d}}\,(ad - bc)\log\left(cd\sqrt{-\frac{1}{c^3d}} + x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)/(c+d/x**2),x)
```

```
[Out] a*x/c + sqrt(-1/(c**3*d))*(a*d - b*c)*log(-c*d*sqrt(-1/(c**3*d)) + x)/2 - s
qrt(-1/(c**3*d))*(a*d - b*c)*log(c*d*sqrt(-1/(c**3*d)) + x)/2
```

Giac [A]
time = 0.57, size = 33, normalized size = 0.85

$$\frac{ax}{c} + \frac{(bc - ad)\arctan\left(\frac{cx}{\sqrt{cd}}\right)}{\sqrt{cd}\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/(c+d/x^2),x, algorithm="giac")
```

```
[Out] a*x/c + (b*c - a*d)*arctan(c*x/sqrt(c*d))/(sqrt(c*d)*c)
```

Mupad [B]
time = 0.07, size = 32, normalized size = 0.82

$$\frac{a\,x}{c}-\frac{\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{d}}\right)\,\left(a\,d-b\,c\right)}{c^{3/2}\,\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x^2)/(c + d/x^2),x)
[Out] (a*x)/c - (atan((c^(1/2)*x)/d^(1/2))*(a*d - b*c))/(c^(3/2)*d^(1/2))
```

3.271 $\int \sqrt{a+\frac{b}{x^2}}\,\sqrt{c+\frac{d}{x^2}}\,dx$

Optimal. Leaf size=233

$$-\frac{2d\sqrt{a+\frac{b}{x^2}}}{\sqrt{c+\frac{d}{x^2}}\,x}+\sqrt{a+\frac{b}{x^2}}\,\sqrt{c+\frac{d}{x^2}}\,x+\frac{2\sqrt{c}\,\sqrt{d}\,\sqrt{a+\frac{b}{x^2}}\,E\left(\cot^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{d}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{\frac{c\left(a+\frac{b}{x^2}\right)}{a\left(c+\frac{d}{x^2}\right)}}\,\sqrt{c+\frac{d}{x^2}}}-\frac{\sqrt{c}\,(bc+ad)\sqrt{a-}}{a\sqrt{d}\,\sqrt{}}$$

```
[Out] -2*d*(a+b/x^2)^(1/2)/x/(c+d/x^2)^(1/2)-(a*d+b*c)*(x^2*c/d/(1+x^2*c/d))^(1/2
)/x*(1+x^2*c/d)^(1/2)*EllipticF(1/(1+x^2*c/d)^(1/2),(1-b*c/a/d)^(1/2))*(a+b
/x^2)^(1/2)/a/(c*(a+b/x^2)/a/(c+d/x^2))^(1/2)/(c+d/x^2)^(1/2)+2*(x^2*c/d/(1
+x^2*c/d))^(1/2)/x*d*(1+x^2*c/d)^(1/2)*EllipticE(1/(1+x^2*c/d)^(1/2),(1-b*c
/a/d)^(1/2))*(a+b/x^2)^(1/2)/(c*(a+b/x^2)/a/(c+d/x^2))^(1/2)/(c+d/x^2)^(1/2
)+x*(a+b/x^2)^(1/2)*(c+d/x^2)^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 233, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {382, 484, 545, 429, 506, 422}

$$-\frac{2d\sqrt{a+\frac{b}{x^2}}}{x\sqrt{c+\frac{d}{x^2}}}+x\sqrt{a+\frac{b}{x^2}}\,\sqrt{c+\frac{d}{x^2}}-\frac{\sqrt{c}\,\sqrt{a+\frac{b}{x^2}}\,(ad+bc)F\left(\cot^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{d}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{d}\,\sqrt{c+\frac{d}{x^2}}\,\sqrt{\frac{c\left(a+\frac{b}{x^2}\right)}{a\left(c+\frac{d}{x^2}\right)}}}+\frac{2\sqrt{c}\,\sqrt{d}\,\sqrt{a+\frac{b}{x^2}}\,E\left(\cot^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{d}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c+\frac{d}{x^2}}\,\sqrt{\frac{c\left(a+\frac{b}{x^2}\right)}{a\left(c+\frac{d}{x^2}\right)}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b/x^2]*Sqrt[c + d/x^2],x]
```

```
[Out] (-2*d*Sqrt[a + b/x^2])/(Sqrt[c + d/x^2]*x) + Sqrt[a + b/x^2]*Sqrt[c + d/x^2
]*x + (2*Sqrt[c]*Sqrt[d]*Sqrt[a + b/x^2]*EllipticE[ArcCot[(Sqrt[c]*x)/Sqrt[
d]], 1 - (b*c)/(a*d)])/(Sqrt[(c*(a + b/x^2))/(a*(c + d/x^2))]*Sqrt[c + d/x^
2]) - (Sqrt[c]*(b*c + a*d)*Sqrt[a + b/x^2]*EllipticF[ArcCot[(Sqrt[c]*x)/Sqr
t[d]], 1 - (b*c)/(a*d)])/(a*Sqrt[d]*Sqrt[(c*(a + b/x^2))/(a*(c + d/x^2))]*S
qrt[c + d/x^2])
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
```

```
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 484

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^p*((c + d*x^n)^q/(e*(m
+ 1))), x] - Dist[n/(e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^(p - 1)*(c
 + d*x^n)^(q - 1)*Simp[b*c*p + a*d*q + b*d*(p + q)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && LtQ
[m, -1] && GtQ[p, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rubi steps

$$\int \sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}\,dx = -\text{Subst}\left(\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^2}\,dx, x, \frac{1}{x}\right)$$

$$= \sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}\,x - 2\text{Subst}\left(\int \frac{\frac{1}{2}(bc+ad)+bdx^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx, x, \frac{1}{x}\right)$$

$$= \sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}\,x - (2bd)\text{Subst}\left(\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx, x, \frac{1}{x}\right) - (bc+$$

$$= -\frac{2d\sqrt{a+\frac{b}{x^2}}}{\sqrt{c+\frac{d}{x^2}}\,x} + \sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}\,x - \frac{\sqrt{c}\,(bc+ad)\sqrt{a+\frac{b}{x^2}}\,F\left(\cot^{-1}\left(\frac{\sqrt{}}{\sqrt{}}\right.\right.}{a\sqrt{d}\sqrt{\frac{c\left(a+\frac{b}{x^2}\right)}{a\left(c+\frac{d}{x^2}\right)}}\sqrt{c+}}$$

$$= -\frac{2d\sqrt{a+\frac{b}{x^2}}}{\sqrt{c+\frac{d}{x^2}}\,x} + \sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}\,x + \frac{2\sqrt{c}\sqrt{d}\sqrt{a+\frac{b}{x^2}}\,E\left(\cot^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{d}}\right.\right.}{\sqrt{\frac{c\left(a+\frac{b}{x^2}\right)}{a\left(c+\frac{d}{x^2}\right)}}\sqrt{c+\frac{d}{x^2}}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.13, size = 205, normalized size = 0.88

$$-\frac{\sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}\,x\left(\sqrt{\frac{a}{b}}\,(b+ax^2)(d+cx^2)+2iadx\sqrt{1+\frac{ax^2}{b}}\sqrt{1+\frac{cx^2}{d}}\,E\left(i\sinh^{-1}\left(\sqrt{\frac{a}{b}}\,x\right)|\frac{bc}{ad}\right)+i(bc-ad)x\sqrt{1+\frac{ax^2}{b}}\sqrt{1+\frac{cx^2}{d}}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{a}{b}}\,x\right)|\frac{bc}{ad}\right)\right)}{\sqrt{\frac{a}{b}}\,(b+ax^2)(d+cx^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b/x^2]*Sqrt[c + d/x^2],x]
```

```
[Out] -((Sqrt[a + b/x^2]*Sqrt[c + d/x^2]*x*(Sqrt[a/b]*(b + a*x^2)*(d + c*x^2) + (
2*I)*a*d*x*Sqrt[1 + (a*x^2)/b]*Sqrt[1 + (c*x^2)/d]*EllipticE[I*ArcSinh[Sqrt
[a/b]*x], (b*c)/(a*d)] + I*(b*c - a*d)*x*Sqrt[1 + (a*x^2)/b]*Sqrt[1 + (c*x^
2)/d]*EllipticF[I*ArcSinh[Sqrt[a/b]*x], (b*c)/(a*d)]))/(Sqrt[a/b]*(b + a*x^
2)*(d + c*x^2)))
```

Maple [A]
time = 0.09, size = 277, normalized size = 1.19

method	result
default	$\frac{\sqrt{\frac{c\,x^2+d}{x^2}}\,x\sqrt{\frac{a\,x^2+b}{x^2}}\left(-\sqrt{-\frac{c}{d}}\,ac\,x^4+2cb\sqrt{\frac{c\,x^2+d}{d}}\sqrt{\frac{a\,x^2+b}{b}}\,x\,\text{EllipticE}\left(x\sqrt{-\frac{c}{d}},\sqrt{\frac{ad}{bc}}\right)+\sqrt{\frac{c\,x^2+d}{d}}\sqrt{\frac{a\,x^2+b}{b}}\right.}{(ac\,x^4+ad}$

risch	$-x\sqrt{\frac{c\,x^2+d}{x^2}}\,\sqrt{\frac{a\,x^2+b}{x^2}} + \frac{\left(-\frac{2cb\sqrt{1+\frac{c\,x^2}{d}}\,\sqrt{1+\frac{a\,x^2}{b}}\left(\mathrm{EllipticF}\left(x\sqrt{-\frac{c}{d}},\sqrt{-1+\frac{ad+bc}{cb}}\right)-\mathrm{EllipticE}\left(x\sqrt{-\right.\right.}{\sqrt{-\frac{c}{d}}\,\sqrt{ac\,x^4+ad\,x^2+c\,x^2b+bd}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d/x^2)^(1/2)*(b/x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((c*x^2+d)/x^2)^(1/2)*x*((a*x^2+b)/x^2)^(1/2)*(-(-c/d)^(1/2)*a*c*x^4+2*c*b*
((c*x^2+d)/d)^(1/2)*((a*x^2+b)/b)^(1/2)*x*EllipticE(x*(-c/d)^(1/2),(a*d/b/c
)^(1/2))+((c*x^2+d)/d)^(1/2)*((a*x^2+b)/b)^(1/2)*EllipticF(x*(-c/d)^(1/2),(
a*d/b/c)^(1/2))*a*d*x-c*b*((c*x^2+d)/d)^(1/2)*((a*x^2+b)/b)^(1/2)*x*Ellipti
cF(x*(-c/d)^(1/2),(a*d/b/c)^(1/2))-(-c/d)^(1/2)*a*d*x^2-(-c/d)^(1/2)*b*c*x^
2-(-c/d)^(1/2)*b*d)/(a*c*x^4+a*d*x^2+b*c*x^2+b*d)/(-c/d)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x^2)^(1/2)*(a+b/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a + b/x^2)*sqrt(c + d/x^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x^2)^(1/2)*(a+b/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt((a*x^2 + b)/x^2)*sqrt((c*x^2 + d)/x^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a+\frac{b}{x^2}}\,\sqrt{c+\frac{d}{x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x**2)**(1/2)*(a+b/x**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b/x**2)*sqrt(c + d/x**2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x^2)^(1/2)*(a+b/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a + b/x^2)*sqrt(c + d/x^2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{a+\frac{b}{x^2}}\,\sqrt{c+\frac{d}{x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x^2)^(1/2)*(c + d/x^2)^(1/2),x)
```

```
[Out] int((a + b/x^2)^(1/2)*(c + d/x^2)^(1/2), x)
```

3.272 $\int \frac{\sqrt{a+\frac{b}{x^2}}}{\sqrt{c+\frac{d}{x^2}}}\,dx$

Optimal. Leaf size=232

$$-\frac{d\sqrt{a+\frac{b}{x^2}}}{c\sqrt{c+\frac{d}{x^2}}\,x}+\frac{\sqrt{a+\frac{b}{x^2}}\,\sqrt{c+\frac{d}{x^2}}\,x}{c}+\frac{\sqrt{d}\,\sqrt{a+\frac{b}{x^2}}\,E\left(\cot^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{d}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\,\sqrt{\frac{c\left(a+\frac{b}{x^2}\right)}{a\left(c+\frac{d}{x^2}\right)}}\,\sqrt{c+\frac{d}{x^2}}}-\frac{b\sqrt{c}\,\sqrt{a+\frac{b}{x^2}}\,F\left(\cot\right.}{a\sqrt{d}\,\sqrt{\frac{c\left(a+\right.}{a\left(c+\right.}}}$$

```
[Out] -d*(a+b/x^2)^(1/2)/c/x/(c+d/x^2)^(1/2)-b*(x^2*c/d/(1+x^2*c/d))^(1/2)/x*(1+x
^2*c/d)^(1/2)*EllipticF(1/(1+x^2*c/d)^(1/2),(1-b*c/a/d)^(1/2))*(a+b/x^2)^(1
/2)/a/(c*(a+b/x^2)/a/(c+d/x^2))^(1/2)/(c+d/x^2)^(1/2)+(x^2*c/d/(1+x^2*c/d))
^(1/2)/x/c*d*(1+x^2*c/d)^(1/2)*EllipticE(1/(1+x^2*c/d)^(1/2),(1-b*c/a/d)^(1
/2))*(a+b/x^2)^(1/2)/(c*(a+b/x^2)/a/(c+d/x^2))^(1/2)/(c+d/x^2)^(1/2)+x*(a+b
/x^2)^(1/2)*(c+d/x^2)^(1/2)/c
```

Rubi [A]

time = 0.15, antiderivative size = 232, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {382, 486, 21, 433, 429, 506, 422}

$$-\frac{d\sqrt{a+\frac{b}{x^2}}}{cx\sqrt{c+\frac{d}{x^2}}}+\frac{x\sqrt{a+\frac{b}{x^2}}\,\sqrt{c+\frac{d}{x^2}}}{c}-\frac{b\sqrt{c}\,\sqrt{a+\frac{b}{x^2}}\,F\left(\cot^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{d}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{d}\,\sqrt{c+\frac{d}{x^2}}\,\sqrt{\frac{c\left(a+\frac{b}{x^2}\right)}{a\left(c+\frac{d}{x^2}\right)}}}+\frac{\sqrt{d}\,\sqrt{a+\frac{b}{x^2}}\,E\left(\cot^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{d}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\,\sqrt{c+\frac{d}{x^2}}\,\sqrt{\frac{c\left(a+\frac{b}{x^2}\right)}{a\left(c+\frac{d}{x^2}\right)}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b/x^2]/Sqrt[c + d/x^2],x]
```

```
[Out] -((d*Sqrt[a + b/x^2])/(c*Sqrt[c + d/x^2]*x)) + (Sqrt[a + b/x^2]*Sqrt[c + d/
x^2]*x)/c + (Sqrt[d]*Sqrt[a + b/x^2]*EllipticE[ArcCot[(Sqrt[c]*x)/Sqrt[d]],
 1 - (b*c)/(a*d)])/(Sqrt[c]*Sqrt[(c*(a + b/x^2))/(a*(c + d/x^2))]*Sqrt[c +
d/x^2]) - (b*Sqrt[c]*Sqrt[a + b/x^2]*EllipticF[ArcCot[(Sqrt[c]*x)/Sqrt[d]],
 1 - (b*c)/(a*d)])/(a*Sqrt[d]*Sqrt[(c*(a + b/x^2))/(a*(c + d/x^2))]*Sqrt[c
+ d/x^2])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 433

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
a, Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Dist[b, Int[x^2/(Sqrt[
a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c]
&& PosQ[b/a]
```

Rule 486

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/
(a*e*(m + 1))), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rubi steps

$$\int \frac{\sqrt{a+\frac{b}{x^2}}}{\sqrt{c+\frac{d}{x^2}}}\,dx = -\text{Subst}\left(\int \frac{\sqrt{a+bx^2}}{x^2\sqrt{c+dx^2}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{\sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}\,x}{c} - \frac{\text{Subst}\left(\int \frac{bc+bdx^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= \frac{\sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}\,x}{c} - \frac{b\text{Subst}\left(\int \frac{\sqrt{c+dx^2}}{\sqrt{a+bx^2}}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= \frac{\sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}\,x}{c} - b\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx, x, \frac{1}{x}\right) - \frac{(bd)\text{Subst}\left(\int \frac{}{\sqrt{c}}\right.}{}$$

$$= -\frac{d\sqrt{a+\frac{b}{x^2}}}{c\sqrt{c+\frac{d}{x^2}}\,x} + \frac{\sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}\,x}{c} - \frac{b\sqrt{c}\sqrt{a+\frac{b}{x^2}}\,F\left(\cot^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{d}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{\frac{c\left(a+\frac{b}{x^2}\right)}{a\left(c+\frac{d}{x^2}\right)}}\sqrt{c+\frac{d}{x^2}}} -$$

$$= -\frac{d\sqrt{a+\frac{b}{x^2}}}{c\sqrt{c+\frac{d}{x^2}}\,x} + \frac{\sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}\,x}{c} + \frac{\sqrt{d}\sqrt{a+\frac{b}{x^2}}\,E\left(\cot^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{d}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{\frac{c\left(a+\frac{b}{x^2}\right)}{a\left(c+\frac{d}{x^2}\right)}}\sqrt{c+\frac{d}{x^2}}} -$$

Mathematica [A]
time = 0.84, size = 86, normalized size = 0.37

$$\frac{\sqrt{a+\frac{b}{x^2}}\sqrt{\frac{d+cx^2}{d}}\,E\left(\sin^{-1}\left(\sqrt{-\frac{c}{d}}\,x\right)|\frac{ad}{bc}\right)}{\sqrt{-\frac{c}{d}}\sqrt{c+\frac{d}{x^2}}\sqrt{\frac{b+ax^2}{b}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b/x^2]/Sqrt[c + d/x^2],x]
```

```
[Out] (Sqrt[a + b/x^2]*Sqrt[(d + c*x^2)/d]*EllipticE[ArcSin[Sqrt[-(c/d)]*x], (a*d
)/(b*c)])/(Sqrt[-(c/d)]*Sqrt[c + d/x^2]*Sqrt[(b + a*x^2)/b])
```

Maple [A]
time = 0.04, size = 94, normalized size = 0.41

method	result	size
default	$\frac{\mathrm{EllipticE}\left(x\sqrt{-\frac{c}{d}},\sqrt{\frac{ad}{bc}}\right)\sqrt{\frac{a\,x^{2}+b}{b}}\,\sqrt{\frac{c\,x^{2}+d}{d}}\,b\sqrt{\frac{a\,x^{2}+b}{x^{2}}}}{\sqrt{-\frac{c}{d}}\,(a\,x^{2}+b)\sqrt{\frac{c\,x^{2}+d}{x^{2}}}}$	94

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/x^2+a)^(1/2)/(c+d/x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] EllipticE(x*(-c/d)^(1/2),(a*d/b/c)^(1/2))*((a*x^2+b)/b)^(1/2)*((c*x^2+d)/d)
^(1/2)*b*((a*x^2+b)/x^2)^(1/2)/(-c/d)^(1/2)/(a*x^2+b)/((c*x^2+d)/x^2)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^(1/2)/(c+d/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a + b/x^2)/sqrt(c + d/x^2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^(1/2)/(c+d/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + \frac{b}{x^{2}}}}{\sqrt{c + \frac{d}{x^{2}}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)**(1/2)/(c+d/x**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b/x**2)/sqrt(c + d/x**2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^(1/2)/(c+d/x^2)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(a + b/x^2)/sqrt(c + d/x^2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{a+\frac{b}{x^2}}}{\sqrt{c+\frac{d}{x^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x^2)^(1/2)/(c + d/x^2)^(1/2),x)
[Out] int((a + b/x^2)^(1/2)/(c + d/x^2)^(1/2), x)
```

3.273 $\int \frac{\sqrt{a+\frac{b}{x^2}}}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx$

Optimal. Leaf size=262

$$-\frac{2d\sqrt{a+\frac{b}{x^2}}}{c^2\sqrt{c+\frac{d}{x^2}}\,x}-\frac{\sqrt{a+\frac{b}{x^2}}\,x}{c\sqrt{c+\frac{d}{x^2}}}+\frac{2\sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}\,x}{c^2}+\frac{2\sqrt{d}\sqrt{a+\frac{b}{x^2}}\,E\left(\cot^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{d}}\right)|1-\frac{bc}{ad}\right)}{c^{3/2}\sqrt{\frac{c\left(a+\frac{b}{x^2}\right)}{a\left(c+\frac{d}{x^2}\right)}}\sqrt{c+\frac{d}{x^2}}}-\frac{b\sqrt{a\cdots}}{a\sqrt{\cdots}}$$

```
[Out] -2*d*(a+b/x^2)^(1/2)/c^2/x/(c+d/x^2)^(1/2)-x*(a+b/x^2)^(1/2)/c/(c+d/x^2)^(1
/2)-b*(x^2*c/d/(1+x^2*c/d))^(1/2)/x/c*(1+x^2*c/d)^(1/2)*EllipticF(1/(1+x^2*
c/d)^(1/2),(1-b*c/a/d)^(1/2))*(a+b/x^2)^(1/2)/a/(c*(a+b/x^2)/a/(c+d/x^2))^(
1/2)/(c+d/x^2)^(1/2)+2*(x^2*c/d/(1+x^2*c/d))^(1/2)/x/c^2*d*(1+x^2*c/d)^(1/2
)*EllipticE(1/(1+x^2*c/d)^(1/2),(1-b*c/a/d)^(1/2))*(a+b/x^2)^(1/2)/(c*(a+b/
x^2)/a/(c+d/x^2))^(1/2)/(c+d/x^2)^(1/2)+2*x*(a+b/x^2)^(1/2)*(c+d/x^2)^(1/2)
/c^2
```

Rubi [A]
time = 0.19, antiderivative size = 262, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$,
Rules used = {382, 480, 597, 545, 429, 506, 422}

$$\frac{2\sqrt{d}\sqrt{a+\frac{b}{x^2}}\,E\left(\cot^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{d}}\right)|1-\frac{bc}{ad}\right)}{c^{3/2}\sqrt{c+\frac{d}{x^2}}\sqrt{\frac{c\left(a+\frac{b}{x^2}\right)}{a\left(c+\frac{d}{x^2}\right)}}}-\frac{2d\sqrt{a+\frac{b}{x^2}}}{c^2x\sqrt{c+\frac{d}{x^2}}}+\frac{2x\sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}}{c^2}-\frac{x\sqrt{a+\frac{b}{x^2}}}{c\sqrt{c+\frac{d}{x^2}}}-\frac{b\sqrt{a+\frac{b}{x^2}}\,F\left(\cot^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{d}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{c}\sqrt{d}\sqrt{c+\frac{d}{x^2}}\sqrt{\frac{c\left(a+\frac{b}{x^2}\right)}{a\left(c+\frac{d}{x^2}\right)}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b/x^2]/(c + d/x^2)^(3/2),x]
```

```
[Out] (-2*d*Sqrt[a + b/x^2])/(c^2*Sqrt[c + d/x^2]*x) - (Sqrt[a + b/x^2]*x)/(c*Sqr
t[c + d/x^2]) + (2*Sqrt[a + b/x^2]*Sqrt[c + d/x^2]*x)/c^2 + (2*Sqrt[d]*Sqrt
[a + b/x^2]*EllipticE[ArcCot[(Sqrt[c]*x)/Sqrt[d]], 1 - (b*c)/(a*d)])/(c^(3/
2)*Sqrt[(c*(a + b/x^2))/(a*(c + d/x^2))]*Sqrt[c + d/x^2]) - (b*Sqrt[a + b/x
^2]*EllipticF[ArcCot[(Sqrt[c]*x)/Sqrt[d]], 1 - (b*c)/(a*d)])/(a*Sqrt[c]*Sqr
t[d]*Sqrt[(c*(a + b/x^2))/(a*(c + d/x^2))]*Sqrt[c + d/x^2])
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 480

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(e*x)^(m + 1))*(a + b*x^n)^(p + 1)*((c + d*x^n
)^q/(a*e*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p
 + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m + n*(p + 1) + 1) + d*(m + n*(p + q + 1)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0]
&& IGtQ[n, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b, c, d, e,
m, n, p, q, x]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
```

```
] && LtQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{a+\frac{b}{x^2}}}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx &= -\text{Subst}\left(\int \frac{\sqrt{a+bx^2}}{x^2\left(c+dx^2\right)^{3/2}}\,dx, x, \frac{1}{x}\right)\\
&= -\frac{\sqrt{a+\frac{b}{x^2}}\,x}{c\sqrt{c+\frac{d}{x^2}}}+\frac{\text{Subst}\left(\int \frac{-2a-bx^2}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx, x, \frac{1}{x}\right)}{c}\\
&= -\frac{\sqrt{a+\frac{b}{x^2}}\,x}{c\sqrt{c+\frac{d}{x^2}}}+\frac{2\sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}\,x}{c^2}-\frac{\text{Subst}\left(\int \frac{abc+2abdx^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx, x, \frac{1}{x}\right)}{ac^2}\\
&= -\frac{\sqrt{a+\frac{b}{x^2}}\,x}{c\sqrt{c+\frac{d}{x^2}}}+\frac{2\sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}\,x}{c^2}-\frac{b\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx, x, \frac{1}{x}\right)}{c}-\\
&= -\frac{2d\sqrt{a+\frac{b}{x^2}}}{c^2\sqrt{c+\frac{d}{x^2}}\,x}-\frac{\sqrt{a+\frac{b}{x^2}}\,x}{c\sqrt{c+\frac{d}{x^2}}}+\frac{2\sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}\,x}{c^2}-\frac{b\sqrt{a+\frac{b}{x^2}}\,F\left(\cot^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{d}}\right.\right.}{a\sqrt{c}\sqrt{d}\sqrt{\frac{c\left(a+\frac{b}{x^2}\right)}{a\left(c+\frac{d}{x^2}\right)}}\ \sqrt{}}\\
&= -\frac{2d\sqrt{a+\frac{b}{x^2}}}{c^2\sqrt{c+\frac{d}{x^2}}\,x}-\frac{\sqrt{a+\frac{b}{x^2}}\,x}{c\sqrt{c+\frac{d}{x^2}}}+\frac{2\sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}\,x}{c^2}+\frac{2\sqrt{d}\sqrt{a+\frac{b}{x^2}}\,E\left(\cot^{-1}\left(\right.\right.}{c^{3/2}\sqrt{\frac{c\left(a+\frac{b}{x^2}\right)}{a\left(c+\frac{d}{x^2}\right)}}\ \sqrt{}}
\end{aligned}
$$

Mathematica [C] Result contains complex when optimal does not.
time = 2.06, size = 191, normalized size = 0.73

$$
-\frac{\sqrt{a+\frac{b}{x^2}}\left(\sqrt{\frac{a}{b}}\,cx(b+ax^2)+2iad\sqrt{1+\frac{ax^2}{b}}\sqrt{1+\frac{cx^2}{d}}\,E\left(i\sinh^{-1}\left(\sqrt{\frac{a}{b}}\,x\right)|\frac{bc}{ad}\right)+i(bc-2ad)\sqrt{1+\frac{ax^2}{b}}\sqrt{1+\frac{cx^2}{d}}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{a}{b}}\,x\right)|\frac{bc}{ad}\right)\right)}{\sqrt{\frac{a}{b}}\,c^2\sqrt{c+\frac{d}{x^2}}\,(b+ax^2)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b/x^2]/(c + d/x^2)^(3/2),x]
```

```
[Out] -((Sqrt[a + b/x^2]*(Sqrt[a/b]*c*x*(b + a*x^2) + (2*I)*a*d*Sqrt[1 + (a*x^2)/
b]*Sqrt[1 + (c*x^2)/d]*EllipticE[I*ArcSinh[Sqrt[a/b]*x], (b*c)/(a*d)] + I*(
b*c - 2*a*d)*Sqrt[1 + (a*x^2)/b]*Sqrt[1 + (c*x^2)/d]*EllipticF[I*ArcSinh[Sq
rt[a/b]*x], (b*c)/(a*d)]))/(Sqrt[a/b]*c^2*Sqrt[c + d/x^2]*(b + a*x^2)))
```

Maple [A]
time = 0.06, size = 185, normalized size = 0.71

method	result
default	$-\frac{\left(\sqrt{-\frac{c}{d}}\, a\, x^{3}+b \sqrt{\frac{c\, x^{2}+d}{d}}\, \sqrt{\frac{a\, x^{2}+b}{b}}\, \mathrm{EllipticF}\left(x \sqrt{-\frac{c}{d}}, \sqrt{\frac{a d}{b c}}\right)-2 b \sqrt{\frac{c\, x^{2}+d}{d}}\, \sqrt{\frac{a\, x^{2}+b}{b}}\, \mathrm{EllipticE}\left(x \sqrt{-\frac{c}{d}}, \sqrt{}\right.\right.}{\sqrt{-\frac{c}{d}}\, \left(a\, x^{2}+b\right) c\, x^{2} \left(\frac{c\, x^{2}+d}{x^{2}}\right)^{\frac{3}{2}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/x^2+a)^(1/2)/(c+d/x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -((-c/d)^(1/2)*a*x^3+b*((c*x^2+d)/d)^(1/2)*((a*x^2+b)/b)^(1/2)*EllipticF(x*
(-c/d)^(1/2),(a*d/b/c)^(1/2))-2*b*((c*x^2+d)/d)^(1/2)*((a*x^2+b)/b)^(1/2)*E
llipticE(x*(-c/d)^(1/2),(a*d/b/c)^(1/2))+(-c/d)^(1/2)*b*x)*((a*x^2+b)/x^2)^
(1/2)*(c*x^2+d)/(-c/d)^(1/2)/(a*x^2+b)/c/x^2/((c*x^2+d)/x^2)^(3/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^(1/2)/(c+d/x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a + b/x^2)/(c + d/x^2)^(3/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^(1/2)/(c+d/x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+\frac{b}{x^{2}}}}{\left(c+\frac{d}{x^{2}}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)**(1/2)/(c+d/x**2)**(3/2),x)
```

```
[Out] Integral(sqrt(a + b/x**2)/(c + d/x**2)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^(1/2)/(c+d/x^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a + b/x^2)/(c + d/x^2)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{a+\frac{b}{x^2}}}{{\left(c+\frac{d}{x^2}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x^2)^(1/2)/(c + d/x^2)^(3/2),x)
```

```
[Out] int((a + b/x^2)^(1/2)/(c + d/x^2)^(3/2), x)
```

3.274 $\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q dx$

Optimal. Leaf size=79

$$\left(a+\frac{b}{x^2}\right)^p \left(1+\frac{b}{ax^2}\right)^{-p} \left(c+\frac{d}{x^2}\right)^q \left(1+\frac{d}{cx^2}\right)^{-q} xF_1\left(-\frac{1}{2};-p,-q;\frac{1}{2};-\frac{b}{ax^2},-\frac{d}{cx^2}\right)$$

```
[Out] (a+b/x^2)^p*(c+d/x^2)^q*x*AppellF1(-1/2,-p,-q,1/2,-b/a/x^2,-d/c/x^2)/((1+b/
a/x^2)^p)/((1+d/c/x^2)^q)
```

Rubi [A]
time = 0.06, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {382, 525, 524}

$$x\left(a+\frac{b}{x^2}\right)^p \left(\frac{b}{ax^2}+1\right)^{-p} \left(c+\frac{d}{x^2}\right)^q \left(\frac{d}{cx^2}+1\right)^{-q} F_1\left(-\frac{1}{2};-p,-q;\frac{1}{2};-\frac{b}{ax^2},-\frac{d}{cx^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)^p*(c + d/x^2)^q,x]
```

```
[Out] ((a + b/x^2)^p*(c + d/x^2)^q*x*AppellF1[-1/2, -p, -q, 1/2, -(b/(a*x^2)), -(
d/(c*x^2))])/((1 + b/(a*x^2))^p*(1 + d/(c*x^2))^q)
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q dx = -\text{Subst}\left(\int \frac{(a+bx^2)^p (c+dx^2)^q}{x^2}\, dx, x, \frac{1}{x}\right)$$

$$= -\left(\left(\left(a+\frac{b}{x^2}\right)^p \left(1+\frac{b}{ax^2}\right)^{-p}\right) \text{Subst}\left(\int \frac{\left(1+\frac{bx^2}{a}\right)^p (c+dx^2)^q}{x^2}\, dx, x, \frac{1}{x}\right)$$

$$= -\left(\left(\left(a+\frac{b}{x^2}\right)^p \left(1+\frac{b}{ax^2}\right)^{-p} \left(c+\frac{d}{x^2}\right)^q \left(1+\frac{d}{cx^2}\right)^{-q}\right) \text{Subst}\left(\int \frac{(1+\cdots}{\cdots}\right.$$

$$= \left(a+\frac{b}{x^2}\right)^p \left(1+\frac{b}{ax^2}\right)^{-p} \left(c+\frac{d}{x^2}\right)^q \left(1+\frac{d}{cx^2}\right)^{-q} xF_1\left(-\frac{1}{2}; -p, -q; \frac{1}{2}; -\frac{\cdots}{a\cdots}\right.$$

Mathematica [A]
time = 0.10, size = 104, normalized size = 1.32

$$-\frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x \left(1+\frac{ax^2}{b}\right)^{-p} \left(1+\frac{cx^2}{d}\right)^{-q} F_1\left(\frac{1}{2}-p-q; -p, -q; \frac{3}{2}-p-q; -\frac{ax^2}{b}, -\frac{cx^2}{d}\right)}{-1+2p+2q}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)^p*(c + d/x^2)^q,x]
[Out] -(((a + b/x^2)^p*(c + d/x^2)^q*x*AppellF1[1/2 - p - q, -p, -q, 3/2 - p - q,
 -((a*x^2)/b), -((c*x^2)/d)])/((-1 + 2*p + 2*q)*(1 + (a*x^2)/b)^p*(1 + (c*x
^2)/d)^q))
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \left(\frac{b}{x^2}+a\right)^p \left(c+\frac{d}{x^2}\right)^q dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/x^2+a)^p*(c+d/x^2)^q,x)
[Out] int((b/x^2+a)^p*(c+d/x^2)^q,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q,x, algorithm="maxima")
```

```
[Out] integrate((a + b/x^2)^p*(c + d/x^2)^q, x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q,x, algorithm="fricas")
```

```
[Out] integral(((a*x^2 + b)/x^2)^p*((c*x^2 + d)/x^2)^q, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)**p*(c+d/x**2)**q,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q,x, algorithm="giac")
```

```
[Out] integrate((a + b/x^2)^p*(c + d/x^2)^q, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a+\frac{b}{x^2}\right)^p\,\left(c+\frac{d}{x^2}\right)^q\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x^2)^p*(c + d/x^2)^q,x)
```

```
[Out] int((a + b/x^2)^p*(c + d/x^2)^q, x)
```

3.275 $\int \frac{a+\frac{b}{x^3}}{c+\frac{d}{x^3}}\,dx$

Optimal. Leaf size=145

$$\frac{ax}{c}-\frac{(bc-ad)\tan^{-1}\left(\frac{\sqrt[3]{d}-2\sqrt[3]{c}\,x}{\sqrt{3}\,\sqrt[3]{d}}\right)}{\sqrt{3}\,c^{4/3}d^{2/3}}+\frac{(bc-ad)\log\left(\sqrt[3]{d}+\sqrt[3]{c}\,x\right)}{3c^{4/3}d^{2/3}}-\frac{(bc-ad)\log\left(d^{2/3}-\sqrt[3]{c}\,\sqrt[3]{d}\,x+c^{2/3}x^2\right)}{6c^{4/3}d^{2/3}}$$

```
[Out] a*x/c+1/3*(-a*d+b*c)*ln(d^(1/3)+c^(1/3)*x)/c^(4/3)/d^(2/3)-1/6*(-a*d+b*c)*l
n(d^(2/3)-c^(1/3)*d^(1/3)*x+c^(2/3)*x^2)/c^(4/3)/d^(2/3)-1/3*(-a*d+b*c)*arc
tan(1/3*(d^(1/3)-2*c^(1/3)*x)/d^(1/3)*3^(1/2))/c^(4/3)/d^(2/3)*3^(1/2)
```

Rubi [A]

time = 0.07, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.471, Rules used = {381, 396, 206, 31, 648, 631, 210, 642}

$$-\frac{(bc-ad)\text{ArcTan}\left(\frac{\sqrt[3]{d}-2\sqrt[3]{c}\,x}{\sqrt{3}\,\sqrt[3]{d}}\right)}{\sqrt{3}\,c^{4/3}d^{2/3}}-\frac{(bc-ad)\log\left(c^{2/3}x^2-\sqrt[3]{c}\,\sqrt[3]{d}\,x+d^{2/3}\right)}{6c^{4/3}d^{2/3}}+\frac{(bc-ad)\log\left(\sqrt[3]{c}\,x+\sqrt[3]{d}\right)}{3c^{4/3}d^{2/3}}+\frac{ax}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^3)/(c + d/x^3),x]
```

```
[Out] (a*x)/c - ((b*c - a*d)*ArcTan[(d^(1/3) - 2*c^(1/3)*x)/(Sqrt[3]*d^(1/3))])/(
Sqrt[3]*c^(4/3)*d^(2/3)) + ((b*c - a*d)*Log[d^(1/3) + c^(1/3)*x])/(3*c^(4/3
)*d^(2/3)) - ((b*c - a*d)*Log[d^(2/3) - c^(1/3)*d^(1/3)*x + c^(2/3)*x^2])/(
6*c^(4/3)*d^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 381

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[x^(n*(p + q))*(b + a/x^n)^p*(d + c/x^n)^q, x] /; FreeQ[{a, b, c, d
, n}, x] && NeQ[b*c - a*d, 0] && IntegersQ[p, q] && NegQ[n]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{a+\frac{b}{x^3}}{c+\frac{d}{x^3}}\,dx = \int \frac{b+ax^3}{d+cx^3}\,dx$$
$$= \frac{ax}{c} - \frac{(-bc+ad)\int \frac{1}{d+cx^3}\,dx}{c}$$
$$= \frac{ax}{c} + \frac{(bc-ad)\int \frac{1}{\sqrt[3]{d}+\sqrt[3]{c}\,x}\,dx}{3cd^{2/3}} + \frac{(bc-ad)\int \frac{2\sqrt[3]{d}-\sqrt[3]{c}\,x}{d^{2/3}-\sqrt[3]{c}\sqrt[3]{d}\,x+c^{2/3}x^2}\,dx}{3cd^{2/3}}$$
$$= \frac{ax}{c} + \frac{(bc-ad)\log\left(\sqrt[3]{d}+\sqrt[3]{c}\,x\right)}{3c^{4/3}d^{2/3}} - \frac{(bc-ad)\int \frac{-\sqrt[3]{c}\sqrt[3]{d}+2c^{2/3}x}{d^{2/3}-\sqrt[3]{c}\sqrt[3]{d}\,x+c^{2/3}x^2}\,dx}{6c^{4/3}d^{2/3}} + \frac{(bc-ad)\int \frac{}{d^{2/3}-}}{2c}$$
$$= \frac{ax}{c} + \frac{(bc-ad)\log\left(\sqrt[3]{d}+\sqrt[3]{c}\,x\right)}{3c^{4/3}d^{2/3}} - \frac{(bc-ad)\log\left(d^{2/3}-\sqrt[3]{c}\sqrt[3]{d}\,x+c^{2/3}x^2\right)}{6c^{4/3}d^{2/3}} + \frac{(bc-ad)\mathrm{S}}{}$$
$$= \frac{ax}{c} - \frac{(bc-ad)\tan^{-1}\left(\frac{\sqrt[3]{d}-2\sqrt[3]{c}\,x}{\sqrt{3}\sqrt[3]{d}}\right)}{\sqrt{3}\,c^{4/3}d^{2/3}} + \frac{(bc-ad)\log\left(\sqrt[3]{d}+\sqrt[3]{c}\,x\right)}{3c^{4/3}d^{2/3}} - \frac{(bc-ad)\log\left(d^{2/3}-\right.}{6c^{4/3}}$$

Mathematica [A]
time = 0.06, size = 129, normalized size = 0.89

$$\frac{6a\sqrt[3]{c}\,d^{2/3}x - 2\sqrt{3}\,(bc-ad)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{c}\,x}{\sqrt[3]{d}}}{\sqrt{3}}\right) + 2(bc-ad)\log\left(\sqrt[3]{d}+\sqrt[3]{c}\,x\right) - (bc-ad)\log\left(d^{2/3}-\sqrt[3]{c}\sqrt[3]{d}\,x+c^{2/3}x^2\right)}{6c^{4/3}d^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^3)/(c + d/x^3),x]
```

```
[Out] (6*a*c^(1/3)*d^(2/3)*x - 2*Sqrt[3]*(b*c - a*d)*ArcTan[(1 - (2*c^(1/3)*x)/d^
(1/3))/Sqrt[3]] + 2*(b*c - a*d)*Log[d^(1/3) + c^(1/3)*x] - (b*c - a*d)*Log[
d^(2/3) - c^(1/3)*d^(1/3)*x + c^(2/3)*x^2])/(6*c^(4/3)*d^(2/3))
```

Maple [A]
time = 0.02, size = 110, normalized size = 0.76

method	result	size
risch	$\frac{ax}{c} + \frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^3+d\right)} \frac{(-ad+bc)\ln\left(x-\_R\right)}{\_R^2}}{3c^2}$	42

default	$\frac{ax}{c}+\frac{\left(\frac{\ln\left(x+\left(\frac{d}{c}\right)^{\frac{1}{3}}\right)}{3c\left(\frac{d}{c}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{d}{c}\right)^{\frac{1}{3}}x+\left(\frac{d}{c}\right)^{\frac{2}{3}}\right)}{6c\left(\frac{d}{c}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{d}{c}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3c\left(\frac{d}{c}\right)^{\frac{2}{3}}}\right)\left(-ad+bc\right)}{c}$	110

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^3)/(c+d/x^3),x,method=_RETURNVERBOSE)
```

```
[Out] a*x/c+(1/3/c/(d/c)^(2/3)*ln(x+(d/c)^(1/3))-1/6/c/(d/c)^(2/3)*ln(x^2-(d/c)^(
1/3)*x+(d/c)^(2/3))+1/3/c/(d/c)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(d/c)^(
1/3)*x-1)))*(-a*d+b*c)/c
```

Maxima [A]
time = 0.50, size = 128, normalized size = 0.88

$$\frac{ax}{c}+\frac{\sqrt{3}\,(bc-ad)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{d}{c}\right)^{\frac{1}{3}}\right)}{3\left(\frac{d}{c}\right)^{\frac{1}{3}}}\right)}{3\,c^{2}\left(\frac{d}{c}\right)^{\frac{2}{3}}}-\frac{(bc-ad)\log\left(x^{2}-x\left(\frac{d}{c}\right)^{\frac{1}{3}}+\left(\frac{d}{c}\right)^{\frac{2}{3}}\right)}{6\,c^{2}\left(\frac{d}{c}\right)^{\frac{2}{3}}}+\frac{(bc-ad)\log\left(x+\left(\frac{d}{c}\right)^{\frac{1}{3}}\right)}{3\,c^{2}\left(\frac{d}{c}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^3)/(c+d/x^3),x, algorithm="maxima")
```

```
[Out] a*x/c + 1/3*sqrt(3)*(b*c - a*d)*arctan(1/3*sqrt(3)*(2*x - (d/c)^(1/3))/(d/c
)^(1/3))/(c^2*(d/c)^(2/3)) - 1/6*(b*c - a*d)*log(x^2 - x*(d/c)^(1/3) + (d/c
)^(2/3))/(c^2*(d/c)^(2/3)) + 1/3*(b*c - a*d)*log(x + (d/c)^(1/3))/(c^2*(d/c
)^(2/3))
```

Fricas [A]
time = 2.32, size = 390, normalized size = 2.69

$$\left[\frac{6\,acd^{2}x-3\,\sqrt{\frac{1}{3}}\,(bc^{2}d-acd^{2})\sqrt{\frac{(-cd^{2})^{\frac{1}{3}}}{c}}\log\left(\frac{2\,cdx^{3}+3\,(-cd^{2})^{\frac{1}{3}}dx-d^{2}-3\,\sqrt{\frac{1}{3}}\left(2\,cdx^{2}+(-cd^{2})^{\frac{2}{3}}x+(-cd^{2})^{\frac{1}{3}}d\right)\sqrt{\frac{(-cd^{2})^{\frac{1}{3}}}{c}}}{cx^{3}+d}\right)-(-cd^{2})^{\frac{2}{3}}(bc-ad)\log\left(cdx^{2}-(-cd^{2})^{\frac{2}{3}}x-(-cd^{2})^{\frac{1}{3}}d\right)+2\,(-cd^{2})^{\frac{2}{3}}(bc-ad)\log\left(cdx+(-cd^{2})^{\frac{2}{3}}\right)}{6\,c^{2}d^{2}},\ \frac{6\,acd^{2}x+6\,\sqrt{\frac{1}{3}}\,(bc^{2}d-acd^{2})\sqrt{-\frac{(-cd^{2})^{\frac{1}{3}}}{c}}\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(2\,(-cd^{2})^{\frac{2}{3}}x+(-cd^{2})^{\frac{1}{3}}d\right)\sqrt{-\frac{(-cd^{2})^{\frac{1}{3}}}{c}}}{d^{2}}\right)-(-cd^{2})^{\frac{2}{3}}(bc-ad)\log\left(cdx^{2}-(-cd^{2})^{\frac{2}{3}}x-(-cd^{2})^{\frac{1}{3}}d\right)+2\,(-cd^{2})^{\frac{2}{3}}(bc-ad)\log\left(cdx+(-cd^{2})^{\frac{2}{3}}\right)}{6\,c^{2}d^{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^3)/(c+d/x^3),x, algorithm="fricas")
```

```
[Out] [1/6*(6*a*c*d^2*x - 3*sqrt(1/3)*(b*c^2*d - a*c*d^2)*sqrt((-c*d^2)^(1/3)/c)*
log((2*c*d*x^3 + 3*(-c*d^2)^(1/3)*d*x - d^2 - 3*sqrt(1/3)*(2*c*d*x^2 + (-c*
d^2)^(2/3)*x + (-c*d^2)^(1/3)*d)*sqrt((-c*d^2)^(1/3)/c))/(c*x^3 + d)) - (-c
```

```
*d^2)^(2/3)*(b*c - a*d)*log(c*d*x^2 - (-c*d^2)^(2/3)*x - (-c*d^2)^(1/3)*d)
+ 2*(-c*d^2)^(2/3)*(b*c - a*d)*log(c*d*x + (-c*d^2)^(2/3)))/(c^2*d^2), 1/6*
(6*a*c*d^2*x + 6*sqrt(1/3)*(b*c^2*d - a*c*d^2)*sqrt(-(-c*d^2)^(1/3)/c)*arct
an(sqrt(1/3)*(2*(-c*d^2)^(2/3)*x + (-c*d^2)^(1/3)*d)*sqrt(-(-c*d^2)^(1/3)/c
)/d^2) - (-c*d^2)^(2/3)*(b*c - a*d)*log(c*d*x^2 - (-c*d^2)^(2/3)*x - (-c*d^
2)^(1/3)*d) + 2*(-c*d^2)^(2/3)*(b*c - a*d)*log(c*d*x + (-c*d^2)^(2/3)))/(c^
2*d^2)]
```

Sympy [A]
time = 0.32, size = 71, normalized size = 0.49

$$\frac{ax}{c} + \operatorname{RootSum}\left(27t^3c^4d^2 + a^3d^3 - 3a^2bcd^2 + 3ab^2c^2d - b^3c^3, \left(t \mapsto t\log\left(-\frac{3tcd}{ad-bc} + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**3)/(c+d/x**3),x)
```

```
[Out] a*x/c + RootSum(27*_t**3*c**4*d**2 + a**3*d**3 - 3*a**2*b*c*d**2 + 3*a*b**2
*c**2*d - b**3*c**3, Lambda(_t, _t*log(-3*_t*c*d/(a*d - b*c) + x)))
```

Giac [A]
time = 1.53, size = 133, normalized size = 0.92

$$-\frac{\sqrt{3}\,(bc-ad)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{d}{c}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{d}{c}\right)^{\frac{1}{3}}}\right)}{3\,(-c^2d)^{\frac{2}{3}}} - \frac{(bc-ad)\log\left(x^2+x\left(-\frac{d}{c}\right)^{\frac{1}{3}}+\left(-\frac{d}{c}\right)^{\frac{2}{3}}\right)}{6\,(-c^2d)^{\frac{2}{3}}} + \frac{ax}{c} - \frac{(bc-ad)\left(-\frac{d}{c}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{d}{c}\right)^{\frac{1}{3}}\right|\right)}{3\,cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^3)/(c+d/x^3),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*(b*c - a*d)*arctan(1/3*sqrt(3)*(2*x + (-d/c)^(1/3))/(-d/c)^(1/
3))/(-c^2*d)^(2/3) - 1/6*(b*c - a*d)*log(x^2 + x*(-d/c)^(1/3) + (-d/c)^(2/3
))/(-c^2*d)^(2/3) + a*x/c - 1/3*(b*c - a*d)*(-d/c)^(1/3)*log(abs(x - (-d/c)
^(1/3)))/(c*d)
```

Mupad [B]
time = 0.27, size = 123, normalized size = 0.85

$$\frac{a\,x}{c} - \frac{\ln\left(c^{1/3}\,x+d^{1/3}\right)\,\left(a\,d-b\,c\right)}{3\,c^{4/3}\,d^{2/3}} + \frac{\ln\left(d^{1/3}-2\,c^{1/3}\,x+\sqrt{3}\,d^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(a\,d-b\,c\right)}{3\,c^{4/3}\,d^{2/3}} - \frac{\ln\left(2\,c^{1/3}\,x-d^{1/3}+\sqrt{3}\,d^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(a\,d-b\,c\right)}{3\,c^{4/3}\,d^{2/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x^3)/(c + d/x^3),x)
```

```
[Out] (a*x)/c - (log(c^(1/3)*x + d^(1/3))*(a*d - b*c))/(3*c^(4/3)*d^(2/3)) + (log
(3^(1/2)*d^(1/3)*1i - 2*c^(1/3)*x + d^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(a*d -
b*c))/(3*c^(4/3)*d^(2/3)) - (log(3^(1/2)*d^(1/3)*1i + 2*c^(1/3)*x - d^(1/3)
)*((3^(1/2)*1i)/2 - 1/2)*(a*d - b*c))/(3*c^(4/3)*d^(2/3))
```

3.276 $\int \frac{a+b\sqrt{x}}{c+d\sqrt{x}}\,dx$

Optimal. Leaf size=49

$$-\frac{2(bc-ad)\sqrt{x}}{d^2}+\frac{bx}{d}+\frac{2c(bc-ad)\log\left(c+d\sqrt{x}\right)}{d^3}$$

```
[Out] b*x/d+2*c*(-a*d+b*c)*ln(c+d*x^(1/2))/d^3-2*(-a*d+b*c)*x^(1/2)/d^2
```

Rubi [A]
time = 0.03, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {383, 78}

$$\frac{2c(bc-ad)\log\left(c+d\sqrt{x}\right)}{d^3}-\frac{2\sqrt{x}\,(bc-ad)}{d^2}+\frac{bx}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sqrt[x])/(c + d*Sqrt[x]),x]
```

```
[Out] (-2*(b*c - a*d)*Sqrt[x])/d^2 + (b*x)/d + (2*c*(b*c - a*d)*Log[c + d*Sqrt[x]
])/d^3
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 383

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(a + b*x^(g*n))
^p*(c + d*x^(g*n))^q, x], x, x^(1/g)], x]] /; FreeQ[{a, b, c, d, p, q}, x]
&& NeQ[b*c - a*d, 0] && FractionQ[n]
```

Rubi steps

$$\int \frac{a+b\sqrt{x}}{c+d\sqrt{x}}\,dx = 2\text{Subst}\left(\int \frac{x(a+bx)}{c+dx}\,dx, x, \sqrt{x}\right)$$
$$= 2\text{Subst}\left(\int \left(\frac{-bc+ad}{d^2}+\frac{bx}{d}+\frac{c(bc-ad)}{d^2(c+dx)}\right)dx, x, \sqrt{x}\right)$$
$$= -\frac{2(bc-ad)\sqrt{x}}{d^2}+\frac{bx}{d}+\frac{2c(bc-ad)\log\left(c+d\sqrt{x}\right)}{d^3}$$

Mathematica [A]
time = 0.03, size = 51, normalized size = 1.04

$$\frac{\left(-2bc+2ad+bd\sqrt{x}\right)\sqrt{x}}{d^2}+\frac{2c(bc-ad)\log\left(c+d\sqrt{x}\right)}{d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sqrt[x])/(c + d*Sqrt[x]),x]
```

```
[Out] ((-2*b*c + 2*a*d + b*d*Sqrt[x])*Sqrt[x])/d^2 + (2*c*(b*c - a*d)*Log[c + d*S
qrt[x]])/d^3
```

Maple [A]
time = 0.25, size = 48, normalized size = 0.98

method	result	size
derivativedivides	$\frac{xbd+2ad\sqrt{x}-2bc\sqrt{x}}{d^2}-\frac{2c(ad-bc)\ln\left(c+d\sqrt{x}\right)}{d^3}$	48
default	$\frac{xbd+2ad\sqrt{x}-2bc\sqrt{x}}{d^2}-\frac{2c(ad-bc)\ln\left(c+d\sqrt{x}\right)}{d^3}$	48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^(1/2))/(c+d*x^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d^2*(1/2*x*b*d+a*d*x^(1/2)-b*c*x^(1/2))-2*c*(a*d-b*c)/d^3*ln(c+d*x^(1/2))
```

Maxima [A]
time = 0.28, size = 47, normalized size = 0.96

$$\frac{bdx-2\,(bc-ad)\sqrt{x}}{d^2}+\frac{2\,(bc^2-acd)\log\left(d\sqrt{x}+c\right)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))/(c+d*x^(1/2)),x, algorithm="maxima")
```

```
[Out] (b*d*x - 2*(b*c - a*d)*sqrt(x))/d^2 + 2*(b*c^2 - a*c*d)*log(d*sqrt(x) + c)/
d^3
```

Fricas [A]
time = 2.54, size = 48, normalized size = 0.98

$$\frac{bd^2x + 2\left(bc^2 - acd\right)\log\left(d\sqrt{x} + c\right) - 2\left(bcd - ad^2\right)\sqrt{x}}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))/(c+d*x^(1/2)),x, algorithm="fricas")
```

```
[Out] (b*d^2*x + 2*(b*c^2 - a*c*d)*log(d*sqrt(x) + c) - 2*(b*c*d - a*d^2)*sqrt(x)
)/d^3
```

Sympy [A]
time = 0.27, size = 82, normalized size = 1.67

$$\begin{cases} -\frac{2ac\log\left(\frac{c}{d}+\sqrt{x}\right)}{d^2} + \frac{2a\sqrt{x}}{d} + \frac{2bc^2\log\left(\frac{c}{d}+\sqrt{x}\right)}{d^3} - \frac{2bc\sqrt{x}}{d^2} + \frac{bx}{d} & \text{for } d \neq 0 \\ \frac{ax+\frac{2bx^{\frac{3}{2}}}{3}}{c} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**(1/2))/(c+d*x**(1/2)),x)
```

```
[Out] Piecewise((-2*a*c*log(c/d + sqrt(x))/d**2 + 2*a*sqrt(x)/d + 2*b*c**2*log(c/
d + sqrt(x))/d**3 - 2*b*c*sqrt(x)/d**2 + b*x/d, Ne(d, 0)), ((a*x + 2*b*x**(
3/2)/3)/c, True))
```

Giac [A]
time = 1.17, size = 49, normalized size = 1.00

$$\frac{bdx - 2\,bc\sqrt{x} + 2\,ad\sqrt{x}}{d^2} + \frac{2\left(bc^2 - acd\right)\log\left(\left|d\sqrt{x} + c\right|\right)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))/(c+d*x^(1/2)),x, algorithm="giac")
```

```
[Out] (b*d*x - 2*b*c*sqrt(x) + 2*a*d*sqrt(x))/d^2 + 2*(b*c^2 - a*c*d)*log(abs(d*s
qrt(x) + c))/d^3
```

Mupad [B]
time = 0.07, size = 49, normalized size = 1.00

$$\sqrt{x}\,\left(\frac{2\,a}{d} - \frac{2\,b\,c}{d^2}\right) + \frac{\ln\left(c + d\,\sqrt{x}\right)\,\left(2\,b\,c^2 - 2\,a\,c\,d\right)}{d^3} + \frac{b\,x}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^(1/2))/(c + d*x^(1/2)),x)
```

```
[Out] x^(1/2)*((2*a)/d - (2*b*c)/d^2) + (log(c + d*x^(1/2))*(2*b*c^2 - 2*a*c*d))/
d^3 + (b*x)/d
```

3.277 $\int \frac{-1+\sqrt[3]{x}}{1+\sqrt[3]{x}}\,dx$

Optimal. Leaf size=26

$$6\sqrt[3]{x}-3x^{2/3}+x-6\log\left(1+\sqrt[3]{x}\right)$$

```
[Out] 6*x^(1/3)-3*x^(2/3)+x-6*ln(1+x^(1/3))
```

Rubi [A]
time = 0.01, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}=0.118$, Rules used = {383, 78}

$$-3x^{2/3}+x+6\sqrt[3]{x}-6\log\left(\sqrt[3]{x}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x^(1/3))/(1 + x^(1/3)),x]
```

```
[Out] 6*x^(1/3) - 3*x^(2/3) + x - 6*Log[1 + x^(1/3)]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

Rule 383

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(a + b*x^(g*n))^p*(c + d*x^(g*n))^q, x], x, x^(1/g)], x]] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && FractionQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{-1+\sqrt[3]{x}}{1+\sqrt[3]{x}}\,dx &= 3\text{Subst}\left(\int \frac{(-1+x)x^2}{1+x}\,dx,x,\sqrt[3]{x}\right)\\ &= 3\text{Subst}\left(\int \left(2-2x+x^2-\frac{2}{1+x}\right)dx,x,\sqrt[3]{x}\right)\\ &= 6\sqrt[3]{x}-3x^{2/3}+x-6\log\left(1+\sqrt[3]{x}\right)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 26, normalized size = 1.00

$$6\sqrt[3]{x}-3x^{2/3}+x-6\log\left(1+\sqrt[3]{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^(1/3))/(1 + x^(1/3)),x]
[Out] 6*x^(1/3) - 3*x^(2/3) + x - 6*Log[1 + x^(1/3)]
```

Maple [A]
time = 0.24, size = 21, normalized size = 0.81

method	result	size
derivativedivides	$6x^{\frac{1}{3}}-3x^{\frac{2}{3}}+x-6\ln\left(1+x^{\frac{1}{3}}\right)$	21
default	$6x^{\frac{1}{3}}-3x^{\frac{2}{3}}+x-6\ln\left(1+x^{\frac{1}{3}}\right)$	21
trager	$x-1+6x^{\frac{1}{3}}-3x^{\frac{2}{3}}-2\ln\left(-3x^{\frac{2}{3}}-3x^{\frac{1}{3}}-x-1\right)$	32
meijerg	$\frac{x^{\frac{1}{3}}\left(4x^{\frac{2}{3}}-6x^{\frac{1}{3}}+12\right)}{4}-6\ln\left(1+x^{\frac{1}{3}}\right)+\frac{x^{\frac{1}{3}}\left(-3x^{\frac{1}{3}}+6\right)}{2}$	39

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(1/3)-1)/(1+x^(1/3)),x,method=_RETURNVERBOSE)
[Out] 6*x^(1/3)-3*x^(2/3)+x-6*ln(1+x^(1/3))
```

Maxima [A]
time = 0.26, size = 20, normalized size = 0.77

$$x-3\,x^{\frac{2}{3}}+6\,x^{\frac{1}{3}}-6\,\log\left(x^{\frac{1}{3}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x^(1/3))/(1+x^(1/3)),x, algorithm="maxima")
[Out] x - 3*x^(2/3) + 6*x^(1/3) - 6*log(x^(1/3) + 1)
```

Fricas [A]
time = 2.09, size = 20, normalized size = 0.77

$$x-3\,x^{\frac{2}{3}}+6\,x^{\frac{1}{3}}-6\,\log\left(x^{\frac{1}{3}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x^(1/3))/(1+x^(1/3)),x, algorithm="fricas")
```

[Out] x - 3*x^(2/3) + 6*x^(1/3) - 6*log(x^(1/3) + 1)

Sympy [A]
time = 0.14, size = 24, normalized size = 0.92

$$-3x^{\frac{2}{3}} + 6\sqrt[3]{x} + x - 6\log\left(\sqrt[3]{x} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1+x**(1/3))/(1+x**(1/3)),x)

[Out] -3*x**(2/3) + 6*x**(1/3) + x - 6*log(x**(1/3) + 1)

Giac [A]
time = 1.43, size = 20, normalized size = 0.77

$$x - 3\,x^{\frac{2}{3}} + 6\,x^{\frac{1}{3}} - 6\,\log\left(x^{\frac{1}{3}} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1+x^(1/3))/(1+x^(1/3)),x, algorithm="giac")

[Out] x - 3*x^(2/3) + 6*x^(1/3) - 6*log(x^(1/3) + 1)

Mupad [B]
time = 0.03, size = 20, normalized size = 0.77

$$x - 6\,\ln\left(x^{1/3} + 1\right) + 6\,x^{1/3} - 3\,x^{2/3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^(1/3) - 1)/(x^(1/3) + 1),x)

[Out] x - 6*log(x^(1/3) + 1) + 6*x^(1/3) - 3*x^(2/3)

3.278 $\int \frac{1+x^{2/3}}{-1+x^{2/3}} dx$

Optimal. Leaf size=17

$$6\sqrt[3]{x}+x-6\tanh^{-1}\left(\sqrt[3]{x}\right)$$

[Out] 6*x^(1/3)+x-6*arctanh(x^(1/3))

Rubi [A]
time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {383, 470, 327, 213}

$$x+6\sqrt[3]{x}-6\tanh^{-1}\left(\sqrt[3]{x}\right)$$

Antiderivative was successfully verified.

[In] Int[(1 + x^(2/3))/(-1 + x^(2/3)),x]

[Out] 6*x^(1/3) + x - 6*ArcTanh[x^(1/3)]

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 383

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(a + b*x^(g*n))
^p*(c + d*x^(g*n))^q, x], x, x^(1/g)], x]] /; FreeQ[{a, b, c, d, p, q}, x]
&& NeQ[b*c - a*d, 0] && FractionQ[n]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
```

```
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1+x^{2/3}}{-1+x^{2/3}}\,dx &= 3\text{Subst}\left(\int \frac{x^2(1+x^2)}{-1+x^2}\,dx, x, \sqrt[3]{x}\right)\\ &= x+6\text{Subst}\left(\int \frac{x^2}{-1+x^2}\,dx, x, \sqrt[3]{x}\right)\\ &= 6\sqrt[3]{x}+x+6\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sqrt[3]{x}\right)\\ &= 6\sqrt[3]{x}+x-6\tanh^{-1}\left(\sqrt[3]{x}\right)\end{aligned}$$

Mathematica [A]
time = 0.02, size = 17, normalized size = 1.00

$$6\sqrt[3]{x}+x-6\tanh^{-1}\left(\sqrt[3]{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^(2/3))/(-1 + x^(2/3)),x]
[Out] 6*x^(1/3) + x - 6*ArcTanh[x^(1/3)]
```

Maple [A]
time = 0.26, size = 24, normalized size = 1.41

method	result	size
derivativedivides	$x+6x^{\frac{1}{3}}+3\ln\left(x^{\frac{1}{3}}-1\right)-3\ln\left(1+x^{\frac{1}{3}}\right)$	24
default	$x+6x^{\frac{1}{3}}+3\ln\left(x^{\frac{1}{3}}-1\right)-3\ln\left(1+x^{\frac{1}{3}}\right)$	24
trager	$x-2+6x^{\frac{1}{3}}-3\ln\left(-\frac{2x^{\frac{2}{3}}+2x^{\frac{1}{3}}+x+1}{x-1}\right)$	32
meijerg	$-\frac{3i\left(2ix^{\frac{1}{3}}-2i\,\mathrm{arctanh}\left(x^{\frac{1}{3}}\right)\right)}{2}+\frac{3i\left(-\frac{2ix^{\frac{1}{3}}\left(5x^{\frac{2}{3}}+15\right)}{15}+2i\,\mathrm{arctanh}\left(x^{\frac{1}{3}}\right)\right)}{2}$	43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x^(2/3))/(-1+x^(2/3)),x,method=_RETURNVERBOSE)
[Out] x+6*x^(1/3)+3*ln(x^(1/3)-1)-3*ln(1+x^(1/3))
```

Maxima [A]
time = 0.28, size = 23, normalized size = 1.35

$$x + 6\,x^{\frac{1}{3}} - 3\,\log\left(x^{\frac{1}{3}} + 1\right) + 3\,\log\left(x^{\frac{1}{3}} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x^(2/3))/(-1+x^(2/3)),x, algorithm="maxima")
[Out] x + 6*x^(1/3) - 3*log(x^(1/3) + 1) + 3*log(x^(1/3) - 1)
```

Fricas [A]
time = 2.64, size = 23, normalized size = 1.35

$$x + 6\,x^{\frac{1}{3}} - 3\,\log\left(x^{\frac{1}{3}} + 1\right) + 3\,\log\left(x^{\frac{1}{3}} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x^(2/3))/(-1+x^(2/3)),x, algorithm="fricas")
[Out] x + 6*x^(1/3) - 3*log(x^(1/3) + 1) + 3*log(x^(1/3) - 1)
```

Sympy [A]
time = 0.19, size = 27, normalized size = 1.59

$$6\sqrt[3]{x} + x + 3\log\left(\sqrt[3]{x} - 1\right) - 3\log\left(\sqrt[3]{x} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x**(2/3))/(-1+x**(2/3)),x)
[Out] 6*x**(1/3) + x + 3*log(x**(1/3) - 1) - 3*log(x**(1/3) + 1)
```

Giac [A]
time = 1.37, size = 24, normalized size = 1.41

$$x + 6\,x^{\frac{1}{3}} - 3\,\log\left(x^{\frac{1}{3}} + 1\right) + 3\,\log\left(\left|x^{\frac{1}{3}} - 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x^(2/3))/(-1+x^(2/3)),x, algorithm="giac")
[Out] x + 6*x^(1/3) - 3*log(x^(1/3) + 1) + 3*log(abs(x^(1/3) - 1))
```

Mupad [B]
time = 1.46, size = 13, normalized size = 0.76

$$x - 6\,\mathrm{atanh}\left(x^{1/3}\right) + 6\,x^{1/3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(2/3) + 1)/(x^(2/3) - 1),x)
[Out] x - 6*atanh(x^(1/3)) + 6*x^(1/3)
```

3.279 $\int \frac{-16+x^{3/4}}{16+x^{3/4}}\,dx$

Optimal. Leaf size=104

$$-128\sqrt[4]{x}+x-\frac{256\sqrt[3]{2}\tan^{-1}\left(\frac{\sqrt[3]{2}-\sqrt[4]{x}}{\sqrt[3]{2}\sqrt{3}}\right)}{\sqrt{3}}+\frac{256}{3}\sqrt[3]{2}\log\left(2\sqrt[3]{2}+\sqrt[4]{x}\right)-\frac{128}{3}\sqrt[3]{2}\log\left(4\ 2^{2/3}-2\sqrt[3]{2}\sqrt[4]{x}+\right.$$

```
[Out] -128*x^(1/4)+x+256/3*2^(1/3)*ln(2*2^(1/3)+x^(1/4))-128/3*2^(1/3)*ln(4*2^(2/
3)-2*2^(1/3)*x^(1/4)+x^(1/2))-256/3*2^(1/3)*arctan(1/6*(2^(1/3)-x^(1/4))*2^
(2/3)*3^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.529$, Rules used = {383, 470, 327, 206, 31, 648, 631, 210, 642}

$$-\frac{256\sqrt[3]{2}\,\text{ArcTan}\left(\frac{\sqrt[3]{2}-\sqrt[4]{x}}{\sqrt[3]{2}\sqrt{3}}\right)}{\sqrt{3}}+x-128\sqrt[4]{x}+\frac{256}{3}\sqrt[3]{2}\log\left(\sqrt[4]{x}+2\sqrt[3]{2}\right)-\frac{128}{3}\sqrt[3]{2}\log\left(\sqrt{x}-2\sqrt[3]{2}\sqrt[4]{x}+4\ 2^{2/3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(-16 + x^(3/4))/(16 + x^(3/4)),x]
```

```
[Out] -128*x^(1/4) + x - (256*2^(1/3)*ArcTan[(2^(1/3) - x^(1/4))/(2^(1/3)*Sqrt[3]
)])/Sqrt[3] + (256*2^(1/3)*Log[2*2^(1/3) + x^(1/4)])/3 - (128*2^(1/3)*Log[4
*2^(2/3) - 2*2^(1/3)*x^(1/4) + Sqrt[x]])/3
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 383

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(a + b*x^(g*n))
^p*(c + d*x^(g*n))^q, x], x, x^(1/g)], x]] /; FreeQ[{a, b, c, d, p, q}, x]
&& NeQ[b*c - a*d, 0] && FractionQ[n]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{-16+x^{3/4}}{16+x^{3/4}}\,dx = 4\text{Subst}\left(\int \frac{x^3(-16+x^3)}{16+x^3}\,dx, x, \sqrt[4]{x}\right)$$
$$= x - 128\text{Subst}\left(\int \frac{x^3}{16+x^3}\,dx, x, \sqrt[4]{x}\right)$$
$$= -128\sqrt[4]{x} + x + 2048\text{Subst}\left(\int \frac{1}{16+x^3}\,dx, x, \sqrt[4]{x}\right)$$
$$= -128\sqrt[4]{x} + x + \frac{1}{3}\left(256\sqrt[3]{2}\right)\text{Subst}\left(\int \frac{1}{2\sqrt[3]{2}+x}\,dx, x, \sqrt[4]{x}\right) + \frac{1}{3}\left(256\sqrt[3]{2}\right)\text{Subst}\Bigg(\int$$
$$= -128\sqrt[4]{x} + x + \frac{256}{3}\sqrt[3]{2}\log\left(2\sqrt[3]{2}+\sqrt[4]{x}\right) - \frac{1}{3}\left(128\sqrt[3]{2}\right)\text{Subst}\Bigg(\int \frac{-2\sqrt[3]{2}+2x}{4\ 2^{2/3}-2\sqrt[3]{2}\,x}$$
$$= -128\sqrt[4]{x} + x + \frac{256}{3}\sqrt[3]{2}\log\left(2\sqrt[3]{2}+\sqrt[4]{x}\right) - \frac{128}{3}\sqrt[3]{2}\log\left(4\ 2^{2/3}-2\sqrt[3]{2}\sqrt[4]{x}+\sqrt{x}\right)$$
$$= -128\sqrt[4]{x} + x - \frac{256\sqrt[3]{2}\tan^{-1}\left(\frac{2-2^{2/3}\sqrt[4]{x}}{2\sqrt{3}}\right)}{\sqrt{3}} + \frac{256}{3}\sqrt[3]{2}\log\left(2\sqrt[3]{2}+\sqrt[4]{x}\right) - \frac{128}{3}\sqrt[3]{2}$$

Mathematica [A]
time = 0.11, size = 104, normalized size = 1.00

$$-128\sqrt[4]{x} + x - \frac{256\sqrt[3]{2}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{\sqrt[4]{x}}{\sqrt[3]{2}\sqrt{3}}\right)}{\sqrt{3}} + \frac{256}{3}\sqrt[3]{2}\log\left(4+2^{2/3}\sqrt[4]{x}\right) - \frac{128}{3}\sqrt[3]{2}\log\left(-8+2\ 2^{2/3}\sqrt[4]{x}-\sqrt[3]{2}\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-16 + x^(3/4))/(16 + x^(3/4)),x]
```

```
[Out] -128*x^(1/4) + x - (256*2^(1/3)*ArcTan[1/Sqrt[3] - x^(1/4)/(2^(1/3)*Sqrt[3]
)])/Sqrt[3] + (256*2^(1/3)*Log[4 + 2^(2/3)*x^(1/4)])/3 - (128*2^(1/3)*Log[-
8 + 2*2^(2/3)*x^(1/4) - 2^(1/3)*Sqrt[x]])/3
```

Maple [A]
time = 56.06, size = 66, normalized size = 0.63

method	result
derivativedivides	$x-128x^{\frac{1}{4}}+\frac{128\,16^{\frac{1}{3}}\ln\left(x^{\frac{1}{4}}+16^{\frac{1}{3}}\right)}{3}-\frac{64\,16^{\frac{1}{3}}\ln\left(\sqrt{x}-16^{\frac{1}{3}}x^{\frac{1}{4}}+16^{\frac{2}{3}}\right)}{3}+\frac{128\,16^{\frac{1}{3}}\sqrt{3}\ \arctan\left(\frac{\sqrt{3}\ \left(\frac{16^{\frac{2}{3}}}{8}\right.}{3}\right.}{3}$

default	$x-128x^{\frac{1}{4}}+\frac{128\,16^{\frac{1}{3}}\ln\left(x^{\frac{1}{4}}+16^{\frac{1}{3}}\right)}{3}-\frac{64\,16^{\frac{1}{3}}\ln\left(\sqrt{x}-16^{\frac{1}{3}}x^{\frac{1}{4}}+16^{\frac{2}{3}}\right)}{3}+\frac{128\,16^{\frac{1}{3}}\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,\left(\frac{16^{\frac{2}{3}}x^{\frac{1}{4}}}{8}\right.}{3}\right.}{3}$
meijerg	$-\frac{128\,2^{\frac{1}{3}}\left(\frac{3x^{\frac{1}{4}}2^{\frac{2}{3}}}{4}-\frac{x^{\frac{1}{4}}2^{\frac{2}{3}}\left(\frac{2\,2^{\frac{1}{3}}\ln\left(1+\frac{x^{\frac{1}{4}}2^{\frac{2}{3}}}{4}\right)}{x^{\frac{1}{4}}}-\frac{2^{\frac{1}{3}}\ln\left(1-\frac{x^{\frac{1}{4}}2^{\frac{2}{3}}}{4}+\frac{2^{\frac{1}{3}}\sqrt{x}}{8}\right)}{x^{\frac{1}{4}}}+\frac{2\,2^{\frac{1}{3}}\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,2^{\frac{2}{3}}x^{\frac{1}{4}}}{8-x^{\frac{1}{4}}2^{\frac{2}{3}}}\right)}{x^{\frac{1}{4}}}\right)}{4}\right)}{3}+$
trager	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-16+x^(3/4))/(16+x^(3/4)),x,method=_RETURNVERBOSE)
```

```
[Out] x-128*x^(1/4)+128/3*16^(1/3)*ln(x^(1/4)+16^(1/3))-64/3*16^(1/3)*ln(x^(1/2)-
16^(1/3)*x^(1/4)+16^(2/3))+128/3*16^(1/3)*3^(1/2)*arctan(1/3*3^(1/2)*(1/8*1
6^(2/3)*x^(1/4)-1))
```

Maxima [A]
time = 0.48, size = 71, normalized size = 0.68

$$\frac{256}{3}\sqrt{3}\,2^{\frac{1}{3}}\arctan\left(-\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}-x^{\frac{1}{4}}\right)\right)-\frac{128}{3}\cdot 2^{\frac{1}{3}}\log\left(4\cdot 2^{\frac{2}{3}}-2\cdot 2^{\frac{1}{3}}x^{\frac{1}{4}}+\sqrt{x}\right)+\frac{256}{3}\cdot 2^{\frac{1}{3}}\log\left(2\cdot 2^{\frac{1}{3}}+x^{\frac{1}{4}}\right)+x-128\,x^{\frac{1}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-16+x^(3/4))/(16+x^(3/4)),x, algorithm="maxima")
```

```
[Out] 256/3*sqrt(3)*2^(1/3)*arctan(-1/6*sqrt(3)*2^(2/3)*(2^(1/3) - x^(1/4))) - 12
8/3*2^(1/3)*log(4*2^(2/3) - 2*2^(1/3)*x^(1/4) + sqrt(x)) + 256/3*2^(1/3)*lo
g(2*2^(1/3) + x^(1/4)) + x - 128*x^(1/4)
```

Fricas [A]
time = 2.84, size = 71, normalized size = 0.68

$$\frac{256}{3}\sqrt{3}\,2^{\frac{1}{3}}\arctan\left(\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}x^{\frac{1}{4}}-\frac{1}{3}\sqrt{3}\right)-\frac{128}{3}\cdot 2^{\frac{1}{3}}\log\left(4\cdot 2^{\frac{2}{3}}-2\cdot 2^{\frac{1}{3}}x^{\frac{1}{4}}+\sqrt{x}\right)+\frac{256}{3}\cdot 2^{\frac{1}{3}}\log\left(2\cdot 2^{\frac{1}{3}}+x^{\frac{1}{4}}\right)+x-128\,x^{\frac{1}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-16+x^(3/4))/(16+x^(3/4)),x, algorithm="fricas")
```

```
[Out] 256/3*sqrt(3)*2^(1/3)*arctan(1/6*sqrt(3)*2^(2/3)*x^(1/4) - 1/3*sqrt(3)) - 1
28/3*2^(1/3)*log(4*2^(2/3) - 2*2^(1/3)*x^(1/4) + sqrt(x)) + 256/3*2^(1/3)*l
og(2*2^(1/3) + x^(1/4)) + x - 128*x^(1/4)
```

Sympy [A]
time = 1.85, size = 102, normalized size = 0.98

$$-128\sqrt[4]{x}+x+\frac{256\cdot\sqrt[3]{2}\log\left(\sqrt[4]{x}+2\cdot\sqrt[3]{2}\right)}{3}-\frac{128\cdot\sqrt[3]{2}\log\left(-2\cdot\sqrt[3]{2}\sqrt[4]{x}+\sqrt{x}+4\cdot2^{\frac{2}{3}}\right)}{3}+\frac{256\cdot\sqrt[3]{2}\sqrt{3}\operatorname{atan}\left(\frac{2^{\frac{2}{3}}\sqrt{3}\sqrt[4]{x}}{6}-\frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-16+x**(3/4))/(16+x**(3/4)),x)
```

```
[Out] -128*x**(1/4) + x + 256*2**(1/3)*log(x**(1/4) + 2*2**(1/3))/3 - 128*2**(1/3
)*log(-2*2**(1/3)*x**(1/4) + sqrt(x) + 4*2**(2/3))/3 + 256*2**(1/3)*sqrt(3)
*atan(2**(2/3)*sqrt(3)*x**(1/4)/6 - sqrt(3)/3)/3
```

Giac [A]
time = 1.22, size = 71, normalized size = 0.68

$$\frac{256}{3}\sqrt{3}\,2^{\frac{1}{3}}\arctan\left(-\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}-x^{\frac{1}{4}}\right)\right)-\frac{128}{3}\cdot2^{\frac{1}{3}}\log\left(4\cdot2^{\frac{2}{3}}-2\cdot2^{\frac{1}{3}}x^{\frac{1}{4}}+\sqrt{x}\right)+\frac{256}{3}\cdot2^{\frac{1}{3}}\log\left(2\cdot2^{\frac{1}{3}}+x^{\frac{1}{4}}\right)+x-128\,x^{\frac{1}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-16+x^(3/4))/(16+x^(3/4)),x, algorithm="giac")
```

```
[Out] 256/3*sqrt(3)*2^(1/3)*arctan(-1/6*sqrt(3)*2^(2/3)*(2^(1/3) - x^(1/4))) - 12
8/3*2^(1/3)*log(4*2^(2/3) - 2*2^(1/3)*x^(1/4) + sqrt(x)) + 256/3*2^(1/3)*lo
g(2*2^(1/3) + x^(1/4)) + x - 128*x^(1/4)
```

Mupad [B]
time = 1.50, size = 90, normalized size = 0.87

$$x+\frac{256\,2^{1/3}\ln\left(12288\,2^{1/3}+6144\,x^{1/4}\right)}{3}-128\,x^{1/4}+\frac{128\,2^{1/3}\ln\left(6144\,x^{1/4}+6144\,2^{1/3}\left(-1+\sqrt{3}\,1\mathrm{i}\right)\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{3}-\frac{128\,2^{1/3}\ln\left(6144\,x^{1/4}-6144\,2^{1/3}\left(1+\sqrt{3}\,1\mathrm{i}\right)\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(3/4) - 16)/(x^(3/4) + 16),x)
```

```
[Out] x + (256*2^(1/3)*log(12288*2^(1/3) + 6144*x^(1/4)))/3 - 128*x^(1/4) + (128*
2^(1/3)*log(6144*x^(1/4) + 6144*2^(1/3)*(3^(1/2)*1i - 1))*(3^(1/2)*1i - 1))
/3 - (128*2^(1/3)*log(6144*x^(1/4) - 6144*2^(1/3)*(3^(1/2)*1i + 1))*(3^(1/2
)*1i + 1))/3
```

3.280 $\int \frac{1+\frac{1}{\sqrt[3]{x}}}{-1+\frac{1}{\sqrt[3]{x}}} \, dx$

Optimal. Leaf size=30

$$-6\sqrt[3]{x}-3x^{2/3}-x-6\log\left(1-\sqrt[3]{x}\right)$$

```
[Out] -6*x^(1/3)-3*x^(2/3)-x-6*ln(1-x^(1/3))
```

Rubi [A]
time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {381, 383, 78}

$$-3x^{2/3}-x-6\sqrt[3]{x}-6\log\left(1-\sqrt[3]{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^(-1/3))/(-1 + x^(-1/3)),x]
```

```
[Out] -6*x^(1/3) - 3*x^(2/3) - x - 6*Log[1 - x^(1/3)]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 381

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[x^(n*(p + q))*(b + a/x^n)^p*(d + c/x^n)^q, x] /; FreeQ[{a, b, c, d
, n}, x] && NeQ[b*c - a*d, 0] && IntegersQ[p, q] && NegQ[n]
```

Rule 383

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(a + b*x^(g*n))
^p*(c + d*x^(g*n))^q, x], x, x^(1/g)], x]] /; FreeQ[{a, b, c, d, p, q}, x]
&& NeQ[b*c - a*d, 0] && FractionQ[n]
```

Rubi steps

$$\int \frac{1+\frac{1}{\sqrt[3]{x}}}{-1+\frac{1}{\sqrt[3]{x}}}\,dx = \int \frac{1+\sqrt[3]{x}}{1-\sqrt[3]{x}}\,dx$$
$$= 3\text{Subst}\left(\int \frac{x^2(1+x)}{1-x}\,dx, x, \sqrt[3]{x}\right)$$
$$= 3\text{Subst}\left(\int \left(-2-\frac{2}{-1+x}-2x-x^2\right)\,dx, x, \sqrt[3]{x}\right)$$
$$= -6\sqrt[3]{x}-3x^{2/3}-x-6\log\left(1-\sqrt[3]{x}\right)$$

Mathematica [A]
time = 0.02, size = 28, normalized size = 0.93

$$-6\sqrt[3]{x}-3x^{2/3}-x-6\log\left(-1+\sqrt[3]{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^(-1/3))/(-1 + x^(-1/3)),x]
```

```
[Out] -6*x^(1/3) - 3*x^(2/3) - x - 6*Log[-1 + x^(1/3)]
```

Maple [A]
time = 0.24, size = 23, normalized size = 0.77

method	result	size
derivativedivides	$-x-3x^{\frac{2}{3}}-6x^{\frac{1}{3}}-6\ln\left(x^{\frac{1}{3}}-1\right)$	23
default	$-x-3x^{\frac{2}{3}}-6x^{\frac{1}{3}}-6\ln\left(x^{\frac{1}{3}}-1\right)$	23
trager	$2-x-6x^{\frac{1}{3}}-3x^{\frac{2}{3}}-2\ln\left(-3x^{\frac{2}{3}}+3x^{\frac{1}{3}}+x-1\right)$	32
meijerg	$-\frac{x^{\frac{1}{3}}\left(4x^{\frac{2}{3}}+6x^{\frac{1}{3}}+12\right)}{4}-6\ln\left(1-x^{\frac{1}{3}}\right)-\frac{x^{\frac{1}{3}}\left(6+3x^{\frac{1}{3}}\right)}{2}$	41

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+1/x^(1/3))/(-1+1/x^(1/3)),x,method=_RETURNVERBOSE)
```

```
[Out] -x-3*x^(2/3)-6*x^(1/3)-6*ln(x^(1/3)-1)
```

Maxima [A]
time = 0.27, size = 22, normalized size = 0.73

$$-x-3\,x^{\frac{2}{3}}-6\,x^{\frac{1}{3}}-6\,\log\left(x^{\frac{1}{3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+1/x^(1/3))/(-1+1/x^(1/3)),x, algorithm="maxima")
```

```
[Out] -x - 3*x^(2/3) - 6*x^(1/3) - 6*log(x^(1/3) - 1)
```

Fricas [A]
time = 2.15, size = 22, normalized size = 0.73

$$-x-3\,x^{\frac{2}{3}}-6\,x^{\frac{1}{3}}-6\,\log\left(x^{\frac{1}{3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+1/x^(1/3))/(-1+1/x^(1/3)),x, algorithm="fricas")
```

```
[Out] -x - 3*x^(2/3) - 6*x^(1/3) - 6*log(x^(1/3) - 1)
```

Sympy [A]
time = 0.05, size = 26, normalized size = 0.87

$$-3x^{\frac{2}{3}}-6\sqrt[3]{x}-x-6\log\left(\sqrt[3]{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+1/x**(1/3))/(-1+1/x**(1/3)),x)
```

```
[Out] -3*x**(2/3) - 6*x**(1/3) - x - 6*log(x**(1/3) - 1)
```

Giac [A]
time = 1.30, size = 23, normalized size = 0.77

$$-x-3\,x^{\frac{2}{3}}-6\,x^{\frac{1}{3}}-6\,\log\left(\left|x^{\frac{1}{3}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+1/x^(1/3))/(-1+1/x^(1/3)),x, algorithm="giac")
```

```
[Out] -x - 3*x^(2/3) - 6*x^(1/3) - 6*log(abs(x^(1/3) - 1))
```

Mupad [B]
time = 0.04, size = 22, normalized size = 0.73

$$-x-6\,\ln\left(x^{1/3}-1\right)-6\,x^{1/3}-3\,x^{2/3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/x^(1/3) + 1)/(1/x^(1/3) - 1),x)
```

```
[Out] - x - 6*log(x^(1/3) - 1) - 6*x^(1/3) - 3*x^(2/3)
```

3.281 $\int (a-bx^n)^{3/2} (a+bx^n)^{3/2} \, dx$

Optimal. Leaf size=79

$$\frac{a^2 x \sqrt{a-bx^n} \sqrt{a+bx^n} \, _2F_1\left(-\frac{3}{2},\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);\frac{b^2x^{2n}}{a^2}\right)}{\sqrt{1-\frac{b^2x^{2n}}{a^2}}}$$

```
[Out] a^2*x*hypergeom([-3/2, 1/2/n],[1+1/2/n],b^2*x^(2*n)/a^2)*(a-b*x^n)^(1/2)*(a
+b*x^n)^(1/2)/(1-b^2*x^(2*n)/a^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {259, 252, 251}

$$\frac{a^2 x \sqrt{a-bx^n} \sqrt{a+bx^n} \, _2F_1\left(-\frac{3}{2},\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);\frac{b^2x^{2n}}{a^2}\right)}{\sqrt{1-\frac{b^2x^{2n}}{a^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^n)^(3/2)*(a + b*x^n)^(3/2),x]
```

```
[Out] (a^2*x*Sqrt[a - b*x^n]*Sqrt[a + b*x^n]*Hypergeometric2F1[-3/2, 1/(2*n), (2
+ n^(-1))/2, (b^2*x^(2*n))/a^2])/Sqrt[1 - (b^2*x^(2*n))/a^2]
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 259

```
Int[((a1_.) + (b1_.)*(x_)^(n_))^(p_)*((a2_.) + (b2_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Dist[(a1 + b1*x^n)^FracPart[p]*((a2 + b2*x^n)^FracPart[p]/(a1*a2 +
 b1*b2*x^(2*n))^FracPart[p]), Int[(a1*a2 + b1*b2*x^(2*n))^p, x], x] /; Free
```

```
Q[{a1, b1, a2, b2, n, p}, x] && EqQ[a2*b1 + a1*b2, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int (a-bx^n)^{3/2}(a+bx^n)^{3/2}\,dx &= \frac{\left(\sqrt{a-bx^n}\,\sqrt{a+bx^n}\right)\int \left(a^2-b^2x^{2n}\right)^{3/2}\,dx}{\sqrt{a^2-b^2x^{2n}}}\\ &= \frac{\left(a^2\sqrt{a-bx^n}\,\sqrt{a+bx^n}\right)\int \left(1-\frac{b^2x^{2n}}{a^2}\right)^{3/2}\,dx}{\sqrt{1-\frac{b^2x^{2n}}{a^2}}}\\ &= \frac{a^2x\sqrt{a-bx^n}\,\sqrt{a+bx^n}\;{}_2F_1\left(-\frac{3}{2},\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);\frac{b^2x^{2n}}{a^2}\right)}{\sqrt{1-\frac{b^2x^{2n}}{a^2}}}\end{aligned}$$

Mathematica [A]
time = 0.24, size = 79, normalized size = 1.00

$$\frac{a^2x\sqrt{a-bx^n}\,\sqrt{a+bx^n}\;{}_2F_1\left(-\frac{3}{2},\frac{1}{2n};1+\frac{1}{2n};\frac{b^2x^{2n}}{a^2}\right)}{\sqrt{1-\frac{b^2x^{2n}}{a^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^n)^(3/2)*(a + b*x^n)^(3/2),x]
[Out] (a^2*x*Sqrt[a - b*x^n]*Sqrt[a + b*x^n]*Hypergeometric2F1[-3/2, 1/(2*n), 1 +
 1/(2*n), (b^2*x^(2*n))/a^2])/Sqrt[1 - (b^2*x^(2*n))/a^2]
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a-b\,x^{n}\right)^{\frac{3}{2}}\left(a+b\,x^{n}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-b*x^n)^(3/2)*(a+b*x^n)^(3/2),x)
[Out] int((a-b*x^n)^(3/2)*(a+b*x^n)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-b*x^n)^(3/2)*(a+b*x^n)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^n + a)^(3/2)*(-b*x^n + a)^(3/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-b*x^n)^(3/2)*(a+b*x^n)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a - bx^n)^{\frac{3}{2}} (a + bx^n)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-b*x**n)**(3/2)*(a+b*x**n)**(3/2),x)
```

```
[Out] Integral((a - b*x**n)**(3/2)*(a + b*x**n)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-b*x^n)^(3/2)*(a+b*x^n)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a)^(3/2)*(-b*x^n + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (a + b\,x^n)^{3/2}\,(a - b\,x^n)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)^(3/2)*(a - b*x^n)^(3/2),x)
```

```
[Out] int((a + b*x^n)^(3/2)*(a - b*x^n)^(3/2), x)
```

3.282 $\int \sqrt{a-bx^n}\,\sqrt{a+bx^n}\,dx$

Optimal. Leaf size=76

$$\frac{x\sqrt{a-bx^n}\,\sqrt{a+bx^n}\,{}_2F_1\left(-\frac{1}{2},\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);\frac{b^2x^{2n}}{a^2}\right)}{\sqrt{1-\frac{b^2x^{2n}}{a^2}}}$$

```
[Out] x*hypergeom([-1/2, 1/2/n],[1+1/2/n],b^2*x^(2*n)/a^2)*(a-b*x^n)^(1/2)*(a+b*x
^n)^(1/2)/(1-b^2*x^(2*n)/a^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {259, 252, 251}

$$\frac{x\sqrt{a-bx^n}\,\sqrt{a+bx^n}\,{}_2F_1\left(-\frac{1}{2},\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);\frac{b^2x^{2n}}{a^2}\right)}{\sqrt{1-\frac{b^2x^{2n}}{a^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a - b*x^n]*Sqrt[a + b*x^n],x]
```

```
[Out] (x*Sqrt[a - b*x^n]*Sqrt[a + b*x^n]*Hypergeometric2F1[-1/2, 1/(2*n), (2 + n^
(-1))/2, (b^2*x^(2*n))/a^2])/Sqrt[1 - (b^2*x^(2*n))/a^2]
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 259

```
Int[((a1_.) + (b1_.)*(x_)^(n_))^(p_)*((a2_.) + (b2_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Dist[(a1 + b1*x^n)^FracPart[p]*((a2 + b2*x^n)^FracPart[p]/(a1*a2 +
 b1*b2*x^(2*n))^FracPart[p]), Int[(a1*a2 + b1*b2*x^(2*n))^p, x], x] /; Free
```

```
Q[{a1, b1, a2, b2, n, p}, x] && EqQ[a2*b1 + a1*b2, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \sqrt{a-bx^n}\sqrt{a+bx^n}\,dx = \frac{\left(\sqrt{a-bx^n}\sqrt{a+bx^n}\right)\int \sqrt{a^2-b^2x^{2n}}\,dx}{\sqrt{a^2-b^2x^{2n}}}$$
$$= \frac{\left(\sqrt{a-bx^n}\sqrt{a+bx^n}\right)\int \sqrt{1-\frac{b^2x^{2n}}{a^2}}\,dx}{\sqrt{1-\frac{b^2x^{2n}}{a^2}}}$$
$$= \frac{x\sqrt{a-bx^n}\sqrt{a+bx^n}\,{}_2F_1\left(-\frac{1}{2},\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);\frac{b^2x^{2n}}{a^2}\right)}{\sqrt{1-\frac{b^2x^{2n}}{a^2}}}$$

Mathematica [A]
time = 0.08, size = 76, normalized size = 1.00

$$\frac{x\sqrt{a-bx^n}\sqrt{a+bx^n}\,{}_2F_1\left(-\frac{1}{2},\frac{1}{2n};1+\frac{1}{2n};\frac{b^2x^{2n}}{a^2}\right)}{\sqrt{1-\frac{b^2x^{2n}}{a^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a - b*x^n]*Sqrt[a + b*x^n],x]
[Out] (x*Sqrt[a - b*x^n]*Sqrt[a + b*x^n]*Hypergeometric2F1[-1/2, 1/(2*n), 1 + 1/(
2*n), (b^2*x^(2*n))/a^2])/Sqrt[1 - (b^2*x^(2*n))/a^2]
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a-b\,x^n}\,\sqrt{a+b\,x^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-b*x^n)^(1/2)*(a+b*x^n)^(1/2),x)
[Out] int((a-b*x^n)^(1/2)*(a+b*x^n)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-b*x^n)^(1/2)*(a+b*x^n)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^n + a)*sqrt(-b*x^n + a), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-b*x^n)^(1/2)*(a+b*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a - bx^n}\,\sqrt{a + bx^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-b*x**n)**(1/2)*(a+b*x**n)**(1/2),x)
```

```
[Out] Integral(sqrt(a - b*x**n)*sqrt(a + b*x**n), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-b*x^n)^(1/2)*(a+b*x^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^n + a)*sqrt(-b*x^n + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{a+b\,x^n}\,\sqrt{a-b\,x^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)^(1/2)*(a - b*x^n)^(1/2),x)
```

```
[Out] int((a + b*x^n)^(1/2)*(a - b*x^n)^(1/2), x)
```

3.283 $\int (a-bx^n)^p (a+bx^n)^p \, dx$

Optimal. Leaf size=72

$$x(a-bx^n)^p (a+bx^n)^p \left(1-\frac{b^2x^{2n}}{a^2}\right)^{-p} {}_2F_1\left(\frac{1}{2n},-p;\frac{1}{2}\left(2+\frac{1}{n}\right);\frac{b^2x^{2n}}{a^2}\right)$$

```
[Out] x*(a-b*x^n)^p*(a+b*x^n)^p*hypergeom([-p, 1/2/n],[1+1/2/n],b^2*x^(2*n)/a^2)/
((1-b^2*x^(2*n)/a^2)^p)
```

Rubi [A]
time = 0.02, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {259, 252, 251}

$$x(a-bx^n)^p (a+bx^n)^p \left(1-\frac{b^2x^{2n}}{a^2}\right)^{-p} {}_2F_1\left(\frac{1}{2n},-p;\frac{1}{2}\left(2+\frac{1}{n}\right);\frac{b^2x^{2n}}{a^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^n)^p*(a + b*x^n)^p,x]
```

```
[Out] (x*(a - b*x^n)^p*(a + b*x^n)^p*Hypergeometric2F1[1/(2*n), -p, (2 + n^(-1))/
2, (b^2*x^(2*n))/a^2])/(1 - (b^2*x^(2*n))/a^2)^p
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 259

```
Int[((a1_.) + (b1_.)*(x_)^(n_))^(p_)*((a2_.) + (b2_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Dist[(a1 + b1*x^n)^FracPart[p]*((a2 + b2*x^n)^FracPart[p]/(a1*a2 +
 b1*b2*x^(2*n))^FracPart[p]), Int[(a1*a2 + b1*b2*x^(2*n))^p, x], x] /; Free
Q[{a1, b1, a2, b2, n, p}, x] && EqQ[a2*b1 + a1*b2, 0] && !IntegerQ[p]
```

Rubi steps

$$\int (a-bx^n)^p (a+bx^n)^p \, dx = \left((a-bx^n)^p (a+bx^n)^p \left(a^2-b^2x^{2n}\right)^{-p}\right) \int \left(a^2-b^2x^{2n}\right)^p \, dx$$
$$= \left((a-bx^n)^p (a+bx^n)^p \left(1-\frac{b^2x^{2n}}{a^2}\right)^{-p}\right) \int \left(1-\frac{b^2x^{2n}}{a^2}\right)^p \, dx$$
$$= x(a-bx^n)^p (a+bx^n)^p \left(1-\frac{b^2x^{2n}}{a^2}\right)^{-p} {}_2F_1\left(\frac{1}{2n},-p;\frac{1}{2}\left(2+\frac{1}{n}\right);\frac{b^2x^{2n}}{a^2}\right)$$

Mathematica [A]
time = 0.04, size = 72, normalized size = 1.00

$$x(a-bx^n)^p (a+bx^n)^p \left(1-\frac{b^2x^{2n}}{a^2}\right)^{-p} {}_2F_1\left(\frac{1}{2n},-p;1+\frac{1}{2n};\frac{b^2x^{2n}}{a^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^n)^p*(a + b*x^n)^p,x]
[Out] (x*(a - b*x^n)^p*(a + b*x^n)^p*Hypergeometric2F1[1/(2*n), -p, 1 + 1/(2*n), 
(b^2*x^(2*n))/a^2])/(1 - (b^2*x^(2*n))/a^2)^p
```

Maple [F]
time = 0.13, size = 0, normalized size = 0.00

$$\int (a-b\,x^n)^p \,(a+b\,x^n)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-b*x^n)^p*(a+b*x^n)^p,x)
[Out] int((a-b*x^n)^p*(a+b*x^n)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-b*x^n)^p*(a+b*x^n)^p,x, algorithm="maxima")
[Out] integrate((b*x^n + a)^p*(-b*x^n + a)^p, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-b*x^n)^p*(a+b*x^n)^p,x, algorithm="fricas")
```

```
[Out] integral((b*x^n + a)^p*(-b*x^n + a)^p, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a - bx^n)^p (a + bx^n)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-b*x**n)**p*(a+b*x**n)**p,x)
```

```
[Out] Integral((a - b*x**n)**p*(a + b*x**n)**p, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-b*x^n)^p*(a+b*x^n)^p,x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a)^p*(-b*x^n + a)^p, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (a + b\,x^n)^p\,(a - b\,x^n)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)^p*(a - b*x^n)^p,x)
```

```
[Out] int((a + b*x^n)^p*(a - b*x^n)^p, x)
```

3.284 $\int (a+bx^n)(c+dx^n)^4\,dx$

Optimal. Leaf size=132

$$ac^4x+\frac{c^3(bc+4ad)x^{1+n}}{1+n}+\frac{2c^2d(2bc+3ad)x^{1+2n}}{1+2n}+\frac{2cd^2(3bc+2ad)x^{1+3n}}{1+3n}+\frac{d^3(4bc+ad)x^{1+4n}}{1+4n}+\frac{bd^4x^{1+5n}}{1+5n}$$

```
[Out] a*c^4*x+c^3*(4*a*d+b*c)*x^(1+n)/(1+n)+2*c^2*d*(3*a*d+2*b*c)*x^(1+2*n)/(1+2*
n)+2*c*d^2*(2*a*d+3*b*c)*x^(1+3*n)/(1+3*n)+d^3*(a*d+4*b*c)*x^(1+4*n)/(1+4*n
)+b*d^4*x^(1+5*n)/(1+5*n)
```

Rubi [A]
time = 0.08, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {380}

$$\frac{c^3x^{n+1}(4ad+bc)}{n+1}+\frac{2c^2dx^{2n+1}(3ad+2bc)}{2n+1}+\frac{d^3x^{4n+1}(ad+4bc)}{4n+1}+\frac{2cd^2x^{3n+1}(2ad+3bc)}{3n+1}+ac^4x+\frac{bd^4x^{5n+1}}{5n+1}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n)*(c + d*x^n)^4,x]
```

```
[Out] a*c^4*x + (c^3*(b*c + 4*a*d)*x^(1 + n))/(1 + n) + (2*c^2*d*(2*b*c + 3*a*d)*
x^(1 + 2*n))/(1 + 2*n) + (2*c*d^2*(3*b*c + 2*a*d)*x^(1 + 3*n))/(1 + 3*n) +
(d^3*(4*b*c + a*d)*x^(1 + 4*n))/(1 + 4*n) + (b*d^4*x^(1 + 5*n))/(1 + 5*n)
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int (a+bx^n)(c+dx^n)^4\,dx &= \int \left(ac^4+c^3(bc+4ad)x^n+2c^2d(2bc+3ad)x^{2n}+2cd^2(3bc+2ad)x^{3n}+d^3(4bc\right.\\ &= ac^4x+\frac{c^3(bc+4ad)x^{1+n}}{1+n}+\frac{2c^2d(2bc+3ad)x^{1+2n}}{1+2n}+\frac{2cd^2(3bc+2ad)x^{1+3n}}{1+3n}+\frac{d}{}\end{aligned}$$

Mathematica [A]
time = 0.49, size = 123, normalized size = 0.93

$$x\left(ac^4+\frac{c^3(bc+4ad)x^n}{1+n}+\frac{2c^2d(2bc+3ad)x^{2n}}{1+2n}+\frac{2cd^2(3bc+2ad)x^{3n}}{1+3n}+\frac{d^3(4bc+ad)x^{4n}}{1+4n}+\frac{bd^4x^{5n}}{1+5n}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^n)*(c + d*x^n)^4,x]
```

```
[Out] x*(a*c^4 + (c^3*(b*c + 4*a*d)*x^n)/(1 + n) + (2*c^2*d*(2*b*c + 3*a*d)*x^(2*
n))/(1 + 2*n) + (2*c*d^2*(3*b*c + 2*a*d)*x^(3*n))/(1 + 3*n) + (d^3*(4*b*c +
 a*d)*x^(4*n))/(1 + 4*n) + (b*d^4*x^(5*n))/(1 + 5*n))
```

Maple [A]
time = 0.26, size = 128, normalized size = 0.97

method	result
risch	$a\,c^4x+\frac{b\,d^4x\,x^{5n}}{1+5n}+\frac{c^3(4ad+bc)x\,x^n}{1+n}+\frac{d^3(ad+4bc)x\,x^{4n}}{1+4n}+\frac{2c\,d^2(2ad+3bc)x\,x^{3n}}{1+3n}+\frac{2c^2d(3ad+2bc)x\,x^{2n}}{1+2n}$
norman	$a\,c^4x+\frac{b\,d^4x\,e^{5n\ln(x)}}{1+5n}+\frac{c^3(4ad+bc)x\,e^{n\ln(x)}}{1+n}+\frac{d^3(ad+4bc)x\,e^{4n\ln(x)}}{1+4n}+\frac{2c\,d^2(2ad+3bc)x\,e^{3n\ln(x)}}{1+3n}+\frac{2c^2d(3ad+2bc)x\,e^{2n\ln(x)}}{1+2n}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^n)*(c+d*x^n)^4,x,method=_RETURNVERBOSE)
```

```
[Out] a*c^4*x+b*d^4/(1+5*n)*x*(x^n)^5+c^3*(4*a*d+b*c)/(1+n)*x*x^n+d^3*(a*d+4*b*c)
/(1+4*n)*x*(x^n)^4+2*c*d^2*(2*a*d+3*b*c)/(1+3*n)*x*(x^n)^3+2*c^2*d*(3*a*d+2
*b*c)/(1+2*n)*x*(x^n)^2
```

Maxima [A]
time = 0.30, size = 186, normalized size = 1.41

$$ac^4x+\frac{bd^4x^{5n+1}}{5n+1}+\frac{4\,bcd^3x^{4n+1}}{4n+1}+\frac{ad^4x^{4n+1}}{4n+1}+\frac{6\,bc^2d^2x^{3n+1}}{3n+1}+\frac{4\,acd^3x^{3n+1}}{3n+1}+\frac{4\,bc^3dx^{2n+1}}{2n+1}+\frac{6\,ac^2d^2x^{2n+1}}{2n+1}+\frac{bc^4x^{n+1}}{n+1}+\frac{4\,ac^3dx^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)*(c+d*x^n)^4,x, algorithm="maxima")
```

```
[Out] a*c^4*x + b*d^4*x^(5*n + 1)/(5*n + 1) + 4*b*c*d^3*x^(4*n + 1)/(4*n + 1) + a
*d^4*x^(4*n + 1)/(4*n + 1) + 6*b*c^2*d^2*x^(3*n + 1)/(3*n + 1) + 4*a*c*d^3*
x^(3*n + 1)/(3*n + 1) + 4*b*c^3*d*x^(2*n + 1)/(2*n + 1) + 6*a*c^2*d^2*x^(2*
n + 1)/(2*n + 1) + b*c^4*x^(n + 1)/(n + 1) + 4*a*c^3*d*x^(n + 1)/(n + 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 527 vs. 2(132) = 264.
time = 2.60, size = 527, normalized size = 3.99

$$\frac{(24\,bd^4n^4+50\,bd^4n^3+35\,bd^4n^2+10\,bd^4n+bd^4)xx^{5n}+(4\,bcd^3+ad^4+30\,(4\,bcd^3+ad^4)n^4+61\,(4\,bcd^3+ad^4)n^3+41\,(4\,bcd^3+ad^4)n^2+11\,(4\,bcd^3+ad^4)n)xx^{4n}+2\,(3\,bc^2d^2+2\,acd^3+40\,(3\,bc^2d^2+2\,acd^3)n^4+78\,(3\,bc^2d^2+2\,acd^3)n^3+49\,(3\,bc^2d^2+2\,acd^3)n^2+12\,(3\,bc^2d^2+2\,acd^3)n)xx^{3n}+2\,(2\,bc^3d+3\,ac^2d^2+60\,(2\,bc^3d+3\,ac^2d^2)n^4+107\,(2\,bc^3d+3\,ac^2d^2)n^3+59\,(2\,bc^3d+3\,ac^2d^2)n^2+13\,(2\,bc^3d+3\,ac^2d^2)n)xx^{2n}+(bc^4+4\,ac^3d+120\,(bc^4+4\,ac^3d)n^4+154\,(bc^4+4\,ac^3d)n^3+71\,(bc^4+4\,ac^3d)n^2+14\,(bc^4+4\,ac^3d)n)xx^n+(120\,ac^4n^5+274\,ac^4n^4+225\,ac^4n^3+85\,ac^4n^2+15\,ac^4n+ac^4)x}{120\,n^5+274\,n^4+225\,n^3+85\,n^2+15\,n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)*(c+d*x^n)^4,x, algorithm="fricas")
```

```
[Out] ((24*b*d^4*n^4 + 50*b*d^4*n^3 + 35*b*d^4*n^2 + 10*b*d^4*n + b*d^4)*x*x^(5*n
) + (4*b*c*d^3 + a*d^4 + 30*(4*b*c*d^3 + a*d^4)*n^4 + 61*(4*b*c*d^3 + a*d^4
```

```
)*n^3 + 41*(4*b*c*d^3 + a*d^4)*n^2 + 11*(4*b*c*d^3 + a*d^4)*n)*x*x^(4*n) +
2*(3*b*c^2*d^2 + 2*a*c*d^3 + 40*(3*b*c^2*d^2 + 2*a*c*d^3)*n^4 + 78*(3*b*c^2
*d^2 + 2*a*c*d^3)*n^3 + 49*(3*b*c^2*d^2 + 2*a*c*d^3)*n^2 + 12*(3*b*c^2*d^2
+ 2*a*c*d^3)*n)*x*x^(3*n) + 2*(2*b*c^3*d + 3*a*c^2*d^2 + 60*(2*b*c^3*d + 3*
a*c^2*d^2)*n^4 + 107*(2*b*c^3*d + 3*a*c^2*d^2)*n^3 + 59*(2*b*c^3*d + 3*a*c^
2*d^2)*n^2 + 13*(2*b*c^3*d + 3*a*c^2*d^2)*n)*x*x^(2*n) + (b*c^4 + 4*a*c^3*d
 + 120*(b*c^4 + 4*a*c^3*d)*n^4 + 154*(b*c^4 + 4*a*c^3*d)*n^3 + 71*(b*c^4 +
4*a*c^3*d)*n^2 + 14*(b*c^4 + 4*a*c^3*d)*n)*x*x^n + (120*a*c^4*n^5 + 274*a*c
^4*n^4 + 225*a*c^4*n^3 + 85*a*c^4*n^2 + 15*a*c^4*n + a*c^4)*x)/(120*n^5 + 2
74*n^4 + 225*n^3 + 85*n^2 + 15*n + 1)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 2744 vs. $2(124) = 248$.

time = 0.82, size = 2744, normalized size = 20.79

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n)*(c+d*x**n)**4,x)
```

```
[Out] Piecewise((a*c**4*x + 4*a*c**3*d*log(x) - 6*a*c**2*d**2/x - 2*a*c*d**3/x**2
 - a*d**4/(3*x**3) + b*c**4*log(x) - 4*b*c**3*d/x - 3*b*c**2*d**2/x**2 - 4*
b*c*d**3/(3*x**3) - b*d**4/(4*x**4), Eq(n, -1)), (a*c**4*x + 8*a*c**3*d*sqr
t(x) + 6*a*c**2*d**2*log(x) - 8*a*c*d**3/sqrt(x) - a*d**4/x + 2*b*c**4*sqrt
(x) + 4*b*c**3*d*log(x) - 12*b*c**2*d**2/sqrt(x) - 4*b*c*d**3/x - 2*b*d**4/
(3*x**(3/2)), Eq(n, -1/2)), (a*c**4*x + 6*a*c**3*d*x**(2/3) + 18*a*c**2*d**
2*x**(1/3) + 4*a*c*d**3*log(x) - 3*a*d**4/x**(1/3) + 3*b*c**4*x**(2/3)/2 +
12*b*c**3*d*x**(1/3) + 6*b*c**2*d**2*log(x) - 12*b*c*d**3/x**(1/3) - 3*b*d*
*4/(2*x**(2/3)), Eq(n, -1/3)), (a*c**4*x + 16*a*c**3*d*x**(3/4)/3 + 12*a*c*
*2*d**2*sqrt(x) + 16*a*c*d**3*x**(1/4) + a*d**4*log(x) + 4*b*c**4*x**(3/4)/
3 + 8*b*c**3*d*sqrt(x) + 24*b*c**2*d**2*x**(1/4) + 4*b*c*d**3*log(x) - 4*b*
d**4/x**(1/4), Eq(n, -1/4)), (a*c**4*x + 5*a*c**3*d*x**(4/5) + 10*a*c**2*d*
*2*x**(3/5) + 10*a*c*d**3*x**(2/5) + 5*a*d**4*x**(1/5) + 5*b*c**4*x**(4/5)/
4 + 20*b*c**3*d*x**(3/5)/3 + 15*b*c**2*d**2*x**(2/5) + 20*b*c*d**3*x**(1/5)
 + b*d**4*log(x), Eq(n, -1/5)), (120*a*c**4*n**5*x/(120*n**5 + 274*n**4 + 2
25*n**3 + 85*n**2 + 15*n + 1) + 274*a*c**4*n**4*x/(120*n**5 + 274*n**4 + 22
5*n**3 + 85*n**2 + 15*n + 1) + 225*a*c**4*n**3*x/(120*n**5 + 274*n**4 + 225
*n**3 + 85*n**2 + 15*n + 1) + 85*a*c**4*n**2*x/(120*n**5 + 274*n**4 + 225*n
**3 + 85*n**2 + 15*n + 1) + 15*a*c**4*n*x/(120*n**5 + 274*n**4 + 225*n**3 +
 85*n**2 + 15*n + 1) + a*c**4*x/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 +
 15*n + 1) + 480*a*c**3*d*n**4*x*x**n/(120*n**5 + 274*n**4 + 225*n**3 + 85*
n**2 + 15*n + 1) + 616*a*c**3*d*n**3*x*x**n/(120*n**5 + 274*n**4 + 225*n**3
 + 85*n**2 + 15*n + 1) + 284*a*c**3*d*n**2*x*x**n/(120*n**5 + 274*n**4 + 22
5*n**3 + 85*n**2 + 15*n + 1) + 56*a*c**3*d*n*x*x**n/(120*n**5 + 274*n**4 +
225*n**3 + 85*n**2 + 15*n + 1) + 4*a*c**3*d*x*x**n/(120*n**5 + 274*n**4 + 2
```

```
25*n**3 + 85*n**2 + 15*n + 1) + 360*a*c**2*d**2*n**4*x*x**(2*n)/(120*n**5 +
 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 642*a*c**2*d**2*n**3*x*x**(2*n
)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 354*a*c**2*d**2*n
**2*x*x**(2*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 78*a
*c**2*d**2*n*x*x**(2*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n +
1) + 6*a*c**2*d**2*x*x**(2*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 1
5*n + 1) + 160*a*c*d**3*n**4*x*x**(3*n)/(120*n**5 + 274*n**4 + 225*n**3 + 8
5*n**2 + 15*n + 1) + 312*a*c*d**3*n**3*x*x**(3*n)/(120*n**5 + 274*n**4 + 22
5*n**3 + 85*n**2 + 15*n + 1) + 196*a*c*d**3*n**2*x*x**(3*n)/(120*n**5 + 274
*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 48*a*c*d**3*n*x*x**(3*n)/(120*n**5
 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 4*a*c*d**3*x*x**(3*n)/(120*n
**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 30*a*d**4*n**4*x*x**(4*n)
/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 61*a*d**4*n**3*x*x
**(4*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 41*a*d**4*n
**2*x*x**(4*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 11*a
*d**4*n*x*x**(4*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) +
a*d**4*x*x**(4*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 1
20*b*c**4*n**4*x*x**n/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1)
 + 154*b*c**4*n**3*x*x**n/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n
+ 1) + 71*b*c**4*n**2*x*x**n/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15
*n + 1) + 14*b*c**4*n*x*x**n/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15
*n + 1) + b*c**4*x*x**n/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n +
1) + 240*b*c**3*d*n**4*x*x**(2*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2
 + 15*n + 1) + 428*b*c**3*d*n**3*x*x**(2*n)/(120*n**5 + 274*n**4 + 225*n**3
 + 85*n**2 + 15*n + 1) + 236*b*c**3*d*n**2*x*x**(2*n)/(120*n**5 + 274*n**4
+ 225*n**3 + 85*n**2 + 15*n + 1) + 52*b*c**3*d*n*x*x**(2*n)/(120*n**5 + 274
*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 4*b*c**3*d*x*x**(2*n)/(120*n**5 +
274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 240*b*c**2*d**2*n**4*x*x**(3*n)
/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 468*b*c**2*d**2*n*
*3*x*x**(3*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 294*b
*c**2*d**2*n**2*x*x**(3*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n
 + 1) + 72*b*c**2*d**2*n*x*x**(3*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n*
*2 + 15*n + 1) + 6*b*c**2*d**2*x*x**(3*n)/(120*n**5 + 274*n**4 + 225*n**3 +
 85*n**2 + 15*n + 1) + 120*b*c*d**3*n**4*x*x**(4*n)/(120*n**5 + 274*n**4 +
225*n**3 + 85*n**2 + 15*n + 1) + 244*b*c*d**3*n**3*x*x**(4*n)/(120*n**5 + 2
74*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 164*b*c*d**3*n**2*x*x**(4*n)/(12
0*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 44*b*c*d**3*n*x*x**(4*
n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 4*b*c*d**3*x*x**
(4*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 24*b*d**4*n**
4*x*x**(5*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 50*b*d
**4*n**3*x*x**(5*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) +
 35*b*d**4*n**2*x*x**(5*n)/(120*n**5 + 274*n**4...
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 740 vs. $2(132) = 264$.

time = 1.81, size = 740, normalized size = 5.61

$$\frac{120\,a c^4 n^5 x + 24\,b d^4 n^4 x x^{5n} + 120\,b c d^3 n^4 x x^{4n} + 30\,a d^4 n^4 x x^{4n} + 240\,b c^2 d^2 n^4 x x^{3n} + 160\,a c d^3 n^4 x x^{3n} + 240\,b c^3 d n^4 x x^{2n} + 360\,a c^2 d^2 n^4 x x^{2n} + 120\,b c^4 n^4 x x^{n} + 480\,a c^3 d n^4 x x^{n} + 274\,a c^4 n^4 x + 50\,b d^4 n^3 x x^{5n} + 244\,b c d^3 n^3 x x^{4n} + 61\,a d^4 n^3 x x^{4n} + 468\,b c^2 d^2 n^3 x x^{3n} + 312\,a c d^3 n^3 x x^{3n} + 428\,b c^3 d n^3 x x^{2n} + 642\,a c^2 d^2 n^3 x x^{2n} + 154\,b c^4 n^3 x x^{n} + 616\,a c^3 d n^3 x x^{n} + 225\,a c^4 n^3 x + 35\,b d^4 n^2 x x^{5n} + 164\,b c d^3 n^2 x x^{4n} + 41\,a d^4 n^2 x x^{4n} + 294\,b c^2 d^2 n^2 x x^{3n} + 196\,a c d^3 n^2 x x^{3n} + 236\,b c^3 d n^2 x x^{2n} + 354\,a c^2 d^2 n^2 x x^{2n} + 71\,b c^4 n^2 x x^{n} + 284\,a c^3 d n^2 x x^{n} + 85\,a c^4 n^2 x + 10\,b d^4 n x x^{5n} + 44\,b c d^3 n x x^{4n} + 11\,a d^4 n x x^{4n} + 72\,b c^2 d^2 n x x^{3n} + 48\,a c d^3 n x x^{3n} + 52\,b c^3 d n x x^{2n} + 78\,a c^2 d^2 n x x^{2n} + 14\,b c^4 n x x^{n} + 56\,a c^3 d n x x^{n} + 15\,a c^4 n x + b d^4 x x^{5n} + 4\,b c d^3 x x^{4n} + a d^4 x x^{4n} + 6\,b c^2 d^2 x x^{3n} + 4\,a c d^3 x x^{3n} + 4\,b c^3 d x x^{2n} + 6\,a c^2 d^2 x x^{2n} + b c^4 x x^{n} + 4\,a c^3 d x x^{n} + a c^4 x}{120\,n^5 + 274\,n^4 + 225\,n^3 + 85\,n^2 + 15\,n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)*(c+d*x^n)^4,x, algorithm="giac")
```

```
[Out] (120*a*c^4*n^5*x + 24*b*d^4*n^4*x*x^(5*n) + 120*b*c*d^3*n^4*x*x^(4*n) + 30*
a*d^4*n^4*x*x^(4*n) + 240*b*c^2*d^2*n^4*x*x^(3*n) + 160*a*c*d^3*n^4*x*x^(3*
n) + 240*b*c^3*d*n^4*x*x^(2*n) + 360*a*c^2*d^2*n^4*x*x^(2*n) + 120*b*c^4*n^
4*x*x^n + 480*a*c^3*d*n^4*x*x^n + 274*a*c^4*n^4*x + 50*b*d^4*n^3*x*x^(5*n)
+ 244*b*c*d^3*n^3*x*x^(4*n) + 61*a*d^4*n^3*x*x^(4*n) + 468*b*c^2*d^2*n^3*x*
x^(3*n) + 312*a*c*d^3*n^3*x*x^(3*n) + 428*b*c^3*d*n^3*x*x^(2*n) + 642*a*c^2
*d^2*n^3*x*x^(2*n) + 154*b*c^4*n^3*x*x^n + 616*a*c^3*d*n^3*x*x^n + 225*a*c^
4*n^3*x + 35*b*d^4*n^2*x*x^(5*n) + 164*b*c*d^3*n^2*x*x^(4*n) + 41*a*d^4*n^2
*x*x^(4*n) + 294*b*c^2*d^2*n^2*x*x^(3*n) + 196*a*c*d^3*n^2*x*x^(3*n) + 236*
b*c^3*d*n^2*x*x^(2*n) + 354*a*c^2*d^2*n^2*x*x^(2*n) + 71*b*c^4*n^2*x*x^n +
284*a*c^3*d*n^2*x*x^n + 85*a*c^4*n^2*x + 10*b*d^4*n*x*x^(5*n) + 44*b*c*d^3*
n*x*x^(4*n) + 11*a*d^4*n*x*x^(4*n) + 72*b*c^2*d^2*n*x*x^(3*n) + 48*a*c*d^3*
n*x*x^(3*n) + 52*b*c^3*d*n*x*x^(2*n) + 78*a*c^2*d^2*n*x*x^(2*n) + 14*b*c^4*
n*x*x^n + 56*a*c^3*d*n*x*x^n + 15*a*c^4*n*x + b*d^4*x*x^(5*n) + 4*b*c*d^3*x
*x^(4*n) + a*d^4*x*x^(4*n) + 6*b*c^2*d^2*x*x^(3*n) + 4*a*c*d^3*x*x^(3*n) +
4*b*c^3*d*x*x^(2*n) + 6*a*c^2*d^2*x*x^(2*n) + b*c^4*x*x^n + 4*a*c^3*d*x*x^n
 + a*c^4*x)/(120*n^5 + 274*n^4 + 225*n^3 + 85*n^2 + 15*n + 1)
```

Mupad [B]
time = 1.64, size = 131, normalized size = 0.99

$$a\,c^4\,x+\frac{x\,x^n\,\left(b\,c^4+4\,a\,d\,c^3\right)}{n+1}+\frac{x\,x^{4\,n}\,\left(a\,d^4+4\,b\,c\,d^3\right)}{4\,n+1}+\frac{b\,d^4\,x\,x^{5\,n}}{5\,n+1}+\frac{2\,c^2\,d\,x\,x^{2\,n}\,\left(3\,a\,d+2\,b\,c\right)}{2\,n+1}+\frac{2\,c\,d^2\,x\,x^{3\,n}\,\left(2\,a\,d+3\,b\,c\right)}{3\,n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)*(c + d*x^n)^4,x)
```

```
[Out] a*c^4*x + (x*x^n*(b*c^4 + 4*a*c^3*d))/(n + 1) + (x*x^(4*n)*(a*d^4 + 4*b*c*d
^3))/(4*n + 1) + (b*d^4*x*x^(5*n))/(5*n + 1) + (2*c^2*d*x*x^(2*n)*(3*a*d +
2*b*c))/(2*n + 1) + (2*c*d^2*x*x^(3*n)*(2*a*d + 3*b*c))/(3*n + 1)
```

3.285 $\int (a+bx^n)(c+dx^n)^3\,dx$

Optimal. Leaf size=99

$$ac^3x+\frac{c^2(bc+3ad)x^{1+n}}{1+n}+\frac{3cd(bc+ad)x^{1+2n}}{1+2n}+\frac{d^2(3bc+ad)x^{1+3n}}{1+3n}+\frac{bd^3x^{1+4n}}{1+4n}$$

```
[Out] a*c^3*x+c^2*(3*a*d+b*c)*x^(1+n)/(1+n)+3*c*d*(a*d+b*c)*x^(1+2*n)/(1+2*n)+d^2
*(a*d+3*b*c)*x^(1+3*n)/(1+3*n)+b*d^3*x^(1+4*n)/(1+4*n)
```

Rubi [A]
time = 0.05, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {380}

$$\frac{c^2x^{n+1}(3ad+bc)}{n+1}+\frac{d^2x^{3n+1}(ad+3bc)}{3n+1}+\frac{3cdx^{2n+1}(ad+bc)}{2n+1}+ac^3x+\frac{bd^3x^{4n+1}}{4n+1}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n)*(c + d*x^n)^3,x]
```

```
[Out] a*c^3*x + (c^2*(b*c + 3*a*d)*x^(1 + n))/(1 + n) + (3*c*d*(b*c + a*d)*x^(1 +
 2*n))/(1 + 2*n) + (d^2*(3*b*c + a*d)*x^(1 + 3*n))/(1 + 3*n) + (b*d^3*x^(1
+ 4*n))/(1 + 4*n)
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int (a+bx^n)(c+dx^n)^3\,dx &= \int \left(ac^3+c^2(bc+3ad)x^n+3cd(bc+ad)x^{2n}+d^2(3bc+ad)x^{3n}+bd^3x^{4n}\right)dx\\ &= ac^3x+\frac{c^2(bc+3ad)x^{1+n}}{1+n}+\frac{3cd(bc+ad)x^{1+2n}}{1+2n}+\frac{d^2(3bc+ad)x^{1+3n}}{1+3n}+\frac{bd^3x^{1+4n}}{1+4n}\end{aligned}$$

Mathematica [A]
time = 0.14, size = 92, normalized size = 0.93

$$x\left(ac^3+\frac{c^2(bc+3ad)x^n}{1+n}+\frac{3cd(bc+ad)x^{2n}}{1+2n}+\frac{d^2(3bc+ad)x^{3n}}{1+3n}+\frac{bd^3x^{4n}}{1+4n}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^n)*(c + d*x^n)^3,x]
```

```
[Out] x*(a*c^3 + (c^2*(b*c + 3*a*d)*x^n)/(1 + n) + (3*c*d*(b*c + a*d)*x^(2*n))/(1
 + 2*n) + (d^2*(3*b*c + a*d)*x^(3*n))/(1 + 3*n) + (b*d^3*x^(4*n))/(1 + 4*n)
)
```

Maple [A]
time = 0.25, size = 96, normalized size = 0.97

method	result	size
risch	$a\,c^{3} x +\frac{b\,d^{3} x\,x^{4 n}}{1+4 n}+\frac{c^{2} \left(3 a d +b c \right) x\,x^{n}}{1+n}+\frac{d^{2} \left(a d +3 b c \right) x\,x^{3 n}}{1+3 n}+\frac{3 c d \left(a d +b c \right) x\,x^{2 n}}{1+2 n}$	96
norman	$a\,c^{3} x +\frac{b\,d^{3} x\,{\mathrm e}^{4 n \ln \left(x \right)}}{1+4 n}+\frac{c^{2} \left(3 a d +b c \right) x\,{\mathrm e}^{n \ln \left(x \right)}}{1+n}+\frac{d^{2} \left(a d +3 b c \right) x\,{\mathrm e}^{3 n \ln \left(x \right)}}{1+3 n}+\frac{3 c d \left(a d +b c \right) x\,{\mathrm e}^{2 n \ln \left(x \right)}}{1+2 n}$	104

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^n)*(c+d*x^n)^3,x,method=_RETURNVERBOSE)
```

```
[Out] a*c^3*x+b*d^3/(1+4*n)*x*(x^n)^4+c^2*(3*a*d+b*c)/(1+n)*x*x^n+d^2*(a*d+3*b*c)
/(1+3*n)*x*(x^n)^3+3*c*d*(a*d+b*c)/(1+2*n)*x*(x^n)^2
```

Maxima [A]
time = 0.30, size = 140, normalized size = 1.41

$$ac^{3}x+\frac{bd^{3}x^{4\,n+1}}{4\,n+1}+\frac{3\,bcd^{2}x^{3\,n+1}}{3\,n+1}+\frac{ad^{3}x^{3\,n+1}}{3\,n+1}+\frac{3\,bc^{2}dx^{2\,n+1}}{2\,n+1}+\frac{3\,acd^{2}x^{2\,n+1}}{2\,n+1}+\frac{bc^{3}x^{n+1}}{n+1}+\frac{3\,ac^{2}dx^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)*(c+d*x^n)^3,x, algorithm="maxima")
```

```
[Out] a*c^3*x + b*d^3*x^(4*n + 1)/(4*n + 1) + 3*b*c*d^2*x^(3*n + 1)/(3*n + 1) + a
*d^3*x^(3*n + 1)/(3*n + 1) + 3*b*c^2*d*x^(2*n + 1)/(2*n + 1) + 3*a*c*d^2*x^
(2*n + 1)/(2*n + 1) + b*c^3*x^(n + 1)/(n + 1) + 3*a*c^2*d*x^(n + 1)/(n + 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 319 vs. 2(99) = 198.
time = 2.63, size = 319, normalized size = 3.22

$$\frac{(6\,bd^{3}n^{3}+11\,bd^{3}n^{2}+6\,bd^{3}n+bd^{3})xx^{4\,n}+(3\,bcd^{2}+ad^{3}+8\,(3\,bcd^{2}+ad^{3})n^{3}+14\,(3\,bcd^{2}+ad^{3})n^{2}+7\,(3\,bcd^{2}+ad^{3})n)xx^{3\,n}+3\,(bc^{2}d+acd^{2}+12\,(bc^{2}d+acd^{2})n^{3}+19\,(bc^{2}d+acd^{2})n^{2}+8\,(bc^{2}d+acd^{2})n)xx^{2\,n}+(bc^{3}+3\,ac^{2}d+24\,(bc^{3}+3\,ac^{2}d)n^{3}+26\,(bc^{3}+3\,ac^{2}d)n^{2}+9\,(bc^{3}+3\,ac^{2}d)n)xx^{n}+(24\,ac^{3}n^{4}+50\,ac^{3}n^{3}+35\,ac^{3}n^{2}+10\,ac^{3}n+ac^{3})x}{24\,n^{4}+50\,n^{3}+35\,n^{2}+10\,n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)*(c+d*x^n)^3,x, algorithm="fricas")
```

```
[Out] ((6*b*d^3*n^3 + 11*b*d^3*n^2 + 6*b*d^3*n + b*d^3)*x*x^(4*n) + (3*b*c*d^2 +
a*d^3 + 8*(3*b*c*d^2 + a*d^3)*n^3 + 14*(3*b*c*d^2 + a*d^3)*n^2 + 7*(3*b*c*d
^2 + a*d^3)*n)*x*x^(3*n) + 3*(b*c^2*d + a*c*d^2 + 12*(b*c^2*d + a*c*d^2)*n^
```

```
3 + 19*(b*c^2*d + a*c*d^2)*n^2 + 8*(b*c^2*d + a*c*d^2)*n)*x*x^(2*n) + (b*c^
3 + 3*a*c^2*d + 24*(b*c^3 + 3*a*c^2*d)*n^3 + 26*(b*c^3 + 3*a*c^2*d)*n^2 + 9
*(b*c^3 + 3*a*c^2*d)*n)*x*x^n + (24*a*c^3*n^4 + 50*a*c^3*n^3 + 35*a*c^3*n^2
 + 10*a*c^3*n + a*c^3)*x)/(24*n^4 + 50*n^3 + 35*n^2 + 10*n + 1)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1540 vs. $2(90) = 180$.
time = 1.21, size = 1540, normalized size = 15.56

$$\begin{cases} ac^3x + 3ac^2d\log(x) - \frac{3acd^2}{x} - \frac{ad^3}{2x^2} + bc^3\log(x) - \frac{3bc^2d}{x} - \frac{3bcd^2}{2x^2} - \frac{bd^3}{3x^3} & \\ ac^3x + 6ac^2d\sqrt{x} + 3acd^2\log(x) - \frac{2ad^3}{\sqrt{x}} + 2bc^3\sqrt{x} + 3bc^2d\log(x) - \frac{6bcd^2}{\sqrt{x}} - \frac{bd^3}{x} & \\ ac^3x + \frac{9ac^2dx^{\frac{2}{3}}}{2} + 9acd^2\sqrt[3]{x} + ad^3\log(x) + \frac{3bc^3x^{\frac{2}{3}}}{2} + 9bc^2d\sqrt[3]{x} + 3bcd^2\log(x) - \frac{3bd^3}{\sqrt[3]{x}} & \\ ac^3x + 4ac^2dx^{\frac{3}{4}} + 6acd^2\sqrt{x} + 4ad^3\sqrt[4]{x} + \frac{4bc^3x^{\frac{3}{4}}}{3} + 6bc^2d\sqrt{x} + 12bcd^2\sqrt[4]{x} + bd^3\log(x) & \\ \frac{24ac^3n^4x}{24n^4+50n^3+35n^2+10n+1} + \frac{50ac^3n^3x}{24n^4+50n^3+35n^2+10n+1} + \frac{35ac^3n^2x}{24n^4+50n^3+35n^2+10n+1} + \frac{10ac^3nx}{24n^4+50n^3+35n^2+10n+1} + \frac{ac^3x}{24n^4+50n^3+35n^2+10n+1} + \frac{72ac^2dn^3xx^n}{24n^4+50n^3+35n^2+10n+1} + \frac{78ac^2dn^2xx^n}{24n^4+50n^3+35n^2+10n+1} + \frac{27ac^2dnxx^n}{24n^4+50n^3+35n^2+10n+1} + \frac{3ac^2dxx^n}{24n^4+50n^3+35n^2+10n+1} + \frac{36acd^2n^3xx^{2n}}{24n^4+50n^3+35n^2+10n+1} + \frac{57acd^2n^2xx^{2n}}{24n^4+50n^3+35n^2+10n+1} + \frac{24acd^2nxx^{2n}}{24n^4+50n^3+35n^2+10n+1} + \frac{3acd^2xx^{2n}}{24n^4+50n^3+35n^2+10n+1} + \text{[illegible]} & \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n)*(c+d*x**n)**3,x)
```

```
[Out] Piecewise((a*c**3*x + 3*a*c**2*d*log(x) - 3*a*c*d**2/x - a*d**3/(2*x**2) +
b*c**3*log(x) - 3*b*c**2*d/x - 3*b*c*d**2/(2*x**2) - b*d**3/(3*x**3), Eq(n,
 -1)), (a*c**3*x + 6*a*c**2*d*sqrt(x) + 3*a*c*d**2*log(x) - 2*a*d**3/sqrt(x
) + 2*b*c**3*sqrt(x) + 3*b*c**2*d*log(x) - 6*b*c*d**2/sqrt(x) - b*d**3/x, E
q(n, -1/2)), (a*c**3*x + 9*a*c**2*d*x**(2/3)/2 + 9*a*c*d**2*x**(1/3) + a*d*
*3*log(x) + 3*b*c**3*x**(2/3)/2 + 9*b*c**2*d*x**(1/3) + 3*b*c*d**2*log(x) -
 3*b*d**3/x**(1/3), Eq(n, -1/3)), (a*c**3*x + 4*a*c**2*d*x**(3/4) + 6*a*c*d
**2*sqrt(x) + 4*a*d**3*x**(1/4) + 4*b*c**3*x**(3/4)/3 + 6*b*c**2*d*sqrt(x)
+ 12*b*c*d**2*x**(1/4) + b*d**3*log(x), Eq(n, -1/4)), (24*a*c**3*n**4*x/(24
*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 50*a*c**3*n**3*x/(24*n**4 + 50*n**3
 + 35*n**2 + 10*n + 1) + 35*a*c**3*n**2*x/(24*n**4 + 50*n**3 + 35*n**2 + 10
*n + 1) + 10*a*c**3*n*x/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + a*c**3*x
/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 72*a*c**2*d*n**3*x*x**n/(24*n**
4 + 50*n**3 + 35*n**2 + 10*n + 1) + 78*a*c**2*d*n**2*x*x**n/(24*n**4 + 50*n
**3 + 35*n**2 + 10*n + 1) + 27*a*c**2*d*n*x*x**n/(24*n**4 + 50*n**3 + 35*n*
*2 + 10*n + 1) + 3*a*c**2*d*x*x**n/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1)
 + 36*a*c*d**2*n**3*x*x**(2*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 5
7*a*c*d**2*n**2*x*x**(2*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 24*a*
c*d**2*n*x*x**(2*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 3*a*c*d**2*x
*x**(2*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 8*a*d**3*n**3*x*x**(3*
n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 14*a*d**3*n**2*x*x**(3*n)/(24
*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 7*a*d**3*n*x*x**(3*n)/(24*n**4 + 50
*n**3 + 35*n**2 + 10*n + 1) + a*d**3*x*x**(3*n)/(24*n**4 + 50*n**3 + 35*n**
2 + 10*n + 1) + 24*b*c**3*n**3*x*x**n/(24*n**4 + 50*n**3 + 35*n**2 + 10*n +
 1) + 26*b*c**3*n**2*x*x**n/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 9*b*
c**3*n*x*x**n/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + b*c**3*x*x**n/(24*
n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 36*b*c**2*d*n**3*x*x**(2*n)/(24*n**4
 + 50*n**3 + 35*n**2 + 10*n + 1) + 57*b*c**2*d*n**2*x*x**(2*n)/(24*n**4 + 5
0*n**3 + 35*n**2 + 10*n + 1) + 24*b*c**2*d*n*x*x**(2*n)/(24*n**4 + 50*n**3
+ 35*n**2 + 10*n + 1) + 3*b*c**2*d*x*x**(2*n)/(24*n**4 + 50*n**3 + 35*n**2
+ 10*n + 1) + 24*b*c*d**2*n**3*x*x**(3*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10
```

```
*n + 1) + 42*b*c*d**2*n**2*x*x**(3*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n +
 1) + 21*b*c*d**2*n*x*x**(3*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 3
*b*c*d**2*x*x**(3*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 6*b*d**3*n*
*3*x*x**(4*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 11*b*d**3*n**2*x*x
**(4*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 6*b*d**3*n*x*x**(4*n)/(2
4*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + b*d**3*x*x**(4*n)/(24*n**4 + 50*n*
*3 + 35*n**2 + 10*n + 1), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 450 vs. 2(99) = 198.
time = 0.91, size = 450, normalized size = 4.55

$$\frac{24\,ac^3n^4x + 6\,bd^3n^3xx^{4\,n} + 24\,bcd^2n^3xx^{3\,n} + 8\,ad^3n^3xx^{3\,n} + 36\,bc^2dn^3xx^{2\,n} + 36\,acd^2n^3xx^{2\,n} + 24\,bc^3n^3xx^n + 72\,ac^2dn^3xx^n + 50\,ac^3n^3x + 11\,bd^3n^2xx^{4\,n} + 42\,bcd^2n^2xx^{3\,n} + 14\,ad^3n^2xx^{3\,n} + 57\,bc^2dn^2xx^{2\,n} + 57\,acd^2n^2xx^{2\,n} + 26\,bc^3n^2xx^n + 78\,ac^2dn^2xx^n + 35\,ac^3n^2x + 6\,bd^3nxx^{4\,n} + 21\,bcd^2nxx^{3\,n} + 7\,ad^3nxx^{3\,n} + 24\,bc^2dnxx^{2\,n} + 24\,acd^2nxx^{2\,n} + 9\,bc^3nxx^n + 27\,ac^2dnxx^n + 10\,ac^3nx + bd^3xx^{4\,n} + 3\,bcd^2xx^{3\,n} + ad^3xx^{3\,n} + 3\,bc^2dxx^{2\,n} + 3\,acd^2xx^{2\,n} + bc^3xx^n + 3\,ac^2dxx^n + ac^3x}{24\,n^4 + 50\,n^3 + 35\,n^2 + 10\,n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)*(c+d*x^n)^3,x, algorithm="giac")
```

```
[Out] (24*a*c^3*n^4*x + 6*b*d^3*n^3*x*x^(4*n) + 24*b*c*d^2*n^3*x*x^(3*n) + 8*a*d^
3*n^3*x*x^(3*n) + 36*b*c^2*d*n^3*x*x^(2*n) + 36*a*c*d^2*n^3*x*x^(2*n) + 24*
b*c^3*n^3*x*x^n + 72*a*c^2*d*n^3*x*x^n + 50*a*c^3*n^3*x + 11*b*d^3*n^2*x*x^
(4*n) + 42*b*c*d^2*n^2*x*x^(3*n) + 14*a*d^3*n^2*x*x^(3*n) + 57*b*c^2*d*n^2*
x*x^(2*n) + 57*a*c*d^2*n^2*x*x^(2*n) + 26*b*c^3*n^2*x*x^n + 78*a*c^2*d*n^2*
x*x^n + 35*a*c^3*n^2*x + 6*b*d^3*n*x*x^(4*n) + 21*b*c*d^2*n*x*x^(3*n) + 7*a
*d^3*n*x*x^(3*n) + 24*b*c^2*d*n*x*x^(2*n) + 24*a*c*d^2*n*x*x^(2*n) + 9*b*c^
3*n*x*x^n + 27*a*c^2*d*n*x*x^n + 10*a*c^3*n*x + b*d^3*x*x^(4*n) + 3*b*c*d^2
*x*x^(3*n) + a*d^3*x*x^(3*n) + 3*b*c^2*d*x*x^(2*n) + 3*a*c*d^2*x*x^(2*n) +
b*c^3*x*x^n + 3*a*c^2*d*x*x^n + a*c^3*x)/(24*n^4 + 50*n^3 + 35*n^2 + 10*n +
 1)
```

Mupad [B]
time = 1.56, size = 99, normalized size = 1.00

$$a\,c^3\,x+\frac{x\,x^n\,\left(b\,c^3+3\,a\,d\,c^2\right)}{n+1}+\frac{x\,x^{3\,n}\,\left(a\,d^3+3\,b\,c\,d^2\right)}{3\,n+1}+\frac{b\,d^3\,x\,x^{4\,n}}{4\,n+1}+\frac{3\,c\,d\,x\,x^{2\,n}\,\left(a\,d+b\,c\right)}{2\,n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)*(c + d*x^n)^3,x)
```

```
[Out] a*c^3*x + (x*x^n*(b*c^3 + 3*a*c^2*d))/(n + 1) + (x*x^(3*n)*(a*d^3 + 3*b*c*d
^2))/(3*n + 1) + (b*d^3*x*x^(4*n))/(4*n + 1) + (3*c*d*x*x^(2*n)*(a*d + b*c)
)/(2*n + 1)
```

3.286 $\int (a+bx^n)(c+dx^n)^2 \, dx$

Optimal. Leaf size=70

$$ac^2x+\frac{c(bc+2ad)x^{1+n}}{1+n}+\frac{d(2bc+ad)x^{1+2n}}{1+2n}+\frac{bd^2x^{1+3n}}{1+3n}$$

```
[Out] a*c^2*x+c*(2*a*d+b*c)*x^(1+n)/(1+n)+d*(a*d+2*b*c)*x^(1+2*n)/(1+2*n)+b*d^2*x
^(1+3*n)/(1+3*n)
```

Rubi [A]
time = 0.03, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {380}

$$\frac{cx^{n+1}(2ad+bc)}{n+1}+\frac{dx^{2n+1}(ad+2bc)}{2n+1}+ac^2x+\frac{bd^2x^{3n+1}}{3n+1}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n)*(c + d*x^n)^2,x]
```

```
[Out] a*c^2*x + (c*(b*c + 2*a*d)*x^(1 + n))/(1 + n) + (d*(2*b*c + a*d)*x^(1 + 2*n
))/(1 + 2*n) + (b*d^2*x^(1 + 3*n))/(1 + 3*n)
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int (a+bx^n)(c+dx^n)^2 \, dx &= \int \left(ac^2+c(bc+2ad)x^n+d(2bc+ad)x^{2n}+bd^2x^{3n}\right) dx\\ &= ac^2x+\frac{c(bc+2ad)x^{1+n}}{1+n}+\frac{d(2bc+ad)x^{1+2n}}{1+2n}+\frac{bd^2x^{1+3n}}{1+3n}\end{aligned}$$

Mathematica [A]
time = 0.10, size = 65, normalized size = 0.93

$$x\left(ac^2+\frac{c(bc+2ad)x^n}{1+n}+\frac{d(2bc+ad)x^{2n}}{1+2n}+\frac{bd^2x^{3n}}{1+3n}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^n)*(c + d*x^n)^2,x]
```

```
[Out] x*(a*c^2 + (c*(b*c + 2*a*d)*x^n)/(1 + n) + (d*(2*b*c + a*d)*x^(2*n))/(1 + 2
*n) + (b*d^2*x^(3*n))/(1 + 3*n))
```

Maple [A]
time = 0.23, size = 68, normalized size = 0.97

method	result	size
risch	$a\,c^{2} x +\frac{b\,d^{2} x\,x^{3 n}}{1+3 n}+\frac{c \left(2 a d +b c \right) x\,x^{n}}{1+n}+\frac{d \left(a d +2 b c \right) x\,x^{2 n}}{1+2 n}$	68
norman	$a\,c^{2} x +\frac{b\,d^{2} x\,{\mathrm e}^{3 n \ln \left(x \right)}}{1+3 n}+\frac{c \left(2 a d +b c \right) x\,{\mathrm e}^{n \ln \left(x \right)}}{1+n}+\frac{d \left(a d +2 b c \right) x\,{\mathrm e}^{2 n \ln \left(x \right)}}{1+2 n}$	74

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^n)*(c+d*x^n)^2,x,method=_RETURNVERBOSE)
```

```
[Out] a*c^2*x+b*d^2/(1+3*n)*x*(x^n)^3+c*(2*a*d+b*c)/(1+n)*x*x^n+d*(a*d+2*b*c)/(1+
2*n)*x*(x^n)^2
```

Maxima [A]
time = 0.29, size = 94, normalized size = 1.34

$$ac^{2} x + \frac{b d^{2} x^{3 \, n + 1}}{3 \, n + 1} + \frac{2 \, b c d x^{2 \, n + 1}}{2 \, n + 1} + \frac{a d^{2} x^{2 \, n + 1}}{2 \, n + 1} + \frac{b c^{2} x^{n + 1}}{n + 1} + \frac{2 \, a c d x^{n + 1}}{n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)*(c+d*x^n)^2,x, algorithm="maxima")
```

```
[Out] a*c^2*x + b*d^2*x^(3*n + 1)/(3*n + 1) + 2*b*c*d*x^(2*n + 1)/(2*n + 1) + a*d
^2*x^(2*n + 1)/(2*n + 1) + b*c^2*x^(n + 1)/(n + 1) + 2*a*c*d*x^(n + 1)/(n +
 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 175 vs. 2(70) = 140.
time = 3.45, size = 175, normalized size = 2.50

$$\frac{{\left(2 \, b d^{2} n^{2} + 3 \, b d^{2} n + b d^{2}\right)} x x^{3 \, n} + {\left(2 \, b c d + a d^{2} + 3 \, {\left(2 \, b c d + a d^{2}\right)} n^{2} + 4 \, {\left(2 \, b c d + a d^{2}\right)} n\right)} x x^{2 \, n} + {\left(b c^{2} + 2 \, a c d + 6 \, {\left(b c^{2} + 2 \, a c d\right)} n^{2} + 5 \, {\left(b c^{2} + 2 \, a c d\right)} n\right)} x x^{n} + {\left(6 \, a c^{2} n^{3} + 11 \, a c^{2} n^{2} + 6 \, a c^{2} n + a c^{2}\right)} x}{6 \, n^{3} + 11 \, n^{2} + 6 \, n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)*(c+d*x^n)^2,x, algorithm="fricas")
```

```
[Out] ((2*b*d^2*n^2 + 3*b*d^2*n + b*d^2)*x*x^(3*n) + (2*b*c*d + a*d^2 + 3*(2*b*c*
d + a*d^2)*n^2 + 4*(2*b*c*d + a*d^2)*n)*x*x^(2*n) + (b*c^2 + 2*a*c*d + 6*(b
*c^2 + 2*a*c*d)*n^2 + 5*(b*c^2 + 2*a*c*d)*n)*x*x^n + (6*a*c^2*n^3 + 11*a*c^
2*n^2 + 6*a*c^2*n + a*c^2)*x)/(6*n^3 + 11*n^2 + 6*n + 1)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 726 vs. $2(63) = 126$.
time = 0.30, size = 726, normalized size = 10.37

$$\begin{cases} ac^2x + 2acd\log(x) - \frac{ad^2}{x} + bc^2\log(x) - \frac{2bcd}{x} - \frac{bd^2}{2x^2} & \text{for } n = -1 \\ ac^2x + 4acd\sqrt{x} + ad^2\log(x) + 2bc^2\sqrt{x} + 2bcd\log(x) - \frac{2bd^2}{\sqrt{x}} & \text{for } n = -\frac{1}{2} \\ ac^2x + 3acdx^{\frac{2}{3}} + 3ad^2\sqrt[3]{x} + \frac{3bc^2x^{\frac{2}{3}}}{2} + 6bcd\sqrt[3]{x} + bd^2\log(x) & \text{for } n = -\frac{1}{3} \\ \frac{6ac^2n^3x}{6n^3+11n^2+6n+1} + \frac{11ac^2n^2x}{6n^3+11n^2+6n+1} + \frac{6ac^2nx}{6n^3+11n^2+6n+1} + \frac{ac^2x}{6n^3+11n^2+6n+1} + \frac{12acdn^2xx^n}{6n^3+11n^2+6n+1} + \frac{10acdnxx^n}{6n^3+11n^2+6n+1} + \frac{2acdxx^n}{6n^3+11n^2+6n+1} + \frac{3ad^2n^2xx^{2n}}{6n^3+11n^2+6n+1} + \frac{4ad^2nxx^{2n}}{6n^3+11n^2+6n+1} + \frac{ad^2xx^{2n}}{6n^3+11n^2+6n+1} + \frac{6bc^2n^2xx^n}{6n^3+11n^2+6n+1} + \frac{5bc^2nxx^n}{6n^3+11n^2+6n+1} + \frac{bc^2xx^n}{6n^3+11n^2+6n+1} + \frac{6bcdn^2xx^{2n}}{6n^3+11n^2+6n+1} + \frac{8bcdnxx^{2n}}{6n^3+11n^2+6n+1} + \frac{2bcdxx^{2n}}{6n^3+11n^2+6n+1} + \frac{2bd^2n^2xx^{3n}}{6n^3+11n^2+6n+1} + \frac{3bd^2nxx^{3n}}{6n^3+11n^2+6n+1} + \frac{bd^2xx^{3n}}{6n^3+11n^2+6n+1} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n)*(c+d*x**n)**2,x)
```

```
[Out] Piecewise((a*c**2*x + 2*a*c*d*log(x) - a*d**2/x + b*c**2*log(x) - 2*b*c*d/x
 - b*d**2/(2*x**2), Eq(n, -1)), (a*c**2*x + 4*a*c*d*sqrt(x) + a*d**2*log(x)
 + 2*b*c**2*sqrt(x) + 2*b*c*d*log(x) - 2*b*d**2/sqrt(x), Eq(n, -1/2)), (a*c
**2*x + 3*a*c*d*x**(2/3) + 3*a*d**2*x**(1/3) + 3*b*c**2*x**(2/3)/2 + 6*b*c*
d*x**(1/3) + b*d**2*log(x), Eq(n, -1/3)), (6*a*c**2*n**3*x/(6*n**3 + 11*n**
2 + 6*n + 1) + 11*a*c**2*n**2*x/(6*n**3 + 11*n**2 + 6*n + 1) + 6*a*c**2*n*x
/(6*n**3 + 11*n**2 + 6*n + 1) + a*c**2*x/(6*n**3 + 11*n**2 + 6*n + 1) + 12*
a*c*d*n**2*x*x**n/(6*n**3 + 11*n**2 + 6*n + 1) + 10*a*c*d*n*x*x**n/(6*n**3
+ 11*n**2 + 6*n + 1) + 2*a*c*d*x*x**n/(6*n**3 + 11*n**2 + 6*n + 1) + 3*a*d*
*2*n**2*x*x**(2*n)/(6*n**3 + 11*n**2 + 6*n + 1) + 4*a*d**2*n*x*x**(2*n)/(6*
n**3 + 11*n**2 + 6*n + 1) + a*d**2*x*x**(2*n)/(6*n**3 + 11*n**2 + 6*n + 1)
+ 6*b*c**2*n**2*x*x**n/(6*n**3 + 11*n**2 + 6*n + 1) + 5*b*c**2*n*x*x**n/(6*
n**3 + 11*n**2 + 6*n + 1) + b*c**2*x*x**n/(6*n**3 + 11*n**2 + 6*n + 1) + 6*
b*c*d*n**2*x*x**(2*n)/(6*n**3 + 11*n**2 + 6*n + 1) + 8*b*c*d*n*x*x**(2*n)/(
6*n**3 + 11*n**2 + 6*n + 1) + 2*b*c*d*x*x**(2*n)/(6*n**3 + 11*n**2 + 6*n +
1) + 2*b*d**2*n**2*x*x**(3*n)/(6*n**3 + 11*n**2 + 6*n + 1) + 3*b*d**2*n*x*x
**(3*n)/(6*n**3 + 11*n**2 + 6*n + 1) + b*d**2*x*x**(3*n)/(6*n**3 + 11*n**2
+ 6*n + 1), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 232 vs. $2(70) = 140$.
time = 0.71, size = 232, normalized size = 3.31

$$\frac{6\,ac^2n^3x + 2\,bd^2n^2xx^{3\,n} + 6\,bcdn^2xx^{2\,n} + 3\,ad^2n^2xx^{2\,n} + 6\,bc^2n^2xx^n + 12\,acdn^2xx^n + 11\,ac^2n^2x + 3\,bd^2nxx^{3\,n} + 8\,bcdnxx^{2\,n} + 4\,ad^2nxx^{2\,n} + 5\,bc^2nxx^n + 10\,acdnxx^n + 6\,ac^2nx + bd^2xx^{3\,n} + 2\,bcdxx^{2\,n} + ad^2xx^{2\,n} + bc^2xx^n + 2\,acdxx^n + ac^2x}{6\,n^3 + 11\,n^2 + 6\,n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)*(c+d*x^n)^2,x, algorithm="giac")
```

```
[Out] (6*a*c^2*n^3*x + 2*b*d^2*n^2*x*x^(3*n) + 6*b*c*d*n^2*x*x^(2*n) + 3*a*d^2*n^
2*x*x^(2*n) + 6*b*c^2*n^2*x*x^n + 12*a*c*d*n^2*x*x^n + 11*a*c^2*n^2*x + 3*b
*d^2*n*x*x^(3*n) + 8*b*c*d*n*x*x^(2*n) + 4*a*d^2*n*x*x^(2*n) + 5*b*c^2*n*x*
x^n + 10*a*c*d*n*x*x^n + 6*a*c^2*n*x + b*d^2*x*x^(3*n) + 2*b*c*d*x*x^(2*n)
+ a*d^2*x*x^(2*n) + b*c^2*x*x^n + 2*a*c*d*x*x^n + a*c^2*x)/(6*n^3 + 11*n^2
+ 6*n + 1)
```

Mupad [B]
time = 1.53, size = 71, normalized size = 1.01

$$a\,c^2\,x+\frac{x\,x^{2\,n}\,\left(a\,d^2+2\,b\,c\,d\right)}{2\,n+1}+\frac{x\,x^n\,\left(b\,c^2+2\,a\,d\,c\right)}{n+1}+\frac{b\,d^2\,x\,x^{3\,n}}{3\,n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)*(c + d*x^n)^2,x)
```

```
[Out] a*c^2*x + (x*x^(2*n)*(a*d^2 + 2*b*c*d))/(2*n + 1) + (x*x^n*(b*c^2 + 2*a*c*d
))/(n + 1) + (b*d^2*x*x^(3*n))/(3*n + 1)
```

3.287 $\int (a+bx^n)(c+dx^n)\,dx$

Optimal. Leaf size=40

$$acx+\frac{(bc+ad)x^{1+n}}{1+n}+\frac{bdx^{1+2n}}{1+2n}$$

```
[Out] a*c*x+(a*d+b*c)*x^(1+n)/(1+n)+b*d*x^(1+2*n)/(1+2*n)
```

Rubi [A]
time = 0.02, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}=0.067$, Rules used = {380}

$$\frac{x^{n+1}(ad+bc)}{n+1}+acx+\frac{bdx^{2n+1}}{2n+1}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n)*(c + d*x^n),x]
```

```
[Out] a*c*x + ((b*c + a*d)*x^(1 + n))/(1 + n) + (b*d*x^(1 + 2*n))/(1 + 2*n)
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int (a+bx^n)(c+dx^n)\,dx&=\int \left(ac+(bc+ad)x^n+bdx^{2n}\right)dx\\&=acx+\frac{(bc+ad)x^{1+n}}{1+n}+\frac{bdx^{1+2n}}{1+2n}\end{aligned}$$

Mathematica [A]
time = 0.08, size = 37, normalized size = 0.92

$$x\left(ac+\frac{(bc+ad)x^n}{1+n}+\frac{bdx^{2n}}{1+2n}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^n)*(c + d*x^n),x]
```

```
[Out] x*(a*c + ((b*c + a*d)*x^n)/(1 + n) + (b*d*x^(2*n))/(1 + 2*n))
```

Maple [A]
time = 0.02, size = 39, normalized size = 0.98

method	result	size
risch	$acx+\frac{(ad+bc)x\,x^{n}}{1+n}+\frac{bdx\,x^{2n}}{1+2n}$	39
norman	$acx+\frac{(ad+bc)x\,\mathrm{e}^{n\ln(x)}}{1+n}+\frac{bdx\,\mathrm{e}^{2n\ln(x)}}{1+2n}$	43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^n)*(c+d*x^n),x,method=_RETURNVERBOSE)
```

```
[Out] a*c*x+(a*d+b*c)/(1+n)*x*x^n+b*d/(1+2*n)*x*(x^n)^2
```

Maxima [A]
time = 0.29, size = 48, normalized size = 1.20

$$acx+\frac{bdx^{2\,n+1}}{2\,n+1}+\frac{bcx^{n+1}}{n+1}+\frac{adx^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)*(c+d*x^n),x, algorithm="maxima")
```

```
[Out] a*c*x + b*d*x^(2*n + 1)/(2*n + 1) + b*c*x^(n + 1)/(n + 1) + a*d*x^(n + 1)/(
n + 1)
```

Fricas [A]
time = 3.37, size = 69, normalized size = 1.72

$$\frac{(bdn+bd)xx^{2\,n}+(bc+ad+2\,(bc+ad)n)xx^{n}+(2\,acn^{2}+3\,acn+ac)x}{2\,n^{2}+3\,n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)*(c+d*x^n),x, algorithm="fricas")
```

```
[Out] ((b*d*n + b*d)*x*x^(2*n) + (b*c + a*d + 2*(b*c + a*d)*n)*x*x^n + (2*a*c*n^2
 + 3*a*c*n + a*c)*x)/(2*n^2 + 3*n + 1)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 236 vs. 2(34) = 68.
time = 0.18, size = 236, normalized size = 5.90

$$\begin{cases} acx+ad\log{(x)}+bc\log{(x)}-\frac{bd}{x} & \text{for}\: n=-1 \\ acx+2ad\sqrt{x}+2bc\sqrt{x}+bd\log{(x)} & \text{for}\: n=-\frac{1}{2} \\ \frac{2acn^{2}x}{2n^{2}+3n+1}+\frac{3acnx}{2n^{2}+3n+1}+\frac{acx}{2n^{2}+3n+1}+\frac{2adnxx^{n}}{2n^{2}+3n+1}+\frac{adxx^{n}}{2n^{2}+3n+1}+\frac{2bcnxx^{n}}{2n^{2}+3n+1}+\frac{bcxx^{n}}{2n^{2}+3n+1}+\frac{bdnxx^{2n}}{2n^{2}+3n+1}+\frac{bdxx^{2n}}{2n^{2}+3n+1} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n)*(c+d*x**n),x)
```

```
[Out] Piecewise((a*c*x + a*d*log(x) + b*c*log(x) - b*d/x, Eq(n, -1)), (a*c*x + 2*
a*d*sqrt(x) + 2*b*c*sqrt(x) + b*d*log(x), Eq(n, -1/2)), (2*a*c*n**2*x/(2*n*
*2 + 3*n + 1) + 3*a*c*n*x/(2*n**2 + 3*n + 1) + a*c*x/(2*n**2 + 3*n + 1) + 2
*a*d*n*x*x**n/(2*n**2 + 3*n + 1) + a*d*x*x**n/(2*n**2 + 3*n + 1) + 2*b*c*n*
x*x**n/(2*n**2 + 3*n + 1) + b*c*x*x**n/(2*n**2 + 3*n + 1) + b*d*n*x*x**(2*n
)/(2*n**2 + 3*n + 1) + b*d*x*x**(2*n)/(2*n**2 + 3*n + 1), True))
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 83 vs. $2(40) = 80$.
time = 0.83, size = 83, normalized size = 2.08

$$\frac{2\,acn^{2}x+bdnxx^{2\,n}+2\,bcnxx^{n}+2\,adnxx^{n}+3\,acnx+bdxx^{2\,n}+bcxx^{n}+adxx^{n}+acx}{2\,n^{2}+3\,n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)*(c+d*x^n),x, algorithm="giac")
```

```
[Out] (2*a*c*n^2*x + b*d*n*x*x^(2*n) + 2*b*c*n*x*x^n + 2*a*d*n*x*x^n + 3*a*c*n*x
+ b*d*x*x^(2*n) + b*c*x*x^n + a*d*x*x^n + a*c*x)/(2*n^2 + 3*n + 1)
```

Mupad [**B**]
time = 1.48, size = 38, normalized size = 0.95

$$a\,c\,x+\frac{x\,x^{n}\,\left(a\,d+b\,c\right)}{n+1}+\frac{b\,d\,x\,x^{2\,n}}{2\,n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)*(c + d*x^n),x)
```

```
[Out] a*c*x + (x*x^n*(a*d + b*c))/(n + 1) + (b*d*x*x^(2*n))/(2*n + 1)
```

3.288 $\int \frac{a+bx^n}{c+dx^n} \, dx$

Optimal. Leaf size=43

$$\frac{bx}{d} - \frac{(bc-ad)x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{cd}$$

```
[Out] b*x/d-(-a*d+b*c)*x*hypergeom([1, 1/n],[1+1/n],-d*x^n/c)/c/d
```

Rubi [A]
time = 0.01, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {396, 251}

$$\frac{bx}{d} - \frac{x(bc-ad)\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{cd}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n)/(c + d*x^n),x]
```

```
[Out] (b*x)/d - ((b*c - a*d)*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((d*x^n)
/c)])/(c*d)
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx^n}{c+dx^n}\,dx &= \frac{bx}{d} - \frac{(bc-ad)\int \frac{1}{c+dx^n}\,dx}{d}\\ &= \frac{bx}{d} - \frac{(bc-ad)x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{cd}\end{aligned}$$

Mathematica [A]
time = 0.08, size = 40, normalized size = 0.93

$$\frac{x\left(bc+(-bc+ad)\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)\right)}{cd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^n)/(c + d*x^n),x]
```

```
[Out] (x*(b*c + (-(b*c) + a*d)*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((d*x^n)
/c)]))/(c*d)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{a+b\,x^n}{c+d\,x^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^n)/(c+d*x^n),x)
```

```
[Out] int((a+b*x^n)/(c+d*x^n),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)/(c+d*x^n),x, algorithm="maxima")
```

```
[Out] -(b*c - a*d)*integrate(1/(d^2*x^n + c*d), x) + b*x/d
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)/(c+d*x^n),x, algorithm="fricas")
```

```
[Out] integral((b*x^n + a)/(d*x^n + c), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.14, size = 73, normalized size = 1.70

$$\frac{ax\Phi\left(\frac{dx^ne^{i\pi}}{c},1,\frac{1}{n}\right)\Gamma\left(\frac{1}{n}\right)}{cn^2\Gamma\left(1+\frac{1}{n}\right)}-\frac{bx\Phi\left(\frac{cx^{-n}e^{i\pi}}{d},1,\frac{e^{i\pi}}{n}\right)\Gamma\left(\frac{1}{n}\right)}{dn^2\Gamma\left(1+\frac{1}{n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n)/(c+d*x**n),x)
```

```
[Out] a*x*lerchphi(d*x**n*exp_polar(I*pi)/c, 1, 1/n)*gamma(1/n)/(c*n**2*gamma(1 +
 1/n)) - b*x*lerchphi(c*exp_polar(I*pi)/(d*x**n), 1, exp_polar(I*pi)/n)*gam
ma(1/n)/(d*n**2*gamma(1 + 1/n))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)/(c+d*x^n),x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a)/(d*x^n + c), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{a+b\,x^n}{c+d\,x^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)/(c + d*x^n),x)
```

```
[Out] int((a + b*x^n)/(c + d*x^n), x)
```

3.289 $\int \frac{a+bx^n}{(c+dx^n)^2} dx$

Optimal. Leaf size=73

$$-\frac{(bc-ad)x}{cdn\,(c+dx^n)}+\frac{(bc-ad(1-n))x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{c^2dn}$$

```
[Out] -(-a*d+b*c)*x/c/d/n/(c+d*x^n)+(b*c-a*d*(1-n))*x*hypergeom([1, 1/n],[1+1/n],
-d*x^n/c)/c^2/d/n
```

Rubi [A]
time = 0.02, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {393, 251}

$$\frac{x(bc-ad(1-n))\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{c^2dn}-\frac{x(bc-ad)}{cdn\,(c+dx^n)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n)/(c + d*x^n)^2,x]
```

```
[Out] -(((b*c - a*d)*x)/(c*d*n*(c + d*x^n))) + ((b*c - a*d*(1 - n))*x*Hypergeomet
ric2F1[1, n^(-1), 1 + n^(-1), -((d*x^n)/c)])/(c^2*d*n)
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx^n}{(c+dx^n)^2}\,dx &= -\frac{(bc-ad)x}{cdn\,(c+dx^n)}+\frac{(bc-ad(1-n))\int \frac{1}{c+dx^n}\,dx}{cdn}\\ &= -\frac{(bc-ad)x}{cdn\,(c+dx^n)}+\frac{(bc-ad(1-n))x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{c^2dn}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 56, normalized size = 0.77

$$\frac{x\left(\frac{b}{c+dx^n}-\frac{(bc+ad(-1+n))\,{}_2F_1\left(2,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{c^2}\right)}{d-dn}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^n)/(c + d*x^n)^2,x]
```

```
[Out] (x*(b/(c + d*x^n) - ((b*c + a*d*(-1 + n))*Hypergeometric2F1[2, n^(-1), 1 +
n^(-1), -((d*x^n)/c)])/c^2))/(d - d*n)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{a+b\,x^n}{\left(c+d\,x^n\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^n)/(c+d*x^n)^2,x)
```

```
[Out] int((a+b*x^n)/(c+d*x^n)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)/(c+d*x^n)^2,x, algorithm="maxima")
```

```
[Out] (a*d*(n - 1) + b*c)*integrate(1/(c*d^2*n*x^n + c^2*d*n), x) - (b*c - a*d)*x
/(c*d^2*n*x^n + c^2*d*n)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)/(c+d*x^n)^2,x, algorithm="fricas")
```

```
[Out] integral((b*x^n + a)/(d^2*x^(2*n) + 2*c*d*x^n + c^2), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 3.49, size = 592, normalized size = 8.11

$$a\left(\frac{nx\Phi\left(\frac{dx^{n}e^{i\pi}}{c},1,\frac{1}{n}\right)\Gamma\left(\frac{1}{n}\right)}{c\left(cn^{3}\Gamma\left(1+\frac{1}{n}\right)+dn^{3}x^{n}\Gamma\left(1+\frac{1}{n}\right)\right)}+\frac{nx\Gamma\left(\frac{1}{n}\right)}{c\left(cn^{3}\Gamma\left(1+\frac{1}{n}\right)+dn^{3}x^{n}\Gamma\left(1+\frac{1}{n}\right)\right)}-\frac{x\Phi\left(\frac{dx^{n}e^{i\pi}}{c},1,\frac{1}{n}\right)\Gamma\left(\frac{1}{n}\right)}{c\left(cn^{3}\Gamma\left(1+\frac{1}{n}\right)+dn^{3}x^{n}\Gamma\left(1+\frac{1}{n}\right)\right)}+\frac{dnxx^{n}\Phi\left(\frac{dx^{n}e^{i\pi}}{c},1,\frac{1}{n}\right)\Gamma\left(\frac{1}{n}\right)}{c^{2}\left(cn^{3}\Gamma\left(1+\frac{1}{n}\right)+dn^{3}x^{n}\Gamma\left(1+\frac{1}{n}\right)\right)}-\frac{dxx^{n}\Phi\left(\frac{dx^{n}e^{i\pi}}{c},1,\frac{1}{n}\right)\Gamma\left(\frac{1}{n}\right)}{c^{2}\left(cn^{3}\Gamma\left(1+\frac{1}{n}\right)+dn^{3}x^{n}\Gamma\left(1+\frac{1}{n}\right)\right)}\right)+b\left(\frac{n^{2}xx^{n}\Gamma\left(1+\frac{1}{n}\right)}{c\left(cn^{3}\Gamma\left(2+\frac{1}{n}\right)+dn^{3}x^{n}\Gamma\left(2+\frac{1}{n}\right)\right)}-\frac{nxx^{n}\Phi\left(\frac{dx^{n}e^{i\pi}}{c},1,1+\frac{1}{n}\right)\Gamma\left(1+\frac{1}{n}\right)}{c\left(cn^{3}\Gamma\left(2+\frac{1}{n}\right)+dn^{3}x^{n}\Gamma\left(2+\frac{1}{n}\right)\right)}+\frac{nxx^{n}\Gamma\left(1+\frac{1}{n}\right)}{c\left(cn^{3}\Gamma\left(2+\frac{1}{n}\right)+dn^{3}x^{n}\Gamma\left(2+\frac{1}{n}\right)\right)}-\frac{xx^{n}\Phi\left(\frac{dx^{n}e^{i\pi}}{c},1,1+\frac{1}{n}\right)\Gamma\left(1+\frac{1}{n}\right)}{c\left(cn^{3}\Gamma\left(2+\frac{1}{n}\right)+dn^{3}x^{n}\Gamma\left(2+\frac{1}{n}\right)\right)}-\frac{dnxx^{2n}\Phi\left(\frac{dx^{n}e^{i\pi}}{c},1,1+\frac{1}{n}\right)\Gamma\left(1+\frac{1}{n}\right)}{c^{2}\left(cn^{3}\Gamma\left(2+\frac{1}{n}\right)+dn^{3}x^{n}\Gamma\left(2+\frac{1}{n}\right)\right)}-\frac{dxx^{2n}\Phi\left(\frac{dx^{n}e^{i\pi}}{c},1,1+\frac{1}{n}\right)\Gamma\left(1+\frac{1}{n}\right)}{c^{2}\left(cn^{3}\Gamma\left(2+\frac{1}{n}\right)+dn^{3}x^{n}\Gamma\left(2+\frac{1}{n}\right)\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n)/(c+d*x**n)**2,x)
```

```
[Out] a*(n*x*lerchphi(d*x**n*exp_polar(I*pi)/c, 1, 1/n)*gamma(1/n)/(c*(c*n**3*gam
ma(1 + 1/n) + d*n**3*x**n*gamma(1 + 1/n))) + n*x*gamma(1/n)/(c*(c*n**3*gamm
a(1 + 1/n) + d*n**3*x**n*gamma(1 + 1/n))) - x*lerchphi(d*x**n*exp_polar(I*p
i)/c, 1, 1/n)*gamma(1/n)/(c*(c*n**3*gamma(1 + 1/n) + d*n**3*x**n*gamma(1 +
1/n))) + d*n*x*x**n*lerchphi(d*x**n*exp_polar(I*pi)/c, 1, 1/n)*gamma(1/n)/(
c**2*(c*n**3*gamma(1 + 1/n) + d*n**3*x**n*gamma(1 + 1/n))) - d*x*x**n*lerch
phi(d*x**n*exp_polar(I*pi)/c, 1, 1/n)*gamma(1/n)/(c**2*(c*n**3*gamma(1 + 1/
n) + d*n**3*x**n*gamma(1 + 1/n)))) + b*(n**2*x*x**n*gamma(1 + 1/n)/(c*(c*n*
*3*gamma(2 + 1/n) + d*n**3*x**n*gamma(2 + 1/n))) - n*x*x**n*lerchphi(d*x**n
*exp_polar(I*pi)/c, 1, 1 + 1/n)*gamma(1 + 1/n)/(c*(c*n**3*gamma(2 + 1/n) +
d*n**3*x**n*gamma(2 + 1/n))) + n*x*x**n*gamma(1 + 1/n)/(c*(c*n**3*gamma(2 +
 1/n) + d*n**3*x**n*gamma(2 + 1/n))) - x*x**n*lerchphi(d*x**n*exp_polar(I*p
i)/c, 1, 1 + 1/n)*gamma(1 + 1/n)/(c*(c*n**3*gamma(2 + 1/n) + d*n**3*x**n*ga
mma(2 + 1/n))) - d*n*x*x**(2*n)*lerchphi(d*x**n*exp_polar(I*pi)/c, 1, 1 + 1
/n)*gamma(1 + 1/n)/(c**2*(c*n**3*gamma(2 + 1/n) + d*n**3*x**n*gamma(2 + 1/n
))) - d*x*x**(2*n)*lerchphi(d*x**n*exp_polar(I*pi)/c, 1, 1 + 1/n)*gamma(1 +
 1/n)/(c**2*(c*n**3*gamma(2 + 1/n) + d*n**3*x**n*gamma(2 + 1/n))))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)/(c+d*x^n)^2,x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a)/(d*x^n + c)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\,x^n}{{\left(c+d\,x^n\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)/(c + d*x^n)^2,x)
```

```
[Out] int((a + b*x^n)/(c + d*x^n)^2, x)
```

3.290 $\int \frac{a+bx^n}{(c+dx^n)^3} \, dx$

Optimal. Leaf size=78

$$-\frac{(bc-ad)x}{2cdn\left(c+dx^n\right)^2}+\frac{(bc-ad(1-2n))x\,{}_2F_1\left(2,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{2c^3dn}$$

```
[Out] -1/2*(-a*d+b*c)*x/c/d/n/(c+d*x^n)^2+1/2*(b*c-a*d*(1-2*n))*x*hypergeom([2, 1
/n],[1+1/n],-d*x^n/c)/c^3/d/n
```

Rubi [A]

time = 0.02, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {393, 251}

$$\frac{x(bc-ad(1-2n))\,{}_2F_1\left(2,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{2c^3dn}-\frac{x(bc-ad)}{2cdn\left(c+dx^n\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n)/(c + d*x^n)^3,x]
```

```
[Out] -1/2*((b*c - a*d)*x)/(c*d*n*(c + d*x^n)^2) + ((b*c - a*d*(1 - 2*n))*x*Hyper
geometric2F1[2, n^(-1), 1 + n^(-1), -((d*x^n)/c)])/(2*c^3*d*n)
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx^n}{\left(c+dx^n\right)^3} \, dx &= -\frac{(bc-ad)x}{2cdn\left(c+dx^n\right)^2}+\frac{(bc-ad(1-2n))\int \frac{1}{\left(c+dx^n\right)^2} \, dx}{2cdn}\\ &= -\frac{(bc-ad)x}{2cdn\left(c+dx^n\right)^2}+\frac{(bc-ad(1-2n))x\,{}_2F_1\left(2,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{2c^3dn}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 58, normalized size = 0.74

$$\frac{x\left(\frac{b}{(c+dx^n)^2}-\frac{(bc+ad(-1+2n))\,{}_2F_1\left(3,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{c^3}\right)}{d-2dn}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^n)/(c + d*x^n)^3,x]
```

```
[Out] (x*(b/(c + d*x^n)^2 - ((b*c + a*d*(-1 + 2*n))*Hypergeometric2F1[3, n^(-1),
1 + n^(-1), -((d*x^n)/c)])/c^3))/(d - 2*d*n)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{a+b\,x^n}{(c+d\,x^n)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^n)/(c+d*x^n)^3,x)
```

```
[Out] int((a+b*x^n)/(c+d*x^n)^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)/(c+d*x^n)^3,x, algorithm="maxima")
```

```
[Out] ((2*n^2 - 3*n + 1)*a*d + b*c*(n - 1))*integrate(1/2/(c^2*d^2*n^2*x^n + c^3*
d*n^2), x) + 1/2*((a*d^2*(2*n - 1) + b*c*d)*x*x^n + (a*c*d*(3*n - 1) - b*c^
2*(n - 1))*x)/(c^2*d^3*n^2*x^(2*n) + 2*c^3*d^2*n^2*x^n + c^4*d*n^2)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)/(c+d*x^n)^3,x, algorithm="fricas")
```

```
[Out] integral((b*x^n + a)/(d^3*x^(3*n) + 3*c*d^2*x^(2*n) + 3*c^2*d*x^n + c^3), x
)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n)/(c+d*x**n)**3,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)/(c+d*x^n)^3,x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a)/(d*x^n + c)^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\,x^n}{{\left(c+d\,x^n\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)/(c + d*x^n)^3,x)
```

```
[Out] int((a + b*x^n)/(c + d*x^n)^3, x)
```

3.291 $\int \frac{a+bx^n}{(c+dx^n)^4}\,dx$

Optimal. Leaf size=78

$$-\frac{(bc-ad)x}{3cdn\,(c+dx^n)^3}+\frac{(bc-ad(1-3n))x\,{}_2F_1\left(3,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{3c^4dn}$$

```
[Out] -1/3*(-a*d+b*c)*x/c/d/n/(c+d*x^n)^3+1/3*(b*c-a*d*(1-3*n))*x*hypergeom([3, 1
/n],[1+1/n],-d*x^n/c)/c^4/d/n
```

Rubi [A]

time = 0.02, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {393, 251}

$$\frac{x(bc-ad(1-3n))\,{}_2F_1\left(3,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{3c^4dn}-\frac{x(bc-ad)}{3cdn\,(c+dx^n)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n)/(c + d*x^n)^4,x]
```

```
[Out] -1/3*((b*c - a*d)*x)/(c*d*n*(c + d*x^n)^3) + ((b*c - a*d*(1 - 3*n))*x*Hyper
geometric2F1[3, n^(-1), 1 + n^(-1), -((d*x^n)/c)])/(3*c^4*d*n)
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx^n}{(c+dx^n)^4}\,dx&=-\frac{(bc-ad)x}{3cdn\,(c+dx^n)^3}+\frac{(bc-ad(1-3n))\int \frac{1}{(c+dx^n)^3}\,dx}{3cdn}\\&=-\frac{(bc-ad)x}{3cdn\,(c+dx^n)^3}+\frac{(bc-ad(1-3n))x\,{}_2F_1\left(3,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{3c^4dn}\end{aligned}$$

Mathematica [A]
time = 0.07, size = 58, normalized size = 0.74

$$\frac{x\left(\frac{b}{(c+dx^n)^3}-\frac{(bc+ad(-1+3n))\,{}_2F_1\left(4,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{c^4}\right)}{d-3dn}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^n)/(c + d*x^n)^4,x]
```

```
[Out] (x*(b/(c + d*x^n)^3 - ((b*c + a*d*(-1 + 3*n))*Hypergeometric2F1[4, n^(-1),
1 + n^(-1), -((d*x^n)/c)])/c^4))/(d - 3*d*n)
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{a+b\,x^n}{\left(c+d\,x^n\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^n)/(c+d*x^n)^4,x)
```

```
[Out] int((a+b*x^n)/(c+d*x^n)^4,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)/(c+d*x^n)^4,x, algorithm="maxima")
```

```
[Out] ((2*n^2 - 3*n + 1)*b*c + (6*n^3 - 11*n^2 + 6*n - 1)*a*d)*integrate(1/6/(c^3
*d^2*n^3*x^n + c^4*d*n^3), x) + 1/6*(((6*n^2 - 5*n + 1)*a*d^3 + b*c*d^2*(2*
n - 1))*x*x^(2*n) + ((15*n^2 - 11*n + 2)*a*c*d^2 + b*c^2*d*(5*n - 2))*x*x^n
 - ((2*n^2 - 3*n + 1)*b*c^3 - (11*n^2 - 6*n + 1)*a*c^2*d)*x)/(c^3*d^4*n^3*x
^(3*n) + 3*c^4*d^3*n^3*x^(2*n) + 3*c^5*d^2*n^3*x^n + c^6*d*n^3)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)/(c+d*x^n)^4,x, algorithm="fricas")
```

```
[Out] integral((b*x^n + a)/(d^4*x^(4*n) + 4*c*d^3*x^(3*n) + 6*c^2*d^2*x^(2*n) + 4
*c^3*d*x^n + c^4), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 190.75, size = 18149, normalized size = 232.68

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n)/(c+d*x**n)**4,x)
```

```
[Out] a*(6*c**5*n**3*x*lerchphi(d*x**n*exp_polar(I*pi)/c, 1, 1/n)*gamma(1/n)/(6*c
**9*n**5*gamma(1 + 1/n) + 36*c**8*d*n**5*x**n*gamma(1 + 1/n) + 90*c**7*d**2
*n**5*x**(2*n)*gamma(1 + 1/n) + 120*c**6*d**3*n**5*x**(3*n)*gamma(1 + 1/n)
+ 90*c**5*d**4*n**5*x**(4*n)*gamma(1 + 1/n) + 36*c**4*d**5*n**5*x**(5*n)*ga
mma(1 + 1/n) + 6*c**3*d**6*n**5*x**(6*n)*gamma(1 + 1/n)) + 11*c**5*n**3*x*g
amma(1/n)/(6*c**9*n**5*gamma(1 + 1/n) + 36*c**8*d*n**5*x**n*gamma(1 + 1/n)
+ 90*c**7*d**2*n**5*x**(2*n)*gamma(1 + 1/n) + 120*c**6*d**3*n**5*x**(3*n)*g
amma(1 + 1/n) + 90*c**5*d**4*n**5*x**(4*n)*gamma(1 + 1/n) + 36*c**4*d**5*n*
*5*x**(5*n)*gamma(1 + 1/n) + 6*c**3*d**6*n**5*x**(6*n)*gamma(1 + 1/n)) - 11
*c**5*n**2*x*lerchphi(d*x**n*exp_polar(I*pi)/c, 1, 1/n)*gamma(1/n)/(6*c**9*
n**5*gamma(1 + 1/n) + 36*c**8*d*n**5*x**n*gamma(1 + 1/n) + 90*c**7*d**2*n**
5*x**(2*n)*gamma(1 + 1/n) + 120*c**6*d**3*n**5*x**(3*n)*gamma(1 + 1/n) + 90
*c**5*d**4*n**5*x**(4*n)*gamma(1 + 1/n) + 36*c**4*d**5*n**5*x**(5*n)*gamma(
1 + 1/n) + 6*c**3*d**6*n**5*x**(6*n)*gamma(1 + 1/n)) - 6*c**5*n**2*x*gamma(
1/n)/(6*c**9*n**5*gamma(1 + 1/n) + 36*c**8*d*n**5*x**n*gamma(1 + 1/n) + 90*
c**7*d**2*n**5*x**(2*n)*gamma(1 + 1/n) + 120*c**6*d**3*n**5*x**(3*n)*gamma(
1 + 1/n) + 90*c**5*d**4*n**5*x**(4*n)*gamma(1 + 1/n) + 36*c**4*d**5*n**5*x*
*(5*n)*gamma(1 + 1/n) + 6*c**3*d**6*n**5*x**(6*n)*gamma(1 + 1/n)) + 6*c**5*
n*x*lerchphi(d*x**n*exp_polar(I*pi)/c, 1, 1/n)*gamma(1/n)/(6*c**9*n**5*gamm
a(1 + 1/n) + 36*c**8*d*n**5*x**n*gamma(1 + 1/n) + 90*c**7*d**2*n**5*x**(2*n
)*gamma(1 + 1/n) + 120*c**6*d**3*n**5*x**(3*n)*gamma(1 + 1/n) + 90*c**5*d**
4*n**5*x**(4*n)*gamma(1 + 1/n) + 36*c**4*d**5*n**5*x**(5*n)*gamma(1 + 1/n)
+ 6*c**3*d**6*n**5*x**(6*n)*gamma(1 + 1/n)) + c**5*n*x*gamma(1/n)/(6*c**9*n
**5*gamma(1 + 1/n) + 36*c**8*d*n**5*x**n*gamma(1 + 1/n) + 90*c**7*d**2*n**5
*x**(2*n)*gamma(1 + 1/n) + 120*c**6*d**3*n**5*x**(3*n)*gamma(1 + 1/n) + 90*
c**5*d**4*n**5*x**(4*n)*gamma(1 + 1/n) + 36*c**4*d**5*n**5*x**(5*n)*gamma(1
 + 1/n) + 6*c**3*d**6*n**5*x**(6*n)*gamma(1 + 1/n)) - c**5*x*lerchphi(d*x**
n*exp_polar(I*pi)/c, 1, 1/n)*gamma(1/n)/(6*c**9*n**5*gamma(1 + 1/n) + 36*c*
*8*d*n**5*x**n*gamma(1 + 1/n) + 90*c**7*d**2*n**5*x**(2*n)*gamma(1 + 1/n) +
 120*c**6*d**3*n**5*x**(3*n)*gamma(1 + 1/n) + 90*c**5*d**4*n**5*x**(4*n)*ga
mma(1 + 1/n) + 36*c**4*d**5*n**5*x**(5*n)*gamma(1 + 1/n) + 6*c**3*d**6*n**5
*x**(6*n)*gamma(1 + 1/n)) + 36*c**4*d*n**3*x*x**n*lerchphi(d*x**n*exp_polar
(I*pi)/c, 1, 1/n)*gamma(1/n)/(6*c**9*n**5*gamma(1 + 1/n) + 36*c**8*d*n**5*x
**n*gamma(1 + 1/n) + 90*c**7*d**2*n**5*x**(2*n)*gamma(1 + 1/n) + 120*c**6*d
```

```
**3*n**5*x**(3*n)*gamma(1 + 1/n) + 90*c**5*d**4*n**5*x**(4*n)*gamma(1 + 1/n
) + 36*c**4*d**5*n**5*x**(5*n)*gamma(1 + 1/n) + 6*c**3*d**6*n**5*x**(6*n)*g
amma(1 + 1/n)) + 48*c**4*d*n**3*x*x**n*gamma(1/n)/(6*c**9*n**5*gamma(1 + 1/
n) + 36*c**8*d*n**5*x**n*gamma(1 + 1/n) + 90*c**7*d**2*n**5*x**(2*n)*gamma(
1 + 1/n) + 120*c**6*d**3*n**5*x**(3*n)*gamma(1 + 1/n) + 90*c**5*d**4*n**5*x
**(4*n)*gamma(1 + 1/n) + 36*c**4*d**5*n**5*x**(5*n)*gamma(1 + 1/n) + 6*c**3
*d**6*n**5*x**(6*n)*gamma(1 + 1/n)) - 66*c**4*d*n**2*x*x**n*lerchphi(d*x**n
*exp_polar(I*pi)/c, 1, 1/n)*gamma(1/n)/(6*c**9*n**5*gamma(1 + 1/n) + 36*c**
8*d*n**5*x**n*gamma(1 + 1/n) + 90*c**7*d**2*n**5*x**(2*n)*gamma(1 + 1/n) +
120*c**6*d**3*n**5*x**(3*n)*gamma(1 + 1/n) + 90*c**5*d**4*n**5*x**(4*n)*gam
ma(1 + 1/n) + 36*c**4*d**5*n**5*x**(5*n)*gamma(1 + 1/n) + 6*c**3*d**6*n**5*
x**(6*n)*gamma(1 + 1/n)) - 29*c**4*d*n**2*x*x**n*gamma(1/n)/(6*c**9*n**5*ga
mma(1 + 1/n) + 36*c**8*d*n**5*x**n*gamma(1 + 1/n) + 90*c**7*d**2*n**5*x**(2
*n)*gamma(1 + 1/n) + 120*c**6*d**3*n**5*x**(3*n)*gamma(1 + 1/n) + 90*c**5*d
**4*n**5*x**(4*n)*gamma(1 + 1/n) + 36*c**4*d**5*n**5*x**(5*n)*gamma(1 + 1/n
) + 6*c**3*d**6*n**5*x**(6*n)*gamma(1 + 1/n)) + 36*c**4*d*n*x*x**n*lerchphi
(d*x**n*exp_polar(I*pi)/c, 1, 1/n)*gamma(1/n)/(6*c**9*n**5*gamma(1 + 1/n) +
 36*c**8*d*n**5*x**n*gamma(1 + 1/n) + 90*c**7*d**2*n**5*x**(2*n)*gamma(1 +
1/n) + 120*c**6*d**3*n**5*x**(3*n)*gamma(1 + 1/n) + 90*c**5*d**4*n**5*x**(4
*n)*gamma(1 + 1/n) + 36*c**4*d**5*n**5*x**(5*n)*gamma(1 + 1/n) + 6*c**3*d**
6*n**5*x**(6*n)*gamma(1 + 1/n)) + 5*c**4*d*n*x*x**n*gamma(1/n)/(6*c**9*n**5
*gamma(1 + 1/n) + 36*c**8*d*n**5*x**n*gamma(1 + 1/n) + 90*c**7*d**2*n**5*x*
*(2*n)*gamma(1 + 1/n) + 120*c**6*d**3*n**5*x**(3*n)*gamma(1 + 1/n) + 90*c**
5*d**4*n**5*x**(4*n)*gamma(1 + 1/n) + 36*c**4*d**5*n**5*x**(5*n)*gamma(1 +
1/n) + 6*c**3*d**6*n**5*x**(6*n)*gamma(1 + 1/n)) - 6*c**4*d*x*x**n*lerchphi
(d*x**n*exp_polar(I*pi)/c, 1, 1/n)*gamma(1/n)/(6*c**9*n**5*gamma(1 + 1/n) +
 36*c**8*d*n**5*x**n*gamma(1 + 1/n) + 90*c**7*d**2*n**5*x**(2*n)*gamma(1 +
1/n) + 120*c**6*d**3*n**5*x**(3*n)*gamma(1 + 1/n) + 90*c**5*d**4*n**5*x**(4
*n)*gamma(1 + 1/n) + 36*c**4*d**5*n**5*x**(5*n)*gamma(1 + 1/n) + 6*c**3*d**
6*n**5*x**(6*n)*gamma(1 + 1/n)) + 90*c**3*d**2*n**3*x*x**(2*n)*lerchphi(d*x
**n*exp_polar(I*pi)/c, 1, 1/n)*gamma(1/n)/(6*c**9*n**5*gamma(1 + 1/n) + 36*
c**8*d*n**5*x**n*gamma(1 + 1/n) + 90*c**7*d**2*n**5*x**(2*n)*gamma(1 + 1/n)
 + 120*c**6*d**3*n**5*x**(3*n)*gamma(1 + 1/n) +...
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)/(c+d*x^n)^4,x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a)/(d*x^n + c)^4, x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a + b\,x^n}{(c + d\,x^n)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)/(c + d*x^n)^4,x)
```

```
[Out] int((a + b*x^n)/(c + d*x^n)^4, x)
```

3.292 $\int (a+bx^n)^2 (d+ex^n)^3 \, dx$

Optimal. Leaf size=158

$$a^2d^3x+\frac{ad^2(2bd+3ae)x^{1+n}}{1+n}+\frac{d(b^2d^2+6abde+3a^2e^2)\,x^{1+2n}}{1+2n}+\frac{e(3b^2d^2+6abde+a^2e^2)\,x^{1+3n}}{1+3n}+\frac{be^2(3bd+2ae)}{1+4n}$$

```
[Out] a^2*d^3*x+a*d^2*(3*a*e+2*b*d)*x^(1+n)/(1+n)+d*(3*a^2*e^2+6*a*b*d*e+b^2*d^2)
*x^(1+2*n)/(1+2*n)+e*(a^2*e^2+6*a*b*d*e+3*b^2*d^2)*x^(1+3*n)/(1+3*n)+b*e^2*
(2*a*e+3*b*d)*x^(1+4*n)/(1+4*n)+b^2*e^3*x^(1+5*n)/(1+5*n)
```

Rubi [A]
time = 0.09, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {380}

$$\frac{dx^{2n+1}(3a^2e^2+6abde+b^2d^2)}{2n+1}+\frac{ex^{3n+1}(a^2e^2+6abde+3b^2d^2)}{3n+1}+a^2d^3x+\frac{ad^2x^{n+1}(3ae+2bd)}{n+1}+\frac{be^2x^{4n+1}(2ae+3bd)}{4n+1}+\frac{b^2e^3x^{5n+1}}{5n+1}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n)^2*(d + e*x^n)^3,x]
```

```
[Out] a^2*d^3*x + (a*d^2*(2*b*d + 3*a*e)*x^(1 + n))/(1 + n) + (d*(b^2*d^2 + 6*a*b
*d*e + 3*a^2*e^2)*x^(1 + 2*n))/(1 + 2*n) + (e*(3*b^2*d^2 + 6*a*b*d*e + a^2*
e^2)*x^(1 + 3*n))/(1 + 3*n) + (b*e^2*(3*b*d + 2*a*e)*x^(1 + 4*n))/(1 + 4*n)
 + (b^2*e^3*x^(1 + 5*n))/(1 + 5*n)
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int (a+bx^n)^2 (d+ex^n)^3 \, dx = \int \left(a^2d^3+ad^2(2bd+3ae)x^n+d\left(b^2d^2+6abde+3a^2e^2\right)x^{2n}+e\left(3b^2d^2+6abd\right.\right.$$

$$=a^2d^3x+\frac{ad^2(2bd+3ae)x^{1+n}}{1+n}+\frac{d(b^2d^2+6abde+3a^2e^2)\,x^{1+2n}}{1+2n}+\frac{e(3b^2d^2+6a}{1}$$

Mathematica [A]
time = 0.57, size = 149, normalized size = 0.94

$$x\left(a^2d^3+\frac{ad^2(2bd+3ae)x^n}{1+n}+\frac{d(b^2d^2+6abde+3a^2e^2)\,x^{2n}}{1+2n}+\frac{e(3b^2d^2+6abde+a^2e^2)\,x^{3n}}{1+3n}+\frac{be^2(3bd+2ae)x^{4n}}{1+4n}+\frac{b^2e^3x^{5n}}{1+5n}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^n)^2*(d + e*x^n)^3,x]
```

```
[Out] x*(a^2*d^3 + (a*d^2*(2*b*d + 3*a*e)*x^n)/(1 + n) + (d*(b^2*d^2 + 6*a*b*d*e
+ 3*a^2*e^2)*x^(2*n))/(1 + 2*n) + (e*(3*b^2*d^2 + 6*a*b*d*e + a^2*e^2)*x^(3
*n))/(1 + 3*n) + (b*e^2*(3*b*d + 2*a*e)*x^(4*n))/(1 + 4*n) + (b^2*e^3*x^(5*
n))/(1 + 5*n))
```

Maple [A]
time = 0.26, size = 154, normalized size = 0.97

method	result
risch	$a^2 d^3 x + \frac{b^2 e^3 x\, x^{5n}}{1+5n} + \frac{d(3a^2e^2+6abde+b^2d^2)x\, x^{2n}}{1+2n} + \frac{e(a^2e^2+6abde+3b^2d^2)x\, x^{3n}}{1+3n} + \frac{a\, d^2(3ae+2bd)x\, x^n}{1+n} + \frac{b\, e^2(2ae+3bd)x}{1+4n}$
norman	$a^2 d^3 x + \frac{b^2 e^3 x\, \mathrm{e}^{5n\ln(x)}}{1+5n} + \frac{d(3a^2e^2+6abde+b^2d^2)x\, \mathrm{e}^{2n\ln(x)}}{1+2n} + \frac{e(a^2e^2+6abde+3b^2d^2)x\, \mathrm{e}^{3n\ln(x)}}{1+3n} + \frac{a\, d^2(3ae+2bd)x\, \mathrm{e}^{n\ln(x)}}{1+n}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^n)^2*(d+e*x^n)^3,x,method=_RETURNVERBOSE)
```

```
[Out] a^2*d^3*x+b^2*e^3/(1+5*n)*x*(x^n)^5+d*(3*a^2*e^2+6*a*b*d*e+b^2*d^2)/(1+2*n)
*x*(x^n)^2+e*(a^2*e^2+6*a*b*d*e+3*b^2*d^2)/(1+3*n)*x*(x^n)^3+a*d^2*(3*a*e+2
*b*d)/(1+n)*x*x^n+b*e^2*(2*a*e+3*b*d)/(1+4*n)*x*(x^n)^4
```

Maxima [A]
time = 0.29, size = 242, normalized size = 1.53

$$a^2d^3x + \frac{b^2e^3x^{5n+1}}{5n+1} + \frac{3b^2de^2x^{4n+1}}{4n+1} + \frac{2abe^3x^{4n+1}}{4n+1} + \frac{3b^2d^2ex^{3n+1}}{3n+1} + \frac{6abde^2x^{3n+1}}{3n+1} + \frac{a^2e^3x^{3n+1}}{3n+1} + \frac{b^2d^3x^{2n+1}}{2n+1} + \frac{6abd^2ex^{2n+1}}{2n+1} + \frac{3a^2de^2x^{2n+1}}{2n+1} + \frac{2abd^3x^{n+1}}{n+1} + \frac{3a^2d^2ex^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^2*(d+e*x^n)^3,x, algorithm="maxima")
```

```
[Out] a^2*d^3*x + b^2*e^3*x^(5*n + 1)/(5*n + 1) + 3*b^2*d*e^2*x^(4*n + 1)/(4*n +
1) + 2*a*b*e^3*x^(4*n + 1)/(4*n + 1) + 3*b^2*d^2*e*x^(3*n + 1)/(3*n + 1) +
6*a*b*d*e^2*x^(3*n + 1)/(3*n + 1) + a^2*e^3*x^(3*n + 1)/(3*n + 1) + b^2*d^3
*x^(2*n + 1)/(2*n + 1) + 6*a*b*d^2*e*x^(2*n + 1)/(2*n + 1) + 3*a^2*d*e^2*x^
(2*n + 1)/(2*n + 1) + 2*a*b*d^3*x^(n + 1)/(n + 1) + 3*a^2*d^2*e*x^(n + 1)/(
n + 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 624 vs. 2(159) = 318.
time = 2.18, size = 624, normalized size = 3.95

$$\frac{(24b^2n^4+50b^2n^3+35b^2n^2+10b^2n+b^2)xx^{5n}e^3+(120a^2d^3n^5+274a^2d^3n^4+225a^2d^3n^3+85a^2d^3n^2+15a^2d^3n+a^2d^3)x+(2(30abn^4+61abn^3+41abn^2+11abn+ab)xe^3+3(30b^2dn^4+61b^2dn^3+41b^2dn^2+11b^2dn+b^2d)xe^2)x^{4n}+((40a^2n^4+78a^2n^3+49a^2n^2+12a^2n+a^2)xe^3+6(40abdn^4+78abdn^3+49abdn^2+12abdn+abd)xe^2+3(40b^2d^2n^4+78b^2d^2n^3+49b^2d^2n^2+12b^2d^2n+b^2d^2)xe)x^{3n}+(3(60a^2dn^4+107a^2dn^3+59a^2dn^2+13a^2dn+a^2d)xe^2+6(60abd^2n^4+107abd^2n^3+59abd^2n^2+13abd^2n+abd^2)xe+(60b^2d^3n^4+107b^2d^3n^3+59b^2d^3n^2+13b^2d^3n+b^2d^3)x)x^{2n}+(3(120a^2d^2n^4+154a^2d^2n^3+71a^2d^2n^2+14a^2d^2n+a^2d^2)xe+2(120abd^3n^4+154abd^3n^3+71abd^3n^2+14abd^3n+abd^3)x)x^n}{120n^5+274n^4+225n^3+85n^2+15n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^2*(d+e*x^n)^3,x, algorithm="fricas")
```

```
[Out] ((24*b^2*n^4 + 50*b^2*n^3 + 35*b^2*n^2 + 10*b^2*n + b^2)*x*x^(5*n)*e^3 + (1
20*a^2*d^3*n^5 + 274*a^2*d^3*n^4 + 225*a^2*d^3*n^3 + 85*a^2*d^3*n^2 + 15*a^
2*d^3*n + a^2*d^3)*x + (2*(30*a*b*n^4 + 61*a*b*n^3 + 41*a*b*n^2 + 11*a*b*n
+ a*b)*x*e^3 + 3*(30*b^2*d*n^4 + 61*b^2*d*n^3 + 41*b^2*d*n^2 + 11*b^2*d*n +
 b^2*d)*x*e^2)*x^(4*n) + ((40*a^2*n^4 + 78*a^2*n^3 + 49*a^2*n^2 + 12*a^2*n
+ a^2)*x*e^3 + 6*(40*a*b*d*n^4 + 78*a*b*d*n^3 + 49*a*b*d*n^2 + 12*a*b*d*n +
 a*b*d)*x*e^2 + 3*(40*b^2*d^2*n^4 + 78*b^2*d^2*n^3 + 49*b^2*d^2*n^2 + 12*b^
2*d^2*n + b^2*d^2)*x*e)*x^(3*n) + (3*(60*a^2*d*n^4 + 107*a^2*d*n^3 + 59*a^2
*d*n^2 + 13*a^2*d*n + a^2*d)*x*e^2 + 6*(60*a*b*d^2*n^4 + 107*a*b*d^2*n^3 +
59*a*b*d^2*n^2 + 13*a*b*d^2*n + a*b*d^2)*x*e + (60*b^2*d^3*n^4 + 107*b^2*d^
3*n^3 + 59*b^2*d^3*n^2 + 13*b^2*d^3*n + b^2*d^3)*x)*x^(2*n) + (3*(120*a^2*d
^2*n^4 + 154*a^2*d^2*n^3 + 71*a^2*d^2*n^2 + 14*a^2*d^2*n + a^2*d^2)*x*e + 2
*(120*a*b*d^3*n^4 + 154*a*b*d^3*n^3 + 71*a*b*d^3*n^2 + 14*a*b*d^3*n + a*b*d
^3)*x)*x^n)/(120*n^5 + 274*n^4 + 225*n^3 + 85*n^2 + 15*n + 1)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 3380 vs. $2(151) = 302$.
time = 20.85, size = 3380, normalized size = 21.39

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n)**2*(d+e*x**n)**3,x)
```

```
[Out] Piecewise((a**2*d**3*x + 3*a**2*d**2*e*log(x) - 3*a**2*d*e**2/x - a**2*e**3
/(2*x**2) + 2*a*b*d**3*log(x) - 6*a*b*d**2*e/x - 3*a*b*d*e**2/x**2 - 2*a*b*
e**3/(3*x**3) - b**2*d**3/x - 3*b**2*d**2*e/(2*x**2) - b**2*d*e**2/x**3 - b
**2*e**3/(4*x**4), Eq(n, -1)), (a**2*d**3*x + 6*a**2*d**2*e*sqrt(x) + 3*a**
2*d*e**2*log(x) - 2*a**2*e**3/sqrt(x) + 4*a*b*d**3*sqrt(x) + 6*a*b*d**2*e*l
og(x) - 12*a*b*d*e**2/sqrt(x) - 2*a*b*e**3/x + b**2*d**3*log(x) - 6*b**2*d*
*2*e/sqrt(x) - 3*b**2*d*e**2/x - 2*b**2*e**3/(3*x**(3/2)), Eq(n, -1/2)), (a
**2*d**3*x + 9*a**2*d**2*e*x**(2/3)/2 + 9*a**2*d*e**2*x**(1/3) + a**2*e**3*
log(x) + 3*a*b*d**3*x**(2/3) + 18*a*b*d**2*e*x**(1/3) + 6*a*b*d*e**2*log(x)
 - 6*a*b*e**3/x**(1/3) + 3*b**2*d**3*x**(1/3) + 3*b**2*d**2*e*log(x) - 9*b*
*2*d*e**2/x**(1/3) - 3*b**2*e**3/(2*x**(2/3)), Eq(n, -1/3)), (a**2*d**3*x +
 4*a*d**2*x**(3/4)*(3*a*e + 2*b*d)/3 - 4*b**2*e**3/x**(1/4) - 4*b*e**2*(2*a
*e + 3*b*d)*log(x**(-1/4)) + 2*d*sqrt(x)*(3*a**2*e**2 + 6*a*b*d*e + b**2*d*
*2) + 4*e*x**(1/4)*(a**2*e**2 + 6*a*b*d*e + 3*b**2*d**2), Eq(n, -1/4)), (a*
*2*d**3*x + 5*a*d**2*x**(4/5)*(3*a*e + 2*b*d)/4 - 5*b**2*e**3*log(x**(-1/5)
) + 5*b*e**2*x**(1/5)*(2*a*e + 3*b*d) + 5*d*x**(3/5)*(3*a**2*e**2 + 6*a*b*d
*e + b**2*d**2)/3 + 5*e*x**(2/5)*(a**2*e**2 + 6*a*b*d*e + 3*b**2*d**2)/2, E
q(n, -1/5)), (120*a**2*d**3*n**5*x/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**
2 + 15*n + 1) + 274*a**2*d**3*n**4*x/(120*n**5 + 274*n**4 + 225*n**3 + 85*n
**2 + 15*n + 1) + 225*a**2*d**3*n**3*x/(120*n**5 + 274*n**4 + 225*n**3 + 85
```

*n**2 + 15*n + 1) + 85*a**2*d**3*n**2*x/(120*n**5 + 274*n**4 + 225*n**3 + 8
5*n**2 + 15*n + 1) + 15*a**2*d**3*n*x/(120*n**5 + 274*n**4 + 225*n**3 + 85*
n**2 + 15*n + 1) + a**2*d**3*x/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 +
15*n + 1) + 360*a**2*d**2*e*n**4*x*x**n/(120*n**5 + 274*n**4 + 225*n**3 + 8
5*n**2 + 15*n + 1) + 462*a**2*d**2*e*n**3*x*x**n/(120*n**5 + 274*n**4 + 225
*n**3 + 85*n**2 + 15*n + 1) + 213*a**2*d**2*e*n**2*x*x**n/(120*n**5 + 274*n
4 + 225*n3 + 85*n**2 + 15*n + 1) + 42*a**2*d**2*e*n*x*x**n/(120*n**5 +
274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 3*a**2*d**2*e*x*x**n/(120*n**5
+ 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 180*a**2*d*e**2*n**4*x*x**(2*
n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 321*a**2*d*e**2*
n**3*x*x**(2*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 177
*a**2*d*e**2*n**2*x*x**(2*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15
*n + 1) + 39*a**2*d*e**2*n*x*x**(2*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*
n**2 + 15*n + 1) + 3*a**2*d*e**2*x*x**(2*n)/(120*n**5 + 274*n**4 + 225*n**3
 + 85*n**2 + 15*n + 1) + 40*a**2*e**3*n**4*x*x**(3*n)/(120*n**5 + 274*n**4
+ 225*n**3 + 85*n**2 + 15*n + 1) + 78*a**2*e**3*n**3*x*x**(3*n)/(120*n**5 +
 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 49*a**2*e**3*n**2*x*x**(3*n)/(
120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 12*a**2*e**3*n*x*x**
(3*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + a**2*e**3*x*x
(3*n)/(120*n5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 240*a*b*d**
3*n**4*x*x**n/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 308*a
*b*d**3*n**3*x*x**n/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) +
 142*a*b*d**3*n**2*x*x**n/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n
+ 1) + 28*a*b*d**3*n*x*x**n/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*
n + 1) + 2*a*b*d**3*x*x**n/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n
 + 1) + 360*a*b*d**2*e*n**4*x*x**(2*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85
*n**2 + 15*n + 1) + 642*a*b*d**2*e*n**3*x*x**(2*n)/(120*n**5 + 274*n**4 + 2
25*n**3 + 85*n**2 + 15*n + 1) + 354*a*b*d**2*e*n**2*x*x**(2*n)/(120*n**5 +
274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 78*a*b*d**2*e*n*x*x**(2*n)/(120
*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 6*a*b*d**2*e*x*x**(2*n)
/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 240*a*b*d*e**2*n**
4*x*x**(3*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 468*a*
b*d*e**2*n**3*x*x**(3*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n +
 1) + 294*a*b*d*e**2*n**2*x*x**(3*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n
2 + 15*n + 1) + 72*a*b*d*e2*n*x*x**(3*n)/(120*n**5 + 274*n**4 + 225*n**
3 + 85*n**2 + 15*n + 1) + 6*a*b*d*e**2*x*x**(3*n)/(120*n**5 + 274*n**4 + 22
5*n**3 + 85*n**2 + 15*n + 1) + 60*a*b*e**3*n**4*x*x**(4*n)/(120*n**5 + 274*
n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 122*a*b*e**3*n**3*x*x**(4*n)/(120*n
5 + 274*n4 + 225*n**3 + 85*n**2 + 15*n + 1) + 82*a*b*e**3*n**2*x*x**(4*
n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 22*a*b*e**3*n*x*
x**(4*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 2*a*b*e**3
*x*x**(4*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 60*b**2
*d**3*n**4*x*x**(2*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1)
 + 107*b**2*d**3*n**3*x*x**(2*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2
+ 15*n + 1) + 59*b**2*d**3*n**2*x*x**(2*n)/(120*n**5 + 274*n**4 + 225*n**3

```
+ 85*n**2 + 15*n + 1) + 13*b**2*d**3*n*x*x**(2*n)/(120*n**5 + 274*n**4 + 22
5*n**3 + 85*n**2 + 15*n + 1) + b**2*d**3*x*x**(...
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 947 vs. 2(159) = 318.
time = 0.57, size = 947, normalized size = 5.99

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^2*(d+e*x^n)^3,x, algorithm="giac")
```

```
[Out] (120*a^2*d^3*n^5*x + 60*b^2*d^3*n^4*x*x^(2*n) + 240*a*b*d^3*n^4*x*x^n + 120
*b^2*d^2*n^4*x*x^(3*n)*e + 360*a*b*d^2*n^4*x*x^(2*n)*e + 360*a^2*d^2*n^4*x*
x^n*e + 274*a^2*d^3*n^4*x + 107*b^2*d^3*n^3*x*x^(2*n) + 308*a*b*d^3*n^3*x*x
^n + 90*b^2*d*n^4*x*x^(4*n)*e^2 + 240*a*b*d*n^4*x*x^(3*n)*e^2 + 180*a^2*d*n
^4*x*x^(2*n)*e^2 + 234*b^2*d^2*n^3*x*x^(3*n)*e + 642*a*b*d^2*n^3*x*x^(2*n)*
e + 462*a^2*d^2*n^3*x*x^n*e + 225*a^2*d^3*n^3*x + 59*b^2*d^3*n^2*x*x^(2*n)
+ 142*a*b*d^3*n^2*x*x^n + 24*b^2*n^4*x*x^(5*n)*e^3 + 60*a*b*n^4*x*x^(4*n)*e
^3 + 40*a^2*n^4*x*x^(3*n)*e^3 + 183*b^2*d*n^3*x*x^(4*n)*e^2 + 468*a*b*d*n^3
*x*x^(3*n)*e^2 + 321*a^2*d*n^3*x*x^(2*n)*e^2 + 147*b^2*d^2*n^2*x*x^(3*n)*e
+ 354*a*b*d^2*n^2*x*x^(2*n)*e + 213*a^2*d^2*n^2*x*x^n*e + 85*a^2*d^3*n^2*x
+ 13*b^2*d^3*n*x*x^(2*n) + 28*a*b*d^3*n*x*x^n + 50*b^2*n^3*x*x^(5*n)*e^3 +
122*a*b*n^3*x*x^(4*n)*e^3 + 78*a^2*n^3*x*x^(3*n)*e^3 + 123*b^2*d*n^2*x*x^(4
*n)*e^2 + 294*a*b*d*n^2*x*x^(3*n)*e^2 + 177*a^2*d*n^2*x*x^(2*n)*e^2 + 36*b^
2*d^2*n*x*x^(3*n)*e + 78*a*b*d^2*n*x*x^(2*n)*e + 42*a^2*d^2*n*x*x^n*e + 15*
a^2*d^3*n*x + b^2*d^3*x*x^(2*n) + 2*a*b*d^3*x*x^n + 35*b^2*n^2*x*x^(5*n)*e^
3 + 82*a*b*n^2*x*x^(4*n)*e^3 + 49*a^2*n^2*x*x^(3*n)*e^3 + 33*b^2*d*n*x*x^(4
*n)*e^2 + 72*a*b*d*n*x*x^(3*n)*e^2 + 39*a^2*d*n*x*x^(2*n)*e^2 + 3*b^2*d^2*x
*x^(3*n)*e + 6*a*b*d^2*x*x^(2*n)*e + 3*a^2*d^2*x*x^n*e + a^2*d^3*x + 10*b^2
*n*x*x^(5*n)*e^3 + 22*a*b*n*x*x^(4*n)*e^3 + 12*a^2*n*x*x^(3*n)*e^3 + 3*b^2*
d*x*x^(4*n)*e^2 + 6*a*b*d*x*x^(3*n)*e^2 + 3*a^2*d*x*x^(2*n)*e^2 + b^2*x*x^(
5*n)*e^3 + 2*a*b*x*x^(4*n)*e^3 + a^2*x*x^(3*n)*e^3)/(120*n^5 + 274*n^4 + 22
5*n^3 + 85*n^2 + 15*n + 1)
```

Mupad [B]
time = 1.71, size = 157, normalized size = 0.99

$$a^2\,d^3\,x+\frac{x\,x^{2\,n}\,\left(3\,a^2\,d\,e^2+6\,a\,b\,d^2\,e+b^2\,d^3\right)}{2\,n+1}+\frac{x\,x^{3\,n}\,\left(a^2\,e^3+6\,a\,b\,d\,e^2+3\,b^2\,d^2\,e\right)}{3\,n+1}+\frac{b^2\,e^3\,x\,x^{5\,n}}{5\,n+1}+\frac{a\,d^2\,x\,x^n\,\left(3\,a\,e+2\,b\,d\right)}{n+1}+\frac{b\,e^2\,x\,x^{4\,n}\,\left(2\,a\,e+3\,b\,d\right)}{4\,n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)^2*(d + e*x^n)^3,x)
```

```
[Out] a^2*d^3*x + (x*x^(2*n)*(b^2*d^3 + 3*a^2*d*e^2 + 6*a*b*d^2*e))/(2*n + 1) + (
x*x^(3*n)*(a^2*e^3 + 3*b^2*d^2*e + 6*a*b*d*e^2))/(3*n + 1) + (b^2*e^3*x*x^(
5*n))/(5*n + 1) + (a*d^2*x*x^n*(3*a*e + 2*b*d))/(n + 1) + (b*e^2*x*x^(4*n)*
(2*a*e + 3*b*d))/(4*n + 1)
```

3.293 $\int (a+bx^n)^2 (d+ex^n)^2 \, dx$

Optimal. Leaf size=112

$$a^2d^2x+\frac{2ad(bd+ae)x^{1+n}}{1+n}+\frac{(b^2d^2+4abde+a^2e^2)\,x^{1+2n}}{1+2n}+\frac{2be(bd+ae)x^{1+3n}}{1+3n}+\frac{b^2e^2x^{1+4n}}{1+4n}$$

```
[Out] a^2*d^2*x+2*a*d*(a*e+b*d)*x^(1+n)/(1+n)+(a^2*e^2+4*a*b*d*e+b^2*d^2)*x^(1+2*
n)/(1+2*n)+2*b*e*(a*e+b*d)*x^(1+3*n)/(1+3*n)+b^2*e^2*x^(1+4*n)/(1+4*n)
```

Rubi [A]
time = 0.06, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {380}

$$\frac{x^{2n+1}(a^2e^2+4abde+b^2d^2)}{2n+1}+a^2d^2x+\frac{2adx^{n+1}(ae+bd)}{n+1}+\frac{2bex^{3n+1}(ae+bd)}{3n+1}+\frac{b^2e^2x^{4n+1}}{4n+1}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n)^2*(d + e*x^n)^2,x]
```

```
[Out] a^2*d^2*x + (2*a*d*(b*d + a*e)*x^(1 + n))/(1 + n) + ((b^2*d^2 + 4*a*b*d*e +
 a^2*e^2)*x^(1 + 2*n))/(1 + 2*n) + (2*b*e*(b*d + a*e)*x^(1 + 3*n))/(1 + 3*n
) + (b^2*e^2*x^(1 + 4*n))/(1 + 4*n)
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int (a+bx^n)^2 (d+ex^n)^2 \, dx = \int \left(a^2d^2+2ad(bd+ae)x^n+\left(b^2d^2+4abde+a^2e^2\right)x^{2n}+2be(bd+ae)x^{3n}+\right.$$

$$=a^2d^2x+\frac{2ad(bd+ae)x^{1+n}}{1+n}+\frac{\left(b^2d^2+4abde+a^2e^2\right)x^{1+2n}}{1+2n}+\frac{2be(bd+ae)x^{1+}}{1+3n}$$

Mathematica [A]
time = 0.59, size = 105, normalized size = 0.94

$$x\left(a^2d^2+\frac{2ad(bd+ae)x^n}{1+n}+\frac{(b^2d^2+4abde+a^2e^2)\,x^{2n}}{1+2n}+\frac{2be(bd+ae)x^{3n}}{1+3n}+\frac{b^2e^2x^{4n}}{1+4n}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^n)^2*(d + e*x^n)^2,x]
```

```
[Out] x*(a^2*d^2 + (2*a*d*(b*d + a*e)*x^n)/(1 + n) + ((b^2*d^2 + 4*a*b*d*e + a^2*
e^2)*x^(2*n))/(1 + 2*n) + (2*b*e*(b*d + a*e)*x^(3*n))/(1 + 3*n) + (b^2*e^2*
x^(4*n))/(1 + 4*n))
```

Maple [A]
time = 0.26, size = 109, normalized size = 0.97

method	result	size
risch	$a^{2} d^{2} x +\frac{\left(a^{2} e^{2}+4 a b d e +b^{2} d^{2}\right) x \,x^{2 n}}{1+2 n}+\frac{b^{2} e^{2} x \,x^{4 n}}{1+4 n}+\frac{2 a d \left(a e +b d \right) x \,x^{n}}{1+n}+\frac{2 b e \left(a e +b d \right) x \,x^{3 n}}{1+3 n}$	109
norman	$a^{2} d^{2} x +\frac{\left(a^{2} e^{2}+4 a b d e +b^{2} d^{2}\right) x \,{\mathrm e}^{2 n \ln \left(x \right)}}{1+2 n}+\frac{b^{2} e^{2} x \,{\mathrm e}^{4 n \ln \left(x \right)}}{1+4 n}+\frac{2 a d \left(a e +b d \right) x \,{\mathrm e}^{n \ln \left(x \right)}}{1+n}+\frac{2 b e \left(a e +b d \right) x \,{\mathrm e}^{3 n \ln \left(x \right)}}{1+3 n}$	117

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^n)^2*(d+e*x^n)^2,x,method=_RETURNVERBOSE)
```

```
[Out] a^2*d^2*x+(a^2*e^2+4*a*b*d*e+b^2*d^2)/(1+2*n)*x*(x^n)^2+b^2*e^2/(1+4*n)*x*(
x^n)^4+2*a*d*(a*e+b*d)/(1+n)*x*x^n+2*b*e*(a*e+b*d)/(1+3*n)*x*(x^n)^3
```

Maxima [A]
time = 0.29, size = 168, normalized size = 1.50

$$a^{2} d^{2} x + \frac{b^{2} e^{2} x^{4 \, n + 1}}{4 \, n + 1} + \frac{2 \, b^{2} d e x^{3 \, n + 1}}{3 \, n + 1} + \frac{2 \, a b e^{2} x^{3 \, n + 1}}{3 \, n + 1} + \frac{b^{2} d^{2} x^{2 \, n + 1}}{2 \, n + 1} + \frac{4 \, a b d e x^{2 \, n + 1}}{2 \, n + 1} + \frac{a^{2} e^{2} x^{2 \, n + 1}}{2 \, n + 1} + \frac{2 \, a b d^{2} x^{n + 1}}{n + 1} + \frac{2 \, a^{2} d e x^{n + 1}}{n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^2*(d+e*x^n)^2,x, algorithm="maxima")
```

```
[Out] a^2*d^2*x + b^2*e^2*x^(4*n + 1)/(4*n + 1) + 2*b^2*d*e*x^(3*n + 1)/(3*n + 1)
 + 2*a*b*e^2*x^(3*n + 1)/(3*n + 1) + b^2*d^2*x^(2*n + 1)/(2*n + 1) + 4*a*b*
d*e*x^(2*n + 1)/(2*n + 1) + a^2*e^2*x^(2*n + 1)/(2*n + 1) + 2*a*b*d^2*x^(n
+ 1)/(n + 1) + 2*a^2*d*e*x^(n + 1)/(n + 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 369 vs. 2(114) = 228.
time = 3.23, size = 369, normalized size = 3.29

$$\frac{{\left(6 \, b^{2} n^{3} + 11 \, b^{2} n^{2} + 6 \, b^{2} n + b^{2}\right)} x x^{4 \, n} e^{2} + {\left(24 \, a^{2} d^{2} n^{4} + 50 \, a^{2} d^{2} n^{3} + 35 \, a^{2} d^{2} n^{2} + 10 \, a^{2} d^{2} n + a^{2} d^{2}\right)} x + 2 \, {\left({\left(8 \, a b n^{3} + 14 \, a b n^{2} + 7 \, a b n + a b\right)} x e^{2} + {\left(8 \, b^{2} d n^{3} + 14 \, b^{2} d n^{2} + 7 \, b^{2} d n + b^{2} d\right)} x e\right)} x^{3 \, n} + {\left({\left(12 \, a^{2} n^{3} + 19 \, a^{2} n^{2} + 8 \, a^{2} n + a^{2}\right)} x e^{2} + 4 \, {\left(12 \, a b d n^{3} + 19 \, a b d n^{2} + 8 \, a b d n + a b d\right)} x e + {\left(12 \, b^{2} d^{2} n^{3} + 19 \, b^{2} d^{2} n^{2} + 8 \, b^{2} d^{2} n + b^{2} d^{2}\right)} x\right)} x^{2 \, n} + 2 \, {\left({\left(24 \, a^{2} d n^{3} + 26 \, a^{2} d n^{2} + 9 \, a^{2} d n + a^{2} d\right)} x e + {\left(24 \, a b d^{2} n^{3} + 26 \, a b d^{2} n^{2} + 9 \, a b d^{2} n + a b d^{2}\right)} x\right)} x^{n}}{24 \, n^{4} + 50 \, n^{3} + 35 \, n^{2} + 10 \, n + 1}$$

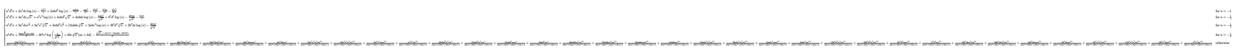
Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^2*(d+e*x^n)^2,x, algorithm="fricas")
```

```
[Out] ((6*b^2*n^3 + 11*b^2*n^2 + 6*b^2*n + b^2)*x*x^(4*n)*e^2 + (24*a^2*d^2*n^4 +
 50*a^2*d^2*n^3 + 35*a^2*d^2*n^2 + 10*a^2*d^2*n + a^2*d^2)*x + 2*((8*a*b*n^
```

3 + 14*a*b*n^2 + 7*a*b*n + a*b)*x*e^2 + (8*b^2*d*n^3 + 14*b^2*d*n^2 + 7*b^2
*d*n + b^2*d)*x*e)*x^(3*n) + ((12*a^2*n^3 + 19*a^2*n^2 + 8*a^2*n + a^2)*x*e
^2 + 4*(12*a*b*d*n^3 + 19*a*b*d*n^2 + 8*a*b*d*n + a*b*d)*x*e + (12*b^2*d^2*
n^3 + 19*b^2*d^2*n^2 + 8*b^2*d^2*n + b^2*d^2)*x)*x^(2*n) + 2*((24*a^2*d*n^3
 + 26*a^2*d*n^2 + 9*a^2*d*n + a^2*d)*x*e + (24*a*b*d^2*n^3 + 26*a*b*d^2*n^2
 + 9*a*b*d^2*n + a*b*d^2)*x)*x^n)/(24*n^4 + 50*n^3 + 35*n^2 + 10*n + 1)

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1765 vs. $2(104) = 208$.
time = 12.36, size = 1765, normalized size = 15.76



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*x**n)**2*(d+e*x**n)**2,x)

[Out] Piecewise((a**2*d**2*x + 2*a**2*d*e*log(x) - a**2*e**2/x + 2*a*b*d**2*log(x
) - 4*a*b*d*e/x - a*b*e**2/x**2 - b**2*d**2/x - b**2*d*e/x**2 - b**2*e**2/(
3*x**3), Eq(n, -1)), (a**2*d**2*x + 4*a**2*d*e*sqrt(x) + a**2*e**2*log(x) +
 4*a*b*d**2*sqrt(x) + 4*a*b*d*e*log(x) - 4*a*b*e**2/sqrt(x) + b**2*d**2*log
(x) - 4*b**2*d*e/sqrt(x) - b**2*e**2/x, Eq(n, -1/2)), (a**2*d**2*x + 3*a**2
*d*e*x**(2/3) + 3*a**2*e**2*x**(1/3) + 3*a*b*d**2*x**(2/3) + 12*a*b*d*e*x**
(1/3) + 2*a*b*e**2*log(x) + 3*b**2*d**2*x**(1/3) + 2*b**2*d*e*log(x) - 3*b*
*2*e**2/x**(1/3), Eq(n, -1/3)), (a**2*d**2*x + 8*a*d*x**(3/4)*(a*e + b*d)/3
 - 4*b**2*e**2*log(x**(-1/4)) + 8*b*e*x**(1/4)*(a*e + b*d) - sqrt(x)*(-4*a*
*2*e**2 - 16*a*b*d*e - 4*b**2*d**2)/2, Eq(n, -1/4)), (24*a**2*d**2*n**4*x/(
24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 50*a**2*d**2*n**3*x/(24*n**4 + 50
*n**3 + 35*n**2 + 10*n + 1) + 35*a**2*d**2*n**2*x/(24*n**4 + 50*n**3 + 35*n
2 + 10*n + 1) + 10*a2*d**2*n*x/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1)
 + a**2*d**2*x/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 48*a**2*d*e*n**3*
x*x**n/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 52*a**2*d*e*n**2*x*x**n/(
24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 18*a**2*d*e*n*x*x**n/(24*n**4 + 5
0*n**3 + 35*n**2 + 10*n + 1) + 2*a**2*d*e*x*x**n/(24*n**4 + 50*n**3 + 35*n*
*2 + 10*n + 1) + 12*a**2*e**2*n**3*x*x**(2*n)/(24*n**4 + 50*n**3 + 35*n**2
 + 10*n + 1) + 19*a**2*e**2*n**2*x*x**(2*n)/(24*n**4 + 50*n**3 + 35*n**2 + 1
0*n + 1) + 8*a**2*e**2*n*x*x**(2*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1
) + a**2*e**2*x*x**(2*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 48*a*b*
d**2*n**3*x*x**n/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 52*a*b*d**2*n**
2*x*x**n/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 18*a*b*d**2*n*x*x**n/(2
4*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 2*a*b*d**2*x*x**n/(24*n**4 + 50*n*
*3 + 35*n**2 + 10*n + 1) + 48*a*b*d*e*n**3*x*x**(2*n)/(24*n**4 + 50*n**3 +
35*n**2 + 10*n + 1) + 76*a*b*d*e*n**2*x*x**(2*n)/(24*n**4 + 50*n**3 + 35*n*
*2 + 10*n + 1) + 32*a*b*d*e*n*x*x**(2*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*
n + 1) + 4*a*b*d*e*x*x**(2*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 16
*a*b*e**2*n**3*x*x**(3*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 28*a*b

```
*e**2*n**2*x*x**(3*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 14*a*b*e**
2*n*x*x**(3*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 2*a*b*e**2*x*x**(
3*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 12*b**2*d**2*n**3*x*x**(2*n
)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 19*b**2*d**2*n**2*x*x**(2*n)/(
24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 8*b**2*d**2*n*x*x**(2*n)/(24*n**4
 + 50*n**3 + 35*n**2 + 10*n + 1) + b**2*d**2*x*x**(2*n)/(24*n**4 + 50*n**3
+ 35*n**2 + 10*n + 1) + 16*b**2*d*e*n**3*x*x**(3*n)/(24*n**4 + 50*n**3 + 35
*n**2 + 10*n + 1) + 28*b**2*d*e*n**2*x*x**(3*n)/(24*n**4 + 50*n**3 + 35*n**
2 + 10*n + 1) + 14*b**2*d*e*n*x*x**(3*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*
n + 1) + 2*b**2*d*e*x*x**(3*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 6
*b**2*e**2*n**3*x*x**(4*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 11*b*
*2*e**2*n**2*x*x**(4*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 6*b**2*e
**2*n*x*x**(4*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + b**2*e**2*x*x**
(4*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 539 vs. $2(114) = 228$.
time = 0.65, size = 539, normalized size = 4.81

$$\frac{24a^2d^2n^4x + 12b^2d^2n^3xx^{2n} + 48abd^2n^3xx^n + 16b^2dn^3xx^{3n}e + 48abdn^3xx^{2n}e + 48a^2dn^3xx^ne + 50a^2d^2n^3x + 19b^2d^2n^2xx^{2n} + 52abd^2n^2xx^n + 6b^2n^3xx^{4n}e^2 + 16abn^3xx^{3n}e^2 + 12a^2n^3xx^{2n}e^2 + 28b^2dn^2xx^{3n}e + 76abdn^2xx^{2n}e + 52a^2dn^2xx^ne + 35a^2d^2n^2x + 8b^2d^2nxx^{2n} + 18abd^2nxx^n + 11b^2n^2xx^{4n}e^2 + 28abn^2xx^{3n}e^2 + 19a^2n^2xx^{2n}e^2 + 14b^2dnxx^{3n}e + 32abdnxx^{2n}e + 18a^2dnxx^ne + 10a^2d^2nx + b^2d^2xx^{2n} + 2abd^2xx^n + 6b^2nxx^{4n}e^2 + 14abnxx^{3n}e^2 + 8a^2nxx^{2n}e^2 + 2b^2dxx^{3n}e + 4abdxx^{2n}e + 2a^2dxx^ne + a^2d^2x + b^2xx^{4n}e^2 + 2abxx^{3n}e^2 + a^2xx^{2n}e^2}{24n^4 + 50n^3 + 35n^2 + 10n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^2*(d+e*x^n)^2,x, algorithm="giac")
```

```
[Out] (24*a^2*d^2*n^4*x + 12*b^2*d^2*n^3*x*x^(2*n) + 48*a*b*d^2*n^3*x*x^n + 16*b^
2*d*n^3*x*x^(3*n)*e + 48*a*b*d*n^3*x*x^(2*n)*e + 48*a^2*d*n^3*x*x^n*e + 50*
a^2*d^2*n^3*x + 19*b^2*d^2*n^2*x*x^(2*n) + 52*a*b*d^2*n^2*x*x^n + 6*b^2*n^3
*x*x^(4*n)*e^2 + 16*a*b*n^3*x*x^(3*n)*e^2 + 12*a^2*n^3*x*x^(2*n)*e^2 + 28*b
^2*d*n^2*x*x^(3*n)*e + 76*a*b*d*n^2*x*x^(2*n)*e + 52*a^2*d*n^2*x*x^n*e + 35
*a^2*d^2*n^2*x + 8*b^2*d^2*n*x*x^(2*n) + 18*a*b*d^2*n*x*x^n + 11*b^2*n^2*x*
x^(4*n)*e^2 + 28*a*b*n^2*x*x^(3*n)*e^2 + 19*a^2*n^2*x*x^(2*n)*e^2 + 14*b^2*
d*n*x*x^(3*n)*e + 32*a*b*d*n*x*x^(2*n)*e + 18*a^2*d*n*x*x^n*e + 10*a^2*d^2*
n*x + b^2*d^2*x*x^(2*n) + 2*a*b*d^2*x*x^n + 6*b^2*n*x*x^(4*n)*e^2 + 14*a*b*
n*x*x^(3*n)*e^2 + 8*a^2*n*x*x^(2*n)*e^2 + 2*b^2*d*x*x^(3*n)*e + 4*a*b*d*x*x
^(2*n)*e + 2*a^2*d*x*x^n*e + a^2*d^2*x + b^2*x*x^(4*n)*e^2 + 2*a*b*x*x^(3*n
)*e^2 + a^2*x*x^(2*n)*e^2)/(24*n^4 + 50*n^3 + 35*n^2 + 10*n + 1)
```

Mupad [B]
time = 1.57, size = 108, normalized size = 0.96

$$a^2\,d^2\,x+\frac{x\,x^{2\,n}\,\left(a^2\,e^2+4\,a\,b\,d\,e+b^2\,d^2\right)}{2\,n+1}+\frac{b^2\,e^2\,x\,x^{4\,n}}{4\,n+1}+\frac{2\,b\,e\,x\,x^{3\,n}\,\left(a\,e+b\,d\right)}{3\,n+1}+\frac{2\,a\,d\,x\,x^n\,\left(a\,e+b\,d\right)}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)^2*(d + e*x^n)^2,x)
```

```
[Out] a^2*d^2*x + (x*x^(2*n)*(a^2*e^2 + b^2*d^2 + 4*a*b*d*e))/(2*n + 1) + (b^2*e^
2*x*x^(4*n))/(4*n + 1) + (2*b*e*x*x^(3*n)*(a*e + b*d))/(3*n + 1) + (2*a*d*x
*x^n*(a*e + b*d))/(n + 1)
```

3.294 $\int (a+bx^n)^2 (c+dx^n) \, dx$

Optimal. Leaf size=70

$$a^2cx+\frac{a(2bc+ad)x^{1+n}}{1+n}+\frac{b(bc+2ad)x^{1+2n}}{1+2n}+\frac{b^2dx^{1+3n}}{1+3n}$$

```
[Out] a^2*c*x+a*(a*d+2*b*c)*x^(1+n)/(1+n)+b*(2*a*d+b*c)*x^(1+2*n)/(1+2*n)+b^2*d*x
^(1+3*n)/(1+3*n)
```

Rubi [A]
time = 0.03, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {380}

$$a^2cx+\frac{ax^{n+1}(ad+2bc)}{n+1}+\frac{bx^{2n+1}(2ad+bc)}{2n+1}+\frac{b^2dx^{3n+1}}{3n+1}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n)^2*(c + d*x^n),x]
```

```
[Out] a^2*c*x + (a*(2*b*c + a*d)*x^(1 + n))/(1 + n) + (b*(b*c + 2*a*d)*x^(1 + 2*n
))/(1 + 2*n) + (b^2*d*x^(1 + 3*n))/(1 + 3*n)
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int (a+bx^n)^2 (c+dx^n) \, dx &= \int \left(a^2c+a(2bc+ad)x^n+b(bc+2ad)x^{2n}+b^2dx^{3n}\right) \, dx\\ &= a^2cx+\frac{a(2bc+ad)x^{1+n}}{1+n}+\frac{b(bc+2ad)x^{1+2n}}{1+2n}+\frac{b^2dx^{1+3n}}{1+3n}\end{aligned}$$

Mathematica [A]
time = 0.10, size = 65, normalized size = 0.93

$$x\left(a^2c+\frac{a(2bc+ad)x^n}{1+n}+\frac{b(bc+2ad)x^{2n}}{1+2n}+\frac{b^2dx^{3n}}{1+3n}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^n)^2*(c + d*x^n),x]
```

```
[Out] x*(a^2*c + (a*(2*b*c + a*d)*x^n)/(1 + n) + (b*(b*c + 2*a*d)*x^(2*n))/(1 + 2
*n) + (b^2*d*x^(3*n))/(1 + 3*n))
```

Maple [A]
time = 0.24, size = 68, normalized size = 0.97

method	result	size
risch	$a^2cx+\frac{a(ad+2bc)x\,x^n}{1+n}+\frac{b(2ad+bc)x\,x^{2n}}{1+2n}+\frac{b^2dx\,x^{3n}}{1+3n}$	68
norman	$a^2cx+\frac{a(ad+2bc)x\,\mathrm{e}^{n\ln(x)}}{1+n}+\frac{b(2ad+bc)x\,\mathrm{e}^{2n\ln(x)}}{1+2n}+\frac{b^2dx\,\mathrm{e}^{3n\ln(x)}}{1+3n}$	74

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^n)^2*(c+d*x^n),x,method=_RETURNVERBOSE)
```

```
[Out] a^2*c*x+a*(a*d+2*b*c)/(1+n)*x*x^n+b*(2*a*d+b*c)/(1+2*n)*x*(x^n)^2+b^2*d/(1+
3*n)*x*(x^n)^3
```

Maxima [A]
time = 0.29, size = 94, normalized size = 1.34

$$a^2cx+\frac{b^2dx^{3n+1}}{3n+1}+\frac{b^2cx^{2n+1}}{2n+1}+\frac{2abdx^{2n+1}}{2n+1}+\frac{2abcx^{n+1}}{n+1}+\frac{a^2dx^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^2*(c+d*x^n),x, algorithm="maxima")
```

```
[Out] a^2*c*x + b^2*d*x^(3*n + 1)/(3*n + 1) + b^2*c*x^(2*n + 1)/(2*n + 1) + 2*a*b
*d*x^(2*n + 1)/(2*n + 1) + 2*a*b*c*x^(n + 1)/(n + 1) + a^2*d*x^(n + 1)/(n +
 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 175 vs. $2(70) = 140$.
time = 3.38, size = 175, normalized size = 2.50

$$\frac{(2b^2dn^2+3b^2dn+b^2d)xx^{3n}+(b^2c+2abd+3(b^2c+2abd)n^2+4(b^2c+2abd)n)xx^{2n}+(2abc+a^2d+6(2abc+a^2d)n^2+5(2abc+a^2d)n)xx^n+(6a^2cn^3+11a^2cn^2+6a^2cn+a^2c)x}{6n^3+11n^2+6n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^2*(c+d*x^n),x, algorithm="fricas")
```

```
[Out] ((2*b^2*d*n^2 + 3*b^2*d*n + b^2*d)*x*x^(3*n) + (b^2*c + 2*a*b*d + 3*(b^2*c
+ 2*a*b*d)*n^2 + 4*(b^2*c + 2*a*b*d)*n)*x*x^(2*n) + (2*a*b*c + a^2*d + 6*(2
*a*b*c + a^2*d)*n^2 + 5*(2*a*b*c + a^2*d)*n)*x*x^n + (6*a^2*c*n^3 + 11*a^2*
c*n^2 + 6*a^2*c*n + a^2*c)*x)/(6*n^3 + 11*n^2 + 6*n + 1)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 726 vs. $2(63) = 126$.
time = 0.30, size = 726, normalized size = 10.37

$$\begin{cases} a^2cx + a^2d\log(x) + 2abc\log(x) - \frac{2abd}{x} - \frac{b^2c}{x} - \frac{b^2d}{2x^2} & \text{for } n = -1 \\ a^2cx + 2a^2d\sqrt{x} + 4abc\sqrt{x} + 2abd\log(x) + b^2c\log(x) - \frac{2b^2d}{\sqrt{x}} & \text{for } n = -\frac{1}{2} \\ a^2cx + \frac{3a^2dx^{\frac{2}{3}}}{2} + 3abcx^{\frac{2}{3}} + 6abd\sqrt[3]{x} + 3b^2c\sqrt[3]{x} + b^2d\log(x) & \text{for } n = -\frac{1}{3} \\ \frac{6a^2cn^3x}{6n^3+11n^2+6n+1} + \frac{11a^2cn^2x}{6n^3+11n^2+6n+1} + \frac{6a^2cnx}{6n^3+11n^2+6n+1} + \frac{a^2cx}{6n^3+11n^2+6n+1} + \frac{6a^2dn^2xx^n}{6n^3+11n^2+6n+1} + \frac{5a^2dnxx^n}{6n^3+11n^2+6n+1} + \frac{a^2dxx^n}{6n^3+11n^2+6n+1} + \frac{12abcn^2xx^n}{6n^3+11n^2+6n+1} + \frac{10abcnxx^n}{6n^3+11n^2+6n+1} + \frac{2abcxx^n}{6n^3+11n^2+6n+1} + \frac{6abdn^2xx^{2n}}{6n^3+11n^2+6n+1} + \frac{8abdnxx^{2n}}{6n^3+11n^2+6n+1} + \frac{2abdxx^{2n}}{6n^3+11n^2+6n+1} + \frac{3b^2cn^2xx^{2n}}{6n^3+11n^2+6n+1} + \frac{4b^2cnxx^{2n}}{6n^3+11n^2+6n+1} + \frac{b^2cxx^{2n}}{6n^3+11n^2+6n+1} + \frac{2b^2dn^2xx^{3n}}{6n^3+11n^2+6n+1} + \frac{3b^2dnxx^{3n}}{6n^3+11n^2+6n+1} + \frac{b^2dxx^{3n}}{6n^3+11n^2+6n+1} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n)**2*(c+d*x**n),x)
```

```
[Out] Piecewise((a**2*c*x + a**2*d*log(x) + 2*a*b*c*log(x) - 2*a*b*d/x - b**2*c/x
 - b**2*d/(2*x**2), Eq(n, -1)), (a**2*c*x + 2*a**2*d*sqrt(x) + 4*a*b*c*sqrt
(x) + 2*a*b*d*log(x) + b**2*c*log(x) - 2*b**2*d/sqrt(x), Eq(n, -1/2)), (a**
2*c*x + 3*a**2*d*x**(2/3)/2 + 3*a*b*c*x**(2/3) + 6*a*b*d*x**(1/3) + 3*b**2*
c*x**(1/3) + b**2*d*log(x), Eq(n, -1/3)), (6*a**2*c*n**3*x/(6*n**3 + 11*n**
2 + 6*n + 1) + 11*a**2*c*n**2*x/(6*n**3 + 11*n**2 + 6*n + 1) + 6*a**2*c*n*x
/(6*n**3 + 11*n**2 + 6*n + 1) + a**2*c*x/(6*n**3 + 11*n**2 + 6*n + 1) + 6*a
**2*d*n**2*x*x**n/(6*n**3 + 11*n**2 + 6*n + 1) + 5*a**2*d*n*x*x**n/(6*n**3
+ 11*n**2 + 6*n + 1) + a**2*d*x*x**n/(6*n**3 + 11*n**2 + 6*n + 1) + 12*a*b*
c*n**2*x*x**n/(6*n**3 + 11*n**2 + 6*n + 1) + 10*a*b*c*n*x*x**n/(6*n**3 + 11
*n**2 + 6*n + 1) + 2*a*b*c*x*x**n/(6*n**3 + 11*n**2 + 6*n + 1) + 6*a*b*d*n*
*2*x*x**(2*n)/(6*n**3 + 11*n**2 + 6*n + 1) + 8*a*b*d*n*x*x**(2*n)/(6*n**3 +
 11*n**2 + 6*n + 1) + 2*a*b*d*x*x**(2*n)/(6*n**3 + 11*n**2 + 6*n + 1) + 3*b
**2*c*n**2*x*x**(2*n)/(6*n**3 + 11*n**2 + 6*n + 1) + 4*b**2*c*n*x*x**(2*n)/
(6*n**3 + 11*n**2 + 6*n + 1) + b**2*c*x*x**(2*n)/(6*n**3 + 11*n**2 + 6*n +
1) + 2*b**2*d*n**2*x*x**(3*n)/(6*n**3 + 11*n**2 + 6*n + 1) + 3*b**2*d*n*x*x
**(3*n)/(6*n**3 + 11*n**2 + 6*n + 1) + b**2*d*x*x**(3*n)/(6*n**3 + 11*n**2
+ 6*n + 1), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 232 vs. $2(70) = 140$.
time = 1.58, size = 232, normalized size = 3.31

$$\frac{6\,a^2cn^3x + 2\,b^2dn^2xx^{3\,n} + 3\,b^2cn^2xx^{2\,n} + 6\,abdn^2xx^{2\,n} + 12\,abcn^2xx^n + 6\,a^2dn^2xx^n + 11\,a^2cn^2x + 3\,b^2dnxx^{3\,n} + 4\,b^2cnxx^{2\,n} + 8\,abdnxx^{2\,n} + 10\,abcnxx^n + 5\,a^2dnxx^n + 6\,a^2cnx + b^2dxx^{3\,n} + b^2cxx^{2\,n} + 2\,abdxx^{2\,n} + 2\,abcxx^n + a^2dxx^n + a^2cx}{6\,n^3 + 11\,n^2 + 6\,n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^2*(c+d*x^n),x, algorithm="giac")
```

```
[Out] (6*a^2*c*n^3*x + 2*b^2*d*n^2*x*x^(3*n) + 3*b^2*c*n^2*x*x^(2*n) + 6*a*b*d*n^
2*x*x^(2*n) + 12*a*b*c*n^2*x*x^n + 6*a^2*d*n^2*x*x^n + 11*a^2*c*n^2*x + 3*b
^2*d*n*x*x^(3*n) + 4*b^2*c*n*x*x^(2*n) + 8*a*b*d*n*x*x^(2*n) + 10*a*b*c*n*x
*x^n + 5*a^2*d*n*x*x^n + 6*a^2*c*n*x + b^2*d*x*x^(3*n) + b^2*c*x*x^(2*n) +
2*a*b*d*x*x^(2*n) + 2*a*b*c*x*x^n + a^2*d*x*x^n + a^2*c*x)/(6*n^3 + 11*n^2
+ 6*n + 1)
```

Mupad [B]
time = 1.53, size = 71, normalized size = 1.01

$$a^2\,c\,x+\frac{x\,x^{2\,n}\,\left(c\,b^2+2\,a\,d\,b\right)}{2\,n+1}+\frac{x\,x^n\,\left(d\,a^2+2\,b\,c\,a\right)}{n+1}+\frac{b^2\,d\,x\,x^{3\,n}}{3\,n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)^2*(c + d*x^n),x)
```

```
[Out] a^2*c*x + (x*x^(2*n)*(b^2*c + 2*a*b*d))/(2*n + 1) + (x*x^n*(a^2*d + 2*a*b*c
))/(n + 1) + (b^2*d*x*x^(3*n))/(3*n + 1)
```

3.295 $\int \frac{(a+bx^n)^2}{c+dx^n}\,dx$

Optimal. Leaf size=84

$$-\frac{b(bc(1+n)-ad(1+2n))x}{d^2(1+n)}+\frac{bx(a+bx^n)}{d(1+n)}+\frac{(bc-ad)^2x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{cd^2}$$

```
[Out] -b*(b*c*(1+n)-a*d*(1+2*n))*x/d^2/(1+n)+b*x*(a+b*x^n)/d/(1+n)+(-a*d+b*c)^2*x
*hypergeom([1, 1/n],[1+1/n],-d*x^n/c)/c/d^2
```

Rubi [A]
time = 0.06, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {427, 396, 251}

$$\frac{x(bc-ad)^2\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{cd^2}-\frac{bx(bc(n+1)-ad(2n+1))}{d^2(n+1)}+\frac{bx(a+bx^n)}{d(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n)^2/(c + d*x^n),x]
```

```
[Out] -((b*(b*c*(1 + n) - a*d*(1 + 2*n))*x)/(d^2*(1 + n))) + (b*x*(a + b*x^n))/(d
*(1 + n)) + ((b*c - a*d)^2*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((d*
x^n)/c)])/(c*d^2)
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
```

```
0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rubi steps

$$\int \frac{(a+bx^n)^2}{c+dx^n}\,dx = \frac{bx(a+bx^n)}{d(1+n)} + \frac{\int \frac{-a(bc-ad(1+n))-b(bc(1+n)-ad(1+2n))x^n}{c+dx^n}\,dx}{d(1+n)}$$
$$= -\frac{b(bc(1+n)-ad(1+2n))x}{d^2(1+n)} + \frac{bx(a+bx^n)}{d(1+n)} + \frac{(bc-ad)^2\int \frac{1}{c+dx^n}\,dx}{d^2}$$
$$= -\frac{b(bc(1+n)-ad(1+2n))x}{d^2(1+n)} + \frac{bx(a+bx^n)}{d(1+n)} + \frac{(bc-ad)^2x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{cd^2}$$

Mathematica [A]
time = 0.14, size = 82, normalized size = 0.98

$$\frac{a^2x}{c} - \frac{(bc-ad)^2x}{cd^2} + \frac{b^2x^{1+n}}{d(1+n)} + \frac{(-bc+ad)^2x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{cd^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^n)^2/(c + d*x^n),x]
```

```
[Out] (a^2*x)/c - ((b*c - a*d)^2*x)/(c*d^2) + (b^2*x^(1 + n))/(d*(1 + n)) + ((-(b
*c) + a*d)^2*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((d*x^n)/c)])/(c*d
^2)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,x^n)^2}{c+d\,x^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^n)^2/(c+d*x^n),x)
```

```
[Out] int((a+b*x^n)^2/(c+d*x^n),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^2/(c+d*x^n),x, algorithm="maxima")
```

```
[Out] (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*integrate(1/(d^3*x^n + c*d^2), x) + (b^2*d*
x*x^n - (b^2*c*(n + 1) - 2*a*b*d*(n + 1))*x)/(d^2*(n + 1))
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^2/(c+d*x^n),x, algorithm="fricas")
```

```
[Out] integral((b^2*x^(2*n) + 2*a*b*x^n + a^2)/(d*x^n + c), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.94, size = 170, normalized size = 2.02

$$\frac{a^{2}x\Phi\left(\frac{dx^{n}e^{i\pi}}{c},1,\frac{1}{n}\right)\Gamma\left(\frac{1}{n}\right)}{cn^{2}\Gamma\left(1+\frac{1}{n}\right)}-\frac{2abx\Phi\left(\frac{cx^{-n}e^{i\pi}}{d},1,\frac{e^{i\pi}}{n}\right)\Gamma\left(\frac{1}{n}\right)}{dn^{2}\Gamma\left(1+\frac{1}{n}\right)}+\frac{2b^{2}xx^{2n}\Phi\left(\frac{dx^{n}e^{i\pi}}{c},1,2+\frac{1}{n}\right)\Gamma\left(2+\frac{1}{n}\right)}{cn\Gamma\left(3+\frac{1}{n}\right)}+\frac{b^{2}xx^{2n}\Phi\left(\frac{dx^{n}e^{i\pi}}{c},1,2+\frac{1}{n}\right)\Gamma\left(2+\frac{1}{n}\right)}{cn^{2}\Gamma\left(3+\frac{1}{n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n)**2/(c+d*x**n),x)
```

```
[Out] a**2*x*lerchphi(d*x**n*exp_polar(I*pi)/c, 1, 1/n)*gamma(1/n)/(c*n**2*gamma(
1 + 1/n)) - 2*a*b*x*lerchphi(c*exp_polar(I*pi)/(d*x**n), 1, exp_polar(I*pi)
/n)*gamma(1/n)/(d*n**2*gamma(1 + 1/n)) + 2*b**2*x*x**(2*n)*lerchphi(d*x**n*
exp_polar(I*pi)/c, 1, 2 + 1/n)*gamma(2 + 1/n)/(c*n*gamma(3 + 1/n)) + b**2*x
*x**(2*n)*lerchphi(d*x**n*exp_polar(I*pi)/c, 1, 2 + 1/n)*gamma(2 + 1/n)/(c*
n**2*gamma(3 + 1/n))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^2/(c+d*x^n),x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a)^2/(d*x^n + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+b\,x^n\right)}^2}{c+d\,x^n} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)^2/(c + d*x^n),x)
```

```
[Out] int((a + b*x^n)^2/(c + d*x^n), x)
```

3.296 $\int \frac{(a+bx^n)^2}{(c+dx^n)^2} \, dx$

Optimal. Leaf size=115

$$-\frac{b(ad-bc(1+n))x}{cd^2n}-\frac{(bc-ad)x(a+bx^n)}{cdn\left(c+dx^n\right)}+\frac{(bc-ad)(ad(1-n)-bc(1+n))x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{c^2d^2n}$$

```
[Out] -b*(a*d-b*c*(1+n))*x/c/d^2/n-(-a*d+b*c)*x*(a+b*x^n)/c/d/n/(c+d*x^n)+(-a*d+b
*c)*(a*d*(1-n)-b*c*(1+n))*x*hypergeom([1, 1/n],[1+1/n],-d*x^n/c)/c^2/d^2/n
```

Rubi [A]
time = 0.06, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {424, 396, 251}

$$\frac{x(bc-ad)(ad(1-n)-bc(n+1))\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{c^2d^2n}-\frac{bx(ad-bc(n+1))}{cd^2n}-\frac{x(bc-ad)\left(a+bx^n\right)}{cdn\left(c+dx^n\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n)^2/(c + d*x^n)^2,x]
```

```
[Out] -((b*(a*d - b*c*(1 + n))*x)/(c*d^2*n)) - ((b*c - a*d)*x*(a + b*x^n))/(c*d*n
*(c + d*x^n)) + ((b*c - a*d)*(a*d*(1 - n) - b*c*(1 + n))*x*Hypergeometric2F
1[1, n^(-1), 1 + n^(-1), -((d*x^n)/c)])/(c^2*d^2*n)
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
```

```
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rubi steps

$$\int \frac{(a+bx^n)^2}{(c+dx^n)^2}\,dx = -\frac{(bc-ad)x(a+bx^n)}{cdn\,(c+dx^n)}+\frac{\int \frac{a(bc-ad(1-n))-b(ad-bc(1+n))x^n}{c+dx^n}\,dx}{cdn}$$
$$= -\frac{b(ad-bc(1+n))x}{cd^2n}-\frac{(bc-ad)x(a+bx^n)}{cdn\,(c+dx^n)}+\frac{((bc-ad)(ad(1-n)-bc(1+n)))\int \frac{1}{c+dx^n}$$
$$= -\frac{b(ad-bc(1+n))x}{cd^2n}-\frac{(bc-ad)x(a+bx^n)}{cdn\,(c+dx^n)}+\frac{(bc-ad)(ad(1-n)-bc(1+n))x\,{}_2F_1(1,}{c^2d^2n}$$

Mathematica [A]
time = 0.17, size = 95, normalized size = 0.83

$$\frac{x\left(\frac{c(-2abcd+a^2d^2+b^2c(c+cn+dnx^n))}{c+dx^n}-(bc-ad)(ad(-1+n)+bc(1+n))\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)\right)}{c^2d^2n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^n)^2/(c + d*x^n)^2,x]
[Out] (x*((c*(-2*a*b*c*d + a^2*d^2 + b^2*c*(c + c*n + d*n*x^n)))/(c + d*x^n) - (b
*c - a*d)*(a*d*(-1 + n) + b*c*(1 + n))*Hypergeometric2F1[1, n^(-1), 1 + n^(
-1), -((d*x^n)/c)]))/(c^2*d^2*n)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,x^n)^2}{(c+d\,x^n)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^n)^2/(c+d*x^n)^2,x)
[Out] int((a+b*x^n)^2/(c+d*x^n)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^2/(c+d*x^n)^2,x, algorithm="maxima")
```

```
[Out] -(b^2*c^2*(n + 1) - a^2*d^2*(n - 1) - 2*a*b*c*d)*integrate(1/(c*d^3*n*x^n +
 c^2*d^2*n), x) + (b^2*c*d*n*x*x^n + (b^2*c^2*(n + 1) - 2*a*b*c*d + a^2*d^2
)*x)/(c*d^3*n*x^n + c^2*d^2*n)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^2/(c+d*x^n)^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*x^(2*n) + 2*a*b*x^n + a^2)/(d^2*x^(2*n) + 2*c*d*x^n + c^2), x
)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + bx^n)^2}{(c + dx^n)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n)**2/(c+d*x**n)**2,x)
```

```
[Out] Integral((a + b*x**n)**2/(c + d*x**n)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^2/(c+d*x^n)^2,x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a)^2/(d*x^n + c)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + b\,x^n)^2}{(c + d\,x^n)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)^2/(c + d*x^n)^2,x)
```

```
[Out] int((a + b*x^n)^2/(c + d*x^n)^2, x)
```

3.297 $\int \frac{(a+bx^n)^2}{(c+dx^n)^3}\,dx$

Optimal. Leaf size=160

$$-\frac{(bc-ad)x(a+bx^n)}{2cdn\,(c+dx^n)^2}+\frac{(bc-ad)(ad(1-2n)-bc(1+n))x}{2c^2d^2n^2\,(c+dx^n)}-\frac{(2abcd(1-n)-b^2c^2(1+n)-a^2d^2(1-3n+2}{2c^3d^2n^2}$$

```
[Out] -1/2*(-a*d+b*c)*x*(a+b*x^n)/c/d/n/(c+d*x^n)^2+1/2*(-a*d+b*c)*(a*d*(1-2*n)-b
*c*(1+n))*x/c^2/d^2/n^2/(c+d*x^n)-1/2*(2*a*b*c*d*(1-n)-b^2*c^2*(1+n)-a^2*d^
2*(2*n^2-3*n+1))*x*hypergeom([1, 1/n],[1+1/n],-d*x^n/c)/c^3/d^2/n^2
```

Rubi [A]

time = 0.11, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {424, 393, 251}

$$-\frac{x\left(-a^2d^2(2n^2-3n+1)+2abcd(1-n)-b^2c^2(n+1)\right)\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{2c^3d^2n^2}+\frac{x(bc-ad)(ad(1-2n)-bc(n+1))}{2c^2d^2n^2\,(c+dx^n)}-\frac{x(bc-ad)\,(a+bx^n)}{2cdn\,(c+dx^n)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n)^2/(c + d*x^n)^3,x]
```

```
[Out] -1/2*((b*c - a*d)*x*(a + b*x^n))/(c*d*n*(c + d*x^n)^2) + ((b*c - a*d)*(a*d*
(1 - 2*n) - b*c*(1 + n))*x)/(2*c^2*d^2*n^2*(c + d*x^n)) - ((2*a*b*c*d*(1 -
n) - b^2*c^2*(1 + n) - a^2*d^2*(1 - 3*n + 2*n^2))*x*Hypergeometric2F1[1, n^
(-1), 1 + n^(-1), -((d*x^n)/c)])/(2*c^3*d^2*n^2)
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
```

```
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^n)^2}{(c+dx^n)^3}\,dx &= -\frac{(bc-ad)x(a+bx^n)}{2cdn\,(c+dx^n)^2}+\frac{\int \frac{a(bc-ad(1-2n))-b(ad(1-n)-bc(1+n))x^n}{(c+dx^n)^2}\,dx}{2cdn}\\ &= -\frac{(bc-ad)x(a+bx^n)}{2cdn\,(c+dx^n)^2}+\frac{(bc-ad)(ad(1-2n)-bc(1+n))x}{2c^2d^2n^2\,(c+dx^n)}-\frac{(2abcd(1-n)-b^2c^2(1\\ &= -\frac{(bc-ad)x(a+bx^n)}{2cdn\,(c+dx^n)^2}+\frac{(bc-ad)(ad(1-2n)-bc(1+n))x}{2c^2d^2n^2\,(c+dx^n)}-\frac{(2abcd(1-n)-b^2c^2(1\end{aligned}$$

Mathematica [A]
time = 0.14, size = 133, normalized size = 0.83

$$\frac{x\left(\frac{c^2(bc-ad)^2n}{(c+dx^n)^2}-\frac{c(bc-ad)(ad(-1+2n)+b(c+2cn))}{c+dx^n}+\left(2abcd(-1+n)+b^2c^2(1+n)+a^2d^2(1-3n+2n^2)\right)\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)\right)}{2c^3d^2n^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^n)^2/(c + d*x^n)^3,x]
```

```
[Out] (x*((c^2*(b*c - a*d)^2*n)/(c + d*x^n)^2 - (c*(b*c - a*d)*(a*d*(-1 + 2*n) +
b*(c + 2*c*n)))/(c + d*x^n) + (2*a*b*c*d*(-1 + n) + b^2*c^2*(1 + n) + a^2*d
^2*(1 - 3*n + 2*n^2))*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((d*x^n)/c)
]))/(2*c^3*d^2*n^2)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,x^n)^2}{(c+d\,x^n)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^n)^2/(c+d*x^n)^3,x)
```

```
[Out] int((a+b*x^n)^2/(c+d*x^n)^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^2/(c+d*x^n)^3,x, algorithm="maxima")
```

```
[Out] ((2*n^2 - 3*n + 1)*a^2*d^2 + b^2*c^2*(n + 1) + 2*a*b*c*d*(n - 1))*integrate
(1/2/(c^2*d^3*n^2*x^n + c^3*d^2*n^2), x) - 1/2*((b^2*c^2*d*(2*n + 1) - a^2*
d^3*(2*n - 1) - 2*a*b*c*d^2)*x*x^n - (a^2*c*d^2*(3*n - 1) - b^2*c^3*(n + 1)
 - 2*a*b*c^2*d*(n - 1))*x)/(c^2*d^4*n^2*x^(2*n) + 2*c^3*d^3*n^2*x^n + c^4*d
^2*n^2)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^2/(c+d*x^n)^3,x, algorithm="fricas")
```

```
[Out] integral((b^2*x^(2*n) + 2*a*b*x^n + a^2)/(d^3*x^(3*n) + 3*c*d^2*x^(2*n) + 3
*c^2*d*x^n + c^3), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n)**2/(c+d*x**n)**3,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^2/(c+d*x^n)^3,x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a)^2/(d*x^n + c)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,x^n)^2}{(c+d\,x^n)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)^2/(c + d*x^n)^3,x)
```

```
[Out] int((a + b*x^n)^2/(c + d*x^n)^3, x)
```

3.298 $\int \frac{(c+dx^n)^4}{a+bx^n}\,dx$

Optimal. Leaf size=310

$$-\frac{d(a^3d^3(1+6n+11n^2+6n^3)-b^3c^3(1+7n+18n^2+24n^3)-a^2bcd^2(3+19n+38n^2+24n^3)+ab^2c^2d(3}{b^4(1+n)(1+2n)(1+3n)}$$

```
[Out] -d*(a^3*d^3*(6*n^3+11*n^2+6*n+1)-b^3*c^3*(24*n^3+18*n^2+7*n+1)-a^2*b*c*d^2*
(24*n^3+38*n^2+19*n+3)+a*b^2*c^2*d*(36*n^3+45*n^2+20*n+3))*x/b^4/(6*n^3+11*
n^2+6*n+1)-d*(2*a*b*c*d*(1+3*n)^2-a^2*d^2*(6*n^2+5*n+1)-b^2*c^2*(18*n^2+7*n
+1))*x*(c+d*x^n)/b^3/(6*n^3+11*n^2+6*n+1)-d*(a*d*(1+3*n)-b*(6*c*n+c))*x*(c+
d*x^n)^2/b^2/(6*n^2+5*n+1)+d*x*(c+d*x^n)^3/b/(1+3*n)+(-a*d+b*c)^4*x*hyperge
om([1, 1/n],[1+1/n],-b*x^n/a)/a/b^4
```

Rubi [A]
time = 0.34, antiderivative size = 310, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {427, 542, 396, 251}

$$-\frac{dx(c+dx^n)\left(-a^2d^2(6n^2+5n+1)+2abcd(3n+1)^2-b^2c^2(18n^2+7n+1)\right)}{b^3(n+1)(2n+1)(3n+1)}-\frac{dx(a^3d^3(6n^3+11n^2+6n+1)-a^2bcd^2(24n^3+38n^2+19n+3)+ab^2c^2d(36n^3+45n^2+20n+3)-b^3c^3(24n^3+18n^2+7n+1))}{b^4(n+1)(2n+1)(3n+1)}+\frac{x(bc-ad)^4\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{ab^4}-\frac{dx(c+dx^n)^2\left(ad(3n+1)-b(6cn+c)\right)}{b^2\left(6n^2+5n+1\right)}+\frac{dx(c+dx^n)^3}{b(3n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^n)^4/(a + b*x^n),x]
```

```
[Out] -((d*(a^3*d^3*(1 + 6*n + 11*n^2 + 6*n^3) - b^3*c^3*(1 + 7*n + 18*n^2 + 24*n
^3) - a^2*b*c*d^2*(3 + 19*n + 38*n^2 + 24*n^3) + a*b^2*c^2*d*(3 + 20*n + 45
*n^2 + 36*n^3))*x)/(b^4*(1 + n)*(1 + 2*n)*(1 + 3*n))) - (d*(2*a*b*c*d*(1 +
3*n)^2 - a^2*d^2*(1 + 5*n + 6*n^2) - b^2*c^2*(1 + 7*n + 18*n^2))*x*(c + d*x
^n))/(b^3*(1 + n)*(1 + 2*n)*(1 + 3*n)) - (d*(a*d*(1 + 3*n) - b*(c + 6*c*n))
*x*(c + d*x^n)^2)/(b^2*(1 + 5*n + 6*n^2)) + (d*x*(c + d*x^n)^3)/(b*(1 + 3*n
)) + ((b*c - a*d)^4*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)
])/(a*b^4)
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
```

```
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rubi steps

$$\int \frac{(c+dx^n)^4}{a+bx^n}\,dx = \frac{dx(c+dx^n)^3}{b(1+3n)} + \frac{\int \frac{(c+dx^n)^2(-c(ad-b(c+3cn))-d(ad(1+3n)-b(c+6cn))x^n)}{a+bx^n}\,dx}{b(1+3n)}$$

$$= -\frac{d(ad(1+3n)-b(c+6cn))x(c+dx^n)^2}{b^2(1+5n+6n^2)} + \frac{dx(c+dx^n)^3}{b(1+3n)} + \int \frac{(c+dx^n)\left(c\left(a^2d^2(1+3n)-2abcd(1+\right.\right.}{}$$

$$= -\frac{d\left(2abcd(1+3n)^2-a^2d^2(1+5n+6n^2)-b^2c^2(1+7n+18n^2)\right)x(c+dx^n)}{b^3(1+n)(1+5n+6n^2)} - \frac{d(ad(1+}{}$$

$$= -\frac{d(a^3d^3(1+6n+11n^2+6n^3)-b^3c^3(1+7n+18n^2+24n^3)-a^2bcd^2(3+19n+38n^2+}{b^4(1+n)(1+5n+6n^2)}$$

$$= -\frac{d(a^3d^3(1+6n+11n^2+6n^3)-b^3c^3(1+7n+18n^2+24n^3)-a^2bcd^2(3+19n+38n^2+}{b^4(1+n)(1+5n+6n^2)}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.
time = 2.34, size = 133, normalized size = 0.43

$$\frac{x\left(4c^3dx^n\Phi\left(-\frac{bx^n}{a},1,1+\frac{1}{n}\right)+6c^2d^2x^{2n}\Phi\left(-\frac{bx^n}{a},1,2+\frac{1}{n}\right)+4cd^3x^{3n}\Phi\left(-\frac{bx^n}{a},1,3+\frac{1}{n}\right)+d^4x^{4n}\Phi\left(-\frac{bx^n}{a},1,4+\frac{1}{n}\right)+c^4\Phi\left(-\frac{bx^n}{a},1,\frac{1}{n}\right)\right)}{an}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^n)^4/(a + b*x^n),x]
```

```
[Out] (x*(4*c^3*d*x^n*HurwitzLerchPhi[-((b*x^n)/a), 1, 1 + n^(-1)] + 6*c^2*d^2*x^
(2*n)*HurwitzLerchPhi[-((b*x^n)/a), 1, 2 + n^(-1)] + 4*c*d^3*x^(3*n)*Hurwit
zLerchPhi[-((b*x^n)/a), 1, 3 + n^(-1)] + d^4*x^(4*n)*HurwitzLerchPhi[-((b*x
^n)/a), 1, 4 + n^(-1)] + c^4*HurwitzLerchPhi[-((b*x^n)/a), 1, n^(-1)]))/(a*
n)
```

Maple [**F**]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{(c+d\,x^n)^4}{a+b\,x^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*x^n)^4/(a+b*x^n),x)
```

```
[Out] int((c+d*x^n)^4/(a+b*x^n),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^n)^4/(a+b*x^n),x, algorithm="maxima")
```

```
[Out] (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*int
egrate(1/(b^5*x^n + a*b^4), x) + ((2*n^2 + 3*n + 1)*b^3*d^4*x*x^(3*n) + (4*
(3*n^2 + 4*n + 1)*b^3*c*d^3 - (3*n^2 + 4*n + 1)*a*b^2*d^4)*x*x^(2*n) + (6*(
6*n^2 + 5*n + 1)*b^3*c^2*d^2 - 4*(6*n^2 + 5*n + 1)*a*b^2*c*d^3 + (6*n^2 + 5
*n + 1)*a^2*b*d^4)*x*x^n + (4*(6*n^3 + 11*n^2 + 6*n + 1)*b^3*c^3*d - 6*(6*n
^3 + 11*n^2 + 6*n + 1)*a*b^2*c^2*d^2 + 4*(6*n^3 + 11*n^2 + 6*n + 1)*a^2*b*c
*d^3 - (6*n^3 + 11*n^2 + 6*n + 1)*a^3*d^4)*x)/((6*n^3 + 11*n^2 + 6*n + 1)*b
^4)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^n)^4/(a+b*x^n),x, algorithm="fricas")
```

```
[Out] integral((d^4*x^(4*n) + 4*c*d^3*x^(3*n) + 6*c^2*d^2*x^(2*n) + 4*c^3*d*x^n +
 c^4)/(b*x^n + a), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 4.81, size = 369, normalized size = 1.19

$$-\frac{4c^3dx\Phi\left(\frac{ax^{-n}e^{i\pi}}{b},1,\frac{e^{i\pi}}{n}\right)\Gamma\left(\frac{1}{n}\right)}{bn^2\Gamma\left(1+\frac{1}{n}\right)}+\frac{c^4x\Phi\left(\frac{bx^ne^{i\pi}}{a},1,\frac{1}{n}\right)\Gamma\left(\frac{1}{n}\right)}{an^2\Gamma\left(1+\frac{1}{n}\right)}+\frac{12c^2d^2xx^{2n}\Phi\left(\frac{bx^ne^{i\pi}}{a},1,2+\frac{1}{n}\right)\Gamma\left(2+\frac{1}{n}\right)}{an\Gamma\left(3+\frac{1}{n}\right)}+\frac{6c^2d^2xx^{2n}\Phi\left(\frac{bx^ne^{i\pi}}{a},1,2+\frac{1}{n}\right)\Gamma\left(2+\frac{1}{n}\right)}{an^2\Gamma\left(3+\frac{1}{n}\right)}+\frac{12cd^3xx^{3n}\Phi\left(\frac{bx^ne^{i\pi}}{a},1,3+\frac{1}{n}\right)\Gamma\left(3+\frac{1}{n}\right)}{an\Gamma\left(4+\frac{1}{n}\right)}+\frac{4cd^3xx^{3n}\Phi\left(\frac{bx^ne^{i\pi}}{a},1,3+\frac{1}{n}\right)\Gamma\left(3+\frac{1}{n}\right)}{an^2\Gamma\left(4+\frac{1}{n}\right)}+\frac{4d^4xx^{4n}\Phi\left(\frac{bx^ne^{i\pi}}{a},1,4+\frac{1}{n}\right)\Gamma\left(4+\frac{1}{n}\right)}{an\Gamma\left(5+\frac{1}{n}\right)}+\frac{d^4xx^{4n}\Phi\left(\frac{bx^ne^{i\pi}}{a},1,4+\frac{1}{n}\right)\Gamma\left(4+\frac{1}{n}\right)}{an^2\Gamma\left(5+\frac{1}{n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x**n)**4/(a+b*x**n),x)
```

```
[Out] -4*c**3*d*x*lerchphi(a*exp_polar(I*pi)/(b*x**n), 1, exp_polar(I*pi)/n)*gamm
a(1/n)/(b*n**2*gamma(1 + 1/n)) + c**4*x*lerchphi(b*x**n*exp_polar(I*pi)/a,
1, 1/n)*gamma(1/n)/(a*n**2*gamma(1 + 1/n)) + 12*c**2*d**2*x*x**(2*n)*lerchp
hi(b*x**n*exp_polar(I*pi)/a, 1, 2 + 1/n)*gamma(2 + 1/n)/(a*n*gamma(3 + 1/n)
) + 6*c**2*d**2*x*x**(2*n)*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 2 + 1/n)*g
amma(2 + 1/n)/(a*n**2*gamma(3 + 1/n)) + 12*c*d**3*x*x**(3*n)*lerchphi(b*x**
n*exp_polar(I*pi)/a, 1, 3 + 1/n)*gamma(3 + 1/n)/(a*n*gamma(4 + 1/n)) + 4*c*
d**3*x*x**(3*n)*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 3 + 1/n)*gamma(3 + 1/
n)/(a*n**2*gamma(4 + 1/n)) + 4*d**4*x*x**(4*n)*lerchphi(b*x**n*exp_polar(I*
pi)/a, 1, 4 + 1/n)*gamma(4 + 1/n)/(a*n*gamma(5 + 1/n)) + d**4*x*x**(4*n)*le
rchphi(b*x**n*exp_polar(I*pi)/a, 1, 4 + 1/n)*gamma(4 + 1/n)/(a*n**2*gamma(5
 + 1/n))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^n)^4/(a+b*x^n),x, algorithm="giac")
```

```
[Out] integrate((d*x^n + c)^4/(b*x^n + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c+d\,x^n)^4}{a+b\,x^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^n)^4/(a + b*x^n),x)
```

```
[Out] int((c + d*x^n)^4/(a + b*x^n), x)
```

3.299 $\int \frac{(c+dx^n)^3}{a+bx^n}\,dx$

Optimal. Leaf size=173

$$\frac{d(a^2d^2(1+3n+2n^2)+b^2c^2(1+4n+6n^2)-abcd(2+7n+6n^2))\,x}{b^3(1+n)(1+2n)}-\frac{d(ad(1+2n)-b(c+4cn))x(c+dx^n)}{b^2(1+n)(1+2n)}$$

```
[Out] d*(a^2*d^2*(2*n^2+3*n+1)+b^2*c^2*(6*n^2+4*n+1)-a*b*c*d*(6*n^2+7*n+2))*x/b^3
/(2*n^2+3*n+1)-d*(a*d*(1+2*n)-b*(4*c*n+c))*x*(c+d*x^n)/b^2/(2*n^2+3*n+1)+d*
x*(c+d*x^n)^2/b/(1+2*n)+(-a*d+b*c)^3*x*hypergeom([1, 1/n],[1+1/n],-b*x^n/a)
/a/b^3
```

Rubi [A]
time = 0.18, antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {427, 542, 396, 251}

$$\frac{dx(a^2d^2(2n^2+3n+1)-abcd(6n^2+7n+2)+b^2c^2(6n^2+4n+1))}{b^3(n+1)(2n+1)}+\frac{x(bc-ad)^3\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{ab^3}-\frac{dx(c+dx^n)\,(ad(2n+1)-b(4cn+c))}{b^2(n+1)(2n+1)}+\frac{dx(c+dx^n)^2}{b(2n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^n)^3/(a + b*x^n),x]
```

```
[Out] (d*(a^2*d^2*(1 + 3*n + 2*n^2) + b^2*c^2*(1 + 4*n + 6*n^2) - a*b*c*d*(2 + 7*
n + 6*n^2))*x)/(b^3*(1 + n)*(1 + 2*n)) - (d*(a*d*(1 + 2*n) - b*(c + 4*c*n))
*x*(c + d*x^n))/(b^2*(1 + n)*(1 + 2*n)) + (d*x*(c + d*x^n)^2)/(b*(1 + 2*n))
 + ((b*c - a*d)^3*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)])
/(a*b^3)
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
```

```
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(c+dx^n)^3}{a+bx^n}\,dx &= \frac{dx(c+dx^n)^2}{b(1+2n)}+\frac{\int \frac{(c+dx^n)(-c(ad-b(c+2cn))-d(ad(1+2n)-b(c+4cn))x^n)}{a+bx^n}\,dx}{b(1+2n)}\\
&= -\frac{d(ad(1+2n)-b(c+4cn))x(c+dx^n)}{b^2(1+n)(1+2n)}+\frac{dx(c+dx^n)^2}{b(1+2n)}+\frac{\int \frac{c(a^2d^2(1+2n)-abcd(2+5n)+b^2c^2(1+}{}}{}\\
&= \frac{d\left(a^2d^2(1+3n+2n^2)+b^2c^2(1+4n+6n^2)-abcd(2+7n+6n^2)\right)x}{b^3(1+n)(1+2n)}-\frac{d(ad(1+2n)-b}{b^2(1+}\\
&= \frac{d\left(a^2d^2(1+3n+2n^2)+b^2c^2(1+4n+6n^2)-abcd(2+7n+6n^2)\right)x}{b^3(1+n)(1+2n)}-\frac{d(ad(1+2n)-b}{b^2(1+}
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.
time = 0.98, size = 104, normalized size = 0.60

$$
\frac{x\left(3c^2dx^n\Phi\left(-\frac{bx^n}{a},1,1+\frac{1}{n}\right)+3cd^2x^{2n}\Phi\left(-\frac{bx^n}{a},1,2+\frac{1}{n}\right)+d^3x^{3n}\Phi\left(-\frac{bx^n}{a},1,3+\frac{1}{n}\right)+c^3\Phi\left(-\frac{bx^n}{a},1,\frac{1}{n}\right)\right)}{an}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^n)^3/(a + b*x^n),x]
```

```
[Out] (x*(3*c^2*d*x^n*HurwitzLerchPhi[-((b*x^n)/a), 1, 1 + n^(-1)] + 3*c*d^2*x^(2
*n)*HurwitzLerchPhi[-((b*x^n)/a), 1, 2 + n^(-1)] + d^3*x^(3*n)*HurwitzLerch
Phi[-((b*x^n)/a), 1, 3 + n^(-1)] + c^3*HurwitzLerchPhi[-((b*x^n)/a), 1, n^(
-1)]))/(a*n)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(c + d\,x^n)^3}{a + b\,x^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*x^n)^3/(a+b*x^n),x)
```

```
[Out] int((c+d*x^n)^3/(a+b*x^n),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^n)^3/(a+b*x^n),x, algorithm="maxima")
```

```
[Out] (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*integrate(1/(b^4*x^n +
a*b^3), x) + (b^2*d^3*(n + 1)*x*x^(2*n) + (3*b^2*c*d^2*(2*n + 1) - a*b*d^3*
(2*n + 1))*x*x^n + (3*(2*n^2 + 3*n + 1)*b^2*c^2*d - 3*(2*n^2 + 3*n + 1)*a*b
*c*d^2 + (2*n^2 + 3*n + 1)*a^2*d^3)*x)/((2*n^2 + 3*n + 1)*b^3)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^n)^3/(a+b*x^n),x, algorithm="fricas")
```

```
[Out] integral((d^3*x^(3*n) + 3*c*d^2*x^(2*n) + 3*c^2*d*x^n + c^3)/(b*x^n + a), x
)
```

Sympy [C] Result contains complex when optimal does not.
time = 2.80, size = 269, normalized size = 1.55

$$-\frac{3c^2dx\Phi\left(\frac{ax^{-n}e^{i\pi}}{b},1,\frac{e^{i\pi}}{n}\right)\Gamma\left(\frac{1}{n}\right)}{bn^2\Gamma\left(1+\frac{1}{n}\right)}+\frac{c^3x\Phi\left(\frac{bx^ne^{i\pi}}{a},1,\frac{1}{n}\right)\Gamma\left(\frac{1}{n}\right)}{an^2\Gamma\left(1+\frac{1}{n}\right)}+\frac{6cd^2xx^{2n}\Phi\left(\frac{bx^ne^{i\pi}}{a},1,2+\frac{1}{n}\right)\Gamma\left(2+\frac{1}{n}\right)}{an\Gamma\left(3+\frac{1}{n}\right)}+\frac{3cd^2xx^{2n}\Phi\left(\frac{bx^ne^{i\pi}}{a},1,2+\frac{1}{n}\right)\Gamma\left(2+\frac{1}{n}\right)}{an^2\Gamma\left(3+\frac{1}{n}\right)}+\frac{3d^3xx^{3n}\Phi\left(\frac{bx^ne^{i\pi}}{a},1,3+\frac{1}{n}\right)\Gamma\left(3+\frac{1}{n}\right)}{an\Gamma\left(4+\frac{1}{n}\right)}+\frac{d^3xx^{3n}\Phi\left(\frac{bx^ne^{i\pi}}{a},1,3+\frac{1}{n}\right)\Gamma\left(3+\frac{1}{n}\right)}{an^2\Gamma\left(4+\frac{1}{n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x**n)**3/(a+b*x**n),x)
```

```
[Out] -3*c**2*d*x*lerchphi(a*exp_polar(I*pi)/(b*x**n), 1, exp_polar(I*pi)/n)*gamm
a(1/n)/(b*n**2*gamma(1 + 1/n)) + c**3*x*lerchphi(b*x**n*exp_polar(I*pi)/a,
1, 1/n)*gamma(1/n)/(a*n**2*gamma(1 + 1/n)) + 6*c*d**2*x*x**(2*n)*lerchphi(b
```

```
*x**n*exp_polar(I*pi)/a, 1, 2 + 1/n)*gamma(2 + 1/n)/(a*n*gamma(3 + 1/n)) +
3*c*d**2*x*x**(2*n)*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 2 + 1/n)*gamma(2
+ 1/n)/(a*n**2*gamma(3 + 1/n)) + 3*d**3*x*x**(3*n)*lerchphi(b*x**n*exp_pola
r(I*pi)/a, 1, 3 + 1/n)*gamma(3 + 1/n)/(a*n*gamma(4 + 1/n)) + d**3*x*x**(3*n
)*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 3 + 1/n)*gamma(3 + 1/n)/(a*n**2*gam
ma(4 + 1/n))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^n)^3/(a+b*x^n),x, algorithm="giac")
[Out] integrate((d*x^n + c)^3/(b*x^n + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c+d\,x^n)^3}{a+b\,x^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^n)^3/(a + b*x^n),x)
[Out] int((c + d*x^n)^3/(a + b*x^n), x)
```

3.300 $\int \frac{(c+dx^n)^2}{a+bx^n}\,dx$

Optimal. Leaf size=84

$$-\frac{d(ad(1+n)-b(c+2cn))x}{b^2(1+n)}+\frac{dx(c+dx^n)}{b(1+n)}+\frac{(bc-ad)^2x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{ab^2}$$

```
[Out] -d*(a*d*(1+n)-b*(2*c*n+c))*x/b^2/(1+n)+d*x*(c+d*x^n)/b/(1+n)+(-a*d+b*c)^2*x
*hypergeom([1, 1/n],[1+1/n],-b*x^n/a)/a/b^2
```

Rubi [A]
time = 0.07, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {427, 396, 251}

$$\frac{x(bc-ad)^2\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{ab^2}-\frac{dx(ad(n+1)-b(2cn+c))}{b^2(n+1)}+\frac{dx(c+dx^n)}{b(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^n)^2/(a + b*x^n),x]
```

```
[Out] -((d*(a*d*(1 + n) - b*(c + 2*c*n))*x)/(b^2*(1 + n))) + (d*x*(c + d*x^n))/(b
*(1 + n)) + ((b*c - a*d)^2*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*
x^n)/a)])/(a*b^2)
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
```

```
0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] && !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx^n)^2}{a+bx^n}\,dx &= \frac{dx(c+dx^n)}{b(1+n)}+\frac{\int \frac{-c(ad-bc(1+n))-d(ad(1+n)-b(c+2cn))x^n}{a+bx^n}\,dx}{b(1+n)}\\ &= -\frac{d(ad(1+n)-b(c+2cn))x}{b^2(1+n)}+\frac{dx(c+dx^n)}{b(1+n)}+\frac{(bc-ad)^2\int \frac{1}{a+bx^n}\,dx}{b^2}\\ &= -\frac{d(ad(1+n)-b(c+2cn))x}{b^2(1+n)}+\frac{dx(c+dx^n)}{b(1+n)}+\frac{(bc-ad)^2x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{ab^2}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.
time = 0.31, size = 75, normalized size = 0.89

$$\frac{x\left(2cdx^n\Phi\left(-\frac{bx^n}{a},1,1+\frac{1}{n}\right)+d^2x^{2n}\Phi\left(-\frac{bx^n}{a},1,2+\frac{1}{n}\right)+c^2\Phi\left(-\frac{bx^n}{a},1,\frac{1}{n}\right)\right)}{an}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^n)^2/(a + b*x^n),x]
[Out] (x*(2*c*d*x^n*HurwitzLerchPhi[-((b*x^n)/a), 1, 1 + n^(-1)] + d^2*x^(2*n)*Hu
rwitzLerchPhi[-((b*x^n)/a), 1, 2 + n^(-1)] + c^2*HurwitzLerchPhi[-((b*x^n)/
a), 1, n^(-1)]))/(a*n)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(c+d\,x^n)^2}{a+b\,x^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*x^n)^2/(a+b*x^n),x)
[Out] int((c+d*x^n)^2/(a+b*x^n),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^n)^2/(a+b*x^n),x, algorithm="maxima")
```

```
[Out] (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*integrate(1/(b^3*x^n + a*b^2), x) + (b*d^2*
x*x^n + (2*b*c*d*(n + 1) - a*d^2*(n + 1))*x)/(b^2*(n + 1))
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^n)^2/(a+b*x^n),x, algorithm="fricas")
```

```
[Out] integral((d^2*x^(2*n) + 2*c*d*x^n + c^2)/(b*x^n + a), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.97, size = 170, normalized size = 2.02

$$-\frac{2cdx\Phi\left(\frac{ax^{-n}e^{i\pi}}{b},1,\frac{e^{i\pi}}{n}\right)\Gamma\left(\frac{1}{n}\right)}{bn^2\Gamma\left(1+\frac{1}{n}\right)}+\frac{c^2x\Phi\left(\frac{bx^ne^{i\pi}}{a},1,\frac{1}{n}\right)\Gamma\left(\frac{1}{n}\right)}{an^2\Gamma\left(1+\frac{1}{n}\right)}+\frac{2d^2xx^{2n}\Phi\left(\frac{bx^ne^{i\pi}}{a},1,2+\frac{1}{n}\right)\Gamma\left(2+\frac{1}{n}\right)}{an\Gamma\left(3+\frac{1}{n}\right)}+\frac{d^2xx^{2n}\Phi\left(\frac{bx^ne^{i\pi}}{a},1,2+\frac{1}{n}\right)\Gamma\left(2+\frac{1}{n}\right)}{an^2\Gamma\left(3+\frac{1}{n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x**n)**2/(a+b*x**n),x)
```

```
[Out] -2*c*d*x*lerchphi(a*exp_polar(I*pi)/(b*x**n), 1, exp_polar(I*pi)/n)*gamma(1
/n)/(b*n**2*gamma(1 + 1/n)) + c**2*x*lerchphi(b*x**n*exp_polar(I*pi)/a, 1,
1/n)*gamma(1/n)/(a*n**2*gamma(1 + 1/n)) + 2*d**2*x*x**(2*n)*lerchphi(b*x**n
*exp_polar(I*pi)/a, 1, 2 + 1/n)*gamma(2 + 1/n)/(a*n*gamma(3 + 1/n)) + d**2*
x*x**(2*n)*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 2 + 1/n)*gamma(2 + 1/n)/(a
*n**2*gamma(3 + 1/n))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^n)^2/(a+b*x^n),x, algorithm="giac")
```

```
[Out] integrate((d*x^n + c)^2/(b*x^n + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c+d\,x^n\right)}^2}{a+b\,x^n} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^n)^2/(a + b*x^n),x)
```

```
[Out] int((c + d*x^n)^2/(a + b*x^n), x)
```

3.301 $\int \frac{c+dx^n}{a+bx^n}\,dx$

Optimal. Leaf size=42

$$\frac{dx}{b}+\frac{(bc-ad)x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{ab}$$

```
[Out] d*x/b+(-a*d+b*c)*x*hypergeom([1, 1/n],[1+1/n],-b*x^n/a)/a/b
```

Rubi [A]
time = 0.01, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}=0.118$, Rules used = {396, 251}

$$\frac{x(bc-ad)\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{ab}+\frac{dx}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^n)/(a + b*x^n),x]
```

```
[Out] (d*x)/b + ((b*c - a*d)*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)
/a)])/(a*b)
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{c+dx^n}{a+bx^n}\,dx&=\frac{dx}{b}-\frac{(-bc+ad)\int \frac{1}{a+bx^n}\,dx}{b}\\&=\frac{dx}{b}+\frac{(bc-ad)x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{ab}\end{aligned}$$

Mathematica [A]
time = 0.09, size = 40, normalized size = 0.95

$$\frac{x\left(ad+(bc-ad)\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)\right)}{ab}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^n)/(a + b*x^n),x]
```

```
[Out] (x*(a*d + (b*c - a*d)*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)
]))/(a*b)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{c+d\,x^n}{a+b\,x^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*x^n)/(a+b*x^n),x)
```

```
[Out] int((c+d*x^n)/(a+b*x^n),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^n)/(a+b*x^n),x, algorithm="maxima")
```

```
[Out] (b*c - a*d)*integrate(1/(b^2*x^n + a*b), x) + d*x/b
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^n)/(a+b*x^n),x, algorithm="fricas")
```

```
[Out] integral((d*x^n + c)/(b*x^n + a), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.99, size = 73, normalized size = 1.74

$$-\frac{dx\Phi\left(\frac{ax^{-n}e^{i\pi}}{b},1,\frac{e^{i\pi}}{n}\right)\Gamma\left(\frac{1}{n}\right)}{bn^2\Gamma\left(1+\frac{1}{n}\right)}+\frac{cx\Phi\left(\frac{bx^ne^{i\pi}}{a},1,\frac{1}{n}\right)\Gamma\left(\frac{1}{n}\right)}{an^2\Gamma\left(1+\frac{1}{n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x**n)/(a+b*x**n),x)
```

```
[Out] -d*x*lerchphi(a*exp_polar(I*pi)/(b*x**n), 1, exp_polar(I*pi)/n)*gamma(1/n)/
(b*n**2*gamma(1 + 1/n)) + c*x*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 1/n)*ga
mma(1/n)/(a*n**2*gamma(1 + 1/n))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^n)/(a+b*x^n),x, algorithm="giac")
```

```
[Out] integrate((d*x^n + c)/(b*x^n + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{c+d\,x^n}{a+b\,x^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^n)/(a + b*x^n),x)
```

```
[Out] int((c + d*x^n)/(a + b*x^n), x)
```

3.302 $\int \frac{1}{(a+bx^n)(c+dx^n)}\,dx$

Optimal. Leaf size=72

$$\frac{bx\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a(bc-ad)}-\frac{dx\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{c(bc-ad)}$$

```
[Out] b*x*hypergeom([1, 1/n],[1+1/n],-b*x^n/a)/a/(-a*d+b*c)-d*x*hypergeom([1, 1/n
],[1+1/n],-d*x^n/c)/c/(-a*d+b*c)
```

Rubi [A]
time = 0.02, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {400, 251}

$$\frac{bx\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a(bc-ad)}-\frac{dx\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{c(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^n)*(c + d*x^n)),x]
```

```
[Out] (b*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/(a*(b*c - a*d)
) - (d*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((d*x^n)/c)])/(c*(b*c -
a*d))
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 400

```
Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[d/(b*c - a*d), Int[1/(c +
 d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+bx^n)(c+dx^n)}\,dx &= \frac{b\int \frac{1}{a+bx^n}\,dx}{bc-ad}-\frac{d\int \frac{1}{c+dx^n}\,dx}{bc-ad}\\ &= \frac{bx\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a(bc-ad)}-\frac{dx\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{c(bc-ad)}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 64, normalized size = 0.89

$$\frac{x\left(-bc\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)+ad\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)\right)}{ac(-bc+ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^n)*(c + d*x^n)),x]
```

```
[Out] (x*(-(b*c*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)]) + a*d*Hyp
ergeometric2F1[1, n^(-1), 1 + n^(-1), -((d*x^n)/c)]))/(a*c*(-(b*c) + a*d))
```

Maple [F]
time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+b\,x^n)\,(c+d\,x^n)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*x^n)/(c+d*x^n),x)
```

```
[Out] int(1/(a+b*x^n)/(c+d*x^n),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^n)/(c+d*x^n),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^n + a)*(d*x^n + c)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^n)/(c+d*x^n),x, algorithm="fricas")
```

```
[Out] integral(1/(b*d*x^(2*n) + a*c + (b*c + a*d)*x^n), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx^n)\,(c+dx^n)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x**n)/(c+d*x**n),x)
```

```
[Out] Integral(1/((a + b*x**n)*(c + d*x**n)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^n)/(c+d*x^n),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^n + a)*(d*x^n + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(a+b\,x^n)\,(c+d\,x^n)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^n)*(c + d*x^n)),x)
```

```
[Out] int(1/((a + b*x^n)*(c + d*x^n)), x)
```

3.303 $\int \frac{1}{(a+bx^n)(c+dx^n)^2} dx$

Optimal. Leaf size=123

$$-\frac{dx}{c(bc-ad)n\left(c+dx^n\right)}+\frac{b^2x\,_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a(bc-ad)^2}+\frac{d(bc(1-2n)-ad(1-n))x\,_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{c^2(bc-ad)^2n}$$

```
[Out] -d*x/c/(-a*d+b*c)/n/(c+d*x^n)+b^2*x*hypergeom([1, 1/n],[1+1/n],-b*x^n/a)/a/
(-a*d+b*c)^2+d*(b*c*(1-2*n)-a*d*(1-n))*x*hypergeom([1, 1/n],[1+1/n],-d*x^n/
c)/c^2/(-a*d+b*c)^2/n
```

Rubi [A]
time = 0.10, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {425, 536, 251}

$$\frac{b^2x\,_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a(bc-ad)^2}+\frac{dx(bc(1-2n)-ad(1-n))\,_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{c^2n(bc-ad)^2}-\frac{dx}{cn(bc-ad)\left(c+dx^n\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^n)*(c + d*x^n)^2),x]
```

```
[Out] -((d*x)/(c*(b*c - a*d)*n*(c + d*x^n))) + (b^2*x*Hypergeometric2F1[1, n^(-1)
, 1 + n^(-1), -((b*x^n)/a)])/(a*(b*c - a*d)^2) + (d*(b*c*(1 - 2*n) - a*d*(1
 - n))*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((d*x^n)/c)])/(c^2*(b*c
 - a*d)^2*n)
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rubi steps

$$\int \frac{1}{(a+bx^n)(c+dx^n)^2}\,dx = -\frac{dx}{c(bc-ad)n(c+dx^n)} + \frac{\int \frac{bcn+a(d-dn)+bd(1-n)x^n}{(a+bx^n)(c+dx^n)}\,dx}{c(bc-ad)n}$$

$$= -\frac{dx}{c(bc-ad)n(c+dx^n)} + \frac{b^2\int \frac{1}{a+bx^n}\,dx}{(bc-ad)^2} - \frac{(d(ad(1-n)-b(c-2cn)))\int \frac{1}{c+dx}}{c(bc-ad)^2n}$$

$$= -\frac{dx}{c(bc-ad)n(c+dx^n)} + \frac{b^2x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a(bc-ad)^2} + \frac{d(bc(1-2n)-ad(1-}{c^2(}$$

Mathematica [A]
time = 0.14, size = 121, normalized size = 0.98

$$\frac{x\left(b^2c^2n(c+dx^n)\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)+ad\left(c(-bc+ad)+(ad(-1+n)+b(c-2cn))(c+dx^n)\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)\right)\right)}{ac^2(bc-ad)^2n(c+dx^n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^n)*(c + d*x^n)^2),x]
```

```
[Out] (x*(b^2*c^2*n*(c + d*x^n)*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n
)/a)] + a*d*(c*(-(b*c) + a*d) + (a*d*(-1 + n) + b*(c - 2*c*n))*(c + d*x^n)*
Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((d*x^n)/c)])))/(a*c^2*(b*c - a*d
)^2*n*(c + d*x^n))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+b\,x^n)\,(c+d\,x^n)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*x^n)/(c+d*x^n)^2,x)
```

```
[Out] int(1/(a+b*x^n)/(c+d*x^n)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^n)/(c+d*x^n)^2,x, algorithm="maxima")
```

```
[Out] b^2*integrate(1/(a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2 + (b^3*c^2 - 2*a*b^2*c*d
 + a^2*b*d^2)*x^n), x) - (b*c*d*(2*n - 1) - a*d^2*(n - 1))*integrate(1/(b^2
*c^4*n - 2*a*b*c^3*d*n + a^2*c^2*d^2*n + (b^2*c^3*d*n - 2*a*b*c^2*d^2*n + a
^2*c*d^3*n)*x^n), x) - d*x/(b*c^3*n - a*c^2*d*n + (b*c^2*d*n - a*c*d^2*n)*x
^n)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^n)/(c+d*x^n)^2,x, algorithm="fricas")
```

```
[Out] integral(1/(b*d^2*x^(3*n) + a*c^2 + (2*b*c*d + a*d^2)*x^(2*n) + (b*c^2 + 2*
a*c*d)*x^n), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x**n)/(c+d*x**n)**2,x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^n)/(c+d*x^n)^2,x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^n + a)*(d*x^n + c)^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(a+b\,x^n\right)\,{\left(c+d\,x^n\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^n)*(c + d*x^n)^2),x)
```

```
[Out] int(1/((a + b*x^n)*(c + d*x^n)^2), x)
```

3.304 $\int \frac{1}{(a+bx^n)(c+dx^n)^3}\,dx$

Optimal. Leaf size=210

$$-\frac{dx}{2c(bc-ad)n\left(c+dx^n\right)^2}-\frac{d(ad(1-2n)-b(c-4cn))x}{2c^2(bc-ad)^2n^2\left(c+dx^n\right)}+\frac{b^3x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a(bc-ad)^3}-\frac{d(a^2d^2(1-3n+2n^2}{}$$

```
[Out] -1/2*d*x/c/(-a*d+b*c)/n/(c+d*x^n)^2-1/2*d*(a*d*(1-2*n)-b*(-4*c*n+c))*x/c^2/
(-a*d+b*c)^2/n^2/(c+d*x^n)+b^3*x*hypergeom([1, 1/n],[1+1/n],-b*x^n/a)/a/(-a
*d+b*c)^3-1/2*d*(a^2*d^2*(2*n^2-3*n+1)-2*a*b*c*d*(3*n^2-4*n+1)+b^2*c^2*(6*n
^2-5*n+1))*x*hypergeom([1, 1/n],[1+1/n],-d*x^n/c)/c^3/(-a*d+b*c)^3/n^2
```

Rubi [A]
time = 0.23, antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {425, 541, 536, 251}

$$-\frac{dx\left(a^2d^2(2n^2-3n+1)-2abcd(3n^2-4n+1)+b^2c^2(6n^2-5n+1)\right)\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{2c^3n^2(bc-ad)^3}+\frac{b^3x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a(bc-ad)^3}+\frac{dx(bc(1-4n)-ad(1-2n))}{2c^2n^2(bc-ad)^2\left(c+dx^n\right)}-\frac{dx}{2cn(bc-ad)\left(c+dx^n\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^n)*(c + d*x^n)^3),x]
```

```
[Out] -1/2*(d*x)/(c*(b*c - a*d)*n*(c + d*x^n)^2) + (d*(b*c*(1 - 4*n) - a*d*(1 - 2
*n))*x)/(2*c^2*(b*c - a*d)^2*n^2*(c + d*x^n)) + (b^3*x*Hypergeometric2F1[1,
 n^(-1), 1 + n^(-1), -((b*x^n)/a)])/(a*(b*c - a*d)^3) - (d*(a^2*d^2*(1 - 3*
n + 2*n^2) - 2*a*b*c*d*(1 - 4*n + 3*n^2) + b^2*c^2*(1 - 5*n + 6*n^2))*x*Hyp
ergeometric2F1[1, n^(-1), 1 + n^(-1), -((d*x^n)/c)])/(2*c^3*(b*c - a*d)^3*n
^2)
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{1}{(a+bx^n)(c+dx^n)^3}\,dx = -\frac{dx}{2c(bc-ad)n(c+dx^n)^2} + \frac{\int \frac{2bcn+a(d-2dn)+bd(1-2n)x^n}{(a+bx^n)(c+dx^n)^2}\,dx}{2c(bc-ad)n}$$

$$= -\frac{dx}{2c(bc-ad)n(c+dx^n)^2} + \frac{d(bc(1-4n)-ad(1-2n))x}{2c^2(bc-ad)^2n^2(c+dx^n)} + \frac{\int \frac{2b^2c^2n^2+a^2d^2(1-3n}{}}{}$$

$$= -\frac{dx}{2c(bc-ad)n(c+dx^n)^2} + \frac{d(bc(1-4n)-ad(1-2n))x}{2c^2(bc-ad)^2n^2(c+dx^n)} + \frac{b^3\int \frac{1}{a+bx^n}\,dx}{(bc-ad)^3} - \frac{(a}{}$$

$$= -\frac{dx}{2c(bc-ad)n(c+dx^n)^2} + \frac{d(bc(1-4n)-ad(1-2n))x}{2c^2(bc-ad)^2n^2(c+dx^n)} + \frac{b^3x\,{}_2F_1\left(1,\frac{1}{n};1+\right.}{a(bc-ad)}$$

Mathematica [A]
time = 0.19, size = 210, normalized size = 1.00

$$\frac{x\left(-ac^2d(bc-ad)^2n+acd(bc-ad)(ad(-1+2n)+b(c-4cn))(c+dx^n)+2b^3c^3n^2(c+dx^n)^2\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)-ad(a^2d^2(1-3n+2n^2)-2abcd(1-4n+3n^2)+b^2c^2(1-5n+6n^2))(c+dx^n)^2\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)\right)}{2ac^3(bc-ad)^3n^2(c+dx^n)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^n)*(c + d*x^n)^3),x]
```

```
[Out] (x*(-(a*c^2*d*(b*c - a*d)^2*n) + a*c*d*(b*c - a*d)*(a*d*(-1 + 2*n) + b*(c -
 4*c*n))*(c + d*x^n) + 2*b^3*c^3*n^2*(c + d*x^n)^2*Hypergeometric2F1[1, n^(
-1), 1 + n^(-1), -((b*x^n)/a)] - a*d*(a^2*d^2*(1 - 3*n + 2*n^2) - 2*a*b*c*d
*(1 - 4*n + 3*n^2) + b^2*c^2*(1 - 5*n + 6*n^2))*(c + d*x^n)^2*Hypergeometri
c2F1[1, n^(-1), 1 + n^(-1), -((d*x^n)/c)]))/(2*a*c^3*(b*c - a*d)^3*n^2*(c +
 d*x^n)^2)
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+b\,x^n)\,(c+d\,x^n)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*x^n)/(c+d*x^n)^3,x)
```

```
[Out] int(1/(a+b*x^n)/(c+d*x^n)^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^n)/(c+d*x^n)^3,x, algorithm="maxima")
```

```
[Out] -b^3*integrate(-1/(a*b^3*c^3 - 3*a^2*b^2*c^2*d + 3*a^3*b*c*d^2 - a^4*d^3 +
(b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*x^n), x) + ((6*n^2
- 5*n + 1)*b^2*c^2*d - 2*(3*n^2 - 4*n + 1)*a*b*c*d^2 + (2*n^2 - 3*n + 1)*a^
2*d^3)*integrate(-1/2/(b^3*c^6*n^2 - 3*a*b^2*c^5*d*n^2 + 3*a^2*b*c^4*d^2*n^
2 - a^3*c^3*d^3*n^2 + (b^3*c^5*d*n^2 - 3*a*b^2*c^4*d^2*n^2 + 3*a^2*b*c^3*d^
3*n^2 - a^3*c^2*d^4*n^2)*x^n), x) - 1/2*((b*c*d^2*(4*n - 1) - a*d^3*(2*n -
1))*x*x^n + (b*c^2*d*(5*n - 1) - a*c*d^2*(3*n - 1))*x)/(b^2*c^6*n^2 - 2*a*b
*c^5*d*n^2 + a^2*c^4*d^2*n^2 + (b^2*c^4*d^2*n^2 - 2*a*b*c^3*d^3*n^2 + a^2*c
^2*d^4*n^2)*x^(2*n) + 2*(b^2*c^5*d*n^2 - 2*a*b*c^4*d^2*n^2 + a^2*c^3*d^3*n^
2)*x^n)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^n)/(c+d*x^n)^3,x, algorithm="fricas")
```

```
[Out] integral(1/(b*d^3*x^(4*n) + a*c^3 + (3*b*c*d^2 + a*d^3)*x^(3*n) + 3*(b*c^2*
d + a*c*d^2)*x^(2*n) + (b*c^3 + 3*a*c^2*d)*x^n), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x**n)/(c+d*x**n)**3,x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^n)/(c+d*x^n)^3,x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^n + a)*(d*x^n + c)^3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(a+b\,x^n\right)\,{\left(c+d\,x^n\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^n)*(c + d*x^n)^3),x)
```

```
[Out] int(1/((a + b*x^n)*(c + d*x^n)^3), x)
```

3.305 $\int \frac{(c+dx^n)^4}{(a+bx^n)^2} \, dx$

Optimal. Leaf size=341

$$-\frac{d(b^3c^3(1+3n+2n^2)-a^3d^3(1+6n+11n^2+6n^3)-ab^2c^2d(3+12n+17n^2+12n^3)+a^2bcd^2(3+15n+}{ab^4n(1+n)(1+2n)}$$

```
[Out] -d*(b^3*c^3*(2*n^2+3*n+1)-a^3*d^3*(6*n^3+11*n^2+6*n+1)-a*b^2*c^2*d*(12*n^3+
17*n^2+12*n+3)+a^2*b*c*d^2*(16*n^3+26*n^2+15*n+3))*x/a/b^4/n/(2*n^2+3*n+1)-
d*(b^2*c^2*(2*n^2+3*n+1)-2*a*b*c*d*(5*n^2+4*n+1)+a^2*d^2*(6*n^2+5*n+1))*x*(
c+d*x^n)/a/b^3/n/(2*n^2+3*n+1)-d*(-3*a*d*n+2*b*c*n-a*d+b*c)*x*(c+d*x^n)^2/a
/b^2/n/(1+2*n)+(-a*d+b*c)*x*(c+d*x^n)^3/a/b/n/(a+b*x^n)-(-a*d+b*c)^3*(b*c*(
1-n)-a*d*(1+3*n))*x*hypergeom([1, 1/n],[1+1/n],-b*x^n/a)/a^2/b^4/n
```

Rubi [A]
time = 0.38, antiderivative size = 341, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {424, 542, 396, 251}

$$-\frac{x(bc-ad)^3(bc(1-n)-ad(3n+1))\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a^2b^4n}-\frac{dx(c+dx^n)\left(a^2d^2(6n^2+5n+1)-2abcd(5n^2+4n+1)+b^2c^2(2n^2+3n+1)\right)}{ab^3n(n+1)(2n+1)}-\frac{dx(-a^3d^3(6n^3+11n^2+6n+1)+a^2bcd^2(16n^3+26n^2+15n+3)-ab^2c^2d(12n^3+17n^2+12n+3)+b^3c^3(2n^2+3n+1))}{ab^4n(n+1)(2n+1)}+\frac{dx(c+dx^n)^2\left(ad(3n+1)-b(2cn+c)\right)}{ab^2n(2n+1)}+\frac{x(bc-ad)\left(c+dx^n\right)^3}{abn\left(a+bx^n\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^n)^4/(a + b*x^n)^2,x]
```

```
[Out] -((d*(b^3*c^3*(1 + 3*n + 2*n^2) - a^3*d^3*(1 + 6*n + 11*n^2 + 6*n^3) - a*b^
2*c^2*d*(3 + 12*n + 17*n^2 + 12*n^3) + a^2*b*c*d^2*(3 + 15*n + 26*n^2 + 16*
n^3))*x)/(a*b^4*n*(1 + n)*(1 + 2*n))) - (d*(b^2*c^2*(1 + 3*n + 2*n^2) - 2*a
*b*c*d*(1 + 4*n + 5*n^2) + a^2*d^2*(1 + 5*n + 6*n^2))*x*(c + d*x^n))/(a*b^3
*n*(1 + n)*(1 + 2*n)) + (d*(a*d*(1 + 3*n) - b*(c + 2*c*n))*x*(c + d*x^n)^2)
/(a*b^2*n*(1 + 2*n)) + ((b*c - a*d)*x*(c + d*x^n)^3)/(a*b*n*(a + b*x^n)) -
((b*c - a*d)^3*(b*c*(1 - n) - a*d*(1 + 3*n))*x*Hypergeometric2F1[1, n^(-1),
 1 + n^(-1), -((b*x^n)/a)])/(a^2*b^4*n)
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
```

```
c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rubi steps

$$\int \frac{(c+dx^n)^4}{(a+bx^n)^2}\,dx = \frac{(bc-ad)x(c+dx^n)^3}{abn\,(a+bx^n)} + \frac{\int \frac{(c+dx^n)^2(c(ad-bc(1-n))+d(ad(1+3n)-b(c+2cn))x^n)}{a+bx^n}\,dx}{abn}$$

$$= \frac{d(ad(1+3n)-b(c+2cn))x(c+dx^n)^2}{ab^2n(1+2n)} + \frac{(bc-ad)x(c+dx^n)^3}{abn\,(a+bx^n)} + \frac{\int \frac{(c+dx^n)(c(2abcd(1+2n)-}{}}{}$$

$$= -\frac{d(b^2c^2(1+3n+2n^2)-2abcd(1+4n+5n^2)+a^2d^2(1+5n+6n^2))\,x(c+dx^n)}{ab^3n(1+n)(1+2n)} + \frac{d(ad}{}$$

$$= -\frac{d(b^3c^3(1+3n+2n^2)-a^3d^3(1+6n+11n^2+6n^3)-ab^2c^2d(3+12n+17n^2+12n^3)+}{ab^4n(1+n)(1+2n)}$$

$$= -\frac{d(b^3c^3(1+3n+2n^2)-a^3d^3(1+6n+11n^2+6n^3)-ab^2c^2d(3+12n+17n^2+12n^3)+}{ab^4n(1+n)(1+2n)}$$

Mathematica [A]
time = 6.10, size = 217, normalized size = 0.64

$$\frac{x\left(\frac{4ab^3c^3d-6a^2b^2c^2d^2+4a^3bcd^3-a^4d^4+b^4c^4(-1+n)}{a^2n}+\frac{(-bc+ad)^3(bc(-1+n)+ad(1+3n))}{a^2n}+\frac{2bd^3(2bc-ad)x^n}{1+n}+\frac{b^2d^4x^{2n}}{1+2n}+\frac{(bc-ad)^4}{an(a+bx^n)}+\frac{(bc-ad)^3(bc(-1+n)+ad(1+3n))\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a^2n}\right)}{b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^n)^4/(a + b*x^n)^2,x]
```

```
[Out] (x*((4*a*b^3*c^3*d - 6*a^2*b^2*c^2*d^2 + 4*a^3*b*c*d^3 - a^4*d^4 + b^4*c^4*
(-1 + n))/(a^2*n) + ((-(b*c) + a*d)^3*(b*c*(-1 + n) + a*d*(1 + 3*n)))/(a^2*
n) + (2*b*d^3*(2*b*c - a*d)*x^n)/(1 + n) + (b^2*d^4*x^(2*n))/(1 + 2*n) + (b
*c - a*d)^4/(a*n*(a + b*x^n)) + ((b*c - a*d)^3*(b*c*(-1 + n) + a*d*(1 + 3*n
))*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/(a^2*n)))/b^4
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\left(c+d\,x^{n}\right)^{4}}{\left(a+b\,x^{n}\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*x^n)^4/(a+b*x^n)^2,x)
```

```
[Out] int((c+d*x^n)^4/(a+b*x^n)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^n)^4/(a+b*x^n)^2,x, algorithm="maxima")
```

```
[Out] -(a^4*d^4*(3*n + 1) - 4*a^3*b*c*d^3*(2*n + 1) + 6*a^2*b^2*c^2*d^2*(n + 1) -
 b^4*c^4*(n - 1) - 4*a*b^3*c^3*d)*integrate(1/(a*b^5*n*x^n + a^2*b^4*n), x)
 + ((n^2 + n)*a*b^3*d^4*x*x^(3*n) + (4*(2*n^2 + n)*a*b^3*c*d^3 - (3*n^2 + n
)*a^2*b^2*d^4)*x*x^(2*n) + (6*(2*n^3 + 3*n^2 + n)*a*b^3*c^2*d^2 - 4*(4*n^3
+ 4*n^2 + n)*a^2*b^2*c*d^3 + (6*n^3 + 5*n^2 + n)*a^3*b*d^4)*x*x^n + ((2*n^2
 + 3*n + 1)*b^4*c^4 - 4*(2*n^2 + 3*n + 1)*a*b^3*c^3*d + 6*(2*n^3 + 5*n^2 +
4*n + 1)*a^2*b^2*c^2*d^2 - 4*(4*n^3 + 8*n^2 + 5*n + 1)*a^3*b*c*d^3 + (6*n^3
 + 11*n^2 + 6*n + 1)*a^4*d^4)*x)/((2*n^3 + 3*n^2 + n)*a*b^5*x^n + (2*n^3 +
3*n^2 + n)*a^2*b^4)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^n)^4/(a+b*x^n)^2,x, algorithm="fricas")
```

```
[Out] integral((d^4*x^(4*n) + 4*c*d^3*x^(3*n) + 6*c^2*d^2*x^(2*n) + 4*c^3*d*x^n +
 c^4)/(b^2*x^(2*n) + 2*a*b*x^n + a^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c + dx^n)^4}{(a + bx^n)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x**n)**4/(a+b*x**n)**2,x)
```

```
[Out] Integral((c + d*x**n)**4/(a + b*x**n)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^n)^4/(a+b*x^n)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x^n + c)^4/(b*x^n + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c + d\,x^n)^4}{(a + b\,x^n)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^n)^4/(a + b*x^n)^2,x)
```

```
[Out] int((c + d*x^n)^4/(a + b*x^n)^2, x)
```

3.306 $\int \frac{(c+dx^n)^3}{(a+bx^n)^2} dx$

Optimal. Leaf size=200

$$-\frac{d(b^2c^2(1+n)+a^2d^2(1+3n+2n^2)-abcd(2+4n+3n^2))\,x}{ab^3n(1+n)}-\frac{d(bc(1+n)-ad(1+2n))x(c+dx^n)}{ab^2n(1+n)}+\frac{(bc$$

```
[Out] -d*(b^2*c^2*(1+n)+a^2*d^2*(2*n^2+3*n+1)-a*b*c*d*(3*n^2+4*n+2))*x/a/b^3/n/(1
+n)-d*(b*c*(1+n)-a*d*(1+2*n))*x*(c+d*x^n)/a/b^2/n/(1+n)+(-a*d+b*c)*x*(c+d*x
^n)^2/a/b/n/(a+b*x^n)-(-a*d+b*c)^2*(b*c*(1-n)-a*d*(1+2*n))*x*hypergeom([1,
1/n],[1+1/n],-b*x^n/a)/a^2/b^3/n
```

Rubi [A]
time = 0.18, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {424, 542, 396, 251}

$$-\frac{x(bc-ad)^2(bc(1-n)-ad(2n+1))\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a^2b^3n}-\frac{dx(a^2d^2(2n^2+3n+1)-abcd(3n^2+4n+2)+b^2c^2(n+1))}{ab^3n(n+1)}-\frac{dx(c+dx^n)\,(bc(n+1)-ad(2n+1))}{ab^2n(n+1)}+\frac{x(bc-ad)\,(c+dx^n)^2}{abn\,(a+bx^n)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^n)^3/(a + b*x^n)^2,x]
```

```
[Out] -((d*(b^2*c^2*(1 + n) + a^2*d^2*(1 + 3*n + 2*n^2) - a*b*c*d*(2 + 4*n + 3*n^
2))*x)/(a*b^3*n*(1 + n))) - (d*(b*c*(1 + n) - a*d*(1 + 2*n))*x*(c + d*x^n))
/(a*b^2*n*(1 + n)) + ((b*c - a*d)*x*(c + d*x^n)^2)/(a*b*n*(a + b*x^n)) - ((
b*c - a*d)^2*(b*c*(1 - n) - a*d*(1 + 2*n))*x*Hypergeometric2F1[1, n^(-1), 1
 + n^(-1), -((b*x^n)/a)])/(a^2*b^3*n)
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
```

```
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rubi steps

$$\int \frac{(c+dx^n)^3}{(a+bx^n)^2}\,dx = \frac{(bc-ad)x(c+dx^n)^2}{abn\,(a+bx^n)} + \frac{\int \frac{(c+dx^n)(c(ad-bc(1-n))-d(bc(1+n)-ad(1+2n))x^n)}{a+bx^n}\,dx}{abn}$$

$$= -\frac{d(bc(1+n)-ad(1+2n))x(c+dx^n)}{ab^2n(1+n)} + \frac{(bc-ad)x(c+dx^n)^2}{abn\,(a+bx^n)} + \frac{\int \frac{c(2abcd(1+n)-a^2d^2(1+2n}{}}{}$$

$$= -\frac{d(b^2c^2(1+n)+a^2d^2(1+3n+2n^2)-abcd(2+4n+3n^2))\,x}{ab^3n(1+n)} - \frac{d(bc(1+n)-ad(1+2}{ab^2n(1+n}$$

$$= -\frac{d(b^2c^2(1+n)+a^2d^2(1+3n+2n^2)-abcd(2+4n+3n^2))\,x}{ab^3n(1+n)} - \frac{d(bc(1+n)-ad(1+2}{ab^2n(1+n}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.
time = 3.35, size = 2050, normalized size = 10.25

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x^n)^3/(a + b*x^n)^2,x]
```

```
[Out] (x*(3*a*(1 + 10*n + 35*n^2 + 50*n^3 + 24*n^4)*(c^3*(1 + n)^4 + 3*c^2*d*(1 +
 4*n + 6*n^2 + 2*n^3 + n^4)*x^n + 3*c*d^2*(1 + n)^4*x^(2*n) + d^3*(1 + n)^4
*x^(3*n))*HurwitzLerchPhi[-((b*x^n)/a), 1, 1 + n^(-1)] - 3*a*(1 + 10*n + 35
*n^2 + 50*n^3 + 24*n^4)*(c^3*(1 + 2*n)^4 + 3*c^2*d*(1 + 2*n)^4*x^n + 3*c*d^
2*(1 + 8*n + 24*n^2 + 34*n^3 + 18*n^4)*x^(2*n) + d^3*(1 + 2*n)^4*x^(3*n))*H
urwitzLerchPhi[-((b*x^n)/a), 1, 2 + n^(-1)] + a*c^3*HurwitzLerchPhi[-((b*x^
```

n)/a), 1, 3 + n^(-1)] + 22*a*c^3*n*HurwitzLerchPhi[-((b*x^n)/a), 1, 3 + n^(
-1)] + 209*a*c^3*n^2*HurwitzLerchPhi[-((b*x^n)/a), 1, 3 + n^(-1)] + 1118*a*
c^3*n^3*HurwitzLerchPhi[-((b*x^n)/a), 1, 3 + n^(-1)] + 3675*a*c^3*n^4*Hurwi
tzLerchPhi[-((b*x^n)/a), 1, 3 + n^(-1)] + 7578*a*c^3*n^5*HurwitzLerchPhi[-(
(b*x^n)/a), 1, 3 + n^(-1)] + 9531*a*c^3*n^6*HurwitzLerchPhi[-((b*x^n)/a), 1
, 3 + n^(-1)] + 6642*a*c^3*n^7*HurwitzLerchPhi[-((b*x^n)/a), 1, 3 + n^(-1)]
 + 1944*a*c^3*n^8*HurwitzLerchPhi[-((b*x^n)/a), 1, 3 + n^(-1)] + 3*a*c^2*d*
x^n*HurwitzLerchPhi[-((b*x^n)/a), 1, 3 + n^(-1)] + 66*a*c^2*d*n*x^n*Hurwitz
LerchPhi[-((b*x^n)/a), 1, 3 + n^(-1)] + 627*a*c^2*d*n^2*x^n*HurwitzLerchPhi
[-((b*x^n)/a), 1, 3 + n^(-1)] + 3354*a*c^2*d*n^3*x^n*HurwitzLerchPhi[-((b*x
^n)/a), 1, 3 + n^(-1)] + 11025*a*c^2*d*n^4*x^n*HurwitzLerchPhi[-((b*x^n)/a)
, 1, 3 + n^(-1)] + 22734*a*c^2*d*n^5*x^n*HurwitzLerchPhi[-((b*x^n)/a), 1, 3
 + n^(-1)] + 28593*a*c^2*d*n^6*x^n*HurwitzLerchPhi[-((b*x^n)/a), 1, 3 + n^(
-1)] + 19926*a*c^2*d*n^7*x^n*HurwitzLerchPhi[-((b*x^n)/a), 1, 3 + n^(-1)] +
 5832*a*c^2*d*n^8*x^n*HurwitzLerchPhi[-((b*x^n)/a), 1, 3 + n^(-1)] + 3*a*c*
d^2*x^(2*n)*HurwitzLerchPhi[-((b*x^n)/a), 1, 3 + n^(-1)] + 66*a*c*d^2*n*x^(
2*n)*HurwitzLerchPhi[-((b*x^n)/a), 1, 3 + n^(-1)] + 627*a*c*d^2*n^2*x^(2*n)
*HurwitzLerchPhi[-((b*x^n)/a), 1, 3 + n^(-1)] + 3354*a*c*d^2*n^3*x^(2*n)*Hu
rwitzLerchPhi[-((b*x^n)/a), 1, 3 + n^(-1)] + 11025*a*c*d^2*n^4*x^(2*n)*Hurw
itzLerchPhi[-((b*x^n)/a), 1, 3 + n^(-1)] + 22734*a*c*d^2*n^5*x^(2*n)*Hurwit
zLerchPhi[-((b*x^n)/a), 1, 3 + n^(-1)] + 28593*a*c*d^2*n^6*x^(2*n)*HurwitzL
erchPhi[-((b*x^n)/a), 1, 3 + n^(-1)] + 19926*a*c*d^2*n^7*x^(2*n)*HurwitzLer
chPhi[-((b*x^n)/a), 1, 3 + n^(-1)] + 5832*a*c*d^2*n^8*x^(2*n)*HurwitzLerchP
hi[-((b*x^n)/a), 1, 3 + n^(-1)] + a*d^3*x^(3*n)*HurwitzLerchPhi[-((b*x^n)/a
), 1, 3 + n^(-1)] + 22*a*d^3*n*x^(3*n)*HurwitzLerchPhi[-((b*x^n)/a), 1, 3 +
 n^(-1)] + 209*a*d^3*n^2*x^(3*n)*HurwitzLerchPhi[-((b*x^n)/a), 1, 3 + n^(-1
)] + 1112*a*d^3*n^3*x^(3*n)*HurwitzLerchPhi[-((b*x^n)/a), 1, 3 + n^(-1)] +
3603*a*d^3*n^4*x^(3*n)*HurwitzLerchPhi[-((b*x^n)/a), 1, 3 + n^(-1)] + 7248*
a*d^3*n^5*x^(3*n)*HurwitzLerchPhi[-((b*x^n)/a), 1, 3 + n^(-1)] + 8811*a*d^3
*n^6*x^(3*n)*HurwitzLerchPhi[-((b*x^n)/a), 1, 3 + n^(-1)] + 5898*a*d^3*n^7*
x^(3*n)*HurwitzLerchPhi[-((b*x^n)/a), 1, 3 + n^(-1)] + 1656*a*d^3*n^8*x^(3*
n)*HurwitzLerchPhi[-((b*x^n)/a), 1, 3 + n^(-1)] - a*c^3*HurwitzLerchPhi[-((
b*x^n)/a), 1, n^(-1)] - 10*a*c^3*n*HurwitzLerchPhi[-((b*x^n)/a), 1, n^(-1)]
 - 35*a*c^3*n^2*HurwitzLerchPhi[-((b*x^n)/a), 1, n^(-1)] - 56*a*c^3*n^3*Hur
witzLerchPhi[-((b*x^n)/a), 1, n^(-1)] - 78*a*c^3*n^4*HurwitzLerchPhi[-((b*x
^n)/a), 1, n^(-1)] - 150*a*c^3*n^5*HurwitzLerchPhi[-((b*x^n)/a), 1, n^(-1)]
 - 90*a*c^3*n^6*HurwitzLerchPhi[-((b*x^n)/a), 1, n^(-1)] + 156*a*c^3*n^7*Hu
rwitzLerchPhi[-((b*x^n)/a), 1, n^(-1)] + 144*a*c^3*n^8*HurwitzLerchPhi[-((b
*x^n)/a), 1, n^(-1)] - 3*a*c^2*d*x^n*HurwitzLerchPhi[-((b*x^n)/a), 1, n^(-1
)] - 30*a*c^2*d*n*x^n*HurwitzLerchPhi[-((b*x^n)/a), 1, n^(-1)] - 105*a*c^2*
d*n^2*x^n*HurwitzLerchPhi[-((b*x^n)/a), 1, n^(-1)] - 150*a*c^2*d*n^3*x^n*Hu
rwitzLerchPhi[-((b*x^n)/a), 1, n^(-1)] - 72*a*c^2*d*n^4*x^n*HurwitzLerchPhi
[-((b*x^n)/a), 1, n^(-1)] - 3*a*c*d^2*x^(2*n)*HurwitzLerchPhi[-((b*x^n)/a),
 1, n^(-1)] - 30*a*c*d^2*n*x^(2*n)*HurwitzLerchPhi[-((b*x^n)/a), 1, n^(-1)]
 - 105*a*c*d^2*n^2*x^(2*n)*HurwitzLerchPhi[-((b*x^n)/a), 1, n^(-1)] - 150*a

```
*c*d^2*n^3*x^(2*n)*HurwitzLerchPhi[-((b*x^n)/a), 1, n^(-1)] - 72*a*c*d^2*n^
4*x^(2*n)*HurwitzLerchPhi[-((b*x^n)/a), 1, n^(-1)] - a*d^3*x^(3*n)*HurwitzL
erchPhi[-((b*x^n)/a), 1, n^(-1)] - 10*a*d^3*n*x^(3*n)*HurwitzLerchPhi[-((b*
x^n)/a), 1, n^(-1)] - 35*a*d^3*n^2*x^(3*n)*HurwitzLerchPhi[-((b*x^n)/a), 1,
 n^(-1)] - 50*a*d^3*n^3*x^(3*n)*HurwitzLerchPhi[-((b*x^n)/a), 1, n^(-1)] -
24*a*d^3*n^4*x^(3*n)*HurwitzLerchPhi[-((b*x^n)/a), 1, n^(-1)] - 6*b*c^3*n^8
*x^n*HypergeometricPFQ[{2, 2, 2, 2, 1 + n^(-1)}, {1, 1, 1, 5 + n^(-1)}, -((
b*x^n)/a)] - 18*b*c^2*d*n^8*x^(2*n)*HypergeometricPFQ[{2, 2, 2, 2, 1 + n^(-
1)}, {1, 1, 1, 5 + n^(-1)}, -((b*x^n)/a)] - 18*b*c*d^2*n^8*x^(3*n)*Hypergeo
metricPFQ[{2, 2, 2, 2, 1 + n^(-1)}, {1, 1, 1, 5 + n^(-1)}, -((b*x^n)/a)] -
6*b*d^3*n^8*x^(4*n)*HypergeometricPFQ[{2, 2, 2, 2, 1 + n^(-1)}, {1, 1, 1, 5
 + n^(-1)}, -((b*x^n)/a)]))/(6*a^3*n^5*(1 + n)*(1 + 2*n)*(1 + 3*n)*(1 + 4*n
))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{\left(c+d\,x^{n}\right)^{3}}{\left(a+b\,x^{n}\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*x^n)^3/(a+b*x^n)^2,x)
```

```
[Out] int((c+d*x^n)^3/(a+b*x^n)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^n)^3/(a+b*x^n)^2,x, algorithm="maxima")
```

```
[Out] (a^3*d^3*(2*n + 1) - 3*a^2*b*c*d^2*(n + 1) + b^3*c^3*(n - 1) + 3*a*b^2*c^2*
d)*integrate(1/(a*b^4*n*x^n + a^2*b^3*n), x) + (a*b^2*d^3*n*x*x^(2*n) + (3*
(n^2 + n)*a*b^2*c*d^2 - (2*n^2 + n)*a^2*b*d^3)*x*x^n + (3*(n^2 + 2*n + 1)*a
^2*b*c*d^2 - (2*n^2 + 3*n + 1)*a^3*d^3 + b^3*c^3*(n + 1) - 3*a*b^2*c^2*d*(n
 + 1))*x)/((n^2 + n)*a*b^4*x^n + (n^2 + n)*a^2*b^3)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^n)^3/(a+b*x^n)^2,x, algorithm="fricas")
```

```
[Out] integral((d^3*x^(3*n) + 3*c*d^2*x^(2*n) + 3*c^2*d*x^n + c^3)/(b^2*x^(2*n) +
 2*a*b*x^n + a^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c + dx^n)^3}{(a + bx^n)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x**n)**3/(a+b*x**n)**2,x)
```

```
[Out] Integral((c + d*x**n)**3/(a + b*x**n)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^n)^3/(a+b*x^n)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x^n + c)^3/(b*x^n + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c + d\, x^n)^3}{(a + b\, x^n)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^n)^3/(a + b*x^n)^2,x)
```

```
[Out] int((c + d*x^n)^3/(a + b*x^n)^2, x)
```

3.307 $\int \frac{(c+dx^n)^2}{(a+bx^n)^2} dx$

Optimal. Leaf size=115

$$-\frac{d(bc-ad(1+n))x}{ab^2n}+\frac{(bc-ad)x(c+dx^n)}{abn(a+bx^n)}-\frac{(bc-ad)(bc(1-n)-ad(1+n))x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a^2b^2n}$$

```
[Out] -d*(b*c-a*d*(1+n))*x/a/b^2/n+(-a*d+b*c)*x*(c+d*x^n)/a/b/n/(a+b*x^n)-(-a*d+b
*c)*(b*c*(1-n)-a*d*(1+n))*x*hypergeom([1, 1/n],[1+1/n],-b*x^n/a)/a^2/b^2/n
```

Rubi [A]
time = 0.06, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {424, 396, 251}

$$-\frac{x(bc-ad)(bc(1-n)-ad(n+1))\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a^2b^2n}-\frac{dx(bc-ad(n+1))}{ab^2n}+\frac{x(bc-ad)(c+dx^n)}{abn(a+bx^n)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^n)^2/(a + b*x^n)^2,x]
```

```
[Out] -((d*(b*c - a*d*(1 + n))*x)/(a*b^2*n)) + ((b*c - a*d)*x*(c + d*x^n))/(a*b*n
*(a + b*x^n)) - ((b*c - a*d)*(b*c*(1 - n) - a*d*(1 + n))*x*Hypergeometric2F
1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/(a^2*b^2*n)
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
```

0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]

Rubi steps

$$\int \frac{(c+dx^n)^2}{(a+bx^n)^2}\,dx = \frac{(bc-ad)x(c+dx^n)}{abn\,(a+bx^n)} + \frac{\int \frac{c(ad-bc(1-n))-d(bc-ad(1+n))x^n}{a+bx^n}\,dx}{abn}$$
$$= -\frac{d(bc-ad(1+n))x}{ab^2n} + \frac{(bc-ad)x(c+dx^n)}{abn\,(a+bx^n)} - \frac{((bc-ad)(bc(1-n)-ad(1+n)))\int \frac{}{a+}}{ab^2n}$$
$$= -\frac{d(bc-ad(1+n))x}{ab^2n} + \frac{(bc-ad)x(c+dx^n)}{abn\,(a+bx^n)} - \frac{(bc-ad)(bc(1-n)-ad(1+n))x\,{}_2F_1(}{a^2b^2n}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.
time = 1.21, size = 666, normalized size = 5.79

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x^n)^2/(a + b*x^n)^2,x]
[Out] (x*(-2*a*(1 + 6*n + 11*n^2 + 6*n^3)*(c^2*(1 + n)^3 + 2*c*d*(1 + 3*n + 4*n^2
 + n^3)*x^n + d^2*(1 + n)^3*x^(2*n))*HurwitzLerchPhi[-((b*x^n)/a), 1, 1 + n
^(-1)] + a*(1 + 6*n + 11*n^2 + 6*n^3)*(c^2*(1 + 2*n)^3 + 2*c*d*(1 + 2*n)^3*
x^n + d^2*(1 + 6*n + 10*n^2 + 6*n^3)*x^(2*n))*HurwitzLerchPhi[-((b*x^n)/a),
 1, 2 + n^(-1)] + a*c^2*HurwitzLerchPhi[-((b*x^n)/a), 1, n^(-1)] + 6*a*c^2*
n*HurwitzLerchPhi[-((b*x^n)/a), 1, n^(-1)] + 9*a*c^2*n^2*HurwitzLerchPhi[-(
(b*x^n)/a), 1, n^(-1)] - 4*a*c^2*n^3*HurwitzLerchPhi[-((b*x^n)/a), 1, n^(-1
)] - 10*a*c^2*n^4*HurwitzLerchPhi[-((b*x^n)/a), 1, n^(-1)] + 10*a*c^2*n^5*H
urwitzLerchPhi[-((b*x^n)/a), 1, n^(-1)] + 12*a*c^2*n^6*HurwitzLerchPhi[-((b
*x^n)/a), 1, n^(-1)] + 2*a*c*d*x^n*HurwitzLerchPhi[-((b*x^n)/a), 1, n^(-1)]
 + 12*a*c*d*n*x^n*HurwitzLerchPhi[-((b*x^n)/a), 1, n^(-1)] + 22*a*c*d*n^2*x
^n*HurwitzLerchPhi[-((b*x^n)/a), 1, n^(-1)] + 12*a*c*d*n^3*x^n*HurwitzLerch
Phi[-((b*x^n)/a), 1, n^(-1)] + a*d^2*x^(2*n)*HurwitzLerchPhi[-((b*x^n)/a),
1, n^(-1)] + 6*a*d^2*n*x^(2*n)*HurwitzLerchPhi[-((b*x^n)/a), 1, n^(-1)] + 1
1*a*d^2*n^2*x^(2*n)*HurwitzLerchPhi[-((b*x^n)/a), 1, n^(-1)] + 6*a*d^2*n^3*
x^(2*n)*HurwitzLerchPhi[-((b*x^n)/a), 1, n^(-1)] - 2*b*c^2*n^6*x^n*Hypergeo
metricPFQ[{2, 2, 2, 1 + n^(-1)}, {1, 1, 4 + n^(-1)}, -((b*x^n)/a)] - 4*b*c*
d*n^6*x^(2*n)*HypergeometricPFQ[{2, 2, 2, 1 + n^(-1)}, {1, 1, 4 + n^(-1)},
-((b*x^n)/a)] - 2*b*d^2*n^6*x^(3*n)*HypergeometricPFQ[{2, 2, 2, 1 + n^(-1)}
, {1, 1, 4 + n^(-1)}, -((b*x^n)/a)]))/(2*a^3*n^4*(1 + 6*n + 11*n^2 + 6*n^3)
)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(c+d\,x^n)^2}{(a+b\,x^n)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*x^n)^2/(a+b*x^n)^2,x)
```

```
[Out] int((c+d*x^n)^2/(a+b*x^n)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^n)^2/(a+b*x^n)^2,x, algorithm="maxima")
```

```
[Out] -(a^2*d^2*(n + 1) - b^2*c^2*(n - 1) - 2*a*b*c*d)*integrate(1/(a*b^3*n*x^n +
 a^2*b^2*n), x) + (a*b*d^2*n*x*x^n + (a^2*d^2*(n + 1) + b^2*c^2 - 2*a*b*c*d
)*x)/(a*b^3*n*x^n + a^2*b^2*n)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^n)^2/(a+b*x^n)^2,x, algorithm="fricas")
```

```
[Out] integral((d^2*x^(2*n) + 2*c*d*x^n + c^2)/(b^2*x^(2*n) + 2*a*b*x^n + a^2), x
)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx^n)^2}{(a+bx^n)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x**n)**2/(a+b*x**n)**2,x)
```

```
[Out] Integral((c + d*x**n)**2/(a + b*x**n)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^n)^2/(a+b*x^n)^2,x, algorithm="giac")
[Out] integrate((d*x^n + c)^2/(b*x^n + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c+d\,x^n)^2}{(a+b\,x^n)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^n)^2/(a + b*x^n)^2,x)
[Out] int((c + d*x^n)^2/(a + b*x^n)^2, x)
```

3.308 $\int \frac{c+dx^n}{(a+bx^n)^2} dx$

Optimal. Leaf size=72

$$\frac{(bc-ad)x}{abn\,(a+bx^n)}+\frac{(ad-bc(1-n))x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a^2bn}$$

```
[Out] (-a*d+b*c)*x/a/b/n/(a+b*x^n)+(a*d-b*c*(1-n))*x*hypergeom([1, 1/n],[1+1/n],-
b*x^n/a)/a^2/b/n
```

Rubi [A]
time = 0.02, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {393, 251}

$$\frac{x(ad-bc(1-n))\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a^2bn}+\frac{x(bc-ad)}{abn\,(a+bx^n)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^n)/(a + b*x^n)^2,x]
```

```
[Out] ((b*c - a*d)*x)/(a*b*n*(a + b*x^n)) + ((a*d - b*c*(1 - n))*x*Hypergeometric
2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/(a^2*b*n)
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{c+dx^n}{(a+bx^n)^2}\,dx &= \frac{(bc-ad)x}{abn\,(a+bx^n)}+\frac{(ad-bc(1-n))\int \frac{1}{a+bx^n}\,dx}{abn}\\ &= \frac{(bc-ad)x}{abn\,(a+bx^n)}+\frac{(ad-bc(1-n))x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a^2bn}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 56, normalized size = 0.78

$$\frac{x\left(\frac{d}{a+bx^n}-\frac{(ad+bc(-1+n))\,{}_2F_1\left(2,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a^2}\right)}{b-bn}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x^n)/(a + b*x^n)^2,x]

[Out] (x*(d/(a + b*x^n) - ((a*d + b*c*(-1 + n))*Hypergeometric2F1[2, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/a^2))/(b - b*n)

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{c+d\,x^n}{\left(a+b\,x^n\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c+d*x^n)/(a+b*x^n)^2,x)

[Out] int((c+d*x^n)/(a+b*x^n)^2,x)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d*x^n)/(a+b*x^n)^2,x, algorithm="maxima")

[Out] (b*c*(n - 1) + a*d)*integrate(1/(a*b^2*n*x^n + a^2*b*n), x) + (b*c - a*d)*x/(a*b^2*n*x^n + a^2*b*n)

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d*x^n)/(a+b*x^n)^2,x, algorithm="fricas")

[Out] integral((d*x^n + c)/(b^2*x^(2*n) + 2*a*b*x^n + a^2), x)

Sympy [C] Result contains complex when optimal does not.
time = 3.47, size = 592, normalized size = 8.22

$$c\left(\frac{nx\Phi\left(\frac{bx^ne^{i\pi}}{a},1,\frac{1}{n}\right)\Gamma\left(\frac{1}{n}\right)}{a\left(an^3\Gamma\left(1+\frac{1}{n}\right)+bn^3x^n\Gamma\left(1+\frac{1}{n}\right)\right)}+\frac{nx\Gamma\left(\frac{1}{n}\right)}{a\left(an^3\Gamma\left(1+\frac{1}{n}\right)+bn^3x^n\Gamma\left(1+\frac{1}{n}\right)\right)}-\frac{x\Phi\left(\frac{bx^ne^{i\pi}}{a},1,\frac{1}{n}\right)\Gamma\left(\frac{1}{n}\right)}{a\left(an^3\Gamma\left(1+\frac{1}{n}\right)+bn^3x^n\Gamma\left(1+\frac{1}{n}\right)\right)}+\frac{bnxx^n\Phi\left(\frac{bx^ne^{i\pi}}{a},1,\frac{1}{n}\right)\Gamma\left(\frac{1}{n}\right)}{a^2\left(an^3\Gamma\left(1+\frac{1}{n}\right)+bn^3x^n\Gamma\left(1+\frac{1}{n}\right)\right)}-\frac{bxx^n\Phi\left(\frac{bx^ne^{i\pi}}{a},1,\frac{1}{n}\right)\Gamma\left(\frac{1}{n}\right)}{a^2\left(an^3\Gamma\left(1+\frac{1}{n}\right)+bn^3x^n\Gamma\left(1+\frac{1}{n}\right)\right)}\right)+d\left(\frac{n^2xx^n\Gamma\left(1+\frac{1}{n}\right)}{a\left(an^3\Gamma\left(2+\frac{1}{n}\right)+bn^3x^n\Gamma\left(2+\frac{1}{n}\right)\right)}-\frac{nxx^n\Phi\left(\frac{bx^ne^{i\pi}}{a},1,1+\frac{1}{n}\right)\Gamma\left(1+\frac{1}{n}\right)}{a\left(an^3\Gamma\left(2+\frac{1}{n}\right)+bn^3x^n\Gamma\left(2+\frac{1}{n}\right)\right)}+\frac{nxx^n\Gamma\left(1+\frac{1}{n}\right)}{a\left(an^3\Gamma\left(2+\frac{1}{n}\right)+bn^3x^n\Gamma\left(2+\frac{1}{n}\right)\right)}-\frac{xx^n\Phi\left(\frac{bx^ne^{i\pi}}{a},1,1+\frac{1}{n}\right)\Gamma\left(1+\frac{1}{n}\right)}{a\left(an^3\Gamma\left(2+\frac{1}{n}\right)+bn^3x^n\Gamma\left(2+\frac{1}{n}\right)\right)}-\frac{bnxx^{2n}\Phi\left(\frac{bx^ne^{i\pi}}{a},1,1+\frac{1}{n}\right)\Gamma\left(1+\frac{1}{n}\right)}{a^2\left(an^3\Gamma\left(2+\frac{1}{n}\right)+bn^3x^n\Gamma\left(2+\frac{1}{n}\right)\right)}-\frac{bxx^{2n}\Phi\left(\frac{bx^ne^{i\pi}}{a},1,1+\frac{1}{n}\right)\Gamma\left(1+\frac{1}{n}\right)}{a^2\left(an^3\Gamma\left(2+\frac{1}{n}\right)+bn^3x^n\Gamma\left(2+\frac{1}{n}\right)\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x**n)/(a+b*x**n)**2,x)
```

```
[Out] c*(n*x*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 1/n)*gamma(1/n)/(a*(a*n**3*gam
ma(1 + 1/n) + b*n**3*x**n*gamma(1 + 1/n))) + n*x*gamma(1/n)/(a*(a*n**3*gamm
a(1 + 1/n) + b*n**3*x**n*gamma(1 + 1/n))) - x*lerchphi(b*x**n*exp_polar(I*p
i)/a, 1, 1/n)*gamma(1/n)/(a*(a*n**3*gamma(1 + 1/n) + b*n**3*x**n*gamma(1 +
1/n))) + b*n*x*x**n*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 1/n)*gamma(1/n)/(
a**2*(a*n**3*gamma(1 + 1/n) + b*n**3*x**n*gamma(1 + 1/n))) - b*x*x**n*lerch
phi(b*x**n*exp_polar(I*pi)/a, 1, 1/n)*gamma(1/n)/(a**2*(a*n**3*gamma(1 + 1/
n) + b*n**3*x**n*gamma(1 + 1/n)))) + d*(n**2*x*x**n*gamma(1 + 1/n)/(a*(a*n*
*3*gamma(2 + 1/n) + b*n**3*x**n*gamma(2 + 1/n))) - n*x*x**n*lerchphi(b*x**n
*exp_polar(I*pi)/a, 1, 1 + 1/n)*gamma(1 + 1/n)/(a*(a*n**3*gamma(2 + 1/n) +
b*n**3*x**n*gamma(2 + 1/n))) + n*x*x**n*gamma(1 + 1/n)/(a*(a*n**3*gamma(2 +
 1/n) + b*n**3*x**n*gamma(2 + 1/n))) - x*x**n*lerchphi(b*x**n*exp_polar(I*p
i)/a, 1, 1 + 1/n)*gamma(1 + 1/n)/(a*(a*n**3*gamma(2 + 1/n) + b*n**3*x**n*ga
mma(2 + 1/n))) - b*n*x*x**(2*n)*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 1 + 1
/n)*gamma(1 + 1/n)/(a**2*(a*n**3*gamma(2 + 1/n) + b*n**3*x**n*gamma(2 + 1/n
))) - b*x*x**(2*n)*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 1 + 1/n)*gamma(1 +
 1/n)/(a**2*(a*n**3*gamma(2 + 1/n) + b*n**3*x**n*gamma(2 + 1/n))))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^n)/(a+b*x^n)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x^n + c)/(b*x^n + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{c+d\,x^n}{{\left(a+b\,x^n\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^n)/(a + b*x^n)^2,x)
```

```
[Out] int((c + d*x^n)/(a + b*x^n)^2, x)
```

3.309 $\int \frac{1}{(a+bx^n)^2(c+dx^n)}\,dx$

Optimal. Leaf size=122

$$\frac{bx}{a(bc-ad)n\,(a+bx^n)}+\frac{b(ad(1-2n)-bc(1-n))x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a^2(bc-ad)^2n}+\frac{d^2x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{c(bc-ad)^2}$$

```
[Out] b*x/a/(-a*d+b*c)/n/(a+b*x^n)+b*(a*d*(1-2*n)-b*c*(1-n))*x*hypergeom([1, 1/n]
,[1+1/n],-b*x^n/a)/a^2/(-a*d+b*c)^2/n+d^2*x*hypergeom([1, 1/n],[1+1/n],-d*x
^n/c)/c/(-a*d+b*c)^2
```

Rubi [A]
time = 0.10, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {425, 536, 251}

$$\frac{bx(ad(1-2n)-bc(1-n))\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a^2n(bc-ad)^2}+\frac{d^2x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{c(bc-ad)^2}+\frac{bx}{an(bc-ad)\,(a+bx^n)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^n)^2*(c + d*x^n)),x]
```

```
[Out] (b*x)/(a*(b*c - a*d)*n*(a + b*x^n)) + (b*(a*d*(1 - 2*n) - b*c*(1 - n))*x*Hy
pergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/(a^2*(b*c - a*d)^2*n)
 + (d^2*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((d*x^n)/c)])/(c*(b*c -
 a*d)^2)
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] && !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rubi steps

$$\int \frac{1}{(a+bx^n)^2(c+dx^n)}\,dx = \frac{bx}{a(bc-ad)n(a+bx^n)} - \frac{\int \frac{adn+b(c-cn)+bd(1-n)x^n}{(a+bx^n)(c+dx^n)}\,dx}{a(bc-ad)n}$$

$$= \frac{bx}{a(bc-ad)n(a+bx^n)} + \frac{d^2\int \frac{1}{c+dx^n}\,dx}{(bc-ad)^2} + \frac{(b(ad(1-2n)-bc(1-n)))\int \frac{1}{a+bx^n}\,d}{a(bc-ad)^2n}$$

$$= \frac{bx}{a(bc-ad)n(a+bx^n)} + \frac{b(ad(1-2n)-bc(1-n))x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a^2(bc-ad)^2n} +$$

Mathematica [A]
time = 0.15, size = 108, normalized size = 0.89

$$\frac{x\left(\frac{b^2c-abd}{a^2n+abnx^n}+\frac{b(ad(1-2n)+bc(-1+n))\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a^2n}+\frac{d^2\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{c}\right)}{(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^n)^2*(c + d*x^n)),x]
```

```
[Out] (x*((b^2*c - a*b*d)/(a^2*n + a*b*n*x^n) + (b*(a*d*(1 - 2*n) + b*c*(-1 + n))
*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/(a^2*n) + (d^2*Hyp
ergeometric2F1[1, n^(-1), 1 + n^(-1), -((d*x^n)/c)])/c))/(b*c - a*d)^2
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+b\,x^n)^2(c+d\,x^n)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*x^n)^2/(c+d*x^n),x)
```

```
[Out] int(1/(a+b*x^n)^2/(c+d*x^n),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^n)^2/(c+d*x^n),x, algorithm="maxima")
```

```
[Out] d^2*integrate(1/(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2 + (b^2*c^2*d - 2*a*b*c*d
^2 + a^2*d^3)*x^n), x) - (a*b*d*(2*n - 1) - b^2*c*(n - 1))*integrate(1/(a^2
*b^2*c^2*n - 2*a^3*b*c*d*n + a^4*d^2*n + (a*b^3*c^2*n - 2*a^2*b^2*c*d*n + a
^3*b*d^2*n)*x^n), x) + b*x/(a^2*b*c*n - a^3*d*n + (a*b^2*c*n - a^2*b*d*n)*x
^n)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^n)^2/(c+d*x^n),x, algorithm="fricas")
```

```
[Out] integral(1/(b^2*d*x^(3*n) + a^2*c + (b^2*c + 2*a*b*d)*x^(2*n) + (2*a*b*c +
a^2*d)*x^n), x)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x**n)**2/(c+d*x**n),x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^n)^2/(c+d*x^n),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^n + a)^2*(d*x^n + c)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(a+b\,x^n)^2\,(c+d\,x^n)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^n)^2*(c + d*x^n)),x)
```

```
[Out] int(1/((a + b*x^n)^2*(c + d*x^n)), x)
```

3.310 $\int \frac{1}{(a+bx^n)^2(c+dx^n)^2}\,dx$

Optimal. Leaf size=193

$$\frac{d(bc+ad)x}{ac(bc-ad)^2n\,(c+dx^n)}+\frac{bx}{a(bc-ad)n\,(a+bx^n)\,(c+dx^n)}+\frac{b^2(ad(1-3n)-b(c-cn))x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a^2(bc-ad)^3n}$$

```
[Out] d*(a*d+b*c)*x/a/c/(-a*d+b*c)^2/n/(c+d*x^n)+b*x/a/(-a*d+b*c)/n/(a+b*x^n)/(c+
d*x^n)+b^2*(a*d*(1-3*n)-b*(-c*n+c))*x*hypergeom([1, 1/n],[1+1/n],-b*x^n/a)/
a^2/(-a*d+b*c)^3/n-d^2*(b*c*(1-3*n)-a*d*(1-n))*x*hypergeom([1, 1/n],[1+1/n]
,-d*x^n/c)/c^2/(-a*d+b*c)^3/n
```

Rubi [A]
time = 0.20, antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {425, 541, 536, 251}

$$\frac{b^2x(ad(1-3n)-b(c-cn))\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a^2n(bc-ad)^3}-\frac{d^2x(bc(1-3n)-ad(1-n))\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{c^2n(bc-ad)^3}+\frac{bx}{an(bc-ad)\,(a+bx^n)\,(c+dx^n)}+\frac{dx(ad+bc)}{acn(bc-ad)^2\,(c+dx^n)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^n)^2*(c + d*x^n)^2),x]
```

```
[Out] (d*(b*c + a*d)*x)/(a*c*(b*c - a*d)^2*n*(c + d*x^n)) + (b*x)/(a*(b*c - a*d)*
n*(a + b*x^n)*(c + d*x^n)) + (b^2*(a*d*(1 - 3*n) - b*(c - c*n))*x*Hypergeom
etric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/(a^2*(b*c - a*d)^3*n) - (d^2
*(b*c*(1 - 3*n) - a*d*(1 - n))*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -
((d*x^n)/c)])/(c^2*(b*c - a*d)^3*n)
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{1}{(a+bx^n)^2(c+dx^n)^2}\,dx = \frac{bx}{a(bc-ad)n(a+bx^n)(c+dx^n)} - \frac{\int \frac{adn+b(c-cn)+bd(1-2n)x^n}{(a+bx^n)(c+dx^n)^2}\,dx}{a(bc-ad)n}$$

$$= \frac{d(bc+ad)x}{ac(bc-ad)^2n(c+dx^n)} + \frac{bx}{a(bc-ad)n(a+bx^n)(c+dx^n)} - \frac{\int \frac{n(b^2c^2(1-n)+a^2 \cdots}{\cdots}}{\cdots}$$

$$= \frac{d(bc+ad)x}{ac(bc-ad)^2n(c+dx^n)} + \frac{bx}{a(bc-ad)n(a+bx^n)(c+dx^n)} + \frac{(d^2(ad(1-n) \cdots}{c \cdots}$$

$$= \frac{d(bc+ad)x}{ac(bc-ad)^2n(c+dx^n)} + \frac{bx}{a(bc-ad)n(a+bx^n)(c+dx^n)} + \frac{b^2(ad(1-3n) \cdots}{\cdots}$$

Mathematica [A]
time = 0.19, size = 147, normalized size = 0.76

$$\frac{x\left(\frac{b^2(bc-ad)}{a(a+bx^n)}+\frac{d^2(bc-ad)}{c(c+dx^n)}+\frac{b^2(ad(1-3n)+bc(-1+n))\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a^2}+\frac{d^2(-ad(-1+n)+bc(-1+3n))\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{c^2}\right)}{(bc-ad)^3n}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^n)^2*(c + d*x^n)^2),x]
```

```
[Out] (x*((b^2*(b*c - a*d))/(a*(a + b*x^n)) + (d^2*(b*c - a*d))/(c*(c + d*x^n)) +
 (b^2*(a*d*(1 - 3*n) + b*c*(-1 + n))*Hypergeometric2F1[1, n^(-1), 1 + n^(-1
), -((b*x^n)/a)])/a^2 + (d^2*(-(a*d*(-1 + n)) + b*c*(-1 + 3*n))*Hypergeomet
ric2F1[1, n^(-1), 1 + n^(-1), -((d*x^n)/c)])/c^2))/((b*c - a*d)^3*n)
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+b\,x^{n})^{2}\,(c+d\,x^{n})^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*x^n)^2/(c+d*x^n)^2,x)
```

```
[Out] int(1/(a+b*x^n)^2/(c+d*x^n)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^n)^2/(c+d*x^n)^2,x, algorithm="maxima")
```

```
[Out] (a*b^2*d*(3*n - 1) - b^3*c*(n - 1))*integrate(-1/(a^2*b^3*c^3*n - 3*a^3*b^2
*c^2*d*n + 3*a^4*b*c*d^2*n - a^5*d^3*n + (a*b^4*c^3*n - 3*a^2*b^3*c^2*d*n +
 3*a^3*b^2*c*d^2*n - a^4*b*d^3*n)*x^n), x) - (b*c*d^2*(3*n - 1) - a*d^3*(n
- 1))*integrate(-1/(b^3*c^5*n - 3*a*b^2*c^4*d*n + 3*a^2*b*c^3*d^2*n - a^3*c
^2*d^3*n + (b^3*c^4*d*n - 3*a*b^2*c^3*d^2*n + 3*a^2*b*c^2*d^3*n - a^3*c*d^4
*n)*x^n), x) + ((b^2*c*d + a*b*d^2)*x*x^n + (b^2*c^2 + a^2*d^2)*x)/(a^2*b^2
*c^4*n - 2*a^3*b*c^3*d*n + a^4*c^2*d^2*n + (a*b^3*c^3*d*n - 2*a^2*b^2*c^2*d
^2*n + a^3*b*c*d^3*n)*x^(2*n) + (a*b^3*c^4*n - a^2*b^2*c^3*d*n - a^3*b*c^2*
d^2*n + a^4*c*d^3*n)*x^n)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^n)^2/(c+d*x^n)^2,x, algorithm="fricas")
```

```
[Out] integral(1/(b^2*d^2*x^(4*n) + a^2*c^2 + 2*(b^2*c*d + a*b*d^2)*x^(3*n) + (b^
2*c^2 + 4*a*b*c*d + a^2*d^2)*x^(2*n) + 2*(a*b*c^2 + a^2*c*d)*x^n), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x**n)**2/(c+d*x**n)**2,x)
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^n)^2/(c+d*x^n)^2,x, algorithm="giac")
[Out] integrate(1/((b*x^n + a)^2*(d*x^n + c)^2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(a+b\,x^n\right)}^2\,{\left(c+d\,x^n\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^n)^2*(c + d*x^n)^2),x)
[Out] int(1/((a + b*x^n)^2*(c + d*x^n)^2), x)
```

3.311 $\int \frac{1}{(a+bx^n)^2(c+dx^n)^3}\,dx$

Optimal. Leaf size=299

$$\frac{d(2bc+ad)x}{2ac(bc-ad)^2n\,(c+dx^n)^2}+\frac{bx}{a(bc-ad)n\,(a+bx^n)\,(c+dx^n)^2}-\frac{d(abcd(1-6n)-a^2d^2(1-2n)-2b^2c^2n)\,x}{2ac^2(bc-ad)^3n^2\,(c+dx^n)}+$$

```
[Out] 1/2*d*(a*d+2*b*c)*x/a/c/(-a*d+b*c)^2/n/(c+d*x^n)^2+b*x/a/(-a*d+b*c)/n/(a+b*
x^n)/(c+d*x^n)^2-1/2*d*(a*b*c*d*(1-6*n)-a^2*d^2*(1-2*n)-2*b^2*c^2*n)*x/a/c^
2/(-a*d+b*c)^3/n^2/(c+d*x^n)+b^3*(a*d*(1-4*n)-b*c*(1-n))*x*hypergeom([1, 1/
n],[1+1/n],-b*x^n/a)/a^2/(-a*d+b*c)^4/n+1/2*d^2*(a^2*d^2*(2*n^2-3*n+1)-2*a*
b*c*d*(4*n^2-5*n+1)+b^2*c^2*(12*n^2-7*n+1))*x*hypergeom([1, 1/n],[1+1/n],-d
*x^n/c)/c^3/(-a*d+b*c)^4/n^2
```

Rubi [A]
time = 0.38, antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {425, 541, 536, 251}

$$\frac{b^3x(ad(1-4n)-bc(1-n))\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a^2n(bc-ad)^4}-\frac{dx(-a^2d^2(1-2n)+abcd(1-6n)-2b^2c^2n)}{2ac^2n^2(bc-ad)^3\,(c+dx^n)}+\frac{d^2x(a^2d^2(2n^2-3n+1)-2abcd(4n^2-5n+1)+b^2c^2(12n^2-7n+1))\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{2c^3n^2(bc-ad)^4}+\frac{bx}{an(bc-ad)\,(a+bx^n)\,(c+dx^n)^2}+\frac{dx(ad+2bc)}{2acn(bc-ad)^2\,(c+dx^n)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^n)^2*(c + d*x^n)^3),x]
```

```
[Out] (d*(2*b*c + a*d)*x)/(2*a*c*(b*c - a*d)^2*n*(c + d*x^n)^2) + (b*x)/(a*(b*c -
 a*d)*n*(a + b*x^n)*(c + d*x^n)^2) - (d*(a*b*c*d*(1 - 6*n) - a^2*d^2*(1 - 2
*n) - 2*b^2*c^2*n)*x)/(2*a*c^2*(b*c - a*d)^3*n^2*(c + d*x^n)) + (b^3*(a*d*(
1 - 4*n) - b*c*(1 - n))*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n
)/a)])/(a^2*(b*c - a*d)^4*n) + (d^2*(a^2*d^2*(1 - 3*n + 2*n^2) - 2*a*b*c*d*
(1 - 5*n + 4*n^2) + b^2*c^2*(1 - 7*n + 12*n^2))*x*Hypergeometric2F1[1, n^(-
1), 1 + n^(-1), -((d*x^n)/c)])/(2*c^3*(b*c - a*d)^4*n^2)
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
```

```
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a+bx^n)^2(c+dx^n)^3}\,dx &= \frac{bx}{a(bc-ad)n(a+bx^n)(c+dx^n)^2} - \frac{\int \frac{adn+b(c-cn)+bd(1-3n)x^n}{(a+bx^n)(c+dx^n)^3}\,dx}{a(bc-ad)n} \\
&= \frac{d(2bc+ad)x}{2ac(bc-ad)^2n(c+dx^n)^2} + \frac{bx}{a(bc-ad)n(a+bx^n)(c+dx^n)^2} - \frac{\int \frac{n(a^2d^2(1-2}{}}{} \\
&= \frac{d(2bc+ad)x}{2ac(bc-ad)^2n(c+dx^n)^2} + \frac{bx}{a(bc-ad)n(a+bx^n)(c+dx^n)^2} - \frac{d(abcd(1-}{2ac} \\
&= \frac{d(2bc+ad)x}{2ac(bc-ad)^2n(c+dx^n)^2} + \frac{bx}{a(bc-ad)n(a+bx^n)(c+dx^n)^2} - \frac{d(abcd(1-}{2ac} \\
&= \frac{d(2bc+ad)x}{2ac(bc-ad)^2n(c+dx^n)^2} + \frac{bx}{a(bc-ad)n(a+bx^n)(c+dx^n)^2} - \frac{d(abcd(1-}{2ac}
\end{aligned}$$

Mathematica [A]
time = 0.28, size = 233, normalized size = 0.78

$$\frac{x\left(\frac{2b^3(bc-ad)n}{a(a+bx^n)}+\frac{d^2(bc-ad)^2n}{c(c+dx^n)^2}+\frac{d^2(-bc+ad)(ad(-1+2n)+b(c-6cn))}{c^2(c+dx^n)}+\frac{2b^3(ad(1-4n)+bc(-1+n))n\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a^2}+\frac{d^2(a^2d^2(1-3n+2n^2)-2abcd(1-5n+4n^2)+b^2c^2(1-7n+12n^2))\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{c^3}\right)}{2(bc-ad)^4n^2}$$

Antiderivative was successfully verified.

[In] Integrate[1/((a + b*x^n)^2*(c + d*x^n)^3),x]

[Out] (x*((2*b^3*(b*c - a*d)*n)/(a*(a + b*x^n)) + (d^2*(b*c - a*d)^2*n)/(c*(c + d*x^n)^2) + (d^2*(-(b*c) + a*d)*(a*d*(-1 + 2*n) + b*(c - 6*c*n)))/(c^2*(c + d*x^n)) + (2*b^3*(a*d*(1 - 4*n) + b*c*(-1 + n))*n*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/a^2 + (d^2*(a^2*d^2*(1 - 3*n + 2*n^2) - 2*a*b*c*d*(1 - 5*n + 4*n^2) + b^2*c^2*(1 - 7*n + 12*n^2))*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((d*x^n)/c)])/c^3))/(2*(b*c - a*d)^4*n^2)

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + b\,x^{n}\right)^{2} \left(c + d\,x^{n}\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+b*x^n)^2/(c+d*x^n)^3,x)

[Out] int(1/(a+b*x^n)^2/(c+d*x^n)^3,x)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*x^n)^2/(c+d*x^n)^3,x, algorithm="maxima")

[Out] ((12*n^2 - 7*n + 1)*b^2*c^2*d^2 - 2*(4*n^2 - 5*n + 1)*a*b*c*d^3 + (2*n^2 - 3*n + 1)*a^2*d^4)*integrate(1/2/(b^4*c^7*n^2 - 4*a*b^3*c^6*d*n^2 + 6*a^2*b^2*c^5*d^2*n^2 - 4*a^3*b*c^4*d^3*n^2 + a^4*c^3*d^4*n^2 + (b^4*c^6*d*n^2 - 4*a*b^3*c^5*d^2*n^2 + 6*a^2*b^2*c^4*d^3*n^2 - 4*a^3*b*c^3*d^4*n^2 + a^4*c^2*d^5*n^2)*x^n), x) - (a*b^3*d*(4*n - 1) - b^4*c*(n - 1))*integrate(1/(a^2*b^4*c^4*n - 4*a^3*b^3*c^3*d*n + 6*a^4*b^2*c^2*d^2*n - 4*a^5*b*c*d^3*n + a^6*d^4*n + (a*b^5*c^4*n - 4*a^2*b^4*c^3*d*n + 6*a^3*b^3*c^2*d^2*n - 4*a^4*b^2*c*d^3*n + a^5*b*d^4*n)*x^n), x) + 1/2*((a*b^2*c*d^3*(6*n - 1) - a^2*b*d^4*(2*n - 1) + 2*b^3*c^2*d^2*n)*x*x^(2*n) + (a*b^2*c^2*d^2*(7*n - 1) - a^3*d^4*(2*n - 1) + 4*b^3*c^3*d*n + 3*a^2*b*c*d^3*n)*x*x^n + (a^2*b*c^2*d^2*(7*n - 1) - a^3*c*d^3*(3*n - 1) + 2*b^3*c^4*n)*x)/(a^2*b^3*c^7*n^2 - 3*a^3*b^2*c^6*d*n^2 + 3*a^4*b*c^5*d^2*n^2 - a^5*c^4*d^3*n^2 + (a*b^4*c^5*d^2*n^2 - 3*a^2*b^3*c^4*d^3*n^2 + 3*a^3*b^2*c^3*d^4*n^2 - a^4*b*c^2*d^5*n^2)*x^(3*n) + (2*a*b^4*c^6*d*n^2 - 5*a^2*b^3*c^5*d^2*n^2 + 3*a^3*b^2*c^4*d^3*n^2 + a^4*b*c^3*d^4*n^2 - a^5*c^2*d^5*n^2)*x^(2*n) + (a*b^4*c^7*n^2 - a^2*b^3*c^6*d*n^2 - 3*a^3*b^2*c^5*d^2*n^2 + 5*a^4*b*c^4*d^3*n^2 - 2*a^5*c^3*d^4*n^2)*x^n)

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^n)^2/(c+d*x^n)^3,x, algorithm="fricas")
```

```
[Out] integral(1/(b^2*d^3*x^(5*n) + a^2*c^3 + (3*b^2*c*d^2 + 2*a*b*d^3)*x^(4*n) +
 (3*b^2*c^2*d + 6*a*b*c*d^2 + a^2*d^3)*x^(3*n) + (b^2*c^3 + 6*a*b*c^2*d + 3
*a^2*c*d^2)*x^(2*n) + (2*a*b*c^3 + 3*a^2*c^2*d)*x^n), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + bx^{n}\right)^{2} \left(c + dx^{n}\right)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x**n)**2/(c+d*x**n)**3,x)
```

```
[Out] Integral(1/((a + b*x**n)**2*(c + d*x**n)**3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^n)^2/(c+d*x^n)^3,x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^n + a)^2*(d*x^n + c)^3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(a+b\,x^n\right)}^2\,{\left(c+d\,x^n\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^n)^2*(c + d*x^n)^3),x)
```

```
[Out] int(1/((a + b*x^n)^2*(c + d*x^n)^3), x)
```

3.312 $\int (a+bx^n)^p (c+dx^n)^q \, dx$

Optimal. Leaf size=81

$$x(a+bx^n)^p \left(1+\frac{bx^n}{a}\right)^{-p} (c+dx^n)^q \left(1+\frac{dx^n}{c}\right)^{-q} F_1\left(\frac{1}{n};-p,-q;1+\frac{1}{n};-\frac{bx^n}{a},-\frac{dx^n}{c}\right)$$

```
[Out] x*(a+b*x^n)^p*(c+d*x^n)^q*AppellF1(1/n,-p,-q,1+1/n,-b*x^n/a,-d*x^n/c)/((1+b
*x^n/a)^p)/((1+d*x^n/c)^q)
```

Rubi [A]
time = 0.04, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {441, 440}

$$x(a+bx^n)^p \left(\frac{bx^n}{a}+1\right)^{-p} (c+dx^n)^q \left(\frac{dx^n}{c}+1\right)^{-q} F_1\left(\frac{1}{n};-p,-q;1+\frac{1}{n};-\frac{bx^n}{a},-\frac{dx^n}{c}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n)^p*(c + d*x^n)^q,x]
```

```
[Out] (x*(a + b*x^n)^p*(c + d*x^n)^q*AppellF1[n^(-1), -p, -q, 1 + n^(-1), -((b*x^
n)/a), -((d*x^n)/c)])/((1 + (b*x^n)/a)^p*(1 + (d*x^n)/c)^q)
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int (a+bx^n)^p (c+dx^n)^q \, dx = \left((a+bx^n)^p \left(1+\frac{bx^n}{a}\right)^{-p}\right) \int \left(1+\frac{bx^n}{a}\right)^p (c+dx^n)^q \, dx$$
$$= \left((a+bx^n)^p \left(1+\frac{bx^n}{a}\right)^{-p} (c+dx^n)^q \left(1+\frac{dx^n}{c}\right)^{-q}\right) \int \left(1+\frac{bx^n}{a}\right)^p \left(1+\frac{d}{}\right.$$
$$= x(a+bx^n)^p \left(1+\frac{bx^n}{a}\right)^{-p} (c+dx^n)^q \left(1+\frac{dx^n}{c}\right)^{-q} F_1\left(\frac{1}{n}; -p, -q; 1+\frac{1}{n}; -\frac{b}{}\right.$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 190 vs. 2(81) = 162.
time = 0.30, size = 190, normalized size = 2.35

$$\frac{ac(1+n)x(a+bx^n)^p (c+dx^n)^q F_1\left(\frac{1}{n}; -p, -q; 1+\frac{1}{n}; -\frac{bx^n}{a}, -\frac{dx^n}{c}\right)}{bcnpx^n F_1\left(1+\frac{1}{n}; 1-p, -q; 2+\frac{1}{n}; -\frac{bx^n}{a}, -\frac{dx^n}{c}\right) + adnqx^n F_1\left(1+\frac{1}{n}; -p, 1-q; 2+\frac{1}{n}; -\frac{bx^n}{a}, -\frac{dx^n}{c}\right) + ac(1+n) F_1\left(\frac{1}{n}; -p, -q; 1+\frac{1}{n}; -\frac{bx^n}{a}, -\frac{dx^n}{c}\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^n)^p*(c + d*x^n)^q,x]
```

```
[Out] (a*c*(1 + n)*x*(a + b*x^n)^p*(c + d*x^n)^q*AppellF1[n^(-1), -p, -q, 1 + n^(
-1), -((b*x^n)/a), -((d*x^n)/c)])/(b*c*n*p*x^n*AppellF1[1 + n^(-1), 1 - p,
-q, 2 + n^(-1), -((b*x^n)/a), -((d*x^n)/c)] + a*d*n*q*x^n*AppellF1[1 + n^(-
1), -p, 1 - q, 2 + n^(-1), -((b*x^n)/a), -((d*x^n)/c)] + a*c*(1 + n)*Appell
F1[n^(-1), -p, -q, 1 + n^(-1), -((b*x^n)/a), -((d*x^n)/c)])
```

Maple [F]
time = 0.12, size = 0, normalized size = 0.00

$$\int (a+b\,x^n)^p (c+d\,x^n)^q \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^n)^p*(c+d*x^n)^q,x)
```

```
[Out] int((a+b*x^n)^p*(c+d*x^n)^q,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^p*(c+d*x^n)^q,x, algorithm="maxima")
```

```
[Out] integrate((b*x^n + a)^p*(d*x^n + c)^q, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^p*(c+d*x^n)^q,x, algorithm="fricas")
[Out] integral((b*x^n + a)^p*(d*x^n + c)^q, x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n)**p*(c+d*x**n)**q,x)
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^p*(c+d*x^n)^q,x, algorithm="giac")
[Out] integrate((b*x^n + a)^p*(d*x^n + c)^q, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (a+b\,x^n)^p\,(c+d\,x^n)^q\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)^p*(c + d*x^n)^q,x)
[Out] int((a + b*x^n)^p*(c + d*x^n)^q, x)
```

3.313 $\int (a+bx^n)^p (c+dx^n)^3 \, dx$

Optimal. Leaf size=402

$$\frac{d(a^2d^2(1+3n+2n^2)-abcd(2+n^2(7+p)+n(9+2p))+b^2c^2(1+2n(3+p)+n^2(11+6p+p^2)))\,x(a+b}{b^3(1+n+np)(1+n(2+p))(1+n(3+p))}$$

```
[Out] d*(a^2*d^2*(2*n^2+3*n+1)-a*b*c*d*(2+n^2*(7+p)+n*(9+2*p))+b^2*c^2*(1+2*n*(3+
p)+n^2*(p^2+6*p+11)))*x*(a+b*x^n)^(1+p)/b^3/(n*p+n+1)/(1+n*(2+p))/(1+n*(3+p
))-d*(a*d*(1+2*n)-b*c*(1+n*(5+p)))*x*(a+b*x^n)^(1+p)*(c+d*x^n)/b^2/(1+n*(2+
p))/(1+n*(3+p))+d*x*(a+b*x^n)^(1+p)*(c+d*x^n)^2/b/(n*p+3*n+1)-(a^3*d^3*(2*n
^2+3*n+1)-3*a^2*b*c*d^2*(1+n)*(1+n*(3+p))+3*a*b^2*c^2*d*(1+n*(5+2*p)+n^2*(p
^2+5*p+6))-b^3*c^3*(1+3*n*(2+p)+n^2*(3*p^2+12*p+11)+n^3*(p^3+6*p^2+11*p+6))
)*x*(a+b*x^n)^p*hypergeom([-p, 1/n],[1+1/n],-b*x^n/a)/b^3/(n*p+n+1)/(1+n*(2
+p))/(1+n*(3+p))/((1+b*x^n/a)^p)
```

Rubi [A]

time = 0.41, antiderivative size = 401, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {427, 542, 396, 252, 251}

$$\frac{dx(a+bx^n)^{p+1}\left(a^2d^2(2n^2+3n+1)-abcd(n^2(p+7)+n(2p+9)+2)+b^2c^2(n^2(p^2+6p+11)+2n(p+3)+1)\right)}{b^3(np+n+1)(n(p+2)+1)(n(p+3)+1)}-\frac{x(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}\left(a^3d^3(2n^2+3n+1)-3a^2bcd^2(n+1)(n(p+3)+1)+3ab^2c^2d(n^2(p^2+5p+6)+n(2p+5)+1)-b^3c^3(n^3(p^3+6p^2+11p+6)+n^2(3p^2+12p+11)+3n(p+2)+1)\right){}_2F_1\left(\frac{1}{n},-p;1+\frac{1}{n};-\frac{bx^n}{a}\right)}{b^3(np+n+1)(n(p+2)+1)(n(p+3)+1)}-\frac{dx(c+dx^n)(a+bx^n)^{p+1}(ad(2n+1)-b(cn(p+5)+c))}{b^2(n(p+2)+1)(n(p+3)+1)}+\frac{dx(c+dx^n)^2(a+bx^n)^{p+1}}{b(n(p+3)+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n)^p*(c + d*x^n)^3,x]
```

```
[Out] (d*(a^2*d^2*(1 + 3*n + 2*n^2) - a*b*c*d*(2 + n^2*(7 + p) + n*(9 + 2*p)) + b
^2*c^2*(1 + 2*n*(3 + p) + n^2*(11 + 6*p + p^2)))*x*(a + b*x^n)^(1 + p))/(b^
3*(1 + n + n*p)*(1 + n*(2 + p))*(1 + n*(3 + p))) - (d*(a*d*(1 + 2*n) - b*(c
 + c*n*(5 + p)))*x*(a + b*x^n)^(1 + p)*(c + d*x^n))/(b^2*(1 + n*(2 + p))*(1
 + n*(3 + p))) + (d*x*(a + b*x^n)^(1 + p)*(c + d*x^n)^2)/(b*(1 + n*(3 + p))
) - ((a^3*d^3*(1 + 3*n + 2*n^2) - 3*a^2*b*c*d^2*(1 + n)*(1 + n*(3 + p)) + 3
*a*b^2*c^2*d*(1 + n*(5 + 2*p) + n^2*(6 + 5*p + p^2)) - b^3*c^3*(1 + 3*n*(2
+ p) + n^2*(11 + 12*p + 3*p^2) + n^3*(6 + 11*p + 6*p^2 + p^3)))*x*(a + b*x^
n)^p*Hypergeometric2F1[n^(-1), -p, 1 + n^(-1), -((b*x^n)/a)])/(b^3*(1 + n +
 n*p)*(1 + n*(2 + p))*(1 + n*(3 + p))*(1 + (b*x^n)/a)^p)
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rubi steps

$$\int (a+bx^n)^p (c+dx^n)^3 \, dx = \frac{dx(a+bx^n)^{1+p}(c+dx^n)^2}{b(1+n(3+p))} + \frac{\int (a+bx^n)^p (c+dx^n)(-c(ad-b(c+cn(3+p}{b(1+n(}$$

$$= -\frac{d(ad(1+2n)-b(c+cn(5+p)))x(a+bx^n)^{1+p}(c+dx^n)}{b^2(1+n(2+p))(1+n(3+p))} + \frac{dx(a+bx^n)^{1+}}{b(1+n(3}$$

$$= \frac{d(a^2d^2(1+3n+2n^2)-abcd(2+n^2(7+p)+n(9+2p))+b^2c^2(1+2n(3+}{b^3(1+n+np)(1+n(2+p))(1+n(3+p}$$

$$= \frac{d(a^2d^2(1+3n+2n^2)-abcd(2+n^2(7+p)+n(9+2p))+b^2c^2(1+2n(3+}{b^3(1+n+np)(1+n(2+p))(1+n(3+p}$$

$$= \frac{d(a^2d^2(1+3n+2n^2)-abcd(2+n^2(7+p)+n(9+2p))+b^2c^2(1+2n(3+}{b^3(1+n+np)(1+n(2+p))(1+n(3+p}$$

Mathematica [A]
time = 5.22, size = 168, normalized size = 0.42

$$x(a+bx^n)^p \left(1+\frac{bx^n}{a}\right)^{-p} \left(\frac{3c^2dx^n\,{}_2F_1\left(1+\frac{1}{n},-p;2+\frac{1}{n};-\frac{bx^n}{a}\right)}{1+n} + \frac{3cd^2x^{2n}\,{}_2F_1\left(2+\frac{1}{n},-p;3+\frac{1}{n};-\frac{bx^n}{a}\right)}{1+2n} + \frac{d^3x^{3n}\,{}_2F_1\left(3+\frac{1}{n},-p;4+\frac{1}{n};-\frac{bx^n}{a}\right)}{1+3n} + c^3\,{}_2F_1\left(\frac{1}{n},-p;1+\frac{1}{n};-\frac{bx^n}{a}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^n)^p*(c + d*x^n)^3,x]
```

```
[Out] (x*(a + b*x^n)^p*((3*c^2*d*x^n*Hypergeometric2F1[1 + n^(-1), -p, 2 + n^(-1)
, -((b*x^n)/a)])/(1 + n) + (3*c*d^2*x^(2*n)*Hypergeometric2F1[2 + n^(-1), -
p, 3 + n^(-1), -((b*x^n)/a)])/(1 + 2*n) + (d^3*x^(3*n)*Hypergeometric2F1[3
+ n^(-1), -p, 4 + n^(-1), -((b*x^n)/a)])/(1 + 3*n) + c^3*Hypergeometric2F1[
n^(-1), -p, 1 + n^(-1), -((b*x^n)/a)]))/(1 + (b*x^n)/a)^p
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int (a+b\,x^n)^p \left(c+d\,x^n\right)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^n)^p*(c+d*x^n)^3,x)
```

```
[Out] int((a+b*x^n)^p*(c+d*x^n)^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^p*(c+d*x^n)^3,x, algorithm="maxima")
```

```
[Out] integrate((d*x^n + c)^3*(b*x^n + a)^p, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^p*(c+d*x^n)^3,x, algorithm="fricas")
```

```
[Out] integral((d^3*x^(3*n) + 3*c*d^2*x^(2*n) + 3*c^2*d*x^n + c^3)*(b*x^n + a)^p,
 x)
```

Sympy [C] Result contains complex when optimal does not.
time = 54.42, size = 199, normalized size = 0.50

$$\frac{a^p c^3 x \Gamma\left(\frac{1}{n}\right) {}_2F_1\left(\begin{matrix} \frac{1}{n}, -p \\ 1+\frac{1}{n} \end{matrix}\middle| \frac{bx^n e^{i\pi}}{a}\right)}{n\Gamma\left(1+\frac{1}{n}\right)} + \frac{3a^p c^2 d x x^n \Gamma\left(1+\frac{1}{n}\right) {}_2F_1\left(\begin{matrix} -p, 1+\frac{1}{n} \\ 2+\frac{1}{n} \end{matrix}\middle| \frac{bx^n e^{i\pi}}{a}\right)}{n\Gamma\left(2+\frac{1}{n}\right)} + \frac{3a^p c d^2 x x^{2n} \Gamma\left(2+\frac{1}{n}\right) {}_2F_1\left(\begin{matrix} -p, 2+\frac{1}{n} \\ 3+\frac{1}{n} \end{matrix}\middle| \frac{bx^n e^{i\pi}}{a}\right)}{n\Gamma\left(3+\frac{1}{n}\right)} + \frac{a^p d^3 x x^{3n} \Gamma\left(3+\frac{1}{n}\right) {}_2F_1\left(\begin{matrix} -p, 3+\frac{1}{n} \\ 4+\frac{1}{n} \end{matrix}\middle| \frac{bx^n e^{i\pi}}{a}\right)}{n\Gamma\left(4+\frac{1}{n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n)**p*(c+d*x**n)**3,x)
```

```
[Out] a**p*c**3*x*gamma(1/n)*hyper((1/n, -p), (1 + 1/n,), b*x**n*exp_polar(I*pi)/
a)/(n*gamma(1 + 1/n)) + 3*a**p*c**2*d*x*x**n*gamma(1 + 1/n)*hyper((-p, 1 +
1/n), (2 + 1/n,), b*x**n*exp_polar(I*pi)/a)/(n*gamma(2 + 1/n)) + 3*a**p*c*d
**2*x*x**(2*n)*gamma(2 + 1/n)*hyper((-p, 2 + 1/n), (3 + 1/n,), b*x**n*exp_p
olar(I*pi)/a)/(n*gamma(3 + 1/n)) + a**p*d**3*x*x**(3*n)*gamma(3 + 1/n)*hype
r((-p, 3 + 1/n), (4 + 1/n,), b*x**n*exp_polar(I*pi)/a)/(n*gamma(4 + 1/n))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^p*(c+d*x^n)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{1,[2,0,6,4,2,4,4,3,0]%%%}+%%%{4,[2,0,6,4,2,3,4,3,0]%%%}+%%%{6
,[2,0,
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (a + b\,x^n)^p\,(c + d\,x^n)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)^p*(c + d*x^n)^3,x)
[Out] int((a + b*x^n)^p*(c + d*x^n)^3, x)
```

3.314 $\int (a+bx^n)^p (c+dx^n)^2 \, dx$

Optimal. Leaf size=202

$$-\frac{d(ad(1+n)-bc(1+n(3+p)))x(a+bx^n)^{1+p}}{b^2(1+n+np)(1+n(2+p))}+\frac{dx(a+bx^n)^{1+p}(c+dx^n)}{b(1+2n+np)}-\frac{(bc(1+n+np)(ad-bc(1+n$$

```
[Out] -d*(a*d*(1+n)-b*c*(1+n*(3+p)))*x*(a+b*x^n)^(1+p)/b^2/(n*p+n+1)/(1+n*(2+p))+
d*x*(a+b*x^n)^(1+p)*(c+d*x^n)/b/(n*p+2*n+1)-(b*c*(n*p+n+1)*(a*d-b*c*(1+n*(2
+p)))-a*d*(a*d*(1+n)-b*c*(1+n*(3+p))))*x*(a+b*x^n)^p*hypergeom([-p, 1/n],[1
+1/n],-b*x^n/a)/b^2/(n*p+n+1)/(1+n*(2+p))/((1+b*x^n/a)^p)
```

Rubi [A]
time = 0.18, antiderivative size = 197, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {427, 396, 252, 251}

$$-\frac{dx(a+bx^n)^{p+1}(ad(n+1)-b(cn(p+3)+c))}{b^2(np+n+1)(n(p+2)+1)}-\frac{x(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}\left(c(ad-b(cn(p+2)+c))-\frac{ad(ad(n+1)-b(cn(p+3)+c))}{b(np+n+1)}\right){}_2F_1\left(\frac{1}{n},-p;1+\frac{1}{n};-\frac{bx^n}{a}\right)}{b(n(p+2)+1)}+\frac{dx(c+dx^n)(a+bx^n)^{p+1}}{b(n(p+2)+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n)^p*(c + d*x^n)^2,x]
```

```
[Out] -((d*(a*d*(1 + n) - b*(c + c*n*(3 + p)))*x*(a + b*x^n)^(1 + p))/(b^2*(1 + n
 + n*p)*(1 + n*(2 + p)))) + (d*x*(a + b*x^n)^(1 + p)*(c + d*x^n))/(b*(1 + n
*(2 + p))) - ((c*(a*d - b*(c + c*n*(2 + p))) - (a*d*(a*d*(1 + n) - b*(c + c
*n*(3 + p))))/(b*(1 + n + n*p)))*x*(a + b*x^n)^p*Hypergeometric2F1[n^(-1),
-p, 1 + n^(-1), -((b*x^n)/a)])/(b*(1 + n*(2 + p))*(1 + (b*x^n)/a)^p)
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
```

```
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rubi steps

$$\int (a+bx^n)^p (c+dx^n)^2 \, dx = \frac{dx(a+bx^n)^{1+p}(c+dx^n)}{b(1+n(2+p))} + \frac{\int (a+bx^n)^p (-c(ad-b(c+cn(2+p)))-d(ad(}{b(1+n(2+p))}$$

$$= -\frac{d(ad(1+n)-b(c+cn(3+p)))x(a+bx^n)^{1+p}}{b^2(1+n+np)(1+n(2+p))} + \frac{dx(a+bx^n)^{1+p}(c+dx^n)}{b(1+n(2+p))}$$

$$= -\frac{d(ad(1+n)-b(c+cn(3+p)))x(a+bx^n)^{1+p}}{b^2(1+n+np)(1+n(2+p))} + \frac{dx(a+bx^n)^{1+p}(c+dx^n)}{b(1+n(2+p))}$$

$$= -\frac{d(ad(1+n)-b(c+cn(3+p)))x(a+bx^n)^{1+p}}{b^2(1+n+np)(1+n(2+p))} + \frac{dx(a+bx^n)^{1+p}(c+dx^n)}{b(1+n(2+p))}$$

Mathematica [A]
time = 5.15, size = 140, normalized size = 0.69

$$\frac{x(a+bx^n)^p \left(1+\frac{bx^n}{a}\right)^{-p} \left(2cd(1+2n)x^n\, {}_2F_1\left(1+\frac{1}{n},-p;2+\frac{1}{n};-\frac{bx^n}{a}\right)+(1+n)\left(d^2x^{2n}\, {}_2F_1\left(2+\frac{1}{n},-p;3+\frac{1}{n};-\frac{bx^n}{a}\right)+c^2(1+2n)\, {}_2F_1\left(\frac{1}{n},-p;1+\frac{1}{n};-\frac{bx^n}{a}\right)\right)\right)}{(1+n)(1+2n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^n)^p*(c + d*x^n)^2,x]
```

```
[Out] (x*(a + b*x^n)^p*(2*c*d*(1 + 2*n)*x^n*Hypergeometric2F1[1 + n^(-1), -p, 2 +
 n^(-1), -((b*x^n)/a)] + (1 + n)*(d^2*x^(2*n)*Hypergeometric2F1[2 + n^(-1),
 -p, 3 + n^(-1), -((b*x^n)/a)] + c^2*(1 + 2*n)*Hypergeometric2F1[n^(-1), -p
, 1 + n^(-1), -((b*x^n)/a)])))/((1 + n)*(1 + 2*n)*(1 + (b*x^n)/a)^p)
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int (a+b\,x^n)^p (c+d\,x^n)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^n)^p*(c+d*x^n)^2,x)
```

```
[Out] int((a+b*x^n)^p*(c+d*x^n)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^p*(c+d*x^n)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*x^n + c)^2*(b*x^n + a)^p, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^p*(c+d*x^n)^2,x, algorithm="fricas")
```

```
[Out] integral((d^2*x^(2*n) + 2*c*d*x^n + c^2)*(b*x^n + a)^p, x)
```

Sympy [C] Result contains complex when optimal does not.
time = 25.05, size = 143, normalized size = 0.71

$$\frac{a^p c^2 x \Gamma\left(\frac{1}{n}\right) {}_2F_1\left(\begin{matrix} \frac{1}{n}, -p \\ 1+\frac{1}{n} \end{matrix}\middle| \frac{bx^n e^{i\pi}}{a}\right)}{n\Gamma\left(1+\frac{1}{n}\right)} + \frac{2a^p c d x x^n \Gamma\left(1+\frac{1}{n}\right) {}_2F_1\left(\begin{matrix} -p, 1+\frac{1}{n} \\ 2+\frac{1}{n} \end{matrix}\middle| \frac{bx^n e^{i\pi}}{a}\right)}{n\Gamma\left(2+\frac{1}{n}\right)} + \frac{a^p d^2 x x^{2n} \Gamma\left(2+\frac{1}{n}\right) {}_2F_1\left(\begin{matrix} -p, 2+\frac{1}{n} \\ 3+\frac{1}{n} \end{matrix}\middle| \frac{bx^n e^{i\pi}}{a}\right)}{n\Gamma\left(3+\frac{1}{n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n)**p*(c+d*x**n)**2,x)
```

```
[Out] a**p*c**2*x*gamma(1/n)*hyper((1/n, -p), (1 + 1/n,), b*x**n*exp_polar(I*pi)/
a)/(n*gamma(1 + 1/n)) + 2*a**p*c*d*x*x**n*gamma(1 + 1/n)*hyper((-p, 1 + 1/n
), (2 + 1/n,), b*x**n*exp_polar(I*pi)/a)/(n*gamma(2 + 1/n)) + a**p*d**2*x*x
**(2*n)*gamma(2 + 1/n)*hyper((-p, 2 + 1/n), (3 + 1/n,), b*x**n*exp_polar(I*
pi)/a)/(n*gamma(3 + 1/n))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^p*(c+d*x^n)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{-1,[1,0,4,3,1,3,3,2,0]%%%}+%%%{-3,[1,0,4,3,1,2,3,2,0]%%%}+%%%
{-3,[1
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (a+b\,x^n)^p\,(c+d\,x^n)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)^p*(c + d*x^n)^2,x)
```

```
[Out] int((a + b*x^n)^p*(c + d*x^n)^2, x)
```

3.315 $\int (a+bx^n)^p (c+dx^n)\, dx$

Optimal. Leaf size=98

$$\frac{dx(a+bx^n)^{1+p}}{b(1+n+np)}-\frac{(ad-bc(1+n+np))x(a+bx^n)^p\left(1+\frac{bx^n}{a}\right)^{-p}{}_2F_1\left(\frac{1}{n},-p;1+\frac{1}{n};-\frac{bx^n}{a}\right)}{b(1+n+np)}$$

```
[Out] d*x*(a+b*x^n)^(1+p)/b/(n*p+n+1)-(a*d-b*c*(n*p+n+1))*x*(a+b*x^n)^p*hypergeom
([-p, 1/n],[1+1/n],-b*x^n/a)/b/(n*p+n+1)/((1+b*x^n/a)^p)
```

Rubi [A]
time = 0.03, antiderivative size = 89, normalized size of antiderivative = 0.91, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {396, 252, 251}

$$x(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}\left(c-\frac{ad}{bnp+bn+b}\right){}_2F_1\left(\frac{1}{n},-p;1+\frac{1}{n};-\frac{bx^n}{a}\right)+\frac{dx(a+bx^n)^{p+1}}{b(np+n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n)^p*(c + d*x^n),x]
```

```
[Out] (d*x*(a + b*x^n)^(1 + p))/(b*(1 + n + n*p)) + ((c - (a*d)/(b + b*n + b*n*p)
)*x*(a + b*x^n)^p*Hypergeometric2F1[n^(-1), -p, 1 + n^(-1), -((b*x^n)/a)])/
(1 + (b*x^n)/a)^p
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rubi steps

$$\int (a+bx^n)^p (c+dx^n)\, dx = \frac{dx(a+bx^n)^{1+p}}{b(1+n+np)} - \left(-c+\frac{ad}{b+bn+bnp}\right)\int (a+bx^n)^p\, dx$$
$$= \frac{dx(a+bx^n)^{1+p}}{b(1+n+np)} - \left(\left(-c+\frac{ad}{b+bn+bnp}\right)(a+bx^n)^p\left(1+\frac{bx^n}{a}\right)^{-p}\right)\int \Big(1+$$
$$= \frac{dx(a+bx^n)^{1+p}}{b(1+n+np)} + \left(c-\frac{ad}{b+bn+bnp}\right)x(a+bx^n)^p\left(1+\frac{bx^n}{a}\right)^{-p} {}_2F_1\Big(\frac{1}{n},-p$$

Mathematica [A]
time = 0.05, size = 94, normalized size = 0.96

$$\frac{x(a+bx^n)^p\left(1+\frac{bx^n}{a}\right)^{-p}\left(d(a+bx^n)\left(1+\frac{bx^n}{a}\right)^p+(-ad+bc(1+n+np))\,{}_2F_1\left(\frac{1}{n},-p;1+\frac{1}{n};-\frac{bx^n}{a}\right)\right)}{b(1+n+np)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^n)^p*(c + d*x^n),x]
```

```
[Out] (x*(a + b*x^n)^p*(d*(a + b*x^n)*(1 + (b*x^n)/a)^p + (-(a*d) + b*c*(1 + n +
n*p))*Hypergeometric2F1[n^(-1), -p, 1 + n^(-1), -((b*x^n)/a)]))/(b*(1 + n +
 n*p)*(1 + (b*x^n)/a)^p)
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int (a+b\,x^n)^p (c+d\,x^n)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^n)^p*(c+d*x^n),x)
```

```
[Out] int((a+b*x^n)^p*(c+d*x^n),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^p*(c+d*x^n),x, algorithm="maxima")
```

```
[Out] integrate((d*x^n + c)*(b*x^n + a)^p, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^p*(c+d*x^n),x, algorithm="fricas")
```

```
[Out] integral((d*x^n + c)*(b*x^n + a)^p, x)
```

Sympy [C] Result contains complex when optimal does not.
time = 3.82, size = 87, normalized size = 0.89

$$\frac{a^{p}cx\Gamma\left(\frac{1}{n}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{n},-p\\1+\frac{1}{n}\end{matrix}\middle|\frac{bx^{n}e^{i\pi}}{a}\right)}{n\Gamma\left(1+\frac{1}{n}\right)}+\frac{a^{p}dxx^{n}\Gamma\left(1+\frac{1}{n}\right){}_{2}F_{1}\left(\begin{matrix}-p,1+\frac{1}{n}\\2+\frac{1}{n}\end{matrix}\middle|\frac{bx^{n}e^{i\pi}}{a}\right)}{n\Gamma\left(2+\frac{1}{n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n)**p*(c+d*x**n),x)
```

```
[Out] a**p*c*x*gamma(1/n)*hyper((1/n, -p), (1 + 1/n,), b*x**n*exp_polar(I*pi)/a)/
(n*gamma(1 + 1/n)) + a**p*d*x*x**n*gamma(1 + 1/n)*hyper((-p, 1 + 1/n), (2 +
 1/n,), b*x**n*exp_polar(I*pi)/a)/(n*gamma(2 + 1/n))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^p*(c+d*x^n),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{1,[0,0,2,2,1,2,1,0,1]%%%}+%%%{2,[0,0,2,2,1,1,1,0,1]%%%}+%%%{1
,[0,0,
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (a+b\,x^n)^p\,(c+d\,x^n)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)^p*(c + d*x^n),x)
```

```
[Out] int((a + b*x^n)^p*(c + d*x^n), x)
```

3.316 $\int (a+bx^n)^p \, dx$

Optimal. Leaf size=46

$$x(a+bx^n)^p \left(1+\frac{bx^n}{a}\right)^{-p} {}_2F_1\left(\frac{1}{n},-p;1+\frac{1}{n};-\frac{bx^n}{a}\right)$$

```
[Out] x*(a+b*x^n)^p*hypergeom([-p, 1/n],[1+1/n],-b*x^n/a)/((1+b*x^n/a)^p)
```

Rubi [A]
time = 0.01, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {252, 251}

$$x(a+bx^n)^p \left(\frac{bx^n}{a}+1\right)^{-p} {}_2F_1\left(\frac{1}{n},-p;1+\frac{1}{n};-\frac{bx^n}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n)^p,x]
```

```
[Out] (x*(a + b*x^n)^p*Hypergeometric2F1[n^(-1), -p, 1 + n^(-1), -((b*x^n)/a)])/(
1 + (b*x^n)/a)^p
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}
\int (a+bx^n)^p \, dx &= \left((a+bx^n)^p \left(1+\frac{bx^n}{a}\right)^{-p}\right) \int \left(1+\frac{bx^n}{a}\right)^p \, dx \\
&= x(a+bx^n)^p \left(1+\frac{bx^n}{a}\right)^{-p} {}_2F_1\left(\frac{1}{n},-p;1+\frac{1}{n};-\frac{bx^n}{a}\right)
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 46, normalized size = 1.00

$$x(a+bx^n)^p\left(1+\frac{bx^n}{a}\right)^{-p}{}_2F_1\left(\frac{1}{n},-p;1+\frac{1}{n};-\frac{bx^n}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^n)^p,x]
[Out] (x*(a + b*x^n)^p*Hypergeometric2F1[n^(-1), -p, 1 + n^(-1), -((b*x^n)/a)])/(
1 + (b*x^n)/a)^p
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\,x^n)^p\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^n)^p,x)
[Out] int((a+b*x^n)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^p,x, algorithm="maxima")
[Out] integrate((b*x^n + a)^p, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^p,x, algorithm="fricas")
[Out] integral((b*x^n + a)^p, x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.80, size = 37, normalized size = 0.80

$$\frac{a^px\Gamma\left(\frac{1}{n}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{n},-p\\1+\frac{1}{n}\end{matrix}\right|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(1+\frac{1}{n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n)**p,x)
```

```
[Out] a**p*x*gamma(1/n)*hyper((1/n, -p), (1 + 1/n,), b*x**n*exp_polar(I*pi)/a)/(n
*gamma(1 + 1/n))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^p,x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a)^p, x)
```

Mupad [B]
time = 2.30, size = 47, normalized size = 1.02

$$\frac{x\,{\left(a+b\,x^n\right)}^p\,{}_2\mathrm{F}_1\left(\frac{1}{n},-p;\ \frac{1}{n}+1;\ -\frac{b\,x^n}{a}\right)}{{\left(\frac{b\,x^n}{a}+1\right)}^p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)^p,x)
```

```
[Out] (x*(a + b*x^n)^p*hypergeom([1/n, -p], 1/n + 1, -(b*x^n)/a))/((b*x^n)/a + 1)
^p
```

3.317 $\int \frac{(a+bx^n)^p}{c+dx^n} \, dx$

Optimal. Leaf size=59

$$\frac{x(a+bx^n)^p \left(1+\frac{bx^n}{a}\right)^{-p} F_1\left(\frac{1}{n};-p,1;1+\frac{1}{n};-\frac{bx^n}{a},-\frac{dx^n}{c}\right)}{c}$$

```
[Out] x*(a+b*x^n)^p*AppellF1(1/n,-p,1,1+1/n,-b*x^n/a,-d*x^n/c)/c/((1+b*x^n/a)^p)
```

Rubi [A]

time = 0.02, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {441, 440}

$$\frac{x(a+bx^n)^p \left(\frac{bx^n}{a}+1\right)^{-p} F_1\left(\frac{1}{n};-p,1;1+\frac{1}{n};-\frac{bx^n}{a},-\frac{dx^n}{c}\right)}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n)^p/(c + d*x^n),x]
```

```
[Out] (x*(a + b*x^n)^p*AppellF1[n^(-1), -p, 1, 1 + n^(-1), -((b*x^n)/a), -((d*x^n
)/c)])/(c*(1 + (b*x^n)/a)^p)
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{(a+bx^n)^p}{c+dx^n} \, dx &= \left((a+bx^n)^p \left(1+\frac{bx^n}{a}\right)^{-p}\right) \int \frac{\left(1+\frac{bx^n}{a}\right)^p}{c+dx^n} \, dx \\ &= \frac{x(a+bx^n)^p \left(1+\frac{bx^n}{a}\right)^{-p} F_1\left(\frac{1}{n};-p,1;1+\frac{1}{n};-\frac{bx^n}{a},-\frac{dx^n}{c}\right)}{c} \end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 180 vs. 2(59) = 118.
time = 0.21, size = 180, normalized size = 3.05

$$\frac{ac(1+n)x(a+bx^n)^p F_1\left(\frac{1}{n};-p,1;1+\frac{1}{n};-\frac{bx^n}{a},-\frac{dx^n}{c}\right)}{(c+dx^n)\left(bcnpx^nF_1\left(1+\frac{1}{n};1-p,1;2+\frac{1}{n};-\frac{bx^n}{a},-\frac{dx^n}{c}\right)-adnx^nF_1\left(1+\frac{1}{n};-p,2;2+\frac{1}{n};-\frac{bx^n}{a},-\frac{dx^n}{c}\right)+ac(1+n)F_1\left(\frac{1}{n};-p,1;1+\frac{1}{n};-\frac{bx^n}{a},-\frac{dx^n}{c}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^n)^p/(c + d*x^n),x]
```

```
[Out] (a*c*(1 + n)*x*(a + b*x^n)^p*AppellF1[n^(-1), -p, 1, 1 + n^(-1), -((b*x^n)/
a), -((d*x^n)/c)])/((c + d*x^n)*(b*c*n*p*x^n*AppellF1[1 + n^(-1), 1 - p, 1,
 2 + n^(-1), -((b*x^n)/a), -((d*x^n)/c)] - a*d*n*x^n*AppellF1[1 + n^(-1), -
p, 2, 2 + n^(-1), -((b*x^n)/a), -((d*x^n)/c)] + a*c*(1 + n)*AppellF1[n^(-1)
, -p, 1, 1 + n^(-1), -((b*x^n)/a), -((d*x^n)/c)]))
```

Maple [F]
time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,x^n)^p}{c+d\,x^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^n)^p/(c+d*x^n),x)
```

```
[Out] int((a+b*x^n)^p/(c+d*x^n),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^p/(c+d*x^n),x, algorithm="maxima")
```

```
[Out] integrate((b*x^n + a)^p/(d*x^n + c), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^p/(c+d*x^n),x, algorithm="fricas")
```

```
[Out] integral((b*x^n + a)^p/(d*x^n + c), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n)**p/(c+d*x**n),x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^p/(c+d*x^n),x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a)^p/(d*x^n + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(a+b\,x^n)^p}{c+d\,x^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)^p/(c + d*x^n),x)
```

```
[Out] int((a + b*x^n)^p/(c + d*x^n), x)
```

3.318 $\int \frac{(a+bx^n)^p}{(c+dx^n)^2}\,dx$

Optimal. Leaf size=59

$$\frac{x(a+bx^n)^p\left(1+\frac{bx^n}{a}\right)^{-p}F_1\left(\frac{1}{n};-p,2;1+\frac{1}{n};-\frac{bx^n}{a},-\frac{dx^n}{c}\right)}{c^2}$$

```
[Out] x*(a+b*x^n)^p*AppellF1(1/n,-p,2,1+1/n,-b*x^n/a,-d*x^n/c)/c^2/((1+b*x^n/a)^p
)
```

Rubi [A]
time = 0.02, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {441, 440}

$$\frac{x(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}F_1\left(\frac{1}{n};-p,2;1+\frac{1}{n};-\frac{bx^n}{a},-\frac{dx^n}{c}\right)}{c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n)^p/(c + d*x^n)^2,x]
```

```
[Out] (x*(a + b*x^n)^p*AppellF1[n^(-1), -p, 2, 1 + n^(-1), -((b*x^n)/a), -((d*x^n
)/c)])/(c^2*(1 + (b*x^n)/a)^p)
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^n)^p}{(c+dx^n)^2}\,dx &= \left((a+bx^n)^p\left(1+\frac{bx^n}{a}\right)^{-p}\right)\int \frac{\left(1+\frac{bx^n}{a}\right)^p}{(c+dx^n)^2}\,dx\\ &= \frac{x(a+bx^n)^p\left(1+\frac{bx^n}{a}\right)^{-p}F_1\left(\frac{1}{n};-p,2;1+\frac{1}{n};-\frac{bx^n}{a},-\frac{dx^n}{c}\right)}{c^2}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 180 vs. $2(59) = 118$.
time = 0.23, size = 180, normalized size = 3.05

$$\frac{ac(1+n)x(a+bx^n)^p F_1\left(\frac{1}{n};-p,2;1+\frac{1}{n};-\frac{bx^n}{a},-\frac{dx^n}{c}\right)}{(c+dx^n)^2\left(bcnpx^nF_1\left(1+\frac{1}{n};1-p,2;2+\frac{1}{n};-\frac{bx^n}{a},-\frac{dx^n}{c}\right)-2adnx^nF_1\left(1+\frac{1}{n};-p,3;2+\frac{1}{n};-\frac{bx^n}{a},-\frac{dx^n}{c}\right)+ac(1+n)F_1\left(\frac{1}{n};-p,2;1+\frac{1}{n};-\frac{bx^n}{a},-\frac{dx^n}{c}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^n)^p/(c + d*x^n)^2,x]
```

```
[Out] (a*c*(1 + n)*x*(a + b*x^n)^p*AppellF1[n^(-1), -p, 2, 1 + n^(-1), -((b*x^n)/
a), -((d*x^n)/c)])/((c + d*x^n)^2*(b*c*n*p*x^n*AppellF1[1 + n^(-1), 1 - p,
2, 2 + n^(-1), -((b*x^n)/a), -((d*x^n)/c)] - 2*a*d*n*x^n*AppellF1[1 + n^(-1
), -p, 3, 2 + n^(-1), -((b*x^n)/a), -((d*x^n)/c)] + a*c*(1 + n)*AppellF1[n^
(-1), -p, 2, 1 + n^(-1), -((b*x^n)/a), -((d*x^n)/c)]))
```

Maple [F]
time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,x^n)^p}{(c+d\,x^n)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^n)^p/(c+d*x^n)^2,x)
```

```
[Out] int((a+b*x^n)^p/(c+d*x^n)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^p/(c+d*x^n)^2,x, algorithm="maxima")
```

```
[Out] integrate((b*x^n + a)^p/(d*x^n + c)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^p/(c+d*x^n)^2,x, algorithm="fricas")
```

```
[Out] integral((b*x^n + a)^p/(d^2*x^(2*n) + 2*c*d*x^n + c^2), x)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n)**p/(c+d*x**n)**2,x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^p/(c+d*x^n)^2,x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a)^p/(d*x^n + c)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(a+b\,x^n)^p}{(c+d\,x^n)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)^p/(c + d*x^n)^2,x)
```

```
[Out] int((a + b*x^n)^p/(c + d*x^n)^2, x)
```

3.319 $\int \frac{(a+bx^n)^p}{(c+dx^n)^3}\,dx$

Optimal. Leaf size=59

$$\frac{x(a+bx^n)^p\left(1+\frac{bx^n}{a}\right)^{-p}F_1\left(\frac{1}{n};-p,3;1+\frac{1}{n};-\frac{bx^n}{a},-\frac{dx^n}{c}\right)}{c^3}$$

```
[Out] x*(a+b*x^n)^p*AppellF1(1/n,-p,3,1+1/n,-b*x^n/a,-d*x^n/c)/c^3/((1+b*x^n/a)^p
)
```

Rubi [A]

time = 0.02, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {441, 440}

$$\frac{x(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}F_1\left(\frac{1}{n};-p,3;1+\frac{1}{n};-\frac{bx^n}{a},-\frac{dx^n}{c}\right)}{c^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n)^p/(c + d*x^n)^3,x]
```

```
[Out] (x*(a + b*x^n)^p*AppellF1[n^(-1), -p, 3, 1 + n^(-1), -((b*x^n)/a), -((d*x^n
)/c)])/(c^3*(1 + (b*x^n)/a)^p)
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^n)^p}{(c+dx^n)^3}\,dx &= \left((a+bx^n)^p\left(1+\frac{bx^n}{a}\right)^{-p}\right)\int \frac{\left(1+\frac{bx^n}{a}\right)^p}{(c+dx^n)^3}\,dx\\ &= \frac{x(a+bx^n)^p\left(1+\frac{bx^n}{a}\right)^{-p}F_1\left(\frac{1}{n};-p,3;1+\frac{1}{n};-\frac{bx^n}{a},-\frac{dx^n}{c}\right)}{c^3}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 180 vs. 2(59) = 118.
time = 0.32, size = 180, normalized size = 3.05

$$\frac{ac(1+n)x(a+bx^n)^p F_1\left(\frac{1}{n};-p,3;1+\frac{1}{n};-\frac{bx^n}{a},-\frac{dx^n}{c}\right)}{(c+dx^n)^3\left(bcnpx^nF_1\left(1+\frac{1}{n};1-p,3;2+\frac{1}{n};-\frac{bx^n}{a},-\frac{dx^n}{c}\right)-3adnx^nF_1\left(1+\frac{1}{n};-p,4;2+\frac{1}{n};-\frac{bx^n}{a},-\frac{dx^n}{c}\right)+ac(1+n)F_1\left(\frac{1}{n};-p,3;1+\frac{1}{n};-\frac{bx^n}{a},-\frac{dx^n}{c}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^n)^p/(c + d*x^n)^3,x]
```

```
[Out] (a*c*(1 + n)*x*(a + b*x^n)^p*AppellF1[n^(-1), -p, 3, 1 + n^(-1), -((b*x^n)/
a), -((d*x^n)/c)])/((c + d*x^n)^3*(b*c*n*p*x^n*AppellF1[1 + n^(-1), 1 - p,
3, 2 + n^(-1), -((b*x^n)/a), -((d*x^n)/c)] - 3*a*d*n*x^n*AppellF1[1 + n^(-1
), -p, 4, 2 + n^(-1), -((b*x^n)/a), -((d*x^n)/c)] + a*c*(1 + n)*AppellF1[n^
(-1), -p, 3, 1 + n^(-1), -((b*x^n)/a), -((d*x^n)/c)]))
```

Maple [F]
time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,x^n)^p}{(c+d\,x^n)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^n)^p/(c+d*x^n)^3,x)
```

```
[Out] int((a+b*x^n)^p/(c+d*x^n)^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^p/(c+d*x^n)^3,x, algorithm="maxima")
```

```
[Out] integrate((b*x^n + a)^p/(d*x^n + c)^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^p/(c+d*x^n)^3,x, algorithm="fricas")
```

```
[Out] integral((b*x^n + a)^p/(d^3*x^(3*n) + 3*c*d^2*x^(2*n) + 3*c^2*d*x^n + c^3),
 x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx^n)^p}{(c+dx^n)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n)**p/(c+d*x**n)**3,x)
```

```
[Out] Integral((a + b*x**n)**p/(c + d*x**n)**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^p/(c+d*x^n)^3,x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a)^p/(d*x^n + c)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(a+b\,x^n)^p}{(c+d\,x^n)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)^p/(c + d*x^n)^3,x)
```

```
[Out] int((a + b*x^n)^p/(c + d*x^n)^3, x)
```

3.320 $\int (a+bx^n)^p (c+dx^n)^{-1-\frac{1}{n}-p} \, dx$

Optimal. Leaf size=93

$$\frac{x(a+bx^n)^p \left(\frac{c(a+bx^n)}{a(c+dx^n)}\right)^{-p} (c+dx^n)^{-\frac{1}{n}-p} \, {}_2F_1\left(\frac{1}{n},-p;1+\frac{1}{n};-\frac{(bc-ad)x^n}{a(c+dx^n)}\right)}{c}$$

```
[Out] x*(a+b*x^n)^p*(c+d*x^n)^(-1/n-p)*hypergeom([-p, 1/n],[1+1/n],-(-a*d+b*c)*x^
n/a/(c+d*x^n))/c/((c*(a+b*x^n)/a/(c+d*x^n))^p)
```

Rubi [A]
time = 0.02, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.036$, Rules used = {388}

$$\frac{x(a+bx^n)^p (c+dx^n)^{-\frac{1}{n}-p} \left(\frac{c(a+bx^n)}{a(c+dx^n)}\right)^{-p} \, {}_2F_1\left(\frac{1}{n},-p;1+\frac{1}{n};-\frac{(bc-ad)x^n}{a(dx^n+c)}\right)}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n)^p*(c + d*x^n)^(-1 - n^(-1) - p),x]
```

```
[Out] (x*(a + b*x^n)^p*(c + d*x^n)^(-n^(-1) - p)*Hypergeometric2F1[n^(-1), -p, 1
+ n^(-1), -(((b*c - a*d)*x^n)/(a*(c + d*x^n)))])/(c*((c*(a + b*x^n))/(a*(c
+ d*x^n)))^p)
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[x*((a + b*x^n)^p/(c*(c*((a + b*x^n)/(a*(c + d*x^n))))^p*(c + d*x^n)
^(1/n + p)))*Hypergeometric2F1[1/n, -p, 1 + 1/n, (-(b*c - a*d))*(x^n/(a*(c
+ d*x^n)))], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] &&
EqQ[n*(p + q + 1) + 1, 0]
```

Rubi steps

$$\int (a+bx^n)^p (c+dx^n)^{-1-\frac{1}{n}-p} \, dx = \frac{x(a+bx^n)^p \left(\frac{c(a+bx^n)}{a(c+dx^n)}\right)^{-p} (c+dx^n)^{-\frac{1}{n}-p} \, {}_2F_1\left(\frac{1}{n},-p;1+\frac{1}{n};-\frac{(bc-ad)x^n}{a(c+dx^n)}\right)}{c}$$

Mathematica [A]
time = 0.11, size = 94, normalized size = 1.01

$$\frac{x(a+bx^n)^p \left(1+\frac{bx^n}{a}\right)^{-p} (c+dx^n)^{-\frac{1+np}{n}} \left(1+\frac{dx^n}{c}\right)^p \, {}_2F_1\left(\frac{1}{n},-p;1+\frac{1}{n};\frac{(-bc+ad)x^n}{a(c+dx^n)}\right)}{c}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^n)^p*(c + d*x^n)^(-1 - n^(-1) - p),x]
```

```
[Out] (x*(a + b*x^n)^p*(1 + (d*x^n)/c)^p*Hypergeometric2F1[n^(-1), -p, 1 + n^(-1)
, ((-(b*c) + a*d)*x^n)/(a*(c + d*x^n))])/(c*(1 + (b*x^n)/a)^p*(c + d*x^n)^(
(1 + n*p)/n))
```

Maple [F]
time = 0.13, size = 0, normalized size = 0.00

$$\int (a + b\,x^{n})^{p} \,(c + d\,x^{n})^{-1-\frac{1}{n}-p}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^n)^p*(c+d*x^n)^(-1-1/n-p),x)
```

```
[Out] int((a+b*x^n)^p*(c+d*x^n)^(-1-1/n-p),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^p*(c+d*x^n)^(-1-1/n-p),x, algorithm="maxima")
```

```
[Out] integrate((b*x^n + a)^p*(d*x^n + c)^(-p - 1/n - 1), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^p*(c+d*x^n)^(-1-1/n-p),x, algorithm="fricas")
```

```
[Out] integral((b*x^n + a)^p/(d*x^n + c)^((n*p + n + 1)/n), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n)**p*(c+d*x**n)**(-1-1/n-p),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 5010 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^p*(c+d*x^n)^(-1-1/n-p),x, algorithm="giac")
[Out] integrate((b*x^n + a)^p*(d*x^n + c)^(-p - 1/n - 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,x^n)^p}{(c+d\,x^n)^{p+\frac{1}{n}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)^p/(c + d*x^n)^(p + 1/n + 1),x)
[Out] int((a + b*x^n)^p/(c + d*x^n)^(p + 1/n + 1), x)
```

3.321 $\int (a+bx^n)^3 (c+dx^n)^{-4-\frac{1}{n}}\, dx$

Optimal. Leaf size=178

$$\frac{x(a+bx^n)^3 (c+dx^n)^{-3-\frac{1}{n}}}{c(1+3n)}+\frac{3anx(a+bx^n)^2 (c+dx^n)^{-2-\frac{1}{n}}}{c^2 (1+5n+6n^2)}+\frac{6a^2n^2x(a+bx^n) (c+dx^n)^{-1-\frac{1}{n}}}{c^3(1+n)(1+2n)(1+3n)}+\frac{6a^3n^3x(c-}{c^4(1+n)(1+}$$

```
[Out] x*(a+b*x^n)^3*(c+d*x^n)^(-3-1/n)/c/(1+3*n)+3*a*n*x*(a+b*x^n)^2*(c+d*x^n)^(-
2-1/n)/c^2/(6*n^2+5*n+1)+6*a^2*n^2*x*(a+b*x^n)*(c+d*x^n)^(-1-1/n)/c^3/(6*n^
3+11*n^2+6*n+1)+6*a^3*n^3*x/c^4/(6*n^3+11*n^2+6*n+1)/((c+d*x^n)^(1/n))
```

Rubi [A]
time = 0.06, antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {386, 197}

$$\frac{6a^3n^3x(c+dx^n)^{-1/n}}{c^4(n+1)(2n+1)(3n+1)}+\frac{6a^2n^2x(a+bx^n)(c+dx^n)^{-\frac{1}{n}-1}}{c^3(n+1)(2n+1)(3n+1)}+\frac{3anx(a+bx^n)^2(c+dx^n)^{-\frac{1}{n}-2}}{c^2(6n^2+5n+1)}+\frac{x(a+bx^n)^3(c+dx^n)^{-\frac{1}{n}-3}}{c(3n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n)^3*(c + d*x^n)^(-4 - n^(-1)),x]
```

```
[Out] (x*(a + b*x^n)^3*(c + d*x^n)^(-3 - n^(-1)))/(c*(1 + 3*n)) + (3*a*n*x*(a + b
*x^n)^2*(c + d*x^n)^(-2 - n^(-1)))/(c^2*(1 + 5*n + 6*n^2)) + (6*a^2*n^2*x*(
a + b*x^n)*(c + d*x^n)^(-1 - n^(-1)))/(c^3*(1 + n)*(1 + 2*n)*(1 + 3*n)) + (
6*a^3*n^3*x)/(c^4*(1 + n)*(1 + 2*n)*(1 + 3*n)*(c + d*x^n)^n^(-1))
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 386

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] - Dist[
c*(q/(a*(p + 1))), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1,
0] && GtQ[q, 0] && NeQ[p, -1]
```

Rubi steps

$$\int (a+bx^n)^3 (c+dx^n)^{-4-\frac{1}{n}}\,dx = \frac{x(a+bx^n)^3 (c+dx^n)^{-3-\frac{1}{n}}}{c(1+3n)} + \frac{(3an)\int (a+bx^n)^2 (c+dx^n)^{-3-\frac{1}{n}}\,dx}{c(1+3n)}$$

$$= \frac{x(a+bx^n)^3 (c+dx^n)^{-3-\frac{1}{n}}}{c(1+3n)} + \frac{3anx(a+bx^n)^2 (c+dx^n)^{-2-\frac{1}{n}}}{c^2(1+5n+6n^2)} + \frac{(6a^2n^2)}{}$$

$$= \frac{x(a+bx^n)^3 (c+dx^n)^{-3-\frac{1}{n}}}{c(1+3n)} + \frac{3anx(a+bx^n)^2 (c+dx^n)^{-2-\frac{1}{n}}}{c^2(1+5n+6n^2)} + \frac{6a^2n^2x(c}{c^3(1-}$$

$$= \frac{x(a+bx^n)^3 (c+dx^n)^{-3-\frac{1}{n}}}{c(1+3n)} + \frac{3anx(a+bx^n)^2 (c+dx^n)^{-2-\frac{1}{n}}}{c^2(1+5n+6n^2)} + \frac{6a^2n^2x(c}{c^3(1-}$$

Mathematica [A]
time = 0.14, size = 218, normalized size = 1.22

$$\frac{x(c+dx^n)^{-3-\frac{1}{n}}\left(b^3c^3(1+3n+2n^2)x^{3n}+3ab^2c^2(1+n)x^{2n}(c+3cn+dnx^n)+3a^2bcx^n(c^2(1+5n+6n^2)+2cdn(1+3n)x^n+2d^2n^2x^{2n})+a^3(c^3(1+6n+11n^2+6n^3)+3c^2dn(1+5n+6n^2)x^n+6cd^2n^2(1+3n)x^{2n}+6d^3n^3x^{3n})\right)}{c^4(1+n)(1+2n)(1+3n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^n)^3*(c + d*x^n)^(-4 - n^(-1)),x]
```

```
[Out] (x*(c + d*x^n)^(-3 - n^(-1))*(b^3*c^3*(1 + 3*n + 2*n^2)*x^(3*n) + 3*a*b^2*c
^2*(1 + n)*x^(2*n)*(c + 3*c*n + d*n*x^n) + 3*a^2*b*c*x^n*(c^2*(1 + 5*n + 6*
n^2) + 2*c*d*n*(1 + 3*n)*x^n + 2*d^2*n^2*x^(2*n)) + a^3*(c^3*(1 + 6*n + 11*
n^2 + 6*n^3) + 3*c^2*d*n*(1 + 5*n + 6*n^2)*x^n + 6*c*d^2*n^2*(1 + 3*n)*x^(2
*n) + 6*d^3*n^3*x^(3*n))))/(c^4*(1 + n)*(1 + 2*n)*(1 + 3*n))
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int (a+b\,x^n)^3 (c+d\,x^n)^{-4-\frac{1}{n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^n)^3*(c+d*x^n)^(-4-1/n),x)
```

```
[Out] int((a+b*x^n)^3*(c+d*x^n)^(-4-1/n),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^3*(c+d*x^n)^(-4-1/n),x, algorithm="maxima")
```

```
[Out] integrate((b*x^n + a)^3*(d*x^n + c)^(-1/n - 4), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 478 vs. $2(174) = 348$.
time = 3.41, size = 478, normalized size = 2.69

$$\frac{(6\,a^3d^4n^3+b^3c^3d+(2\,b^3c^3d+3\,ab^2c^2d^2+6\,a^2bcd^3)n^2+3\,(b^3c^3d+ab^2c^2d^2)n)xx^{4\,n}+(24\,a^3cd^3n^3+b^3c^4+3\,ab^2c^3d+2\,(b^3c^4+6\,ab^2c^3d+12\,a^2bc^2d^2+3\,a^3cd^3)n^2+3\,(b^3c^4+5\,ab^2c^3d+2\,a^2bc^2d^2)n)xx^{3\,n}+3\,(12\,a^3c^2d^2n^3+ab^2c^4+a^2bc^3d+(3\,ab^2c^4+12\,a^2bc^3d+7\,a^3c^2d^2)n^2+(4\,ab^2c^4+7\,a^2bc^3d+a^3c^2d^2)n)xx^{2\,n}+(24\,a^3c^3dn^3+3\,a^2bc^4+a^3c^3d+2\,(9\,a^2bc^4+13\,a^3c^3d)n^2+3\,(5\,a^2bc^4+3\,a^3c^3d)n)xx^n+(6\,a^3c^4n^3+11\,a^3c^4n^2+6\,a^3c^4n+a^3c^4)x}{(6\,c^4n^3+11\,c^4n^2+6\,c^4n+c^4)(dx^n+c)^{\frac{4\,n+1}{n}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^3*(c+d*x^n)^(-4-1/n),x, algorithm="fricas")
```

```
[Out] ((6*a^3*d^4*n^3 + b^3*c^3*d + (2*b^3*c^3*d + 3*a*b^2*c^2*d^2 + 6*a^2*b*c*d^
3)*n^2 + 3*(b^3*c^3*d + a*b^2*c^2*d^2)*n)*x*x^(4*n) + (24*a^3*c*d^3*n^3 + b
^3*c^4 + 3*a*b^2*c^3*d + 2*(b^3*c^4 + 6*a*b^2*c^3*d + 12*a^2*b*c^2*d^2 + 3*
a^3*c*d^3)*n^2 + 3*(b^3*c^4 + 5*a*b^2*c^3*d + 2*a^2*b*c^2*d^2)*n)*x*x^(3*n)
 + 3*(12*a^3*c^2*d^2*n^3 + a*b^2*c^4 + a^2*b*c^3*d + (3*a*b^2*c^4 + 12*a^2*
b*c^3*d + 7*a^3*c^2*d^2)*n^2 + (4*a*b^2*c^4 + 7*a^2*b*c^3*d + a^3*c^2*d^2)*
n)*x*x^(2*n) + (24*a^3*c^3*d*n^3 + 3*a^2*b*c^4 + a^3*c^3*d + 2*(9*a^2*b*c^4
 + 13*a^3*c^3*d)*n^2 + 3*(5*a^2*b*c^4 + 3*a^3*c^3*d)*n)*x*x^n + (6*a^3*c^4*
n^3 + 11*a^3*c^4*n^2 + 6*a^3*c^4*n + a^3*c^4)*x)/((6*c^4*n^3 + 11*c^4*n^2 +
 6*c^4*n + c^4)*(d*x^n + c)^((4*n + 1)/n))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n)**3*(c+d*x**n)**(-4-1/n),x)
```

```
[Out] Timed out
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^3*(c+d*x^n)^(-4-1/n),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{81,[2,0,6,4,2,4,3,0]%%%}+%%%{108,[2,0,6,3,2,4,3,0]%%%}+%%%{54
,[2,0,
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,x^n)^3}{(c+d\,x^n)^{\frac{1}{n}+4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)^3/(c + d*x^n)^(1/n + 4),x)
```

```
[Out] int((a + b*x^n)^3/(c + d*x^n)^(1/n + 4), x)
```

3.322 $\int (a+bx^n)^2 (c+dx^n)^{-3-\frac{1}{n}} \, dx$

Optimal. Leaf size=116

$$\frac{x(a+bx^n)^2 (c+dx^n)^{-2-\frac{1}{n}}}{c(1+2n)}+\frac{2anx(a+bx^n) (c+dx^n)^{-1-\frac{1}{n}}}{c^2(1+n)(1+2n)}+\frac{2a^2n^2x(c+dx^n)^{-1/n}}{c^3(1+n)(1+2n)}$$

```
[Out] x*(a+b*x^n)^2*(c+d*x^n)^(-2-1/n)/c/(1+2*n)+2*a*n*x*(a+b*x^n)*(c+d*x^n)^(-1-
1/n)/c^2/(2*n^2+3*n+1)+2*a^2*n^2*x/c^3/(2*n^2+3*n+1)/((c+d*x^n)^(1/n))
```

Rubi [A]
time = 0.02, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {386, 197}

$$\frac{2a^2n^2x(c+dx^n)^{-1/n}}{c^3(n+1)(2n+1)}+\frac{2anx(a+bx^n) (c+dx^n)^{-\frac{1}{n}-1}}{c^2(n+1)(2n+1)}+\frac{x(a+bx^n)^2 (c+dx^n)^{-\frac{1}{n}-2}}{c(2n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n)^2*(c + d*x^n)^(-3 - n^(-1)),x]
```

```
[Out] (x*(a + b*x^n)^2*(c + d*x^n)^(-2 - n^(-1)))/(c*(1 + 2*n)) + (2*a*n*x*(a + b
*x^n)*(c + d*x^n)^(-1 - n^(-1)))/(c^2*(1 + n)*(1 + 2*n)) + (2*a^2*n^2*x)/(c
^3*(1 + n)*(1 + 2*n)*(c + d*x^n)^n^(-1))
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 386

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] - Dist[
c*(q/(a*(p + 1))), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1,
0] && GtQ[q, 0] && NeQ[p, -1]
```

Rubi steps

$$\int (a+bx^n)^2 (c+dx^n)^{-3-\frac{1}{n}} dx = \frac{x(a+bx^n)^2 (c+dx^n)^{-2-\frac{1}{n}}}{c(1+2n)} + \frac{(2an)\int (a+bx^n)(c+dx^n)^{-2-\frac{1}{n}} dx}{c(1+2n)}$$

$$= \frac{x(a+bx^n)^2 (c+dx^n)^{-2-\frac{1}{n}}}{c(1+2n)} + \frac{2anx(a+bx^n)(c+dx^n)^{-1-\frac{1}{n}}}{c^2(1+n)(1+2n)} + \frac{(2a^2n^2)\int}{c^2(1}$$

$$= \frac{x(a+bx^n)^2 (c+dx^n)^{-2-\frac{1}{n}}}{c(1+2n)} + \frac{2anx(a+bx^n)(c+dx^n)^{-1-\frac{1}{n}}}{c^2(1+n)(1+2n)} + \frac{2a^2n^2x(c}{c^3(1+n}$$

Mathematica [A]
time = 0.09, size = 113, normalized size = 0.97

$$\frac{x(c+dx^n)^{-2-\frac{1}{n}} (b^2c^2(1+n)x^{2n} + 2abcx^n(c+2cn+dnx^n) + a^2(c^2(1+3n+2n^2) + 2cdn(1+2n)x^n + 2d^2n^2x^{2n}))}{c^3(1+n)(1+2n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^n)^2*(c + d*x^n)^(-3 - n^(-1)),x]
```

```
[Out] (x*(c + d*x^n)^(-2 - n^(-1))*(b^2*c^2*(1 + n)*x^(2*n) + 2*a*b*c*x^n*(c + 2*
c*n + d*n*x^n) + a^2*(c^2*(1 + 3*n + 2*n^2) + 2*c*d*n*(1 + 2*n)*x^n + 2*d^2
*n^2*x^(2*n))))/(c^3*(1 + n)*(1 + 2*n))
```

Maple [F]
time = 0.09, size = 0, normalized size = 0.00

$$\int (a+b\,x^n)^2 (c+d\,x^n)^{-3-\frac{1}{n}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^n)^2*(c+d*x^n)^(-3-1/n),x)
```

```
[Out] int((a+b*x^n)^2*(c+d*x^n)^(-3-1/n),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^2*(c+d*x^n)^(-3-1/n),x, algorithm="maxima")
```

```
[Out] integrate((b*x^n + a)^2*(d*x^n + c)^(-1/n - 3), x)
```

Fricas [A]
time = 2.47, size = 231, normalized size = 1.99

$$\frac{\left(2 a^{2} d^{3} n^{2} + b^{2} c^{2} d + \left(b^{2} c^{2} d + 2 a b c d^{2}\right) n\right) x x^{3 n} + \left(6 a^{2} c d^{2} n^{2} + b^{2} c^{3} + 2 a b c^{2} d + \left(b^{2} c^{3} + 6 a b c^{2} d + 2 a^{2} c d^{2}\right) n\right) x x^{2 n} + \left(6 a^{2} c^{2} d n^{2} + 2 a b c^{3} + a^{2} c^{2} d + \left(4 a b c^{3} + 5 a^{2} c^{2} d\right) n\right) x x^{n} + \left(2 a^{2} c^{3} n^{2} + 3 a^{2} c^{3} n + a^{2} c^{3}\right) x}{\left(2 c^{3} n^{2} + 3 c^{3} n + c^{3}\right) \left(d x^{n} + c\right)^{\frac{3 n + 1}{n}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^2*(c+d*x^n)^(-3-1/n),x, algorithm="fricas")
```

```
[Out] ((2*a^2*d^3*n^2 + b^2*c^2*d + (b^2*c^2*d + 2*a*b*c*d^2)*n)*x*x^(3*n) + (6*a
^2*c*d^2*n^2 + b^2*c^3 + 2*a*b*c^2*d + (b^2*c^3 + 6*a*b*c^2*d + 2*a^2*c*d^2
)*n)*x*x^(2*n) + (6*a^2*c^2*d*n^2 + 2*a*b*c^3 + a^2*c^2*d + (4*a*b*c^3 + 5*
a^2*c^2*d)*n)*x*x^n + (2*a^2*c^3*n^2 + 3*a^2*c^3*n + a^2*c^3)*x)/((2*c^3*n^
2 + 3*c^3*n + c^3)*(d*x^n + c)^((3*n + 1)/n))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n)**2*(c+d*x**n)**(-3-1/n),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3007 deep
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^2*(c+d*x^n)^(-3-1/n),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{8,[1,0,4,3,1,3,2,0]%%%}+%%%{12,[1,0,4,2,1,3,2,0]%%%}+%%%{6,[1
,0,4,1
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+b\,x^n\right)}^2}{{\left(c+d\,x^n\right)}^{\frac{1}{n}+3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)^2/(c + d*x^n)^(1/n + 3),x)
```

```
[Out] int((a + b*x^n)^2/(c + d*x^n)^(1/n + 3), x)
```

3.323 $\int (a+bx^n) (c+dx^n)^{-2-\frac{1}{n}} \, dx$

Optimal. Leaf size=58

$$\frac{x(a+bx^n) (c+dx^n)^{-1-\frac{1}{n}}}{c(1+n)}+\frac{anx(c+dx^n)^{-1/n}}{c^2(1+n)}$$

```
[Out] x*(a+b*x^n)*(c+d*x^n)^(-1-1/n)/c/(1+n)+a*n*x/c^2/(1+n)/((c+d*x^n)^(1/n))
```

Rubi [A]
time = 0.01, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {386, 197}

$$\frac{x(a+bx^n) (c+dx^n)^{-\frac{1}{n}-1}}{c(n+1)}+\frac{anx(c+dx^n)^{-1/n}}{c^2(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n)*(c + d*x^n)^(-2 - n^(-1)),x]
```

```
[Out] (x*(a + b*x^n)*(c + d*x^n)^(-1 - n^(-1)))/(c*(1 + n)) + (a*n*x)/(c^2*(1 + n
)*(c + d*x^n)^n^(-1))
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 386

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] - Dist[
c*(q/(a*(p + 1))), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1,
0] && GtQ[q, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int (a+bx^n) (c+dx^n)^{-2-\frac{1}{n}} \, dx &= \frac{x(a+bx^n) (c+dx^n)^{-1-\frac{1}{n}}}{c(1+n)}+\frac{(an) \int (c+dx^n)^{-1-\frac{1}{n}} \, dx}{c(1+n)}\\ &= \frac{x(a+bx^n) (c+dx^n)^{-1-\frac{1}{n}}}{c(1+n)}+\frac{anx(c+dx^n)^{-1/n}}{c^2(1+n)}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.11, size = 82, normalized size = 1.41

$$\frac{x(c+dx^n)^{-\frac{1+n}{n}}\left(bcx^n+a(1+n)\left(c+dx^n\right)\left(1+\frac{dx^n}{c}\right)^{\frac{1}{n}}{}_2F_1\left(2+\frac{1}{n},\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)\right)}{c^2(1+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^n)*(c + d*x^n)^(-2 - n^(-1)),x]
```

```
[Out] (x*(b*c*x^n + a*(1 + n)*(c + d*x^n)*(1 + (d*x^n)/c)^n^(-1)*Hypergeometric2F
1[2 + n^(-1), n^(-1), 1 + n^(-1), -((d*x^n)/c)]))/(c^2*(1 + n)*(c + d*x^n)^
((1 + n)/n))
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int\left(a+b\,x^{n}\right)\left(c+d\,x^{n}\right)^{-2-\frac{1}{n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^n)*(c+d*x^n)^(-2-1/n),x)
```

```
[Out] int((a+b*x^n)*(c+d*x^n)^(-2-1/n),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)*(c+d*x^n)^(-2-1/n),x, algorithm="maxima")
```

```
[Out] integrate((b*x^n + a)*(d*x^n + c)^(-1/n - 2), x)
```

Fricas [A]
time = 3.01, size = 85, normalized size = 1.47

$$\frac{(ad^2n+bcd)xx^{2\,n}+(2\,acdn+bc^2+acd)xx^n+(ac^2n+ac^2)x}{(c^2n+c^2)(dx^n+c)^{\frac{2\,n+1}{n}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)*(c+d*x^n)^(-2-1/n),x, algorithm="fricas")
```

```
[Out] ((a*d^2*n + b*c*d)*x*x^(2*n) + (2*a*c*d*n + b*c^2 + a*c*d)*x*x^n + (a*c^2*n
 + a*c^2)*x)/((c^2*n + c^2)*(d*x^n + c)^((2*n + 1)/n))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n)*(c+d*x**n)**(-2-1/n),x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)*(c+d*x^n)^(-2-1/n),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{1,[0,0,2,2,1,1,0,1]%%%}+%%%{1,[0,0,2,1,1,1,0,1]%%%}+%%%{1,[0,
0,2,1,
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{a+b\,x^n}{{\left(c+d\,x^n\right)}^{\frac{1}{n}+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)/(c + d*x^n)^(1/n + 2),x)
```

```
[Out] int((a + b*x^n)/(c + d*x^n)^(1/n + 2), x)
```

3.324 $\int (c+dx^n)^{-1-\frac{1}{n}}\,dx$

Optimal. Leaf size=18

$$\frac{x(c+dx^n)^{-1/n}}{c}$$

```
[Out] x/c/((c+d*x^n)^(1/n))
```

Rubi [A]
time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {197}

$$\frac{x(c+dx^n)^{-1/n}}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^n)^(-1 - n^(-1)),x]
[Out] x/(c*(c + d*x^n)^n^(-1))
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int (c+dx^n)^{-1-\frac{1}{n}}\,dx = \frac{x(c+dx^n)^{-1/n}}{c}$$

Mathematica [A]
time = 0.03, size = 18, normalized size = 1.00

$$\frac{x(c+dx^n)^{-1/n}}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^n)^(-1 - n^(-1)),x]
[Out] x/(c*(c + d*x^n)^n^(-1))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 52 vs. 2(18) = 36.
time = 0.32, size = 53, normalized size = 2.94

method	result	size
norman	$x\,\mathrm{e}^{\left(-1-\frac{1}{n}\right)\ln\left(c+d\,\mathrm{e}^{n\ln(x)}\right)}+\frac{xd\,\mathrm{e}^{n\ln(x)}\mathrm{e}^{\left(-1-\frac{1}{n}\right)\ln\left(c+d\,\mathrm{e}^{n\ln(x)}\right)}}{c}$	53

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*x^n)^(-1-1/n),x,method=_RETURNVERBOSE)
```

```
[Out] x*exp((-1-1/n)*ln(c+d*exp(n*ln(x))))+1/c*x*d*exp(n*ln(x))*exp((-1-1/n)*ln(c
+d*exp(n*ln(x))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^n)^(-1-1/n),x, algorithm="maxima")
```

```
[Out] integrate((d*x^n + c)^(-1/n - 1), x)
```

Fricas [A]
time = 3.08, size = 31, normalized size = 1.72

$$\frac{dxx^n+cx}{(dx^n+c)^{\frac{n+1}{n}}c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^n)^(-1-1/n),x, algorithm="fricas")
```

```
[Out] (d*x*x^n + c*x)/((d*x^n + c)^((n + 1)/n)*c)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 207 vs. $2(12) = 24$.
time = 11.38, size = 207, normalized size = 11.50

$$\begin{cases} -\frac{xx^{-n}(dx^n)^{-\frac{1}{n}}}{dn} & \text{for } c=0 \\ 0^{-1-\frac{1}{n}}x & \text{for } c=-dx^n \\ x(0^n)^{-1-\frac{1}{n}} & \text{for } c=0^n-dx^n \\ \frac{c^2x}{c^3(c+dx^n)^{\frac{1}{n}}+2c^2dx^n(c+dx^n)^{\frac{1}{n}}+cd^2x^{2n}(c+dx^n)^{\frac{1}{n}}}+\frac{cdxx^n}{c^3(c+dx^n)^{\frac{1}{n}}+2c^2dx^n(c+dx^n)^{\frac{1}{n}}+cd^2x^{2n}(c+dx^n)^{\frac{1}{n}}}+\frac{dxx^n}{c^2(c+dx^n)^{\frac{1}{n}}+cdx^n(c+dx^n)^{\frac{1}{n}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x**n)**(-1-1/n),x)
```

```
[Out] Piecewise((-x/(d*n*x**n*(d*x**n)**(1/n)), Eq(c, 0)), (0**(-1 - 1/n)*x, Eq(c
, -d*x**n)), (x*(0**n)**(-1 - 1/n), Eq(c, 0**n - d*x**n)), (c**2*x/(c**3*(c
```

```
 + d*x**n)**(1/n) + 2*c**2*d*x**n*(c + d*x**n)**(1/n) + c*d**2*x**(2*n)*(c
+ d*x**n)**(1/n)) + c*d*x*x**n/(c**3*(c + d*x**n)**(1/n) + 2*c**2*d*x**n*(c
 + d*x**n)**(1/n) + c*d**2*x**(2*n)*(c + d*x**n)**(1/n)) + d*x*x**n/(c**2*(
c + d*x**n)**(1/n) + c*d*x**n*(c + d*x**n)**(1/n)), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^n)^(-1-1/n),x, algorithm="giac")
[Out] integrate((d*x^n + c)^(-1/n - 1), x)
```

Mupad [B]
time = 1.76, size = 75, normalized size = 4.17

$$\frac{d\,x^{n+1}\,\left(\frac{c}{d\,x^n}-{\left(\frac{c}{d\,x^n}+1\right)}^{\frac{n+1}{n}}+1\right)}{c\,n\,\left(\frac{n+1}{n}-1\right)\,{\left(c+d\,x^n\right)}^{\frac{n+1}{n}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c + d*x^n)^(1/n + 1),x)
[Out] (d*x^(n + 1)*(c/(d*x^n) - (c/(d*x^n) + 1)^((n + 1)/n) + 1))/(c*n*((n + 1)/n
 - 1)*(c + d*x^n)^((n + 1)/n))
```

3.325 $\int \frac{(c+dx^n)^{-1/n}}{a+bx^n} dx$

Optimal. Leaf size=53

$$\frac{x(c+dx^n)^{-1/n}\ {}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{(bc-ad)x^n}{a(c+dx^n)}\right)}{a}$$

```
[Out] x*hypergeom([1, 1/n],[1+1/n],-(-a*d+b*c)*x^n/a/(c+d*x^n))/a/((c+d*x^n)^(1/n
))
```

Rubi [A]
time = 0.01, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {387}

$$\frac{x(c+dx^n)^{-1/n}\ {}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{(bc-ad)x^n}{a(dx^n+c)}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^n)*(c + d*x^n)^n^(-1)),x]
```

```
[Out] (x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -(((b*c - a*d)*x^n)/(a*(c + d*x
^n)))])/(a*(c + d*x^n)^n^(-1))
```

Rule 387

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*(x/(c^(p + 1)*(c + d*x^n)^(1/n)))*Hypergeometric2F1[1/n, -p, 1
+ 1/n, (-(b*c - a*d))*(x^n/(a*(c + d*x^n)))], x] /; FreeQ[{a, b, c, d, n, q
}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0] && ILtQ[p, 0]
```

Rubi steps

$$\int \frac{(c+dx^n)^{-1/n}}{a+bx^n} dx = \frac{x(c+dx^n)^{-1/n}\ {}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{(bc-ad)x^n}{a(c+dx^n)}\right)}{a}$$

Mathematica [A]
time = 0.04, size = 52, normalized size = 0.98

$$\frac{x(c+dx^n)^{-1/n}\ {}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};\frac{(-bc+ad)x^n}{a(c+dx^n)}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^n)*(c + d*x^n)^n^(-1)),x]
```

```
[Out] (x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), ((-(b*c) + a*d)*x^n)/(a*(c + d*
x^n))])/(a*(c + d*x^n)^n^(-1))
```

Maple [F]
time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{(c+d\,x^{n})^{-\frac{1}{n}}}{a+b\,x^{n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*x^n)/((c+d*x^n)^(1/n)),x)
```

```
[Out] int(1/(a+b*x^n)/((c+d*x^n)^(1/n)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^n)/((c+d*x^n)^(1/n)),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^n + a)*(d*x^n + c)^(1/n)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^n)/((c+d*x^n)^(1/n)),x, algorithm="fricas")
```

```
[Out] integral(1/((b*x^n + a)*(d*x^n + c)^(1/n)), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x**n)/((c+d*x**n)**(1/n)),x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^n)/((c+d*x^n)^(1/n)),x, algorithm="giac")
[Out] integrate(1/((b*x^n + a)*(d*x^n + c)^(1/n)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{(a+b\,x^n)\,\left(c+d\,x^n\right)^{1/n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^n)*(c + d*x^n)^(1/n)),x)
[Out] int(1/((a + b*x^n)*(c + d*x^n)^(1/n)), x)
```

3.326 $\int \frac{(c+dx^n)^{1-\frac{1}{n}}}{(a+bx^n)^2} \, dx$

Optimal. Leaf size=54

$$\frac{cx(c+dx^n)^{-1/n} \ {}_2F_1\left(2,\frac{1}{n};1+\frac{1}{n};-\frac{(bc-ad)x^n}{a(c+dx^n)}\right)}{a^2}$$

```
[Out] c*x*hypergeom([2, 1/n],[1+1/n],-(-a*d+b*c)*x^n/a/(c+d*x^n))/a^2/((c+d*x^n)^
(1/n))
```

Rubi [A]
time = 0.01, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = $\{387\}$

$$\frac{cx(c+dx^n)^{-1/n} \ {}_2F_1\left(2,\frac{1}{n};1+\frac{1}{n};-\frac{(bc-ad)x^n}{a(dx^n+c)}\right)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^n)^(1 - n^(-1))/(a + b*x^n)^2,x]
```

```
[Out] (c*x*Hypergeometric2F1[2, n^(-1), 1 + n^(-1), -(((b*c - a*d)*x^n)/(a*(c + d
*x^n)))])/(a^2*(c + d*x^n)^n^(-1))
```

Rule 387

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*(x/(c^(p + 1)*(c + d*x^n)^(1/n)))*Hypergeometric2F1[1/n, -p, 1
+ 1/n, (-(b*c - a*d))*(x^n/(a*(c + d*x^n)))], x] /; FreeQ[{a, b, c, d, n, q
}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0] && ILtQ[p, 0]
```

Rubi steps

$$\int \frac{(c+dx^n)^{1-\frac{1}{n}}}{(a+bx^n)^2} \, dx = \frac{cx(c+dx^n)^{-1/n} \ {}_2F_1\left(2,\frac{1}{n};1+\frac{1}{n};-\frac{(bc-ad)x^n}{a(c+dx^n)}\right)}{a^2}$$

Mathematica [A]
time = 0.06, size = 53, normalized size = 0.98

$$\frac{cx(c+dx^n)^{-1/n} \ {}_2F_1\left(2,\frac{1}{n};1+\frac{1}{n};\frac{(-bc+ad)x^n}{a(c+dx^n)}\right)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^n)^(1 - n^(-1))/(a + b*x^n)^2,x]
```

```
[Out] (c*x*Hypergeometric2F1[2, n^(-1), 1 + n^(-1), ((-(b*c) + a*d)*x^n)/(a*(c + d*x^n))])/(a^2*(c + d*x^n)^n^(-1))
```

Maple [F]
time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{\left(c+d\,x^{n}\right)^{1-\frac{1}{n}}}{\left(a+b\,x^{n}\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*x^n)^(1-1/n)/(a+b*x^n)^2,x)
```

```
[Out] int((c+d*x^n)^(1-1/n)/(a+b*x^n)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^n)^(1-1/n)/(a+b*x^n)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*x^n + c)^(-1/n + 1)/(b*x^n + a)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^n)^(1-1/n)/(a+b*x^n)^2,x, algorithm="fricas")
```

```
[Out] integral((d*x^n + c)^((n - 1)/n)/(b^2*x^(2*n) + 2*a*b*x^n + a^2), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x**n)**(1-1/n)/(a+b*x**n)**2,x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^n)^(1-1/n)/(a+b*x^n)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x^n + c)^(-1/n + 1)/(b*x^n + a)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(c+d\,x^n)^{1-\frac{1}{n}}}{(a+b\,x^n)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^n)^(1 - 1/n)/(a + b*x^n)^2,x)
```

```
[Out] int((c + d*x^n)^(1 - 1/n)/(a + b*x^n)^2, x)
```

3.327 $\int \frac{(c+dx^n)^{2-\frac{1}{n}}}{(a+bx^n)^3} dx$

Optimal. Leaf size=56

$$\frac{c^2x(c+dx^n)^{-1/n} \, {}_2F_1\left(3,\frac{1}{n};1+\frac{1}{n};-\frac{(bc-ad)x^n}{a(c+dx^n)}\right)}{a^3}$$

```
[Out] c^2*x*hypergeom([3, 1/n],[1+1/n],-(-a*d+b*c)*x^n/a/(c+d*x^n))/a^3/((c+d*x^n
)^(1/n))
```

Rubi [A]
time = 0.01, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {387}

$$\frac{c^2x(c+dx^n)^{-1/n} \, {}_2F_1\left(3,\frac{1}{n};1+\frac{1}{n};-\frac{(bc-ad)x^n}{a(dx^n+c)}\right)}{a^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^n)^(2 - n^(-1))/(a + b*x^n)^3,x]
```

```
[Out] (c^2*x*Hypergeometric2F1[3, n^(-1), 1 + n^(-1), -(((b*c - a*d)*x^n)/(a*(c +
 d*x^n)))])/(a^3*(c + d*x^n)^n^(-1))
```

Rule 387

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*(x/(c^(p + 1)*(c + d*x^n)^(1/n)))*Hypergeometric2F1[1/n, -p, 1
+ 1/n, (-(b*c - a*d))*(x^n/(a*(c + d*x^n)))], x] /; FreeQ[{a, b, c, d, n, q
}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0] && ILtQ[p, 0]
```

Rubi steps

$$\int \frac{(c+dx^n)^{2-\frac{1}{n}}}{(a+bx^n)^3} dx = \frac{c^2x(c+dx^n)^{-1/n} \, {}_2F_1\left(3,\frac{1}{n};1+\frac{1}{n};-\frac{(bc-ad)x^n}{a(c+dx^n)}\right)}{a^3}$$

Mathematica [A]
time = 0.05, size = 55, normalized size = 0.98

$$\frac{c^2x(c+dx^n)^{-1/n} \, {}_2F_1\left(3,\frac{1}{n};1+\frac{1}{n};\frac{(-bc+ad)x^n}{a(c+dx^n)}\right)}{a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^n)^(2 - n^(-1))/(a + b*x^n)^3,x]
```

```
[Out] (c^2*x*Hypergeometric2F1[3, n^(-1), 1 + n^(-1), ((-(b*c) + a*d)*x^n)/(a*(c
+ d*x^n))])/(a^3*(c + d*x^n)^n^(-1))
```

Maple [F]
time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{(c+d\,x^{n})^{2-\frac{1}{n}}}{(a+b\,x^{n})^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*x^n)^(2-1/n)/(a+b*x^n)^3,x)
```

```
[Out] int((c+d*x^n)^(2-1/n)/(a+b*x^n)^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^n)^(2-1/n)/(a+b*x^n)^3,x, algorithm="maxima")
```

```
[Out] integrate((d*x^n + c)^(-1/n + 2)/(b*x^n + a)^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^n)^(2-1/n)/(a+b*x^n)^3,x, algorithm="fricas")
```

```
[Out] integral((d*x^n + c)^((2*n - 1)/n)/(b^3*x^(3*n) + 3*a*b^2*x^(2*n) + 3*a^2*b
*x^n + a^3), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x**n)**(2-1/n)/(a+b*x**n)**3,x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^n)^(2-1/n)/(a+b*x^n)^3,x, algorithm="giac")
[Out] integrate((d*x^n + c)^(-1/n + 2)/(b*x^n + a)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(c+d\,x^n)^{2-\frac{1}{n}}}{(a+b\,x^n)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^n)^(2 - 1/n)/(a + b*x^n)^3,x)
[Out] int((c + d*x^n)^(2 - 1/n)/(a + b*x^n)^3, x)
```

3.328 $\int (a+bx^n)^p (c+dx^n)^{-2-\frac{1}{n}-p} \, dx$

Optimal. Leaf size=193

$$-\frac{bx(a+bx^n)^{1+p}(c+dx^n)^{-1-\frac{1}{n}-p}}{a(bc-ad)n(1+p)}+\frac{(bc+(bc-ad)n(1+p))x(a+bx^n)^{1+p}\left(\frac{c(a+bx^n)}{a(c+dx^n)}\right)^{-1-p}(c+dx^n)^{-1-\frac{1}{n}-p}\ {}_2}{ac(bc-ad)n(1+p)}$$

```
[Out] -b*x*(a+b*x^n)^(1+p)*(c+d*x^n)^(-1-1/n-p)/a/(-a*d+b*c)/n/(1+p)+(b*c+(-a*d+b
*c)*n*(1+p))*x*(a+b*x^n)^(1+p)*(c*(a+b*x^n)/a/(c+d*x^n))^(-1-p)*(c+d*x^n)^(
-1-1/n-p)*hypergeom([1/n, -1-p],[1+1/n],-(-a*d+b*c)*x^n/a/(c+d*x^n))/a/c/(-
a*d+b*c)/n/(1+p)
```

Rubi [A]
time = 0.06, antiderivative size = 179, normalized size of antiderivative = 0.93, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {390, 388}

$$\frac{x(a+bx^n)^{p+1}(c+dx^n)^{-\frac{1}{n}-p-1}\left(\frac{b}{n(p+1)(bc-ad)}+\frac{1}{c}\right)\left(\frac{c(a+bx^n)}{a(c+dx^n)}\right)^{-p-1}{}_2F_1\left(\frac{1}{n},-p-1;1+\frac{1}{n};-\frac{(bc-ad)x^n}{a(dx^n+c)}\right)}{a}-\frac{bx(a+bx^n)^{p+1}(c+dx^n)^{-\frac{1}{n}-p-1}}{an(p+1)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n)^p*(c + d*x^n)^(-2 - n^(-1) - p),x]
```

```
[Out] -((b*x*(a + b*x^n)^(1 + p)*(c + d*x^n)^(-1 - n^(-1) - p))/(a*(b*c - a*d)*n*
(1 + p))) + ((c^(-1) + b/((b*c - a*d)*n*(1 + p)))*x*(a + b*x^n)^(1 + p)*((c
*(a + b*x^n))/(a*(c + d*x^n)))^(-1 - p)*(c + d*x^n)^(-1 - n^(-1) - p)*Hyper
geometric2F1[n^(-1), -1 - p, 1 + n^(-1), -(((b*c - a*d)*x^n)/(a*(c + d*x^n)
))])/a
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[x*((a + b*x^n)^p/(c*(c*((a + b*x^n)/(a*(c + d*x^n))))^p*(c + d*x^n)
^(1/n + p)))*Hypergeometric2F1[1/n, -p, 1 + 1/n, (-(b*c - a*d))*(x^n/(a*(c
+ d*x^n)))], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] &&
EqQ[n*(p + q + 1) + 1, 0]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)),
 Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q},
 x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !L
tQ[q, -1]) && NeQ[p, -1]
```

Rubi steps

$$\int (a+bx^n)^p (c+dx^n)^{-2-\frac{1}{n}-p}\,dx = -\frac{bx(a+bx^n)^{1+p}(c+dx^n)^{-1-\frac{1}{n}-p}}{a(bc-ad)n(1+p)} + \frac{\left(1+\frac{bc}{(bc-ad)n(1+p)}\right)\int (a+bx^n)^{1+p}}{a}$$

$$= -\frac{bx(a+bx^n)^{1+p}(c+dx^n)^{-1-\frac{1}{n}-p}}{a(bc-ad)n(1+p)} + \frac{\left(1+\frac{bc}{(bc-ad)n(1+p)}\right)x(a+bx^n)^{1+p}}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 1414 vs. $2(193) = 386$.
time = 37.64, size = 1414, normalized size = 7.33

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^n)^p*(c + d*x^n)^(-2 - n^(-1) - p),x]
```

```
[Out] (c^4*(1 + n)*(1 + 2*n)*(1 + 3*n)*x*(a + b*x^n)^(3 + p)*(c + d*x^n)^(-2 - n^
(-1) - p)*(1 + (d*x^n)/c)*Gamma[2 + n^(-1)]*Gamma[-p]*(Hypergeometric2F1[1,
 -p, 1 + n^(-1), ((b*c - a*d)*x^n)/(c*(a + b*x^n))] + (d*n*x^n*((c*Hypergeo
metric2F1[1, -p, 2 + n^(-1), ((b*c - a*d)*x^n)/(c*(a + b*x^n))])/(1 + n) +
((b*c - a*d)*x^n*Gamma[1 + n^(-1)]*Gamma[1 - p]*Hypergeometric2F1[2, 1 - p,
 3 + n^(-1), ((b*c - a*d)*x^n)/(c*(a + b*x^n))])/((1 + 2*n)*(a + b*x^n)*Gam
ma[2 + n^(-1)]*Gamma[-p])))/c^2))/(-(c*d*(1 + 3*n)*(1 + n + n*p)*x^n*(a + b
*x^n)^2*(c^2*(1 + n)*(1 + 2*n)*(a + b*x^n)*Gamma[2 + n^(-1)]*Gamma[-p]*Hype
rgeometric2F1[1, -p, 1 + n^(-1), ((b*c - a*d)*x^n)/(c*(a + b*x^n))] + d*n*x
^n*(c*(1 + 2*n)*(a + b*x^n)*Gamma[2 + n^(-1)]*Gamma[-p]*Hypergeometric2F1[1
, -p, 2 + n^(-1), ((b*c - a*d)*x^n)/(c*(a + b*x^n))] + (b*c - a*d)*(1 + n)*
x^n*Gamma[1 + n^(-1)]*Gamma[1 - p]*Hypergeometric2F1[2, 1 - p, 3 + n^(-1),
((b*c - a*d)*x^n)/(c*(a + b*x^n))]))) + b*c*n*(1 + 3*n)*p*x^n*(a + b*x^n)*(
c + d*x^n)*(c^2*(1 + n)*(1 + 2*n)*(a + b*x^n)*Gamma[2 + n^(-1)]*Gamma[-p]*H
ypergeometric2F1[1, -p, 1 + n^(-1), ((b*c - a*d)*x^n)/(c*(a + b*x^n))] + d*
n*x^n*(c*(1 + 2*n)*(a + b*x^n)*Gamma[2 + n^(-1)]*Gamma[-p]*Hypergeometric2F
1[1, -p, 2 + n^(-1), ((b*c - a*d)*x^n)/(c*(a + b*x^n))] + (b*c - a*d)*(1 +
n)*x^n*Gamma[1 + n^(-1)]*Gamma[1 - p]*Hypergeometric2F1[2, 1 - p, 3 + n^(-1
), ((b*c - a*d)*x^n)/(c*(a + b*x^n))])) + c*(1 + 3*n)*(a + b*x^n)^2*(c + d*
x^n)*(c^2*(1 + n)*(1 + 2*n)*(a + b*x^n)*Gamma[2 + n^(-1)]*Gamma[-p]*Hyperge
ometric2F1[1, -p, 1 + n^(-1), ((b*c - a*d)*x^n)/(c*(a + b*x^n))] + d*n*x^n*
(c*(1 + 2*n)*(a + b*x^n)*Gamma[2 + n^(-1)]*Gamma[-p]*Hypergeometric2F1[1, -
p, 2 + n^(-1), ((b*c - a*d)*x^n)/(c*(a + b*x^n))] + (b*c - a*d)*(1 + n)*x^n
*Gamma[1 + n^(-1)]*Gamma[1 - p]*Hypergeometric2F1[2, 1 - p, 3 + n^(-1), ((b
*c - a*d)*x^n)/(c*(a + b*x^n))])) + n^2*x^n*(c + d*x^n)*(a*c^2*(-(b*c) + a*
```

```
d)*(1 + 2*n)*(1 + 3*n)*p*(a + b*x^n)*Gamma[2 + n^(-1)]*Gamma[-p]*Hypergeome
tric2F1[2, 1 - p, 2 + n^(-1), ((b*c - a*d)*x^n)/(c*(a + b*x^n))] + c*d*(1 +
 3*n)*(a + b*x^n)^2*(c*(1 + 2*n)*(a + b*x^n)*Gamma[2 + n^(-1)]*Gamma[-p]*Hy
pergeometric2F1[1, -p, 2 + n^(-1), ((b*c - a*d)*x^n)/(c*(a + b*x^n))] + (b*
c - a*d)*(1 + n)*x^n*Gamma[1 + n^(-1)]*Gamma[1 - p]*Hypergeometric2F1[2, 1
- p, 3 + n^(-1), ((b*c - a*d)*x^n)/(c*(a + b*x^n))]) - d*(b*c - a*d)*x^n*(b
*c*(1 + n)*(1 + 3*n)*x^n*(a + b*x^n)*Gamma[1 + n^(-1)]*Gamma[1 - p]*Hyperge
ometric2F1[2, 1 - p, 3 + n^(-1), ((b*c - a*d)*x^n)/(c*(a + b*x^n))] - c*(1
+ n)*(1 + 3*n)*(a + b*x^n)^2*Gamma[1 + n^(-1)]*Gamma[1 - p]*Hypergeometric2
F1[2, 1 - p, 3 + n^(-1), ((b*c - a*d)*x^n)/(c*(a + b*x^n))] + a*c*n*(1 + 3*
n)*p*(a + b*x^n)*Gamma[2 + n^(-1)]*Gamma[-p]*Hypergeometric2F1[2, 1 - p, 3
+ n^(-1), ((b*c - a*d)*x^n)/(c*(a + b*x^n))] - 2*a*(-(b*c) + a*d)*n*(1 + n)
*(-1 + p)*x^n*Gamma[1 + n^(-1)]*Gamma[1 - p]*Hypergeometric2F1[3, 2 - p, 4
+ n^(-1), ((b*c - a*d)*x^n)/(c*(a + b*x^n))])))
```

Maple [F]
time = 0.12, size = 0, normalized size = 0.00

$$\int (a+b\,x^{n})^{p}\,(c+d\,x^{n})^{-2-\frac{1}{n}-p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^n)^p*(c+d*x^n)^(-2-1/n-p),x)
```

```
[Out] int((a+b*x^n)^p*(c+d*x^n)^(-2-1/n-p),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^p*(c+d*x^n)^(-2-1/n-p),x, algorithm="maxima")
```

```
[Out] integrate((b*x^n + a)^p*(d*x^n + c)^(-p - 1/n - 2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^p*(c+d*x^n)^(-2-1/n-p),x, algorithm="fricas")
```

```
[Out] integral((b*x^n + a)^p/(d*x^n + c)^((n*p + 2*n + 1)/n), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n)**p*(c+d*x**n)**(-2-1/n-p),x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^p*(c+d*x^n)^(-2-1/n-p),x, algorithm="giac")
[Out] integrate((b*x^n + a)^p*(d*x^n + c)^(-p - 1/n - 2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,x^n)^p}{(c+d\,x^n)^{p+\frac{1}{n}+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)^p/(c + d*x^n)^(p + 1/n + 2),x)
[Out] int((a + b*x^n)^p/(c + d*x^n)^(p + 1/n + 2), x)
```

3.329 $\int (a+bx^n)^{\frac{adn-bc(1+n)}{(bc-ad)n}} (c+dx^n)^{\frac{ad-bcn+adn}{bcn-adn}} dx$

Optimal. Leaf size=57

$$\frac{x(a+bx^n)^{-\frac{bc}{(bc-ad)n}} (c+dx^n)^{\frac{ad}{(bc-ad)n}}}{ac}$$

```
[Out] x*(c+d*x^n)^(a*d/(-a*d+b*c)/n)/a/c/((a+b*x^n)^(b*c/(-a*d+b*c)/n))
```

Rubi [A]
time = 0.02, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 69, $\frac{\text{number of rules}}{\text{integrand size}} = 0.014$, Rules used = {389}

$$\frac{x(a+bx^n)^{-\frac{bc}{n(bc-ad)}} (c+dx^n)^{\frac{ad}{n(bc-ad)}}}{ac}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n)^((a*d*n - b*c*(1 + n))/((b*c - a*d)*n))*(c + d*x^n)^((a*d -
 b*c*n + a*d*n)/(b*c*n - a*d*n)),x]
```

```
[Out] (x*(c + d*x^n)^((a*d)/((b*c - a*d)*n)))/(a*c*(a + b*x^n)^((b*c)/((b*c - a*d
)*n)))
```

Rule 389

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c)), x] /; FreeQ[{a,
b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] &&
EqQ[a*d*(p + 1) + b*c*(q + 1), 0]
```

Rubi steps

$$\int (a+bx^n)^{\frac{adn-bc(1+n)}{(bc-ad)n}} (c+dx^n)^{\frac{ad-bcn+adn}{bcn-adn}} dx = \frac{x(a+bx^n)^{-\frac{bc}{(bc-ad)n}} (c+dx^n)^{\frac{ad}{(bc-ad)n}}}{ac}$$

Mathematica [A]
time = 0.38, size = 55, normalized size = 0.96

$$\frac{x(a+bx^n)^{-\frac{bc}{bcn-adn}} (c+dx^n)^{\frac{ad}{bcn-adn}}}{ac}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^n)^((a*d*n - b*c*(1 + n))/((b*c - a*d)*n))*(c + d*x^n)^(
(a*d - b*c*n + a*d*n)/(b*c*n - a*d*n)),x]
```

```
[Out] (x*(c + d*x^n)^((a*d)/(b*c*n - a*d*n)))/(a*c*(a + b*x^n)^((b*c)/(b*c*n - a*
d*n)))
```

Maple [F]
time = 0.16, size = 0, normalized size = 0.00

$$\int \left(a + b\,x^{n}\right)^{\frac{adn - bc(1+n)}{(-ad+bc)n}} \left(c + d\,x^{n}\right)^{\frac{adn - bcn + ad}{-adn + bcn}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^n)^((a*d*n-b*c*(1+n))/(-a*d+b*c)/n)*(c+d*x^n)^((a*d*n-b*c*n+a*d)
/(-a*d*n+b*c*n)),x)
```

```
[Out] int((a+b*x^n)^((a*d*n-b*c*(1+n))/(-a*d+b*c)/n)*(c+d*x^n)^((a*d*n-b*c*n+a*d)
/(-a*d*n+b*c*n)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^((a*d*n-b*c*(1+n))/(-a*d+b*c)/n)*(c+d*x^n)^((a*d*n-b*c*
n+a*d)/(-a*d*n+b*c*n)),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^n + a)^((b*c*(n + 1) - a*d*n)/((b*c - a*d)*n))*(d*x^n + c
)^((b*c*n - a*d*n - a*d)/(b*c*n - a*d*n))), x)
```

Fricas [A]
time = 2.85, size = 108, normalized size = 1.89

$$\frac{\left(bdxx^{2\,n} + acx + (bc + ad)xx^{n}\right)\left(dx^{n} + c\right)^{\frac{ad - (bc - ad)n}{(bc - ad)n}}}{\left(bx^{n} + a\right)^{\frac{bc + (bc - ad)n}{(bc - ad)n}}\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^((a*d*n-b*c*(1+n))/(-a*d+b*c)/n)*(c+d*x^n)^((a*d*n-b*c*
n+a*d)/(-a*d*n+b*c*n)),x, algorithm="fricas")
```

```
[Out] (b*d*x*x^(2*n) + a*c*x + (b*c + a*d)*x*x^n)*(d*x^n + c)^((a*d - (b*c - a*d)
*n)/((b*c - a*d)*n))/((b*x^n + a)^((b*c + (b*c - a*d)*n)/((b*c - a*d)*n))*a
*c)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n)**((a*d*n-b*c*(1+n))/(-a*d+b*c)/n)*(c+d*x**n)**((a*d*n-
b*c*n+a*d)/(-a*d*n+b*c*n)),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^((a*d*n-b*c*(1+n))/(-a*d+b*c)/n)*(c+d*x^n)^((a*d*n-b*c*
n+a*d)/(-a*d*n+b*c*n)),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^n + a)^((b*c*(n + 1) - a*d*n)/((b*c - a*d)*n))*(d*x^n + c
)^((b*c*n - a*d*n - a*d)/(b*c*n - a*d*n))), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{{\left(a+b\,x^n\right)}^{\frac{a\,d\,n-b\,c\,\left(n+1\right)}{n\,\left(a\,d-b\,c\right)}}\,{\left(c+d\,x^n\right)}^{\frac{a\,d+a\,d\,n-b\,c\,n}{a\,d\,n-b\,c\,n}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^n)^((a*d*n - b*c*(n + 1))/(n*(a*d - b*c)))*(c + d*x^n)^((a*
d + a*d*n - b*c*n)/(a*d*n - b*c*n))),x)
```

```
[Out] int(1/((a + b*x^n)^((a*d*n - b*c*(n + 1))/(n*(a*d - b*c)))*(c + d*x^n)^((a*
d + a*d*n - b*c*n)/(a*d*n - b*c*n))), x)
```

3.330 $\int (a+bx^n)^2 (c+dx^n)^{-4-\frac{1}{n}} \, dx$

Optimal. Leaf size=327

$$-\frac{bx(a+bx^n)^3 (c+dx^n)^{-3-\frac{1}{n}}}{3a(bc-ad)n} - \frac{(3adn-b(c+3cn))x(a+bx^n)^3 (c+dx^n)^{-3-\frac{1}{n}}}{3ac(bc-ad)n(1+3n)} - \frac{(3adn-b(c+3cn))x(a+}{c^2(bc-ad)(1+}$$

```
[Out] -1/3*b*x*(a+b*x^n)^3*(c+d*x^n)^(-3-1/n)/a/(-a*d+b*c)/n-1/3*(3*a*d*n-b*(3*c*
n+c))*x*(a+b*x^n)^3*(c+d*x^n)^(-3-1/n)/a/c/(-a*d+b*c)/n/(1+3*n)-(3*a*d*n-b*
(3*c*n+c))*x*(a+b*x^n)^2*(c+d*x^n)^(-2-1/n)/c^2/(-a*d+b*c)/(6*n^2+5*n+1)-2*
a*n*(3*a*d*n-b*(3*c*n+c))*x*(a+b*x^n)*(c+d*x^n)^(-1-1/n)/c^3/(-a*d+b*c)/(6*
n^3+11*n^2+6*n+1)-2*a^2*n^2*(3*a*d*n-b*(3*c*n+c))*x/c^4/(-a*d+b*c)/(6*n^3+1
1*n^2+6*n+1)/((c+d*x^n)^(1/n))
```

Rubi [A]

time = 0.13, antiderivative size = 327, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {390, 386, 197}

$$-\frac{2a^2n^2x(c+dx^n)^{-1/n}(3adn-b(3cn+c))}{c^4(n+1)(2n+1)(3n+1)(bc-ad)} - \frac{2anx(a+bx^n)(c+dx^n)^{-\frac{1}{n}-1}(3adn-b(3cn+c))}{c^3(n+1)(2n+1)(3n+1)(bc-ad)} - \frac{x(a+bx^n)^2(c+dx^n)^{-\frac{1}{n}-2}(3adn-b(3cn+c))}{c^2(6n^2+5n+1)(bc-ad)} - \frac{x(a+bx^n)^3(c+dx^n)^{-\frac{1}{n}-3}(3adn-b(3cn+c))}{3acn(3n+1)(bc-ad)} - \frac{bx(a+bx^n)^3(c+dx^n)^{-\frac{1}{n}-3}}{3an(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n)^2*(c + d*x^n)^(-4 - n^(-1)),x]
```

```
[Out] -1/3*(b*x*(a + b*x^n)^3*(c + d*x^n)^(-3 - n^(-1)))/(a*(b*c - a*d)*n) - ((3*
a*d*n - b*(c + 3*c*n))*x*(a + b*x^n)^3*(c + d*x^n)^(-3 - n^(-1)))/(3*a*c*(b
*c - a*d)*n*(1 + 3*n)) - ((3*a*d*n - b*(c + 3*c*n))*x*(a + b*x^n)^2*(c + d*
x^n)^(-2 - n^(-1)))/(c^2*(b*c - a*d)*(1 + 5*n + 6*n^2)) - (2*a*n*(3*a*d*n -
 b*(c + 3*c*n))*x*(a + b*x^n)*(c + d*x^n)^(-1 - n^(-1)))/(c^3*(b*c - a*d)*(
1 + n)*(1 + 2*n)*(1 + 3*n)) - (2*a^2*n^2*(3*a*d*n - b*(c + 3*c*n))*x)/(c^4*
(b*c - a*d)*(1 + n)*(1 + 2*n)*(1 + 3*n)*(c + d*x^n)^n^(-1))
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 386

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] - Dist[
c*(q/(a*(p + 1))), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1,
0] && GtQ[q, 0] && NeQ[p, -1]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)),
 Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q},
 x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !L
tQ[q, -1]) && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int (a+bx^n)^2 (c+dx^n)^{-4-\frac{1}{n}}\,dx &= -\frac{bx(a+bx^n)^3 (c+dx^n)^{-3-\frac{1}{n}}}{3a(bc-ad)n}+\frac{\left(3+\frac{bc}{bcn-adn}\right)\int (a+bx^n)^3 (c+dx^n)^{-4-\frac{1}{n}}}{3a}\\ &= -\frac{bx(a+bx^n)^3 (c+dx^n)^{-3-\frac{1}{n}}}{3a(bc-ad)n}+\frac{\left(3+\frac{bc}{bcn-adn}\right)x(a+bx^n)^3 (c+dx^n)^{-3-\frac{1}{n}}}{3ac(1+3n)}\\ &= -\frac{bx(a+bx^n)^3 (c+dx^n)^{-3-\frac{1}{n}}}{3a(bc-ad)n}+\frac{\left(3+\frac{bc}{bcn-adn}\right)x(a+bx^n)^3 (c+dx^n)^{-3-\frac{1}{n}}}{3ac(1+3n)}\\ &= -\frac{bx(a+bx^n)^3 (c+dx^n)^{-3-\frac{1}{n}}}{3a(bc-ad)n}+\frac{\left(3+\frac{bc}{bcn-adn}\right)x(a+bx^n)^3 (c+dx^n)^{-3-\frac{1}{n}}}{3ac(1+3n)}\\ &= -\frac{bx(a+bx^n)^3 (c+dx^n)^{-3-\frac{1}{n}}}{3a(bc-ad)n}+\frac{\left(3+\frac{bc}{bcn-adn}\right)x(a+bx^n)^3 (c+dx^n)^{-3-\frac{1}{n}}}{3ac(1+3n)}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.31, size = 136, normalized size = 0.42

$$\frac{x(c+dx^n)^{-1/n}\left(1+\frac{dx^n}{c}\right)^{\frac{1}{n}}\left(b^2c^2\,{}_2F_1\left(2+\frac{1}{n},\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)-(bc-ad)\left(2bc\,{}_2F_1\left(3+\frac{1}{n},\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)+(-bc+ad)\,{}_2F_1\left(4+\frac{1}{n},\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)\right)\right)}{c^4d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^n)^2*(c + d*x^n)^(-4 - n^(-1)),x]
```

```
[Out] (x*(1 + (d*x^n)/c)^n^(-1)*(b^2*c^2*Hypergeometric2F1[2 + n^(-1), n^(-1), 1
+ n^(-1), -((d*x^n)/c)] - (b*c - a*d)*(2*b*c*Hypergeometric2F1[3 + n^(-1),
n^(-1), 1 + n^(-1), -((d*x^n)/c)] + (-(b*c) + a*d)*Hypergeometric2F1[4 + n^
(-1), n^(-1), 1 + n^(-1), -((d*x^n)/c)])))/(c^4*d^2*(c + d*x^n)^n^(-1))
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int (a+b\,x^n)^2 (c+d\,x^n)^{-4-\frac{1}{n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^n)^2*(c+d*x^n)^(-4-1/n),x)
[Out] int((a+b*x^n)^2*(c+d*x^n)^(-4-1/n),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^2*(c+d*x^n)^(-4-1/n),x, algorithm="maxima")
[Out] integrate((b*x^n + a)^2*(d*x^n + c)^(-1/n - 4), x)
```

Fricas [A]
time = 2.20, size = 400, normalized size = 1.22

$$\frac{(6\,a^2d^4n^3+b^2c^2d^2n+(b^2c^2d^2+4\,abcd^3)n^2)xx^{4\,n}+(24\,a^2cd^3n^3+b^2c^3d+2\,(2\,b^2c^3d+8\,abc^2d^2+3\,a^2cd^3)n^2+(5\,b^2c^3d+4\,abc^2d^2)n)xx^{3\,n}+(36\,a^2c^2d^2n^3+b^2c^4+2\,abc^3d+3\,(b^2c^4+8\,abc^3d+7\,a^2c^2d^2)n^2+(4\,b^2c^4+14\,abc^3d+3\,a^2c^2d^2)n)xx^{2\,n}+(24\,a^2c^3dn^3+2\,abc^4+a^2c^3d+2\,(6\,abc^4+13\,a^2c^3d)n^2+(10\,abc^4+9\,a^2c^3d)n)xx^n+(6\,a^2c^4n^3+11\,a^2c^4n^2+6\,a^2c^4n+a^2c^4)x}{(6\,c^4n^3+11\,c^4n^2+6\,c^4n+c^4)(dx^n+c)^{\frac{4\,n+1}{n}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^2*(c+d*x^n)^(-4-1/n),x, algorithm="fricas")
[Out] ((6*a^2*d^4*n^3 + b^2*c^2*d^2*n + (b^2*c^2*d^2 + 4*a*b*c*d^3)*n^2)*x*x^(4*n
) + (24*a^2*c*d^3*n^3 + b^2*c^3*d + 2*(2*b^2*c^3*d + 8*a*b*c^2*d^2 + 3*a^2*
c*d^3)*n^2 + (5*b^2*c^3*d + 4*a*b*c^2*d^2)*n)*x*x^(3*n) + (36*a^2*c^2*d^2*n
^3 + b^2*c^4 + 2*a*b*c^3*d + 3*(b^2*c^4 + 8*a*b*c^3*d + 7*a^2*c^2*d^2)*n^2
+ (4*b^2*c^4 + 14*a*b*c^3*d + 3*a^2*c^2*d^2)*n)*x*x^(2*n) + (24*a^2*c^3*d*n
^3 + 2*a*b*c^4 + a^2*c^3*d + 2*(6*a*b*c^4 + 13*a^2*c^3*d)*n^2 + (10*a*b*c^4
 + 9*a^2*c^3*d)*n)*x*x^n + (6*a^2*c^4*n^3 + 11*a^2*c^4*n^2 + 6*a^2*c^4*n +
a^2*c^4)*x)/((6*c^4*n^3 + 11*c^4*n^2 + 6*c^4*n + c^4)*(d*x^n + c)^((4*n + 1
)/n))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n)**2*(c+d*x**n)**(-4-1/n),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 6439 deep
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^2*(c+d*x^n)^(-4-1/n),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{27,[1,0,4,3,1,3,2,0]%%%}+%%%{27,[1,0,4,2,1,3,2,0]%%%}+%%%{9,[
1,0,4,
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,x^n)^2}{(c+d\,x^n)^{\frac{1}{n}+4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)^2/(c + d*x^n)^(1/n + 4),x)
```

```
[Out] int((a + b*x^n)^2/(c + d*x^n)^(1/n + 4), x)
```

3.331 $\int (a+bx^n)(c+dx^n)^{-3-\frac{1}{n}}\,dx$

Optimal. Leaf size=127

$$-\frac{(bc-ad)x(c+dx^n)^{-2-\frac{1}{n}}}{cd(1+2n)}+\frac{(bc+2adn)x(c+dx^n)^{-1-\frac{1}{n}}}{c^2d(1+n)(1+2n)}+\frac{n(bc+2adn)x(c+dx^n)^{-1/n}}{c^3d(1+n)(1+2n)}$$

```
[Out] -(-a*d+b*c)*x*(c+d*x^n)^(-2-1/n)/c/d/(1+2*n)+(2*a*d*n+b*c)*x*(c+d*x^n)^(-1-
1/n)/c^2/d/(1+n)/(1+2*n)+n*(2*a*d*n+b*c)*x/c^3/d/(1+n)/(1+2*n)/((c+d*x^n)^(
1/n))
```

Rubi [A]
time = 0.04, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {393, 198, 197}

$$\frac{nx(c+dx^n)^{-1/n}(2adn+bc)}{c^3d(n+1)(2n+1)}+\frac{x(c+dx^n)^{-\frac{1}{n}-1}(2adn+bc)}{c^2d(n+1)(2n+1)}-\frac{x(bc-ad)(c+dx^n)^{-\frac{1}{n}-2}}{cd(2n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n)*(c + d*x^n)^(-3 - n^(-1)),x]
```

```
[Out] -(((b*c - a*d)*x*(c + d*x^n)^(-2 - n^(-1)))/(c*d*(1 + 2*n))) + ((b*c + 2*a*
d*n)*x*(c + d*x^n)^(-1 - n^(-1)))/(c^2*d*(1 + n)*(1 + 2*n)) + (n*(b*c + 2*a
*d*n)*x)/(c^3*d*(1 + n)*(1 + 2*n)*(c + d*x^n)^n^(-1))
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rubi steps

$$\int (a+bx^n)(c+dx^n)^{-3-\frac{1}{n}}\,dx = -\frac{(bc-ad)x(c+dx^n)^{-2-\frac{1}{n}}}{cd(1+2n)} + \frac{(bc+2adn)\int (c+dx^n)^{-2-\frac{1}{n}}\,dx}{cd(1+2n)}$$
$$= -\frac{(bc-ad)x(c+dx^n)^{-2-\frac{1}{n}}}{cd(1+2n)} + \frac{(bc+2adn)x(c+dx^n)^{-1-\frac{1}{n}}}{c^2d(1+n)(1+2n)} + \frac{(n(bc+2adn)}{c^2d(1}$$
$$= -\frac{(bc-ad)x(c+dx^n)^{-2-\frac{1}{n}}}{cd(1+2n)} + \frac{(bc+2adn)x(c+dx^n)^{-1-\frac{1}{n}}}{c^2d(1+n)(1+2n)} + \frac{n(bc+2adn)}{c^3d(1+r}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.13, size = 94, normalized size = 0.74

$$\frac{x(c+dx^n)^{-1/n}\left(1+\frac{dx^n}{c}\right)^{\frac{1}{n}}\left(bc\,{}_2F_1\left(2+\frac{1}{n},\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)+(-bc+ad)\,{}_2F_1\left(3+\frac{1}{n},\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)\right)}{c^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^n)*(c + d*x^n)^(-3 - n^(-1)),x]
```

```
[Out] (x*(1 + (d*x^n)/c)^n^(-1)*(b*c*Hypergeometric2F1[2 + n^(-1), n^(-1), 1 + n^
(-1), -((d*x^n)/c)] + (-(b*c) + a*d)*Hypergeometric2F1[3 + n^(-1), n^(-1),
1 + n^(-1), -((d*x^n)/c)]))/(c^3*d*(c + d*x^n)^n^(-1))
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int (a+b\,x^n)(c+d\,x^n)^{-3-\frac{1}{n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^n)*(c+d*x^n)^(-3-1/n),x)
```

```
[Out] int((a+b*x^n)*(c+d*x^n)^(-3-1/n),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)*(c+d*x^n)^(-3-1/n),x, algorithm="maxima")
```

```
[Out] integrate((b*x^n + a)*(d*x^n + c)^(-1/n - 3), x)
```

Fricas [A]
time = 2.47, size = 173, normalized size = 1.36

$$\frac{(2\,ad^3n^2+bcd^2n)xx^{3\,n}+(6\,acd^2n^2+bc^2d+(3\,bc^2d+2\,acd^2)n)xx^{2\,n}+(6\,ac^2dn^2+bc^3+ac^2d+(2\,bc^3+5\,ac^2d)n)xx^n+(2\,ac^3n^2+3\,ac^3n+ac^3)x}{(2\,c^3n^2+3\,c^3n+c^3)(dx^n+c)^{\frac{3\,n+1}{n}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)*(c+d*x^n)^(-3-1/n),x, algorithm="fricas")
```

```
[Out] ((2*a*d^3*n^2 + b*c*d^2*n)*x*x^(3*n) + (6*a*c*d^2*n^2 + b*c^2*d + (3*b*c^2*
d + 2*a*c*d^2)*n)*x*x^(2*n) + (6*a*c^2*d*n^2 + b*c^3 + a*c^2*d + (2*b*c^3 +
 5*a*c^2*d)*n)*x*x^n + (2*a*c^3*n^2 + 3*a*c^3*n + a*c^3)*x)/((2*c^3*n^2 + 3
*c^3*n + c^3)*(d*x^n + c)^((3*n + 1)/n))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n)*(c+d*x**n)**(-3-1/n),x)
```

```
[Out] Timed out
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)*(c+d*x^n)^(-3-1/n),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{4,[0,0,2,2,1,1,0,1]%%%}+%%%{2,[0,0,2,1,1,1,0,1]%%%}+%%%{2,[0,
0,2,1,
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\,x^n}{{\left(c+d\,x^n\right)}^{\frac{1}{n}+3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)/(c + d*x^n)^(1/n + 3),x)
```

```
[Out] int((a + b*x^n)/(c + d*x^n)^(1/n + 3), x)
```

3.332 $\int (c+dx^n)^{-2-\frac{1}{n}}\,dx$

Optimal. Leaf size=50

$$\frac{x(c+dx^n)^{-1-\frac{1}{n}}}{c(1+n)}+\frac{nx(c+dx^n)^{-1/n}}{c^2(1+n)}$$

```
[Out] x*(c+d*x^n)^(-1-1/n)/c/(1+n)+n*x/c^2/(1+n)/((c+d*x^n)^(1/n))
```

Rubi [A]
time = 0.01, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {198, 197}

$$\frac{nx(c+dx^n)^{-1/n}}{c^2(n+1)}+\frac{x(c+dx^n)^{-\frac{1}{n}-1}}{c(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^n)^(-2 - n^(-1)),x]
```

```
[Out] (x*(c + d*x^n)^(-1 - n^(-1)))/(c*(1 + n)) + (n*x)/(c^2*(1 + n)*(c + d*x^n)^
n^(-1))
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int (c+dx^n)^{-2-\frac{1}{n}}\,dx &= \frac{x(c+dx^n)^{-1-\frac{1}{n}}}{c(1+n)}+\frac{n\int (c+dx^n)^{-1-\frac{1}{n}}\,dx}{c(1+n)}\\ &= \frac{x(c+dx^n)^{-1-\frac{1}{n}}}{c(1+n)}+\frac{nx(c+dx^n)^{-1/n}}{c^2(1+n)}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.04, size = 55, normalized size = 1.10

$$\frac{x(c+dx^n)^{-1/n}\left(1+\frac{dx^n}{c}\right)^{\frac{1}{n}}{}_2F_1\left(2+\frac{1}{n},\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^n)^(-2 - n^(-1)),x]
```

```
[Out] (x*(1 + (d*x^n)/c)^n^(-1)*Hypergeometric2F1[2 + n^(-1), n^(-1), 1 + n^(-1),
 -((d*x^n)/c)])/(c^2*(c + d*x^n)^n^(-1))
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int \left(c+d\,x^{n}\right)^{-2-\frac{1}{n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*x^n)^(-2-1/n),x)
```

```
[Out] int((c+d*x^n)^(-2-1/n),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^n)^(-2-1/n),x, algorithm="maxima")
```

```
[Out] integrate((d*x^n + c)^(-1/n - 2), x)
```

Fricas [A]
time = 3.36, size = 68, normalized size = 1.36

$$\frac{d^2nxx^{2\,n}+(2\,cdn+cd)xx^n+(c^2n+c^2)x}{(c^2n+c^2)(dx^n+c)^{\frac{2\,n+1}{n}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^n)^(-2-1/n),x, algorithm="fricas")
```

```
[Out] (d^2*n*x*x^(2*n) + (2*c*d*n + c*d)*x*x^n + (c^2*n + c^2)*x)/((c^2*n + c^2)*
(d*x^n + c)^((2*n + 1)/n))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x**n)**(-2-1/n),x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^n)^(-2-1/n),x, algorithm="giac")
```

```
[Out] integrate((d*x^n + c)^(-1/n - 2), x)
```

Mupad [B]
time = 1.76, size = 64, normalized size = 1.28

$$-\frac{x^{1-2\,n}\,{\left(\frac{c}{d\,x^n}+1\right)}^{1/n}\,{}_2\mathrm{F}_1\left(2,\frac{1}{n}+2;\ 3;\ -\frac{c}{d\,x^n}\right)}{2\,d^2\,n\,{\left(c+d\,x^n\right)}^{1/n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c + d*x^n)^(1/n + 2),x)
```

```
[Out] -(x^(1 - 2*n)*(c/(d*x^n) + 1)^(1/n)*hypergeom([2, 1/n + 2], 3, -c/(d*x^n)))
/(2*d^2*n*(c + d*x^n)^(1/n))
```

3.333 $\int \frac{(c+dx^n)^{-1-\frac{1}{n}}}{a+bx^n} dx$

Optimal. Leaf size=95

$$-\frac{dx(c+dx^n)^{-1/n}}{c(bc-ad)}+\frac{bx(c+dx^n)^{-1/n}\ {}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{(bc-ad)x^n}{a(c+dx^n)}\right)}{a(bc-ad)}$$

```
[Out] -d*x/c/(-a*d+b*c)/((c+d*x^n)^(1/n))+b*x*hypergeom([1, 1/n],[1+1/n],-(-a*d+b
*c)*x^n/a/(c+d*x^n))/a/(-a*d+b*c)/((c+d*x^n)^(1/n))
```

Rubi [A]
time = 0.03, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {390, 387}

$$\frac{bx(c+dx^n)^{-1/n}\ {}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{(bc-ad)x^n}{a(dx^n+c)}\right)}{a(bc-ad)}-\frac{dx(c+dx^n)^{-1/n}}{c(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^n)^(-1 - n^(-1))/(a + b*x^n),x]
```

```
[Out] -((d*x)/(c*(b*c - a*d)*(c + d*x^n)^n^(-1))) + (b*x*Hypergeometric2F1[1, n^(
-1), 1 + n^(-1), -(((b*c - a*d)*x^n)/(a*(c + d*x^n)))])/(a*(b*c - a*d)*(c +
 d*x^n)^n^(-1))
```

Rule 387

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*(x/(c^(p + 1)*(c + d*x^n)^(1/n)))*Hypergeometric2F1[1/n, -p, 1
+ 1/n, (-(b*c - a*d))*(x^n/(a*(c + d*x^n)))], x] /; FreeQ[{a, b, c, d, n, q
}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0] && ILtQ[p, 0]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)),
 Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q},
 x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !L
tQ[q, -1]) && NeQ[p, -1]
```

Rubi steps

$$\int \frac{(c+dx^n)^{-1-\frac{1}{n}}}{a+bx^n}\,dx = -\frac{dx(c+dx^n)^{-1/n}}{c(bc-ad)} + \frac{b\int \frac{(c+dx^n)^{-1/n}}{a+bx^n}\,dx}{bc-ad}$$
$$= -\frac{dx(c+dx^n)^{-1/n}}{c(bc-ad)} + \frac{bx(c+dx^n)^{-1/n}\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{(bc-ad)x^n}{a(c+dx^n)}\right)}{a(bc-ad)}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.
time = 6.01, size = 153, normalized size = 1.61

$$\frac{x(c+dx^n)^{-\frac{1+n}{n}}\left(\frac{a(c+dx^n)}{c(a+bx^n)} + \frac{bx^n\Phi\left(\frac{(-bc+ad)x^n}{a(c+dx^n)},1,1+\frac{1}{n}\right)}{a} + \frac{b(-bc+ad)nx^{2n}\,{}_2F_1\left(2,2+\frac{1}{n};3+\frac{1}{n};\frac{(-bc+ad)x^n}{a(c+dx^n)}\right)}{a^2(1+2n)(c+dx^n)}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^n)^(-1 - n^(-1))/(a + b*x^n),x]
```

```
[Out] (x*((a*(c + d*x^n))/(c*(a + b*x^n)) + (b*x^n*HurwitzLerchPhi[((-(b*c) + a*d
)*x^n)/(a*(c + d*x^n)), 1, 1 + n^(-1)])/a + (b*(-(b*c) + a*d)*n*x^(2*n)*Hyp
ergeometric2F1[2, 2 + n^(-1), 3 + n^(-1), ((-(b*c) + a*d)*x^n)/(a*(c + d*x^
n))])/(a^2*(1 + 2*n)*(c + d*x^n))))/(a*(c + d*x^n)^((1 + n)/n))
```

Maple [F]
time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{(c+d\,x^n)^{-1-\frac{1}{n}}}{a+b\,x^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*x^n)^(-1-1/n)/(a+b*x^n),x)
```

```
[Out] int((c+d*x^n)^(-1-1/n)/(a+b*x^n),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^n)^(-1-1/n)/(a+b*x^n),x, algorithm="maxima")
```

```
[Out] integrate((d*x^n + c)^(-1/n - 1)/(b*x^n + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^n)^(-1-1/n)/(a+b*x^n),x, algorithm="fricas")
[Out] integral(1/((b*x^n + a)*(d*x^n + c)^((n + 1)/n)), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x**n)**(-1-1/n)/(a+b*x**n),x)
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^n)^(-1-1/n)/(a+b*x^n),x, algorithm="giac")
[Out] integrate((d*x^n + c)^(-1/n - 1)/(b*x^n + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(a+b\,x^n)\,\left(c+d\,x^n\right)^{\frac{1}{n}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^n)*(c + d*x^n)^(1/n + 1)),x)
[Out] int(1/((a + b*x^n)*(c + d*x^n)^(1/n + 1)), x)
```

3.334 $\int \frac{(c+dx^n)^{-1/n}}{(a+bx^n)^2}\,dx$

Optimal. Leaf size=127

$$\frac{bx(c+dx^n)^{-\frac{1-n}{n}}}{a(bc-ad)n\,(a+bx^n)}-\frac{(bc(1-n)+adn)x(c+dx^n)^{-1/n}\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{(bc-ad)x^n}{a(c+dx^n)}\right)}{a^2(bc-ad)n}$$

```
[Out] b*x/a/(-a*d+b*c)/n/(a+b*x^n)/((c+d*x^n)^((1-n)/n))-(b*c*(1-n)+a*d*n)*x*hype
rgeom([1, 1/n],[1+1/n],-(-a*d+b*c)*x^n/a/(c+d*x^n))/a^2/(-a*d+b*c)/n/((c+d*
x^n)^(1/n))
```

Rubi [A]
time = 0.04, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {390, 387}

$$\frac{bx(c+dx^n)^{-\frac{1-n}{n}}}{an(bc-ad)\,(a+bx^n)}-\frac{x(c+dx^n)^{-1/n}\,(adn+bc(1-n))\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{(bc-ad)x^n}{a(dx^n+c)}\right)}{a^2n(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^n)^2*(c + d*x^n)^n^(-1)),x]
```

```
[Out] (b*x)/(a*(b*c - a*d)*n*(a + b*x^n)*(c + d*x^n)^((1 - n)/n)) - ((b*c*(1 - n)
 + a*d*n)*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -(((b*c - a*d)*x^n)/(a
*(c + d*x^n)))])/(a^2*(b*c - a*d)*n*(c + d*x^n)^n^(-1))
```

Rule 387

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*(x/(c^(p + 1)*(c + d*x^n)^(1/n)))*Hypergeometric2F1[1/n, -p, 1
+ 1/n, (-(b*c - a*d))*(x^n/(a*(c + d*x^n)))], x] /; FreeQ[{a, b, c, d, n, q
}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0] && ILtQ[p, 0]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)),
 Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q},
 x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !L
tQ[q, -1]) && NeQ[p, -1]
```

Rubi steps

$$\int \frac{(c+dx^n)^{-1/n}}{(a+bx^n)^2}\,dx = \frac{bx(c+dx^n)^{-\frac{1-n}{n}}}{a(bc-ad)n\,(a+bx^n)} - \frac{(bc-(bc-ad)n)\int \frac{(c+dx^n)^{-1/n}}{a+bx^n}\,dx}{a(bc-ad)n}$$

$$= \frac{bx(c+dx^n)^{-\frac{1-n}{n}}}{a(bc-ad)n\,(a+bx^n)} - \frac{(bc(1-n)+adn)x(c+dx^n)^{-1/n}\ {}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{(bc-ad)x}{a(c+dx^n}\right.}{a^2(bc-ad)n}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 1070 vs. $2(127) = 254$.
time = 36.73, size = 1070, normalized size = 8.43

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^n)^2*(c + d*x^n)^n^(-1)),x]
```

```
[Out] (c^2*(1 + 2*n)*(1 + 3*n)*x*(a + b*x^n)*(1 + (d*x^n)/c)*Gamma[2 + n^(-1)]*Ga
mma[3 + n^(-1)]*((c*(c + c*n + d*n*x^n)*Hypergeometric2F1[1, 2, 2 + n^(-1),
 ((b*c - a*d)*x^n)/(c*(a + b*x^n))])/Gamma[2 + n^(-1)] + (2*(b*c - a*d)*n*x
^n*(c + d*x^n)*Hypergeometric2F1[2, 3, 3 + n^(-1), ((b*c - a*d)*x^n)/(c*(a
+ b*x^n))])/((a + b*x^n)*Gamma[3 + n^(-1)])))/((c + d*x^n)^n^(-1)*(-(c*d*(1
 - n)*(1 + 2*n)*(1 + 3*n)*x^n*(a + b*x^n)^2*(c*(a + b*x^n)*(c + c*n + d*n*x
^n)*Gamma[3 + n^(-1)]*Hypergeometric2F1[1, 2, 2 + n^(-1), ((b*c - a*d)*x^n)
/(c*(a + b*x^n))] + 2*(b*c - a*d)*n*x^n*(c + d*x^n)*Gamma[2 + n^(-1)]*Hyper
geometric2F1[2, 3, 3 + n^(-1), ((b*c - a*d)*x^n)/(c*(a + b*x^n))])) - 2*b*c
*n*(1 + 2*n)*(1 + 3*n)*x^n*(a + b*x^n)*(c + d*x^n)*(c*(a + b*x^n)*(c + c*n
+ d*n*x^n)*Gamma[3 + n^(-1)]*Hypergeometric2F1[1, 2, 2 + n^(-1), ((b*c - a*
d)*x^n)/(c*(a + b*x^n))] + 2*(b*c - a*d)*n*x^n*(c + d*x^n)*Gamma[2 + n^(-1)
]*Hypergeometric2F1[2, 3, 3 + n^(-1), ((b*c - a*d)*x^n)/(c*(a + b*x^n))]) +
 c*(1 + 2*n)*(1 + 3*n)*(a + b*x^n)^2*(c + d*x^n)*(c*(a + b*x^n)*(c + c*n +
d*n*x^n)*Gamma[3 + n^(-1)]*Hypergeometric2F1[1, 2, 2 + n^(-1), ((b*c - a*d)
*x^n)/(c*(a + b*x^n))] + 2*(b*c - a*d)*n*x^n*(c + d*x^n)*Gamma[2 + n^(-1)]*
Hypergeometric2F1[2, 3, 3 + n^(-1), ((b*c - a*d)*x^n)/(c*(a + b*x^n))]) + n
^2*x^n*(c + d*x^n)*(c^2*d*(1 + 2*n)*(1 + 3*n)*(a + b*x^n)^3*Gamma[3 + n^(-1
)]*Hypergeometric2F1[1, 2, 2 + n^(-1), ((b*c - a*d)*x^n)/(c*(a + b*x^n))] +
 2*c*d*(b*c - a*d)*(1 + 2*n)*(1 + 3*n)*x^n*(a + b*x^n)^2*Gamma[2 + n^(-1)]*
Hypergeometric2F1[2, 3, 3 + n^(-1), ((b*c - a*d)*x^n)/(c*(a + b*x^n))] - 2*
b*c*(b*c - a*d)*(1 + 2*n)*(1 + 3*n)*x^n*(a + b*x^n)*(c + d*x^n)*Gamma[2 + n
^(-1)]*Hypergeometric2F1[2, 3, 3 + n^(-1), ((b*c - a*d)*x^n)/(c*(a + b*x^n)
)] + 2*c*(b*c - a*d)*(1 + 2*n)*(1 + 3*n)*(a + b*x^n)^2*(c + d*x^n)*Gamma[2
+ n^(-1)]*Hypergeometric2F1[2, 3, 3 + n^(-1), ((b*c - a*d)*x^n)/(c*(a + b*x
^n))] + 2*a*c*(b*c - a*d)*(1 + 3*n)*(a + b*x^n)*(c + c*n + d*n*x^n)*Gamma[3
```

```
 + n^(-1)]*Hypergeometric2F1[2, 3, 3 + n^(-1), ((b*c - a*d)*x^n)/(c*(a + b*
x^n))] + 12*a*(b*c - a*d)^2*n*(1 + 2*n)*x^n*(c + d*x^n)*Gamma[2 + n^(-1)]*H
ypergeometric2F1[3, 4, 4 + n^(-1), ((b*c - a*d)*x^n)/(c*(a + b*x^n))]))
```

Maple [F]
time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{(c+d\,x^{n})^{-\frac{1}{n}}}{(a+b\,x^{n})^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*x^n)^2/((c+d*x^n)^(1/n)),x)
[Out] int(1/(a+b*x^n)^2/((c+d*x^n)^(1/n)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^n)^2/((c+d*x^n)^(1/n)),x, algorithm="maxima")
[Out] integrate(1/((b*x^n + a)^2*(d*x^n + c)^(1/n)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^n)^2/((c+d*x^n)^(1/n)),x, algorithm="fricas")
[Out] integral(1/((b^2*x^(2*n) + 2*a*b*x^n + a^2)*(d*x^n + c)^(1/n)), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x**n)**2/((c+d*x**n)**(1/n)),x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^n)^2/((c+d*x^n)^(1/n)),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^n + a)^2*(d*x^n + c)^(1/n)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(a+b\,x^n)^2\,(c+d\,x^n)^{1/n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^n)^2*(c + d*x^n)^(1/n)),x)
```

```
[Out] int(1/((a + b*x^n)^2*(c + d*x^n)^(1/n)), x)
```

3.335 $\int \frac{(c+dx^n)^{1-\frac{1}{n}}}{(a+bx^n)^3}\,dx$

Optimal. Leaf size=131

$$\frac{bx(c+dx^n)^{2-\frac{1}{n}}}{2a(bc-ad)n\,(a+bx^n)^2}-\frac{c(bc(1-2n)+2adn)x(c+dx^n)^{-1/n}\,{}_2F_1\left(2,\frac{1}{n};1+\frac{1}{n};-\frac{(bc-ad)x^n}{a(c+dx^n)}\right)}{2a^3(bc-ad)n}$$

```
[Out] 1/2*b*x*(c+d*x^n)^(2-1/n)/a/(-a*d+b*c)/n/(a+b*x^n)^2-1/2*c*(b*c*(1-2*n)+2*a
*d*n)*x*hypergeom([2, 1/n],[1+1/n],-(-a*d+b*c)*x^n/a/(c+d*x^n))/a^3/(-a*d+b
*c)/n/((c+d*x^n)^(1/n))
```

Rubi [A]
time = 0.04, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {390, 387}

$$\frac{bx(c+dx^n)^{2-\frac{1}{n}}}{2an(bc-ad)\,(a+bx^n)^2}-\frac{cx(c+dx^n)^{-1/n}\,(2adn+bc(1-2n))\,{}_2F_1\left(2,\frac{1}{n};1+\frac{1}{n};-\frac{(bc-ad)x^n}{a(dx^n+c)}\right)}{2a^3n(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^n)^(1 - n^(-1))/(a + b*x^n)^3,x]
```

```
[Out] (b*x*(c + d*x^n)^(2 - n^(-1)))/(2*a*(b*c - a*d)*n*(a + b*x^n)^2) - (c*(b*c*
(1 - 2*n) + 2*a*d*n)*x*Hypergeometric2F1[2, n^(-1), 1 + n^(-1), -(((b*c - a
*d)*x^n)/(a*(c + d*x^n)))])/(2*a^3*(b*c - a*d)*n*(c + d*x^n)^n^(-1))
```

Rule 387

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*(x/(c^(p + 1)*(c + d*x^n)^(1/n)))*Hypergeometric2F1[1/n, -p, 1
+ 1/n, (-(b*c - a*d))*(x^n/(a*(c + d*x^n)))], x] /; FreeQ[{a, b, c, d, n, q
}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0] && ILtQ[p, 0]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)),
 Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q},
 x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !L
tQ[q, -1]) && NeQ[p, -1]
```

Rubi steps

$$\int \frac{(c+dx^n)^{1-\frac{1}{n}}}{(a+bx^n)^3}\,dx = \frac{bx(c+dx^n)^{2-\frac{1}{n}}}{2a(bc-ad)n\,(a+bx^n)^2} - \frac{(bc-2(bc-ad)n)\int \frac{(c+dx^n)^{1-\frac{1}{n}}}{(a+bx^n)^2}\,dx}{2a(bc-ad)n}$$

$$= \frac{bx(c+dx^n)^{2-\frac{1}{n}}}{2a(bc-ad)n\,(a+bx^n)^2} - \frac{c(bc(1-2n)+2adn)x(c+dx^n)^{-1/n}\,{}_2F_1\left(2,\frac{1}{n};1+\frac{1}{n};-\frac{(b\ldots}{a(\ldots}\right.}{2a^3(bc-ad)n}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 1241 vs. $2(131) = 262$.

time = 40.12, size = 1241, normalized size = 9.47

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x^n)^(1 - n^(-1))/(a + b*x^n)^3,x]
```

```
[Out] -((c^3*(1 + n)*(1 + 2*n)*(1 + 3*n)*x*(c + d*x^n)^(2 - n^(-1))*Gamma[2 + n^(
-1)]*(Hypergeometric2F1[1, 3, 1 + n^(-1), ((b*c - a*d)*x^n)/(c*(a + b*x^n))
] + (d*n*x^n*((c*Hypergeometric2F1[1, 3, 2 + n^(-1), ((b*c - a*d)*x^n)/(c*(
a + b*x^n))])/(1 + n) + (3*(b*c - a*d)*x^n*Gamma[1 + n^(-1)]*Hypergeometric
2F1[2, 4, 3 + n^(-1), ((b*c - a*d)*x^n)/(c*(a + b*x^n))])/((1 + 2*n)*(a + b
*x^n)*Gamma[2 + n^(-1)])))/c^2))/(c*d*(1 - 2*n)*(1 + 3*n)*x^n*(a + b*x^n)^2
*(c^2*(1 + n)*(1 + 2*n)*(a + b*x^n)*Gamma[2 + n^(-1)]*Hypergeometric2F1[1,
3, 1 + n^(-1), ((b*c - a*d)*x^n)/(c*(a + b*x^n))] + d*n*x^n*(c*(1 + 2*n)*(a
 + b*x^n)*Gamma[2 + n^(-1)]*Hypergeometric2F1[1, 3, 2 + n^(-1), ((b*c - a*d
)*x^n)/(c*(a + b*x^n))] + 3*(b*c - a*d)*(1 + n)*x^n*Gamma[1 + n^(-1)]*Hyper
geometric2F1[2, 4, 3 + n^(-1), ((b*c - a*d)*x^n)/(c*(a + b*x^n))])) + 3*b*c
*n*(1 + 3*n)*x^n*(a + b*x^n)*(c + d*x^n)*(c^2*(1 + n)*(1 + 2*n)*(a + b*x^n)
*Gamma[2 + n^(-1)]*Hypergeometric2F1[1, 3, 1 + n^(-1), ((b*c - a*d)*x^n)/(c
*(a + b*x^n))] + d*n*x^n*(c*(1 + 2*n)*(a + b*x^n)*Gamma[2 + n^(-1)]*Hyperge
ometric2F1[1, 3, 2 + n^(-1), ((b*c - a*d)*x^n)/(c*(a + b*x^n))] + 3*(b*c -
a*d)*(1 + n)*x^n*Gamma[1 + n^(-1)]*Hypergeometric2F1[2, 4, 3 + n^(-1), ((b*
c - a*d)*x^n)/(c*(a + b*x^n))])) - c*(1 + 3*n)*(a + b*x^n)^2*(c + d*x^n)*(c
^2*(1 + n)*(1 + 2*n)*(a + b*x^n)*Gamma[2 + n^(-1)]*Hypergeometric2F1[1, 3,
1 + n^(-1), ((b*c - a*d)*x^n)/(c*(a + b*x^n))] + d*n*x^n*(c*(1 + 2*n)*(a +
b*x^n)*Gamma[2 + n^(-1)]*Hypergeometric2F1[1, 3, 2 + n^(-1), ((b*c - a*d)*x
^n)/(c*(a + b*x^n))] + 3*(b*c - a*d)*(1 + n)*x^n*Gamma[1 + n^(-1)]*Hypergeo
metric2F1[2, 4, 3 + n^(-1), ((b*c - a*d)*x^n)/(c*(a + b*x^n))])) + n^2*x^n*
(c + d*x^n)*(3*a*c^2*(-(b*c) + a*d)*(1 + 2*n)*(1 + 3*n)*(a + b*x^n)*Gamma[2
 + n^(-1)]*Hypergeometric2F1[2, 4, 2 + n^(-1), ((b*c - a*d)*x^n)/(c*(a + b*
x^n))] - c*d*(1 + 3*n)*(a + b*x^n)^2*(c*(1 + 2*n)*(a + b*x^n)*Gamma[2 + n^(
-1)]*Hypergeometric2F1[1, 3, 2 + n^(-1), ((b*c - a*d)*x^n)/(c*(a + b*x^n))]
```

```
 + 3*(b*c - a*d)*(1 + n)*x^n*Gamma[1 + n^(-1)]*Hypergeometric2F1[2, 4, 3 +
n^(-1), ((b*c - a*d)*x^n)/(c*(a + b*x^n))]) + 3*d*(b*c - a*d)*x^n*(b*c*(1 +
 n)*(1 + 3*n)*x^n*(a + b*x^n)*Gamma[1 + n^(-1)]*Hypergeometric2F1[2, 4, 3 +
 n^(-1), ((b*c - a*d)*x^n)/(c*(a + b*x^n))] - c*(1 + n)*(1 + 3*n)*(a + b*x^
n)^2*Gamma[1 + n^(-1)]*Hypergeometric2F1[2, 4, 3 + n^(-1), ((b*c - a*d)*x^n
)/(c*(a + b*x^n))] - a*c*n*(1 + 3*n)*(a + b*x^n)*Gamma[2 + n^(-1)]*Hypergeo
metric2F1[2, 4, 3 + n^(-1), ((b*c - a*d)*x^n)/(c*(a + b*x^n))] + 8*a*(-(b*c
) + a*d)*n*(1 + n)*x^n*Gamma[1 + n^(-1)]*Hypergeometric2F1[3, 5, 4 + n^(-1)
, ((b*c - a*d)*x^n)/(c*(a + b*x^n))]))))
```

Maple [F]
time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{(c+d\,x^{n})^{1-\frac{1}{n}}}{(a+b\,x^{n})^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*x^n)^(1-1/n)/(a+b*x^n)^3,x)
```

```
[Out] int((c+d*x^n)^(1-1/n)/(a+b*x^n)^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^n)^(1-1/n)/(a+b*x^n)^3,x, algorithm="maxima")
```

```
[Out] integrate((d*x^n + c)^(-1/n + 1)/(b*x^n + a)^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^n)^(1-1/n)/(a+b*x^n)^3,x, algorithm="fricas")
```

```
[Out] integral((d*x^n + c)^((n - 1)/n)/(b^3*x^(3*n) + 3*a*b^2*x^(2*n) + 3*a^2*b*x
^n + a^3), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x**n)**(1-1/n)/(a+b*x**n)**3,x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^n)^(1-1/n)/(a+b*x^n)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x^n + c)^(-1/n + 1)/(b*x^n + a)^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c+d\,x^n\right)^{1-\frac{1}{n}}}{\left(a+b\,x^n\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^n)^(1 - 1/n)/(a + b*x^n)^3,x)
```

```
[Out] int((c + d*x^n)^(1 - 1/n)/(a + b*x^n)^3, x)
```

3.336 $\int \frac{(c+dx^n)^{2-\frac{1}{n}}}{(a+bx^n)^4} dx$

Optimal. Leaf size=133

$$\frac{bx(c+dx^n)^{3-\frac{1}{n}}}{3a(bc-ad)n(a+bx^n)^3}-\frac{c^2(bc(1-3n)+3adn)x(c+dx^n)^{-1/n}\,{}_2F_1\left(3,\frac{1}{n};1+\frac{1}{n};-\frac{(bc-ad)x^n}{a(c+dx^n)}\right)}{3a^4(bc-ad)n}$$

```
[Out] 1/3*b*x*(c+d*x^n)^(3-1/n)/a/(-a*d+b*c)/n/(a+b*x^n)^3-1/3*c^2*(b*c*(1-3*n)+3
*a*d*n)*x*hypergeom([3, 1/n],[1+1/n],-(-a*d+b*c)*x^n/a/(c+d*x^n))/a^4/(-a*d
+b*c)/n/((c+d*x^n)^(1/n))
```

Rubi [A]
time = 0.04, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {390, 387}

$$\frac{bx(c+dx^n)^{3-\frac{1}{n}}}{3an(bc-ad)(a+bx^n)^3}-\frac{c^2x(c+dx^n)^{-1/n}(3adn+bc(1-3n))\,{}_2F_1\left(3,\frac{1}{n};1+\frac{1}{n};-\frac{(bc-ad)x^n}{a(dx^n+c)}\right)}{3a^4n(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^n)^(2 - n^(-1))/(a + b*x^n)^4,x]
```

```
[Out] (b*x*(c + d*x^n)^(3 - n^(-1)))/(3*a*(b*c - a*d)*n*(a + b*x^n)^3) - (c^2*(b*
c*(1 - 3*n) + 3*a*d*n)*x*Hypergeometric2F1[3, n^(-1), 1 + n^(-1), -(((b*c -
 a*d)*x^n)/(a*(c + d*x^n)))])/(3*a^4*(b*c - a*d)*n*(c + d*x^n)^n^(-1))
```

Rule 387

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*(x/(c^(p + 1)*(c + d*x^n)^(1/n)))*Hypergeometric2F1[1/n, -p, 1
+ 1/n, (-(b*c - a*d))*(x^n/(a*(c + d*x^n)))], x] /; FreeQ[{a, b, c, d, n, q
}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0] && ILtQ[p, 0]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)),
 Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q},
 x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !L
tQ[q, -1]) && NeQ[p, -1]
```

Rubi steps

$$\int \frac{(c+dx^n)^{2-\frac{1}{n}}}{(a+bx^n)^4}\,dx = \frac{bx(c+dx^n)^{3-\frac{1}{n}}}{3a(bc-ad)n\,(a+bx^n)^3} - \frac{(bc-3(bc-ad)n)\int \frac{(c+dx^n)^{2-\frac{1}{n}}}{(a+bx^n)^3}\,dx}{3a(bc-ad)n}$$

$$= \frac{bx(c+dx^n)^{3-\frac{1}{n}}}{3a(bc-ad)n\,(a+bx^n)^3} - \frac{c^2(bc(1-3n)+3adn)x(c+dx^n)^{-1/n}\ {}_2F_1\left(3,\frac{1}{n};1+\frac{1}{n};-\right.}{3a^4(bc-ad)n}$$

Mathematica [F]
time = 180.01, size = 0, normalized size = 0.00

$Aborted

Verification is not applicable to the result.

```
[In] Integrate[(c + d*x^n)^(2 - n^(-1))/(a + b*x^n)^4,x]
[Out] $Aborted
```

Maple [F]
time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{(c+d\,x^n)^{2-\frac{1}{n}}}{(a+b\,x^n)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*x^n)^(2-1/n)/(a+b*x^n)^4,x)
[Out] int((c+d*x^n)^(2-1/n)/(a+b*x^n)^4,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^n)^(2-1/n)/(a+b*x^n)^4,x, algorithm="maxima")
[Out] integrate((d*x^n + c)^(-1/n + 2)/(b*x^n + a)^4, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^n)^(2-1/n)/(a+b*x^n)^4,x, algorithm="fricas")
```

```
[Out] integral((d*x^n + c)^((2*n - 1)/n)/(b^4*x^(4*n) + 4*a*b^3*x^(3*n) + 6*a^2*b
^2*x^(2*n) + 4*a^3*b*x^n + a^4), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x**n)**(2-1/n)/(a+b*x**n)**4,x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^n)^(2-1/n)/(a+b*x^n)^4,x, algorithm="giac")
```

```
[Out] integrate((d*x^n + c)^(-1/n + 2)/(b*x^n + a)^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c+d\,x^n)^{2-\frac{1}{n}}}{(a+b\,x^n)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^n)^(2 - 1/n)/(a + b*x^n)^4,x)
```

```
[Out] int((c + d*x^n)^(2 - 1/n)/(a + b*x^n)^4, x)
```

3.337 $\int x^5 \sqrt{-c+dx}\, \sqrt{c+dx}\, (a+bx^2)\, dx$

Optimal. Leaf size=152

$$\frac{c^4(bc^2+ad^2)\,(-c+dx)^{3/2}(c+dx)^{3/2}}{3d^8}+\frac{c^2(3bc^2+2ad^2)\,(-c+dx)^{5/2}(c+dx)^{5/2}}{5d^8}+\frac{(3bc^2+ad^2)\,(-c+dx)^{7/2}}{7d^8}$$

```
[Out] 1/3*c^4*(a*d^2+b*c^2)*(d*x-c)^(3/2)*(d*x+c)^(3/2)/d^8+1/5*c^2*(2*a*d^2+3*b*
c^2)*(d*x-c)^(5/2)*(d*x+c)^(5/2)/d^8+1/7*(a*d^2+3*b*c^2)*(d*x-c)^(7/2)*(d*x
+c)^(7/2)/d^8+1/9*b*(d*x-c)^(9/2)*(d*x+c)^(9/2)/d^8
```

Rubi [A]
time = 0.08, antiderivative size = 164, normalized size of antiderivative = 1.08, number of steps used = 6, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {471, 102, 12, 75}

$$\frac{4c^2x^2(dx-c)^{3/2}(c+dx)^{3/2}\,(3ad^2+2bc^2)}{105d^6}+\frac{x^4(dx-c)^{3/2}(c+dx)^{3/2}\,(3ad^2+2bc^2)}{21d^4}+\frac{8c^4(dx-c)^{3/2}(c+dx)^{3/2}\,(3ad^2+2bc^2)}{315d^8}+\frac{bx^6(dx-c)^{3/2}(c+dx)^{3/2}}{9d^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*Sqrt[-c + d*x]*Sqrt[c + d*x]*(a + b*x^2),x]
```

```
[Out] (8*c^4*(2*b*c^2 + 3*a*d^2)*(-c + d*x)^(3/2)*(c + d*x)^(3/2))/(315*d^8) + (4
*c^2*(2*b*c^2 + 3*a*d^2)*x^2*(-c + d*x)^(3/2)*(c + d*x)^(3/2))/(105*d^6) +
((2*b*c^2 + 3*a*d^2)*x^4*(-c + d*x)^(3/2)*(c + d*x)^(3/2))/(21*d^4) + (b*x^
6*(-c + d*x)^(3/2)*(c + d*x)^(3/2))/(9*d^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 75

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 102

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/(d*f*(m + n + p + 1))), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
```

```
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[d*(e*x)^(
m + 1)*(a1 + b1*x^(n/2))^(p + 1)*((a2 + b2*x^(n/2))^(p + 1)/(b1*b2*e*(m + n
*(p + 1) + 1))), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\int x^5\sqrt{-c+dx}\,\sqrt{c+dx}\,\left(a+bx^2\right)\,dx = \frac{bx^6(-c+dx)^{3/2}(c+dx)^{3/2}}{9d^2}+\frac{1}{3}\left(3a+\frac{2bc^2}{d^2}\right)\int x^5\sqrt{-c+dx}\,\sqrt{c\cdot}$$
$$=\frac{(2bc^2+3ad^2)\,x^4(-c+dx)^{3/2}(c+dx)^{3/2}}{21d^4}+\frac{bx^6(-c+dx)^{3/2}(c+dx}{9d^2}$$
$$=\frac{(2bc^2+3ad^2)\,x^4(-c+dx)^{3/2}(c+dx)^{3/2}}{21d^4}+\frac{bx^6(-c+dx)^{3/2}(c+dx}{9d^2}$$
$$=\frac{4c^2(2bc^2+3ad^2)\,x^2(-c+dx)^{3/2}(c+dx)^{3/2}}{105d^6}+\frac{(2bc^2+3ad^2)\,x^4(-c}{21}$$
$$=\frac{4c^2(2bc^2+3ad^2)\,x^2(-c+dx)^{3/2}(c+dx)^{3/2}}{105d^6}+\frac{(2bc^2+3ad^2)\,x^4(-c}{21}$$
$$=\frac{8c^4(2bc^2+3ad^2)\,(-c+dx)^{3/2}(c+dx)^{3/2}}{315d^8}+\frac{4c^2(2bc^2+3ad^2)\,x^2(-}{105}$$

Mathematica [A]
time = 0.17, size = 97, normalized size = 0.64

$$\frac{(-c+dx)^{3/2}(c+dx)^{3/2}\,(3ad^2(8c^4+12c^2d^2x^2+15d^4x^4)+b(16c^6+24c^4d^2x^2+30c^2d^4x^4+35d^6x^6))}{315d^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*Sqrt[-c + d*x]*Sqrt[c + d*x]*(a + b*x^2),x]
```

```
[Out] ((-c + d*x)^(3/2)*(c + d*x)^(3/2)*(3*a*d^2*(8*c^4 + 12*c^2*d^2*x^2 + 15*d^4
*x^4) + b*(16*c^6 + 24*c^4*d^2*x^2 + 30*c^2*d^4*x^4 + 35*d^6*x^6)))/(315*d^
8)
```

Maple [A]
time = 0.28, size = 104, normalized size = 0.68

method	result	size
gosper	$\frac{(dx+c)^{\frac{3}{2}}(35b\,x^6d^6+45a\,d^6x^4+30b\,c^2d^4x^4+36a\,c^2d^4x^2+24b\,c^4d^2x^2+24a\,c^4d^2+16b\,c^6)(dx-c)^{\frac{3}{2}}}{315d^8}$	92
default	$-\frac{\sqrt{dx-c}\,\sqrt{dx+c}\,(-d^2x^2+c^2)(35b\,x^6d^6+45a\,d^6x^4+30b\,c^2d^4x^4+36a\,c^2d^4x^2+24b\,c^4d^2x^2+24a\,c^4d^2+16b\,c^6)}{315d^8}$	104
risch	$\frac{\sqrt{dx+c}\,(-35b\,x^8d^8-45a\,d^8x^6+5b\,c^2d^6x^6+9a\,c^2d^6x^4+6b\,c^4d^4x^4+12a\,c^4d^4x^2+8b\,c^6d^2x^2+24a\,c^6d^2+16b\,c^8)(-dx+c)}{315\sqrt{dx-c}\,d^8}$	122

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/315*(d*x-c)^(1/2)*(d*x+c)^(1/2)*(-d^2*x^2+c^2)*(35*b*d^6*x^6+45*a*d^6*x^
4+30*b*c^2*d^4*x^4+36*a*c^2*d^4*x^2+24*b*c^4*d^2*x^2+24*a*c^4*d^2+16*b*c^6)
/d^8
```

Maxima [A]
time = 0.33, size = 178, normalized size = 1.17

$$\frac{(d^2x^2-c^2)^{\frac{3}{2}}bx^6}{9\,d^2}+\frac{2\,(d^2x^2-c^2)^{\frac{3}{2}}bc^2x^4}{21\,d^4}+\frac{(d^2x^2-c^2)^{\frac{3}{2}}ax^4}{7\,d^2}+\frac{8\,(d^2x^2-c^2)^{\frac{3}{2}}bc^4x^2}{105\,d^6}+\frac{4\,(d^2x^2-c^2)^{\frac{3}{2}}ac^2x^2}{35\,d^4}+\frac{16\,(d^2x^2-c^2)^{\frac{3}{2}}bc^6}{315\,d^8}+\frac{8\,(d^2x^2-c^2)^{\frac{3}{2}}ac^4}{105\,d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/9*(d^2*x^2 - c^2)^(3/2)*b*x^6/d^2 + 2/21*(d^2*x^2 - c^2)^(3/2)*b*c^2*x^4/
d^4 + 1/7*(d^2*x^2 - c^2)^(3/2)*a*x^4/d^2 + 8/105*(d^2*x^2 - c^2)^(3/2)*b*c
^4*x^2/d^6 + 4/35*(d^2*x^2 - c^2)^(3/2)*a*c^2*x^2/d^4 + 16/315*(d^2*x^2 - c
^2)^(3/2)*b*c^6/d^8 + 8/105*(d^2*x^2 - c^2)^(3/2)*a*c^4/d^6
```

Fricas [A]
time = 3.72, size = 114, normalized size = 0.75

$$\frac{(35\,bd^8x^8-16\,bc^8-24\,ac^6d^2-5\,(bc^2d^6-9\,ad^8)x^6-3\,(2\,bc^4d^4+3\,ac^2d^6)x^4-4\,(2\,bc^6d^2+3\,ac^4d^4)x^2)\sqrt{dx+c}\,\sqrt{dx-c}}{315\,d^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/315*(35*b*d^8*x^8 - 16*b*c^8 - 24*a*c^6*d^2 - 5*(b*c^2*d^6 - 9*a*d^8)*x^6
 - 3*(2*b*c^4*d^4 + 3*a*c^2*d^6)*x^4 - 4*(2*b*c^6*d^2 + 3*a*c^4*d^4)*x^2)*s
qrt(d*x + c)*sqrt(d*x - c)/d^8
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^5\left(a+bx^2\right)\sqrt{-c+dx}\,\sqrt{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b*x**2+a)*(d*x-c)**(1/2)*(d*x+c)**(1/2),x)
```

```
[Out] Integral(x**5*(a + b*x**2)*sqrt(-c + d*x)*sqrt(c + d*x), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 621 vs. 2(128) = 256.
time = 0.67, size = 621, normalized size = 4.09

$$\frac{168\left(\left(\left(2\left((dx+c)\left(4(dx+c)\left(\frac{5(dx+c)}{d^5}-\frac{31c}{d^5}\right)+\frac{321c^2}{d^5}\right)-\frac{451c^3}{d^5}\right)(dx+c)+\frac{745c^4}{d^5}\right)(dx+c)-\frac{405c^5}{d^5}\right)\sqrt{dx+c}\sqrt{dx-c}-\frac{150c^6\log\left(\left|-\sqrt{dx+c}+\sqrt{dx-c}\right|\right)}{d^5}\right)ac+3\left(\left(\left(2\left(\left(4\left(5(dx+c)\left(6(dx+c)\left(\frac{7(dx+c)}{d^7}-\frac{57c}{d^7}\right)+\frac{1219c^2}{d^7}\right)-\frac{12463c^3}{d^7}\right)(dx+c)+\frac{64233c^4}{d^7}\right)(dx+c)-\frac{53963c^5}{d^7}\right)(dx+c)+\frac{59465c^6}{d^7}\right)(dx+c)-\frac{23205c^7}{d^7}\right)\sqrt{dx+c}\sqrt{dx-c}-\frac{7350c^8\log\left(\left|-\sqrt{dx+c}+\sqrt{dx-c}\right|\right)}{d^7}\right)bc+24\left(\left(\left(2\left(\left(4(dx+c)\left(5(dx+c)\left(\frac{6(dx+c)}{d^6}-\frac{43c}{d^6}\right)+\frac{661c^2}{d^6}\right)-\frac{4551c^3}{d^6}\right)(dx+c)+\frac{4781c^4}{d^6}\right)(dx+c)-\frac{6335c^5}{d^6}\right)(dx+c)+\frac{2835c^6}{d^6}\right)\sqrt{dx+c}\sqrt{dx-c}+\frac{1050c^7\log\left(\left|-\sqrt{dx+c}+\sqrt{dx-c}\right|\right)}{d^6}\right)ad+\left(\left(\left(2\left(\left(4\left(5\left(2(dx+c)\left(7(dx+c)\left(\frac{8(dx+c)}{d^8}-\frac{73c}{d^8}\right)+\frac{2073c^2}{d^8}\right)-\frac{9833c^3}{d^8}\right)(dx+c)+\frac{75293c^4}{d^8}\right)(dx+c)-\frac{310203c^5}{d^8}\right)(dx+c)+\frac{216993c^6}{d^8}\right)(dx+c)-\frac{205275c^7}{d^8}\right)(dx+c)+\frac{69615c^8}{d^8}\right)\sqrt{dx+c}\sqrt{dx-c}+\frac{22050c^9\log\left(\left|-\sqrt{dx+c}+\sqrt{dx-c}\right|\right)}{d^8}\right)bd}{40320\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/40320*(168*(((2*((d*x + c)*(4*(d*x + c)*(5*(d*x + c)/d^5 - 31*c/d^5) + 32
1*c^2/d^5) - 451*c^3/d^5)*(d*x + c) + 745*c^4/d^5)*(d*x + c) - 405*c^5/d^5)
*sqrt(d*x + c)*sqrt(d*x - c) - 150*c^6*log(abs(-sqrt(d*x + c) + sqrt(d*x -
c)))/d^5)*a*c + 3*(((2*((4*(5*(d*x + c)*(6*(d*x + c)*(7*(d*x + c)/d^7 - 57*
c/d^7) + 1219*c^2/d^7) - 12463*c^3/d^7)*(d*x + c) + 64233*c^4/d^7)*(d*x + c
) - 53963*c^5/d^7)*(d*x + c) + 59465*c^6/d^7)*(d*x + c) - 23205*c^7/d^7)*sq
rt(d*x + c)*sqrt(d*x - c) - 7350*c^8*log(abs(-sqrt(d*x + c) + sqrt(d*x - c)
))/d^7)*b*c + 24*(((2*((4*(d*x + c)*(5*(d*x + c)*(6*(d*x + c)/d^6 - 43*c/d^
6) + 661*c^2/d^6) - 4551*c^3/d^6)*(d*x + c) + 4781*c^4/d^6)*(d*x + c) - 633
5*c^5/d^6)*(d*x + c) + 2835*c^6/d^6)*sqrt(d*x + c)*sqrt(d*x - c) + 1050*c^7
*log(abs(-sqrt(d*x + c) + sqrt(d*x - c)))/d^6)*a*d + (((2*((4*(5*(2*(d*x +
c)*(7*(d*x + c)*(8*(d*x + c)/d^8 - 73*c/d^8) + 2073*c^2/d^8) - 9833*c^3/d^8
)*(d*x + c) + 75293*c^4/d^8)*(d*x + c) - 310203*c^5/d^8)*(d*x + c) + 216993
*c^6/d^8)*(d*x + c) - 205275*c^7/d^8)*(d*x + c) + 69615*c^8/d^8)*sqrt(d*x +
 c)*sqrt(d*x - c) + 22050*c^9*log(abs(-sqrt(d*x + c) + sqrt(d*x - c)))/d^8)
*b*d)/d
```

Mupad [B]
time = 1.76, size = 152, normalized size = 1.00

$$-\sqrt{d\,x-c}\,\left(\frac{\left(16\,b\,c^8+24\,a\,c^6\,d^2\right)\,\sqrt{c+d\,x}}{315\,d^8}-\frac{b\,x^8\,\sqrt{c+d\,x}}{9}+\frac{x^4\,\left(6\,b\,c^4\,d^4+9\,a\,c^2\,d^6\right)\,\sqrt{c+d\,x}}{315\,d^8}+\frac{x^2\,\left(8\,b\,c^6\,d^2+12\,a\,c^4\,d^4\right)\,\sqrt{c+d\,x}}{315\,d^8}-\frac{x^6\,\left(45\,a\,d^8-5\,b\,c^2\,d^6\right)\,\sqrt{c+d\,x}}{315\,d^8}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a + b*x^2)*(c + d*x)^(1/2)*(d*x - c)^(1/2),x)
```

```
[Out] -(d*x - c)^(1/2)*(((16*b*c^8 + 24*a*c^6*d^2)*(c + d*x)^(1/2))/(315*d^8) - (
b*x^8*(c + d*x)^(1/2))/9 + (x^4*(9*a*c^2*d^6 + 6*b*c^4*d^4)*(c + d*x)^(1/2)
)/(315*d^8) + (x^2*(12*a*c^4*d^4 + 8*b*c^6*d^2)*(c + d*x)^(1/2))/(315*d^8)
- (x^6*(45*a*d^8 - 5*b*c^2*d^6)*(c + d*x)^(1/2))/(315*d^8))
```

3.338 $\int x^3 \sqrt{-c+dx} \sqrt{c+dx} (a+bx^2) \, dx$

Optimal. Leaf size=109

$$\frac{c^2 (bc^2+ad^2) (-c+dx)^{3/2} (c+dx)^{3/2}}{3d^6}+\frac{(2bc^2+ad^2) (-c+dx)^{5/2} (c+dx)^{5/2}}{5d^6}+\frac{b(-c+dx)^{7/2} (c+dx)^{7/2}}{7d^6}$$

```
[Out] 1/3*c^2*(a*d^2+b*c^2)*(d*x-c)^(3/2)*(d*x+c)^(3/2)/d^6+1/5*(a*d^2+2*b*c^2)*(
d*x-c)^(5/2)*(d*x+c)^(5/2)/d^6+1/7*b*(d*x-c)^(7/2)*(d*x+c)^(7/2)/d^6
```

Rubi [A]
time = 0.05, antiderivative size = 118, normalized size of antiderivative = 1.08, number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {471, 102, 12, 75}

$$\frac{2c^2 (dx-c)^{3/2} (c+dx)^{3/2} (7ad^2+4bc^2)}{105d^6}+\frac{x^2 (dx-c)^{3/2} (c+dx)^{3/2} (7ad^2+4bc^2)}{35d^4}+\frac{bx^4 (dx-c)^{3/2} (c+dx)^{3/2}}{7d^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[-c + d*x]*Sqrt[c + d*x]*(a + b*x^2),x]
```

```
[Out] (2*c^2*(4*b*c^2 + 7*a*d^2)*(-c + d*x)^(3/2)*(c + d*x)^(3/2))/(105*d^6) + ((
4*b*c^2 + 7*a*d^2)*x^2*(-c + d*x)^(3/2)*(c + d*x)^(3/2))/(35*d^4) + (b*x^4*
(-c + d*x)^(3/2)*(c + d*x)^(3/2))/(7*d^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 75

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 102

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/(d*f*(m + n + p + 1))), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[d*(e*x)^(
m + 1)*(a1 + b1*x^(n/2))^(p + 1)*((a2 + b2*x^(n/2))^(p + 1)/(b1*b2*e*(m + n
*(p + 1) + 1))), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\int x^3\sqrt{-c+dx}\,\sqrt{c+dx}\,\left(a+bx^2\right)\,dx=\frac{bx^4(-c+dx)^{3/2}(c+dx)^{3/2}}{7d^2}-\frac{1}{7}\left(-7a-\frac{4bc^2}{d^2}\right)\int x^3\sqrt{-c+dx}\,\sqrt{}$$

$$=\frac{(4bc^2+7ad^2)\,x^2(-c+dx)^{3/2}(c+dx)^{3/2}}{35d^4}+\frac{bx^4(-c+dx)^{3/2}(c+dx}{7d^2}$$

$$=\frac{(4bc^2+7ad^2)\,x^2(-c+dx)^{3/2}(c+dx)^{3/2}}{35d^4}+\frac{bx^4(-c+dx)^{3/2}(c+dx}{7d^2}$$

$$=\frac{2c^2(4bc^2+7ad^2)\,(-c+dx)^{3/2}(c+dx)^{3/2}}{105d^6}+\frac{(4bc^2+7ad^2)\,x^2(-c+}{35d^4}$$

Mathematica [A]
time = 0.13, size = 75, normalized size = 0.69

$$\frac{(-c+dx)^{3/2}(c+dx)^{3/2}\,(7ad^2(2c^2+3d^2x^2)+b(8c^4+12c^2d^2x^2+15d^4x^4))}{105d^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sqrt[-c + d*x]*Sqrt[c + d*x]*(a + b*x^2),x]
[Out] ((-c + d*x)^(3/2)*(c + d*x)^(3/2)*(7*a*d^2*(2*c^2 + 3*d^2*x^2) + b*(8*c^4 +
 12*c^2*d^2*x^2 + 15*d^4*x^4)))/(105*d^6)
```

Maple [A]
time = 0.31, size = 80, normalized size = 0.73

method	result	size
gosper	$\frac{(dx+c)^{\frac{3}{2}}\left(15b\,d^4x^4+21a\,d^4x^2+12b\,c^2d^2x^2+14a\,c^2d^2+8b\,c^4\right)(dx-c)^{\frac{3}{2}}}{105d^6}$	68
default	$-\frac{\sqrt{dx-c}\,\sqrt{dx+c}\,\left(-d^2x^2+c^2\right)\left(15b\,d^4x^4+21a\,d^4x^2+12b\,c^2d^2x^2+14a\,c^2d^2+8b\,c^4\right)}{105d^6}$	80

risch	$\frac{\sqrt{dx+c}\,(-15b\,x^6d^6-21a\,d^6x^4+3b\,c^2d^4x^4+7a\,c^2d^4x^2+4b\,c^4d^2x^2+14a\,c^4d^2+8b\,c^6)(-dx+c)}{105\sqrt{dx-c}\,d^6}$	98

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/105*(d*x-c)^(1/2)*(d*x+c)^(1/2)*(-d^2*x^2+c^2)*(15*b*d^4*x^4+21*a*d^4*x^
2+12*b*c^2*d^2*x^2+14*a*c^2*d^2+8*b*c^4)/d^6
```

Maxima [A]
time = 0.27, size = 124, normalized size = 1.14

$$\frac{(d^2x^2-c^2)^{\frac{3}{2}}bx^4}{7\,d^2}+\frac{4\,(d^2x^2-c^2)^{\frac{3}{2}}bc^2x^2}{35\,d^4}+\frac{(d^2x^2-c^2)^{\frac{3}{2}}ax^2}{5\,d^2}+\frac{8\,(d^2x^2-c^2)^{\frac{3}{2}}bc^4}{105\,d^6}+\frac{2\,(d^2x^2-c^2)^{\frac{3}{2}}ac^2}{15\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/7*(d^2*x^2 - c^2)^(3/2)*b*x^4/d^2 + 4/35*(d^2*x^2 - c^2)^(3/2)*b*c^2*x^2/
d^4 + 1/5*(d^2*x^2 - c^2)^(3/2)*a*x^2/d^2 + 8/105*(d^2*x^2 - c^2)^(3/2)*b*c
^4/d^6 + 2/15*(d^2*x^2 - c^2)^(3/2)*a*c^2/d^4
```

Fricas [A]
time = 3.65, size = 90, normalized size = 0.83

$$\frac{(15\,bd^6x^6-8\,bc^6-14\,ac^4d^2-3\,(bc^2d^4-7\,ad^6)x^4-(4\,bc^4d^2+7\,ac^2d^4)x^2)\sqrt{dx+c}\,\sqrt{dx-c}}{105\,d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/105*(15*b*d^6*x^6 - 8*b*c^6 - 14*a*c^4*d^2 - 3*(b*c^2*d^4 - 7*a*d^6)*x^4
- (4*b*c^4*d^2 + 7*a*c^2*d^4)*x^2)*sqrt(d*x + c)*sqrt(d*x - c)/d^6
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^3\left(a+bx^2\right)\sqrt{-c+dx}\,\sqrt{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**2+a)*(d*x-c)**(1/2)*(d*x+c)**(1/2),x)
```

```
[Out] Integral(x**3*(a + b*x**2)*sqrt(-c + d*x)*sqrt(c + d*x), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 495 vs. $2(91) = 182$.
time = 0.76, size = 495, normalized size = 4.54

$$\frac{70\left(\left((dx+c)\left(2(dx+c)\left(\frac{3(dx+c)}{d^3}-\frac{13c}{d^3}\right)+\frac{43c^2}{d^3}\right)-\frac{39c^3}{d^3}\right)\sqrt{dx+c}\,\sqrt{dx-c}-\frac{18c^4\log\left(\left|-\sqrt{dx+c}+\sqrt{dx-c}\right|\right)}{d^3}\right)ac+7\left(\left(\left(2\left((dx+c)\left(4(dx+c)\left(\frac{5(dx+c)}{d^5}-\frac{31c}{d^5}\right)+\frac{321c^2}{d^5}\right)-\frac{451c^3}{d^5}\right)(dx+c)+\frac{745c^4}{d^5}\right)(dx+c)-\frac{405c^5}{d^5}\right)\sqrt{dx+c}\,\sqrt{dx-c}-\frac{150c^6\log\left(\left|-\sqrt{dx+c}+\sqrt{dx-c}\right|\right)}{d^5}\right)bc+14\left(\left(\left(2(dx+c)\left(3(dx+c)\left(\frac{4(dx+c)}{d^4}-\frac{21c}{d^4}\right)+\frac{133c^2}{d^4}\right)-\frac{295c^3}{d^4}\right)(dx+c)+\frac{195c^4}{d^4}\right)\sqrt{dx+c}\,\sqrt{dx-c}+\frac{90c^5\log\left(\left|-\sqrt{dx+c}+\sqrt{dx-c}\right|\right)}{d^4}\right)ad+\left(\left(\left(2\left(\left(4(dx+c)\left(5(dx+c)\left(\frac{6(dx+c)}{d^6}-\frac{43c}{d^6}\right)+\frac{661c^2}{d^6}\right)-\frac{4551c^3}{d^6}\right)(dx+c)+\frac{4781c^4}{d^6}\right)(dx+c)-\frac{6335c^5}{d^6}\right)(dx+c)+\frac{2835c^6}{d^6}\right)\sqrt{dx+c}\,\sqrt{dx-c}+\frac{1050c^7\log\left(\left|-\sqrt{dx+c}+\sqrt{dx-c}\right|\right)}{d^6}\right)bd}{1680\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/1680*(70*(((d*x + c)*(2*(d*x + c)*(3*(d*x + c)/d^3 - 13*c/d^3) + 43*c^2/d
^3) - 39*c^3/d^3)*sqrt(d*x + c)*sqrt(d*x - c) - 18*c^4*log(abs(-sqrt(d*x +
c) + sqrt(d*x - c)))/d^3)*a*c + 7*(((2*((d*x + c)*(4*(d*x + c)*(5*(d*x + c)
/d^5 - 31*c/d^5) + 321*c^2/d^5) - 451*c^3/d^5)*(d*x + c) + 745*c^4/d^5)*(d*
x + c) - 405*c^5/d^5)*sqrt(d*x + c)*sqrt(d*x - c) - 150*c^6*log(abs(-sqrt(d
*x + c) + sqrt(d*x - c)))/d^5)*b*c + 14*(((2*(d*x + c)*(3*(d*x + c)*(4*(d*x
 + c)/d^4 - 21*c/d^4) + 133*c^2/d^4) - 295*c^3/d^4)*(d*x + c) + 195*c^4/d^4
)*sqrt(d*x + c)*sqrt(d*x - c) + 90*c^5*log(abs(-sqrt(d*x + c) + sqrt(d*x -
c)))/d^4)*a*d + (((2*((4*(d*x + c)*(5*(d*x + c)*(6*(d*x + c)/d^6 - 43*c/d^6
) + 661*c^2/d^6) - 4551*c^3/d^6)*(d*x + c) + 4781*c^4/d^6)*(d*x + c) - 6335
*c^5/d^6)*(d*x + c) + 2835*c^6/d^6)*sqrt(d*x + c)*sqrt(d*x - c) + 1050*c^7*
log(abs(-sqrt(d*x + c) + sqrt(d*x - c)))/d^6)*b*d)/d
```

Mupad [B]
time = 1.74, size = 118, normalized size = 1.08

$$-\sqrt{d\,x-c}\,\left(\frac{\left(8\,b\,c^6+14\,a\,c^4\,d^2\right)\,\sqrt{c+d\,x}}{105\,d^6}-\frac{b\,x^6\,\sqrt{c+d\,x}}{7}+\frac{x^2\,\left(4\,b\,c^4\,d^2+7\,a\,c^2\,d^4\right)\,\sqrt{c+d\,x}}{105\,d^6}-\frac{x^4\,\left(21\,a\,d^6-3\,b\,c^2\,d^4\right)\,\sqrt{c+d\,x}}{105\,d^6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x^2)*(c + d*x)^(1/2)*(d*x - c)^(1/2),x)
```

```
[Out] -(d*x - c)^(1/2)*(((8*b*c^6 + 14*a*c^4*d^2)*(c + d*x)^(1/2))/(105*d^6) - (b
*x^6*(c + d*x)^(1/2))/7 + (x^2*(7*a*c^2*d^4 + 4*b*c^4*d^2)*(c + d*x)^(1/2))
/(105*d^6) - (x^4*(21*a*d^6 - 3*b*c^2*d^4)*(c + d*x)^(1/2))/(105*d^6))
```

3.339 $\int x\sqrt{-c+dx}\,\sqrt{c+dx}\,(a+bx^2)\,dx$

Optimal. Leaf size=67

$$\frac{(bc^2+ad^2)\,(-c+dx)^{3/2}(c+dx)^{3/2}}{3d^4}+\frac{b(-c+dx)^{5/2}(c+dx)^{5/2}}{5d^4}$$

```
[Out] 1/3*(a*d^2+b*c^2)*(d*x-c)^(3/2)*(d*x+c)^(3/2)/d^4+1/5*b*(d*x-c)^(5/2)*(d*x+
c)^(5/2)/d^4
```

Rubi [A]

time = 0.03, antiderivative size = 72, normalized size of antiderivative = 1.07, number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}=0.069$, Rules used = {471, 75}

$$\frac{(dx-c)^{3/2}(c+dx)^{3/2}\,(5ad^2+2bc^2)}{15d^4}+\frac{bx^2(dx-c)^{3/2}(c+dx)^{3/2}}{5d^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[-c + d*x]*Sqrt[c + d*x]*(a + b*x^2),x]
```

```
[Out] ((2*b*c^2 + 5*a*d^2)*(-c + d*x)^(3/2)*(c + d*x)^(3/2))/(15*d^4) + (b*x^2*(-
c + d*x)^(3/2)*(c + d*x)^(3/2))/(5*d^2)
```

Rule 75

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[d*(e*x)^(
m + 1)*(a1 + b1*x^(n/2))^(p + 1)*((a2 + b2*x^(n/2))^(p + 1)/(b1*b2*e*(m + n
*(p + 1) + 1))), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}\int x\sqrt{-c+dx}\,\sqrt{c+dx}\,(a+bx^2)\,dx &= \frac{bx^2(-c+dx)^{3/2}(c+dx)^{3/2}}{5d^2}-\frac{1}{5}\left(-5a-\frac{2bc^2}{d^2}\right)\int x\sqrt{-c+dx}\,\sqrt{c\\ &= \frac{(2bc^2+5ad^2)\,(-c+dx)^{3/2}(c+dx)^{3/2}}{15d^4}+\frac{bx^2(-c+dx)^{3/2}(c+dx)^{3}}{5d^2}\end{aligned}$$

Mathematica [A]
time = 0.10, size = 49, normalized size = 0.73

$$\frac{(-c+dx)^{3/2}(c+dx)^{3/2}\,(2bc^2+5ad^2+3bd^2x^2)}{15d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[-c + d*x]*Sqrt[c + d*x]*(a + b*x^2),x]
```

```
[Out] ((-c + d*x)^(3/2)*(c + d*x)^(3/2)*(2*b*c^2 + 5*a*d^2 + 3*b*d^2*x^2))/(15*d^
4)
```

Maple [A]
time = 0.27, size = 56, normalized size = 0.84

method	result	size
gosper	$\frac{(dx+c)^{\frac{3}{2}}\left(3b\,d^2x^2+5a\,d^2+2b\,c^2\right)(dx-c)^{\frac{3}{2}}}{15d^4}$	44
default	$-\frac{\sqrt{dx-c}\,\sqrt{dx+c}\,\left(-d^2x^2+c^2\right)\left(3b\,d^2x^2+5a\,d^2+2b\,c^2\right)}{15d^4}$	56
risch	$\frac{\sqrt{dx+c}\,\left(-3b\,d^4x^4-5a\,d^4x^2+b\,c^2d^2x^2+5a\,c^2d^2+2b\,c^4\right)(-dx+c)}{15\sqrt{dx-c}\,d^4}$	73

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/15*(d*x-c)^(1/2)*(d*x+c)^(1/2)*(-d^2*x^2+c^2)*(3*b*d^2*x^2+5*a*d^2+2*b*c
^2)/d^4
```

Maxima [A]
time = 0.26, size = 70, normalized size = 1.04

$$\frac{(d^2x^2-c^2)^{\frac{3}{2}}bx^2}{5\,d^2}+\frac{2\,(d^2x^2-c^2)^{\frac{3}{2}}bc^2}{15\,d^4}+\frac{(d^2x^2-c^2)^{\frac{3}{2}}a}{3\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/5*(d^2*x^2 - c^2)^(3/2)*b*x^2/d^2 + 2/15*(d^2*x^2 - c^2)^(3/2)*b*c^2/d^4
+ 1/3*(d^2*x^2 - c^2)^(3/2)*a/d^2
```

Fricas [A]
time = 2.65, size = 66, normalized size = 0.99

$$\frac{(3\,bd^4x^4-2\,bc^4-5\,ac^2d^2-(bc^2d^2-5\,ad^4)x^2)\sqrt{dx+c}\,\sqrt{dx-c}}{15\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/15*(3*b*d^4*x^4 - 2*b*c^4 - 5*a*c^2*d^2 - (b*c^2*d^2 - 5*a*d^4)*x^2)*sqrt
(d*x + c)*sqrt(d*x - c)/d^4
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x(a + bx^2)\sqrt{-c + dx}\,\sqrt{c + dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**2+a)*(d*x-c)**(1/2)*(d*x+c)**(1/2),x)
```

```
[Out] Integral(x*(a + b*x**2)*sqrt(-c + d*x)*sqrt(c + d*x), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 361 vs. 2(55) = 110.
time = 0.74, size = 361, normalized size = 5.39

$$\frac{5\left(\left((dx+c)\left(2\,(dx+c)\left(\frac{3\,(dx+c)}{d^3}-\frac{13\,c}{d^3}\right)+\frac{43\,c^2}{d^3}\right)-\frac{39\,c^3}{d^3}\right)\sqrt{dx+c}\,\sqrt{dx-c}-\frac{18\,c^4\log\left(\left|-\sqrt{dx+c}+\sqrt{dx-c}\right|\right)}{d^3}\right)bc+20\left(\sqrt{dx+c}\,\sqrt{dx-c}\left((dx+c)\left(\frac{2\,(dx+c)}{d^2}-\frac{7\,c}{d^2}\right)+\frac{9\,c^2}{d^2}\right)+\frac{6\,c^3\log\left(\left|-\sqrt{dx+c}+\sqrt{dx-c}\right|\right)}{d^2}\right)ad+\left(\left(\left(2\,(dx+c)\left(3\,(dx+c)\left(\frac{4\,(dx+c)}{d^4}-\frac{21\,c}{d^4}\right)+\frac{133\,c^2}{d^4}\right)-\frac{295\,c^3}{d^4}\right)(dx+c)+\frac{195\,c^4}{d^4}\right)\sqrt{dx+c}\,\sqrt{dx-c}+\frac{90\,c^5\log\left(\left|-\sqrt{dx+c}+\sqrt{dx-c}\right|\right)}{d^4}\right)bd-\frac{60\left(2\,c^2\log\left(\left|-\sqrt{dx+c}+\sqrt{dx-c}\right|\right)-\sqrt{dx+c}\,\sqrt{dx-c}\,(dx-2\,c)\right)ac}{d}}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/120*(5*(((d*x + c)*(2*(d*x + c)*(3*(d*x + c)/d^3 - 13*c/d^3) + 43*c^2/d^3
) - 39*c^3/d^3)*sqrt(d*x + c)*sqrt(d*x - c) - 18*c^4*log(abs(-sqrt(d*x + c)
 + sqrt(d*x - c)))/d^3)*b*c + 20*(sqrt(d*x + c)*sqrt(d*x - c)*((d*x + c)*(2
*(d*x + c)/d^2 - 7*c/d^2) + 9*c^2/d^2) + 6*c^3*log(abs(-sqrt(d*x + c) + sqr
t(d*x - c)))/d^2)*a*d + (((2*(d*x + c)*(3*(d*x + c)*(4*(d*x + c)/d^4 - 21*c
/d^4) + 133*c^2/d^4) - 295*c^3/d^4)*(d*x + c) + 195*c^4/d^4)*sqrt(d*x + c)*
sqrt(d*x - c) + 90*c^5*log(abs(-sqrt(d*x + c) + sqrt(d*x - c)))/d^4)*b*d -
60*(2*c^2*log(abs(-sqrt(d*x + c) + sqrt(d*x - c))) - sqrt(d*x + c)*sqrt(d*x
 - c)*(d*x - 2*c))*a*c/d)/d
```

Mupad [B]
time = 1.64, size = 83, normalized size = 1.24

$$\sqrt{d\,x-c}\,\left(\frac{b\,x^4\,\sqrt{c+d\,x}}{5}-\frac{\left(2\,b\,c^4+5\,a\,c^2\,d^2\right)\,\sqrt{c+d\,x}}{15\,d^4}+\frac{x^2\,\left(5\,a\,d^4-b\,c^2\,d^2\right)\,\sqrt{c+d\,x}}{15\,d^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x^2)*(c + d*x)^(1/2)*(d*x - c)^(1/2),x)
```

```
[Out] (d*x - c)^(1/2)*((b*x^4*(c + d*x)^(1/2))/5 - ((2*b*c^4 + 5*a*c^2*d^2)*(c +
d*x)^(1/2))/(15*d^4) + (x^2*(5*a*d^4 - b*c^2*d^2)*(c + d*x)^(1/2))/(15*d^4)
)
```

3.340 $\int \frac{\sqrt{-c+dx}\,\sqrt{c+dx}\,(a+bx^2)}{x}\,dx$

Optimal. Leaf size=80

$$a\sqrt{-c+dx}\,\sqrt{c+dx}+\frac{b(-c+dx)^{3/2}(c+dx)^{3/2}}{3d^2}-ac\tan^{-1}\left(\frac{\sqrt{-c+dx}\,\sqrt{c+dx}}{c}\right)$$

```
[Out] 1/3*b*(d*x-c)^(3/2)*(d*x+c)^(3/2)/d^2-a*c*arctan((d*x-c)^(1/2)*(d*x+c)^(1/2
)/c)+a*(d*x-c)^(1/2)*(d*x+c)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {471, 103, 12, 94, 211}

$$-ac\text{ArcTan}\left(\frac{\sqrt{dx-c}\,\sqrt{c+dx}}{c}\right)+a\sqrt{dx-c}\,\sqrt{c+dx}+\frac{b(dx-c)^{3/2}(c+dx)^{3/2}}{3d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[-c + d*x]*Sqrt[c + d*x]*(a + b*x^2))/x,x]
```

```
[Out] a*Sqrt[-c + d*x]*Sqrt[c + d*x] + (b*(-c + d*x)^(3/2)*(c + d*x)^(3/2))/(3*d^
2) - a*c*ArcTan[(Sqrt[-c + d*x]*Sqrt[c + d*x])/c]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Simp[(a + b*x)^m*(c + d*x)^n*((e + f*x)^(p + 1)/(f*
(m + n + p + 1))), x] - Dist[1/(f*(m + n + p + 1)), Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1)*(e + f*x)^p*Simp[c*m*(b*e - a*f) + a*n*(d*e - c*f) + (d*m*
(b*e - a*f) + b*n*(d*e - c*f))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}
, x] && GtQ[m, 0] && GtQ[n, 0] && NeQ[m + n + p + 1, 0] && (IntegersQ[2*m,
```

```
2*n, 2*p] || (IntegersQ[m, n + p] || IntegersQ[p, m + n]))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[d*(e*x)^(
m + 1)*(a1 + b1*x^(n/2))^(p + 1)*((a2 + b2*x^(n/2))^(p + 1)/(b1*b2*e*(m + n
*(p + 1) + 1))), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\int \frac{\sqrt{-c+dx}\,\sqrt{c+dx}\,(a+bx^2)}{x}\,dx = \frac{b(-c+dx)^{3/2}(c+dx)^{3/2}}{3d^2} + a\int \frac{\sqrt{-c+dx}\,\sqrt{c+dx}}{x}\,dx$$
$$= a\sqrt{-c+dx}\,\sqrt{c+dx} + \frac{b(-c+dx)^{3/2}(c+dx)^{3/2}}{3d^2} - a\int \frac{\ldots}{x\sqrt{-c+\ldots}}$$
$$= a\sqrt{-c+dx}\,\sqrt{c+dx} + \frac{b(-c+dx)^{3/2}(c+dx)^{3/2}}{3d^2} - \left(ac^2\right)\int \frac{\ldots}{x\sqrt{-\ldots}}$$
$$= a\sqrt{-c+dx}\,\sqrt{c+dx} + \frac{b(-c+dx)^{3/2}(c+dx)^{3/2}}{3d^2} - \left(ac^2d\right)\text{Subst}\Big(\ldots$$
$$= a\sqrt{-c+dx}\,\sqrt{c+dx} + \frac{b(-c+dx)^{3/2}(c+dx)^{3/2}}{3d^2} - ac\tan^{-1}\left(\frac{\sqrt{-\ldots}}{\ldots}\right.$$

Mathematica [A]
time = 0.14, size = 75, normalized size = 0.94

$$\frac{\sqrt{-c+dx}\,\sqrt{c+dx}\,(-bc^2+3ad^2+bd^2x^2)}{3d^2} - 2ac\tan^{-1}\left(\frac{\sqrt{-c+dx}}{\sqrt{c+dx}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[-c + d*x]*Sqrt[c + d*x]*(a + b*x^2))/x,x]
```

```
[Out] (Sqrt[-c + d*x]*Sqrt[c + d*x]*(-(b*c^2) + 3*a*d^2 + b*d^2*x^2))/(3*d^2) - 2
*a*c*ArcTan[Sqrt[-c + d*x]/Sqrt[c + d*x]]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 173 vs. $2(66) = 132$.
time = 0.28, size = 174, normalized size = 2.18

method	result
default	$\frac{\sqrt{dx-c}\,\sqrt{dx+c}\,\left(b\,d^{2}x^{2}\sqrt{-c^{2}}\,\sqrt{d^{2}x^{2}-c^{2}}+3\ln\left(-\frac{2\left(c^{2}-\sqrt{-c^{2}}\,\sqrt{d^{2}x^{2}-c^{2}}\right)}{x}\right)a\,c^{2}d^{2}+3\sqrt{d^{2}x^{2}-c^{2}}}{3\sqrt{d^{2}x^{2}-c^{2}}\,d^{2}\sqrt{-c^{2}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(d*x-c)^(1/2)*(d*x+c)^(1/2)*(b*d^2*x^2*(-c^2)^(1/2)*(d^2*x^2-c^2)^(1/2)
+3*ln(-2*(c^2-(-c^2)^(1/2)*(d^2*x^2-c^2)^(1/2))/x)*a*c^2*d^2+3*(d^2*x^2-c^2
)^(1/2)*(-c^2)^(1/2)*a*d^2-b*c^2*(-c^2)^(1/2)*(d^2*x^2-c^2)^(1/2))/(d^2*x^2
-c^2)^(1/2)/d^2/(-c^2)^(1/2)
```

Maxima [A]
time = 0.51, size = 52, normalized size = 0.65

$$ac\arcsin\left(\frac{c}{d|x|}\right)+\sqrt{d^{2}x^{2}-c^{2}}\,a+\frac{\left(d^{2}x^{2}-c^{2}\right)^{\frac{3}{2}}b}{3\,d^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] a*c*arcsin(c/(d*abs(x))) + sqrt(d^2*x^2 - c^2)*a + 1/3*(d^2*x^2 - c^2)^(3/2
)*b/d^2
```

Fricas [A]
time = 1.89, size = 80, normalized size = 1.00

$$-\frac{6\,acd^{2}\arctan\left(-\frac{dx-\sqrt{dx+c}\,\sqrt{dx-c}}{c}\right)-\left(bd^{2}x^{2}-bc^{2}+3\,ad^{2}\right)\sqrt{dx+c}\,\sqrt{dx-c}}{3\,d^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] -1/3*(6*a*c*d^2*arctan(-(d*x - sqrt(d*x + c)*sqrt(d*x - c))/c) - (b*d^2*x^2
 - b*c^2 + 3*a*d^2)*sqrt(d*x + c)*sqrt(d*x - c))/d^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(a+bx^{2}\right)\sqrt{-c+dx}\,\sqrt{c+dx}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(d*x-c)**(1/2)*(d*x+c)**(1/2)/x,x)
```

```
[Out] Integral((a + b*x**2)*sqrt(-c + d*x)*sqrt(c + d*x)/x, x)
```

Giac [A]
time = 0.63, size = 78, normalized size = 0.98

$$2\,ac\arctan\left(\frac{\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^2}{2\,c}\right)+\frac{1}{3}\sqrt{dx+c}\,\sqrt{dx-c}\,\left((dx+c)\left(\frac{(dx+c)b}{d^2}-\frac{2\,bc}{d^2}\right)+3\,a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2)/x,x, algorithm="giac")
```

```
[Out] 2*a*c*arctan(1/2*(sqrt(d*x + c) - sqrt(d*x - c))^2/c) + 1/3*sqrt(d*x + c)*s
qrt(d*x - c)*((d*x + c)*((d*x + c)*b/d^2 - 2*b*c/d^2) + 3*a)
```

Mupad [B]
time = 3.60, size = 248, normalized size = 3.10

$$a\sqrt{-c}\,\sqrt{c}\,\ln\left(\frac{\left(\sqrt{c+dx}-\sqrt{c}\right)^2}{\left(\sqrt{-c}-\sqrt{dx-c}\right)^2}+1\right)-a\sqrt{-c}\,\sqrt{c}\,\ln\left(\frac{\sqrt{c+dx}-\sqrt{c}}{\sqrt{-c}-\sqrt{dx-c}}\right)-\frac{b\,(c^2-d^2x^2)\,\sqrt{c+dx}\,\sqrt{dx-c}}{3\,d^2}-\frac{8\,a\sqrt{-c}\,\sqrt{c}\,\left(\sqrt{c+dx}-\sqrt{c}\right)^2}{\left(\sqrt{-c}-\sqrt{dx-c}\right)^2\left(\frac{\left(\sqrt{c+dx}-\sqrt{c}\right)^4}{\left(\sqrt{-c}-\sqrt{dx-c}\right)^4}-\frac{2\left(\sqrt{c+dx}-\sqrt{c}\right)^2}{\left(\sqrt{-c}-\sqrt{dx-c}\right)^2}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)*(c + d*x)^(1/2)*(d*x - c)^(1/2))/x,x)
```

```
[Out] a*(-c)^(1/2)*c^(1/2)*log(((c + d*x)^(1/2) - c^(1/2))^2/((-c)^(1/2) - (d*x -
 c)^(1/2))^2 + 1) - a*(-c)^(1/2)*c^(1/2)*log(((c + d*x)^(1/2) - c^(1/2))/((
-c)^(1/2) - (d*x - c)^(1/2))) - (b*(c^2 - d^2*x^2)*(c + d*x)^(1/2)*(d*x - c
)^(1/2))/(3*d^2) - (8*a*(-c)^(1/2)*c^(1/2)*((c + d*x)^(1/2) - c^(1/2))^2)/(
((-c)^(1/2) - (d*x - c)^(1/2))^2*(((c + d*x)^(1/2) - c^(1/2))^4/((-c)^(1/2)
 - (d*x - c)^(1/2))^4 - (2*((c + d*x)^(1/2) - c^(1/2))^2)/((-c)^(1/2) - (d*
x - c)^(1/2))^2 + 1))
```

3.341 $\int \frac{\sqrt{-c+dx}\,\sqrt{c+dx}\,(a+bx^2)}{x^3}\,dx$

Optimal. Leaf size=96

$$b\sqrt{-c+dx}\,\sqrt{c+dx}-\frac{a\sqrt{-c+dx}\,\sqrt{c+dx}}{2x^2}-\frac{(2bc^2-ad^2)\tan^{-1}\left(\frac{\sqrt{-c+dx}\,\sqrt{c+dx}}{c}\right)}{2c}$$

```
[Out] -1/2*(-a*d^2+2*b*c^2)*arctan((d*x-c)^(1/2)*(d*x+c)^(1/2)/c)/c+b*(d*x-c)^(1/
2)*(d*x+c)^(1/2)-1/2*a*(d*x-c)^(1/2)*(d*x+c)^(1/2)/x^2
```

Rubi [A]
time = 0.06, antiderivative size = 114, normalized size of antiderivative = 1.19, number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {465, 103, 12, 94, 211}

$$-\frac{(2bc^2-ad^2)\,\text{ArcTan}\left(\frac{\sqrt{dx-c}\,\sqrt{c+dx}}{c}\right)}{2c}+\frac{1}{2}\sqrt{dx-c}\,\sqrt{c+dx}\left(2b-\frac{ad^2}{c^2}\right)+\frac{a(dx-c)^{3/2}(c+dx)^{3/2}}{2c^2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[-c + d*x]*Sqrt[c + d*x]*(a + b*x^2))/x^3,x]
```

```
[Out] ((2*b - (a*d^2)/c^2)*Sqrt[-c + d*x]*Sqrt[c + d*x])/2 + (a*(-c + d*x)^(3/2)*
(c + d*x)^(3/2))/(2*c^2*x^2) - ((2*b*c^2 - a*d^2)*ArcTan[(Sqrt[-c + d*x]*Sq
rt[c + d*x])/c])/(2*c)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Simp[(a + b*x)^m*(c + d*x)^n*((e + f*x)^(p + 1)/(f*
(m + n + p + 1))), x] - Dist[1/(f*(m + n + p + 1)), Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1)*(e + f*x)^p*Simp[c*m*(b*e - a*f) + a*n*(d*e - c*f) + (d*m*
(b*e - a*f) + b*n*(d*e - c*f))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}
```

```
, x] && GtQ[m, 0] && GtQ[n, 0] && NeQ[m + n + p + 1, 0] && (IntegersQ[2*m,
2*n, 2*p] || (IntegersQ[m, n + p] || IntegersQ[p, m + n]))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 465

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[c*(e*x)^(
m + 1)*(a1 + b1*x^(n/2))^(p + 1)*((a2 + b2*x^(n/2))^(p + 1)/(a1*a2*e*(m + 1
))), x] + Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/(a1*a2*e^n*(
m + 1)), Int[(e*x)^(m + n)*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/2))^p, x], x]
/; FreeQ[{a1, b1, a2, b2, c, d, e, p}, x] && EqQ[non2, n/2] && EqQ[a2*b1 +
a1*b2, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (L
tQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{-c+dx}\,\sqrt{c+dx}\,(a+bx^2)}{x^3}\,dx &= \frac{a(-c+dx)^{3/2}(c+dx)^{3/2}}{2c^2x^2} + \frac{1}{2}\left(2b-\frac{ad^2}{c^2}\right)\int \frac{\sqrt{-c+dx}\,\sqrt{c+dx}}{x} \\
&= \frac{1}{2}\left(2b-\frac{ad^2}{c^2}\right)\sqrt{-c+dx}\,\sqrt{c+dx} + \frac{a(-c+dx)^{3/2}(c+dx)^{3/2}}{2c^2x^2} + \\
&= \frac{1}{2}\left(2b-\frac{ad^2}{c^2}\right)\sqrt{-c+dx}\,\sqrt{c+dx} + \frac{a(-c+dx)^{3/2}(c+dx)^{3/2}}{2c^2x^2} + \\
&= \frac{1}{2}\left(2b-\frac{ad^2}{c^2}\right)\sqrt{-c+dx}\,\sqrt{c+dx} + \frac{a(-c+dx)^{3/2}(c+dx)^{3/2}}{2c^2x^2} - \\
&= \frac{1}{2}\left(2b-\frac{ad^2}{c^2}\right)\sqrt{-c+dx}\,\sqrt{c+dx} + \frac{a(-c+dx)^{3/2}(c+dx)^{3/2}}{2c^2x^2} -
\end{aligned}$$

Mathematica [A]
time = 0.18, size = 74, normalized size = 0.77

$$\frac{\sqrt{-c+dx}\,\sqrt{c+dx}\,(-a+2bx^2)}{2x^2} + \left(-2bc+\frac{ad^2}{c}\right)\tan^{-1}\left(\frac{\sqrt{-c+dx}}{\sqrt{c+dx}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[-c + d*x]*Sqrt[c + d*x]*(a + b*x^2))/x^3,x]
```

```
[Out] (Sqrt[-c + d*x]*Sqrt[c + d*x]*(-a + 2*b*x^2))/(2*x^2) + (-2*b*c + (a*d^2)/c
)*ArcTan[Sqrt[-c + d*x]/Sqrt[c + d*x]]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 181 vs. $2(80) = 160$.
time = 0.32, size = 182, normalized size = 1.90

method	result
risch	$\frac{a(-dx+c)\sqrt{dx+c}}{2x^2\sqrt{dx-c}} - \frac{\left(-b\sqrt{(dx-c)(dx+c)} + \frac{\ln\left(\frac{-2c^2+2\sqrt{-c^2}\sqrt{d^2x^2-c^2}}{x}\right)a\,d^2}{2\sqrt{-c^2}} - \frac{\ln\left(\frac{-2c^2+2\sqrt{-c^2}\sqrt{d}}{x}\right.}{\sqrt{-c^2}}\right.}{\sqrt{dx-c}\,\sqrt{dx+c}}$
default	$-\frac{\sqrt{dx-c}\,\sqrt{dx+c}\left(\ln\left(-\frac{2\left(c^2-\sqrt{-c^2}\,\sqrt{d^2x^2-c^2}\right)}{x}\right)a\,d^2x^2-2\ln\left(-\frac{2\left(c^2-\sqrt{-c^2}\,\sqrt{d^2x^2-c^2}\right)}{x}\right)b\,c^2x^2-\right.}{2\sqrt{d^2x^2-c^2}\,x^2\sqrt{-c^2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(d*x-c)^(1/2)*(d*x+c)^(1/2)*(ln(-2*(c^2-(-c^2)^(1/2)*(d^2*x^2-c^2)^(1/
2))/x)*a*d^2*x^2-2*ln(-2*(c^2-(-c^2)^(1/2)*(d^2*x^2-c^2)^(1/2))/x)*b*c^2*x^
2-2*b*x^2*(-c^2)^(1/2)*(d^2*x^2-c^2)^(1/2)+(d^2*x^2-c^2)^(1/2)*(-c^2)^(1/2)
*a)/(d^2*x^2-c^2)^(1/2)/x^2/(-c^2)^(1/2)
```

Maxima [A]
time = 0.48, size = 98, normalized size = 1.02

$$bc\arcsin\left(\frac{c}{d|x|}\right) - \frac{ad^2\arcsin\left(\frac{c}{d|x|}\right)}{2\,c} + \sqrt{d^2x^2-c^2}\,b - \frac{\sqrt{d^2x^2-c^2}\,ad^2}{2\,c^2} + \frac{(d^2x^2-c^2)^{\frac{3}{2}}a}{2\,c^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] b*c*arcsin(c/(d*abs(x))) - 1/2*a*d^2*arcsin(c/(d*abs(x)))/c + sqrt(d^2*x^2
- c^2)*b - 1/2*sqrt(d^2*x^2 - c^2)*a*d^2/c^2 + 1/2*(d^2*x^2 - c^2)^(3/2)*a/
(c^2*x^2)
```

Fricas [A]
time = 2.39, size = 85, normalized size = 0.89

$$-\frac{2\,(2\,bc^2-ad^2)x^2\arctan\left(-\frac{dx-\sqrt{dx+c}\,\sqrt{dx-c}}{c}\right)-(2\,bcx^2-ac)\sqrt{dx+c}\,\sqrt{dx-c}}{2\,cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] -1/2*(2*(2*b*c^2 - a*d^2)*x^2*arctan(-(d*x - sqrt(d*x + c)*sqrt(d*x - c))/c
) - (2*b*c*x^2 - a*c)*sqrt(d*x + c)*sqrt(d*x - c))/(c*x^2)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx^2)\sqrt{-c+dx}\,\sqrt{c+dx}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(d*x-c)**(1/2)*(d*x+c)**(1/2)/x**3,x)
```

```
[Out] Integral((a + b*x**2)*sqrt(-c + d*x)*sqrt(c + d*x)/x**3, x)
```

Giac [**A**]
time = 0.64, size = 157, normalized size = 1.64

$$\frac{\sqrt{dx+c}\,\sqrt{dx-c}\,bd+\frac{(2\,bc^2d-ad^3)\arctan\left(\frac{\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^2}{2\,c}\right)}{c}+\frac{2\left(ad^3\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^6-4\,ac^2d^3\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^2\right)}{\left(\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^4+4\,c^2\right)^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] (sqrt(d*x + c)*sqrt(d*x - c)*b*d + (2*b*c^2*d - a*d^3)*arctan(1/2*(sqrt(d*x
 + c) - sqrt(d*x - c))^2/c)/c + 2*(a*d^3*(sqrt(d*x + c) - sqrt(d*x - c))^6
- 4*a*c^2*d^3*(sqrt(d*x + c) - sqrt(d*x - c))^2)/((sqrt(d*x + c) - sqrt(d*x
 - c))^4 + 4*c^2)^2)/d
```

Mupad [**B**]
time = 6.89, size = 584, normalized size = 6.08

$$b\sqrt{-c}\,\sqrt{c}\,\ln\left(\frac{\left(\sqrt{c+dx}-\sqrt{c}\right)^2}{\left(\sqrt{-c}-\sqrt{dx-c}\right)^2}+1\right)-\frac{\frac{a\sqrt{-c}\,d^2}{32\,c^{3/2}}+\frac{a\sqrt{-c}\,d^2\left(\sqrt{c+dx}-\sqrt{c}\right)^2}{16\,c^{3/2}\left(\sqrt{-c}-\sqrt{dx-c}\right)^2}-\frac{15\,a\sqrt{-c}\,d^2\left(\sqrt{c+dx}-\sqrt{c}\right)^4}{32\,c^{3/2}\left(\sqrt{-c}-\sqrt{dx-c}\right)^4}}{\frac{\left(\sqrt{c+dx}-\sqrt{c}\right)^2}{\left(\sqrt{-c}-\sqrt{dx-c}\right)^2}+\frac{2\left(\sqrt{c+dx}-\sqrt{c}\right)^4}{\left(\sqrt{-c}-\sqrt{dx-c}\right)^4}+\frac{\left(\sqrt{c+dx}-\sqrt{c}\right)^6}{\left(\sqrt{-c}-\sqrt{dx-c}\right)^6}}-b\sqrt{-c}\,\sqrt{c}\,\ln\left(\frac{\sqrt{c+dx}-\sqrt{c}}{\sqrt{-c}-\sqrt{dx-c}}\right)+\frac{a\sqrt{-c}\,d^2\ln\left(\frac{\sqrt{c+dx}-\sqrt{c}}{\sqrt{-c}-\sqrt{dx-c}}\right)}{2\,c^{3/2}}-\frac{a\sqrt{-c}\,d^2\ln\left(\frac{\left(\sqrt{c+dx}-\sqrt{c}\right)^2}{\left(\sqrt{-c}-\sqrt{dx-c}\right)^2}+1\right)}{2\,c^{3/2}}-\frac{a\sqrt{-c}\,d^2\left(\sqrt{c+dx}-\sqrt{c}\right)^2}{32\,c^{3/2}\left(\sqrt{-c}-\sqrt{dx-c}\right)^2}-\frac{8\,b\sqrt{-c}\,\sqrt{c}\left(\sqrt{c+dx}-\sqrt{c}\right)^2}{\left(\sqrt{-c}-\sqrt{dx-c}\right)^2\left(\frac{\left(\sqrt{c+dx}-\sqrt{c}\right)^4}{\left(\sqrt{-c}-\sqrt{dx-c}\right)^4}-\frac{2\left(\sqrt{c+dx}-\sqrt{c}\right)^2}{\left(\sqrt{-c}-\sqrt{dx-c}\right)^2}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)*(c + d*x)^(1/2)*(d*x - c)^(1/2))/x^3,x)
```

```
[Out] b*(-c)^(1/2)*c^(1/2)*log(((c + d*x)^(1/2) - c^(1/2))^2/((-c)^(1/2) - (d*x -
 c)^(1/2))^2 + 1) - ((a*(-c)^(1/2)*d^2)/(32*c^(3/2)) + (a*(-c)^(1/2)*d^2*((
c + d*x)^(1/2) - c^(1/2))^2)/(16*c^(3/2)*((-c)^(1/2) - (d*x - c)^(1/2))^2)
- (15*a*(-c)^(1/2)*d^2*((c + d*x)^(1/2) - c^(1/2))^4)/(32*c^(3/2)*((-c)^(1/
2) - (d*x - c)^(1/2))^4))/(((c + d*x)^(1/2) - c^(1/2))^2/((-c)^(1/2) - (d*x
```

```
 - c)^(1/2))^2 + (2*((c + d*x)^(1/2) - c^(1/2))^4)/((-c)^(1/2) - (d*x - c)^
(1/2))^4 + ((c + d*x)^(1/2) - c^(1/2))^6/((-c)^(1/2) - (d*x - c)^(1/2))^6)
- b*(-c)^(1/2)*c^(1/2)*log(((c + d*x)^(1/2) - c^(1/2))/((-c)^(1/2) - (d*x -
 c)^(1/2))) + (a*(-c)^(1/2)*d^2*log(((c + d*x)^(1/2) - c^(1/2))/((-c)^(1/2)
 - (d*x - c)^(1/2))))/(2*c^(3/2)) - (a*(-c)^(1/2)*d^2*log(((c + d*x)^(1/2)
- c^(1/2))^2/((-c)^(1/2) - (d*x - c)^(1/2))^2 + 1))/(2*c^(3/2)) - (a*(-c)^(
1/2)*d^2*((c + d*x)^(1/2) - c^(1/2))^2)/(32*c^(3/2)*((-c)^(1/2) - (d*x - c)
^(1/2))^2) - (8*b*(-c)^(1/2)*c^(1/2)*((c + d*x)^(1/2) - c^(1/2))^2)/(((-c)^
(1/2) - (d*x - c)^(1/2))^2*(((c + d*x)^(1/2) - c^(1/2))^4/((-c)^(1/2) - (d*
x - c)^(1/2))^4 - (2*((c + d*x)^(1/2) - c^(1/2))^2)/((-c)^(1/2) - (d*x - c)
^(1/2))^2 + 1))
```

3.342 $\int \frac{\sqrt{-c+dx}\,\sqrt{c+dx}\,(a+bx^2)}{x^5}\,dx$

Optimal. Leaf size=121

$$-\frac{(4bc^2+ad^2)\sqrt{-c+dx}\,\sqrt{c+dx}}{8c^2x^2}+\frac{a(-c+dx)^{3/2}(c+dx)^{3/2}}{4c^2x^4}+\frac{d^2(4bc^2+ad^2)\tan^{-1}\left(\frac{\sqrt{-c+dx}\,\sqrt{c+dx}}{c}\right)}{8c^3}$$

```
[Out] 1/4*a*(d*x-c)^(3/2)*(d*x+c)^(3/2)/c^2/x^4+1/8*d^2*(a*d^2+4*b*c^2)*arctan((d
*x-c)^(1/2)*(d*x+c)^(1/2)/c)/c^3-1/8*(a*d^2+4*b*c^2)*(d*x-c)^(1/2)*(d*x+c)^
(1/2)/c^2/x^2
```

Rubi [A]
time = 0.07, antiderivative size = 164, normalized size of antiderivative = 1.36, number of steps used = 5, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {465, 96, 94, 211}

$$\frac{d^2(ad^2+4bc^2)\,\text{ArcTan}\left(\frac{\sqrt{dx-c}\,\sqrt{c+dx}}{c}\right)}{8c^3}-\frac{\sqrt{dx-c}\,(c+dx)^{3/2}\,(ad^2+4bc^2)}{8c^3x^2}+\frac{d\sqrt{dx-c}\,\sqrt{c+dx}\,(ad^2+4bc^2)}{8c^3x}+\frac{a(dx-c)^{3/2}(c+dx)^{3/2}}{4c^2x^4}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[-c + d*x]*Sqrt[c + d*x]*(a + b*x^2))/x^5,x]
```

```
[Out] (d*(4*b*c^2 + a*d^2)*Sqrt[-c + d*x]*Sqrt[c + d*x])/(8*c^3*x) - ((4*b*c^2 +
a*d^2)*Sqrt[-c + d*x]*(c + d*x)^(3/2))/(8*c^3*x^2) + (a*(-c + d*x)^(3/2)*(c
 + d*x)^(3/2))/(4*c^2*x^4) + (d^2*(4*b*c^2 + a*d^2)*ArcTan[(Sqrt[-c + d*x]*
Sqrt[c + d*x])/c])/(8*c^3)
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 465

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[c*(e*x)^(
m + 1)*(a1 + b1*x^(n/2))^(p + 1)*((a2 + b2*x^(n/2))^(p + 1)/(a1*a2*e*(m + 1
))), x] + Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/(a1*a2*e^n*(
m + 1)), Int[(e*x)^(m + n)*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/2))^p, x], x]
/; FreeQ[{a1, b1, a2, b2, c, d, e, p}, x] && EqQ[non2, n/2] && EqQ[a2*b1 +
a1*b2, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (L
tQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\int \frac{\sqrt{-c+dx}\,\sqrt{c+dx}\,(a+bx^2)}{x^5}\,dx = \frac{a(-c+dx)^{3/2}(c+dx)^{3/2}}{4c^2x^4} + \frac{1}{4}\left(4b+\frac{ad^2}{c^2}\right)\int \frac{\sqrt{-c+dx}\,\sqrt{c+dx}}{x^3}\,d$$

$$= -\frac{(4bc^2+ad^2)\sqrt{-c+dx}\,(c+dx)^{3/2}}{8c^3x^2} + \frac{a(-c+dx)^{3/2}(c+dx)^{3/2}}{4c^2x^4} +$$

$$= \frac{d(4bc^2+ad^2)\sqrt{-c+dx}\,\sqrt{c+dx}}{8c^3x} - \frac{(4bc^2+ad^2)\sqrt{-c+dx}\,(c+d}{8c^3x^2}$$

$$= \frac{d(4bc^2+ad^2)\sqrt{-c+dx}\,\sqrt{c+dx}}{8c^3x} - \frac{(4bc^2+ad^2)\sqrt{-c+dx}\,(c+d}{8c^3x^2}$$

$$= \frac{d(4bc^2+ad^2)\sqrt{-c+dx}\,\sqrt{c+dx}}{8c^3x} - \frac{(4bc^2+ad^2)\sqrt{-c+dx}\,(c+d}{8c^3x^2}$$

Mathematica [**A**]
time = 0.17, size = 99, normalized size = 0.82

$$\frac{c\sqrt{-c+dx}\,\sqrt{c+dx}\,(-2ac^2-4bc^2x^2+ad^2x^2)+2d^2(4bc^2+ad^2)\,x^4\tan^{-1}\left(\frac{\sqrt{-c+dx}}{\sqrt{c+dx}}\right)}{8c^3x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[-c + d*x]*Sqrt[c + d*x]*(a + b*x^2))/x^5,x]
```

```
[Out] (c*Sqrt[-c + d*x]*Sqrt[c + d*x]*(-2*a*c^2 - 4*b*c^2*x^2 + a*d^2*x^2) + 2*d^
2*(4*b*c^2 + a*d^2)*x^4*ArcTan[Sqrt[-c + d*x]/Sqrt[c + d*x]])/(8*c^3*x^4)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 225 vs. $2(103) = 206$.
time = 0.30, size = 226, normalized size = 1.87

method	result
risch	$\frac{\sqrt{dx+c}\,(-dx+c)(-a\,d^2x^2+4b\,c^2x^2+2c^2a)}{8x^4c^2\sqrt{dx-c}} - \frac{\left(\frac{d^4\ln\left(\frac{-2c^2+2\sqrt{-c^2}\,\sqrt{d^2x^2-c^2}}{x}\right)a}{8c^2\sqrt{-c^2}} + \frac{d^2\ln\left(\frac{-2c^2+2\sqrt{-c^2}\,\sqrt{d^2x \cdots}}{x}\right)\cdots}{2\sqrt{-c^2}}\cdots\right)}{\sqrt{dx-c}\,\sqrt{dx+c}\cdots}$
default	$-\frac{\sqrt{dx-c}\,\sqrt{dx+c}\,\left(\ln\left(-\frac{2\left(c^2-\sqrt{-c^2}\,\sqrt{d^2x^2-c^2}\right)}{x}\right)a\,d^4x^4+4\ln\left(-\frac{2\left(c^2-\sqrt{-c^2}\,\sqrt{d^2x^2-c^2}\right)}{x}\right)b\,c^2d\cdots\right)}{8c^2\sqrt{d^2x^2-c^2}\,x^4\sqrt{\cdots}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2)/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*(d*x-c)^(1/2)*(d*x+c)^(1/2)/c^2*(ln(-2*(c^2-(-c^2)^(1/2)*(d^2*x^2-c^2)
^(1/2))/x)*a*d^4*x^4+4*ln(-2*(c^2-(-c^2)^(1/2)*(d^2*x^2-c^2)^(1/2))/x)*b*c^
2*d^2*x^4-(d^2*x^2-c^2)^(1/2)*(-c^2)^(1/2)*a*d^2*x^2+4*(d^2*x^2-c^2)^(1/2)*
(-c^2)^(1/2)*b*c^2*x^2+2*(d^2*x^2-c^2)^(1/2)*(-c^2)^(1/2)*a*c^2)/(d^2*x^2-c
^2)^(1/2)/x^4/(-c^2)^(1/2)
```

Maxima [A]
time = 0.52, size = 162, normalized size = 1.34

$$-\frac{bd^2\arcsin\left(\frac{c}{d|x|}\right)}{2\,c}-\frac{ad^4\arcsin\left(\frac{c}{d|x|}\right)}{8\,c^3}-\frac{\sqrt{d^2x^2-c^2}\,bd^2}{2\,c^2}-\frac{\sqrt{d^2x^2-c^2}\,ad^4}{8\,c^4}+\frac{(d^2x^2-c^2)^{\frac{3}{2}}b}{2\,c^2x^2}+\frac{(d^2x^2-c^2)^{\frac{3}{2}}ad^2}{8\,c^4x^2}+\frac{(d^2x^2-c^2)^{\frac{3}{2}}a}{4\,c^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2)/x^5,x, algorithm="maxima")
```

```
[Out] -1/2*b*d^2*arcsin(c/(d*abs(x)))/c - 1/8*a*d^4*arcsin(c/(d*abs(x)))/c^3 - 1/
2*sqrt(d^2*x^2 - c^2)*b*d^2/c^2 - 1/8*sqrt(d^2*x^2 - c^2)*a*d^4/c^4 + 1/2*(
d^2*x^2 - c^2)^(3/2)*b/(c^2*x^2) + 1/8*(d^2*x^2 - c^2)^(3/2)*a*d^2/(c^4*x^2
) + 1/4*(d^2*x^2 - c^2)^(3/2)*a/(c^2*x^4)
```

Fricas [A]
time = 2.64, size = 100, normalized size = 0.83

$$\frac{2\,(4\,bc^2d^2+ad^4)x^4\arctan\left(-\frac{dx-\sqrt{dx+c}\,\sqrt{dx-c}}{c}\right)-(2\,ac^3+(4\,bc^3-acd^2)x^2)\sqrt{dx+c}\,\sqrt{dx-c}}{8\,c^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2)/x^5,x, algorithm="fricas")
```

```
[Out] 1/8*(2*(4*b*c^2*d^2 + a*d^4)*x^4*arctan(-(d*x - sqrt(d*x + c)*sqrt(d*x - c)
)/c) - (2*a*c^3 + (4*b*c^3 - a*c*d^2)*x^2)*sqrt(d*x + c)*sqrt(d*x - c))/(c^
3*x^4)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(d*x-c)**(1/2)*(d*x+c)**(1/2)/x**5,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 324 vs. 2(103) = 206.
time = 0.64, size = 324, normalized size = 2.68

$$-\frac{\frac{(4bc^2d^3+ad^5)\arctan\left(\frac{\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^2}{2c}\right)}{c^3}-\frac{2\left(4bc^2d^3\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^{14}-ad^5\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^{14}+16bc^4d^3\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^{10}+28ac^2d^5\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^{10}-64bc^6d^3\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^{6}-112ac^4d^5\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^{6}-256bc^8d^3\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^{2}+64ac^6d^5\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^{2}\right)}{\left(\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^4+4c^2\right)^4c^2}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2)/x^5,x, algorithm="giac")
```

```
[Out] -1/4*((4*b*c^2*d^3 + a*d^5)*arctan(1/2*(sqrt(d*x + c) - sqrt(d*x - c))^2/c)
/c^3 - 2*(4*b*c^2*d^3*(sqrt(d*x + c) - sqrt(d*x - c))^14 - a*d^5*(sqrt(d*x
+ c) - sqrt(d*x - c))^14 + 16*b*c^4*d^3*(sqrt(d*x + c) - sqrt(d*x - c))^10
+ 28*a*c^2*d^5*(sqrt(d*x + c) - sqrt(d*x - c))^10 - 64*b*c^6*d^3*(sqrt(d*x
+ c) - sqrt(d*x - c))^6 - 112*a*c^4*d^5*(sqrt(d*x + c) - sqrt(d*x - c))^6 -
 256*b*c^8*d^3*(sqrt(d*x + c) - sqrt(d*x - c))^2 + 64*a*c^6*d^5*(sqrt(d*x +
 c) - sqrt(d*x - c))^2)/(((sqrt(d*x + c) - sqrt(d*x - c))^4 + 4*c^2)^4*c^2)
)/d
```

Mupad [B]
time = 15.56, size = 1004, normalized size = 8.30

$$\frac{\frac{a\sqrt{-c}\,d^4}{1024c^{7/2}}+\frac{a\sqrt{-c}\,d^4\left(\sqrt{c+dx}-\sqrt{c}\right)^2}{128c^{7/2}\left(\sqrt{-c}-\sqrt{dx-c}\right)^2}+\frac{11a\sqrt{-c}\,d^4\left(\sqrt{c+dx}-\sqrt{c}\right)^4}{512c^{7/2}\left(\sqrt{-c}-\sqrt{dx-c}\right)^4}+\frac{7a\sqrt{-c}\,d^4\left(\sqrt{c+dx}-\sqrt{c}\right)^6}{256c^{7/2}\left(\sqrt{-c}-\sqrt{dx-c}\right)^6}-\frac{239a\sqrt{-c}\,d^4\left(\sqrt{c+dx}-\sqrt{c}\right)^8}{1024c^{7/2}\left(\sqrt{-c}-\sqrt{dx-c}\right)^8}+\frac{a\sqrt{-c}\,d^4\left(\sqrt{c+dx}-\sqrt{c}\right)^{10}}{256c^{7/2}\left(\sqrt{-c}-\sqrt{dx-c}\right)^{10}}}{\frac{\left(\sqrt{c+dx}-\sqrt{c}\right)^4}{\left(\sqrt{-c}-\sqrt{dx-c}\right)^4}+\frac{4\left(\sqrt{c+dx}-\sqrt{c}\right)^6}{\left(\sqrt{-c}-\sqrt{dx-c}\right)^6}+\frac{6\left(\sqrt{c+dx}-\sqrt{c}\right)^8}{\left(\sqrt{-c}-\sqrt{dx-c}\right)^8}+\frac{4\left(\sqrt{c+dx}-\sqrt{c}\right)^{10}}{\left(\sqrt{-c}-\sqrt{dx-c}\right)^{10}}+\frac{\left(\sqrt{c+dx}-\sqrt{c}\right)^{12}}{\left(\sqrt{-c}-\sqrt{dx-c}\right)^{12}}}-\frac{\frac{b\sqrt{-c}\,d^2}{32c^{3/2}}+\frac{b\sqrt{-c}\,d^2\left(\sqrt{c+dx}-\sqrt{c}\right)^2}{16c^{3/2}\left(\sqrt{-c}-\sqrt{dx-c}\right)^2}-\frac{15b\sqrt{-c}\,d^2\left(\sqrt{c+dx}-\sqrt{c}\right)^4}{32c^{3/2}\left(\sqrt{-c}-\sqrt{dx-c}\right)^4}}{\frac{\left(\sqrt{c+dx}-\sqrt{c}\right)^2}{\left(\sqrt{-c}-\sqrt{dx-c}\right)^2}+\frac{2\left(\sqrt{c+dx}-\sqrt{c}\right)^4}{\left(\sqrt{-c}-\sqrt{dx-c}\right)^4}+\frac{\left(\sqrt{c+dx}-\sqrt{c}\right)^6}{\left(\sqrt{-c}-\sqrt{dx-c}\right)^6}}+\frac{a\sqrt{-c}\,d^4\ln\left(\frac{\sqrt{c+dx}-\sqrt{c}}{\sqrt{-c}-\sqrt{dx-c}}\right)}{8c^{7/2}}+\frac{b\sqrt{-c}\,d^2\ln\left(\frac{\sqrt{c+dx}-\sqrt{c}}{\sqrt{-c}-\sqrt{dx-c}}\right)}{2c^{3/2}}-\frac{a\sqrt{-c}\,d^4\ln\left(\frac{\left(\sqrt{c+dx}-\sqrt{c}\right)^2}{\left(\sqrt{-c}-\sqrt{dx-c}\right)^2}+1\right)}{8c^{7/2}}-\frac{b\sqrt{-c}\,d^2\ln\left(\frac{\left(\sqrt{c+dx}-\sqrt{c}\right)^2}{\left(\sqrt{-c}-\sqrt{dx-c}\right)^2}+1\right)}{2c^{3/2}}+\frac{a\sqrt{-c}\,d^4\left(\sqrt{c+dx}-\sqrt{c}\right)^2}{256c^{7/2}\left(\sqrt{-c}-\sqrt{dx-c}\right)^2}+\frac{a\sqrt{-c}\,d^4\left(\sqrt{c+dx}-\sqrt{c}\right)^4}{1024c^{7/2}\left(\sqrt{-c}-\sqrt{dx-c}\right)^4}-\frac{b\sqrt{-c}\,d^2\left(\sqrt{c+dx}-\sqrt{c}\right)^2}{32c^{3/2}\left(\sqrt{-c}-\sqrt{dx-c}\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)*(c + d*x)^(1/2)*(d*x - c)^(1/2))/x^5,x)
```

```
[Out] ((a*(-c)^(1/2)*d^4)/(1024*c^(7/2)) + (a*(-c)^(1/2)*d^4*((c + d*x)^(1/2) - c
^(1/2))^2)/(128*c^(7/2)*((-c)^(1/2) - (d*x - c)^(1/2))^2) + (11*a*(-c)^(1/2
)*d^4*((c + d*x)^(1/2) - c^(1/2))^4)/(512*c^(7/2)*((-c)^(1/2) - (d*x - c)^(
1/2))^4) + (7*a*(-c)^(1/2)*d^4*((c + d*x)^(1/2) - c^(1/2))^6)/(256*c^(7/2)*
```

((-c)^(1/2) - (d*x - c)^(1/2))^6) - (239*a*(-c)^(1/2)*d^4*((c + d*x)^(1/2) - c^(1/2))^8)/(1024*c^(7/2)*((-c)^(1/2) - (d*x - c)^(1/2))^8) + (a*(-c)^(1/2)*d^4*((c + d*x)^(1/2) - c^(1/2))^10)/(256*c^(7/2)*((-c)^(1/2) - (d*x - c)^(1/2))^10))/(((c + d*x)^(1/2) - c^(1/2))^4/((-c)^(1/2) - (d*x - c)^(1/2))^4 + (4*((c + d*x)^(1/2) - c^(1/2))^6)/((-c)^(1/2) - (d*x - c)^(1/2))^6 + (6*((c + d*x)^(1/2) - c^(1/2))^8)/((-c)^(1/2) - (d*x - c)^(1/2))^8 + (4*((c + d*x)^(1/2) - c^(1/2))^10)/((-c)^(1/2) - (d*x - c)^(1/2))^10 + ((c + d*x)^(1/2) - c^(1/2))^12/((-c)^(1/2) - (d*x - c)^(1/2))^12) - ((b*(-c)^(1/2)*d^2)/(32*c^(3/2)) + (b*(-c)^(1/2)*d^2*((c + d*x)^(1/2) - c^(1/2))^2)/(16*c^(3/2)*((-c)^(1/2) - (d*x - c)^(1/2))^2) - (15*b*(-c)^(1/2)*d^2*((c + d*x)^(1/2) - c^(1/2))^4)/(32*c^(3/2)*((-c)^(1/2) - (d*x - c)^(1/2))^4))/(((c + d*x)^(1/2) - c^(1/2))^2/((-c)^(1/2) - (d*x - c)^(1/2))^2 + (2*((c + d*x)^(1/2) - c^(1/2))^4)/((-c)^(1/2) - (d*x - c)^(1/2))^4 + ((c + d*x)^(1/2) - c^(1/2))^6/((-c)^(1/2) - (d*x - c)^(1/2))^6) + (a*(-c)^(1/2)*d^4*log(((c + d*x)^(1/2) - c^(1/2))/((-c)^(1/2) - (d*x - c)^(1/2))))/(8*c^(7/2)) + (b*(-c)^(1/2)*d^2*log(((c + d*x)^(1/2) - c^(1/2))/((-c)^(1/2) - (d*x - c)^(1/2))))/(2*c^(3/2)) - (a*(-c)^(1/2)*d^4*log(((c + d*x)^(1/2) - c^(1/2))^2/((-c)^(1/2) - (d*x - c)^(1/2))^2 + 1))/(8*c^(7/2)) - (b*(-c)^(1/2)*d^2*log(((c + d*x)^(1/2) - c^(1/2))^2/((-c)^(1/2) - (d*x - c)^(1/2))^2 + 1))/(2*c^(3/2)) + (a*(-c)^(1/2)*d^4*((c + d*x)^(1/2) - c^(1/2))^2)/(256*c^(7/2)*((-c)^(1/2) - (d*x - c)^(1/2))^2) + (a*(-c)^(1/2)*d^4*((c + d*x)^(1/2) - c^(1/2))^4)/(1024*c^(7/2)*((-c)^(1/2) - (d*x - c)^(1/2))^4) - (b*(-c)^(1/2)*d^2*((c + d*x)^(1/2) - c^(1/2))^2)/(32*c^(3/2)*((-c)^(1/2) - (d*x - c)^(1/2))^2)

3.343 $\int x^4\sqrt{-c+dx}\,\sqrt{c+dx}\,(a+bx^2)\,dx$

Optimal. Leaf size=208

$$\frac{c^4(5bc^2+8ad^2)\,x\sqrt{-c+dx}\,\sqrt{c+dx}}{128d^6}+\frac{c^2(5bc^2+8ad^2)\,x(-c+dx)^{3/2}(c+dx)^{3/2}}{64d^6}+\frac{(5bc^2+8ad^2)\,x^3(-c+dx}{48d^4}$$

```
[Out] 1/64*c^2*(8*a*d^2+5*b*c^2)*x*(d*x-c)^(3/2)*(d*x+c)^(3/2)/d^6+1/48*(8*a*d^2+
5*b*c^2)*x^3*(d*x-c)^(3/2)*(d*x+c)^(3/2)/d^4+1/8*b*x^5*(d*x-c)^(3/2)*(d*x+c
)^(3/2)/d^2-1/64*c^6*(8*a*d^2+5*b*c^2)*arctanh((d*x-c)^(1/2)/(d*x+c)^(1/2))
/d^7+1/128*c^4*(8*a*d^2+5*b*c^2)*x*(d*x-c)^(1/2)*(d*x+c)^(1/2)/d^6
```

Rubi [A]
time = 0.09, antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.258$, Rules used = {471, 102, 12, 92, 38, 65, 223, 212}

$$\frac{c^2x(dx-c)^{3/2}(c+dx)^{3/2}\,(8ad^2+5bc^2)}{64d^6}+\frac{x^3(dx-c)^{3/2}(c+dx)^{3/2}\,(8ad^2+5bc^2)}{48d^4}-\frac{c^6(8ad^2+5bc^2)\tanh^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)}{64d^7}+\frac{c^4x\sqrt{dx-c}\,\sqrt{c+dx}\,(8ad^2+5bc^2)}{128d^6}+\frac{bx^5(dx-c)^{3/2}(c+dx)^{3/2}}{8d^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*Sqrt[-c + d*x]*Sqrt[c + d*x]*(a + b*x^2),x]
```

```
[Out] (c^4*(5*b*c^2 + 8*a*d^2)*x*Sqrt[-c + d*x]*Sqrt[c + d*x])/(128*d^6) + (c^2*(
5*b*c^2 + 8*a*d^2)*x*(-c + d*x)^(3/2)*(c + d*x)^(3/2))/(64*d^6) + ((5*b*c^2
 + 8*a*d^2)*x^3*(-c + d*x)^(3/2)*(c + d*x)^(3/2))/(48*d^4) + (b*x^5*(-c + d
*x)^(3/2)*(c + d*x)^(3/2))/(8*d^2) - (c^6*(5*b*c^2 + 8*a*d^2)*ArcTanh[Sqrt[
-c + d*x]/Sqrt[c + d*x]])/(64*d^7)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[x*
(a + b*x)^m*((c + d*x)^m/(2*m + 1)), x] + Dist[2*a*c*(m/(2*m + 1)), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 92

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(
d*f*(n + p + 3))), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 102

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/(d*f*(m + n + p + 1))), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[d*(e*x)^(
m + 1)*(a1 + b1*x^(n/2))^(p + 1)*((a2 + b2*x^(n/2))^(p + 1)/(b1*b2*e*(m + n
*(p + 1) + 1))), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\int x^4\sqrt{-c+dx}\,\sqrt{c+dx}\,(a+bx^2)\,dx = \frac{bx^5(-c+dx)^{3/2}(c+dx)^{3/2}}{8d^2} - \frac{1}{8}\left(-8a-\frac{5bc^2}{d^2}\right)\int x^4\sqrt{-c+dx}\,\sqrt{}$$

$$= \frac{(5bc^2+8ad^2)\,x^3(-c+dx)^{3/2}(c+dx)^{3/2}}{48d^4} + \frac{bx^5(-c+dx)^{3/2}(c+dx}{8d^2}$$

$$= \frac{(5bc^2+8ad^2)\,x^3(-c+dx)^{3/2}(c+dx)^{3/2}}{48d^4} + \frac{bx^5(-c+dx)^{3/2}(c+dx}{8d^2}$$

$$= \frac{c^2(5bc^2+8ad^2)\,x(-c+dx)^{3/2}(c+dx)^{3/2}}{64d^6} + \frac{(5bc^2+8ad^2)\,x^3(-c+}{48d^4}$$

$$= \frac{c^2(5bc^2+8ad^2)\,x(-c+dx)^{3/2}(c+dx)^{3/2}}{64d^6} + \frac{(5bc^2+8ad^2)\,x^3(-c+}{48d^4}$$

$$= \frac{c^4(5bc^2+8ad^2)\,x\sqrt{-c+dx}\,\sqrt{c+dx}}{128d^6} + \frac{c^2(5bc^2+8ad^2)\,x(-c+d}{64d^6}$$

$$= \frac{c^4(5bc^2+8ad^2)\,x\sqrt{-c+dx}\,\sqrt{c+dx}}{128d^6} + \frac{c^2(5bc^2+8ad^2)\,x(-c+d}{64d^6}$$

$$= \frac{c^4(5bc^2+8ad^2)\,x\sqrt{-c+dx}\,\sqrt{c+dx}}{128d^6} + \frac{c^2(5bc^2+8ad^2)\,x(-c+d}{64d^6}$$

$$= \frac{c^4(5bc^2+8ad^2)\,x\sqrt{-c+dx}\,\sqrt{c+dx}}{128d^6} + \frac{c^2(5bc^2+8ad^2)\,x(-c+d}{64d^6}$$

Mathematica [A]
time = 0.30, size = 142, normalized size = 0.68

$$\frac{dx\sqrt{-c+dx}\,\sqrt{c+dx}\,(8ad^2(-3c^4-2c^2d^2x^2+8d^4x^4)-b(15c^6+10c^4d^2x^2+8c^2d^4x^4-48d^6x^6))-6c^6(5bc^2+8ad^2)\tanh^{-1}\left(\frac{\sqrt{-c+dx}}{\sqrt{c+dx}}\right)}{384d^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*Sqrt[-c + d*x]*Sqrt[c + d*x]*(a + b*x^2),x]
```

```
[Out] (d*x*Sqrt[-c + d*x]*Sqrt[c + d*x]*(8*a*d^2*(-3*c^4 - 2*c^2*d^2*x^2 + 8*d^4*
x^4) - b*(15*c^6 + 10*c^4*d^2*x^2 + 8*c^2*d^4*x^4 - 48*d^6*x^6)) - 6*c^6*(5
*b*c^2 + 8*a*d^2)*ArcTanh[Sqrt[-c + d*x]/Sqrt[c + d*x]])/(384*d^7)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.29, size = 298, normalized size = 1.43

method	result

risch	$\frac{x\left(-48b\,x^{6}d^{6}-64a\,d^{6}x^{4}+8b\,c^{2}d^{4}x^{4}+16a\,c^{2}d^{4}x^{2}+10b\,c^{4}d^{2}x^{2}+24a\,c^{4}d^{2}+15b\,c^{6}\right)(-dx+c)\sqrt{dx+c}}{384d^{6}\sqrt{dx-c}}-\frac{\left(\frac{c^{6}\ln\left(\frac{d^{2}x}{\sqrt{d^{2}}}+\sqrt{d^{2}x^{2}}\right.}{16d^{4}\sqrt{d^{2}}}\right.}{}$
default	$-\frac{\sqrt{dx-c}\,\sqrt{dx+c}\,\left(-48\,\mathrm{csgn}(d)b\,d^{7}x^{7}\sqrt{d^{2}x^{2}-c^{2}}-64\,\mathrm{csgn}(d)a\,d^{7}x^{5}\sqrt{d^{2}x^{2}-c^{2}}+8\,\mathrm{csgn}(d)b\,c^{2}d^{5}x^{5}\sqrt{d^{2}x^{2}-\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/384*(d*x-c)^(1/2)*(d*x+c)^(1/2)*(-48*csgn(d)*b*d^7*x^7*(d^2*x^2-c^2)^(1/
2)-64*csgn(d)*a*d^7*x^5*(d^2*x^2-c^2)^(1/2)+8*csgn(d)*b*c^2*d^5*x^5*(d^2*x^
2-c^2)^(1/2)+16*csgn(d)*a*c^2*d^5*x^3*(d^2*x^2-c^2)^(1/2)+10*csgn(d)*b*c^4*
d^3*x^3*(d^2*x^2-c^2)^(1/2)+24*(d^2*x^2-c^2)^(1/2)*csgn(d)*d^3*a*c^4*x+15*(
d^2*x^2-c^2)^(1/2)*csgn(d)*d*b*c^6*x+24*ln(((d^2*x^2-c^2)^(1/2)*csgn(d)+d*x
)*csgn(d))*a*c^6*d^2+15*ln(((d^2*x^2-c^2)^(1/2)*csgn(d)+d*x)*csgn(d))*b*c^8
)*csgn(d)/(d^2*x^2-c^2)^(1/2)/d^7
```

Maxima [A]
time = 0.28, size = 246, normalized size = 1.18

$$\frac{(d^2x^2-c^2)^{\frac{3}{2}}bx^5}{8\,d^2}+\frac{5\,(d^2x^2-c^2)^{\frac{3}{2}}bc^2x^3}{48\,d^4}+\frac{(d^2x^2-c^2)^{\frac{3}{2}}ax^3}{6\,d^2}-\frac{5\,bc^8\log\left(2\,d^2x+2\,\sqrt{d^2x^2-c^2}\,d\right)}{128\,d^7}-\frac{ac^6\log\left(2\,d^2x+2\,\sqrt{d^2x^2-c^2}\,d\right)}{16\,d^5}+\frac{5\,\sqrt{d^2x^2-c^2}\,bc^6x}{128\,d^6}+\frac{\sqrt{d^2x^2-c^2}\,ac^4x}{16\,d^4}+\frac{5\,(d^2x^2-c^2)^{\frac{3}{2}}bc^4x}{64\,d^6}+\frac{(d^2x^2-c^2)^{\frac{3}{2}}ac^2x}{8\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/8*(d^2*x^2 - c^2)^(3/2)*b*x^5/d^2 + 5/48*(d^2*x^2 - c^2)^(3/2)*b*c^2*x^3/
d^4 + 1/6*(d^2*x^2 - c^2)^(3/2)*a*x^3/d^2 - 5/128*b*c^8*log(2*d^2*x + 2*sqr
t(d^2*x^2 - c^2)*d)/d^7 - 1/16*a*c^6*log(2*d^2*x + 2*sqrt(d^2*x^2 - c^2)*d)
/d^5 + 5/128*sqrt(d^2*x^2 - c^2)*b*c^6*x/d^6 + 1/16*sqrt(d^2*x^2 - c^2)*a*c
^4*x/d^4 + 5/64*(d^2*x^2 - c^2)^(3/2)*b*c^4*x/d^6 + 1/8*(d^2*x^2 - c^2)^(3/
2)*a*c^2*x/d^4
```

Fricas [A]
time = 3.86, size = 138, normalized size = 0.66

$$\frac{(48\,bd^7x^7-8\,(bc^2d^5-8\,ad^7)x^5-2\,(5\,bc^4d^3+8\,ac^2d^5)x^3-3\,(5\,bc^6d+8\,ac^4d^3)x)\sqrt{dx+c}\,\sqrt{dx-c}+3\,(5\,bc^8+8\,ac^6d^2)\log\left(-dx+\sqrt{dx+c}\,\sqrt{dx-c}\right)}{384\,d^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/384*((48*b*d^7*x^7 - 8*(b*c^2*d^5 - 8*a*d^7)*x^5 - 2*(5*b*c^4*d^3 + 8*a*c
^2*d^5)*x^3 - 3*(5*b*c^6*d + 8*a*c^4*d^3)*x)*sqrt(d*x + c)*sqrt(d*x - c) +
3*(5*b*c^8 + 8*a*c^6*d^2)*log(-d*x + sqrt(d*x + c)*sqrt(d*x - c)))/d^7
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^4(a+bx^2)\sqrt{-c+dx}\,\sqrt{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**2+a)*(d*x-c)**(1/2)*(d*x+c)**(1/2),x)
```

```
[Out] Integral(x**4*(a + b*x**2)*sqrt(-c + d*x)*sqrt(c + d*x), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 558 vs. 2(178) = 356.
time = 0.67, size = 558, normalized size = 2.68

$$\frac{112\left(\left(\left(2\,(dx+c)\left(3\,(dx+c)\left(\frac{4\,(dx+c)}{d^4}-\frac{21\,c}{d^4}\right)+\frac{133\,c^2}{d^4}\right)-\frac{295\,c^3}{d^4}\right)(dx+c)+\frac{195\,c^4}{d^4}\right)\sqrt{dx+c}\,\sqrt{dx-c}+\frac{90\,c^5\log\left(\left|-\sqrt{dx+c}+\sqrt{dx-c}\right|\right)}{d^4}\right)ac+8\left(\left(\left(2\left(\left(4\,(dx+c)\left(5\,(dx+c)\left(\frac{6\,(dx+c)}{d^6}-\frac{43\,c}{d^6}\right)+\frac{661\,c^2}{d^6}\right)-\frac{4551\,c^3}{d^6}\right)(dx+c)+\frac{4781\,c^4}{d^6}\right)(dx+c)-\frac{6335\,c^5}{d^6}\right)(dx+c)+\frac{2835\,c^6}{d^6}\right)\sqrt{dx+c}\,\sqrt{dx-c}+\frac{1050\,c^7\log\left(\left|-\sqrt{dx+c}+\sqrt{dx-c}\right|\right)}{d^6}\right)bc+56\left(\left(\left(2\left((dx+c)\left(4\,(dx+c)\left(\frac{5\,(dx+c)}{d^5}-\frac{31\,c}{d^5}\right)+\frac{321\,c^2}{d^5}\right)-\frac{451\,c^3}{d^5}\right)(dx+c)+\frac{745\,c^4}{d^5}\right)(dx+c)-\frac{405\,c^5}{d^5}\right)\sqrt{dx+c}\,\sqrt{dx-c}-\frac{150\,c^6\log\left(\left|-\sqrt{dx+c}+\sqrt{dx-c}\right|\right)}{d^5}\right)ad+\left(\left(\left(2\left(\left(4\left(5\,(dx+c)\left(6\,(dx+c)\left(\frac{7\,(dx+c)}{d^7}-\frac{57\,c}{d^7}\right)+\frac{1219\,c^2}{d^7}\right)-\frac{12463\,c^3}{d^7}\right)(dx+c)+\frac{64233\,c^4}{d^7}\right)(dx+c)-\frac{53963\,c^5}{d^7}\right)(dx+c)+\frac{59465\,c^6}{d^7}\right)(dx+c)-\frac{23205\,c^7}{d^7}\right)\sqrt{dx+c}\,\sqrt{dx-c}-\frac{7350\,c^8\log\left(\left|-\sqrt{dx+c}+\sqrt{dx-c}\right|\right)}{d^7}\right)bd}{13440\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/13440*(112*(((2*(d*x + c)*(3*(d*x + c)*(4*(d*x + c)/d^4 - 21*c/d^4) + 133
*c^2/d^4) - 295*c^3/d^4)*(d*x + c) + 195*c^4/d^4)*sqrt(d*x + c)*sqrt(d*x -
c) + 90*c^5*log(abs(-sqrt(d*x + c) + sqrt(d*x - c)))/d^4)*a*c + 8*(((2*((4*
(d*x + c)*(5*(d*x + c)*(6*(d*x + c)/d^6 - 43*c/d^6) + 661*c^2/d^6) - 4551*c
^3/d^6)*(d*x + c) + 4781*c^4/d^6)*(d*x + c) - 6335*c^5/d^6)*(d*x + c) + 283
5*c^6/d^6)*sqrt(d*x + c)*sqrt(d*x - c) + 1050*c^7*log(abs(-sqrt(d*x + c) +
sqrt(d*x - c)))/d^6)*b*c + 56*(((2*((d*x + c)*(4*(d*x + c)*(5*(d*x + c)/d^5
 - 31*c/d^5) + 321*c^2/d^5) - 451*c^3/d^5)*(d*x + c) + 745*c^4/d^5)*(d*x +
c) - 405*c^5/d^5)*sqrt(d*x + c)*sqrt(d*x - c) - 150*c^6*log(abs(-sqrt(d*x +
 c) + sqrt(d*x - c)))/d^5)*a*d + (((2*((4*(5*(d*x + c)*(6*(d*x + c)*(7*(d*x
 + c)/d^7 - 57*c/d^7) + 1219*c^2/d^7) - 12463*c^3/d^7)*(d*x + c) + 64233*c^
4/d^7)*(d*x + c) - 53963*c^5/d^7)*(d*x + c) + 59465*c^6/d^7)*(d*x + c) - 23
205*c^7/d^7)*sqrt(d*x + c)*sqrt(d*x - c) - 7350*c^8*log(abs(-sqrt(d*x + c)
+ sqrt(d*x - c)))/d^7)*b*d)/d
```

Mupad [B]
time = 39.15, size = 2314, normalized size = 11.12

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*x^2)*(c + d*x)^(1/2)*(d*x - c)^(1/2),x)
```

```
[Out] ((35*a*c^6*((c + d*x)^(1/2) - c^(1/2))^3)/(12*((-c)^(1/2) - (d*x - c)^(1/2)
)^3) - (a*c^6*((c + d*x)^(1/2) - c^(1/2)))/(4*((-c)^(1/2) - (d*x - c)^(1/2)
)) + (757*a*c^6*((c + d*x)^(1/2) - c^(1/2))^5)/(4*((-c)^(1/2) - (d*x - c)^(
```

1/2))^5) + (7339*a*c^6*((c + d*x)^(1/2) - c^(1/2))^7)/(4*((-c)^(1/2) - (d*x - c)^(1/2))^7) + (41929*a*c^6*((c + d*x)^(1/2) - c^(1/2))^9)/(6*((-c)^(1/2) - (d*x - c)^(1/2))^9) + (25661*a*c^6*((c + d*x)^(1/2) - c^(1/2))^11)/(2*((-c)^(1/2) - (d*x - c)^(1/2))^11) + (25661*a*c^6*((c + d*x)^(1/2) - c^(1/2))^13)/(2*((-c)^(1/2) - (d*x - c)^(1/2))^13) + (41929*a*c^6*((c + d*x)^(1/2) - c^(1/2))^15)/(6*((-c)^(1/2) - (d*x - c)^(1/2))^15) + (7339*a*c^6*((c + d*x)^(1/2) - c^(1/2))^17)/(4*((-c)^(1/2) - (d*x - c)^(1/2))^17) + (757*a*c^6*((c + d*x)^(1/2) - c^(1/2))^19)/(4*((-c)^(1/2) - (d*x - c)^(1/2))^19) + (35*a*c^6*((c + d*x)^(1/2) - c^(1/2))^21)/(12*((-c)^(1/2) - (d*x - c)^(1/2))^21) - (a*c^6*((c + d*x)^(1/2) - c^(1/2))^23)/(4*((-c)^(1/2) - (d*x - c)^(1/2))^23))/(d^5 - (12*d^5*((c + d*x)^(1/2) - c^(1/2))^2)/((-c)^(1/2) - (d*x - c)^(1/2))^2 + (66*d^5*((c + d*x)^(1/2) - c^(1/2))^4)/((-c)^(1/2) - (d*x - c)^(1/2))^4 - (220*d^5*((c + d*x)^(1/2) - c^(1/2))^6)/((-c)^(1/2) - (d*x - c)^(1/2))^6 + (495*d^5*((c + d*x)^(1/2) - c^(1/2))^8)/((-c)^(1/2) - (d*x - c)^(1/2))^8 - (792*d^5*((c + d*x)^(1/2) - c^(1/2))^10)/((-c)^(1/2) - (d*x - c)^(1/2))^10 + (924*d^5*((c + d*x)^(1/2) - c^(1/2))^12)/((-c)^(1/2) - (d*x - c)^(1/2))^12 - (792*d^5*((c + d*x)^(1/2) - c^(1/2))^14)/((-c)^(1/2) - (d*x - c)^(1/2))^14 + (495*d^5*((c + d*x)^(1/2) - c^(1/2))^16)/((-c)^(1/2) - (d*x - c)^(1/2))^16 - (220*d^5*((c + d*x)^(1/2) - c^(1/2))^18)/((-c)^(1/2) - (d*x - c)^(1/2))^18 + (66*d^5*((c + d*x)^(1/2) - c^(1/2))^20)/((-c)^(1/2) - (d*x - c)^(1/2))^20 - (12*d^5*((c + d*x)^(1/2) - c^(1/2))^22)/((-c)^(1/2) - (d*x - c)^(1/2))^22 + (d^5*((c + d*x)^(1/2) - c^(1/2))^24)/((-c)^(1/2) - (d*x - c)^(1/2))^24) - ((5*b*c^8*((c + d*x)^(1/2) - c^(1/2)))/(32*((-c)^(1/2) - (d*x - c)^(1/2))) - (235*b*c^8*((c + d*x)^(1/2) - c^(1/2))^3)/(96*((-c)^(1/2) - (d*x - c)^(1/2))^3) + (1723*b*c^8*((c + d*x)^(1/2) - c^(1/2))^5)/(96*((-c)^(1/2) - (d*x - c)^(1/2))^5) + (72283*b*c^8*((c + d*x)^(1/2) - c^(1/2))^7)/(32*((-c)^(1/2) - (d*x - c)^(1/2))^7) + (848801*b*c^8*((c + d*x)^(1/2) - c^(1/2))^9)/(32*((-c)^(1/2) - (d*x - c)^(1/2))^9) + (4181067*b*c^8*((c + d*x)^(1/2) - c^(1/2))^11)/(32*((-c)^(1/2) - (d*x - c)^(1/2))^11) + (10994181*b*c^8*((c + d*x)^(1/2) - c^(1/2))^13)/(32*((-c)^(1/2) - (d*x - c)^(1/2))^13) + (17457599*b*c^8*((c + d*x)^(1/2) - c^(1/2))^15)/(32*((-c)^(1/2) - (d*x - c)^(1/2))^15) + (17457599*b*c^8*((c + d*x)^(1/2) - c^(1/2))^17)/(32*((-c)^(1/2) - (d*x - c)^(1/2))^17) + (10994181*b*c^8*((c + d*x)^(1/2) - c^(1/2))^19)/(32*((-c)^(1/2) - (d*x - c)^(1/2))^19) + (4181067*b*c^8*((c + d*x)^(1/2) - c^(1/2))^21)/(32*((-c)^(1/2) - (d*x - c)^(1/2))^21) + (848801*b*c^8*((c + d*x)^(1/2) - c^(1/2))^23)/(32*((-c)^(1/2) - (d*x - c)^(1/2))^23) + (72283*b*c^8*((c + d*x)^(1/2) - c^(1/2))^25)/(32*((-c)^(1/2) - (d*x - c)^(1/2))^25) + (1723*b*c^8*((c + d*x)^(1/2) - c^(1/2))^27)/(96*((-c)^(1/2) - (d*x - c)^(1/2))^27) - (235*b*c^8*((c + d*x)^(1/2) - c^(1/2))^29)/(96*((-c)^(1/2) - (d*x - c)^(1/2))^29) + (5*b*c^8*((c + d*x)^(1/2) - c^(1/2))^31)/(32*((-c)^(1/2) - (d*x - c)^(1/2))^31))/(d^7 - (16*d^7*((c + d*x)^(1/2) - c^(1/2))^2)/((-c)^(1/2) - (d*x - c)^(1/2))^2 + (120*d^7*((c + d*x)^(1/2) - c^(1/2))^4)/((-c)^(1/2) - (d*x - c)^(1/2))^4 - (560*d^7*((c + d*x)^(1/2) - c^(1/2))^6)/((-c)^(1/2) - (d*x - c)^(1/2))^6 + (1820*d^7*((c + d*x)^(1/2) - c^(1/2))^8)/((-c)^(1/2) - (d*x - c)^(1/2))^8 - (4368*d^7*((c + d*x)^(1/2)

- c^(1/2))^10)/((-c)^(1/2) - (d*x - c)^(1/2))^10 + (8008*d^7*((c + d*x)^(1/2) - c^(1/2))^12)/((-c)^(1/2) - (d*x - c)^(1/2))^12 - (11440*d^7*((c + d*x)^(1/2) - c^(1/2))^14)/((-c)^(1/2) - (d*x - c)^(1/2))^14 + (12870*d^7*((c + d*x)^(1/2) - c^(1/2))^16)/((-c)^(1/2) - (d*x - c)^(1/2))^16 - (11440*d^7*((c + d*x)^(1/2) - c^(1/2))^18)/((-c)^(1/2) - (d*x - c)^(1/2))^18 + (8008*d^7*((c + d*x)^(1/2) - c^(1/2))^20)/((-c)^(1/2) - (d*x - c)^(1/2))^20 - (4368*d^7*((c + d*x)^(1/2) - c^(1/2))^22)/((-c)^(1/2) - (d*x - c)^(1/2))^22 + (1820*d^7*((c + d*x)^(1/2) - c^(1/2))^24)/((-c)^(1/2) - (d*x - c)^(1/2))^24 - (560*d^7*((c + d*x)^(1/2) - c^(1/2))^26)/((-c)^(1/2) - (d*x - c)^(1/2))^26 + (120*d^7*((c + d*x)^(1/2) - c^(1/2))^28)/((-c)^(1/2) - (d*x - c)^(1/2))^28 - (16*d^7*((c + d*x)^(1/2) - c^(1/2))^30)/((-c)^(1/2) - (d*x - c)^(1/2))^30 + (d^7*((c + d*x)^(1/2) - c^(1/2))^32)/((-c)^(1/2) - (d*x - c)^(1/2))^32) + (a*c^6*atanh(((c + d*x)^(1/2) - c^(1/2))/((-c)^(1/2) - (d*x - c)^(1/2))))/(4*d^5) + (5*b*c^8*atanh(((c + d*x)^(1/2) - c^(1/2))/((-c)^(1/2) - (d*x - c)^(1/2))))/(32*d^7)

3.344 $\int x^2 \sqrt{-c+dx}\,\sqrt{c+dx}\,(a+bx^2)\,dx$

Optimal. Leaf size=159

$$\frac{c^2(bc^2+2ad^2)\,x\sqrt{-c+dx}\,\sqrt{c+dx}}{16d^4}+\frac{(bc^2+2ad^2)\,x(-c+dx)^{3/2}(c+dx)^{3/2}}{8d^4}+\frac{bx^3(-c+dx)^{3/2}(c+dx)^{3/2}}{6d^2}$$

```
[Out] 1/8*(2*a*d^2+b*c^2)*x*(d*x-c)^(3/2)*(d*x+c)^(3/2)/d^4+1/6*b*x^3*(d*x-c)^(3/
2)*(d*x+c)^(3/2)/d^2-1/8*c^4*(2*a*d^2+b*c^2)*arctanh((d*x-c)^(1/2)/(d*x+c)^
(1/2))/d^5+1/16*c^2*(2*a*d^2+b*c^2)*x*(d*x-c)^(1/2)*(d*x+c)^(1/2)/d^4
```

Rubi [A]
time = 0.08, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {471, 92, 12, 38, 65, 223, 212}

$$\frac{c^2x\sqrt{dx-c}\,\sqrt{c+dx}\,(2ad^2+bc^2)}{16d^4}+\frac{x(dx-c)^{3/2}(c+dx)^{3/2}\,(2ad^2+bc^2)}{8d^4}-\frac{c^4(2ad^2+bc^2)\tanh^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)}{8d^5}+\frac{bx^3(dx-c)^{3/2}(c+dx)^{3/2}}{6d^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[-c + d*x]*Sqrt[c + d*x]*(a + b*x^2),x]
```

```
[Out] (c^2*(b*c^2 + 2*a*d^2)*x*Sqrt[-c + d*x]*Sqrt[c + d*x])/(16*d^4) + ((b*c^2 +
 2*a*d^2)*x*(-c + d*x)^(3/2)*(c + d*x)^(3/2))/(8*d^4) + (b*x^3*(-c + d*x)^(
3/2)*(c + d*x)^(3/2))/(6*d^2) - (c^4*(b*c^2 + 2*a*d^2)*ArcTanh[Sqrt[-c + d*
x]/Sqrt[c + d*x]])/(8*d^5)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[x*
(a + b*x)^m*((c + d*x)^m/(2*m + 1)), x] + Dist[2*a*c*(m/(2*m + 1)), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 92

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(
d*f*(n + p + 3))), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[d*(e*x)^(
m + 1)*(a1 + b1*x^(n/2))^(p + 1)*((a2 + b2*x^(n/2))^(p + 1)/(b1*b2*e*(m + n
*(p + 1) + 1))), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\int x^2\sqrt{-c+dx}\,\sqrt{c+dx}\,(a+bx^2)\,dx = \frac{bx^3(-c+dx)^{3/2}(c+dx)^{3/2}}{6d^2} + \frac{1}{2}\left(2a+\frac{bc^2}{d^2}\right)\int x^2\sqrt{-c+dx}\,\sqrt{c}$$

$$= \frac{(bc^2+2ad^2)\,x(-c+dx)^{3/2}(c+dx)^{3/2}}{8d^4} + \frac{bx^3(-c+dx)^{3/2}(c+dx}{6d^2}$$

$$= \frac{(bc^2+2ad^2)\,x(-c+dx)^{3/2}(c+dx)^{3/2}}{8d^4} + \frac{bx^3(-c+dx)^{3/2}(c+dx}{6d^2}$$

$$= \frac{c^2(bc^2+2ad^2)\,x\sqrt{-c+dx}\,\sqrt{c+dx}}{16d^4} + \frac{(bc^2+2ad^2)\,x(-c+dx)^{3}}{8d^4}$$

$$= \frac{c^2(bc^2+2ad^2)\,x\sqrt{-c+dx}\,\sqrt{c+dx}}{16d^4} + \frac{(bc^2+2ad^2)\,x(-c+dx)^{3}}{8d^4}$$

$$= \frac{c^2(bc^2+2ad^2)\,x\sqrt{-c+dx}\,\sqrt{c+dx}}{16d^4} + \frac{(bc^2+2ad^2)\,x(-c+dx)^{3}}{8d^4}$$

$$= \frac{c^2(bc^2+2ad^2)\,x\sqrt{-c+dx}\,\sqrt{c+dx}}{16d^4} + \frac{(bc^2+2ad^2)\,x(-c+dx)^{3}}{8d^4}$$

Mathematica [A]
time = 0.24, size = 116, normalized size = 0.73

$$\frac{dx\sqrt{-c+dx}\,\sqrt{c+dx}\,(-6ad^2(c^2-2d^2x^2)+b(-3c^4-2c^2d^2x^2+8d^4x^4))-6c^4(bc^2+2ad^2)\tanh^{-1}\left(\frac{\sqrt{-c+dx}}{\sqrt{c+dx}}\right)}{48d^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[-c + d*x]*Sqrt[c + d*x]*(a + b*x^2),x]
```

```
[Out] (d*x*Sqrt[-c + d*x]*Sqrt[c + d*x]*(-6*a*d^2*(c^2 - 2*d^2*x^2) + b*(-3*c^4 -
 2*c^2*d^2*x^2 + 8*d^4*x^4)) - 6*c^4*(b*c^2 + 2*a*d^2)*ArcTanh[Sqrt[-c + d*
x]/Sqrt[c + d*x]])/(48*d^5)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.29, size = 240, normalized size = 1.51

method	result
risch	$\frac{x(-8b\,d^4x^4-12a\,d^4x^2+2b\,c^2d^2x^2+6a\,c^2d^2+3b\,c^4)(-dx+c)\sqrt{dx+c}}{48d^4\sqrt{dx-c}} - \frac{\left(\frac{c^4\ln\left(\frac{d^2x}{\sqrt{d^2}}+\sqrt{d^2x^2-c^2}\right)a}{8d^2\sqrt{d^2}}+\frac{c^6\ln\left(\frac{d^2x}{\sqrt{d^2}}+\right.}{16d}\right.}{\sqrt{dx-c}\ \sqrt{}}$

default	$-\frac{\sqrt{dx-c}\,\sqrt{dx+c}\,\left(-8\,\mathrm{csgn}(d)b\,d^5x^5\sqrt{d^2x^2-c^2}-12\,\mathrm{csgn}(d)a\,d^5x^3\sqrt{d^2x^2-c^2}+2\,\mathrm{csgn}(d)b\,c^2d^3x^3\sqrt{d^2x^2-c^2}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/48*(d*x-c)^(1/2)*(d*x+c)^(1/2)*(-8*csgn(d)*b*d^5*x^5*(d^2*x^2-c^2)^(1/2)
-12*csgn(d)*a*d^5*x^3*(d^2*x^2-c^2)^(1/2)+2*csgn(d)*b*c^2*d^3*x^3*(d^2*x^2-
c^2)^(1/2)+6*(d^2*x^2-c^2)^(1/2)*csgn(d)*d^3*a*c^2*x+3*(d^2*x^2-c^2)^(1/2)*
csgn(d)*d*b*c^4*x+6*ln(((d^2*x^2-c^2)^(1/2)*csgn(d)+d*x)*csgn(d))*a*c^4*d^2
+3*ln(((d^2*x^2-c^2)^(1/2)*csgn(d)+d*x)*csgn(d))*b*c^6)*csgn(d)/(d^2*x^2-c^
2)^(1/2)/d^5
```

Maxima [A]
time = 0.27, size = 192, normalized size = 1.21

$$\frac{(d^2x^2-c^2)^{\frac{3}{2}}bx^3}{6\,d^2}-\frac{bc^6\log\left(2\,d^2x+2\,\sqrt{d^2x^2-c^2}\,d\right)}{16\,d^5}-\frac{ac^4\log\left(2\,d^2x+2\,\sqrt{d^2x^2-c^2}\,d\right)}{8\,d^3}+\frac{\sqrt{d^2x^2-c^2}\,bc^4x}{16\,d^4}+\frac{\sqrt{d^2x^2-c^2}\,ac^2x}{8\,d^2}+\frac{(d^2x^2-c^2)^{\frac{3}{2}}bc^2x}{8\,d^4}+\frac{(d^2x^2-c^2)^{\frac{3}{2}}ax}{4\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/6*(d^2*x^2 - c^2)^(3/2)*b*x^3/d^2 - 1/16*b*c^6*log(2*d^2*x + 2*sqrt(d^2*x
^2 - c^2)*d)/d^5 - 1/8*a*c^4*log(2*d^2*x + 2*sqrt(d^2*x^2 - c^2)*d)/d^3 + 1
/16*sqrt(d^2*x^2 - c^2)*b*c^4*x/d^4 + 1/8*sqrt(d^2*x^2 - c^2)*a*c^2*x/d^2 +
 1/8*(d^2*x^2 - c^2)^(3/2)*b*c^2*x/d^4 + 1/4*(d^2*x^2 - c^2)^(3/2)*a*x/d^2
```

Fricas [A]
time = 3.44, size = 112, normalized size = 0.70

$$\frac{(8\,bd^5x^5-2\,(bc^2d^3-6\,ad^5)x^3-3\,(bc^4d+2\,ac^2d^3)x)\sqrt{dx+c}\,\sqrt{dx-c}\,+3\,(bc^6+2\,ac^4d^2)\log\left(-dx+\sqrt{dx+c}\,\sqrt{dx-c}\right)}{48\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/48*((8*b*d^5*x^5 - 2*(b*c^2*d^3 - 6*a*d^5)*x^3 - 3*(b*c^4*d + 2*a*c^2*d^3
)*x)*sqrt(d*x + c)*sqrt(d*x - c) + 3*(b*c^6 + 2*a*c^4*d^2)*log(-d*x + sqrt(
d*x + c)*sqrt(d*x - c)))/d^5
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\left(a+bx^2\right)\sqrt{-c+dx}\,\sqrt{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*(b*x**2+a)*(d*x-c)**(1/2)*(d*x+c)**(1/2),x)

[Out] Integral(x**2*(a + b*x**2)*sqrt(-c + d*x)*sqrt(c + d*x), x)

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 432 vs. 2(135) = 270.
time = 0.63, size = 432, normalized size = 2.72

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2),x, algorithm="giac")

[Out] 1/240*(40*(sqrt(d*x + c)*sqrt(d*x - c)*((d*x + c)*(2*(d*x + c)/d^2 - 7*c/d^2) + 9*c^2/d^2) + 6*c^3*log(abs(-sqrt(d*x + c) + sqrt(d*x - c)))/d^2)*a*c + 2*(((2*(d*x + c)*(3*(d*x + c)*(4*(d*x + c)/d^4 - 21*c/d^4) + 133*c^2/d^4) - 295*c^3/d^4)*(d*x + c) + 195*c^4/d^4)*sqrt(d*x + c)*sqrt(d*x - c) + 90*c^5*log(abs(-sqrt(d*x + c) + sqrt(d*x - c)))/d^4)*b*c + 10*(((d*x + c)*(2*(d*x + c)*(3*(d*x + c)/d^3 - 13*c/d^3) + 43*c^2/d^3) - 39*c^3/d^3)*sqrt(d*x + c)*sqrt(d*x - c) - 18*c^4*log(abs(-sqrt(d*x + c) + sqrt(d*x - c)))/d^3)*a*d + (((2*((d*x + c)*(4*(d*x + c)*(5*(d*x + c)/d^5 - 31*c/d^5) + 321*c^2/d^5) - 451*c^3/d^5)*(d*x + c) + 745*c^4/d^5)*(d*x + c) - 405*c^5/d^5)*sqrt(d*x + c)*sqrt(d*x - c) - 150*c^6*log(abs(-sqrt(d*x + c) + sqrt(d*x - c)))/d^5)*b*d)/d

Mupad [B]
time = 42.57, size = 1681, normalized size = 10.57

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(a + b*x^2)*(c + d*x)^(1/2)*(d*x - c)^(1/2),x)

[Out] ((35*b*c^6*((c + d*x)^(1/2) - c^(1/2))^3)/(12*((-c)^(1/2) - (d*x - c)^(1/2))^3) - (b*c^6*((c + d*x)^(1/2) - c^(1/2)))/(4*((-c)^(1/2) - (d*x - c)^(1/2))) + (757*b*c^6*((c + d*x)^(1/2) - c^(1/2))^5)/(4*((-c)^(1/2) - (d*x - c)^(1/2))^5) + (7339*b*c^6*((c + d*x)^(1/2) - c^(1/2))^7)/(4*((-c)^(1/2) - (d*x - c)^(1/2))^7) + (41929*b*c^6*((c + d*x)^(1/2) - c^(1/2))^9)/(6*((-c)^(1/2) - (d*x - c)^(1/2))^9) + (25661*b*c^6*((c + d*x)^(1/2) - c^(1/2))^11)/(2*((-c)^(1/2) - (d*x - c)^(1/2))^11) + (25661*b*c^6*((c + d*x)^(1/2) - c^(1/2))^13)/(2*((-c)^(1/2) - (d*x - c)^(1/2))^13) + (41929*b*c^6*((c + d*x)^(1/2) - c^(1/2))^15)/(6*((-c)^(1/2) - (d*x - c)^(1/2))^15) + (7339*b*c^6*((c + d*x)^(1/2) - c^(1/2))^17)/(4*((-c)^(1/2) - (d*x - c)^(1/2))^17) + (757*b*c^6*((c + d*x)^(1/2) - c^(1/2))^19)/(4*((-c)^(1/2) - (d*x - c)^(1/2))^19) + (35*b*c^6*((c + d*x)^(1/2) - c^(1/2))^21)/(12*((-c)^(1/2) - (d*x - c)^(1/2))^21) - (b*c^6*((c + d*x)^(1/2) - c^(1/2))^23)/(4*((-c)^(1/2) - (d*x - c)^(1/

2))^23))/(d^5 - (12*d^5*((c + d*x)^(1/2) - c^(1/2))^2)/((-c)^(1/2) - (d*x - c)^(1/2))^2 + (66*d^5*((c + d*x)^(1/2) - c^(1/2))^4)/((-c)^(1/2) - (d*x - c)^(1/2))^4 - (220*d^5*((c + d*x)^(1/2) - c^(1/2))^6)/((-c)^(1/2) - (d*x - c)^(1/2))^6 + (495*d^5*((c + d*x)^(1/2) - c^(1/2))^8)/((-c)^(1/2) - (d*x - c)^(1/2))^8 - (792*d^5*((c + d*x)^(1/2) - c^(1/2))^10)/((-c)^(1/2) - (d*x - c)^(1/2))^10 + (924*d^5*((c + d*x)^(1/2) - c^(1/2))^12)/((-c)^(1/2) - (d*x - c)^(1/2))^12 - (792*d^5*((c + d*x)^(1/2) - c^(1/2))^14)/((-c)^(1/2) - (d*x - c)^(1/2))^14 + (495*d^5*((c + d*x)^(1/2) - c^(1/2))^16)/((-c)^(1/2) - (d*x - c)^(1/2))^16 - (220*d^5*((c + d*x)^(1/2) - c^(1/2))^18)/((-c)^(1/2) - (d*x - c)^(1/2))^18 + (66*d^5*((c + d*x)^(1/2) - c^(1/2))^20)/((-c)^(1/2) - (d*x - c)^(1/2))^20 - (12*d^5*((c + d*x)^(1/2) - c^(1/2))^22)/((-c)^(1/2) - (d*x - c)^(1/2))^22 + (d^5*((c + d*x)^(1/2) - c^(1/2))^24)/((-c)^(1/2) - (d*x - c)^(1/2))^24) - ((a*c^4*((c + d*x)^(1/2) - c^(1/2)))/(2*((-c)^(1/2) - (d*x - c)^(1/2))) + (35*a*c^4*((c + d*x)^(1/2) - c^(1/2))^3)/(2*((-c)^(1/2) - (d*x - c)^(1/2))^3) + (273*a*c^4*((c + d*x)^(1/2) - c^(1/2))^5)/(2*((-c)^(1/2) - (d*x - c)^(1/2))^5) + (715*a*c^4*((c + d*x)^(1/2) - c^(1/2))^7)/(2*((-c)^(1/2) - (d*x - c)^(1/2))^7) + (715*a*c^4*((c + d*x)^(1/2) - c^(1/2))^9)/(2*((-c)^(1/2) - (d*x - c)^(1/2))^9) + (273*a*c^4*((c + d*x)^(1/2) - c^(1/2))^11)/(2*((-c)^(1/2) - (d*x - c)^(1/2))^11) + (35*a*c^4*((c + d*x)^(1/2) - c^(1/2))^13)/(2*((-c)^(1/2) - (d*x - c)^(1/2))^13) + (a*c^4*((c + d*x)^(1/2) - c^(1/2))^15)/(2*((-c)^(1/2) - (d*x - c)^(1/2))^15))/(d^3 - (8*d^3*((c + d*x)^(1/2) - c^(1/2))^2)/((-c)^(1/2) - (d*x - c)^(1/2))^2 + (28*d^3*((c + d*x)^(1/2) - c^(1/2))^4)/((-c)^(1/2) - (d*x - c)^(1/2))^4 - (56*d^3*((c + d*x)^(1/2) - c^(1/2))^6)/((-c)^(1/2) - (d*x - c)^(1/2))^6 + (70*d^3*((c + d*x)^(1/2) - c^(1/2))^8)/((-c)^(1/2) - (d*x - c)^(1/2))^8 - (56*d^3*((c + d*x)^(1/2) - c^(1/2))^10)/((-c)^(1/2) - (d*x - c)^(1/2))^10 + (28*d^3*((c + d*x)^(1/2) - c^(1/2))^12)/((-c)^(1/2) - (d*x - c)^(1/2))^12 - (8*d^3*((c + d*x)^(1/2) - c^(1/2))^14)/((-c)^(1/2) - (d*x - c)^(1/2))^14 + (d^3*((c + d*x)^(1/2) - c^(1/2))^16)/((-c)^(1/2) - (d*x - c)^(1/2))^16) + (a*c^4*atanh(((c + d*x)^(1/2) - c^(1/2))/((-c)^(1/2) - (d*x - c)^(1/2))))/(2*d^3) + (b*c^6*atanh(((c + d*x)^(1/2) - c^(1/2))/((-c)^(1/2) - (d*x - c)^(1/2))))/(4*d^5)

3.345 $\int \sqrt{-c+dx}\,\sqrt{c+dx}\,(a+bx^2)\,dx$

Optimal. Leaf size=114

$$\frac{(bc^2+4ad^2)\,x\sqrt{-c+dx}\,\sqrt{c+dx}}{8d^2}+\frac{bx(-c+dx)^{3/2}(c+dx)^{3/2}}{4d^2}-\frac{c^2(bc^2+4ad^2)\tanh^{-1}\left(\frac{\sqrt{-c+dx}}{\sqrt{c+dx}}\right)}{4d^3}$$

```
[Out] 1/4*b*x*(d*x-c)^(3/2)*(d*x+c)^(3/2)/d^2-1/4*c^2*(4*a*d^2+b*c^2)*arctanh((d*
x-c)^(1/2)/(d*x+c)^(1/2))/d^3+1/8*(4*a*d^2+b*c^2)*x*(d*x-c)^(1/2)*(d*x+c)^(
1/2)/d^2
```

Rubi [A]
time = 0.03, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {397, 38, 65, 223, 212}

$$\frac{x\sqrt{dx-c}\,\sqrt{c+dx}\,(4ad^2+bc^2)}{8d^2}-\frac{c^2(4ad^2+bc^2)\tanh^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)}{4d^3}+\frac{bx(dx-c)^{3/2}(c+dx)^{3/2}}{4d^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-c + d*x]*Sqrt[c + d*x]*(a + b*x^2),x]
```

```
[Out] ((b*c^2 + 4*a*d^2)*x*Sqrt[-c + d*x]*Sqrt[c + d*x])/(8*d^2) + (b*x*(-c + d*x
)^(3/2)*(c + d*x)^(3/2))/(4*d^2) - (c^2*(b*c^2 + 4*a*d^2)*ArcTanh[Sqrt[-c +
 d*x]/Sqrt[c + d*x]])/(4*d^3)
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[x*
(a + b*x)^m*((c + d*x)^m/(2*m + 1)), x] + Dist[2*a*c*(m/(2*m + 1)), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 397

```
Int[((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_
.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[d*x*(a1 + b1*x^(n/2))^(p + 1
)*((a2 + b2*x^(n/2))^(p + 1)/(b1*b2*(n*(p + 1) + 1))), x] - Dist[(a1*a2*d -
 b1*b2*c*(n*(p + 1) + 1))/(b1*b2*(n*(p + 1) + 1)), Int[(a1 + b1*x^(n/2))^p*
(a2 + b2*x^(n/2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, n, p}, x] && Eq
Q[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rubi steps

$$\int \sqrt{-c+dx}\,\sqrt{c+dx}\,\left(a+bx^2\right)\,dx = \frac{bx(-c+dx)^{3/2}(c+dx)^{3/2}}{4d^2} - \frac{\left(-bc^2-4ad^2\right)\int \sqrt{-c+dx}\,\sqrt{c+dx}\,d}{4d^2}$$

$$= \frac{\left(bc^2+4ad^2\right)x\sqrt{-c+dx}\,\sqrt{c+dx}}{8d^2} + \frac{bx(-c+dx)^{3/2}(c+dx)^{3/2}}{4d^2} + \cdots$$

$$= \frac{\left(bc^2+4ad^2\right)x\sqrt{-c+dx}\,\sqrt{c+dx}}{8d^2} + \frac{bx(-c+dx)^{3/2}(c+dx)^{3/2}}{4d^2} - \cdots$$

$$= \frac{\left(bc^2+4ad^2\right)x\sqrt{-c+dx}\,\sqrt{c+dx}}{8d^2} + \frac{bx(-c+dx)^{3/2}(c+dx)^{3/2}}{4d^2} - \cdots$$

$$= \frac{\left(bc^2+4ad^2\right)x\sqrt{-c+dx}\,\sqrt{c+dx}}{8d^2} + \frac{bx(-c+dx)^{3/2}(c+dx)^{3/2}}{4d^2} - \cdots$$

Mathematica [A]
time = 0.18, size = 92, normalized size = 0.81

$$\frac{dx\sqrt{-c+dx}\,\sqrt{c+dx}\,\left(-bc^2+4ad^2+2bd^2x^2\right)-2c^2\left(bc^2+4ad^2\right)\tanh^{-1}\left(\frac{\sqrt{-c+dx}}{\sqrt{c+dx}}\right)}{8d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-c + d*x]*Sqrt[c + d*x]*(a + b*x^2),x]
```

[Out] (d*x*Sqrt[-c + d*x]*Sqrt[c + d*x]*(-(b*c^2) + 4*a*d^2 + 2*b*d^2*x^2) - 2*c^2*(b*c^2 + 4*a*d^2)*ArcTanh[Sqrt[-c + d*x]/Sqrt[c + d*x]])/(8*d^3)

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.29, size = 182, normalized size = 1.60

method	result
risch	$-\frac{x\left(2b\,d^{2}x^{2}+4a\,d^{2}-b\,c^{2}\right)\left(-dx+c\right)\sqrt{dx+c}}{8d^{2}\sqrt{dx-c}}-\frac{\left(\frac{c^{2}\ln\left(\frac{d^{2}x}{\sqrt{d^{2}}}+\sqrt{d^{2}x^{2}-c^{2}}\right)a}{2\sqrt{d^{2}}}+\frac{c^{4}\ln\left(\frac{d^{2}x}{\sqrt{d^{2}}}+\sqrt{d^{2}x^{2}-c^{2}}\right)b}{8d^{2}\sqrt{d^{2}}}\right)\sqrt{}}{\sqrt{dx-c}\,\sqrt{dx+c}}$
default	$\frac{\sqrt{dx-c}\,\sqrt{dx+c}\,\left(2\,\mathrm{csgn}(d)b\,d^{3}x^{3}\sqrt{d^{2}x^{2}-c^{2}}+4\sqrt{d^{2}x^{2}-c^{2}}\,\mathrm{csgn}(d)d^{3}ax-\sqrt{d^{2}x^{2}-c^{2}}\,\mathrm{csgn}(d)db\,c^{2}x-4\ln\right.}{8\sqrt{d^{2}x^{2}-c^{2}}\,d^{3}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/8*(d*x-c)^(1/2)*(d*x+c)^(1/2)*(2*csgn(d)*b*d^3*x^3*(d^2*x^2-c^2)^(1/2)+4*(d^2*x^2-c^2)^(1/2)*csgn(d)*d^3*a*x-(d^2*x^2-c^2)^(1/2)*csgn(d)*d*b*c^2*x-4*ln(((d^2*x^2-c^2)^(1/2)*csgn(d)+d*x)*csgn(d))*a*c^2*d^2-ln(((d^2*x^2-c^2)^(1/2)*csgn(d)+d*x)*csgn(d))*b*c^4)*csgn(d)/(d^2*x^2-c^2)^(1/2)/d^3

Maxima [A]
time = 0.26, size = 137, normalized size = 1.20

$$-\frac{bc^{4}\log\left(2\,d^{2}x+2\,\sqrt{d^{2}x^{2}-c^{2}}\,d\right)}{8\,d^{3}}-\frac{ac^{2}\log\left(2\,d^{2}x+2\,\sqrt{d^{2}x^{2}-c^{2}}\,d\right)}{2\,d}+\frac{1}{2}\,\sqrt{d^{2}x^{2}-c^{2}}\,ax+\frac{\sqrt{d^{2}x^{2}-c^{2}}\,bc^{2}x}{8\,d^{2}}+\frac{\left(d^{2}x^{2}-c^{2}\right)^{\frac{3}{2}}bx}{4\,d^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2),x, algorithm="maxima")

[Out] -1/8*b*c^4*log(2*d^2*x + 2*sqrt(d^2*x^2 - c^2)*d)/d^3 - 1/2*a*c^2*log(2*d^2*x + 2*sqrt(d^2*x^2 - c^2)*d)/d + 1/2*sqrt(d^2*x^2 - c^2)*a*x + 1/8*sqrt(d^2*x^2 - c^2)*b*c^2*x/d^2 + 1/4*(d^2*x^2 - c^2)^(3/2)*b*x/d^2

Fricas [A]
time = 3.13, size = 88, normalized size = 0.77

$$\frac{\left(2\,bd^{3}x^{3}-\left(bc^{2}d-4\,ad^{3}\right)x\right)\sqrt{dx+c}\,\sqrt{dx-c}+\left(bc^{4}+4\,ac^{2}d^{2}\right)\log\left(-dx+\sqrt{dx+c}\,\sqrt{dx-c}\right)}{8\,d^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2),x, algorithm="fricas")

```
[Out] 1/8*((2*b*d^3*x^3 - (b*c^2*d - 4*a*d^3)*x)*sqrt(d*x + c)*sqrt(d*x - c) + (b
*c^4 + 4*a*c^2*d^2)*log(-d*x + sqrt(d*x + c)*sqrt(d*x - c)))/d^3
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + bx^2\right)\sqrt{-c + dx}\,\sqrt{c + dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(d*x-c)**(1/2)*(d*x+c)**(1/2),x)
```

```
[Out] Integral((a + b*x**2)*sqrt(-c + d*x)*sqrt(c + d*x), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 288 vs. 2(96) = 192.
time = 0.72, size = 288, normalized size = 2.53

$$\frac{24\left(2\,c\log\left(\left|-\sqrt{dx+c}+\sqrt{dx-c}\right|\right)+\sqrt{dx+c}\,\sqrt{dx-c}\right)ac+4\left(\sqrt{dx+c}\,\sqrt{dx-c}\left((dx+c)\left(\frac{2\,(dx+c)}{d^2}-\frac{7\,c}{d^2}\right)+\frac{9\,c^2}{d^2}\right)+\frac{6\,c^3\log\left(\left|-\sqrt{dx+c}+\sqrt{dx-c}\right|\right)}{d^2}\right)bc+\left(\left((dx+c)\left(2\,(dx+c)\left(\frac{3\,(dx+c)}{d^3}-\frac{13\,c}{d^3}\right)+\frac{43\,c^2}{d^3}\right)-\frac{39\,c^3}{d^3}\right)\sqrt{dx+c}\,\sqrt{dx-c}-\frac{18\,c^4\log\left(\left|-\sqrt{dx+c}+\sqrt{dx-c}\right|\right)}{d^3}\right)bd-12\left(2\,c^2\log\left(\left|-\sqrt{dx+c}+\sqrt{dx-c}\right|\right)-\sqrt{dx+c}\,\sqrt{dx-c}\,(dx-2\,c)\right)a}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/24*(24*(2*c*log(abs(-sqrt(d*x + c) + sqrt(d*x - c))) + sqrt(d*x + c)*sqrt
(d*x - c))*a*c + 4*(sqrt(d*x + c)*sqrt(d*x - c)*((d*x + c)*(2*(d*x + c)/d^2
 - 7*c/d^2) + 9*c^2/d^2) + 6*c^3*log(abs(-sqrt(d*x + c) + sqrt(d*x - c)))/d
^2)*b*c + (((d*x + c)*(2*(d*x + c)*(3*(d*x + c)/d^3 - 13*c/d^3) + 43*c^2/d^
3) - 39*c^3/d^3)*sqrt(d*x + c)*sqrt(d*x - c) - 18*c^4*log(abs(-sqrt(d*x + c
) + sqrt(d*x - c)))/d^3)*b*d - 12*(2*c^2*log(abs(-sqrt(d*x + c) + sqrt(d*x
- c))) - sqrt(d*x + c)*sqrt(d*x - c)*(d*x - 2*c))*a)/d
```

Mupad [B]
time = 17.43, size = 734, normalized size = 6.44

$$\frac{a\,x\,\sqrt{c+d\,x}\,\sqrt{d\,x-c}}{2}-\frac{\frac{b\,c^4\left(\sqrt{c+d\,x}-\sqrt{c}\right)}{2\left(\sqrt{-c}-\sqrt{d\,x-c}\right)}+\frac{35\,b\,c^4\left(\sqrt{c+d\,x}-\sqrt{c}\right)^3}{2\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^3}+\frac{273\,b\,c^4\left(\sqrt{c+d\,x}-\sqrt{c}\right)^5}{2\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^5}+\frac{715\,b\,c^4\left(\sqrt{c+d\,x}-\sqrt{c}\right)^7}{2\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^7}+\frac{715\,b\,c^4\left(\sqrt{c+d\,x}-\sqrt{c}\right)^9}{2\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^9}+\frac{273\,b\,c^4\left(\sqrt{c+d\,x}-\sqrt{c}\right)^{11}}{2\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^{11}}+\frac{35\,b\,c^4\left(\sqrt{c+d\,x}-\sqrt{c}\right)^{13}}{2\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^{13}}+\frac{b\,c^4\left(\sqrt{c+d\,x}-\sqrt{c}\right)^{15}}{2\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^{15}}}{d^3-\frac{8\,d^3\left(\sqrt{c+d\,x}-\sqrt{c}\right)^2}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^2}+\frac{28\,d^3\left(\sqrt{c+d\,x}-\sqrt{c}\right)^4}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^4}-\frac{56\,d^3\left(\sqrt{c+d\,x}-\sqrt{c}\right)^6}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^6}+\frac{70\,d^3\left(\sqrt{c+d\,x}-\sqrt{c}\right)^8}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^8}-\frac{56\,d^3\left(\sqrt{c+d\,x}-\sqrt{c}\right)^{10}}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^{10}}+\frac{28\,d^3\left(\sqrt{c+d\,x}-\sqrt{c}\right)^{12}}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^{12}}-\frac{8\,d^3\left(\sqrt{c+d\,x}-\sqrt{c}\right)^{14}}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^{14}}+\frac{d^3\left(\sqrt{c+d\,x}-\sqrt{c}\right)^{16}}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^{16}}}-\frac{a\,c^2\ln\left(d\,x+\sqrt{c+d\,x}\,\sqrt{d\,x-c}\right)}{2\,d}+\frac{b\,c^4\,\mathrm{atanh}\left(\frac{\sqrt{c+d\,x}-\sqrt{c}}{\sqrt{-c}-\sqrt{d\,x-c}}\right)}{2\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)*(c + d*x)^(1/2)*(d*x - c)^(1/2),x)
```

```
[Out] (a*x*(c + d*x)^(1/2)*(d*x - c)^(1/2))/2 - ((b*c^4*((c + d*x)^(1/2) - c^(1/2
)))/(2*((-c)^(1/2) - (d*x - c)^(1/2))) + (35*b*c^4*((c + d*x)^(1/2) - c^(1/
2))^3)/(2*((-c)^(1/2) - (d*x - c)^(1/2))^3) + (273*b*c^4*((c + d*x)^(1/2) -
 c^(1/2))^5)/(2*((-c)^(1/2) - (d*x - c)^(1/2))^5) + (715*b*c^4*((c + d*x)^(
1/2) - c^(1/2))^7)/(2*((-c)^(1/2) - (d*x - c)^(1/2))^7) + (715*b*c^4*((c +
d*x)^(1/2) - c^(1/2))^9)/(2*((-c)^(1/2) - (d*x - c)^(1/2))^9) + (273*b*c^4*
((c + d*x)^(1/2) - c^(1/2))^11)/(2*((-c)^(1/2) - (d*x - c)^(1/2))^11) + (35
```

```
*b*c^4*((c + d*x)^(1/2) - c^(1/2))^13)/(2*((-c)^(1/2) - (d*x - c)^(1/2))^13
) + (b*c^4*((c + d*x)^(1/2) - c^(1/2))^15)/(2*((-c)^(1/2) - (d*x - c)^(1/2)
)^15))/(d^3 - (8*d^3*((c + d*x)^(1/2) - c^(1/2))^2)/((-c)^(1/2) - (d*x - c)
^(1/2))^2 + (28*d^3*((c + d*x)^(1/2) - c^(1/2))^4)/((-c)^(1/2) - (d*x - c)^
(1/2))^4 - (56*d^3*((c + d*x)^(1/2) - c^(1/2))^6)/((-c)^(1/2) - (d*x - c)^(
1/2))^6 + (70*d^3*((c + d*x)^(1/2) - c^(1/2))^8)/((-c)^(1/2) - (d*x - c)^(1
/2))^8 - (56*d^3*((c + d*x)^(1/2) - c^(1/2))^10)/((-c)^(1/2) - (d*x - c)^(1
/2))^10 + (28*d^3*((c + d*x)^(1/2) - c^(1/2))^12)/((-c)^(1/2) - (d*x - c)^(
1/2))^12 - (8*d^3*((c + d*x)^(1/2) - c^(1/2))^14)/((-c)^(1/2) - (d*x - c)^(
1/2))^14 + (d^3*((c + d*x)^(1/2) - c^(1/2))^16)/((-c)^(1/2) - (d*x - c)^(1/
2))^16) - (a*c^2*log(d*x + (c + d*x)^(1/2)*(d*x - c)^(1/2)))/(2*d) + (b*c^4
*atanh(((c + d*x)^(1/2) - c^(1/2))/((-c)^(1/2) - (d*x - c)^(1/2))))/(2*d^3)
```

3.346 $\int \frac{\sqrt{-c+dx}\,\sqrt{c+dx}\,(a+bx^2)}{x^2}\,dx$

Optimal. Leaf size=104

$$\frac{1}{2}\left(b-\frac{2ad^2}{c^2}\right)x\sqrt{-c+dx}\,\sqrt{c+dx}+\frac{a(-c+dx)^{3/2}(c+dx)^{3/2}}{c^2x}-\frac{(bc^2-2ad^2)\tanh^{-1}\left(\frac{\sqrt{-c+dx}}{\sqrt{c+dx}}\right)}{d}$$

```
[Out] a*(d*x-c)^(3/2)*(d*x+c)^(3/2)/c^2/x-(-2*a*d^2+b*c^2)*arctanh((d*x-c)^(1/2)/
(d*x+c)^(1/2))/d+1/2*(b-2*a*d^2/c^2)*x*(d*x-c)^(1/2)*(d*x+c)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {465, 38, 65, 223, 212}

$$\frac{1}{2}x\sqrt{dx-c}\,\sqrt{c+dx}\left(b-\frac{2ad^2}{c^2}\right)-\frac{(bc^2-2ad^2)\tanh^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)}{d}+\frac{a(dx-c)^{3/2}(c+dx)^{3/2}}{c^2x}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[-c + d*x]*Sqrt[c + d*x]*(a + b*x^2))/x^2,x]
```

```
[Out] ((b - (2*a*d^2)/c^2)*x*Sqrt[-c + d*x]*Sqrt[c + d*x])/2 + (a*(-c + d*x)^(3/2
)*(c + d*x)^(3/2))/(c^2*x) - ((b*c^2 - 2*a*d^2)*ArcTanh[Sqrt[-c + d*x]/Sqrt
[c + d*x]])/d
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[x*
(a + b*x)^m*((c + d*x)^m/(2*m + 1)), x] + Dist[2*a*c*(m/(2*m + 1)), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 465

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[c*(e*x)^(
m + 1)*(a1 + b1*x^(n/2))^(p + 1)*((a2 + b2*x^(n/2))^(p + 1)/(a1*a2*e*(m + 1
))), x] + Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/(a1*a2*e^n*(
m + 1)), Int[(e*x)^(m + n)*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/2))^p, x], x]
/; FreeQ[{a1, b1, a2, b2, c, d, e, p}, x] && EqQ[non2, n/2] && EqQ[a2*b1 +
a1*b2, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (L
tQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\int \frac{\sqrt{-c+dx}\,\sqrt{c+dx}\,(a+bx^2)}{x^2}\,dx = \frac{a(-c+dx)^{3/2}(c+dx)^{3/2}}{c^2x} + \left(b-\frac{2ad^2}{c^2}\right)\int \sqrt{-c+dx}\,\sqrt{c+dx}\,dx$$

$$= \frac{1}{2}\left(b-\frac{2ad^2}{c^2}\right)x\sqrt{-c+dx}\,\sqrt{c+dx} + \frac{a(-c+dx)^{3/2}(c+dx)^{3/2}}{c^2x} +$$

$$= \frac{1}{2}\left(b-\frac{2ad^2}{c^2}\right)x\sqrt{-c+dx}\,\sqrt{c+dx} + \frac{a(-c+dx)^{3/2}(c+dx)^{3/2}}{c^2x} +$$

$$= \frac{1}{2}\left(b-\frac{2ad^2}{c^2}\right)x\sqrt{-c+dx}\,\sqrt{c+dx} + \frac{a(-c+dx)^{3/2}(c+dx)^{3/2}}{c^2x} +$$

$$= \frac{1}{2}\left(b-\frac{2ad^2}{c^2}\right)x\sqrt{-c+dx}\,\sqrt{c+dx} + \frac{a(-c+dx)^{3/2}(c+dx)^{3/2}}{c^2x} -$$

Mathematica [A]
time = 0.20, size = 74, normalized size = 0.71

$$\frac{\sqrt{-c+dx}\,\sqrt{c+dx}\,(-2a+bx^2)}{2x} + \left(-\frac{bc^2}{d}+2ad\right)\tanh^{-1}\left(\frac{\sqrt{-c+dx}}{\sqrt{c+dx}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[-c + d*x]*Sqrt[c + d*x]*(a + b*x^2))/x^2,x]
```

```
[Out] (Sqrt[-c + d*x]*Sqrt[c + d*x]*(-2*a + b*x^2))/(2*x) + (-((b*c^2)/d) + 2*a*d
)*ArcTanh[Sqrt[-c + d*x]/Sqrt[c + d*x]]
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.31, size = 153, normalized size = 1.47

method	result
risch	$\frac{\sqrt{dx+c}\,(-dx+c)(-b x^2+2a)}{2x\sqrt{dx-c}} - \frac{\left(-\frac{\ln\left(\frac{d^2 x}{\sqrt{d^2}}+\sqrt{d^2x^2-c^2}\right) a d^2}{\sqrt{d^2}} + \frac{\ln\left(\frac{d^2 x}{\sqrt{d^2}}+\sqrt{d^2x^2-c^2}\right) b c^2}{2\sqrt{d^2}}\right)\sqrt{(dx-c)(}}{\sqrt{dx-c}\,\sqrt{dx+c}}$
default	$-\frac{\sqrt{dx-c}\,\sqrt{dx+c}\,\left(-\mathrm{csgn}(d) b d x^2\sqrt{d^2x^2-c^2}-2\ln\left(\left(\sqrt{d^2x^2-c^2}\,\mathrm{csgn}(d)+dx\right)\mathrm{csgn}(d)\right) a d^2 x+\ln\left(\left(\sqrt{d^2x^2}\right.\right.\right.}{2\sqrt{d^2x^2-c^2}\,xd}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(d*x-c)^(1/2)*(d*x+c)^(1/2)*(-csgn(d)*b*d*x^2*(d^2*x^2-c^2)^(1/2)-2*ln
(((d^2*x^2-c^2)^(1/2)*csgn(d)+d*x)*csgn(d))*a*d^2*x+ln(((d^2*x^2-c^2)^(1/2)
*csgn(d)+d*x)*csgn(d))*b*c^2*x+2*(d^2*x^2-c^2)^(1/2)*csgn(d)*d*a)*csgn(d)/(
d^2*x^2-c^2)^(1/2)/x/d
```

Maxima [A]
time = 0.50, size = 105, normalized size = 1.01

$$-\frac{bc^2\log\left(2\,d^2x+2\,\sqrt{d^2x^2-c^2}\,d\right)}{2\,d}+ad\log\left(2\,d^2x+2\,\sqrt{d^2x^2-c^2}\,d\right)+\frac{1}{2}\,\sqrt{d^2x^2-c^2}\,bx-\frac{\sqrt{d^2x^2-c^2}\,a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] -1/2*b*c^2*log(2*d^2*x + 2*sqrt(d^2*x^2 - c^2)*d)/d + a*d*log(2*d^2*x + 2*s
qrt(d^2*x^2 - c^2)*d) + 1/2*sqrt(d^2*x^2 - c^2)*b*x - sqrt(d^2*x^2 - c^2)*a
/x
```

Fricas [A]
time = 2.85, size = 83, normalized size = 0.80

$$-\frac{2\,ad^2x-(bc^2-2\,ad^2)x\log\left(-dx+\sqrt{dx+c}\,\sqrt{dx-c}\right)-(bdx^2-2\,ad)\sqrt{dx+c}\,\sqrt{dx-c}}{2\,dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*a*d^2*x - (b*c^2 - 2*a*d^2)*x*log(-d*x + sqrt(d*x + c)*sqrt(d*x - c
)) - (b*d*x^2 - 2*a*d)*sqrt(d*x + c)*sqrt(d*x - c))/(d*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + bx^2)\sqrt{-c + dx}\,\sqrt{c + dx}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(d*x-c)**(1/2)*(d*x+c)**(1/2)/x**2,x)
```

```
[Out] Integral((a + b*x**2)*sqrt(-c + d*x)*sqrt(c + d*x)/x**2, x)
```

Giac [A]
time = 0.58, size = 110, normalized size = 1.06

$$-\frac{\frac{32\,ac^2d^2}{\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^4+4\,c^2} - 2\left((dx+c)b - bc\right)\sqrt{dx+c}\,\sqrt{dx-c} - (bc^2 - 2\,ad^2)\log\left(\left(\sqrt{dx+c} - \sqrt{dx-c}\right)^4\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] -1/4*(32*a*c^2*d^2/((sqrt(d*x + c) - sqrt(d*x - c))^4 + 4*c^2) - 2*((d*x +
c)*b - b*c)*sqrt(d*x + c)*sqrt(d*x - c) - (b*c^2 - 2*a*d^2)*log((sqrt(d*x +
 c) - sqrt(d*x - c))^4))/d
```

Mupad [B]
time = 3.49, size = 243, normalized size = 2.34

$$\frac{a\,d+\frac{5\,a\,d\,\left(\sqrt{c+d\,x}-\sqrt{c}\right)^2}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^2}}{\frac{4\left(\sqrt{c+d\,x}-\sqrt{c}\right)}{\sqrt{-c}-\sqrt{d\,x-c}}+\frac{4\left(\sqrt{c+d\,x}-\sqrt{c}\right)^3}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^3}} - 4\,a\,d\,\mathrm{atanh}\left(\frac{\sqrt{c+d\,x}-\sqrt{c}}{\sqrt{-c}-\sqrt{d\,x-c}}\right) + \frac{b\,x\,\sqrt{c+d\,x}\,\sqrt{d\,x-c}}{2} - \frac{b\,c^2\,\ln\left(d\,x+\sqrt{c+d\,x}\,\sqrt{d\,x-c}\right)}{2\,d} + \frac{a\,d\left(\sqrt{c+d\,x}-\sqrt{c}\right)}{4\left(\sqrt{-c}-\sqrt{d\,x-c}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)*(c + d*x)^(1/2)*(d*x - c)^(1/2))/x^2,x)
```

```
[Out] (a*d + (5*a*d*((c + d*x)^(1/2) - c^(1/2))^2)/((-c)^(1/2) - (d*x - c)^(1/2))
^2)/((4*((c + d*x)^(1/2) - c^(1/2)))/((-c)^(1/2) - (d*x - c)^(1/2)) + (4*((
c + d*x)^(1/2) - c^(1/2))^3)/((-c)^(1/2) - (d*x - c)^(1/2))^3) - 4*a*d*atan
h(((c + d*x)^(1/2) - c^(1/2))/((-c)^(1/2) - (d*x - c)^(1/2))) + (b*x*(c + d
*x)^(1/2)*(d*x - c)^(1/2))/2 - (b*c^2*log(d*x + (c + d*x)^(1/2)*(d*x - c)^(
1/2)))/(2*d) + (a*d*((c + d*x)^(1/2) - c^(1/2)))/(4*((-c)^(1/2) - (d*x - c)
^(1/2)))
```

3.347 $\int \frac{\sqrt{-c+dx}\,\sqrt{c+dx}\,(a+bx^2)}{x^4}\,dx$

Optimal. Leaf size=84

$$-\frac{b\sqrt{-c+dx}\,\sqrt{c+dx}}{x}+\frac{a(-c+dx)^{3/2}(c+dx)^{3/2}}{3c^2x^3}+2bd\tanh^{-1}\left(\frac{\sqrt{-c+dx}}{\sqrt{c+dx}}\right)$$

```
[Out] 1/3*a*(d*x-c)^(3/2)*(d*x+c)^(3/2)/c^2/x^3+2*b*d*arctanh((d*x-c)^(1/2)/(d*x+
c)^(1/2))-b*(d*x-c)^(1/2)*(d*x+c)^(1/2)/x
```

Rubi [A]
time = 0.05, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {465, 99, 12, 65, 223, 212}

$$\frac{a(dx-c)^{3/2}(c+dx)^{3/2}}{3c^2x^3}-\frac{b\sqrt{dx-c}\,\sqrt{c+dx}}{x}+2bd\tanh^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[-c + d*x]*Sqrt[c + d*x]*(a + b*x^2))/x^4,x]
```

```
[Out] -((b*Sqrt[-c + d*x]*Sqrt[c + d*x])/x) + (a*(-c + d*x)^(3/2)*(c + d*x)^(3/2)
)/(3*c^2*x^3) + 2*b*d*ArcTanh[Sqrt[-c + d*x]/Sqrt[c + d*x]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^p/(b*(
m + 1))), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
```

```
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 465

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[c*(e*x)^(
m + 1)*(a1 + b1*x^(n/2))^(p + 1)*((a2 + b2*x^(n/2))^(p + 1)/(a1*a2*e*(m + 1
))), x] + Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/(a1*a2*e^n*(
m + 1)), Int[(e*x)^(m + n)*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/2))^p, x], x]
/; FreeQ[{a1, b1, a2, b2, c, d, e, p}, x] && EqQ[non2, n/2] && EqQ[a2*b1 +
a1*b2, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (L
tQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{-c+dx}\,\sqrt{c+dx}\,(a+bx^2)}{x^4}\,dx &= \frac{a(-c+dx)^{3/2}(c+dx)^{3/2}}{3c^2x^3} + b\int \frac{\sqrt{-c+dx}\,\sqrt{c+dx}}{x^2}\,dx \\
&= -\frac{b\sqrt{-c+dx}\,\sqrt{c+dx}}{x} + \frac{a(-c+dx)^{3/2}(c+dx)^{3/2}}{3c^2x^3} + b\int \frac{}{\sqrt{-c+}} \\
&= -\frac{b\sqrt{-c+dx}\,\sqrt{c+dx}}{x} + \frac{a(-c+dx)^{3/2}(c+dx)^{3/2}}{3c^2x^3} + \left(bd^2\right)\int \frac{}{\sqrt{-}} \\
&= -\frac{b\sqrt{-c+dx}\,\sqrt{c+dx}}{x} + \frac{a(-c+dx)^{3/2}(c+dx)^{3/2}}{3c^2x^3} + (2bd)\text{Subst}\Big(\\
&= -\frac{b\sqrt{-c+dx}\,\sqrt{c+dx}}{x} + \frac{a(-c+dx)^{3/2}(c+dx)^{3/2}}{3c^2x^3} + (2bd)\text{Subst}\Big(\\
&= -\frac{b\sqrt{-c+dx}\,\sqrt{c+dx}}{x} + \frac{a(-c+dx)^{3/2}(c+dx)^{3/2}}{3c^2x^3} + 2bd\tanh^{-1}\Big(
\end{aligned}$$

Mathematica [A]

time = 0.16, size = 81, normalized size = 0.96

$$-\frac{\sqrt{-c+dx}\,\sqrt{c+dx}\,(3bc^2x^2+a(c^2-d^2x^2))}{3c^2x^3}+2bd\tanh^{-1}\left(\frac{\sqrt{-c+dx}}{\sqrt{c+dx}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[-c + d*x]*Sqrt[c + d*x]*(a + b*x^2))/x^4,x]
```

```
[Out] -1/3*(Sqrt[-c + d*x]*Sqrt[c + d*x]*(3*b*c^2*x^2 + a*(c^2 - d^2*x^2)))/(c^2*
x^3) + 2*b*d*ArcTanh[Sqrt[-c + d*x]/Sqrt[c + d*x]]
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.30, size = 153, normalized size = 1.82

method	result
risch	$\frac{\sqrt{dx+c}\,(-dx+c)(-a\,d^2x^2+3b\,c^2x^2+c^2a)}{3x^3c^2\sqrt{dx-c}}+\frac{b\,d^2\ln\left(\frac{d^2x}{\sqrt{d^2}}+\sqrt{d^2x^2-c^2}\right)\sqrt{(dx-c)(dx+c)}}{\sqrt{d^2}\,\sqrt{dx-c}\,\sqrt{dx+c}}$
default	$-\frac{\sqrt{dx-c}\,\sqrt{dx+c}\,\left(-3\ln\left(\left(\sqrt{d^2x^2-c^2}\,\mathrm{csgn}(d)+dx\right)\mathrm{csgn}(d)\right)b\,c^2d\,x^3-\mathrm{csgn}(d)a\,d^2x^2\sqrt{d^2x^2-c^2}+3\,\mathrm{csgn}(d)b\,c\right.}{3\sqrt{d^2x^2-c^2}\,c^2x^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*(d*x-c)^(1/2)*(d*x+c)^(1/2)*(-3*ln(((d^2*x^2-c^2)^(1/2)*csgn(d)+d*x)*c
sgn(d))*b*c^2*d*x^3-csgn(d)*a*d^2*x^2*(d^2*x^2-c^2)^(1/2)+3*csgn(d)*b*c^2*x
^2*(d^2*x^2-c^2)^(1/2)+csgn(d)*a*c^2*(d^2*x^2-c^2)^(1/2))*csgn(d)/(d^2*x^2-
c^2)^(1/2)/c^2/x^3
```

Maxima [A]
time = 0.56, size = 75, normalized size = 0.89

$$bd\log\left(2\,d^2x+2\,\sqrt{d^2x^2-c^2}\,d\right)-\frac{\sqrt{d^2x^2-c^2}\,b}{x}+\frac{(d^2x^2-c^2)^{\frac{3}{2}}a}{3\,c^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] b*d*log(2*d^2*x + 2*sqrt(d^2*x^2 - c^2)*d) - sqrt(d^2*x^2 - c^2)*b/x + 1/3*
(d^2*x^2 - c^2)^(3/2)*a/(c^2*x^3)
```

Fricas [A]
time = 2.35, size = 100, normalized size = 1.19

$$-\frac{3\,bc^2dx^3\log\left(-dx+\sqrt{dx+c}\,\sqrt{dx-c}\right)+(3\,bc^2d-ad^3)x^3+(ac^2+(3\,bc^2-ad^2)x^2)\sqrt{dx+c}\,\sqrt{dx-c}}{3\,c^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] -1/3*(3*b*c^2*d*x^3*log(-d*x + sqrt(d*x + c)*sqrt(d*x - c)) + (3*b*c^2*d -
a*d^3)*x^3 + (a*c^2 + (3*b*c^2 - a*d^2)*x^2)*sqrt(d*x + c)*sqrt(d*x - c))/(
c^2*x^3)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: MellinTransformStripError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(d*x-c)**(1/2)*(d*x+c)**(1/2)/x**4,x)
```

```
[Out] Exception raised: MellinTransformStripError >> Pole inside critical strip?
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 171 vs. 2(70) = 140.
time = 0.63, size = 171, normalized size = 2.04

$$-\frac{3\,bd^2\log\left(\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^4\right)+\frac{16\left(3\,bc^2d^2\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^8-3\,ad^4\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^8+24\,bc^4d^2\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^4+48\,bc^6d^2-16\,ac^4d^4\right)}{\left(\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^4+4\,c^2\right)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] -1/6*(3*b*d^2*log((sqrt(d*x + c) - sqrt(d*x - c))^4) + 16*(3*b*c^2*d^2*(sqr
t(d*x + c) - sqrt(d*x - c))^8 - 3*a*d^4*(sqrt(d*x + c) - sqrt(d*x - c))^8 +
 24*b*c^4*d^2*(sqrt(d*x + c) - sqrt(d*x - c))^4 + 48*b*c^6*d^2 - 16*a*c^4*d
^4)/((sqrt(d*x + c) - sqrt(d*x - c))^4 + 4*c^2)^3)/d
```

Mupad [B]
time = 3.44, size = 236, normalized size = 2.81

$$\frac{b\,d+\frac{5\,b\,d\left(\sqrt{c+d\,x}-\sqrt{c}\right)^2}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^2}}{\frac{4\left(\sqrt{c+d\,x}-\sqrt{c}\right)}{\sqrt{-c}-\sqrt{d\,x-c}}+\frac{4\left(\sqrt{c+d\,x}-\sqrt{c}\right)^3}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^3}}-4\,b\,d\,\mathrm{atanh}\left(\frac{\sqrt{c+d\,x}-\sqrt{c}}{\sqrt{-c}-\sqrt{d\,x-c}}\right)-\frac{\left(\frac{a\sqrt{c+d\,x}}{3}-\frac{a\,d^2\,x^2\sqrt{c+d\,x}}{3\,c^2}\right)\sqrt{d\,x-c}}{x^3}+\frac{b\,d\left(\sqrt{c+d\,x}-\sqrt{c}\right)}{4\left(\sqrt{-c}-\sqrt{d\,x-c}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)*(c + d*x)^(1/2)*(d*x - c)^(1/2))/x^4,x)
```

```
[Out] (b*d + (5*b*d*((c + d*x)^(1/2) - c^(1/2))^2)/((-c)^(1/2) - (d*x - c)^(1/2))
^2)/((4*((c + d*x)^(1/2) - c^(1/2)))/((-c)^(1/2) - (d*x - c)^(1/2)) + (4*((
c + d*x)^(1/2) - c^(1/2))^3)/((-c)^(1/2) - (d*x - c)^(1/2))^3) - 4*b*d*atan
h(((c + d*x)^(1/2) - c^(1/2))/((-c)^(1/2) - (d*x - c)^(1/2))) - (((a*(c + d
*x)^(1/2))/3 - (a*d^2*x^2*(c + d*x)^(1/2))/(3*c^2))*(d*x - c)^(1/2))/x^3 +
(b*d*((c + d*x)^(1/2) - c^(1/2)))/(4*((-c)^(1/2) - (d*x - c)^(1/2)))
```

3.348 $\int \frac{x^4(a+bx^2)}{\sqrt{-1+cx}\sqrt{1+cx}} dx$

Optimal. Leaf size=125

$$\frac{(5b+6ac^2)\,x\sqrt{-1+cx}\,\sqrt{1+cx}}{16c^6}+\frac{(5b+6ac^2)\,x^3\sqrt{-1+cx}\,\sqrt{1+cx}}{24c^4}+\frac{bx^5\sqrt{-1+cx}\,\sqrt{1+cx}}{6c^2}+\frac{(5b+6ac$$

```
[Out] 1/16*(6*a*c^2+5*b)*arccosh(c*x)/c^7+1/16*(6*a*c^2+5*b)*x*(c*x-1)^(1/2)*(c*x
+1)^(1/2)/c^6+1/24*(6*a*c^2+5*b)*x^3*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^4+1/6*b*
x^5*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^2
```

Rubi [A]

time = 0.06, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {471, 102, 12, 92, 54}

$$\frac{(6ac^2+5b)\cosh^{-1}(cx)}{16c^7}+\frac{x\sqrt{cx-1}\,\sqrt{cx+1}\,(6ac^2+5b)}{16c^6}+\frac{x^3\sqrt{cx-1}\,\sqrt{cx+1}\,(6ac^2+5b)}{24c^4}+\frac{bx^5\sqrt{cx-1}\,\sqrt{cx+1}}{6c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(a + b*x^2))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]),x]
```

```
[Out] ((5*b + 6*a*c^2)*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(16*c^6) + ((5*b + 6*a*c^2
)*x^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(24*c^4) + (b*x^5*Sqrt[-1 + c*x]*Sqrt[1
 + c*x])/(6*c^2) + ((5*b + 6*a*c^2)*ArcCosh[c*x])/(16*c^7)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 54

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[b*(x/a)]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 92

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(
d*f*(n + p + 3))), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 102

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/(d*f*(m + n + p + 1))), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[d*(e*x)^(
m + 1)*(a1 + b1*x^(n/2))^(p + 1)*((a2 + b2*x^(n/2))^(p + 1)/(b1*b2*e*(m + n
*(p + 1) + 1))), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\int \frac{x^4(a+bx^2)}{\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx = \frac{bx^5\sqrt{-1+cx}\,\sqrt{1+cx}}{6c^2} - \frac{1}{6}\left(-6a-\frac{5b}{c^2}\right)\int \frac{x^4}{\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx$$

$$= \frac{(5b+6ac^2)\,x^3\sqrt{-1+cx}\,\sqrt{1+cx}}{24c^4} + \frac{bx^5\sqrt{-1+cx}\,\sqrt{1+cx}}{6c^2} + \frac{(5b+6ac^2)}{}$$

$$= \frac{(5b+6ac^2)\,x^3\sqrt{-1+cx}\,\sqrt{1+cx}}{24c^4} + \frac{bx^5\sqrt{-1+cx}\,\sqrt{1+cx}}{6c^2} + \frac{(5b+6ac^2)}{}$$

$$= \frac{(5b+6ac^2)\,x\sqrt{-1+cx}\,\sqrt{1+cx}}{16c^6} + \frac{(5b+6ac^2)\,x^3\sqrt{-1+cx}\,\sqrt{1+cx}}{24c^4} + \frac{bx}{}$$

$$= \frac{(5b+6ac^2)\,x\sqrt{-1+cx}\,\sqrt{1+cx}}{16c^6} + \frac{(5b+6ac^2)\,x^3\sqrt{-1+cx}\,\sqrt{1+cx}}{24c^4} + \frac{bx}{}$$

Mathematica [A]
time = 0.21, size = 96, normalized size = 0.77

$$\frac{cx\sqrt{-1+cx}\,\sqrt{1+cx}\,(6ac^2(3+2c^2x^2)+b(15+10c^2x^2+8c^4x^4))+6(5b+6ac^2)\tanh^{-1}\left(\sqrt{\frac{-1+cx}{1+cx}}\right)}{48c^7}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^4*(a + b*x^2))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]),x]
```

```
[Out] (c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(6*a*c^2*(3 + 2*c^2*x^2) + b*(15 + 10*c^2
*x^2 + 8*c^4*x^4)) + 6*(5*b + 6*a*c^2)*ArcTanh[Sqrt[(-1 + c*x)/(1 + c*x)]])
/(48*c^7)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.29, size = 191, normalized size = 1.53

method	result
risch	$\frac{x\left(8b\,x^4c^4+12a\,c^4x^2+10b\,c^2x^2+18c^2a+15b\right)\sqrt{cx+1}\,\sqrt{cx-1}}{48c^6}+\frac{\left(\frac{3\ln\left(\frac{c^2x}{\sqrt{c^2}}+\sqrt{c^2x^2-1}\right)a}{8c^4\sqrt{c^2}}+\frac{5\ln\left(\frac{c^2x}{\sqrt{c^2}}+\sqrt{c^2x^2\right.}}{16c^6\sqrt{c^2}}\right.}{\sqrt{cx-1}\,\sqrt{cx+}}$
default	$\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,\left(8\,\mathrm{csgn}(c)b\,c^5x^5\sqrt{c^2x^2-1}+12\,\mathrm{csgn}(c)a\,c^5x^3\sqrt{c^2x^2-1}+10\sqrt{c^2x^2-1}\,\mathrm{csgn}(c)c^3b\,x^3+18\sqrt{c}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^2+a)/(c*x-1)^(1/2)/(c*x+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/48*(c*x-1)^(1/2)*(c*x+1)^(1/2)*(8*csgn(c)*b*c^5*x^5*(c^2*x^2-1)^(1/2)+12*
csgn(c)*a*c^5*x^3*(c^2*x^2-1)^(1/2)+10*(c^2*x^2-1)^(1/2)*csgn(c)*c^3*b*x^3+
18*(c^2*x^2-1)^(1/2)*csgn(c)*c^3*a*x+15*(c^2*x^2-1)^(1/2)*csgn(c)*c*b*x+18*
ln(((c^2*x^2-1)^(1/2)*csgn(c)+c*x)*csgn(c))*a*c^2+15*ln(((c^2*x^2-1)^(1/2)*
csgn(c)+c*x)*csgn(c))*b)*csgn(c)/c^7/(c^2*x^2-1)^(1/2)
```

Maxima [A]
time = 0.28, size = 153, normalized size = 1.22

$$\frac{\sqrt{c^2x^2-1}\,bx^5}{6\,c^2}+\frac{\sqrt{c^2x^2-1}\,ax^3}{4\,c^2}+\frac{5\,\sqrt{c^2x^2-1}\,bx^3}{24\,c^4}+\frac{3\,\sqrt{c^2x^2-1}\,ax}{8\,c^4}+\frac{3\,a\log\left(2\,c^2x+2\,\sqrt{c^2x^2-1}\,c\right)}{8\,c^5}+\frac{5\,\sqrt{c^2x^2-1}\,bx}{16\,c^6}+\frac{5\,b\log\left(2\,c^2x+2\,\sqrt{c^2x^2-1}\,c\right)}{16\,c^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(c^2*x^2 - 1)*b*x^5/c^2 + 1/4*sqrt(c^2*x^2 - 1)*a*x^3/c^2 + 5/24*sq
rt(c^2*x^2 - 1)*b*x^3/c^4 + 3/8*sqrt(c^2*x^2 - 1)*a*x/c^4 + 3/8*a*log(2*c^2
*x + 2*sqrt(c^2*x^2 - 1)*c)/c^5 + 5/16*sqrt(c^2*x^2 - 1)*b*x/c^6 + 5/16*b*l
og(2*c^2*x + 2*sqrt(c^2*x^2 - 1)*c)/c^7
```

Fricas [A]
time = 3.28, size = 96, normalized size = 0.77

$$\frac{\left(8\,bc^5x^5+2\,(6\,ac^5+5\,bc^3)x^3+3\,(6\,ac^3+5\,bc)x\right)\sqrt{cx+1}\,\sqrt{cx-1}\,-3\,(6\,ac^2+5\,b)\log\left(-cx+\sqrt{cx+1}\,\sqrt{cx-1}\right)}{48\,c^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/48*((8*b*c^5*x^5 + 2*(6*a*c^5 + 5*b*c^3)*x^3 + 3*(6*a*c^3 + 5*b*c)*x)*sqr
t(c*x + 1)*sqrt(c*x - 1) - 3*(6*a*c^2 + 5*b)*log(-c*x + sqrt(c*x + 1)*sqrt(
c*x - 1)))/c^7
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**2+a)/(c*x-1)**(1/2)/(c*x+1)**(1/2),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.65, size = 172, normalized size = 1.38

$$\frac{\left(\left(2\left((cx+1)\left(4(cx+1)\left(\frac{(cx+1)b}{c^6}-\frac{5b}{c^6}\right)+\frac{3(2ac^{38}+15bc^{36})}{c^{42}}\right)-\frac{18ac^{38}+55bc^{36}}{c^{42}}\right)(cx+1)+\frac{54ac^{38}+85bc^{36}}{c^{42}}\right)(cx+1)-\frac{3(10ac^{38}+11bc^{36})}{c^{42}}\right)\sqrt{cx+1}\sqrt{cx-1}-\frac{6(6ac^2+5b)\log\left(\sqrt{cx+1}-\sqrt{cx-1}\right)}{c^6}}{48c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/48*(((2*((c*x + 1)*(4*(c*x + 1)*((c*x + 1)*b/c^6 - 5*b/c^6) + 3*(2*a*c^38
 + 15*b*c^36)/c^42) - (18*a*c^38 + 55*b*c^36)/c^42)*(c*x + 1) + (54*a*c^38
+ 85*b*c^36)/c^42)*(c*x + 1) - 3*(10*a*c^38 + 11*b*c^36)/c^42)*sqrt(c*x + 1
)*sqrt(c*x - 1) - 6*(6*a*c^2 + 5*b)*log(sqrt(c*x + 1) - sqrt(c*x - 1))/c^6)
/c
```

Mupad [**B**]
time = 32.63, size = 1154, normalized size = 9.23

$$-\frac{-\frac{175b\left(\sqrt{cx-1}-i\right)^3}{12\left(\sqrt{cx+1}-1\right)^3}+\frac{311b\left(\sqrt{cx-1}-i\right)^5}{4\left(\sqrt{cx+1}-1\right)^5}+\frac{8361b\left(\sqrt{cx-1}-i\right)^7}{4\left(\sqrt{cx+1}-1\right)^7}+\frac{42259b\left(\sqrt{cx-1}-i\right)^9}{6\left(\sqrt{cx+1}-1\right)^9}+\frac{25295b\left(\sqrt{cx-1}-i\right)^{11}}{2\left(\sqrt{cx+1}-1\right)^{11}}+\frac{25295b\left(\sqrt{cx-1}-i\right)^{13}}{2\left(\sqrt{cx+1}-1\right)^{13}}+\frac{42259b\left(\sqrt{cx-1}-i\right)^{15}}{6\left(\sqrt{cx+1}-1\right)^{15}}+\frac{8361b\left(\sqrt{cx-1}-i\right)^{17}}{4\left(\sqrt{cx+1}-1\right)^{17}}+\frac{311b\left(\sqrt{cx-1}-i\right)^{19}}{4\left(\sqrt{cx+1}-1\right)^{19}}-\frac{175b\left(\sqrt{cx-1}-i\right)^{21}}{12\left(\sqrt{cx+1}-1\right)^{21}}+\frac{5b\left(\sqrt{cx-1}-i\right)^{23}}{4\left(\sqrt{cx+1}-1\right)^{23}}+\frac{5b\left(\sqrt{cx-1}-i\right)}{4\left(\sqrt{cx+1}-1\right)}}{c^7-\frac{12c^7\left(\sqrt{cx-1}-i\right)^2}{\left(\sqrt{cx+1}-1\right)^2}+\frac{66c^7\left(\sqrt{cx-1}-i\right)^4}{\left(\sqrt{cx+1}-1\right)^4}-\frac{220c^7\left(\sqrt{cx-1}-i\right)^6}{\left(\sqrt{cx+1}-1\right)^6}+\frac{495c^7\left(\sqrt{cx-1}-i\right)^8}{\left(\sqrt{cx+1}-1\right)^8}-\frac{792c^7\left(\sqrt{cx-1}-i\right)^{10}}{\left(\sqrt{cx+1}-1\right)^{10}}+\frac{924c^7\left(\sqrt{cx-1}-i\right)^{12}}{\left(\sqrt{cx+1}-1\right)^{12}}-\frac{792c^7\left(\sqrt{cx-1}-i\right)^{14}}{\left(\sqrt{cx+1}-1\right)^{14}}+\frac{495c^7\left(\sqrt{cx-1}-i\right)^{16}}{\left(\sqrt{cx+1}-1\right)^{16}}-\frac{220c^7\left(\sqrt{cx-1}-i\right)^{18}}{\left(\sqrt{cx+1}-1\right)^{18}}+\frac{66c^7\left(\sqrt{cx-1}-i\right)^{20}}{\left(\sqrt{cx+1}-1\right)^{20}}-\frac{12c^7\left(\sqrt{cx-1}-i\right)^{22}}{\left(\sqrt{cx+1}-1\right)^{22}}+\frac{c^7\left(\sqrt{cx-1}-i\right)^{24}}{\left(\sqrt{cx+1}-1\right)^{24}}}+\frac{\frac{23a\left(\sqrt{cx-1}-i\right)^3}{2\left(\sqrt{cx+1}-1\right)^3}+\frac{333a\left(\sqrt{cx-1}-i\right)^5}{2\left(\sqrt{cx+1}-1\right)^5}+\frac{671a\left(\sqrt{cx-1}-i\right)^7}{2\left(\sqrt{cx+1}-1\right)^7}+\frac{671a\left(\sqrt{cx-1}-i\right)^9}{2\left(\sqrt{cx+1}-1\right)^9}+\frac{333a\left(\sqrt{cx-1}-i\right)^{11}}{2\left(\sqrt{cx+1}-1\right)^{11}}+\frac{23a\left(\sqrt{cx-1}-i\right)^{13}}{2\left(\sqrt{cx+1}-1\right)^{13}}-\frac{3a\left(\sqrt{cx-1}-i\right)^{15}}{2\left(\sqrt{cx+1}-1\right)^{15}}-\frac{3a\left(\sqrt{cx-1}-i\right)}{2\left(\sqrt{cx+1}-1\right)}}{c^5-\frac{8c^5\left(\sqrt{cx-1}-i\right)^2}{\left(\sqrt{cx+1}-1\right)^2}+\frac{28c^5\left(\sqrt{cx-1}-i\right)^4}{\left(\sqrt{cx+1}-1\right)^4}-\frac{56c^5\left(\sqrt{cx-1}-i\right)^6}{\left(\sqrt{cx+1}-1\right)^6}+\frac{70c^5\left(\sqrt{cx-1}-i\right)^8}{\left(\sqrt{cx+1}-1\right)^8}-\frac{56c^5\left(\sqrt{cx-1}-i\right)^{10}}{\left(\sqrt{cx+1}-1\right)^{10}}+\frac{28c^5\left(\sqrt{cx-1}-i\right)^{12}}{\left(\sqrt{cx+1}-1\right)^{12}}-\frac{8c^5\left(\sqrt{cx-1}-i\right)^{14}}{\left(\sqrt{cx+1}-1\right)^{14}}+\frac{c^5\left(\sqrt{cx-1}-i\right)^{16}}{\left(\sqrt{cx+1}-1\right)^{16}}}+\frac{3a\,\mathrm{atanh}\left(\frac{\sqrt{cx-1}-i}{\sqrt{cx+1}-1}\right)}{2c^5}+\frac{5b\,\mathrm{atanh}\left(\frac{\sqrt{cx-1}-i}{\sqrt{cx+1}-1}\right)}{4c^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(a + b*x^2))/((c*x - 1)^(1/2)*(c*x + 1)^(1/2)),x)
```

```
[Out] ((23*a*((c*x - 1)^(1/2) - 1i)^3)/(2*((c*x + 1)^(1/2) - 1)^3) + (333*a*((c*x
 - 1)^(1/2) - 1i)^5)/(2*((c*x + 1)^(1/2) - 1)^5) + (671*a*((c*x - 1)^(1/2)
- 1i)^7)/(2*((c*x + 1)^(1/2) - 1)^7) + (671*a*((c*x - 1)^(1/2) - 1i)^9)/(2*
((c*x + 1)^(1/2) - 1)^9) + (333*a*((c*x - 1)^(1/2) - 1i)^11)/(2*((c*x + 1)^
(1/2) - 1)^11) + (23*a*((c*x - 1)^(1/2) - 1i)^13)/(2*((c*x + 1)^(1/2) - 1)^
13) - (3*a*((c*x - 1)^(1/2) - 1i)^15)/(2*((c*x + 1)^(1/2) - 1)^15) - (3*a*(
```

```
(c*x - 1)^(1/2) - 1i))/(2*((c*x + 1)^(1/2) - 1)))/(c^5 - (8*c^5*((c*x - 1)^
(1/2) - 1i)^2)/((c*x + 1)^(1/2) - 1)^2 + (28*c^5*((c*x - 1)^(1/2) - 1i)^4)/
((c*x + 1)^(1/2) - 1)^4 - (56*c^5*((c*x - 1)^(1/2) - 1i)^6)/((c*x + 1)^(1/2
) - 1)^6 + (70*c^5*((c*x - 1)^(1/2) - 1i)^8)/((c*x + 1)^(1/2) - 1)^8 - (56*
c^5*((c*x - 1)^(1/2) - 1i)^10)/((c*x + 1)^(1/2) - 1)^10 + (28*c^5*((c*x - 1
)^(1/2) - 1i)^12)/((c*x + 1)^(1/2) - 1)^12 - (8*c^5*((c*x - 1)^(1/2) - 1i)^
14)/((c*x + 1)^(1/2) - 1)^14 + (c^5*((c*x - 1)^(1/2) - 1i)^16)/((c*x + 1)^(
1/2) - 1)^16) - ((311*b*((c*x - 1)^(1/2) - 1i)^5)/(4*((c*x + 1)^(1/2) - 1)^
5) - (175*b*((c*x - 1)^(1/2) - 1i)^3)/(12*((c*x + 1)^(1/2) - 1)^3) + (8361*
b*((c*x - 1)^(1/2) - 1i)^7)/(4*((c*x + 1)^(1/2) - 1)^7) + (42259*b*((c*x -
1)^(1/2) - 1i)^9)/(6*((c*x + 1)^(1/2) - 1)^9) + (25295*b*((c*x - 1)^(1/2) -
 1i)^11)/(2*((c*x + 1)^(1/2) - 1)^11) + (25295*b*((c*x - 1)^(1/2) - 1i)^13)
/(2*((c*x + 1)^(1/2) - 1)^13) + (42259*b*((c*x - 1)^(1/2) - 1i)^15)/(6*((c*
x + 1)^(1/2) - 1)^15) + (8361*b*((c*x - 1)^(1/2) - 1i)^17)/(4*((c*x + 1)^(1
/2) - 1)^17) + (311*b*((c*x - 1)^(1/2) - 1i)^19)/(4*((c*x + 1)^(1/2) - 1)^1
9) - (175*b*((c*x - 1)^(1/2) - 1i)^21)/(12*((c*x + 1)^(1/2) - 1)^21) + (5*b
*((c*x - 1)^(1/2) - 1i)^23)/(4*((c*x + 1)^(1/2) - 1)^23) + (5*b*((c*x - 1)^
(1/2) - 1i))/(4*((c*x + 1)^(1/2) - 1)))/(c^7 - (12*c^7*((c*x - 1)^(1/2) - 1
i)^2)/((c*x + 1)^(1/2) - 1)^2 + (66*c^7*((c*x - 1)^(1/2) - 1i)^4)/((c*x + 1
)^(1/2) - 1)^4 - (220*c^7*((c*x - 1)^(1/2) - 1i)^6)/((c*x + 1)^(1/2) - 1)^6
 + (495*c^7*((c*x - 1)^(1/2) - 1i)^8)/((c*x + 1)^(1/2) - 1)^8 - (792*c^7*((
c*x - 1)^(1/2) - 1i)^10)/((c*x + 1)^(1/2) - 1)^10 + (924*c^7*((c*x - 1)^(1/
2) - 1i)^12)/((c*x + 1)^(1/2) - 1)^12 - (792*c^7*((c*x - 1)^(1/2) - 1i)^14)
/((c*x + 1)^(1/2) - 1)^14 + (495*c^7*((c*x - 1)^(1/2) - 1i)^16)/((c*x + 1)^
(1/2) - 1)^16 - (220*c^7*((c*x - 1)^(1/2) - 1i)^18)/((c*x + 1)^(1/2) - 1)^1
8 + (66*c^7*((c*x - 1)^(1/2) - 1i)^20)/((c*x + 1)^(1/2) - 1)^20 - (12*c^7*(
(c*x - 1)^(1/2) - 1i)^22)/((c*x + 1)^(1/2) - 1)^22 + (c^7*((c*x - 1)^(1/2)
- 1i)^24)/((c*x + 1)^(1/2) - 1)^24) + (3*a*atanh(((c*x - 1)^(1/2) - 1i)/((c
*x + 1)^(1/2) - 1)))/(2*c^5) + (5*b*atanh(((c*x - 1)^(1/2) - 1i)/((c*x + 1)
^(1/2) - 1)))/(4*c^7)
```

3.349 $\int \frac{x^3(a+bx^2)}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx$

Optimal. Leaf size=103

$$\frac{2(4b+5ac^2)\sqrt{-1+cx}\sqrt{1+cx}}{15c^6}+\frac{(4b+5ac^2)\,x^2\sqrt{-1+cx}\sqrt{1+cx}}{15c^4}+\frac{bx^4\sqrt{-1+cx}\sqrt{1+cx}}{5c^2}$$

```
[Out] 2/15*(5*a*c^2+4*b)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^6+1/15*(5*a*c^2+4*b)*x^2*(
c*x-1)^(1/2)*(c*x+1)^(1/2)/c^4+1/5*b*x^4*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^2
```

Rubi [A]

time = 0.08, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {471, 102, 12, 75}

$$\frac{2\sqrt{cx-1}\sqrt{cx+1}\,(5ac^2+4b)}{15c^6}+\frac{x^2\sqrt{cx-1}\sqrt{cx+1}\,(5ac^2+4b)}{15c^4}+\frac{bx^4\sqrt{cx-1}\sqrt{cx+1}}{5c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*x^2))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]),x]
```

```
[Out] (2*(4*b + 5*a*c^2)*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(15*c^6) + ((4*b + 5*a*c^2
)*x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(15*c^4) + (b*x^4*Sqrt[-1 + c*x]*Sqrt[1
 + c*x])/(5*c^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 75

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 102

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/(d*f*(m + n + p + 1))), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
```

```
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[d*(e*x)^(
m + 1)*(a1 + b1*x^(n/2))^(p + 1)*((a2 + b2*x^(n/2))^(p + 1)/(b1*b2*e*(m + n
*(p + 1) + 1))), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3(a+bx^2)}{\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx &= \frac{bx^4\sqrt{-1+cx}\,\sqrt{1+cx}}{5c^2} - \frac{1}{5}\left(-5a-\frac{4b}{c^2}\right)\int \frac{x^3}{\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx \\ &= \frac{(4b+5ac^2)\,x^2\sqrt{-1+cx}\,\sqrt{1+cx}}{15c^4} + \frac{bx^4\sqrt{-1+cx}\,\sqrt{1+cx}}{5c^2} + \frac{(4b+5ac^2)\int}{} \\ &= \frac{(4b+5ac^2)\,x^2\sqrt{-1+cx}\,\sqrt{1+cx}}{15c^4} + \frac{bx^4\sqrt{-1+cx}\,\sqrt{1+cx}}{5c^2} + \frac{(2(4b+5ac^2)}{} \\ &= \frac{2(4b+5ac^2)\,\sqrt{-1+cx}\,\sqrt{1+cx}}{15c^6} + \frac{(4b+5ac^2)\,x^2\sqrt{-1+cx}\,\sqrt{1+cx}}{15c^4} + \frac{bx^4}{}\end{aligned}$$

Mathematica [A]
time = 0.10, size = 61, normalized size = 0.59

$$\frac{\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(5ac^2(2+c^2x^2)+b(8+4c^2x^2+3c^4x^4)\right)}{15c^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*x^2))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]),x]
```

```
[Out] (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(5*a*c^2*(2 + c^2*x^2) + b*(8 + 4*c^2*x^2 + 3
*c^4*x^4)))/(15*c^6)
```

Maple [A]
time = 0.30, size = 57, normalized size = 0.55

method	result	size
gosper	$\frac{\sqrt{cx+1}\,\sqrt{cx-1}\,(3b\,x^4c^4+5a\,c^4x^2+4b\,c^2x^2+10c^2a+8b)}{15c^6}$	57

default	$\frac{\sqrt{cx+1}\,\sqrt{cx-1}\,(3b\,x^{4}c^{4}+5a\,c^{4}x^{2}+4b\,c^{2}x^{2}+10c^{2}a+8b)}{15c^{6}}$	57
risch	$\frac{\sqrt{cx+1}\,\sqrt{cx-1}\,(3b\,x^{4}c^{4}+5a\,c^{4}x^{2}+4b\,c^{2}x^{2}+10c^{2}a+8b)}{15c^{6}}$	57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^2+a)/(c*x-1)^(1/2)/(c*x+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/15*(c*x+1)^(1/2)*(c*x-1)^(1/2)*(3*b*c^4*x^4+5*a*c^4*x^2+4*b*c^2*x^2+10*a*
c^2+8*b)/c^6
```

Maxima [A]
time = 0.27, size = 95, normalized size = 0.92

$$\frac{\sqrt{c^{2}x^{2}-1}\,bx^{4}}{5\,c^{2}}+\frac{\sqrt{c^{2}x^{2}-1}\,ax^{2}}{3\,c^{2}}+\frac{4\,\sqrt{c^{2}x^{2}-1}\,bx^{2}}{15\,c^{4}}+\frac{2\,\sqrt{c^{2}x^{2}-1}\,a}{3\,c^{4}}+\frac{8\,\sqrt{c^{2}x^{2}-1}\,b}{15\,c^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/5*sqrt(c^2*x^2 - 1)*b*x^4/c^2 + 1/3*sqrt(c^2*x^2 - 1)*a*x^2/c^2 + 4/15*sq
rt(c^2*x^2 - 1)*b*x^2/c^4 + 2/3*sqrt(c^2*x^2 - 1)*a/c^4 + 8/15*sqrt(c^2*x^2
 - 1)*b/c^6
```

Fricas [A]
time = 2.71, size = 55, normalized size = 0.53

$$\frac{(3\,bc^{4}x^{4}+10\,ac^{2}+(5\,ac^{4}+4\,bc^{2})x^{2}+8\,b)\sqrt{cx+1}\,\sqrt{cx-1}}{15\,c^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/15*(3*b*c^4*x^4 + 10*a*c^2 + (5*a*c^4 + 4*b*c^2)*x^2 + 8*b)*sqrt(c*x + 1)
*sqrt(c*x - 1)/c^6
```

Sympy [C] Result contains complex when optimal does not.
time = 42.07, size = 216, normalized size = 2.10

$$\frac{aG_{6,6}^{6,2}\left(\begin{matrix}-\frac{5}{4},-\frac{3}{4} & -1,-1,-\frac{1}{2},1\\ -\frac{3}{2},-\frac{5}{4},-1,-\frac{3}{4},-\frac{1}{2},0 & \end{matrix}\middle|\,\frac{1}{c^{2}x^{2}}\right)}{4\pi^{\frac{3}{2}}c^{4}}+\frac{iaG_{6,6}^{2,6}\left(\begin{matrix}-2,-\frac{7}{4},-\frac{3}{2},-\frac{5}{4},-1,1 & \\ -\frac{7}{4},-\frac{5}{4} & -2,-\frac{3}{2},-\frac{3}{2},0\end{matrix}\middle|\,\frac{e^{2i\pi}}{c^{2}x^{2}}\right)}{4\pi^{\frac{3}{2}}c^{4}}+\frac{bG_{6,6}^{6,2}\left(\begin{matrix}-\frac{9}{4},-\frac{7}{4} & -2,-2,-\frac{3}{2},1\\ -\frac{5}{2},-\frac{9}{4},-2,-\frac{7}{4},-\frac{3}{2},0 & \end{matrix}\middle|\,\frac{1}{c^{2}x^{2}}\right)}{4\pi^{\frac{3}{2}}c^{6}}+\frac{ibG_{6,6}^{2,6}\left(\begin{matrix}-3,-\frac{11}{4},-\frac{5}{2},-\frac{9}{4},-2,1 & \\ -\frac{11}{4},-\frac{9}{4} & -3,-\frac{5}{2},-\frac{5}{2},0\end{matrix}\middle|\,\frac{e^{2i\pi}}{c^{2}x^{2}}\right)}{4\pi^{\frac{3}{2}}c^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**2+a)/(c*x-1)**(1/2)/(c*x+1)**(1/2),x)
```

```
[Out] a*meijerg(((-5/4, -3/4), (-1, -1, -1/2, 1)), ((-3/2, -5/4, -1, -3/4, -1/2,
0), ()), 1/(c**2*x**2))/(4*pi**(3/2)*c**4) + I*a*meijerg(((-2, -7/4, -3/2,
```

```
-5/4, -1, 1), ()), ((-7/4, -5/4), (-2, -3/2, -3/2, 0)), exp_polar(2*I*pi)/(
c**2*x**2))/(4*pi**(3/2)*c**4) + b*meijerg(((-9/4, -7/4), (-2, -2, -3/2, 1)
), ((-5/2, -9/4, -2, -7/4, -3/2, 0), ()), 1/(c**2*x**2))/(4*pi**(3/2)*c**6)
 + I*b*meijerg(((-3, -11/4, -5/2, -9/4, -2, 1), ()), ((-11/4, -9/4), (-3, -
5/2, -5/2, 0)), exp_polar(2*I*pi)/(c**2*x**2))/(4*pi**(3/2)*c**6)
```

Giac [A]
time = 0.76, size = 108, normalized size = 1.05

$$\frac{\left(\left((cx+1)\left(3\,(cx+1)\left(\frac{(cx+1)b}{c^5}-\frac{4\,b}{c^5}\right)+\frac{5\,ac^{27}+22\,bc^{25}}{c^{30}}\right)-\frac{10\,(ac^{27}+2\,bc^{25})}{c^{30}}\right)(cx+1)+\frac{15\,(ac^{27}+bc^{25})}{c^{30}}\right)\sqrt{cx+1}\,\sqrt{cx-1}}{15\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/15*(((c*x + 1)*(3*(c*x + 1)*((c*x + 1)*b/c^5 - 4*b/c^5) + (5*a*c^27 + 22*
b*c^25)/c^30) - 10*(a*c^27 + 2*b*c^25)/c^30)*(c*x + 1) + 15*(a*c^27 + b*c^2
5)/c^30)*sqrt(c*x + 1)*sqrt(c*x - 1)/c
```

Mupad [B]
time = 2.44, size = 108, normalized size = 1.05

$$\frac{\sqrt{c\,x-1}\,\left(\frac{10\,a\,c^2+8\,b}{15\,c^6}+\frac{b\,x^5}{5\,c}+\frac{b\,x^4}{5\,c^2}+\frac{x^2\,\left(5\,a\,c^4+4\,b\,c^2\right)}{15\,c^6}+\frac{x^3\,\left(5\,a\,c^5+4\,b\,c^3\right)}{15\,c^6}+\frac{x\,\left(10\,a\,c^3+8\,b\,c\right)}{15\,c^6}\right)}{\sqrt{c\,x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*x^2))/((c*x - 1)^(1/2)*(c*x + 1)^(1/2)),x)
```

```
[Out] ((c*x - 1)^(1/2)*((8*b + 10*a*c^2)/(15*c^6) + (b*x^5)/(5*c) + (b*x^4)/(5*c^
2) + (x^2*(5*a*c^4 + 4*b*c^2))/(15*c^6) + (x^3*(5*a*c^5 + 4*b*c^3))/(15*c^6
) + (x*(8*b*c + 10*a*c^3))/(15*c^6)))/(c*x + 1)^(1/2)
```

3.350 $\int \frac{x^2(a+bx^2)}{\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx$

Optimal. Leaf size=87

$$\frac{(3b+4ac^2)\,x\sqrt{-1+cx}\,\sqrt{1+cx}}{8c^4}+\frac{bx^3\sqrt{-1+cx}\,\sqrt{1+cx}}{4c^2}+\frac{(3b+4ac^2)\cosh^{-1}(cx)}{8c^5}$$

```
[Out] 1/8*(4*a*c^2+3*b)*arccosh(c*x)/c^5+1/8*(4*a*c^2+3*b)*x*(c*x-1)^(1/2)*(c*x+1
)^(1/2)/c^4+1/4*b*x^3*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^2
```

Rubi [A]
time = 0.10, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {471, 92, 54}

$$\frac{(4ac^2+3b)\cosh^{-1}(cx)}{8c^5}+\frac{x\sqrt{cx-1}\,\sqrt{cx+1}\,(4ac^2+3b)}{8c^4}+\frac{bx^3\sqrt{cx-1}\,\sqrt{cx+1}}{4c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*x^2))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]),x]
```

```
[Out] ((3*b + 4*a*c^2)*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(8*c^4) + (b*x^3*Sqrt[-1 +
 c*x]*Sqrt[1 + c*x])/(4*c^2) + ((3*b + 4*a*c^2)*ArcCosh[c*x])/(8*c^5)
```

Rule 54

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[b*(x/a)]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 92

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(
d*f*(n + p + 3))), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[d*(e*x)^(
m + 1)*(a1 + b1*x^(n/2))^(p + 1)*((a2 + b2*x^(n/2))^(p + 1)/(b1*b2*e*(m + n
*(p + 1) + 1))), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
```

```
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2(a+bx^2)}{\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx &= \frac{bx^3\sqrt{-1+cx}\,\sqrt{1+cx}}{4c^2} - \frac{1}{4}\left(-4a-\frac{3b}{c^2}\right)\int \frac{x^2}{\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx \\ &= \frac{(3b+4ac^2)\,x\sqrt{-1+cx}\,\sqrt{1+cx}}{8c^4} + \frac{bx^3\sqrt{-1+cx}\,\sqrt{1+cx}}{4c^2} + \frac{(3b+4ac^2)\int \\ &= \frac{(3b+4ac^2)\,x\sqrt{-1+cx}\,\sqrt{1+cx}}{8c^4} + \frac{bx^3\sqrt{-1+cx}\,\sqrt{1+cx}}{4c^2} + \frac{(3b+4ac^2)\,\mathrm{co}}{8c^5}\end{aligned}$$

Mathematica [A]
time = 0.16, size = 77, normalized size = 0.89

$$\frac{cx\sqrt{-1+cx}\,\sqrt{1+cx}\,(4ac^2+b(3+2c^2x^2))+(6b+8ac^2)\tanh^{-1}\left(\sqrt{\frac{-1+cx}{1+cx}}\right)}{8c^5}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*(a + b*x^2))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]),x]
```

```
[Out] (c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(4*a*c^2 + b*(3 + 2*c^2*x^2)) + (6*b + 8*
a*c^2)*ArcTanh[Sqrt[(-1 + c*x)/(1 + c*x)]])/(8*c^5)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.28, size = 147, normalized size = 1.69

method	result
risch	$\frac{x(2b\,c^2x^2+4c^2a+3b)\sqrt{cx+1}\,\sqrt{cx-1}}{8c^4} + \frac{\left(\frac{\ln\left(\frac{c^2x}{\sqrt{c^2}}+\sqrt{c^2x^2-1}\right)a}{2c^2\sqrt{c^2}} + \frac{3\ln\left(\frac{c^2x}{\sqrt{c^2}}+\sqrt{c^2x^2-1}\right)b}{8c^4\sqrt{c^2}}\right)\sqrt{(cx+}}{\sqrt{cx-1}\,\sqrt{cx+1}}$
default	$\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,\left(2\sqrt{c^2x^2-1}\,\mathrm{csgn}(c)c^3b\,x^3+4\sqrt{c^2x^2-1}\,\mathrm{csgn}(c)c^3ax+3\sqrt{c^2x^2-1}\,\mathrm{csgn}(c)cbx+4\ln\left(\left(\sqrt{c^2}\right.\right.\right.}{8c^5\sqrt{c^2x^2-1}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2+a)/(c*x-1)^(1/2)/(c*x+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*(c*x-1)^(1/2)*(c*x+1)^(1/2)*(2*(c^2*x^2-1)^(1/2)*csgn(c)*c^3*b*x^3+4*(c
^2*x^2-1)^(1/2)*csgn(c)*c^3*a*x+3*(c^2*x^2-1)^(1/2)*csgn(c)*c*b*x+4*ln(((c^
2*x^2-1)^(1/2)*csgn(c)+c*x)*csgn(c))*a*c^2+3*ln(((c^2*x^2-1)^(1/2)*csgn(c)+
c*x)*csgn(c))*b)*csgn(c)/c^5/(c^2*x^2-1)^(1/2)
```

Maxima [A]
time = 0.31, size = 113, normalized size = 1.30

$$\frac{\sqrt{c^2x^2-1}\,bx^3}{4\,c^2}+\frac{\sqrt{c^2x^2-1}\,ax}{2\,c^2}+\frac{a\log\left(2\,c^2x+2\,\sqrt{c^2x^2-1}\,c\right)}{2\,c^3}+\frac{3\,\sqrt{c^2x^2-1}\,bx}{8\,c^4}+\frac{3\,b\log\left(2\,c^2x+2\,\sqrt{c^2x^2-1}\,c\right)}{8\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(c^2*x^2 - 1)*b*x^3/c^2 + 1/2*sqrt(c^2*x^2 - 1)*a*x/c^2 + 1/2*a*log
(2*c^2*x + 2*sqrt(c^2*x^2 - 1)*c)/c^3 + 3/8*sqrt(c^2*x^2 - 1)*b*x/c^4 + 3/8
*b*log(2*c^2*x + 2*sqrt(c^2*x^2 - 1)*c)/c^5
```

Fricas [A]
time = 2.89, size = 77, normalized size = 0.89

$$\frac{\left(2\,bc^3x^3+\left(4\,ac^3+3\,bc\right)x\right)\sqrt{cx+1}\,\sqrt{cx-1}-\left(4\,ac^2+3\,b\right)\log\left(-cx+\sqrt{cx+1}\,\sqrt{cx-1}\right)}{8\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/8*((2*b*c^3*x^3 + (4*a*c^3 + 3*b*c)*x)*sqrt(c*x + 1)*sqrt(c*x - 1) - (4*a
*c^2 + 3*b)*log(-c*x + sqrt(c*x + 1)*sqrt(c*x - 1)))/c^5
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**2+a)/(c*x-1)**(1/2)/(c*x+1)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.95, size = 121, normalized size = 1.39

$$\frac{\left((cx+1)\left(2\,(cx+1)\left(\frac{(cx+1)b}{c^4}-\frac{3\,b}{c^4}\right)+\frac{4\,ac^{18}+9\,bc^{16}}{c^{20}}\right)-\frac{4\,ac^{18}+5\,bc^{16}}{c^{20}}\right)\sqrt{cx+1}\,\sqrt{cx-1}-\frac{2\,(4\,ac^2+3\,b)\log\left(\sqrt{cx+1}-\sqrt{cx-1}\right)}{c^4}}{8\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*(((c*x + 1)*(2*(c*x + 1)*((c*x + 1)*b/c^4 - 3*b/c^4) + (4*a*c^18 + 9*b*
c^16)/c^20) - (4*a*c^18 + 5*b*c^16)/c^20)*sqrt(c*x + 1)*sqrt(c*x - 1) - 2*(
4*a*c^2 + 3*b)*log(sqrt(c*x + 1) - sqrt(c*x - 1))/c^4)/c
```

Mupad [B]
time = 22.50, size = 720, normalized size = 8.28

$$\frac{\frac{23b\left(\sqrt{cx-1}-\mathrm{i}\right)^3}{2\left(\sqrt{cx+1}-1\right)^3}+\frac{333b\left(\sqrt{cx-1}-\mathrm{i}\right)^5}{2\left(\sqrt{cx+1}-1\right)^5}+\frac{671b\left(\sqrt{cx-1}-\mathrm{i}\right)^7}{2\left(\sqrt{cx+1}-1\right)^7}+\frac{671b\left(\sqrt{cx-1}-\mathrm{i}\right)^9}{2\left(\sqrt{cx+1}-1\right)^9}+\frac{333b\left(\sqrt{cx-1}-\mathrm{i}\right)^{11}}{2\left(\sqrt{cx+1}-1\right)^{11}}+\frac{23b\left(\sqrt{cx-1}-\mathrm{i}\right)^{13}}{2\left(\sqrt{cx+1}-1\right)^{13}}-\frac{3b\left(\sqrt{cx-1}-\mathrm{i}\right)^{15}}{2\left(\sqrt{cx+1}-1\right)^{15}}-\frac{3b\left(\sqrt{cx-1}-\mathrm{i}\right)}{2\left(\sqrt{cx+1}-1\right)}}{c^5-\frac{8c^5\left(\sqrt{cx-1}-\mathrm{i}\right)^2}{\left(\sqrt{cx+1}-1\right)^2}+\frac{28c^5\left(\sqrt{cx-1}-\mathrm{i}\right)^4}{\left(\sqrt{cx+1}-1\right)^4}-\frac{56c^5\left(\sqrt{cx-1}-\mathrm{i}\right)^6}{\left(\sqrt{cx+1}-1\right)^6}+\frac{70c^5\left(\sqrt{cx-1}-\mathrm{i}\right)^8}{\left(\sqrt{cx+1}-1\right)^8}-\frac{56c^5\left(\sqrt{cx-1}-\mathrm{i}\right)^{10}}{\left(\sqrt{cx+1}-1\right)^{10}}+\frac{28c^5\left(\sqrt{cx-1}-\mathrm{i}\right)^{12}}{\left(\sqrt{cx+1}-1\right)^{12}}-\frac{8c^5\left(\sqrt{cx-1}-\mathrm{i}\right)^{14}}{\left(\sqrt{cx+1}-1\right)^{14}}+\frac{c^5\left(\sqrt{cx-1}-\mathrm{i}\right)^{16}}{\left(\sqrt{cx+1}-1\right)^{16}}}-\frac{\frac{14a\left(\sqrt{cx-1}-\mathrm{i}\right)^3}{\left(\sqrt{cx+1}-1\right)^3}+\frac{14a\left(\sqrt{cx-1}-\mathrm{i}\right)^5}{\left(\sqrt{cx+1}-1\right)^5}+\frac{2a\left(\sqrt{cx-1}-\mathrm{i}\right)^7}{\left(\sqrt{cx+1}-1\right)^7}+\frac{2a\left(\sqrt{cx-1}-\mathrm{i}\right)}{\sqrt{cx+1}-1}}{c^3-\frac{4c^3\left(\sqrt{cx-1}-\mathrm{i}\right)^2}{\left(\sqrt{cx+1}-1\right)^2}+\frac{6c^3\left(\sqrt{cx-1}-\mathrm{i}\right)^4}{\left(\sqrt{cx+1}-1\right)^4}-\frac{4c^3\left(\sqrt{cx-1}-\mathrm{i}\right)^6}{\left(\sqrt{cx+1}-1\right)^6}+\frac{c^3\left(\sqrt{cx-1}-\mathrm{i}\right)^8}{\left(\sqrt{cx+1}-1\right)^8}}+\frac{2a\,\mathrm{atanh}\left(\frac{\sqrt{cx-1}-\mathrm{i}}{\sqrt{cx+1}-1}\right)}{c^3}+\frac{3b\,\mathrm{atanh}\left(\frac{\sqrt{cx-1}-\mathrm{i}}{\sqrt{cx+1}-1}\right)}{2c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*x^2))/((c*x - 1)^(1/2)*(c*x + 1)^(1/2)),x)
```

```
[Out] ((23*b*((c*x - 1)^(1/2) - 1i)^3)/(2*((c*x + 1)^(1/2) - 1)^3) + (333*b*((c*x
 - 1)^(1/2) - 1i)^5)/(2*((c*x + 1)^(1/2) - 1)^5) + (671*b*((c*x - 1)^(1/2)
- 1i)^7)/(2*((c*x + 1)^(1/2) - 1)^7) + (671*b*((c*x - 1)^(1/2) - 1i)^9)/(2*
((c*x + 1)^(1/2) - 1)^9) + (333*b*((c*x - 1)^(1/2) - 1i)^11)/(2*((c*x + 1)^
(1/2) - 1)^11) + (23*b*((c*x - 1)^(1/2) - 1i)^13)/(2*((c*x + 1)^(1/2) - 1)^
13) - (3*b*((c*x - 1)^(1/2) - 1i)^15)/(2*((c*x + 1)^(1/2) - 1)^15) - (3*b*(
(c*x - 1)^(1/2) - 1i))/(2*((c*x + 1)^(1/2) - 1)))/(c^5 - (8*c^5*((c*x - 1)^
(1/2) - 1i)^2)/((c*x + 1)^(1/2) - 1)^2 + (28*c^5*((c*x - 1)^(1/2) - 1i)^4)/
((c*x + 1)^(1/2) - 1)^4 - (56*c^5*((c*x - 1)^(1/2) - 1i)^6)/((c*x + 1)^(1/2
) - 1)^6 + (70*c^5*((c*x - 1)^(1/2) - 1i)^8)/((c*x + 1)^(1/2) - 1)^8 - (56*
c^5*((c*x - 1)^(1/2) - 1i)^10)/((c*x + 1)^(1/2) - 1)^10 + (28*c^5*((c*x - 1
)^(1/2) - 1i)^12)/((c*x + 1)^(1/2) - 1)^12 - (8*c^5*((c*x - 1)^(1/2) - 1i)^
14)/((c*x + 1)^(1/2) - 1)^14 + (c^5*((c*x - 1)^(1/2) - 1i)^16)/((c*x + 1)^(
1/2) - 1)^16) - ((14*a*((c*x - 1)^(1/2) - 1i)^3)/((c*x + 1)^(1/2) - 1)^3 +
(14*a*((c*x - 1)^(1/2) - 1i)^5)/((c*x + 1)^(1/2) - 1)^5 + (2*a*((c*x - 1)^(
1/2) - 1i)^7)/((c*x + 1)^(1/2) - 1)^7 + (2*a*((c*x - 1)^(1/2) - 1i))/((c*x
+ 1)^(1/2) - 1))/(c^3 - (4*c^3*((c*x - 1)^(1/2) - 1i)^2)/((c*x + 1)^(1/2) -
 1)^2 + (6*c^3*((c*x - 1)^(1/2) - 1i)^4)/((c*x + 1)^(1/2) - 1)^4 - (4*c^3*(
(c*x - 1)^(1/2) - 1i)^6)/((c*x + 1)^(1/2) - 1)^6 + (c^3*((c*x - 1)^(1/2) -
1i)^8)/((c*x + 1)^(1/2) - 1)^8) + (2*a*atanh(((c*x - 1)^(1/2) - 1i)/((c*x +
 1)^(1/2) - 1)))/c^3 + (3*b*atanh(((c*x - 1)^(1/2) - 1i)/((c*x + 1)^(1/2) -
 1)))/(2*c^5)
```

3.351 $\int \frac{x(a+bx^2)}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx$

Optimal. Leaf size=65

$$\frac{(2b+3ac^2)\sqrt{-1+cx}\sqrt{1+cx}}{3c^4}+\frac{bx^2\sqrt{-1+cx}\sqrt{1+cx}}{3c^2}$$

```
[Out] 1/3*(3*a*c^2+2*b)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^4+1/3*b*x^2*(c*x-1)^(1/2)*(
c*x+1)^(1/2)/c^2
```

Rubi [A]
time = 0.06, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {471, 75}

$$\frac{\sqrt{cx-1}\sqrt{cx+1}(3ac^2+2b)}{3c^4}+\frac{bx^2\sqrt{cx-1}\sqrt{cx+1}}{3c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*x^2))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]),x]
```

```
[Out] ((2*b + 3*a*c^2)*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(3*c^4) + (b*x^2*Sqrt[-1 + c
*x]*Sqrt[1 + c*x])/(3*c^2)
```

Rule 75

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[d*(e*x)^(
m + 1)*(a1 + b1*x^(n/2))^(p + 1)*((a2 + b2*x^(n/2))^(p + 1)/(b1*b2*e*(m + n
*(p + 1) + 1))), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\int \frac{x(a+bx^2)}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx = \frac{bx^2\sqrt{-1+cx}\sqrt{1+cx}}{3c^2} - \frac{1}{3}\left(-3a-\frac{2b}{c^2}\right)\int \frac{x}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx$$
$$= \frac{(2b+3ac^2)\sqrt{-1+cx}\sqrt{1+cx}}{3c^4} + \frac{bx^2\sqrt{-1+cx}\sqrt{1+cx}}{3c^2}$$

Mathematica [A]
time = 0.07, size = 43, normalized size = 0.66

$$\frac{\sqrt{-1+cx}\sqrt{1+cx}\,(3ac^2+b(2+c^2x^2))}{3c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*x^2))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]),x]
```

```
[Out] (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(3*a*c^2 + b*(2 + c^2*x^2)))/(3*c^4)
```

Maple [A]
time = 0.27, size = 38, normalized size = 0.58

method	result	size
gosper	$\frac{\sqrt{cx+1}\,\sqrt{cx-1}\,(b\,c^2x^2+3c^2a+2b)}{3c^4}$	38
default	$\frac{\sqrt{cx+1}\,\sqrt{cx-1}\,(b\,c^2x^2+3c^2a+2b)}{3c^4}$	38
risch	$\frac{\sqrt{cx+1}\,\sqrt{cx-1}\,(b\,c^2x^2+3c^2a+2b)}{3c^4}$	38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2+a)/(c*x-1)^(1/2)/(c*x+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*(b*c^2*x^2+3*a*c^2+2*b)/c^4
```

Maxima [A]
time = 0.29, size = 54, normalized size = 0.83

$$\frac{\sqrt{c^2x^2-1}\,bx^2}{3\,c^2} + \frac{\sqrt{c^2x^2-1}\,a}{c^2} + \frac{2\sqrt{c^2x^2-1}\,b}{3\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(c^2*x^2 - 1)*b*x^2/c^2 + sqrt(c^2*x^2 - 1)*a/c^2 + 2/3*sqrt(c^2*x^
2 - 1)*b/c^4
```

Fricas [A]
time = 2.85, size = 37, normalized size = 0.57

$$\frac{\left(bc^2x^2+3\,ac^2+2\,b\right)\sqrt{cx+1}\,\sqrt{cx-1}}{3\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*(b*c^2*x^2 + 3*a*c^2 + 2*b)*sqrt(c*x + 1)*sqrt(c*x - 1)/c^4
```

Sympy [C] Result contains complex when optimal does not.
time = 27.30, size = 202, normalized size = 3.11

$$\frac{aG_{6,6}^{6,2}\left(\begin{matrix} -\frac{1}{4},\frac{1}{4} & 0,0,\frac{1}{2},1 \\ -\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},0 & \end{matrix}\middle|\frac{1}{c^2x^2}\right)}{4\pi^{\frac{3}{2}}c^2}+\frac{iaG_{6,6}^{2,6}\left(\begin{matrix} -1,-\frac{3}{4},-\frac{1}{2},-\frac{1}{4},0,1 & \\ -\frac{3}{4},-\frac{1}{4} & -1,-\frac{1}{2},-\frac{1}{2},0 \end{matrix}\middle|\frac{e^{2i\pi}}{c^2x^2}\right)}{4\pi^{\frac{3}{2}}c^2}+\frac{bG_{6,6}^{6,2}\left(\begin{matrix} -\frac{5}{4},-\frac{3}{4} & -1,-1,-\frac{1}{2},1 \\ -\frac{3}{2},-\frac{5}{4},-1,-\frac{3}{4},-\frac{1}{2},0 & \end{matrix}\middle|\frac{1}{c^2x^2}\right)}{4\pi^{\frac{3}{2}}c^4}+\frac{ibG_{6,6}^{2,6}\left(\begin{matrix} -2,-\frac{7}{4},-\frac{3}{2},-\frac{5}{4},-1,1 & \\ -\frac{7}{4},-\frac{5}{4} & -2,-\frac{3}{2},-\frac{3}{2},0 \end{matrix}\middle|\frac{e^{2i\pi}}{c^2x^2}\right)}{4\pi^{\frac{3}{2}}c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**2+a)/(c*x-1)**(1/2)/(c*x+1)**(1/2),x)
```

```
[Out] a*meijerg(((-1/4, 1/4), (0, 0, 1/2, 1)), ((-1/2, -1/4, 0, 1/4, 1/2, 0), ())
, 1/(c**2*x**2))/(4*pi**(3/2)*c**2) + I*a*meijerg(((-1, -3/4, -1/2, -1/4, 0
, 1), ()), ((-3/4, -1/4), (-1, -1/2, -1/2, 0)), exp_polar(2*I*pi)/(c**2*x**
2))/(4*pi**(3/2)*c**2) + b*meijerg(((-5/4, -3/4), (-1, -1, -1/2, 1)), ((-3/
2, -5/4, -1, -3/4, -1/2, 0), ()), 1/(c**2*x**2))/(4*pi**(3/2)*c**4) + I*b*m
eijerg(((-2, -7/4, -3/2, -5/4, -1, 1), ()), ((-7/4, -5/4), (-2, -3/2, -3/2,
 0)), exp_polar(2*I*pi)/(c**2*x**2))/(4*pi**(3/2)*c**4)
```

Giac [A]
time = 0.69, size = 59, normalized size = 0.91

$$\frac{\sqrt{cx+1}\,\sqrt{cx-1}\,\left((cx+1)\left(\frac{(cx+1)b}{c^3}-\frac{2\,b}{c^3}\right)+\frac{3\,\left(ac^{11}+bc^9\right)}{c^{12}}\right)}{3\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(c*x + 1)*sqrt(c*x - 1)*((c*x + 1)*((c*x + 1)*b/c^3 - 2*b/c^3) + 3*
(a*c^11 + b*c^9)/c^12)/c
```

Mupad [B]
time = 2.36, size = 66, normalized size = 1.02

$$\frac{\sqrt{c\,x-1}\,\left(\frac{3\,a\,c^2+2\,b}{3\,c^4}+\frac{b\,x^3}{3\,c}+\frac{b\,x^2}{3\,c^2}+\frac{x\,\left(3\,a\,c^3+2\,b\,c\right)}{3\,c^4}\right)}{\sqrt{c\,x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*x^2))/((c*x - 1)^(1/2)*(c*x + 1)^(1/2)),x)
```

```
[Out] ((c*x - 1)^(1/2)*((2*b + 3*a*c^2)/(3*c^4) + (b*x^3)/(3*c) + (b*x^2)/(3*c^2)
 + (x*(2*b*c + 3*a*c^3))/(3*c^4)))/(c*x + 1)^(1/2)
```

3.352 $\int \frac{a+bx^2}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx$

Optimal. Leaf size=47

$$\frac{bx\sqrt{-1+cx}\,\sqrt{1+cx}}{2c^2}+\frac{(b+2ac^2)\cosh^{-1}(cx)}{2c^3}$$

```
[Out] 1/2*(2*a*c^2+b)*arccosh(c*x)/c^3+1/2*b*x*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^2
```

Rubi [A]
time = 0.03, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {397, 54}

$$\frac{(2ac^2+b)\cosh^{-1}(cx)}{2c^3}+\frac{bx\sqrt{cx-1}\,\sqrt{cx+1}}{2c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]),x]
```

```
[Out] (b*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(2*c^2) + ((b + 2*a*c^2)*ArcCosh[c*x])/(
2*c^3)
```

Rule 54

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[b*(x/a)]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 397

```
Int[((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_
.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[d*x*(a1 + b1*x^(n/2))^(p + 1
)*((a2 + b2*x^(n/2))^(p + 1)/(b1*b2*(n*(p + 1) + 1))), x] - Dist[(a1*a2*d -
 b1*b2*c*(n*(p + 1) + 1))/(b1*b2*(n*(p + 1) + 1)), Int[(a1 + b1*x^(n/2))^p*
(a2 + b2*x^(n/2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, n, p}, x] && Eq
Q[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx^2}{\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx &= \frac{bx\sqrt{-1+cx}\,\sqrt{1+cx}}{2c^2}-\frac{(-b-2ac^2)\int \frac{1}{\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx}{2c^2}\\ &= \frac{bx\sqrt{-1+cx}\,\sqrt{1+cx}}{2c^2}+\frac{(b+2ac^2)\cosh^{-1}(cx)}{2c^3}\end{aligned}$$

Mathematica [A]
time = 0.10, size = 58, normalized size = 1.23

$$\frac{bcx\sqrt{-1+cx}\,\sqrt{1+cx}+2(b+2ac^2)\tanh^{-1}\left(\sqrt{\frac{-1+cx}{1+cx}}\right)}{2c^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^2)/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]),x]
```

```
[Out] (b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x] + 2*(b + 2*a*c^2)*ArcTanh[Sqrt[(-1 + c*
x)/(1 + c*x)]])/(2*c^3)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.30, size = 103, normalized size = 2.19

method	result
default	$\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,\left(\sqrt{c^2x^2-1}\,\mathrm{csgn}(c)cbx+2\ln\left(\left(\sqrt{c^2x^2-1}\,\mathrm{csgn}(c)+cx\right)\mathrm{csgn}(c)\right)a\,c^2+\ln\left(\left(\sqrt{c^2x^2-1}\,\mathrm{csg}\right.\right.\right.}{2c^3\sqrt{c^2x^2-1}}$
risch	$\frac{bx\sqrt{cx-1}\,\sqrt{cx+1}}{2c^2}+\frac{\left(\frac{\ln\left(\frac{c^2x}{\sqrt{c^2}}+\sqrt{c^2x^2-1}\right)a}{\sqrt{c^2}}+\frac{\ln\left(\frac{c^2x}{\sqrt{c^2}}+\sqrt{c^2x^2-1}\right)b}{2c^2\sqrt{c^2}}\right)\sqrt{(cx+1)\,(cx-1)}}{\sqrt{cx-1}\,\sqrt{cx+1}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)/(c*x-1)^(1/2)/(c*x+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^3*((c^2*x^2-1)^(1/2)*csgn(c)*c*b*x+2*ln((
(c^2*x^2-1)^(1/2)*csgn(c)+c*x)*csgn(c))*a*c^2+ln(((c^2*x^2-1)^(1/2)*csgn(c)
+c*x)*csgn(c))*b)/(c^2*x^2-1)^(1/2)*csgn(c)
```

Maxima [A]
time = 0.28, size = 74, normalized size = 1.57

$$\frac{a\log\left(2\,c^2x+2\,\sqrt{c^2x^2-1}\,c\right)}{c}+\frac{\sqrt{c^2x^2-1}\,bx}{2\,c^2}+\frac{b\log\left(2\,c^2x+2\,\sqrt{c^2x^2-1}\,c\right)}{2\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] a*log(2*c^2*x + 2*sqrt(c^2*x^2 - 1)*c)/c + 1/2*sqrt(c^2*x^2 - 1)*b*x/c^2 +
1/2*b*log(2*c^2*x + 2*sqrt(c^2*x^2 - 1)*c)/c^3
```

Fricas [A]
time = 2.97, size = 55, normalized size = 1.17

$$\frac{\sqrt{cx+1}\,\sqrt{cx-1}\,bcx-(2\,ac^2+b)\log\left(-cx+\sqrt{cx+1}\,\sqrt{cx-1}\right)}{2\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(c*x + 1)*sqrt(c*x - 1)*b*c*x - (2*a*c^2 + b)*log(-c*x + sqrt(c*x
+ 1)*sqrt(c*x - 1)))/c^3
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/(c*x-1)**(1/2)/(c*x+1)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.69, size = 69, normalized size = 1.47

$$\frac{\sqrt{cx+1}\,\sqrt{cx-1}\left(\frac{(cx+1)b}{c^2}-\frac{b}{c^2}\right)-\frac{2\,(2\,ac^2+b)\log\left(\sqrt{cx+1}-\sqrt{cx-1}\right)}{c^2}}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*(sqrt(c*x + 1)*sqrt(c*x - 1)*((c*x + 1)*b/c^2 - b/c^2) - 2*(2*a*c^2 + b
)*log(sqrt(c*x + 1) - sqrt(c*x - 1))/c^2)/c
```

Mupad [B]
time = 12.69, size = 293, normalized size = 6.23

$$-\frac{\frac{14\,b\,\left(\sqrt{cx-1}-\mathrm{i}\right)^3}{\left(\sqrt{cx+1}-1\right)^3}+\frac{14\,b\,\left(\sqrt{cx-1}-\mathrm{i}\right)^5}{\left(\sqrt{cx+1}-1\right)^5}+\frac{2\,b\,\left(\sqrt{cx-1}-\mathrm{i}\right)^7}{\left(\sqrt{cx+1}-1\right)^7}+\frac{2\,b\,\left(\sqrt{cx-1}-\mathrm{i}\right)}{\sqrt{cx+1}-1}}{c^3-\frac{4\,c^3\,\left(\sqrt{cx-1}-\mathrm{i}\right)^2}{\left(\sqrt{cx+1}-1\right)^2}+\frac{6\,c^3\,\left(\sqrt{cx-1}-\mathrm{i}\right)^4}{\left(\sqrt{cx+1}-1\right)^4}-\frac{4\,c^3\,\left(\sqrt{cx-1}-\mathrm{i}\right)^6}{\left(\sqrt{cx+1}-1\right)^6}+\frac{c^3\,\left(\sqrt{cx-1}-\mathrm{i}\right)^8}{\left(\sqrt{cx+1}-1\right)^8}}+\frac{2\,b\,\mathrm{atanh}\left(\frac{\sqrt{cx-1}-\mathrm{i}}{\sqrt{cx+1}-1}\right)}{c^3}-\frac{4\,a\,\mathrm{atan}\left(\frac{c\,\left(\sqrt{cx-1}-\mathrm{i}\right)}{\left(\sqrt{cx+1}-1\right)\,\sqrt{-c^2}}\right)}{\sqrt{-c^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)/((c*x - 1)^(1/2)*(c*x + 1)^(1/2)),x)
```

```
[Out] (2*b*atanh(((c*x - 1)^(1/2) - 1i)/((c*x + 1)^(1/2) - 1)))/c^3 - ((14*b*((c*
x - 1)^(1/2) - 1i)^3)/((c*x + 1)^(1/2) - 1)^3 + (14*b*((c*x - 1)^(1/2) - 1i
)^5)/((c*x + 1)^(1/2) - 1)^5 + (2*b*((c*x - 1)^(1/2) - 1i)^7)/((c*x + 1)^(1
/2) - 1)^7 + (2*b*((c*x - 1)^(1/2) - 1i))/((c*x + 1)^(1/2) - 1))/(c^3 - (4*
c^3*((c*x - 1)^(1/2) - 1i)^2)/((c*x + 1)^(1/2) - 1)^2 + (6*c^3*((c*x - 1)^(
1/2) - 1i)^4)/((c*x + 1)^(1/2) - 1)^4 - (4*c^3*((c*x - 1)^(1/2) - 1i)^6)/((
c*x + 1)^(1/2) - 1)^6 + (c^3*((c*x - 1)^(1/2) - 1i)^8)/((c*x + 1)^(1/2) - 1
)^8) - (4*a*atan((c*((c*x - 1)^(1/2) - 1i))/(((c*x + 1)^(1/2) - 1)*(-c^2)^(
1/2))))/(-c^2)^(1/2)
```

3.353 $\int \frac{a+bx^2}{x\sqrt{-1+cx}\sqrt{1+cx}}\,dx$

Optimal. Leaf size=46

$$\frac{b\sqrt{-1+cx}\sqrt{1+cx}}{c^2}+a\tan^{-1}\left(\sqrt{-1+cx}\sqrt{1+cx}\right)$$

```
[Out] a*arctan((c*x-1)^(1/2)*(c*x+1)^(1/2))+b*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^2
```

Rubi [A]
time = 0.07, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}=0.103$, Rules used = {471, 94, 211}

$$a\text{ArcTan}\left(\sqrt{cx-1}\sqrt{cx+1}\right)+\frac{b\sqrt{cx-1}\sqrt{cx+1}}{c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)/(x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]),x]
```

```
[Out] (b*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/c^2 + a*ArcTan[Sqrt[-1 + c*x]*Sqrt[1 + c*x
]]
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[d*(e*x)^(
m + 1)*(a1 + b1*x^(n/2))^(p + 1)*((a2 + b2*x^(n/2))^(p + 1)/(b1*b2*e*(m + n
*(p + 1) + 1))), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx^2}{x\sqrt{-1+cx}\sqrt{1+cx}}\,dx &= \frac{b\sqrt{-1+cx}\sqrt{1+cx}}{c^2}+a\int \frac{1}{x\sqrt{-1+cx}\sqrt{1+cx}}\,dx\\ &= \frac{b\sqrt{-1+cx}\sqrt{1+cx}}{c^2}+(ac)\text{Subst}\left(\int \frac{1}{c+cx^2}\,dx, x, \sqrt{-1+cx}\sqrt{1+cx}\right)\\ &= \frac{b\sqrt{-1+cx}\sqrt{1+cx}}{c^2}+a\tan^{-1}\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\end{aligned}$$

Mathematica [**A**]
time = 0.08, size = 45, normalized size = 0.98

$$\frac{b\sqrt{-1+cx}\sqrt{1+cx}}{c^2}+2a\tan^{-1}\left(\sqrt{\frac{-1+cx}{1+cx}}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^2)/(x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]),x]
[Out] (b*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/c^2 + 2*a*ArcTan[Sqrt[(-1 + c*x)/(1 + c*x)
]]
```

Maple [**A**]
time = 0.27, size = 62, normalized size = 1.35

method	result	size
default	$\frac{\left(-\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)a\,c^2+b\sqrt{c^2x^2-1}\right)\sqrt{cx-1}\,\sqrt{cx+1}}{\sqrt{c^2x^2-1}\,c^2}$	62

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)/x/(c*x-1)^(1/2)/(c*x+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] (-arctan(1/(c^2*x^2-1)^(1/2))*a*c^2+b*(c^2*x^2-1)^(1/2))*(c*x-1)^(1/2)*(c*x
+1)^(1/2)/(c^2*x^2-1)^(1/2)/c^2
```

Maxima [**A**]
time = 0.60, size = 29, normalized size = 0.63

$$-a\arcsin\left(\frac{1}{c|x|}\right)+\frac{\sqrt{c^2x^2-1}\,b}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -a*arcsin(1/(c*abs(x))) + sqrt(c^2*x^2 - 1)*b/c^2
```

Fricas [A]
time = 4.06, size = 48, normalized size = 1.04

$$\frac{2\,ac^2\arctan\left(-cx+\sqrt{cx+1}\,\sqrt{cx-1}\right)+\sqrt{cx+1}\,\sqrt{cx-1}\,b}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] (2*a*c^2*arctan(-c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) + sqrt(c*x + 1)*sqrt(c*
x - 1)*b)/c^2
```

Sympy [C] Result contains complex when optimal does not.
time = 38.99, size = 162, normalized size = 3.52

$$-\frac{aG_{6,6}^{5,3}\left(\begin{matrix}\frac{3}{4},\frac{5}{4},1 & 1,1,\frac{3}{2}\\ \frac{1}{2},\frac{3}{4},1,\frac{5}{4},\frac{3}{2} & 0\end{matrix}\middle|\,\frac{1}{c^2x^2}\right)}{4\pi^{\frac{3}{2}}}+\frac{iaG_{6,6}^{2,6}\left(\begin{matrix}0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,1 & \\ \frac{1}{4},\frac{3}{4} & 0,\frac{1}{2},\frac{1}{2},0\end{matrix}\middle|\,\frac{e^{2i\pi}}{c^2x^2}\right)}{4\pi^{\frac{3}{2}}}+\frac{bG_{6,6}^{6,2}\left(\begin{matrix}-\frac{1}{4},\frac{1}{4} & 0,0,\frac{1}{2},1\\ -\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},0 & \end{matrix}\middle|\,\frac{1}{c^2x^2}\right)}{4\pi^{\frac{3}{2}}c^2}+\frac{ibG_{6,6}^{2,6}\left(\begin{matrix}-1,-\frac{3}{4},-\frac{1}{2},-\frac{1}{4},0,1 & \\ -\frac{3}{4},-\frac{1}{4} & -1,-\frac{1}{2},-\frac{1}{2},0\end{matrix}\middle|\,\frac{e^{2i\pi}}{c^2x^2}\right)}{4\pi^{\frac{3}{2}}c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/x/(c*x-1)**(1/2)/(c*x+1)**(1/2),x)
```

```
[Out] -a*meijerg(((3/4, 5/4, 1), (1, 1, 3/2)), ((1/2, 3/4, 1, 5/4, 3/2), (0,)), 1
/(c**2*x**2))/(4*pi**(3/2)) + I*a*meijerg(((0, 1/4, 1/2, 3/4, 1, 1), ()), (
(1/4, 3/4), (0, 1/2, 1/2, 0)), exp_polar(2*I*pi)/(c**2*x**2))/(4*pi**(3/2))
 + b*meijerg(((-1/4, 1/4), (0, 0, 1/2, 1)), ((-1/2, -1/4, 0, 1/4, 1/2, 0),
()), 1/(c**2*x**2))/(4*pi**(3/2)*c**2) + I*b*meijerg(((-1, -3/4, -1/2, -1/4
, 0, 1), ()), ((-3/4, -1/4), (-1, -1/2, -1/2, 0)), exp_polar(2*I*pi)/(c**2*
x**2))/(4*pi**(3/2)*c**2)
```

Giac [A]
time = 0.61, size = 45, normalized size = 0.98

$$-2\,a\arctan\left(\frac{1}{2}\left(\sqrt{cx+1}-\sqrt{cx-1}\right)^2\right)+\frac{\sqrt{cx+1}\,\sqrt{cx-1}\,b}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] -2*a*arctan(1/2*(sqrt(c*x + 1) - sqrt(c*x - 1))^2) + sqrt(c*x + 1)*sqrt(c*x
 - 1)*b/c^2
```

Mupad [B]
time = 3.86, size = 77, normalized size = 1.67

$$\frac{b\sqrt{c\,x-1}\,\sqrt{c\,x+1}}{c^2}-a\left(\ln\left(\frac{{\left(\sqrt{c\,x-1}-\mathrm{i}\right)}^2}{{\left(\sqrt{c\,x+1}-1\right)}^2}+1\right)-\ln\left(\frac{\sqrt{c\,x-1}-\mathrm{i}}{\sqrt{c\,x+1}-1}\right)\right)\,1{}\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)/(x*(c*x - 1)^(1/2)*(c*x + 1)^(1/2)),x)
```

```
[Out] (b*(c*x - 1)^(1/2)*(c*x + 1)^(1/2))/c^2 - a*(log(((c*x - 1)^(1/2) - 1i)^2/(
(c*x + 1)^(1/2) - 1)^2 + 1) - log(((c*x - 1)^(1/2) - 1i)/((c*x + 1)^(1/2) -
 1)))*1i
```

3.354 $\int \frac{a+bx^2}{x^2\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx$

Optimal. Leaf size=33

$$\frac{a\sqrt{-1+cx}\,\sqrt{1+cx}}{x}+\frac{b\cosh^{-1}(cx)}{c}$$

```
[Out] b*arccosh(c*x)/c+a*(c*x-1)^(1/2)*(c*x+1)^(1/2)/x
```

Rubi [A]
time = 0.04, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {465, 54}

$$\frac{a\sqrt{cx-1}\,\sqrt{cx+1}}{x}+\frac{b\cosh^{-1}(cx)}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)/(x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]),x]
```

```
[Out] (a*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/x + (b*ArcCosh[c*x])/c
```

Rule 54

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[b*(x/a)]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 465

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[c*(e*x)^(
m + 1)*(a1 + b1*x^(n/2))^(p + 1)*((a2 + b2*x^(n/2))^(p + 1)/(a1*a2*e*(m + 1
))), x] + Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/(a1*a2*e^n*(
m + 1)), Int[(e*x)^(m + n)*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/2))^p, x], x]
/; FreeQ[{a1, b1, a2, b2, c, d, e, p}, x] && EqQ[non2, n/2] && EqQ[a2*b1 +
a1*b2, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (L
tQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx^2}{x^2\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx&=\frac{a\sqrt{-1+cx}\,\sqrt{1+cx}}{x}+b\int \frac{1}{\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx\\&=\frac{a\sqrt{-1+cx}\,\sqrt{1+cx}}{x}+\frac{b\cosh^{-1}(cx)}{c}\end{aligned}$$

Mathematica [A]
time = 0.09, size = 48, normalized size = 1.45

$$\frac{a\sqrt{-1+cx}\,\sqrt{1+cx}}{x}+\frac{2b\tanh^{-1}\left(\sqrt{\frac{-1+cx}{1+cx}}\right)}{c}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^2)/(x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]),x]
```

```
[Out] (a*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/x + (2*b*ArcTanh[Sqrt[(-1 + c*x)/(1 + c*x)
]])/c
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.29, size = 77, normalized size = 2.33

method	result	size
default	$\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,\left(\sqrt{c^2x^2-1}\,\mathrm{csgn}(c)ca+\ln\left(\left(\sqrt{c^2x^2-1}\,\mathrm{csgn}(c)+cx\right)\mathrm{csgn}(c)\right)bx\right)\mathrm{csgn}(c)}{\sqrt{c^2x^2-1}\,cx}$	77
risch	$\frac{a\sqrt{cx-1}\,\sqrt{cx+1}}{x}+\frac{b\ln\left(\frac{c^2x}{\sqrt{c^2}}+\sqrt{c^2x^2-1}\right)\sqrt{(cx+1)(cx-1)}}{\sqrt{c^2}\,\sqrt{cx-1}\,\sqrt{cx+1}}$	78

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)/x^2/(c*x-1)^(1/2)/(c*x+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (c*x-1)^(1/2)*(c*x+1)^(1/2)*((c^2*x^2-1)^(1/2)*csgn(c)*c*a+ln(((c^2*x^2-1)^
(1/2)*csgn(c)+c*x)*csgn(c))*b*x)*csgn(c)/(c^2*x^2-1)^(1/2)/c/x
```

Maxima [A]
time = 0.50, size = 44, normalized size = 1.33

$$\frac{b\log\left(2\,c^2x+2\,\sqrt{c^2x^2-1}\,c\right)}{c}+\frac{\sqrt{c^2x^2-1}\,a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^2/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] b*log(2*c^2*x + 2*sqrt(c^2*x^2 - 1)*c)/c + sqrt(c^2*x^2 - 1)*a/x
```

Fricas [A]
time = 3.32, size = 56, normalized size = 1.70

$$\frac{ac^2x+\sqrt{cx+1}\,\sqrt{cx-1}\,ac-bx\log\left(-cx+\sqrt{cx+1}\,\sqrt{cx-1}\right)}{cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^2/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] (a*c^2*x + sqrt(c*x + 1)*sqrt(c*x - 1)*a*c - b*x*log(-c*x + sqrt(c*x + 1)*s
qrt(c*x - 1)))/(c*x)
```

Sympy [C] Result contains complex when optimal does not.
time = 38.91, size = 148, normalized size = 4.48

$$-\frac{acG_{6,6}^{5,3}\left(\begin{matrix} \frac{5}{4}, \frac{7}{4}, 1 & \frac{3}{2}, \frac{3}{2}, 2 \\ 1, \frac{5}{4}, \frac{3}{2}, \frac{7}{4}, 2 & 0 \end{matrix} \middle| {\frac{1}{c^{2}x^{2}}} \right)}{4\pi^{\frac{3}{2}}} - \frac{iacG_{6,6}^{2,6}\left(\begin{matrix} \frac{1}{2}, \frac{3}{4}, 1, \frac{5}{4}, \frac{3}{2}, 1 & \\ \frac{3}{4}, \frac{5}{4} & \frac{1}{2}, 1, 1, 0 \end{matrix} \middle| {\frac{e^{2i\pi}}{c^{2}x^{2}}} \right)}{4\pi^{\frac{3}{2}}} + \frac{bG_{6,6}^{6,2}\left(\begin{matrix} \frac{1}{4}, \frac{3}{4} & \frac{1}{2}, \frac{1}{2}, 1, 1 \\ 0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1, 0 & \end{matrix} \middle| {\frac{1}{c^{2}x^{2}}} \right)}{4\pi^{\frac{3}{2}}c} - \frac{ibG_{6,6}^{2,6}\left(\begin{matrix} -\frac{1}{2}, -\frac{1}{4}, 0, \frac{1}{4}, \frac{1}{2}, 1 & \\ -\frac{1}{4}, \frac{1}{4} & -\frac{1}{2}, 0, 0, 0 \end{matrix} \middle| {\frac{e^{2i\pi}}{c^{2}x^{2}}} \right)}{4\pi^{\frac{3}{2}}c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/x**2/(c*x-1)**(1/2)/(c*x+1)**(1/2),x)
```

```
[Out] -a*c*meijerg(((5/4, 7/4, 1), (3/2, 3/2, 2)), ((1, 5/4, 3/2, 7/4, 2), (0,)),
 1/(c**2*x**2))/(4*pi**(3/2)) - I*a*c*meijerg(((1/2, 3/4, 1, 5/4, 3/2, 1),
()), ((3/4, 5/4), (1/2, 1, 1, 0)), exp_polar(2*I*pi)/(c**2*x**2))/(4*pi**(3
/2)) + b*meijerg(((1/4, 3/4), (1/2, 1/2, 1, 1)), ((0, 1/4, 1/2, 3/4, 1, 0),
 ()), 1/(c**2*x**2))/(4*pi**(3/2)*c) - I*b*meijerg(((-1/2, -1/4, 0, 1/4, 1/
2, 1), ()), ((-1/4, 1/4), (-1/2, 0, 0, 0)), exp_polar(2*I*pi)/(c**2*x**2))/
(4*pi**(3/2)*c)
```

Giac [A]
time = 0.68, size = 58, normalized size = 1.76

$$\frac{\frac{16\,ac^2}{\left(\sqrt{cx+1}-\sqrt{cx-1}\right)^4+4} - b\log\left(\left(\sqrt{cx+1}-\sqrt{cx-1}\right)^4\right)}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^2/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*(16*a*c^2/((sqrt(c*x + 1) - sqrt(c*x - 1))^4 + 4) - b*log((sqrt(c*x + 1
) - sqrt(c*x - 1))^4))/c
```

Mupad [B]
time = 2.59, size = 61, normalized size = 1.85

$$\frac{a\,\sqrt{c\,x-1}\,\sqrt{c\,x+1}}{x} - \frac{4\,b\,\mathrm{atan}\left(\frac{c\,\left(\sqrt{c\,x-1}-\mathrm{i}\right)}{\left(\sqrt{c\,x+1}-1\right)\,\sqrt{-c^2}}\right)}{\sqrt{-c^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)/(x^2*(c*x - 1)^(1/2)*(c*x + 1)^(1/2)),x)
```

```
[Out] (a*(c*x - 1)^(1/2)*(c*x + 1)^(1/2))/x - (4*b*atan((c*((c*x - 1)^(1/2) - 1i)
)/(((c*x + 1)^(1/2) - 1)*(-c^2)^(1/2))))/(-c^2)^(1/2)
```

3.355 $\int \frac{a+bx^2}{x^3\sqrt{-1+cx}\sqrt{1+cx}}\,dx$

Optimal. Leaf size=60

$$\frac{a\sqrt{-1+cx}\sqrt{1+cx}}{2x^2}+\frac{1}{2}(2b+ac^2)\tan^{-1}\left(\sqrt{-1+cx}\sqrt{1+cx}\right)$$

```
[Out] 1/2*(a*c^2+2*b)*arctan((c*x-1)^(1/2)*(c*x+1)^(1/2))+1/2*a*(c*x-1)^(1/2)*(c*
x+1)^(1/2)/x^2
```

Rubi [A]
time = 0.05, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {465, 94, 211}

$$\frac{1}{2}(ac^2+2b)\,\text{ArcTan}\left(\sqrt{cx-1}\sqrt{cx+1}\right)+\frac{a\sqrt{cx-1}\sqrt{cx+1}}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)/(x^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]),x]
```

```
[Out] (a*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(2*x^2) + ((2*b + a*c^2)*ArcTan[Sqrt[-1 +
c*x]*Sqrt[1 + c*x]])/2
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 465

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[c*(e*x)^(
m + 1)*(a1 + b1*x^(n/2))^(p + 1)*((a2 + b2*x^(n/2))^(p + 1)/(a1*a2*e*(m + 1
))), x] + Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/(a1*a2*e^n*(
m + 1)), Int[(e*x)^(m + n)*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/2))^p, x], x]
/; FreeQ[{a1, b1, a2, b2, c, d, e, p}, x] && EqQ[non2, n/2] && EqQ[a2*b1 +
a1*b2, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (L
```

```
tQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx^2}{x^3\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx &= \frac{a\sqrt{-1+cx}\,\sqrt{1+cx}}{2x^2}+\frac{1}{2}\left(2b+ac^2\right)\int \frac{1}{x\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx\\ &= \frac{a\sqrt{-1+cx}\,\sqrt{1+cx}}{2x^2}+\frac{1}{2}\left(c\left(2b+ac^2\right)\right)\text{Subst}\left(\int \frac{1}{c+cx^2}\,dx,x,\sqrt{-1+cx}\right.\\ &= \frac{a\sqrt{-1+cx}\,\sqrt{1+cx}}{2x^2}+\frac{1}{2}\left(2b+ac^2\right)\tan^{-1}\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\end{aligned}$$

Mathematica [A]
time = 0.10, size = 55, normalized size = 0.92

$$\frac{a\sqrt{-1+cx}\,\sqrt{1+cx}}{2x^2}+\left(2b+ac^2\right)\tan^{-1}\left(\sqrt{\frac{-1+cx}{1+cx}}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^2)/(x^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]),x]
```

```
[Out] (a*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(2*x^2) + (2*b + a*c^2)*ArcTan[Sqrt[(-1 +
c*x)/(1 + c*x)]]
```

Maple [A]
time = 0.31, size = 84, normalized size = 1.40

method	result	size
default	$-\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,\left(\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)a\,c^2x^2+2\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)b\,x^2-\sqrt{c^2x^2-1}\,a\right)}{2\sqrt{c^2x^2-1}\,x^2}$	84
risch	$\frac{a\sqrt{cx-1}\,\sqrt{cx+1}}{2x^2}+\frac{\left(-\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)b-\frac{\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)a\,c^2}{2}\right)\sqrt{(cx+1)\,(cx-1)}}{\sqrt{cx-1}\,\sqrt{cx+1}}$	84

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)/x^3/(c*x-1)^(1/2)/(c*x+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(c*x-1)^(1/2)*(c*x+1)^(1/2)*(arctan(1/(c^2*x^2-1)^(1/2))*a*c^2*x^2+2*a
rctan(1/(c^2*x^2-1)^(1/2))*b*x^2-(c^2*x^2-1)^(1/2)*a)/(c^2*x^2-1)^(1/2)/x^2
```

Maxima [A]
time = 0.56, size = 45, normalized size = 0.75

$$-\frac{1}{2}ac^2 \arcsin\left(\frac{1}{c|x|}\right) - b\arcsin\left(\frac{1}{c|x|}\right) + \frac{\sqrt{c^2x^2-1}\,a}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^3/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*a*c^2*arcsin(1/(c*abs(x))) - b*arcsin(1/(c*abs(x))) + 1/2*sqrt(c^2*x^2
 - 1)*a/x^2
```

Fricas [A]
time = 3.09, size = 57, normalized size = 0.95

$$\frac{2\,(ac^2+2\,b)x^2 \arctan\left(-cx+\sqrt{cx+1}\,\sqrt{cx-1}\right)+\sqrt{cx+1}\,\sqrt{cx-1}\,a}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^3/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(2*(a*c^2 + 2*b)*x^2*arctan(-c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) + sqrt(
c*x + 1)*sqrt(c*x - 1)*a)/x^2
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/x**3/(c*x-1)**(1/2)/(c*x+1)**(1/2),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 114 vs. 2(48) = 96.
time = 0.61, size = 114, normalized size = 1.90

$$-\frac{(ac^3+2\,bc)\arctan\left(\frac{1}{2}\left(\sqrt{cx+1}-\sqrt{cx-1}\right)^2\right)+\frac{2\left(ac^3\left(\sqrt{cx+1}-\sqrt{cx-1}\right)^6-4\,ac^3\left(\sqrt{cx+1}-\sqrt{cx-1}\right)^2\right)}{\left(\left(\sqrt{cx+1}-\sqrt{cx-1}\right)^4+4\right)^2}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^3/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] -((a*c^3 + 2*b*c)*arctan(1/2*(sqrt(c*x + 1) - sqrt(c*x - 1))^2) + 2*(a*c^3*
(sqrt(c*x + 1) - sqrt(c*x - 1))^6 - 4*a*c^3*(sqrt(c*x + 1) - sqrt(c*x - 1))
^2)/((sqrt(c*x + 1) - sqrt(c*x - 1))^4 + 4)^2)/c
```

Mupad [B]
time = 8.67, size = 297, normalized size = 4.95

$$\frac{\frac{a\,c^2\,1\mathrm{i}}{32}+\frac{a\,c^2\left(\sqrt{c\,x-1}-\mathrm{i}\right)^2\,1\mathrm{i}}{16\left(\sqrt{c\,x+1}-1\right)^2}-\frac{a\,c^2\left(\sqrt{c\,x-1}-\mathrm{i}\right)^4\,15\mathrm{i}}{32\left(\sqrt{c\,x+1}-1\right)^4}}{\frac{\left(\sqrt{c\,x-1}-\mathrm{i}\right)^2}{\left(\sqrt{c\,x+1}-1\right)^2}+\frac{2\left(\sqrt{c\,x-1}-\mathrm{i}\right)^4}{\left(\sqrt{c\,x+1}-1\right)^4}+\frac{\left(\sqrt{c\,x-1}-\mathrm{i}\right)^6}{\left(\sqrt{c\,x+1}-1\right)^6}}-b\left(\ln\left(\frac{\left(\sqrt{c\,x-1}-\mathrm{i}\right)^2}{\left(\sqrt{c\,x+1}-1\right)^2}+1\right)-\ln\left(\frac{\sqrt{c\,x-1}-\mathrm{i}}{\sqrt{c\,x+1}-1}\right)\right)1\mathrm{i}-\frac{a\,c^2\ln\left(\frac{\left(\sqrt{c\,x-1}-\mathrm{i}\right)^2}{\left(\sqrt{c\,x+1}-1\right)^2}+1\right)1\mathrm{i}}{2}+\frac{a\,c^2\ln\left(\frac{\sqrt{c\,x-1}-\mathrm{i}}{\sqrt{c\,x+1}-1}\right)1\mathrm{i}}{2}+\frac{a\,c^2\left(\sqrt{c\,x-1}-\mathrm{i}\right)^2\,1\mathrm{i}}{32\left(\sqrt{c\,x+1}-1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)/(x^3*(c*x - 1)^(1/2)*(c*x + 1)^(1/2)),x)
```

```
[Out] ((a*c^2*1i)/32 + (a*c^2*((c*x - 1)^(1/2) - 1i)^2*1i)/(16*((c*x + 1)^(1/2) -
 1)^2) - (a*c^2*((c*x - 1)^(1/2) - 1i)^4*15i)/(32*((c*x + 1)^(1/2) - 1)^4))
/((((c*x - 1)^(1/2) - 1i)^2/((c*x + 1)^(1/2) - 1)^2 + (2*((c*x - 1)^(1/2) -
1i)^4)/((c*x + 1)^(1/2) - 1)^4 + ((c*x - 1)^(1/2) - 1i)^6/((c*x + 1)^(1/2)
- 1)^6) - b*(log(((c*x - 1)^(1/2) - 1i)^2/((c*x + 1)^(1/2) - 1)^2 + 1) - lo
g(((c*x - 1)^(1/2) - 1i)/((c*x + 1)^(1/2) - 1)))*1i - (a*c^2*log(((c*x - 1)
^(1/2) - 1i)^2/((c*x + 1)^(1/2) - 1)^2 + 1)*1i)/2 + (a*c^2*log(((c*x - 1)^(
1/2) - 1i)/((c*x + 1)^(1/2) - 1))*1i)/2 + (a*c^2*((c*x - 1)^(1/2) - 1i)^2*1
i)/(32*((c*x + 1)^(1/2) - 1)^2)
```

3.356 $\int \frac{a+bx^2}{x^4\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx$

Optimal. Leaf size=62

$$\frac{a\sqrt{-1+cx}\,\sqrt{1+cx}}{3x^3}+\frac{(3b+2ac^2)\,\sqrt{-1+cx}\,\sqrt{1+cx}}{3x}$$

```
[Out] 1/3*a*(c*x-1)^(1/2)*(c*x+1)^(1/2)/x^3+1/3*(2*a*c^2+3*b)*(c*x-1)^(1/2)*(c*x+
1)^(1/2)/x
```

Rubi [A]
time = 0.04, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {465, 97}

$$\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,(2ac^2+3b)}{3x}+\frac{a\sqrt{cx-1}\,\sqrt{cx+1}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)/(x^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x]),x]
```

```
[Out] (a*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(3*x^3) + ((3*b + 2*a*c^2)*Sqrt[-1 + c*x]*
Sqrt[1 + c*x])/(3*x)
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] /; FreeQ[{a, b, c, d, e, f
, m, n, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && EqQ[a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1), 0] && NeQ[m, -1]
```

Rule 465

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[c*(e*x)^(
m + 1)*(a1 + b1*x^(n/2))^(p + 1)*((a2 + b2*x^(n/2))^(p + 1)/(a1*a2*e*(m + 1
))), x] + Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/(a1*a2*e^n*(
m + 1)), Int[(e*x)^(m + n)*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/2))^p, x], x]
/; FreeQ[{a1, b1, a2, b2, c, d, e, p}, x] && EqQ[non2, n/2] && EqQ[a2*b1 +
a1*b2, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (L
tQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\int \frac{a+bx^2}{x^4\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx = \frac{a\sqrt{-1+cx}\,\sqrt{1+cx}}{3x^3} + \frac{1}{3}(3b+2ac^2)\int \frac{1}{x^2\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx$$
$$= \frac{a\sqrt{-1+cx}\,\sqrt{1+cx}}{3x^3} + \frac{(3b+2ac^2)\sqrt{-1+cx}\,\sqrt{1+cx}}{3x}$$

Mathematica [A]
time = 0.09, size = 42, normalized size = 0.68

$$\frac{\sqrt{-1+cx}\,\sqrt{1+cx}\,(a+3bx^2+2ac^2x^2)}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)/(x^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x]),x]
```

```
[Out] (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + 3*b*x^2 + 2*a*c^2*x^2))/(3*x^3)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 2.
time = 0.28, size = 41, normalized size = 0.66

method	result	size
gosper	$\frac{\sqrt{cx+1}\,\sqrt{cx-1}\,(2a\,c^2x^2+3b\,x^2+a)}{3x^3}$	37
risch	$\frac{\sqrt{cx+1}\,\sqrt{cx-1}\,(2a\,c^2x^2+3b\,x^2+a)}{3x^3}$	37
default	$\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,\mathrm{csgn}(c)^2(2a\,c^2x^2+3b\,x^2+a)}{3x^3}$	41

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)/x^4/(c*x-1)^(1/2)/(c*x+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(c*x-1)^(1/2)*(c*x+1)^(1/2)*csgn(c)^2*(2*a*c^2*x^2+3*b*x^2+a)/x^3
```

Maxima [A]
time = 0.51, size = 54, normalized size = 0.87

$$\frac{2\sqrt{c^2x^2-1}\,ac^2}{3\,x} + \frac{\sqrt{c^2x^2-1}\,b}{x} + \frac{\sqrt{c^2x^2-1}\,a}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^4/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3*sqrt(c^2*x^2 - 1)*a*c^2/x + sqrt(c^2*x^2 - 1)*b/x + 1/3*sqrt(c^2*x^2 -
1)*a/x^3
```

Fricas [A]
time = 3.35, size = 52, normalized size = 0.84

$$\frac{(2\,ac^{3}+3\,bc)x^{3}+((2\,ac^{2}+3\,b)x^{2}+a)\sqrt{cx+1}\,\sqrt{cx-1}}{3\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^4/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*((2*a*c^3 + 3*b*c)*x^3 + ((2*a*c^2 + 3*b)*x^2 + a)*sqrt(c*x + 1)*sqrt(c
*x - 1))/x^3
```

Sympy [C] Result contains complex when optimal does not.
time = 26.80, size = 146, normalized size = 2.35

$$-\frac{ac^{3}G_{6,6}^{5,3}\left(\begin{matrix}\frac{9}{4},\frac{11}{4},1 & \frac{5}{2},\frac{5}{2},3\\ 2,\frac{9}{4},\frac{5}{2},\frac{11}{4},3 & 0\end{matrix}\middle|\,\frac{1}{c^{2}x^{2}}\right)}{4\pi^{\frac{3}{2}}}-\frac{iac^{3}G_{6,6}^{2,6}\left(\begin{matrix}\frac{3}{2},\frac{7}{4},2,\frac{9}{4},\frac{5}{2},1 & \\ \frac{7}{4},\frac{9}{4} & \frac{3}{2},2,2,0\end{matrix}\middle|\,\frac{e^{2i\pi}}{c^{2}x^{2}}\right)}{4\pi^{\frac{3}{2}}}-\frac{bcG_{6,6}^{5,3}\left(\begin{matrix}\frac{5}{4},\frac{7}{4},1 & \frac{3}{2},\frac{3}{2},2\\ 1,\frac{5}{4},\frac{3}{2},\frac{7}{4},2 & 0\end{matrix}\middle|\,\frac{1}{c^{2}x^{2}}\right)}{4\pi^{\frac{3}{2}}}-\frac{ibcG_{6,6}^{2,6}\left(\begin{matrix}\frac{1}{2},\frac{3}{4},1,\frac{5}{4},\frac{3}{2},1 & \\ \frac{3}{4},\frac{5}{4} & \frac{1}{2},1,1,0\end{matrix}\middle|\,\frac{e^{2i\pi}}{c^{2}x^{2}}\right)}{4\pi^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/x**4/(c*x-1)**(1/2)/(c*x+1)**(1/2),x)
```

```
[Out] -a*c**3*meijerg(((9/4, 11/4, 1), (5/2, 5/2, 3)), ((2, 9/4, 5/2, 11/4, 3), (
0,)), 1/(c**2*x**2))/(4*pi**(3/2)) - I*a*c**3*meijerg(((3/2, 7/4, 2, 9/4, 5
/2, 1), ()), ((7/4, 9/4), (3/2, 2, 2, 0)), exp_polar(2*I*pi)/(c**2*x**2))/(
4*pi**(3/2)) - b*c*meijerg(((5/4, 7/4, 1), (3/2, 3/2, 2)), ((1, 5/4, 3/2, 7
/4, 2), (0,)), 1/(c**2*x**2))/(4*pi**(3/2)) - I*b*c*meijerg(((1/2, 3/4, 1,
5/4, 3/2, 1), ()), ((3/4, 5/4), (1/2, 1, 1, 0)), exp_polar(2*I*pi)/(c**2*x*
*2))/(4*pi**(3/2))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 116 vs. 2(50) = 100.
time = 0.63, size = 116, normalized size = 1.87

$$\frac{8\left(3\,bc^{2}\left(\sqrt{cx+1}-\sqrt{cx-1}\right)^{8}+24\,ac^{4}\left(\sqrt{cx+1}-\sqrt{cx-1}\right)^{4}+24\,bc^{2}\left(\sqrt{cx+1}-\sqrt{cx-1}\right)^{4}+32\,ac^{4}+48\,bc^{2}\right)}{3\left(\left(\sqrt{cx+1}-\sqrt{cx-1}\right)^{4}+4\right)^{3}c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^4/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] 8/3*(3*b*c^2*(sqrt(c*x + 1) - sqrt(c*x - 1))^8 + 24*a*c^4*(sqrt(c*x + 1) -
sqrt(c*x - 1))^4 + 24*b*c^2*(sqrt(c*x + 1) - sqrt(c*x - 1))^4 + 32*a*c^4 +
48*b*c^2)/(((sqrt(c*x + 1) - sqrt(c*x - 1))^4 + 4)^3*c)
```

Mupad [B]
time = 2.44, size = 53, normalized size = 0.85

$$\frac{\sqrt{c\,x-1}\,\left(\left(\frac{2\,a\,c^3}{3}+b\,c\right)\,x^3+\left(\frac{2\,a\,c^2}{3}+b\right)\,x^2+\frac{a\,c\,x}{3}+\frac{a}{3}\right)}{x^3\,\sqrt{c\,x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)/(x^4*(c*x - 1)^(1/2)*(c*x + 1)^(1/2)),x)
```

```
[Out] ((c*x - 1)^(1/2)*(a/3 + x^3*(b*c + (2*a*c^3)/3) + x^2*(b + (2*a*c^2)/3) + (
a*c*x)/3))/(x^3*(c*x + 1)^(1/2))
```

3.357 $\int \frac{a+bx^2}{x^5\sqrt{-1+cx}\sqrt{1+cx}}\,dx$

Optimal. Leaf size=99

$$\frac{a\sqrt{-1+cx}\sqrt{1+cx}}{4x^4}+\frac{(4b+3ac^2)\sqrt{-1+cx}\sqrt{1+cx}}{8x^2}+\frac{1}{8}c^2(4b+3ac^2)\tan^{-1}\left(\sqrt{-1+cx}\sqrt{1+cx}\right)$$

```
[Out] 1/8*c^2*(3*a*c^2+4*b)*arctan((c*x-1)^(1/2)*(c*x+1)^(1/2))+1/4*a*(c*x-1)^(1/
2)*(c*x+1)^(1/2)/x^4+1/8*(3*a*c^2+4*b)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/x^2
```

Rubi [A]
time = 0.05, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {465, 105, 12, 94, 211}

$$\frac{1}{8}c^2(3ac^2+4b)\,\text{ArcTan}\left(\sqrt{cx-1}\sqrt{cx+1}\right)+\frac{\sqrt{cx-1}\sqrt{cx+1}(3ac^2+4b)}{8x^2}+\frac{a\sqrt{cx-1}\sqrt{cx+1}}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)/(x^5*Sqrt[-1 + c*x]*Sqrt[1 + c*x]),x]
```

```
[Out] (a*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(4*x^4) + ((4*b + 3*a*c^2)*Sqrt[-1 + c*x]*
Sqrt[1 + c*x])/(8*x^2) + (c^2*(4*b + 3*a*c^2)*ArcTan[Sqrt[-1 + c*x]*Sqrt[1
+ c*x]])/8
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 465

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[c*(e*x)^(
m + 1)*(a1 + b1*x^(n/2))^(p + 1)*((a2 + b2*x^(n/2))^(p + 1)/(a1*a2*e*(m + 1
))), x] + Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/(a1*a2*e^n*(
m + 1)), Int[(e*x)^(m + n)*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/2))^p, x], x]
/; FreeQ[{a1, b1, a2, b2, c, d, e, p}, x] && EqQ[non2, n/2] && EqQ[a2*b1 +
a1*b2, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (L
tQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\int \frac{a+bx^2}{x^5\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx = \frac{a\sqrt{-1+cx}\,\sqrt{1+cx}}{4x^4} + \frac{1}{4}\left(4b+3ac^2\right)\int \frac{1}{x^3\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx$$
$$= \frac{a\sqrt{-1+cx}\,\sqrt{1+cx}}{4x^4} + \frac{\left(4b+3ac^2\right)\sqrt{-1+cx}\,\sqrt{1+cx}}{8x^2} + \frac{1}{8}\left(4b+3ac^2\right)\int$$
$$= \frac{a\sqrt{-1+cx}\,\sqrt{1+cx}}{4x^4} + \frac{\left(4b+3ac^2\right)\sqrt{-1+cx}\,\sqrt{1+cx}}{8x^2} + \frac{1}{8}\left(c^2\left(4b+3ac^2\right.\right.$$
$$= \frac{a\sqrt{-1+cx}\,\sqrt{1+cx}}{4x^4} + \frac{\left(4b+3ac^2\right)\sqrt{-1+cx}\,\sqrt{1+cx}}{8x^2} + \frac{1}{8}\left(c^3\left(4b+3ac^2\right.\right.$$
$$= \frac{a\sqrt{-1+cx}\,\sqrt{1+cx}}{4x^4} + \frac{\left(4b+3ac^2\right)\sqrt{-1+cx}\,\sqrt{1+cx}}{8x^2} + \frac{1}{8}c^2\left(4b+3ac^2\right)$$

Mathematica [**A**]
time = 0.18, size = 78, normalized size = 0.79

$$\frac{1}{8}\left(\frac{\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(4bx^2+a\left(2+3c^2x^2\right)\right)}{x^4}+\left(8bc^2+6ac^4\right)\tan^{-1}\left(\sqrt{\frac{-1+cx}{1+cx}}\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^2)/(x^5*Sqrt[-1 + c*x]*Sqrt[1 + c*x]),x]
```

```
[Out] ((Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(4*b*x^2 + a*(2 + 3*c^2*x^2)))/x^4 + (8*b*c^
2 + 6*a*c^4)*ArcTan[Sqrt[(-1 + c*x)/(1 + c*x)]])/8
```

Maple [A]
time = 0.30, size = 125, normalized size = 1.26

method	result
risch	$\frac{\sqrt{cx+1}\,\sqrt{cx-1}\,(3a\,c^2x^2+4b\,x^2+2a)}{8x^4}+\frac{\left(-\frac{3\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)a\,c^4}{8}-\frac{\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)b\,c^2}{2}\right)\sqrt{(cx+}{\sqrt{cx-1}\,\sqrt{cx+1}}$
default	$-\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,\left(3\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)a\,c^4x^4+4\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)b\,c^2x^4-3\sqrt{c^2x^2-1}\,a\,c^2x^2-4\sqrt{}\right.}{8\sqrt{c^2x^2-1}\,x^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)/x^5/(c*x-1)^(1/2)/(c*x+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*(c*x-1)^(1/2)*(c*x+1)^(1/2)*(3*arctan(1/(c^2*x^2-1)^(1/2))*a*c^4*x^4+4
*arctan(1/(c^2*x^2-1)^(1/2))*b*c^2*x^4-3*(c^2*x^2-1)^(1/2)*a*c^2*x^2-4*(c^2
*x^2-1)^(1/2)*b*x^2-2*(c^2*x^2-1)^(1/2)*a)/(c^2*x^2-1)^(1/2)/x^4
```

Maxima [A]
time = 0.49, size = 85, normalized size = 0.86

$$-\frac{3}{8}\,ac^4\arcsin\left(\frac{1}{c|x|}\right)-\frac{1}{2}\,bc^2\arcsin\left(\frac{1}{c|x|}\right)+\frac{3\,\sqrt{c^2x^2-1}\,ac^2}{8\,x^2}+\frac{\sqrt{c^2x^2-1}\,b}{2\,x^2}+\frac{\sqrt{c^2x^2-1}\,a}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^5/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -3/8*a*c^4*arcsin(1/(c*abs(x))) - 1/2*b*c^2*arcsin(1/(c*abs(x))) + 3/8*sqrt
(c^2*x^2 - 1)*a*c^2/x^2 + 1/2*sqrt(c^2*x^2 - 1)*b/x^2 + 1/4*sqrt(c^2*x^2 -
1)*a/x^4
```

Fricas [A]
time = 3.49, size = 78, normalized size = 0.79

$$\frac{2\,(3\,ac^4+4\,bc^2)x^4\arctan\left(-cx+\sqrt{cx+1}\,\sqrt{cx-1}\right)+((3\,ac^2+4\,b)x^2+2\,a)\sqrt{cx+1}\,\sqrt{cx-1}}{8\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^5/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/8*(2*(3*a*c^4 + 4*b*c^2)*x^4*arctan(-c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) +
 ((3*a*c^2 + 4*b)*x^2 + 2*a)*sqrt(c*x + 1)*sqrt(c*x - 1))/x^4
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/x**5/(c*x-1)**(1/2)/(c*x+1)**(1/2),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 268 vs. 2(81) = 162.
time = 0.80, size = 268, normalized size = 2.71

$$-\frac{\left(3\,a c^{5}+4\,b c^{3}\right)\arctan\left(\frac{1}{2}\left(\sqrt{cx+1}-\sqrt{cx-1}\right)^{2}\right)+\frac{2\left(3\,a c^{5}\left(\sqrt{cx+1}-\sqrt{cx-1}\right)^{14}+4\,b c^{3}\left(\sqrt{cx+1}-\sqrt{cx-1}\right)^{14}+44\,a c^{5}\left(\sqrt{cx+1}-\sqrt{cx-1}\right)^{10}+16\,b c^{3}\left(\sqrt{cx+1}-\sqrt{cx-1}\right)^{10}-176\,a c^{5}\left(\sqrt{cx+1}-\sqrt{cx-1}\right)^{6}-64\,b c^{3}\left(\sqrt{cx+1}-\sqrt{cx-1}\right)^{6}-192\,a c^{5}\left(\sqrt{cx+1}-\sqrt{cx-1}\right)^{2}-256\,b c^{3}\left(\sqrt{cx+1}-\sqrt{cx-1}\right)^{2}\right)}{\left(\left(\sqrt{cx+1}-\sqrt{cx-1}\right)^{4}+4\right)^{4}}}{4\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^5/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] -1/4*((3*a*c^5 + 4*b*c^3)*arctan(1/2*(sqrt(c*x + 1) - sqrt(c*x - 1))^2) + 2
*(3*a*c^5*(sqrt(c*x + 1) - sqrt(c*x - 1))^14 + 4*b*c^3*(sqrt(c*x + 1) - sqr
t(c*x - 1))^14 + 44*a*c^5*(sqrt(c*x + 1) - sqrt(c*x - 1))^10 + 16*b*c^3*(sq
rt(c*x + 1) - sqrt(c*x - 1))^10 - 176*a*c^5*(sqrt(c*x + 1) - sqrt(c*x - 1))
^6 - 64*b*c^3*(sqrt(c*x + 1) - sqrt(c*x - 1))^6 - 192*a*c^5*(sqrt(c*x + 1)
- sqrt(c*x - 1))^2 - 256*b*c^3*(sqrt(c*x + 1) - sqrt(c*x - 1))^2)/((sqrt(c*
x + 1) - sqrt(c*x - 1))^4 + 4)^4)/c
```

Mupad [B]
time = 21.45, size = 650, normalized size = 6.57

$$\frac{\frac{b c^{2}\,1\mathrm{i}}{32}+\frac{b c^{2}\left(\sqrt{cx-1}-\mathrm{i}\right)^{2}1\mathrm{i}}{16\left(\sqrt{cx+1}-1\right)^{2}}-\frac{b c^{2}\left(\sqrt{cx-1}-\mathrm{i}\right)^{4}15\mathrm{i}}{32\left(\sqrt{cx+1}-1\right)^{4}}}{\frac{\left(\sqrt{cx-1}-\mathrm{i}\right)^{2}}{\left(\sqrt{cx+1}-1\right)^{2}}+\frac{2\left(\sqrt{cx-1}-\mathrm{i}\right)^{4}}{\left(\sqrt{cx+1}-1\right)^{4}}+\frac{\left(\sqrt{cx-1}-\mathrm{i}\right)^{6}}{\left(\sqrt{cx+1}-1\right)^{6}}}-\frac{\frac{a c^{4}\,1\mathrm{i}}{1024}-\frac{a c^{4}\left(\sqrt{cx-1}-\mathrm{i}\right)^{2}3\mathrm{i}}{128\left(\sqrt{cx+1}-1\right)^{2}}-\frac{a c^{4}\left(\sqrt{cx-1}-\mathrm{i}\right)^{4}53\mathrm{i}}{512\left(\sqrt{cx+1}-1\right)^{4}}+\frac{a c^{4}\left(\sqrt{cx-1}-\mathrm{i}\right)^{6}87\mathrm{i}}{256\left(\sqrt{cx+1}-1\right)^{6}}+\frac{a c^{4}\left(\sqrt{cx-1}-\mathrm{i}\right)^{8}657\mathrm{i}}{1024\left(\sqrt{cx+1}-1\right)^{8}}+\frac{a c^{4}\left(\sqrt{cx-1}-\mathrm{i}\right)^{10}121\mathrm{i}}{256\left(\sqrt{cx+1}-1\right)^{10}}}{\frac{\left(\sqrt{cx-1}-\mathrm{i}\right)^{4}}{\left(\sqrt{cx+1}-1\right)^{4}}+\frac{4\left(\sqrt{cx-1}-\mathrm{i}\right)^{6}}{\left(\sqrt{cx+1}-1\right)^{6}}+\frac{6\left(\sqrt{cx-1}-\mathrm{i}\right)^{8}}{\left(\sqrt{cx+1}-1\right)^{8}}+\frac{4\left(\sqrt{cx-1}-\mathrm{i}\right)^{10}}{\left(\sqrt{cx+1}-1\right)^{10}}+\frac{\left(\sqrt{cx-1}-\mathrm{i}\right)^{12}}{\left(\sqrt{cx+1}-1\right)^{12}}}-\frac{a c^{4}\ln\left(\frac{\left(\sqrt{cx-1}-\mathrm{i}\right)^{2}}{\left(\sqrt{cx+1}-1\right)^{2}}+1\right)3\mathrm{i}}{8}-\frac{b c^{2}\ln\left(\frac{\left(\sqrt{cx-1}-\mathrm{i}\right)^{2}}{\left(\sqrt{cx+1}-1\right)^{2}}+1\right)1\mathrm{i}}{2}+\frac{a c^{4}\ln\left(\frac{\sqrt{cx-1}-\mathrm{i}}{\sqrt{cx+1}-1}\right)3\mathrm{i}}{8}+\frac{b c^{2}\ln\left(\frac{\sqrt{cx-1}-\mathrm{i}}{\sqrt{cx+1}-1}\right)1\mathrm{i}}{2}+\frac{a c^{4}\left(\sqrt{cx-1}-\mathrm{i}\right)^{2}7\mathrm{i}}{256\left(\sqrt{cx+1}-1\right)^{2}}-\frac{a c^{4}\left(\sqrt{cx-1}-\mathrm{i}\right)^{4}1\mathrm{i}}{1024\left(\sqrt{cx+1}-1\right)^{4}}+\frac{b c^{2}\left(\sqrt{cx-1}-\mathrm{i}\right)^{2}1\mathrm{i}}{32\left(\sqrt{cx+1}-1\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)/(x^5*(c*x - 1)^(1/2)*(c*x + 1)^(1/2)),x)
```

```
[Out] ((b*c^2*1i)/32 + (b*c^2*((c*x - 1)^(1/2) - 1i)^2*1i)/(16*((c*x + 1)^(1/2) -
 1)^2) - (b*c^2*((c*x - 1)^(1/2) - 1i)^4*15i)/(32*((c*x + 1)^(1/2) - 1)^4))
/((((c*x - 1)^(1/2) - 1i)^2/((c*x + 1)^(1/2) - 1)^2 + (2*((c*x - 1)^(1/2) -
1i)^4)/((c*x + 1)^(1/2) - 1)^4 + ((c*x - 1)^(1/2) - 1i)^6/((c*x + 1)^(1/2)
- 1)^6) - ((a*c^4*1i)/1024 - (a*c^4*((c*x - 1)^(1/2) - 1i)^2*3i)/(128*((c*x
 + 1)^(1/2) - 1)^2) - (a*c^4*((c*x - 1)^(1/2) - 1i)^4*53i)/(512*((c*x + 1)^
(1/2) - 1)^4) + (a*c^4*((c*x - 1)^(1/2) - 1i)^6*87i)/(256*((c*x + 1)^(1/2)
- 1)^6) + (a*c^4*((c*x - 1)^(1/2) - 1i)^8*657i)/(1024*((c*x + 1)^(1/2) - 1)
^8) + (a*c^4*((c*x - 1)^(1/2) - 1i)^10*121i)/(256*((c*x + 1)^(1/2) - 1)^10)
)/((((c*x - 1)^(1/2) - 1i)^4/((c*x + 1)^(1/2) - 1)^4 + (4*((c*x - 1)^(1/2) -
```

```
 1i)^6)/((c*x + 1)^(1/2) - 1)^6 + (6*((c*x - 1)^(1/2) - 1i)^8)/((c*x + 1)^(
1/2) - 1)^8 + (4*((c*x - 1)^(1/2) - 1i)^10)/((c*x + 1)^(1/2) - 1)^10 + ((c*
x - 1)^(1/2) - 1i)^12/((c*x + 1)^(1/2) - 1)^12) - (a*c^4*log(((c*x - 1)^(1/
2) - 1i)^2/((c*x + 1)^(1/2) - 1)^2 + 1)*3i)/8 - (b*c^2*log(((c*x - 1)^(1/2)
 - 1i)^2/((c*x + 1)^(1/2) - 1)^2 + 1)*1i)/2 + (a*c^4*log(((c*x - 1)^(1/2) -
 1i)/((c*x + 1)^(1/2) - 1))*3i)/8 + (b*c^2*log(((c*x - 1)^(1/2) - 1i)/((c*x
 + 1)^(1/2) - 1))*1i)/2 + (a*c^4*((c*x - 1)^(1/2) - 1i)^2*7i)/(256*((c*x +
1)^(1/2) - 1)^2) - (a*c^4*((c*x - 1)^(1/2) - 1i)^4*1i)/(1024*((c*x + 1)^(1/
2) - 1)^4) + (b*c^2*((c*x - 1)^(1/2) - 1i)^2*1i)/(32*((c*x + 1)^(1/2) - 1)^
2)
```

3.358 $\int \frac{x^4(a+bx^2)}{\sqrt{-c+dx}\,\sqrt{c+dx}}\,dx$

Optimal. Leaf size=164

$$\frac{c^2(5bc^2+6ad^2)\,x\sqrt{-c+dx}\,\sqrt{c+dx}}{16d^6}+\frac{(5bc^2+6ad^2)\,x^3\sqrt{-c+dx}\,\sqrt{c+dx}}{24d^4}+\frac{bx^5\sqrt{-c+dx}\,\sqrt{c+dx}}{6d^2}+\frac{c^4(\ldots}{\ldots}$$

```
[Out] 1/8*c^4*(6*a*d^2+5*b*c^2)*arctanh((d*x-c)^(1/2)/(d*x+c)^(1/2))/d^7+1/16*c^2
*(6*a*d^2+5*b*c^2)*x*(d*x-c)^(1/2)*(d*x+c)^(1/2)/d^6+1/24*(6*a*d^2+5*b*c^2)
*x^3*(d*x-c)^(1/2)*(d*x+c)^(1/2)/d^4+1/6*b*x^5*(d*x-c)^(1/2)*(d*x+c)^(1/2)/
d^2
```

Rubi [A]
time = 0.08, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {471, 102, 12, 92, 65, 223, 212}

$$\frac{c^2x\sqrt{dx-c}\,\sqrt{c+dx}\,(6ad^2+5bc^2)}{16d^6}+\frac{x^3\sqrt{dx-c}\,\sqrt{c+dx}\,(6ad^2+5bc^2)}{24d^4}+\frac{c^4(6ad^2+5bc^2)\tanh^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)}{8d^7}+\frac{bx^5\sqrt{dx-c}\,\sqrt{c+dx}}{6d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(a + b*x^2))/(Sqrt[-c + d*x]*Sqrt[c + d*x]),x]
```

```
[Out] (c^2*(5*b*c^2 + 6*a*d^2)*x*Sqrt[-c + d*x]*Sqrt[c + d*x])/(16*d^6) + ((5*b*c
^2 + 6*a*d^2)*x^3*Sqrt[-c + d*x]*Sqrt[c + d*x])/(24*d^4) + (b*x^5*Sqrt[-c +
 d*x]*Sqrt[c + d*x])/(6*d^2) + (c^4*(5*b*c^2 + 6*a*d^2)*ArcTanh[Sqrt[-c + d
*x]/Sqrt[c + d*x]])/(8*d^7)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 92

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(
```

```
d*f*(n + p + 3))), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 102

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/(d*f*(m + n + p + 1))), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[d*(e*x)^(
m + 1)*(a1 + b1*x^(n/2))^(p + 1)*((a2 + b2*x^(n/2))^(p + 1)/(b1*b2*e*(m + n
*(p + 1) + 1))), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\int \frac{x^4(a+bx^2)}{\sqrt{-c+dx}\sqrt{c+dx}}\,dx = \frac{bx^5\sqrt{-c+dx}\sqrt{c+dx}}{6d^2} - \frac{1}{6}\left(-6a-\frac{5bc^2}{d^2}\right)\int \frac{x^4}{\sqrt{-c+dx}\sqrt{c+dx}}\,dx$$

$$= \frac{(5bc^2+6ad^2)\,x^3\sqrt{-c+dx}\sqrt{c+dx}}{24d^4} + \frac{bx^5\sqrt{-c+dx}\sqrt{c+dx}}{6d^2} + \frac{(5bc^2+6a}{}$$

$$= \frac{(5bc^2+6ad^2)\,x^3\sqrt{-c+dx}\sqrt{c+dx}}{24d^4} + \frac{bx^5\sqrt{-c+dx}\sqrt{c+dx}}{6d^2} + \frac{(c^2(5bc^2+}{}$$

$$= \frac{c^2(5bc^2+6ad^2)\,x\sqrt{-c+dx}\sqrt{c+dx}}{16d^6} + \frac{(5bc^2+6ad^2)\,x^3\sqrt{-c+dx}\sqrt{c+dx}}{24d^4}$$

$$= \frac{c^2(5bc^2+6ad^2)\,x\sqrt{-c+dx}\sqrt{c+dx}}{16d^6} + \frac{(5bc^2+6ad^2)\,x^3\sqrt{-c+dx}\sqrt{c+dx}}{24d^4}$$

$$= \frac{c^2(5bc^2+6ad^2)\,x\sqrt{-c+dx}\sqrt{c+dx}}{16d^6} + \frac{(5bc^2+6ad^2)\,x^3\sqrt{-c+dx}\sqrt{c+dx}}{24d^4}$$

$$= \frac{c^2(5bc^2+6ad^2)\,x\sqrt{-c+dx}\sqrt{c+dx}}{16d^6} + \frac{(5bc^2+6ad^2)\,x^3\sqrt{-c+dx}\sqrt{c+dx}}{24d^4}$$

$$= \frac{c^2(5bc^2+6ad^2)\,x\sqrt{-c+dx}\sqrt{c+dx}}{16d^6} + \frac{(5bc^2+6ad^2)\,x^3\sqrt{-c+dx}\sqrt{c+dx}}{24d^4}$$

Mathematica [A]
time = 0.25, size = 119, normalized size = 0.73

$$\frac{dx\sqrt{-c+dx}\sqrt{c+dx}\,(6ad^2(3c^2+2d^2x^2)+b(15c^4+10c^2d^2x^2+8d^4x^4))+6c^4(5bc^2+6ad^2)\tanh^{-1}\left(\frac{\sqrt{-c+dx}}{\sqrt{c+dx}}\right)}{48d^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(a + b*x^2))/(Sqrt[-c + d*x]*Sqrt[c + d*x]),x]
```

```
[Out] (d*x*Sqrt[-c + d*x]*Sqrt[c + d*x]*(6*a*d^2*(3*c^2 + 2*d^2*x^2) + b*(15*c^4
+ 10*c^2*d^2*x^2 + 8*d^4*x^4)) + 6*c^4*(5*b*c^2 + 6*a*d^2)*ArcTanh[Sqrt[-c
+ d*x]/Sqrt[c + d*x]])/(48*d^7)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.33, size = 240, normalized size = 1.46

method	result

risch	$-\frac{x(8b\,d^4x^4+12a\,d^4x^2+10b\,c^2d^2x^2+18a\,c^2d^2+15b\,c^4)(-dx+c)\sqrt{dx+c}}{48d^6\sqrt{dx-c}}+\frac{\left(\frac{3c^4\ln\left(\frac{d^2x}{\sqrt{d^2}}+\sqrt{d^2x^2-c^2}\right)a}{8d^4\sqrt{d^2}}+\frac{5c^6\ln\left(\frac{d}{\sqrt{}}\right.}{}\right.}{\sqrt{dx-}}$
default	$\frac{\sqrt{dx-c}\,\sqrt{dx+c}\,\left(8\,\mathrm{csgn}(d)b\,d^5x^5\sqrt{d^2x^2-c^2}+12\,\mathrm{csgn}(d)a\,d^5x^3\sqrt{d^2x^2-c^2}+10\,\mathrm{csgn}(d)b\,c^2d^3x^3\sqrt{d^2x^2-c^2}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4*(b*x^2+a)/(d*x-c)^(1/2)/(d*x+c)^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/48*(d*x-c)^(1/2)*(d*x+c)^(1/2)*(8*csgn(d)*b*d^5*x^5*(d^2*x^2-c^2)^(1/2)+12*csgn(d)*a*d^5*x^3*(d^2*x^2-c^2)^(1/2)+10*csgn(d)*b*c^2*d^3*x^3*(d^2*x^2-c^2)^(1/2)+18*(d^2*x^2-c^2)^(1/2)*csgn(d)*d^3*a*c^2*x+15*(d^2*x^2-c^2)^(1/2)*csgn(d)*d*b*c^4*x+18*ln(((d^2*x^2-c^2)^(1/2)*csgn(d)+d*x)*csgn(d))*a*c^4*d^2+15*ln(((d^2*x^2-c^2)^(1/2)*csgn(d)+d*x)*csgn(d))*b*c^6)*csgn(d)/d^7/(d^2*x^2-c^2)^(1/2)

Maxima [A]
time = 0.28, size = 196, normalized size = 1.20

$$\frac{\sqrt{d^2x^2-c^2}\,bx^5}{6\,d^2}+\frac{5\,\sqrt{d^2x^2-c^2}\,bc^2x^3}{24\,d^4}+\frac{\sqrt{d^2x^2-c^2}\,ax^3}{4\,d^2}+\frac{5\,bc^6\log\left(2\,d^2x+2\,\sqrt{d^2x^2-c^2}\,d\right)}{16\,d^7}+\frac{3\,ac^4\log\left(2\,d^2x+2\,\sqrt{d^2x^2-c^2}\,d\right)}{8\,d^5}+\frac{5\,\sqrt{d^2x^2-c^2}\,bc^4x}{16\,d^6}+\frac{3\,\sqrt{d^2x^2-c^2}\,ac^2x}{8\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*(b*x^2+a)/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="maxima")

[Out] 1/6*sqrt(d^2*x^2 - c^2)*b*x^5/d^2 + 5/24*sqrt(d^2*x^2 - c^2)*b*c^2*x^3/d^4 + 1/4*sqrt(d^2*x^2 - c^2)*a*x^3/d^2 + 5/16*b*c^6*log(2*d^2*x + 2*sqrt(d^2*x^2 - c^2)*d)/d^7 + 3/8*a*c^4*log(2*d^2*x + 2*sqrt(d^2*x^2 - c^2)*d)/d^5 + 5/16*sqrt(d^2*x^2 - c^2)*b*c^4*x/d^6 + 3/8*sqrt(d^2*x^2 - c^2)*a*c^2*x/d^4

Fricas [A]
time = 3.42, size = 115, normalized size = 0.70

$$\frac{(8\,bd^5x^5+2\,(5\,bc^2d^3+6\,ad^5)x^3+3\,(5\,bc^4d+6\,ac^2d^3)x)\sqrt{dx+c}\,\sqrt{dx-c}\,-3\,(5\,bc^6+6\,ac^4d^2)\log\left(-dx+\sqrt{dx+c}\,\sqrt{dx-c}\,\right)}{48\,d^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*(b*x^2+a)/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="fricas")

[Out] 1/48*((8*b*d^5*x^5 + 2*(5*b*c^2*d^3 + 6*a*d^5)*x^3 + 3*(5*b*c^4*d + 6*a*c^2*d^3)*x)*sqrt(d*x + c)*sqrt(d*x - c) - 3*(5*b*c^6 + 6*a*c^4*d^2)*log(-d*x + sqrt(d*x + c)*sqrt(d*x - c)))/d^7

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**2+a)/(d*x-c)**(1/2)/(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.65, size = 203, normalized size = 1.24

$$\frac{\left(\left(2\left((dx+c)\left(4(dx+c)\left(\frac{(dx+c)b}{d^6}-\frac{5bc}{d^6}\right)+\frac{3(15bc^2d^{36}+2ad^{38})}{d^{42}}\right)-\frac{55bc^3d^{36}+18acd^{38}}{d^{42}}\right)(dx+c)+\frac{85bc^4d^{36}+54ac^2d^{38}}{d^{42}}\right)(dx+c)-\frac{3(11bc^5d^{36}+10ac^3d^{38})}{d^{42}}\right)\sqrt{dx+c}\sqrt{dx-c}-\frac{6(5bc^6+6ac^4d^2)\log\left(\left|-\sqrt{dx+c}+\sqrt{dx-c}\right|\right)}{d^6}}{48d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/48*(((2*((d*x + c)*(4*(d*x + c)*((d*x + c)*b/d^6 - 5*b*c/d^6) + 3*(15*b*c
^2*d^36 + 2*a*d^38)/d^42) - (55*b*c^3*d^36 + 18*a*c*d^38)/d^42)*(d*x + c) +
 (85*b*c^4*d^36 + 54*a*c^2*d^38)/d^42)*(d*x + c) - 3*(11*b*c^5*d^36 + 10*a*
c^3*d^38)/d^42)*sqrt(d*x + c)*sqrt(d*x - c) - 6*(5*b*c^6 + 6*a*c^4*d^2)*log
(abs(-sqrt(d*x + c) + sqrt(d*x - c)))/d^6)/d
```

Mupad [B]
time = 42.66, size = 1682, normalized size = 10.26

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(a + b*x^2))/((c + d*x)^(1/2)*(d*x - c)^(1/2)),x)
```

```
[Out] ((5*b*c^6*((c + d*x)^(1/2) - c^(1/2)))/(4*((-c)^(1/2) - (d*x - c)^(1/2))) -
 (175*b*c^6*((c + d*x)^(1/2) - c^(1/2))^3)/(12*((-c)^(1/2) - (d*x - c)^(1/2
))^3) + (311*b*c^6*((c + d*x)^(1/2) - c^(1/2))^5)/(4*((-c)^(1/2) - (d*x - c
)^(1/2))^5) + (8361*b*c^6*((c + d*x)^(1/2) - c^(1/2))^7)/(4*((-c)^(1/2) - (
d*x - c)^(1/2))^7) + (42259*b*c^6*((c + d*x)^(1/2) - c^(1/2))^9)/(6*((-c)^(
1/2) - (d*x - c)^(1/2))^9) + (25295*b*c^6*((c + d*x)^(1/2) - c^(1/2))^11)/(
2*((-c)^(1/2) - (d*x - c)^(1/2))^11) + (25295*b*c^6*((c + d*x)^(1/2) - c^(1
/2))^13)/(2*((-c)^(1/2) - (d*x - c)^(1/2))^13) + (42259*b*c^6*((c + d*x)^(1
/2) - c^(1/2))^15)/(6*((-c)^(1/2) - (d*x - c)^(1/2))^15) + (8361*b*c^6*((c
+ d*x)^(1/2) - c^(1/2))^17)/(4*((-c)^(1/2) - (d*x - c)^(1/2))^17) + (311*b*
c^6*((c + d*x)^(1/2) - c^(1/2))^19)/(4*((-c)^(1/2) - (d*x - c)^(1/2))^19) -
 (175*b*c^6*((c + d*x)^(1/2) - c^(1/2))^21)/(12*((-c)^(1/2) - (d*x - c)^(1/
2))^21) + (5*b*c^6*((c + d*x)^(1/2) - c^(1/2))^23)/(4*((-c)^(1/2) - (d*x -
```

```
c)^(1/2))^23))/(d^7 - (12*d^7*((c + d*x)^(1/2) - c^(1/2))^2)/((-c)^(1/2) -
(d*x - c)^(1/2))^2 + (66*d^7*((c + d*x)^(1/2) - c^(1/2))^4)/((-c)^(1/2) - (
d*x - c)^(1/2))^4 - (220*d^7*((c + d*x)^(1/2) - c^(1/2))^6)/((-c)^(1/2) - (
d*x - c)^(1/2))^6 + (495*d^7*((c + d*x)^(1/2) - c^(1/2))^8)/((-c)^(1/2) - (
d*x - c)^(1/2))^8 - (792*d^7*((c + d*x)^(1/2) - c^(1/2))^10)/((-c)^(1/2) -
(d*x - c)^(1/2))^10 + (924*d^7*((c + d*x)^(1/2) - c^(1/2))^12)/((-c)^(1/2)
- (d*x - c)^(1/2))^12 - (792*d^7*((c + d*x)^(1/2) - c^(1/2))^14)/((-c)^(1/2
) - (d*x - c)^(1/2))^14 + (495*d^7*((c + d*x)^(1/2) - c^(1/2))^16)/((-c)^(1
/2) - (d*x - c)^(1/2))^16 - (220*d^7*((c + d*x)^(1/2) - c^(1/2))^18)/((-c)^
(1/2) - (d*x - c)^(1/2))^18 + (66*d^7*((c + d*x)^(1/2) - c^(1/2))^20)/((-c)
^(1/2) - (d*x - c)^(1/2))^20 - (12*d^7*((c + d*x)^(1/2) - c^(1/2))^22)/((-c
)^(1/2) - (d*x - c)^(1/2))^22 + (d^7*((c + d*x)^(1/2) - c^(1/2))^24)/((-c)^
(1/2) - (d*x - c)^(1/2))^24) - ((23*a*c^4*((c + d*x)^(1/2) - c^(1/2))^3)/(2
*((-c)^(1/2) - (d*x - c)^(1/2))^3) - (3*a*c^4*((c + d*x)^(1/2) - c^(1/2)))/
(2*((-c)^(1/2) - (d*x - c)^(1/2))) + (333*a*c^4*((c + d*x)^(1/2) - c^(1/2))
^5)/(2*((-c)^(1/2) - (d*x - c)^(1/2))^5) + (671*a*c^4*((c + d*x)^(1/2) - c^
(1/2))^7)/(2*((-c)^(1/2) - (d*x - c)^(1/2))^7) + (671*a*c^4*((c + d*x)^(1/2
) - c^(1/2))^9)/(2*((-c)^(1/2) - (d*x - c)^(1/2))^9) + (333*a*c^4*((c + d*x
)^(1/2) - c^(1/2))^11)/(2*((-c)^(1/2) - (d*x - c)^(1/2))^11) + (23*a*c^4*((
c + d*x)^(1/2) - c^(1/2))^13)/(2*((-c)^(1/2) - (d*x - c)^(1/2))^13) - (3*a*
c^4*((c + d*x)^(1/2) - c^(1/2))^15)/(2*((-c)^(1/2) - (d*x - c)^(1/2))^15))/
(d^5 - (8*d^5*((c + d*x)^(1/2) - c^(1/2))^2)/((-c)^(1/2) - (d*x - c)^(1/2))
^2 + (28*d^5*((c + d*x)^(1/2) - c^(1/2))^4)/((-c)^(1/2) - (d*x - c)^(1/2))^
4 - (56*d^5*((c + d*x)^(1/2) - c^(1/2))^6)/((-c)^(1/2) - (d*x - c)^(1/2))^6
 + (70*d^5*((c + d*x)^(1/2) - c^(1/2))^8)/((-c)^(1/2) - (d*x - c)^(1/2))^8
- (56*d^5*((c + d*x)^(1/2) - c^(1/2))^10)/((-c)^(1/2) - (d*x - c)^(1/2))^10
 + (28*d^5*((c + d*x)^(1/2) - c^(1/2))^12)/((-c)^(1/2) - (d*x - c)^(1/2))^1
2 - (8*d^5*((c + d*x)^(1/2) - c^(1/2))^14)/((-c)^(1/2) - (d*x - c)^(1/2))^1
4 + (d^5*((c + d*x)^(1/2) - c^(1/2))^16)/((-c)^(1/2) - (d*x - c)^(1/2))^16)
 - (3*a*c^4*atanh(((c + d*x)^(1/2) - c^(1/2))/((-c)^(1/2) - (d*x - c)^(1/2)
)))/(2*d^5) - (5*b*c^6*atanh(((c + d*x)^(1/2) - c^(1/2))/((-c)^(1/2) - (d*x
 - c)^(1/2))))/(4*d^7)
```

3.359 $\int \frac{x^3(a+bx^2)}{\sqrt{-c+dx}\sqrt{c+dx}}\,dx$

Optimal. Leaf size=118

$$\frac{2c^2(4bc^2+5ad^2)\sqrt{-c+dx}\sqrt{c+dx}}{15d^6}+\frac{(4bc^2+5ad^2)x^2\sqrt{-c+dx}\sqrt{c+dx}}{15d^4}+\frac{bx^4\sqrt{-c+dx}\sqrt{c+dx}}{5d^2}$$

```
[Out] 2/15*c^2*(5*a*d^2+4*b*c^2)*(d*x-c)^(1/2)*(d*x+c)^(1/2)/d^6+1/15*(5*a*d^2+4*
b*c^2)*x^2*(d*x-c)^(1/2)*(d*x+c)^(1/2)/d^4+1/5*b*x^4*(d*x-c)^(1/2)*(d*x+c)^
(1/2)/d^2
```

Rubi [A]
time = 0.06, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {471, 102, 12, 75}

$$\frac{2c^2\sqrt{dx-c}\sqrt{c+dx}(5ad^2+4bc^2)}{15d^6}+\frac{x^2\sqrt{dx-c}\sqrt{c+dx}(5ad^2+4bc^2)}{15d^4}+\frac{bx^4\sqrt{dx-c}\sqrt{c+dx}}{5d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*x^2))/(Sqrt[-c + d*x]*Sqrt[c + d*x]),x]
```

```
[Out] (2*c^2*(4*b*c^2 + 5*a*d^2)*Sqrt[-c + d*x]*Sqrt[c + d*x])/(15*d^6) + ((4*b*c
^2 + 5*a*d^2)*x^2*Sqrt[-c + d*x]*Sqrt[c + d*x])/(15*d^4) + (b*x^4*Sqrt[-c +
 d*x]*Sqrt[c + d*x])/(5*d^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 75

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 102

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/(d*f*(m + n + p + 1))), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
```

```
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[d*(e*x)^(
m + 1)*(a1 + b1*x^(n/2))^(p + 1)*((a2 + b2*x^(n/2))^(p + 1)/(b1*b2*e*(m + n
*(p + 1) + 1))), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3(a+bx^2)}{\sqrt{-c+dx}\,\sqrt{c+dx}}\,dx &= \frac{bx^4\sqrt{-c+dx}\,\sqrt{c+dx}}{5d^2}-\frac{1}{5}\left(-5a-\frac{4bc^2}{d^2}\right)\int \frac{x^3}{\sqrt{-c+dx}\,\sqrt{c+dx}}\,dx\\ &= \frac{(4bc^2+5ad^2)\,x^2\sqrt{-c+dx}\,\sqrt{c+dx}}{15d^4}+\frac{bx^4\sqrt{-c+dx}\,\sqrt{c+dx}}{5d^2}+\frac{(4bc^2+5}{}\\ &= \frac{(4bc^2+5ad^2)\,x^2\sqrt{-c+dx}\,\sqrt{c+dx}}{15d^4}+\frac{bx^4\sqrt{-c+dx}\,\sqrt{c+dx}}{5d^2}+\frac{(2c^2(4bc^2}{}\\ &= \frac{2c^2(4bc^2+5ad^2)\,\sqrt{-c+dx}\,\sqrt{c+dx}}{15d^6}+\frac{(4bc^2+5ad^2)\,x^2\sqrt{-c+dx}\,\sqrt{c+dx}}{15d^4}\end{aligned}$$

Mathematica [A]
time = 0.12, size = 74, normalized size = 0.63

$$\frac{\sqrt{-c+dx}\,\sqrt{c+dx}\,(5ad^2(2c^2+d^2x^2)+b(8c^4+4c^2d^2x^2+3d^4x^4))}{15d^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*x^2))/(Sqrt[-c + d*x]*Sqrt[c + d*x]),x]
[Out] (Sqrt[-c + d*x]*Sqrt[c + d*x]*(5*a*d^2*(2*c^2 + d^2*x^2) + b*(8*c^4 + 4*c^2
*d^2*x^2 + 3*d^4*x^4)))/(15*d^6)
```

Maple [A]
time = 0.28, size = 68, normalized size = 0.58

method	result	size
gosper	$\frac{\sqrt{dx+c}\,(3b\,d^4x^4+5a\,d^4x^2+4b\,c^2d^2x^2+10a\,c^2d^2+8b\,c^4)\sqrt{dx-c}}{15d^6}$	68

default	$\frac{\sqrt{dx+c}\,(3b\,d^4x^4+5a\,d^4x^2+4b\,c^2d^2x^2+10a\,c^2d^2+8b\,c^4)\sqrt{dx-c}}{15d^6}$	68
risch	$-\frac{\sqrt{dx+c}\,(-dx+c)(3b\,d^4x^4+5a\,d^4x^2+4b\,c^2d^2x^2+10a\,c^2d^2+8b\,c^4)}{15d^6\sqrt{dx-c}}$	74

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^2+a)/(d*x-c)^(1/2)/(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/15*(d*x+c)^(1/2)*(3*b*d^4*x^4+5*a*d^4*x^2+4*b*c^2*d^2*x^2+10*a*c^2*d^2+8*
b*c^4)/d^6*(d*x-c)^(1/2)
```

Maxima [A]
time = 0.32, size = 124, normalized size = 1.05

$$\frac{\sqrt{d^2x^2-c^2}\,bx^4}{5\,d^2}+\frac{4\,\sqrt{d^2x^2-c^2}\,bc^2x^2}{15\,d^4}+\frac{\sqrt{d^2x^2-c^2}\,ax^2}{3\,d^2}+\frac{8\,\sqrt{d^2x^2-c^2}\,bc^4}{15\,d^6}+\frac{2\,\sqrt{d^2x^2-c^2}\,ac^2}{3\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/5*sqrt(d^2*x^2 - c^2)*b*x^4/d^2 + 4/15*sqrt(d^2*x^2 - c^2)*b*c^2*x^2/d^4
+ 1/3*sqrt(d^2*x^2 - c^2)*a*x^2/d^2 + 8/15*sqrt(d^2*x^2 - c^2)*b*c^4/d^6 +
2/3*sqrt(d^2*x^2 - c^2)*a*c^2/d^4
```

Fricas [A]
time = 3.32, size = 66, normalized size = 0.56

$$\frac{(3\,bd^4x^4+8\,bc^4+10\,ac^2d^2+(4\,bc^2d^2+5\,ad^4)x^2)\sqrt{dx+c}\,\sqrt{dx-c}}{15\,d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/15*(3*b*d^4*x^4 + 8*b*c^4 + 10*a*c^2*d^2 + (4*b*c^2*d^2 + 5*a*d^4)*x^2)*s
qrt(d*x + c)*sqrt(d*x - c)/d^6
```

Sympy [C] Result contains complex when optimal does not.
time = 30.64, size = 240, normalized size = 2.03

$$\frac{ac^3G_{6,6}^{6,2}\left(\begin{matrix}-\frac{5}{4},-\frac{3}{4} & -1,-1,-\frac{1}{2},1\\ -\frac{3}{2},-\frac{5}{4},-1,-\frac{3}{4},-\frac{1}{2},0 & \end{matrix}\middle|\,\frac{c^2}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d^4}+\frac{iac^3G_{6,6}^{2,6}\left(\begin{matrix}-2,-\frac{7}{4},-\frac{3}{2},-\frac{5}{4},-1,1 & \\ -\frac{7}{4},-\frac{5}{4} & -2,-\frac{3}{2},-\frac{3}{2},0\end{matrix}\middle|\,\frac{c^2e^{2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d^4}+\frac{bc^5G_{6,6}^{6,2}\left(\begin{matrix}-\frac{9}{4},-\frac{7}{4} & -2,-2,-\frac{3}{2},1\\ -\frac{5}{2},-\frac{9}{4},-2,-\frac{7}{4},-\frac{3}{2},0 & \end{matrix}\middle|\,\frac{c^2}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d^6}+\frac{ibc^5G_{6,6}^{2,6}\left(\begin{matrix}-3,-\frac{11}{4},-\frac{5}{2},-\frac{9}{4},-2,1 & \\ -\frac{11}{4},-\frac{9}{4} & -3,-\frac{5}{2},-\frac{5}{2},0\end{matrix}\middle|\,\frac{c^2e^{2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**2+a)/(d*x-c)**(1/2)/(d*x+c)**(1/2),x)
```

```
[Out] a*c**3*meijerg(((-5/4, -3/4), (-1, -1, -1/2, 1)), ((-3/2, -5/4, -1, -3/4, -
1/2, 0), ()), c**2/(d**2*x**2))/(4*pi**(3/2)*d**4) + I*a*c**3*meijerg(((-2,
```

```
-7/4, -3/2, -5/4, -1, 1), ()), ((-7/4, -5/4), (-2, -3/2, -3/2, 0)), c**2*e
xp_polar(2*I*pi)/(d**2*x**2))/(4*pi**(3/2)*d**4) + b*c**5*meijerg(((-9/4, -
7/4), (-2, -2, -3/2, 1)), ((-5/2, -9/4, -2, -7/4, -3/2, 0), ()), c**2/(d**2
*x**2))/(4*pi**(3/2)*d**6) + I*b*c**5*meijerg(((-3, -11/4, -5/2, -9/4, -2,
1), ()), ((-11/4, -9/4), (-3, -5/2, -5/2, 0)), c**2*exp_polar(2*I*pi)/(d**2
*x**2))/(4*pi**(3/2)*d**6)
```

Giac [A]
time = 0.67, size = 124, normalized size = 1.05

$$\frac{\left(\left(\left(dx+c\right)\left(3\left(dx+c\right)\left(\frac{(dx+c)b}{d^5}-\frac{4\,bc}{d^5}\right)+\frac{22\,bc^2d^{25}+5\,ad^{27}}{d^{30}}\right)-\frac{10\left(2\,bc^3d^{25}+acd^{27}\right)}{d^{30}}\right)\left(dx+c\right)+\frac{15\left(bc^4d^{25}+ac^2d^{27}\right)}{d^{30}}\right)\sqrt{dx+c}\,\sqrt{dx-c}}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/15*(((d*x + c)*(3*(d*x + c)*((d*x + c)*b/d^5 - 4*b*c/d^5) + (22*b*c^2*d^2
5 + 5*a*d^27)/d^30) - 10*(2*b*c^3*d^25 + a*c*d^27)/d^30)*(d*x + c) + 15*(b*
c^4*d^25 + a*c^2*d^27)/d^30)*sqrt(d*x + c)*sqrt(d*x - c)/d
```

Mupad [B]
time = 2.70, size = 130, normalized size = 1.10

$$\frac{\sqrt{d\,x-c}\,\left(\frac{8\,b\,c^5+10\,a\,c^3\,d^2}{15\,d^6}+\frac{x^3\,\left(4\,b\,c^2\,d^3+5\,a\,d^5\right)}{15\,d^6}+\frac{x\,\left(8\,b\,c^4\,d+10\,a\,c^2\,d^3\right)}{15\,d^6}+\frac{b\,x^5}{5\,d}+\frac{x^2\,\left(4\,b\,c^3\,d^2+5\,a\,c\,d^4\right)}{15\,d^6}+\frac{b\,c\,x^4}{5\,d^2}\right)}{\sqrt{c+d\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*x^2))/((c + d*x)^(1/2)*(d*x - c)^(1/2)),x)
```

```
[Out] ((d*x - c)^(1/2)*((8*b*c^5 + 10*a*c^3*d^2)/(15*d^6) + (x^3*(5*a*d^5 + 4*b*c
^2*d^3))/(15*d^6) + (x*(10*a*c^2*d^3 + 8*b*c^4*d))/(15*d^6) + (b*x^5)/(5*d)
 + (x^2*(4*b*c^3*d^2 + 5*a*c*d^4))/(15*d^6) + (b*c*x^4)/(5*d^2)))/(c + d*x)
^(1/2)
```

3.360 $\int \frac{x^2(a+bx^2)}{\sqrt{-c+dx}\,\sqrt{c+dx}}\,dx$

Optimal. Leaf size=118

$$\frac{(3bc^2+4ad^2)\,x\sqrt{-c+dx}\,\sqrt{c+dx}}{8d^4}+\frac{bx^3\sqrt{-c+dx}\,\sqrt{c+dx}}{4d^2}+\frac{c^2(3bc^2+4ad^2)\tanh^{-1}\left(\frac{\sqrt{-c+dx}}{\sqrt{c+dx}}\right)}{4d^5}$$

```
[Out] 1/4*c^2*(4*a*d^2+3*b*c^2)*arctanh((d*x-c)^(1/2)/(d*x+c)^(1/2))/d^5+1/8*(4*a
*d^2+3*b*c^2)*x*(d*x-c)^(1/2)*(d*x+c)^(1/2)/d^4+1/4*b*x^3*(d*x-c)^(1/2)*(d*
x+c)^(1/2)/d^2
```

Rubi [A]
time = 0.07, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {471, 92, 12, 65, 223, 212}

$$\frac{c^2(4ad^2+3bc^2)\tanh^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)}{4d^5}+\frac{x\sqrt{dx-c}\,\sqrt{c+dx}\,(4ad^2+3bc^2)}{8d^4}+\frac{bx^3\sqrt{dx-c}\,\sqrt{c+dx}}{4d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*x^2))/(Sqrt[-c + d*x]*Sqrt[c + d*x]),x]
```

```
[Out] ((3*b*c^2 + 4*a*d^2)*x*Sqrt[-c + d*x]*Sqrt[c + d*x])/(8*d^4) + (b*x^3*Sqrt[
-c + d*x]*Sqrt[c + d*x])/(4*d^2) + (c^2*(3*b*c^2 + 4*a*d^2)*ArcTanh[Sqrt[-c
 + d*x]/Sqrt[c + d*x]])/(4*d^5)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 92

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(
d*f*(n + p + 3))), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
```

```
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[d*(e*x)^(
m + 1)*(a1 + b1*x^(n/2))^(p + 1)*((a2 + b2*x^(n/2))^(p + 1)/(b1*b2*e*(m + n
*(p + 1) + 1))), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\int \frac{x^2(a+bx^2)}{\sqrt{-c+dx}\,\sqrt{c+dx}}\,dx = \frac{bx^3\sqrt{-c+dx}\,\sqrt{c+dx}}{4d^2} - \frac{1}{4}\left(-4a-\frac{3bc^2}{d^2}\right)\int \frac{x^2}{\sqrt{-c+dx}\,\sqrt{c+dx}}\,dx$$

$$= \frac{(3bc^2+4ad^2)\,x\sqrt{-c+dx}\,\sqrt{c+dx}}{8d^4} + \frac{bx^3\sqrt{-c+dx}\,\sqrt{c+dx}}{4d^2} + \frac{(3bc^2+4a\cdots}{\cdots}$$

$$= \frac{(3bc^2+4ad^2)\,x\sqrt{-c+dx}\,\sqrt{c+dx}}{8d^4} + \frac{bx^3\sqrt{-c+dx}\,\sqrt{c+dx}}{4d^2} + \frac{(c^2(3bc^2+\cdots}{\cdots}$$

$$= \frac{(3bc^2+4ad^2)\,x\sqrt{-c+dx}\,\sqrt{c+dx}}{8d^4} + \frac{bx^3\sqrt{-c+dx}\,\sqrt{c+dx}}{4d^2} + \frac{(c^2(3bc^2+\cdots}{\cdots}$$

$$= \frac{(3bc^2+4ad^2)\,x\sqrt{-c+dx}\,\sqrt{c+dx}}{8d^4} + \frac{bx^3\sqrt{-c+dx}\,\sqrt{c+dx}}{4d^2} + \frac{(c^2(3bc^2+\cdots}{\cdots}$$

$$= \frac{(3bc^2+4ad^2)\,x\sqrt{-c+dx}\,\sqrt{c+dx}}{8d^4} + \frac{bx^3\sqrt{-c+dx}\,\sqrt{c+dx}}{4d^2} + \frac{c^2(3bc^2+\cdots}{\cdots}$$

Mathematica [A]
time = 0.18, size = 92, normalized size = 0.78

$$\frac{dx\sqrt{-c+dx}\,\sqrt{c+dx}\,(3bc^2+4ad^2+2bd^2x^2)+(6bc^4+8ac^2d^2)\tanh^{-1}\left(\frac{\sqrt{-c+dx}}{\sqrt{c+dx}}\right)}{8d^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*x^2))/(Sqrt[-c + d*x]*Sqrt[c + d*x]),x]
```

```
[Out] (d*x*Sqrt[-c + d*x]*Sqrt[c + d*x]*(3*b*c^2 + 4*a*d^2 + 2*b*d^2*x^2) + (6*b*
c^4 + 8*a*c^2*d^2)*ArcTanh[Sqrt[-c + d*x]/Sqrt[c + d*x]])/(8*d^5)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.29, size = 182, normalized size = 1.54

method	result
risch	$-\frac{x(2b\,d^2x^2+4a\,d^2+3b\,c^2)(-dx+c)\sqrt{dx+c}}{8d^4\sqrt{dx-c}}+\frac{\left(\frac{c^2\ln\left(\frac{d^2x}{\sqrt{d^2}}+\sqrt{d^2x^2-c^2}\right)a}{2d^2\sqrt{d^2}}+\frac{3c^4\ln\left(\frac{d^2x}{\sqrt{d^2}}+\sqrt{d^2x^2-c^2}\right)b}{8d^4\sqrt{d^2}}\right)\sqrt{}}{\sqrt{dx-c}\,\sqrt{dx+c}}$
default	$\frac{\sqrt{dx-c}\,\sqrt{dx+c}\,\left(2\,\mathrm{csgn}(d)b\,d^3x^3\sqrt{d^2x^2-c^2}+4\sqrt{d^2x^2-c^2}\,\mathrm{csgn}(d)d^3ax+3\sqrt{d^2x^2-c^2}\,\mathrm{csgn}(d)db\,c^2x+4\ln\left(\right.\right.}{8d^5\sqrt{d^2x^2-c^2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2+a)/(d*x-c)^(1/2)/(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*(d*x-c)^(1/2)*(d*x+c)^(1/2)*(2*csgn(d)*b*d^3*x^3*(d^2*x^2-c^2)^(1/2)+4*
(d^2*x^2-c^2)^(1/2)*csgn(d)*d^3*a*x+3*(d^2*x^2-c^2)^(1/2)*csgn(d)*d*b*c^2*x
+4*ln(((d^2*x^2-c^2)^(1/2)*csgn(d)+d*x)*csgn(d))*a*c^2*d^2+3*ln(((d^2*x^2-c
^2)^(1/2)*csgn(d)+d*x)*csgn(d))*b*c^4)*csgn(d)/d^5/(d^2*x^2-c^2)^(1/2)
```

Maxima [A]
time = 0.31, size = 142, normalized size = 1.20

$$\frac{\sqrt{d^2x^2-c^2}\,bx^3}{4\,d^2}+\frac{3\,bc^4\log\left(2\,d^2x+2\,\sqrt{d^2x^2-c^2}\,d\right)}{8\,d^5}+\frac{ac^2\log\left(2\,d^2x+2\,\sqrt{d^2x^2-c^2}\,d\right)}{2\,d^3}+\frac{3\,\sqrt{d^2x^2-c^2}\,bc^2x}{8\,d^4}+\frac{\sqrt{d^2x^2-c^2}\,ax}{2\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(d^2*x^2 - c^2)*b*x^3/d^2 + 3/8*b*c^4*log(2*d^2*x + 2*sqrt(d^2*x^2
- c^2)*d)/d^5 + 1/2*a*c^2*log(2*d^2*x + 2*sqrt(d^2*x^2 - c^2)*d)/d^3 + 3/8*
sqrt(d^2*x^2 - c^2)*b*c^2*x/d^4 + 1/2*sqrt(d^2*x^2 - c^2)*a*x/d^2
```

Fricas [A]
time = 3.38, size = 90, normalized size = 0.76

$$\frac{(2\,bd^3x^3+(3\,bc^2d+4\,ad^3)x)\sqrt{dx+c}\,\sqrt{dx-c}-(3\,bc^4+4\,ac^2d^2)\log\left(-dx+\sqrt{dx+c}\,\sqrt{dx-c}\right)}{8\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/8*((2*b*d^3*x^3 + (3*b*c^2*d + 4*a*d^3)*x)*sqrt(d*x + c)*sqrt(d*x - c) -
(3*b*c^4 + 4*a*c^2*d^2)*log(-d*x + sqrt(d*x + c)*sqrt(d*x - c)))/d^5
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**2+a)/(d*x-c)**(1/2)/(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.65, size = 140, normalized size = 1.19

$$\frac{\left((dx+c)\left(2\,(dx+c)\left(\frac{(dx+c)b}{d^4}-\frac{3\,bc}{d^4}\right)+\frac{9\,bc^2d^{16}+4\,ad^{18}}{d^{20}}\right)-\frac{5\,bc^3d^{16}+4\,acd^{18}}{d^{20}}\right)\sqrt{dx+c}\,\sqrt{dx-c}-\frac{2\,(3\,bc^4+4\,ac^2d^2)\log\left(\left|-\sqrt{dx+c}+\sqrt{dx-c}\right|\right)}{d^4}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*(((d*x + c)*(2*(d*x + c)*((d*x + c)*b/d^4 - 3*b*c/d^4) + (9*b*c^2*d^16
+ 4*a*d^18)/d^20) - (5*b*c^3*d^16 + 4*a*c*d^18)/d^20)*sqrt(d*x + c)*sqrt(d*
x - c) - 2*(3*b*c^4 + 4*a*c^2*d^2)*log(abs(-sqrt(d*x + c) + sqrt(d*x - c)))
/d^4)/d
```

Mupad [B]
time = 25.51, size = 1048, normalized size = 8.88

$$\frac{\frac{2\,a\,c^2\left(\sqrt{c+d\,x}-\sqrt{c}\right)}{\sqrt{-c}-\sqrt{d\,x-c}}+\frac{14\,a\,c^2\left(\sqrt{c+d\,x}-\sqrt{c}\right)^3}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^3}+\frac{14\,a\,c^2\left(\sqrt{c+d\,x}-\sqrt{c}\right)^5}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^5}+\frac{2\,a\,c^2\left(\sqrt{c+d\,x}-\sqrt{c}\right)^7}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^7}}{d^3-\frac{4\,d^3\left(\sqrt{c+d\,x}-\sqrt{c}\right)^2}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^2}+\frac{6\,d^3\left(\sqrt{c+d\,x}-\sqrt{c}\right)^4}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^4}-\frac{4\,d^3\left(\sqrt{c+d\,x}-\sqrt{c}\right)^6}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^6}+\frac{d^3\left(\sqrt{c+d\,x}-\sqrt{c}\right)^8}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^8}}-\frac{\frac{23\,b\,c^4\left(\sqrt{c+d\,x}-\sqrt{c}\right)^3}{2\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^3}-\frac{3\,b\,c^4\left(\sqrt{c+d\,x}-\sqrt{c}\right)}{2\left(\sqrt{-c}-\sqrt{d\,x-c}\right)}+\frac{333\,b\,c^4\left(\sqrt{c+d\,x}-\sqrt{c}\right)^5}{2\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^5}+\frac{671\,b\,c^4\left(\sqrt{c+d\,x}-\sqrt{c}\right)^7}{2\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^7}+\frac{671\,b\,c^4\left(\sqrt{c+d\,x}-\sqrt{c}\right)^9}{2\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^9}+\frac{333\,b\,c^4\left(\sqrt{c+d\,x}-\sqrt{c}\right)^{11}}{2\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^{11}}+\frac{23\,b\,c^4\left(\sqrt{c+d\,x}-\sqrt{c}\right)^{13}}{2\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^{13}}-\frac{3\,b\,c^4\left(\sqrt{c+d\,x}-\sqrt{c}\right)^{15}}{2\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^{15}}}{d^5-\frac{8\,d^5\left(\sqrt{c+d\,x}-\sqrt{c}\right)^2}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^2}+\frac{28\,d^5\left(\sqrt{c+d\,x}-\sqrt{c}\right)^4}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^4}-\frac{56\,d^5\left(\sqrt{c+d\,x}-\sqrt{c}\right)^6}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^6}+\frac{70\,d^5\left(\sqrt{c+d\,x}-\sqrt{c}\right)^8}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^8}-\frac{56\,d^5\left(\sqrt{c+d\,x}-\sqrt{c}\right)^{10}}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^{10}}+\frac{28\,d^5\left(\sqrt{c+d\,x}-\sqrt{c}\right)^{12}}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^{12}}-\frac{8\,d^5\left(\sqrt{c+d\,x}-\sqrt{c}\right)^{14}}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^{14}}+\frac{d^5\left(\sqrt{c+d\,x}-\sqrt{c}\right)^{16}}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^{16}}}-\frac{2\,a\,c^2\,\mathrm{atanh}\left(\frac{\sqrt{c+d\,x}-\sqrt{c}}{\sqrt{-c}-\sqrt{d\,x-c}}\right)}{d^3}-\frac{3\,b\,c^4\,\mathrm{atanh}\left(\frac{\sqrt{c+d\,x}-\sqrt{c}}{\sqrt{-c}-\sqrt{d\,x-c}}\right)}{2\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*x^2))/((c + d*x)^(1/2)*(d*x - c)^(1/2)),x)
```

```
[Out] ((2*a*c^2*((c + d*x)^(1/2) - c^(1/2)))/((-c)^(1/2) - (d*x - c)^(1/2)) + (14
*a*c^2*((c + d*x)^(1/2) - c^(1/2))^3)/((-c)^(1/2) - (d*x - c)^(1/2))^3 + (1
```

```
4*a*c^2*((c + d*x)^(1/2) - c^(1/2))^5)/((-c)^(1/2) - (d*x - c)^(1/2))^5 + (
2*a*c^2*((c + d*x)^(1/2) - c^(1/2))^7)/((-c)^(1/2) - (d*x - c)^(1/2))^7)/(d
^3 - (4*d^3*((c + d*x)^(1/2) - c^(1/2))^2)/((-c)^(1/2) - (d*x - c)^(1/2))^2
 + (6*d^3*((c + d*x)^(1/2) - c^(1/2))^4)/((-c)^(1/2) - (d*x - c)^(1/2))^4 -
 (4*d^3*((c + d*x)^(1/2) - c^(1/2))^6)/((-c)^(1/2) - (d*x - c)^(1/2))^6 + (
d^3*((c + d*x)^(1/2) - c^(1/2))^8)/((-c)^(1/2) - (d*x - c)^(1/2))^8) - ((23
*b*c^4*((c + d*x)^(1/2) - c^(1/2))^3)/(2*((-c)^(1/2) - (d*x - c)^(1/2))^3)
- (3*b*c^4*((c + d*x)^(1/2) - c^(1/2)))/(2*((-c)^(1/2) - (d*x - c)^(1/2)))
+ (333*b*c^4*((c + d*x)^(1/2) - c^(1/2))^5)/(2*((-c)^(1/2) - (d*x - c)^(1/2
))^5) + (671*b*c^4*((c + d*x)^(1/2) - c^(1/2))^7)/(2*((-c)^(1/2) - (d*x - c
)^(1/2))^7) + (671*b*c^4*((c + d*x)^(1/2) - c^(1/2))^9)/(2*((-c)^(1/2) - (d
*x - c)^(1/2))^9) + (333*b*c^4*((c + d*x)^(1/2) - c^(1/2))^11)/(2*((-c)^(1/
2) - (d*x - c)^(1/2))^11) + (23*b*c^4*((c + d*x)^(1/2) - c^(1/2))^13)/(2*((
-c)^(1/2) - (d*x - c)^(1/2))^13) - (3*b*c^4*((c + d*x)^(1/2) - c^(1/2))^15)
/(2*((-c)^(1/2) - (d*x - c)^(1/2))^15))/(d^5 - (8*d^5*((c + d*x)^(1/2) - c^
(1/2))^2)/((-c)^(1/2) - (d*x - c)^(1/2))^2 + (28*d^5*((c + d*x)^(1/2) - c^(
1/2))^4)/((-c)^(1/2) - (d*x - c)^(1/2))^4 - (56*d^5*((c + d*x)^(1/2) - c^(1
/2))^6)/((-c)^(1/2) - (d*x - c)^(1/2))^6 + (70*d^5*((c + d*x)^(1/2) - c^(1/
2))^8)/((-c)^(1/2) - (d*x - c)^(1/2))^8 - (56*d^5*((c + d*x)^(1/2) - c^(1/2
))^10)/((-c)^(1/2) - (d*x - c)^(1/2))^10 + (28*d^5*((c + d*x)^(1/2) - c^(1/
2))^12)/((-c)^(1/2) - (d*x - c)^(1/2))^12 - (8*d^5*((c + d*x)^(1/2) - c^(1/
2))^14)/((-c)^(1/2) - (d*x - c)^(1/2))^14 + (d^5*((c + d*x)^(1/2) - c^(1/2)
)^16)/((-c)^(1/2) - (d*x - c)^(1/2))^16) - (2*a*c^2*atanh(((c + d*x)^(1/2)
- c^(1/2))/((-c)^(1/2) - (d*x - c)^(1/2))))/d^3 - (3*b*c^4*atanh(((c + d*x)
^(1/2) - c^(1/2))/((-c)^(1/2) - (d*x - c)^(1/2))))/(2*d^5)
```

3.361 $\int \frac{x(a+bx^2)}{\sqrt{-c+dx}\,\sqrt{c+dx}}\,dx$

Optimal. Leaf size=72

$$\frac{(2bc^2+3ad^2)\sqrt{-c+dx}\,\sqrt{c+dx}}{3d^4}+\frac{bx^2\sqrt{-c+dx}\,\sqrt{c+dx}}{3d^2}$$

```
[Out] 1/3*(3*a*d^2+2*b*c^2)*(d*x-c)^(1/2)*(d*x+c)^(1/2)/d^4+1/3*b*x^2*(d*x-c)^(1/
2)*(d*x+c)^(1/2)/d^2
```

Rubi [A]
time = 0.03, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {471, 75}

$$\frac{\sqrt{dx-c}\,\sqrt{c+dx}\,(3ad^2+2bc^2)}{3d^4}+\frac{bx^2\sqrt{dx-c}\,\sqrt{c+dx}}{3d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*x^2))/(Sqrt[-c + d*x]*Sqrt[c + d*x]),x]
```

```
[Out] ((2*b*c^2 + 3*a*d^2)*Sqrt[-c + d*x]*Sqrt[c + d*x])/(3*d^4) + (b*x^2*Sqrt[-c
 + d*x]*Sqrt[c + d*x])/(3*d^2)
```

Rule 75

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[d*(e*x)^(
m + 1)*(a1 + b1*x^(n/2))^(p + 1)*((a2 + b2*x^(n/2))^(p + 1)/(b1*b2*e*(m + n
*(p + 1) + 1))), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\int \frac{x(a+bx^2)}{\sqrt{-c+dx}\,\sqrt{c+dx}}\,dx = \frac{bx^2\sqrt{-c+dx}\,\sqrt{c+dx}}{3d^2} - \frac{1}{3}\left(-3a-\frac{2bc^2}{d^2}\right)\int \frac{x}{\sqrt{-c+dx}\,\sqrt{c+dx}}\,dx$$
$$= \frac{(2bc^2+3ad^2)\sqrt{-c+dx}\,\sqrt{c+dx}}{3d^4} + \frac{bx^2\sqrt{-c+dx}\,\sqrt{c+dx}}{3d^2}$$

Mathematica [A]
time = 0.09, size = 48, normalized size = 0.67

$$\frac{\sqrt{-c+dx}\,\sqrt{c+dx}\,(2bc^2+3ad^2+bd^2x^2)}{3d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*x^2))/(Sqrt[-c + d*x]*Sqrt[c + d*x]),x]
[Out] (Sqrt[-c + d*x]*Sqrt[c + d*x]*(2*b*c^2 + 3*a*d^2 + b*d^2*x^2))/(3*d^4)
```

Maple [A]
time = 0.30, size = 43, normalized size = 0.60

method	result	size
gosper	$\frac{\sqrt{dx+c}\,\left(b\,d^2x^2+3a\,d^2+2b\,c^2\right)\sqrt{dx-c}}{3d^4}$	43
default	$\frac{\sqrt{dx+c}\,\left(b\,d^2x^2+3a\,d^2+2b\,c^2\right)\sqrt{dx-c}}{3d^4}$	43
risch	$-\frac{\sqrt{dx+c}\,(-dx+c)\left(b\,d^2x^2+3a\,d^2+2b\,c^2\right)}{3d^4\sqrt{dx-c}}$	49

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2+a)/(d*x-c)^(1/2)/(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/3*(d*x+c)^(1/2)*(b*d^2*x^2+3*a*d^2+2*b*c^2)/d^4*(d*x-c)^(1/2)
```

Maxima [A]
time = 0.33, size = 69, normalized size = 0.96

$$\frac{\sqrt{d^2x^2-c^2}\,bx^2}{3\,d^2} + \frac{2\,\sqrt{d^2x^2-c^2}\,bc^2}{3\,d^4} + \frac{\sqrt{d^2x^2-c^2}\,a}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="maxima")
[Out] 1/3*sqrt(d^2*x^2 - c^2)*b*x^2/d^2 + 2/3*sqrt(d^2*x^2 - c^2)*b*c^2/d^4 + sqr
t(d^2*x^2 - c^2)*a/d^2
```

Fricas [A]
time = 2.84, size = 42, normalized size = 0.58

$$\frac{\left(bd^{2}x^{2}+2\,bc^{2}+3\,ad^{2}\right)\sqrt{dx+c}\,\sqrt{dx-c}}{3\,d^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*(b*d^2*x^2 + 2*b*c^2 + 3*a*d^2)*sqrt(d*x + c)*sqrt(d*x - c)/d^4
```

Sympy [C] Result contains complex when optimal does not.
time = 19.78, size = 223, normalized size = 3.10

$$\frac{acG_{6,6}^{6,2}\left(\begin{matrix} -\frac{1}{4},\frac{1}{4} & 0,0,\frac{1}{2},1 \\ -\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},0 & \end{matrix}\middle|{\frac{c^{2}}{d^{2}x^{2}}}\right)}{4\pi^{\frac{3}{2}}d^{2}}+\frac{iacG_{6,6}^{2,6}\left(\begin{matrix} -1,-\frac{3}{4},-\frac{1}{2},-\frac{1}{4},0,1 & \\ -\frac{3}{4},-\frac{1}{4} & -1,-\frac{1}{2},-\frac{1}{2},0 \end{matrix}\middle|{\frac{c^{2}e^{2i\pi}}{d^{2}x^{2}}}\right)}{4\pi^{\frac{3}{2}}d^{2}}+\frac{bc^{3}G_{6,6}^{6,2}\left(\begin{matrix} -\frac{5}{4},-\frac{3}{4} & -1,-1,-\frac{1}{2},1 \\ -\frac{3}{2},-\frac{5}{4},-1,-\frac{3}{4},-\frac{1}{2},0 & \end{matrix}\middle|{\frac{c^{2}}{d^{2}x^{2}}}\right)}{4\pi^{\frac{3}{2}}d^{4}}+\frac{ibc^{3}G_{6,6}^{2,6}\left(\begin{matrix} -2,-\frac{7}{4},-\frac{3}{2},-\frac{5}{4},-1,1 & \\ -\frac{7}{4},-\frac{5}{4} & -2,-\frac{3}{2},-\frac{3}{2},0 \end{matrix}\middle|{\frac{c^{2}e^{2i\pi}}{d^{2}x^{2}}}\right)}{4\pi^{\frac{3}{2}}d^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**2+a)/(d*x-c)**(1/2)/(d*x+c)**(1/2),x)
```

```
[Out] a*c*meijerg(((-1/4, 1/4), (0, 0, 1/2, 1)), ((-1/2, -1/4, 0, 1/4, 1/2, 0), (
)), c**2/(d**2*x**2))/(4*pi**(3/2)*d**2) + I*a*c*meijerg(((-1, -3/4, -1/2,
-1/4, 0, 1), ()), ((-3/4, -1/4), (-1, -1/2, -1/2, 0)), c**2*exp_polar(2*I*p
i)/(d**2*x**2))/(4*pi**(3/2)*d**2) + b*c**3*meijerg(((-5/4, -3/4), (-1, -1,
 -1/2, 1)), ((-3/2, -5/4, -1, -3/4, -1/2, 0), ()), c**2/(d**2*x**2))/(4*pi*
*(3/2)*d**4) + I*b*c**3*meijerg(((-2, -7/4, -3/2, -5/4, -1, 1), ()), ((-7/4
, -5/4), (-2, -3/2, -3/2, 0)), c**2*exp_polar(2*I*pi)/(d**2*x**2))/(4*pi**(
3/2)*d**4)
```

Giac [A]
time = 0.61, size = 65, normalized size = 0.90

$$\frac{\sqrt{dx+c}\,\sqrt{dx-c}\,\left((dx+c)\left(\frac{(dx+c)b}{d^{3}}-\frac{2\,bc}{d^{3}}\right)+\frac{3\,\left(bc^{2}d^{9}+ad^{11}\right)}{d^{12}}\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(d*x + c)*sqrt(d*x - c)*((d*x + c)*((d*x + c)*b/d^3 - 2*b*c/d^3) +
3*(b*c^2*d^9 + a*d^11)/d^12)/d
```

Mupad [B]
time = 2.66, size = 76, normalized size = 1.06

$$\frac{\sqrt{d\,x-c}\,\left(\frac{2\,b\,c^{3}+3\,a\,c\,d^{2}}{3\,d^{4}}+\frac{b\,x^{3}}{3\,d}+\frac{x\,\left(2\,b\,c^{2}\,d+3\,a\,d^{3}\right)}{3\,d^{4}}+\frac{b\,c\,x^{2}}{3\,d^{2}}\right)}{\sqrt{c+d\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*x^2))/((c + d*x)^(1/2)*(d*x - c)^(1/2)),x)
```

```
[Out] ((d*x - c)^(1/2)*((2*b*c^3 + 3*a*c*d^2)/(3*d^4) + (b*x^3)/(3*d) + (x*(3*a*d
^3 + 2*b*c^2*d))/(3*d^4) + (b*c*x^2)/(3*d^2)))/(c + d*x)^(1/2)
```

3.362 $\int \frac{a+bx^2}{\sqrt{-c+dx}\sqrt{c+dx}} dx$

Optimal. Leaf size=68

$$\frac{bx\sqrt{-c+dx}\sqrt{c+dx}}{2d^2}+\frac{(bc^2+2ad^2)\tanh^{-1}\left(\frac{\sqrt{-c+dx}}{\sqrt{c+dx}}\right)}{d^3}$$

```
[Out] (2*a*d^2+b*c^2)*arctanh((d*x-c)^(1/2)/(d*x+c)^(1/2))/d^3+1/2*b*x*(d*x-c)^(1
/2)*(d*x+c)^(1/2)/d^2
```

Rubi [A]
time = 0.02, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {397, 65, 223, 212}

$$\frac{(2ad^2+bc^2)\tanh^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)}{d^3}+\frac{bx\sqrt{dx-c}\sqrt{c+dx}}{2d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)/(Sqrt[-c + d*x]*Sqrt[c + d*x]),x]
```

```
[Out] (b*x*Sqrt[-c + d*x]*Sqrt[c + d*x])/(2*d^2) + ((b*c^2 + 2*a*d^2)*ArcTanh[Sqr
t[-c + d*x]/Sqrt[c + d*x]])/d^3
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 397

```
Int[((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_
.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[d*x*(a1 + b1*x^(n/2))^(p + 1
)*((a2 + b2*x^(n/2))^(p + 1)/(b1*b2*(n*(p + 1) + 1))), x] - Dist[(a1*a2*d -
 b1*b2*c*(n*(p + 1) + 1))/(b1*b2*(n*(p + 1) + 1)), Int[(a1 + b1*x^(n/2))^p*
(a2 + b2*x^(n/2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, n, p}, x] && Eq
Q[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx^2}{\sqrt{-c+dx}\,\sqrt{c+dx}}\,dx &= \frac{bx\sqrt{-c+dx}\,\sqrt{c+dx}}{2d^2}-\frac{(-bc^2-2ad^2)\int \frac{1}{\sqrt{-c+dx}\,\sqrt{c+dx}}\,dx}{2d^2}\\ &= \frac{bx\sqrt{-c+dx}\,\sqrt{c+dx}}{2d^2}+\frac{(bc^2+2ad^2)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{2c+x^2}}\,dx,x,\sqrt{-c+dx}\right.}{d^3}\\ &= \frac{bx\sqrt{-c+dx}\,\sqrt{c+dx}}{2d^2}+\frac{(bc^2+2ad^2)\,\mathrm{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\frac{\sqrt{-c+dx}}{\sqrt{c+dx}}\right)}{d^3}\\ &= \frac{bx\sqrt{-c+dx}\,\sqrt{c+dx}}{2d^2}+\frac{(bc^2+2ad^2)\tanh^{-1}\left(\frac{\sqrt{-c+dx}}{\sqrt{c+dx}}\right)}{d^3}\end{aligned}$$

Mathematica [A]
time = 0.12, size = 68, normalized size = 1.00

$$\frac{bdx\sqrt{-c+dx}\,\sqrt{c+dx}+2(bc^2+2ad^2)\tanh^{-1}\left(\frac{\sqrt{-c+dx}}{\sqrt{c+dx}}\right)}{2d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)/(Sqrt[-c + d*x]*Sqrt[c + d*x]),x]
[Out] (b*d*x*Sqrt[-c + d*x]*Sqrt[c + d*x] + 2*(b*c^2 + 2*a*d^2)*ArcTanh[Sqrt[-c +
 d*x]/Sqrt[c + d*x]])/(2*d^3)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.29, size = 124, normalized size = 1.82

method	result
default	$\frac{\sqrt{dx-c}\,\sqrt{dx+c}\,\left(\sqrt{d^2x^2-c^2}\,\mathrm{csgn}(d)dbx+\ln\left(\left(\sqrt{d^2x^2-c^2}\,\mathrm{csgn}(d)+dx\right)\mathrm{csgn}(d)\right)b\,c^2+2\ln\left(\left(\sqrt{d^2x^2-c^2}\right.\right.\right.}{2d^3\sqrt{d^2x^2-c^2}}$

risch	$-\frac{(-dx+c)\sqrt{dx+c}\, bx}{2d^2\sqrt{dx-c}}+\frac{\left(\frac{\ln\left(\frac{d^2x}{\sqrt{d^2}}+\sqrt{d^2x^2-c^2}\right)a}{\sqrt{d^2}}+\frac{\ln\left(\frac{d^2x}{\sqrt{d^2}}+\sqrt{d^2x^2-c^2}\right)b\,c^2}{2d^2\sqrt{d^2}}\right)\sqrt{(dx-c)(dx+c}}{\sqrt{dx-c}\,\sqrt{dx+c}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)/(d*x-c)^(1/2)/(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(d*x-c)^(1/2)*(d*x+c)^(1/2)/d^3*((d^2*x^2-c^2)^(1/2)*csgn(d)*d*b*x+ln((
(d^2*x^2-c^2)^(1/2)*csgn(d)+d*x)*csgn(d))*b*c^2+2*ln(((d^2*x^2-c^2)^(1/2)*c
sgn(d)+d*x)*csgn(d))*a*d^2)/(d^2*x^2-c^2)^(1/2)*csgn(d)
```

Maxima [A]
time = 0.31, size = 89, normalized size = 1.31

$$\frac{bc^2\log\left(2\,d^2x+2\,\sqrt{d^2x^2-c^2}\,d\right)}{2\,d^3}+\frac{a\log\left(2\,d^2x+2\,\sqrt{d^2x^2-c^2}\,d\right)}{d}+\frac{\sqrt{d^2x^2-c^2}\,bx}{2\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*b*c^2*log(2*d^2*x + 2*sqrt(d^2*x^2 - c^2)*d)/d^3 + a*log(2*d^2*x + 2*sq
rt(d^2*x^2 - c^2)*d)/d + 1/2*sqrt(d^2*x^2 - c^2)*b*x/d^2
```

Fricas [A]
time = 2.72, size = 63, normalized size = 0.93

$$\frac{\sqrt{dx+c}\,\sqrt{dx-c}\,bdx-(bc^2+2\,ad^2)\log\left(-dx+\sqrt{dx+c}\,\sqrt{dx-c}\right)}{2\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(d*x + c)*sqrt(d*x - c)*b*d*x - (b*c^2 + 2*a*d^2)*log(-d*x + sqrt(
d*x + c)*sqrt(d*x - c)))/d^3
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/(d*x-c)**(1/2)/(d*x+c)**(1/2),x)
```

[Out] Timed out

Giac [A]
time = 0.56, size = 79, normalized size = 1.16

$$\frac{\sqrt{dx+c}\,\sqrt{dx-c}\left(\frac{(dx+c)b}{d^2}-\frac{bc}{d^2}\right)-\frac{2\,(bc^2+2\,ad^2)\log\left(\left|-\sqrt{dx+c}+\sqrt{dx-c}\right|\right)}{d^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*(sqrt(d*x + c)*sqrt(d*x - c)*((d*x + c)*b/d^2 - b*c/d^2) - 2*(b*c^2 + 2
*a*d^2)*log(abs(-sqrt(d*x + c) + sqrt(d*x - c)))/d^2)/d
```

Mupad [B]
time = 10.80, size = 417, normalized size = 6.13

$$\frac{\frac{2\,b\,c^2\,\left(\sqrt{c+d\,x}-\sqrt{c}\right)}{\sqrt{-c}-\sqrt{d\,x-c}}+\frac{14\,b\,c^2\,{\left(\sqrt{c+d\,x}-\sqrt{c}\right)}^3}{{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)}^3}+\frac{14\,b\,c^2\,{\left(\sqrt{c+d\,x}-\sqrt{c}\right)}^5}{{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)}^5}+\frac{2\,b\,c^2\,{\left(\sqrt{c+d\,x}-\sqrt{c}\right)}^7}{{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)}^7}}{d^3-\frac{4\,d^3\,{\left(\sqrt{c+d\,x}-\sqrt{c}\right)}^2}{{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)}^2}+\frac{6\,d^3\,{\left(\sqrt{c+d\,x}-\sqrt{c}\right)}^4}{{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)}^4}-\frac{4\,d^3\,{\left(\sqrt{c+d\,x}-\sqrt{c}\right)}^6}{{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)}^6}+\frac{d^3\,{\left(\sqrt{c+d\,x}-\sqrt{c}\right)}^8}{{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)}^8}}+\frac{4\,a\,\mathrm{atan}\left(\frac{d\,\left(\sqrt{-c}-\sqrt{d\,x-c}\right)}{\sqrt{-d^2}\,\left(\sqrt{c+d\,x}-\sqrt{c}\right)}\right)}{\sqrt{-d^2}}-\frac{2\,b\,c^2\,\mathrm{atanh}\left(\frac{\sqrt{c+d\,x}-\sqrt{c}}{\sqrt{-c}-\sqrt{d\,x-c}}\right)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)/((c + d*x)^(1/2)*(d*x - c)^(1/2)),x)
```

```
[Out] ((2*b*c^2*((c + d*x)^(1/2) - c^(1/2)))/((-c)^(1/2) - (d*x - c)^(1/2)) + (14
*b*c^2*((c + d*x)^(1/2) - c^(1/2))^3)/((-c)^(1/2) - (d*x - c)^(1/2))^3 + (1
4*b*c^2*((c + d*x)^(1/2) - c^(1/2))^5)/((-c)^(1/2) - (d*x - c)^(1/2))^5 + (
2*b*c^2*((c + d*x)^(1/2) - c^(1/2))^7)/((-c)^(1/2) - (d*x - c)^(1/2))^7)/(d
^3 - (4*d^3*((c + d*x)^(1/2) - c^(1/2))^2)/((-c)^(1/2) - (d*x - c)^(1/2))^2
 + (6*d^3*((c + d*x)^(1/2) - c^(1/2))^4)/((-c)^(1/2) - (d*x - c)^(1/2))^4 -
 (4*d^3*((c + d*x)^(1/2) - c^(1/2))^6)/((-c)^(1/2) - (d*x - c)^(1/2))^6 + (
d^3*((c + d*x)^(1/2) - c^(1/2))^8)/((-c)^(1/2) - (d*x - c)^(1/2))^8) + (4*a
*atan((d*((-c)^(1/2) - (d*x - c)^(1/2)))/((-d^2)^(1/2)*((c + d*x)^(1/2) - c
^(1/2)))))/(-d^2)^(1/2) - (2*b*c^2*atanh(((c + d*x)^(1/2) - c^(1/2))/((-c)^
(1/2) - (d*x - c)^(1/2))))/d^3
```

3.363 $\int \frac{a+bx^2}{x\sqrt{-c+dx}\,\sqrt{c+dx}}\,dx$

Optimal. Leaf size=56

$$\frac{b\sqrt{-c+dx}\,\sqrt{c+dx}}{d^2}+\frac{a\tan^{-1}\left(\frac{\sqrt{-c+dx}\,\sqrt{c+dx}}{c}\right)}{c}$$

```
[Out] a*arctan((d*x-c)^(1/2)*(d*x+c)^(1/2)/c)/c+b*(d*x-c)^(1/2)*(d*x+c)^(1/2)/d^2
```

Rubi [A]
time = 0.05, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {471, 94, 211}

$$\frac{a\text{ArcTan}\left(\frac{\sqrt{dx-c}\,\sqrt{c+dx}}{c}\right)}{c}+\frac{b\sqrt{dx-c}\,\sqrt{c+dx}}{d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)/(x*Sqrt[-c + d*x]*Sqrt[c + d*x]),x]
```

```
[Out] (b*Sqrt[-c + d*x]*Sqrt[c + d*x])/d^2 + (a*ArcTan[(Sqrt[-c + d*x]*Sqrt[c + d
*x])/c])/c
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[d*(e*x)^(
m + 1)*(a1 + b1*x^(n/2))^(p + 1)*((a2 + b2*x^(n/2))^(p + 1)/(b1*b2*e*(m + n
*(p + 1) + 1))), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
```

```
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+bx^2}{x\sqrt{-c+dx}\,\sqrt{c+dx}}\,dx &= \frac{b\sqrt{-c+dx}\,\sqrt{c+dx}}{d^2}+a\int \frac{1}{x\sqrt{-c+dx}\,\sqrt{c+dx}}\,dx\\
&= \frac{b\sqrt{-c+dx}\,\sqrt{c+dx}}{d^2}+(ad)\text{Subst}\left(\int \frac{1}{c^2d+dx^2}\,dx, x, \sqrt{-c+dx}\,\sqrt{c+dx}\right)\\
&= \frac{b\sqrt{-c+dx}\,\sqrt{c+dx}}{d^2}+\frac{a\tan^{-1}\left(\frac{\sqrt{-c+dx}\,\sqrt{c+dx}}{c}\right)}{c}
\end{aligned}$$

Mathematica [A]
time = 0.09, size = 54, normalized size = 0.96

$$\frac{b\sqrt{-c+dx}\,\sqrt{c+dx}}{d^2}+\frac{2a\tan^{-1}\left(\frac{\sqrt{-c+dx}}{\sqrt{c+dx}}\right)}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)/(x*Sqrt[-c + d*x]*Sqrt[c + d*x]),x]
```

```
[Out] (b*Sqrt[-c + d*x]*Sqrt[c + d*x])/d^2 + (2*a*ArcTan[Sqrt[-c + d*x]/Sqrt[c +
d*x]])/c
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 107 vs. $2(48) = 96$.
time = 0.30, size = 108, normalized size = 1.93

method	result	size
default	$\frac{\left(-\ln\left(-\frac{2\left(c^2-\sqrt{-c^2}\,\sqrt{d^2x^2-c^2}\right)}{x}\right)a\,d^2+b\sqrt{-c^2}\,\sqrt{d^2x^2-c^2}\right)\sqrt{dx-c}\,\sqrt{dx+c}}{\sqrt{d^2x^2-c^2}\,d^2\sqrt{-c^2}}$	108

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)/x/(d*x-c)^(1/2)/(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-ln(-2*(c^2-(-c^2)^(1/2)*(d^2*x^2-c^2)^(1/2))/x)*a*d^2+b*(-c^2)^(1/2)*(d^2
*x^2-c^2)^(1/2))*(d*x-c)^(1/2)*(d*x+c)^(1/2)/(d^2*x^2-c^2)^(1/2)/d^2/(-c^2)
^(1/2)
```

Maxima [A]
time = 0.49, size = 37, normalized size = 0.66

$$-\frac{a \arcsin\left(\frac{c}{d|x|}\right)}{c} + \frac{\sqrt{d^2x^2 - c^2}\, b}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] -a*arcsin(c/(d*abs(x)))/c + sqrt(d^2*x^2 - c^2)*b/d^2
```

Fricas [A]
time = 2.66, size = 61, normalized size = 1.09

$$\frac{2\, a d^2 \arctan\left(-\frac{dx - \sqrt{dx + c}\,\sqrt{dx - c}}{c}\right) + \sqrt{dx + c}\,\sqrt{dx - c}\, bc}{cd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] (2*a*d^2*arctan(-(d*x - sqrt(d*x + c)*sqrt(d*x - c))/c) + sqrt(d*x + c)*sqr
t(d*x - c)*b*c)/(c*d^2)
```

Sympy [C] Result contains complex when optimal does not.
time = 28.48, size = 178, normalized size = 3.18

$$-\frac{aG_{6,6}^{5,3}\left(\begin{matrix} \frac{3}{4}, \frac{5}{4}, 1 & 1, 1, \frac{3}{2} \\ \frac{1}{2}, \frac{3}{4}, 1, \frac{5}{4}, \frac{3}{2} & 0 \end{matrix} \middle| \frac{c^2}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}c} + \frac{iaG_{6,6}^{2,6}\left(\begin{matrix} 0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1, 1 & \\ \frac{1}{4}, \frac{3}{4} & 0, \frac{1}{2}, \frac{1}{2}, 0 \end{matrix} \middle| \frac{c^2e^{2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}c} + \frac{bcG_{6,6}^{6,2}\left(\begin{matrix} -\frac{1}{4}, \frac{1}{4} & 0, 0, \frac{1}{2}, 1 \\ -\frac{1}{2}, -\frac{1}{4}, 0, \frac{1}{4}, \frac{1}{2}, 0 & \end{matrix} \middle| \frac{c^2}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d^2} + \frac{ibcG_{6,6}^{2,6}\left(\begin{matrix} -1, -\frac{3}{4}, -\frac{1}{2}, -\frac{1}{4}, 0, 1 & \\ -\frac{3}{4}, -\frac{1}{4} & -1, -\frac{1}{2}, -\frac{1}{2}, 0 \end{matrix} \middle| \frac{c^2e^{2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/x/(d*x-c)**(1/2)/(d*x+c)**(1/2),x)
```

```
[Out] -a*meijerg(((3/4, 5/4, 1), (1, 1, 3/2)), ((1/2, 3/4, 1, 5/4, 3/2), (0,)), c
**2/(d**2*x**2))/(4*pi**(3/2)*c) + I*a*meijerg(((0, 1/4, 1/2, 3/4, 1, 1), (
)), ((1/4, 3/4), (0, 1/2, 1/2, 0)), c**2*exp_polar(2*I*pi)/(d**2*x**2))/(4*
pi**(3/2)*c) + b*c*meijerg(((-1/4, 1/4), (0, 0, 1/2, 1)), ((-1/2, -1/4, 0,
1/4, 1/2, 0), ()), c**2/(d**2*x**2))/(4*pi**(3/2)*d**2) + I*b*c*meijerg(((-
1, -3/4, -1/2, -1/4, 0, 1), ()), ((-3/4, -1/4), (-1, -1/2, -1/2, 0)), c**2*
exp_polar(2*I*pi)/(d**2*x**2))/(4*pi**(3/2)*d**2)
```

Giac [A]
time = 0.62, size = 55, normalized size = 0.98

$$-\frac{2\, a \arctan\left(\frac{\left(\sqrt{dx + c} - \sqrt{dx - c}\right)^2}{2\, c}\right)}{c} + \frac{\sqrt{dx + c}\,\sqrt{dx - c}\, b}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] -2*a*arctan(1/2*(sqrt(d*x + c) - sqrt(d*x - c))^2/c)/c + sqrt(d*x + c)*sqrt
(d*x - c)*b/d^2
```

Mupad [B]
time = 3.97, size = 108, normalized size = 1.93

$$\frac{b\sqrt{c+d\,x}\ \sqrt{d\,x-c}}{d^2}-\frac{a\sqrt{-c}\ \left(\ln\left(\frac{{\left(\sqrt{c+d\,x}-\sqrt{c}\right)}^2}{{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)}^2}+1\right)-\ln\left(\frac{\sqrt{c+d\,x}-\sqrt{c}}{\sqrt{-c}-\sqrt{d\,x-c}}\right)\right)}{c^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)/(x*(c + d*x)^(1/2)*(d*x - c)^(1/2)),x)
```

```
[Out] (b*(c + d*x)^(1/2)*(d*x - c)^(1/2))/d^2 - (a*(-c)^(1/2)*(log(((c + d*x)^(1/
2) - c^(1/2))^2/((-c)^(1/2) - (d*x - c)^(1/2))^2 + 1) - log(((c + d*x)^(1/2
) - c^(1/2))/((-c)^(1/2) - (d*x - c)^(1/2)))))/c^(3/2)
```

3.364 $\int \frac{a+bx^2}{x^2\sqrt{-c+dx}\sqrt{c+dx}}\,dx$

Optimal. Leaf size=57

$$\frac{a\sqrt{-c+dx}\,\sqrt{c+dx}}{c^2x}+\frac{2b\tanh^{-1}\left(\frac{\sqrt{-c+dx}}{\sqrt{c+dx}}\right)}{d}$$

```
[Out] 2*b*arctanh((d*x-c)^(1/2)/(d*x+c)^(1/2))/d+a*(d*x-c)^(1/2)*(d*x+c)^(1/2)/c^
2/x
```

Rubi [A]
time = 0.05, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {465, 65, 223, 212}

$$\frac{a\sqrt{dx-c}\,\sqrt{c+dx}}{c^2x}+\frac{2b\tanh^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)/(x^2*Sqrt[-c + d*x]*Sqrt[c + d*x]),x]
```

```
[Out] (a*Sqrt[-c + d*x]*Sqrt[c + d*x])/(c^2*x) + (2*b*ArcTanh[Sqrt[-c + d*x]/Sqrt
[c + d*x]])/d
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 465

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[c*(e*x)^(
m + 1)*(a1 + b1*x^(n/2))^(p + 1)*((a2 + b2*x^(n/2))^(p + 1)/(a1*a2*e*(m + 1
))), x] + Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/(a1*a2*e^n*(
m + 1)), Int[(e*x)^(m + n)*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/2))^p, x], x]
/; FreeQ[{a1, b1, a2, b2, c, d, e, p}, x] && EqQ[non2, n/2] && EqQ[a2*b1 +
a1*b2, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (L
tQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{a+bx^2}{x^2\sqrt{-c+dx}\sqrt{c+dx}}\,dx &= \frac{a\sqrt{-c+dx}\sqrt{c+dx}}{c^2x}+b\int \frac{1}{\sqrt{-c+dx}\sqrt{c+dx}}\,dx\\
&= \frac{a\sqrt{-c+dx}\sqrt{c+dx}}{c^2x}+\frac{(2b)\text{Subst}\left(\int \frac{1}{\sqrt{2c+x^2}}\,dx,x,\sqrt{-c+dx}\right)}{d}\\
&= \frac{a\sqrt{-c+dx}\sqrt{c+dx}}{c^2x}+\frac{(2b)\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\frac{\sqrt{-c+dx}}{\sqrt{c+dx}}\right)}{d}\\
&= \frac{a\sqrt{-c+dx}\sqrt{c+dx}}{c^2x}+\frac{2b\tanh^{-1}\left(\frac{\sqrt{-c+dx}}{\sqrt{c+dx}}\right)}{d}
\end{aligned}
$$

Mathematica [A]
time = 0.10, size = 57, normalized size = 1.00

$$
\frac{a\sqrt{-c+dx}\sqrt{c+dx}}{c^2x}+\frac{2b\tanh^{-1}\left(\frac{\sqrt{-c+dx}}{\sqrt{c+dx}}\right)}{d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)/(x^2*Sqrt[-c + d*x]*Sqrt[c + d*x]),x]
```

```
[Out] (a*Sqrt[-c + d*x]*Sqrt[c + d*x])/(c^2*x) + (2*b*ArcTanh[Sqrt[-c + d*x]/Sqrt
[c + d*x]])/d
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.30, size = 97, normalized size = 1.70

method	result	size

default	$\frac{\sqrt{dx-c}\,\sqrt{dx+c}\,\left(\ln\left(\left(\sqrt{d^2x^2-c^2}\,\mathrm{csgn}(d)+dx\right)\mathrm{csgn}(d)\right)b\,c^2x+\sqrt{d^2x^2-c^2}\,\mathrm{csgn}(d)da\right)\mathrm{csgn}(d)}{c^2\sqrt{d^2x^2-c^2}\,dx}$	97
risch	$-\frac{a(-dx+c)\sqrt{dx+c}}{c^2x\sqrt{dx-c}}+\frac{b\ln\left(\frac{d^2x}{\sqrt{d^2}}+\sqrt{d^2x^2-c^2}\right)\sqrt{(dx-c)\,(dx+c)}}{\sqrt{d^2}\,\sqrt{dx-c}\,\sqrt{dx+c}}$	98

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)/x^2/(d*x-c)^(1/2)/(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (d*x-c)^(1/2)*(d*x+c)^(1/2)/c^2*(ln(((d^2*x^2-c^2)^(1/2)*csgn(d)+d*x)*csgn(
d))*b*c^2*x+(d^2*x^2-c^2)^(1/2)*csgn(d)*d*a)*csgn(d)/(d^2*x^2-c^2)^(1/2)/d/
x
```

Maxima [A]
time = 0.50, size = 55, normalized size = 0.96

$$\frac{b\log\left(2\,d^2x+2\,\sqrt{d^2x^2-c^2}\,d\right)}{d}+\frac{\sqrt{d^2x^2-c^2}\,a}{c^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^2/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] b*log(2*d^2*x + 2*sqrt(d^2*x^2 - c^2)*d)/d + sqrt(d^2*x^2 - c^2)*a/(c^2*x)
```

Fricas [A]
time = 2.18, size = 68, normalized size = 1.19

$$-\frac{bc^2x\log\left(-dx+\sqrt{dx+c}\,\sqrt{dx-c}\right)-ad^2x-\sqrt{dx+c}\,\sqrt{dx-c}\,ad}{c^2dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^2/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] -(b*c^2*x*log(-d*x + sqrt(d*x + c)*sqrt(d*x - c)) - a*d^2*x - sqrt(d*x + c)
*sqrt(d*x - c)*a*d)/(c^2*d*x)
```

Sympy [C] Result contains complex when optimal does not.
time = 27.99, size = 165, normalized size = 2.89

$$-\frac{adG_{6,6}^{5,3}\left(\begin{matrix}\frac{5}{4},\frac{7}{4},1 & \frac{3}{2},\frac{3}{2},2\\ 1,\frac{5}{4},\frac{3}{2},\frac{7}{4},2 & 0\end{matrix}\middle|\frac{c^2}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}c^2}-\frac{iadG_{6,6}^{2,6}\left(\begin{matrix}\frac{1}{2},\frac{3}{4},1,\frac{5}{4},\frac{3}{2},1 & \\ \frac{3}{4},\frac{5}{4} & \frac{1}{2},1,1,0\end{matrix}\middle|\frac{c^2e^{2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}c^2}+\frac{bG_{6,6}^{6,2}\left(\begin{matrix}\frac{1}{4},\frac{3}{4} & \frac{1}{2},\frac{1}{2},1,1\\ 0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,0 & \end{matrix}\middle|\frac{c^2}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d}-\frac{ibG_{6,6}^{2,6}\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},1 & \\ -\frac{1}{4},\frac{1}{4} & -\frac{1}{2},0,0,0\end{matrix}\middle|\frac{c^2e^{2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/x**2/(d*x-c)**(1/2)/(d*x+c)**(1/2),x)
```

```
[Out] -a*d*meijerg(((5/4, 7/4, 1), (3/2, 3/2, 2)), ((1, 5/4, 3/2, 7/4, 2), (0,)),
 c**2/(d**2*x**2))/(4*pi**(3/2)*c**2) - I*a*d*meijerg(((1/2, 3/4, 1, 5/4, 3
/2, 1), ()), ((3/4, 5/4), (1/2, 1, 1, 0)), c**2*exp_polar(2*I*pi)/(d**2*x**
2))/(4*pi**(3/2)*c**2) + b*meijerg(((1/4, 3/4), (1/2, 1/2, 1, 1)), ((0, 1/4
, 1/2, 3/4, 1, 0), ()), c**2/(d**2*x**2))/(4*pi**(3/2)*d) - I*b*meijerg(((-
1/2, -1/4, 0, 1/4, 1/2, 1), ()), ((-1/4, 1/4), (-1/2, 0, 0, 0)), c**2*exp_p
olar(2*I*pi)/(d**2*x**2))/(4*pi**(3/2)*d)
```

Giac [A]
time = 0.67, size = 66, normalized size = 1.16

$$\frac{\frac{16\,ad^2}{\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^4+4\,c^2}-b\log\left(\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^4\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^2/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*(16*a*d^2/((sqrt(d*x + c) - sqrt(d*x - c))^4 + 4*c^2) - b*log((sqrt(d*x
 + c) - sqrt(d*x - c))^4))/d
```

Mupad [B]
time = 2.94, size = 77, normalized size = 1.35

$$\frac{4\,b\,\mathrm{atan}\left(\frac{d\left(\sqrt{-c}-\sqrt{d\,x-c}\right)}{\sqrt{-d^2}\,\left(\sqrt{c+d\,x}-\sqrt{c}\right)}\right)}{\sqrt{-d^2}}+\frac{a\,\sqrt{c+d\,x}\,\sqrt{d\,x-c}}{c^2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)/(x^2*(c + d*x)^(1/2)*(d*x - c)^(1/2)),x)
```

```
[Out] (4*b*atan((d*((-c)^(1/2) - (d*x - c)^(1/2)))/((-d^2)^(1/2)*((c + d*x)^(1/2)
 - c^(1/2)))))/(-d^2)^(1/2) + (a*(c + d*x)^(1/2)*(d*x - c)^(1/2))/(c^2*x)
```

3.365 $\int \frac{a+bx^2}{x^3\sqrt{-c+dx}\,\sqrt{c+dx}}\,dx$

Optimal. Leaf size=76

$$\frac{a\sqrt{-c+dx}\,\sqrt{c+dx}}{2c^2x^2}+\frac{(2bc^2+ad^2)\tan^{-1}\left(\frac{\sqrt{-c+dx}\,\sqrt{c+dx}}{c}\right)}{2c^3}$$

```
[Out] 1/2*(a*d^2+2*b*c^2)*arctan((d*x-c)^(1/2)*(d*x+c)^(1/2)/c)/c^3+1/2*a*(d*x-c)
^(1/2)*(d*x+c)^(1/2)/c^2/x^2
```

Rubi [A]
time = 0.05, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {465, 94, 211}

$$\frac{(ad^2+2bc^2)\,\text{ArcTan}\left(\frac{\sqrt{dx-c}\,\sqrt{c+dx}}{c}\right)}{2c^3}+\frac{a\sqrt{dx-c}\,\sqrt{c+dx}}{2c^2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)/(x^3*Sqrt[-c + d*x]*Sqrt[c + d*x]),x]
```

```
[Out] (a*Sqrt[-c + d*x]*Sqrt[c + d*x])/(2*c^2*x^2) + ((2*b*c^2 + a*d^2)*ArcTan[(S
qrt[-c + d*x]*Sqrt[c + d*x])/c])/(2*c^3)
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 465

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[c*(e*x)^(
m + 1)*(a1 + b1*x^(n/2))^(p + 1)*((a2 + b2*x^(n/2))^(p + 1)/(a1*a2*e*(m + 1
))), x] + Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/(a1*a2*e^n*(
m + 1)), Int[(e*x)^(m + n)*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/2))^p, x], x]
/; FreeQ[{a1, b1, a2, b2, c, d, e, p}, x] && EqQ[non2, n/2] && EqQ[a2*b1 +
```

```
a1*b2, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (L
tQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

Rubi steps

$$\int \frac{a+bx^2}{x^3\sqrt{-c+dx}\,\sqrt{c+dx}}\,dx = \frac{a\sqrt{-c+dx}\,\sqrt{c+dx}}{2c^2x^2} + \frac{1}{2}\left(2b+\frac{ad^2}{c^2}\right)\int \frac{1}{x\sqrt{-c+dx}\,\sqrt{c+dx}}\,dx$$

$$= \frac{a\sqrt{-c+dx}\,\sqrt{c+dx}}{2c^2x^2} + \frac{1}{2}\left(d\left(2b+\frac{ad^2}{c^2}\right)\right)\text{Subst}\left(\int \frac{1}{c^2d+dx^2}\,dx, x, \sqrt{-c\right.$$

$$= \frac{a\sqrt{-c+dx}\,\sqrt{c+dx}}{2c^2x^2} + \frac{(2bc^2+ad^2)\tan^{-1}\left(\frac{\sqrt{-c+dx}\,\sqrt{c+dx}}{c}\right)}{2c^3}$$

Mathematica [A]
time = 0.11, size = 70, normalized size = 0.92

$$\frac{\frac{ac\sqrt{-c+dx}\,\sqrt{c+dx}}{x^2} + 2(2bc^2+ad^2)\tan^{-1}\left(\frac{\sqrt{-c+dx}}{\sqrt{c+dx}}\right)}{2c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)/(x^3*Sqrt[-c + d*x]*Sqrt[c + d*x]),x]
```

```
[Out] ((a*c*Sqrt[-c + d*x]*Sqrt[c + d*x])/x^2 + 2*(2*b*c^2 + a*d^2)*ArcTan[Sqrt[-
c + d*x]/Sqrt[c + d*x]])/(2*c^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 157 vs. 2(64) = 128.
time = 0.30, size = 158, normalized size = 2.08

method	result
default	$-\frac{\sqrt{dx-c}\,\sqrt{dx+c}\left(\ln\left(-\frac{2\left(c^2-\sqrt{-c^2}\,\sqrt{d^2x^2-c^2}\right)}{x}\right)a\,d^2x^2+2\ln\left(-\frac{2\left(c^2-\sqrt{-c^2}\,\sqrt{d^2x^2-c^2}\right)}{x}\right)b\,c^2x^2\right.}{2c^2\sqrt{d^2x^2-c^2}\,x^2\sqrt{-c^2}}$
risch	$-\frac{a(-dx+c)\sqrt{dx+c}}{2c^2x^2\sqrt{dx-c}} + \frac{\left(-\frac{\ln\left(\frac{-2c^2+2\sqrt{-c^2}\,\sqrt{d^2x^2-c^2}}{x}\right)a\,d^2}{2c^2\sqrt{-c^2}} - \frac{\ln\left(\frac{-2c^2+2\sqrt{-c^2}\,\sqrt{d^2x^2-c^2}}{x}\right)b}{\sqrt{-c^2}}\right)\sqrt{(dx-}}{\sqrt{dx-c}\,\sqrt{dx+c}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)/x^3/(d*x-c)^(1/2)/(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(d*x-c)^(1/2)*(d*x+c)^(1/2)/c^2*(ln(-2*(c^2-(-c^2)^(1/2)*(d^2*x^2-c^2)
^(1/2))/x)*a*d^2*x^2+2*ln(-2*(c^2-(-c^2)^(1/2)*(d^2*x^2-c^2)^(1/2))/x)*b*c^
2*x^2-(d^2*x^2-c^2)^(1/2)*(-c^2)^(1/2)*a)/(d^2*x^2-c^2)^(1/2)/x^2/(-c^2)^(1
/2)
```

Maxima [A]
time = 0.49, size = 60, normalized size = 0.79

$$-\frac{b\arcsin\left(\frac{c}{d|x|}\right)}{c}-\frac{ad^2\arcsin\left(\frac{c}{d|x|}\right)}{2\,c^3}+\frac{\sqrt{d^2x^2-c^2}\,a}{2\,c^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^3/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] -b*arcsin(c/(d*abs(x)))/c - 1/2*a*d^2*arcsin(c/(d*abs(x)))/c^3 + 1/2*sqrt(d
^2*x^2 - c^2)*a/(c^2*x^2)
```

Fricas [A]
time = 2.41, size = 73, normalized size = 0.96

$$\frac{2\,(2\,bc^2+ad^2)x^2\arctan\left(-\frac{dx-\sqrt{dx+c}\,\sqrt{dx-c}}{c}\right)+\sqrt{dx+c}\,\sqrt{dx-c}\,ac}{2\,c^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^3/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(2*(2*b*c^2 + a*d^2)*x^2*arctan(-(d*x - sqrt(d*x + c)*sqrt(d*x - c))/c)
 + sqrt(d*x + c)*sqrt(d*x - c)*a*c)/(c^3*x^2)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/x**3/(d*x-c)**(1/2)/(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 141 vs. $2(64) = 128$.
time = 0.60, size = 141, normalized size = 1.86

$$-\frac{\frac{(2\,bc^2d+ad^3)\arctan\left(\frac{\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^2}{2\,c}\right)}{c^3}+\frac{2\left(ad^3\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^6-4\,ac^2d^3\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^2\right)}{\left(\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^4+4\,c^2\right)^2c^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^3/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] -((2*b*c^2*d + a*d^3)*arctan(1/2*(sqrt(d*x + c) - sqrt(d*x - c))^2/c)/c^3 +
 2*(a*d^3*(sqrt(d*x + c) - sqrt(d*x - c))^6 - 4*a*c^2*d^3*(sqrt(d*x + c) -
sqrt(d*x - c))^2)/(((sqrt(d*x + c) - sqrt(d*x - c))^4 + 4*c^2)^2*c^2))/d
```

Mupad [B]
time = 7.50, size = 457, normalized size = 6.01

$$\frac{a\,{\left(-c\right)}^{3/2}\,d^2\,\ln\left(\frac{{\left(\sqrt{c+d\,x}-\sqrt{c}\right)}^2}{{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)}^2}+1\right)}{2\,c^{9/2}}-\frac{b\,\sqrt{-c}\,\left(\ln\left(\frac{{\left(\sqrt{c+d\,x}-\sqrt{c}\right)}^2}{{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)}^2}+1\right)-\ln\left(\frac{\sqrt{c+d\,x}-\sqrt{c}}{\sqrt{-c}-\sqrt{d\,x-c}}\right)\right)}{c^{3/2}}-\frac{a\,{\left(-c\right)}^{3/2}\,d^2\,\ln\left(\frac{\sqrt{c+d\,x}-\sqrt{c}}{\sqrt{-c}-\sqrt{d\,x-c}}\right)}{2\,c^{9/2}}-\frac{\frac{a\,{\left(-c\right)}^{3/2}\,d^2}{32\,c^{9/2}}+\frac{a\,{\left(-c\right)}^{3/2}\,d^2\,{\left(\sqrt{c+d\,x}-\sqrt{c}\right)}^2}{16\,c^{9/2}\,{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)}^2}-\frac{15\,a\,{\left(-c\right)}^{3/2}\,d^2\,{\left(\sqrt{c+d\,x}-\sqrt{c}\right)}^4}{32\,c^{9/2}\,{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)}^4}}{\frac{{\left(\sqrt{c+d\,x}-\sqrt{c}\right)}^2}{{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)}^2}+\frac{2\,{\left(\sqrt{c+d\,x}-\sqrt{c}\right)}^4}{{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)}^4}+\frac{{\left(\sqrt{c+d\,x}-\sqrt{c}\right)}^6}{{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)}^6}}+\frac{a\,d^2\,{\left(\sqrt{c+d\,x}-\sqrt{c}\right)}^2}{32\,{\left(-c\right)}^{3/2}\,c^{3/2}\,{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)/(x^3*(c + d*x)^(1/2)*(d*x - c)^(1/2)),x)
```

```
[Out] (a*(-c)^(3/2)*d^2*log(((c + d*x)^(1/2) - c^(1/2))^2/((-c)^(1/2) - (d*x - c)
^(1/2))^2 + 1))/(2*c^(9/2)) - (b*(-c)^(1/2)*(log(((c + d*x)^(1/2) - c^(1/2)
)^2/((-c)^(1/2) - (d*x - c)^(1/2))^2 + 1) - log(((c + d*x)^(1/2) - c^(1/2))
/((-c)^(1/2) - (d*x - c)^(1/2)))))/c^(3/2) - (a*(-c)^(3/2)*d^2*log(((c + d*
x)^(1/2) - c^(1/2))/((-c)^(1/2) - (d*x - c)^(1/2))))/(2*c^(9/2)) - ((a*(-c)
^(3/2)*d^2)/(32*c^(9/2)) + (a*(-c)^(3/2)*d^2*((c + d*x)^(1/2) - c^(1/2))^2)
/(16*c^(9/2)*((-c)^(1/2) - (d*x - c)^(1/2))^2) - (15*a*(-c)^(3/2)*d^2*((c +
 d*x)^(1/2) - c^(1/2))^4)/(32*c^(9/2)*((-c)^(1/2) - (d*x - c)^(1/2))^4))/((
(c + d*x)^(1/2) - c^(1/2))^2/((-c)^(1/2) - (d*x - c)^(1/2))^2 + (2*((c + d*
x)^(1/2) - c^(1/2))^4)/((-c)^(1/2) - (d*x - c)^(1/2))^4 + ((c + d*x)^(1/2)
- c^(1/2))^6/((-c)^(1/2) - (d*x - c)^(1/2))^6) + (a*d^2*((c + d*x)^(1/2) -
c^(1/2))^2)/(32*(-c)^(3/2)*c^(3/2)*((-c)^(1/2) - (d*x - c)^(1/2))^2)
```

3.366 $\int \frac{a+bx^2}{x^4\sqrt{-c+dx}\sqrt{c+dx}}\,dx$

Optimal. Leaf size=75

$$\frac{a\sqrt{-c+dx}\sqrt{c+dx}}{3c^2x^3}+\frac{(3bc^2+2ad^2)\sqrt{-c+dx}\sqrt{c+dx}}{3c^4x}$$

```
[Out] 1/3*a*(d*x-c)^(1/2)*(d*x+c)^(1/2)/c^2/x^3+1/3*(2*a*d^2+3*b*c^2)*(d*x-c)^(1/
2)*(d*x+c)^(1/2)/c^4/x
```

Rubi [A]
time = 0.05, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {465, 97}

$$\frac{\sqrt{dx-c}\sqrt{c+dx}\,(2ad^2+3bc^2)}{3c^4x}+\frac{a\sqrt{dx-c}\sqrt{c+dx}}{3c^2x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)/(x^4*Sqrt[-c + d*x]*Sqrt[c + d*x]),x]
```

```
[Out] (a*Sqrt[-c + d*x]*Sqrt[c + d*x])/(3*c^2*x^3) + ((3*b*c^2 + 2*a*d^2)*Sqrt[-c
 + d*x]*Sqrt[c + d*x])/(3*c^4*x)
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] /; FreeQ[{a, b, c, d, e, f
, m, n, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && EqQ[a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1), 0] && NeQ[m, -1]
```

Rule 465

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[c*(e*x)^(
m + 1)*(a1 + b1*x^(n/2))^(p + 1)*((a2 + b2*x^(n/2))^(p + 1)/(a1*a2*e*(m + 1
))), x] + Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/(a1*a2*e^n*(
m + 1)), Int[(e*x)^(m + n)*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/2))^p, x], x]
/; FreeQ[{a1, b1, a2, b2, c, d, e, p}, x] && EqQ[non2, n/2] && EqQ[a2*b1 +
a1*b2, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (L
tQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\int \frac{a+bx^2}{x^4\sqrt{-c+dx}\,\sqrt{c+dx}}\,dx = \frac{a\sqrt{-c+dx}\,\sqrt{c+dx}}{3c^2x^3} + \frac{1}{3}\left(3b+\frac{2ad^2}{c^2}\right)\int \frac{1}{x^2\sqrt{-c+dx}\,\sqrt{c+dx}}\,dx$$
$$= \frac{a\sqrt{-c+dx}\,\sqrt{c+dx}}{3c^2x^3} + \frac{(3bc^2+2ad^2)\sqrt{-c+dx}\,\sqrt{c+dx}}{3c^4x}$$

Mathematica [A]
time = 0.10, size = 54, normalized size = 0.72

$$\frac{\sqrt{-c+dx}\,\sqrt{c+dx}\,(3bc^2x^2+a(c^2+2d^2x^2))}{3c^4x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)/(x^4*Sqrt[-c + d*x]*Sqrt[c + d*x]),x]
```

```
[Out] (Sqrt[-c + d*x]*Sqrt[c + d*x]*(3*b*c^2*x^2 + a*(c^2 + 2*d^2*x^2)))/(3*c^4*x
^3)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 2.
time = 0.29, size = 53, normalized size = 0.71

method	result	size
gosper	$\frac{\sqrt{dx+c}\,(2a\,d^2x^2+3b\,c^2x^2+c^2a)\sqrt{dx-c}}{3x^3c^4}$	49
default	$\frac{\sqrt{dx-c}\,\sqrt{dx+c}\,\mathrm{csgn}(d)^2(2a\,d^2x^2+3b\,c^2x^2+c^2a)}{3c^4x^3}$	53
risch	$-\frac{\sqrt{dx+c}\,(-dx+c)(2a\,d^2x^2+3b\,c^2x^2+c^2a)}{3x^3c^4\sqrt{dx-c}}$	55

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)/x^4/(d*x-c)^(1/2)/(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(d*x-c)^(1/2)*(d*x+c)^(1/2)*csgn(d)^2/c^4*(2*a*d^2*x^2+3*b*c^2*x^2+a*c^
2)/x^3
```

Maxima [A]
time = 0.48, size = 75, normalized size = 1.00

$$\frac{\sqrt{d^2x^2-c^2}\,b}{c^2x} + \frac{2\,\sqrt{d^2x^2-c^2}\,ad^2}{3\,c^4x} + \frac{\sqrt{d^2x^2-c^2}\,a}{3\,c^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^4/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(d^2*x^2 - c^2)*b/(c^2*x) + 2/3*sqrt(d^2*x^2 - c^2)*a*d^2/(c^4*x) + 1/3
*sqrt(d^2*x^2 - c^2)*a/(c^2*x^3)
```

Fricas [A]
time = 2.61, size = 67, normalized size = 0.89

$$\frac{(3\,bc^2d + 2\,ad^3)x^3 + (ac^2 + (3\,bc^2 + 2\,ad^2)x^2)\sqrt{dx + c}\,\sqrt{dx - c}}{3\,c^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^4/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*((3*b*c^2*d + 2*a*d^3)*x^3 + (a*c^2 + (3*b*c^2 + 2*a*d^2)*x^2)*sqrt(d*x
 + c)*sqrt(d*x - c))/(c^4*x^3)
```

Sympy [C] Result contains complex when optimal does not.
time = 32.12, size = 170, normalized size = 2.27

$$-\frac{ad^3G^{5,3}_{6,6}\left(\begin{matrix}\frac{9}{4},\frac{11}{4},1 & \frac{5}{2},\frac{5}{2},3\\ 2,\frac{9}{4},\frac{5}{2},\frac{11}{4},3 & 0\end{matrix}\middle|\frac{c^2}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}c^4} - \frac{iad^3G^{2,6}_{6,6}\left(\begin{matrix}\frac{3}{2},\frac{7}{4},2,\frac{9}{4},\frac{5}{2},1 & \\ \frac{7}{4},\frac{9}{4} & \frac{3}{2},2,2,0\end{matrix}\middle|\frac{c^2e^{2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}c^4} - \frac{bdG^{5,3}_{6,6}\left(\begin{matrix}\frac{5}{4},\frac{7}{4},1 & \frac{3}{2},\frac{3}{2},2\\ 1,\frac{5}{4},\frac{3}{2},\frac{7}{4},2 & 0\end{matrix}\middle|\frac{c^2}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}c^2} - \frac{ibdG^{2,6}_{6,6}\left(\begin{matrix}\frac{1}{2},\frac{3}{4},1,\frac{5}{4},\frac{3}{2},1 & \\ \frac{3}{4},\frac{5}{4} & \frac{1}{2},1,1,0\end{matrix}\middle|\frac{c^2e^{2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/x**4/(d*x-c)**(1/2)/(d*x+c)**(1/2),x)
```

```
[Out] -a*d**3*meijerg(((9/4, 11/4, 1), (5/2, 5/2, 3)), ((2, 9/4, 5/2, 11/4, 3), (
0,)), c**2/(d**2*x**2))/(4*pi**(3/2)*c**4) - I*a*d**3*meijerg(((3/2, 7/4, 2
, 9/4, 5/2, 1), ()), ((7/4, 9/4), (3/2, 2, 2, 0)), c**2*exp_polar(2*I*pi)/(
d**2*x**2))/(4*pi**(3/2)*c**4) - b*d*meijerg(((5/4, 7/4, 1), (3/2, 3/2, 2))
, ((1, 5/4, 3/2, 7/4, 2), (0,)), c**2/(d**2*x**2))/(4*pi**(3/2)*c**2) - I*b
*d*meijerg(((1/2, 3/4, 1, 5/4, 3/2, 1), ()), ((3/4, 5/4), (1/2, 1, 1, 0)),
c**2*exp_polar(2*I*pi)/(d**2*x**2))/(4*pi**(3/2)*c**2)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 137 vs. 2(63) = 126.
time = 0.57, size = 137, normalized size = 1.83

$$\frac{8\left(3\,bd^2\left(\sqrt{dx+c} - \sqrt{dx-c}\right)^8 + 24\,bc^2d^2\left(\sqrt{dx+c} - \sqrt{dx-c}\right)^4 + 24\,ad^4\left(\sqrt{dx+c} - \sqrt{dx-c}\right)^4 + 48\,bc^4d^2 + 32\,ac^2d^4\right)}{3\left(\left(\sqrt{dx+c} - \sqrt{dx-c}\right)^4 + 4\,c^2\right)^3 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^4/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 8/3*(3*b*d^2*(sqrt(d*x + c) - sqrt(d*x - c))^8 + 24*b*c^2*d^2*(sqrt(d*x + c
) - sqrt(d*x - c))^4 + 24*a*d^4*(sqrt(d*x + c) - sqrt(d*x - c))^4 + 48*b*c^
4*d^2 + 32*a*c^2*d^4)/(((sqrt(d*x + c) - sqrt(d*x - c))^4 + 4*c^2)^3*d)
```

Mupad [B]
time = 2.77, size = 79, normalized size = 1.05

$$\frac{\sqrt{d\,x-c}\,\left(\frac{a}{3\,c}+\frac{x^2\,\left(3\,b\,c^3+2\,a\,c\,d^2\right)}{3\,c^4}+\frac{x^3\,\left(3\,b\,c^2\,d+2\,a\,d^3\right)}{3\,c^4}+\frac{a\,d\,x}{3\,c^2}\right)}{x^3\,\sqrt{c+d\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)/(x^4*(c + d*x)^(1/2)*(d*x - c)^(1/2)),x)
```

```
[Out] ((d*x - c)^(1/2)*(a/(3*c) + (x^2*(3*b*c^3 + 2*a*c*d^2))/(3*c^4) + (x^3*(2*a
*d^3 + 3*b*c^2*d))/(3*c^4) + (a*d*x)/(3*c^2)))/(x^3*(c + d*x)^(1/2))
```

3.367 $\int \frac{a+bx^2}{x^5\sqrt{-c+dx}\sqrt{c+dx}}\,dx$

Optimal. Leaf size=123

$$\frac{a\sqrt{-c+dx}\sqrt{c+dx}}{4c^2x^4}+\frac{(4bc^2+3ad^2)\sqrt{-c+dx}\sqrt{c+dx}}{8c^4x^2}+\frac{d^2(4bc^2+3ad^2)\tan^{-1}\left(\frac{\sqrt{-c+dx}\sqrt{c+dx}}{c}\right)}{8c^5}$$

```
[Out] 1/8*d^2*(3*a*d^2+4*b*c^2)*arctan((d*x-c)^(1/2)*(d*x+c)^(1/2)/c)/c^5+1/4*a*(
d*x-c)^(1/2)*(d*x+c)^(1/2)/c^2/x^4+1/8*(3*a*d^2+4*b*c^2)*(d*x-c)^(1/2)*(d*x
+c)^(1/2)/c^4/x^2
```

Rubi [A]
time = 0.06, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {465, 105, 12, 94, 211}

$$\frac{d^2(3ad^2+4bc^2)\,\text{ArcTan}\left(\frac{\sqrt{dx-c}\sqrt{c+dx}}{c}\right)}{8c^5}+\frac{\sqrt{dx-c}\sqrt{c+dx}\,(3ad^2+4bc^2)}{8c^4x^2}+\frac{a\sqrt{dx-c}\sqrt{c+dx}}{4c^2x^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)/(x^5*Sqrt[-c + d*x]*Sqrt[c + d*x]),x]
```

```
[Out] (a*Sqrt[-c + d*x]*Sqrt[c + d*x])/(4*c^2*x^4) + ((4*b*c^2 + 3*a*d^2)*Sqrt[-c
 + d*x]*Sqrt[c + d*x])/(8*c^4*x^2) + (d^2*(4*b*c^2 + 3*a*d^2)*ArcTan[(Sqrt[
-c + d*x]*Sqrt[c + d*x])/c])/(8*c^5)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
```

```
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 465

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[c*(e*x)^(
m + 1)*(a1 + b1*x^(n/2))^(p + 1)*((a2 + b2*x^(n/2))^(p + 1)/(a1*a2*e*(m + 1
))), x] + Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/(a1*a2*e^n*(
m + 1)), Int[(e*x)^(m + n)*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/2))^p, x], x]
/; FreeQ[{a1, b1, a2, b2, c, d, e, p}, x] && EqQ[non2, n/2] && EqQ[a2*b1 +
a1*b2, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (L
tQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

Rubi steps

$$\int \frac{a+bx^2}{x^5\sqrt{-c+dx}\,\sqrt{c+dx}}\,dx = \frac{a\sqrt{-c+dx}\,\sqrt{c+dx}}{4c^2x^4} + \frac{1}{4}\left(4b+\frac{3ad^2}{c^2}\right)\int \frac{1}{x^3\sqrt{-c+dx}\,\sqrt{c+dx}}\,dx$$

$$= \frac{a\sqrt{-c+dx}\,\sqrt{c+dx}}{4c^2x^4} + \frac{(4bc^2+3ad^2)\sqrt{-c+dx}\,\sqrt{c+dx}}{8c^4x^2} + \frac{(4bc^2+3ad^2}{}$$

$$= \frac{a\sqrt{-c+dx}\,\sqrt{c+dx}}{4c^2x^4} + \frac{(4bc^2+3ad^2)\sqrt{-c+dx}\,\sqrt{c+dx}}{8c^4x^2} + \frac{(d^2(4bc^2+3}{}$$

$$= \frac{a\sqrt{-c+dx}\,\sqrt{c+dx}}{4c^2x^4} + \frac{(4bc^2+3ad^2)\sqrt{-c+dx}\,\sqrt{c+dx}}{8c^4x^2} + \frac{(d^3(4bc^2+3}{}$$

$$= \frac{a\sqrt{-c+dx}\,\sqrt{c+dx}}{4c^2x^4} + \frac{(4bc^2+3ad^2)\sqrt{-c+dx}\,\sqrt{c+dx}}{8c^4x^2} + \frac{d^2(4bc^2+3a}{}$$

Mathematica [A]
time = 0.18, size = 101, normalized size = 0.82

$$\frac{c\sqrt{-c+dx}\,\sqrt{c+dx}\,(2ac^2+4bc^2x^2+3ad^2x^2)+2d^2(4bc^2+3ad^2)\,x^4\tan^{-1}\left(\frac{\sqrt{-c+dx}}{\sqrt{c+dx}}\right)}{8c^5x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)/(x^5*Sqrt[-c + d*x]*Sqrt[c + d*x]),x]
```

```
[Out] (c*Sqrt[-c + d*x]*Sqrt[c + d*x]*(2*a*c^2 + 4*b*c^2*x^2 + 3*a*d^2*x^2) + 2*d
^2*(4*b*c^2 + 3*a*d^2)*x^4*ArcTan[Sqrt[-c + d*x]/Sqrt[c + d*x]])/(8*c^5*x^4
)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 226 vs. $2(105) = 210$.
time = 0.30, size = 227, normalized size = 1.85

method	result
risch	$-\frac{\sqrt{dx+c}\,(-dx+c)(3a\,d^2x^2+4b\,c^2x^2+2c^2a)}{8c^4x^4\sqrt{dx-c}}+\frac{\left(-\frac{3d^4\ln\left(\frac{-2c^2+2\sqrt{-c^2}\,\sqrt{d^2x^2-c^2}}{x}\right)a}{8c^4\sqrt{-c^2}}-\frac{d^2\ln\left(\frac{-2c^2+2\sqrt{-c^2}\,\sqrt{}}{x}\right.}{2c^2\sqrt{-c^2}}\right.}{\sqrt{dx-c}\,\sqrt{dx+c}}$
default	$-\frac{\sqrt{dx-c}\,\sqrt{dx+c}\,\left(3\ln\left(-\frac{2\left(c^2-\sqrt{-c^2}\,\sqrt{d^2x^2-c^2}\right)}{x}\right)a\,d^4x^4+4\ln\left(-\frac{2\left(c^2-\sqrt{-c^2}\,\sqrt{d^2x^2-c^2}\right)}{x}\right)b\,c^2\right.}{8c^4\sqrt{d^2x^2-c^2}\,x^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)/x^5/(d*x-c)^(1/2)/(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*(d*x-c)^(1/2)*(d*x+c)^(1/2)/c^4*(3*ln(-2*(c^2-(-c^2)^(1/2)*(d^2*x^2-c^
2)^(1/2))/x)*a*d^4*x^4+4*ln(-2*(c^2-(-c^2)^(1/2)*(d^2*x^2-c^2)^(1/2))/x)*b*
c^2*d^2*x^4-3*(d^2*x^2-c^2)^(1/2)*(-c^2)^(1/2)*a*d^2*x^2-4*(d^2*x^2-c^2)^(1
/2)*(-c^2)^(1/2)*b*c^2*x^2-2*(d^2*x^2-c^2)^(1/2)*(-c^2)^(1/2)*a*c^2)/(d^2*x
^2-c^2)^(1/2)/x^4/(-c^2)^(1/2)
```

Maxima [A]
time = 0.53, size = 114, normalized size = 0.93

$$-\frac{bd^2\arcsin\left(\frac{c}{d|x|}\right)}{2\,c^3}-\frac{3\,ad^4\arcsin\left(\frac{c}{d|x|}\right)}{8\,c^5}+\frac{\sqrt{d^2x^2-c^2}\,b}{2\,c^2x^2}+\frac{3\,\sqrt{d^2x^2-c^2}\,ad^2}{8\,c^4x^2}+\frac{\sqrt{d^2x^2-c^2}\,a}{4\,c^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^5/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*b*d^2*arcsin(c/(d*abs(x)))/c^3 - 3/8*a*d^4*arcsin(c/(d*abs(x)))/c^5 +
1/2*sqrt(d^2*x^2 - c^2)*b/(c^2*x^2) + 3/8*sqrt(d^2*x^2 - c^2)*a*d^2/(c^4*x^
2) + 1/4*sqrt(d^2*x^2 - c^2)*a/(c^2*x^4)
```

Fricas [A]
time = 2.35, size = 100, normalized size = 0.81

$$\frac{2\,(4\,bc^2d^2+3\,ad^4)x^4\arctan\left(-\frac{dx-\sqrt{dx+c}\,\sqrt{dx-c}}{c}\right)+(2\,ac^3+(4\,bc^3+3\,acd^2)x^2)\sqrt{dx+c}\,\sqrt{dx-c}}{8\,c^5x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^5/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/8*(2*(4*b*c^2*d^2 + 3*a*d^4)*x^4*arctan(-(d*x - sqrt(d*x + c)*sqrt(d*x -
c))/c) + (2*a*c^3 + (4*b*c^3 + 3*a*c*d^2)*x^2)*sqrt(d*x + c)*sqrt(d*x - c))
/(c^5*x^4)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/x**5/(d*x-c)**(1/2)/(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 325 vs. 2(105) = 210.
time = 0.58, size = 325, normalized size = 2.64

$$-\frac{\frac{(4\,bc^2d^3+3\,ad^5)\arctan\left(\frac{\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^2}{2\,c}\right)}{c^5}+\frac{2\left(4\,bc^2d^3\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^{14}+3\,ad^5\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^{14}+16\,bc^4d^3\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^{10}+44\,ac^2d^5\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^{10}-64\,bc^6d^3\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^{6}-176\,ac^4d^5\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^{6}-256\,bc^8d^3\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^{2}-192\,ac^6d^5\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^{2}\right)}{\left(\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^4+4\,c^2\right)^4c^4}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^5/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/4*((4*b*c^2*d^3 + 3*a*d^5)*arctan(1/2*(sqrt(d*x + c) - sqrt(d*x - c))^2/
c)/c^5 + 2*(4*b*c^2*d^3*(sqrt(d*x + c) - sqrt(d*x - c))^14 + 3*a*d^5*(sqrt(
d*x + c) - sqrt(d*x - c))^14 + 16*b*c^4*d^3*(sqrt(d*x + c) - sqrt(d*x - c))
^10 + 44*a*c^2*d^5*(sqrt(d*x + c) - sqrt(d*x - c))^10 - 64*b*c^6*d^3*(sqrt(
d*x + c) - sqrt(d*x - c))^6 - 176*a*c^4*d^5*(sqrt(d*x + c) - sqrt(d*x - c))
^6 - 256*b*c^8*d^3*(sqrt(d*x + c) - sqrt(d*x - c))^2 - 192*a*c^6*d^5*(sqrt(
d*x + c) - sqrt(d*x - c))^2)/(((sqrt(d*x + c) - sqrt(d*x - c))^4 + 4*c^2)^4
*c^4))/d
```

Mupad [B]
time = 19.13, size = 1005, normalized size = 8.17

$$\frac{3\,a\,\sqrt{-c}\,d^4\ln\left(\frac{\sqrt{c+d\,x}-\sqrt{c}}{\sqrt{-c}-\sqrt{d\,x-c}}\right)}{8\,c^{11/2}}-\frac{\frac{b\,(-c)^{3/2}\,d^2}{32\,c^{9/2}}+\frac{b\,(-c)^{3/2}\,d^2\left(\sqrt{c+d\,x}-\sqrt{c}\right)^2}{16\,c^{9/2}\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^2}-\frac{15\,b\,(-c)^{3/2}\,d^2\left(\sqrt{c+d\,x}-\sqrt{c}\right)^4}{32\,c^{9/2}\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^4}}{\frac{\left(\sqrt{c+d\,x}-\sqrt{c}\right)^2}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^2}+\frac{2\left(\sqrt{c+d\,x}-\sqrt{c}\right)^4}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^4}+\frac{\left(\sqrt{c+d\,x}-\sqrt{c}\right)^6}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^6}}-\frac{\frac{a\,\sqrt{-c}\,d^4}{1024\,c^{11/2}}-\frac{3\,a\,\sqrt{-c}\,d^4\left(\sqrt{c+d\,x}-\sqrt{c}\right)^2}{128\,c^{11/2}\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^2}-\frac{53\,a\,\sqrt{-c}\,d^4\left(\sqrt{c+d\,x}-\sqrt{c}\right)^4}{512\,c^{11/2}\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^4}+\frac{87\,a\,\sqrt{-c}\,d^4\left(\sqrt{c+d\,x}-\sqrt{c}\right)^6}{256\,c^{11/2}\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^6}+\frac{657\,a\,\sqrt{-c}\,d^4\left(\sqrt{c+d\,x}-\sqrt{c}\right)^8}{1024\,c^{11/2}\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^8}+\frac{121\,a\,\sqrt{-c}\,d^4\left(\sqrt{c+d\,x}-\sqrt{c}\right)^{10}}{256\,c^{11/2}\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^{10}}}{\frac{\left(\sqrt{c+d\,x}-\sqrt{c}\right)^4}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^4}+\frac{4\left(\sqrt{c+d\,x}-\sqrt{c}\right)^6}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^6}+\frac{6\left(\sqrt{c+d\,x}-\sqrt{c}\right)^8}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^8}+\frac{4\left(\sqrt{c+d\,x}-\sqrt{c}\right)^{10}}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^{10}}+\frac{\left(\sqrt{c+d\,x}-\sqrt{c}\right)^{12}}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^{12}}}-\frac{b\,(-c)^{3/2}\,d^2\ln\left(\frac{\sqrt{c+d\,x}-\sqrt{c}}{\sqrt{-c}-\sqrt{d\,x-c}}\right)}{2\,c^{9/2}}-\frac{3\,a\,\sqrt{-c}\,d^4\ln\left(\frac{\left(\sqrt{c+d\,x}-\sqrt{c}\right)^2}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^2}+1\right)}{8\,c^{11/2}}+\frac{b\,(-c)^{3/2}\,d^2\ln\left(\frac{\left(\sqrt{c+d\,x}-\sqrt{c}\right)^2}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^2}+1\right)}{2\,c^{9/2}}-\frac{7\,a\,d^4\left(\sqrt{c+d\,x}-\sqrt{c}\right)^2}{256\,\sqrt{-c}\,c^{9/2}\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^2}+\frac{a\,d^4\left(\sqrt{c+d\,x}-\sqrt{c}\right)^4}{1024\,\sqrt{-c}\,c^{9/2}\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^4}+\frac{b\,d^2\left(\sqrt{c+d\,x}-\sqrt{c}\right)^2}{32\,(-c)^{3/2}\,c^{3/2}\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)/(x^5*(c + d*x)^(1/2)*(d*x - c)^(1/2)),x)
```

```
[Out] (3*a*(-c)^(1/2)*d^4*log(((c + d*x)^(1/2) - c^(1/2))/((-c)^(1/2) - (d*x - c)
^(1/2))))/(8*c^(11/2)) - ((b*(-c)^(3/2)*d^2)/(32*c^(9/2)) + (b*(-c)^(3/2)*d
^2*((c + d*x)^(1/2) - c^(1/2))^2)/(16*c^(9/2)*((-c)^(1/2) - (d*x - c)^(1/2)
)^2) - (15*b*(-c)^(3/2)*d^2*((c + d*x)^(1/2) - c^(1/2))^4)/(32*c^(9/2)*((-c
)^(1/2) - (d*x - c)^(1/2))^4))/(((c + d*x)^(1/2) - c^(1/2))^2/((-c)^(1/2) -
 (d*x - c)^(1/2))^2 + (2*((c + d*x)^(1/2) - c^(1/2))^4)/((-c)^(1/2) - (d*x
- c)^(1/2))^4 + ((c + d*x)^(1/2) - c^(1/2))^6/((-c)^(1/2) - (d*x - c)^(1/2)
)^6) - ((a*(-c)^(1/2)*d^4)/(1024*c^(11/2)) - (3*a*(-c)^(1/2)*d^4*((c + d*x)
^(1/2) - c^(1/2))^2)/(128*c^(11/2)*((-c)^(1/2) - (d*x - c)^(1/2))^2) - (53*
a*(-c)^(1/2)*d^4*((c + d*x)^(1/2) - c^(1/2))^4)/(512*c^(11/2)*((-c)^(1/2) -
 (d*x - c)^(1/2))^4) + (87*a*(-c)^(1/2)*d^4*((c + d*x)^(1/2) - c^(1/2))^6)/
(256*c^(11/2)*((-c)^(1/2) - (d*x - c)^(1/2))^6) + (657*a*(-c)^(1/2)*d^4*((c
 + d*x)^(1/2) - c^(1/2))^8)/(1024*c^(11/2)*((-c)^(1/2) - (d*x - c)^(1/2))^8
) + (121*a*(-c)^(1/2)*d^4*((c + d*x)^(1/2) - c^(1/2))^10)/(256*c^(11/2)*((-
c)^(1/2) - (d*x - c)^(1/2))^10))/(((c + d*x)^(1/2) - c^(1/2))^4/((-c)^(1/2)
 - (d*x - c)^(1/2))^4 + (4*((c + d*x)^(1/2) - c^(1/2))^6)/((-c)^(1/2) - (d*
x - c)^(1/2))^6 + (6*((c + d*x)^(1/2) - c^(1/2))^8)/((-c)^(1/2) - (d*x - c)
^(1/2))^8 + (4*((c + d*x)^(1/2) - c^(1/2))^10)/((-c)^(1/2) - (d*x - c)^(1/2
))^10 + ((c + d*x)^(1/2) - c^(1/2))^12/((-c)^(1/2) - (d*x - c)^(1/2))^12) -
 (b*(-c)^(3/2)*d^2*log(((c + d*x)^(1/2) - c^(1/2))/((-c)^(1/2) - (d*x - c)^
(1/2))))/(2*c^(9/2)) - (3*a*(-c)^(1/2)*d^4*log(((c + d*x)^(1/2) - c^(1/2))^
2/((-c)^(1/2) - (d*x - c)^(1/2))^2 + 1))/(8*c^(11/2)) + (b*(-c)^(3/2)*d^2*l
og(((c + d*x)^(1/2) - c^(1/2))^2/((-c)^(1/2) - (d*x - c)^(1/2))^2 + 1))/(2*
c^(9/2)) - (7*a*d^4*((c + d*x)^(1/2) - c^(1/2))^2)/(256*(-c)^(1/2)*c^(9/2)*
((-c)^(1/2) - (d*x - c)^(1/2))^2) + (a*d^4*((c + d*x)^(1/2) - c^(1/2))^4)/(
1024*(-c)^(1/2)*c^(9/2)*((-c)^(1/2) - (d*x - c)^(1/2))^4) + (b*d^2*((c + d*
x)^(1/2) - c^(1/2))^2)/(32*(-c)^(3/2)*c^(3/2)*((-c)^(1/2) - (d*x - c)^(1/2)
)^2)
```

3.368 $\int \frac{x^4(a+bx^2)}{(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx$

Optimal. Leaf size=161

$$-\frac{(5bc^2+4ad^2)\,x^3}{4d^4\sqrt{-c+dx}\,\sqrt{c+dx}}+\frac{bx^5}{4d^2\sqrt{-c+dx}\,\sqrt{c+dx}}+\frac{3(5bc^2+4ad^2)\,x\sqrt{-c+dx}\,\sqrt{c+dx}}{8d^6}+\frac{3c^2(5bc^2+4ad^2)}{}$$

[Out] 3/4*c^2*(4*a*d^2+5*b*c^2)*arctanh((d*x-c)^(1/2)/(d*x+c)^(1/2))/d^7-1/4*(4*a*d^2+5*b*c^2)*x^3/d^4/(d*x-c)^(1/2)/(d*x+c)^(1/2)+1/4*b*x^5/d^2/(d*x-c)^(1/2)/(d*x+c)^(1/2)+3/8*(4*a*d^2+5*b*c^2)*x*(d*x-c)^(1/2)*(d*x+c)^(1/2)/d^6

Rubi [A]
time = 0.08, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.258$, Rules used = {471, 100, 21, 92, 12, 65, 223, 212}

$$\frac{3c^2(4ad^2+5bc^2)\tanh^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)}{4d^7}+\frac{3x\sqrt{dx-c}\,\sqrt{c+dx}\,(4ad^2+5bc^2)}{8d^6}-\frac{x^3(4ad^2+5bc^2)}{4d^4\sqrt{dx-c}\,\sqrt{c+dx}}+\frac{bx^5}{4d^2\sqrt{dx-c}\,\sqrt{c+dx}}$$

Antiderivative was successfully verified.

[In] Int[(x^4*(a + b*x^2))/((-c + d*x)^(3/2)*(c + d*x)^(3/2)),x]

[Out] -1/4*((5*b*c^2 + 4*a*d^2)*x^3)/(d^4*Sqrt[-c + d*x]*Sqrt[c + d*x]) + (b*x^5)/(4*d^2*Sqrt[-c + d*x]*Sqrt[c + d*x]) + (3*(5*b*c^2 + 4*a*d^2)*x*Sqrt[-c + d*x]*Sqrt[c + d*x])/(8*d^6) + (3*c^2*(5*b*c^2 + 4*a*d^2)*ArcTanh[Sqrt[-c + d*x]/Sqrt[c + d*x]])/(4*d^7)

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 92

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(
d*f*(n + p + 3))), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[d*(e*x)^(
m + 1)*(a1 + b1*x^(n/2))^(p + 1)*((a2 + b2*x^(n/2))^(p + 1)/(b1*b2*e*(m + n
*(p + 1) + 1))), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\int \frac{x^4(a+bx^2)}{(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx = \frac{bx^5}{4d^2\sqrt{-c+dx}\,\sqrt{c+dx}} - \frac{1}{4}\left(-4a-\frac{5bc^2}{d^2}\right)\int \frac{x^4}{(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx$$

$$= -\frac{(5bc^2+4ad^2)\,x^3}{4d^4\sqrt{-c+dx}\,\sqrt{c+dx}} + \frac{bx^5}{4d^2\sqrt{-c+dx}\,\sqrt{c+dx}} - \frac{\left(4a+\frac{5bc^2}{d^2}\right)\int \frac{}{\sqrt{-}}}{4c}$$

$$= -\frac{(5bc^2+4ad^2)\,x^3}{4d^4\sqrt{-c+dx}\,\sqrt{c+dx}} + \frac{bx^5}{4d^2\sqrt{-c+dx}\,\sqrt{c+dx}} + \frac{(3(5bc^2+4ad^2))\int}{}$$

$$= -\frac{(5bc^2+4ad^2)\,x^3}{4d^4\sqrt{-c+dx}\,\sqrt{c+dx}} + \frac{bx^5}{4d^2\sqrt{-c+dx}\,\sqrt{c+dx}} + \frac{3(5bc^2+4ad^2)\,x\sqrt{}}{8}$$

$$= -\frac{(5bc^2+4ad^2)\,x^3}{4d^4\sqrt{-c+dx}\,\sqrt{c+dx}} + \frac{bx^5}{4d^2\sqrt{-c+dx}\,\sqrt{c+dx}} + \frac{3(5bc^2+4ad^2)\,x\sqrt{}}{8}$$

$$= -\frac{(5bc^2+4ad^2)\,x^3}{4d^4\sqrt{-c+dx}\,\sqrt{c+dx}} + \frac{bx^5}{4d^2\sqrt{-c+dx}\,\sqrt{c+dx}} + \frac{3(5bc^2+4ad^2)\,x\sqrt{}}{8}$$

$$= -\frac{(5bc^2+4ad^2)\,x^3}{4d^4\sqrt{-c+dx}\,\sqrt{c+dx}} + \frac{bx^5}{4d^2\sqrt{-c+dx}\,\sqrt{c+dx}} + \frac{3(5bc^2+4ad^2)\,x\sqrt{}}{8}$$

$$= -\frac{(5bc^2+4ad^2)\,x^3}{4d^4\sqrt{-c+dx}\,\sqrt{c+dx}} + \frac{bx^5}{4d^2\sqrt{-c+dx}\,\sqrt{c+dx}} + \frac{3(5bc^2+4ad^2)\,x\sqrt{}}{8}$$

Mathematica [**A**]
time = 0.25, size = 137, normalized size = 0.85

$$\frac{4ad^3x(-3c^2+d^2x^2)+bdx(-15c^4+5c^2d^2x^2+2d^4x^4)+6c^2(5bc^2+4ad^2)\sqrt{-c+dx}\,\sqrt{c+dx}\,\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{-c+dx}}\right)}{8d^7\sqrt{-c+dx}\,\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(a + b*x^2))/((-c + d*x)^(3/2)*(c + d*x)^(3/2)),x]
```

```
[Out] (4*a*d^3*x*(-3*c^2 + d^2*x^2) + b*d*x*(-15*c^4 + 5*c^2*d^2*x^2 + 2*d^4*x^4)
 + 6*c^2*(5*b*c^2 + 4*a*d^2)*Sqrt[-c + d*x]*Sqrt[c + d*x]*ArcTanh[Sqrt[c +
d*x]/Sqrt[-c + d*x]])/(8*d^7*Sqrt[-c + d*x]*Sqrt[c + d*x])
```

Maple [**C**] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.32, size = 324, normalized size = 2.01

---

method	result
default	$\frac{\sqrt{dx-c}\left(-2\,\mathrm{csgn}(d) b\,d^5 x^5 \sqrt{d^2x^2-c^2}-4\,\mathrm{csgn}(d) a\,d^5 x^3 \sqrt{d^2x^2-c^2}-5\,\mathrm{csgn}(d) b\,c^2 d^3 x^3 \sqrt{d^2x^2-c^2}-12\ln\left(\left(\sqrt{\right.\right.}{}$
risch	$-\frac{x(2b\,d^2x^2+4a\,d^2+7b\,c^2)(-dx+c)\sqrt{dx+c}}{8d^6\sqrt{dx-c}}-\frac{\left(-\frac{3c^2\ln\left(\frac{d^2x}{\sqrt{d^2}}+\sqrt{d^2x^2-c^2}\right)a}{2d^4\sqrt{d^2}}-\frac{15c^4\ln\left(\frac{d^2x}{\sqrt{d^2}}+\sqrt{d^2x^2-c^2}\right)}{8d^6\sqrt{d^2}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^2+a)/(d*x-c)^(3/2)/(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*(d*x-c)^(1/2)*(-2*csgn(d)*b*d^5*x^5*(d^2*x^2-c^2)^(1/2)-4*csgn(d)*a*d^5
*x^3*(d^2*x^2-c^2)^(1/2)-5*csgn(d)*b*c^2*d^3*x^3*(d^2*x^2-c^2)^(1/2)-12*ln(
((d^2*x^2-c^2)^(1/2)*csgn(d)+d*x)*csgn(d))*a*c^2*d^4*x^2-15*ln(((d^2*x^2-c^
2)^(1/2)*csgn(d)+d*x)*csgn(d))*b*c^4*d^2*x^2+12*(d^2*x^2-c^2)^(1/2)*csgn(d)
*d^3*a*c^2*x+15*(d^2*x^2-c^2)^(1/2)*csgn(d)*d*b*c^4*x+12*ln(((d^2*x^2-c^2)^
(1/2)*csgn(d)+d*x)*csgn(d))*a*c^4*d^2+15*ln(((d^2*x^2-c^2)^(1/2)*csgn(d)+d*
x)*csgn(d))*b*c^6)*csgn(d)/(-d*x+c)/(d^2*x^2-c^2)^(1/2)/d^7/(d*x+c)^(1/2)
```

Maxima [A]

time = 0.31, size = 196, normalized size = 1.22

$$\frac{bx^5}{4\sqrt{d^2x^2-c^2}\,d^2}+\frac{5\,bc^2x^3}{8\sqrt{d^2x^2-c^2}\,d^4}+\frac{ax^3}{2\sqrt{d^2x^2-c^2}\,d^2}-\frac{15\,bc^4x}{8\sqrt{d^2x^2-c^2}\,d^6}-\frac{3\,ac^2x}{2\sqrt{d^2x^2-c^2}\,d^4}+\frac{15\,bc^4\log\left(2\,d^2x+2\sqrt{d^2x^2-c^2}\,d\right)}{8\,d^7}+\frac{3\,ac^2\log\left(2\,d^2x+2\sqrt{d^2x^2-c^2}\,d\right)}{2\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*b*x^5/(sqrt(d^2*x^2 - c^2)*d^2) + 5/8*b*c^2*x^3/(sqrt(d^2*x^2 - c^2)*d^
4) + 1/2*a*x^3/(sqrt(d^2*x^2 - c^2)*d^2) - 15/8*b*c^4*x/(sqrt(d^2*x^2 - c^2
)*d^6) - 3/2*a*c^2*x/(sqrt(d^2*x^2 - c^2)*d^4) + 15/8*b*c^4*log(2*d^2*x + 2
*sqrt(d^2*x^2 - c^2)*d)/d^7 + 3/2*a*c^2*log(2*d^2*x + 2*sqrt(d^2*x^2 - c^2)
*d)/d^5
```

Fricas [A]

time = 3.14, size = 190, normalized size = 1.18

$$\frac{8\,bc^6+8\,ac^4d^2-8\,(bc^4d^2+ac^2d^4)x^2+(2\,bd^5x^5+(5\,bc^2d^3+4\,ad^5)x^3-3\,(5\,bc^4d+4\,ac^2d^3)x)\sqrt{dx+c}\,\sqrt{dx-c}+3\,(5\,bc^6+4\,ac^4d^2-(5\,bc^4d^2+4\,ac^2d^4)x^2)\log\left(-dx+\sqrt{dx+c}\,\sqrt{dx-c}\right)}{8\,(d^9x^2-c^2d^7)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/8*(8*b*c^6 + 8*a*c^4*d^2 - 8*(b*c^4*d^2 + a*c^2*d^4)*x^2 + (2*b*d^5*x^5 +
 (5*b*c^2*d^3 + 4*a*d^5)*x^3 - 3*(5*b*c^4*d + 4*a*c^2*d^3)*x)*sqrt(d*x + c)
```

```
*sqrt(d*x - c) + 3*(5*b*c^6 + 4*a*c^4*d^2 - (5*b*c^4*d^2 + 4*a*c^2*d^4)*x^2
)*log(-d*x + sqrt(d*x + c)*sqrt(d*x - c)))/(d^9*x^2 - c^2*d^7)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**2+a)/(d*x-c)**(3/2)/(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.60, size = 214, normalized size = 1.33

$$\frac{\left(\left(\left(dx+c\right)\left(2\left(dx+c\right)\left(\frac{(dx+c)b}{d^7}-\frac{5\,bc}{d^7}\right)+\frac{25\,bc^2d^{35}+4\,ad^{37}}{d^{42}}\right)-\frac{35\,bc^3d^{35}+12\,acd^{37}}{d^{42}}\right)(dx+c)+\frac{2\left(7\,bc^4d^{35}+2\,ac^2d^{37}\right)}{d^{42}}\right)\sqrt{dx+c}}{8\sqrt{dx-c}}-\frac{3\left(5\,bc^4+4\,ac^2d^2\right)\log\left(\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^2\right)}{8\,d^7}-\frac{2\left(bc^5+ac^3d^2\right)}{\left(\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^2+2\,c\right)d^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] 1/8*(((d*x + c)*(2*(d*x + c)*((d*x + c)*b/d^7 - 5*b*c/d^7) + (25*b*c^2*d^35
 + 4*a*d^37)/d^42) - (35*b*c^3*d^35 + 12*a*c*d^37)/d^42)*(d*x + c) + 2*(7*b
*c^4*d^35 + 2*a*c^2*d^37)/d^42)*sqrt(d*x + c)/sqrt(d*x - c) - 3/8*(5*b*c^4
+ 4*a*c^2*d^2)*log((sqrt(d*x + c) - sqrt(d*x - c))^2)/d^7 - 2*(b*c^5 + a*c^
3*d^2)/(((sqrt(d*x + c) - sqrt(d*x - c))^2 + 2*c)*d^7)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4\,\left(b\,x^2+a\right)}{{\left(c+d\,x\right)}^{3/2}\,{\left(d\,x-c\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(a + b*x^2))/((c + d*x)^(3/2)*(d*x - c)^(3/2)),x)
```

```
[Out] int((x^4*(a + b*x^2))/((c + d*x)^(3/2)*(d*x - c)^(3/2)), x)
```

3.369 $\int \frac{x^3(a+bx^2)}{(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx$

Optimal. Leaf size=115

$$-\frac{(4bc^2+3ad^2)\,x^2}{3d^4\sqrt{-c+dx}\,\sqrt{c+dx}}+\frac{bx^4}{3d^2\sqrt{-c+dx}\,\sqrt{c+dx}}+\frac{2(4bc^2+3ad^2)\sqrt{-c+dx}\,\sqrt{c+dx}}{3d^6}$$

```
[Out] -1/3*(3*a*d^2+4*b*c^2)*x^2/d^4/(d*x-c)^(1/2)/(d*x+c)^(1/2)+1/3*b*x^4/d^2/(d
*x-c)^(1/2)/(d*x+c)^(1/2)+2/3*(3*a*d^2+4*b*c^2)*(d*x-c)^(1/2)*(d*x+c)^(1/2)
/d^6
```

Rubi [A]
time = 0.06, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {471, 100, 21, 75}

$$\frac{2\sqrt{dx-c}\,\sqrt{c+dx}\,(3ad^2+4bc^2)}{3d^6}-\frac{x^2(3ad^2+4bc^2)}{3d^4\sqrt{dx-c}\,\sqrt{c+dx}}+\frac{bx^4}{3d^2\sqrt{dx-c}\,\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*x^2))/((-c + d*x)^(3/2)*(c + d*x)^(3/2)),x]
```

```
[Out] -1/3*((4*b*c^2 + 3*a*d^2)*x^2)/(d^4*Sqrt[-c + d*x]*Sqrt[c + d*x]) + (b*x^4)
/(3*d^2*Sqrt[-c + d*x]*Sqrt[c + d*x]) + (2*(4*b*c^2 + 3*a*d^2)*Sqrt[-c + d*
x]*Sqrt[c + d*x])/(3*d^6)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 75

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
```

```
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[d*(e*x)^(
m + 1)*(a1 + b1*x^(n/2))^(p + 1)*((a2 + b2*x^(n/2))^(p + 1)/(b1*b2*e*(m + n
*(p + 1) + 1))), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\int \frac{x^3(a+bx^2)}{(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx = \frac{bx^4}{3d^2\sqrt{-c+dx}\,\sqrt{c+dx}} - \frac{1}{3}\left(-3a-\frac{4bc^2}{d^2}\right)\int \frac{x^3}{(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx$$

$$= -\frac{(4bc^2+3ad^2)\,x^2}{3d^4\sqrt{-c+dx}\,\sqrt{c+dx}} + \frac{bx^4}{3d^2\sqrt{-c+dx}\,\sqrt{c+dx}} - \frac{\left(3a+\frac{4bc^2}{d^2}\right)\int \frac{}{\sqrt{-}}}{3c}$$

$$= -\frac{(4bc^2+3ad^2)\,x^2}{3d^4\sqrt{-c+dx}\,\sqrt{c+dx}} + \frac{bx^4}{3d^2\sqrt{-c+dx}\,\sqrt{c+dx}} + \frac{\left(2\left(3a+\frac{4bc^2}{d^2}\right)\right)\int}{}$$

$$= -\frac{(4bc^2+3ad^2)\,x^2}{3d^4\sqrt{-c+dx}\,\sqrt{c+dx}} + \frac{bx^4}{3d^2\sqrt{-c+dx}\,\sqrt{c+dx}} + \frac{2(4bc^2+3ad^2)\sqrt{-}}{3a}$$

Mathematica [A]
time = 0.12, size = 72, normalized size = 0.63

$$\frac{-8bc^4-6ac^2d^2+4bc^2d^2x^2+3ad^4x^2+bd^4x^4}{3d^6\sqrt{-c+dx}\,\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*x^2))/((-c + d*x)^(3/2)*(c + d*x)^(3/2)),x]
[Out] (-8*b*c^4 - 6*a*c^2*d^2 + 4*b*c^2*d^2*x^2 + 3*a*d^4*x^2 + b*d^4*x^4)/(3*d^6
*Sqrt[-c + d*x]*Sqrt[c + d*x])
```

Maple [A]
time = 0.33, size = 76, normalized size = 0.66

method	result	size
gosper	$-\frac{-b\,d^{4}x^{4}-3a\,d^{4}x^{2}-4b\,c^{2}d^{2}x^{2}+6a\,c^{2}d^{2}+8b\,c^{4}}{3\sqrt{dx+c}\,d^{6}\sqrt{dx-c}}$	68
default	$\frac{\sqrt{dx-c}\,\left(-b\,d^{4}x^{4}-3a\,d^{4}x^{2}-4b\,c^{2}d^{2}x^{2}+6a\,c^{2}d^{2}+8b\,c^{4}\right)}{3(-dx+c)d^{6}\sqrt{dx+c}}$	76
risch	$-\frac{\left(b\,d^{2}x^{2}+3a\,d^{2}+5b\,c^{2}\right)\left(-dx+c\right)\sqrt{dx+c}}{3d^{6}\sqrt{dx-c}}-\frac{c^{2}\left(a\,d^{2}+b\,c^{2}\right)\sqrt{\left(dx-c\right)\left(dx+c\right)}}{d^{6}\sqrt{-\left(dx+c\right)\left(-dx+c\right)}\,\sqrt{dx-c}\,\sqrt{dx+c}}$	115

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^2+a)/(d*x-c)^(3/2)/(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(d*x-c)^(1/2)*(-b*d^4*x^4-3*a*d^4*x^2-4*b*c^2*d^2*x^2+6*a*c^2*d^2+8*b*c
^4)/(-d*x+c)/d^6/(d*x+c)^(1/2)
```

Maxima [A]
time = 0.27, size = 123, normalized size = 1.07

$$\frac{bx^4}{3\sqrt{d^2x^2-c^2}\,d^2}+\frac{4\,bc^2x^2}{3\sqrt{d^2x^2-c^2}\,d^4}+\frac{ax^2}{\sqrt{d^2x^2-c^2}\,d^2}-\frac{8\,bc^4}{3\sqrt{d^2x^2-c^2}\,d^6}-\frac{2\,ac^2}{\sqrt{d^2x^2-c^2}\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/3*b*x^4/(sqrt(d^2*x^2 - c^2)*d^2) + 4/3*b*c^2*x^2/(sqrt(d^2*x^2 - c^2)*d^
4) + a*x^2/(sqrt(d^2*x^2 - c^2)*d^2) - 8/3*b*c^4/(sqrt(d^2*x^2 - c^2)*d^6)
 - 2*a*c^2/(sqrt(d^2*x^2 - c^2)*d^4)
```

Fricas [A]
time = 3.33, size = 80, normalized size = 0.70

$$\frac{\left(bd^4x^4-8\,bc^4-6\,ac^2d^2+\left(4\,bc^2d^2+3\,ad^4\right)x^2\right)\sqrt{dx+c}\,\sqrt{dx-c}}{3\left(d^8x^2-c^2d^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*(b*d^4*x^4 - 8*b*c^4 - 6*a*c^2*d^2 + (4*b*c^2*d^2 + 3*a*d^4)*x^2)*sqrt(
d*x + c)*sqrt(d*x - c)/(d^8*x^2 - c^2*d^6)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**2+a)/(d*x-c)**(3/2)/(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 200 vs. 2(97) = 194.
time = 0.57, size = 200, normalized size = 1.74

$$\frac{\left(2\,(dx+c)\left((dx+c)\left(\frac{(dx+c)b}{d^6}-\frac{4\,bc}{d^6}\right)+\frac{10\,bc^2d^{24}+3\,ad^{26}}{d^{30}}\right)-\frac{3\,(9\,bc^3d^{24}+5\,acd^{26})}{d^{30}}\right)\sqrt{dx+c}}{6\,\sqrt{dx-c}}+\frac{2\,(b^2c^8+2\,abc^6d^2+a^2c^4d^4)}{\left(bc^4\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^2+ac^2d^2\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^2+2\,bc^5+2\,ac^3d^2\right)d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] 1/6*(2*(d*x + c)*((d*x + c)*((d*x + c)*b/d^6 - 4*b*c/d^6) + (10*b*c^2*d^24
+ 3*a*d^26)/d^30) - 3*(9*b*c^3*d^24 + 5*a*c*d^26)/d^30)*sqrt(d*x + c)/sqrt(
d*x - c) + 2*(b^2*c^8 + 2*a*b*c^6*d^2 + a^2*c^4*d^4)/((b*c^4*(sqrt(d*x + c)
 - sqrt(d*x - c))^2 + a*c^2*d^2*(sqrt(d*x + c) - sqrt(d*x - c))^2 + 2*b*c^5
 + 2*a*c^3*d^2)*d^6)
```

Mupad [B]
time = 2.80, size = 90, normalized size = 0.78

$$\frac{\sqrt{d\,x-c}\,\left(\frac{x^2\,\left(4\,b\,c^2\,d^2+3\,a\,d^4\right)}{3\,d^7}-\frac{8\,b\,c^4+6\,a\,c^2\,d^2}{3\,d^7}+\frac{b\,x^4}{3\,d^3}\right)}{x\,\sqrt{c+d\,x}-\frac{c\,\sqrt{c+d\,x}}{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*x^2))/((c + d*x)^(3/2)*(d*x - c)^(3/2)),x)
```

```
[Out] ((d*x - c)^(1/2)*((x^2*(3*a*d^4 + 4*b*c^2*d^2))/(3*d^7) - (8*b*c^4 + 6*a*c^
2*d^2)/(3*d^7) + (b*x^4)/(3*d^3)))/(x*(c + d*x)^(1/2) - (c*(c + d*x)^(1/2))
/d)
```

3.370 $\int \frac{x^2(a+bx^2)}{(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx$

Optimal. Leaf size=152

$$-\frac{c(3bc^2+2ad^2)}{2d^5\sqrt{-c+dx}\,\sqrt{c+dx}}+\frac{bx^3}{2d^2\sqrt{-c+dx}\,\sqrt{c+dx}}-\frac{(3bc^2+2ad^2)\sqrt{-c+dx}}{2d^5\sqrt{c+dx}}+\frac{(3bc^2+2ad^2)\tanh^{-1}\left(\ldots\right.}{d^5}$$

```
[Out] (2*a*d^2+3*b*c^2)*arctanh((d*x-c)^(1/2)/(d*x+c)^(1/2))/d^5-1/2*c*(2*a*d^2+3
*b*c^2)/d^5/(d*x-c)^(1/2)/(d*x+c)^(1/2)+1/2*b*x^3/d^2/(d*x-c)^(1/2)/(d*x+c)
^(1/2)-1/2*(2*a*d^2+3*b*c^2)*(d*x-c)^(1/2)/d^5/(d*x+c)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {471, 91, 12, 79, 65, 223, 212}

$$-\frac{\sqrt{dx-c}\,(2ad^2+3bc^2)}{2d^5\sqrt{c+dx}}-\frac{c(2ad^2+3bc^2)}{2d^5\sqrt{dx-c}\,\sqrt{c+dx}}+\frac{(2ad^2+3bc^2)\tanh^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)}{d^5}+\frac{bx^3}{2d^2\sqrt{dx-c}\,\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*x^2))/((-c + d*x)^(3/2)*(c + d*x)^(3/2)),x]
```

```
[Out] -1/2*(c*(3*b*c^2 + 2*a*d^2))/(d^5*Sqrt[-c + d*x]*Sqrt[c + d*x]) + (b*x^3)/(
2*d^2*Sqrt[-c + d*x]*Sqrt[c + d*x]) - ((3*b*c^2 + 2*a*d^2)*Sqrt[-c + d*x])/
(2*d^5*Sqrt[c + d*x]) + ((3*b*c^2 + 2*a*d^2)*ArcTanh[Sqrt[-c + d*x]/Sqrt[c
+ d*x]])/d^5
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
```

```
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[d*(e*x)^(
m + 1)*(a1 + b1*x^(n/2))^(p + 1)*((a2 + b2*x^(n/2))^(p + 1)/(b1*b2*e*(m + n
*(p + 1) + 1))), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\int \frac{x^2(a+bx^2)}{(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx = \frac{bx^3}{2d^2\sqrt{-c+dx}\,\sqrt{c+dx}} - \frac{1}{2}\left(-2a-\frac{3bc^2}{d^2}\right)\int \frac{x^2}{(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx$$

$$= -\frac{c(3bc^2+2ad^2)}{2d^5\sqrt{-c+dx}\,\sqrt{c+dx}} + \frac{bx^3}{2d^2\sqrt{-c+dx}\,\sqrt{c+dx}} - \frac{\left(-2a-\frac{3bc^2}{d^2}\right)\int}{}$$

$$= -\frac{c(3bc^2+2ad^2)}{2d^5\sqrt{-c+dx}\,\sqrt{c+dx}} + \frac{bx^3}{2d^2\sqrt{-c+dx}\,\sqrt{c+dx}} + \frac{(3bc^2+2ad^2)\int}{}$$

$$= -\frac{c(3bc^2+2ad^2)}{2d^5\sqrt{-c+dx}\,\sqrt{c+dx}} + \frac{bx^3}{2d^2\sqrt{-c+dx}\,\sqrt{c+dx}} - \frac{(3bc^2+2ad^2)\sqrt{-}}{2d^5\sqrt{c+d}}$$

$$= -\frac{c(3bc^2+2ad^2)}{2d^5\sqrt{-c+dx}\,\sqrt{c+dx}} + \frac{bx^3}{2d^2\sqrt{-c+dx}\,\sqrt{c+dx}} - \frac{(3bc^2+2ad^2)\sqrt{-}}{2d^5\sqrt{c+d}}$$

$$= -\frac{c(3bc^2+2ad^2)}{2d^5\sqrt{-c+dx}\,\sqrt{c+dx}} + \frac{bx^3}{2d^2\sqrt{-c+dx}\,\sqrt{c+dx}} - \frac{(3bc^2+2ad^2)\sqrt{-}}{2d^5\sqrt{c+d}}$$

$$= -\frac{c(3bc^2+2ad^2)}{2d^5\sqrt{-c+dx}\,\sqrt{c+dx}} + \frac{bx^3}{2d^2\sqrt{-c+dx}\,\sqrt{c+dx}} - \frac{(3bc^2+2ad^2)\sqrt{-}}{2d^5\sqrt{c+d}}$$

Mathematica [A]
time = 0.18, size = 108, normalized size = 0.71

$$\frac{-3bc^2dx-2ad^3x+bd^3x^3+2(3bc^2+2ad^2)\sqrt{-c+dx}\,\sqrt{c+dx}\,\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{-c+dx}}\right)}{2d^5\sqrt{-c+dx}\,\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*x^2))/((-c + d*x)^(3/2)*(c + d*x)^(3/2)),x]
```

```
[Out] (-3*b*c^2*d*x - 2*a*d^3*x + b*d^3*x^3 + 2*(3*b*c^2 + 2*a*d^2)*Sqrt[-c + d*x
]*Sqrt[c + d*x]*ArcTanh[Sqrt[c + d*x]/Sqrt[-c + d*x]])/(2*d^5*Sqrt[-c + d*x
]*Sqrt[c + d*x])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.32, size = 263, normalized size = 1.73

method	result

default	$\frac{\sqrt{dx-c}\,\left(-\mathrm{csgn}(d) b\, d^3 x^3 \sqrt{d^2x^2-c^2}-2\ln\left(\left(\sqrt{d^2x^2-c^2}\,\mathrm{csgn}(d)+dx\right)\mathrm{csgn}(d)\right) a\, d^4 x^2-3\ln\left(\left(\sqrt{d^2x^2-c^2}\,\mathrm{csgn}\right.\right.\right.}{}$
risch	$-\frac{bx(-dx+c)\sqrt{dx+c}}{2d^4\sqrt{dx-c}}-\frac{\left(-\frac{\ln\left(\frac{d^2x}{\sqrt{d^2}}+\sqrt{d^2x^2-c^2}\right)a}{d^2\sqrt{d^2}}-\frac{3\ln\left(\frac{d^2x}{\sqrt{d^2}}+\sqrt{d^2x^2-c^2}\right)b\,c^2}{2d^4\sqrt{d^2}}+\frac{\sqrt{d^2\left(x+\frac{c}{d}\right)^2-2}}{2d^4\left(x+\frac{c}{d}\right.}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2+a)/(d*x-c)^(3/2)/(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(d*x-c)^(1/2)*(-csgn(d)*b*d^3*x^3*(d^2*x^2-c^2)^(1/2)-2*ln(((d^2*x^2-c^
2)^(1/2)*csgn(d)+d*x)*csgn(d))*a*d^4*x^2-3*ln(((d^2*x^2-c^2)^(1/2)*csgn(d)+
d*x)*csgn(d))*b*c^2*d^2*x^2+2*(d^2*x^2-c^2)^(1/2)*csgn(d)*d^3*a*x+3*(d^2*x^
2-c^2)^(1/2)*csgn(d)*d*b*c^2*x+2*ln(((d^2*x^2-c^2)^(1/2)*csgn(d)+d*x)*csgn(
d))*a*c^2*d^2+3*ln(((d^2*x^2-c^2)^(1/2)*csgn(d)+d*x)*csgn(d))*b*c^4)*csgn(d
)/(-d*x+c)/(d^2*x^2-c^2)^(1/2)/d^5/(d*x+c)^(1/2)
```

Maxima [A]
time = 0.30, size = 138, normalized size = 0.91

$$\frac{bx^3}{2\sqrt{d^2x^2-c^2}\,d^2}-\frac{3\,bc^2x}{2\sqrt{d^2x^2-c^2}\,d^4}-\frac{ax}{\sqrt{d^2x^2-c^2}\,d^2}+\frac{3\,bc^2\log\left(2\,d^2x+2\sqrt{d^2x^2-c^2}\,d\right)}{2\,d^5}+\frac{a\log\left(2\,d^2x+2\sqrt{d^2x^2-c^2}\,d\right)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*b*x^3/(sqrt(d^2*x^2 - c^2)*d^2) - 3/2*b*c^2*x/(sqrt(d^2*x^2 - c^2)*d^4)
 - a*x/(sqrt(d^2*x^2 - c^2)*d^2) + 3/2*b*c^2*log(2*d^2*x + 2*sqrt(d^2*x^2 -
 c^2)*d)/d^5 + a*log(2*d^2*x + 2*sqrt(d^2*x^2 - c^2)*d)/d^3
```

Fricas [A]
time = 3.71, size = 159, normalized size = 1.05

$$\frac{2\,bc^4+2\,ac^2d^2-2\,(bc^2d^2+ad^4)x^2+(bd^3x^3-(3\,bc^2d+2\,ad^3)x)\sqrt{dx+c}\,\sqrt{dx-c}+(3\,bc^4+2\,ac^2d^2-(3\,bc^2d^2+2\,ad^4)x^2)\log\left(-dx+\sqrt{dx+c}\,\sqrt{dx-c}\right)}{2\,(d^7x^2-c^2d^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b*c^4 + 2*a*c^2*d^2 - 2*(b*c^2*d^2 + a*d^4)*x^2 + (b*d^3*x^3 - (3*b*
c^2*d + 2*a*d^3)*x)*sqrt(d*x + c)*sqrt(d*x - c) + (3*b*c^4 + 2*a*c^2*d^2 -
(3*b*c^2*d^2 + 2*a*d^4)*x^2)*log(-d*x + sqrt(d*x + c)*sqrt(d*x - c)))/(d^7*
x^2 - c^2*d^5)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**2+a)/(d*x-c)**(3/2)/(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.60, size = 147, normalized size = 0.97

$$\frac{\sqrt{dx+c}\left((dx+c)\left(\frac{(dx+c)b}{d^5}-\frac{3\,bc}{d^5}\right)+\frac{bc^2d^{15}-ad^{17}}{d^{20}}\right)}{2\sqrt{dx-c}}-\frac{(3\,bc^2+2\,ad^2)\log\left(\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^2\right)}{2\,d^5}-\frac{2\,(bc^3+acd^2)}{\left(\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^2+2\,c\right)d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(d*x + c)*((d*x + c)*((d*x + c)*b/d^5 - 3*b*c/d^5) + (b*c^2*d^15 -
a*d^17)/d^20)/sqrt(d*x - c) - 1/2*(3*b*c^2 + 2*a*d^2)*log((sqrt(d*x + c) -
sqrt(d*x - c))^2)/d^5 - 2*(b*c^3 + a*c*d^2)/(((sqrt(d*x + c) - sqrt(d*x - c
))^2 + 2*c)*d^5)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,\left(b\,x^2+a\right)}{{\left(c+d\,x\right)}^{3/2}\,{\left(d\,x-c\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*x^2))/((c + d*x)^(3/2)*(d*x - c)^(3/2)),x)
```

```
[Out] int((x^2*(a + b*x^2))/((c + d*x)^(3/2)*(d*x - c)^(3/2)), x)
```

3.371 $\int \frac{x(a+bx^2)}{(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx$

Optimal. Leaf size=76

$$-\frac{\left(\frac{a}{c^2}+\frac{b}{d^2}\right)x^2}{\sqrt{-c+dx}\,\sqrt{c+dx}}+\frac{(2bc^2+ad^2)\sqrt{-c+dx}\,\sqrt{c+dx}}{c^2d^4}$$

```
[Out] -(a/c^2+b/d^2)*x^2/(d*x-c)^(1/2)/(d*x+c)^(1/2)+(a*d^2+2*b*c^2)*(d*x-c)^(1/2
)*(d*x+c)^(1/2)/c^2/d^4
```

Rubi [A]
time = 0.04, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {469, 75}

$$\frac{\sqrt{dx-c}\,\sqrt{c+dx}\,(ad^2+2bc^2)}{c^2d^4}-\frac{x^2\left(\frac{a}{c^2}+\frac{b}{d^2}\right)}{\sqrt{dx-c}\,\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*x^2))/((-c + d*x)^(3/2)*(c + d*x)^(3/2)),x]
```

```
[Out] -(((a/c^2 + b/d^2)*x^2)/(Sqrt[-c + d*x]*Sqrt[c + d*x])) + ((2*b*c^2 + a*d^2
)*Sqrt[-c + d*x]*Sqrt[c + d*x])/(c^2*d^4)
```

Rule 75

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 469

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b1*b2*
c - a1*a2*d))*(e*x)^(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*((a2 + b2*x^(n/2))^(p
 + 1)/(a1*a2*b1*b2*e*n*(p + 1))), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m +
 n*(p + 1) + 1))/(a1*a2*b1*b2*n*(p + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^(p
+ 1)*(a2 + b2*x^(n/2))^(p + 1), x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m
, n}, x] && EqQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] && LtQ[p, -1] && (( !In
tegerQ[p + 1/2] && NeQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p
 + 1/2, 0] && LeQ[-1, m, (-n)*(p + 1)]))
```

Rubi steps

$$\int \frac{x(a+bx^2)}{(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx = -\frac{\left(\frac{a}{c^2}+\frac{b}{d^2}\right)x^2}{\sqrt{-c+dx}\,\sqrt{c+dx}} - \left(-\frac{a}{c^2}-\frac{2b}{d^2}\right)\int \frac{x}{\sqrt{-c+dx}\,\sqrt{c+dx}}\,dx$$
$$= -\frac{\left(\frac{a}{c^2}+\frac{b}{d^2}\right)x^2}{\sqrt{-c+dx}\,\sqrt{c+dx}} + \frac{\left(\frac{a}{c^2}+\frac{2b}{d^2}\right)\sqrt{-c+dx}\,\sqrt{c+dx}}{d^2}$$

Mathematica [A]
time = 0.10, size = 45, normalized size = 0.59

$$\frac{-2bc^2-ad^2+bd^2x^2}{d^4\sqrt{-c+dx}\,\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*x^2))/((-c + d*x)^(3/2)*(c + d*x)^(3/2)),x]
[Out] (-2*b*c^2 - a*d^2 + b*d^2*x^2)/(d^4*Sqrt[-c + d*x]*Sqrt[c + d*x])
```

Maple [A]
time = 0.29, size = 50, normalized size = 0.66

method	result	size
gosper	$-\frac{-b\,d^2x^2+a\,d^2+2b\,c^2}{\sqrt{dx+c}\,d^4\sqrt{dx-c}}$	43
default	$\frac{\sqrt{dx-c}\,\left(-b\,d^2x^2+a\,d^2+2b\,c^2\right)}{\sqrt{dx+c}\,d^4(-dx+c)}$	50
risch	$-\frac{b(-dx+c)\sqrt{dx+c}}{d^4\sqrt{dx-c}}-\frac{\left(a\,d^2+b\,c^2\right)\sqrt{(dx-c)\,(dx+c)}}{d^4\sqrt{-\left(dx+c\right)\left(-dx+c\right)}\,\sqrt{dx-c}\,\sqrt{dx+c}}$	92

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2+a)/(d*x-c)^(3/2)/(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
[Out] (d*x-c)^(1/2)*(-b*d^2*x^2+a*d^2+2*b*c^2)/(d*x+c)^(1/2)/d^4/(-d*x+c)
```

Maxima [A]
time = 0.28, size = 69, normalized size = 0.91

$$\frac{bx^2}{\sqrt{d^2x^2-c^2}\,d^2}-\frac{2\,bc^2}{\sqrt{d^2x^2-c^2}\,d^4}-\frac{a}{\sqrt{d^2x^2-c^2}\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] b*x^2/(sqrt(d^2*x^2 - c^2)*d^2) - 2*b*c^2/(sqrt(d^2*x^2 - c^2)*d^4) - a/(sq
rt(d^2*x^2 - c^2)*d^2)
```

Fricas [A]
time = 3.00, size = 56, normalized size = 0.74

$$\frac{(bd^2x^2 - 2\,bc^2 - ad^2)\sqrt{dx+c}\,\sqrt{dx-c}}{d^6x^2 - c^2d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] (b*d^2*x^2 - 2*b*c^2 - a*d^2)*sqrt(d*x + c)*sqrt(d*x - c)/(d^6*x^2 - c^2*d^
4)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**2+a)/(d*x-c)**(3/2)/(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 152 vs. 2(68) = 136.
time = 0.55, size = 152, normalized size = 2.00

$$\frac{\sqrt{dx+c}\left(\frac{2\,(dx+c)b}{d^4} - \frac{5\,bc^2d^8+ad^{10}}{cd^{12}}\right)}{2\,\sqrt{dx-c}} + \frac{2\,(b^2c^4 + 2\,abc^2d^2 + a^2d^4)}{\left(bc^2\left(\sqrt{dx+c} - \sqrt{dx-c}\right)^2 + ad^2\left(\sqrt{dx+c} - \sqrt{dx-c}\right)^2 + 2\,bc^3 + 2\,acd^2\right)d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(d*x + c)*(2*(d*x + c)*b/d^4 - (5*b*c^2*d^8 + a*d^10)/(c*d^12))/sqr
t(d*x - c) + 2*(b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d^4)/((b*c^2*(sqrt(d*x + c) -
 sqrt(d*x - c))^2 + a*d^2*(sqrt(d*x + c) - sqrt(d*x - c))^2 + 2*b*c^3 + 2*a
*c*d^2)*d^4)
```

Mupad [B]
time = 2.75, size = 67, normalized size = 0.88

$$\frac{a\,d^2\,\sqrt{d\,x-c} + 2\,b\,c^2\,\sqrt{d\,x-c} - b\,d^2\,x^2\,\sqrt{d\,x-c}}{d^4\,\sqrt{c+d\,x}\,\left(c-d\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*x^2))/((c + d*x)^(3/2)*(d*x - c)^(3/2)),x)
```

```
[Out] (a*d^2*(d*x - c)^(1/2) + 2*b*c^2*(d*x - c)^(1/2) - b*d^2*x^2*(d*x - c)^(1/2
))/(d^4*(c + d*x)^(1/2)*(c - d*x))
```

3.372 $\int \frac{a+bx^2}{(-c+dx)^{3/2}(c+dx)^{3/2}} \, dx$

Optimal. Leaf size=63

$$-\frac{\left(\frac{a}{c^2}+\frac{b}{d^2}\right)x}{\sqrt{-c+dx}\,\sqrt{c+dx}}+\frac{2b\tanh^{-1}\left(\frac{\sqrt{-c+dx}}{\sqrt{c+dx}}\right)}{d^3}$$

[Out] 2*b*arctanh((d*x-c)^(1/2)/(d*x+c)^(1/2))/d^3-(a/c^2+b/d^2)*x/(d*x-c)^(1/2)/(d*x+c)^(1/2)

Rubi [A]
time = 0.02, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {394, 65, 223, 212}

$$\frac{2b\tanh^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)}{d^3}-\frac{x\left(\frac{a}{c^2}+\frac{b}{d^2}\right)}{\sqrt{dx-c}\,\sqrt{c+dx}}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x^2)/((-c + d*x)^(3/2)*(c + d*x)^(3/2)),x]

[Out] -(((a/c^2 + b/d^2)*x)/(Sqrt[-c + d*x]*Sqrt[c + d*x])) + (2*b*ArcTanh[Sqrt[-c + d*x]/Sqrt[c + d*x]])/d^3

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 394

```
Int[((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_
.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b1*b2*c - a1*a2*d))*x*(a1
 + b1*x^(n/2))^(p + 1)*((a2 + b2*x^(n/2))^(p + 1)/(a1*a2*b1*b2*n*(p + 1))),
 x] - Dist[(a1*a2*d - b1*b2*c*(n*(p + 1) + 1))/(a1*a2*b1*b2*n*(p + 1)), Int
[(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1), x], x] /; FreeQ[{a1,
b1, a2, b2, c, d, n}, x] && EqQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] && (LtQ
[p, -1] || ILtQ[1/n + p, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{a+bx^2}{(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx &= -\frac{\left(\frac{a}{c^2}+\frac{b}{d^2}\right)x}{\sqrt{-c+dx}\,\sqrt{c+dx}}+\frac{b\int \frac{1}{\sqrt{-c+dx}\,\sqrt{c+dx}}\,dx}{d^2}\\
&= -\frac{\left(\frac{a}{c^2}+\frac{b}{d^2}\right)x}{\sqrt{-c+dx}\,\sqrt{c+dx}}+\frac{(2b)\mathrm{Subst}\left(\int \frac{1}{\sqrt{2c+x^2}}\,dx,x,\sqrt{-c+dx}\right)}{d^3}\\
&= -\frac{\left(\frac{a}{c^2}+\frac{b}{d^2}\right)x}{\sqrt{-c+dx}\,\sqrt{c+dx}}+\frac{(2b)\mathrm{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\frac{\sqrt{-c+dx}}{\sqrt{c+dx}}\right)}{d^3}\\
&= -\frac{\left(\frac{a}{c^2}+\frac{b}{d^2}\right)x}{\sqrt{-c+dx}\,\sqrt{c+dx}}+\frac{2b\tanh^{-1}\left(\frac{\sqrt{-c+dx}}{\sqrt{c+dx}}\right)}{d^3}
\end{aligned}$$

Mathematica [A]
time = 0.17, size = 71, normalized size = 1.13

$$\frac{-\frac{2(bc^2d+ad^3)x}{c^2\sqrt{-c+dx}\,\sqrt{c+dx}}+4b\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{-c+dx}}\right)}{2d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)/((-c + d*x)^(3/2)*(c + d*x)^(3/2)),x]
```

```
[Out] ((-2*(b*c^2*d + a*d^3)*x)/(c^2*Sqrt[-c + d*x]*Sqrt[c + d*x]) + 4*b*ArcTanh[
Sqrt[c + d*x]/Sqrt[-c + d*x]])/(2*d^3)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.30, size = 166, normalized size = 2.63

method	result

default	$\frac{\sqrt{dx-c}\,\left(-\ln\left(\left(\operatorname{csgn}(d)\sqrt{-(dx+c)(-dx+c)}+dx\right)\operatorname{csgn}(d)\right)b\,c^2d^2x^2+\sqrt{d^2x^2-c^2}\,\operatorname{csgn}(d)d^3ax+\sqrt{d^2x^2-}}{(-dx+c)\sqrt{d^2x^2-c^2}\,c^2d^3\sqrt{dx-}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)/(d*x-c)^(3/2)/(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] (d*x-c)^(1/2)*(-ln((csgn(d)*(-(d*x+c)*(-d*x+c))^(1/2)+d*x)*csgn(d))*b*c^2*d
^2*x^2+(d^2*x^2-c^2)^(1/2)*csgn(d)*d^3*a*x+(d^2*x^2-c^2)^(1/2)*csgn(d)*d*b*
c^2*x+ln((csgn(d)*(-(d*x+c)*(-d*x+c))^(1/2)+d*x)*csgn(d))*b*c^4)*csgn(d)/(-
d*x+c)/(d^2*x^2-c^2)^(1/2)/c^2/d^3/(d*x+c)^(1/2)
```

Maxima [A]
time = 0.29, size = 76, normalized size = 1.21

$$-\frac{ax}{\sqrt{d^2x^2-c^2}\,c^2}-\frac{bx}{\sqrt{d^2x^2-c^2}\,d^2}+\frac{b\log\left(2\,d^2x+2\,\sqrt{d^2x^2-c^2}\,d\right)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] -a*x/(sqrt(d^2*x^2 - c^2)*c^2) - b*x/(sqrt(d^2*x^2 - c^2)*d^2) + b*log(2*d^
2*x + 2*sqrt(d^2*x^2 - c^2)*d)/d^3
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 129 vs. $2(55) = 110$.
time = 3.52, size = 129, normalized size = 2.05

$$\frac{bc^4+ac^2d^2-(bc^2d+ad^3)\sqrt{dx+c}\,\sqrt{dx-c}\,x-(bc^2d^2+ad^4)x^2-(bc^2d^2x^2-bc^4)\log\left(-dx+\sqrt{dx+c}\,\sqrt{dx-c}\right)}{c^2d^5x^2-c^4d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] (b*c^4 + a*c^2*d^2 - (b*c^2*d + a*d^3)*sqrt(d*x + c)*sqrt(d*x - c)*x - (b*c
^2*d^2 + a*d^4)*x^2 - (b*c^2*d^2*x^2 - b*c^4)*log(-d*x + sqrt(d*x + c)*sqrt
(d*x - c)))/(c^2*d^5*x^2 - c^4*d^3)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/(d*x-c)**(3/2)/(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 113 vs. 2(55) = 110.
time = 0.58, size = 113, normalized size = 1.79

$$-\frac{b\log\left(\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^2\right)}{d^3}-\frac{2\,(bc^2+ad^2)}{\left(\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^2+2\,c\right)cd^3}-\frac{(bc^2d^3+ad^5)\sqrt{dx+c}}{2\,\sqrt{dx-c}\,c^2d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] -b*log((sqrt(d*x + c) - sqrt(d*x - c))^2)/d^3 - 2*(b*c^2 + a*d^2)/(((sqrt(d
*x + c) - sqrt(d*x - c))^2 + 2*c)*c*d^3) - 1/2*(b*c^2*d^3 + a*d^5)*sqrt(d*x
 + c)/(sqrt(d*x - c)*c^2*d^6)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{b\,x^2+a}{{\left(c+d\,x\right)}^{3/2}\,{\left(d\,x-c\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)/((c + d*x)^(3/2)*(d*x - c)^(3/2)),x)
```

```
[Out] int((a + b*x^2)/((c + d*x)^(3/2)*(d*x - c)^(3/2)), x)
```

3.373 $\int \frac{a+bx^2}{x(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx$

Optimal. Leaf size=65

$$-\frac{\frac{a}{c^2}+\frac{b}{d^2}}{\sqrt{-c+dx}\,\sqrt{c+dx}}-\frac{a\tan^{-1}\left(\frac{\sqrt{-c+dx}\,\sqrt{c+dx}}{c}\right)}{c^3}$$

```
[Out] -a*arctan((d*x-c)^(1/2)*(d*x+c)^(1/2)/c)/c^3+(-a/c^2-b/d^2)/(d*x-c)^(1/2)/(
d*x+c)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {469, 94, 211}

$$-\frac{a\text{ArcTan}\left(\frac{\sqrt{dx-c}\,\sqrt{c+dx}}{c}\right)}{c^3}-\frac{\frac{a}{c^2}+\frac{b}{d^2}}{\sqrt{dx-c}\,\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)/(x*(-c + d*x)^(3/2)*(c + d*x)^(3/2)),x]
```

```
[Out] -((a/c^2 + b/d^2)/(Sqrt[-c + d*x]*Sqrt[c + d*x])) - (a*ArcTan[(Sqrt[-c + d*
x]*Sqrt[c + d*x])/c])/c^3
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 469

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b1*b2*
c - a1*a2*d))*(e*x)^(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*((a2 + b2*x^(n/2))^(p
 + 1)/(a1*a2*b1*b2*e*n*(p + 1))), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m +
 n*(p + 1) + 1))/(a1*a2*b1*b2*n*(p + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^(p
+ 1)*(a2 + b2*x^(n/2))^(p + 1), x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m
```

```
, n}, x] && EqQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] && LtQ[p, -1] && (( !In
tegerQ[p + 1/2] && NeQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p
 + 1/2, 0] && LeQ[-1, m, (-n)*(p + 1)]))
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx^2}{x(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx &= -\frac{\frac{a}{c^2}+\frac{b}{d^2}}{\sqrt{-c+dx}\,\sqrt{c+dx}}-\frac{a\int \frac{1}{x\sqrt{-c+dx}\,\sqrt{c+dx}}\,dx}{c^2}\\ &= -\frac{\frac{a}{c^2}+\frac{b}{d^2}}{\sqrt{-c+dx}\,\sqrt{c+dx}}-\frac{(ad)\text{Subst}\left(\int \frac{1}{c^2d+dx^2}\,dx,x,\sqrt{-c+dx}\,\sqrt{c+dx}\right)}{c^2}\\ &= -\frac{\frac{a}{c^2}+\frac{b}{d^2}}{\sqrt{-c+dx}\,\sqrt{c+dx}}-\frac{a\tan^{-1}\left(\frac{\sqrt{-c+dx}\,\sqrt{c+dx}}{c}\right)}{c^3}\end{aligned}$$

Mathematica [A]
time = 0.15, size = 70, normalized size = 1.08

$$\frac{-\frac{2(bc^3+acd^2)}{d^2\sqrt{-c+dx}\,\sqrt{c+dx}}+4a\tan^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{-c+dx}}\right)}{2c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)/(x*(-c + d*x)^(3/2)*(c + d*x)^(3/2)),x]
```

```
[Out] ((-2*(b*c^3 + a*c*d^2))/(d^2*Sqrt[-c + d*x]*Sqrt[c + d*x]) + 4*a*ArcTan[Sqr
t[c + d*x]/Sqrt[-c + d*x]])/(2*c^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 193 vs. 2(58) = 116.
time = 0.30, size = 194, normalized size = 2.98

method	result
default	$\frac{\sqrt{dx-c}\left(-\ln\left(-\frac{2\left(c^2-\sqrt{-c^2}\,\sqrt{d^2x^2-c^2}\right)}{x}\right)a\,d^4x^2+\ln\left(-\frac{2\left(c^2-\sqrt{-c^2}\,\sqrt{d^2x^2-c^2}\right)}{x}\right)a\,c^2d^2+\sqrt{d^2x^2-c}\right.}{c^2\sqrt{-c^2}\,(-dx+c)\sqrt{d^2x^2-c^2}\,d^2\sqrt{dx+c}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)/x/(d*x-c)^(3/2)/(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] (d*x-c)^(1/2)/c^2*(-ln(-2*(c^2-(-c^2)^(1/2)*(d^2*x^2-c^2)^(1/2))/x)*a*d^4*x
^2+ln(-2*(c^2-(-c^2)^(1/2)*(d^2*x^2-c^2)^(1/2))/x)*a*c^2*d^2+(d^2*x^2-c^2)^
(1/2)*(-c^2)^(1/2)*a*d^2+(d^2*x^2-c^2)^(1/2)*(-c^2)^(1/2)*b*c^2)/(-c^2)^(1/
2)/(-d*x+c)/(d^2*x^2-c^2)^(1/2)/d^2/(d*x+c)^(1/2)
```

Maxima [A]
time = 0.53, size = 58, normalized size = 0.89

$$\frac{a\arcsin\left(\frac{c}{d|x|}\right)}{c^3}-\frac{a}{\sqrt{d^2x^2-c^2}\,c^2}-\frac{b}{\sqrt{d^2x^2-c^2}\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] a*arcsin(c/(d*abs(x)))/c^3 - a/(sqrt(d^2*x^2 - c^2)*c^2) - b/(sqrt(d^2*x^2
- c^2)*d^2)
```

Fricas [A]
time = 3.13, size = 101, normalized size = 1.55

$$-\frac{(bc^3+acd^2)\sqrt{dx+c}\,\sqrt{dx-c}\,+2\,(ad^4x^2-ac^2d^2)\arctan\left(-\frac{dx-\sqrt{dx+c}\,\sqrt{dx-c}}{c}\right)}{c^3d^4x^2-c^5d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] -((b*c^3 + a*c*d^2)*sqrt(d*x + c)*sqrt(d*x - c) + 2*(a*d^4*x^2 - a*c^2*d^2)
*arctan(-(d*x - sqrt(d*x + c)*sqrt(d*x - c))/c))/(c^3*d^4*x^2 - c^5*d^2)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/x/(d*x-c)**(3/2)/(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 115 vs. 2(57) = 114.
time = 0.59, size = 115, normalized size = 1.77

$$\frac{2\,a\arctan\left(\frac{\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^2}{2\,c}\right)}{c^3}-\frac{(bc^2+ad^2)\sqrt{dx+c}}{2\,\sqrt{dx-c}\,c^3d^2}+\frac{2\,(bc^2+ad^2)}{\left(\left(\sqrt{dx+c}\,-\sqrt{dx-c}\,\right)^2+2\,c\right)c^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] 2*a*arctan(1/2*(sqrt(d*x + c) - sqrt(d*x - c))^2/c)/c^3 - 1/2*(b*c^2 + a*d^
2)*sqrt(d*x + c)/(sqrt(d*x - c)*c^3*d^2) + 2*(b*c^2 + a*d^2)/(((sqrt(d*x +
c) - sqrt(d*x - c))^2 + 2*c)*c^2*d^2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{b\,x^2+a}{x\,{\left(c+d\,x\right)}^{3/2}\,{\left(d\,x-c\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)/(x*(c + d*x)^(3/2)*(d*x - c)^(3/2)),x)
```

```
[Out] int((a + b*x^2)/(x*(c + d*x)^(3/2)*(d*x - c)^(3/2)), x)
```

3.374 $\int \frac{a+bx^2}{x^2(-c+dx)^{3/2}(c+dx)^{3/2}} dx$

Optimal. Leaf size=67

$$\frac{a}{c^2x\sqrt{-c+dx}\sqrt{c+dx}}-\frac{(bc^2+2ad^2)x}{c^4\sqrt{-c+dx}\sqrt{c+dx}}$$

```
[Out] a/c^2/x/(d*x-c)^(1/2)/(d*x+c)^(1/2)-(2*a*d^2+b*c^2)*x/c^4/(d*x-c)^(1/2)/(d*
x+c)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {465, 39}

$$\frac{a}{c^2x\sqrt{dx-c}\sqrt{c+dx}}-\frac{x(2ad^2+bc^2)}{c^4\sqrt{dx-c}\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)/(x^2*(-c + d*x)^(3/2)*(c + d*x)^(3/2)),x]
```

```
[Out] a/(c^2*x*Sqrt[-c + d*x]*Sqrt[c + d*x]) - ((b*c^2 + 2*a*d^2)*x)/(c^4*Sqrt[-c
 + d*x]*Sqrt[c + d*x])
```

Rule 39

```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

Rule 465

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[c*(e*x)^(
m + 1)*(a1 + b1*x^(n/2))^(p + 1)*((a2 + b2*x^(n/2))^(p + 1)/(a1*a2*e*(m + 1
))), x] + Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/(a1*a2*e^n*(
m + 1)), Int[(e*x)^(m + n)*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/2))^p, x], x]
/; FreeQ[{a1, b1, a2, b2, c, d, e, p}, x] && EqQ[non2, n/2] && EqQ[a2*b1 +
a1*b2, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (L
tQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

Rubi steps

$$\int \frac{a+bx^2}{x^2(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx = \frac{a}{c^2x\sqrt{-c+dx}\,\sqrt{c+dx}} + \left(b+\frac{2ad^2}{c^2}\right)\int \frac{1}{(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx$$
$$= \frac{a}{c^2x\sqrt{-c+dx}\,\sqrt{c+dx}} - \frac{(bc^2+2ad^2)\,x}{c^4\sqrt{-c+dx}\,\sqrt{c+dx}}$$

Mathematica [A]
time = 0.11, size = 51, normalized size = 0.76

$$\frac{-bc^2x^2+a(c^2-2d^2x^2)}{c^4x\sqrt{-c+dx}\,\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)/(x^2*(-c + d*x)^(3/2)*(c + d*x)^(3/2)),x]
[Out] (-(b*c^2*x^2) + a*(c^2 - 2*d^2*x^2))/(c^4*x*Sqrt[-c + d*x]*Sqrt[c + d*x])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 2.
time = 0.30, size = 60, normalized size = 0.90

method	result	size
gosper	$\frac{-2a\,d^2x^2-b\,c^2x^2+c^2a}{\sqrt{dx+c}\;x\,c^4\sqrt{dx-c}}$	48
default	$\frac{(2a\,d^2x^2+b\,c^2x^2-c^2a)\sqrt{dx-c}\;\mathrm{csgn}(d)^2}{c^4x(-dx+c)\sqrt{dx+c}}$	60
risch	$\frac{a(-dx+c)\sqrt{dx+c}}{c^4x\sqrt{dx-c}} - \frac{(a\,d^2+b\,c^2)x\sqrt{(dx-c)\,(dx+c)}}{\sqrt{-(dx+c)\,(-dx+c)}\;c^4\sqrt{dx-c}\;\sqrt{dx+c}}$	95

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)/x^2/(d*x-c)^(3/2)/(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
[Out] (2*a*d^2*x^2+b*c^2*x^2-a*c^2)*(d*x-c)^(1/2)*csgn(d)^2/c^4/x/(-d*x+c)/(d*x+c
)^(1/2)
```

Maxima [A]
time = 0.50, size = 71, normalized size = 1.06

$$-\frac{bx}{\sqrt{d^2x^2-c^2}\,c^2} - \frac{2\,ad^2x}{\sqrt{d^2x^2-c^2}\,c^4} + \frac{a}{\sqrt{d^2x^2-c^2}\,c^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^2/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="maxima")
```

[Out] -b*x/(sqrt(d^2*x^2 - c^2)*c^2) - 2*a*d^2*x/(sqrt(d^2*x^2 - c^2)*c^4) + a/(sqrt(d^2*x^2 - c^2)*c^2*x)

Fricas [**A**]
time = 4.11, size = 103, normalized size = 1.54

$$-\frac{(bc^2d^2+2\,ad^4)x^3-(ac^2d-(bc^2d+2\,ad^3)x^2)\sqrt{dx+c}\,\sqrt{dx-c}\,-(bc^4+2\,ac^2d^2)x}{c^4d^3x^3-c^6dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^2/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] -((b*c^2*d^2 + 2*a*d^4)*x^3 - (a*c^2*d - (b*c^2*d + 2*a*d^3)*x^2)*sqrt(d*x + c)*sqrt(d*x - c) - (b*c^4 + 2*a*c^2*d^2)*x)/(c^4*d^3*x^3 - c^6*d*x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/x**2/(d*x-c)**(3/2)/(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 219 vs. 2(59) = 118.
time = 0.88, size = 219, normalized size = 3.27

$$-\frac{(bc^2+ad^2)\sqrt{dx+c}}{2\sqrt{dx-c}\,c^4d}-\frac{2\left(bc^2\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^4+ad^2\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^4+4\,acd^2\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^2+4\,bc^4+12\,ac^2d^2\right)}{\left(\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^6+2\,c\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^4+4\,c^2\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^2+8\,c^3\right)c^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^2/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] -1/2*(b*c^2 + a*d^2)*sqrt(d*x + c)/(sqrt(d*x - c)*c^4*d) - 2*(b*c^2*(sqrt(d*x + c) - sqrt(d*x - c))^4 + a*d^2*(sqrt(d*x + c) - sqrt(d*x - c))^4 + 4*a*c*d^2*(sqrt(d*x + c) - sqrt(d*x - c))^2 + 4*b*c^4 + 12*a*c^2*d^2)/(((sqrt(d*x + c) - sqrt(d*x - c))^6 + 2*c*(sqrt(d*x + c) - sqrt(d*x - c))^4 + 4*c^2*(sqrt(d*x + c) - sqrt(d*x - c))^2 + 8*c^3)*c^3*d)
```

Mupad [**B**]
time = 2.87, size = 73, normalized size = 1.09

$$\frac{2\,a\,d^2\,x^2\,\sqrt{d\,x-c}\,-a\,c^2\,\sqrt{d\,x-c}\,+b\,c^2\,x^2\,\sqrt{d\,x-c}}{c^4\,x\,\sqrt{c+d\,x}\,\,(c-d\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)/(x^2*(c + d*x)^(3/2)*(d*x - c)^(3/2)),x)
```

```
[Out] (2*a*d^2*x^2*(d*x - c)^(1/2) - a*c^2*(d*x - c)^(1/2) + b*c^2*x^2*(d*x - c)^
(1/2))/(c^4*x*(c + d*x)^(1/2)*(c - d*x))
```

3.375 $\int \frac{a+bx^2}{x^3(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx$

Optimal. Leaf size=117

$$-\frac{2bc^2+3ad^2}{2c^4\sqrt{-c+dx}\,\sqrt{c+dx}}+\frac{a}{2c^2x^2\sqrt{-c+dx}\,\sqrt{c+dx}}-\frac{(2bc^2+3ad^2)\tan^{-1}\left(\frac{\sqrt{-c+dx}\,\sqrt{c+dx}}{c}\right)}{2c^5}$$

```
[Out] -1/2*(3*a*d^2+2*b*c^2)*arctan((d*x-c)^(1/2)*(d*x+c)^(1/2)/c)/c^5+1/2*(-3*a*
d^2-2*b*c^2)/c^4/(d*x-c)^(1/2)/(d*x+c)^(1/2)+1/2*a/c^2/x^2/(d*x-c)^(1/2)/(d
*x+c)^(1/2)
```

Rubi [A]

time = 0.07, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {465, 106, 21, 94, 211}

$$-\frac{(3ad^2+2bc^2)\,\text{ArcTan}\left(\frac{\sqrt{dx-c}\,\sqrt{c+dx}}{c}\right)}{2c^5}-\frac{3ad^2+2bc^2}{2c^4\sqrt{dx-c}\,\sqrt{c+dx}}+\frac{a}{2c^2x^2\sqrt{dx-c}\,\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)/(x^3*(-c + d*x)^(3/2)*(c + d*x)^(3/2)),x]
```

```
[Out] -1/2*(2*b*c^2 + 3*a*d^2)/(c^4*Sqrt[-c + d*x]*Sqrt[c + d*x]) + a/(2*c^2*x^2*
Sqrt[-c + d*x]*Sqrt[c + d*x]) - ((2*b*c^2 + 3*a*d^2)*ArcTan[(Sqrt[-c + d*x]
*Sqrt[c + d*x])/c])/(2*c^5)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 106

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
```

```
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegersQ
[2*m, 2*n, 2*p]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 465

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[c*(e*x)^(
m + 1)*(a1 + b1*x^(n/2))^(p + 1)*((a2 + b2*x^(n/2))^(p + 1)/(a1*a2*e*(m + 1
))), x] + Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/(a1*a2*e^n*(
m + 1)), Int[(e*x)^(m + n)*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/2))^p, x], x]
/; FreeQ[{a1, b1, a2, b2, c, d, e, p}, x] && EqQ[non2, n/2] && EqQ[a2*b1 +
a1*b2, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (L
tQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\int \frac{a+bx^2}{x^3(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx = \frac{a}{2c^2x^2\sqrt{-c+dx}\,\sqrt{c+dx}} + \frac{1}{2}\left(2b+\frac{3ad^2}{c^2}\right)\int \frac{1}{x(-c+dx)^{3/2}(c+dx)^{3/2}}$$

$$= -\frac{2bc^2+3ad^2}{2c^4\sqrt{-c+dx}\,\sqrt{c+dx}} + \frac{a}{2c^2x^2\sqrt{-c+dx}\,\sqrt{c+dx}} + \frac{\left(-2b-\frac{3ad^2}{c^2}\right)}{}$$

$$= -\frac{2bc^2+3ad^2}{2c^4\sqrt{-c+dx}\,\sqrt{c+dx}} + \frac{a}{2c^2x^2\sqrt{-c+dx}\,\sqrt{c+dx}} - \frac{(2bc^2+3ad^2)}{}$$

$$= -\frac{2bc^2+3ad^2}{2c^4\sqrt{-c+dx}\,\sqrt{c+dx}} + \frac{a}{2c^2x^2\sqrt{-c+dx}\,\sqrt{c+dx}} - \frac{(d(2bc^2+3ad}{}$$

$$= -\frac{2bc^2+3ad^2}{2c^4\sqrt{-c+dx}\,\sqrt{c+dx}} + \frac{a}{2c^2x^2\sqrt{-c+dx}\,\sqrt{c+dx}} - \frac{(2bc^2+3ad^2)}{}$$

Mathematica [**A**]

time = 0.20, size = 93, normalized size = 0.79

$$\frac{\frac{-2bc^3x^2+a(c^3-3cd^2x^2)}{x^2\sqrt{-c+dx}\,\sqrt{c+dx}}+(4bc^2+6ad^2)\tan^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{-c+dx}}\right)}{2c^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)/(x^3*(-c + d*x)^(3/2)*(c + d*x)^(3/2)),x]
```

```
[Out] ((-2*b*c^3*x^2 + a*(c^3 - 3*c*d^2*x^2))/(x^2*Sqrt[-c + d*x]*Sqrt[c + d*x])
+ (4*b*c^2 + 6*a*d^2)*ArcTan[Sqrt[c + d*x]/Sqrt[-c + d*x]])/(2*c^5)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 323 vs. 2(99) = 198.
time = 0.34, size = 324, normalized size = 2.77

method	result
default	$\frac{\sqrt{dx-c}\left(-3\ln\left(-\frac{2\left(c^2-\sqrt{-c^2}\,\sqrt{d^2x^2-c^2}\right)}{x}\right)a\,d^4x^4-2\ln\left(-\frac{2\left(c^2-\sqrt{-c^2}\,\sqrt{d^2x^2-c^2}\right)}{x}\right)b\,c^2d^2x^4+3\ln\left(-\right.\right.}{}$
risch	$\frac{a(-dx+c)\sqrt{dx+c}}{2c^4x^2\sqrt{dx-c}}-\frac{\left(-\frac{d\sqrt{d^2\left(x+\frac{c}{d}\right)^2-2cd\left(x+\frac{c}{d}\right)}\,a}{2c^5\left(x+\frac{c}{d}\right)}-\frac{\sqrt{d^2\left(x+\frac{c}{d}\right)^2-2cd\left(x+\frac{c}{d}\right)}\,b}{2c^3d\left(x+\frac{c}{d}\right)}+\frac{d\sqrt{d^2\left(\right.}}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)/x^3/(d*x-c)^(3/2)/(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(d*x-c)^(1/2)/c^4*(-3*ln(-2*(c^2-(-c^2)^(1/2)*(d^2*x^2-c^2)^(1/2))/x)*a
*d^4*x^4-2*ln(-2*(c^2-(-c^2)^(1/2)*(d^2*x^2-c^2)^(1/2))/x)*b*c^2*d^2*x^4+3*
ln(-2*(c^2-(-c^2)^(1/2)*(d^2*x^2-c^2)^(1/2))/x)*a*c^2*d^2*x^2+2*ln(-2*(c^2-
(-c^2)^(1/2)*(d^2*x^2-c^2)^(1/2))/x)*b*c^4*x^2+3*(d^2*x^2-c^2)^(1/2)*(-c^2)
^(1/2)*a*d^2*x^2+2*(d^2*x^2-c^2)^(1/2)*(-c^2)^(1/2)*b*c^2*x^2-(d^2*x^2-c^2)
^(1/2)*(-c^2)^(1/2)*a*c^2)/(-c^2)^(1/2)/x^2/(-d*x+c)/(d^2*x^2-c^2)^(1/2)/(d
*x+c)^(1/2)
```

Maxima [A]
time = 0.50, size = 104, normalized size = 0.89

$$\frac{b\arcsin\left(\frac{c}{d|x|}\right)}{c^3}+\frac{3\,ad^2\arcsin\left(\frac{c}{d|x|}\right)}{2\,c^5}-\frac{b}{\sqrt{d^2x^2-c^2}\,c^2}-\frac{3\,ad^2}{2\,\sqrt{d^2x^2-c^2}\,c^4}+\frac{a}{2\,\sqrt{d^2x^2-c^2}\,c^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^3/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] b*arcsin(c/(d*abs(x)))/c^3 + 3/2*a*d^2*arcsin(c/(d*abs(x)))/c^5 - b/(sqrt(d
^2*x^2 - c^2)*c^2) - 3/2*a*d^2/(sqrt(d^2*x^2 - c^2)*c^4) + 1/2*a/(sqrt(d^2*
x^2 - c^2)*c^2*x^2)
```

Fricas [**A**]
time = 4.14, size = 138, normalized size = 1.18

$$\frac{(ac^3 - (2\,bc^3 + 3\,acd^2)x^2)\sqrt{dx+c}\,\sqrt{dx-c} - 2\,((2\,bc^2d^2 + 3\,ad^4)x^4 - (2\,bc^4 + 3\,ac^2d^2)x^2)\arctan\left(-\frac{dx - \sqrt{dx+c}\,\sqrt{dx-c}}{c}\right)}{2\,(c^5d^2x^4 - c^7x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^3/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*((a*c^3 - (2*b*c^3 + 3*a*c*d^2)*x^2)*sqrt(d*x + c)*sqrt(d*x - c) - 2*((
2*b*c^2*d^2 + 3*a*d^4)*x^4 - (2*b*c^4 + 3*a*c^2*d^2)*x^2)*arctan(-(d*x - sq
rt(d*x + c)*sqrt(d*x - c))/c))/(c^5*d^2*x^4 - c^7*x^2)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/x**3/(d*x-c)**(3/2)/(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 211 vs. 2(99) = 198.
time = 0.71, size = 211, normalized size = 1.80

$$\frac{(2\,bc^2 + 3\,ad^2)\arctan\left(\frac{\left(\sqrt{dx+c} - \sqrt{dx-c}\right)^2}{2\,c}\right)}{c^5} - \frac{(bc^2 + ad^2)\sqrt{dx+c}}{2\,\sqrt{dx-c}\,c^5} + \frac{2\,(bc^2 + ad^2)}{\left(\left(\sqrt{dx+c} - \sqrt{dx-c}\right)^2 + 2\,c\right)c^4} + \frac{2\left(ad^2\left(\sqrt{dx+c} - \sqrt{dx-c}\right)^6 - 4\,ac^2d^2\left(\sqrt{dx+c} - \sqrt{dx-c}\right)^2\right)}{\left(\left(\sqrt{dx+c} - \sqrt{dx-c}\right)^4 + 4\,c^2\right)^2 c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^3/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] (2*b*c^2 + 3*a*d^2)*arctan(1/2*(sqrt(d*x + c) - sqrt(d*x - c))^2/c)/c^5 - 1
/2*(b*c^2 + a*d^2)*sqrt(d*x + c)/(sqrt(d*x - c)*c^5) + 2*(b*c^2 + a*d^2)/((
(sqrt(d*x + c) - sqrt(d*x - c))^2 + 2*c)*c^4) + 2*(a*d^2*(sqrt(d*x + c) - s
qrt(d*x - c))^6 - 4*a*c^2*d^2*(sqrt(d*x + c) - sqrt(d*x - c))^2)/(((sqrt(d*
x + c) - sqrt(d*x - c))^4 + 4*c^2)^2*c^4)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{b\,x^2+a}{x^3\,{\left(c+d\,x\right)}^{3/2}\,{\left(d\,x-c\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)/(x^3*(c + d*x)^(3/2)*(d*x - c)^(3/2)),x)
[Out] int((a + b*x^2)/(x^3*(c + d*x)^(3/2)*(d*x - c)^(3/2)), x)
```

3.376 $\int \frac{a+bx^2}{x^4(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx$

Optimal. Leaf size=119

$$\frac{a}{3c^2x^3\sqrt{-c+dx}\,\sqrt{c+dx}}+\frac{3bc^2+4ad^2}{3c^4x\sqrt{-c+dx}\,\sqrt{c+dx}}-\frac{2d^2(3bc^2+4ad^2)\,x}{3c^6\sqrt{-c+dx}\,\sqrt{c+dx}}$$

```
[Out] 1/3*a/c^2/x^3/(d*x-c)^(1/2)/(d*x+c)^(1/2)+1/3*(4*a*d^2+3*b*c^2)/c^4/x/(d*x-
c)^(1/2)/(d*x+c)^(1/2)-2/3*d^2*(4*a*d^2+3*b*c^2)*x/c^6/(d*x-c)^(1/2)/(d*x+c
)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {465, 105, 12, 39}

$$-\frac{2d^2x(4ad^2+3bc^2)}{3c^6\sqrt{dx-c}\,\sqrt{c+dx}}+\frac{4ad^2+3bc^2}{3c^4x\sqrt{dx-c}\,\sqrt{c+dx}}+\frac{a}{3c^2x^3\sqrt{dx-c}\,\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)/(x^4*(-c + d*x)^(3/2)*(c + d*x)^(3/2)),x]
```

```
[Out] a/(3*c^2*x^3*Sqrt[-c + d*x]*Sqrt[c + d*x]) + (3*b*c^2 + 4*a*d^2)/(3*c^4*x*S
qrt[-c + d*x]*Sqrt[c + d*x]) - (2*d^2*(3*b*c^2 + 4*a*d^2)*x)/(3*c^6*Sqrt[-c
 + d*x]*Sqrt[c + d*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 39

```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 465

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[c*(e*x)^(
m + 1)*(a1 + b1*x^(n/2))^(p + 1)*((a2 + b2*x^(n/2))^(p + 1)/(a1*a2*e*(m + 1
))), x] + Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/(a1*a2*e^n*(
m + 1)), Int[(e*x)^(m + n)*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/2))^p, x], x]
/; FreeQ[{a1, b1, a2, b2, c, d, e, p}, x] && EqQ[non2, n/2] && EqQ[a2*b1 +
a1*b2, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (L
tQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx^2}{x^4(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx &= \frac{a}{3c^2x^3\sqrt{-c+dx}\,\sqrt{c+dx}}+\frac{1}{3}\left(3b+\frac{4ad^2}{c^2}\right)\int \frac{1}{x^2(-c+dx)^{3/2}(c+dx)\\
&= \frac{a}{3c^2x^3\sqrt{-c+dx}\,\sqrt{c+dx}}+\frac{3bc^2+4ad^2}{3c^4x\sqrt{-c+dx}\,\sqrt{c+dx}}+\frac{\left(3b+\frac{4ad^2}{c^2}\right)\int}{}\\
&= \frac{a}{3c^2x^3\sqrt{-c+dx}\,\sqrt{c+dx}}+\frac{3bc^2+4ad^2}{3c^4x\sqrt{-c+dx}\,\sqrt{c+dx}}+\frac{\left(2d^2\left(3b+\frac{4a}{c}\right.\right.}{}\\
&= \frac{a}{3c^2x^3\sqrt{-c+dx}\,\sqrt{c+dx}}+\frac{3bc^2+4ad^2}{3c^4x\sqrt{-c+dx}\,\sqrt{c+dx}}-\frac{2d^2(3bc^2+}{3c^6\sqrt{-c+dx}}\end{aligned}$$

Mathematica [A]
time = 0.13, size = 77, normalized size = 0.65

$$\frac{3bc^2x^2(c^2-2d^2x^2)+a(c^4+4c^2d^2x^2-8d^4x^4)}{3c^6x^3\sqrt{-c+dx}\,\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)/(x^4*(-c + d*x)^(3/2)*(c + d*x)^(3/2)),x]
```

```
[Out] (3*b*c^2*x^2*(c^2 - 2*d^2*x^2) + a*(c^4 + 4*c^2*d^2*x^2 - 8*d^4*x^4))/(3*c^
6*x^3*Sqrt[-c + d*x]*Sqrt[c + d*x])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 2.
time = 0.30, size = 85, normalized size = 0.71

method	result	size
gosper	$\frac{-8a\,d^4x^4-6b\,c^2d^2x^4+4a\,c^2d^2x^2+3b\,c^4x^2+a\,c^4}{3\sqrt{dx+c}\;x^3c^6\sqrt{dx-c}}$	73

default	$-\frac{\sqrt{dx-c}\,\mathrm{csgn}(d)^{2}\left(-8a\,d^{4}x^{4}-6b\,c^{2}d^{2}x^{4}+4a\,c^{2}d^{2}x^{2}+3b\,c^{4}x^{2}+a\,c^{4}\right)}{3c^{6}x^{3}(-dx+c)\sqrt{dx+c}}$	85
risch	$\frac{\sqrt{dx+c}\,(-dx+c)\left(5a\,d^{2}x^{2}+3b\,c^{2}x^{2}+c^{2}a\right)}{3c^{6}x^{3}\sqrt{dx-c}}-\frac{d^{2}\left(a\,d^{2}+b\,c^{2}\right)x\sqrt{(dx-c)\,(dx+c)}}{\sqrt{-(dx+c)\,(-dx+c)}\,c^{6}\sqrt{dx-c}\,\sqrt{dx+c}}$	122

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)/x^4/(d*x-c)^(3/2)/(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*(d*x-c)^(1/2)*csgn(d)^2/c^6*(-8*a*d^4*x^4-6*b*c^2*d^2*x^4+4*a*c^2*d^2*
x^2+3*b*c^4*x^2+a*c^4)/x^3/(-d*x+c)/(d*x+c)^(1/2)
```

Maxima [A]
time = 0.55, size = 125, normalized size = 1.05

$$-\frac{2\,bd^{2}x}{\sqrt{d^{2}x^{2}-c^{2}}\,c^{4}}-\frac{8\,ad^{4}x}{3\,\sqrt{d^{2}x^{2}-c^{2}}\,c^{6}}+\frac{b}{\sqrt{d^{2}x^{2}-c^{2}}\,c^{2}x}+\frac{4\,ad^{2}}{3\,\sqrt{d^{2}x^{2}-c^{2}}\,c^{4}x}+\frac{a}{3\,\sqrt{d^{2}x^{2}-c^{2}}\,c^{2}x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^4/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] -2*b*d^2*x/(sqrt(d^2*x^2 - c^2)*c^4) - 8/3*a*d^4*x/(sqrt(d^2*x^2 - c^2)*c^6
) + b/(sqrt(d^2*x^2 - c^2)*c^2*x) + 4/3*a*d^2/(sqrt(d^2*x^2 - c^2)*c^4*x) +
 1/3*a/(sqrt(d^2*x^2 - c^2)*c^2*x^3)
```

Fricas [A]
time = 3.95, size = 132, normalized size = 1.11

$$-\frac{2\,(3\,bc^{2}d^{3}+4\,ad^{5})x^{5}-2\,(3\,bc^{4}d+4\,ac^{2}d^{3})x^{3}-(ac^{4}-2\,(3\,bc^{2}d^{2}+4\,ad^{4})x^{4}+(3\,bc^{4}+4\,ac^{2}d^{2})x^{2})\sqrt{dx+c}\,\sqrt{dx-c}}{3\,(c^{6}d^{2}x^{5}-c^{8}x^{3})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^4/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/3*(2*(3*b*c^2*d^3 + 4*a*d^5)*x^5 - 2*(3*b*c^4*d + 4*a*c^2*d^3)*x^3 - (a*
c^4 - 2*(3*b*c^2*d^2 + 4*a*d^4)*x^4 + (3*b*c^4 + 4*a*c^2*d^2)*x^2)*sqrt(d*x
 + c)*sqrt(d*x - c))/(c^6*d^2*x^5 - c^8*x^3)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/x**4/(d*x-c)**(3/2)/(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 242 vs. 2(101) = 202.
time = 0.87, size = 242, normalized size = 2.03

$$-\frac{(bc^2d+ad^3)\sqrt{dx+c}}{2\sqrt{dx-c}\,c^6}-\frac{2\,(bc^2d+ad^3)}{\left(\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^2+2\,c\right)c^5}-\frac{8\left(3\,bc^2d\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^8+3\,ad^3\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^8+24\,bc^4d\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^4+48\,ac^2d^3\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^4+48\,bc^6d+80\,ac^4d^3\right)}{3\left(\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^4+4\,c^2\right)^3c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^4/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] -1/2*(b*c^2*d + a*d^3)*sqrt(d*x + c)/(sqrt(d*x - c)*c^6) - 2*(b*c^2*d + a*d
^3)/(((sqrt(d*x + c) - sqrt(d*x - c))^2 + 2*c)*c^5) - 8/3*(3*b*c^2*d*(sqrt(
d*x + c) - sqrt(d*x - c))^8 + 3*a*d^3*(sqrt(d*x + c) - sqrt(d*x - c))^8 + 2
4*b*c^4*d*(sqrt(d*x + c) - sqrt(d*x - c))^4 + 48*a*c^2*d^3*(sqrt(d*x + c) -
 sqrt(d*x - c))^4 + 48*b*c^6*d + 80*a*c^4*d^3)/(((sqrt(d*x + c) - sqrt(d*x
- c))^4 + 4*c^2)^3*c^4)
```

Mupad [B]
time = 2.90, size = 104, normalized size = 0.87

$$\frac{\sqrt{d\,x-c}\,\left(\frac{a}{3\,c^2\,d}+\frac{x^2\,\left(3\,b\,c^4+4\,a\,c^2\,d^2\right)}{3\,c^6\,d}-\frac{x^4\,\left(6\,b\,c^2\,d^2+8\,a\,d^4\right)}{3\,c^6\,d}\right)}{x^4\,\sqrt{c+d\,x}-\frac{c\,x^3\,\sqrt{c+d\,x}}{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)/(x^4*(c + d*x)^(3/2)*(d*x - c)^(3/2)),x)
```

```
[Out] ((d*x - c)^(1/2)*(a/(3*c^2*d) + (x^2*(3*b*c^4 + 4*a*c^2*d^2))/(3*c^6*d) - (
x^4*(8*a*d^4 + 6*b*c^2*d^2))/(3*c^6*d)))/(x^4*(c + d*x)^(1/2) - (c*x^3*(c +
 d*x)^(1/2))/d)
```

3.377 $\int \frac{a+bx^2}{x^5(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx$

Optimal. Leaf size=166

$$-\frac{3d^2(4bc^2+5ad^2)}{8c^6\sqrt{-c+dx}\,\sqrt{c+dx}}+\frac{a}{4c^2x^4\sqrt{-c+dx}\,\sqrt{c+dx}}+\frac{4bc^2+5ad^2}{8c^4x^2\sqrt{-c+dx}\,\sqrt{c+dx}}-\frac{3d^2(4bc^2+5ad^2)\tan^{-1}$$

```
[Out] -3/8*d^2*(5*a*d^2+4*b*c^2)*arctan((d*x-c)^(1/2)*(d*x+c)^(1/2)/c)/c^7-3/8*d^
2*(5*a*d^2+4*b*c^2)/c^6/(d*x-c)^(1/2)/(d*x+c)^(1/2)+1/4*a/c^2/x^4/(d*x-c)^(
1/2)/(d*x+c)^(1/2)+1/8*(5*a*d^2+4*b*c^2)/c^4/x^2/(d*x-c)^(1/2)/(d*x+c)^(1/2
)
```

Rubi [A]
time = 0.08, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {465, 105, 12, 106, 21, 94, 211}

$$-\frac{3d^2(5ad^2+4bc^2)\,\text{ArcTan}\left(\frac{\sqrt{dx-c}\,\sqrt{c+dx}}{c}\right)}{8c^7}-\frac{3d^2(5ad^2+4bc^2)}{8c^6\sqrt{dx-c}\,\sqrt{c+dx}}+\frac{5ad^2+4bc^2}{8c^4x^2\sqrt{dx-c}\,\sqrt{c+dx}}+\frac{a}{4c^2x^4\sqrt{dx-c}\,\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)/(x^5*(-c + d*x)^(3/2)*(c + d*x)^(3/2)),x]
```

```
[Out] (-3*d^2*(4*b*c^2 + 5*a*d^2))/(8*c^6*Sqrt[-c + d*x]*Sqrt[c + d*x]) + a/(4*c^
2*x^4*Sqrt[-c + d*x]*Sqrt[c + d*x]) + (4*b*c^2 + 5*a*d^2)/(8*c^4*x^2*Sqrt[-
c + d*x]*Sqrt[c + d*x]) - (3*d^2*(4*b*c^2 + 5*a*d^2)*ArcTan[(Sqrt[-c + d*x]
*Sqrt[c + d*x])/c])/(8*c^7)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
```

```
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 106

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegersQ
[2*m, 2*n, 2*p]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 465

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[c*(e*x)^(
m + 1)*(a1 + b1*x^(n/2))^(p + 1)*((a2 + b2*x^(n/2))^(p + 1)/(a1*a2*e*(m + 1
))), x] + Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/(a1*a2*e^n*(
m + 1)), Int[(e*x)^(m + n)*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/2))^p, x], x]
/; FreeQ[{a1, b1, a2, b2, c, d, e, p}, x] && EqQ[non2, n/2] && EqQ[a2*b1 +
a1*b2, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (L
tQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\int \frac{a+bx^2}{x^5(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx = \frac{a}{4c^2x^4\sqrt{-c+dx}\,\sqrt{c+dx}} + \frac{1}{4}\left(4b+\frac{5ad^2}{c^2}\right)\int \frac{1}{x^3(-c+dx)^{3/2}(c+dx)^{3/}}$$

$$= \frac{a}{4c^2x^4\sqrt{-c+dx}\,\sqrt{c+dx}} + \frac{4bc^2+5ad^2}{8c^4x^2\sqrt{-c+dx}\,\sqrt{c+dx}} + \frac{(4bc^2+5ad^2)}{}$$

$$= \frac{a}{4c^2x^4\sqrt{-c+dx}\,\sqrt{c+dx}} + \frac{4bc^2+5ad^2}{8c^4x^2\sqrt{-c+dx}\,\sqrt{c+dx}} + \frac{(3d^2(4bc^2+5}{}$$

$$= -\frac{3d^2(4bc^2+5ad^2)}{8c^6\sqrt{-c+dx}\,\sqrt{c+dx}} + \frac{a}{4c^2x^4\sqrt{-c+dx}\,\sqrt{c+dx}} + \frac{4bc^2+}{8c^4x^2\sqrt{-c+d}}$$

$$= -\frac{3d^2(4bc^2+5ad^2)}{8c^6\sqrt{-c+dx}\,\sqrt{c+dx}} + \frac{a}{4c^2x^4\sqrt{-c+dx}\,\sqrt{c+dx}} + \frac{4bc^2+}{8c^4x^2\sqrt{-c+d}}$$

$$= -\frac{3d^2(4bc^2+5ad^2)}{8c^6\sqrt{-c+dx}\,\sqrt{c+dx}} + \frac{a}{4c^2x^4\sqrt{-c+dx}\,\sqrt{c+dx}} + \frac{4bc^2+}{8c^4x^2\sqrt{-c+d}}$$

$$= -\frac{3d^2(4bc^2+5ad^2)}{8c^6\sqrt{-c+dx}\,\sqrt{c+dx}} + \frac{a}{4c^2x^4\sqrt{-c+dx}\,\sqrt{c+dx}} + \frac{4bc^2+}{8c^4x^2\sqrt{-c+d}}$$

Mathematica [A]
time = 0.28, size = 122, normalized size = 0.73

$$\frac{\frac{4bc^3x^2(c^2-3d^2x^2)+a(2c^5+5c^3d^2x^2-15cd^4x^4)}{x^4\sqrt{-c+dx}\,\sqrt{c+dx}} + 6d^2(4bc^2+5ad^2)\tan^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{-c+dx}}\right)}{8c^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)/(x^5*(-c + d*x)^(3/2)*(c + d*x)^(3/2)),x]
```

```
[Out] ((4*b*c^3*x^2*(c^2 - 3*d^2*x^2) + a*(2*c^5 + 5*c^3*d^2*x^2 - 15*c*d^4*x^4))
/(x^4*Sqrt[-c + d*x]*Sqrt[c + d*x]) + 6*d^2*(4*b*c^2 + 5*a*d^2)*ArcTan[Sqrt
[c + d*x]/Sqrt[-c + d*x]])/(8*c^7)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 394 vs. 2(142) = 284.
time = 0.34, size = 395, normalized size = 2.38

method	result

risch	$\frac{\sqrt{dx+c}\,(-dx+c)(7a\,d^2x^2+4b\,c^2x^2+2c^2a)}{8c^6x^4\sqrt{dx-c}} - \frac{\left(-\frac{d^3\sqrt{d^2\left(x+\frac{c}{d}\right)^2-2cd\left(x+\frac{c}{d}\right)}\,a}{2c^7\left(x+\frac{c}{d}\right)} - \frac{d\sqrt{d^2\left(x+\frac{c}{d}\right)^2-2cd}}{2c^5\left(x+\frac{c}{d}\right)}\right.}{}$
default	$\frac{\sqrt{dx-c}\left(-15\ln\left(-\frac{2\left(c^2-\sqrt{-c^2}\,\sqrt{d^2x^2-c^2}\right)}{x}\right)a\,d^6x^6-12\ln\left(-\frac{2\left(c^2-\sqrt{-c^2}\,\sqrt{d^2x^2-c^2}\right)}{x}\right)b\,c^2d^4x^6+15\ln\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)/x^5/(d*x-c)^(3/2)/(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*(d*x-c)^(1/2)/c^6*(-15*ln(-2*(c^2-(-c^2)^(1/2)*(d^2*x^2-c^2)^(1/2))/x)*
a*d^6*x^6-12*ln(-2*(c^2-(-c^2)^(1/2)*(d^2*x^2-c^2)^(1/2))/x)*b*c^2*d^4*x^6+
15*ln(-2*(c^2-(-c^2)^(1/2)*(d^2*x^2-c^2)^(1/2))/x)*a*c^2*d^4*x^4+12*ln(-2*(
c^2-(-c^2)^(1/2)*(d^2*x^2-c^2)^(1/2))/x)*b*c^4*d^2*x^4+15*(d^2*x^2-c^2)^(1/
2)*(-c^2)^(1/2)*a*d^4*x^4+12*(d^2*x^2-c^2)^(1/2)*(-c^2)^(1/2)*b*c^2*d^2*x^4
-5*a*c^2*d^2*x^2*(d^2*x^2-c^2)^(1/2)*(-c^2)^(1/2)-4*b*c^4*x^2*(d^2*x^2-c^2)
^(1/2)*(-c^2)^(1/2)-2*a*c^4*(d^2*x^2-c^2)^(1/2)*(-c^2)^(1/2))/(-c^2)^(1/2)/
x^4/(-d*x+c)/(d^2*x^2-c^2)^(1/2)/(d*x+c)^(1/2)
```

Maxima [A]

time = 0.50, size = 162, normalized size = 0.98

$$\frac{3\,bd^2\arcsin\left(\frac{c}{d|x|}\right)}{2\,c^5}+\frac{15\,ad^4\arcsin\left(\frac{c}{d|x|}\right)}{8\,c^7}-\frac{3\,bd^2}{2\,\sqrt{d^2x^2-c^2}\,c^4}-\frac{15\,ad^4}{8\,\sqrt{d^2x^2-c^2}\,c^6}+\frac{b}{2\,\sqrt{d^2x^2-c^2}\,c^2x^2}+\frac{5\,ad^2}{8\,\sqrt{d^2x^2-c^2}\,c^4x^2}+\frac{a}{4\,\sqrt{d^2x^2-c^2}\,c^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^5/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] 3/2*b*d^2*arcsin(c/(d*abs(x)))/c^5 + 15/8*a*d^4*arcsin(c/(d*abs(x)))/c^7 -
3/2*b*d^2/(sqrt(d^2*x^2 - c^2)*c^4) - 15/8*a*d^4/(sqrt(d^2*x^2 - c^2)*c^6)
+ 1/2*b/(sqrt(d^2*x^2 - c^2)*c^2*x^2) + 5/8*a*d^2/(sqrt(d^2*x^2 - c^2)*c^4*
x^2) + 1/4*a/(sqrt(d^2*x^2 - c^2)*c^2*x^4)
```

Fricas [A]

time = 3.79, size = 165, normalized size = 0.99

$$\frac{(2\,ac^5-3\,(4\,bc^3d^2+5\,acd^4)x^4+(4\,bc^5+5\,ac^3d^2)x^2)\sqrt{dx+c}\,\sqrt{dx-c}-6\,((4\,bc^2d^4+5\,ad^6)x^6-(4\,bc^4d^2+5\,ac^2d^4)x^4)\arctan\left(-\frac{dx-\sqrt{dx+c}\,\sqrt{dx-c}}{c}\right)}{8\,(c^7d^2x^6-c^9x^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^5/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/8*((2*a*c^5 - 3*(4*b*c^3*d^2 + 5*a*c*d^4)*x^4 + (4*b*c^5 + 5*a*c^3*d^2)*x
^2)*sqrt(d*x + c)*sqrt(d*x - c) - 6*((4*b*c^2*d^4 + 5*a*d^6)*x^6 - (4*b*c^4
*d^2 + 5*a*c^2*d^4)*x^4)*arctan(-(d*x - sqrt(d*x + c)*sqrt(d*x - c))/c))/(c
^7*d^2*x^6 - c^9*x^4)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/x**5/(d*x-c)**(3/2)/(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 402 vs. 2(142) = 284.
time = 1.04, size = 402, normalized size = 2.42

$$\frac{3\left(4bc^2d^2+5ad^4\right)\arctan\left(\frac{\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^2}{2c}\right)}{4c^7}-\frac{\left(bc^2d^2+ad^4\right)\sqrt{dx+c}}{2\sqrt{dx-c}\,c^7}+\frac{2\left(bc^2d^2+ad^4\right)}{\left(\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^2+2c\right)c^6}+\frac{4bc^2d^2\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^{14}+7ad^4\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^{14}+16bc^4d^2\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^{10}+60ac^2d^4\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^{10}-64bc^6d^2\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^{6}-240ac^4d^4\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^{6}-256bc^8d^2\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^{2}-448ac^6d^4\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^{2}}{2\left(\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^4+4c^2\right)^4c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^5/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] 3/4*(4*b*c^2*d^2 + 5*a*d^4)*arctan(1/2*(sqrt(d*x + c) - sqrt(d*x - c))^2/c)
/c^7 - 1/2*(b*c^2*d^2 + a*d^4)*sqrt(d*x + c)/(sqrt(d*x - c)*c^7) + 2*(b*c^2
*d^2 + a*d^4)/(((sqrt(d*x + c) - sqrt(d*x - c))^2 + 2*c)*c^6) + 1/2*(4*b*c^
2*d^2*(sqrt(d*x + c) - sqrt(d*x - c))^14 + 7*a*d^4*(sqrt(d*x + c) - sqrt(d*
x - c))^14 + 16*b*c^4*d^2*(sqrt(d*x + c) - sqrt(d*x - c))^10 + 60*a*c^2*d^4
*(sqrt(d*x + c) - sqrt(d*x - c))^10 - 64*b*c^6*d^2*(sqrt(d*x + c) - sqrt(d*
x - c))^6 - 240*a*c^4*d^4*(sqrt(d*x + c) - sqrt(d*x - c))^6 - 256*b*c^8*d^2
*(sqrt(d*x + c) - sqrt(d*x - c))^2 - 448*a*c^6*d^4*(sqrt(d*x + c) - sqrt(d*
x - c))^2)/(((sqrt(d*x + c) - sqrt(d*x - c))^4 + 4*c^2)^4*c^6)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{b\,x^2+a}{x^5\,{\left(c+d\,x\right)}^{3/2}\,{\left(d\,x-c\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)/(x^5*(c + d*x)^(3/2)*(d*x - c)^(3/2)),x)
```

```
[Out] int((a + b*x^2)/(x^5*(c + d*x)^(3/2)*(d*x - c)^(3/2)), x)
```

3.378 $\int \frac{1+c^2x^2}{x\sqrt{-1+cx}\sqrt{1+cx}}\,dx$

Optimal. Leaf size=40

$$\sqrt{-1+cx}\sqrt{1+cx}+\tan^{-1}\left(\sqrt{-1+cx}\sqrt{1+cx}\right)$$

```
[Out] arctan((c*x-1)^(1/2)*(c*x+1)^(1/2))+(c*x-1)^(1/2)*(c*x+1)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {471, 94, 211}

$$\text{ArcTan}\left(\sqrt{cx-1}\sqrt{cx+1}\right)+\sqrt{cx-1}\sqrt{cx+1}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + c^2*x^2)/(x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]),x]
```

```
[Out] Sqrt[-1 + c*x]*Sqrt[1 + c*x] + ArcTan[Sqrt[-1 + c*x]*Sqrt[1 + c*x]]
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[d*(e*x)^(
m + 1)*(a1 + b1*x^(n/2))^(p + 1)*((a2 + b2*x^(n/2))^(p + 1)/(b1*b2*e*(m + n
*(p + 1) + 1))), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1+c^2x^2}{x\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx &= \sqrt{-1+cx}\,\sqrt{1+cx}+\int \frac{1}{x\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx\\ &= \sqrt{-1+cx}\,\sqrt{1+cx}+c\text{Subst}\left(\int \frac{1}{c+cx^2}\,dx,x,\sqrt{-1+cx}\,\sqrt{1+cx}\right)\\ &= \sqrt{-1+cx}\,\sqrt{1+cx}+\tan^{-1}\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\end{aligned}$$

Mathematica [A]
time = 0.06, size = 40, normalized size = 1.00

$$\sqrt{-1+cx}\,\sqrt{1+cx}+2\tan^{-1}\left(\sqrt{\frac{-1+cx}{1+cx}}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 + c^2*x^2)/(x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]),x]
```

```
[Out] Sqrt[-1 + c*x]*Sqrt[1 + c*x] + 2*ArcTan[Sqrt[(-1 + c*x)/(1 + c*x)]]
```

Maple [A]
time = 0.28, size = 53, normalized size = 1.32

method	result	size
default	$\frac{\left(\sqrt{c^2x^2-1}-\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)\right)\sqrt{cx-1}\,\sqrt{cx+1}}{\sqrt{c^2x^2-1}}$	53

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*x^2+1)/x/(c*x-1)^(1/2)/(c*x+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((c^2*x^2-1)^(1/2)-arctan(1/(c^2*x^2-1)^(1/2)))*(c*x-1)^(1/2)*(c*x+1)^(1/2)
/(c^2*x^2-1)^(1/2)
```

Maxima [A]
time = 0.49, size = 23, normalized size = 0.58

$$\sqrt{c^2x^2-1}-\arcsin\left(\frac{1}{c|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)/x/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(c^2*x^2 - 1) - arcsin(1/(c*abs(x)))
```

Fricas [A]
time = 3.19, size = 39, normalized size = 0.98

$$\sqrt{cx+1}\,\sqrt{cx-1} + 2\,\arctan\left(-cx + \sqrt{cx+1}\,\sqrt{cx-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)/x/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(c*x + 1)*sqrt(c*x - 1) + 2*arctan(-c*x + sqrt(c*x + 1)*sqrt(c*x - 1))
```

Sympy [C] Result contains complex when optimal does not.
time = 26.88, size = 148, normalized size = 3.70

$$\frac{G_{6,6}^{6,2}\left(\begin{matrix} -\frac{1}{4}, \frac{1}{4} & 0, 0, \frac{1}{2}, 1 \\ -\frac{1}{2}, -\frac{1}{4}, 0, \frac{1}{4}, \frac{1}{2}, 0 & \end{matrix} \middle| {\frac{1}{c^{2} x^{2}}} \right)}{4 \pi^{\frac{3}{2}}} - \frac{G_{6,6}^{5,3}\left(\begin{matrix} \frac{3}{4}, \frac{5}{4}, 1 & 1, 1, \frac{3}{2} \\ \frac{1}{2}, \frac{3}{4}, 1, \frac{5}{4}, \frac{3}{2} & 0 \end{matrix} \middle| {\frac{1}{c^{2} x^{2}}} \right)}{4 \pi^{\frac{3}{2}}} + \frac{i G_{6,6}^{2,6}\left(\begin{matrix} -1, -\frac{3}{4}, -\frac{1}{2}, -\frac{1}{4}, 0, 1 & \\ -\frac{3}{4}, -\frac{1}{4} & -1, -\frac{1}{2}, -\frac{1}{2}, 0 \end{matrix} \middle| {\frac{e^{2 i \pi}}{c^{2} x^{2}}} \right)}{4 \pi^{\frac{3}{2}}} + \frac{i G_{6,6}^{2,6}\left(\begin{matrix} 0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1, 1 & \\ \frac{1}{4}, \frac{3}{4} & 0, \frac{1}{2}, \frac{1}{2}, 0 \end{matrix} \middle| {\frac{e^{2 i \pi}}{c^{2} x^{2}}} \right)}{4 \pi^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*x**2+1)/x/(c*x-1)**(1/2)/(c*x+1)**(1/2),x)
```

```
[Out] meijerg(((-1/4, 1/4), (0, 0, 1/2, 1)), ((-1/2, -1/4, 0, 1/4, 1/2, 0), ()),
1/(c**2*x**2))/(4*pi**(3/2)) - meijerg(((3/4, 5/4, 1), (1, 1, 3/2)), ((1/2,
 3/4, 1, 5/4, 3/2), (0,)), 1/(c**2*x**2))/(4*pi**(3/2)) + I*meijerg(((-1, -
3/4, -1/2, -1/4, 0, 1), ()), ((-3/4, -1/4), (-1, -1/2, -1/2, 0)), exp_polar
(2*I*pi)/(c**2*x**2))/(4*pi**(3/2)) + I*meijerg(((0, 1/4, 1/2, 3/4, 1, 1),
()), ((1/4, 3/4), (0, 1/2, 1/2, 0)), exp_polar(2*I*pi)/(c**2*x**2))/(4*pi**
(3/2))
```

Giac [A]
time = 0.68, size = 40, normalized size = 1.00

$$\sqrt{cx+1}\,\sqrt{cx-1} - 2\,\arctan\left(\frac{1}{2}\left(\sqrt{cx+1} - \sqrt{cx-1}\right)^{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)/x/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] sqrt(c*x + 1)*sqrt(c*x - 1) - 2*arctan(1/2*(sqrt(c*x + 1) - sqrt(c*x - 1))^
2)
```

Mupad [B]
time = 3.65, size = 72, normalized size = 1.80

$$\sqrt{c\,x-1}\,\sqrt{c\,x+1} - \ln\left(\frac{\left(\sqrt{c\,x-1}-\mathrm{i}\right)^2}{\left(\sqrt{c\,x+1}-1\right)^2}+1\right)\,1{}\mathrm{i} + \ln\left(\frac{\sqrt{c\,x-1}-\mathrm{i}}{\sqrt{c\,x+1}-1}\right)\,1{}\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*x^2 + 1)/(x*(c*x - 1)^(1/2)*(c*x + 1)^(1/2)),x)
```

```
[Out] log((((c*x - 1)^(1/2) - 1i)/((c*x + 1)^(1/2) - 1))*1i - log((((c*x - 1)^(1/2)
 - 1i)^2/((c*x + 1)^(1/2) - 1)^2 + 1)*1i + (c*x - 1)^(1/2)*(c*x + 1)^(1/2)
```

3.379 $\int \frac{x^{-\frac{2b^2c+a^2d}{b^2c+a^2d}}(c+dx^2)}{\sqrt{-a+bx}\sqrt{a+bx}}\,dx$

Optimal. Leaf size=53

$$\left(\frac{c}{a^2}+\frac{d}{b^2}\right)x^{-\frac{b^2c}{b^2c+a^2d}}\sqrt{-a+bx}\,\sqrt{a+bx}$$

```
[Out] (c/a^2+d/b^2)*(b*x-a)^(1/2)*(b*x+a)^(1/2)/(x^(b^2*c/(a^2*d+b^2*c)))
```

Rubi [A]
time = 0.06, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 57, $\frac{\text{number of rules}}{\text{integrand size}} = 0.018$, Rules used = $\{461\}$

$$\sqrt{bx-a}\,\sqrt{a+bx}\left(\frac{c}{a^2}+\frac{d}{b^2}\right)x^{-\frac{b^2c}{a^2d+b^2c}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)/(x^((2*b^2*c + a^2*d)/(b^2*c + a^2*d))*Sqrt[-a + b*x]*Sqrt[
a + b*x]),x]
```

```
[Out] ((c/a^2 + d/b^2)*Sqrt[-a + b*x]*Sqrt[a + b*x])/x^((b^2*c)/(b^2*c + a^2*d))
```

Rule 461

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[c*(e*x)^(
m + 1)*(a1 + b1*x^(n/2))^(p + 1)*((a2 + b2*x^(n/2))^(p + 1)/(a1*a2*e*(m + 1
))), x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2, n/2] &
& EqQ[a2*b1 + a1*b2, 0] && EqQ[a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1
), 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{x^{-\frac{2b^2c+a^2d}{b^2c+a^2d}}\left(c+dx^2\right)}{\sqrt{-a+bx}\,\sqrt{a+bx}}\,dx=\left(\frac{c}{a^2}+\frac{d}{b^2}\right)x^{-\frac{b^2c}{b^2c+a^2d}}\sqrt{-a+bx}\,\sqrt{a+bx}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.
time = 10.33, size = 296, normalized size = 5.58

$$\frac{(b^2c+a^2d)\,x^{-\frac{b^2c}{b^2c+a^2d}}\sqrt{-a+bx}\,\sqrt{a+bx}\,\sqrt{1-\frac{b^2x^2}{a^2}}\left((b^2c+a^2d)\,F_1\left(-\frac{b^2c}{b^2c+a^2d};-\frac{1}{2},\frac{1}{2};\frac{a^2d}{b^2c+a^2d};\frac{bx}{a},-\frac{bx}{a}\right)+(b^2c+a^2d)\,F_1\left(-\frac{b^2c}{b^2c+a^2d};\frac{1}{2},-\frac{1}{2};\frac{a^2d}{b^2c+a^2d};\frac{bx}{a},-\frac{bx}{a}\right)-2a^2d\,{}_2F_1\left(-\frac{1}{2},-\frac{b^2c}{2(b^2c+a^2d)};1-\frac{b^2c}{2(b^2c+a^2d)};\frac{b^2x^2}{a^2}\right)\right)}{2b^4c\,(a^2-b^2x^2)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x^2)/(x^((2*b^2*c + a^2*d)/(b^2*c + a^2*d))*Sqrt[-a + b*x]
*Sqrt[a + b*x]),x]
```

```
[Out] ((b^2*c + a^2*d)*Sqrt[-a + b*x]*Sqrt[a + b*x]*Sqrt[1 - (b^2*x^2)/a^2]*((b^2
*c + a^2*d)*AppellF1[-((b^2*c)/(b^2*c + a^2*d)), -1/2, 1/2, (a^2*d)/(b^2*c
+ a^2*d), (b*x)/a, -((b*x)/a)] + (b^2*c + a^2*d)*AppellF1[-((b^2*c)/(b^2*c
+ a^2*d)), 1/2, -1/2, (a^2*d)/(b^2*c + a^2*d), (b*x)/a, -((b*x)/a)] - 2*a^2
*d*Hypergeometric2F1[-1/2, -1/2*(b^2*c)/(b^2*c + a^2*d), 1 - (b^2*c)/(2*(b^
2*c + a^2*d)), (b^2*x^2)/a^2]))/(2*b^4*c*x^((b^2*c)/(b^2*c + a^2*d))*(a^2 -
 b^2*x^2))
```

Maple [A]
time = 0.29, size = 66, normalized size = 1.25

method	result	size
gosper	$\frac{x\left(a^{2} d+b^{2} c\right) \sqrt{b x+a}\, x^{-\frac{a^{2} d+2 b^{2} c}{a^{2} d+b^{2} c}} \sqrt{b x-a}}{a^{2} b^{2}}$	66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)/(x^((a^2*d+2*b^2*c)/(a^2*d+b^2*c)))/(b*x-a)^(1/2)/(b*x+a)^(1/
2),x,method=_RETURNVERBOSE)
```

```
[Out] x*(a^2*d+b^2*c)*(b*x+a)^(1/2)/a^2/b^2/(x^((a^2*d+2*b^2*c)/(a^2*d+b^2*c)))*(
b*x-a)^(1/2)
```

Maxima [A]
time = 0.37, size = 79, normalized size = 1.49

$$\frac{\left(b^{2} c+a^{2} d\right) \sqrt{b x+a} \sqrt{b x-a}\, x e^{\left(-\frac{2 b^{2} c \log(x)}{b^{2} c+a^{2} d}-\frac{a^{2} d \log(x)}{b^{2} c+a^{2} d}\right)}}{a^{2} b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(x^((a^2*d+2*b^2*c)/(a^2*d+b^2*c)))/(b*x-a)^(1/2)/(b*x+
a)^(1/2),x, algorithm="maxima")
```

```
[Out] (b^2*c + a^2*d)*sqrt(b*x + a)*sqrt(b*x - a)*x*e^(-2*b^2*c*log(x)/(b^2*c + a
^2*d) - a^2*d*log(x)/(b^2*c + a^2*d))/(a^2*b^2)
```

Fricas [A]
time = 2.72, size = 65, normalized size = 1.23

$$\frac{\left(b^{2} c+a^{2} d\right) \sqrt{b x+a} \sqrt{b x-a}\, x}{a^{2} b^{2} x^{\frac{2 b^{2} c+a^{2} d}{b^{2} c+a^{2} d}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(x^((a^2*d+2*b^2*c)/(a^2*d+b^2*c)))/(b*x-a)^(1/2)/(b*x+
a)^(1/2),x, algorithm="fricas")
```

```
[Out] (b^2*c + a^2*d)*sqrt(b*x + a)*sqrt(b*x - a)*x/(a^2*b^2*x^((2*b^2*c + a^2*d)
/(b^2*c + a^2*d)))
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)/(x**((a**2*d+2*b**2*c)/(a**2*d+b**2*c)))/(b*x-a)**(1/2
)/(b*x+a)**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3437 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(x^((a^2*d+2*b^2*c)/(a^2*d+b^2*c)))/(b*x-a)^(1/2)/(b*x+
a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)/(sqrt(b*x + a)*sqrt(b*x - a)*x^((2*b^2*c + a^2*d)/(b^
2*c + a^2*d))), x)
```

Mupad [**B**]
time = 3.27, size = 96, normalized size = 1.81

$$-\frac{\frac{x\,\left(d\,a^4+c\,a^2\,b^2\right)}{a^2\,b^2}-\frac{x^3\,\left(d\,a^2\,b^2+c\,b^4\right)}{a^2\,b^2}}{x^{\frac{d\,a^2+2\,c\,b^2}{d\,a^2+c\,b^2}}\,\sqrt{a+b\,x}\,\sqrt{b\,x-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)/(x^((a^2*d + 2*b^2*c)/(a^2*d + b^2*c))*(a + b*x)^(1/2)*(b*x
 - a)^(1/2)),x)
```

```
[Out] -((x*(a^4*d + a^2*b^2*c))/(a^2*b^2) - (x^3*(b^4*c + a^2*b^2*d))/(a^2*b^2))/
(x^((a^2*d + 2*b^2*c)/(a^2*d + b^2*c))*(a + b*x)^(1/2)*(b*x - a)^(1/2))
```

3.380 $\int \frac{1}{\sqrt{-1-\sqrt{x}}\,\sqrt{-1+\sqrt{x}}\,\sqrt{1+x}}\,dx$

Optimal. Leaf size=36

$$\frac{\sqrt{1-x}\,\sin^{-1}(x)}{\sqrt{-1-\sqrt{x}}\,\sqrt{-1+\sqrt{x}}}$$

```
[Out] arcsin(x)*(1-x)^(1/2)/(-1-x^(1/2))^(1/2)/(-1+x^(1/2))^(1/2)
```

Rubi [A]

time = 0.01, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$, Rules used = {533, 41, 222}

$$\frac{\sqrt{1-x}\,\text{ArcSin}(x)}{\sqrt{-\sqrt{x}-1}\,\sqrt{\sqrt{x}-1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-1 - Sqrt[x]]*Sqrt[-1 + Sqrt[x]]*Sqrt[1 + x]),x]
```

```
[Out] (Sqrt[1 - x]*ArcSin[x])/(Sqrt[-1 - Sqrt[x]]*Sqrt[-1 + Sqrt[x]])
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 533

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_), x_Symbol] :> Dist[(a1 + b1*x^(n/2))
^FracPart[p]*((a2 + b2*x^(n/2))^FracPart[p]/(a1*a2 + b1*b2*x^n)^FracPart[p]
), Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a1, b1, a2,
b2, c, d, n, p, q}, x] && EqQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] &&  !(EqQ
[n, 2] && IGtQ[q, 0])
```

Rubi steps

$$\int \frac{1}{\sqrt{-1-\sqrt{x}}\,\sqrt{-1+\sqrt{x}}\,\sqrt{1+x}}\,dx = \frac{\sqrt{1-x}\int \frac{1}{\sqrt{1-x}\,\sqrt{1+x}}\,dx}{\sqrt{-1-\sqrt{x}}\,\sqrt{-1+\sqrt{x}}}$$
$$= \frac{\sqrt{1-x}\int \frac{1}{\sqrt{1-x^2}}\,dx}{\sqrt{-1-\sqrt{x}}\,\sqrt{-1+\sqrt{x}}}$$
$$= \frac{\sqrt{1-x}\,\sin^{-1}(x)}{\sqrt{-1-\sqrt{x}}\,\sqrt{-1+\sqrt{x}}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 7.18, size = 44, normalized size = 1.22

$$-i\log\left(-x+i\sqrt{-1-\sqrt{x}}\,\sqrt{-1+\sqrt{x}}\,\sqrt{1+x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-1 - Sqrt[x]]*Sqrt[-1 + Sqrt[x]]*Sqrt[1 + x]),x]
[Out] (-I)*Log[-x + I*Sqrt[-1 - Sqrt[x]]*Sqrt[-1 + Sqrt[x]]*Sqrt[1 + x]]
```

Maple [F]
time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x+1}\,\sqrt{-1-\sqrt{x}}\,\sqrt{-1+\sqrt{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x+1)^(1/2)/(-1-x^(1/2))^(1/2)/(-1+x^(1/2))^(1/2),x)
[Out] int(1/(x+1)^(1/2)/(-1-x^(1/2))^(1/2)/(-1+x^(1/2))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)^(1/2)/(-1-x^(1/2))^(1/2)/(-1+x^(1/2))^(1/2),x, algorithm=
"maxima")
```

```
[Out] integrate(1/(sqrt(x + 1)*sqrt(sqrt(x) - 1)*sqrt(-sqrt(x) - 1)), x)
```

Fricas [C] Result contains complex when optimal does not.
time = 1.90, size = 69, normalized size = 1.92

$$-i \log\left(\frac{\sqrt{x+1}\sqrt{\sqrt{x}-1}\sqrt{-\sqrt{x}-1}+ix-1}{x}\right)+i \log\left(\frac{\sqrt{x+1}\sqrt{\sqrt{x}-1}\sqrt{-\sqrt{x}-1}-ix-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)^(1/2)/(-1-x^(1/2))^(1/2)/(-1+x^(1/2))^(1/2),x, algorithm=
"fricas")
```

```
[Out] -I*log((sqrt(x + 1)*sqrt(sqrt(x) - 1)*sqrt(-sqrt(x) - 1) + I*x - 1)/x) + I*
log((sqrt(x + 1)*sqrt(sqrt(x) - 1)*sqrt(-sqrt(x) - 1) - I*x - 1)/x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-\sqrt{x}-1}\sqrt{\sqrt{x}-1}\sqrt{x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)**(1/2)/(-1-x**(1/2))**(1/2)/(-1+x**(1/2))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-sqrt(x) - 1)*sqrt(sqrt(x) - 1)*sqrt(x + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)^(1/2)/(-1-x^(1/2))^(1/2)/(-1+x^(1/2))^(1/2),x, algorithm=
"giac")
```

```
[Out] integrate(1/(sqrt(x + 1)*sqrt(sqrt(x) - 1)*sqrt(-sqrt(x) - 1)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\sqrt{\sqrt{x}-1}\sqrt{-\sqrt{x}-1}\sqrt{x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x^(1/2) - 1)^(1/2)*(- x^(1/2) - 1)^(1/2)*(x + 1)^(1/2)),x)
```

```
[Out] int(1/((x^(1/2) - 1)^(1/2)*(- x^(1/2) - 1)^(1/2)*(x + 1)^(1/2)), x)
```

3.381 $\int \frac{1}{\sqrt{a-b\sqrt{x}}\,\sqrt{a+b\sqrt{x}}\,\sqrt{a^2+b^2x}}\,dx$

Optimal. Leaf size=75

$$-\frac{2\sqrt{a^2-b^2x}\,\tan^{-1}\left(\frac{\sqrt{a^2-b^2x}}{\sqrt{a^2+b^2x}}\right)}{b^2\sqrt{a-b\sqrt{x}}\,\sqrt{a+b\sqrt{x}}}$$

```
[Out] -2*arctan((-b^2*x+a^2)^(1/2)/(b^2*x+a^2)^(1/2))*(-b^2*x+a^2)^(1/2)/b^2/(a-b
*x^(1/2))^(1/2)/(a+b*x^(1/2))^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.098$, Rules used = {533, 65, 223, 209}

$$-\frac{2\sqrt{a^2-b^2x}\,\text{ArcTan}\left(\frac{\sqrt{a^2-b^2x}}{\sqrt{a^2+b^2x}}\right)}{b^2\sqrt{a-b\sqrt{x}}\,\sqrt{a+b\sqrt{x}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a - b*Sqrt[x]]*Sqrt[a + b*Sqrt[x]]*Sqrt[a^2 + b^2*x]),x]
```

```
[Out] (-2*Sqrt[a^2 - b^2*x]*ArcTan[Sqrt[a^2 - b^2*x]/Sqrt[a^2 + b^2*x]])/(b^2*Sqr
t[a - b*Sqrt[x]]*Sqrt[a + b*Sqrt[x]])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 533

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_), x_Symbol] :> Dist[(a1 + b1*x^(n/2))
^FracPart[p]*((a2 + b2*x^(n/2))^FracPart[p]/(a1*a2 + b1*b2*x^n)^FracPart[p]
), Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a1, b1, a2,
b2, c, d, n, p, q}, x] && EqQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] &&  !(EqQ
[n, 2] && IGtQ[q, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{a-b\sqrt{x}}\,\sqrt{a+b\sqrt{x}}\,\sqrt{a^2+b^2x}}\,dx &= \frac{\sqrt{a^2-b^2x}\,\int \frac{1}{\sqrt{a^2-b^2x}\,\sqrt{a^2+b^2x}}\,dx}{\sqrt{a-b\sqrt{x}}\,\sqrt{a+b\sqrt{x}}}\\ &= -\frac{\left(2\sqrt{a^2-b^2x}\right)\text{Subst}\left(\int \frac{1}{\sqrt{2a^2-x^2}}\,dx,x,\sqrt{a^2-b^2x}\right)}{b^2\sqrt{a-b\sqrt{x}}\,\sqrt{a+b\sqrt{x}}}\\ &= -\frac{\left(2\sqrt{a^2-b^2x}\right)\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\frac{\sqrt{a^2-b^2x}}{\sqrt{a^2+b^2x}}\right)}{b^2\sqrt{a-b\sqrt{x}}\,\sqrt{a+b\sqrt{x}}}\\ &= -\frac{2\sqrt{a^2-b^2x}\,\tan^{-1}\left(\frac{\sqrt{a^2-b^2x}}{\sqrt{a^2+b^2x}}\right)}{b^2\sqrt{a-b\sqrt{x}}\,\sqrt{a+b\sqrt{x}}}\end{aligned}$$

Mathematica [A]
time = 10.02, size = 75, normalized size = 1.00

$$-\frac{2\sqrt{a^2-b^2x}\,\tan^{-1}\left(\frac{\sqrt{a^2-b^2x}}{\sqrt{a^2+b^2x}}\right)}{b^2\sqrt{a-b\sqrt{x}}\,\sqrt{a+b\sqrt{x}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a - b*Sqrt[x]]*Sqrt[a + b*Sqrt[x]]*Sqrt[a^2 + b^2*x]),x]
[Out] (-2*Sqrt[a^2 - b^2*x]*ArcTan[Sqrt[a^2 - b^2*x]/Sqrt[a^2 + b^2*x]])/(b^2*Sqr
t[a - b*Sqrt[x]]*Sqrt[a + b*Sqrt[x]])
```

Maple [F]
time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b^2x+a^2}\,\sqrt{a-b\sqrt{x}}\,\sqrt{a+b\sqrt{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b^2*x+a^2)^(1/2)/(a-b*x^(1/2))^(1/2)/(a+b*x^(1/2))^(1/2),x)
```

```
[Out] int(1/(b^2*x+a^2)^(1/2)/(a-b*x^(1/2))^(1/2)/(a+b*x^(1/2))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x+a^2)^(1/2)/(a-b*x^(1/2))^(1/2)/(a+b*x^(1/2))^(1/2),x, al
gorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b^2*x + a^2)*sqrt(b*sqrt(x) + a)*sqrt(-b*sqrt(x) + a)), x
)
```

Fricas [A]
time = 2.08, size = 50, normalized size = 0.67

$$-\frac{2\arctan\left(-\frac{a^2-\sqrt{b^2x+a^2}\sqrt{b\sqrt{x}+a}\sqrt{-b\sqrt{x}+a}}{b^2x}\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x+a^2)^(1/2)/(a-b*x^(1/2))^(1/2)/(a+b*x^(1/2))^(1/2),x, al
gorithm="fricas")
```

```
[Out] -2*arctan(-(a^2 - sqrt(b^2*x + a^2)*sqrt(b*sqrt(x) + a)*sqrt(-b*sqrt(x) + a
))/(b^2*x))/b^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{a-b\sqrt{x}}\sqrt{a+b\sqrt{x}}\sqrt{a^2+b^2x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b**2*x+a**2)**(1/2)/(a-b*x**(1/2))**(1/2)/(a+b*x**(1/2))**(1/2
),x)
```

```
[Out] Integral(1/(sqrt(a - b*sqrt(x))*sqrt(a + b*sqrt(x))*sqrt(a**2 + b**2*x)), x
)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x+a^2)^(1/2)/(a-b*x^(1/2))^(1/2)/(a+b*x^(1/2))^(1/2),x, al
gorithm="giac")
```

```
[Out] integrate(1/(sqrt(b^2*x + a^2)*sqrt(b*sqrt(x) + a)*sqrt(-b*sqrt(x) + a)), x
)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{a+b\sqrt{x}}\ \sqrt{a-b\sqrt{x}}\ \sqrt{a^2+x\,b^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^(1/2))^(1/2)*(a - b*x^(1/2))^(1/2)*(b^2*x + a^2)^(1/2)),x)
```

```
[Out] int(1/((a + b*x^(1/2))^(1/2)*(a - b*x^(1/2))^(1/2)*(b^2*x + a^2)^(1/2)), x)
```

3.382 $\int (a-bx^n)^p (a+bx^n)^p (c+dx^{2n})^q \, dx$

Optimal. Leaf size=113

$$x(a-bx^n)^p (a+bx^n)^p \left(1-\frac{b^2x^{2n}}{a^2}\right)^{-p} (c+dx^{2n})^q \left(1+\frac{dx^{2n}}{c}\right)^{-q} F_1\left(\frac{1}{2n};-p,-q;\frac{1}{2}\left(2+\frac{1}{n}\right);\frac{b^2x^{2n}}{a^2},-\frac{dx^{2n}}{c}\right)$$

```
[Out] x*(a-b*x^n)^p*(a+b*x^n)^p*(c+d*x^(2*n))^q*AppellF1(1/2/n,-p,-q,1+1/2/n,b^2*
x^(2*n)/a^2,-d*x^(2*n)/c)/((1-b^2*x^(2*n)/a^2)^p)/((1+d*x^(2*n)/c)^q)
```

Rubi [A]
time = 0.06, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {533, 441, 440}

$$x(a-bx^n)^p (a+bx^n)^p \left(1-\frac{b^2x^{2n}}{a^2}\right)^{-p} (c+dx^{2n})^q \left(\frac{dx^{2n}}{c}+1\right)^{-q} F_1\left(\frac{1}{2n};-p,-q;\frac{1}{2}\left(2+\frac{1}{n}\right);\frac{b^2x^{2n}}{a^2},-\frac{dx^{2n}}{c}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^n)^p*(a + b*x^n)^p*(c + d*x^(2*n))^q,x]
```

```
[Out] (x*(a - b*x^n)^p*(a + b*x^n)^p*(c + d*x^(2*n))^q*AppellF1[1/(2*n), -p, -q,
(2 + n^(-1))/2, (b^2*x^(2*n))/a^2, -((d*x^(2*n))/c)])/((1 - (b^2*x^(2*n))/a
^2)^p*(1 + (d*x^(2*n))/c)^q)
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 533

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_), x_Symbol] :> Dist[(a1 + b1*x^(n/2))
^FracPart[p]*((a2 + b2*x^(n/2))^FracPart[p]/(a1*a2 + b1*b2*x^n)^FracPart[p]
), Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a1, b1, a2,
b2, c, d, n, p, q}, x] && EqQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] &&  !(EqQ
```

```
[n, 2] && IGtQ[q, 0])
```

Rubi steps

$$\int (a-bx^n)^p (a+bx^n)^p \left(c+dx^{2n}\right)^q dx = \left((a-bx^n)^p (a+bx^n)^p \left(a^2-b^2x^{2n}\right)^{-p}\right) \int \left(a^2-b^2x^{2n}\right)^p \left(c+dx^{2n}\right.$$
$$= \left((a-bx^n)^p (a+bx^n)^p \left(1-\frac{b^2x^{2n}}{a^2}\right)^{-p}\right) \int \left(1-\frac{b^2x^{2n}}{a^2}\right)^p \left(c+dx\right.$$
$$= \left((a-bx^n)^p (a+bx^n)^p \left(1-\frac{b^2x^{2n}}{a^2}\right)^{-p} \left(c+dx^{2n}\right)^q \left(1+\frac{dx^{2n}}{c}\right)^{-q}\right.$$
$$= x(a-bx^n)^p (a+bx^n)^p \left(1-\frac{b^2x^{2n}}{a^2}\right)^{-p} \left(c+dx^{2n}\right)^q \left(1+\frac{dx^{2n}}{c}\right)^{-q}$$

Mathematica [F]
time = 0.21, size = 0, normalized size = 0.00

$$\int (a-bx^n)^p (a+bx^n)^p \left(c+dx^{2n}\right)^q dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a - b*x^n)^p*(a + b*x^n)^p*(c + d*x^(2*n))^q,x]
[Out] Integrate[(a - b*x^n)^p*(a + b*x^n)^p*(c + d*x^(2*n))^q, x]
```

Maple [F]
time = 0.37, size = 0, normalized size = 0.00

$$\int (a-b\,x^n)^p (a+b\,x^n)^p \left(c+d\,x^{2n}\right)^q dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-b*x^n)^p*(a+b*x^n)^p*(c+d*x^(2*n))^q,x)
[Out] int((a-b*x^n)^p*(a+b*x^n)^p*(c+d*x^(2*n))^q,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-b*x^n)^p*(a+b*x^n)^p*(c+d*x^(2*n))^q,x, algorithm="maxima")
[Out] integrate((d*x^(2*n) + c)^q*(b*x^n + a)^p*(-b*x^n + a)^p, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-b*x^n)^p*(a+b*x^n)^p*(c+d*x^(2*n))^q,x, algorithm="fricas")
[Out] integral((d*x^(2*n) + c)^q*(b*x^n + a)^p*(-b*x^n + a)^p, x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-b*x**n)**p*(a+b*x**n)**p*(c+d*x**(2*n))**q,x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3007 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-b*x^n)^p*(a+b*x^n)^p*(c+d*x^(2*n))^q,x, algorithm="giac")
[Out] integrate((d*x^(2*n) + c)^q*(b*x^n + a)^p*(-b*x^n + a)^p, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(c+d\,x^{2\,n}\right)^q\,{\left(a+b\,x^n\right)}^p\,{\left(a-b\,x^n\right)}^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^(2*n))^q*(a + b*x^n)^p*(a - b*x^n)^p,x)
[Out] int((c + d*x^(2*n))^q*(a + b*x^n)^p*(a - b*x^n)^p, x)
```

3.383 $\int (a-bx^n)^p (a+bx^n)^p (a^2+b^2x^{2n})^p \, dx$

Optimal. Leaf size=87

$$x(a-bx^n)^p (a+bx^n)^p (a^2+b^2x^{2n})^p \left(1-\frac{b^4x^{4n}}{a^4}\right)^{-p} {}_2F_1\left(\frac{1}{4n},-p;\frac{1}{4}\left(4+\frac{1}{n}\right);\frac{b^4x^{4n}}{a^4}\right)$$

```
[Out] x*(a-b*x^n)^p*(a+b*x^n)^p*(a^2+b^2*x^(2*n))^p*hypergeom([-p, 1/4/n],[1+1/4/
n],b^4*x^(4*n)/a^4)/((1-b^4*x^(4*n)/a^4)^p)
```

Rubi [A]
time = 0.04, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {533, 259, 252, 251}

$$x(a-bx^n)^p (a+bx^n)^p (a^2+b^2x^{2n})^p \left(1-\frac{b^4x^{4n}}{a^4}\right)^{-p} {}_2F_1\left(\frac{1}{4n},-p;\frac{1}{4}\left(4+\frac{1}{n}\right);\frac{b^4x^{4n}}{a^4}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^n)^p*(a + b*x^n)^p*(a^2 + b^2*x^(2*n))^p,x]
```

```
[Out] (x*(a - b*x^n)^p*(a + b*x^n)^p*(a^2 + b^2*x^(2*n))^p*Hypergeometric2F1[1/(4
*n), -p, (4 + n^(-1))/4, (b^4*x^(4*n))/a^4])/(1 - (b^4*x^(4*n))/a^4)^p
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 259

```
Int[((a1_.) + (b1_.)*(x_)^(n_))^(p_)*((a2_.) + (b2_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Dist[(a1 + b1*x^n)^FracPart[p]*((a2 + b2*x^n)^FracPart[p]/(a1*a2 +
 b1*b2*x^(2*n))^FracPart[p]), Int[(a1*a2 + b1*b2*x^(2*n))^p, x], x] /; Free
Q[{a1, b1, a2, b2, n, p}, x] && EqQ[a2*b1 + a1*b2, 0] &&  !IntegerQ[p]
```

Rule 533

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_), x_Symbol] :> Dist[(a1 + b1*x^(n/2))
^FracPart[p]*((a2 + b2*x^(n/2))^FracPart[p]/(a1*a2 + b1*b2*x^n)^FracPart[p]
), Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a1, b1, a2,
b2, c, d, n, p, q}, x] && EqQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] &&  !(EqQ
[n, 2] && IGtQ[q, 0])
```

Rubi steps

$$\int (a-bx^n)^p (a+bx^n)^p \left(a^2+b^2x^{2n}\right)^p \, dx = \left((a-bx^n)^p (a+bx^n)^p \left(a^2-b^2x^{2n}\right)^{-p}\right) \int \left(a^2-b^2x^{2n}\right)^p \left(a^2+\right.$$

$$= \left((a-bx^n)^p (a+bx^n)^p \left(a^2+b^2x^{2n}\right)^p \left(a^4-b^4x^{4n}\right)^{-p}\right) \int \left(a^4-\right.$$

$$= \left((a-bx^n)^p (a+bx^n)^p \left(a^2+b^2x^{2n}\right)^p \left(1-\frac{b^4x^{4n}}{a^4}\right)^{-p}\right) \int \left(1-\right.$$

$$= x(a-bx^n)^p (a+bx^n)^p \left(a^2+b^2x^{2n}\right)^p \left(1-\frac{b^4x^{4n}}{a^4}\right)^{-p} {}_2F_1\left(\frac{1}{4n},\right.$$

Mathematica [A]
time = 0.08, size = 87, normalized size = 1.00

$$x(a-bx^n)^p (a+bx^n)^p \left(a^2+b^2x^{2n}\right)^p \left(1-\frac{b^4x^{4n}}{a^4}\right)^{-p} {}_2F_1\left(\frac{1}{4n},-p;1+\frac{1}{4n};\frac{b^4x^{4n}}{a^4}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^n)^p*(a + b*x^n)^p*(a^2 + b^2*x^(2*n))^p,x]
```

```
[Out] (x*(a - b*x^n)^p*(a + b*x^n)^p*(a^2 + b^2*x^(2*n))^p*Hypergeometric2F1[1/(4
*n), -p, 1 + 1/(4*n), (b^4*x^(4*n))/a^4])/(1 - (b^4*x^(4*n))/a^4)^p
```

Maple [F]
time = 0.35, size = 0, normalized size = 0.00

$$\int (a-b\,x^n)^p (a+b\,x^n)^p \left(a^2+b^2x^{2n}\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-b*x^n)^p*(a+b*x^n)^p*(a^2+b^2*x^(2*n))^p,x)
```

```
[Out] int((a-b*x^n)^p*(a+b*x^n)^p*(a^2+b^2*x^(2*n))^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-b*x^n)^p*(a+b*x^n)^p*(a^2+b^2*x^(2*n))^p,x, algorithm="maxima"
)
[Out] integrate((b^2*x^(2*n) + a^2)^p*(b*x^n + a)^p*(-b*x^n + a)^p, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-b*x^n)^p*(a+b*x^n)^p*(a^2+b^2*x^(2*n))^p,x, algorithm="fricas"
)
[Out] integral((b^2*x^(2*n) + a^2)^p*(b*x^n + a)^p*(-b*x^n + a)^p, x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-b*x**n)**p*(a+b*x**n)**p*(a**2+b**2*x**(2*n))**p,x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3007 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-b*x^n)^p*(a+b*x^n)^p*(a^2+b^2*x^(2*n))^p,x, algorithm="giac")
[Out] integrate((b^2*x^(2*n) + a^2)^p*(b*x^n + a)^p*(-b*x^n + a)^p, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (a+b\,x^n)^p\,(a-b\,x^n)^p\,\left(a^2+b^2\,x^{2\,n}\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)^p*(a - b*x^n)^p*(a^2 + b^2*x^(2*n))^p,x)
[Out] int((a + b*x^n)^p*(a - b*x^n)^p*(a^2 + b^2*x^(2*n))^p, x)
```

3.384 $\int \frac{(c+dx^{2n})^p}{(a-bx^n)(a+bx^n)}\,dx$

Optimal. Leaf size=76

$$\frac{x(c+dx^{2n})^p\left(1+\frac{dx^{2n}}{c}\right)^{-p}F_1\left(\frac{1}{2n};1,-p;\frac{1}{2}\left(2+\frac{1}{n}\right);\frac{b^2x^{2n}}{a^2},-\frac{dx^{2n}}{c}\right)}{a^2}$$

```
[Out] x*(c+d*x^(2*n))^p*AppellF1(1/2/n,1,-p,1+1/2/n,b^2*x^(2*n)/a^2,-d*x^(2*n)/c)
/a^2/((1+d*x^(2*n)/c)^p)
```

Rubi [A]
time = 0.04, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {531, 441, 440}

$$\frac{x(c+dx^{2n})^p\left(\frac{dx^{2n}}{c}+1\right)^{-p}F_1\left(\frac{1}{2n};1,-p;\frac{1}{2}\left(2+\frac{1}{n}\right);\frac{b^2x^{2n}}{a^2},-\frac{dx^{2n}}{c}\right)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^(2*n))^p/((a - b*x^n)*(a + b*x^n)),x]
```

```
[Out] (x*(c + d*x^(2*n))^p*AppellF1[1/(2*n), 1, -p, (2 + n^(-1))/2, (b^2*x^(2*n))
/a^2, -((d*x^(2*n))/c)])/(a^2*(1 + (d*x^(2*n))/c)^p)
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 531

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :> Int[u*(a1*a2 + b1*b2
*x^n)^p*(c + d*x^n)^q, x] /; FreeQ[{a1, b1, a2, b2, c, d, n, p, q}, x] && E
qQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p] || (GtQ[a1, 0] && Gt
```

```
Q[a2, 0]))
```

Rubi steps

$$\int \frac{(c+dx^{2n})^p}{(a-bx^n)(a+bx^n)}\,dx = \int \frac{(c+dx^{2n})^p}{a^2-b^2x^{2n}}\,dx$$
$$= \left((c+dx^{2n})^p\left(1+\frac{dx^{2n}}{c}\right)^{-p}\right)\int \frac{\left(1+\frac{dx^{2n}}{c}\right)^p}{a^2-b^2x^{2n}}\,dx$$
$$= \frac{x(c+dx^{2n})^p\left(1+\frac{dx^{2n}}{c}\right)^{-p}F_1\left(\frac{1}{2n};1,-p;\frac{1}{2}\left(2+\frac{1}{n}\right);\frac{b^2x^{2n}}{a^2},-\frac{dx^{2n}}{c}\right)}{a^2}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 258 vs. 2(76) = 152.
time = 0.32, size = 258, normalized size = 3.39

$$\frac{a^2c(1+2n)x(c+dx^{2n})^pF_1\left(\frac{1}{2n};-p,1;1+\frac{1}{2n};-\frac{dx^{2n}}{c},\frac{b^2x^{2n}}{a^2}\right)}{(a^2-b^2x^{2n})\left(2a^2dnpx^{2n}F_1\left(1+\frac{1}{2n};1-p,1;2+\frac{1}{2n};-\frac{dx^{2n}}{c},\frac{b^2x^{2n}}{a^2}\right)+2b^2cnx^{2n}F_1\left(1+\frac{1}{2n};-p,2;2+\frac{1}{2n};-\frac{dx^{2n}}{c},\frac{b^2x^{2n}}{a^2}\right)+a^2c(1+2n)F_1\left(\frac{1}{2n};-p,1;1+\frac{1}{2n};-\frac{dx^{2n}}{c},\frac{b^2x^{2n}}{a^2}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x^(2*n))^p/((a - b*x^n)*(a + b*x^n)),x]
```

```
[Out] (a^2*c*(1 + 2*n)*x*(c + d*x^(2*n))^p*AppellF1[1/(2*n), -p, 1, 1 + 1/(2*n),
-((d*x^(2*n))/c), (b^2*x^(2*n))/a^2])/((a^2 - b^2*x^(2*n))*(2*a^2*d*n*p*x^(
2*n)*AppellF1[1 + 1/(2*n), 1 - p, 1, 2 + 1/(2*n), -((d*x^(2*n))/c), (b^2*x^
(2*n))/a^2] + 2*b^2*c*n*x^(2*n)*AppellF1[1 + 1/(2*n), -p, 2, 2 + 1/(2*n), -
((d*x^(2*n))/c), (b^2*x^(2*n))/a^2] + a^2*c*(1 + 2*n)*AppellF1[1/(2*n), -p,
 1, 1 + 1/(2*n), -((d*x^(2*n))/c), (b^2*x^(2*n))/a^2]))
```

Maple [F]
time = 0.16, size = 0, normalized size = 0.00

$$\int \frac{(c+d\,x^{2n})^p}{(a-b\,x^n)(a+b\,x^n)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*x^(2*n))^p/(a-b*x^n)/(a+b*x^n),x)
```

```
[Out] int((c+d*x^(2*n))^p/(a-b*x^n)/(a+b*x^n),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^(2*n))^p/(a-b*x^n)/(a+b*x^n),x, algorithm="maxima")
[Out] -integrate((d*x^(2*n) + c)^p/((b*x^n + a)*(b*x^n - a)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^(2*n))^p/(a-b*x^n)/(a+b*x^n),x, algorithm="fricas")
[Out] integral(-(d*x^(2*n) + c)^p/(b^2*x^(2*n) - a^2), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x**(2*n))**p/(a-b*x**n)/(a+b*x**n),x)
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^(2*n))^p/(a-b*x^n)/(a+b*x^n),x, algorithm="giac")
[Out] integrate(-(d*x^(2*n) + c)^p/((b*x^n + a)*(b*x^n - a)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$-\int -\frac{(c+d\,x^{2\,n})^p}{a^2-b^2\,x^{2\,n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^(2*n))^p/((a + b*x^n)*(a - b*x^n)),x)
[Out] -int(-(c + d*x^(2*n))^p/(a^2 - b^2*x^(2*n)), x)
```

3.385 $\int \left(a-bx^{n/2}\right)^p \left(a+bx^{n/2}\right)^p \left(\frac{a^2d(1+p)}{b^2\left(1+\frac{-1-2n-np}{n}\right)}+dx^n\right)^{-}$

Optimal. Leaf size=96

$$-\frac{b^2(1+n+np)x\left(a-bx^{n/2}\right)^{1+p}\left(a+bx^{n/2}\right)^{1+p}\left(-\frac{a^2dn(1+p)}{b^2(1+n+np)}+dx^n\right)^{-\frac{1+n+np}{n}}}{a^4dn(1+p)}$$

```
[Out] -b^2*(n*p+n+1)*x*(a-b*x^(1/2*n))^(1+p)*(a+b*x^(1/2*n))^(1+p)/a^4/d/n/(1+p)/
((-a^2*d*n*(1+p)/b^2/(n*p+n+1)+d*x^n)^((n*p+n+1)/n))
```

Rubi [A]
time = 0.09, antiderivative size = 104, normalized size of antiderivative = 1.08, number of steps used = 2, number of rules used = 2, integrand size = 76, $\frac{\text{number of rules}}{\text{integrand size}} = 0.026$, Rules used = {533, 389}

$$-\frac{b^2x(np+n+1)\left(a^2-b^2x^n\right)\left(a-bx^{n/2}\right)^p\left(a+bx^{n/2}\right)^p\left(dx^n-\frac{a^2dn(p+1)}{b^2(np+n+1)}\right)^{-\frac{np+n+1}{n}}}{a^4dn(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^(n/2))^p*(a + b*x^(n/2))^p*((a^2*d*(1 + p))/(b^2*(1 + (-1 - 2*
n - n*p)/n)) + d*x^n)^((-1 - 2*n - n*p)/n),x]
```

```
[Out] -((b^2*(1 + n + n*p)*x*(a - b*x^(n/2))^p*(a + b*x^(n/2))^p*(a^2 - b^2*x^n))
/(a^4*d*n*(1 + p)*(-((a^2*d*n*(1 + p))/(b^2*(1 + n + n*p))) + d*x^n)^((1 +
n + n*p)/n)))
```

Rule 389

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c)), x] /; FreeQ[{a,
b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] &&
EqQ[a*d*(p + 1) + b*c*(q + 1), 0]
```

Rule 533

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_), x_Symbol] :> Dist[(a1 + b1*x^(n/2))
^FracPart[p]*((a2 + b2*x^(n/2))^FracPart[p]/(a1*a2 + b1*b2*x^n)^FracPart[p]
), Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a1, b1, a2,
b2, c, d, n, p, q}, x] && EqQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] &&  !(EqQ
[n, 2] && IGtQ[q, 0])
```

Rubi steps

$$\int \left(a-bx^{n/2}\right)^p \left(a+bx^{n/2}\right)^p \left(\frac{a^2d(1+p)}{b^2\left(1+\frac{-1-2n-np}{n}\right)}+dx^n\right)^{\frac{-1-2n-np}{n}} dx = \left(\left(a-bx^{n/2}\right)^p \left(a+bx^{n/2}\right)^p \left(a^2-b^2\right.\right.$$

$$= -\frac{b^2(1+n+np)x\left(a-bx^{n/2}\right)^p \left(a+\right.}{}$$

Mathematica [A]
time = 0.90, size = 103, normalized size = 1.07

$$-\frac{b^2(1+n+np)x\left(a-bx^{n/2}\right)^p \left(a+bx^{n/2}\right)^p \left(d\left(-\frac{a^2n(1+p)}{b^2(1+n+np)}+x^n\right)\right)^{-\frac{1+n+np}{n}} \left(a^2-b^2x^n\right)}{a^4dn(1+p)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^(n/2))^p*(a + b*x^(n/2))^p*((a^2*d*(1 + p))/(b^2*(1 + (-
1 - 2*n - n*p)/n)) + d*x^n)^((-1 - 2*n - n*p)/n),x]
```

```
[Out] -((b^2*(1 + n + n*p)*x*(a - b*x^(n/2))^p*(a + b*x^(n/2))^p*(a^2 - b^2*x^n))
/(a^4*d*n*(1 + p)*(d*(-((a^2*n*(1 + p))/(b^2*(1 + n + n*p))) + x^n))^((1 +
n + n*p)/n)))
```

Maple [F]
time = 0.39, size = 0, normalized size = 0.00

$$\int \left(a-b\,x^{\frac{n}{2}}\right)^p \left(a+b\,x^{\frac{n}{2}}\right)^p \left(\frac{a^2d(1+p)}{b^2\left(1+\frac{-np-2n-1}{n}\right)}+d\,x^n\right)^{\frac{-np-2n-1}{n}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-b*x^(1/2*n))^p*(a+b*x^(1/2*n))^p*(a^2*d*(1+p)/b^2/(1+(-n*p-2*n-1)/n)
+d*x^n)^((-n*p-2*n-1)/n),x)
```

```
[Out] int((a-b*x^(1/2*n))^p*(a+b*x^(1/2*n))^p*(a^2*d*(1+p)/b^2/(1+(-n*p-2*n-1)/n)
+d*x^n)^((-n*p-2*n-1)/n),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-b*x^(1/2*n))^p*(a+b*x^(1/2*n))^p*(a^2*d*(1+p)/b^2/(1+(-n*p-2*n
-1)/n)+d*x^n)^((-n*p-2*n-1)/n),x, algorithm="maxima")
```

```
[Out] integrate((b*x^(1/2*n) + a)^p*(-b*x^(1/2*n) + a)^p/(d*x^n - a^2*d*(p + 1)/(
b^2*((n*p + 2*n + 1)/n - 1)))^((n*p + 2*n + 1)/n), x)
```

Fricas [**A**]
time = 3.27, size = 180, normalized size = 1.88

$$\frac{((b^4np+b^4n+b^4)xx^{2n}-(2a^2b^2np+2a^2b^2n+a^2b^2)xx^n+(a^4np+a^4n)x)\left(bx^{\frac{1}{2}n}+a\right)^p\left(-bx^{\frac{1}{2}n}+a\right)^p}{(a^4np+a^4n)\left(-\frac{a^2dnp+a^2dn-(b^2dnp+b^2dn+b^2d)x^n}{b^2np+b^2n+b^2}\right)^{\frac{np+2n+1}{n}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-b*x^(1/2*n))^p*(a+b*x^(1/2*n))^p*(a^2*d*(1+p)/b^2/(1+(-n*p-2*n
-1)/n)+d*x^n)^((-n*p-2*n-1)/n),x, algorithm="fricas")
```

```
[Out] ((b^4*n*p + b^4*n + b^4)*x*x^(2*n) - (2*a^2*b^2*n*p + 2*a^2*b^2*n + a^2*b^2
)*x*x^n + (a^4*n*p + a^4*n)*x)*(b*x^(1/2*n) + a)^p*(-b*x^(1/2*n) + a)^p/((a
^4*n*p + a^4*n)*(-(a^2*d*n*p + a^2*d*n - (b^2*d*n*p + b^2*d*n + b^2*d)*x^n)
/(b^2*n*p + b^2*n + b^2))^((n*p + 2*n + 1)/n))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-b*x**(1/2*n))**p*(a+b*x**(1/2*n))**p*(a**2*d*(1+p)/b**2/(1+(-n
*p-2*n-1)/n)+d*x**n)**((-n*p-2*n-1)/n),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-b*x^(1/2*n))^p*(a+b*x^(1/2*n))^p*(a^2*d*(1+p)/b^2/(1+(-n*p-2*n
-1)/n)+d*x^n)^((-n*p-2*n-1)/n),x, algorithm="giac")
```

```
[Out] integrate((b*x^(1/2*n) + a)^p*(-b*x^(1/2*n) + a)^p/(d*x^n - a^2*d*(p + 1)/(
b^2*((n*p + 2*n + 1)/n - 1)))^((n*p + 2*n + 1)/n), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+b\,x^{n/2}\right)^p\,\left(a-b\,x^{n/2}\right)^p}{{\left(d\,x^n-\frac{a^2\,d\,\left(p+1\right)}{b^2\,\left(\frac{2\,n+n\,p+1}{n}-1\right)}\right)}^{\frac{2\,n+n\,p+1}{n}}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^(n/2))^p*(a - b*x^(n/2))^p)/(d*x^n - (a^2*d*(p + 1))/(b^2*((2
*n + n*p + 1)/n - 1)))^((2*n + n*p + 1)/n),x)
[Out] int(((a + b*x^(n/2))^p*(a - b*x^(n/2))^p)/(d*x^n - (a^2*d*(p + 1))/(b^2*((2
*n + n*p + 1)/n - 1)))^((2*n + n*p + 1)/n), x)
```

Chapter 4

Appendix

Local contents

4.1 Download section

The following zip files contain the raw integrals used in this test.

Mathematica format Mathematica_syntax.zip

Maple and Mupad format Maple_syntax.zip

Sympy format SYMPY_syntax.zip

Sage math format SAGE_syntax.zip

4.2 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.2.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                        Small rewrite of logic in main function to make it*)
(*                        match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*           GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)
```

```
GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A","none"}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A","none"}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
```

```
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
```

```
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.2.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);
```

```
        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A","";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of re
                                convert(leaf_count_result,string)," vs. $2 (",
                                convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
```

```
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well"
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A","";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of o
                               convert(leaf_count_result,string),"$ vs. $2(",
                               convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
```

```
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
```

```
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.2.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]

def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_re
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

4.2.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#          Albert Rich to use with Sagemath. This is used to
#          grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#          'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#          issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']    #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
      return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if  type(expn.operands()[0])==Rational:  #type(isinstance(expn.args[0],Rational):
         return 1
      else:
         return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
   elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
      if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
         return expnType(expn.operands()[0])   #expnType(expn.args[0])
      elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
         if type(expn.operands()[0])==Rational:  #isinstance(expn.args[0],Rational)
```

```
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    else:
        return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.op
elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstan
    m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
    m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
    return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
    return max(3,expnType(expn.operands()[0]))
elif is_special_function(expn.operator()): #is_special_function(expn.func)
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(4,m1)  #max(4,m1)
elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(5,m1)  #max(5,m1)
elif is_appell_function(expn.operator()):
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(6,m1)  #max(6,m1)
elif str(expn).find("Integral") != -1: #this will never happen, since it
        #is checked before calling the grading function that is passed.
        #but kept it here.
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(8,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
else:
    return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```